

Algebraic Riemann manifolds

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We give a criterion by which we decide whether two given Riemann manifolds $M, \overline{M}$ are isometric or not. We recall the following classical theorem.

THEOREM (C^ω ISOMETRY THEOREM). *Let $M, \overline{M}$ be real analytic Riemann manifolds of dimension n . Let $p \in M, \bar{p} \in \overline{M}$. Suppose that there exists a linear isometry $I : T_p(M) \rightarrow T_{\bar{p}}(\overline{M})$ which preserves the curvature tensors $R, \overline{R}$, and their covariant differentials $\nabla^k R, \nabla^k \overline{R}$ of any order k . Then the mapping I can be extended to an isometry h between neighborhoods of $p, \bar{p}$. Hence in particular if $M, \overline{M}$ are complete, connected, and simply connected, then $M, \overline{M}$ are isometric.*

By replacing C^ω with the Nash category C^Ω , and introducing the notion "minimal differential polynomial" ϕ_M of a C^Ω Riemann manifold M , we observe that the proof of this theorem implies the following criterion.

THEOREM 1. *Let $M, \overline{M}$ be C^Ω Riemann manifolds of dimension n . Let $p \in M, \bar{p} \in \overline{M}$. Suppose that*

- (1) *the minimal differential polynomials $\phi_M, \phi_{\overline{M}}$ coincide,*
- (2) *the two point $p, \bar{p}$ are "nonsingular" with respect to $\phi_M, \phi_{\overline{M}}$, respectively, and*
- (3) *there exists a linear isometry $I : T_p(M) \rightarrow T_{\bar{p}}(\overline{M})$ which preserves the curvature tensors $R, \overline{R}$, and their first $4n - 5$ covariant differentials $\nabla^k R, \nabla^k \overline{R}$.*

Then the mapping I can be extended to an isometry h between neighborhoods of $p, \bar{p}$.

As an application we obtain

THEOREM 2. *Let M be a compact C^Ω Riemann manifold of dimension n . Suppose that M is nowhere homogeneous, i.e. for any distinct points p, q of M , there exists no isometry $h, h(p) = q$, between neighborhoods of p, q . Then M is C^Ω embeddable, and the embedding is given by means of general scalar curvatures. If any point of M is nonsingular with respect to ϕ_M , then some finite number of general scalar curvatures of order at most $4n-5$ give a one to one mapping of M into a vector space.*

REFERENCE

K.Yamato, *Algebraic Riemann manifolds*, Nagoya Math. J. 115 (1989) (to appear).