

**Optical Interferometric Measurements
Inspired by Time-Reversal Symmetry
of Quantum Mechanics**

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Chapter 1

Introduction

1.1 Background

Interference is a characteristic feature of light as waves and has been utilized for a wide range of optical measurements. Such optical measurements, called *optical interferometric measurements*, are roughly classified into (i) measurements of internal parameters of light and (ii) measurements of parameters of an external object interacting with the light. We now consider Young's double-slit interferometry as examples of these measurements. When the wavelength of the light λ is unknown but the slit separation d and the distance between the slit and the screen L are known, measuring the spacing of the interference fringes Δx can determine the wavelength as $\lambda = d\Delta x/L$. This measurement is included in the measurements (i). The measurements (i) have been made for investigation of fundamental properties of light, including temporal and spatial coherence, phase difference, frequency difference, wavelength, and orbital angular momentum [1–3]. On the other hand, when the slit separation d is unknown but the other parameters are known, measuring the spacing of the fringes Δx can determine the slit separation as $d = L\lambda/\Delta x$. This measurement is included in the measurements (ii). The measurements (ii) are implemented by transferring the parameter of the external object to an internal parameter of light and measuring the internal parameter by an optical interferometer. The measurements (ii) provide precise measurements of various physical quantities including length and time, and have benefited a variety of fields such as

biomedicine [4–6], industry [7,8], and astronomy [9–11].

Light is generally described by quantum theory. The quantum properties of light provide novel measurement techniques that have never performed in classical optics, called *quantum optical measurements*. As the measurements (i), quantum optical measurements can measure more general concepts of light, such as photon antibunching [12], wave functions [13], and a degree of quantum entanglement [14]. In addition, as the measurement (ii), quantum optical measurement can measure parameters of an external object more precisely by employing quantum interference phenomena; the examples include entanglement-enhanced microscope [15] via phase supersensitivity [16,17], and dispersion-insensitive optical coherence tomography (OCT) [18–20] using Hong–Ou–Mandel (HOM) interference [21] and automatic dispersion cancellation [22,23]. Most of the quantum optical measurements exploit nonclassical correlations of time-frequency-entangled photon pairs and are called *two-photon quantum optical measurements*. Two-photon quantum optical measurements, however, often suffer from difficulties in generating, operating, and detecting the entangled photon pairs. The entangled photon pairs are typically generated by spontaneous parametric down-conversion (SPDC), a nonlinear optical effect, and detected by coincidence counting. The generation rate of the photon pairs by SPDC is generally low ($\lesssim 10^6$ Hz [24]), and the coincidence counting rate is limited by the processing speed of the electric circuit. The low output signals of the two-photon quantum optical measurements lead long measurement time, which will yield barriers to real-time measurements.

Recently, a new classical optical measurement technique called chirped-pulse interferometry (CPI) was proposed and demonstrated [25–29]. CPI can achieve some of the advantages of two-photon quantum optical measurements with much stronger signals. CPI is implemented by a time-reversed version of the optical systems for two-photon quantum optical measurements and employs pairs of strongly oppositely-chirped laser pulses as its input light. The optical system implementing CPI is constructed by a completely classical optical system, requiring neither entangled photon pairs nor coincidence counting. Several experiments using CPI have been reported to reproduce various quantum interference patterns with intense signals, such as dispersion-cancelled HOM interference [25], several kinds of two-

photon interference [28], and dispersion-insensitive OCT of a coverglass [27] and an onion piece [29].

The construction method of CPI is based on the following time-reversal symmetry of quantum mechanics. When an initial state $|i\rangle$ evolves with a unitary operation $\hat{U}$ and is projected onto a final state $|f\rangle$ (we call this process *the time-forward process*), the success probability is given by $|\langle f|\hat{U}|i\rangle|^2$. Because the identity

$$|\langle f|\hat{U}|i\rangle|^2 = |\langle i|\hat{U}^{-1}|f\rangle|^2 \quad (1.1)$$

holds, the success probability of the time-forward process is equal to that of the process where the initial state $|f\rangle$ evolves with the unitary operation $\hat{U}^{-1}$ and is projected onto the final state $|i\rangle$ (we call this process *the time-reversed process*). Due to the above relation, the time-reversed quantum optical interferometer in CPI can reproduce the same interference patterns as those of the time-forward quantum optical interferometer. The input light in CPI, pairs of oppositely-chirped laser pulses, corresponds to the frequency-correlated photon pairs detected by coincidence counting in the time-forward process. CPI is just an example of classical optical technique that reproduces quantum optical measurements inspired by the time-reversal symmetry of quantum mechanics; however, examples other than CPI have been limited [30, 31].

1.2 Objective and outline of this thesis

The objective of this thesis is to develop the classical optical technique to reproduce quantum optical measurements using time-reversed optical systems. For this purpose, we first present a construction method of classical optical systems that reproduce quantum optical interference patterns on the basis of the time-reversal symmetry of quantum mechanics; we call this method *the time-reversal method*. Then, by using the time-reversal method, we propose a new classical optical measurement technique to achieve some of the advantages of two-photon quantum optical measurements, which we call the *time-decomposed pulse interferometry (TDPI)*. TDPI is implemented by a time-reversed optical system same as CPI, but requires no chirped laser pulses. Instead, TDPI employs pairs of transform-limited (i.e.,

unmodified) laser pulses with various time differences as its input light, which correspond to the time-correlated photon pairs detected by coincidence counting in the time-forward process. TDPI can achieve much higher signal strength than the two-photon quantum optical systems and be implemented by a simpler optical system than CPI. We experimentally demonstrate that TDPI can perform both kinds of quantum optical measurements: a measurement of internal parameters of quantum light [the measurements (i)] and a more precise measurement of parameters of an external object [the measurements (ii)].

This thesis is organized as shown in Fig. 1.1. We first introduce several fundamental concepts regarding two-photon interferometers and the time-reversal method that leads CPI and TDPI in Chap. 2. In Chaps. 3 and 5, we describe the experiments that classically reproduce the interferograms of two-photon phase super-resolution [32] and dispersion-cancelled Hong–Ou–Mandel interference [21–23] by TDPI, respectively. These experiments are bases for the applications to the optical interferometric measurements described in Chaps. 4 and 6. In Chap. 4, we observe nonlinear variations in the geometric phase in a two-photon polarization qutrit, by using the time-reversed interferometer reproducing two-photon phase superresolution, which is constructed in Chap. 3. This experiment demonstrates the measurement of internal structure of quantum light. In Chap. 6, we demonstrate dispersion-cancelled OCT by using the time-reversed interferometer reproducing dispersion-cancelled HOM interference, which is constructed in Chap. 5. This experiment demonstrates the more precise measurement of parameters of an external object. Finally, we conclude the thesis and remark the future prospects of the time-reversal method and TDPI in Chap. 7.

In the following, we describe the abstract of each chapter.

Chapter 2: Fundamentals of the time-reversal method

In this chapter, we first review two-photon interference and its examples: two-photon phase superresolution [32] (relevant to Chap. 3), HOM interference [21], and automatic dispersion cancellation [22,23] (relevant to Chap. 5); then, we present the time-reversal method for the two-photon interferometers. According to the time-reversal method, a time-reversed optical system for a two-photon interferometer

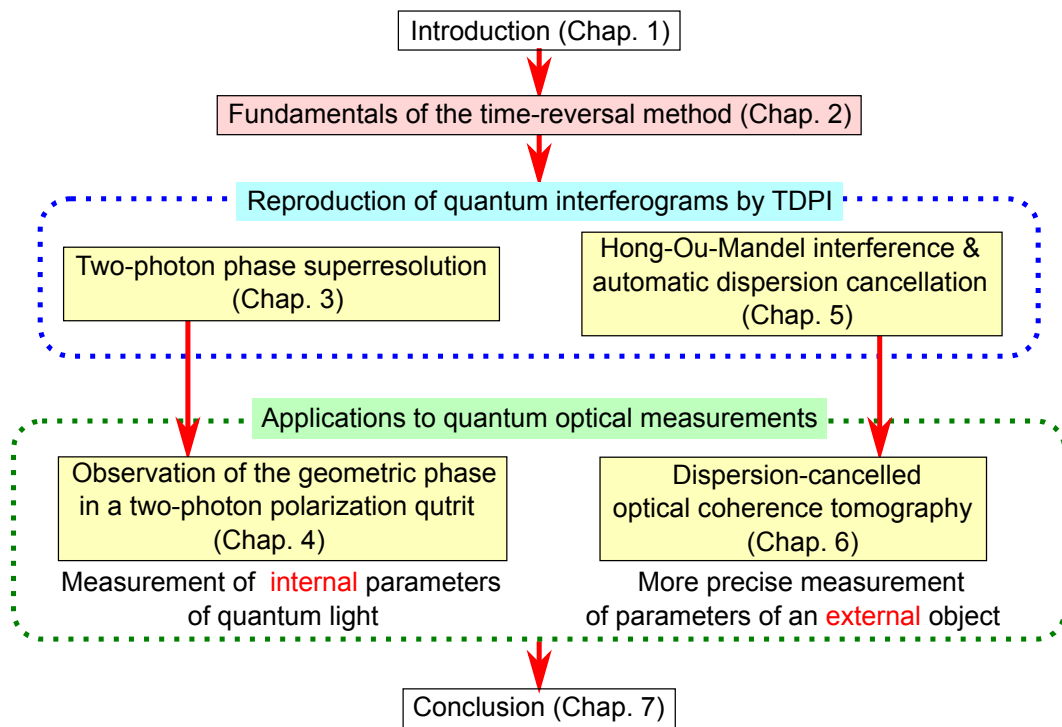


Figure 1.1: Outline of this thesis.

can be constructed in two manners: considering the interferometer in the frequency domain or the time domain. In the former case, the time-reversal method will lead CPI, and in the latter case, TDPI. We also mention a technique to implement the time-reversed process by a completely classical optical system using intense pulsed laser light.

Chapter 3: Classical reproduction of two-photon phase superresolution by TDPI

In this chapter, we describe the experimental demonstration reproducing two-photon phase superresolution in a classical optical interferometer that implements TDPI. The experimental setup is a time-reversed version of the two-photon interferometer proposed by Brendel *et al.* [32]. A measured interferogram exhibited two-photon phase superresolution with a high visibility of $97.9\% \pm 0.4\%$. The maximum optical power of the interference signal was $2.8 \mu\text{W}$, which is about 10^{11} times higher than that in the previous time-forward experiment [32]. The coherence length of the interferogram was about 22 times longer than that of the input laser pulses, showing a classical analog to the large difference between the one- and two-photon coherence lengths of entangled photon pairs.

Chapter 4: Observation of the geometric phase in a two-photon polarization qutrit by TDPI

In this chapter, we describe the experiment that classically observes nonlinear variations in the three-vertex geometric phase in a two-photon polarization qutrit by TDPI. The three-vertex geometric phase is defined by three quantum states, which generally form a three-state (qutrit) system. By changing one of the three constituent states, we observed two rapid increases in the three-vertex geometric phase. The observed variations in the geometric phase are inherent in a three-state quantum system and cannot be observed in a classical light [33]. We reproduced those variations in the geometric phase by using the time-reversed two-photon interferometer for phase superresolution constructed in Chap. 3. Even the minimum optical power of the interference signal exhibited three orders of magnitude greater than that in the previous experiments measuring the geometric phase in entangled

photon pairs [14, 34, 35]. This study exhibits an application of TDPI to a measurement of internal parameters of quantum light.

Chapter 5: Classical reproduction of Hong–Ou–Mandel interference with automatic dispersion cancellation by TDPI

In this chapter, we describe the experimental demonstration reproducing HOM interference [21] with automatic dispersion cancellation [22, 23] in a classical optical interferometer that implements TDPI. The experimental setup is a time-reversed version of a HOM interferometer with pairs of orthogonally-polarized input laser pulses. The results showed that the interferometer can obtain high-visibility ($89.9\%\pm0.7\%$) dispersion-insensitive HOM interferograms with intense optical power ($0.57\text{ }\mu\text{W}$).

Chapter 6: Dispersion-cancelled optical coherence tomography by TDPI

In this chapter, we describe the experiment demonstrating dispersion-insensitive OCT as a more precise measurement of parameters of an external object. In this experiment, we used the time-reversed HOM interferometer constructed in Chap. 5. Our measurement system can achieve much higher signal than quantum-OCT [18–20], and can be implemented by a simpler optical system than CPI [27, 29]. Furthermore, we employ a new technique that makes the output signal patterns free of artifacts and background, which disturb the measured OCT images. We demonstrated dispersion-cancelled, artifact-free, and background-free tomographic imaging of the axial structure of a coverglass and cross-sectional imaging of a 100-yen coin.

Chapter 2

Fundamentals of the time-reversal method

2.1 Introduction

This chapter provides several fundamental concepts regarding two-photon interference and the time-reversal method. In Sec. 2.2, we first describe time-frequency-entangled photon pairs and their two-photon interference. As examples of the two-photon interference, we introduce two-photon phase superresolution, Hong–Ou–Mandel (HOM) interference, and automatic dispersion cancellation. These quantum interference phenomena are relevant to the studies in Chaps. 3 and 5. In Sec. 2.3, we describe the time-reversal method, which is a classical optical technique to reproduce the interferograms of an quantum optical system by temporally reversing the original system. We show that the time-reversal method leads two interferometric techniques: chirped-pulse interferometry (CPI) and time-decomposed pulse interferometry (TDPI). TDPI is our new proposal and the main topic throughout this thesis. We also mention a technique to implement the time-reversed process by a completely classical optical system using intense pulsed laser light.

2.2 Two-photon interference

Two-photon interference is nonclassical interference of two photons, most of which are demonstrated by time-frequency-entangled photon pairs. For thirty years, various kinds of two-photon interference have been proposed and experimentally demonstrated, and have been bases of recent quantum information technologies. In Sec. 2.2.1, we first describe the generation method of entangled photon pairs and the general properties of their entanglement. In Sec. 2.2.2–2.2.4, we illustrate the following three examples of the two-photon interference: two-photon phase superresolution, HOM interference, and automatic dispersion cancellation. The interferograms of these two-photon interference phenomena are reproduced by TDPI in Chaps. 3 and 5.

2.2.1 Time-frequency-entangled photon pairs

Quantum entanglement is the nonclassical correlation that particular quantum states have. The concept of entanglement was first discussed by Einstein *et al.* in 1935 [36], and nowadays, plays a central role in wide range of physics including quantum information science [37]. In this section, we focus on the time-frequency entanglement of two photons and describe their generation method and typical properties (other kinds of entanglement like position-wavevector entanglement can also be discussed in the same manner).

The most typical method to generate time-frequency-entangled photon pairs is spontaneous parametric down-conversion (SPDC) shown in Fig. 2.1. SPDC is a second-order nonlinear optical process occurring in birefringent crystals where high energy photons (pump photons) are converted into pairs of low energy photons (signal and idler photons). In SPDC, the total energy and momentum of the photons must be conserved:

$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i, \quad \hbar\mathbf{k}_p = \hbar\mathbf{k}_s + \hbar\mathbf{k}_i, \quad (2.1)$$

where the subscripts p, s, and i respectively mean pump, signal, and idler photons. Because of these relations, the frequencies and wavevectors of the photon pairs are highly correlated. If polarizations of the signal and idler photons are same,

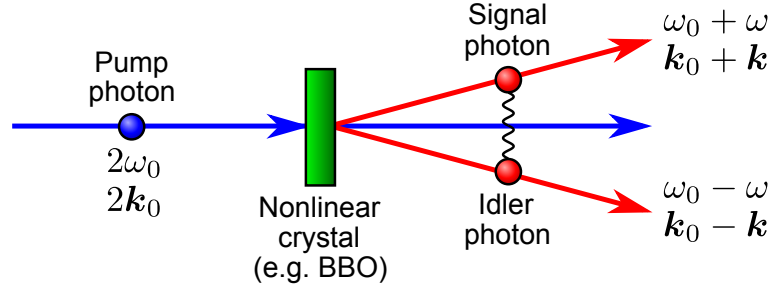


Figure 2.1: Schematic of SPDC. When a pump photon enters a nonlinear crystal, the pump photon is stochastically converted into a pair of signal and idler photons, with the total energy and momentum of the photons conserved.

the SPDC is referred to as type-I; if they are orthogonal, type-II. For the type-II SPDC, the photon pairs are also entangled with respect to their polarizations. The generation rate of the entangled photon pairs is proportional to the pump power but typically low ($\lesssim 10^6$ Hz [24]).

The time-frequency entanglement of the photon pairs is essential to realization of various quantum-optical phenomena including phase superresolution and HOM interference with automatic dispersion cancellation. The quantum state of a time-frequency-entangled photon pair $|\Psi\rangle$ is described as

$$|\Psi\rangle = \int_{-\infty}^{\infty} d\omega_s \int_{-\infty}^{\infty} d\omega_i f(\omega_s, \omega_i) \hat{a}^\dagger(\omega_s) \hat{b}^\dagger(\omega_i) |0\rangle, \quad (2.2)$$

where $\hat{a}^\dagger(\omega_s)$ and $\hat{b}^\dagger(\omega_i)$ are respectively the creation operators of mode A with frequency ω_s and mode B with frequency ω_i , and $|0\rangle$ is the vacuum state. $f(\omega_s, \omega_i)$ is the two-photon wave function in the frequency domain. When the spectrum width of the pump light is sufficiently narrow and that of SPDC is sufficiently broad, the frequencies of the two photons are strongly correlated and the two-photon wave function in the frequency domain forms as shown in Fig. 2.2(a). This correlation means that the sum of the frequencies of the two photons is almost constant although their individual frequencies are considerably uncertain. In the time domain, on the other hand, $|\Psi\rangle$ is represented as

$$|\Psi\rangle = \int_{-\infty}^{\infty} dt_s \int_{-\infty}^{\infty} dt_i f(t_s, t_i) \hat{a}^\dagger(t_s) \hat{b}^\dagger(t_i) |0\rangle, \quad (2.3)$$

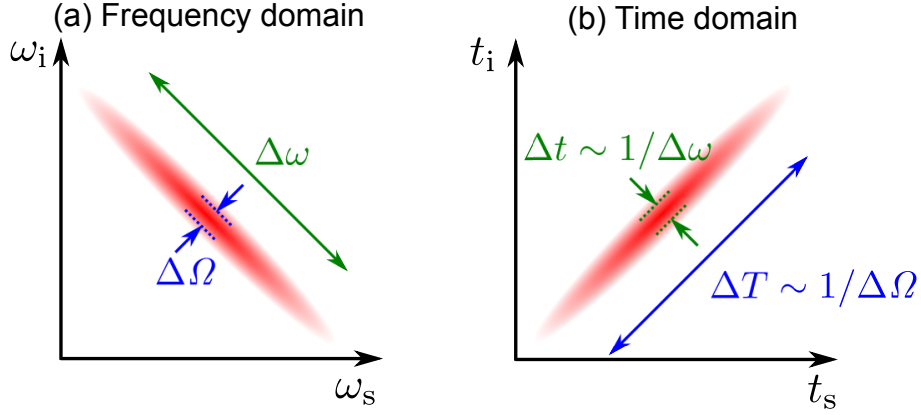


Figure 2.2: Absolute values of the two-photon wave function of the strongly correlated photon pairs (a) in the frequency domain and (b) in the time domain. $\Delta\Omega$ and $\Delta\omega$ respectively represent the spectrum widths of the sum and difference of the frequencies of the two photons, which respectively correspond to the spectrum widths of the pump light and SPDC. ΔT and Δt represent the time width of the sum and difference of the detection time of the two photons, respectively.

where $f(t_s, t_i)$, $\hat{a}^\dagger(t_s)$, and $\hat{b}^\dagger(t_i)$ are the Fourier transforms of $f(\omega_s, \omega_i)$, $\hat{a}^\dagger(\omega_s)$, and $\hat{b}^\dagger(\omega_i)$, respectively. In the strong correlation condition, the two-photon wave function in the time domain forms as shown in Fig. 2.2(b), which means that the two photons are detected at almost the same time although the time when the two photons are detected is considerably uncertain.

2.2.2 Two-photon phase superresolution

As the first example of two-photon interference, we describe two-photon phase superresolution. The phase superresolution is a phenomenon that shows an interference oscillation with a smaller period than that of classical optical interference. We now consider the following interferometer: an input light or photon is first split into two modes A and B; the light or photon in mode B receives a relative phase shift of $e^{i\phi}$; then the two modes are recombined into a detected mode. When we use classical input light, the output intensity shows an interference oscillation with respect to ϕ with the period of 2π . On the other hand, when we use N photons

in the mode-entangled number state $(|N\rangle_A|0\rangle_B + |0\rangle_A|N\rangle_B)/\sqrt{2}$, which is called NOON-state [38], the second term $|0\rangle_A|N\rangle_B$ receives a relative phase shift of $e^{iN\phi}$ in the interferometer, and the N -photon detection probability shows an interference oscillation with the N -times smaller period than that of the classical optical interferometer. Phase superresolution using the entangled photons have been demonstrated for two-photon [32, 39–41], three-photon [42], and four-photon [16, 17, 43] states. Phase superresolution is applied to sub-Rayleigh resolution in quantum lithography [44, 45].

We here describe the experiment for two-photon phase superresolution demonstrated by Brendel *et al.* [32]. The experiment uses an unbalanced Michelson interferometer shown in Fig. 2.3(a). First, input narrow band pump photons are down-converted into time-frequency-entangled photon pairs by SPDC, whose two-photon wave function is shown in Fig. 2.4(a). The photon pairs next enter the unbalanced Michelson interferometer. The optical path difference of the interferometer can be changed by x to add the relative phase shift of $e^{i\omega_0 x/c}$ to each photon, where ω_0 is the central frequency of the down-converted photons and c is the speed of light. After the interferometer, there are four possible states of the photon pairs: LL, LS, SL, and SS, where the letters L and S denote the photons from the longer and shorter arms of the interferometer, respectively. Their two-photon wave functions are represented as Fig. 2.4(b). Finally, the photon pairs are detected by coincidence counting with a narrow time window. When x is large enough, only photon pairs LL and SS contribute to the coincidence counting. Because the two-photon wave functions of the states LL and SS are almost overlapped as Fig. 2.4(c) shows, these terms fully interfere with each other. The success probability $P(x)$ of the coincidence counting is calculated as

$$P(x) \propto \left\| e^{i2\omega_0 x/c} |2\rangle_L |0\rangle_S + |0\rangle_L |2\rangle_S \right\|^2 \propto 1 + \cos(2\omega_0 x/c) \quad (2.4)$$

and shown in Fig. 2.3(b). This interferogram shows two-photon phase superresolution, that is, the period $\pi c/\omega_0$ is twice shorter than that of the classical interferometer $2\pi c/\omega_0$, which is equal to the central wavelength of the down-converted photons.

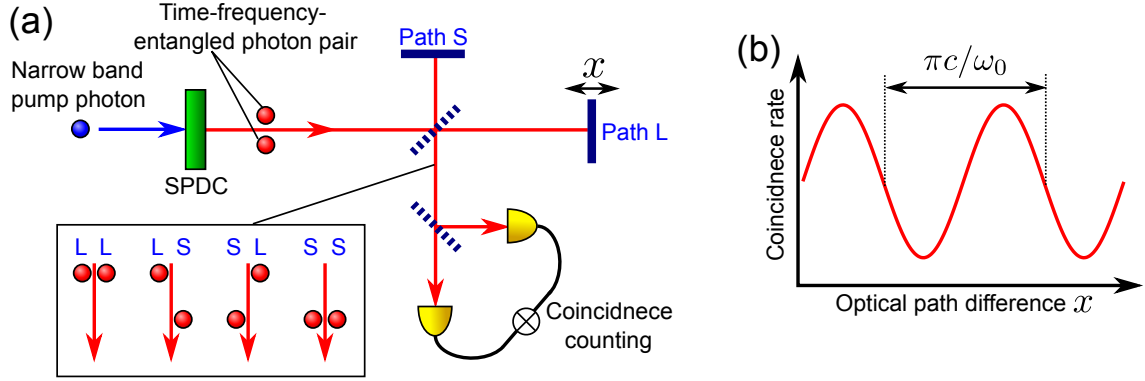


Figure 2.3: (a) Schematic setup of an unbalanced Michelson interferometer for two-photon phase superresolution. (inset) Snapshots of the four possible processes of the photon-pair after the interferometer. The letters L and S denote the photons from the longer and shorter arms of the interferometer, respectively. (b) Coincidence rate $P(x)$ of this interferometer. The period $\pi c/\omega_0$ is twice shorter than the central wavelength of the down-converted photons.

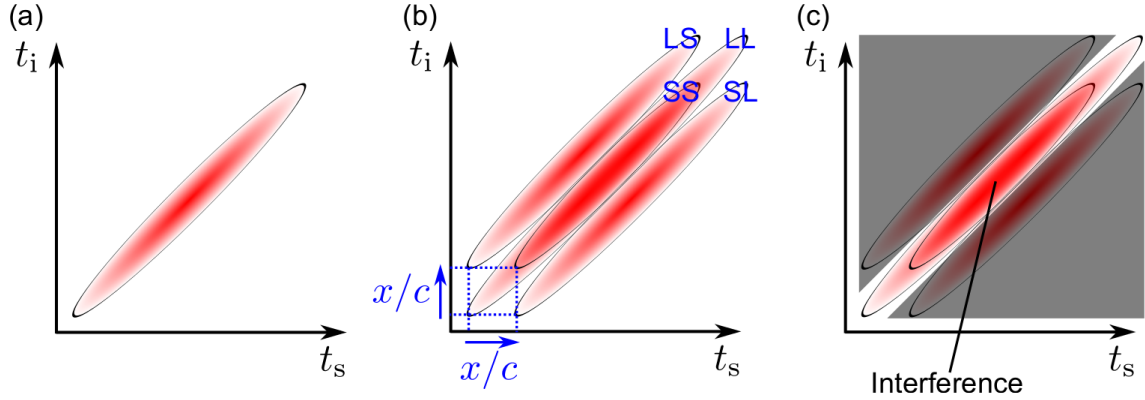


Figure 2.4: Two-photon wave functions in the experiment for two-photon phase superresolution. (a) Initial wave function. (b) After the unbalanced Michelson interferometer, the wave function shifts longitudinally or/and transversely. (c) The coincidence counting extracts the wave functions in the narrow region satisfying $t_s \simeq t_i$. The extracted wave functions, LL and SS, are almost overlapped and fully interfere with each other.

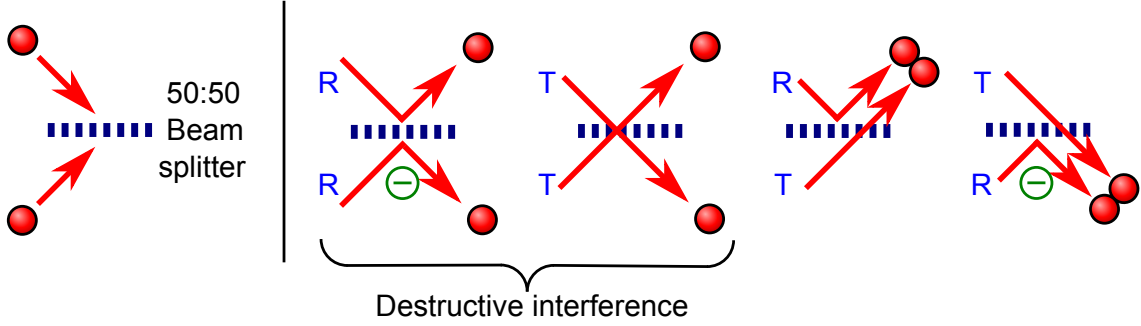


Figure 2.5: Sketch of HOM interference. There are four possibilities of the output states. The RR and TT terms destructively interfere and only the RT and TR terms are observed.

2.2.3 Hong–Ou–Mandel interference

Hong–Ou–Mandel (HOM) interference is one of the most fundamental quantum-optical interference and widely used in quantum information technology including quantum computation [46], quantum cryptography [47], and quantum metrology [16]. HOM interference occurs when two photons enter a 50:50 beam splitter, one in each input port, as shown in Fig. 2.5. There are four possible output states: reflection-reflection (RR), transmission-transmission (TT), reflection-transmission (RT), and transmission-reflection (TR). If the two photons are identical and enter the beam splitter simultaneously, the two states RR and TT are indistinguishable and interfere with each other. In addition, the photons reflected on the bottom side of the beam splitter receive a relative phase shift of -1 , due to the unitarity of the operation of the beam splitter. Therefore, the two states RR and TT are destructively interfere with each other. Totally, we can interpret that the two photons bunch and come out from one of the output ports together.

HOM interference is experimentally demonstrated by using the HOM interferometer shown in Fig. 2.6(a). The input two photons, which are typically generated by SPDC but not necessarily time-frequency-entangled, are emitted into the two path modes. One of the modes has a delay line to add an optical delay x . After the delay, the two photons are recombined at the beam splitter. Finally, the output two photons are detected by coincidence counting, one in each output port. The

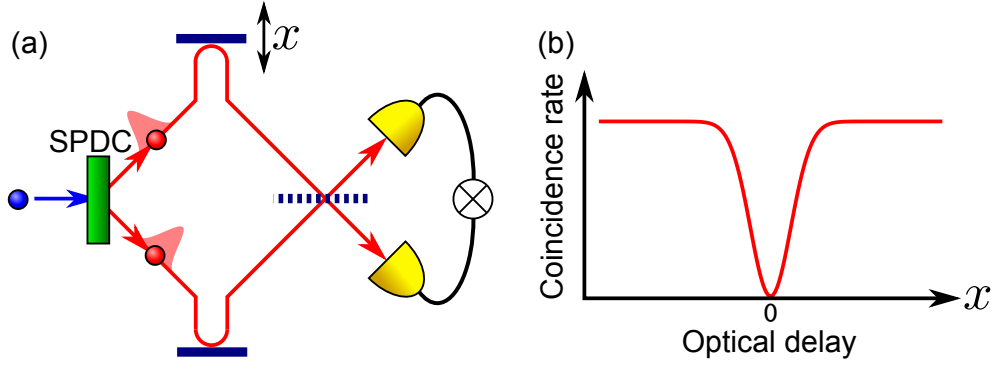


Figure 2.6: (a) HOM interferometer. The two photons are not necessarily time-frequency-entangled. (b) Interferogram of the HOM interferometer, showing HOM dip.

coincidence counting rate drops to zero when the optical delay x is zero, as shown in Fig. 2.6(b). This interferogram is called HOM dip.

We describe a brief calculation of the HOM interferometer. In the frequency domain, the initial state $|\Psi\rangle$ of the two photons with finite spectra is expressed as

$$|\Psi\rangle = \int_{-\infty}^{\infty} d\omega_s f_s(\omega_s) \hat{a}^\dagger(\omega_s) \int_{-\infty}^{\infty} d\omega_i f_i(\omega_i) \hat{b}^\dagger(\omega_i) |0\rangle. \quad (2.5)$$

The coincidence counting rate $C(x)$ of the HOM interferometer is calculated as

$$C(x) \propto \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\infty} d\omega_2 |A_{RR}(\omega_1, \omega_2, x) + A_{TT}(\omega_1, \omega_2, x)|^2, \quad (2.6)$$

where $A_{RR}(\omega_1, \omega_2, x)$ and $A_{TT}(\omega_1, \omega_2, x)$ are the amplitudes where both photons are reflected and transmitted at the beam splitter, respectively. In this case, these amplitudes are given by

$$A_{RR}(\omega_1, \omega_2, x) = -f_s(\omega_1) f_i(\omega_2) e^{i\omega_1 x/c}, \quad (2.7)$$

$$A_{TT}(\omega_1, \omega_2, x) = f_s(\omega_2) f_i(\omega_1) e^{i\omega_2 x/c}. \quad (2.8)$$

When both spectral functions $f_s(\omega)$ and $f_i(\omega)$ are given by $\exp[-(\omega - \omega_0)^2/(2\Delta\omega^2)]$ (ω_0 : the central frequency, $\Delta\omega$: the RMS width of the spectrum), the coincidence counting rate $C(x)$ is calculated as

$$C(x) \propto 1 - \exp \frac{-(x/c)^2}{2(1/\Delta\omega)^2}, \quad (2.9)$$

which exhibits the HOM dip with a RMS width of $c/\Delta\omega$.

2.2.4 Automatic dispersion cancellation

HOM interference of time-frequency-entangled photon pairs exhibits an additional quantum optical effect: automatic dispersion cancellation. When the light propagates through a dispersive medium like a glass, the light receives a nonlinear frequency-dependent phase $\phi(\omega)$. $\phi(\omega)$ is expanded about a frequency ω_0 as

$$\phi(\omega) = \phi(\omega_0) + \alpha \cdot (\omega - \omega_0) + \beta \cdot (\omega - \omega_0)^2 + \dots, \quad (2.10)$$

where the second and third terms are describing the group delay and quadratic group delay dispersion (GDD), respectively. The second- and higher-order terms modify the wave shape of the light in the time domain. In a white-light or low-coherence light interferometer, the dispersion in one of the arms of the interferometer gives rise to broadening of the interferogram. On the other hand, the HOM dip is not affected by even-order dispersion due to the frequency correlation of the entangled photons. Because the third- and higher-order dispersion is negligibly small in general, the dispersion can be substantially cancelled in the HOM interferometer, as shown in Fig. 2.7

The automatic dispersion cancellation is mathematically described as follows. The initial state $|\Psi\rangle$ of the time-frequency-entangled photon pairs is expressed as

$$|\Psi\rangle = \int_{-\infty}^{\infty} d\omega_s \int_{-\infty}^{\infty} d\omega_i f(\omega_s, \omega_i) \hat{a}^\dagger(\omega_s) \hat{b}^\dagger(\omega_i) |0\rangle, \quad (2.11)$$

where $f(\omega_s, \omega_i)$ is the strongly time-frequency-correlated two-photon wave function. The coincidence counting rate $C(x)$ is the same formula as Eq. (2.6), but the amplitudes $A_{RR}(\omega_1, \omega_2, x)$ and $A_{TT}(\omega_1, \omega_2, x)$ are given by

$$A_{RR}(\omega_1, \omega_2, x) = -f(\omega_1, \omega_2) e^{i\omega_1 x/c} e^{i\beta \cdot (\omega_2 - \omega_0)^2}, \quad (2.12)$$

$$A_{TT}(\omega_1, \omega_2, x) = f(\omega_2, \omega_1) e^{i\omega_2 x/c} e^{i\beta \cdot (\omega_1 - \omega_0)^2}, \quad (2.13)$$

where we consider only the second-order dispersion. Because of the correlation $\omega_1 + \omega_2 = 2\omega_0$, $e^{i\beta \cdot (\omega_1 - \omega_0)^2} = e^{i\beta \cdot (\omega_2 - \omega_0)^2}$ and thus the effect of the second-order

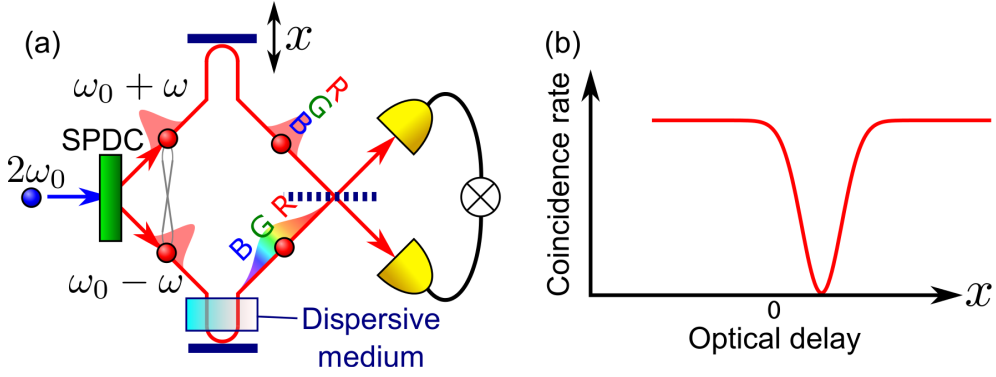


Figure 2.7: (a) HOM interferometer with time-frequency-entangled input photon pairs. The dispersive medium introduces GDD to the photons in the lower path and broadens their wave shape in time domain. (b) The width of the HOM dip is not broadened by the second-order dispersion while the interferogram is moved transversely by the first-order dispersion.

dispersion vanishes. If the two-photon wave function $f(\omega_1, \omega_2)$ is expressed as

$$f(\omega_1, \omega_2) = \exp \frac{-(\omega_1 - \omega_0)^2}{2\Delta\omega^2} \exp \frac{-(\omega_2 - \omega_0)^2}{2\Delta\omega^2} \delta(\omega_1 - \omega_2), \quad (2.14)$$

the coincidence counting rate $C(x)$ is given by the same formula as Eq. (2.9).

2.3 Time-reversal method

In this section, we present the time-reversal method, a construction method of the optical systems that reproduce quantum optical interference patterns on the basis of the time-reversal symmetry of quantum mechanics. In Sec. 2.3.1, we briefly review the time-reversal symmetry of quantum mechanics. In Sec. 2.3.2, we show a common way to calculate the success probabilities of typical time-forward (conventional) two-photon interferometers. In Sec. 2.3.3, we describe the time-reversal method to derive two different time-reversed optical systems, CPI and TDPI, the latter of which is our new proposal. We also mention a technique to implement CPI and TDPI by completely classical optical systems using intense pulsed laser light.

2.3.1 Time-reversal symmetry in quantum mechanics

We first review the time-reversal symmetry of quantum mechanics. For a unitary operator $\hat{U}(x)$ parametrized by x and a pair of states $|i\rangle$ and $|f\rangle$, the relation

$$|\langle f|\hat{U}(x)|i\rangle|^2 = |\langle i|\hat{U}^{-1}(x)|f\rangle|^2 \quad (2.15)$$

holds. The relation is easily derived from conjugate transposition $\langle f|\hat{U}(x)|i\rangle = \langle i|\hat{U}^\dagger(x)|f\rangle^*$ and unitarity $\hat{U}^\dagger(x) = \hat{U}^{-1}(x)$. The left-hand side of Eq. (2.15) corresponds to the success probability of the projection onto the final state $|f\rangle$ for the system evolved with $\hat{U}(x)$ from the initial state $|i\rangle$. We call this process the time-forward process. By utilizing Eq. (2.15), we can exchange the roles of $|i\rangle$ and $|f\rangle$ if we can physically realize the time evolution $\hat{U}^{-1}(x)$. We call the evolution from the initial state $|f\rangle$ by $\hat{U}^{-1}(x)$ projected onto the final state $|i\rangle$ the time-reversed process. The time-reversed process has the same success probability as the corresponding time-forward process.

2.3.2 Time-forward two-photon interferometer

We next consider typical time-forward (conventional) two-photon interferometers using time-frequency-entangled photon pairs. Such optical systems are commonly illustrated as Fig. 2.8. In these systems, the initial state $|i\rangle$ is a single pump photon state with the central frequency $2\omega_0$, which must have a narrow spectrum (i.e., a long coherence time) to generate strongly time-frequency-entangled photon pairs. The unitary evolution $\hat{U}(x)$ in this system consists of SPDC in a nonlinear optical crystal and optical propagation in the other linear optical elements. SPDC is regarded as a unitary process for the pump and down-converted fields. The photon pairs generated by SPDC are finally detected by coincidence counting, that is, one of the photon pair is detected in one mode (mode A), and the other photon is detected in the other mode (mode B).

The success probability $P(x)$ of the coincidence counting is calculated in the following two manners. In the frequency domain, $P(x)$ is represented by the following

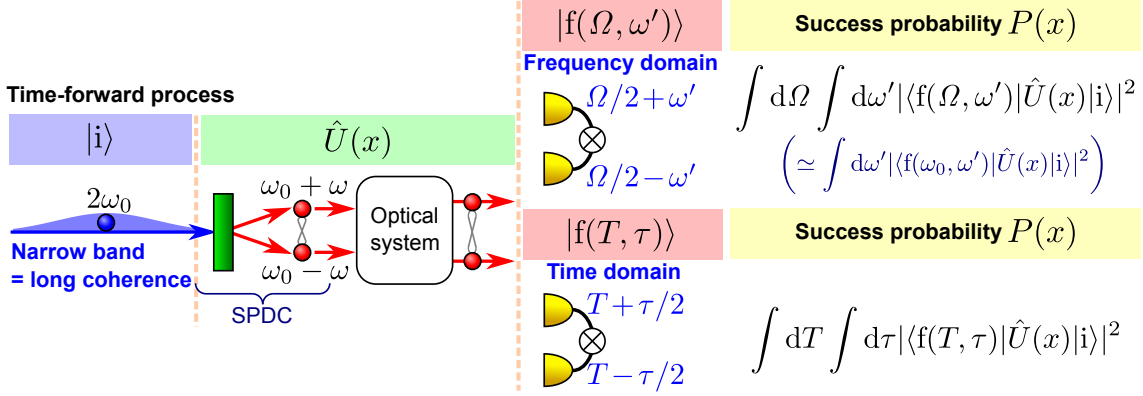


Figure 2.8: Schematic of common two-photon interferometers using time-frequency-entangled photon pairs. There are two representations of coincidence counting: that in the frequency domain or time domain.

integral:

$$P(x) \propto \int_{-\infty}^{\infty} d\Omega \int_{-\infty}^{\infty} d\omega' |\langle f(\Omega, \omega') | \hat{U}(x) | i \rangle|^2, \quad (2.16)$$

where $|f(\Omega, \omega')\rangle := \hat{a}^\dagger(\Omega/2 + \omega') \hat{b}^\dagger(\Omega/2 - \omega') |0\rangle$ means that one photon with frequency $\Omega/2 + \omega'$ is in the mode A, and the other with frequency $\Omega/2 - \omega'$ is in the mode B. For the initial state $|i\rangle$ with a sufficiently narrow spectrum, $P(x)$ can be approximately simplified as

$$P(x) \propto \int_{-\infty}^{\infty} d\omega' |\langle f(\omega_0, \omega') | \hat{U}(x) | i \rangle|^2. \quad (2.17)$$

On the other hand, in the time domain, $P(x)$ is also represented by the following integral:

$$P(x) \propto \int_{-\infty}^{\infty} dT \int_{-\infty}^{\infty} d\tau |\langle f(T, \tau) | \hat{U}(x) | i \rangle|^2, \quad (2.18)$$

where $|f(T, \tau)\rangle := \hat{a}^\dagger(T + \tau/2) \hat{b}^\dagger(T - \tau/2) |0\rangle$ means that one photon is in the mode A at time $T + \tau/2$, and the other is in the mode B at time $T - \tau/2$. Because $|f(T, \tau)\rangle$ is the Fourier transforms of $|f(\Omega, \omega')\rangle$, both integrals in Eqs. (2.16) and (2.18) have the same value due to Parseval's theorem.

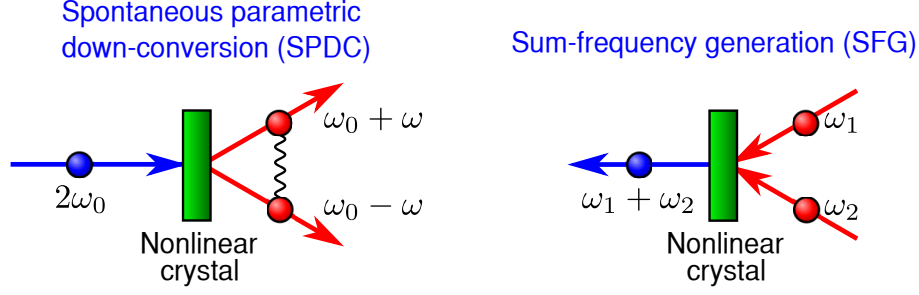


Figure 2.9: SPDC and SFG.

2.3.3 Time-reversed two-photon interferometer

In this section, we apply the time-reversal symmetry of quantum mechanics to the two-photon interferometer to obtain the two time-reversed two-photon interferometry, CPI and TDPI. The outline of this approach in the frequency (time) domain is as follows. The success probability density $|\langle f(\omega_0, \omega') | \hat{U}(x) | i \rangle|^2 = |\langle i | \hat{U}^{-1}(x) | f(\omega_0, \omega') \rangle|^2$ in Eq. (2.17) [$|\langle f(T, \tau) | \hat{U}(x) | i \rangle|^2 = |\langle i | \hat{U}^{-1}(x) | f(T, \tau) \rangle|^2$ in Eq. (2.18)] can also be realized in the system where the initial state $|f(\omega_0, \omega')\rangle$ ($|f(T, \tau)\rangle$) evolves with the unitary evolution $\hat{U}^{-1}(x)$ and is projected onto the final state $|i\rangle$. We observe the success probability density $|\langle i | \hat{U}^{-1}(x) | f(\omega_0, \omega') \rangle|^2$ ($|\langle i | \hat{U}^{-1}(x) | f(T, \tau) \rangle|^2$) by using such a time-reversed system, and we sum up this success probability density over all ω' (T and τ) to reproduce the success probability in the time-forward system $P(x)$.

The time-reversed system realizing the success probability $|\langle i | \hat{U}^{-1}(x) | f(\omega_0, \omega') \rangle|^2$ ($|\langle i | \hat{U}^{-1}(x) | f(T, \tau) \rangle|^2$) is constructed as follows. The initial state $|f(\omega_0, \omega')\rangle$ ($|f(T, \tau)\rangle$) is a two-photon state with single frequency $\omega_0 \pm \omega'$ (localized at time $T \pm \tau/2$). The details of the way to prepare these states are described later in the following sections regarding CPI and TDPI. The time-reversed unitary evolution $\hat{U}^{-1}(x)$ consists of optical propagation in the optical system in the reverse manner and sum-frequency generation (SFG) in the nonlinear optical crystal. SFG, shown in Fig. 2.9, is a second-order nonlinear optical process where two input photons are converted into a sum-frequency single photon. SFG is regarded to be the time-reversed process of SPDC. The projection onto $|i\rangle$ is approximately realized by filter-

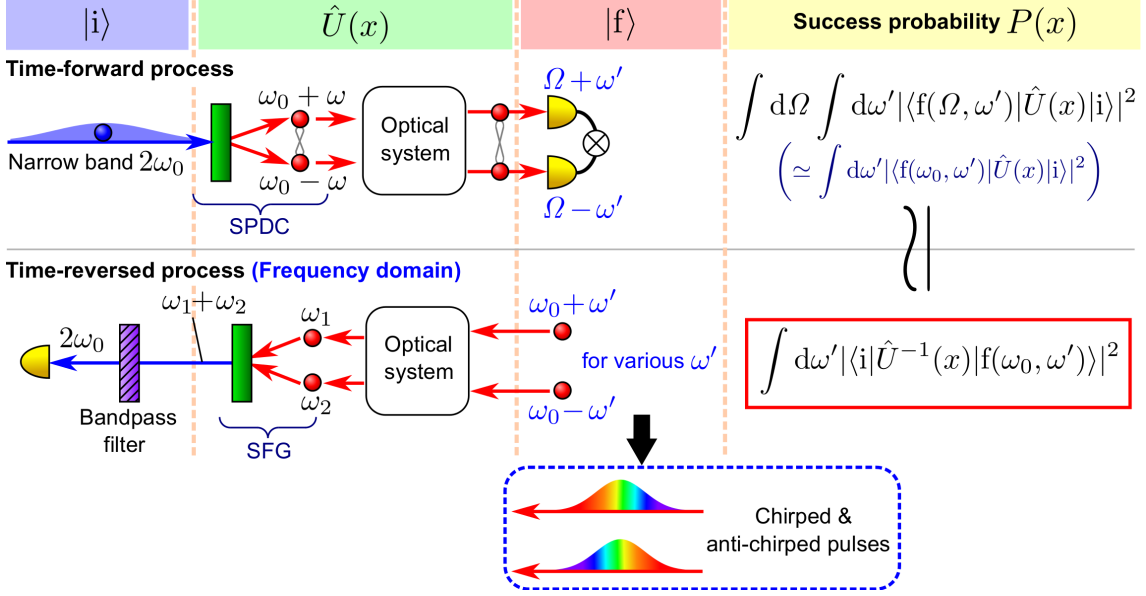


Figure 2.10: Construction of a time-reversed two-photon interferometer in the frequency domain. CPI is a technique realizing this approach. CPI uses a pair of strongly oppositely-chirped pulse state as the input state, which implements a sum of the two-photon states with frequency $\omega_0 + \omega'$ and $\omega_0 - \omega'$ for various ω' .

ing a sufficiently narrow spectrum around $2\omega_0$ with a bandpass filter and detecting a single photon by a photodetector. This detection means is regarded as a projection onto a single frequency state $|i(2\omega_0)\rangle := \hat{a}^\dagger(2\omega_0)|0\rangle$ in the frequency domain, and the projection probability $P(x)$ is given by $P(x) \propto |\langle i(2\omega_0)|\hat{U}^{-1}(x)|f(\omega_0, \omega')\rangle|^2$. In the time domain, on the other hand, this projection probability $P(x)$ is given by the integral of the detection probability density over all the detection time:

$$P(x) \propto \int_{-\infty}^{\infty} dt |\langle i(t)|\hat{U}^{-1}(x)|f(T, \tau)\rangle|^2, \quad (2.19)$$

where $|i(t)\rangle := \hat{a}^\dagger(t)|0\rangle$ is a temporally localized state.

Chirped-pulse interferometry (CPI)

As mentioned above, in the frequency domain, we must prepare the initial states $|f(\omega_0, \omega')\rangle$ for various frequency ω' and sum up the success probability densities over

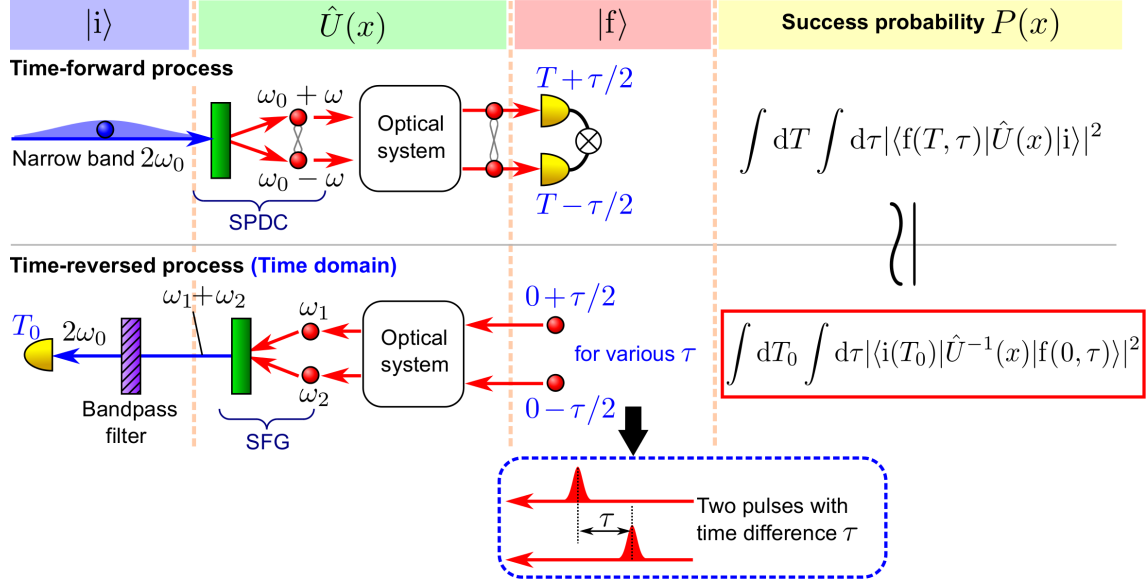


Figure 2.11: Construction of a time-reversed two-photon interferometer in the time domain, which is realized by TDPI. TDPI uses a pair of laser pulses with various time differences τ as its input light. We measure each success probabilities for various differences τ and sum up the measured data over all τ by post-processing.

all ω' . A technique realizing this procedure is chirped-pulse interferometry (CPI), proposed by Kaltenbaek *et al.* [25]. The schematic of CPI is shown in Fig. 2.10. The initial state in CPI is a pair of strongly oppositely-chirped pulse state, that is, a chirped pulse state in the mode A and an anti-chirped pulse state in the mode B, which correspond to the frequency-correlated photon pairs detected by coincidence counting in the time-forward process. This initial state can be interpreted as a sum of the states $|f(\omega_0, \omega')\rangle$ for various frequency ω' . Due to this initial state, the measured optical intensity in CPI directly exhibits the integration of $|\langle i | \hat{U}^{-1}(x) | f(\omega_0, \omega') \rangle|^2$ over all ω' . Several experiments using CPI have demonstrated reproduction of two-photon interference such as dispersion-cancelled HOM interference [25], HOM peak, quantum beating, and phase superresolution [28].

Time-decomposed pulse interferometry (TDPI)

We next consider the time-reversed two-photon interferometer in the time domain. According to the time-reversal method mentioned above, we must prepare the initial states $|f(T, \tau)\rangle$ for various time T and τ and sum up the success probability densities over all T and τ . Nevertheless, we actually only have to sum up the success probability densities over τ , and T can be a fixed value, as shown below. For the time-reversed system with an initial state $|f(T = 0, \tau)\rangle$, the projection probability density onto $|i(T_0)\rangle$ is given by $|\langle i(T_0)|\hat{U}^{-1}(x)|f(0, \tau)\rangle|^2$, which is equal to $|\langle i(0)|\hat{U}^{-1}(x)|f(-T_0, \tau)\rangle|^2$ due to the time-translation symmetry. The projection probability $P(x)$ in Eq. (2.19) is given by

$$\begin{aligned} P(x) &\propto \int_{-\infty}^{\infty} dT_0 |\langle i(0)|\hat{U}^{-1}(x)|f(-T_0, \tau)\rangle|^2 \\ &= \int_{-\infty}^{\infty} dT_0 |\langle i(0)|\hat{U}^{-1}(x)|f(T_0, \tau)\rangle|^2, \end{aligned} \quad (2.20)$$

which means that $|\langle i(0)|\hat{U}^{-1}(x)|f(T_0, \tau)\rangle|^2$ is automatically integrated over T_0 by this projection measurement. Therefore, we only have to prepare the initial states $|f(0, \tau)\rangle$ with various time differences τ and sum up the success probabilities over all τ to reproduce $P(x)$ in Eq. (2.18). A technique realizing this procedure is time-decomposed pulse interferometry (TDPI), which is our new proposal, and its schematic is shown in Fig. 2.11. TDPI employs pairs of transform-limited (unchirped) laser pulses with various time differences τ as its input light, which correspond to the time-correlated photon pairs detected by coincidence counting in the time-forward process. Because the summation of the success probabilities over all τ is done by post-processing, TDPI requires more number of processes compared to CPI; however, TDPI requires no chirped laser pulses and can be implemented by a simpler optical system than CPI.

Classical realization of CPI and TDPI

In the above discussion, the input states in CPI and TDPI are assumed to be two-photon states. We here consider replacing the two-photon input state with classical pulsed light and note a technique to implement CPI and TDPI classically.

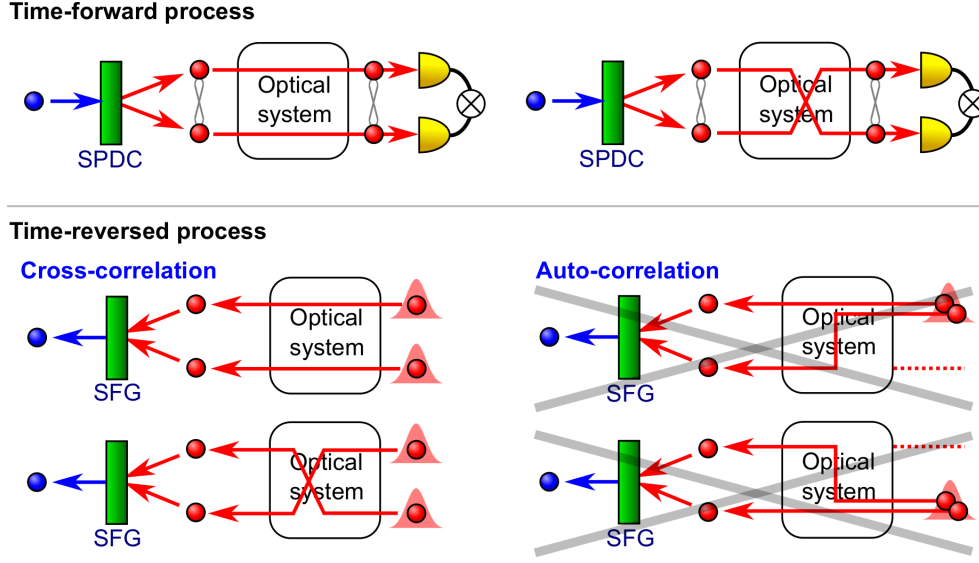


Figure 2.12: (upper panel) Two-photon terms that causes two-photon interference in the time-forward process. (lower panel) Two-photon terms that can contribute the interference signal in the time-reversed process with classical input light. We have to eliminate the contribution from the auto-correlation terms to reproduce the same interferograms as the time-forward process.

Because the output power of SFG is proportional to the product of the instantaneous intensity of two input light, we can easily achieve intense output signals in CPI and TDPI with classical pulsed input light.

As the upper panel of Fig. 2.12 shows, the interference signal of time-forward two-photon interferometer arises from the two-photon terms where one photon is detected by the upper detector and the other by the lower. In the time-reversed process shown in the lower panel of Fig. 2.12, these terms correspond to the cross-correlation terms, where one photon in the upper pulse and one in the lower are up-converted into a sum-frequency photon. The time-reversed process using classical pulsed light, however, includes the undesired auto-correlation terms, where two photons in a same pulse are up-converted into a sum-frequency photon. To eliminate the auto-correlation terms, we must label the two input pulses and exclude the contribution from the two photons with the same label in some way.

In CPI, the input chirped and anti-chirped pulses are already labeled with the frequency modulation. Their cross-correlation terms have narrow spectral width; on the other hand, their auto-correlation terms have broad one. Therefore, the narrow band detection in CPI can eliminate the auto-correlation terms and derive only the cross-correlation terms.

In TDPI, we need to label the input two pulses with an additional degree of freedom. We employ the polarization degree of freedom and label one of the pulses with the horizontal (H) polarization, and the other with the vertical (V). We also employ type-II SFG, where only pairs of orthogonally polarized photons (HV and VH) are up-converted into sum-frequency photons. In this manner, we can eliminate the contribution from the auto-correlation terms in TDPI.

Chapter 3

Classical reproduction of two-photon phase superresolution by TDPI

3.1 Introduction

Entangled photon pairs generated by spontaneous parametric down-conversion (SPDC) have peculiar characteristics that have never been seen in classical optics. The time-frequency correlation is one such characteristic. Two time-frequency-correlated photons tend to be detected simultaneously, and the sum of their frequencies is constant. The two-photon coherence length of time-frequency-entangled photon pairs is much larger than that of individual photons. By utilizing these properties, various two-photon interference phenomena have been observed, such as automatic dispersion cancellation [22, 23], nonlocal interference [48, 49], and two-photon phase superresolution in an unbalanced Michelson interferometer [32, 50]. The demonstration of these quantum optical phenomena suffers from the low efficiency of generating entangled photon pairs; because of low output signals, long-term stability is required for this demonstration.

As described in Chap. 2, two-photon interference with time-frequency-entangled photon pairs, including the two-photon phase superresolution, can be classically re-

produced by time-reversed interferometries, such as chirped-pulse interferometry (CPI) and time-decomposed pulse interferometry (TDPI). Because these interferometries can be implemented by completely classical optical systems, they can achieve intense output signals and thus address the difficulty in the time-forward two-photon interferometers.

In this chapter, we experimentally demonstrate classical reproduction of two-photon phase superresolution in an unbalanced Michelson interferometer [32, 50] by TDPI. We can achieve high-visibility two-photon interferograms with vastly intense signals owing to the simplicity of the experimental setup. In addition, we observe a classical counterpart of the large difference between the one- and two-photon coherence lengths of entangled photon pairs.

This chapter is organized as follows. In Sec. 4.2, we describe a theory of conventional and time-reversed experiments for observing two-photon phase superresolution in an unbalanced Michelson interferometer. In Sec. 3.3, we demonstrate time-reversed experiments for observing two-photon phase super-resolution. We also experimentally confirm a classical counterpart of the large difference between the one- and two-photon coherence lengths of entangled photon pairs. In Sec. 4.4, we summarize the findings of our study and discuss the advantages of our method.

3.2 Theory

In this section, we illustrate the time-forward experiment observing one-photon interference and two-photon phase superresolution in an unbalanced Michelson interferometer, and also, its time-reversed experiment by TDPI.

3.2.1 Time-forward process

The schematic setup of the time-forward experiment is shown in Fig. 3.1(a). The input is narrow-band pump photons with center frequency $2\omega_0$. The nonlinear optical crystal for SPDC converts the input photons into time-frequency-entangled photon pairs with center frequency ω_0 . After passing through the interferometer, the photon pairs are simultaneously detected by the two detectors.

The measured interference pattern varies depending on optical-path difference x of the interferometer. Due to the large difference between one- and two-photon coherence lengths l_1 and l_2 of the entangled photon pairs, one-photon interference occurs for $|x| < l_1$, but two-photon interference occurs for $l_1 < |x| < l_2$. In the latter case, we observed two-photon phase superresolution.

When $|x| < l_1$, the coincidence counting probability per photon pair is given by

$$p(x) = \frac{1}{2} \left[\frac{1 + \cos(\omega_0 x/c)}{2} \right]^2, \quad (3.1)$$

where c is the speed of light in a vacuum. This interference pattern is the square of the one-photon interference pattern.

On the other hand, when $l_1 < |x| < l_2$, the two photons from the shorter and longer arms (S and L) are substantially separated in time. By setting the time window of the coincidence counting shorter than the arrival-time difference between photons S and L, the only photon pairs from the same arms (SS and LL) can be extracted by the coincidence counting. Since $l_1 \ll l_2$ in time-frequency entangled photon pairs, photon pairs SS and LL interfere with each other and generate two-photon interference fringes. The coincidence counting probability per photon pair is given by

$$p(z) = \frac{1}{8} \frac{1 + \cos(2\omega_0 x/c)}{2}, \quad (3.2)$$

which indicates two-photon phase superresolution with perfect visibility. The maximum value of Eq. (3.2) is a quarter of the maximum value of Eq. (3.1).

3.2.2 Time-reversed process

We next present the time-reversal counterpart of the experiment described in the previous subsection. Figure 3.1(b) shows the schematic setup of the time-reversed interferometer. According to the time-reversal method introduced in Sec. 2.3.3, input light of a time-reversed system implementing TDPI must be two-photon states with various time differences. In this case, however, the time-reversed system requires only temporally close two-photon states as input light, which correspond to the two-photon states detected by the coincidence counting with a narrow time

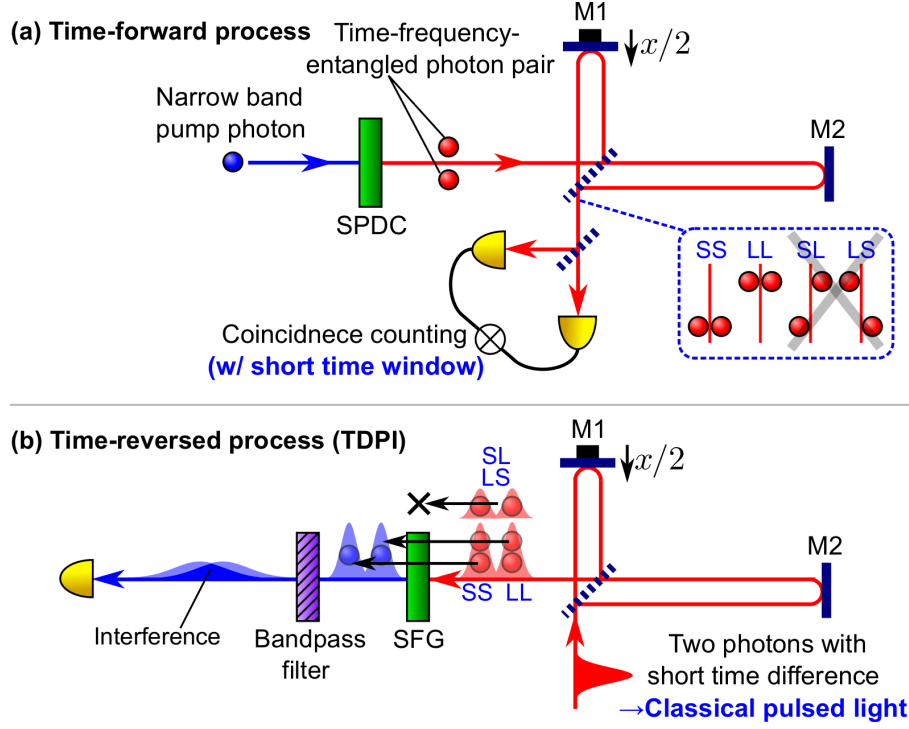


Figure 3.1: Schematic setups for observing two-photon phase superresolution. The Michelson interferometer is composed of a 50:50 beam splitter and mirrors (M1 and M2). M1 is displaced by $x/2$ to provide optical-path difference x between the two arms of the interferometer. (a) Time-forward process. This setup is composed of a nonlinear optical crystal for SPDC, an unbalanced Michelson interferometer, and two detectors for coincidence counting. The dashed-line box describes the possible states of the photon pair. Letters S and L denote the photons from the shorter and longer arms of the interferometer, respectively. When x is large enough, only photon pairs SS and LL contribute to the coincidence counts. (b) Time-reversed process. This setup is composed of an unbalanced Michelson interferometer, a nonlinear optical crystal for SFG, a bandpass filter, and a photodetector. When x is large enough, two pulses from different arms are individually converted into sum-frequency pulses by SFG. After the bandpass filter stretches the pulse widths, these pulses interfere with each other. This interference corresponds to that of photon pairs SS and LL in the time-forward process.

window in the time-forward interferometer. The temporally close two-photon input state can be replaced with a single classical laser pulse; therefore, the time-reversed interferometer can be prepared by a simple setup and requires no post-processing.

In the following, we show the calculation of the interference pattern of this interferometer in the time domain; the calculation in the frequency domain is shown in Appendix A.1.

The input light is a transform-limited laser pulse with center frequency ω_0 and coherence length $l'_1 = c\tau_1$, where τ_1 is the pulse duration. We describe the complex electric field amplitude of the input light as $E_1(t) = f(t)e^{-i\omega_0 t}$, where $f(t)$ is the pulse envelope function. The field amplitude after passing through the interferometer is given by

$$\begin{aligned} E_2(t) &= \frac{1}{2}[E_1(t) + E_1(t + x/c)] \\ &= \frac{1}{2}[f(t) + f(t + x/c)e^{-i\omega_0 x/c}]e^{-i\omega_0 t}, \end{aligned} \quad (3.3)$$

where x is the optical-path difference of the interferometer. The nonlinear optical crystal for SFG converts the field amplitude into

$$\begin{aligned} E_3(t) &= \alpha E_2(t)^2 \\ &= \frac{\alpha}{4}[f(t) + f(t + x/c)e^{-i\omega_0 x/c}]^2 e^{-i2\omega_0 t}, \end{aligned} \quad (3.4)$$

where α is a constant characterizing the SFG efficiency. These pulses pass through the bandpass filter followed by a detector. The bandpass filter narrows the light's bandwidth and broadens its pulse duration. Assuming $g(t)$ denotes an envelope function of the input light, the effect of a bandpass filter is represented by convolution integral $(g * h)(t)$, where $h(t)$ is the Fourier transform of the transmission spectrum function of the bandpass filter. Especially when the bandwidth of the transmission spectrum is narrow enough, that is, time width τ_2 of $h(t)$ is much larger than that of input envelope function $g(t)$, envelope function $g(t)$ is approximated as unnormalized delta function $a[g]\delta(t)$, where coefficient $a[g]$ is defined as $a[g] := \int_{-\infty}^{\infty} dt g(t)$. Thus the field amplitude after the bandpass filter is described as $(g * h)(t) \approx a[g]h(t)$. Coherence length $l'_2 = c\tau_2$ of the converted pulses is usually much longer than l'_1 .

Owing to the time-reversal symmetry of quantum mechanics, we expect to observe a similar interferogram as in the time-forward process. As calculated below, interference of the pump light occurs when $|x| < l'_1$, but the interference of the sum-frequency light occurs when $l'_1 < |x| < l'_2$.

When $|x| < l'_1$, $f(t)$ and $f(t + x/c)$ almost overlap: $f(t) \approx f(t + x/c)$. Thus we obtain

$$E_3(t) \approx \frac{\alpha}{4} f(t)^2 (1 + e^{-i\omega_0 x/c})^2 e^{-i2\omega_0 t}. \quad (3.5)$$

The bandpass filter converts $f(t)^2$ into widely spread envelope function $a[f^2]h(t)$. The detected signal is described as

$$\begin{aligned} I(x) &\approx \int dt \left| \frac{\alpha}{4} a[f^2]h(t) (1 + e^{-i\omega_0 x/c})^2 e^{-i2\omega_0 t} \right|^2 \\ &= |\alpha|^2 \left[\frac{1 + \cos(\omega_0 x/c)}{2} \right]^2 \int dt |a[f^2]h(t)|^2, \end{aligned} \quad (3.6)$$

which reproduces the same interference pattern as the one-photon interference in the time-forward process, expressed as Eq. (3.1). This interference is the square of the white-light interference of the input laser pulses.

On the other hand, when $l'_1 < |x| < l'_2$, $f(t)$ and $f(t + x/c)$ only slightly overlap, $f(t)f(t + x/c) \approx 0$. Thus we obtain

$$E_3(t) \approx \frac{\alpha}{4} [f(t)^2 + f(t + x/c)^2] e^{-i2\omega_0 x/c} e^{-i2\omega_0 t}. \quad (3.7)$$

The bandpass filter converts $f(t)^2$ and $f(t + x/c)^2$ into widely spread envelope functions $a[f^2]h(t)$ and $a[f^2]h(t + x/c)$, respectively. If $l'_2 \gg |x|$, $h(t)$ and $h(t + x/c)$ greatly overlap and are approximated as $h(t) \approx h(t + x/c)$. The detected signal is described as

$$\begin{aligned} I(x) &\approx \int dt \left| \frac{\alpha}{4} a[f^2]h(t) (1 + e^{-i2\omega_0 x/c}) e^{-i2\omega_0 t} \right|^2 \\ &= \frac{|\alpha|^2}{4} \frac{1 + \cos(2\omega_0 x/c)}{2} \int dt |a[f^2]h(t)|^2, \end{aligned} \quad (3.8)$$

which reproduces the same interference pattern as the two-photon interference in the time-forward process, expressed as Eq. (3.2). In this case, the white-light interference of the input pulsed light does not occur due to the large optical path difference

of the interferometer. After the bandpass filter broadens the pulse widths of the sum-frequency light, these pulses interfere with each other. We call this interference *sum-frequency light interference*, which exhibits a classical analogue to two-photon phase superresolution with perfect visibility. We can also see that the maximum intensity of the sum-frequency light interference fringes is a quarter of that of the white-light interference fringes.

As seen in the above discussion, the interferogram's shape in the time-reversed process resembles that in the time-forward process. The difference between l'_1 and l'_2 is a classical analogue to the large difference between one- and two-photon coherence lengths l_1 and l_2 .

3.3 Experiments and results

We demonstrate a time-reversed experiment for observing two-photon phase superresolution. The experimental setup is shown in Fig. 3.2. We used a femtosecond fiber laser (Menlo Systems, T-Light 780) with a center wavelength of 782 nm, a pulse duration of 74.5 fs FWHM, and an average power of 54.1 mW. The coherence length of the laser pulse was $l'_1 = c\tau_1 = 22.3 \mu\text{m}$, where τ_1 is the pulse duration of 74.5 fs FWHM. The Michelson interferometer is composed of a 50:50 nonpolarizing beam splitter for ultrashort pulses and silver mirrors (M1 and M2). M1 can be translated by several μm by a piezoelectric actuator. The output beam from the interferometer is introduced to the flipper mirror (FM). To observe the white-light interference, optical-path difference x of the interferometer is adjusted to about zero. The output beam of the interferometer is reflected onto FM and detected by a photodiode (PD1; Thorlabs, SM05PD1A). On the other hand, for observing sum-frequency light interference, x is adjusted to about 100 μm . FM is removed, and the beam is focused by a lens into a 1 mm-length β -barium borate (BBO) crystal for a collinear type-I SFG. The sum-frequency beam is then collimated by another lens and filtered to pass a 0.039-nm bandwidth centered around 391 nm by a 3,600 lines/mm aluminum-coated diffraction grating followed by a slit. The optical power is measured by a GaP photodiode (PD2; Thorlabs, PDA25K).

Figure 3.3 shows the measured interference fringes for (a) white-light and (b)

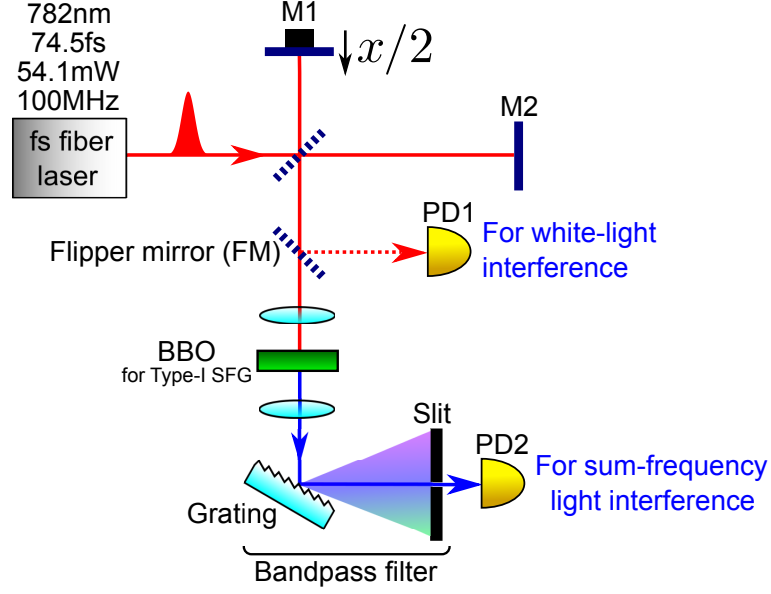


Figure 3.2: Time-reversed experimental setup for observing two-photon phase superresolution in an unbalanced Michelson interferometer. A coherent laser pulse from a femtosecond fiber laser enters the Michelson interferometer. To observe the white-light interference fringes, optical-path difference x of the interferometer is adjusted to about zero and the output light is detected by photodiode PD1. For observing the sum-frequency light interference, x is adjusted to about $100\text{ }\mu\text{m}$ and the output light is converted into sum-frequency light by a BBO crystal. A grating and a slit, both of which function as a bandpass filter, pass the sum-frequency light within a narrow band (0.039 nm) before the light is detected by a photodiode PD2.

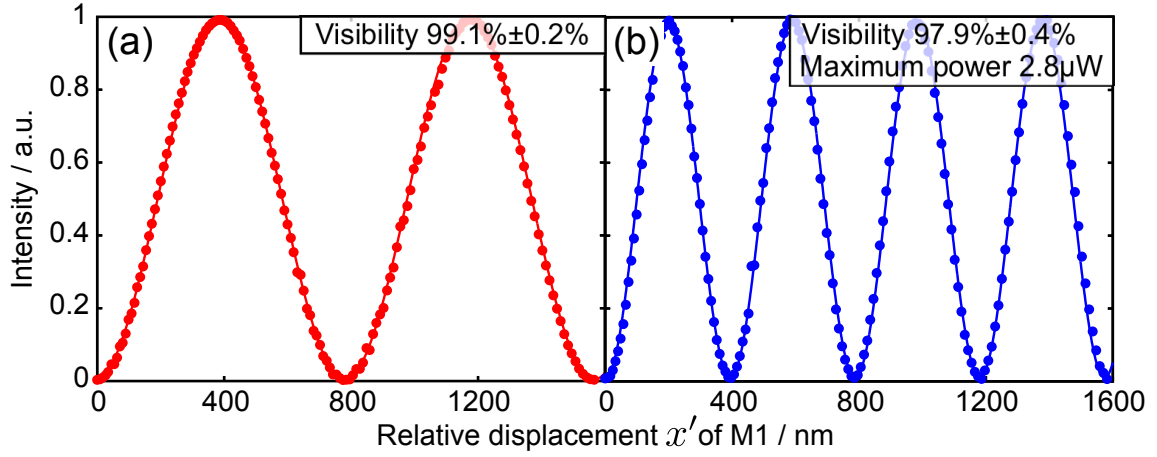


Figure 3.3: Measured interference fringes: (a) white-light and (b) sum-frequency light interference. From the fitted curves in solid lines, visibilities are estimated to be $99.1\% \pm 0.2\%$ and $97.9\% \pm 0.4\%$, respectively. Relative displacement x' of M1 depends nonlinearly on piezo voltage V . Assuming that x' is approximated by a quadratic function of V , we fitted $x'(V)$ to the measured white-light interference fringes (a). The calibrated function $x'(V)$ is also applied to the sum-frequency light interference fringes (b). Note that x' is the relative displacement of M1 from where optical-path difference x is about zero in (a) and about $100\text{ }\mu\text{m}$ in (b).

sum-frequency light as functions of relative displacement x' of M1. The period of the sum-frequency interference is half of the period of the white-light interference. This is the classical analogue to the two-photon phase superresolution. The visibilities of the interference fringes were (a) $99.1\% \pm 0.2\%$ and (b) $97.9\% \pm 0.4\%$, respectively. The maximum optical power of the sum-frequency light interference signal was $2.8 \mu\text{W}$, which corresponds to about 10^{13} photons/s. This count rate is about 10^{11} times higher than the two-photon interference signal in the previous time-forward experiments for observing two-photon phase superresolution in an unbalanced Michelson interferometer [32, 50].

Next we measured an interferogram detected by PD2 for a wide range of x , which is shown in Fig. 3.4. This experiment confirms the difference between the coherence lengths of the white-light and sum-frequency light interference l'_1 and l'_2 . In this measurement, M1 is moved by a DC servo motor instead of a piezoelectric actuator in the previous experiment. We also changed the bandwidth filtered by the grating and the slit to 0.093 nm . For x near zero, we observed a light signal proportional to the square of the white-light interference signal. The measured coherence length of the white-light interference pattern was $23.3 \pm 0.4 \mu\text{m}$ FWHM, which is in good agreement with theoretical coherence length $l'_1 = 22.3 \mu\text{m}$. On the other hand, when $|x|$ is larger than about $100 \mu\text{m}$, the light signal exhibits two-photon phase superresolution. The measured coherence length of the sum-frequency light interference was $510 \pm 20 \mu\text{m}$ FWHM, which is about 22 times larger than that of the white-light interference. The theoretical coherence length is calculated to be $l'_2 = (4 \ln 2 / \pi) \lambda^2 / \Delta\lambda = 1.5 \text{ mm}$, where $\lambda = 391 \text{ nm}$ is the central wavelength of the sum-frequency light and $\Delta\lambda = 0.093 \text{ nm}$ is the bandwidth filtered by the grating and the slit. The measured maximum value of the white-light interference signal is 3.8 times larger than that of the sum-frequency light interference signal. The slight shortage compared with the theoretical value of 4 is due to a misalignment of the interferometer. This interferogram indicates that our experiment demonstrated a classical analogue to the large difference between the one- and two-photon coherence lengths of entangled photon pairs.

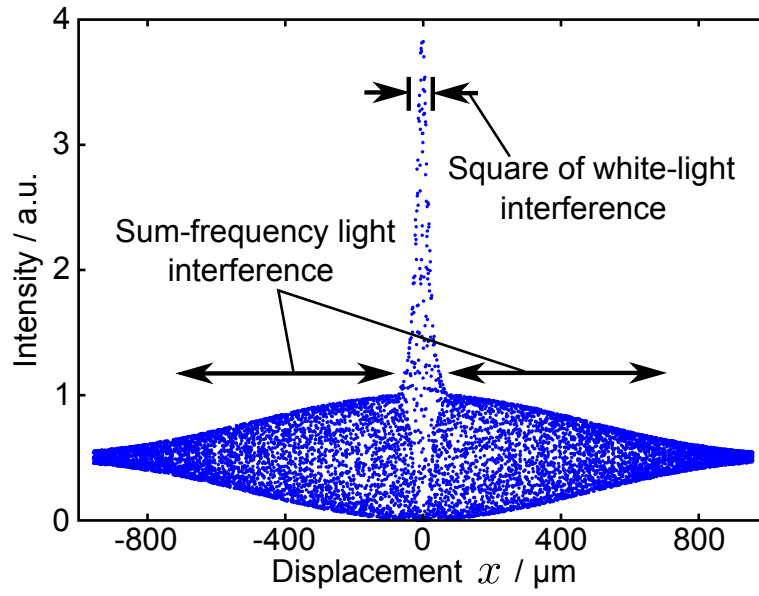


Figure 3.4: Interferogram detected by PD2 for wide range of x . When x is about zero, the light signal is proportional to the square of the white-light interference signal. On the other hand, when $|x|$ is larger than about $100\,\mu\text{m}$, the light signal exhibits two-photon phase superresolution. The coherence length of the sum-frequency interference is about 22 times larger than that of the white-light interference. This difference of coherence lengths is a classical analogue to the large difference between one- and two-photon coherence lengths of entangled photon pairs.

3.4 Summary and Discussion

We observed two-photon phase superresolution in an unbalanced Michelson interferometer with classical transform-limited laser pulses. We used a time-reversed interferometer constructed on the basis of TDPI, which does not require pairs of pulsed light with various time differences, but only single pulsed light. The measured interferogram of the experiment exhibits sum-frequency light interference with about 22 times longer coherence length than that of the input laser light. It is a classical analogue to the large difference between the one- and two-photon coherence lengths of entangled photon pairs.

Kaltenbaek *et al.* [28] first observed a classical analogue to the large difference between one- and two-photon coherence lengths using CPI. They used a pulsed light source with an average power of 2.8 W to observe the white-light and sum-frequency light interferences with visibilities of $87.1\% \pm 0.2\%$ and $84.5\% \pm 0.5\%$, respectively. The maximum optical power of the sum-frequency light interference was about $3.5 \mu\text{W}$. In our experiment the visibilities of the white-light and sum-frequency light interference were $99.1\% \pm 0.2\%$ and $97.9\% \pm 0.4\%$, respectively. The maximum optical power of the sum-frequency light interference was $2.8 \mu\text{W}$, which is comparable to that of the experiment by Kaltenbaek *et al.*, in spite of our low-power pulsed light source (average power 54.1 mW). Such high visibility and high efficiency are due to the simplicity of our experimental setup which requires no chirped laser pulses.

Some previous investigations using coincidence counting for detecting correlated N photons demonstrated N -photon phase superresolution [30, 51]. Coincidence counting, however, cannot detect frequency-correlated photons. For this reason, these previous investigations using coincidence counting did not demonstrate the quantum optical phenomena induced by the frequency correlation of photons, such as the difference between one- and two-photon coherence lengths.

We also mention the relation between our experiments and optical lithography. Phase superresolution using entangled photons has been applied to sub-Rayleigh resolution lithography [44, 45], which is called quantum lithography. Sub-Rayleigh resolution lithography was also proposed and demonstrated by a classical optical

setup with multiphoton absorption [52, 53]. Pe'er *et al.* [54] demonstrated sub-Rayleigh resolution lithography using two-photon absorption. Their experiment resembles ours except it used a Young-like interferometer and detected frequency-correlated photons by two-photon absorption, suggesting that some findings about the time-reversal method can be utilized for sub-Rayleigh resolution optical lithography.

Our study revealed that two-photon phase superresolution can be realized in a simple classical system with transform-limited laser pulses. This simplification enabled us to achieve high-visibility interference with high efficiency. We expect this technique to open up new practical applications of quantum optical technologies.

Chapter 4

Observation of the geometric phase in a two-photon polarization qutrit by TDPI

4.1 Introduction

The geometric phase is a fundamental concept in many areas of physics. It was discovered by Berry [55] as an additional phase factor that emerges in adiabatic and cyclic evolution of a quantum state. The definition of the geometric phase was extended to the non-adiabatic [56] and non-cyclic [57] cases and was finally generalized on the basis of kinematic ideas by Mukunda and Simon [58]. In their formulation, the geometric phase is defined by a trajectory on the quantum state space and is represented as a sum of the following *three-vertex geometric phases*:

$$\gamma(\psi_1, \psi_2, \psi_3) := \arg\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle, \quad (4.1)$$

which is defined by three quantum states [59]. Therefore, the three-vertex geometric phase is regarded as a fundamental building block of an arbitrary geometric phase (for details, see Appendix B.1.1).

The three-vertex geometric phase is ubiquitous in various physical systems involving three different states. In optical systems, the three-vertex geometric phase appears in an additional phase factor after three polarization projections [59] or

three reflections [60], and in the interference patterns of three differently polarized beams [61]. In the problem distinguishing three quantum states, the three-vertex geometric phase is an important factor characterizing their distinguishability [62–64]. In addition, the quantum eraser [65] and weak-value amplification [66] are related to the three-vertex geometric phase defined by the initial, intermediate, and final states in the systems [67, 68].

The three-vertex geometric phase has been widely studied in a two-state (qubit) system such as optical polarization. In a two-state system, the three-vertex geometric phase is geometrically represented as the area of a spherical triangle formed by the three constituent states on the Bloch (Poincaré) sphere [59]. Various nonlinear behaviors of the three-vertex geometric phase in a two-state system have been investigated using the Bloch sphere representation and observed in several optical experiments [67, 69–72]. Appendix B.1.2 describes an example of the nonlinear variation in the three-vertex geometric phase in a two-state system and its Bloch sphere representation.

However, the three arbitrary states that define a three-vertex geometric phase generally span a three-dimensional Hilbert space. To investigate the general properties of the three-vertex geometric phase, we need to treat a three-state (qutrit) system, such as the polarizations of two photons in the same spatiotemporal mode, called a *two-photon polarization qutrit* [73–76]. In our previous study [33], we constructed a geometric representation of the three-vertex geometric phase in a three-state system on the Bloch sphere. Using our Bloch sphere representation, we predicted some nonlinear variations in the three-vertex geometric phase inherent in a three-state system.

In this chapter, we experimentally observe the nonlinear variations in the three-vertex geometric phase in a two-photon polarization qutrit, which is inherent in a three-state system. We utilize the time-reversed two-photon interferometer for phase superresolution constructed in Chap. 3 for this measurement. Unlike the typical quantum interferometers measuring the geometric phase in two-photon polarization [14, 34, 35], our setup can obtain vastly more intense signals and can be implemented using classical light. We observe two rapid increases in the three-vertex geometric phase with respect to a change in one of the three constituent

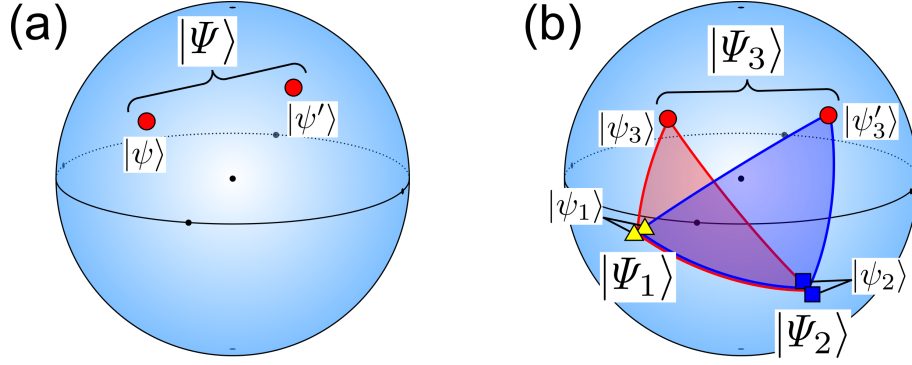


Figure 4.1: (a) Bloch sphere representation of the symmetrized two-qubit state $|\Psi\rangle$. (b) Bloch sphere representation of the three-vertex geometric phase of the standard triplet $\gamma(\Psi_1, \Psi_2, \Psi_3)$, which is proportional to the sum of the area of the two spherical triangles.

states, as predicted by our Bloch sphere representation [33]. This study exhibits an application of TDPI to a measurement of internal parameters of quantum light.

This chapter is organized as follows. In Sec. 4.2, we briefly review the theory of the three-vertex geometric phase in a three-state system investigated in Ref. [33]. In Sec. 4.3, we describe the experimental observation of the nonlinear variations in the three-vertex geometric phase. We also discuss the advantages of using a time-reversed two-photon interferometer for these experiments. Finally, we summarize the findings of our study in Sec. 4.4.

4.2 Theory

We describe the Bloch sphere representation of the three-vertex geometric phase in a three-state system [33]. We also derive the nonlinear variations in the three-vertex geometric phase inherent in a three-state system, which are experimentally observed in Sec. 4.3.

A three-state system can be identified in terms of a symmetrized two-qubit system, such as a two-photon polarization qutrit (see Appendix B.2 for the proofs).

The symmetrized two-qubit state $|\Psi\rangle$ is described as

$$|\Psi\rangle = k(|\psi\rangle|\psi'\rangle + |\psi'\rangle|\psi\rangle), \quad (4.2)$$

where $|\psi\rangle$ and $|\psi'\rangle$ are qubit states, and k is a normalization factor (in what follows, we omit k for simplicity). For a two-photon polarization qutrit, $|\psi\rangle$ and $|\psi'\rangle$ correspond to single-photon polarization states. Equation (4.2) means that $|\Psi\rangle$ can be uniquely depicted as the two points corresponding to $|\psi\rangle$ and $|\psi'\rangle$ on the Bloch sphere (Majorana's stellar representation [77–80]), as shown in Fig. 4.1(a).

To visualize the three-vertex geometric phase on the Bloch sphere, we consider the following *standard triplet*:

$$\begin{aligned} |\Psi_1\rangle &= |\psi_1\rangle|\psi_1\rangle, & |\Psi_2\rangle &= |\psi_2\rangle|\psi_2\rangle, \\ |\Psi_3\rangle &= |\psi_3\rangle|\psi'_3\rangle + |\psi'_3\rangle|\psi_3\rangle, \end{aligned} \quad (4.3)$$

where $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are product states of two identical qubit states, and $|\Psi_3\rangle$ is an arbitrary symmetrized two-qubit state. Although the standard triplet is a special set of three states, any set of three states can be mapped onto a standard triplet by applying the proper unitary transformation (see Appendix B.2 for the proof). The three-vertex geometric phase of the standard triplet is expressed as the sum of two three-vertex geometric phases in two-state systems:

$$\gamma(\Psi_1, \Psi_2, \Psi_3) = \gamma(\psi_1, \psi_2, \psi_3) + \gamma(\psi_1, \psi_2, \psi'_3). \quad (4.4)$$

Because a three-vertex geometric phase in a two-state system is equal to $1/2$ times the area of a spherical triangle on the Bloch sphere [59], the three-vertex geometric phase of the standard triplet can be depicted as the area of two spherical triangles on the Bloch sphere, as shown in Fig. 4.1(b). In this manner, we can represent an arbitrary three-vertex geometric phase in a three-state system on the Bloch sphere.

We next derive the nonlinear variations in the three-vertex geometric phase inherent in a three-state system from the Bloch sphere representation. We employ a two-photon polarization qutrit as a three-state system. $|H\rangle$ and $|V\rangle$ denote the horizontal and vertical polarization states, respectively.

We now consider the following standard triplet of two-photon polarization

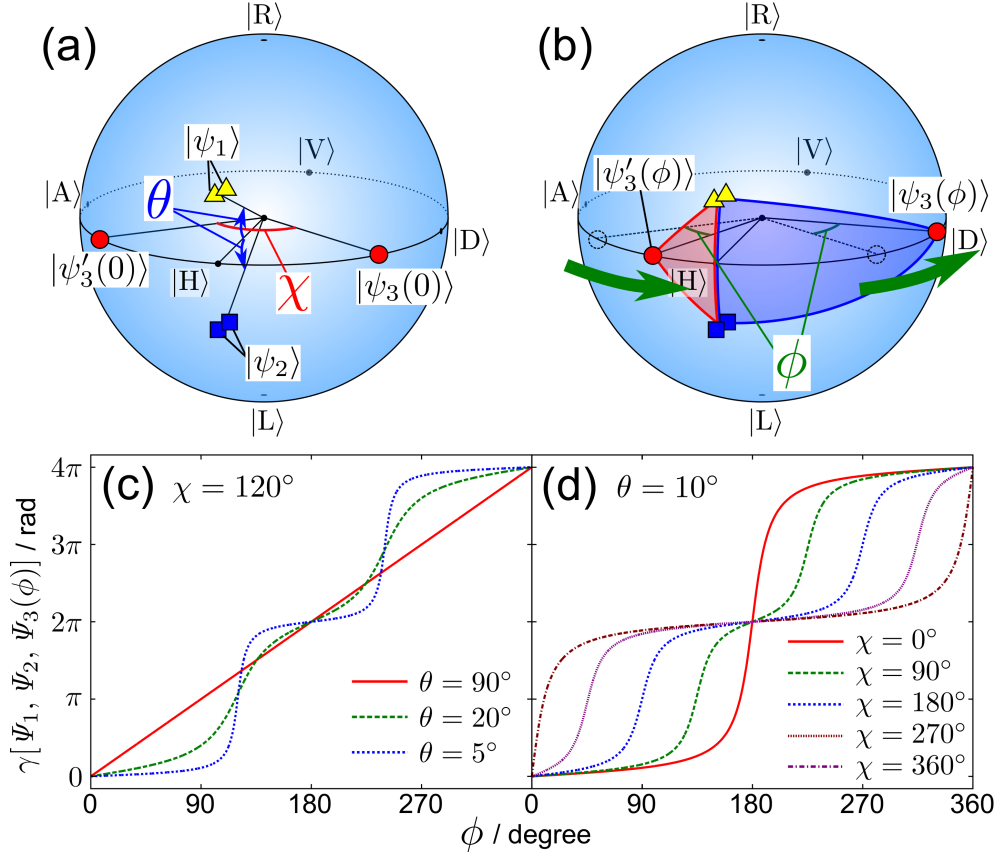


Figure 4.2: (a) Bloch sphere representation of the states given by Eqs. (4.7)–(4.10) when $\phi = 0^\circ$. θ is the half-angle between the states $|\psi_1\rangle$ and $|\psi_2\rangle$, and χ is the angle between the states $|\psi_3(\phi)\rangle$ and $|\psi'_3(\phi)\rangle$. (b) Bloch sphere representation of the geometric phase $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ when the two red circles $|\psi_3(\phi)\rangle$ and $|\psi'_3(\phi)\rangle$ are rotated along the equator. When the red circles pass through the reverse side of the Bloch sphere, the area of the spherical triangles increases rapidly. In addition, as the angle between the yellow triangles and the blue squares decreases, the area of the two spherical triangles increases more rapidly. (c), (d) The variations in $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ with respect to ϕ , for several values of θ when $\chi = 120^\circ$ (c) and for several values of χ when $\theta = 10^\circ$ (d).

qutrits:

$$|\Psi_1\rangle = |\psi_1\rangle|\psi_1\rangle, \quad |\Psi_2\rangle = |\psi_2\rangle|\psi_2\rangle, \quad (4.5)$$

$$|\Psi_3(\phi)\rangle = |\psi_3(\phi)\rangle|\psi'_3(\phi)\rangle + |\psi'_3(\phi)\rangle|\psi_3(\phi)\rangle, \quad (4.6)$$

where

$$|\psi_1\rangle := \cos \frac{\theta}{2} |H\rangle + i \sin \frac{\theta}{2} |V\rangle, \quad (4.7)$$

$$|\psi_2\rangle := \cos \frac{\theta}{2} |H\rangle - i \sin \frac{\theta}{2} |V\rangle, \quad (4.8)$$

$$|\psi_3(\phi)\rangle := \cos \left(\frac{\chi}{4} + \frac{\phi}{2} \right) |H\rangle + \sin \left(\frac{\chi}{4} + \frac{\phi}{2} \right) |V\rangle, \quad (4.9)$$

$$|\psi'_3(\phi)\rangle := \cos \left(\frac{\chi}{4} - \frac{\phi}{2} \right) |H\rangle - \sin \left(\frac{\chi}{4} - \frac{\phi}{2} \right) |V\rangle. \quad (4.10)$$

This standard triplet is depicted in Fig. 4.2(a). The parameters θ and χ are fixed at certain values. We change ϕ to rotate the two red circles $|\psi_3(\phi)\rangle$ and $|\psi'_3(\phi)\rangle$ along the equator on the Bloch sphere, as shown in Fig. 4.2(b). The three-vertex geometric phase $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ is calculated as

$$\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)] = \gamma[\psi_1, \psi_2, \psi_3(\phi)] + \gamma[\psi_1, \psi_2, \psi'_3(\phi)], \quad (4.11)$$

$$\gamma[\psi_1, \psi_2, \psi_3(\phi)] = -2 \tan^{-1} \left[\tan \frac{\theta}{2} \cdot \tan \left(\frac{\chi}{4} + \frac{\phi}{2} \right) \right], \quad (4.12)$$

$$\gamma[\psi_1, \psi_2, \psi'_3(\phi)] = 2 \tan^{-1} \left[\tan \frac{\theta}{2} \cdot \tan \left(\frac{\chi}{4} - \frac{\phi}{2} \right) \right]. \quad (4.13)$$

The variations in $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ with respect to ϕ for several values of θ and χ are shown in Fig. 4.2(c) and (d). These figures indicate that the variations in $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ exhibit two rapid increases by 2π at the angles $\phi = 180^\circ \pm \chi/2$, and as the angle θ decreases, the geometric phase increases more rapidly. These rapid variations in $\gamma[\Psi_1, \Psi_2, \Psi_3(\phi)]$ are interpreted as nonlinear variations in the area of the two spherical triangles on the Bloch sphere.

4.3 Experiments

We next describe our experimental observation of the nonlinear variations in the three-vertex geometric phase in a two-photon polarization qutrit derived in

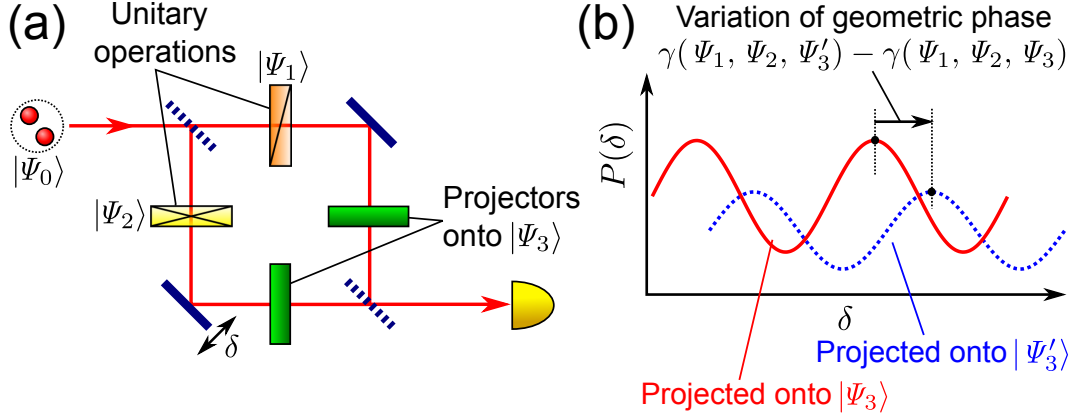


Figure 4.3: (a) Schematic illustration of setup for measuring the three-vertex geometric phase in a quantum eraser. (b) Projection probability $P(\delta)$ for different final internal states $|\Psi_3\rangle$ and $|\Psi'_3\rangle$. From the phase shift of the interference fringes, we can measure the variation in the three-vertex geometric phase $\gamma(\Psi_1, \Psi_2, \Psi'_3) - \gamma(\Psi_1, \Psi_2, \Psi_3)$.

Sec. 4.2. In this experiment, we utilize the time-reversed two-photon interferometer for phase superresolution. In Sec. 4.3.1, we describe our experimental setup for measuring the geometric phase using an optical interferometer. In Sec. 4.3.2, we show the measured nonlinear variations in the geometric phase. In Sec. 4.3.3, we discuss the advantages of using the time-reversed two-photon interferometer for the measurement of the geometric phase in two-photon polarization.

4.3.1 Experimental setup

In this experiment, we measure the three-vertex geometric phase using a quantum eraser [35, 67]. Let us consider the two-photon interferometer shown in Fig. 4.3(a). The input photon pair with the initial two-photon polarization qutrit state $|\Psi_0\rangle$ is first split into two arms by a beam splitter. The two-photon polarization qutrit states of the upper and lower mode are transformed into $|\Psi_1\rangle$ and $|\Psi_2\rangle$, respectively, by unitary operations (e.g. half- or quarter-wave plates). Subsequently, both of the two-photon polarization qutrit states are projected onto $|\Psi_3\rangle$, and the two path modes are combined by another beam splitter. By changing the

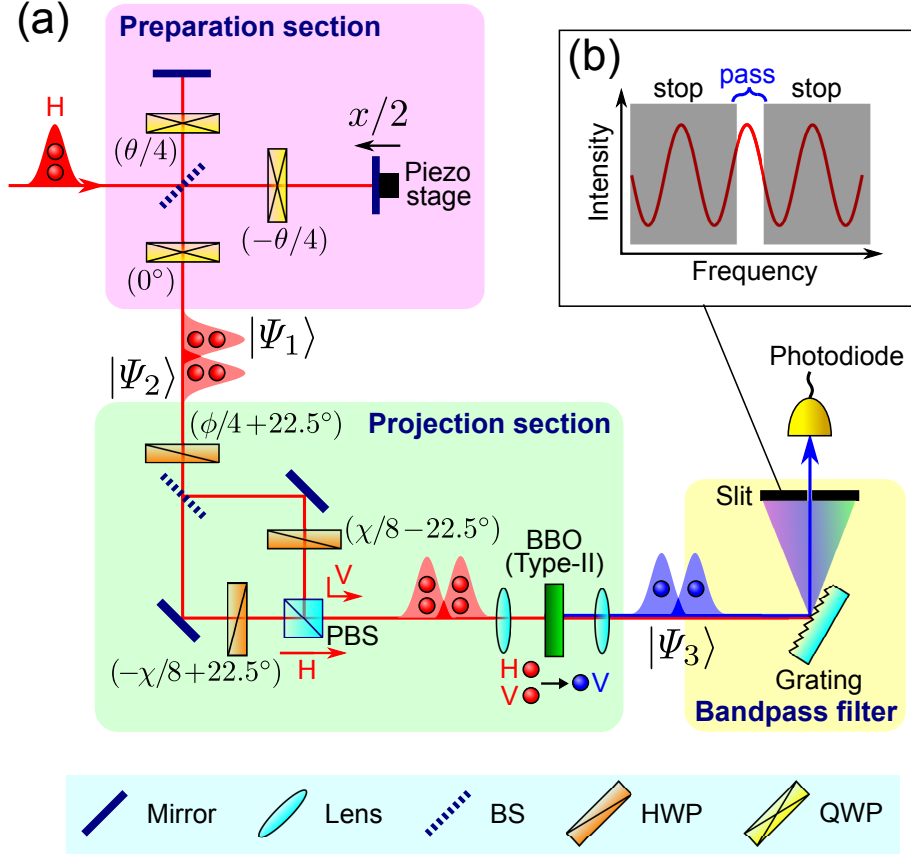


Figure 4.4: (a) Experimental setup for measuring the three-vertex geometric phase in two-photon polarization qutrit. QWP, quarter-wave plate; HWP, half-wave plate; BS, (non-polarizing) beam splitter; PBS, polarizing beam splitter; BBO, β -barium borate crystal. The values in the parentheses next to the QWPs and HWPs denote the angles of their fast axes from the horizontal axis. The parameter θ is adjusted by changing the angles of the QWPs. The parameters χ and ϕ are adjusted by changing the angles of the HWPs. (b) Spectral intensity of the light after the BBO crystal. When the optical path difference x is changed, the distribution shifts transversely. By extracting the light in a narrow frequency region, we can observe the interference fringes with high visibility.

relative phase δ between the two path modes, we can observe interference fringes in the projection probability $P(\delta)$. When the final state $|\Psi_3\rangle$ varies, the variation in the three-vertex geometric phase $\gamma(\Psi_1, \Psi_2, \Psi_3)$ can be measured from a phase shift of the interference fringes, as shown in Fig. 4.3(b).

The actual experimental setup is shown in Fig. 4.4(a). This setup is a modified version of the time-reversed two-photon interferometer for phase superresolution, which includes additional polarization elements. This setup implements the measurement method using a quantum eraser.

We used a femtosecond fiber laser (center wavelength 782 nm, pulse duration 74.5 fs, average power 54 mW, repetition rate 100 MHz) to create transform-limited pulsed light with horizontal polarization. The input pulse enters the preparation section, which forms an unbalanced Michelson interferometer including three quarter-wave plates (QWPs). The optical path difference x between the two arms of the interferometer can be changed by a piezoelectric actuator and is adjusted to about 100 μm . The two output pulses of the interferometer are substantially separated in time and hardly interfere with each other. After passing through the third QWP, the two-photon polarization qutrit states of the later and earlier pulses are transformed into $|\Psi_1\rangle = |\psi_1\rangle|\psi_1\rangle$ and $|\Psi_2\rangle = |\psi_2\rangle|\psi_2\rangle$ in Eq. (4.5), respectively.

The pulses next pass through the projection section, which consists of three half-wave plates (HWPs), polarizing and non-polarizing beam splitters (PBS and BS), and a 1-mm-long β -barium borate (BBO) crystal for collinear type-II sum-frequency generation (SFG). The HWPs, PBS, and BS convert the polarizations $|\psi_3(\phi)\rangle$ and $|\psi'_3(\phi)\rangle$ in Eqs. (4.9) and (4.10) into $|H\rangle$ and $|V\rangle$, respectively. Subsequently, the BBO crystal converts only two photons with the two-photon polarization qutrit state $|H\rangle|V\rangle + |V\rangle|H\rangle$ into a sum-frequency photon. Therefore, the entire section projects the two-photon polarization qutrit state onto $|\Psi_3(\phi)\rangle$ in Eq. (4.6).

The two sum-frequency pulses interfere with each other with respect to each frequency component as shown in Fig. 4.4(b). We filter the two pulses to pass a 0.23-nm bandwidth centered at around 391 nm by a 1,200-lines/mm aluminum-coated diffraction grating followed by a slit. The extracted two pulses produce interference with high visibility. The optical power is measured by a Si photodiode (New Focus, Model 2151).

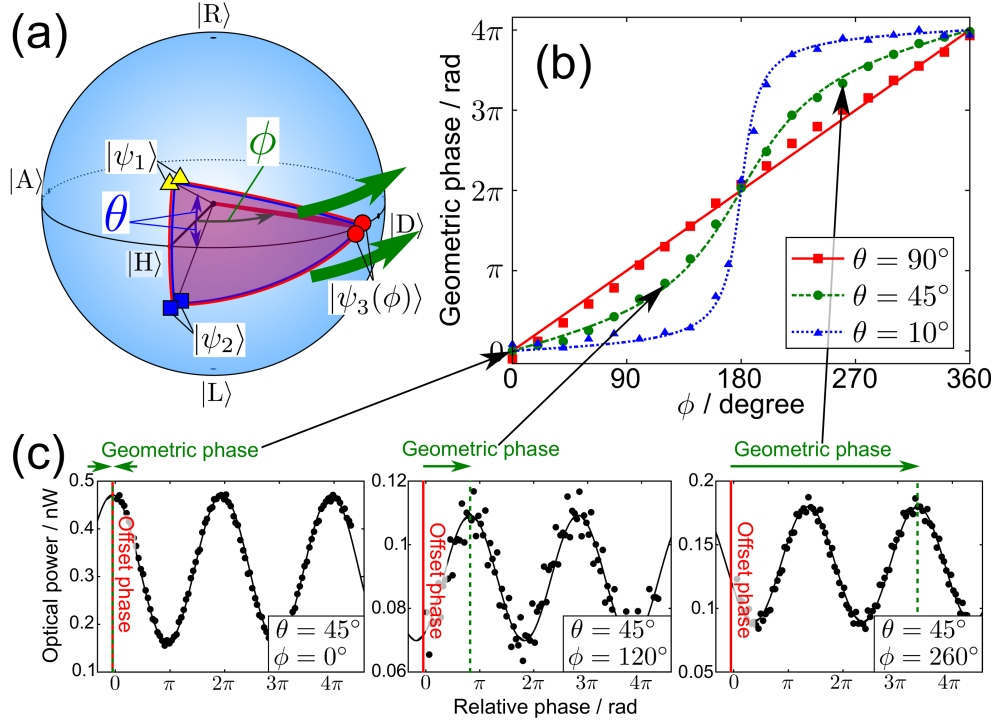


Figure 4.5: Variations in the three-vertex geometric phase for $\chi = 0^\circ$. (a) Bloch sphere representation of the three-vertex geometric phase. (b) Measured variations in the geometric phase with respect to ϕ . (c) Some instances of the measured interference fringes for several values of ϕ when $\theta = 45^\circ$. As the Bloch sphere representation predicts, the geometric phase increases rapidly by 4π at $\phi = 180^\circ$.

We measured the interference fringes as a function of the optical path difference x for various values of θ , χ , and ϕ , and derived the variations in the three-vertex geometric phase from the shifts of the fringes.

4.3.2 Results

We first measured the variations in the three-vertex geometric phase with respect to ϕ for several values of θ when $\chi = 0^\circ$ [Figs. 4.5(a)–(c)] and $\chi = 180^\circ$ [Figs. 4.6(a), (b)]. From the Bloch sphere representation [Figs. 4.5(a), 4.6(a)], we can predict that the geometric phase increases rapidly by 4π at $\phi = 180^\circ$ when $\chi = 0^\circ$, and by 2π at $\phi = 90^\circ$ and 270° when $\chi = 180^\circ$. Figures 4.5(b) and 4.6(b) show the

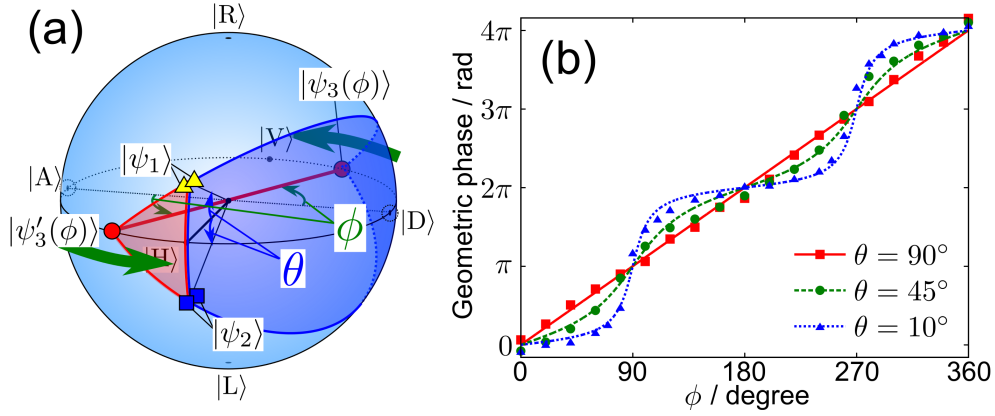


Figure 4.6: Variations in the three-vertex geometric phase for $\chi = 180^\circ$. (a) Bloch sphere representation of the three-vertex geometric phase. (b) Measured variations in the geometric phase with respect to ϕ . As the Bloch sphere representation predicts, the geometric phase increases rapidly by 2π at $\phi = 90^\circ$ and 270° . We can see that as the angle θ decreases, the geometric phase increases more rapidly.

measurement results, where the dots and lines denote the measurement data and the theoretical lines, respectively. Figure 4.5(c) shows some instances of the measured interference fringes for several values of ϕ when $\theta = 45^\circ$. By comparing the phase shifts among these fringes, we measured the relative variation in the three-vertex geometric phase with respect to a certain offset phase. We determined the offset phase by fitting the measurement data of the relative variation in the three-vertex geometric phase to the theoretical lines. The offset phase was determined for every setting of the parameters θ and χ . The measurement results agree well with the theoretical prediction. As the angle θ decreases, the geometric phase increases more rapidly.

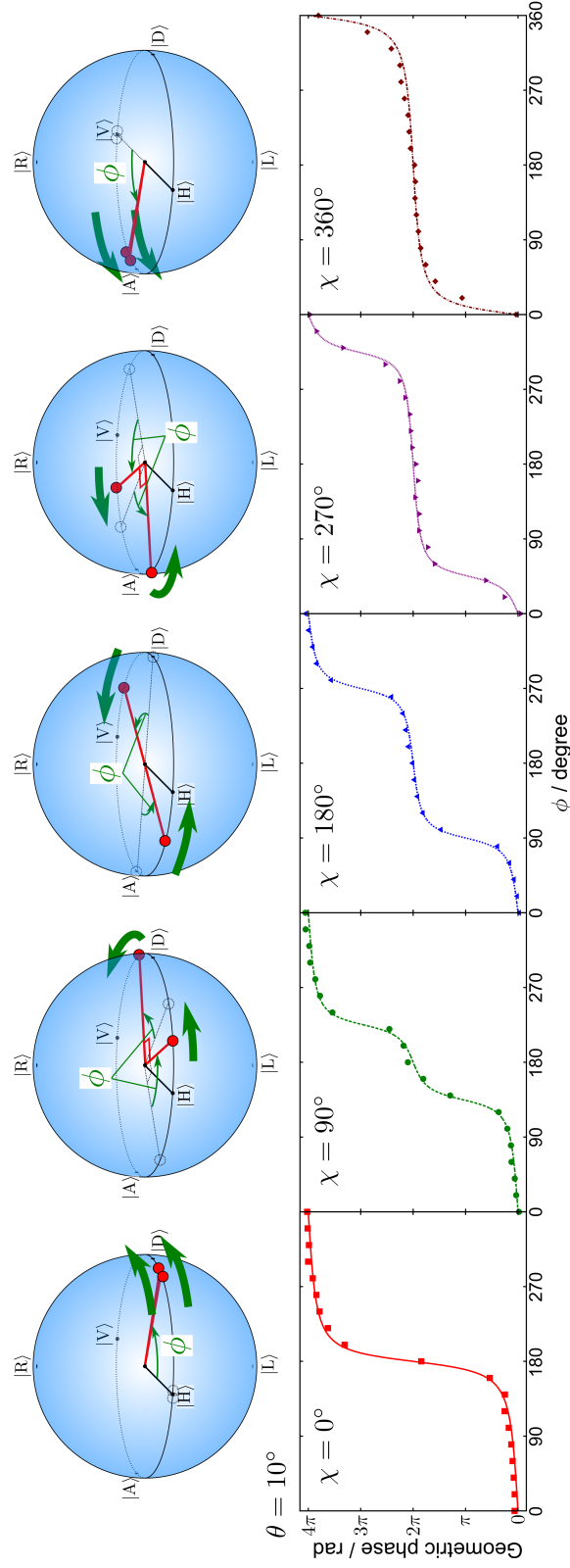


Figure 4.7: Variations in the three-vertex geometric phase with respect to ϕ for several values of χ when $\theta = 10^\circ$. The upper panels are the Bloch sphere representations of $|\psi_3(\phi)\rangle$ and $|\psi'_3(\phi)\rangle$. The graphs in the lower panels show the measurement results corresponding to each Bloch sphere. As the Bloch sphere representation predicts, the locations of the two jumping points of the geometric phase depend on χ .

We also measured the variations in the three-vertex geometric phase with respect to ϕ for several values of χ when $\theta = 10^\circ$. From the Bloch sphere representation (the upper panels in Fig. 4.7), we can predict that the two values of ϕ at which the geometric phase increases rapidly depend on χ : $\phi = 180^\circ \pm \chi/2$. The graphs in the lower panels in Fig. 4.7 show the measurement results, where the dots and lines denote the measurement data and the theoretical lines, respectively, and we determined the offset phase of the measurement data in the same manner as described above. We can see that the measurement results agree well with the theoretical prediction by our Bloch sphere representation, and the locations of the two jumping points depend on χ .

4.3.3 Discussion

Here we discuss the advantages of using the time-reversed two-photon interferometer for the measurement of the geometric phase in two-photon polarization.

The measurements of the geometric phase in two-photon polarization have typically been performed using a (time-forward) two-photon interferometer [14, 34, 35]. In the two-photon interferometer, we need to generate photon pairs by spontaneous parametric down-conversion (SPDC) and to detect them by coincidence counting. Because of the low efficiency of generation and detection of entangled photon pairs, the two-photon interferometer often suffers from weak output signals, which makes our estimation of the geometric phase uncertain.

In contrast, the time-reversed two-photon interferometer can achieve vastly more intense interference signals and be implemented by a simpler experimental setup, as mentioned in Chap. 3. We note that this technique cannot be used for realizing a genuine non-classical effect of photon pairs [14, 35], because the technique is actually implemented using completely classical light and we can just reproduce the same interference pattern as the quantum effect. However, for the purpose of this study, that is, to verify that the geometric phases in a two-photon polarization qutrit change as predicted by the Bloch sphere representation, this technique is more advantageous than the conventional method using a two-photon interferometer.

In our experiment, the output signal power is minimized when $\theta = 10^\circ$, $\chi =$

0° , and $\phi = 180^\circ$. Even in this condition, the measured average power of the interference fringes is 1.1 pW, which corresponds to 2.1×10^6 photons/s. This output power is three orders of magnitude greater than that in previous experiments using two-photon interferometers [14, 34, 35].

4.4 Conclusion

We presented classical optical experiments for measuring the three-vertex geometric phase in a two-photon polarization qutrit by TDPI. We experimentally demonstrated the nonlinear variations in the three-vertex geometric phase, which is inherent in a three-state system such as a two-photon polarization qutrit. The nonlinear variations are predicted by the Bloch sphere representation, and all the measurement results agree well with the theoretical prediction. We noted that our measurement method for the geometric phase using a time-reversed two-photon interferometer enables us to obtain vastly more intense output signals. These results manifest that TDPI is useful for a measurement of internal parameters of quantum light. This measurement method can be used for high-intensity measurement of other properties of the geometric phase. We expect that the measurement technique by TDPI will motivate the investigation of a new nonlinear variation of the geometric phase in higher-dimensional systems, and will enable new quantum optical technology using the geometric phase.

Chapter 5

Classical reproduction of Hong–Ou–Mandel interference with automatic dispersion cancellation by TDPI

5.1 Introduction

Hong–Ou–Mandel (HOM) interference [21] is a fundamental two-photon interference phenomenon in quantum optics. The study of HOM interference has inspired a variety of studies on two-photon interference phenomena, such as quantum beating [81], phase superresolution [39–41, 43], and phase supersensitivity [16, 17]. Due to the time-frequency correlation of photon pairs, HOM interferograms are insensitive to even-order dispersion. This property is utilized in automatic dispersion cancellation [22, 23, 82] and has been applied to precise measurements of length or time in dispersive media such as biological samples and optical fibers. These applications include quantum-optical coherence tomography [18–20], quantum clock synchronization [83], and the measurement of photon tunneling time [84].

Automatic dispersion cancellation in HOM interferometers often suffers from weak output signals due to low efficiency in the generation and detection of entan-

gled photon pairs. To address this, several schemes have been proposed to achieve automatic dispersion cancellation with intense classical input light [25, 26, 85–88]. One of the most successful schemes is chirped-pulse interferometry (CPI) [25, 26, 28], described in Chap. 2. CPI can reproduce dispersion-insensitive HOM interferograms with much stronger signals, and has been applied to dispersion-insensitive optical coherence tomography [27, 29].

According to the time-reversal symmetry of quantum mechanics, two-photon interference with time-frequency-entangled photon pairs can be classically reproduced by not only CPI but also time-decomposed pulse interferometry (TDPI), as described in Chap. 2. In this chapter, we describe the experimental demonstration reproducing HOM interference with automatic dispersion cancellation by a classical optical interferometer that implements TDPI. The results showed that the interferometer can obtain high-visibility dispersion-insensitive HOM interferograms with high signal conversion efficiency owing to its simplicity.

This chapter is organized as follows. In Sec. 5.2, we describe a construction theory of the time-reversed HOM interferometer with classical pulsed input light implementing TDPI. In Sec. 5.3, we illustrate the experiments for observing dispersion-cancelled HOM interferograms using the time-reversed HOM interferometer. In Sec. 5.4, we mention the relation to another type of dispersion cancellation, and then summarize the findings of this study.

5.2 Theory

5.2.1 Time-forward and time-reversed HOM interferometer

In the following, we apply the time-reversal method described in Chap. 2 to an HOM interferometer and construct a time-reversed system with a two-photon input state reproducing dispersion-insensitive HOM interferograms.

We first review the time-forward HOM interferometer with time-frequency-entangled photon pairs, shown in Fig. 5.1(a). The initial state is a narrow-band pump photon state $|i(0)\rangle$, represented by a wave function centered at $t_c = 0$ in

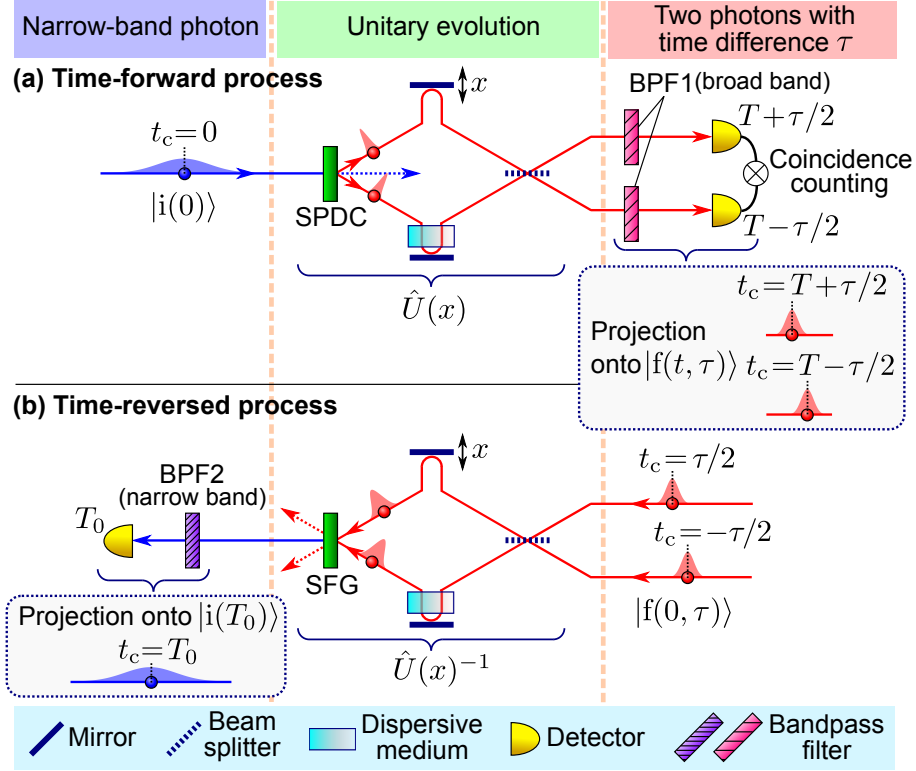


Figure 5.1: (a) Time-forward and (b) time-reversed HOM interferometer with time-frequency-entangled photon pairs. BPF1 and BPF2 are bandpass filters with relatively broad and narrow transmission spectra, respectively. The center frequency of the transmission spectrum of BPF2 is twice as large as that of BPF1.

the time domain. The input photon is converted into a time-frequency-entangled photon pair in a nonlinear crystal through spontaneous parametric down-conversion (SPDC). One of the photons travels the upper arm, including a delay mirror with displacement x . The other travels the lower arm, including passing through a dispersive medium. The two photons are combined at a beam splitter. The overall time evolution is denoted by the unitary operator $\hat{U}(x)$. After passing through the bandpass filters BPF1s, the photon pair is detected in the upper and lower arms at time $T \pm \tau/2$, respectively. These detections are interpreted as the projection onto the final state $|f(T, \tau)\rangle$, which is represented by a pair of wave functions centered at $t_c = T \pm \tau/2$ in the time domain, respectively. The widths of these wave functions are determined by the transmission spectra of the BPF1s. The detection

probability density is given by $|\langle f(T, \tau) | \hat{U}(x) | i(0) \rangle|^2$. The success probability $P(x)$ of the coincidence counting is given by the integration of the detection probability density over T and τ :

$$P(x) = \int_{-\infty}^{\infty} dT \int_{-\infty}^{\infty} d\tau |\langle f(T, \tau) | \hat{U}(x) | i(0) \rangle|^2. \quad (5.1)$$

By changing the displacement x , the success probability sharply drops only when the lengths of the two arms are balanced; this interferogram is called the HOM dip. The width of the HOM dip is determined by the phase-matching condition of SPDC and the transmission spectrum of BPF1.

Based on the time-reversal method, the time-reversed HOM interferometer is constructed as shown in Fig. 5.1(b). The initial state is the two-photon state denoted by $|f(0, \tau)\rangle$, which is represented by a pair of wave functions centered at $t_c = \pm\tau/2$ in the time domain, respectively. This state is the time-reversal counterpart of the final state in the time-forward process. The pair of photons passes through the optical system in the reverse manner to the time-forward case and are converted into a single photon by SFG. We finally detect the up-converted photon at time T_0 through the bandpass filter BPF2. This detection is interpreted as the projection onto the final state $|i(T_0)\rangle$, represented by a wave function centered at $t_c = T_0$ in the time domain. This state is a time-reversal counterpart of the initial state in the time-forward process. The detection probability density is given by $|\langle i(T_0) | \hat{U}(x)^{-1} | f(0, \tau) \rangle|^2$, which is equal to $|\langle i(0) | \hat{U}(x)^{-1} | f(-T_0, \tau) \rangle|^2$ due to the time-translation symmetry. The success probability $P_r(x, \tau)$ is given by the integration of the detection probability density over only T_0 , not τ :

$$\begin{aligned} P_r(x, \tau) &= \int_{-\infty}^{\infty} dT_0 |\langle i(0) | \hat{U}(x)^{-1} | f(-T_0, \tau) \rangle|^2 \\ &= \int_{-\infty}^{\infty} dT_0 |\langle i(0) | \hat{U}(x)^{-1} | f(T_0, \tau) \rangle|^2. \end{aligned} \quad (5.2)$$

By integrating $P_r(x, \tau)$ over all delays τ , we can reproduce the same interferogram as the time-forward HOM interferometer $P(x)$. In our scheme, we measure each success probability $P_r(x, \tau)$ for various delays τ and sum the measured data over all delays τ by post-processing. The width of the HOM dip is determined by the spectrum of the input pulse and the phase-matching condition of SFG.

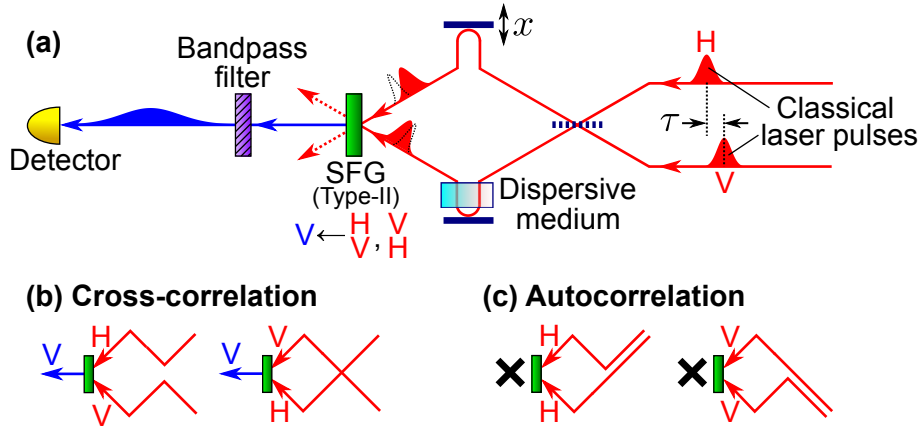


Figure 5.2: (a) Time-reversed HOM interferometer with intense coherent input light. The labels H and V denote horizontal and vertical polarization, respectively. (b), (c) Feynman paths leading to SFG in this setup. Type-II SFG extracts only pairs of orthogonally polarized photons (HV and VH) and thus eliminates the contribution from the autocorrelation terms in (c).

5.2.2 Classical realization of automatic dispersion cancellation by using the time-reversed HOM interferometer

We next consider replacing the two-photon input state with intense coherent light as shown in Fig. 5.2(a). The time-reversed system with intense coherent input light has four Feynman paths leading to SFG, shown in Figs. 5.2(b) and (c). Since only the cross-correlation terms (b) lead to HOM interference, we have to eliminate the contribution from the autocorrelation terms (c), as noted in Chap. 2. In our system, we use pairs of orthogonally polarized (H and V) laser pulses as input light and employ a type-II nonlinear optical crystal for SFG. Type-II SFG extracts only pairs of orthogonally polarized photons and thus eliminates the contribution from the autocorrelation terms (c).

For intense input light, we can calculate the interferogram of the time-reversed HOM interferometer by using only classical electromagnetics, as detailed below. We denote the spectra of the two input pulses by $E(\omega)$ and $E(\omega)e^{i\omega\tau}$, and the effect of the dispersion medium is modeled by a linear transfer function $H(\omega)$. The spectrum

of mixed light through SFG, $E_{\text{SFG}}(\omega)$, is given by the following convolution integral:

$$E_{\text{SFG}}(\omega) \propto \int_{-\infty}^{\infty} d\omega' E(\omega') E(\omega - \omega') H(\omega') e^{i(\omega - \omega')x/c} \left[e^{i\omega'\tau} - e^{i(\omega - \omega')\tau} \right], \quad (5.3)$$

where c is the speed of light in a vacuum. Assuming that the transmission spectrum of the bandpass filter is sufficiently narrow and its center frequency is $2\omega_0$, the measured intensity $I_r(x, \tau)$ after the bandpass filter is given by $I_r(x, \tau) = |E_{\text{SFG}}(2\omega_0)|^2$. By integrating $I_r(x, \tau)$ over all delays τ , the post-processed interferogram $S(x) := \int_{-\infty}^{\infty} d\tau I_r(x, \tau)$ is calculated as

$$S(x) \propto \int_{-\infty}^{\infty} d\omega |E(\omega_0 + \omega)|^2 |E(\omega_0 - \omega)|^2 |H(\omega_0 + \omega)|^2 - \int_{-\infty}^{\infty} d\omega |E(\omega_0 + \omega)|^2 |E(\omega_0 - \omega)|^2 H(\omega_0 + \omega) H(\omega_0 - \omega)^* e^{-i2\omega x/c}. \quad (5.4)$$

We can see that $S(x)$ is insensitive to even-order dispersion, which is represented as $H(\omega) = e^{i[\phi_0 + \phi_2 \cdot (\omega - \omega_0)^2 + \dots]}$. For a Gaussian frequency spectrum $E(\omega) = \exp[-(\omega - \omega_0)^2 / (2\sigma^2)]$, $S(x)$ is calculated as $S(x) \propto 1 - \exp[-\sigma^2(x/c)^2 / 2]$, which is identical to the time-forward HOM interferogram.

5.3 Experiments and Results

We next experimentally generate dispersion-insensitive HOM interferograms with the time-reversed HOM interferometer. Our experimental setup is shown in Fig. 5.3(a). A femtosecond fiber laser (center wavelength 782 nm, pulse duration 74.5 fs FWHM, average power 54 mW) was used to create transform-limited pulsed light with horizontal polarization. In the preparation stage, each pulse is divided into a pair of pulses at the first non-polarizing beam splitter. One of the pair experiences a relative path difference $y := c\tau$ and the other is rotated into vertical polarization by a half-wave plate (HWP). The pair of pulses is introduced into the cross-correlator with path difference y . The cross-correlator introduces a relative path difference x in one of its arms and includes a dispersive medium in the other. We use a 5-mm-thick zinc selenide (ZnSe) plate as a dispersive medium. After propagating in each arm, the pulses are focused into a 1-mm-length β -barium

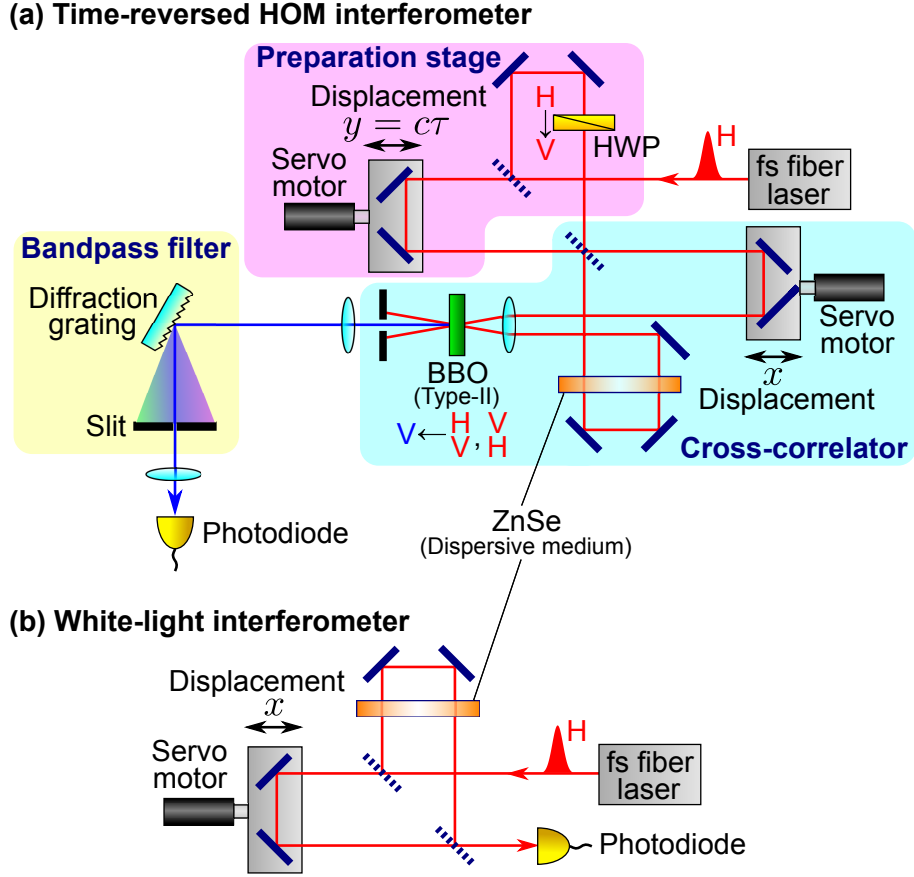


Figure 5.3: (a) Experimental setup of the time-reversed HOM interferometer. Pairs of orthogonally polarized pulses with path difference $y = c\tau$ are prepared in the preparation stage composed of a beam splitter, a half-wave plate (HWP), and a delay mirror moved by a DC servo motor. The cross-correlator is composed of a beam splitter, a delay mirror moved by a DC servo motor, and a BBO crystal for Type-II non-collinear SFG. The bandpass filter passing narrow-band (0.43 nm) light is composed of a grating and a slit. (b) Experimental setup of the white-light interferometer. We used the same light source and dispersive medium as for the time-reversed HOM interferometer.

borate (BBO) crystal for type-II non-collinear SFG. The sum-frequency light is filtered into a 0.43-nm bandwidth around 391 nm by a 1,200-lines/mm aluminum-coated diffraction grating followed by a slit, and then detected by a GaP photodiode equipped with an amplifier. We measured the intensity $I_r(x, y)$ for various

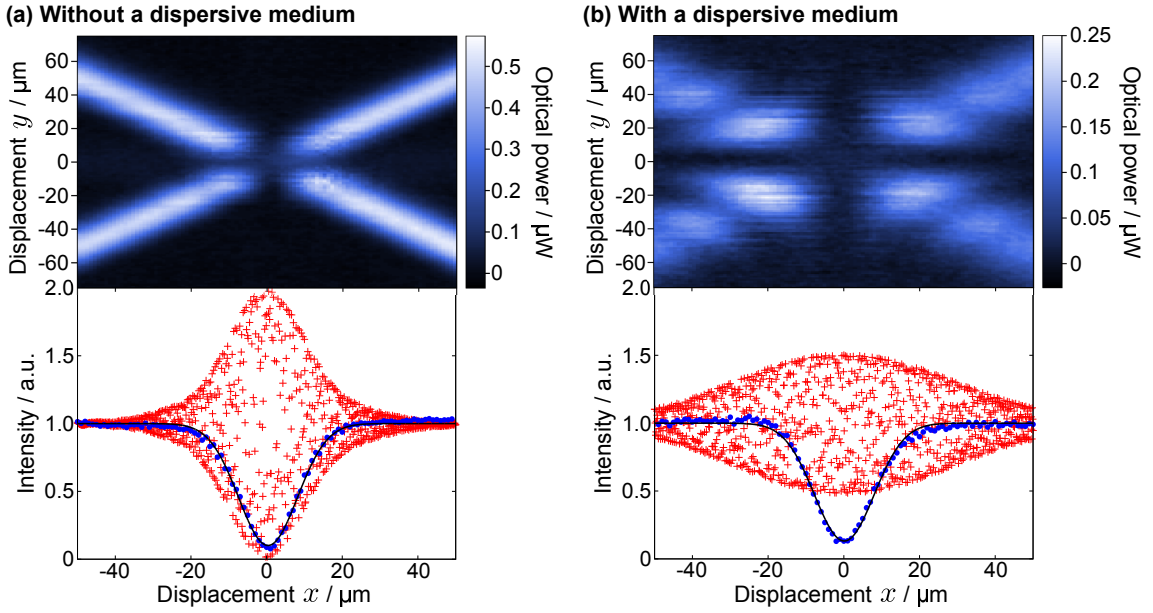


Figure 5.4: Experimental results of the time-reversed HOM interferometer and the white-light interferometer (a) without and (b) with a dispersive medium. The upper color maps show the intensity distribution for all x and y . [Note that the color scales in (a) and (b) differ.] In the lower plots, the blue circles show the time-reversed HOM interferograms, derived by integrating the upper data over all y . The red crosses show the white-light interferograms for comparison.

displacements y and integrated $I_r(x, y)$ for all displacements y to obtain the HOM interferograms. We also observed white-light interferograms for comparison, using an ordinary Mach–Zehnder interferometer with the same light source shown in Fig. 5.3(b).

The experimental results without and with a dispersive medium are shown in Figs. 5.4 (a) and (b), respectively. The upper color maps show the intensity distribution versus displacement x and y in the time-reversed HOM interferometer. Both x and y are spaced at intervals of $1\text{ }\mu\text{m}$. The lower graphs show time-reversed HOM interferograms (blue circles) and white-light interferograms (red crosses). The time-reversed HOM interferograms are derived by vertically integrating the upper graphs over all y . Without a dispersive medium, we observed $18.3 \pm 0.2\text{ }\mu\text{m}$ FWHM for the time-reversed HOM dip and $24.63 \pm 0.04\text{ }\mu\text{m}$ FWHM for the white-light in-

interference pattern; the former had a narrower width by a factor of 1.35. This factor is slightly smaller than the theoretical value $\sqrt{2}$ for input light with a Gaussian spectrum [82] due to the lost bandwidth in SFG. With the dispersive medium, we observe $18.6 \pm 0.2 \mu\text{m}$ FWHM for the time-reversal HOM dip and $69.5 \pm 0.3 \mu\text{m}$ FWHM for the white-light interference pattern. The width of the time-reversed HOM dip remains essentially unchanged owing to automatic dispersion cancellation, whereas that of the white-light interference pattern is clearly increased by a factor of 1.8.

We note that our results by TDPI achieve high-visibility HOM interferograms with high signal conversion efficiency. The measured visibilities of the time-reversed HOM dips are $89.9 \pm 0.7\%$ without dispersion and $86.8 \pm 0.8\%$ with dispersion. From the maximum signal power $0.57 \mu\text{W}$ and the light source power 54 mW , the maximum signal conversion efficiency is estimated to be 1.1×10^{-5} . This value is about one order of magnitude greater than that for CPI, 1.6×10^{-6} , which is estimated from the previous experiment [25]. These improvements are due to the simplicity of the setup of TDPI, which does not require the generation of chirped laser pulses, unlike CPI.

5.4 Discussion and Conclusion

We here mention the time-reversed experiment of the other type of dispersion cancellation, *nonlocal* dispersion cancellation [89]. The property of nonlocal dispersion cancellation manifests the quantum nonlocality of the entangled photons. The time-reversal method can also be used to classically reproduce the dispersion-insensitive measurement results similar to that of the time-forward nonlocal dispersion cancellation experiment [31]. However, the time-reversed experiment does not manifest quantum nonlocality like the time-forward one because the nonlocal two-photon detection used in the time-forward one is replaced by the local detection in the time-reversed one. Note that the physical phenomenon occurring in the time-reversed process is different from that in the time-forward one.

In conclusion, we have constructed a classical optical system reproducing dispersion-insensitive HOM interferograms by TDPI. We have also experimentally

demonstrated automatic dispersion cancellation using this interferometer. Our results exhibit high-visibility interference dips with high signal conversion efficiency, demonstrating that TDPI is appropriate for metrological applications such as dispersion-cancelled optical coherence tomography. Furthermore, the time-reversal method illustrated here is expected to be a step toward seeking a new boundary between quantum and classical physics.

Chapter 6

Dispersion-cancelled optical coherence tomography by TDPI

6.1 Introduction

Optical coherence tomography (OCT) [4] is a non-invasive axial-imaging technique that uses white-light (i.e., light with short coherence length) interference. OCT has been widely used for various imaging applications including medical [90–96] and industrial applications [97–102]. The axial resolution of OCT is ultimately limited by the coherence length of the optical source; however, dispersion of the sample increases the width of the interference signals and results in a reduction of the axial resolution [103]. Various techniques that have been used for depth-dependent dispersion compensation [104–107] require *a priori* knowledge of the dispersion of the sample.

Quantum-optical coherence tomography (Q-OCT) [18–20] is one solution approach to the resolution reduction by dispersion without *a priori* knowledge of the sample properties. Q-OCT employs Hong–Ou–Mandel (HOM) interference [21] of time-frequency-entangled photon pairs, which is insensitive to even-order dispersion such as group-velocity dispersion (GVD) [22, 23], instead of white-light interference. By Q-OCT, dispersion-cancelled axial imaging of a coverglass [19] and surface topography of a gold-coated onion skin [20] have been demonstrated. However, practical applications of Q-OCT are limited by the two obstacles: weak output signals

and unwanted artifacts. The weak output signals stem from the low efficiency in the generation of entangled photon pairs by spontaneous parametric down-conversion (SPDC) and the necessity of coincidence counting of these photon pairs. The weak output signals require long-term measurements, which prevent Q-OCT from imaging of living samples. The unwanted artifacts are observed between each pair of HOM signals corresponding to interfaces of the sample (we call these HOM signals main signals in order to distinguish them from the artifacts). Because the number of artifacts increases with the square of the number of interfaces, the artifacts clutter the images of complex samples.

Recent studies have reported that dispersion-insensitive HOM interferograms can be reproduced by the time-reversed HOM interferometer using pairs of oppositely chirped laser pulses (chirped-pulse interferometry, CPI [25]) or pairs of laser pulses with various time differences (time-decomposed pulse interferometry, TDPI; see Chap. 5). Both techniques are implemented by completely classical optical systems and therefore achieve intense output signals. CPI has been applied for dispersion-cancelled axial imaging of a coverglass [27] and cross-sectional imaging of an onion piece [29]; remarkably, the latter experiment [29] has achieved artifact-free, dispersion-cancelled imaging. There have been reported other classical techniques showing automatic dispersion cancellation [85–88, 108, 109], but none of them have reached practical axial imaging so far and they still suffer from unwanted artifacts.

In this chapter, we use TDPI to experimentally demonstrate dispersion-cancelled axial imaging of a coverglass and cross-sectional imaging of a 100-yen coin. TDPI requires more scanning steps than CPI, but no laser pulse shaping; therefore, the TDPI-based OCT is implemented by a simpler optical system than the CPI-based one. Furthermore, in this experiment we employ a new technique, which we call *subtraction method*, to produce completely background-free OCT images. In the previous experiment using CPI-based OCT [29], there must remain background noise around the main signals. Even though the remained background noise is relatively low, it leads to blurred images in log-scale representation, which is usually used to exhibit practical OCT images [4, 90–102]. The subtraction method extracts only the main signals from the background noise and therefore exhibits clear OCT images even in log-scale representation.

This chapter is organized as follows. In Sec. 6.2, we provide a brief description of OCT and Q-OCT. In Sec. 6.3, we describe the theory of the TDPI-based OCT system including the subtraction method, our new proposal to remove both the artifacts and background noise from OCT images. In Sec. 6.4, we experimentally demonstrate dispersion-cancelled and artifact-background-free OCT for a coverglass and a 100-yen coin by means of TDPI and the subtraction method. In Sec. 6.5, we make a comparison between CPI-based and TDPI-based OCT and show advantages and drawbacks of TDPI compared with CPI. Finally, we summarize the findings of our study and mention future works in Sec. 6.6.

6.2 Fundamentals of OCT and Q-OCT

Before the main study, in this section we briefly review the fundamentals of OCT (Sec. 6.2.1) and Q-OCT (Sec. 6.2.2). We indicate the problems of the resolution degradation by dispersion in OCT, and by weak output signals and artifacts in Q-OCT.

6.2.1 OCT

Here we describe the time-domain (TD) OCT. The schematic is shown in Fig. 6.1(a). The input white light has the wide spectrum $E(\omega) = \exp[-(\omega - \omega_0)^2/(2\Delta\omega^2)]$, where ω_0 and $\Delta\omega$ are the central frequency and the RMS width of the spectrum, respectively. In the upper arm of the interferometer (the reference arm), the light propagate a delay line to receive a relative phase shift of $e^{i\omega x/c}$. In the lower arm (the sample arm), the light propagate through a measured sample, which is dispersive and has multiple interfaces. The effect of the sample is modeled by a linear transfer function $H(\omega)$ such as

$$H(\omega) = r_1(\omega) + r_2(\omega)e^{i\phi_2(\omega)} + r_3(\omega)e^{i\phi_3(\omega)} + \dots, \quad (6.1)$$

where $r_1(\omega), r_2(\omega), \dots$ are the spectral reflectivities and $\phi_2(\omega), \phi_3(\omega), \dots$ are the spectral phases added by the material. For simplicity, we here focus attention on one of the interface and assume the linear transfer function as $H(\omega) = e^{i\beta \cdot (\omega - \omega_0)^2}$

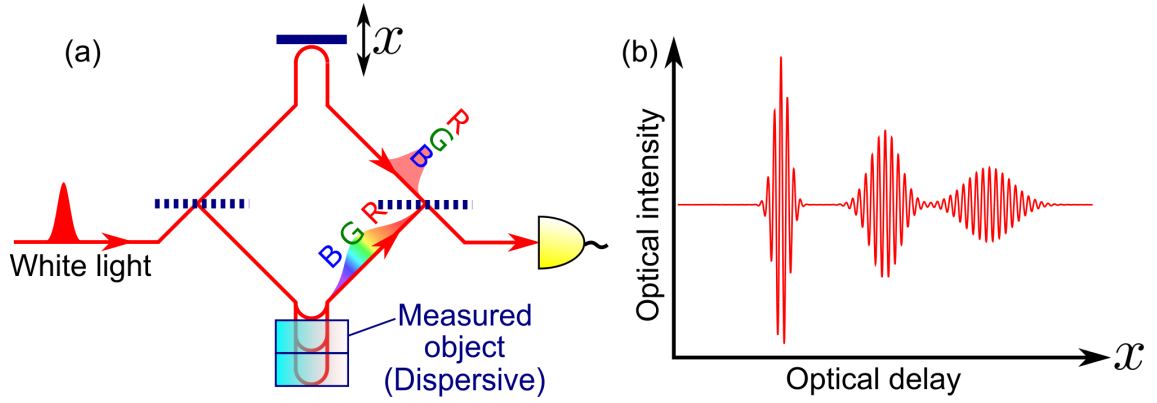


Figure 6.1: (a) Schematic setup for OCT. (b) Interference signal of OCT. Each signal corresponds to each reflective surface of the measured object. The interferogram of the light which propagates inside the measured object suffers broadening by the dispersion.

(i.e., we assume that the reflectivity is unity and the spectral phase has only the second-order dispersion). The interferogram $I(x)$ corresponding to the reflection by one interface of the sample is written by the following form:

$$I(x) \propto 1 + A(\beta) \exp \frac{-x^2}{4c^2 (1/\Delta\omega^2 + \beta^2 \Delta\omega^2)} \cos \left[\frac{\beta x^2}{4c^2 (1/\Delta\omega^4 + \beta^2)} + \omega_0 x/c \right], \quad (6.2)$$

where $A(\beta)$ is a real function of β satisfying $0 \leq A(\beta) \leq 1$. The width of the envelope function $\exp\{-x^2/[4c^2(1/\Delta\omega^2 + \beta^2 \Delta\omega^2)]\}$ corresponds to the resolution of OCT.

When β is nonzero, the envelope function is broadened and the resolution of OCT is degraded. This form of the envelope function also indicates that the broader the spectral width $\Delta\omega$ is (i.e., the shorter the coherence length is), the more sensitively the envelope function is broadened by dispersion. Therefore, for the typical use of OCT such as biological imaging, the resolution is virtually limited to about $20\mu\text{m}$ even though very short-coherence light is used.

6.2.2 Q-OCT

The problem of OCT, the resolution degradation by dispersion, can be overcome by Q-OCT. Q-OCT employs the HOM interference of the time-frequency-entangled photon pairs instead of the white-light interference. The schematic is shown in Fig. 6.2(a). As shown in Fig. 6.2(b), in the interference pattern of Q-OCT, the HOM dips indicate the main signals corresponding to the interfaces of the sample. Because of the property of automatic dispersion cancellation, Q-OCT can achieve dispersion-insensitive tomographic imaging.

However, Q-OCT has other problems for practical imaging application. One problem is the low output signals. Because Q-OCT is based on HOM interference of time-frequency-entangled photon pairs, the amount of the output signals of Q-OCT is limited by the generation rate of photon pairs in SPDC and the time resolution of the coincidence counting circuit. The low output signals cause long measurement time, which is undesirable for *in vivo* imaging applications.

Another problem of Q-OCT is the emergence of artifacts in the interferograms. The artifacts are caused by interference between two terms where the photons are reflected by different interfaces of the sample, as shown in the inset of Fig. 6.2. The artifacts can be both dip- and peak-shapes, because the relative phase between these two terms depends on the width between the two interfaces, the refractive index of the sample, and the central frequency of the pump light. The dip-shape artifacts are indistinguishable from the main signals. In addition, the number of the artifacts increases with the square of the number of the reflective surfaces; therefore, the artifacts can be a serious problem for imaging of a sample with a complicated structure.

6.3 Theory

In this section, we describe the theory of the TDPI-based OCT system that we newly proposed. We provide its description in the following three subsections. First, in Sec. 6.3.1, we introduce the OCT system employing the HOM dips of the time-reversed HOM interferometer by TDPI as the main signal. We note that this OCT

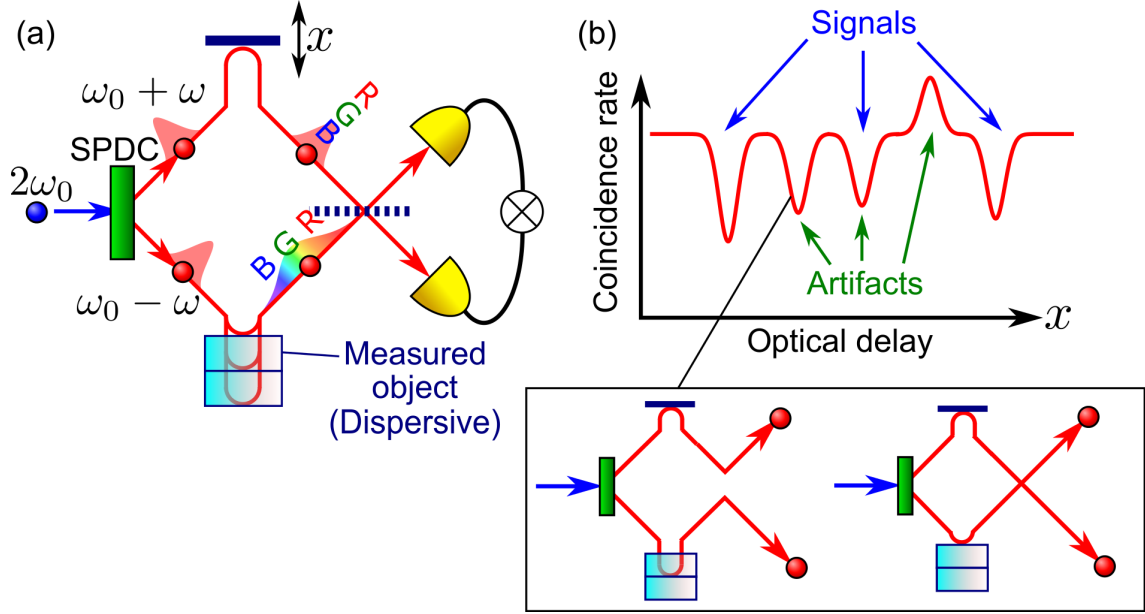


Figure 6.2: (a) Schematic setup for Q-OCT. (b) Interference signals of Q-OCT. Each of the three HOM-dip signals corresponding to each reflective surface of the measured object is not broadened by the dispersion of the measured object. There are also the artifacts between each pair of the main signals (the central artifact overlaps with the central HOM-dip main signal). The artifacts can be both dip- and peak-shapes depending on the width between the two interfaces, the refractive index of the measured sample, and the central frequency of the pump light. (inset) The two Feynman paths leading to the left artifact.

system also suffers from the undesired artifacts. Next, in Sec. 6.3.2, we describe a technique to remove the artifacts, which leaves background noise around the main signals, however. Finally, in Sec. 6.3.3, we propose the subtraction method, a new TDPI-based OCT technique to remove both the artifacts and background noise.

6.3.1 Dispersion-cancelled OCT by TDPI

We first consider a dispersion-cancelled OCT system that utilizes HOM dips reproduced by the time-reversed HOM interferometer implementing TDPI described in Chap. 5. The schematic of this OCT system is shown in Fig. 6.3(a). This system has the same composition as the time-reversed HOM interferometer implementing TDPI, except that the mirror in the lower arm is replaced with a measured sample. We here assume a coverglass as the measured sample. The input light is orthogonally polarized two pulses with various distance y . The two pulses enter the cross-correlator, which includes a delay line to introduce a relative path difference x in the reference arm and a dispersive medium in the sample arm. The pulses are converted into sum-frequency light by type-II SFG. The sum-frequency light is filtered into a narrow band by a bandpass filter, and then detected by a photodiode. Because the coverglass has two reflection surfaces, this system has four Feynman paths (i)–(iv) leading to output signals shown in Fig. 6.3(b).

Figure 6.3(c) shows theoretical calculations of the intensity distribution $I(x, y)$ without and with a dispersive medium. In the calculation, we assume the following condition: the center wavelength and the pulse duration of the input light are respectively 782 nm and 74.5 fs FWHM; the dispersive medium is a 5-mm-thick zinc selenide (ZnSe) plate, which has GVD of 1075 fs²/mm at wavelength of 782 nm; the thickness and the refractive index of the coverglass are respectively 200.8 μm and 1.5; we ignore multiple reflection in the coverglass. In the each intensity distribution, we can see four bright lines, which correspond to the four Feynman paths (i)–(iv) in Fig. 6.3(b). The pair of the terms (i) and (ii), and also the pair of the terms (iii) and (iv), have the phase difference π and destructively interfere. On the other hand, the pair of the terms (i) and (iv), and also the pair of the terms (ii) and (iii), have the phase difference $\pi + k(\omega_0)2d$, where $k(\omega_0)$ is the wavevector of the light with

frequency ω_0 in the coverglass, and d is the thickness of the coverglass. Because $\pi + k(\omega_0)2d \approx \pi/3$ in this condition, these pairs interfere somewhat constructively.

We integrate the each intensity distribution over all y to derive the interference pattern shown in Fig. 6.3(d), which is the tomographic image of the coverglass. Due to dispersion-insensitivity of the HOM interference, we obtain the same interference pattern as Fig. 6.3(d) even in the presence of dispersion. This interference pattern has two dips corresponding to the main signals and a peak artifact at the center of the two dips, which is the similar shape as those of Q-OCT [19] and CPI [27].

6.3.2 Removal of artifacts

We next describe a technique to remove the artifacts from the intensity pattern of the OCT system introduced in Sec. 6.3.1. As seen in the intensity distributions of Fig. 6.3(c), the artifacts are attributed to the interference between the lines (i) and (iv) and between (ii) and (iii). Therefore, we can remove the artifacts by integrating the each intensity distribution over a small range of y around zero. This integration range of y includes the interference between (i) and (ii) and that between (iii) and (iv), which lead the main signals, but avoids these interference between (i) and (iv) and that between (ii) and (iii), which lead the artifacts. We note that this integration range of y must cover most part of the intersection area of (i) and (ii) and that of (iii) and (iv) for dispersion cancellation. In the presence of dispersion, because these intersection areas are broadened as seen in the right panel of Fig. 6.3(c), the integration range of y must be slightly broader. We also note that, when the distance of the two interfaces of the sample is very close, the integration range of y may include the intersection areas that lead artifacts. Such artifacts cannot be removed by this technique. In addition, in this technique, the interference between (i) and (ii) and that between (iii) and (iv) should be arranged to be constructive interference, in order to clarify the main signals in the integrated interference pattern.

The schematic setup implementing this technique is shown in Fig. 6.4(a). We additionally put a half-wave plate (HWP) into the reference arm. The HWP multiplies only the terms including vertically-polarized light in the reference arm, such

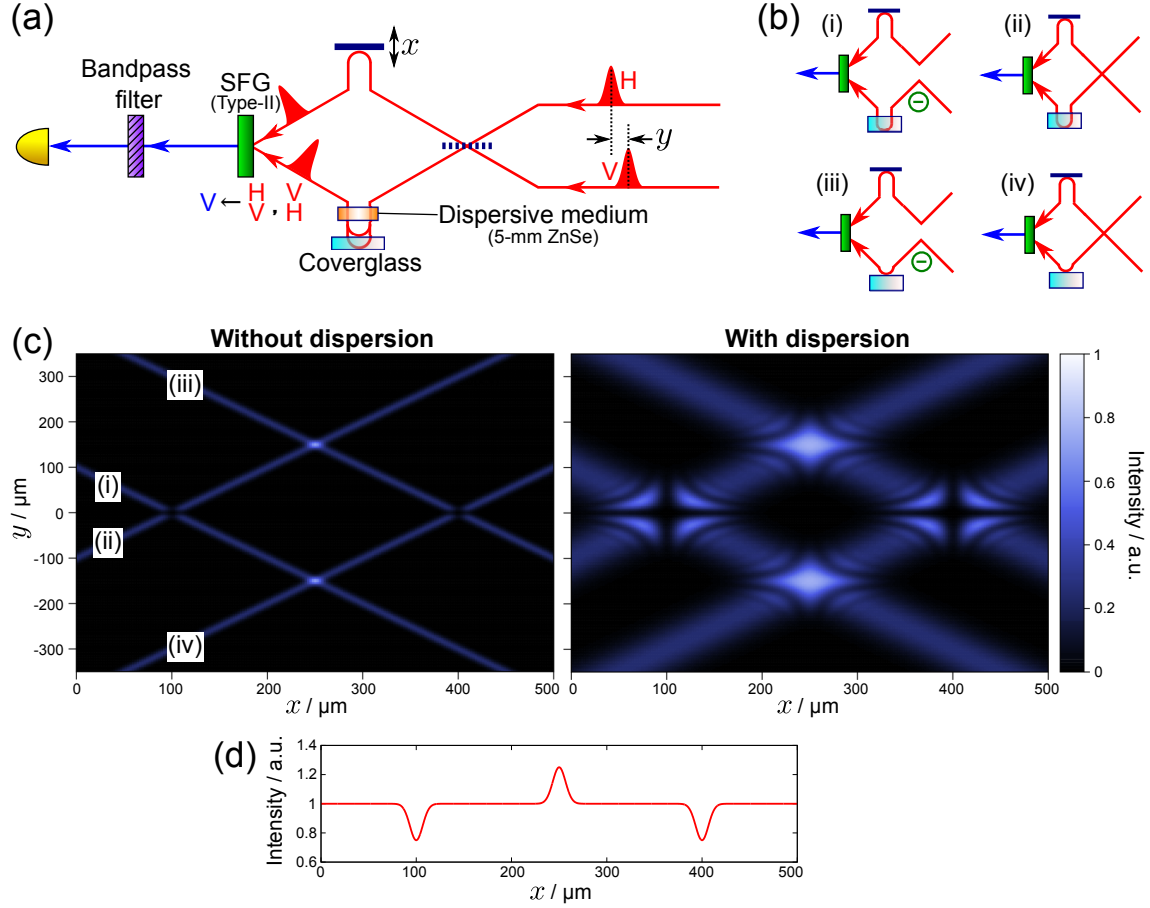


Figure 6.3: (a) Schematic setup for the OCT system using the time-reversed HOM interferometer implementing TDPI. (b) Feynman paths leading to output signals in this setup. (c) Theoretical calculations of the intensity distribution $I(x, y)$ without and with a dispersive medium. The four bright lines (i)–(iv) correspond to the four Feynman paths (i)–(iv) in (b). (d) Interference pattern derived by integrating the each intensity distribution over all y , which shows the tomographic image of the coverglass. This interference pattern is dispersion-insensitive but has an artifact at the center of the two main signals.

as the terms (ii) and (iv) of Fig. 6.4(b), by the factor -1 . Due to the HWP, the terms (i) and (ii), and the terms (iii) and (iv) constructively interfere with each other, as shown in the intensity distributions of Fig. 6.4(c). We integrate the each intensity distribution from $y = -50\ \mu\text{m}$ to $50\ \mu\text{m}$ to derive the each interference pattern shown in Fig. 6.4(d), where the artifact is removed. The interference patterns in linear-scale representation [the upper panels of Fig. 6.4(d)] show that the widths of the main signals are essentially unchanged in the presence of dispersion, while the background noise are remained around the main signals in the cases both without and with dispersion.

This background noise is enhanced in log-scale representation [the lower panels of Fig. 6.4(d)] and can blur the main signals. The log-scale representation is commonly used to enhance the main signals in practical OCT of complex samples [4, 90–102]. Therefore, the background noise around the main signals must be removed for practical OCT.

6.3.3 Subtraction method

We finally propose the subtraction method, a new TDPI-based OCT technique to remove both the artifacts and background noise. Let us consider the schematic setup shown in Fig. 6.5(a), where two quarter-wave plates (QWPs) are inserted into the reference arm instead of a HWP. When the two QWPs' fast axes are parallel, the pair of QWPs performs in the same way as a HWP and multiply vertically-polarized light by the factor -1 . In this condition, which we call peak condition, the intensity distributions $I(x, y)$ show the same patterns as Fig. 6.4(c), and the integrated intensity patterns from $y = -50\ \mu\text{m}$ to $50\ \mu\text{m}$ show two peak-shaped main signals, as shown in the upper panels of Fig. 6.5(b). On the other hand, when the two QWPs' fast axes are orthogonal, the effects of the two QWPs cancel each other out. In this condition, which we call dip condition, the intensity distributions $I(x, y)$ show the same patterns as Fig. 6.3(c), and the integrated intensity patterns from $y = -50\ \mu\text{m}$ to $50\ \mu\text{m}$ show two dip-shaped main signals, as shown in the lower panels of Fig. 6.5(b). These two conditions can be switched by rotating one of the QWPs by 90° . By subtracting the each interference pattern in dip condition

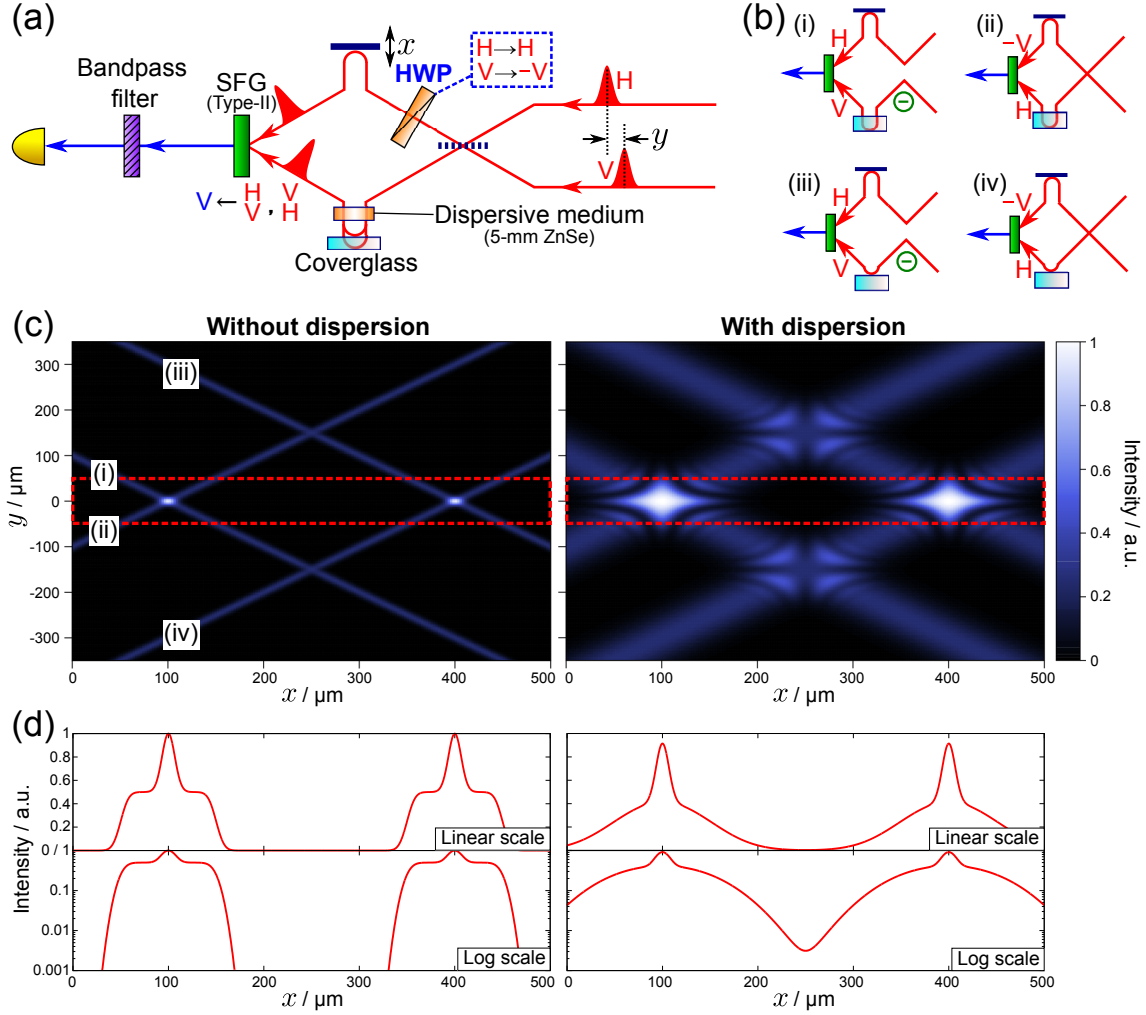


Figure 6.4: (a) Schematic setup implementing the technique to remove the artifacts. (b) Feynman paths leading to output signals in this setup. The terms (ii) and (iv) are multiplied by the factor -1 due to the HWP. (c) The intensity distributions $I(x, y)$ without and with a dispersive medium. (d) Interference patterns in linear-scale (the upper panels) and log-scale representation (the lower panels) derived by integrating the intensity distributions from $y = -50 \mu\text{m}$ to $50 \mu\text{m}$. The each integrated domain is indicated by the area enclosed by red dashed line in (c).

from that in peak condition, we can obtain the artifact-background-free interference pattern shown in the upper panel of Fig. 6.5(c), which is also dispersion-insensitive. As the lower panel of Fig. 6.5(c) shows, the main signals are not blurred even in log-scale representation.

6.4 Experiments and Results

In this section, we experimentally demonstrate dispersion-cancelled and artifact-background-free OCT by means of TDPI and the subtraction method. In Sec. 6.4.1, we demonstrate axial imaging of a coverglass to evaluate this system performance. In Sec. 6.4.2, we demonstrate cross-sectional imaging of 100-yen coin as a practical use of this system.

The experimental setup used in both demonstrations is shown in Fig. 6.6. We used a femtosecond fiber laser (center wavelength 782 nm, pulse duration 74.5 fs FWHM, average power 54 mW, repetition rate 100 MHz) to create transform-limited pulsed light with horizontal polarization. The input pulse is divided into a pair of pulses at the first beam splitter. The HWP in one arm rotates the beam's polarization into vertical, and the delay line in the other arm adds a relative path difference y . The pair of pulses is again divided into two at the second beam splitter. One arm (sample arm) includes a 5-mm-thick ZnSe plate as a dispersive medium and a measured sample (coverglass or 100-yen coin). The sample is mounted on a motorized stage so that we can scan the sample transversely by z in the cross-sectional imaging of a 100-yen coin. The other arm (reference arm) includes two QWPs (QWP1 and QWP2) and a delay line. The angles of QWP1 and QWP2 are swivable and fixed, respectively; we can switch the peak and dip condition by rotating QWP1 by 90° . The delay line adds a relative path difference x . After propagating in each arm, the pulses are focused into a 1-mm-length β -barium borate (BBO) crystal for type-II noncollinear SFG. The sum-frequency light is filtered into a 0.40-nm bandwidth around 391 nm by a 1200-lines/mm aluminum-coated diffraction grating followed by a slit, and then detected by a Si photodiode (New Focus, Model 2151).

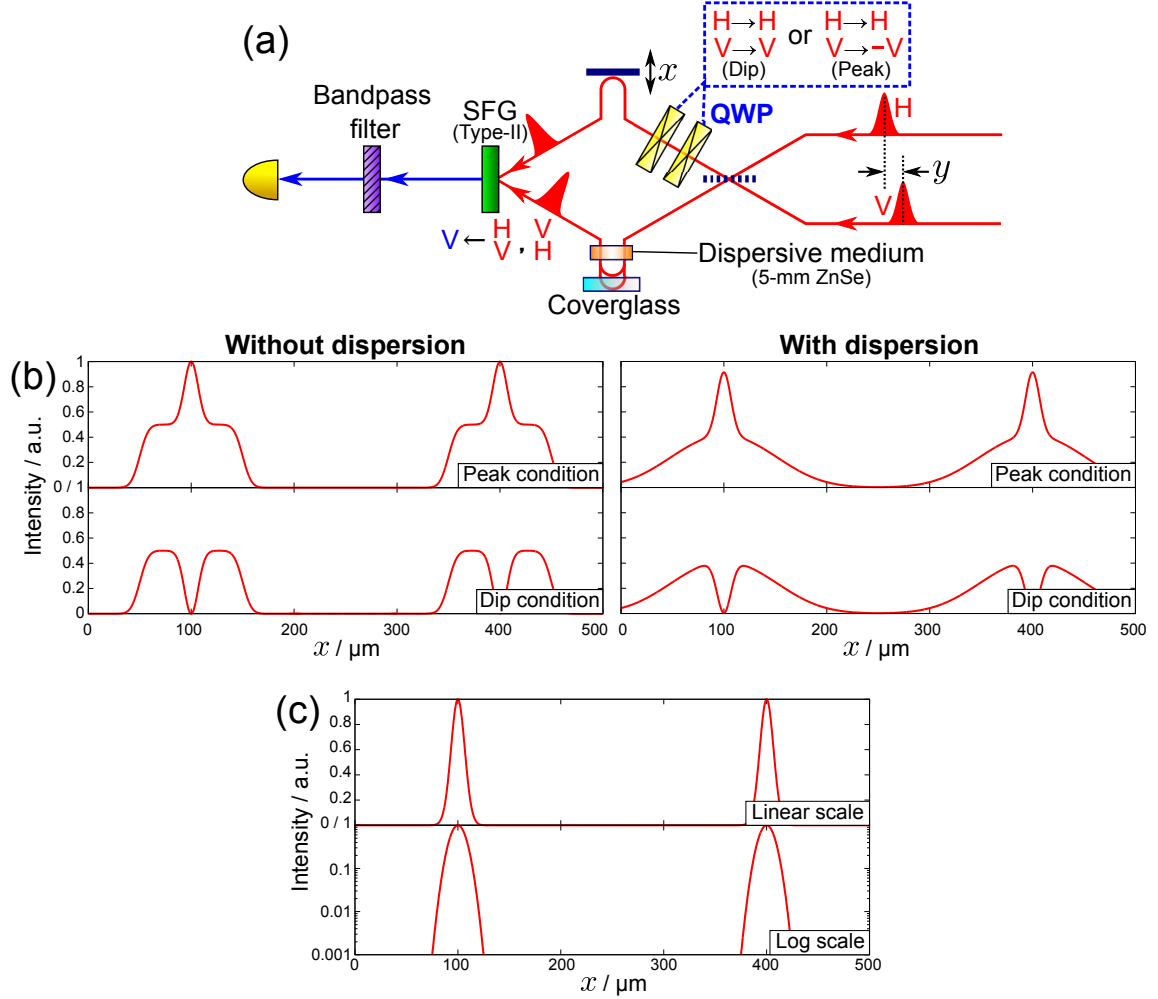


Figure 6.5: (a) Schematic setup implementing the subtraction method. (b) Interference patterns in peak condition (the upper panels) and dip condition (the lower panels) derived by integrating the intensity distributions $I(x, y)$ from $y = -50 \mu\text{m}$ to $50 \mu\text{m}$. (c) Interference pattern derived by subtracting the each interference pattern in dip condition from that in peak condition, which shows artifact-background-free, dispersion-cancelled main signals.

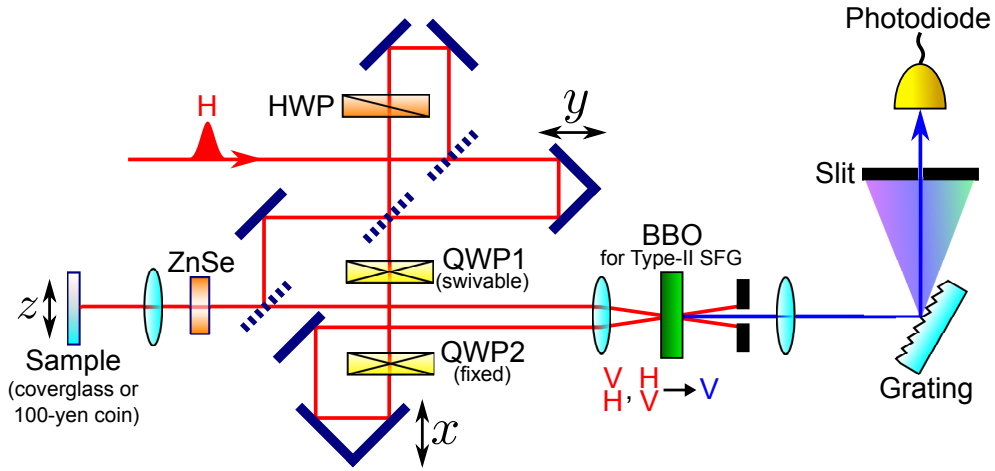


Figure 6.6: Experimental setup for demonstrating dispersion-cancelled, artifact-background-free OCT by TDPI and the subtraction method. HWP, half-wave plate; QWP, quarter-wave plate; BBO, β -barium borate crystal. The translation x , y , z , and the rotation of QWP1 are controlled via a stage controller.

6.4.1 Axial imaging of a coverglass

We demonstrated axial imaging of a coverglass to evaluate this system performance.

We first consider the case without a dispersive medium. The measurement results of the intensity distributions $I(x, y)$ in peak and dip conditions are shown in Fig. 6.7. We took x -direction scans for every $5 \mu\text{m}$ in y -directions. The measured intensity distributions respectively have peak- and dip-shaped intersections as predicted by the theoretical calculations in Secs. 6.3.1 and 6.3.2, while the four bright lines are slightly broadened due to dispersion of the optical elements in the setup.

Figure 6.8(a) shows the measurement results of the interference patterns manifesting the axial image of the coverglass. The first and second panels (“Peak condition” and “Dip condition”) are derived by integrating the intensity distributions in Fig. 6.7 from $y = -50 \mu\text{m}$ to $50 \mu\text{m}$. The third panel (“Peak–Dip”) is our main result and shows an artifact-background-free interference pattern, which is derived by subtracting the second panel from the first panel. The widths of the measured two peaks are $19.4 \mu\text{m}$ and $18.4 \mu\text{m}$ FWHM, which are slightly larger

than the theoretical value $15.8\text{ }\mu\text{m}$ for input light with the pulse duration 74.5 fs FWHM. The differences between experiment and theory are mainly due to the lost bandwidth in SFG. The distance between the two peaks is $229.3\text{ }\mu\text{m}$. The thickness of the coverglass is obtained by dividing the measured distance $229.3\text{ }\mu\text{m}$ by the group index $n_g(\lambda) = n(\lambda) - \lambda \frac{dn}{d\lambda}\big|_\lambda = 1.5273$ of BK7 at $\lambda = 782\text{ nm}$, which results in $150.1\text{ }\mu\text{m}$ in good agreement with $150\text{ }\mu\text{m}$ measured with a micrometer. The forth panel (“Auto-correlation”) shows the interference pattern of auto-correlation corresponding to Fig. 5.2(c) in Chap. 5, which is an artifact-background-free but dispersion-sensitive OCT image [110]. This interference pattern is derived from the intensity distribution in peak condition at $y = 0\text{ }\mu\text{m}$ in Fig. 6.7. The widths of the two peaks are $23.9\text{ }\mu\text{m}$ FWHM together, which are slightly broadened due to dispersion of the optical elements in the setup. The distance between the two peaks $229.5\text{ }\mu\text{m}$ is nearly equal to that of the interference pattern of “Peak–Dip”. Figure 6.8(b) shows the interference patterns of “Peak–Dip” and “Auto-correlation” in log-scale representation. The background noise in both panels is held below 0.01 and main signals can be found obviously.

We next consider the case with the ZnSe plate as a dispersive medium. The measurement results of the intensity distributions $I(x, y)$ in peak and dip conditions are shown in Fig. 6.9, where x -direction scans were taken for every $5\text{ }\mu\text{m}$ in y -directions. The measured intensity distributions have peak- and dip-shaped intersections as expected, and the four bright lines are much broadened due to dispersion of the ZnSe plate.

Figure 6.10(a) shows the measurement results of the interference patterns manifesting the axial image of the coverglass. The first and second panels (“Peak condition” and “Dip condition”) show the integrals of the intensity distributions in Fig. 6.9 from $y = -50\text{ }\mu\text{m}$ to $50\text{ }\mu\text{m}$. The third panel (“Peak–Dip”) is the difference between the first and second panels and shows an artifact-background-free and dispersion-cancelled interference pattern. The widths of the measured two peaks are $20.9\text{ }\mu\text{m}$ and $21.1\text{ }\mu\text{m}$ FWHM, substantially showing dispersion cancellation. The distance between the two peaks is $228.9\text{ }\mu\text{m}$, from which we obtain the thickness of the coverglass $149.9\text{ }\mu\text{m}$, in good agreement with the measured value $150\text{ }\mu\text{m}$. The forth panel (“Auto-correlation”) shows the interference pattern of auto-correlation.

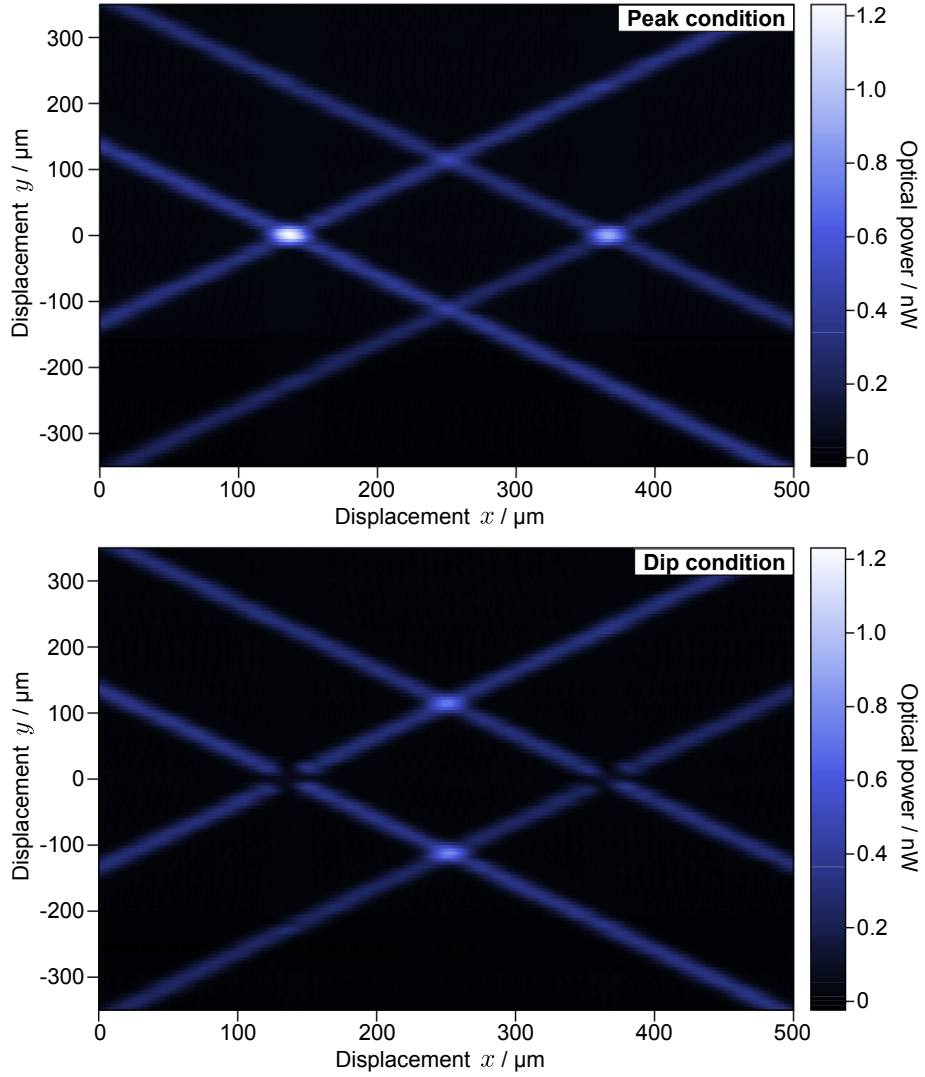


Figure 6.7: Measured intensity distributions $I(x, y)$ in peak and dip conditions, in the case without a dispersive medium.

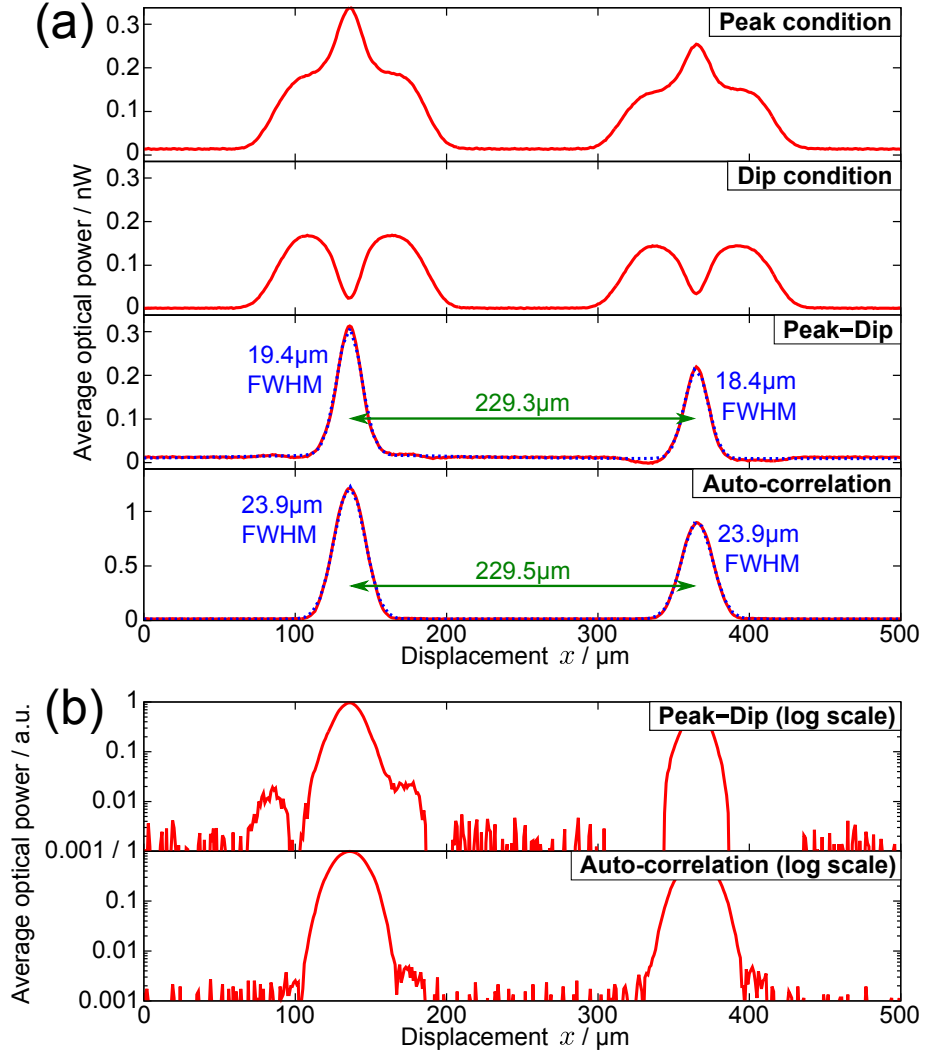


Figure 6.8: Measured interference patterns manifesting the axial image of the coverglass in linear-scale (a) and in log-scale representation (b), in the case without a dispersive medium. The blue dotted lines indicate the Gaussian fitting. In the log-scale representation, the optical power is normalized by the maximum values.

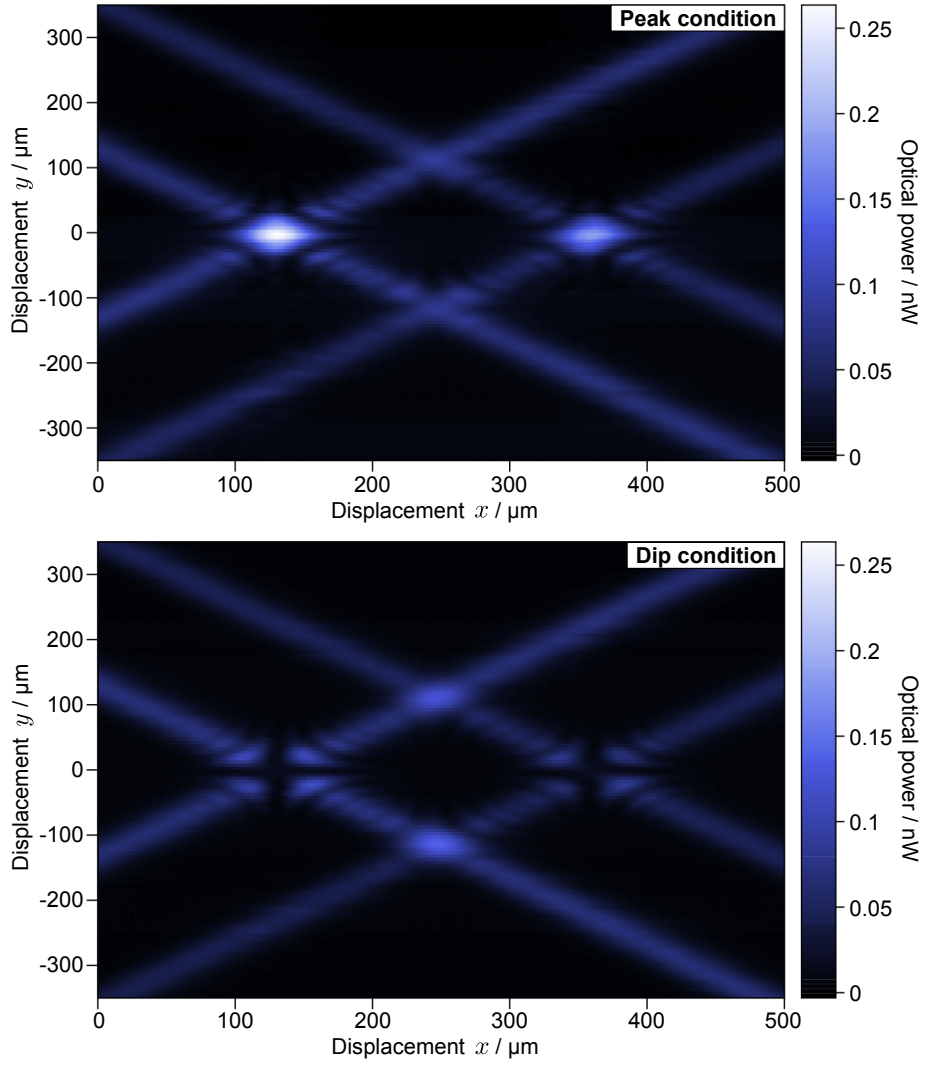


Figure 6.9: Measured intensity distributions $I(x, y)$ in peak and dip conditions, in the case with a dispersive medium.

Because the interference pattern of auto-correlation is sensitive to dispersion, the widths of their two peaks are broadened by 97.5% and 95.4% to $47.2\ \mu\text{m}$ and $46.7\ \mu\text{m}$ FWHM, respectively. This broadening of the peak widths becomes prominent in log-scale representation, as shown in the lower panel [“Auto-correlation (log scale)”] of Fig 6.10(b), while the upper panel [“Peak–Dip (log scale)”] shows that the main signals are still obvious and the background noise is held below 0.01.

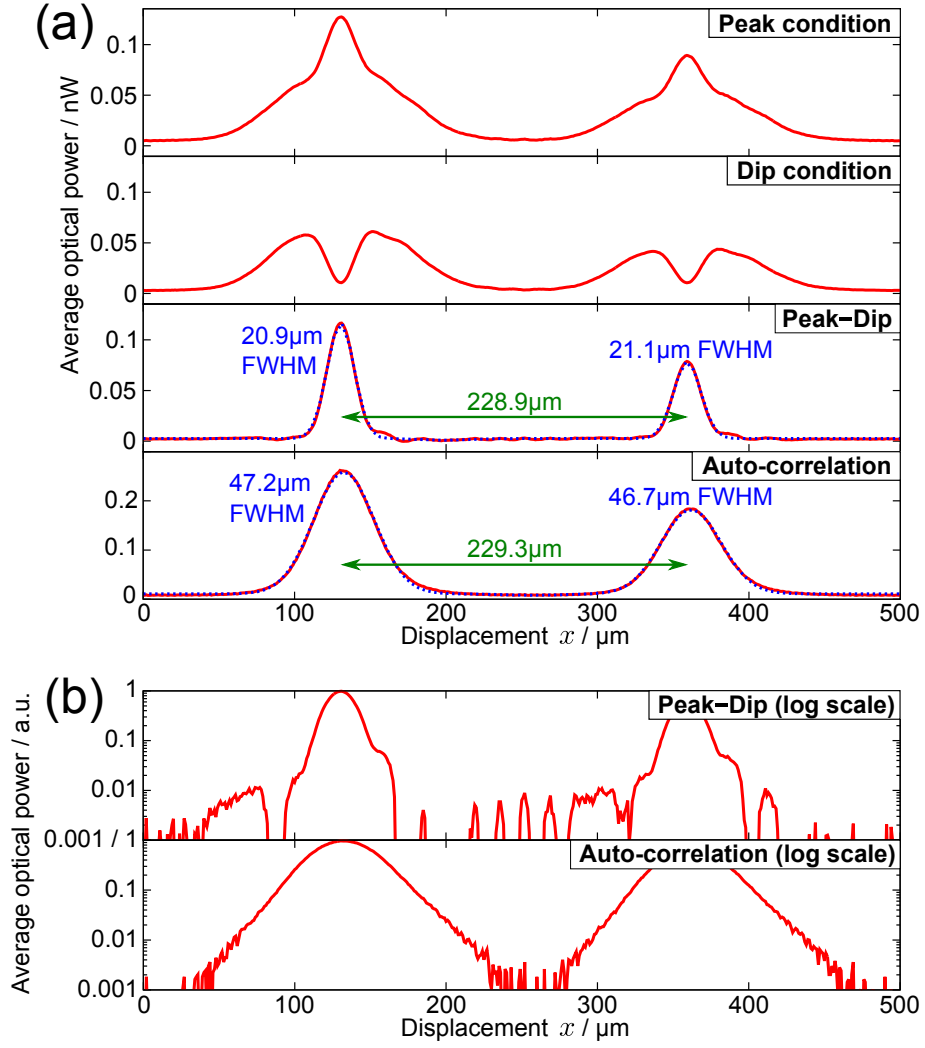


Figure 6.10: Measured interference patterns manifesting the axial image of the coverglass in linear-scale (a) and in log-scale representation (b), in the case with a dispersive medium. The blue dotted lines indicate the Gaussian fitting. In the log-scale representation, the optical power is normalized by the maximum values.

6.4.2 Cross-sectional imaging of 100-yen coin

We demonstrated cross-sectional imaging of a 100-yen coin as practical use of this system. We use the experimental setup shown in Fig. 6.6 with a 100-yen coin shown in the inset of Fig. 6.11. We took x -direction scans (speed 1 mm/s, range 400 μm) for every 10 μm in y -directions from $y = -30 \mu\text{m}$ to 30 μm and summed the scanned data to obtain a single axial scan in peak or dip condition. Subtracting the axial scan in dip condition from that in peak condition gives a single artifact-background-free and dispersion-cancelled axial scan at a single z -position. We took the artifact-background-free and dispersion-cancelled axial scans for every 0.1 mm in z -directions to obtain a cross-sectional image of the 100-yen coin. The acquisition time per TDPI-image was 75 minutes, which was limited by the performance of the motorized stages and data acquisition system that we used. With an optimized system, the acquisition time can be much shorter in principle.

The measurement results are displayed in Fig. 6.11, where the four panels show the cases for both auto-correlation and TDPI (with the subtraction method) without and with dispersion. All the images are shown in log-scale representation. These images exhibit the corrugated structure of the coin's surface. Whereas the signal peaks of auto-correlation are much broadened in the presence of dispersion, those of TDPI remain essentially unchanged and little blurred by background noise even in log-scale representation.

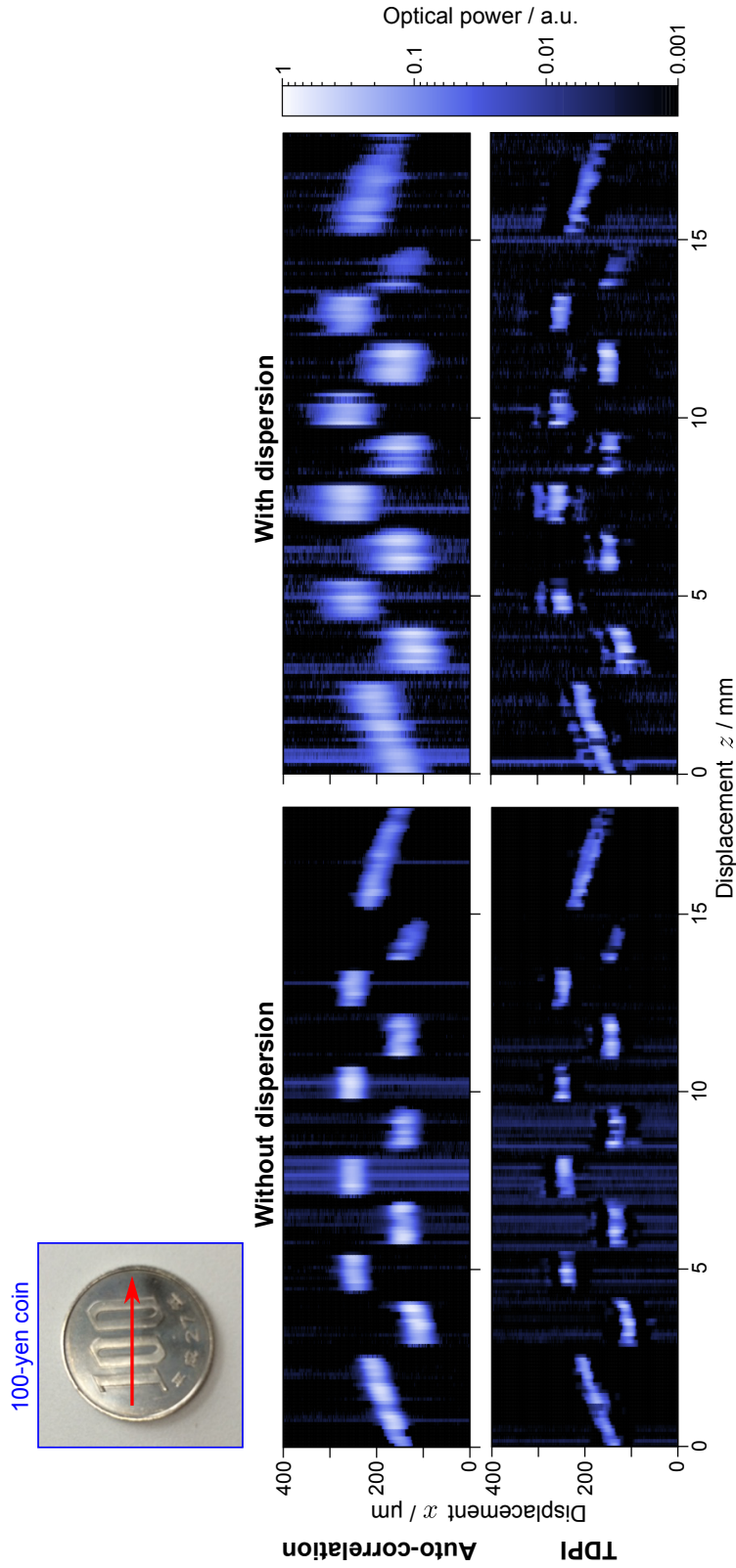


Figure 6.11: Cross-sectional images of a 100-yen coin shown in the inset. The upper and lower panels show the cases for auto-correlation and TDPI with the subtraction method, and the left- and right-sided panels show the cases without and with dispersion, respectively. In all the panels, the coin's head is oriented downward. The color-bar represents the logarithm of the optical power normalized by the maximum value in each panel; these maximum values of optical power are 0.586 nW (upper left), 0.240 nW (lower left), 0.102 nW (upper right), 0.058 nW (lower right).

6.5 Discussion

In this section, we make a comparison between CPI-based and TDPI-based OCT. As described in Chap. 2, both CPI and TDPI are implemented by a time-reversed two-photon interferometer on the basis of the time-reversal symmetry of quantum mechanics. The difference between CPI and TDPI is how to interpret coincidence counting in time-forward two-photon interferometers; CPI is constructed by interpreting coincidence counting in the frequency domain while TDPI is in the time domain. Because both the interpretations are equivalent, interferograms of CPI and TDPI have essentially the same properties. The first experiment of CPI-based OCT [27] have demonstrated dispersion-cancelled axial imaging of a cover-glass. In TDPI, this experiment corresponds to the method described in Sec. 6.3.1. Another experiment of CPI-based OCT [29] have demonstrated not only dispersion-cancelled but also artifact-free cross-sectional imaging of a onion piece. In TDPI, this experiment corresponds to the method described in Sec. 6.3.2. This experiment of CPI-based OCT [29] also states that the background noise is removed; however, it is not quite proper and there remains background noise around the main signals. We can indeed see that the main signals of Fig. 3 in Ref. [29] are blurred by the background noise.

The subtraction method, which we proposed in this study, can achieve artifact-background-free and dispersion-cancelled imaging, as shown by our experiments. While we proposed and demonstrated the subtraction method for TDPI-based OCT, this method can also be applied to CPI-based OCT (and also Q-OCT). In CPI-based OCT using the subtraction method, we have to label the input chirped and anti-chirped pulses with the polarization degree of freedom, and insert two QWPs into the reference arm to switch the peak and dip conditions, in a similar manner to our experimental setup.

Other advantage and drawback of TDPI compared with CPI are as follows. TDPI-based OCT has the advantage of being able to be implemented by a simpler optical system than CPI. Whereas CPI need to prepare pairs of strongly oppositely-chirped laser pulses with a pulse modulation system, TDPI only has to prepare pairs of transform-limited (i.e., unmodified) laser pulses with various time differences.

The drawback of TDPI is that TDPI requires more number of processes compared to CPI due to the multiple x -direction scans for several values of y to obtain a single dispersion-cancelled x -direction scan. The number of additional scans is, however, constant (seven, in our experiment in Sec. 6.4.2) with the size of the measured sample; therefore, the number of processes in TDPI and CPI are regarded to be in the same order.

We also add the possibility of applying the technique removing the artifacts and background noise to Q-OCT. In TDPI, the artifacts can be removed by integrating the intensity distribution over a small range of y around zero, as described in Sec. 6.3.2. In Q-OCT, this integration over a small range of y corresponds to coincidence counting with very narrow coincidence time window. The integration range of $100\text{ }\mu\text{m}$ in TDPI corresponds to the coincidence time window of 333 fs in Q-OCT. However, the timing resolution of currently available coincidence counters is as low as tens of picoseconds (e.g., ID800-TDC, ID Quantique); therefore, this technique in Q-OCT leaves a number of artifacts unremoved. On the other hand, the subtraction method, removing the background noise, can also be realized in Q-OCT, where polarization-entangled photon pairs and two QWPs are employed to switch the peak- and dip-conditions.

6.6 Conclusion

We have proposed TDPI-based OCT with the subtraction method and demonstrated artifact-background-free and dispersion-cancelled OCT for axial imaging of a coverglass and cross-sectional imaging of a 100-yen coin. The measured results exhibit clear OCT images even in log-scale representation due to removal of background noise by the subtraction method. These results manifest that TDPI is useful for a more precise measurement of parameters of an external object. In this experiment, we could not demonstrate cross-sectional imaging of biological samples like a onion piece because of low reflected powers. Incorporating more nonlinear and broadband materials [110–112] and more sensitive detectors will be a solution approach. Employing a laser source with higher power and shorter pulse duration will be another solution, but the optical power density in the sample arm should be

lowered for safe *in vivo* imaging. TDPI-based OCT is expected to be an option for practical OCT appropriate to different purposes, in addition to conventional OCT, Q-OCT, and CPI-based OCT.

Chapter 7

Conclusion

This thesis has investigated the classical optical techniques that reproduce quantum optical interferograms on the basis of the time-reversal symmetry of quantum mechanics, and their application to optical interferometric measurements. We show the summary of each chapter in the following.

In Chap. 2, we have presented the time-reversal method, a classical optical technique to reproduce the two-photon interference phenomena by temporally reversing the original two-photon interferometers. Via the time-reversal method, we have proposed time-decomposed pulse interferometry (TDPI). TDPI is implemented by a time-reversed optical system using pairs of laser pulses with various time differences as its input light. TDPI can achieve much higher signal strength than the quantum optical systems using entangled photons.

In Chap. 3, we experimentally demonstrated reproduction of two-photon phase superresolution by a classical optical interferometer that implements TDPI. The measured interference exhibited high-visibility ($97.9\% \pm 0.4\%$) fringes with much intense optical power ($2.8 \mu\text{W}$). We also demonstrated a classical analog to the large difference between the one- and two-photon coherence lengths of entangled photon pairs.

In Chap. 4, We demonstrated classical optical experiments for measuring the three-vertex geometric phase in a two-photon polarization qutrit by TDPI. In this experiment, we used the time-reversed two-photon interferometer for phase superresolution constructed in Chap. 3. We observed nonlinear variations in the geo-

metric phase inherent in a two-photon polarization qutrit. The measured output signals showed three orders of magnitude larger optical power than that in typical two-photon interferometers measuring the geometric phase. The results of this study manifest that TDPI is useful for a measurement of internal parameters of quantum light.

In Chap. 5, we experimentally demonstrated reproduction of Hong–Ou–Mandel (HOM) interference with automatic dispersion cancellation by a classical optical interferometer that implements TDPI. The measured results showed high-visibility ($89.9\%\pm0.7\%$) dispersion-insensitive HOM interferograms with intense optical power ($0.57\text{ }\mu\text{W}$).

In Chap. 6, we demonstrated artifact-background-free and dispersion-cancelled optical coherence tomography (OCT) for axial imaging of a coverglass and cross-sectional imaging of a 100-yen coin by TDPI. In this experiment, we used the time-reversed HOM interferometer constructed in Chap. 5 and the subtraction method, which we newly proposed. The measured results showed clear OCT images even in the presence of dispersion and in log-scale representation. The results of this study manifest that TDPI is useful for a more precise measurement of parameters of an external object.

As described above, we have developed TDPI and performed both kinds of quantum optical measurements, a measurement of internal parameters of quantum light and a more precise measurement of parameters of an external object, by TDPI. We expect that this study will motivate further research to practical realization of quantum optical technologies.

Finally, we remark the future prospects of this study. For TDPI-based OCT, future direction is to demonstrate cross-sectional imaging of biological samples like an onion piece, to manifest a possibility of more practical application of TDPI. With optimized mechanical system, more nonlinear and broadband materials, and more sensitive detectors, we will be able to achieve fast, highly-resolved, and high-power OCT imaging. Another future direction for this study is investigation of other classical optical measurement techniques inspired by quantum optical phenomena. For example, two-photon quantum optical phenomena such as ghost imaging [113] and automatic aberration cancellation [114] can be reproduced by the time-reversed

optical system in principle. Incorporating these new ideas and the findings in this thesis, more sensitive, or more functional optical measurement techniques are expected to be realized in classical optical systems.

Appendix A

Appendix for Chapter 3

A.1 Calculation of time-reversed two-photon interferometer for phase superresolution in the frequency domain

We show the calculation of the time-reversed two-photon interferometer for phase superresolution in the frequency domain. We consider the experimental setup shown in Fig. 3.1(b) in Chap. 3.

We describe the complex electric-field amplitude of the input light as $E(\omega) := \exp[-(\omega - \omega_0)^2/(2\Delta\omega^2)]$, where ω_0 and $\Delta\omega$ are the central frequency and the RMS width of the spectrum, respectively. The field amplitude after passing through the unbalanced Michelson interferometer is given by $E(\omega)(1 + e^{i\omega x/c})/2$, where x is the optical path difference between the two arms of the interferometer and c is the speed of light. The BBO crystal for SFG converts the field amplitude into the following convolution integral $E_{\text{SFG}}(\omega)$:

$$\begin{aligned} E_{\text{SFG}}(\omega) &\propto \int_{-\infty}^{\infty} d\omega' E(\omega') \left[1 + e^{i\omega' x/c}\right] E(\omega - \omega') \left[1 + e^{i(\omega - \omega') x/c}\right] \\ &\propto \exp \frac{-(\omega - 2\omega_0)^2}{4\Delta\omega^2} \cdot \left[1 + 2 \exp \frac{\Delta\omega^2 x^2}{2c^2} e^{i\omega x/(2c)} + e^{i\omega x/c}\right]. \end{aligned} \quad (\text{A.1})$$

The sum-frequency pulses pass through a bandpass filter centered around $2\omega_0$ followed by a detector. If the transmission spectrum of the bandpass filter is sufficiently

narrow, the measured intensity $I(x)$ after the bandpass filter is given by

$$I(x) = |E_{\text{SFG}}(2\omega_0)|^2 \propto \left| 1 + 2 \exp \frac{\Delta\omega^2 x^2}{2c^2} e^{i\omega_0 x/c} + e^{i2\omega_0 x/c} \right|^2. \quad (\text{A.2})$$

When $|x| \ll c/\Delta\omega$ ($\sim$ the coherence length of the input pulsed light), the second term in Eq. (A.2) much contributes the output intensity:

$$I(x) \propto \left| (1 + e^{i\omega_0 x/c})^2 \right|^2 \propto [1 + \cos(\omega_0 x/c)]^2, \quad (\text{A.3})$$

which shows the square of the white-light interference of the input laser pulses.

When $|x| \gg c/\Delta\omega$, the second term in Eq. (A.2) is negligibly small and the output intensity is calculated as

$$I(x) \propto |1 + e^{i2\omega_0 x/c}|^2 \propto 1 + \cos(2\omega_0 x/c), \quad (\text{A.4})$$

which is the same interference pattern as the two-photon phase superresolution. In this manner, we can observe the two-photon phase superresolution by using the time-reversed two-photon interferometer.

Appendix B

Appendix for Chapter 4

B.1 Brief summary of the geometric phase

This section provides a brief summary of the geometric phase for better understanding of the study in Chap. 4. In Sec. B.1.1, we describe the general theory of the geometric phase and show that the three-vertex geometric phase [Eq. (4.1) in Chap. 4] is regarded as a fundamental building block of an arbitrary geometric phase. In Sec. B.1.2, we provide an example of the geometric phase in a two-state system and show that the variation in the geometric phase in a two-state system can be interpreted geometrically on the Bloch sphere.

B.1.1 General theory of the geometric phase

Quantum mechanics postulates that a pure physical state is described by a unit vector in a complex Hilbert space $\mathcal{H}$. From this postulate, a unit vector $|\psi\rangle$ and the same vector except a global phase $e^{i\phi}|\psi\rangle$ represent an identical physical state. This redundancy of global phases in quantum states is called the gauge degree of freedom.

Relative phase between two different nonorthogonal states $|\psi_1\rangle$ and $|\psi_2\rangle$ is defined by $\arg\langle\psi_1|\psi_2\rangle$ [59]. The relative phase $\arg\langle\psi_1|\psi_2\rangle$ is varied according to the gauge. When the gauge is chosen such that $\arg\langle\psi_1|\psi_2\rangle = 0$, the relationship between the two states $|\psi_1\rangle$ and $|\psi_2\rangle$ are called *in phase* and expressed as $|\psi_1\rangle \frown |\psi_2\rangle$.

The in-phase relationship generally does not satisfy the transitive law; that is, even if $|\psi_1\rangle \frown |\psi_2\rangle$ and $|\psi_2\rangle \frown |\psi_3\rangle$, $|\psi_1\rangle$ and $|\psi_3\rangle$ are not always in phase. As a consequence, among the three states $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$, there exists a gauge-invariant phase factor:

$$\gamma(\psi_1, \psi_2, \psi_3) := \arg\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle. \quad (\text{B.1})$$

Such a gauge-invariant phase factor defined by three or more states is called the geometric phase. The geometric phase for three states was introduced by Pancharatnam [59], and we call it *the three-vertex geometric phase*. The geometric phase for more than three states is defined in the same manner as

$$\gamma(\psi_1, \dots, \psi_n) := \arg\langle\psi_1|\psi_n\rangle\langle\psi_n|\psi_{n-1}\rangle \cdots \langle\psi_2|\psi_1\rangle. \quad (\text{B.2})$$

In general, the geometric phase is defined for a curve $\mathcal{C}$ in the *state space (ray space, projective Hilbert space)* $\mathcal{P}$. The state space $\mathcal{P}$ is the set of normalized non-zero states in $\mathcal{H}$ where the states differing only by a global phase are identified; $\mathcal{P}$ is denoted by $\mathcal{P} = \mathcal{N} / \sim$, where $\mathcal{N} := \{|\psi\rangle \in \mathcal{H} \mid \langle\psi|\psi\rangle = 1\}$ and $\sim$ is an equivalence relation. The curve $\mathcal{C}$ is represented as a projection of a curve $\tilde{\mathcal{C}} := \{|\psi(s)\rangle \in \mathcal{N} \mid s_i \leq s \leq s_f\}$ onto $\mathcal{P}$. The geometric phase $\gamma(\mathcal{C})$ is defined as

$$\gamma(\mathcal{C}) := \arg\langle\psi(s_i)|\psi(s_f)\rangle - \text{Im} \int_{s_i}^{s_f} ds \langle\psi(s)|\dot{\psi}(s)\rangle. \quad (\text{B.3})$$

A geometrical interpretation is shown in Fig. B.1(a).

As shown below, it is proved that the definition of $\gamma(\mathcal{C})$ is equivalent to that of $\gamma(\psi_1, \dots, \psi_n)$ in the limit as $n \rightarrow \infty$.

Proposition. We define real parameters s_j ($j = 1, \dots, n$) as $s_j := s_i + \Delta s(j - 1)$ ($\Delta s := \frac{s_f - s_i}{n-1}$). For an ordered set $\{|\psi(s_1)\rangle, |\psi(s_2)\rangle, \dots, |\psi(s_n)\rangle\}$, which is a subset of the curve $\mathcal{C} := \{|\psi(s)\rangle \in \mathcal{P} \mid s_i \leq s \leq s_f\}$ [depicted in Fig. B.1(b)], the geometric phase $\gamma[\psi(s_1), \dots, \psi(s_n)]$ is equivalent to $\gamma(\mathcal{C})$ in the limit as $n \rightarrow \infty$.

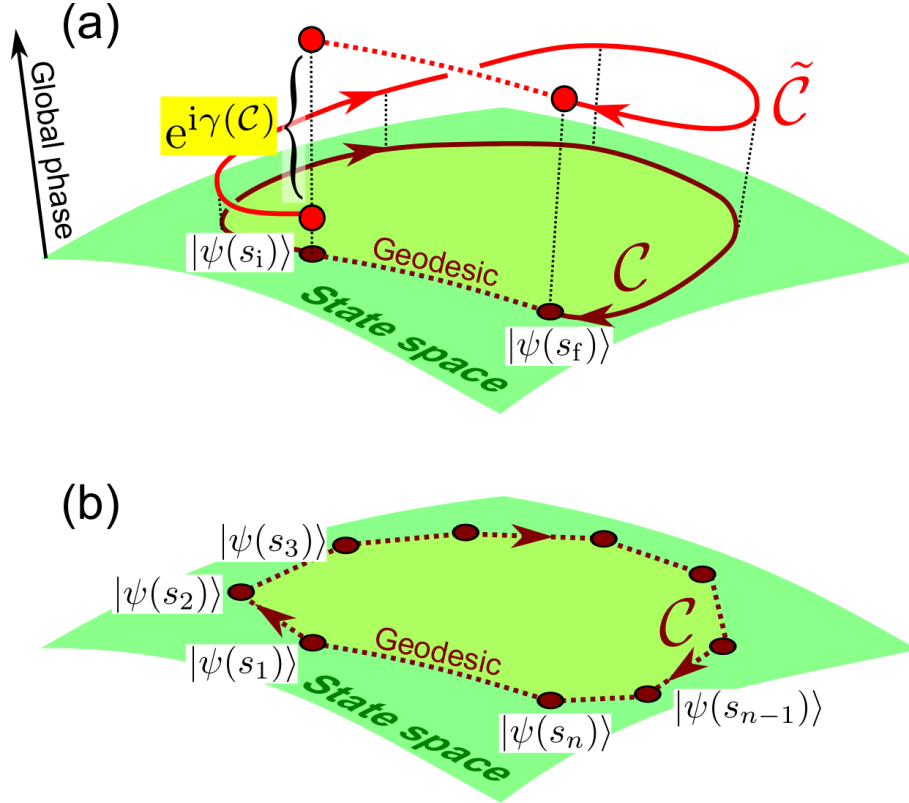


Figure B.1: (a) Geometrical interpretation of the curves $\tilde{\mathcal{C}}$ and $\mathcal{C}$ and the geometric phase $\gamma(\mathcal{C})$. The geodesic is defined as a set of linear combinations of two states at the both end. Whereas the first term $\arg\langle\psi(s_i)|\psi(s_f)\rangle$ and the second term $-\text{Im} \int_{s_i}^{s_f} ds \langle\psi(s)|\dot{\psi}(s)\rangle$ in Eq. (B.3) are gauge-dependent, the sum of them, $\gamma(\mathcal{C})$, is determined by only the curve $\mathcal{C}$ in the state space. (b) The discrete set $\{|\psi(s_j)\rangle\}_{j=1}^n$ on the curve $\mathcal{C}$. In the limit of $n \rightarrow \infty$, $\{|\psi(s_j)\rangle\}_{j=1}^n$ goes to the curve $\mathcal{C}$.

Proof. From the definition, $\gamma[\psi(s_1), \dots, \psi(s_n)]$ is calculated as

$$\begin{aligned}
\gamma[\psi(s_1), \dots, \psi(s_n)] &= \arg \langle \psi(s_1) | \psi(s_n) \rangle \langle \psi(s_n) | \psi(s_{n-1}) \rangle \cdots \langle \psi(s_2) | \psi(s_1) \rangle \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \prod_{j=1}^{n-1} \langle \psi(s_j) | \psi(s_{j+1}) \rangle \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \prod_{j=1}^{n-1} \langle \psi(s_j) | [\psi(s_j) + \Delta s \dot{\psi}(s_j) + O(\Delta s^2)] \rangle \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \prod_{j=1}^{n-1} [1 + \Delta s \langle \psi(s_j) | \dot{\psi}(s_j) \rangle + O(\Delta s^2)] \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \prod_{j=1}^{n-1} \left[\exp \left(\Delta s \langle \psi(s_j) | \dot{\psi}(s_j) \rangle \right) + O(\Delta s^2) \right] \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \left[\exp \left(\sum_{j=1}^{n-1} \Delta s \langle \psi(s_j) | \dot{\psi}(s_j) \rangle \right) + O(\Delta s^2) \right].
\end{aligned} \tag{B.4}$$

In the limit as $n \rightarrow \infty$, Δs goes to zero and the limit of $\gamma[\psi(s_1), \dots, \psi(s_n)]$ is calculated as

$$\begin{aligned}
\lim_{n \rightarrow \infty} \gamma[\psi(s_1), \dots, \psi(s_n)] &= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \arg \exp \left[\int_{s_i}^{s_f} ds \langle \psi(s) | \dot{\psi}(s) \rangle \right] \\
&= \arg \langle \psi(s_i) | \psi(s_f) \rangle - \text{Im} \int_{s_i}^{s_f} ds \langle \psi(s) | \dot{\psi}(s) \rangle \\
&= \gamma(\mathcal{C}),
\end{aligned} \tag{B.5}$$

where we used the fact that $\langle \psi(s) | \dot{\psi}(s) \rangle$ is purely imaginary because $\frac{d}{ds} \langle \psi(s) | \psi(s) \rangle = \langle \dot{\psi}(s) | \psi(s) \rangle + \langle \psi(s) | \dot{\psi}(s) \rangle = 0$. $\square$

Any geometric phases for a discrete set of quantum states $\gamma(\psi_1, \dots, \psi_n)$ can be

decomposed into a sum of the three-vertex geometric phases:

$$\begin{aligned}
\gamma(\psi_1, \dots, \psi_n) &= \arg \langle \psi_1 | \psi_n \rangle \langle \psi_n | \psi_{n-1} \rangle \cdots \langle \psi_2 | \psi_1 \rangle \\
&= \sum_{i=2}^{n-1} \arg \langle \psi_1 | \psi_{i+1} \rangle \langle \psi_{i+1} | \psi_i \rangle \langle \psi_i | \psi_1 \rangle \\
&= \sum_{i=2}^{n-1} \gamma(\psi_1, \psi_i, \psi_{i+1}).
\end{aligned} \tag{B.6}$$

Because the geometric phase for a continuous set of quantum states $\gamma(\mathcal{C})$ is equivalent to $\gamma(\psi_1, \dots, \psi_n)$ in the limit as $n \rightarrow \infty$ as seen above, $\gamma(\mathcal{C})$ can also be decomposed into a sum of the three-vertex geometric phases. Therefore, the three-vertex geometric phase is regarded as a fundamental building block of an arbitrary geometric phase.

B.1.2 Geometric phase in a two-state system

The three-vertex geometric phase has been widely studied in a two-state (qubit) system such as optical polarization. The three-vertex geometric phase in a two-state system has a geometrical representation on the Bloch (Poincaré) sphere. We define $\Omega(\psi_1, \psi_2, \psi_3)$ as a signed area of the spherical triangle on the Bloch sphere enclosed by the three states in a two-state system $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$. We assume that $\Omega(\psi_1, \psi_2, \psi_3)$ is positive when $|\psi_1\rangle \rightarrow |\psi_2\rangle \rightarrow |\psi_3\rangle$ is counterclockwise from the outside of the Bloch sphere. It is known that the three-vertex geometric phase $\gamma(\psi_1, \psi_2, \psi_3)$ and the area of the spherical triangle $\Omega(\psi_1, \psi_2, \psi_3)$ hold the following proportional relationship [59]:

$$\gamma(\psi_1, \psi_2, \psi_3) = \frac{1}{2} \Omega(\psi_1, \psi_2, \psi_3). \tag{B.7}$$

The Bloch sphere representation of the geometric phase is helpful to intuitively understand variations in the geometric phase. In the following, we show an example of the nonlinear variations in the geometric phase in a two-state system and its Bloch sphere representation.

We now consider the following three states in a two-state system:

$$|\psi_1\rangle := \cos \frac{\theta}{2} |H\rangle + i \sin \frac{\theta}{2} |V\rangle, \quad (\text{B.8})$$

$$|\psi_2\rangle := \cos \frac{\theta}{2} |H\rangle - i \sin \frac{\theta}{2} |V\rangle, \quad (\text{B.9})$$

$$|\psi_3(\phi)\rangle := \cos \frac{\phi}{2} |H\rangle + \sin \frac{\phi}{2} |V\rangle, \quad (\text{B.10})$$

where we employ polarization of a photon as a two-state system, and $|H\rangle$ and $|V\rangle$ denote the horizontal and vertical polarization states, respectively. These states $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3(\phi)\rangle$ are represented on the Bloch sphere as shown in Fig. B.2 (a). As the parameter ϕ increases, the point corresponding to $|\psi_3(\phi)\rangle$ rotates along the equator on the Bloch sphere. We can predict from the Bloch sphere representation that the variation of the area of the spherical triangle $\Omega[\psi_1, \psi_2, \psi_3(\phi)]$ exhibits a rapid increase by 4π at the angle $\phi = 180^\circ$, and this increase becomes more rapid as the angle θ decreases. From the proportional relationship of Eq. (B.7), we can predict that the geometric phase $\gamma[\psi_1, \psi_2, \psi_3(\phi)]$ also has the similar variation with respect to ϕ . The geometric phase $\gamma[\psi_1, \psi_2, \psi_3(\phi)]$ is in fact calculated as

$$\gamma[\psi_1, \psi_2, \psi_3(\phi)] = -2 \tan^{-1} \left(\tan \frac{\theta}{2} \cdot \tan \frac{\phi}{2} \right). \quad (\text{B.11})$$

The variation of $\gamma[\psi_1, \psi_2, \psi_3(\phi)]$ with respect to ϕ is shown in Fig. B.2 (b). The calculated variation indicates that, as predicted by the Bloch sphere representation, $\gamma[\psi_1, \psi_2, \psi_3(\phi)]$ rapidly increases by 2π at $\phi = 180^\circ$, and this increase becomes more rapid as the angle θ decreases.

B.2 Proofs of the propositions in Chapter 4

This section provides some proofs of the propositions given in Chap. 4. These proofs are based on our previous paper [33].

We first prove that a three-state system can be identified in terms of a symmetrized two-qubit system. In general, it is proved that an N -dimensional quantum system is isomorphic to a symmetrized $(N - 1)$ -qubit system.

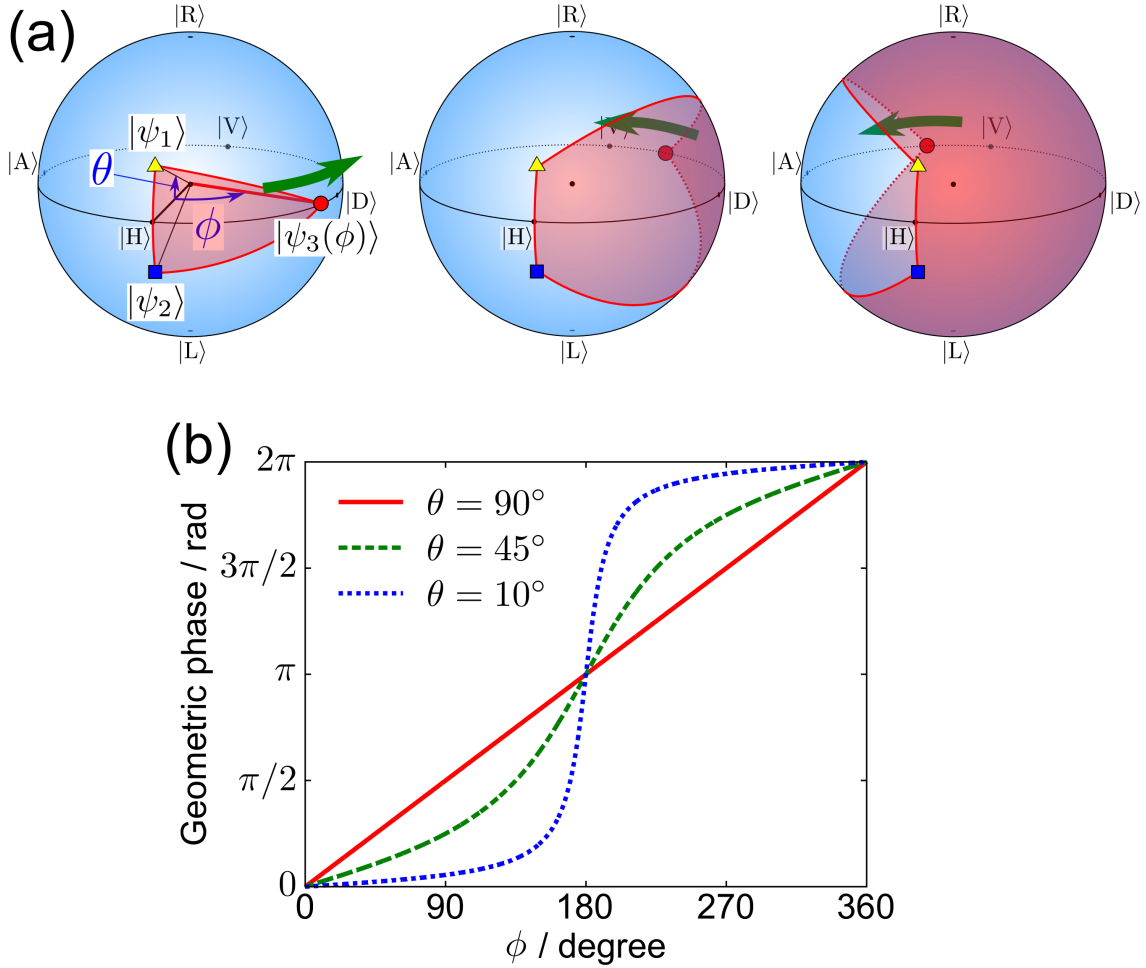


Figure B.2: (a) Bloch sphere representation of the states given by Eqs. (B.8)–(B.10). θ is the half-angle between the states $|\psi_1\rangle$ and $|\psi_2\rangle$. The red circle corresponding to $|\psi_3(\phi)\rangle$ rotates along the equator as the parameter ϕ increases. When the red circle passes through the reverse side of the Bloch sphere, the area of the spherical triangle increases rapidly. This increase becomes more rapid as the angle between the yellow triangle and the blue square 2θ decreases. (b) The variations in the geometric phase $\gamma[\psi_1, \psi_2, \psi_3(\phi)]$ with respect to ϕ , for several values of θ .

Proposition. An N -dimensional Hilbert space $\mathcal{H}_N$ is isomorphic to a symmetrized $(N - 1)$ -qubit Hilbert space:

$$\mathcal{H}_N \simeq \mathcal{S}(\mathcal{H}_2^{\otimes N-1}), \quad (\text{B.12})$$

where $\mathcal{S}$ denotes the projection operator onto the permutation-symmetric subspace.

Proof. We introduce the following $(N - 1)$ -qubit symmetric Dicke state $|S_{N-1,i-1}\rangle$ where $i - 1$ qubits are $|1\rangle$ and the others are $|0\rangle$:

$$|S_{N-1,i-1}\rangle := k \mathcal{S}(\underbrace{|0\rangle \cdots |0\rangle}_{N-i} \underbrace{|1\rangle \cdots |1\rangle}_{i-1}), \quad (\text{B.13})$$

where $\mathcal{S}(|0\rangle \cdots |0\rangle |1\rangle \cdots |1\rangle)$ is the sum of all the permutations of $|0\rangle$ and $|1\rangle$, and k is a normalization factor. $\{|S_{N-1,i-1}\rangle\}_{i=1}^N$ forms an orthonormal basis of $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$. We can therefore construct an isomorphism map from $\mathcal{H}_N$ to $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$ by identifying $|i\rangle$, the i th basis state of $\mathcal{H}_N$, with $|S_{N-1,i-1}\rangle$, the i th basis state of $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$. $\square$

For example, for $N = 3$, the three basis states $|1\rangle$, $|2\rangle$, and $|3\rangle$ in $\mathcal{H}_3$ are identified with the states $|S_{2,0}\rangle = |00\rangle$, $|S_{2,1}\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$, and $|S_{2,2}\rangle = |11\rangle$ in $\mathcal{S}(\mathcal{H}_2 \otimes \mathcal{H}_2)$.

In addition, we prove that a state in a symmetrized $(N - 1)$ -qubit Hilbert space $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$ is represented by a symmetrized $(N - 1)$ -qubit product states.

Proposition. An arbitrary state $|\Psi\rangle$ in $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$ is represented by a symmetrized $(N - 1)$ -qubit product states $k \mathcal{S}(|\psi_1\rangle \cdots |\psi_{N-1}\rangle)$, where $|\psi_i\rangle$ ($i = 1, \dots, N - 1$) is a qubit state.

Proof. We define $|S'_{N-1,i-1}\rangle$ as the following unnormalized $(N - 1)$ -qubit symmetric Dicke state:

$$|S'_{N-1,i-1}\rangle := \mathcal{S}(\underbrace{|0\rangle \cdots |0\rangle}_{N-i} \underbrace{|1\rangle \cdots |1\rangle}_{i-1}). \quad (\text{B.14})$$

A symmetrized $(N - 1)$ -qubit product state $|\Phi\rangle := \mathcal{S}(|\psi_1\rangle \cdots |\psi_{N-1}\rangle)$ is expanded as follows:

$$\begin{aligned} |\Phi\rangle &= k\mathcal{S}[(|0\rangle - \lambda_1|1\rangle) \cdots (|0\rangle - \lambda_{N-1}|1\rangle)] \\ &= k \left[|S'_{N-1,0}\rangle - (\lambda_1 + \cdots + \lambda_{N-1})|S'_{N-1,1}\rangle + \right. \\ &\quad \cdots + (-1)^{i-1} \sum_{1 \leq k_1 < \cdots < k_{i-1} \leq N-1} \lambda_{k_1} \cdots \lambda_{k_{i-1}} |S'_{N-1,i-1}\rangle + \\ &\quad \left. \cdots + (-1)^{N-1} \lambda_1 \cdots \lambda_{N-1} |S'_{N-1,N-1}\rangle \right]. \end{aligned} \quad (\text{B.15})$$

This equation indicates that the relation between $\lambda_1, \dots, \lambda_{N-1}$ and the coefficients of $|S_{N-1,1}\rangle, \dots, |S_{N-1,N-1}\rangle$ is same as the relation between the roots and the coefficients of $(N - 1)$ th-order polynomial. Therefore, if an arbitrary state in $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$

$$|\Psi\rangle = k(|S_{N-1,0}\rangle + a_1|S_{N-1,1}\rangle + \cdots + a_{N-1}|S_{N-1,N-1}\rangle) \quad (\text{B.16})$$

is given, we can represent $|\Psi\rangle$ as $|\Psi\rangle = k\mathcal{S}[(|0\rangle - \lambda_1|1\rangle) \cdots (|0\rangle - \lambda_{N-1}|1\rangle)]$, where $\lambda_1, \dots, \lambda_{N-1}$ are the roots of a $(N - 1)$ th-order polynomial $x^{N-1} + a_1x^{N-2} + \cdots + a_{N-2}x + a_{N-1}$. $\square$

In the above manner, an arbitrary state in $\mathcal{H}_N$ can be uniquely represented by a symmetrized $(N - 1)$ -qubit product states $k\mathcal{S}(|\psi_1\rangle \cdots |\psi_{N-1}\rangle)$ in $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$. For example, a state $|\Psi\rangle := a|1\rangle + b|2\rangle + c|3\rangle$ in $\mathcal{H}_3$ is identified with a state $a|00\rangle + b(|01\rangle + |10\rangle)/\sqrt{2} + c|11\rangle$ in $\mathcal{S}(\mathcal{H}_2 \otimes \mathcal{H}_2)$ and moreover represented by the following symmetrized two-qubit product state:

$$\begin{aligned} |\Psi\rangle &= a|00\rangle + b \frac{|01\rangle + |10\rangle}{\sqrt{2}} + c|11\rangle \\ &= \frac{a}{2}(|\psi_+\rangle|\psi_-\rangle + |\psi_-\rangle|\psi_+\rangle), \end{aligned} \quad (\text{B.17})$$

$$|\psi_{\pm}\rangle := |0\rangle + \left(\frac{b}{\sqrt{2}a} \pm \sqrt{\frac{b^2}{2a^2} - \frac{c}{a}} \right) |1\rangle. \quad (\text{B.18})$$

We next prove that any set of three states can be mapped onto a standard triplet shown in Eq. (4.3) by applying a proper unitary transformation. The standard

triplet in an N -dimensional quantum system is defined as the set of the three states $\{|\Psi_1\rangle, |\Psi_2\rangle, |\Psi_3\rangle\}$ where

$$\begin{aligned} |\Psi_1\rangle &= |\psi_1\rangle \cdots |\psi_1\rangle, \quad |\Psi_2\rangle = |\psi_2\rangle \cdots |\psi_2\rangle, \\ |\Psi_3\rangle &= k \mathcal{S} \left(|\psi_3^{(1)}\rangle \cdots |\psi_3^{(N-1)}\rangle \right). \end{aligned} \quad (\text{B.19})$$

Proposition. For an arbitrary set of three states $\{|\Phi_1\rangle, |\Phi_2\rangle, |\Phi_3\rangle\}$, there exists a unitary operation $\hat{U}$ such that $\{\hat{U}|\Phi_1\rangle, \hat{U}|\Phi_2\rangle, \hat{U}|\Phi_3\rangle\}$ is a standard triplet.

Proof. The inner product of the two state $|\Psi_1\rangle$ and $|\Psi_2\rangle$ in a standard triplet, $\langle\Psi_1|\Psi_2\rangle = \langle\psi_1|\psi_2\rangle^{N-1}$, can be an arbitrary complex number the absolute value of which is one or less. We now choose $|\Psi_1\rangle$ and $|\Psi_2\rangle$ such that the relation $\langle\Psi_1|\Psi_2\rangle = \langle\Phi_1|\Phi_2\rangle$ holds.

We then define the following unitary operator $\hat{U}_1$:

$$\hat{U}_1 := |\Psi_1\rangle\langle\Phi_1| + |\Psi_1^\perp\rangle\langle\Phi_1^\perp| + \sum_{|i\rangle \in \mathcal{H}'} |i\rangle\langle i|, \quad (\text{B.20})$$

where $|\Psi_1^\perp\rangle$ and $|\Phi_1^\perp\rangle$ are respectively the states perpendicular to $|\Psi_1\rangle$ and $|\Phi_1\rangle$ in $\text{span}(|\Psi_1\rangle, |\Phi_1\rangle)$, and $\mathcal{H}'$ is the orthocomplement of $\text{span}(|\Psi_1\rangle, |\Phi_1\rangle)$. $\hat{U}_1$ transforms $|\Phi_1\rangle$ and $|\Phi_2\rangle$ into $\hat{U}_1|\Phi_1\rangle = |\Psi_1\rangle$ and $\hat{U}_1|\Phi_2\rangle =: |\Phi'_2\rangle$, respectively, as shown in Fig. B.3(a).

We next define $\mathcal{H}''$ as the subspace in $\text{Span}(|\Psi_1\rangle, |\Psi_2\rangle, |\Phi'_2\rangle)$ perpendicular to $|\Psi_1\rangle$ and $\hat{P}$ as the projection operator onto $\mathcal{H}''$. $\hat{P}$ projects $|\Psi_2\rangle$ and $|\Phi'_2\rangle$ onto $\hat{P}|\Psi_2\rangle =: |\tilde{\Xi}\rangle$ and $\hat{P}|\Phi'_2\rangle =: |\tilde{\Pi}\rangle \in \mathcal{H}''$, respectively. We also define $|\Xi\rangle$ and $|\Pi\rangle$ as the normalized states of $|\tilde{\Xi}\rangle$ and $|\tilde{\Pi}\rangle$, and $|\Xi^\perp\rangle$ and $|\Pi^\perp\rangle$ as the states perpendicular to $|\Xi\rangle$ and $|\Pi\rangle$ in $\mathcal{H}''$, respectively. Then we introduce the following unitary operator $\hat{U}_2$:

$$\hat{U}_2 := |\Xi\rangle\langle\Pi| + |\Xi^\perp\rangle\langle\Pi^\perp| + \sum_{|i\rangle \in \mathcal{H}'''} |i\rangle\langle i|, \quad (\text{B.21})$$

where $\mathcal{H}'''$ is the orthocomplement of $\mathcal{H}''$. $\hat{U}_2$ transforms $|\Phi'_2\rangle$ into $\hat{U}_2|\Phi'_2\rangle = |\Psi_2\rangle$ while $|\Psi_1\rangle$ is unchanged, as shown in Fig. B.3(b).

Finally we define $|\Psi_3\rangle := \hat{U}_2\hat{U}_1|\Phi_3\rangle$ since $|\Psi_3\rangle$ can be an arbitrary state. In the above manner, the three states $|\Phi_1\rangle$, $|\Phi_2\rangle$, and $|\Phi_3\rangle$ can be transformed into

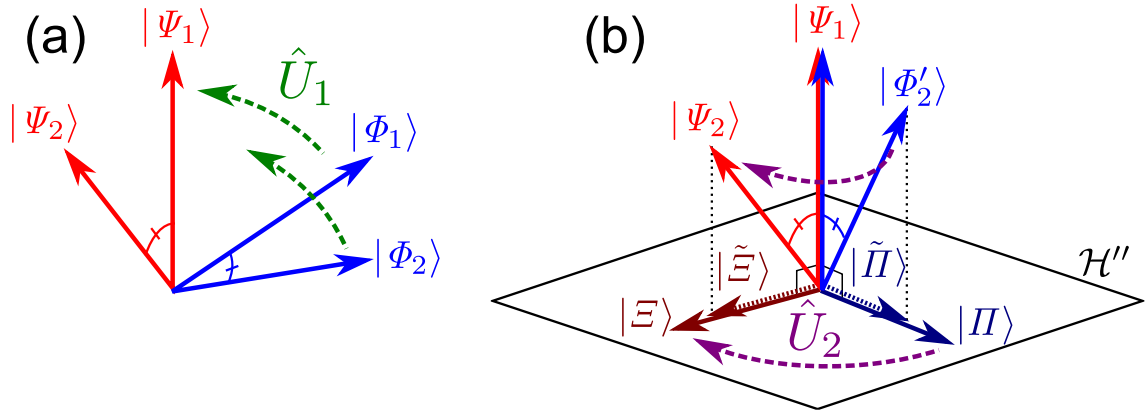


Figure B.3: Conceptual diagrams of the vector space used in the proof. The angle between the two arrows in the diagram corresponds to the inner product of the two vectors. (a) The rotating operation $\hat{U}_1$ transforms $|\Phi_1\rangle$ into $|\Psi_1\rangle$. (b) The rotating operation $\hat{U}_2$ transforms $|\Phi'_2\rangle$ into $|\Psi_2\rangle$ while $|\Psi_1\rangle$ is unchanged.

the three states $|\Psi_1\rangle$, $|\Psi_2\rangle$, and $|\Psi_3\rangle$ in a standard triplet by the unitary operator $\hat{U} := \hat{U}_2\hat{U}_1$. $\square$

Appendix C

Appendix for Chapter 6

C.1 Chirped-pulse interferometry for dispersion-cancelled optical coherence tomography

This section provides a brief review of chirped-pulse interferometry (CPI) for dispersion-cancelled optical coherence tomography (OCT). There have been proposed two types of CPI: a method using a pair of oppositely chirped laser pulses [25,27] and that using a single beam of shaped laser pulses [29]. In Secs. C.1.1 and C.1.2, we introduce the former and latter methods, respectively.

C.1.1 CPI using a pair of oppositely chirped laser pulses

In this section, we introduce CPI using a pair of oppositely chirped laser pulses [25, 27]. The schematic of this type of CPI is shown in Fig. C.1. The input light is a chirped laser pulse in the upper mode and an anti-chirped laser pulse in the lower mode. The chirped laser pulse is prepared by a stretcher, which is constructed by two lenses and two gratings and introduces positive group-delay dispersion. The anti-chirped laser pulse is, on the other hand, prepared by a compressor, which is constructed by two gratings and introduces negative group-delay dispersion. In the frequency domain, the complex electric-field amplitude of the chirped and anti-chirped laser pulses, $E_+(\omega)$ and $E_-(\omega)$, are respectively described

as

$$E_{\pm}(\omega) := \exp \frac{-(\omega - \omega_0)^2}{2\Delta\omega^2} e^{\pm iA(\omega - \omega_0)^2}, \quad (\text{C.1})$$

where ω_0 , $\Delta\omega$, and $\pm A$ are the central frequency, the RMS width of the spectrum, and the strength and sign of the chirp, respectively. The input light enters the cross-correlator. In the upper arm, the delay line introduces a relative phase shift of $e^{i\omega x/c}$. In the lower arm, the light propagates through a measured sample, which is dispersive and has multiple interfaces. For simplicity, we here focus attention on one of the interface, and assume that the reflectivity is unity and the spectral phase has only the second-order dispersion. Then the linear transfer function in the lower arm is described as $e^{i\epsilon(\omega - \omega_0)^2}$. The spectrum of mixed light through SFG, $E_{\text{SFG}}(x, \omega)$, is given by the following convolution integral:

$$E_{\text{SFG}}(x, \omega) = \int_{-\infty}^{\infty} d\omega' [E_+(\omega')E_-(\omega - \omega') - E_-(\omega')E_+(\omega - \omega')] e^{i\epsilon(\omega' - \omega_0)^2} e^{i(\omega - \omega')x/c}. \quad (\text{C.2})$$

The intensity spectrum $|E_{\text{SFG}}(x, \omega)|^2$ is calculated as

$$\begin{aligned} |E_{\text{SFG}}(x, \omega)|^2 \propto & \exp \left[\frac{(C^2 - D_+^2)/\Delta\omega^2 - 2\epsilon C D_+}{2(1/\Delta\omega^4 + \epsilon^2)} + 2E \right] \\ & + \exp \left[\frac{(C^2 - D_-^2)/\Delta\omega^2 - 2\epsilon C D_-}{2(1/\Delta\omega^4 + \epsilon^2)} + 2E \right] \\ & - \exp \left[\frac{(2C^2 - D_+^2 - D_-^2)/\Delta\omega^2 - 2\epsilon C(D_+ + D_-)}{4(1/\Delta\omega^4 + \epsilon^2)} + 2E \right] \\ & \times 2 \cos \left[\frac{\epsilon(-D_+^2 + D_-^2) + 2C(D_+ + D_-)/\Delta\omega^2}{4(1/\Delta\omega^4 + \epsilon^2)} + F_+ - F_- \right], \end{aligned} \quad (\text{C.3})$$

where

$$C = \omega/\Delta\omega^2, \quad (\text{C.4})$$

$$D_{\pm} = 2\epsilon(\omega_0 - \omega) + x/c \pm A(4\omega_0 - 2\omega), \quad (\text{C.5})$$

$$E = (-\omega^2 + 2\omega\omega_0 - 2\omega_0^2)/(2\Delta\omega^2), \quad (\text{C.6})$$

$$F_{\pm} = \epsilon(\omega - \omega_0)^2 \pm A\omega(\omega - 2\omega_0). \quad (\text{C.7})$$

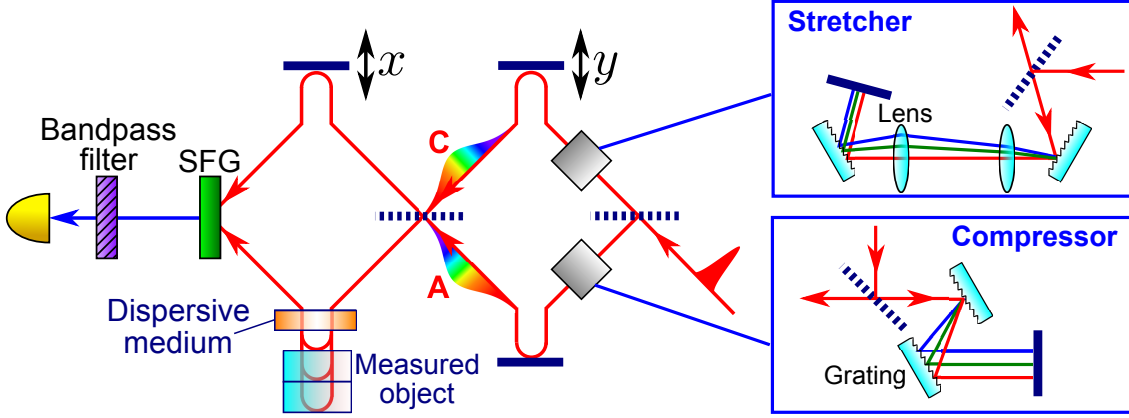


Figure C.1: Schematic of CPI using a pair of oppositely chirped laser pulses. The two insets show schematics of the stretcher and compressor, which prepare chirped and anti-chirped input laser pulses, respectively.

In the above calculation, we used the following integral formula:

$$\begin{aligned}
 & \int_{-\infty}^{\infty} d\omega \exp [(-A + iB)\omega^2 + (C + iD)\omega + (E + iF)] \\
 &= f(A, B) \exp \left[\frac{A(C^2 - D^2) - 2BCD}{4(A^2 + B^2)} + E \right] \exp \left[i \frac{B(C^2 - D^2) + 2ACD}{4(A^2 + B^2)} + iF \right] \\
 &= f(A, B) \exp \left[\frac{(A + iB)(C + iD)^2}{4(A^2 + B^2)} + (E + iF) \right], \tag{C.8}
 \end{aligned}$$

where the parameters $A, B, \dots, F$ are real numbers and $f(A, B)$ is a complex function of A and B .

Figure C.2 shows theoretical calculations of the intensity distributions $|E_{\text{SFG}}(x, \omega)|^2$ without and with dispersion. In the calculation, we assume the following parameter conditions: $A = 2500 \text{ fs}^2$, $\Delta\omega = 104 \text{ THz}$, $\omega_0 = 2.3 \text{ PHz}$, and $\epsilon = 264 \text{ fs}^2$. These intensity patterns are similar to those of the time-decomposed pulse interferometry (TDPI) except that the vertical axis is not the displacement y but the frequency ω of the sum-frequency light. Integral of $|E_{\text{SFG}}(x, \omega)|^2$ over all frequency ω , which is performed by detecting the optical power in wide range of spectrum, reproduces the dispersion-cancelled HOM dip. The bandpass filter before the photodetector in Fig. C.1 is used to eliminate auto-correlation terms of the input chirped and anti-chirped laser pulses. So that most of the spectrum of

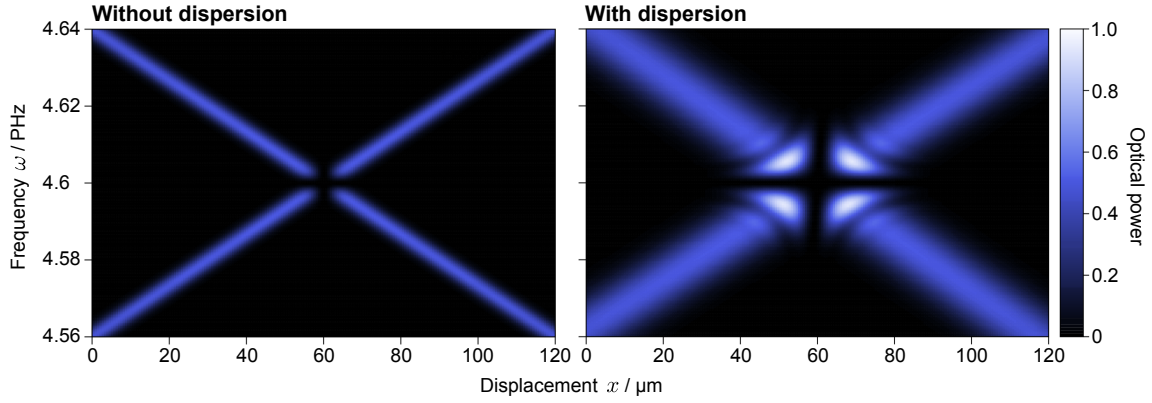


Figure C.2: Theoretical calculations of the intensity distribution $|E_{\text{SFG}}(x, \omega)|^2$ without and with a dispersive medium. These patterns are similar to those of TDPI except that the vertical axis is not the displacement y but the frequency ω of the sum-frequency light. By vertically integrating these distributions over all ω , the dispersion-cancelled HOM dips are derived.

the sum-frequency light leading to the HOM dip passes through the bandpass filter, the strength of the chirp must be sufficiently strong.

The subtraction method, which we proposed in Chap. 6 to achieve background-free imaging, can be applied to this type of CPI. In that case, we label the input chirped and anti-chirped pulses with the polarization degree of freedom, and insert two QWPs into the reference arm to switch the peak and dip conditions, in a similar manner to the case of TDPI.

C.1.2 CPI using a single beam of shaped laser pulses

We next introduce CPI using a single beam of shaped laser pulses [29]. The schematic of this type of CPI is shown in Fig. C.3. The input light is a single beam of laser pulses to which a frequency-dependent phase shift $\phi(\omega) := -A(\omega - \omega_0)|\omega - \omega_0|$ is applied by a 4-F pulse shaper with a spatial light modulator (SLM). This phase shift $\phi(\omega)$ introduces positive dispersion to the lower frequencies ($\omega < \omega_0$) and equal, negative dispersion to the higher frequencies ($\omega > \omega_0$). This shaped laser

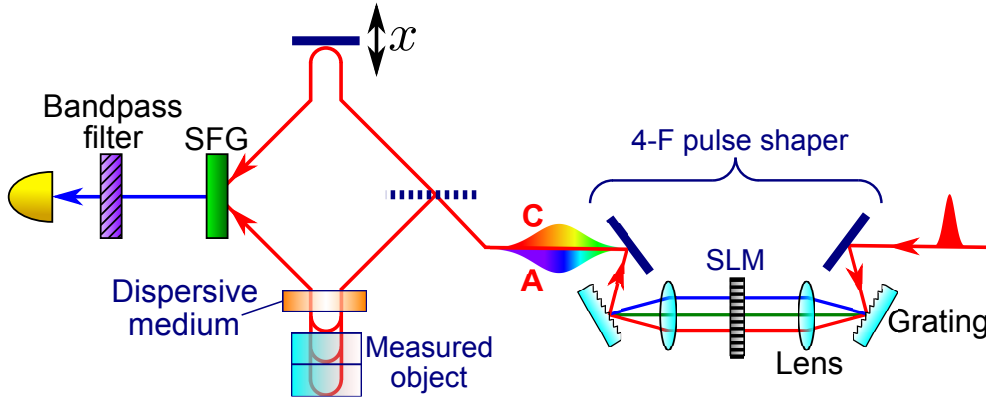


Figure C.3: Schematic of CPI using a single beam of shaped (BARC) laser pulses. The 4-F pulse shaper, composed of an SLM, two gratings and two lenses, creates chirped and anti-chirped laser pulses simultaneously.

pulse is referred as a blue-antichirped-red-chirped (BARC) pulse and described as

$$E_{\text{BARC}}(\omega) := \exp \frac{-(\omega - \omega_0)^2}{2\Delta\omega^2} e^{-iA(\omega - \omega_0)|\omega - \omega_0|}. \quad (\text{C.9})$$

This technique preparing the shaped laser pulses is easier to create complex pulse shapes, more stable and efficient than the technique mentioned in the previous section.

Intensity distribution of this system after SFG exhibits a similar pattern to Fig. C.2 except that the destructive interference at the intersection of the two bright lines is replaced with constructive interference. Integral of this distribution over all frequency ω reproduces the dispersion-cancelled HOM peak. In the experiment in Ref. [29], the spectral intensity was integrated over a narrow range of spectrum around $2\omega_0$ by a narrow bandpass filter, and artifact-free, dispersion-cancelled OCT imaging of an onion piece was achieved. In this method, however, there must remain background noise around the main signals, as noted in Chap. 6. In addition, the subtraction method cannot be applied to this type of OCT because both the chirped and anti-chirped laser pulses enter the same input port.

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- S. Tamate, K. Ogawa, and M. Kitano, “Bloch-sphere representation of three-vertex geometric phases,” *Physical Review A* **84**, 052114-1–5 (2011).
- K. Ogawa, S. Tamate, H. Kobayashi, T. Nakanishi, and M. Kitano, “Time-reversed two-photon interferometry for phase superresolution,” *Physical Review A* **88**, 063813-1–5 (2013).
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(Talk, refereed)

- K. Ogawa, S. Tamate, T. Nakanishi, H. Kobayashi, and M. Kitano, “Classical Realization of Dispersion Cancellation by Using Transform-limited Pulses,” CLEO:2014, FW1A.3, the San Jose Convention Center, USA, June 2014.

(Poster, refereed)

- K. Ogawa, S. Tamate, H. Kobayashi, T. Nakanishi, and M. Kitano, “Observation of Geometric Phases in Three-State Systems for Two-Photon Polarization,” QCMC2014, The Institute of Advanced Technology of University of Science and Technology of China (USTC), China, November 2014.

(Poster, non-refereed)

- S. Tamate, K. Ogawa, and M. Kitano, “Bloch-sphere representation of three-vertex geometric phases,” Global COE 5th International Symposium on Photonics and Electronics Science and Engineering, P-35, Kyoto University, Japan, March 2012.
- K. Ogawa, S. Tamate, H. Kobayashi, and M. Kitano, “Two-Photon Phase Super-Resolution with High Visibility Using Time-Reversed Method,” The International Workshop on New Science and Technologies using Entangled Photons (NSTEP), P21, Osaka University, Japan, July 2013.

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(Talk, non-refereed)

- K. Ogawa, S. Tamate, H. Kobayashi, and M. Kitano, “Majorana Representation of Geometric Phases,” 22pEB-2, Fall Meeting of the Physical Society of Japan, Toyama University, September 2011.
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- K. Ogawa, S. Tamate, T. Nakanishi, H. Kobayashi, and M. Kitano, “Time-reversed dispersion cancellation by using transform-limited pulses,” 28aCC-12, Annual Meeting of the Physical Society of Japan, Tokai University, March 2014.

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- K. Ogawa, “Majorana Representation of Three-Vertex Geometric Phases,” FIRST-QIPP/DYCE Summer School 2011, Kyoto University, August 2011.
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