

Reduced Representation of Spatial Harmonics for Motor Model Order Reduction Using Block Arnoldi Method

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A motor model order reduction (MOR) based on the decomposition into stator and rotor domains requires a Fourier expansion of the gap field with a large of number harmonic components, which are handled by the multiport Cauer ladder network (CLN) method. This study proposes a block Arnoldi method to obtain a reduced representation of spatial harmonics. Efficient bases for reduced harmonic representation were generated based on the inductance matrices of CLNs of stator and rotor domains. The interaction between the stator and rotor domains was considered by exchanging the basis vectors. Simulation results show that the proposed method yields an efficient and accurate MOR of the induction motor.

Index Terms—Block Arnoldi process, Cauer ladder network, induction motor, model order reduction, spatial harmonics.

I. INTRODUCTION

STUDIES on the model order reduction (MOR) of motors [1–3] are ongoing for an efficient coupled analysis of motors and their control systems. Based on the Cauer ladder network (CLN) method [4], an accurate MOR of an induction motor was realized [2], which can simulate motor responses under a wide range of operating conditions without using snapshots of the motor operations. The motor MOR consists of the decomposed stator and rotor domains represented by multiport CLNs. To connect the two domains, the transformation between the stator and rotor coordinates is included in the boundary condition at the air gap, which naturally induces a motional electromotive force in the rotor domain.

An exact connection is realized by choosing all the finite element (FE) edge variables as bases of the airgap field, as was proposed for the full FE analysis of an induction motor [5]; however, this requires a very large number of variables for the reduced system. Ref. [6] and [2] used spatial harmonic components for connecting airgap fields. However, when the numbers of slots and rotor bars are large, a large number of spatial harmonics are required to accurately connect the two domains [7], which increases the computational cost. Ref. [7] showed that an appropriate choice of dominant harmonics can reduce computational costs without losing accuracy.

Several studies (cf. [8]) have discussed possible sets of dominant spatial harmonics owing to combinations of multiples of slot and rotor-bar numbers. However, the number of combinations may be too large for practical applications [7].

The dominant spatial harmonics depend on the stator and

rotor structures. They are expected to be known from the inductance matrices generated by the multiport CLN method because the inductance matrices contain information on the spatial dependence of the reluctance.

This study proposes a method to systematically select the dominant components based on the application of the block Arnoldi process [9] to the inductance matrices of the stator and rotor domains for the efficient representation of spatial harmonics.

II. MOTOR MOR

The CLN method [4] is an efficient and accurate MOR, for which the multiport version has successfully realized the MOR of an induction motor [2]. In this method, reduced stator and rotor models are constructed independently and connected at the airgap interface with the spatial harmonic components of the magnetic field. Matrix Cauer network representations of the stator and rotor domains are shown in Figs. 1(a)(b).

A. Stator CLN

In the stator CLN, I_s and V_s are the source current and voltage vectors, respectively. I and V are the spatial harmonic components of the circumferential magnetic field (H_θ) and axial electric field (E_z) at the stator side of the airgap interface.

The airgap magnetic field H_θ is expanded as

$$H_\theta(\theta, t) = \sqrt{2} \sum_{k=1}^K [H_{ck}(t) \cos(kp\theta) + H_{sk}(t) \sin(kp\theta)] \quad (1)$$

where p is the number of pole pairs and K is the highest order of harmonics considered. Factor $\sqrt{2}$ appears because coefficients H_{ck} and H_{sk} are spatial RMS values, even though they are instantaneous values along the time direction.

From the coefficients, I is given as

$$I = [H_{c1}, H_{s1}, \dots, H_{cK}, H_{sK}]. \quad (2)$$

The air gap electric field is expanded in a manner similar to (2), and the array of its coefficients is denoted by V .

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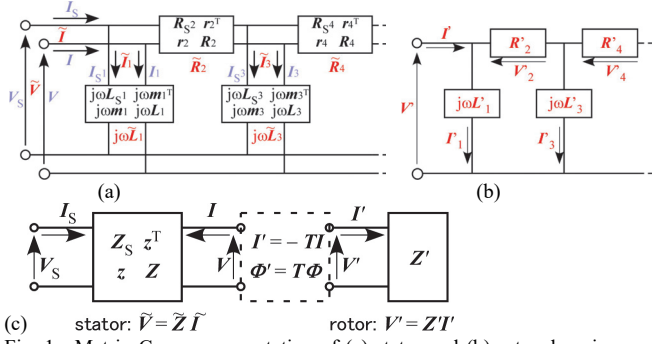


Fig. 1. Matrix Cauer representation of (a) stator and (b) rotor domains, and (c) their connection

For the convenience of basis vector generation in the following section, we provide the source current and voltage vectors based on the Clarke transformation as

$$I_s = \begin{bmatrix} I_0 \\ I_\alpha \\ I_\beta \end{bmatrix} = C_{0\alpha\beta} \begin{bmatrix} I_U \\ I_V \\ I_W \end{bmatrix}, \quad V_s = \begin{bmatrix} V_0 \\ V_\alpha \\ V_\beta \end{bmatrix} = C_{0\alpha\beta} \begin{bmatrix} V_U \\ V_V \\ V_W \end{bmatrix} \quad (3)$$

$$C_{0\alpha\beta} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \quad (4)$$

where (I_U, I_V, I_W) and (V_U, V_V, V_W) are the 3-phase current and voltage components, respectively. The constants in (4) are chosen such that $C_{0\alpha\beta}^{-1} = C_{0\alpha\beta}^T$.

In Fig. 1(a), $\tilde{I}$ and $\tilde{V}$ represent the current and voltage vectors, respectively, and $\tilde{L}_{2n-1}$ represents the inductance matrix [2]. They are given as

$$\tilde{V} = \begin{bmatrix} V_s \\ V \end{bmatrix}, \quad \tilde{I} = \begin{bmatrix} I_s \\ I \end{bmatrix}, \quad \tilde{L}_{2n-1} = \begin{bmatrix} L_{s2n-1} & m_{2n-1}^T \\ m_{2n-1} & L_{2n-1} \end{bmatrix}. \quad (5)$$

Similarly, $\tilde{R}_{2n}$ denotes the resistance matrix.

B. Rotor CLN and connection to Stator CLN

In the rotor CLN, I' and V' are the spatial harmonic components at the rotor side and L'_{2n-1} and R'_{2n} are inductance and resistance matrices.

The boundary conditions at the interface are derived from the relations $H = H'$ for the magnetic field and $\int E dt = \int E' dt'$ for the electric field, where the left-hand side and right-hand side values are those on the stator and rotor sides, respectively. The boundary condition with the relation $\theta' = \theta - \int \omega_m dt$ is represented as

$$I' = -T I, \quad \Phi' = T \Phi \quad (6)$$

where ω_m is the mechanical angular velocity, $\Phi = \int V dt$, $\Phi' = \int V' dt'$ and

$$T = \text{blockdiag}(T_1, \dots, T_K) \quad (7)$$

$$T_k = \begin{bmatrix} \cos(kp \int \omega_m dt) & \sin(kp \int \omega_m dt) \\ -\sin(kp \int \omega_m dt) & \cos(kp \int \omega_m dt) \end{bmatrix} \quad (k = 1, 2, \dots). \quad (8)$$

The stator and rotor CLNs are connected using T , as shown

in Fig. 1(c). Therein, the responses of the reduced stator and rotor systems are represented by the multiport transfer functions $\tilde{Z}$ and Z' , which are written as

$$\tilde{V} = \tilde{Z} \tilde{I}, \quad V' = Z' I', \quad \tilde{Z} = \begin{bmatrix} Z_s & z^T \\ z & Z \end{bmatrix}. \quad (9)$$

III. REDUCED REPRESENTATION OF SPATIAL HARMONICS

A. Block Arnoldi process for rotor side

For an explanation of the basic concept of the Arnoldi process, the reduced representation of rotor side harmonics is derived when the first inductance matrix L'_1 is available explicitly. The magnetostatic relationship is given as

$$\Phi' = L'_1 I'. \quad (10)$$

Let $u'_1, u'_2, \dots, u'_{K_R}$ be the orthonormal basis of I' and Φ' of dimension K_R . I' and Φ' are expanded as:

$$I' = i'_1 u'_1 + i'_2 u'_2 + \dots + i'_{K_R} u'_{K_R} = U' i',$$

$$\Phi' = \phi'_1 u'_1 + \phi'_2 u'_2 + \dots + \phi'_{K_R} u'_{K_R} = U' \phi' \quad (11)$$

where

$$U' = [u'_1, u'_2, \dots], \quad i' = [i'_1, i'_2, \dots]^T, \quad \phi' = [\phi'_1, \phi'_2, \dots]^T. \quad (12)$$

Because U must contain the dominant harmonics, the fundamental cosine and sine components are chosen as

$$u'_1 = [1, 0, 0, \dots]^T, \quad u'_2 = [0, 1, 0, \dots]^T. \quad (13)$$

The other basis vectors are generated using the block Arnoldi method with block size $b = 2$ as follows:

set u'_1 and u'_2 as (13).

for $k = b+1, b+2, \dots$ do

$$\alpha_{k,j} = u_j'^T L'_1 u'_{k-b} \quad (j = 1, 2, \dots, k-1)$$

$$v'_k = L'_1 u'_{k-b} - \sum_j \alpha_{k,j} u'_{k-b}, \quad u'_k = v'_k / |v'_k|.$$

Because L'_1 is symmetric, the procedure above can be reduced to the block Lanczos method.

The procedure above can be implemented without computing L'_1 explicitly beforehand as follows. We denote the basis vector as $u'_{k-b} = [u_{c1}, u_{s1}, \dots]$. Using u'_{k-b} , the airgap magnetic field is given as

$$H'_\theta(\theta') = \sqrt{2} \sum_{m=1} [u_{cm} \cos(mp\theta') + u_{sm} \sin(mp\theta')]. \quad (14)$$

By executing the magnetostatic analysis of the rotor with the Neumann boundary condition (14), the vector potential at the airgap interface is computed, which is expanded as

$$A(\theta') = \sqrt{2} \sum_{m=1} [v_{cm} \cos(mp\theta') + v_{sm} \sin(mp\theta')]. \quad (15)$$

Its coefficients are denoted as $v' = [v_{c1}, v_{s1}, \dots]$. This expression can be regarded as $v' = L'_1 u'_{k-b}$. Although v' is a constant multiple of $L'_1 u'_{k-b}$, it is inconsequential because the basis vector is normalized later. In the case without an explicit L'_1 , the block Arnoldi method is better than the block Lanczos method, where L'_1 denotes the operator above and is not strictly symmetric.

B. Block Arnoldi process for stator side

On the stator side, to generate basis vectors simply, only I and V are reduced independently of I_s and V_s . By replacing I' , Φ' , and L'_1 on the rotor side by I , Φ , and L_1 on the stator side, I and Φ are similarly expanded as

$$I = U\mathbf{i}, \quad \Phi = U\boldsymbol{\varphi}, \quad \mathbf{i} = [i_1, i_2, \dots]^T, \quad \boldsymbol{\varphi} = [\varphi_1, \varphi_2, \dots]^T \quad (16)$$

$$U = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{K_S}]. \quad (17)$$

where K_S represents the reduced dimensions of the stator. A similar block Arnoldi process yields a reduced basis (17) with $b = 2$, and the initial basis vectors

$$\mathbf{u}_1 = [1, 0, 0, \dots]^T, \quad \mathbf{u}_2 = [0, 1, 0, \dots]^T. \quad (18)$$

The basis generated as above is called source-independent basis hereafter.

The boundary condition at the airgap interface is given as

$$\mathbf{i}' = \mathbf{U}^T \mathbf{T} \mathbf{U} \mathbf{i}, \quad \boldsymbol{\varphi}' = \mathbf{U}^T \mathbf{T} \mathbf{U} \boldsymbol{\varphi}. \quad (19)$$

It may be more efficient if the basis vectors are generated for I_s and I in a unified manner as follows:

$$\tilde{I} = \tilde{U}\tilde{\mathbf{i}}, \quad \tilde{\Phi} = \tilde{U}\tilde{\boldsymbol{\varphi}}, \quad \tilde{U} = [\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \dots] = \begin{bmatrix} \mathbf{u}_{S1} & \mathbf{u}_{S2} & \dots \\ \mathbf{u}_1 & \mathbf{u}_2 & \dots \end{bmatrix}. \quad (20)$$

Instead of L'_1 or L_1 , $\tilde{L}_1$ is used to generate a basis vector $\tilde{U}$ using the block Arnoldi process.

Because the source currents I_a and I_b yield spatial fundamental components of airgap flux Φ_{c1} and Φ_{s1} , the spatially fundamental components in (18) do not have to be provided explicitly as the initial basis vectors. The first three basis vectors are expressed as

$$\begin{aligned} \mathbf{u}_{S1} &= [1, 0, 0]^T, \quad \mathbf{u}_1 = 0, \quad \mathbf{u}_{S2} = [0, 1, 0]^T, \quad \mathbf{u}_2 = 0 \\ \mathbf{u}_{S3} &= [0, 0, 1]^T, \quad \mathbf{u}_3 = 0. \end{aligned} \quad (21)$$

The block Arnoldi process is executed with block size $b = 2$ and initial vectors $\tilde{\mathbf{u}}_2$ and $\tilde{\mathbf{u}}_3$ to generate $\tilde{\mathbf{u}}_4$, $\tilde{\mathbf{u}}_5$, Hereafter, the basis generated as described above is referred to as a source-dependent basis.

C. Interaction between Stator and Rotor

Although Ref. [7] suggests that the spatial harmonics due to the interaction between the stator and rotor should be included, the basis generation in the preceding subsections does not consider this. When the basis vector is given as (17) on the stator side, the interaction is included by giving its 3rd and 4th basis vectors to the rotor domain as (22) to be used in the block Arnoldi process with $b = 4$:

$$\mathbf{u}'_3 = \mathbf{u}_3, \quad \mathbf{u}'_4 = \mathbf{u}_4. \quad (22)$$

When the basis vector is given as (20) on the stator side, the interaction is included by setting $\mathbf{u}'_3$ and $\mathbf{u}'_4$ yielded from the orthogonalization of $\mathbf{u}_4$ and $\mathbf{u}_5$ as below:

$$\mathbf{v}'_3 = \mathbf{u}_4 - \alpha_{3,1}\mathbf{u}'_1 - \alpha_{3,2}\mathbf{u}'_2, \quad \mathbf{u}'_3 = \mathbf{v}'_3 / \|\mathbf{v}'_3\| \quad (23)$$

$$\mathbf{v}'_4 = \mathbf{u}_5 - \alpha_{4,1}\mathbf{u}'_1 - \alpha_{4,2}\mathbf{u}'_2 - \alpha_{4,3}\mathbf{u}'_3, \quad \mathbf{u}'_4 = \mathbf{v}'_4 / \|\mathbf{v}'_4\| \quad (24)$$

where $\alpha_{3,j} = \mathbf{u}_4^T \mathbf{u}'_j$, $\alpha_{4,j} = \mathbf{u}_5^T \mathbf{u}'_j$. The block Arnoldi process is

executed with block size $b = 4$ and initial basis vectors $\mathbf{u}'_1, \dots, \mathbf{u}'_4$.

IV. COMPUTATIONAL RESULTS

The induction motor analyzed in [7], which had two pole pairs, 24 stator slots, and 34 rotor bars, was simulated. The simulated transient torque waveforms with a fixed slip of 0.15 at 50 Hz under a sinusoidal three-phase current are shown in Fig. 2(a), where the spatial harmonic components (1) with $K = 32$ and 45 ($K_S = K_R = 64, 90$) are used in the motor MOR. When $K = 32$, the MOR method fails to reconstruct the torque oscillation computed by the FE analysis because the 33-th and 35-th harmonics are not considered. When $K = 45$, a slight discrepancy remains, although the accuracy is improved. Figs. 2(b) and (c) show the torque waveforms obtained using the reduced representation of spatial harmonics with 64 and 90 source-independent bases ($K_S = K_R = 64, 90$) without and with interaction, respectively. Torque oscillation was approximately represented by 64 bases. Bases with interactions provided a more accurate waveform than those without interactions. The waveform reconstructed using 90 bases was very accurate. Figs. 3(a) and (b) show the speed dependence of the average torque reconstructed using the MOR method, corresponding to Figs. 2(a) and (c). The average torque was accurately represented by the original implementation without a reduced representation of harmonics, whereas the source-independent bases resulted in an underestimation of the average torque when only 64 bases were used. Figs. 2(d), (e), and 4(c) show the torque waveform and average torque, respectively, when source-dependent bases were used with and without interaction. The source-dependent bases improved the accuracy of the average torque. The root mean square (RMS) of differences of torque waveforms during the 5th time-period between the FE and reduced computations with 64 basis vectors are indicated in Fig. 2. The waveform was accurately reconstructed using source-dependent bases considering the interaction. Fig. 4 shows the radial magnetic flux density along the airgap, where the RMS of differences from the FE analysis are also indicated. The source-dependent basis with interaction gives more accurate magnetic field than the original implementation.

A free software FreeFEM++ was used to execute the FE computations above by a desktop PC. The generation of stator and motor CLNs requires 379 and 719 sec with 64 and 90 basis vectors, respectively. The FE time-dependent computation of 900 time-step costs 9614 sec whereas the time-dependent computation of reduced model needs 8.5 and 15.5 sec with 64 and 90 bases using MATLAB. The computational cost of the reduced system can be approximately evaluated using K_S^2 and K_R^2 . Accordingly, the reduced harmonic representation effectively reduces the computational cost of the reduced motor system.

V. CONCLUSION

In this study, the block Arnoldi process was applied to the reduced representation of the spatial harmonic components at

the airgap interface for an induction-motor MOR. The torque waveform can be accurately reproduced with reduced harmonic components by exchanging the basis vectors between the stator and rotor domains. A nonlinearization method [10] will be implemented in the near future.

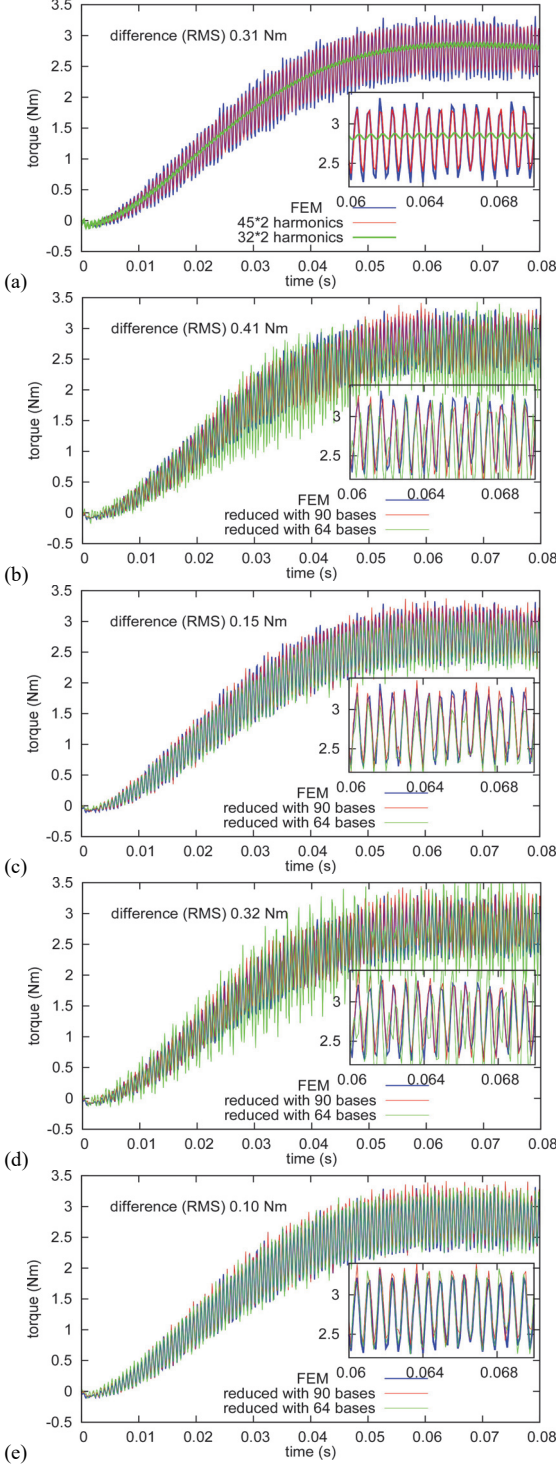


Fig. 2. Torque waveforms computed (a) using original implementation, (b) using source-independent bases without interaction, (c) using source-independent bases with interaction, (d) using source-dependent bases without interaction, and (e) using source-dependent bases with interaction.

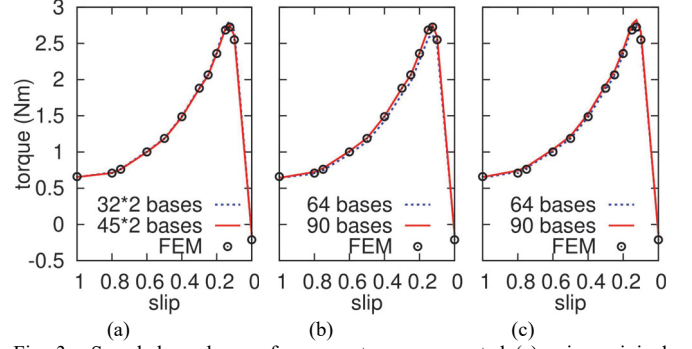


Fig. 3. Speed dependence of average torque computed (a) using original implementation, (b) using source-independent bases with interaction, and (c) using source-dependent bases with interaction.

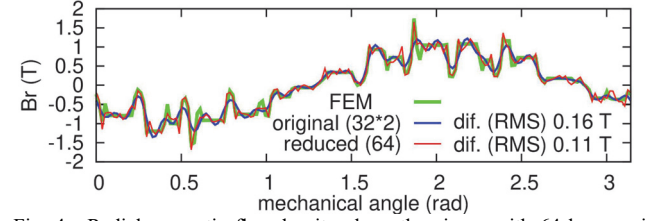


Fig. 4. Radial magnetic flux density along the airgap with 64 bases using original implementation and source-dependent bases with interaction.

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