

Observational and theoretical studies on dwarf-nova outbursts

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Abstract

Dwarf novae are close binary systems composed of a white dwarf (the primary) and a low-mass star (the secondary). The mass-losing secondary forms an accretion disk around the primary. This kind of systems exhibit transient events called outbursts due to the sudden brightening of the accretion disk, and are the best targets for studying physical processes in accretion disks because of short timescales of outbursts and a huge number of samples. Dwarf-nova outbursts are basically explained by the disk instability model in which the disk alternates between the high-viscosity hot state and the low-viscosity cool state with a constant mass transfer rate from the secondary due to partial ionization of hydrogen. This framework seems to have been established until 2000s. However, more and more light variations deviated from the classical picture of dwarf-nova outbursts are discovered these days thanks to the rapid development of optical transient surveys. The aim of this study is to investigate whether the disk instability model can explain a rich variety in dwarf-nova outbursts. This dissertation explored totally four different kinds of unexpected light variations in dwarf novae, and tries to test the disk instability model via optical observations and numerical simulations.

Chapter 2 presents the unexpected superoutburst and rebrightening in 2015 in AL Com, one of WZ Sge-type dwarf novae. The rebrightening type was the same as that in its previous superoutburst, in spite of the significant difference in the main superoutburst. This suggests that rebrightening is inherent to each system, although the physical origin of rebrightenings in WZ Sge stars is still puzzling.

Chapter 3 reports on superoutbursts in three WZ Sge stars. All of these superoutbursts showed a luminosity dip in the plateau stage. The timing analyses of superhumps proved that these systems have extremely low binary mass ratios, which suggests that the tidal instability slowly develops in these systems. The delay of the growth of eccentricity in the disk would stop the efficient removal of the angular momentum at the disk rim, and would make the disk drops off the hot state. This phenomenon can be the source of the luminosity dip in the middle of superoutbursts in WZ Sge stars.

Chapter 4 deals with low-amplitude and infrequent outbursts in dwarf novae. The photometric and spectroscopic observational data showed that the systems entering this kind of outbursts are classified into the following two types: high-inclination systems with low mass-transfer rates and long-period objects with massive white dwarfs and high-temperature secondaries. The outburst properties are consistent with the disk-instability model if the nature of these two types of dwarf novae are considered. The latter type is hard to be explained by the standard scenario of binary evolution, and might evolve towards type Ia supernovae.

Chapter 5 is dedicated to numerical simulations about thermal instability in tilted accretion disks. This study is motivated by anomalous light variations in IW And-type dwarf novae. When the disk is tilted, the gas stream from the secondary penetrates to the inner part of the disk. Some of the numerical simulations in which this effect is taken into account reproduced repetition of a mid-brightness interval with oscillatory variations and brightening, which is a reminiscent of the IW And-type phenomenon. The difference in mass supply patterns can generate a wide variety in the reproduced light curves as well without fluctuations in mass transfer rates. The disk instability in tilted disks could be a plausible model of the IW And-type phenomenon.

These studies provide physical interpretations of some of new types of light variations in dwarf novae by taking into account new aspects in the disk instability model. Here, the new aspects indicate extremely slow development of the tidal instability, very high inclination angles, very long-period objects with high-temperature secondaries, and various mass input patterns by the disk tilt, for examples. These light variations seemed hard to be understood by using the disk instability at first, but I figured out that no instabilities outside of the disk are required to explain them. I conclude that the disk instability is potentially responsible for all kinds of dwarf-nova outbursts throughout the studies in this dissertation. Moreover, the research on dwarf-nova outbursts sometimes give us helpful information about the binary evolution. The studies in chapter 3 and 4 obtained new knowledge about the final stage of the binary evolution and the evolutionary path of dwarf novae with F-type secondaries as by-products. In the future, it would be important to understand the essence of the disk instability in order to explain dwarf-nova outbursts systematically. Analyzing big time-series data obtained in the era of time-domain astronomy and performing numerical simulations by using the code that I built will enable to solve the difficulties that many people faced before, and to investigate both observationally and theoretically the accretion physics generating unexpected light variations.

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Declaration

The works in this thesis are based on our research carried out at Department of Astronomy, Kyoto University, Japan. No part of this thesis has been submitted elsewhere for any other degree or qualification, and it is the sole work of the author unless referenced to the contrary in the text.

The works presented in this thesis have been published or accepted for publication in journals - the relevant publications are listed below.

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General introduction

1.1 Accretion disks in our universe

Accretion disks play an important role in many kinds of astronomical objects. This section is devoted to their basic physics and associated phenomena in our universe before going in depth of the particular subject of this dissertation: dwarf novae.

1.1.1 Gravitational power house

First of all, the gravity governs our universe, while the nuclear power is the most energetic on the ground. Astronomers noticed this in 1960's by the discovery of quasars, and began seeking the source transforming the gravitational energy of gas into huge radiation and kinematic energy. Finally, they found that accretion disks are the central engine of this kind of energy conversion (see Chap. 1 of Kato et al., 2008, for details).

Accretion disks are rotating gaseous flows with accretion, and formed around gravitational objects like main-sequence stars (MSs), white dwarfs (WDs), neutron stars (NSs), and black holes (BHs). If the gas rotates around the central object eternally, it never accretes. Accretion takes place due to friction/viscosity. It works between adjacent concentric layers in a disk, and produces electromagnetic radiation. Simultaneously, the outer layer gains the angular momentum of the inner layer, and the inner layer falls towards the central object. Thus the friction/viscosity realizes inward accretion by transferring outwards the angular momentum of gas.

The disk luminosity is estimated as follows. If a particle has no velocity at infinity, and falls to the radius r , and rotates with Keplerian velocity, the excess of the total of the kinetic energy (K) and the potential energy (U) is radiated. The kinetic

energy and the potential energy are described as

$$K = \frac{1}{2}v^2 = \frac{GM}{2r}, \quad (1.1)$$

$$U = -\frac{GM}{r} = -2K, \quad (1.2)$$

where v is the Keplerian velocity, G is the gravitational constant, and M is the mass of the central source, respectively. The energy excess is

$$0 - (U + K) = K = \frac{GM}{2r}. \quad (1.3)$$

The disk luminosity is given by

$$L_{\text{disk}} = \frac{GM\dot{M}}{2r_{\text{in}}}, \quad (1.4)$$

where $\dot{M}$ is the mass accretion rate. This is half of the gravitational energy of the gas at r_{in} . When the black hole has an accretion disk, the released gravitational energy is $\sim 0.1Mc^2$, where c is the light speed, since r_{in} can be close to $6GM/c^2$, which is defined as the innermost stable circular orbit (ISCO) for a test particle. Compared with the released energy by nuclear fusion ($0.007Mc^2$), the accretion disk releases enormous energy and becomes super luminous.

1.1.2 Astronomical objects with accretion disks

There are mainly four kinds of systems containing accretion disks in our universe: young stellar objects (YSOs), cataclysmic variables (CVs), X-ray binaries (XBs), and active galactic nuclei (AGNs). They are more or less powered by accretion and release a lot of gravitational energy as radiation and/or outflows.

Every time when stars are born, a gaseous disk surrounds a new star and planets are created in it. The disk is named a protoplanetary disk. YSOs are classified into 5 categories by optical and infrared (IR) spectral energy distributions (SEDs), and protoplanetary disks exist in T-Tauri stars. The disk size is typically ~ 100 AU. Some of them show eruptions lasting decades and called FU Ori-type stars (Hartmann, Kenyon, 1996). Since the gravitational potential of MSs is weaker than those of WDs, NSs, and BHs, the temperature of the disk is basically low in comparison with the disk in other kinds of objects. Only the inner portion of the disk is the gaseous disk and it emits optical and NIR photons, while the central MS radiates UV photons (see the upper left panel of Figure 1.1).

CVs are close binary systems composed of a WD (the primary) and a MS (the secondary). Here the more massive star is defined as the primary and the less

massive star is defined as the secondary. In CVs, the secondary star is typically a low-mass star (a K or M-type star), and hence, the primary almost always indicates the WD. Other names of the secondary star are, for example, the donor star and the companion star. The secondary star fills its Roche-lobe, and then, an accretion disk is formed around the primary WD by the Roche-lobe overflow. CVs have many subclasses like novae, dwarf novae, polars, and so on (Warner, 1995; Hellier, 2001a). They suddenly brighten at optical wavelengths, but the origin of brightening differs between those subclasses. For example, the eruptions in novae are thought to be triggered by sudden nuclear fusion on the surface of the WD. They suddenly brightens by ~ 10 magnitudes at optical wavelengths and slowly decays during several tens of days. In contrast, dwarf novae show eruptions called outbursts because of the sudden brightening of their accretion disks. The detailed explanations are described afterwards. If the accretion rate onto the WD is high enough to cause steady nuclear burning, the surface of the WD constantly radiates soft X-rays. This type of objects are called supersoft X-ray sources (Kahabka, van den Heuvel, 1997). The disk in CVs is smallest among YSOs, CVs, XBs, and AGNs. Its size is typically less than 10^{11} cm. The inner disk emits UV photons, but the major part of the disk is observable by optical telescopes (see the upper right panel of Figure 1.1).

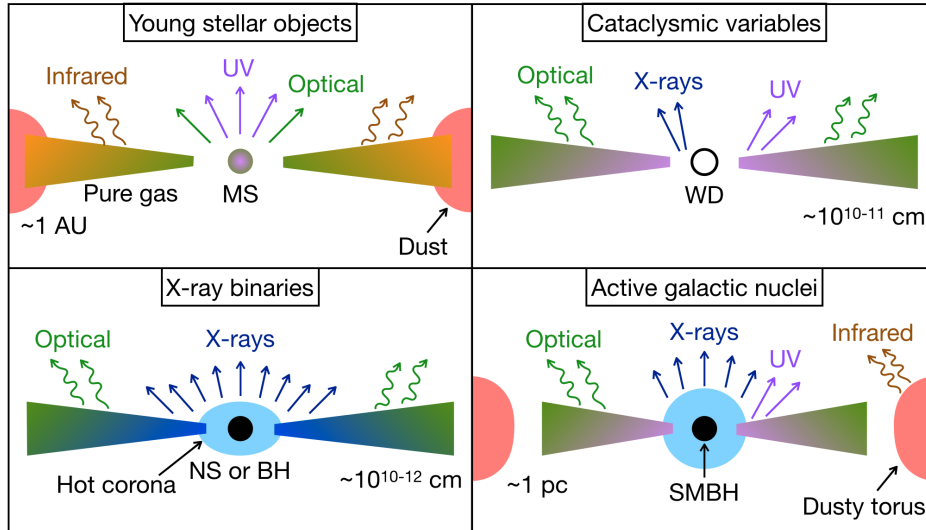


Figure 1.1: Schematic pictures of accretion disks in young stellar objects, cataclysmic variables, X-ray binaries, and active galactic nuclei.

XBs are close binary systems as well as CVs, but the primary star is not a WD but a NS/BH. The central object harbors an accretion disk as in CVs. The innermost region of the disk is much hotter than that in CVs, and XBs are bright at X-ray

wavelengths (see the lower left panel of Figure 1.1). XBs are roughly classified into two types: high-mass XBs (HMXBs) harboring a high-mass donor star and low-mass XBs (LMXBs) containing a low-mass donor star. In the former object, the mass transfer from the donor star takes place by stellar winds. A part of massive stellar winds (typically less than 0.1%) is gravitationally captured by the central compact object. In the latter object, the mass transfer occurs by the Roche-lobe overflow as in CVs. Phenomenologically, XBs are categorized into many subclasses. For example, X-ray bursters exhibit sudden brightening and/or X-ray bursts. The bursts are classified into Type I bursts originating from thermonuclear flares and Type II bursts caused by accretion instability (Galloway, Keek, 2017; Bagnoli et al., 2015). Type I X-ray bursts are similar to novae in CVs in terms of their origin. X-ray transients, most of LMXBs containing black holes as the primary, show outbursts by the brightening of their accretion disks, so they resemble dwarf novae in CVs. Also, ultraluminous X-ray sources (ULXs), new type of XBs, draw attention these days. In these systems, the disk luminosity exceeds the Eddington luminosity, although some people think that they have intermediate-mass black hole and that the phenomena can be explained by sub-Eddington accretion (Kaaret et al., 2017). Although the mass transfer in HMXBs is basically understood by wind accretion, the Roche-lobe overflow would be needed to realize high mass transfer rates in ULXs.

AGNs contain supermassive black holes (SMBHs) fed by accretion disks, and the extremely luminous nuclei in galaxies. They are classified into many subclasses by their radio properties and optical spectroscopy, and so on. The unified model of AGNs interprets all of the subclasses have the same structure like SMBHs surrounded by accretion disks with obscuring molecular torus and sporadic jet ejections but look different because of the orientation angle. AGNs are observable at X-ray wavelengths but its X-ray emission does not come from the inner accretion disk (see the lower right panel of Figure 1.1). The innermost region of the disk emits UV photons as in CVs, because the temperature of the innermost region in the disk decreases as $\sim M^{-1/4}$. The Schwarzschild radius is proportional to the mass of the central object, and the accretion rate scales linearly with mass. By substituting these in equation (1.33), the temperature becomes proportional to $\sim M^{-1/4}$.

1.2 Dwarf novae for the study of accretion physics

As explained in the preceding section, accretion disks are universal and produce the most active phenomena in our universe. Interestingly, many kinds of accretion systems share similar observational features like eruption events called outbursts.

In order to understand systematically the transient events, it is a nearer way to seek the common accretion physics in many kinds of accretion systems.

Dwarf novae (DNe), one class of non-magnetic CVs, exhibit sporadic outbursts, during which the accretion disk brightens mainly at optical wavelengths as mentioned above. DNe are well-investigated objects, and their dramatic light variations have attracted not only professional scientists but also amateur astronomers. Actually, amateur astronomers contribute a lot in the study of DNe. The CVs whose binary parameters have been investigated for long time, and are summarized in the Ritter & Kolb catalog*. Currently more than 700 objects whose orbital periods were measured are known†. The development of time-domain astronomy thus furnishes us plenty of opportunities for observing outbursts in DNe.

As mentioned in Sec. 1.1.2, DNe have the smallest accretion disks amongst the objects showing outbursts. Namely, the timescale of dwarf-nova outbursts is the shortest and it is not time-consuming to observe their overall light variations in comparison with other kinds of accretion systems. For instance, the accretion disk in YSOs and AGNs is quite large and it is difficult to observe all of their years-timescale physical phenomena. Moreover, the accretion disk in DNe is not affected very much by gravitational and magnetic fields of the central compact object. Some of accretion disks in XBs are as small as those in DNe, but the number of their observable samples is much smaller than that in DNe. Also, their inner accretion disks are influenced by strong gravitational and magnetic fields. Therefore DNe are the best target for studying pure accretion physics.

1.3 Basic knowledge on cataclysmic variables

1.3.1 Structure

This subsection introduces the composition and the geometry of a CV. CVs are the members of close binary systems. The most important concept about the geometry of close binary systems is the Roche potential. In Cartesian coordinates (x, y, z) rotating with the binary with origin at the primary, the Roche potential is expressed as

$$\Phi_R = -\frac{GM_1}{(x^2 + y^2 + z^2)^{1/2}} - \frac{GM_2}{((x - a)^2 + y^2 + z^2)^{1/2}} - \frac{1}{2}\Omega_{\text{orb}}^2[(x - \mu a)^2 + y^2], \quad (1.5)$$

where M_1 and M_2 are the masses of the primary and the secondary, $\mu = M_2/(M_1 + M_2)$, $\Omega_{\text{orb}} = 2\pi/P_{\text{orb}}$, where P_{orb} is the orbital period, and a is the binary sep-

* <<https://wwwmpa.mpa-garching.mpg.de/RKcat/>>

† <<https://www.aavso.org/vsx/index.php?view=search.top>>

aration, respectively (Pringle, Wade, 1985). Equation (1.5) draws the shape of Roche equipotentials by $\Phi_R = \text{const}$ (see Figure 1.2). The Roche lobe is defined as the equipotential passing through one of the saddle points of the Roche potential called the inner Lagrangian point (L_1). The Roche radii of the primary and the secondary can be calculated by using the following approximate equations (Silber, 1992; Eggleton, 1983):

$$R_{L_1}(1) = a(1.0015 + q^{0.4056})^{-1} \quad (0.04 \leq q \leq 1), \quad (1.6)$$

$$R_{L_1}(2) = a \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{-1/3})} \quad (0 < q < \infty), \quad (1.7)$$

where q is the ratio of the secondary mass with respect to the primary mass. Since the secondary has a distorted shape, $R_{L_1}(2)$ is defined so that the volume of the secondary is equal to $4/3\pi(R_{L_1}(2))^3$ (Paczynski, 1971).

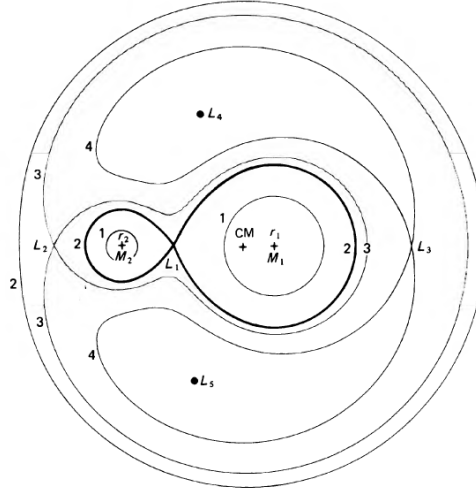


Figure 1.2: Equipotentials in the orbital plane of a binary system consisting of two point masses, which are taken from Iben, Livio (1993). The mass ratio assumed in this figure is 0.25. Here L_i indicates each saddle point.

The close binary systems are classified into three subclasses in accordance with their geometry. If both of the primary and the secondary are apart from their Roche lobes like the left panel of Figure 1.3, the system is called a detached binary. If either of them fills its Roche lobe like the middle panel of Figure 1.3, the system is called a semi-detached binary. In this system, mass transfer from the Roche-filling star occurs. If both of them fill their Roche lobes like the right panel of Figure 1.3, the system is called a contact binary. In a contact binary, mass of the two stars overflows from the outer Lagrangian point (L_2) and a common envelope is formed.

CVs have short orbital periods typically less than several hours. Here the binary

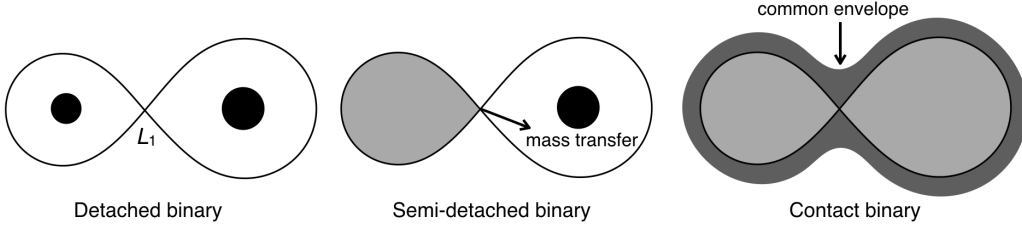


Figure 1.3: Classification of close binary systems. The left, middle, right panels represent a detached binary, a semi-detached binary, and a contact binary, respectively.

separation (a) of the system is derived from Kepler's law as follows:

$$a^3 = \frac{G(M_1 + M_2)P_{\text{orb}}^2}{4\pi^2}. \quad (1.8)$$

If one CV includes two solar-size MSs and substitute 4 hrs as P_{orb} into equation (1.8), a is estimated to $\sim 10^{11}$ cm, which is comparable with the solar radius. However, CVs are not contact binaries. This means CVs are supposed to harbor very compact objects. Actually, one CV contains a WD (the primary), a companion star (the secondary), and an accretion disk. WDs are one class of compact objects and their radii are as small as that of Earth.

A companion star is mostly a low-mass MS like our sun, and CVs are semi-detached binary systems. Since the secondary star fills its Roche lobe, it is distorted as the shape of its Roche lobe. If the white dwarf and the accretion disk are relatively faint, ellipsoidal variations are observed by the orbital motion of the distorted shape of the secondary star (see also Figure 4.8 in chapter 4).

The gas stream from the secondary flows into the Roche lobe of the primary star via the L_1 point. The mass loss rate is defined as $\dot{M} = Q\rho_{L_1}c_s$. Here Q is the effective cross section of the gas stream, ρ_{L_1} is the density at the L_1 point, and c_s is the averaged isothermal sound velocity, respectively. The stream line is determined by the Coriolis force and the gravitational forces of the primary and the secondary (Flannery, 1975). Since the gas stream has the angular momentum at the L_1 point, the gas stream at first rotates at the Lubow-Shu radius (Lubow, Shu, 1975) defined as follows:

$$r_{\text{LS}} = a0.0859q^{-0.426}, \quad (1.9)$$

when there is no mass in the disk. After the stream makes an annulus at the Lubow-Shu radius, the viscosity works and the annulus expands inwards and outwards. Finally, an accretion disk is formed around the primary. The stream continues to impact at the outer rim of the accretion disk and creates a shock-heated area called

a bright spot. The configuration of one CV including the bright spot, is displayed in Figure 1.4.

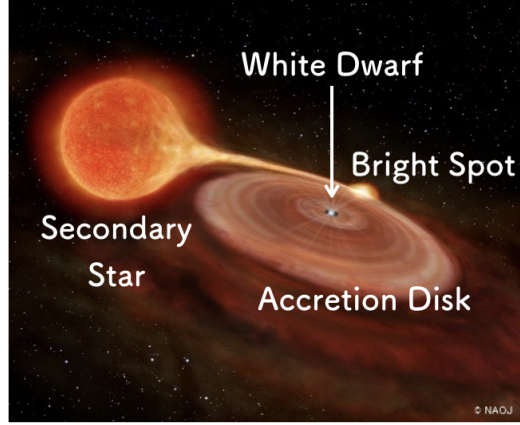


Figure 1.4: Schematic picture of a cataclysmic variable. The figure provided by NAOJ is modified.

The disk cannot expand eternally within the primary Roche lobe. There is a tidal truncation radius where the disk outer edge is truncated by the gravitational influence of the secondary star. That radius is regarded to be the last non-interacting orbit at which two orbits intersect one another. The fluid flow cannot exist stably beyond that radius. Paczyński (1977) computed that radius, which is approximately expressed as

$$r_{\text{tidal}} = a \frac{0.60}{1 + q}. \quad (1.10)$$

However, the disk sometimes expands a little beyond r_{tidal} (Papaloizou, Pringle, 1977). If the disk matter exceeds this radius, its angular momentum is returned back to the secondary star.

– Eclipse –

This is a brief introduction of the light curve of eclipsing CVs. The phase-averaged light curve of the eclipse is given in the left panel of Figure 1.5. Generally, the WD and the disk with a bright spot is hotter than the secondary. If the inclination angle of the binary system is higher than $\sim 70^\circ$, the bright components are occulted by the secondary when the rotating secondary intervene between them and observers.

The light variations of eclipses are resolved into three components: occultations of a WD, a disk, and a bright spot. Since the WD radius is much smaller than the radius of the secondary, the secondary fully eclipses the WD within short time, and the duration depends on the size of the secondary. The light curve has an asymmetric

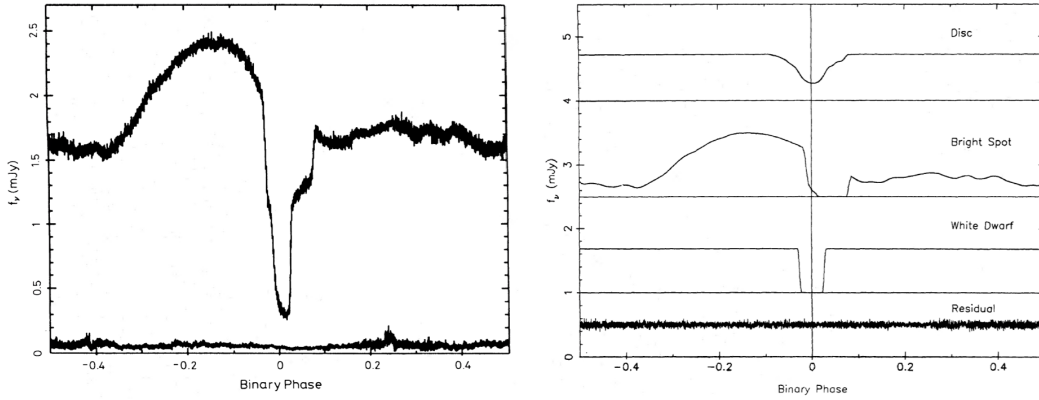


Figure 1.5: Phase-averaged profile of eclipses in CVs and its decomposition, which are taken from Wood et al. (1986). (Left) Mean light curve and uncertainty of eclipses in Z Cha. (Right) Decomposition of the mean light curve. The secondary is located in front of the disk center at the phase zero.

box-shaped depression (see the 3rd panel of the right column in Figure 1.5). The disk is larger than the secondary, and hence, the secondary cannot eclipse the entire disk. The eclipse of the disk becomes asymmetric and smoothly depressed light curve (see the top panel of the right column in Figure 1.5). The phase of the deepest eclipse for a disk matches with that for a WD if the temperature distribution of the disk is axisymmetric. In contrast, the bright spot is always precedes the secondary. The deepest eclipse is delayed in comparison with the other two cases. Also, a bump feature appears when the bright spot emerge to observers with the rotation of the disk (see the 2nd panel of the right column in Figure 1.5).

1.3.2 Spectra

The spectra from one CV is superimposed of the components from the WD, the secondary star, and the accretion disk. The spectrum of a WD is approximately a blackbody with a single temperature. The Balmer absorption lines are simultaneously observed, since the photons from the hot core bounce the electrons in the thin outer layer of hydrogen from one orbit to another orbit. The line width is broader than that from the MS because of the pressure broadening. The Ly α absorption is also commonly observed at ultraviolet (UV) wavelengths (Verbunt, 1987).

As mentioned above, the secondary star of CVs are typically a late-type MS (a K-type/M-type star). The continuum emission is expressed as a single-temperature blackbody (3,000–5,000 K) having a peak at near-infrared (near-IR) wavelengths. Absorption lines of molecules like titanium oxide (TiO) and NaI are observable in

near-IR spectra of the secondary.

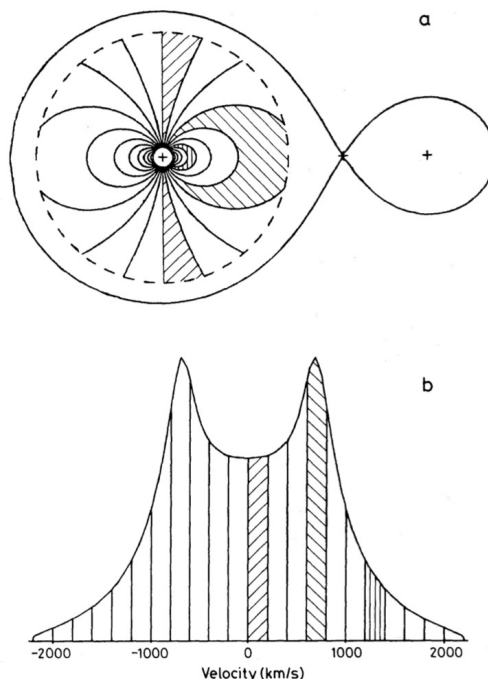


Figure 1.6: Double-peaked emission line profile from the accretion disk with Keplerian rotation. This figure is derived from Horne, Marsh (1986) and they assume the binary mass ratio is 0.15. (a) The configuration of a pole-on accretion disk, and (b) The expected emission line profile.

The spectra from accretion disks are more complex. The outer edge of the disk has low-temperature around 5,000 K, but the inner disk is sometimes heated up to $\sim 30,000$ K. If the disk is axisymmetric and each annulus emits a blackbody with each different temperature, the continuum spectrum becomes the superposition of blackbody emissions with different temperature, i.e., multi-color blackbody. Emission/absorption lines from the disk are superimposed on continuum spectra. The line profiles depend on the temperature distribution in the vertical direction of the disk.

The most prominent line spectrum from accretion disks is double-peaked $H\alpha$ emission line originating from the entire disk each region of which has different velocity to the line of sight. The line profile depends on the inclination angle of the binary. Generally, double-peaked $H\alpha$ emission lines are observed when the inclination is larger than ~ 60 deg. If the disk matter rotates with Keplerian velocity, the emission line profile is like Figure 1.6 (Horne, Marsh, 1986).

The orbital motions of the WD and the secondary can be measured by using spectral lines. Since the binary moves with a few hundreds of km s^{-1} , the line is shifted

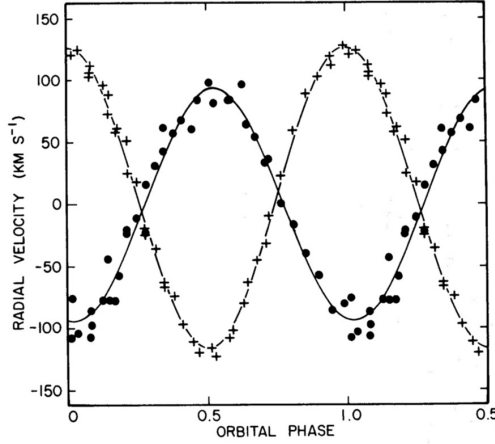


Figure 1.7: The RV curves of RU Peg in quiescence, which is taken from Stover (1981). The pluses and dots are the RVs estimated from the absorption and emission lines, respectively.

by a few to 10 angstroms, and the deviation of the central wavelength of the line varies with the orbital phase. S-wave curves are observable in spectra continuously taken from one CV during one orbital period, which represent the changes of radial velocities (RVs) of the two components. For example, Stover (1981) derived the RV curves from the absorption lines and the emission lines of RU Peg. The RV estimated from the absorption lines (the emission lines) represents the orbital motion of the secondary (the primary) (see Figure 1.7). For instance, $H\alpha$ and/or He II (4686) optical lines are frequently used as the track of the primary, although $H\alpha$ line comes from broad region of the accretion disk. In contrast, metal absorption lines like Na I absorption and/or Ca II triplet lines are popular as the track of the secondary.

If the RV curves of the WD and the secondary are obtained in one system, the binary mass ratio can be dynamically estimated. The ratio of the amplitude of the RV curve of the primary (K_1) against that of the secondary (K_2) represents the binary mass ratio ($q \equiv M_2/M_1$). However, it is difficult to get spectra from the secondary star in short-period CVs because the secondary star is faint. In this case, the mass function of binary systems which is formulated in equation (1.11) helps us to estimate q , if the orbital period is known.

$$f = \frac{P_{\text{orb}} K_1^3}{2\pi G} = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = M_2 \left(\frac{q}{1+q} \right)^2 \sin^3 i, \quad (1.11)$$

where i is the inclination angle of the binary. This estimation always has the degeneracy of the inclination angle.

Although the spectra from only three components are introduced here, other com-

ponents can make real spectra more complex. One of the candidates is outflows from accretion disks called disk winds. The main feature imprinted by disk winds is P-Cyg profiles in UV spectra, but some characteristics for distinguishing disk winds are observable in optical spectra. Some people found extended He II emission line and/or stationary components in H α line in some CVs when the accretion rate onto the WD is high (e.g., Honeycutt et al. (1986); Szkody, Wade (1981)). This kind of line profiles are considered to be generated by recombination of UV photons in disk winds (Matthews et al., 2015).

1.3.3 Evolution

– Mechanisms for angular momentum loss –

In the evolution of binary systems, the angular momentum loss (AML) plays an active role. Since the angular momentum of the binary system (J) is $J = M_1 M_2 \sqrt{\frac{Ga}{M}}$, the AML is represented as follows:

$$\frac{\dot{J}}{J} = \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} - \frac{1}{2} \frac{\dot{M}}{M} + \frac{1}{2} \frac{\dot{a}}{a}, \quad (1.12)$$

where $M = M_1 + M_2$. When the total mass M in the binary is conserved as,

$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}}{J} + 2 \frac{-\dot{M}_2}{M_2} \left(1 - \frac{M_2}{M_1} \right). \quad (1.13)$$

As a result, the binary separation shrinks when $M_2 > M_1$ and $\dot{J} = 0$. Since R_2 is proportional to $a(q/(1+q))^{1/3}$, i.e., $a(M_2/M)^{1/3}$ (see equation (1.7)), the following relation is derived as,

$$\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{1}{3} \frac{\dot{M}_2}{M_2} - \frac{1}{3} \frac{\dot{M}}{M}. \quad (1.14)$$

By combining equations (1.13) and (1.14),

$$\frac{\dot{R}_2}{R_2} = 2 \frac{\dot{J}}{J} + 2 \frac{-\dot{M}_2}{M_2} \left(\frac{5}{6} - \frac{M_2}{M_1} \right). \quad (1.15)$$

If $\dot{J} = 0$, $q < 5/6$ is required to prevent the system from unstable mass transfer from the secondary star, and most CVs satisfy this condition.

The mass transfer steadily occurs in CVs, but if $\dot{J} = 0$ holds, the secondary does not keep in contact with its Roche lobe because of the expansion of the binary separation (see equation (1.13)). Actually, $\dot{J}$ is not zero, and there are main two mechanisms of AML: magnetic braking and gravitational-wave radiation.

Strong magnetic fields are generated by dynamo action in low-mass stars which have an interior radiative layer and an exterior convective layer. The magnetic

field lines are coupled with ionized interstellar medium and small blobs from the secondary star go away having some angular momentum along the field lines. This decelerates the rotation of the low-mass star. This is called magnetic breaking. In the close binary system like CVs, the low-mass secondary star is tidally locked to the binary orbit, and thereby, the spin of the secondary synchronizes with the orbital motion of the binary system. The slower the rotation of the secondary becomes, the less the binary angular momentum becomes, and the binary orbit eventually shrinks.

General relativity says the quadrupole radiation extracts the angular momentum in binary systems at a rate given by (Paczynski, 1967)

$$\frac{\dot{J}_{\text{GR}}}{J} = -\frac{32G^3}{5c^5} \frac{M_1 M_2 (M_1 + M_2)}{a^4}. \quad (1.16)$$

This is efficient in short-period systems because of the short binary separation.

– Pre-cataclysmic evolution –

CVs are originally binary systems with separation of a few hundreds solar radii and with orbital periods of ~ 10 yrs. This subsection briefly reviews how these wide binaries become CVs. In a binary system, the more massive primary star evolves faster than the less massive secondary star. The primary star becomes a red giant after $\sim 10^9$ yrs. At that time, the outer thin layer is transferred towards the secondary star, and the binary separation decreases because of equation (1.13). At some point, the primary fills its Roche lobe, and the mass transfer is accelerated.

In this case, the mass of the primary is larger than that of the secondary, and the mass transfer from the primary becomes unstable and proceeds on the dynamical timescale proportional to $\sqrt{R_1^3/GM_1}$. The secondary star cannot adjust its structure at such a very high mass transfer rate ($\sim 0.1 M_\odot/\text{yr}$), because the thermal timescale is much longer than the dynamical timescale. The transferred material overfills both Roche lobes, and forms a common envelope surrounding the two stars.

The energy in the envelope will exceed the binding energy of the binary orbit, and the mass and angular momentum are ejected. The binary separation rapidly shrinks to $\sim 1 R_\odot$ within $\sim 1,000$ yrs (see Iben, Livio, 1993, for a review). A planetary nebula surrounding the close binary is formed, and if it is expelled, a detached binary including a WD is unveiled. The detached binary loses its angular momentum gradually by magnetic breaking and gravitational-wave radiation, and finally the secondary contacts its Roche lobe, and the mass transfer to the primary starts, which is the birth of a CV.

– **Standard evolution of cataclysmic variables** –

Once the mass transfer from the secondary star towards the WD is triggered, the orbital period of the binary gradually decreases. There are two important timescales for CV evolution: the timescale on which the secondary is losing mass ($\tau_{\dot{M}_2} \sim M_2/\dot{M}_2$) and the thermal (Kelvin-Helmholtz) timescale ($\tau_{\text{KH}} \sim GM_2^2/R_2L_2$). In CVs, the two timescales are comparable, and the secondary star does not satisfy thermal equilibrium (Patterson et al., 2005; Knigge, 2006). When the mass-radius index of the secondary is defined as $\zeta = d \ln R_2 / d \ln M_2$, the ζ value in CVs is smaller than the equilibrium value, i.e., the secondary inflates in comparison with the single star having the same mass.

Just after the secondary loses its mass, the binary separation expands a little in accordance with equation (1.13). While the secondary tries to adjust to the thermal equilibrium on the thermal timescale, i.e., the star size becomes smaller, the angular momentum is removed and the binary separation decreases and the secondary star keeps in contact with its Roche lobe. Although the secondary is deviated from thermal equilibrium, the mass loss continues. Since the secondary star and the binary separation shrinks on the course of evolution, the orbital period becomes smaller.

In addition, the timescale on which the system parameters evolve is

$$\tau_{\text{ev}} \sim \frac{J}{\dot{J}_{\text{sys}}} \sim \frac{M_2}{\dot{M}_2} \sim \frac{P_{\text{orb}}}{\dot{P}_{\text{orb}}}. \quad (1.17)$$

Here, the timescale on which the secondary adjusts its radius to its mass loss is written as τ_{adj} . Since τ_{adj} is much smaller than τ_{ev} (Stehle et al., 1996), CVs have a unique evolutionary track especially in short-period systems.

As long as the secondary star has the radiative zone at the center, the main cause of AML is magnetic breaking. However, if the secondary star becomes fully convective at some point (when the orbital period is around 3 hrs), magnetic breaking no longer works. The secondary loses contact with its Roche lobe and the mass transfer ceases. However, the angular momentum of the binary continues to decrease because of gravitational-wave radiation. When the orbital period decreases to ~ 2 hrs, the mass transfer restarts, since the secondary fills its Roche lobe again. The population of CVs is very small in the range of the orbital period (~ 2 – 3 hrs) like Figure 1.8. This is called the period gap (see Figure 1.8).

After the period gap, the AML proceeds by gravitational-wave radiation and the secondary star becomes smaller and smaller, and the mass-loss rate becomes lower. As the ratio of $\tau_{\dot{M}_2}/\tau_{\text{KH}}$ increases, the secondary becomes able to maintain thermal

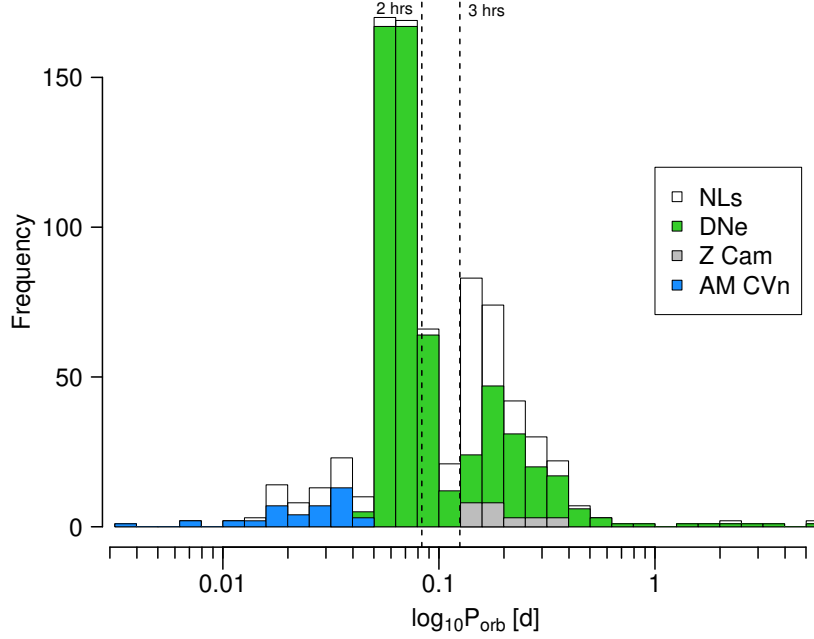


Figure 1.8: The orbital distribution of CVs in observations. The data are taken from the American Association of Variable Star Observers (AAVSO) VSX. The white, green, grey, and blue distributions represent nova-like stars (NLs), dwarf novae (DNe), Z Cam stars, and AM CVn stars, respectively. The region sandwiched by dashed lines indicates the period gap.

equilibrium. Then the mass-radius index becomes the adiabatic value, i.e., $\zeta_{\text{ad}} = -1/3$ (Rappaport et al., 1982). Also, the mass of the secondary becomes below the hydrogen-burning limit mass ($\sim 0.07 M_{\odot}$) at some point. Then the mass-radius index ζ becomes $-1/3$ (Chabrier et al., 2009), which is the same as ζ_{ad} . As a result, the secondary star expands when it loses its mass, and the orbital period becomes longer in the evolution. The evolutionary path bounces on the $M_2(q) - P_{\text{orb}}$ plane. There is thus the period minimum, which is ~ 65 min by theoretical prediction (Kolb, 1993). The system after the period minimum is called period bouncers. However, the period minimum in observations is ~ 80 min. In addition, since the period change is decelerated around the period minimum, many objects will be found around the period minimum. Theoretical model predicts the population ratio is 1:30:70 for long-period CVs, short-period CVs, and period bouncers, respectively. However, the observational population near the period minimum is small, and there is a significant gap between the observations and the theoretical predictions. This is called the missing-population problem (see also Figure 1.8).

Also, the population of the CVs having very long orbital periods ($P_{\text{orb}} > 12$ hrs) is extremely small. This is because the WD mass must be less than the Chandrasekhar

limit of $1.4 M_{\odot}$. The secondary star should be smaller than the primary. Considering the secondary fills its Roche lobe, the orbital period cannot become very long.

AM CVn stars whose distributions are plotted by the blue color in Figure 1.8 have extremely short orbital periods less than 65 min and helium-rich accretion disks, while normal CVs have hydrogen-rich accretion disks. Their evolutionary paths are of course different from the standard evolutionary path of normal CVs. When some binaries enter the common-envelope phase, the hydrogen outer layer of the secondary star is considered to be somehow expelled, and three possible channels are proposed (see Solheim, 2010, for a review).

– **Long-term variations in mass-transfer rates** –

There is clearly a scatter in mass-transfer rates from the secondary star at the same range of P_{orb} especially above the period gap (see Figure 1.8), since NLs and DNe coexist in the same range of orbital periods for example. Actually, a statistical study about the mean magnitude and P_{orb} in DNe suggests this (Warner, 1987). However, all CVs have to join a unique evolutionary track in order to explain the period gap and the period minimum reviewed above. Namely, observations implies the variation in $\dot{M}_2$, but it may not be inherent to each system. In one system, the mass-transfer rate can vary on timescale much shorter than τ_{ev} .

The timescale of variations in mass-transfer rates would be longer than our life timescales. Many people have discussed about the mechanisms triggering that variation, but it is still puzzling. One of possible mechanisms is nova-induced hibernation.

Shara et al. (1986) proposed hibernation scenario for the first time and the comprehensive review is given by Shara (1989). All CVs experience nova eruptions many times in their lives. Once one CV passes through a nova eruption, the hydrogen-rich envelope of a WD is ejected by a thermonuclear runaway. Just after the mass ejection, the binary separation increases, since the secondary is pushed by the pressure of ejected materials. The secondary star seems to become apart from its Roche lobe, and the mass transfer seems to cease. However, the WD has high temperature and irradiates the secondary star. The irradiation continues for a few centuries and the mass-transfer rate keeps higher before an eruption. When the irradiation becomes ineffective, the transfer rate decreases (maybe becomes much lower because of the increase of binary separation), and the system repeats dwarf-nova outbursts. Actually, Livio et al. (1988) found several novae entering outbursts. The outburst frequency will become lower and outbursts even may stop. This phenomenon is

called hibernation. The AML returns the mass-transfer rate back to a high value after long hibernation. The system undergoes the next nova eruption.

1.4 Viscous accretion disks

1.4.1 Basic equations

The main topic in this dissertation is dwarf-nova outbursts, and dwarf-nova outbursts are regarded as the brightening of accretion disks. This section introduces how the accretion disk with viscosity evolves by using some equations.

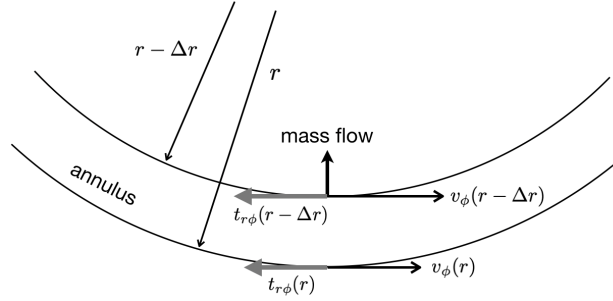


Figure 1.9: Schematic picture of an annulus in a disk and the torque exerted on it.

The viscous torque working between two adjacent axisymmetric rotating annuli in the disk is depicted in Figure 1.9. Here cylindrical coordinates with the origin at the central star are adopted and the z -axis is perpendicular to the plane of the disk. In this case, the rotational velocity has a gradient in the radial direction. The viscous force per unit area exerted in the ϕ -direction at the interface is

$$t_{r\phi} = \eta \left(\frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r} \right) = \eta r \frac{d\Omega}{dr}, \quad (1.18)$$

where v_ϕ is the rotational velocity, $\eta = \rho\nu$ is the dynamical viscosity, and Ω is the angular velocity, respectively. This is the $r\phi$ -component of the viscous stress tensor. The total torque working on the entire surface of the inner annulus is

$$G(r) = 2\pi r \int_{-\infty}^{\infty} r t_{r\phi} dz \approx 4\pi r^3 \eta H \frac{d\Omega}{dr}, \quad (1.19)$$

where H is a half-thickness of the disk.

Here, the disk material is assumed to be in the Keplerian rotation, $|v_r| \ll v_\phi$, and to be axisymmetric and geometrically thin ($H \ll r$). Since the viscous timescale is much longer than the hydrostatic timescale and the thermal timescale, vertically integrated variables are useful, and the basic equations are replaced by one-dimensional ones. The surface density and the vertically integrated viscous stress

are defined as

$$\Sigma = \int_{-\infty}^{\infty} \rho dz, \quad (1.20)$$

$$T_{r\phi} = \int_{-\infty}^{\infty} t_{r\phi} dz = \nu \Sigma r \frac{d\Omega}{dr}, \quad (1.21)$$

where ν is the kinematic viscosity.

The mass conservation for the annulus in Figure 1.9 is formulated as follows. The mass in the annulus is $2\pi r \Delta r \Sigma$ and the inflow and the outflow are written as $(-v_r \cdot 2\pi r \Sigma)_r$ and $(-v_r \cdot 2\pi r \Sigma)_{r-\Delta r}$, respectively. When $\Delta r \ll r$, the continuity equation representing the mass conservation is obtained as,

$$\frac{\partial}{\partial t}(2\pi r \Sigma) + \frac{\partial}{\partial r}(2\pi r \Sigma v_r) = 0. \quad (1.22)$$

When introducing the mass-flow rate ($\dot{M} \equiv -2\pi r v_r \Sigma$), equation (1.22) can be rewritten into the following equation,

$$\frac{\partial}{\partial t}(2\pi r \Sigma) = \frac{\partial \dot{M}}{\partial r}. \quad (1.23)$$

The angular-momentum transfer includes the loss and the gain of angular momentum with the inflow and the outflow ($(-v_r \cdot 2\pi r \Sigma \cdot r^2 \Omega)_r$ and $(-v_r \cdot 2\pi r \Sigma \cdot r^2 \Omega)_{r-\Delta r}$), and the viscous torque. The total torque exerted on the annulus in

$$G(r, t) = -2\pi r^2 T_{r\phi} = 2\pi r^3 \nu \Sigma \frac{d\Omega}{dr}. \quad (1.24)$$

As the mass conservation is derived, $\Delta r \ll r$, and hence, the conservation of angular momentum is

$$\frac{\partial}{\partial t}(2\pi r \Sigma r^2 \Omega) + \frac{\partial}{\partial r}(2\pi r \Sigma v_r r^2 \Omega) = \frac{\partial G}{\partial r}. \quad (1.25)$$

If v_r is replaced by $\dot{M}$ and equation (1.22) is substituted into equation (1.25), the following relation is obtained as

$$\dot{M} \left[\frac{d}{dr}(r^2 \Omega) \right] = -2\pi \frac{\partial}{\partial r}(r^2 T_{r\phi}) = 2\pi \frac{\partial}{\partial r} \left(r^3 \nu \Sigma \frac{d\Omega}{dr} \right). \quad (1.26)$$

By combining equation (1.23) and equation (1.26), the following differential equation for Σ is finally obtained as

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[r^{1/2} \frac{\partial}{\partial r} \left(\nu \Sigma r^{1/2} \right) \right]. \quad (1.27)$$

In general, the energy equation has to be prepared to solve the time evolution of the accretion disk, but it is skipped here. This is because the conservation of energy depends on the temperature and/or the density of the disk. Its formulation is not always the same.

1.4.2 Standard disk

To solve the basic equations in the previous subsection, some assumptions especially for the viscosity are required. The important framework of the accretion disk is the standard disk proposed by Shakura, Sunyaev (1973). Six of the assumptions for the standard disk are introduced in the preceding subsection: (1) the gravitational field is determined by the primary star, and the self-gravity of the disk matter is negligible, (2) the disk lies in the equatorial plane of the primary, (the disk is aligned with the orbital plane in binaries), (3) the disk is axisymmetric, (4) the disk is geometrically thin ($H/r \ll 1$), and (5) rotational motion is dominant ($|v_r| \ll v_\phi$), and Keplerian rotation is adopted ($\Omega = \Omega_K, \equiv \sqrt{\frac{GM}{r^3}}$). The other assumptions are (6) hydrostatic balance holds in the vertical direction, (7) the disk is steady ($\dot{M}$ is constant in space and time), (8) the disk is optically thick in the vertical direction, (9) the viscous stress tensor is proportional to the pressure, and (10) global magnetic fields are negligible.

Under these assumptions, the surface density is rewritten as $\Sigma = 2\rho H$ by the one-zone approximation. By integrating equation (1.26) under $\dot{M} = \text{constant}$ and by determining the numerical constant by the boundary condition ($T_{r\phi} = 0$ at the inner edge of the disk (r_{in})), the angular-momentum conservation is

$$\nu\Sigma = \frac{\dot{M}}{3\pi} \left(1 - \sqrt{\frac{r_{\text{in}}}{r}}\right). \quad (1.28)$$

– Energetics –

In the disk, the viscous heating rate per unit volume ($\rho\nu(r d\Omega/dr)^2$) is balanced with the radiative cooling. The viscous heating is expressed as

$$Q_{\text{vis}}^+ = \int_{-\infty}^{\infty} \nu\rho \left(r \frac{d\Omega}{dr}\right)^2 dz = \frac{9}{4} \nu\Sigma\Omega^2 = \frac{3}{2} T_{r\phi}\Omega. \quad (1.29)$$

The optical depth in the optically-thick disk is

$$\tau = \kappa\rho H, \quad (1.30)$$

where κ is the Rosseland-mean opacity. The radiative flux in the z -direction is defined as

$$F(z) = -\frac{4ac[T(z)]^3}{3\kappa(z)\rho(z)} \frac{\partial T}{\partial z}, \quad (1.31)$$

where a is the radiative constant ($\sigma = ac/4$) and T is the temperature. When the temperature of the disk is defined at the equatorial plane as T_c , the cooling rate is

$$Q_{\text{rad}}^- = 2F = \frac{32\sigma T_c^4}{3\tau}, \quad (1.32)$$

where σ is the Stefan-Boltzmann constant.

The cooling rate in the standard disk is written as $Q_{\text{rad}}^- = 2F = 2\sigma T_{\text{eff}}^4$ by using the effective temperature (T_{eff}). By considering the local thermal equilibrium ($Q_{\text{vis}}^+ = Q_{\text{rad}}^-$), the following relation is derived:

$$F = \sigma T_{\text{eff}}^4 = \frac{3GM\dot{M}}{8\pi r^3} \left(1 - \sqrt{\frac{r_{\text{in}}}{r}}\right). \quad (1.33)$$

The temperature of the disk is proportional to $r^{-3/4}$.

By using equation (1.33), the disk luminosity is derived as

$$L_{\text{disk}} = \int_{r_{\text{in}}}^{\infty} 2F 2\pi r dr = \int_{r_{\text{in}}}^{\infty} \frac{3GM\dot{M}}{2r^2} \left(1 - \sqrt{\frac{r_{\text{in}}}{r}}\right) dr = \frac{GM\dot{M}}{2r_{\text{in}}}. \quad (1.34)$$

This is the same as the luminosity calculated by equation (1.4). When the disk matter falls in the potential well from r to $r - \Delta r$, it releases the local potential energy expressed as

$$\Delta L_{\text{pot}} = \left(-\frac{GM}{r} + \frac{GM}{r - \Delta r}\right) \dot{M} \simeq \frac{GM\dot{M}}{r^2} \Delta r. \quad (1.35)$$

Since half of this energy goes to kinetic energy (rotation of the disk material), the local radiation loss is

$$\Delta L_{\text{local}} \simeq \frac{1}{2} \frac{GM\dot{M}}{r^2} \Delta r. \quad (1.36)$$

The actual energy loss per unit time from an annulus is

$$\Delta L_{\text{rad}} = 2\pi r \Delta r \cdot 2F \simeq \frac{3}{2} \frac{GM\dot{M}}{r^2} \Delta r, \quad (1.37)$$

when $r \gg r_{\text{in}}$. This seems three times larger than ΔL_{local} , but the total energy balance holds as in equation (1.34).

– Viscosity prescription –

In the standard disk, a specialized viscous law is adopted. The $r\phi$ -component of the shear stress tensor is

$$t_{r\phi} = \rho \nu r \frac{d\Omega}{dr} = -\alpha p, \quad (1.38)$$

where α is the viscosity parameter, and p is the total pressure. In this prescription, ν is defined as $\frac{2}{3} \alpha c_s H$, where c_s is the sound speed.

– Timescales –

The three representative timescales of the accretion disk, i.e., the dynamical timescale, the thermal timescale, and the viscous timescale are expressed as follows.

$$t_{\text{dyn}} \equiv \frac{H}{c_s} \sim \frac{1}{\Omega}, \quad (1.39)$$

$$t_{\text{th}} = \left| \frac{\Sigma C_P T_c}{T_{r\phi} \Omega_K} \right| \sim \frac{1}{\alpha \Omega_K}, \quad (1.40)$$

$$t_{\text{vis}} = \frac{r^2}{\nu} \sim \frac{1}{\alpha \Omega_K} \left(\frac{r}{H} \right)^2. \quad (1.41)$$

Here α is less than 1 and normally ranges between 0.01 and 0.3. According to the assumption 5, the thickness of the disk is much less than r . Here $t_{\text{vis}} \gg t_{\text{dyn}}$ and $t_{\text{vis}} \gg t_{\text{th}}$.

1.5 Normal dwarf-nova outbursts

1.5.1 Discovery

Some of DNe exhibiting normal outbursts with amplitudes of ~ 2 –6-mag at optical wavelengths and with duration of a few tens of days are called U Gem-type stars. This type of ordinary dwarf-nova outbursts have been well known since long ago. An outburst in U Gem was discovered in 1855, and this source has been monitored for long time as well as SS Cyg. The recent optical light curves of SS Cyg are given in Figure 1.10.

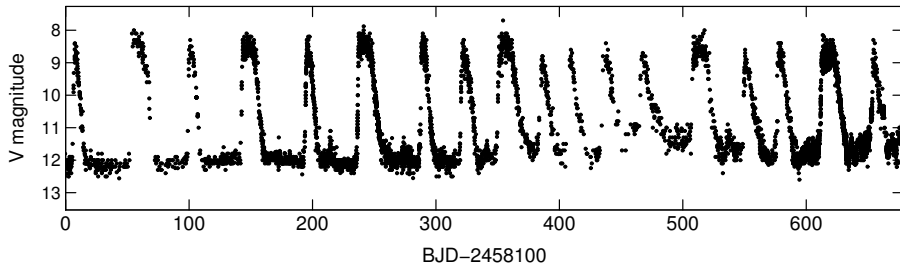


Figure 1.10: Optical V -band light curves of SS Cyg, one of the most famous DNe, which are taken from the AAVSO archive*. Here, BJD is Barycentric Julian dates.

The observations of eclipsing light curves of U Gem led to wrong interpretation about the outburst mechanism: the secondary star is a seat of outbursts (Krzeminski, 1965; Paczyński, 1965; Osaki, 1970). However, other eclipsing U Gem-type

stars were observed, and it was made clear that the eclipse of the disk becomes dominant during outburst (e.g., Rutten et al., 1992). This means the disk is a seat of outbursts.

U Gem is one of rare objects which show grazing eclipses (Zhang, Robinson, 1987). If the inclination is not so high but still enough to cause the eclipse ($\sim 70^\circ$), only the outer region of the accretion disk is occulted by the secondary star. When the disk is in quiescence, the bright spot at the outer rim of the disk is prominent, but when the disk enters an outburst, the outer part of the disk is not remarkable compared with the other regions of the disk, because the temperature distribution of the disk is close to that of the standard disk (see also equation (1.33) and Horne, Cook (1985)). This is the reason why the eclipse becomes shallower during outburst than during quiescence in U Gem.

1.5.2 Mechanism for dwarf-nova outbursts

– History –

As described in the previous subsection, observational studies made it clear the source of outbursts in DNe is the brightening of accretion disks. What kind of physical mechanism can reproduce the sudden brightening of accretion disks? Many theorists have tried to answer this question, and two different scenarios were proposed. One is mass-transfer-burst model (the MTB model) proposed by Bath (1973), in which enhanced mass transfer from the secondary star causes outbursts. The other one is the disk-instability model (the DI model) proposed by Osaki (1974), in which the mass-transfer rate from the secondary is constant. In the DI model, the mass from the secondary star accumulates in the disk during quiescence. When the amount of the stored mass exceeds the critical value for triggering outbursts, the stored mass suddenly accretes onto the primary WD.

Osaki (1974) came up with the DI model by simple calculations on the basis of the observations of U Gem. First of all, he noticed that there is no accretion during quiescence, because the bright spot seems to contribute the half brightness of the system. The shoulder-like curve in the light variations averaged with the orbital period originates from the eclipse of the bright spot and has 0.5-mag amplitudes in U Gem (Krzeminski, 1965). This naturally suggests the mass transferred from the secondary is stored in the disk. In the quiescent state, the brightness of the bright spot dominates optical light curves. Its luminosity is

$$L_{\text{spot}} = \frac{GM_1 \dot{M}_{\text{tr}}}{2R_{\text{disk}}}, \quad (1.42)$$

where $\dot{M}_{\text{tr}}$ is the mass transfer rate from the secondary star and R_{disk} is the disk size, respectively. In contrast, provided that the disk mass accretes onto a WD during outburst, the luminosity of the disk is

$$L_{\text{out}} = \frac{GM_1\dot{M}}{2R_{\text{WD}}}, \quad (1.43)$$

where $\dot{M}$ represents the accretion rate onto a WD and R_{WD} is the WD radius, respectively. Here, $R_{\text{disk}}/R_{\text{WD}}$ is about 32 in U Gem. Also, $\dot{M}/\dot{M}_{\text{tr}}$ is about 10 according to the duration and the interval between outbursts. Finally,

$$L_{\text{out}}/L_{\text{spot}} \approx 320, \quad (1.44)$$

is obtained. This is consistent with the outburst amplitude in U Gem, and good evidence that the accumulated mass in quiescence is drained to a WD during outburst.

The MTB model and the DI model have contested fiercely in late 1970's, but now, the DI model is believed as the most plausible model for dwarf-nova outbursts, since this model is supported by both of observational and theoretical works. As for the MTB model, the expected brightening of the bright spot has not been detected and there are not any good physical reasons for enhancing mass transfer from the secondary.

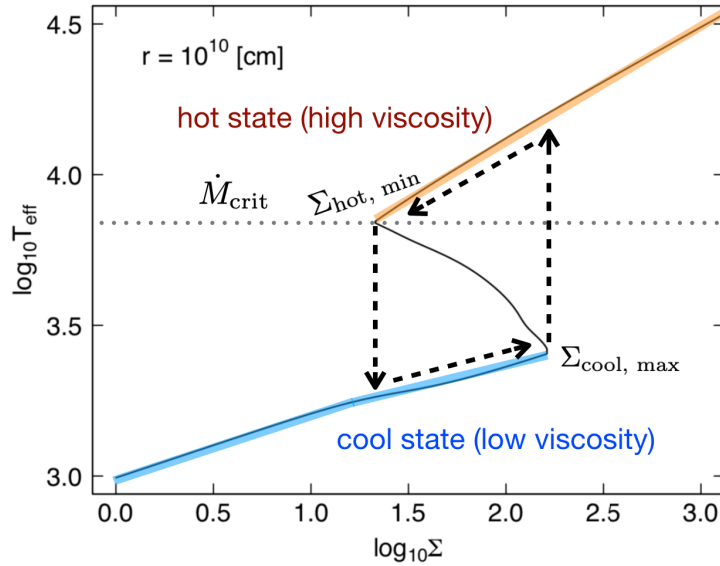


Figure 1.11: An example of S-shaped thermal equilibrium curves at $r = 10^{10}$ cm. This curve is calculated on the basis on the method in Ichikawa, Osaki (1992). The arrows represent the direction of time evolution.

On the other hand, there is a physical mechanism triggering the instability in accretion disks. Hōshi (1979) found the thermal instability is triggered by partial

ionization of hydrogen. When hydrogen is neutral, the disk stays in the cool and low-viscosity state. When hydrogen becomes ionized, the disk is switched to the hot and high-viscosity state. Eventually he was not able to reproduce limit-cycle oscillations necessary for generating repetitive outbursts only by this effect. However, Meyer, Meyer-Hofmeister (1981) took into account the convection in the vertical structure of the disk in addition to partial ionization of hydrogen and reproduced S-shaped thermal equilibrium curve like Figure 1.11. With this equilibrium curve, the disk jumps between the hot and cool states and enters a beautiful limit cycle. Here the S-shaped curve in Figure 1.11 is calculated under some simplified assumptions. The detailed form of the equilibrium curve is more like ξ -shaped (e.g., Cannizzo, Wheeler, 1984; Mineshige, Osaki, 1985).

After this discovery, many researchers performed numerical simulations and reproduced repetitive outbursts as observed (e.g., Mineshige, Osaki, 1983; Cannizzo et al., 1982). Also, many observations favor the DI model. The key observation is the variation of the disk radius during outburst. the DI model predicts the expansion of the disk at the onset of outbursts and the gradual shrinkage after that and during quiescence, while the disk radius suddenly shrinks at the beginning of outbursts and expands at the light maximum in the MTB model (Ichikawa, Osaki, 1992). The typical variation in disk radii in the DI model is displayed in Figure 5.10 in chapter 5. The disk-radius variations predicted by the DI model are consistent with the observational ones (e.g., Smak, 1984). It is also important feature that the disk radius gradually shrinks during quiescence in the DI model. Since the MTB model assumes the standard disk even in the cool state, the predicted shrinkage of the disk radius in quiescence is trivial. In contrast, the disk in the cool state is not steady in the DI model.

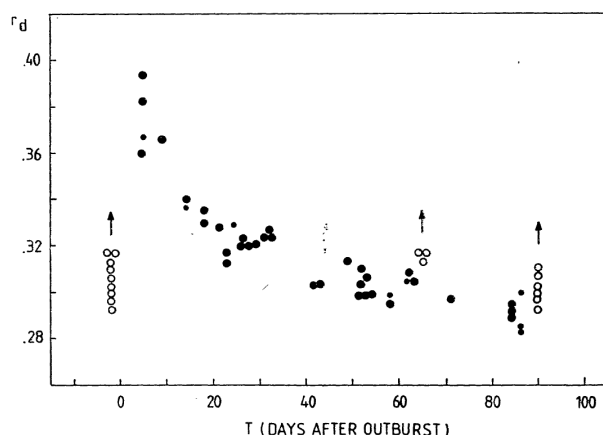


Figure 1.12: Variations of the disk radius during outburst in U Gem, which are taken from Smak (1984). The variations were estimated from the eclipsing light curves.

– Global time evolution of the disk during outburst –

In considering the global time evolution of the accretion disk, three conservation laws: the mass conservation, the angular-momentum conservation, and the energy conservation, and the viscosity prescription of the standard disk, which are mentioned in Sec. 1.4, are necessary. The energy equation is roughly represented as $c\Sigma\partial T/\partial t = Q^+ - Q^-$, where c is the specific heat of matter in the disk. If the right-hand side of this equation is equal to zero, the thermal equilibrium curve is obtained. In the time evolution of the disk in DNe, the S-shaped equilibrium curve determines the overall behavior.

In the thermal equilibrium curve for a given radius of the disk, there are bi-stable states sandwiching an unstable state (see also Figure 1.11). In quiescent state, the temperature of the disk is low and the transferred mass from the secondary star continues to accumulate in the disk. When the surface density in the disk exceeds the critical point ($\Sigma_{\text{cool, max}}$ in Figure 1.11), the disk becomes heated to $\sim 7,000$ K around which hydrogen starts to be ionized. Some free electrons are combined with hydrogen atoms and H^- ions are produced. These ions are the source of the big opacity and absorb photons efficiently. Thus the opacity suddenly increases while hydrogen is partially ionized. At this time, the temperature rapidly increases once the matter is deviated from the stable cool branch, and the disk jumps up to the hot stable branch. This is the onset of an outburst. Then a lot of mass is drained onto the white dwarf during outburst, and the surface density gradually decreases. If the surface density reaches the critical value on the hot state ($\Sigma_{\text{hot, min}}$ in Figure 1.11), the disk jumps back to the cool state, since there is no stable solution below this value at the hot stable branch. In Figure 1.11, the left side of the equilibrium line is cooling-dominated region, and the right side of that line is heating-dominated region, respectively.

From the above explanations, it may seem that the limit-cycle instability is limited to one particular radius. If so, the outburst amplitude will be much smaller than the observed ones. It is partially correct because the thermal instability works locally if the α value is constant. However, the transition between the cool and hot states propagates over the entire disk as cooling and heating waves, if different α values for the hot state and the cool state are adopted. It is required that $\Sigma_{\text{cool, max}}/\Sigma_{\text{hot, min}}$ is larger than 2 to reproduce the observed outburst amplitude (Osaki, 1989; Mineshige, Shields, 1990). The specific formula are for instance described in Sec. 5.3.4. The mass in a given radius flows to adjacent radii when the region at that radius goes up to the hot state, and in contrast, the region absorbs matter from adjacent regions if it drops down to the cool state. Thus the jumps between two stable states occur in unison.

– Source of viscosity in accretion disks –

A lot of material accretes onto the WD if the thermal instability is triggered, since the disk goes to the hot state in which the viscosity of the disk is much higher than that in the cool state. However, molecular viscosity is too feeble to realize accretion. Here magnetic turbulence in accretion disks plays an important role (Balbus, Hawley, 1991).

If hydrogen is ionized, the matter is coupled with magnetic fields. Provided that a field line connects two gas blobs in a disk, the inner blob rotates faster than the outer blob. Then the field line is stretched, and tries to pull the blobs back. It makes the outer blob rotate faster and the inner blob rotate slower, by conveying the angular momentum from the inner one to the outer one. As a result, the outer blob moves outwards and the inner blob falls inwards. The field line becomes stronger and the flows are disordered (see also Figure 1.13). Thus the perturbation of the flow is amplified once it happens in the disk. This effect is called the Balbus-Hawley instability or magnetorotational instability (MRI), and is the source of high viscosity in the hot state.

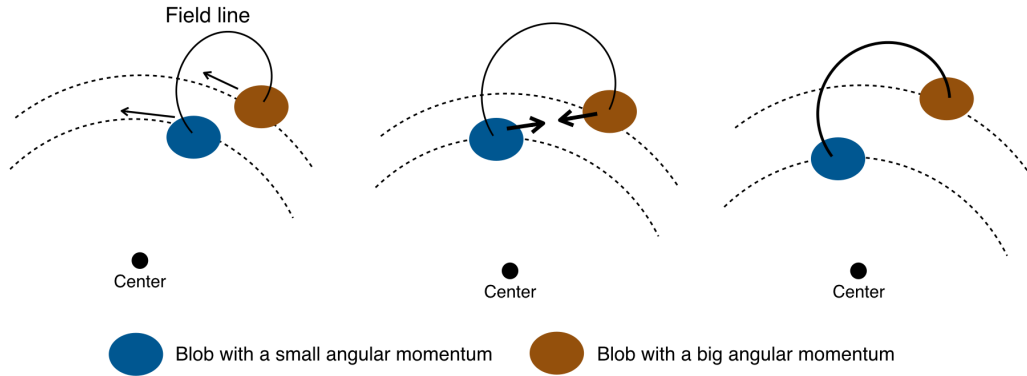


Figure 1.13: Schematic picture of the amplified magnetic field by the Balbus-Hawley instability. The time evolves from the left panel to the right panel.

Actually, the α value larger than 0.1 in the hot state is required to reproduce the large amplitude of the observational outbursts. This value is explained by the MRI (Hirose, 2015). On the other hand, the source of viscosity in the cool state is still unanswered. In the quiescent state, the turbulence by MRI becomes inefficient because of the poor conductivity of the cool disk (Gammie, Menou, 1998), though the magnetic fields coming from the secondary could induce weak turbulence. Also, it is required that α values in the cool state are smaller at least by a few factors than those in the hot state to reproduce dwarf-nova outbursts as mentioned above, but there is no convincing theory about the difference. Extremely low α values

like ~ 0.001 are necessary to explain very long intervals between outbursts in WZ Sge stars which are introduced later, but there is no theoretical reason about this extreme value. One possible explanation may be weak magnetic fields of the secondary, if the source of the viscosity is the MRI (Osaki et al., 2001).

1.5.3 Properties of normal outbursts

Normal dwarf-nova outbursts are classified into three types by their morphology. For example, SS Cyg, one of the most famous and bright DNe which shows long, short, and slow-rise outbursts like Figure 1.14. Long and short outbursts have rapid rises and tend to alternate at least at some epochs. They are regarded as outside-in outbursts in which outbursts are triggered at the outer region of the disk. Basically, most of outbursts are outside-in outbursts in most of DNe. In outside-in outbursts, the ignition to the hot state starts at the outer part of the disk, and the heating front propagates inwards from the outer disk (e.g., Fig. 10 in Mineshige, Osaki, 1985). In the long outburst, a plateau stage represents that the entire disk keeps in the hot state.

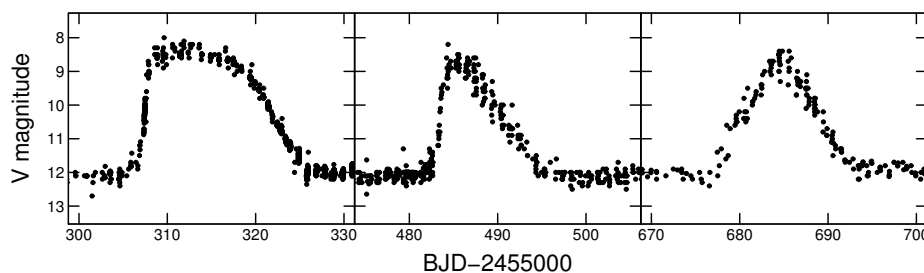


Figure 1.14: Three different kinds of outbursts in SS Cyg. The left, middle, and right panels represent a long, a short, and a slow-rise outbursts, respectively. The data are taken from the AAVSO archive.

Slow-rise outbursts represent inside-out outbursts in which the ignition to the cool state occurs at the inner part of the disk, and the heating front propagates from the inner disk outwards. In this case, it takes longer time for heating waves to propagate outwards, since they go against the inward mass flow, which is the reason why slow rises are observed at the beginning (e.g., Fig. 14 on Mineshige, Osaki, 1985).

Since the outbursts are always quenched at the outer disk, and since the disk has to drain a lot of mass in the disk onto the white dwarf, the decline rate is almost the same among these three types of outbursts. The timescale of the decline is the viscous timescale at the hot state, because the disk has to drain a lot of mass into the WD in order to push inwards the cooling front.

DNe have been well studied for long time, and there are some statistical studies. For example, it is known that the outburst amplitude is correlated with the interval between outbursts, which is named the Kukarkin-Parenago relation (van Paradijs, 1985). The larger the amplitude is, the longer the interval is. Besides this relation, significant correlations were detected between the the V -band maximum absolute magnitude and the orbital period, and between the decay timescale and the orbital period (Warner, 1987; Bailey, 1975).

1.6 Superoutbursts and superhumps

1.6.1 Observations

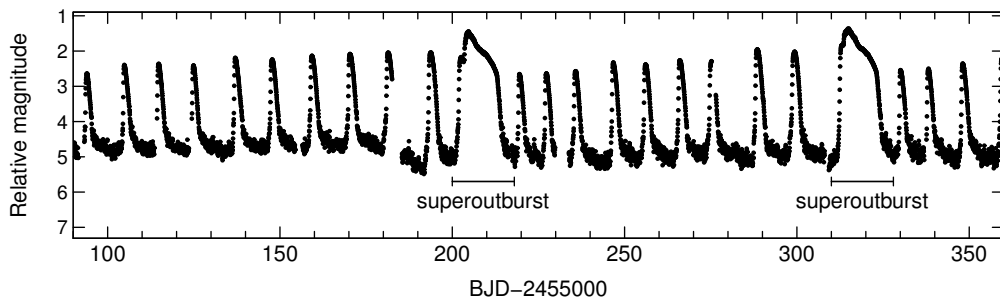


Figure 1.15: Optical light curves of V1504 Cyg, a SU UMa-type DNe. The data are taken from the Kepler archive, and the data are averaged per 0.05 days. This system repeats several normal outbursts and a superoutburst.

Although almost all of DNe above the period gap show only normal dwarf-nova outbursts, DNe below the period gap show superoutbursts in addition to normal outbursts. Superoutbursts are large-amplitude and long-duration outbursts like Figure 1.15. Typically, the amplitude is ~ 6 – 8 mag and the duration is about 2 weeks. During superoutburst, superhumps having periods a few percents longer than the orbital period are observed (see Figure 1.16). This subclass of DNe exhibiting superoutbursts is called SU UMa-type DNe.

Kato et al. (2009) performed statistical studies about the optical variability during superoutbursts of many SU UMa-type DNe, and found that superhumps are classified into three stages on the basis of their period changes and amplitudes. At the beginning of superoutbursts, the period of superhumps is constant and the amplitude increases. This type of superhumps is called stage A superhumps. After stage A, the period shortens and gradually vary with time. The amplitude decreases. This part is called stage B. The stage B is terminated by sudden decrease

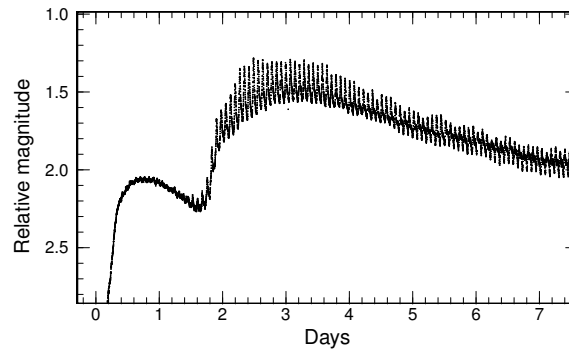


Figure 1.16: An enlarged view of the Kepler light curves of V1504 Cyg near the beginning of a superoutburst. The horizontal axis represents the days from the onset of the superoutburst. Superhumps are growing with time.

of the period. This is the onset of the stage C superhumps. In this stage, the period is short and constant but is still longer than the orbital period. The typical time evolution of superhumps during superoutburst is shown in Figure 1.17.

1.6.2 Mechanism

Many theorists were struggling to explain the curious behavior of superoutbursts and superhumps. Even the MTB model was revived temporarily. For instance, Vogt (1982) proposed that enhanced mass transfer from the L_1 point could occur and that an eccentric ring is formed. Osaki (1985) considered irradiation-induced instability of mass overflow. In this confusion, the hydrodynamic simulations by Whitehurst (1988) brought a breakthrough. He proved that the accretion disk can become tidally unstable if the system has a mass ratio smaller than 0.25, and the disk develops into a precessing eccentric disk. Hirose, Osaki (1990) performed similar simulations and confirmed the tidal instability as did in Whitehurst (1988).

Whitehurst, King (1991) noticed the 3:1 resonance is important to cause eccentricity in the disk. At 3:1 resonance radius, a disk particle rotates around the WD three times while the secondary rotates once, and perturbations of flows in the radial direction are amplified. Lubow (1991) has shown that the 3:1 resonance drives eccentric instability in fluid disks. Here it is considered that one-armed oscillations are excited in the disk by 3:1 resonance (Hirose, Osaki, 1993). They correspond to the oscillations of the disk with $m = 1$, where m stands for the azimuthal index of the eigenfrequency of the mode. Once the disk becomes elliptical, the eccentric disk goes prograde precession. The superhump phenomenon is produced by tidal dissipation in the eccentric disk. Here the superhump period is the synodic period between the precession period of the eccentric disk and the orbital period, and it

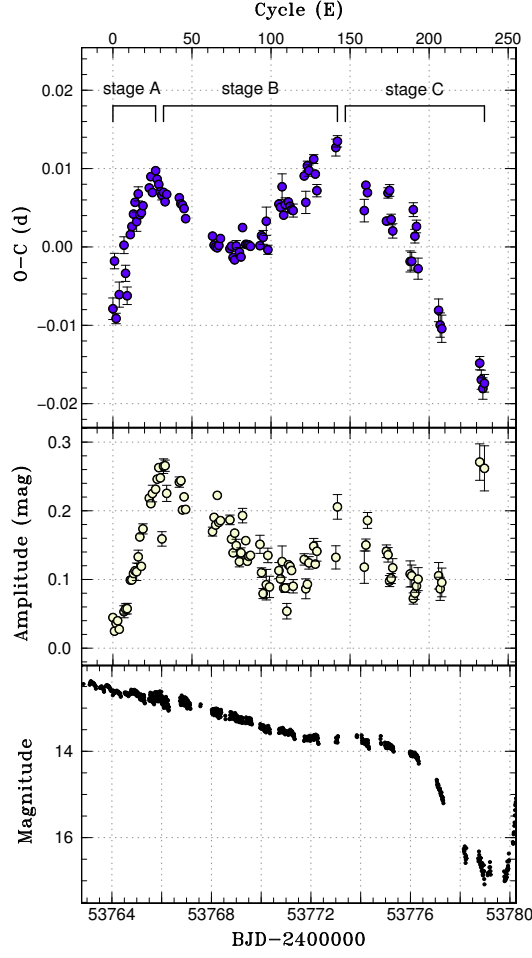


Figure 1.17: An example of the classification of superhumps by the variations of the period and amplitude with the actual observational data of the 2006 outburst in ASAS J102522–1542.4, a WZ Sge-type star, derived from Fig. 24 of Kato et al. (2012). Upper panel: $O - C$ curve of the times of superhump maxima. Middle panel: amplitude of superhumps. Lower panel: light curve. The horizontal axis in units of BJD and cycle number of superhumps is common to these three panels.

is given by

$$\frac{1}{P_{\text{sh}}} = \frac{1}{P_{\text{orb}}} - \frac{1}{P_{\text{pr}}}, \quad (1.45)$$

where P_{sh} and P_{pr} are the superhump period and the precession period, respectively.

Osaki (1989) obtained a hint from this work and explained the main feature of superoutburst by the thermal-tidal instability model (the TTI model). The thermal-tidal instability is the combination of the thermal instability for explaining normal outbursts and the tidal instability proposed by Whitehurst (1988). A schematic picture that helps us to understand the behavior of the thermal-tidal instability is given in Figure 1.18.

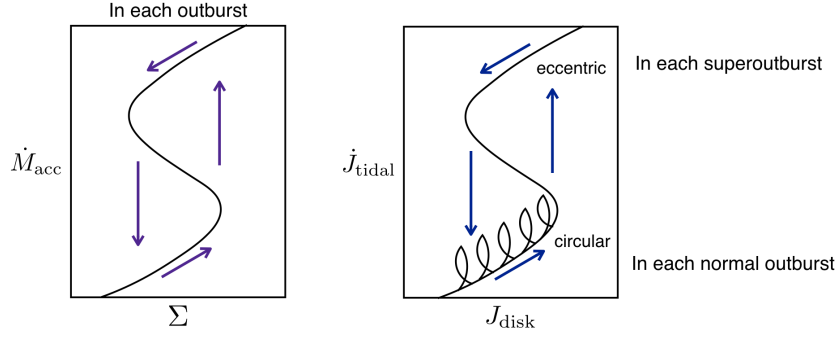


Figure 1.18: Schematic picture about the thermal-tidal instability. The left panel is the schematic figure of the thermal equilibrium curve like Figure 1.11 in normal dwarf-nova outbursts. Here $\dot{M}_{\text{acc}}$ is the mass accretion rate onto the central object. The right panel shows the relation between the total disk angular momentum (J_{disk}) and the angular momentum loss by tidal forces exerted by the secondary ($\dot{J}_{\text{tidal}}$).

If the disk is smaller than the 3:1 resonance radius defined as $a3^{(-2/3)}(1+q)^{-1/3}$, the disk is tidally stable. Then the disk stays at the bottom in the right plane in Figure 1.18, and the system enters normal outbursts by the thermal limit-cycle instability (see the left plane of Figure 1.18). The time evolution of the disk draws small loops around the lower branch in the right panel of Figure 1.18. Since normal outbursts drain small amount of mass from the disk, the leftover material is just stored in the disk. The disk mass and the disk angular momentum continue to increase while the system repeats normal outbursts. Naturally, the disk radius also increases gradually. If the accumulated mass becomes huge, the disk radius exceeds the 3:1 resonance radius when the disk enters next outburst. Although the disk has the tidal truncation radius, in the systems having mass ratios smaller than 0.25, the 3:1 resonance radius is located at the inside of the tidal truncation radius (Osaki, Meyer, 2002). The disk therefore can expand beyond the 3:1 resonance radius.

If the disk exceeds the 3:1 resonance radius, the eccentricity wave develops in the disk, since the tidal instability is triggered in addition to the thermal instability. Once the disk becomes elliptical, the removal of the angular momentum from the disk is enhanced by tidal forces exerted by the secondary star. The disk loses a lot of angular momentum and goes to the upper branch in the right panel of Figure 1.18. The disk is rapidly contracted, and a lot of mass at the outer disk is piled up, which keeps the outer disk in the hot state. That would give a good reason why the superoutburst is long-lasting and has large amplitude. During superoutburst, most of mass stored in the disk during normal outbursts accretes onto the white dwarf, which initializes the disk. The cycle of several normal outbursts and one superoutburst can be completed. Ichikawa et al. (1993) and Buat-Ménard, Hameury

(2002) performed one-dimensional simulations on the basis of this scenario and reproduced SU UMa-like behavior.

1.6.3 Refinements in the TTI model

The TTI model was not easily accepted at first because of one or two days delay of superhumps against the superoutburst maximum. Smak (1996, 2000) criticized that the TTI model cannot predict the delay of the appearance of superhumps, because he thought that superhumps should emerge in the very early stage of superoutbursts in the predictions by the TTI model. Osaki, Meyer (2003) proposed some refinements about the beginning of superoutbursts, and they are summarized in Osaki (2005). To come to the point, it takes around a few days to grow an eccentric disk after the disk radius reaches the 3:1 resonance radius.

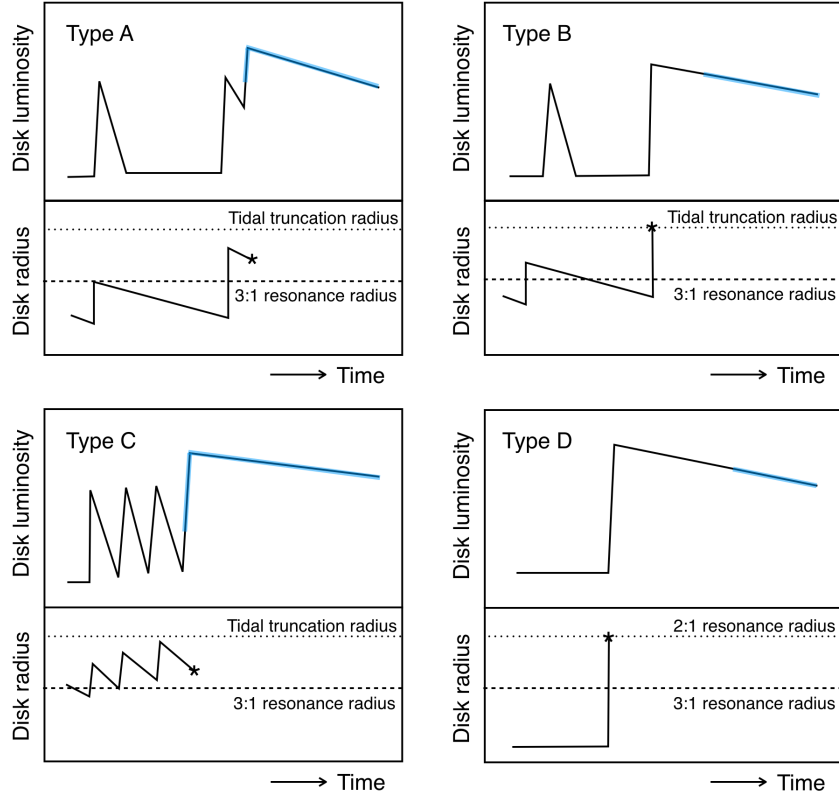


Figure 1.19: Schematic picture about 4 types of superoutbursts. The mark * indicates the start of superoutbursts. The blue color represents the epoch during which superhumps are observable.

Lubow (1991) showed the growth rate of an eccentric disk is inversely proportional to q^2 . Osaki (2005) classifies superoutbursts into 4 types according to the growth rate of the tidal instability. If the system has relatively high mass ratio (but smaller

than 0.25), the tidal instability wins to the propagation of the cooling wave when the disk radius exceeds the 3:1 resonance radius at the trigger of an outburst. The eccentric disk develops rapidly and the strong tidal dissipation brings the outer part of the disk to the hot state after it tends to drop to the cool state. The schematic picture about the time evolution of the disk radius and the disk luminosity around the early stage of superoutbursts in this case is displayed in the upper left panel of Figure 1.19. In this case, a precursor outburst is observed. This case is named Type A superoutbursts.

If the growth rate is not so large, the disk returns back to the cool state once even after the disk radius expands beyond the 3:1 resonance radius. Then the normal outburst fails to trigger a superoutburst. However, in the next normal outburst, the disk expands beyond again the 3:1 resonance radius and even reaches the tidal truncation radius. Then the tidal torques enhanced by the tidal truncation remove a lot of angular momentum from the outer part of the disk and keep the region hot. The system enters a plateau stage during which the eccentricity begins to grow. An eccentric disk fully develops a few days after the outburst maximum and the system experiences a long-lasting viscous plateau stage due to the tidal torques enhanced by the tidal instability (see the upper right panel of Figure 1.19). This case is called Type B superoutbursts.

When the growth rate is small, normal outbursts are barely switched to superoutbursts. The disk repeats a few normal outbursts, while the disk radius exceeds the 3:1 resonance radius. However, the disk finally cannot shrink below that radius even in quiescence, and hence, the tidal instability continues to operate. The disk becomes elliptical during the quiescent state and superhumps are excited. In this case, superhumps are growing at the rising part to a superoutburst. The lower left panel of Figure 1.19 illustrate Type C superoutbursts. The last normal outburst just before the superoutburst is like a big precursor.

If the mass ratio is very small and smaller than 0.08, the disk radius reaches the 2:1 resonance radius passing the 3:1 resonance radius (Osaki, Meyer, 2002). This case is called Type D superoutbursts, and the systems entering this type of superoutbursts are WZ Sge-type stars. In this case, superhumps appear at the late stage of superoutbursts (see the lower right panel of Figure 1.19).

1.6.4 WZ Sge-type dwarf novae

The extreme subclass of SU UMa-type DNe which shows only superoutbursts is called WZ Sge-type DNe, and these systems are considered to have very small mass ratios as mentioned above. Their outbursts have the following characteristic fea-

tures: (1) the amplitude is typically larger than that of superoutbursts in SU UMa stars, exceeding ~ 6 mag (Howell et al., 1995), (2) the interval between superoutbursts (supercycle) is unusually long, typically with a time scale of decades (Kato et al., 2001), (3) modulations with double-peaked humps called early superhumps are observed in the early phase of superoutbursts (Kato, 2002), and (4) after the end of the main superoutburst, single or multiple rebrightenings are observed (Imada et al., 2006). The outburst of the prototype object “WZ Sge” was discovered in 1913 (Leavitt, Mackie, 1919). The light curve of the 2001 superoutburst of WZ Sge is exhibited in Figure 1.20. The outburst interval in WZ Sge stars is very long, and hence, very low α values in the cool state and the very low mass transfer rate around 10^{15} g s^{-1} in comparison with 10^{16} g s^{-1} for ordinary SU UMa stars are needed. Recently, Kato (2015) provided a comprehensive review.

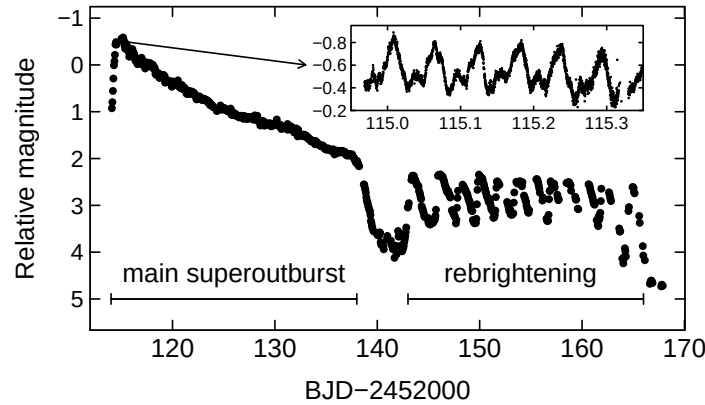


Figure 1.20: Overall light curve of the 2001 superoutburst of WZ Sge. In the small window, a part of early superhumps in the very early stage of the main outburst are plotted. The data was provided by the Variable Star Network (VSNET) cooperation.

Among the properties described above, early superhumps and rebrightenings ((3) and (4)) are considered to be unique to WZ Sge-type DNe (see Kato, 2015). An example of early superhumps is given in the small window in Figure 1.20. Osaki, Meyer (2002) proposed that early superhumps are the manifestation of the 2:1 resonance. The 2:1 resonance itself and its effect to the accretion disk in binary systems were already studied by Lin, Papaloizou (1979). Another name of the 2:1 resonance is the inner Lindblad resonance. If the binary mass ratio is extremely small (< 0.08), the tidal truncation radius is larger than the 2:1 resonance radius defined as $a(2^2(1+q))^{-1/3}$ (see also Figure 1.21). The 2:1 resonance is the eigenmode of the disk with $m = 2$ and excites two-armed spiral density waves in the disk. Then, strong dissipation will work in the disk twice while the secondary orbits around the primary once, which may produce the double-humped modulations. Thus the 2:1 resonance generates strong energy dissipation by tidal forces exerted

by the secondary as well as the 2:1 resonance. In this case, ordinary superhumps appear around two weeks after the start of superoutbursts, since the 2:1 resonance is considered to suppress the 3:1 resonance. Maehara et al. (2007) modeled two-armed spiral structures expected by the 2:1 resonance, and reproduced the light variations of early superhumps.

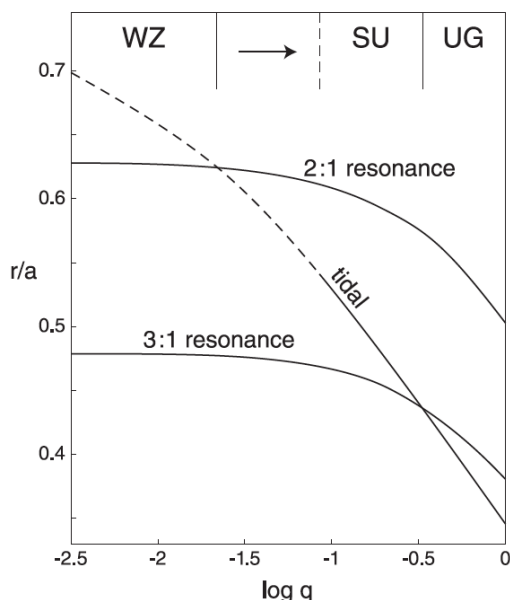


Figure 1.21: Relation between the tidal truncation radius, the 3:1 resonance radius, and the 2:1 resonance radius as a function of q , which is taken from Osaki, Meyer (2002). Here, UG, SU, and WZ represent U Gem-type stars, SU UMa-type stars, and WZ Sge-type stars, respectively. If the tidal force is somehow weak, the border between SU and WZ may move to the dashed line.

The mechanism for rebrightening is puzzling. Hameury et al. (2000) insisted that enhanced mass transfer due to the irradiated secondary makes rebrightening via their numerical simulations, but there is no evidence about enhanced mass transfer after the main superoutburst (Sec. 2.3 in Osaki, Meyer, 2003). Osaki et al. (1997) confirmed that the rebrightening phenomenon in EG Cnc, a WZ Sge star, can be achieved if the viscosity in the cool state of the disk temporarily rises for some reasons by some numerical simulations. In Osaki et al. (2001), they suggested that the temporarily increased viscosity decreases because of stochastic fluctuations of magnetic fields. Also, they proposed that the leftover mass at the outer disk beyond the 2:1 resonance radius feeds the inner disk due to the tidal removal of the angular momentum of that mass, if the eccentric form of the disk survives even after the main superoutburst, as well as Hellier (2001b). Uemura et al. (2008a) performed NIR and optical photometry and found the NIR excess from the standard

disk, which may be evidence for the mass reservoir. Recently, Meyer, Meyer-Hofmeister (2015) proposed that a part of various rebrightening phenomena can be explained by the hypothesis in which the inner disk is permanently hot after the main superoutburst together with the temporal increase of the viscosity is made possible by dynamo-created magnetic fields.

In WZ Sge-type stars, no precursor outburst is observed basically, while precursors easily appear in almost all of ordinary SU UMa-type stars. In ordinary SU UMa-type stars, the cooling wave propagates in the disk until the tidal instability fully develops. On the other hand, the mass stored in quiescence would be large enough to prevent the cooling front from being generated in WZ Sge-type stars. The 2:1 resonance may grow faster compared with the 3:1 resonance.

1.7 Classification of dwarf novae

Sec. 1.5 and Sec. 1.6 introduce only two types of DNe: U Gem stars above the period gap and SU UMa stars including WZ Sge stars below the period gap, but there are many other subclasses of DNe. The important parameters for classification of DNe are the orbital period and the mass transfer rate. The basic classification on the basis of the two parameters is given in Figure 1.22. Although the origin is unclear, there is a diversity in mass-transfer rates from the secondary star even for the group of DNe having similar orbital periods, which can be seen in Figure 1.8. Here the typical mass transfer rate above the period gap is in the range of $10^{-8} - 10^{-9} M_{\odot} \text{ yr}^{-1}$ and that below the period gap is less than $10^{-10} M_{\odot} \text{ yr}^{-1}$, respectively.

Above the period gap, the disk stays in the hot state when the mass transfer rate is higher the critical value (M_{crit}), which is indicated as the dot line in Figure 1.11. The disk luminosity keeps constant, and this kind of objects is called nova-like stars (NLs). If the mass transfer rate is intermediate between that of NLs and that of U Gem stars, the light curve alternates between frequent outbursts and the almost constant luminosity (see Figure 1.23). This subclass is called Z Cam-type DNe. These systems are interpreted to have mass transfer rates close to M_{crit} , and the systems are switched on and off NL states by fluctuations in mass transfer rates (Meyer, Meyer-Hofmeister, 1983), although there is no positive observational evidence.

Below the period gap, the objects show superoutbursts with superhumps. If the mass transfer rate is high enough to keep the disk hot, the disk radius always exceeds the 3:1 resonance radius and exhibit superhumps eternally. This type of

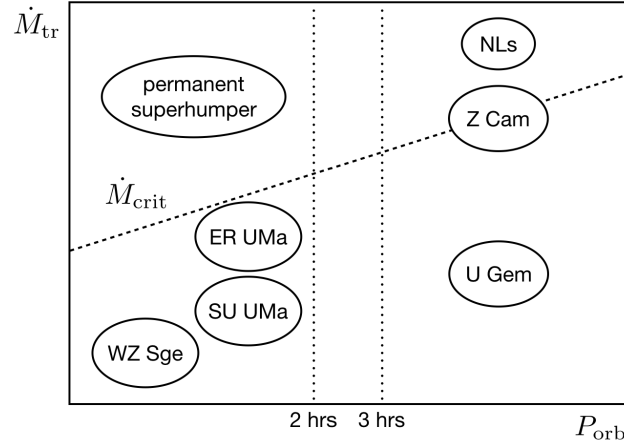


Figure 1.22: Classification of DNe. The dashed line represents the critical mass transfer rate above which the disk always stays in the hot state. The region sandwiched by dot lines is the period gap.

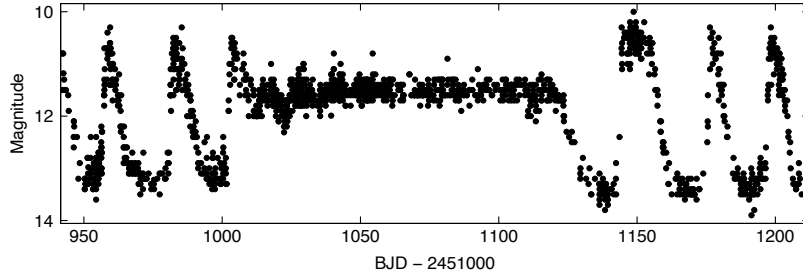


Figure 1.23: Visual observations of Z Cam taken by the AAVSO cooperation.

objects is called permanent superhumpers, corresponding to NLs having small mass ratios. There is a subclass called ER UMa-type DNe. They frequently show superhumps and the interval between superhumps is only 20–50 days (Kato, Kunjaya, 1995). The superhumps are observed even in quiescence. This group is regarded to have the mass transfer rate slightly lower than the critical rate (Osaki, 1995b).

1.8 Tilted accretion disks

1.8.1 Discovery

Before this section, the accretion disk is assumed to be parallel with the binary orbital plane. However, the disk is sometimes misaligned to the binary orbit and has tilted and/or warped structures. This has been confirmed in some of astronomical objects through observations. The observational evidence of tilted and/or warped

accretion disks was discovered in HZ Her, an XB, for the first time. Katz (1973) analyzed X-ray timing data of this object, and reported that it showed 35-days periodicity in the light variations, which is much longer than its orbital period (1.7 days). In this paper, it is suggested that the accretion disk in this object has a warped structure like Figure 1.24, and that the 35-day periodic light variations appear because of complex occultation of the central source radiating X-rays by the precessed outer disk misaligned against the orbital plane. The 35-days period corresponds to the period of the nodal and retrograde precession which the tilted disk experiences. This kind of light variations is called superorbital variations.

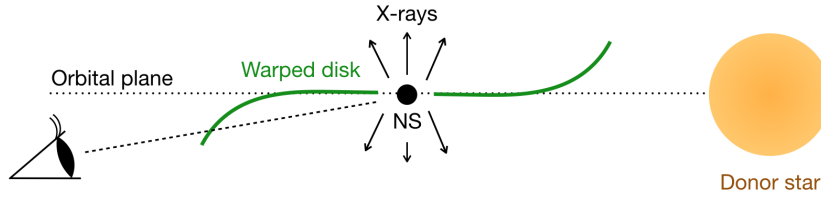


Figure 1.24: Schematic figure of the warped accretion disk in an XB, which is proposed by Katz (1973).

The mechanism with which the tilted and/or warped disk enters the nodal and retrograde precession is the same as that of the nodal precession of Moon, which is triggered by our sun. Provided that one particle rotates around the central source in a binary system, and that it is forced to oscillate in the vertical direction, the particle receives restoring force, which is the gravitational force of the central object and the secondary star. In this case, the total gravitational force is deviated from and a little stronger than that only by the central object, which is proportional to r^2 . As a result, the particle returns back to the orbital plane with a period slightly shorter than the orbital period. Here the precession period of the disk in the rest frame is much longer than the period with which the particle rotates around the primary star in the co-rotating frame of the binary.

After this pilot study, long-period variations similar to those in HZ Her were detected in some XBs (Lang et al., 1981; Margon, 1984), and Gerend, Boynton (1976) and Leibowitz (1984) built detailed model of geometrically tilted disks and applied for those systems for instance.

1.8.2 Negative superhumps

In the field of CVs, the observational features suggesting tilted accretion disks was discovered in TV Col for the first time by Motch (1981). This study detected 5.2-hrs and 4-days X-ray periodic variations in this object, while its orbital period is 5.5

hrs (Hutchings et al., 1981). The 5.2-hrs period is slightly shorter than the orbital period, this type of short-term periodic variations is called negative superhumps. This name originates from the comparison with ordinary superhumps observable in superoutbursts of SU UMa stars (Harvey et al., 1995), which are also called positive superhumps. After this discovery, negative superhumps were detected also at optical wavelengths in TT Ari, an NL (Semeniuk et al., 1987b,a). Negative superhumps are now detected in some CVs having relatively high mass transfer rates (e.g., Patterson et al., 1995; Pavlenko et al., 2010; Armstrong et al., 2013; Ohshima et al., 2014).

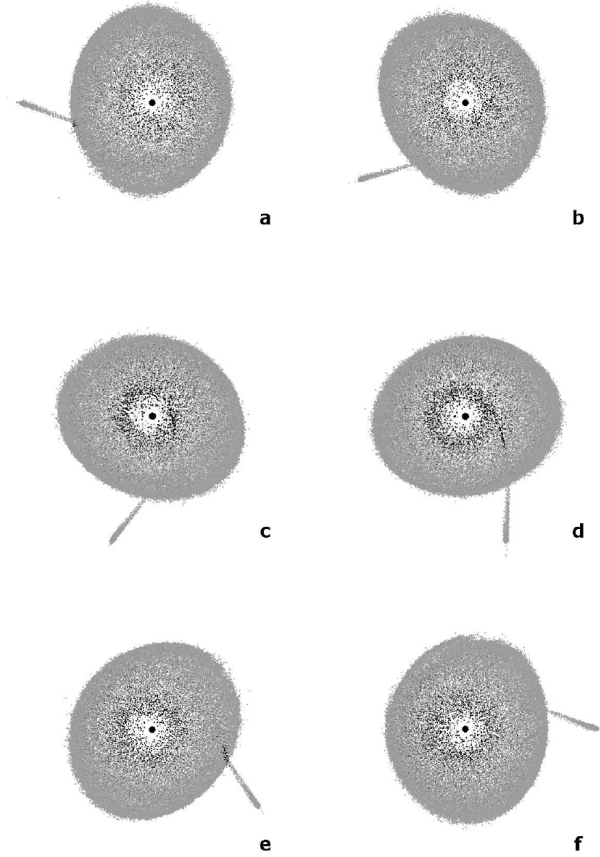


Figure 1.25: Schematic picture of the tilted accretion disk and the gas stream flowing into it in the inertial frame, which is taken from Wood, Burke (2007). During one-half period of negative superhumps, the bright spot moves from the outer disk edge to the inner part of the disk.

Bonnet-Bidaud et al. (1985) suggested that a retrogradely precessed disk is associated with the origin of negative superhumps, motivated by the works in XBs, which are introduced in the preceding subsection. The most plausible explanation for negative superhumps is currently that the transit of the bright spot on the

nodally precessed tilted and/or warped disk with the synodic period between the orbital period and the precession period of the disk (Wood et al., 2000). In this idea, the period of negative superhumps is given as

$$\frac{1}{P_{\text{nsh}}} = \frac{1}{P_{\text{orb}}} + \frac{1}{P_{\text{pr}}}, \quad (1.46)$$

where P_{nsh} is the period of negative superhumps. In the tilted disk, the position of the bright spot varies depending on the geometrical relation between the tilted disk and the secondary like Figure 1.25.

According to this interpretation, the 4-days light variations in TV Col are regarded as the variations of the projection area of the tilted disk to the line of sight. That long-term variability is the same as superorbital variations described in the previous subsection in that both of them originate from the nodal precession of the tilted disk, but the former comes from the tilted disk itself, while the latter is the occultation of the high-temperature source located in the vicinity of the central object.

Osaki, Kato (2013a) and Ohshima et al. (2014) found that negative superhumps coexist with positive superhumps in some superoutbursts in SU UMa-type stars. It is evident that negative superhumps are different from positive superhumps. Their results suggest that the geometry of accretion disks would be really complex if the tilted disk undergoes superoutbursts. Simply, their results mean that the disk can be tilted and eccentric. However, it would not be straightforward to cause retrograde and prograde precessions at the same time.

1.8.3 Various observational studies about tilted disks

After the discovery of several kinds of light variations suggesting tilted and/or warped accretion disks, many people have included weird geometry when they interpret their observations. The tilted and/or warped structures may not be persistent. It is suggested that the disk geometry including warped structures can vary on long timescales from appearance/disappearance of negative superhumps (Gies et al., 2013; Mason, Howell, 2016; Kato et al., 2019a). The changes in the phase-averaged profile of superorbital variations are also observed, but they seem to be interpreted as the changes in the illumination of the accretion disk by the warped inner part (Hung et al., 2010).

In addition, some people tried to interpret not only their photometric light curves but also their spectroscopic observational results by warped structures in the disk. For instance, Hynes et al. (2002) considered the changes in the projection area of warped accretion disks as one of possible interpretations for the abnormal behavior

of optical spectra during the 1999–2000 outburst of XTE J1859+226, a black-hole LMXB.

In the field of black-hole XBs, one particular type of quasi-periodic oscillations (QPOs) are regarded as the evidence of tilted hot coronae located at the innermost part of accretion disks (Ingram et al., 2009), although the case in which the outer part of the disk is misaligned to the orbital plane is introduced so far. In the vicinity of the black holes, Lense-Thirring precession, a kind of relativistic effects is efficient. If the spin axis of the black hole is misaligned from the rotational axis of the disk matter, the disk matter is dragged to the spin axis, which is a mechanism for the tilted hot coronae.

Accretion disks having warped structures are not limited to binary systems. There is evidence for warped accretion disks in some AGNs via the observations of water masers and radio spectral lines. (Herrnstein et al., 1996; Schinnerer et al., 2000). Also, warped structures are also detected in YSOs by direct NIR and/or radio imaging of the disk (Hashimoto et al., 2011; Sakai et al., 2019).

1.8.4 Possible mechanisms for disk tilt

As described in the precious three subsections, some observations highly suggest that a part of the disk or the entire disk in some binary systems is misaligned to the binary orbital plane. On the other hand, the physical mechanism for explaining how tilted and/or warped structures emerge is still unknown. In the case of XBs, instabilities in the vertical structure triggered by strong tidal forces by high-mass companion stars and/or the strong radiation pressure of the inner part of the disk and/or some relativistic effects are possible (Lubow et al., 1993; Pringle, 1996; Ingram et al., 2009). However, the secondary star is not a massive star, the radiation pressure is not dominant because of the lower-temperature inner disk, and relativistic effects are weak in CVs. The following three hypotheses survive now as the origin of disk tilt in the case of CVs, but they are not conclusive.

The 3:1 resonance which causes positive superhumps may amplify oscillations of a disk particle not only in the radial direction but also in the vertical direction. This idea was proposed by Lubow (1992b). However, smoothed particle hydrodynamics (SPH) simulations showed that this instability in the vertical direction cannot grow fast enough (Murray, Armitage, 1998).

Smak (2009) advocated that the stream-disk interaction maintains instability in the vertical direction. Once the disk is tilted against the orbital plane, the irradiated surfaces of the secondary below the plane and above the plane become

asymmetry. Then the gas stream from the secondary has non-zero velocity in the vertical direction. Since the tilted disk enters retrograde precessions, the impact of the gas stream on the disk surface is periodically variable and accelerates the disk matter.

Montgomery, Martin (2010) proposed that the disk is forced to be leaned by a kind of buoyancy, once it is tilted. If the gas stream reaches the disk, it flows over and under the disk. In the tilted disk, the pressure on the front face of the disk differs to that on the back face. The wider the stream line is, the slower its velocity is. Then the pressure becomes greater due to Bernoulli's theorem. The disk receives forces called "lift". The lift is the same force as that working on the wings of airplanes. Montgomery (2012) examined this idea by SPH simulations and reproduced negative superhumps.

1.8.5 Development of warped structures

Besides the mechanism for disk tilt, it is also unclear whether the disk is rigidly tilted or not. Some theorists have studied how warped structures develop. For instance, Papaloizou, Terquem (1995) investigated the perturbation in the protoplanetary disk around a T Tauri star in a young binary system, which is induced by tidal forces exerted by the companion star. They introduced the condition for rigid body precession as follows:

$$\frac{r\omega_P}{\frac{1}{3}c_s} < 1, \quad (1.47)$$

which means that bending waves crossing the disk propagate faster than precession waves. The disk sometimes has a steep temperature distribution in the radial direction, and then, all of the regions of the disk may not satisfy this condition. Basically, the outer part of the disk is harder to meet this condition.

If the disk is deviated from rigid tilt, the warped structures emerge and develop in the disk. Once warped structures are formed, the viscosity by radial pressure gradients is excited. Then, the warps propagate as bending waves when $\alpha < H/r$, and propagate diffusively when $\alpha > H/r$ (Nixon, King, 2016, for a review). This kind of study began in 1980s (Papaloizou, Pringle, 1983), and continues still now (Martin et al., 2019), since the evidence for tilted and/or warped accretion disks has been detected in many kinds of objects as mentioned in Sec. 1.8.3.

1.9 Recent progress in the study of dwarf novae

The development of optical telescopes and wide-field monitoring surveys, which started in 2010's, brings big progress in the study of DNe. Big data taken by them allow us to analyze the optical light variations in detail. There are some important topics for understanding the recent progress in the field of DNe.

1.9.1 Observational tests of the TTI model

Kepler space telescope launched in 2009 took high time-cadence photometric data (the sampling is surprisingly 30 seconds) of a lot of CVs, and advanced the study about SU UMa-type DNe, by enabling us to analyze short-term variations like superhumps in detail (Osaki, Kato, 2013a,b). One of their most important results is that they ruled out the EMT model and the pure thermal limit cycle model which some people still believe as the explanation of superoutbursts.

In the EMT model, the mass transfer rate increases because of irradiation on the surface of the secondary star by UV radiation from the WD fed by accretion. In this model, superhumps are thought to be produced by thermal dissipation of the gas stream because the mass transfer rate periodically changes (e.g., Smak (1991)). The pure thermal instability model was proposed by Cannizzo et al. (2010, 2012). They regard superoutbursts as the same phenomena as long outbursts in U Gem-type stars, and think that the tidal instability is not necessary in order to reproduce superoutbursts.

Osaki, Kato (2013a) found that superoutbursts and superhumps are entwined in V1504 Cyg, and that superhumps developed at the end of a normal outburst prior to the next superoutburst. These observational facts suggest no enhancement of mass transfer and favor the TTI model. Also, they estimated the variations of the disk radius by using negative superhumps which are observed in several outbursts of V1504 Cyg, and proved that they are consistent with the disk-radius variations predicted by the TTI model. Soon after this paper, Smak (2013) criticized these results, but Osaki, Kato (2014) addressed all of his criticisms.

1.9.2 Statistically-sophisticated methods for data analyses

Some sophisticated statistical methods began developping also in the field of astronomy during the same period, as the big data has been provided by new telescopes. Kato, Uemura (2012) applied the method called least absolute shrinkage and selection operator (LASSO) for the period analyses of the optical light curves of CVs.

This was the first practical application of LASSO in Astronomy. The LASSO analyses enable us to resolve several periods even from sparse time series data, even if the periods are close to each other. Currently, many people use LASSO in the study of CVs (Osaki, Kato, 2013b; Ohshima et al., 2014; Pavlenko et al., 2014). The key of the LASSO analyses is to adopt L1 norm as the penalty term, and surprisingly, this term is also implemented in the newest method for the imaging of black-hole shadows (Event Horizon Telescope Collaboration et al., 2019).

Also, it is worth noting that Bayesian inferences are useful these days thanks to the improvements of computers. For example, Uemura et al. (2010) estimated the intrinsic period distribution of WZ Sge-type DNe by applying Bayesian estimations for the observational population. They found that the estimated population has a spike-like feature and that the minimum orbital period is ~ 70 min, and proposed that WZ Sge stars are good candidates for the missing population in CVs. Moreover, Uemura et al. (2012) tried reconstructing the vertical structure of accretion disks by Bayesian approach at the early phase of superoutbursts in WZ Sge stars. They extracted the information about the disk structure only from optical light variations of early superhumps by assuming a smooth height distribution. Their reconstructed disks have two flaring arm-like structures, though one of them seems to be barely deformed by tidal effects.

1.9.3 New dynamical method for estimating binary mass ratios

After Kato et al. (2009), many statistical studies on the time evolution of superhumps were published (Kato et al. (2010, 2012), and so on). They enabled us to consider detailed interpretation of superhumps (e.g., Osaki, Kato (2013b)). (Kato, Osaki, 2013) finally provided the new method for estimating the mass ratio dynamically only by photometry (Kato, Osaki, 2013) on the basis of these works. They regard that the stage A superhumps are the representation of dynamical precession at the 3:1 resonance radius. In this hypothesis, the superhump period is determined only by the dynamical precession rate ω_{dyn} (Hirose, Osaki, 1990) expressed as

$$\omega_{\text{dyn}}/\omega_{\text{orb}} = \frac{q}{\sqrt{1+q}} \left[\frac{1}{4} \frac{1}{\sqrt{r}} b_{3/2}^{(1)} \right], \quad (1.48)$$

where $\frac{1}{2}b_{3/2}^{(1)}$ is the Laplace coefficient defined as

$$\frac{1}{2}b_{3/2}^{(1)}(r) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos(j\phi)d\phi}{(1+r^2-2r\cos\phi)^{3/2}}. \quad (1.49)$$

By substituting the formulation of the 3:1 resonance radius into equation (1.48), $\omega_{\text{dyn}}/\omega_{\text{orb}}$ becomes the function of q . The mass ratio can be derived from the

period of stage A superhumps and the orbital period. In the case of WZ Sge stars, early superhumps are useful for measuring the orbital period because their period is almost equal to the orbital period (Kato, 2015).

Before this method was developed, the period of stage B superhumps which varies with time and is shorter than the period of stage A superhumps was utilized for the mass-ratio estimation, so these estimations were underestimated. Nakata et al. (2013) pointed out that and re-estimated the mass ratio of several systems by using the new method by Kato, Osaki (2013). As a result, some objected are refused to be the candidates for period bouncers. Stage B superhumps do not represent dynamical precession and are regarded to be suffered from the pressure effect, which reduces the apsidal precession rate of the eccentric disk (e.g., Lubow (1992a); Hirose, Osaki (1993)). In other words, the precession rate of the disk is expressed as $\omega_{\text{pr}} = \omega_{\text{dyn}} + \omega_{\text{pressure}} + \omega_{\text{stress}}$ (Lubow, 1992a). Here, ω_{stress} is the contribution by minor wave-wave interaction. In this equation, ω_{pressure} is always negative.

1.9.4 Discovery of a lot of WZ Sge stars

Thanks to large optical surveys like All-Sky Automated Survey for Supernovae (ASAS-SN) survey, Catalina Real-Time Transient survey (CRTS), and so on, more than 100 WZ Sge-type DNe were identified (Kato, 2015). For example, it was unveiled that the rebrighenings peculiar to WZ Sge-type DNe are classified into 5 types: type-A (long-lasting rebrighening), type-B (multiple rebrighenings), type-C (single and short rebrighening), type-D (no rebrighening), and type-E (double superoutbursts) as shown in Figure 1.26.

It was also suggested that the rebrighening types are associated with the evolutionary path of WZ Sge-type stars. As mentioned above, WZ Sge-type stars are important objects to understand the final stage of the CV evolution, since their population likely explain the missing population of CVs. For example, Nakata et al. (2014) suggested the WZ Sge-type DNe entering superoutbursts with type-B rebrighening may be the best candidates for the missing population. Kato (2015) proposed that the rebrighening type indicates a kind of evolutionary sequence (type C $\rightarrow$ D $\rightarrow$ A $\rightarrow$ B $\rightarrow$ E), although there are some outliers in the objects exhibiting type-B rebrighening (see also Fig. 17 in Kato (2015)).

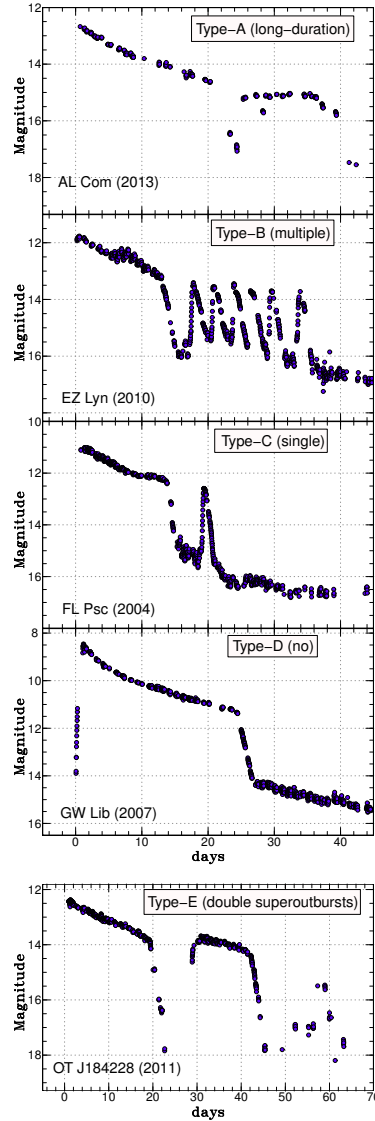


Figure 1.26: Classification of rebrightenings in WZ Sge-type DNe by their morphology. The observational data are taken from Fig. 6, 7, 8 and 9 of Kato (2015). The horizontal axis represents days from the starting date of their outbursts.

1.10 Aim of this study

As reviewed above, the mechanism of normal outbursts and superoutbursts are basically explained in the frame of disk-instability model. Hereafter the term “the disk instability model” includes the thermal-instability model and the TTI model. The rush in the study of DNe seems to have passed in 1990’s, and the disk instability model seemed to be accepted as the unified model for dwarf-nova outbursts. However, more and more light variations unexpected from the past studies about

the disk instability have been discovered simultaneously as the number of the identified CVs increases thanks to the development of time-domain astronomy. Currently, there are even some people who disagree with the disk instability model and propose other models instead of it. Can the disk instability model explain a rich variety of dwarf-nova outbursts? In order to answer this question, I have aimed at testing the disk instability model via observations and numerical simulations by examining whether unexpected light variations in DNe, which were recently detected, can be reproduced by using that model.

As described in Sec. 1.2, DNe have the simplest accretion disks among many kinds of objects harboring accretion disks, and transient events like outbursts are universal among them. The fact that existing models cannot explain all kinds of dwarf-nova outbursts means that nobody understands well the accretion physics potentially common in many objects even for the simplest systems. Studying dwarf-nova outbursts is also important to comprehend accretion events in our universe systematically.

In addition, there is another benefit in studying dwarf-nova outbursts. It is known that some of DNe like WZ Sge-type stars and outlier objects are closely related to unsolved problems in the CV evolution as mentioned before. Estimating their binary parameters and/or identifying their evolutionary paths through observations of their outbursts are helpful for discussing widely the binary evolution. Actually, the components of DNe would be the same as those of the progenitor of type-Ia supernovae, and DNe cannot avoid entering nova eruptions in their long-term evolution. In the long history where one CV evolves during 10^9 – 10^{10} yrs, DNe would experience several subclasses of CVs. Thus the study of dwarf-nova outbursts may give some impacts beyond the research field on DNe.

This dissertation addresses totally four kinds of unexpected light variations in DNe in order to understand better the accretion physics in DNe and the CV evolution. Chapter 2 treats the nature of rebrightening on the basis on the published paper Kimura et al. (2016b). Chapter 3 is integrated of the two publications Kimura et al. (2016a) and Kimura et al. (2018a), and deals with the outburst properties of the candidates for period bouncers. Chapter 4 handles the nature of rare outbursts in long-period objects on the basis of the published paper Kimura et al. (2018b). Chapter 5 is based on the very recently accepted paper Kimura et al. (2019), and is dedicated to examine the behavior of the thermal-viscous instability in tilted accretion disks and discuss the mechanism of IW And-type light variations. The studies in chapters 2, 3, and 4 are observational works and the research in chapter 5 is simulation works. Chapter 6 presents general discussion about what I achieved through these studies, implications about accretion physics in other kinds of ob-

jects, and future perspective. Chapter 7 draws conclusions. Since chapters 2–5 are based on my first-author papers including many co-authors, “we” is used as the term for the first person in these chapters.

In our observational studies, my role was to conduct the observational campaigns of our targets and to gather the observational data, and to perform data analyses and to interpret the results. Many observers in the Variable Star Network (VSNET) team helped me a lot. Daisaku Nogami and Taichi Kato discussed with me and taught me how to analyze photometric and spectroscopic data as my supervisors. In our simulation work, my role was to build numerical codes, to perform simulations, to interpret the results, and to compare them with the recent observations. Yoji Osaki and Taichi Kato shared with me a lot of insightful ideas, which are the starting point of our study. Shin Mineshige gave me many advises about coding.

Unexpected superoutburst and rebrightening of AL Comae Berenices in 2015

2.1 Introduction

In 2015, we detected one outburst deviated from the standard picture of WZ Sge-type DNe, which we described above, in AL Com, a WZ Sge-type DN, which is known as “a perfect twin” of WZ Sge. On 2015 March 4.582 UT, Kevin Hills reported an outburst of AL Com at a magnitude of $V = 14.528$ (vsnet-alert 18377). Immediately after this report, the Variable Star Network (VSNET) collaboration team (Kato et al., 2004) started a worldwide photometric campaign. Surprisingly, the 2015 outburst turned out to be a superoutburst, despite the fact that the brightness was much fainter than those in past superoutbursts and that the previous superoutburst occurred only ~ 450 d before. This feature is deviated from the basic properties of WZ Sge-type superoutbursts explained in Sec. 1.6.4. Also, the rebrightening was the same as that in the previous outburst of this object in 2013.

We thought the peculiar behavior described as above may be a hint for comprehending the nature of rebrightening whose mechanism is still unclear, and launched a world-wide observational campaign. In Sec. 2.2, we give the details of our observations. In Sec. 2.3, we furnish our methods for data analyses. The results are shown in Sec. 2.4, and we discuss them in Sec. 2.5. Finally we summarize our work in Sec. 2.6.

2.2 Observations

Time-resolved CCD photometry was carried out by the VSNET collaboration team at seven sites. Table A1 shows the log of photometric observations. We also used the data downloaded from the AAVSO archive. All of the observation times were converted to Barycentric Julian Date (BJD). The magnitude scales of each site were adjusted to that of the Kolonica Saddle system (DPV in Table A1). The constancy of the comparison star was checked by nearby stars in the same images.

2.3 Methods for Period Analyses

We performed a period analysis by using the Phase Dispersion Minimization method (PDM: Stellingwerf, 1978). In order to subtract a global trend from the light curve, we used locally weighted polynomial regression (LOWESS: Cleveland, 1979). The 1σ errors for the PDM analysis were calculated by the methods in Fernie (1989) and Kato et al. (2010). In estimating the robustness of the PDM result, we used a variety of bootstraps. We made 100 samples, each of which includes randomly the 50% of observations, and performed PDM analyses for the samples. The result of the bootstrap is represented in the form of 90% confidence intervals in the resultant θ statistics.

Also, we calculated $O - C$ of the maxima of superhumps. The $O - C$ curve represents the time difference of superhump periods. We can calculate the $O - C$ as subtracting the predicted timing of superhump maxima defined as the pre-specified constant superhump period multiplied by the number of superhump cycle from the observational superhump time of superhump maxima.

2.4 Results

Figure 2.1(a) represents the overall light curve of the 2015 outburst of AL Com. As can be seen in Figure 2.1(a), the duration of the plateau stage is ~ 10 d. This duration is significantly shorter than those of previous superoutbursts, for which the plateau stages lasted for 20 d or more (Kato et al., 1996; Ishioka et al., 2002; Uemura et al., 2008b; Kato et al., 2014a). In addition, the maximum magnitude was 14.1 mag, about 2 mag fainter than those of previous superoutbursts. It also should be noted that the superoutburst was accompanied by a precursor (BJD 2457086; see Figure 2.1(b)). The 2015 superoutburst of AL Com is the first case that a precursor was observed in WZ Sge-type DNe. After the termination of the

plateau stage, AL Com underwent a rebrightening. The present rebrightening was well classified as the type-A rebrightening (see also Kato, 2015).

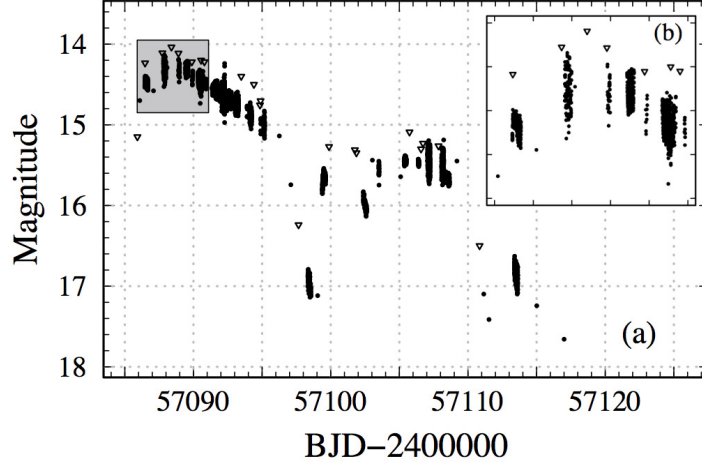


Figure 2.1: (a) Overall light curve of AL Com (BJD 2457085 – 2457120). (b) Enlargement light curve around the precursor (BJD 2457086 – 2457091). It is the enlarged view of the shaded box in panel (a). The black circles and inverted triangles represent CCD photometric observations and visual observations, respectively.

Figure 2.2(a) shows an enlarged light curve on BJD 2457089, on the first night when superhumps appeared. In combination with Figure 2.1(a), the growth time scale of the superhumps can be determined as ~ 3 d, an unprecedentedly short value among WZ Sge-type dwarf novae. Despite careful analyses before the appearance of the superhumps, no evidence for early superhumps was present. The present superoutburst of AL Com is also the first example in which early superhumps are absent in a superoutburst of WZ Sge-type DNe. In Figure 2.2(b), an enlarged light curve on BJD 2457105 is depicted. We can see the regrowth of superhumps after the temporary disappearance of superhumps around the dip in the rebrightening phase in Figure 2.2(b).

The superhump period (P_{sh}) during the plateau stage was 0.0573185(11) d which well agrees with those of previous superoutbursts (Nogami et al., 1997; Kato et al., 2014a). The resultant Θ diagram is shown in the upper panel of Figure 2.3. Also, we show the $O - C$ curve of superhump-maximum timings in the upper panel of Figure 2.4. The light curve during BJD 2457089.3 – 2457095.5 is shown in the lower panel of Figure 2.4. In the upper panel of Figure 2.4, the $O - C$ curve does not seem to be linear. Therefore, we regard superhumps in the plateau stage as stage B superhumps (Kato et al., 2009). We could not find stage A and stage C superhumps in the upper panel of Figure 2.4. The time derivative of the superhump

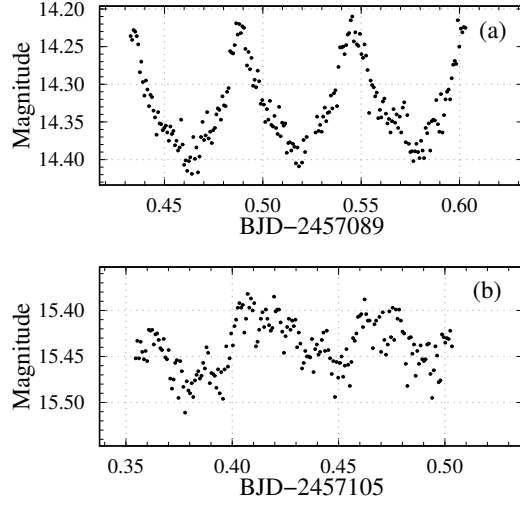


Figure 2.2: Light curves of superhumps: (a) superhumps in the main outburst (BJD 2457089.40 – 2457089.65) and (b) those in the rebrightening (BJD 2457105.35 – 2457105.50).

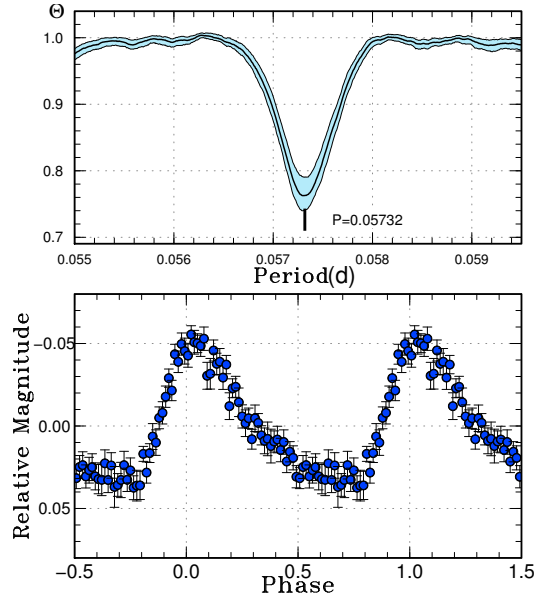


Figure 2.3: Superhumps in the plateau stage of the 2015 outburst of AL Com (BJD 2457089.3 – 2457095.5). The upper panel represents Θ -diagram of our PDM analysis. The lower panel represents a phase-averaged profile.

period $P_{\text{dot}} = +1.5(3.1) \times 10^{-5} \text{ s s}^{-1}$ was recorded in stage B. The estimated values of P_{sh} and P_{dot} agree with those obtained in previous superoutbursts (Nogami et al., 1997; Kato et al., 2009).

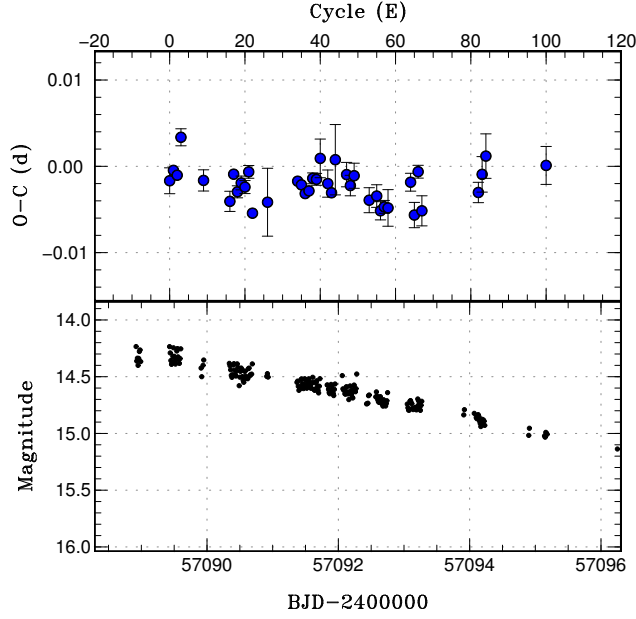


Figure 2.4: The upper panel represents the $O - C$ curve of the superhump-maximum timings of AL Com (BJD 2457089.3 – 2457095.5). An ephemeris of BJD 2457089.432681+0.057318 E was used for drawing this figure. The lower panel represents the light curve during BJD 2457089.3 – 2457095.5. The horizontal axis in units of BJD and cycle number is common to both of upper and lower panels.

2.5 Discussion

2.5.1 Shortest interval between superoutbursts

One of the most surprising results is that AL Com exhibited a superoutburst with a recurrence time of ~ 450 d, despite the fact that the AL Com is known as a WZ Sge-type DN having ~ 6 -yr intervals between superoutbursts over the past 20 years. WZ Sge stars having a short supercycle include AL Com and EZ Lyn. EZ Lyn had the shortest recorded interval between superoutbursts of ~ 4 yr on the basis of the reported superoutbursts (Pavlenko et al., 2012; Isogai et al., 2015). The 2015 superoutburst of AL Com broke and set a new shortest record of intervals between superoutbursts among WZ Sge-type DNe. At present, the reason why AL Com underwent a superoutburst with such a short interval is unclear; it was likely disk instability was somehow triggered earlier than in past superoutbursts. The disk in quiescence is considered to have very low viscosity because of the very long recurrence time of outbursts. For example, the viscosity of the disk may remain high after the 2013 superoutburst for a long time.

2.5.2 Absence of precursor and early superhumps

It is worth noting that a precursor was observed for the first time in WZ Sge-type DNe (see Figure 2.1(b)). In spite of good coverage around the onset of outbursts, the absence of a precursor in WZ Sge-type DNe is confirmed in e.g., GW Lib (Hiroi et al., 2009), V455 And (Matsui et al., 2009), and WZ Sge itself (Ishioka et al., 2002). These systems show large-amplitude and long-lasting superoutbursts compared with the present superoutburst of AL Com. According to the thermal-tidal instability model, the disk radius of the main superoutburst in which precursors are observed critically exceeds the 3:1 resonance radius during the precursor outburst (see, figure 4 of Osaki, Meyer, 2003).

It is also noteworthy that the present superoutburst lacked early superhumps. In other words, superhumps appeared ~ 3 d from the onset of the superoutburst. The appearance of the superhumps in the present superoutburst was about a week earlier than those in previous superoutbursts. In fact, Howell et al. (1996) reported that it took 10 d for superhumps to develop during the 1995 superoutburst. According to Lubow (1991), the growth time of the tidal instability is inversely proportional to the square of the mass ratio as mentioned in Sec. 1.6.3. Osaki (1995a) also followed this interpretation. Instead, Osaki, Meyer (2002) and Osaki, Meyer (2003) proposed that early superhumps are manifestation of the 2:1 resonance radius and that the 2:1 resonance suppresses the development of superhumps as described in Sec. 1.6.4. The observed precursor and the lack of early superhumps can be explained as the result of condition that the maximum radius is below the 2:1 resonance. The striking difference of the time lag until ordinary superhumps appear on the same object (AL Com) between the 2013 and 2015 superoutbursts indicates that the mass ratio cannot be the main cause of the long delay of appearance of ordinary superhumps in WZ Sge-type DNe. We can interpret that the 2:1 resonance (and its manifestation, early superhumps) suppresses the development of ordinary superhumps.

2.5.3 Same type of rebrightening in the same object

Although the plateau stage of the present superoutburst was similar to those of SU UMa-type DNe, a long rebrightening (type-A rebrightening) was observed in the present outburst of AL Com. AL Com showed the type-A (or type A/B) rebrightening also in all of the previous superoutbursts (Nogami et al., 1997; Ishioka et al., 2002; Uemura et al., 2008b; Kato et al., 2014a). In the 2015 outburst, the rebrightening type was the same as previous ones in spite that the stored disk mass just before the outburst should have been much less than previous ones,

which means that the rebrightening type of AL Com is the same regardless of the extent of outburst. It may be possible that the matter remained outside the 3:1 resonance if the radius of the 3:1 resonance is well inside the Roche lobe. The uniqueness of the type of the rebrightening within the same object is recorded not only in AL Com but also in other WZ Sge-type DNe. For example, EZ Lyn showed only type-B rebrightening. Another example is the 2003–2004 and 2013 superoutbursts of UZ Boo, which showed only type-B rebrightening (Kato, 2015). Kato et al. (2014a) studied the relation between P_{dot} and P_{orb} in terms of the type of the rebrightening, in which each type of the rebrightening pattern is clustered on the $P_{\text{dot}} - P_{\text{orb}}$ diagram (see figure 83 of Kato et al. (2014a)). In conjunction with the present study and the $P_{\text{dot}} - P_{\text{orb}}$ diagram, the type of the rebrightening seems to be inherent in the same system.

2.6 Summary

We have reported unusual superoutburst and rebrightening of AL Com in 2015 March. We summarize the most important results of our observations as follows:

- AL Com showed a superoutburst with an interval of ~ 450 d. This interval is the shortest record among WZ Sge-type DNe.
- The light curve showed a precursor and lacked early superhumps, which is the first example in the superoutbursts of WZ Sge-type dwarf novae. The presence of the precursor and the lack of early superhumps suggest that the maximum disk radius of this superoutburst barely exceeded the 3:1 resonance and did not reach the 2:1 resonance radius following the theory proposed by Osaki, Meyer (2002) and Osaki, Meyer (2003).
- AL Com showed the type-A rebrightening in the present superoutburst, despite the fact that the main superoutburst mimicked the typical one in ordinary SU UMa-type dwarf novae, and despite that the mass stored in the accretion disk at the onset of the superoutburst is supposed to be much smaller than those in previous superoutbursts of AL Com.

EG Cnc, one of the famous WZ Sge-type stars (Patterson et al., 1998), entered an outburst in 2018 after the 22 years of dormancy, and it showed completely the same rebrightening as that in the previous superoutburst in 1996 in spite of the shorter plateau stage of the main superoutburst. The idea proposed above is thus strengthened by latest observations.

Outburst properties of possible candidates for period bouncers

3.1 Introduction

As described in Sec. 1.3.3, the CV evolution has some unsolved problems, and one of them is the gap between the observational population and the theoretical prediction of period bouncers, which is called the missing population problem. In the study for seeking the missing population of post-period minimum CVs, it is worth noting that Littlefair et al. (2006) demonstrated with their spectroscopy that the donor star in the eclipsing CV SDSS J103533.03+055158.4 with a period close to the minimum one was a brown dwarf, suggesting that this object is a period bouncer. Moreover, Littlefair et al. (2008) found three more eclipsing CVs possess brown dwarfs as the secondary star using high-speed, three-color photometry. It was demonstrated that one of the systems has a very low-mass brown-dwarf companion by spectroscopic observations (Hernández Santisteban et al., 2016). Besides these works, Unda-Sanzana et al. (2008) found a CV which would have a brown-dwarf companion and a very low mass ratio. Aviles et al. (2010) also detected a brown-dwarf binary with a likely small mass ratio.

Recently, other several good candidates for a period bouncer have been discovered among WZ Sge-type DNe with type-B rebrightening or type-E rebrightening or slow fading rate (the objects and references are summarized in Table 3.1) through optical photometry using the new method of estimating the mass ratio (q) of the secondary to the primary with the period of stage A superhumps and the orbital period (Kato, Osaki, 2013). These period-bouncer candidates have some common properties: (1) long-lasting stage A superhumps, (2) a large decrease of the superhump period between stage A and stage B, (3) a small superhump amplitude ($\lesssim 0.1$ mag), (4) long-lasting early superhumps, i.e., a long delay of ordinary superhump apparance

($\gtrsim 10$ d), (5) slow fading rates at the plateau stage of superoutburst with ordinary superhumps, and (6) a large outburst amplitude at the time of appearance of ordinary superhumps (Table 3.1; subsection 7.8 of Kato (2015)). Kato (2015) also identified the relation between the rebrightening types and the CV evolutionary stage (WZ Sge-type stars would evolve in the order of type C $\rightarrow$ D $\rightarrow$ A $\rightarrow$ B $\rightarrow$ E) and the two objects with type-E rebrightening (see Table 3.1) are the best candidates for a period bouncer.

Here we present the observational results of ASASSN-15jd, ASASSN-16dt, and ASASSN-16hg, newly discovered possible candidates for the period bouncer in 2015–2016, and consider the relation between their evolutionary paths and outburst properties. Also, we summarize the properties of outbursts in the candidates including the objects discovered before this work. We corrected optical photometric data of the WZ Sge-type superoutbursts in the three systems and analyzed them as performed in the previous study in chapter 2. In Sec. 3.2, we explain our methods for observations and analyses. We report the results for each three object in Sec. 3.3–3.5. In Sec. 3.6, we discuss them including the outburst properties of other candidates for period bouncers, and in Sec. 3.7, we put a summary.

3.2 Observations and analyses

Time-resolved CCD photometric observations were performed by the Variable Star Network (VSNET) collaboration team. The logs of the observations of ASASSN-15jd, ASASSN-16dt, and ASASSN-16hg are given in Tables A2, A8, and A5, respectively. In this study, the data from the American Association of Variable Star Observers (AAVSO) archive are also contained. We converted all of the observation times to barycentric Julian date (BJD). We applied zero-point corrections to each observer by adding constants before making the analyses. The constancy of each comparison star was checked by nearby stars in the same images. The data reduction and the calibration of the comparison stars were performed by each observer. The magnitude of each comparison star was measured by the AAVSO Photometric All-Sky Survey (APASS Henden et al., 2016) from the AAVSO Variable Star Database*. As for the data analyses, we used the same method as described in Sec. 2.3.

*<http://www.aavso.org/vsp>

3.3 ASASSN-15jd

3.3.1 Overall light curve

We found the intermediate superoutburst between single and double ones for the first time in 2015 in ASASSN-15jd. The overall light curve of the 2015 superoutburst of ASASSN-15jd is displayed in Figure 3.1. The superoutburst probably began on BJD 2457155 and the object showed a rapid rise at the very early stage. The first plateau stage continued for about ten days during BJD 2457155–2457164.4 and a small dip whose depth is about 1 mag was observed on BJD 2457165. Immediately after the dip, the object showed a rapid rise again and the second plateau stage started. It continued for about a week during BJD 2457165.2–2457172.4 and a rapid fading was seen on 2457174. There were no observations during BJD 2457178–2457186. On BJD 2457186, the onset of a rebrightening was detected (vsnet-alert 18815). The duration of the rebrightening was short, a few days.

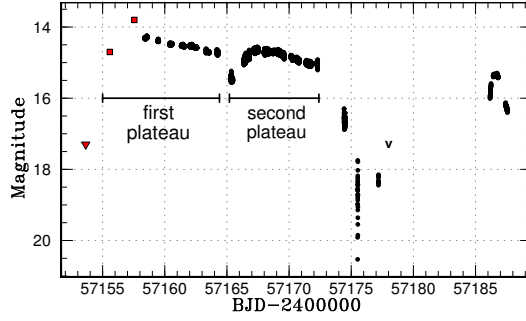


Figure 3.1: Overall light curve of the 2015 superoutburst of ASASSN-15jd (BJD 2457153–2457188). The circles and ‘V’-shapes represent CCD photometric observations and upper limits by KU1, respectively. The quadrangles and inverted triangles represent the detection and upper limits by ASAS-SN, respectively.

3.3.2 Superhumps

During the dip (on BJD 2457165), ordinary superhumps started to develop. The $O - C$ curve of times of superhump maxima, the amplitude of superhumps and the light curve during BJD 2457165.2–2457172.4 are shown in the upper panel, the middle panel and the lower panel of Figure 3.2, respectively. We determined the times of maxima of ordinary superhumps in the same way as in Kato et al. (2009). Some points with large errors were removed in calculating the $O - C$ and amplitude. The resultant times are given in Table A3. Although we could not clearly identify the term of stage A only from the $O - C$ curve, we regarded BJD

2457166.2–2457167.6 ($0 \leq E \leq 16$) as the final part of stage A, judging from the variations of the superhump amplitudes. Also, we considered that the term of stage B was BJD 2457167.9–2457172.4 ($24 \leq E \leq 90$) from the nonlinear behavior on the $O - C$ plane and the slow decrease of superhump amplitudes. The stage B superhumps might continue after BJD 2457172.4 although we could not detect it because of the sparse coverage. The superhumps disappeared at the fading stage on BJD 2457174 and did not develop again in the rebrightening at least in our low accuracy observations and we could not find stage C during the second plateau stage and after that.

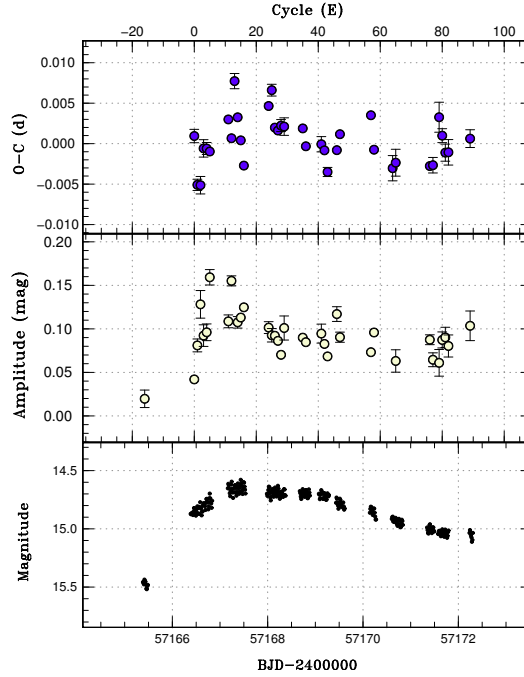


Figure 3.2: Upper panel: $O - C$ curve of the times of superhump maxima of during BJD 2457165.2–2457172.4 (the second plateau stage of ASASSN-15jd). An ephemeris of BJD 2457166.459117+0.0650258 E was used for drawing this figure. Middle panel: amplitude of superhumps. Lower panel: light curve. The horizontal axis in units of BJD and cycle number is common to these three panels.

We applied a period analysis by the PDM method for stage B (BJD 2457167.9–2457172.4) and obtained a period of $P_{\text{sh}} = 0.0649810(78)$ d (see upper panel of Figure 3.3). Here, the data having low accuracy were excluded from the light curve when we performed our PDM analysis. The derivative of the superhump period during stage B was $P_{\text{dot}} (\equiv \dot{P}_{\text{sh}}/P_{\text{sh}}) = 10.8(3.8) \times 10^{-5} \text{ s s}^{-1}$. The mean profile of stage B superhumps is also shown in the lower panel of Figure 3.3.

Early superhumps, double-wave modulations with a period almost the same as the orbital one usually appear prior to superhumps during superoutbursts of WZ Sge-

type DNe (Kato, 2002; Ishioka et al., 2002). However, during the first plateau stage of the 2015 outburst of ASASSN-15jd (during BJD 2457155–2457164.4), any humps with an amplitude of >0.02 mag were not observed in the range of 97.5–99.5% of the estimated superhump period in the previous paragraph.

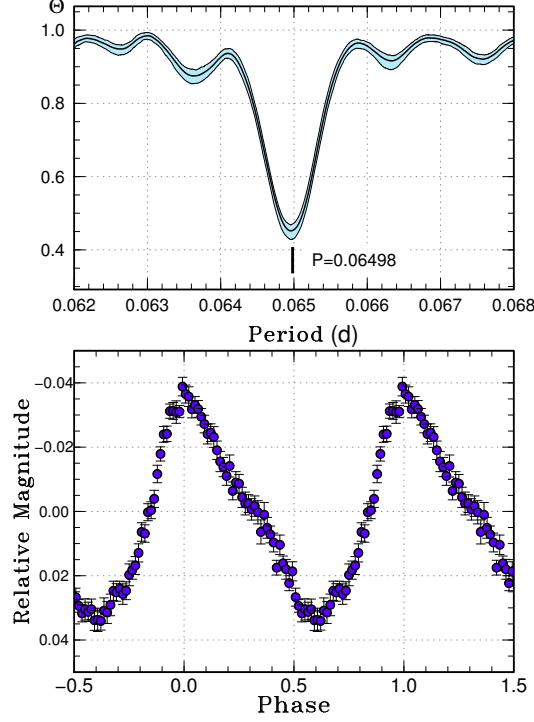


Figure 3.3: Superhumps in the second plateau stage of the 2015 outburst of ASASSN-15jd. Upper: Θ -diagram of our PDM analysis of stage B superhumps (BJD 2457167.9–2457172.4). Lower: Phase-averaged profile of stage B superhumps.

3.4 ASASSN-16dt

3.4.1 Overall light curve

We show the overall light curve of the 2016 superoutburst of ASASSN-16dt in Figure 3.4. The superoutburst probably began on BJD 2457479 and the object showed a rapid rise at the very early stage. A first plateau stage continued for at least 15 days during BJD 2457482.1–2457497.1. A dip of brightness was observed in the days BJD 2457499 and BJD 2457500. A rapid increase in brightness was observed for the following ~ 2 days, and the second plateau stage continued for about two weeks during BJD 2457504.0–2457516.8. A rapid fading was seen on BJD 2457518.

There were no observations during BJD 2457524–2457530. A rebrightening was detected for a few days during BJD 2457531.8–2457534.5.

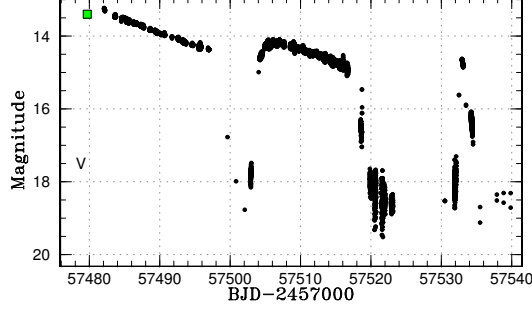


Figure 3.4: Overall light curve of the 2016 superoutburst of ASASSN-16dt (BJD 2457478–2457540). The ‘V’-shape and quadrangle represent the upper limit and the detection by ASAS-SN, respectively.

3.4.2 Early superhumps

Before the rapid decrease on BJD 2457499, double-waved modulations with a constant period, 0.06420(2) d, were detected. They lasted during BJD 2457482.1–2457493.0. After BJD 2457483, the humps became noisy. We regard them as early superhumps. Figure 3.5 represents the results of the PDM analysis and the phase-averaged profile of the early superhumps.

3.4.3 Ordinary superhumps

During the dip (on BJD 2457502), ordinary superhumps started to develop. The $O - C$ curve of times of superhump maxima, the amplitudes of superhumps, and the light curves during BJD 2457502.8–2457522.1 are shown in the upper panel, the middle panel and the lower panel of Figure 3.6, respectively. We determined the times of maxima and amplitudes of ordinary superhumps in the same way as in Kato et al. (2009). Some points with large errors were removed in calculating the $O - C$ and amplitudes. The resultant times are given in Table A6. We regarded the term of stage A as BJD 2457502.8–2457506.8 ($0 \leq E \leq 58$) from both the $O - C$ curve and the variations of the superhump amplitudes. We determine the term of stage B as being BJD 2457506.8–2457516.8 ($62 \leq E \leq 214$), from the nonlinear behavior on the $O - C$ curve and the decreasing amplitudes of superhumps. No stage C superhumps were found. The superhumps continued after the termination of the main superoutburst. At the post-superoutburst stage during BJD 2457519.9–2457522.0 ($263 \leq E \leq 295$), the superhumps having a longer period than the stage

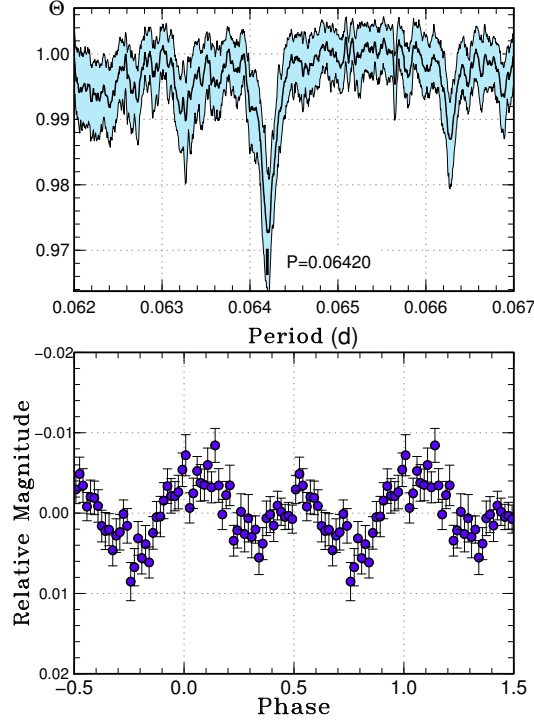


Figure 3.5: Early superhumps in the 2016 superoutburst of ASASSN-16dt. The area of gray scale means 1σ errors. Upper: Θ -diagram of our PDM analysis (BJD 2457482.1–2457493.0). Lower: Phase-averaged profile.

B superhumps were detected. Some modulations were seen at the rebrightening; however, we were not able to detect the superhump maxima and periods.

We applied period analyses by the PDM method for stage A and stage B, and obtained periods of $P_{\text{shA}} = 0.06512(1)$ d and $P_{\text{shB}} = 0.064507(5)$ d (see the upper panels of Figure 3.7). Here, the data having low accuracy were excluded from the light curve when we performed our PDM analyses. The derivative of the superhump period during stage B was $P_{\text{dot}}(\equiv \dot{P}_{\text{sh}}/P_{\text{sh}}) = -1.6(0.5) \times 10^{-5} \text{ s}^{-1}$. The mean profiles of superhumps are also shown in the lower panels of Figure 3.7. In addition, the estimated period of the superhumps at the post-superoutburst stage was 0.06493(4) d.

3.5 ASASSN-16hg

3.5.1 Overall light curve

The overall light curve of the 2016 superoutburst in ASASSN-16hg is shown in Figure 3.8. The first plateau stage continued for more than 6 days before a dip in

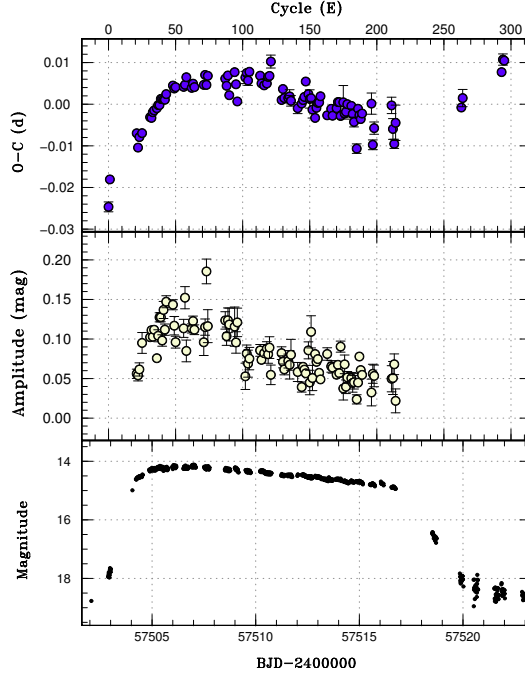


Figure 3.6: Upper panel: $O - C$ curve of the times of superhump maxima during BJD 2457502.8–2457522.1 (the second plateau stage of the main superoutburst in ASASSN-16dt). An ephemeris of BJD 2457502.925071+0.0653055 E was used for drawing this figure. Middle panel: amplitudes of superhumps. Lower panel: light curves. The horizontal axis in units of BJD and cycle number is common to these three panels.

brightness on BJD 2457590. Soon after the dip, the system became bright again and the second plateau stage began and continued for a week during BJD 2457591.6–2457597.9. We detected two rebrightenings after the main superoutburst.

3.5.2 Ordinary superhumps

We have found ordinary superhumps in the second plateau stage and at the beginning of the next decay. The $O - C$ curve of times of superhump maxima, the amplitudes of superhumps and the light curves during the interval are displayed in the upper, middle and lower panels of Figure 3.9, respectively. We determined the times of maxima of ordinary superhumps in the same way as in Sec. 2.3. The resultant times are given in Table A7. We regarded the term during BJD 2457591.6–2457598.8 ($0 \leq E \leq 100$) as stage B judging from the trend of the $O - C$ curve and the decreasing amplitudes. Although there is a possibility that the stage A superhumps appeared between BJD 2457590 (the dip) and BJD 2457591.6 (the initial part of the stage B superhumps), they were not detected due to the

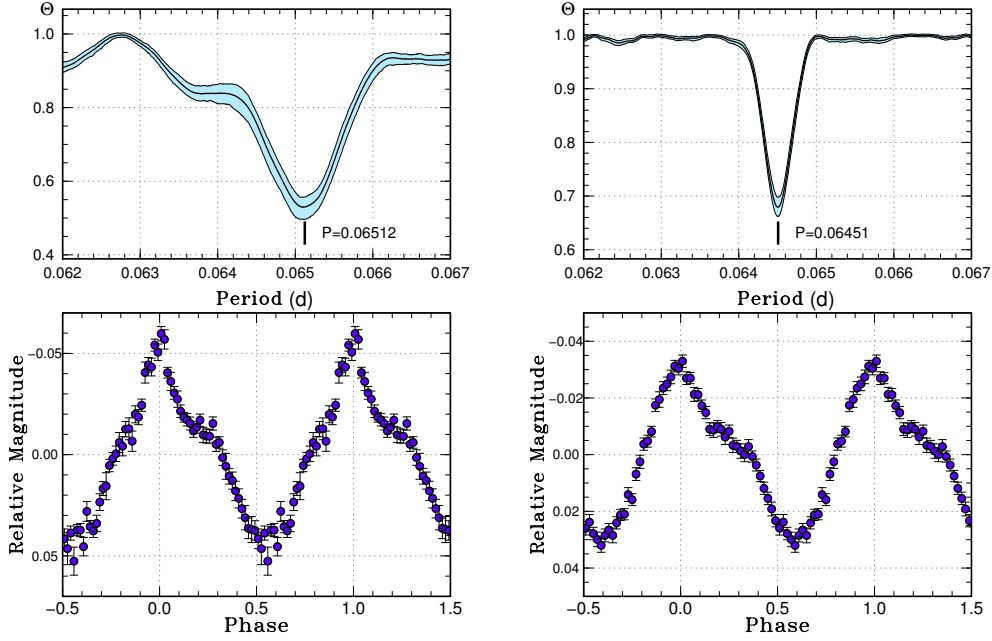


Figure 3.7: Stage A and B superhumps in the second plateau stage of the 2016 superoutburst of ASASSN-16dt are represented in the left and right panels, respectively. The area of gray scale means 1σ errors. Upper: Θ -diagrams of our PDM analyses. Lower: Phase-averaged profiles.

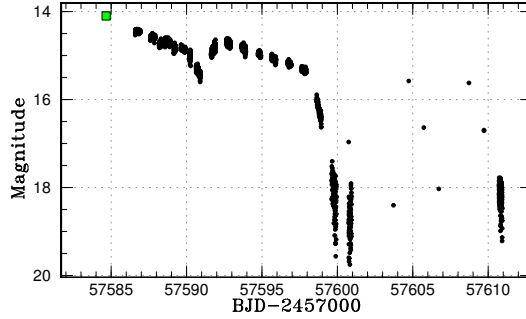


Figure 3.8: Overall light curve of the 2016 superoutburst of ASASSN-16hg (BJD 2457584–2457611). The quadrangle represents the detection by ASAS-SN.

low sampling rate of the data and the lack of observations. We were not able to determine whether some modulations before the dip on BJD 2457590 were early superhumps or not since their small amplitudes and the sparse data prevented us from confirming double-waved variations. We note that the different superoutburst stages were not clearly distinguished in the $O - C$ curve.

We applied a period analysis by the PDM method for stage B and obtained the period of $P_{\text{shB}} = 0.06237(1)$ d (see the upper panel of Figure 3.10). The derivative of the superhump period during stage B was $P_{\text{dot}} = 0.6(1.7) \times 10^{-5} \text{ s}^{-1}$. The

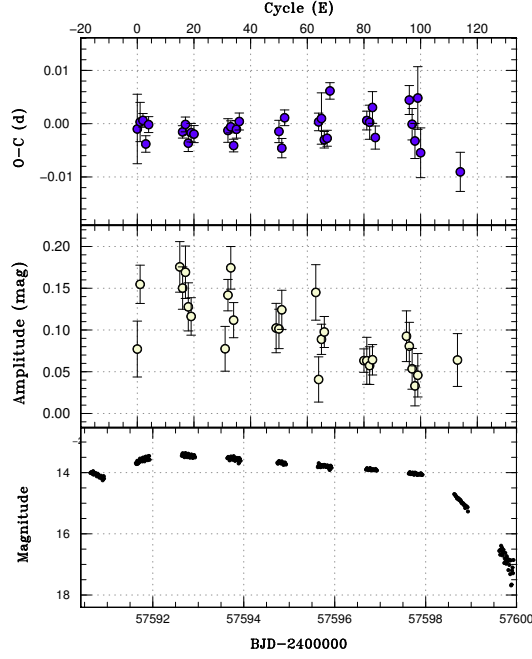


Figure 3.9: Upper panel: $O - C$ curve of the times of superhump maxima during BJD 2457591.6–2457598.8 (the second plateau stage of the main superoutburst in ASASSN-16hg). An ephemeris of BJD 57591.6610+0.0623475 E was used for drawing this figure. Middle panel: amplitudes of superhumps. Lower panel: light curves. The horizontal axis in units of BJD and cycle number is common to these three panels.

mean profile of the stage B superhumps is also shown in the lower panel of Figure 3.10.

3.6 Discussion

3.6.1 Mass ratio estimation from stage A superhumps

We can estimate the mass ratio in ASASSN-16dt using the method proposed by Kato, Osaki (2013), assuming that the early superhump period is identical to the orbital period (Kato, 2002; Ishioka et al., 2002) by the method proposed by Kato, Osaki (2013) as described in Sec. 1.9.3.

The estimated mass ratio with this method is $q = 0.036(2)$ as for ASASSN-16dt. This is shown on the $q - P_{\text{orb}}$ plane in Figure 3.11 with the mass ratios of other period-bouncer candidates and ordinary SU UMa-type DNe derived from Kato et al. (2017). The derived errors originate from the errors of period estimations. The very small mass ratio suggests ASASSN-16dt is one of the best candidates for

Table 3.1: Properties of candidates for a period bouncer (The candidates are limited to the DNe which have been through outbursts).

Object*	P_{shB} (d) [†]	Amp [‡]	Delay [§]	Decrease [#]	Profile [¶]	Decline**	Ref ^{††}
MASTER J2112	0.060221(9)	0.10	~12	2.2%	B	0.127(1)	1
MASTER J2037	0.061307(9)	0.11	–	2.2%	B, slow	0.052(1)	1
SSS J1222	0.07649(1)	0.12	≥9	0.93%	E, slow	0.020(1)	2, 3
OT J1842	0.07234	0.08	~30	–	E, slow	0.045(1)	2, 4
OT J1735	–	–	–	–	slow	0.038(1)	5
OT J0754	0.070758(6)	0.05	–	2.0%	slow	0.0189(3)	6
OT J2304	0.06635(1)	0.13	–	1.3%	slow	0.0340(4)	6
ASASSN-14cv	0.06045(1)	0.07	14	2.0%	B	0.087(1)	7, 8
PNV J1714	0.060084(4)	0.09	11	2.0%	B	0.108(1)	7, 8
OT J0600	0.063310(4)	0.06	–	2.1%	B	0.080(1)	7, 8
PNV J172929	0.06028(2)	0.12	11	1.7%	D	0.094(1)	8
ASASSN-15jd	0.064981(8)	0.09	10	–	e	0.088(2)	9
ASASSN-15gn	0.06364(3)	0.10	11	–	–	0.0635(7)	10
ASASSN-15hn	0.06183(2)	0.10	12	2.2%	–	0.080(3)	10
ASASSN-15kh	0.06048(2)	0.08	≥13	1.7%	–	0.0601(6)	10
ASASSN-16bu	0.06051(7)	0.10	9	0.62%	slow	0.024(1)	10
ASASSN-16js	0.06093(2)	0.23	10	1.2%	–	0.085(1)	11
ASASSN-16dt	0.064610(1)	0.08	~23	0.79%	E, slow	0.0282(6)	12
ASASSN-16hg	0.062371(14)	0.12	≥6	–	e, B	0.090(2)	12

*Objects' name; MASTER J2112, MASTER J2037, SSS J1222, OT J1842, OT J1735, OT J0754, OT J2304, PNV J1714, OT J0600 and PNV J172929 represent MASTER OT J211258.65+242145.4, MASTER OT J203749.39+552210.3, SSS J122221.7–311523, OT J184228.1+483742, OT J173516.9+154708, OT J075418.7+381225, OT J230425.8+062546, PNV J17144255–2943481, OT J060009.9+142615 and PNV J17292916+0054043, respectively.

[†]Period of stage B superhumps.

[‡]Mean amplitude of superhumps. Unit of mag.

[§]Delay time of ordinary superhump appearance. Unit of days.

[#]Decrease rate of stage B superhump period in comparison with stage A superhump period.

[¶]Characteristic shapes of light curves. B: multiple rebrightenings (type-B), D: no rebrightening (type-D), E: double superoutbursts (type-E), e: a small dip in the middle of the plateau, slow: extremely slow fading rate less than ~ 0.05 [mag d^{−1}].

**Fading rate of plateau stage. Unit of mag d^{−1}.

^{††}References. 1: Nakata et al. (2013), 2: Kato et al. (2013b), 3: Neustroev et al. (2017), 4: Katysheva et al. (2013), 5: Kato et al. (2014b), 6: Nakata et al. (2014), 7: Nakata et al. in preparation, 8: Kato et al. (2015), 9: Kimura et al. (2016a), 10: Kato et al. (2016), 11: Kato et al. (2017), 12: Kimura et al. (2018a).

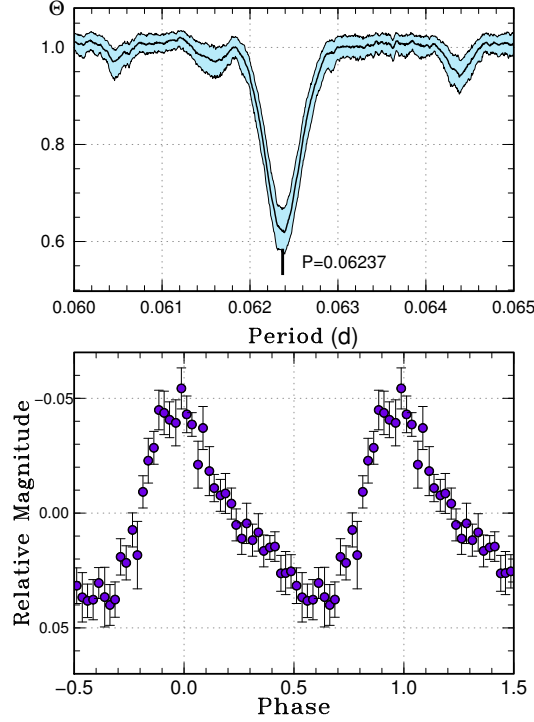


Figure 3.10: Stage B superhumps in the second plateau stage of the 2016 superoutburst of ASASSN-16hg are represented. The area of gray scale means 1σ errors. Upper: Θ -diagram of our PDM analysis. Lower: Phase-averaged profile.

a period bouncer. Our result agrees with the empirical relation between rebrightening types and mass ratios, which was suggested in Kato (2015). The q values of candidates for a period bouncer are displayed in Table 3.2 with those of ordinary WZ Sge-type dwarf novae. Several objects close to the period minimum, which showed repeating (type-B) rebrightenings, have been regarded as promising period-bouncer candidates because they share the outburst properties with the extremely low- q period-bouncer candidates, and have longer orbital periods than the group of WZ Sge-type DNe whose orbital periods are close to the theoretical period minimum (see also Table 3.1). The three objects deviated from the promising period-bouncer candidates in Figure 3.11, which are indicated as light blue circles, are not regarded as being of this kind, since they do not share the aforementioned properties (see also the 2nd paragraph in Sec. 3.1).

It is shown that some of the objects given in Table 3.2 would have brown-dwarf secondaries (Littlefair et al., 2008; Savoury et al., 2011; Hernández Santisteban et al., 2016; Neustroev et al., 2017). There may be a possibility that these objects come from zero-age detached white-dwarf and brown-dwarf binaries. Actually, one object which seems to be a pre-CV candidate having a brown-dwarf secondary has

Table 3.2: Mass ratios of candidates for a period bouncer.

Object*	P_{orb} (d) [†]	P_{shA} (d) [‡]	q [§]	References
SDSS J1507	0.046258	–	0.0625(4)	1
SDSS J1433	0.054241	–	0.069(3)	2, 3
SDSS J1501	0.056841	–	0.067(3)	1
SDSS J1035	0.057007	–	0.057(1)	4
ASASSN-16bu	0.0593(1)	0.06089(7)	0.10(1)*	5
PNV J1714	0.059558(3)	0.06130(2)	0.076(1)*	6, 7
PNV J172929	0.05973	0.06133(7)	0.073(2)*	7
MASTER J2112	0.059732(3)	0.06158(5)	0.081(2)*	8
ASASSN-14cv	0.059917(4)	0.06168(2)	0.077(1)*	6, 7
ASASSN-16js	0.060337(5)	0.0617(1)	0.056(5)*	9
MASTER J2037	0.0605(2)	0.0627(1)	0.097(8)*	8
SDSS J1057	0.062792	–	0.055(2)	1
ASASSN-16dt	0.06420(3)	0.06512(1)	0.036(2)*	13
OT J1842	0.07168(1)	0.07287(8)	0.042(3)*	10
SSS J1222	0.07625(5)	0.07721(1)	0.032(2)*	11, 12

*Objects' name; MASTER J2112, MASTER J2037, SSS J1222, OT J1842, PNV J1714, PNV J172929, SDSS J1057, SDSS J1035, SDSS J1433, SDSS J1501, SDSS 1433, and SDSS 1507 represent MASTER OT J211258.65+242145.4, MASTER OT J203749.39+552210.3, SSS J122221.7–311523, OT J184228.1+483742, PNV J17144255–2943481, PNV J17292916+0054043, SDSS J105754.25+275947.5, SDSS J103533.02+055158.3, SDSS J143317.78+101123.3, SDSS J150137.22+550123.4, SDSS J143317.78+101123.37, and SDSS J150722.30+523039.8, respectively.

[†]Orbital period.

[‡]Period of stage A superhumps.

[§]Mass ratio. The index * represents the mass ratio derived by the method in Kato, Osaki (2013).

^{||}1: McAllister et al. (2017), 2: Littlefair et al. (2008), 3: Hernández Santisteban et al. (2016), 4: Savoury et al. (2011), 5: Kato et al. (2016), 6: Nakata et al. in preparation, 7: Kato et al. (2015), 8: Nakata et al. (2013), 9: Kato et al. (2017), 10: Kato, Osaki (2013), 11: Kato et al. (2013b), 12: Neustroev et al. (2017), 13: Kimura et al. (2018a).

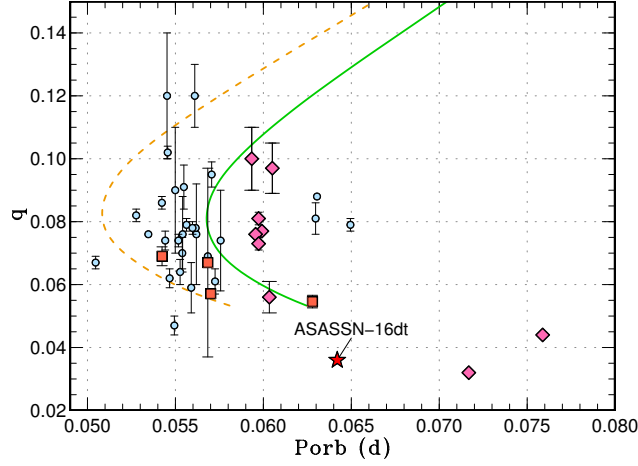


Figure 3.11: $q - P_{\text{orb}}$ relation of the candidates for the period bouncer and ordinary WZ Sge-type DNe. The star, diamonds, rectangles and circles represent ASASSN-16dt, other candidates for a period bouncer among the identified WZ Sge-type DNe, the candidates for a period bouncer among eclipsing CVs, and ordinary WZ Sge-type DNe. The dash and solid lines represent an evolutionary track of the standard evolutionary theory and that of the modified evolutionary theory, respectively, which are derived from Knigge et al. (2011).

recently found (Rappaport et al., 2017). According to Politano (2004), however, $\sim 80\%$ of this kind of detached binaries have small orbital periods less than 0.054 d, and the fraction of them to the total population of zero-age CVs seems to be smaller than a half. Thus some objects in Table 3.2, which we discuss here, would be formed from the zero-age CVs with main-sequence companions via the mass loss of the companions rather than the zero-age CVs with brown-dwarf companions. These are pointed out also in Neustroev et al. (2017).

3.6.2 Dip in brightness during the main superoutburst

ASASSN-15jd is the first example among WZ Sge-type DNe, which showed a small dip of brightness in the middle of the superoutburst. It has been known that the plateau stage of superoutbursts in WZ Sge-type objects can be classified into two types: single plateau stage observed in the objects with type-A, B, C and D rebrightening (except for type-E rebrightening) and double plateau stages observed only in the objects with type-E rebrightening. The schematic diagrams of the light curves of these two plateau types are depicted in Figure 3.12. The left and right panels of Figure 3.12 represent single plateau stage and double plateau stages, respectively. In the single plateau stage, ordinary superhumps are seen as soon as early superhumps disappear, while in the double plateau stages, ordinary

superhumps begin to develop in the second plateau stage a few days after early superhumps disappear. The plateau stage in ASASSN-15jd cannot be classified as one of these two types of the plateau stage. There would be a possibility that the dip in ASASSN-15jd was deeper since we did not have adequate observations around the observed dip on BJD 2457165; nevertheless, the duration of the dip in this object was ~ 2 days and shorter than ~ 10 -days duration in the objects with type-E rebrightening (Kato et al. (2013b); Katysheva et al. (2013) and see also the bottom panel of Figure 1.26). Hence, we can regard that ASASSN-15jd showed an intermediate light curve between the single plateau stage and the double ones.

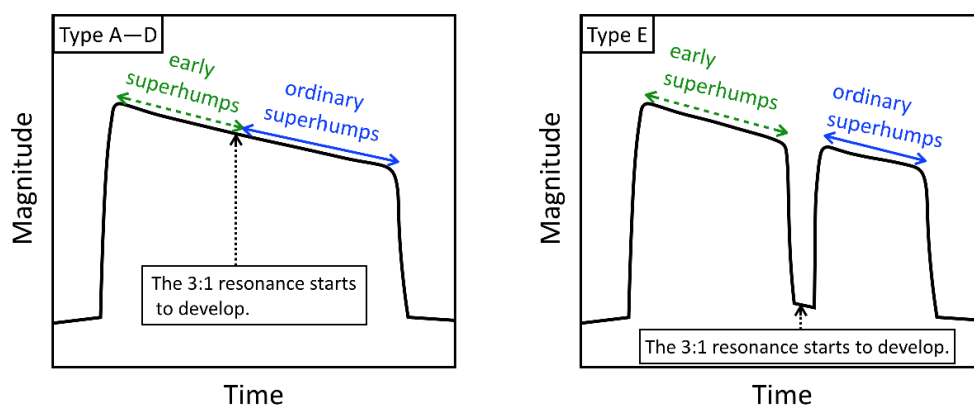


Figure 3.12: Classification of plateau stages observed in WZ Sge-type DNe. Left: single plateau stage observed in objects with type-A, B, C and D rebrightenings. Right: double plateau stages observed in objects with type-E rebrightening.

Ordinary superhumps are considered to develop together with the 3:1 resonance tidal instability occurring in accretion disks (see e.g., Osaki, 1996, for a theoretical review). We can therefore regard that the shapes of plateau stages in WZ Sge-type DNe reflect the speed at which the 3:1 resonance develops. In other words, it is considered that we cannot observe the dip of brightness in the plateau in the objects with the single plateau stage since the 3:1 resonance grows up as soon as the 2:1 resonance finishes working. In the cases when the 3:1 resonance does not grow sufficiently quickly, the cooling wave propagates in the accretion disk before the 3:1 resonance fully builds up in the objects with the double plateau stages, and hence, these have a dip of brightness. The 3:1 resonance in ASASSN-15jd seems to develop slower than that in the former objects (single plateau) and faster than that in the latter objects (double plateaus).

There is a theory explaining the reason why the delay in development of superhumps appears. The growth time of the 3:1 resonance tidal instability is inversely proportional to the square of the mass ratio (Lubow, 1991). The type-E rebrightening was observed in the outbursts of SSS J122221.7–311523 and OT

J184228.1+483742 in the past (Kato et al., 2013b; Katysheva et al., 2013) and these systems are considered to be promising candidates for a period bouncer because of their small estimated mass ratios (Kato, 2015). Their mass ratios are smaller than those of other WZ Sge-type stars with the single plateau, and the behavior is consistent with the slow growth of the 3:1 resonance. Although we could not estimate the accurate value of the mass ratio of ASASSN-15jd, the 3:1 resonance seems to develop slower in this system than in WZ Sge-type stars with the single plateau stage judging from the shape of the plateau stage and it is expected that this object has a small q value. Combined with the relatively long P_{sh} for a WZ Sge-type star, ASASSN-15jd is likely to be a promising candidate for a period bouncer.

The rebrightening type after the second plateau stage in this outburst is probably C (single rebrightening) although there remain a possibility of multiple rebrightenings due to the lack of observations. This rebrightening type has been observed in many other WZ Sge-type objects.

As for the other two objects, ASASSN-16dt belongs to WZ Sge-type objects with double superoutbursts. This object is an analogue of SSS J122221.7–311523 and OT J184228.1+483742 (Kato et al., 2013b; Katysheva et al., 2013; Neustroev et al., 2017). ASASSN-16hg is the second object showing an intermediate light curve between the single plateau stage and the double ones. In addition, the duration of stage A superhumps is normally long in the candidates (Kato et al., 2013b; Nakata et al., 2014), and also in ASASSN-16dt, the stage A superhumps continued for a long interval, ~ 4 days. The small mass ratio and the morphology of the 2016 superoutburst in ASASSN-16dt are in good agreement with this theory. The slow growth of the 3:1 resonance is expected to produce the long-lasting stage A superhumps in small- q systems. Although the small dip during the main superoutburst in ASASSN-16hg may suggest that this system likely has a small q value, it is unclear whether the ordinary superhumps slowly developed, due to the lack of detection of the initial part of the developing ordinary superhumps (stage A superhumps).

3.6.3 Absence of early superhumps

In ASASSN-15jd, no ordinary superhumps were observed for about ten days from the onset of the outburst to the small dip of brightness (on BJD 2457165). In addition, we could not detect early superhumps for that term although they are usually observed in outbursts of WZ Sge-type stars for about a week (Kato, 2015) before ordinary superhumps develop. Uemura et al. (2012) proposed that early

superhumps are caused by the rotation effect of non-axisymmetrically flaring accretion disks (see also Nogami et al., 1997; Kato, 2002). High-inclination (edge-on) systems thus tend to show large-amplitude early superhumps. In addition, early superhumps are regarded as the manifestation of the 2:1 resonance tidal instability and the 2:1 resonance is considered to suppress the development of ordinary superhumps (Lubow, 1991; Osaki, Meyer, 2002, 2003). Based on the above theories, we suggest that the inclination angle of ASASSN-15jd is low and we observed this system at nearly face-on from our results: 1) the superhumps start to develop at the dip, 2) neither early superhumps nor ordinary ones were observed before the dip.

Note that the duration for which the 2:1 resonance supposed to have been dominant before the superhumps developed in ASASSN-15jd is longer than the average duration of early superhumps in ordinary WZ Sge-type DNe. In the superoutbursts of the promising candidates for a period bouncer, the durations of the early superhumps were usually long and this can be one of characters of period bouncers (Kato et al. (2013b); Nakata et al. (2013); Nakata et al. in preparation). Therefore, this finding suggests that ASASSN-15jd has yet another property common to period bouncers. There may be a possibility that ASASSN-16hg has a low inclination angle, since early superhumps in its 2016 superoutburst were ambiguous.

3.6.4 Long Delay of superhump appearance

The delays of ordinary superhump appearance are typically long in the candidates for a period bouncer (see Table 3.1), whilst those in ordinary WZ Sge-type stars are concentrated between 5–10 [d] (Kato, 2015). This feature is also clearly confirmed in the 2016 superoutburst of ASASSN-16dt. Figure 3.13 shows the relation between superhump period and delay time in WZ Sge-type DNe. On the other hand, it is uncertain whether the delay of the superhump appearance was long in ASASSN-16hg since there was no observation between the upper limit by ASAS-SN on BJD 2457573 and the detection by the same survey on BJD 2457584.

In ASASSN-16dt, early superhumps were clearly detected at the early stage of its superoutburst. Osaki, Meyer (2003) proposed that the 2:1 resonance causing early superhumps suppresses the 3:1 resonance. The presence of early superhumps lasting for at least ~ 10 days in ASASSN-16dt support this explanation. As for the objects having very small mass ratios, the disk radius would expand far beyond the 2:1 resonance radius when an outburst is triggered. This is because the stored disk mass naturally become large due to extremely low viscosity in the quiescent

disk, while the tidal effect by the secondary is weak in these objects (Osaki, Meyer, 2002). Collectively, these effects sustain the 2:1 resonance for a long interval.

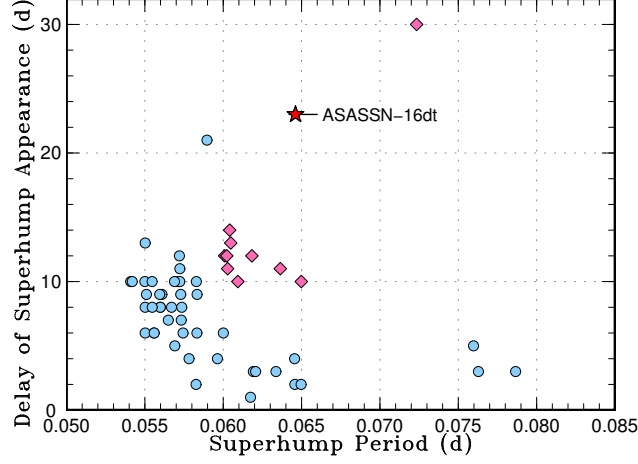


Figure 3.13: P_{sh} vs. delay time of ordinary superhump appearance. The circles and diamonds indicate ordinary WZ Sge-type stars derived from Fig. 19 in Kato (2015) and the candidates for a period bouncer. The star represents ASASSN-16dt.

3.6.5 Small Amplitude of Superhumps

The average superhump amplitudes are small, less than 0.1 mag, in most of the candidates for a period bouncer (Table 3.1). Our results on ASASSN-15jd, ASASSN-16dt, and ASASSN-16hg reinforce this observation (see also Figures 3.2, 3.6, and 3.9). We compare the variation of superhump amplitudes of the candidates for a period bouncer and those of ordinary SU UMa-type systems having orbital periods ranging between 0.06–0.07 d in Figure 3.14*. In plotting this figure, we measured the amplitudes using the template fitting method described in Kato et al. (2009) and took the starting point of the cycle count from the start of stage B. Since superhump amplitudes are known to depend on the orbital periods and the inclination angles (Kato et al., 2012), the data of ASASSN-16js are excluded from this figure. This object may have high inclination, which is judged from the large amplitudes of its early superhumps, ~ 0.2 mag (Kato et al., 2017). It is known that the higher the inclination is, the larger the amplitudes of early superhumps are (Kato, 2015), and the ~ 0.2 -mag amplitudes of early superhumps are comparable to those of WZ Sge having the inclination of 77 ± 2 deg (Steeghs et al., 2007). The median value of the amplitudes between $-3 < E < 5$ in the period-bouncer candidates is significantly smaller, 0.074 mag, than that in ordinary SU UMa-type DNe, 0.22 mag.

*We excluded the data with large errors more than 0.03 mag and of eclipsing systems.

During the 3:1 resonance, the disk becomes elliptical due to the tidal force of the secondary; the orbiting secondary passes the major axis of the disk with the superhump period when the ordinary superhumps are observed. Some particles in the tail of the eccentric disk are periodically absorbed to the secondary, and the time variations of the released energy by the viscous dissipation in the outer disk corresponds the superhump variations (Hirose, Osaki, 1990). It has been proposed that the tidal torques exerted by the secondary significantly affect viscous dissipation in the outer disk (Ichikawa, Osaki, 1994). In the small- q objects, the tidal force from the secondary would be less than that in the large- q objects. The disk would be less elliptical and the liberated energy by the tidal dissipation would be small. Thus the reason why period-bouncer candidates show small-amplitude superhumps seems to be related to the weak tidal effect by the secondary in small- q objects.

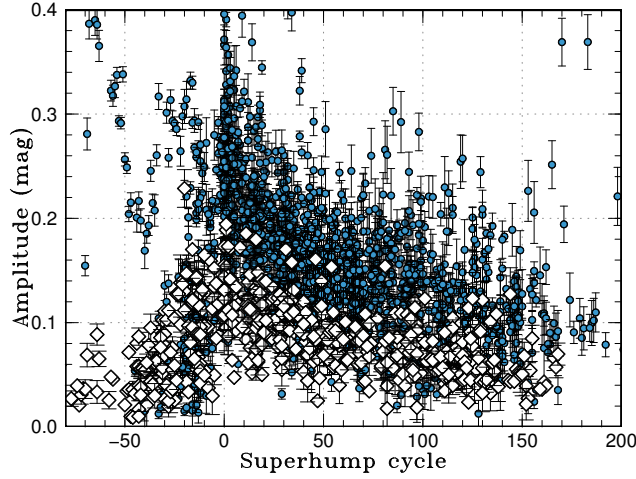


Figure 3.14: Variation of superhump amplitudes in the SU UMa-type objects with $0.06 \text{ d} < P_{\text{orb}} \leq 0.07 \text{ d}$. The diamonds and circles represent the candidates for a period bouncer and ordinary SU UMa-type DNe, respectively. The data of the period-bouncer candidates are derived from Nakata et al. (2013); Kato et al. (2013b); Nakata et al. (2014); Kato et al. (2015, 2016); Kimura et al. (2016a); Kato et al. (2017), and those of ordinary SU UMa-type DNe are derived from Kato et al. (2009, 2010, 2012, 2013a, 2014b,a, 2015, 2016, 2017).

3.6.6 Slow Fading Rate of Plateau Stage

The fading rates of the plateau stage with ordinary superhumps in the superoutbursts of period-bouncer candidates are often small (Sec. 7.8 in Kato, 2015). In particular, all of the three period-bouncer candidates including ASASSN-16dt, which showed double superoutbursts, had extremely low decline rates (see Table 3.1).

They also have very small mass ratios, and the durations of the superoutbursts are long – more than 40 days (see also Kato et al., 2013b). One period-bouncer candidate with type-B rebrightenings and four other candidates whose rebrightening types have not yet been identified showed slow declines (see also Table 3.1). The relation between the fading rate and the superhump period is shown in Figure 3.15. The median value of the fading rates in the period-bouncer candidates is 0.06 mag d^{-1} , while that in ordinary WZ Sge-type stars is 0.10 mag d^{-1} .

The decline timescale is proportional to $\alpha^{-0.7}$ (equation (49) in Osaki (1989)). Here α represents the viscous parameter in the hot state, and the combination of ordinary α due to magnetorotational instability plus the viscosity resulting from the tidal torque (Balbus, Hawley, 1991; Ichikawa, Osaki, 1994). The slow fading rate in period bouncers would therefore be attributed to the weaker tidal torque in low- q objects.

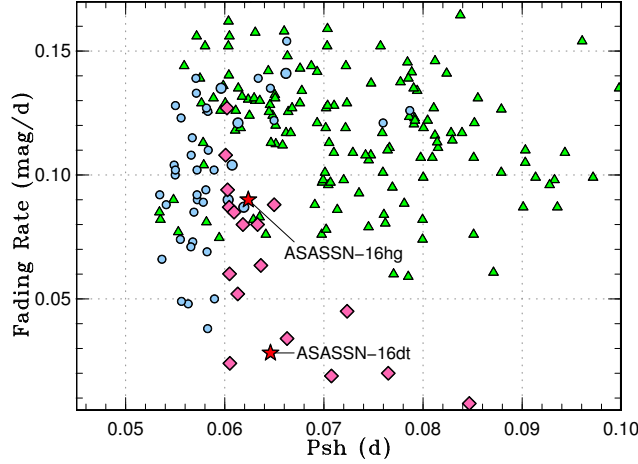


Figure 3.15: Fading rate vs. superhump period in stage B. The circles, triangles, and diamonds represent ordinary SU UMa-type DNe, WZ Sge-type DNe, and candidates for the period bouncer, respectively. The stars indicate ASASSN-16dt and ASASSN-16hg. The data of the ordinary SU UMa-type DNe and WZ Sge-type DNe are derived from Kato et al. (2014b).

3.7 Summary

We have reported on our photometric observations of three WZ Sge-type DNe, ASASSN-15jd, ASASSN-16dt and ASASSN-16hg, and discussed their similar properties to those of period-bouncer candidates. The important findings are summarized as follows:

- ASASSN-15jd is the first object show a superoutburst intermediate between single and double plateaus. ASASSN-16dt and ASASSN-16hg also underwent outbursts with a dip in brightness at their main superoutbursts. The dips imply that the 3:1 resonance grew slowly in their outbursts, and that these objects have low mass ratios.
- The mass ratio in ASASSN-16dt, estimated from the method of Kato, Osaki (2013) via the stage A superhump period (0.06512(1) d) is 0.036(2), which is much lower than the theoretically expected mass ratio at the period minimum. The relatively long orbital period estimated from the early superhumps and the very low mass ratio are enough to judge that this object is one of the best period-bouncer candidates.
- The three objects showed many features similar to those in other candidates for a period bouncer, like long-lasting stage A superhumps and early superhumps, small-amplitude superhumps, and a slow decline rate at the plateau stage.
- Although it is uncertain whether the development of the superhumps in ASASSN-16hg was slow due to no detection of the stage A superhumps, this object might be a possible period-bouncer candidate on the basis of the morphology of the plateau stage which resembles that during the 2015 superoutburst in ASASSN-15jd (Kimura et al., 2016a) and its small superhump amplitude.
- Many outburst properties of the period-bouncer candidates would be interpreted by the small tidal effect by the secondary in small- q systems.

On the nature of long-period DNe with rare and low-amplitude outbursts

4.1 Introduction

It is widely accepted that DNe showing low outburst amplitudes undergo outbursts more frequently (Warner, 1987) as introduced Sec. 1.5.3. Statistical studies on DNe show many DNe having long orbital periods (more than 3 hours) go through outbursts with typical amplitudes of 2–5 mag, and that the intervals between their outbursts are usually less than 1 year (see Warner, 1995, for a review). A very low-amplitude and rare outburst was, however, discovered for the first time in a dwarf-nova like object, ASAS 150946–2147.7 = V364 Lib, (Pojmanski et al., 2009). This object has a long orbital period of 16.86 hours, and enters its outbursts with amplitudes of ~ 1 mag every few years (Wils (2011); vsnet-alert 14271; vsnet-alert 20877). In addition, highly ionized emission lines (He II 4686 and C III/N III) were detected in the 2009 outburst. Although this led Kinugasa et al. (2009) to propose that this system is a black-hole binary, Uemura et al. (2009) reported that the X-ray luminosity during the outburst was comparable to the quiescent X-ray luminosity of black-hole binaries. Thus this object contradicts the general rule of DNe, and the origin of the peculiar outburst and highly ionized emission lines remains a mystery.

After this discovery, this object has been away from the interests of astronomers. However, on June 3.45, 2016 (UT), a small-amplitude and very rare outburst was discovered in another object by Catalina Real-Time Transient Survey (CRTS) (Drake et al., 2016, 2014a). Its name is 1SWASP J162117+441254 (hereafter 1SWASP J1621), and this object also has a long orbital period of 4.99 hours (Lohr

et al., 2013a). The outburst amplitude was ~ 2 mag and only one outburst was detected over ten years before the 2016 outburst in this system (Kjurkchieva et al., 2017). The strong He II 4686 emission line observed in the outburst was similar to that in the 2009 outburst of V364 Lib (Scaringi et al., 2016). Moreover, this object had been previously identified as a W UMa-type system on the basis of SDSS colors and double-waved orbital variations in quiescence (Palaversa et al., 2013; Lohr et al., 2013b; Drake et al., 2014b); hence, the 2016 outburst was expected to be the onset of a merger of two main-sequence (MS) stars, which was reminiscent of the 2008 eruption in V1309 Sco. Scaringi et al. (2016), however, confirmed by their spectroscopic observations that this outburst occurred in an accreting compact binary rather than a MS+MS merger. In addition, this system showed much deeper eclipses in outburst than in quiescence (Zejda, Pejcha, 2016; Kjurkchieva et al., 2017; Zola et al., 2017). This phenomenon reminds us the similarity between this object and the almost forgotten dwarf nova BD Pav, which has an orbital period of 4.3 hours (Barwig, Schoembs, 1983). BD Pav has also shown relatively low-amplitude and rare outbursts, and the He II 4686 emission line was observed in its 1985 outburst. The emission line was not clearly double-peaked, in spite of the deep eclipses in outburst suggesting its high inclination (Barwig, Schoembs, 1987).

Recently, the two objects GY Hya and V1129 Cen, which showed outbursts, spectra, and orbital variability similar to those in 1SWASP J1621, BD Pav and V364 Lib, have been studied (Bruch, Monard, 2017; Bruch, 2017). In addition, HS 0218+3229 (hereafter HS 0218) shows rare outbursts and has a long orbital period, although the outburst amplitude is not so small, ~ 4 mag in the V band (Golysheva et al., 2012, 2013; Katysheva et al., 2015), and it was pointed out that this object is similar to 1SWASP J1621 (Katysheva et al., 2017).

Thus this kind of object recently has started to attract attention, but there is not yet any coherent explanation to produce its peculiar outburst and spectral behavior. Moreover, some of these objects were not clearly identified with normal DNe, and it is even doubted whether the disk instability model, which is believed to be the most plausible model of outbursts in non-magnetic DNe (Osaki, 1996, for a review), can explain the outbursts in this kind of object. For example, Dr. Breedt claims that 1SWASP J1621 has a hot companion star (Waagen, 2017). Although this assumption would be true in V1129 Cen according to Bruch (2017), it is not consistent with the spectroscopic observations of 1SWASP J1621 and BD Pav (Thorstensen, 2016; Sion et al., 2008; Friend et al., 1990). In addition, Qian et al. (2017) even argued that the eruption in 1SWASP J1621 was not a dwarf-nova outburst. They considered the accretion disk is barely formed due to the strong magnetic activity of the companion star in this system, and that a sudden increase

of mass transfer from the companion causes the eruption.

We therefore aimed to investigate the nature of 1SWASP J1621, BD Pav, and V364 Lib, and to examine by analyzing our photometric and spectroscopic observational data and modeling their orbital variations, whether some special feature which is not presented in normal DNe is necessary to explain their peculiar outbursts. In Sec. 4.2, we give the details of our observations. We present the results of our optical photometry of 1SWASP J1621, BD Pav, and V364 Lib in Sec. 4.3, and optical spectroscopy of V364 Lib in Sec. 4.4. In addition, we perform numerical modeling to explain the orbital variations in Sec. 4.5. In Sec. 4.6, we discuss our results, and in Sec. 4.7, we provide a brief summary.

4.2 Observations

4.2.1 Photometry

Time-resolved CCD photometry was carried out by the VSNET collaboration team. Tables A8, A9, and A10 show the logs of photometric observations of 1SWASP J1621, BD Pav, and V364 Lib, respectively. We also used the data downloaded from the AAVSO archive, the ASAS-3 data archive (Pojmanski, Maciejewski, 2004) and the ASAS-SN data archive (Davis et al., 2015). All of the observation times were converted to barycentric Julian date (BJD). Before making the analyses, we applied zero-point corrections to each observer by adding constants as for 1SWASP J1621 and V364 Lib. We did not do that as for BD Pav, since all of the observations were performed by one observer. The magnitude scales of each site were adjusted to that of the Crimean Observatory system (CRI in Table A8). The constancy of the comparison star was checked by nearby stars in the same images. The magnitude of the comparison star was measured by the AAVSO Photometric All-Sky Survey (APASS: Henden et al., 2016) from the AAVSO Variable Star Database. As for V364 Lib, we adjusted the magnitude of our observational data to that of the ASAS data, assuming the magnitude in a clear filter is almost the same as that in the *V* band. The exposure times of the observations by the VSNET team were 30–300 s as for 1SWASP J1621 and V364 Lib, and 30 s as for BD Pav.

4.2.2 Spectroscopy

Our spectroscopic observations of V364 Lib were carried out by using the Gunma Astronomical Observatory’s 1.5-m telescope equipped with Gunma LOW resolution Spectrograph and imager (GLOWS) in the 2009 outburst and using the

High Dispersion Spectrograph (HDS: Noguchi et al., 2002) attached to the 8.2-m Subaru telescope of National Astronomical Observatory of Japan (NAOJ) in quiescence just after the outburst. The spectral coverage of the two spectrographs was about 4000–8000 and 3000–10000 Å, respectively, and the spectroscopic resolutions ($R = \lambda/\Delta\lambda$) are 400–500 for GLOWS and 90,000 for HDS. Data reduction was conducted using IRAF* in the standard manner (bias subtraction, flat fielding, aperture determination, scattered light subtraction, spectral extraction, wavelength calibration, normalization by the continuum, and heliocentric radial-velocity correction). We measured the radial velocities of the object by fitting gaussians with the task SPLOT in IRAF. The observation dates, exposure times, and observatories are summarized in Table A11. The estimated radial velocities are given in Tables A12 and A13.

4.3 Photometric Light Curves

4.3.1 1SWASP J1621

In the 2016 outburst, the amplitude of its outburst was relatively small, ~ 2 mag. The duration of the outburst was probably about two weeks, and the brightness rose slowly in the early stage of the event. The overall optical light curve of the 2016 outburst in 1SWASP J1621 with a clear filter is displayed in Figure 4.1. This system showed deep eclipsing variations on its light curve. As this system was fading, the primary minima became shallower and the secondary minima became deeper. We display in Figure 4.2 the nightly averaged phase profiles in a clear filter during the fading stage in the outburst. This system showed W UMa-type orbital variations with amplitudes of ~ 0.6 mag in quiescence and deep primary minima in outburst. The phase-averaged profiles around the outburst maximum and in quiescence during BJD 2457552–2457559 in the R_C , V , and B bands are exhibited in Figure 4.3. The profile around the outburst maximum was derived from the multi-color observational data obtained by Mic during BJD 2457542.8–2457546.8 (see the annotations in Table A8), which include 3 cycles of the eclipses. When folding the outburst light curve, we subtracted the long-term trend by using locally weighted polynomial regression (LOWESS: Cleveland, 1979) after removing the part of the phase representing the deep eclipse. The derived outburst amplitude is ~ 1 mag in the R_C band from the phase-averaged light curves. During the outburst, we observed colors of $V - R \sim 0.4$ mag and $B - V \sim 0.1$ mag outside eclipses; in

*IRAF is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc., under cooperate agreement with the National Science Foundation.

quiescence, those colors were $V - R \sim 0.6$ mag and $B - V \sim 1.0$ mag. In the B band, the flux of the maxima around phase 0.25 was a little lower than that of the maxima around phase 0.75.

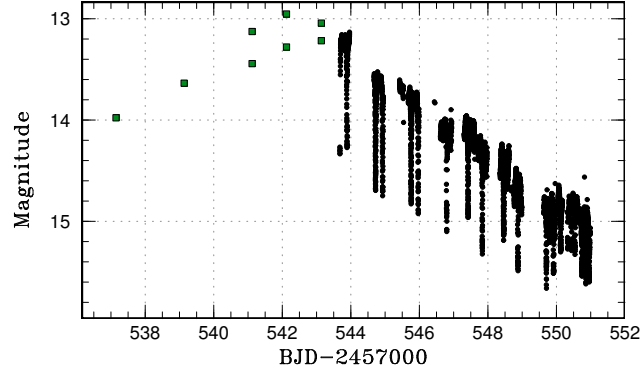


Figure 4.1: Overall light curve of the 2016 outburst in 1SWASP J1621 (BJD 2457537–2457551). The filled rectangles represent the snap-shot observations by Hiroyuki Maehara (Maehara, 2016).

4.3.2 *BD Pav*

We performed the optical photometry of the 2006 outburst of *BD Pav* in the past. The outburst amplitude was ~ 2.5 mag. The overall light curve of this outburst is shown in Figure 4.4. In addition, we confirmed the deep primary minima in this outburst similar to those in the 2016 outburst of 1SWASP J1621. In Figure 4.5, we show the phase-averaged profiles both in the 2006 outburst and in quiescence. In folding the outburst light curve, we subtracted the long-term trend using the same method described for 1SWASP J1621. We can see that this object showed the much deeper primary minima in the outburst than in quiescence and that double-waved orbital modulations with amplitudes of ~ 0.3 mag were observed in quiescence. The main characteristics of these eclipsing profiles are consistent with the results in Figures 2 and 3 in Barwig, Schoembs (1987). Although the rising phase in the 2006 outburst was not observed in our photometry, many other outbursts in *BD Pav* showed slow rises. We show measurements in the V band by ASAS during the recent 2015 and 2017 outbursts in Figure 4.6 as examples.

4.3.3 *V364 Lib*

We carried out the optical photometry of the 2009 outburst of *V364 Lib* in the past. The duration was about 35 days and the outburst amplitude was ~ 1 mag. A slow rise during the early stage of the outburst was observed, as in the outbursts

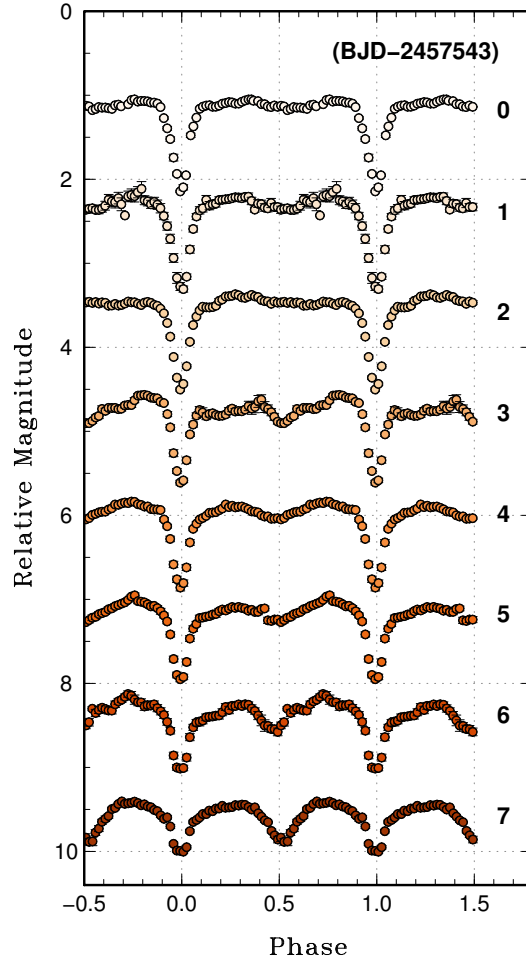


Figure 4.2: Nightly eclipsing variations in magnitudes with clear filter in 1SWASP J1621 during the 2016 outburst (BJD 2457543–2457550). The numbers at the right end represent the days from the beginning of the outburst.

of 1SWASP J1621 and BD Pav. Its overall light curve is displayed in Figure 4.7. Small-amplitude (~ 0.07 mag) and double-waved orbital variations were detected in quiescence, as displayed in Figure 4.8. In outburst, these orbital variations were expected to be observed with an amplitude of ~ 0.025 mag; however they were not detected due to other small-amplitude variations on time scales of ~ 0.1 d, which are shorter than the orbital period (and below the limits of our precision). A part of the variations in outburst is displayed in Figure 4.9.

4.4 Spectroscopy in V364 Lib

We detected absorption lines in the Balmer series and strong emission lines of He II 4686 and C III/N III during the early stage of the 2009 outburst in V364 Lib. An

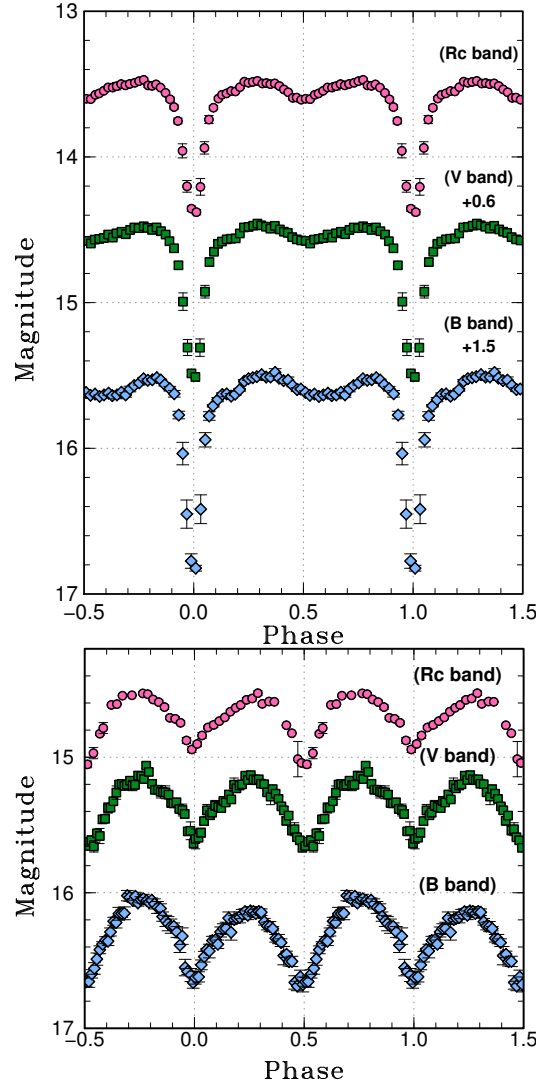


Figure 4.3: Phase-averaged light curves of orbital variations in the outburst state during BJD 2457542.8–2457546.8 (the upper panel) and in quiescence (the lower panel) during BJD 2457552–2457559 in 1SWASP J1621. Diamonds, rectangles, and circles represent the B , V and R_C -bands phase profiles, respectively. In the upper panel, the V -band and B -band magnitudes are offset by 0.6 and 1.5, respectively, for visibility. The folding period is 0.207852 d, which was reported by Drake et al. (2014a). The epochs in the outburst maximum and in quiescence are BJD 2457546.59 and BJD 2457549.7079, respectively.

example of optical spectra in the early stage of the outburst is given in Figure 4.10 with the spectrum in quiescence for comparison. In quiescence, the contribution of the companion star was dominant in the optical spectra. The companion star was determined to be an F-type star with surface temperature between 6,500 K and 6,750 K by comparing the observational spectra with the synthetic spectra which are computed with a synthetic stellar atmosphere interpolated from the Castelli,

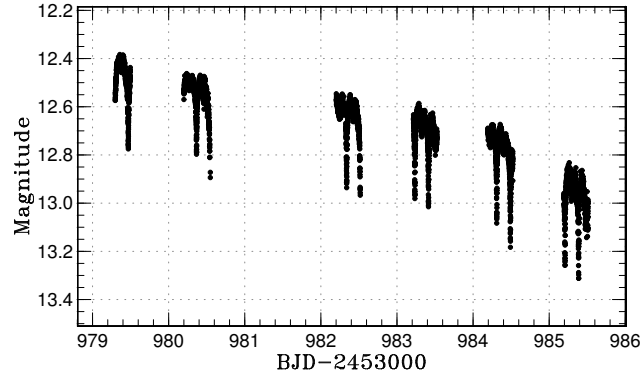


Figure 4.4: Overall light curve of the 2006 outburst in BD Pav with a clear filter (BJD 2453979–2453986).

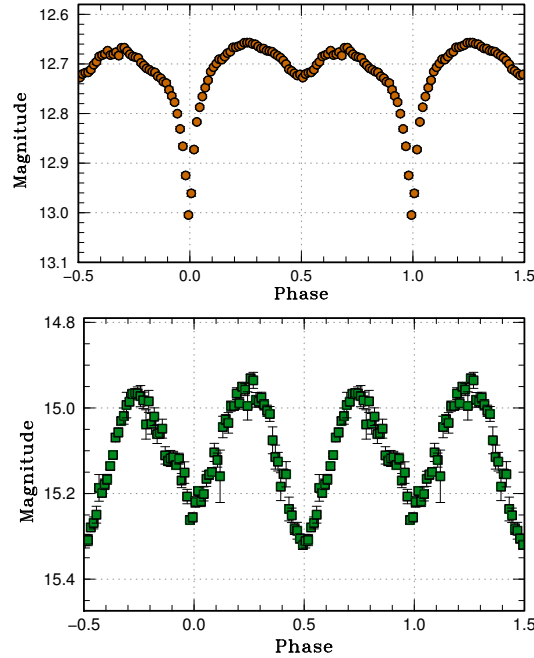


Figure 4.5: Phase-averaged profiles of orbital variations of BD Pav in the 2006 outburst during BJD 2453979–2453986 (the upper panel) and in quiescence during BJD 2456454–2456466 (the lower panel). The folding period is 0.17930, which is reported in Barwig, Schoembs (1983). The epochs are BJD 2453982.339 in the outburst and BJD 2456454.725 in quiescence.

Kurucz (2004) grid*. An example of optical spectra in quiescence is given in Figure 4.11 with some synthetic spectra.

We deduced that the central compact object in this system is a white dwarf as follows. The estimated value of the radial velocity of the companion star (K2),

*<https://www.oact.inaf.it/castelli/castelli/grids.html>

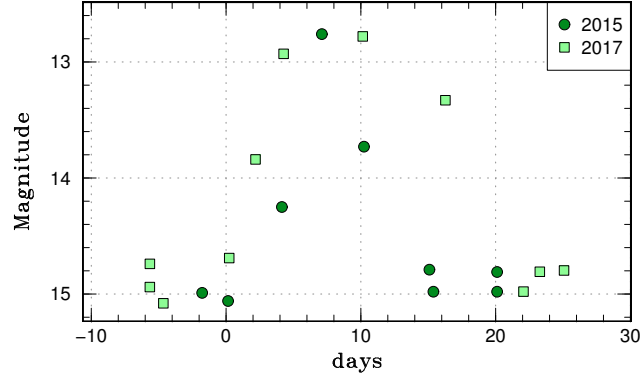


Figure 4.6: Monitoring BD Pav in the V band by ASAS during the 2015 and 2017 outbursts. The horizontal axis represents time in days from each outburst. The circles and squares represent the 2015 and 2017 outbursts, respectively.

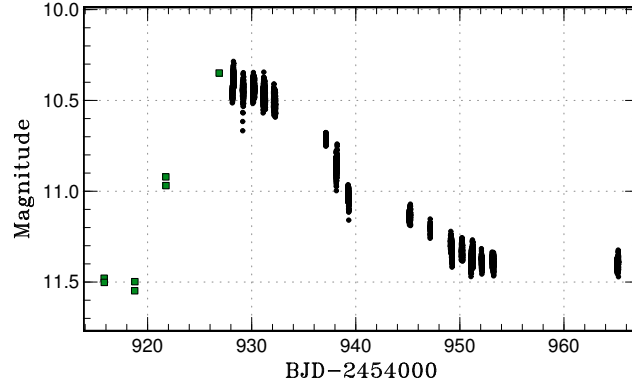


Figure 4.7: Overall light curve of the 2009 outburst in V364 Lib with a clear filter (BJD 2454928–2454954). The squares represent observations in the V band by ASAS.

using the average value of $H\beta$ and Mg absorption lines in quiescence, was 74.1 ± 1.3 km/s (see Figure 4.12). Although the error was somewhat large, the derived value of the radial velocity of the central compact object (K1) from the He II emission line in the outburst state was 107.8 ± 12.6 km/s (see Figure 4.13). We then assumed that the systemic velocity is the same as that in quiescence, and that the orbital period is 0.7024293 d. Thus the mass ratio ($q \equiv M_2/M_1$)* was estimated to be 1.5 ± 0.2 . The surface temperature of the companion suggests that its mass is close to $1.4 M_\odot$ (Allen, 1973), and hence, the mass of the compact object was estimated to be $1.0 \pm 0.2 M_\odot$ from the mass ratio. This value and the faint X-ray luminosity during outburst (Uemura et al., 2009) indicate that the compact object in this system should be a white dwarf. In addition, the inclination angle was

*Here, M_1 and M_2 represent the mass of the primary star and that of the secondary star, respectively.

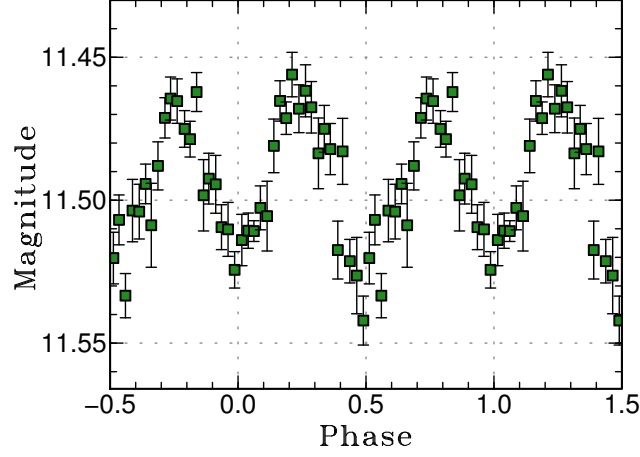


Figure 4.8: Phase-averaged profile of orbital variations in quiescence in V364 Lib. The folding orbital period estimated by the PDM method is 0.7024293(1053) d. The epoch is BJD 2453880.244654.

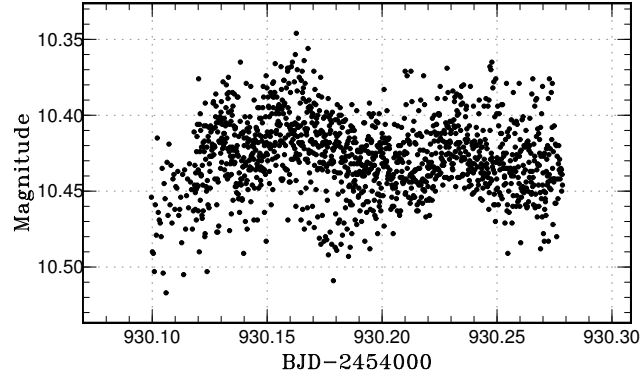


Figure 4.9: An example of the variations on timescales shorter than the orbital period in the 2009 outburst in V364 Lib.

estimated to be $\sim 35^\circ$ from the equation $\sin^3 i = PK_2^3(1+q)^2/(2\pi GM_1)$, using the estimated values of K_2 , mass ratio, and mass of the primary, which is consistent with a $v \sin i$ value of 85 km/s estimated from the absorption lines in quiescence.

4.5 Numerical modeling of orbital variations

4.5.1 Premises

As described in the introduction, the observational W UMa-type orbital variations in quiescence are sometimes ascribed to their hot companions in 1SWASP J1621 and BD Pav (Waagen, 2017; Barwig, Schoembs, 1983), but there is evidence that

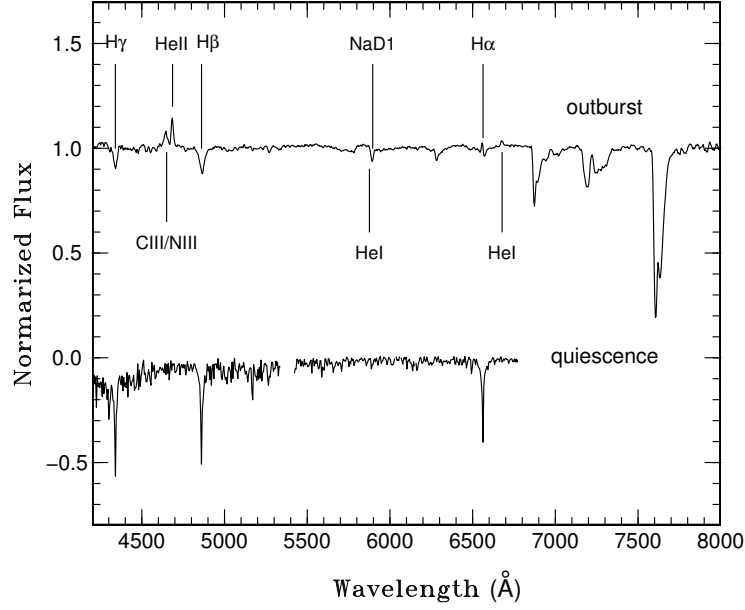


Figure 4.10: Spectrum of V364 Lib on April 7th, 2009 in the outburst state. For reference, an example of the spectra in quiescence on May 5th, 2009 is also displayed. For visibility, an offset of -1.0 is added to the quiescent spectrum.

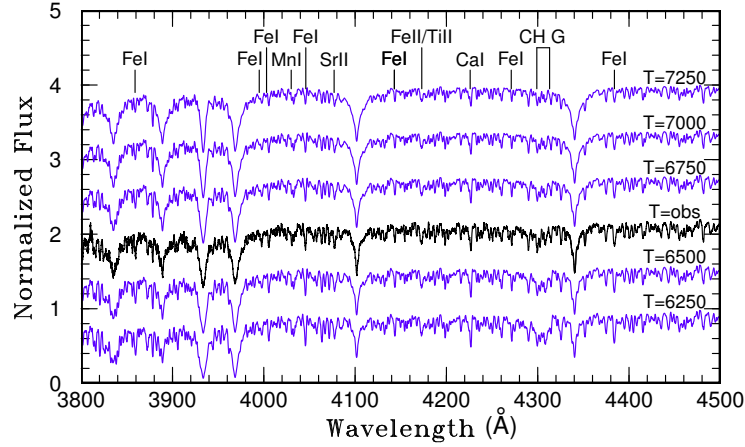


Figure 4.11: Spectrum of V364 Lib in quiescence. The black line represents observations on May 5th, 2009. The blue lines represent the broadened synthetic spectra in Castelli, Kurucz (2004).

1SWASP J1621 and BD Pav have cool companion stars (Thorstensen, 2016; Sion et al., 2008; Friend et al., 1990). In addition, our spectral analyses in Sec. 4.4 suggest that V364 Lib has a hot companion star and a low inclination. We thus wondered if the double-waved modulations in quiescence and the outburst amplitudes are explained without hot companions as for 1SWASP J1621 and BD Pav, and with the nature suggested by our results in Sec. 4.4 in V364 Lib, via the mod-

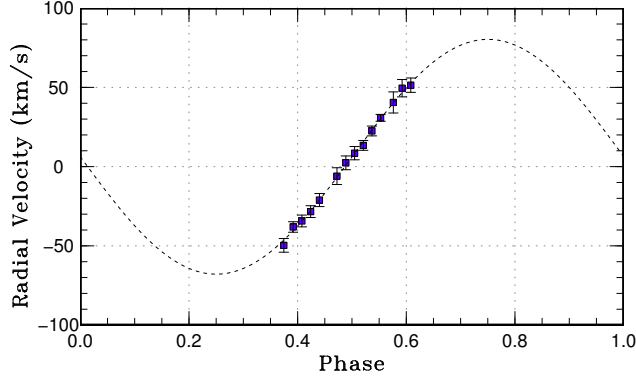


Figure 4.12: Radial velocity extracted from the average value of $H\beta$ and Mg absorption lines in quiescence under the assumption that the orbital period is 0.7024293 d. The rectangles represent the observations. The dashed line is the best fitted sine curve with the semi-amplitude of 74.1 ± 1.1 km/s and the systemic velocity of 6.2 ± 5.4 km/s.

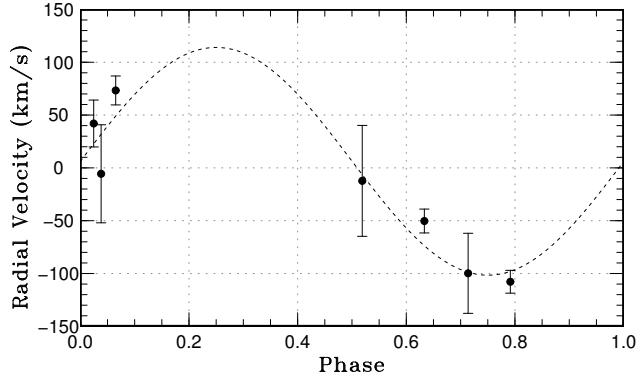


Figure 4.13: Radial velocity extracted from He II 4686 emission line in the 2009 outburst under the assumption that the orbital period is 0.7024293 d. The points represent the observations. The dashed line is the best fitted sine curve with the semi-amplitude of 107.8 ± 12.6 km/s and the systemic velocity of 6.2 km/s.

eling of the orbital phase profiles. Since we have the R_C , V , and B -band phase profiles in outburst (see Figure 4.3) in 1SWASP J1621, we considered reproducing their characteristics in this object.

Before presenting our modeling, we summarize the information derived from our observations. According to Horne (1985), the width of the eclipse of a white dwarf is determined by the mass ratio and inclination of its system. The width is approximately estimated as the width of eclipse at half depth if the disk is axisymmetric. Regarding the inflection phases in outburst in the R_C -band profile for 1SWASP J1621 and in the clear-filter profile for BD Pav as the ends of eclipses, the rough estimates of the width of eclipse of the white dwarf are ~ 0.10 in 1SWASP J1621

and ~ 0.06 in BD Pav, respectively. Substituting the half values of them and the values of mass ratios to equation (3) in Horne et al. (1982), which represents the relation between the inclination, mass ratio, and width of eclipse of a white dwarf, we obtain inclinations of ~ 87 -deg in 1SWASP J1621 and ~ 75 -deg in BD Pav, respectively. These values are consistent with those extracted by Kjurkchieva et al. (2017); Zola et al. (2017); Friend et al. (1990); Sion et al. (2008). In addition, the almost symmetric phase-averaged profiles around the primary minima in these two objects seem to imply a dim hot spot (see Figures 4.3 and 4.5).

4.5.2 Method

4.5.2.1 Code

We used the numerical code which is described in detail by Hachisu, Kato (2001), Hachisu, Kato (2003a), and Hachisu, Kato (2003b). In this code, the companion star fills its Roche lobe and a circular orbit is assumed. The surfaces of the white dwarf, companion star, and accretion disk are divided into patches, and each patch emits photons by black body radiation at the local temperature. The patches of the disk and companion are irradiated by the front-side patches of the white dwarf if there is no patch between them. The non-irradiated surface temperature is determined by viscous heating of the standard disk model (Shakura, Sunyaev, 1973). The total luminosity of the binary system is calculated by summing up the luminosity from all visible patches. The surface patch elements are 32×64 ($\theta \times \phi$) for the companion, 32×64 ($\theta \times \phi \times$ up and down side) for the accretion disk, and 16×32 for the white dwarf.

4.5.2.2 Parameters

Using the code described in Sec. 4.5.2.1, we computed eclipsing light variations with various test parameters: the inclination angle, temperature distribution, height and radius of the accretion disk. Inclinations were varied over ranges of 84–90 deg, 72–78 deg, and 30–40 deg by steps of 1 deg for 1SWASP J1621 and BD Pav, V364 Lib, respectively, by considering the estimation in Sec. 4.4 and Sec. 4.5.1. Tested disk radii were 0.5, 0.6, 0.7, 0.8, 0.9 R_{L1} . Here, R_{L1} is defined as $R_{L1} = a/(1.0015 + q^{0.4056})$, where a is the binary separation (equation (2.4c) Warner, 1995).

We gave the temperature distribution of the accretion disk in outburst, postulating

that the disk is steady at the outburst maximum (Wood et al., 1986) as follows:

$$T = T_* \left(\frac{r}{r_{\text{in}}} \right)^{-3/4} \left[1 - \left(\frac{r_{\text{in}}}{r} \right)^{1/2} \right]^{1/4}, T_* = \left(\frac{3GM\dot{M}}{8\pi\sigma r_{\text{in}}^3} \right)^{1/4}. \quad (4.1)$$

Here, r , r_{in} , G , M , $\dot{M}$ and σ represent the distance from the central object to a given ring of the disk, inner disk radius, constant of gravitation, mass of the primary star, mass accretion rate at a given radius, and Stefan-Boltzmann constant, respectively (Shakura, Sunyaev, 1973), and $\dot{M}$ is different from the mass-transfer rate from a companion star. The value of r_{in} is set to 0.02 in units of the binary separation. The α -value is also fixed to be 0.1. The critical value of $\dot{M}$ at a given radius, at which the disk returns in quiescence, depends on the radius in the accretion disk (equation (39) in Hameury et al., 1998), and the mass accretion rate in outburst would become large in long orbital period systems. We set values of $\dot{M}$ as 2×10^{-9} , 3×10^{-9} , 4×10^{-9} , 5×10^{-9} , 6×10^{-9} $M_{\odot} \text{ yr}^{-1}$ for 1SWASP J1621 and BD Pav* and 6×10^{-8} , 8×10^{-8} , 1×10^{-7} $M_{\odot} \text{ yr}^{-1}$ for V364 Lib, considering their orbital period. Also, we assumed the flat temperature distribution of a disk in quiescence (3,000 or 3,500 or 4,000 or 4,500K), referring to Wood et al. (1986).

On the basis of the assumption of the steady state, we set the thickness of accretion disk in outburst as follows:

$$\frac{H}{r} = 1.72 \times 10^{-2} \alpha^{-1/10} \dot{M}_{16}^{3/20} \frac{M}{M_{\odot}}^{3/8} r_{10}^{1/8} \left[1 - \left(\frac{r_{\text{in}}}{r} \right)^{1/2} \right]^{3/5}, \quad (4.2)$$

where H , $\dot{M}_{16}$ and r_{10} represent the height of the disk, $\dot{M}/10^{16} \text{ g s}^{-1}$ and $r/10^{10} \text{ cm}$, respectively (Shakura, Sunyaev, 1973). On the other hand, we assumed that the disk is flat in quiescence for simplicity. The tested parameters of the height of the disk were 0.003, 0.005, 0.007, 0.009 in units of the binary separation.

The masses of the primary and the companion are fixed. We used the values in Table 4.1. The radii of the white dwarfs in 1SWASP J1621, BD Pav, and V364 Lib are estimated to be 0.009, 0.008, and 0.008 $R_{\odot}$, respectively (Nauenberg, 1972). We assumed that the temperature of the companion star is 4,300 K for 1SWASP J1621, 3,500 K for BD Pav, and 6,600 K for V364 Lib, considering the spectral types of the companions (Allen, 1973) and the reasoning in Sec. 4.4. The temperature of the white dwarfs is fixed to 20,000 K. In addition, we need to set the distance from Earth to the object in order to determine the apparent magnitude.

*If $\dot{M}$ is less than $1 \times 10^{-9} M_{\odot} \text{ yr}^{-1}$, the outer disk cannot be in the outburst state (equation (39) in Hameury et al., 1998).

Table 4.1: Properties of 1SWASP J1621, BD Pav, and V364 Lib.

Object	P_{orb}^*	$M_1^\dagger$	$q^\ddagger$	Outburst History [§]	Reference
1SWASP J1621	0.207852(1)	0.9	0.44	2006, 2016	1, 2, 3
BD Pav	0.17930	1.0	0.44	1938, 1985, 1996, 1997, 1998, 2000 2006, 2015, 2017	4–10 18
V364 Lib	0.70243(11)	1.0 ± 0.2	1.5 ± 0.3	2003, 2006, 2009, 2012, 2017, 2019	11–17 18

*Orbital period in units of d.

[†]Mass of the primary white dwarf in units of $M_\odot$.

[‡]Mass ratio of the companion star to the primary star ($q \equiv M_2/M_1$).

[§]Years when outbursts were recorded.

^{||}1: Thorstensen (2016), 2: Drake et al. (2014a), 3: Kjurkchieva et al. (2017), 4: Sion et al. (2008), 5: Friend et al. (1990), 6: Barwig, Schoembs (1987), 7: vsnet-alert 1008, 8: vsnet-alert 2388, 9: vsnet-alert 4888, 10: vsnet-alert 18762, 11: Uemura et al. (2009), 12: vsnet-alert 14271, 13: Wils (2011), 14: Pojmanski et al. (2009), 15: Kinugasa et al. (2009), 16: vsnet-alert 20877, 17: vsnet-alert 23397, 18: Kimura et al. (2018b).

4.5.2.3 Limitation of our model and how to determine the best model parameters

Our modeling deals with only the emission from optically-thick regions and assumes a simple geometry, an axisymmetric disk, no hot spot and no hot corona. The hot spot is located near the disk edge, and an optically-thin hot corona expanding around the disk is produced by the evaporation of the inner disk at low accretion rate. These components, which are missing from our models, have high temperature ranging between $\sim 5,000$ – $20,000$ K (Liu et al., 1995; Stanishev et al., 2001). In quiescence and/or in high inclination systems, therefore, these optically-thin hot regions are likely dominant, and hence, our models have some difficulties reproducing the observations, especially in the B band. In other words, the observations in outburst in the R_C band are expected to be reproduced more easily in our models than those at shorter wavelengths. In addition, we assume that the distribution of the disk temperature in outburst follows that in the steady state, but the real distribution may not. The lack of a hot spot, simple disk geometry, and simple distribution of the disk temperature in the models are also raised in Zola et al. (2017) as the factors responsible for not completely explaining the observations.

On the basis of the limitations described in the previous paragraph, we gave priority to reproducing the R_C -band phase-averaged profile when determining the best models of 1SWASP J1621. In addition, we note that the inclination must have

the same value in outburst and quiescence. We thus chose the best models as for 1SWASP J1621 in the following way.

1. We estimated at first each χ^2 value between the observational and calculated orbital phases in the outburst state with various tested parameters specified in Sec. 4.5.2.2, and determined the best model in that state by the minimum χ^2 estimations in the R_C band.
2. With the inclination fixed to its value in the best model in outburst, we estimated each χ^2 value between the observational and calculated orbital phases in quiescence. We determined the best model by the minimum χ^2 estimations in the R_C band.

As for BD Pav and V364 Lib, we chose the best model in quiescence by the minimum χ^2 values between the observed V -band phase-averaged profile and the calculated V -band one. In addition, we chose the best mass accretion rate, which can best reproduce the outburst amplitude in the V band, with the disk radius and inclination fixed as the best-fit parameters in quiescence.

It is difficult to employ sophisticated methods to calculate error bars due to the long computational times; therefore, we estimated the model profiles with the rough grids described in Sec. 4.5.2.2, and determined the 90% confidence intervals as the range in which $\Delta\chi^2$ from the minimum χ^2 is within 2.706, by computing the models in the finer grids than those described in Sec. 4.5.2.2. Since accurate constraints of the inclination and disk height in quiescence is beyond our current goals for 1SWASP J1621 and BD Pav, we fixed the inclination to the value which produces the minimum χ^2 in these two objects. Additionally, it is difficult to restrict the ranges of the parameters which are related to the accretion disk in V364 Lib because of the bright companion; hence, we only provided an error bar for the inclination.

4.5.3 Results

Our modeling suggests the inclination is close to 90 deg in 1SWASP J1621 (see Table 4.2). The best-fit models are displayed in Figure 4.14. The very high inclination has been already noticed by Kjurkchieva et al. (2017) and Zola et al. (2017). Our models do not accurately fit the phase profiles in quiescence and in the V and B bands in outburst, but they do reproduce the main characteristics of quiescent double-waved modulations without a hot companion. The models in Kjurkchieva et al. (2017) and Zola et al. (2017) for 1SWASP J1621 also did not reproduce the observations in all of the bands. Although our model includes more reasonable

thicknesses of disks and temperature distributions of quiescent disks, compared to their models, it still had difficulty reproducing the quiescent profiles. This would be due to the limitations of our modeling, which are described in the first paragraph in Sec. 4.5.2.3. In particular, our model does not include any optically-thin region which is observable in high-inclination systems and in quiescence. In addition, a hot spot would be dominant around the phase 0.75 in the B band. Also, the calculated depth of the eclipse in the B band in outburst is ~ 0.5 -mag deeper than the observed one. Moreover, the calculated $B - V$ color is larger by ~ 0.1 mag, and the calculated $V - R$ color was smaller by ~ 0.15 mag than the observed one. Some fine-tuning of the disk temperature and structure may help to reproduce completely the observational depth of the eclipse and colors, but this is beyond our purpose. Since this system seems to have a very high inclination, there is a possibility that a flared disk structure may contribute to the emission. If the inclination is close to 90 deg, the hot white dwarf and the inner disk are hidden by the outer disk, as shown in the upper panel of Figure 4.17. In this case, the radiation from the hot white dwarf and the innermost region of the disk hardly contributes the overall emission from the object. This situation will easily produce the shallow primary minima in quiescence.

As for BD Pav, the inclination seems to be close to 75 deg (see Table 4.2), which is consistent with previous studies (Sion et al., 2008; Friend et al., 1990) and the estimation in Sec. 4.5.1. Since our modeling does not explain the observations well enough to estimate error bars, we just provide the best model parameter in the rough grids. The reasons why our model does not completely reproduce the observations are the limitations described in Sec. 4.5.2.3; in addition, our sparse profile may make it difficult to recognize the eclipse of the white dwarf in this object. The best-fit model in quiescence which is given in Figure 4.15, however, reproduces the W UMa-like orbital modulations without a hot companion as in 1SWASP J1621. The mass accretion rate to reproduce the outburst amplitude is in the range expected by the disk-instability model. In addition, according to Barwig, Schoembs (1987), the $B - V$ colors in the outburst and quiescent states are ~ 0.1 and ~ 0.6 mag, and the $V - R$ colors in the outburst and quiescent states are ~ 0.0 and ~ 0.5 mag, respectively; our modeling reproduces well the $V - R$ colors. Although the calculated $B - V$ color in quiescence was ~ 0.6 mag larger than the observational one, the reported colors in Barwig, Schoembs (1987) were measured at the end of the outburst. Some time after the outburst, the $B - V$ color might become large as the disk becomes cool. Though the grazing eclipses in the systems having ~ 70 -deg inclinations are characterized by deep primary minima in quiescence as in U Gem (chapter 2.4.3 Hellier, 2001a) due to bright hot spots, the eclipse in BD Pav does not agree with this behavior. This would be because of

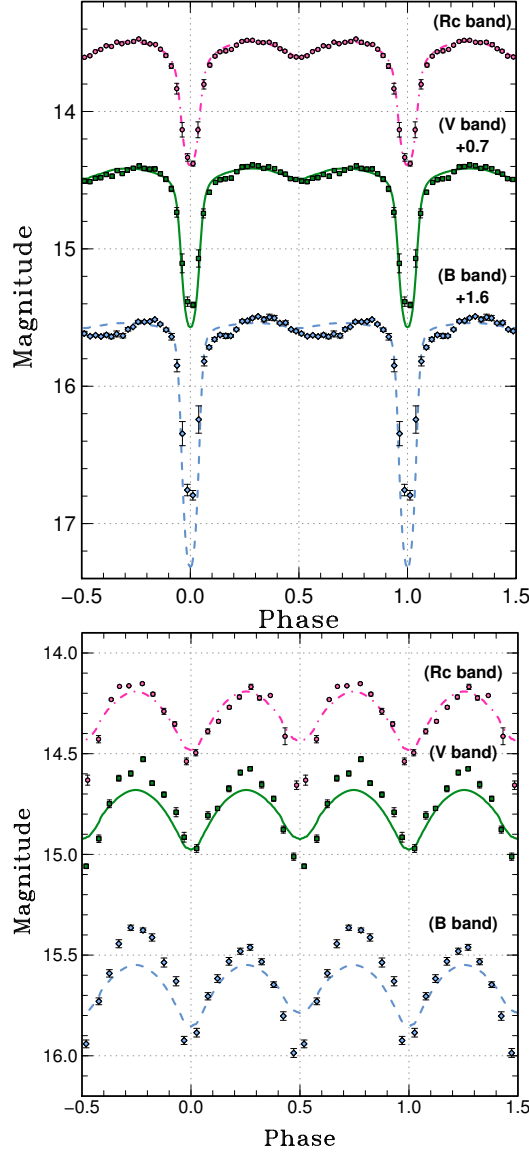


Figure 4.14: Best models of the phase profiles of 1SWASP J1621 in outburst (upper panel) and quiescence (lower panel). The dot-dash, solid and dash lines represent the calculated phase profiles in the R_C , V , and B bands. The points with error bars are the observational phase profiles. The magnitudes of the observational phase profiles are offset for visibility except for those in the R_C -band ones in outburst.

the dim hot spot as suggested in Sec. 4.5.1. If the inclination is 75 deg, the white dwarf is barely eclipsed (see also the middle panel of Figure 4.17).

As for V364 Lib, we confirmed that the ellipsoidal variations of the hot companion are dominant in quiescence, and that the inclination is low (see Table 4.2). The estimated inclination is consistent with that in Sec. 4.4. The best-fit model is shown in Figure 4.16. The accretion disk is not eclipsed, but the ellipsoidal variations are observable due to the hot companion star (see the lower panel of Figure 4.17).

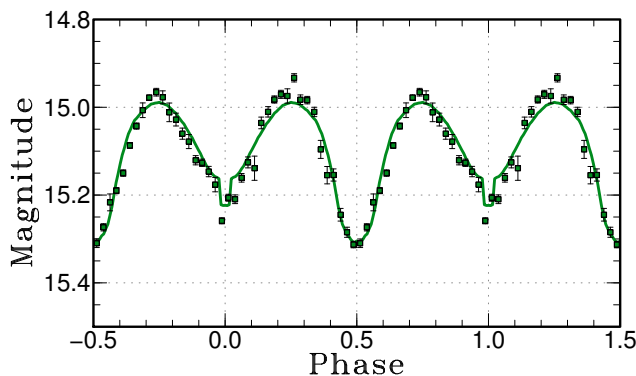


Figure 4.15: Best model of the phase profile of BD Pav in quiescence. The solid line represents the calculated phase profile in the V band. The points with error bars are the observational phase profile.

As for the outburst state, the value of $\dot{M}$ is chosen to reproduce the outburst amplitude. The $\dot{M}$ value much larger than those in the other two objects seem to originate from its long orbital period. The critical value of $\dot{M}$ at which the transition from the hot branch to the cool branch on the thermal equilibrium curve is triggered, is approximately proportional to $r^{2.7}$ (Hameury et al., 1998)*. If the hot state extends to the outer disk in its outbursts, the mass accretion rate in outburst is likely much higher than those in shorter-period DNe, in spite of our rough estimations. We also confirmed very weak ellipsoidal variations with amplitudes of less than 0.03 mag via the model with the parameters given in Table 4.2; this is consistent with our observations. The reason for the lack of prominent ellipsoidal variations in outburst is that the disk becomes much brighter in the outburst than that in quiescence. We consider that the bright disk in outburst reduces the relative contribution of the ellipsoidal variations of its companion star, and that some variations hidden by the bright companion during quiescence become visible in outburst. In addition, the outburst amplitude is reproduced with the parameter in Table 4.2. The colors of $V - R$ and $B - V$ in quiescence are reported to be 0.1 and 0.5 mag, respectively, by Henden et al. (2016), and our calculated colors were almost identical. In the systems having a high-temperature companion, its contribution dominates the whole emission from the dwarf nova. If these systems have high inclinations, the eclipses of companions may become deeper than those of the disks and white dwarfs in quiescence, and the eclipses of disks and white dwarfs are prominent in outburst. These are inconsistent with the observations of V364 Lib.

*Here, r is the same one as defined in Sec. 4.5.2.2.

Table 4.2: Model parameters to reproduce the eclipsing light variations and computed outburst amplitudes and colors in 1SWASP J1621, BD Pav, and V364 Lib.

Parameters	1SWASP J1621	BD Pav	V364 Lib
i^*	87	75	37^{+1}_{-4}
$R_{\text{disk}}^{\text{burst} \dagger}$	$0.89^{+0.01}_{-0.04}$	0.90 (fix)	0.80 (fix)
$\dot{M}^{\ddagger}$	$5.4^{+0.1}_{-0.3} \times 10^{-9}$	2×10^{-9}	1×10^{-7}
$R_{\text{disk}}^{\text{qui} \S}$	$0.86^{+0.04}_{-0.01}$	0.90	0.80
$T_{\text{disk}}^{\parallel}$	$4,470^{+30}_{-70}$	3,000	4,500
$h_{\text{disk}}^{\#}$	0.009	0.007	0.003
$d^{\P}$	166	253	536
Amp ^{**}	0.70	3.11	0.85
$V - R_{\text{out}}^{\dagger\dagger}$	0.23	0.10	0.06
$B - V_{\text{out}}^{\ddagger\dagger}$	0.23	0.04	0.05
$V - R_{\text{qui}}^{\S\S}$	0.49	0.73	0.20
$B - V_{\text{qui}}^{\parallel\parallel\parallel}$	0.87	1.12	0.36

*Inclination angle in units of deg.

$\dagger$ Disk size in the outburst state in units of R_{L1} .

$\ddagger$ Mass accretion rate in outburst in units of $M_{\odot} \text{ yr}^{-1}$. We assume the steady state in outburst.

$\S$ Disk size in quiescence in units of R_{L1} .

$\parallel$ Temperature of a disk in quiescence in Kelvin.

$\#$ Thickness of a disk in units of its binary separation.

$\P$ Distance from Earth to the object, which reproduces the apparent magnitude in the R_{C} band in outburst as for 1SWASP J1621, and in the V band in quiescence as for BD Pav and V364 Lib, in units of pc.

**Outburst amplitude in the R_{C} band at phase 0.25 in units of magnitude. It is derived from the best models in outburst and quiescence.

$\dagger\dagger$ $V - R$ color in outburst at phase 0.25 in units of magnitude.

$\ddagger\dagger$ $B - V$ color in outburst at phase 0.25 in units of magnitude.

$\S\S$ $V - R$ color in quiescence at phase 0.25 in units of magnitude.

$\parallel\parallel\parallel$ $B - V$ color in quiescence at phase 0.25 in units of magnitude.

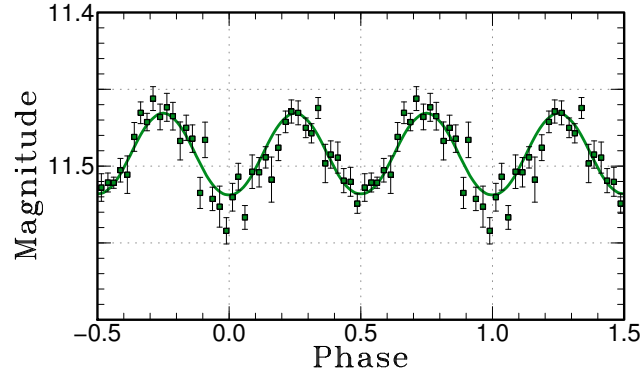


Figure 4.16: Best model of the phase profile of V364 Lib in quiescence. The solid line represents the calculated phase profile in the V band. The points with error bars are the observational phase profile.

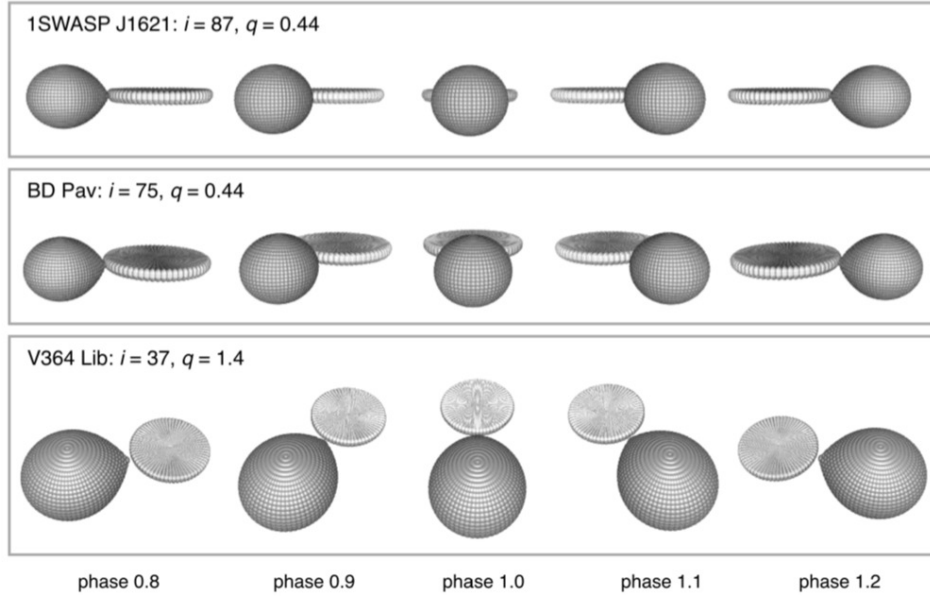


Figure 4.17: Model configurations around the eclipses of the accretion disks in 1SWASP J1621, BD Pav, and V364 Lib in the outburst state.

4.6 Discussion

4.6.1 Nature of the three objects and their low-amplitude outbursts

Although 1SWASP J1621, BD Pav, and V364 Lib show similar outburst behavior, their natures are not necessary the same. As mentioned in the introduction, some people consider 1SWASP J1621 to have an unusually hotter companion star than

normal K or M-type companions in DNe (see Figure 2.45 in Warner, 1995) according to Waagen (2017), but our results show that the characteristics of the orbital phase profiles of 1SWASP J1621 can be explained by its very high inclination and faint hot spot, without a hot companion star. This condition is similar to that in BD Pav. On the other hand, for V364 Lib, our results support that this system has a low inclination and an F-type hot companion star, which is expected to be bright (large) by the long orbital period and the large mass ratio (see also Sec. 4.4).

Their properties are likely relevant to their small-amplitude ($\sim 1\text{--}2$ mag) outbursts (see Sec. 4.3). In the cases of 1SWASP J1621 and BD Pav, the critical conditions are their high inclinations and inside-out outbursts. In high-inclination systems, the observable disk surface is small. In addition, if a system has a very high inclination, as in 1SWASP J1621, the hot inner region of the accretion disk is hidden by the outer disk located along the line of sight. Although the inner disk becomes bright in the outburst state, it cannot be observed. The inside-out outbursts expected from the slow-rise shapes of the outbursts (see Sec. 4.3) would be another key factor of the low-amplitude outbursts. The outburst amplitudes of inside-out outbursts are often smaller than those of outside-in outbursts, since the slowly propagating heating front prevents the whole disk from entering the hot state (Smak, 1984; Cannizzo et al., 1986). On the other hand, in V364 Lib, its hot companion star is the main factor in its low-amplitude outbursts, though inside-out outbursts may also contribute to the small outburst amplitudes. If the companion is unusually hot and big, its contribution to the whole brightness is large at optical wavelengths. The change of the disk brightness between the quiescence and outburst states would be unremarkable at optical wavelengths even if the total luminosity emitted from the disk is high in outburst. We suggest that astronomical surveys might easily overlook such a low-amplitude outburst.

4.6.2 Mass-transfer bursts ?

We believe that the outbursts in the three objects are explained well by the disk-instability model. Qian et al. (2017) claimed the mass transfer ceases because of the star spot on the surface of the companion star, which is located at the L_1 point in quiescence, and the outbursts are caused by an abrupt mass transfer because of the expanding companion in 1SWASP J1621. However, our results show the existence of the large accretion disk in the outburst state in this object, and this is inconsistent with their argument. If a lot of matter is transferred to the primary side at the onset of the outburst, the radius of the accretion disk becomes close to the Lubow-Shu radius (Lubow, Shu, 1975), which is approximately expressed by $0.0859q^{-0.426}$ in units of the binary separation (Hessman, Hopp, 1990), because

of the conservation of angular momentum (Ichikawa, Osaki, 1992). This value is much smaller than the estimated disk size in our modeling. When a small disk is occulted by the companion, the shape of the bottom of primary minima becomes flat, and this phenomenon disagrees with the morphology of the phase profiles observed in the outburst state. In addition, continuous accretion during quiescence may help to build a large disk in outburst by removing matter with small specific angular momentum. Zola et al. (2017) also concluded that their modeling of the eclipsing profile in quiescence suggests a white dwarf plus a large accretion disk, and it is consistent with our results in Sec. 4.5.3. Their results, and ours, argue against the halting of mass transfer in quiescence, which is a key element in Qian et al. (2017). Therefore, there is no need for introducing mass transfer bursts to explain the outbursts in this kind of object. Qian et al. (2017) proposed their interpretation based in part on the observations of Pavlenko et al. (2016), but the short-term periodic variations reported in this reference were not confirmed in our observations.

Actually, there are some contradictions between the observations and the discussion in Qian et al. (2017). For example, they considered that only the eclipse of the white dwarf was observed in quiescence, but the width of the eclipse that they regard as the ingress of the white dwarf, about 0.007 d, indicates that the eclipsed object must have a width of more than 10 times of the radius of a $0.9-M_{\odot}$ white dwarf.* In addition, the depth of the eclipse, about 0.2 mag, is not consistent with the eclipse of a white dwarf. If the temperature of the white dwarf is 20,000 K, the contribution of the white dwarf to the brightness of the entire system is about 10 %. The depth would then be less than 0.1 mag in their observations. Since they consider that accretion onto the white dwarf is stopped during quiescence, the white dwarf should have a low-temperature in their discussion. If the temperature of the white dwarf is only 10,000 K, the depth of eclipses would be less than 0.01 mag. Moreover, they insisted no lasting disk in this object. If the contribution of the disk is very weak and the disk size is small in outburst, the flux at the primary minima in outburst would be the same as those in quiescence; however, this is inconsistent with Figure 3 in Qian et al. (2017).

4.6.3 Infrequent, long-lasting and inside-out outbursts

Though we may overlook the outbursts due to their small amplitudes, the frequency of the outbursts in these systems is relatively low in comparison with many other DNe above the period gap (see also Table 4.1), and their outburst durations are

*The width of the ingress of the white dwarf is derived from the data in Figure 2 in Qian et al. (2017).

relatively long. We discuss below the origin of the infrequent, long-lasting, and inside-out outbursts.

As for the origin of the infrequent outbursts in 1SWASP J1621 and BD Pav, we consider that it is attributed to low mass-transfer rates. This is because 1SWASP J1621 and BD Pav have clearly longer outburst intervals than many other DNe having 4–5 hours orbital periods. In addition, the almost symmetrical profiles with respect to the primary minima in these two objects (see Figures 4.3 and 4.5) suggest that the hot spots are faint. This implies that the mass transfer rate is low, since the brightness of a hot spot depends on it.

On the other hand, the origin of the infrequent outbursts in V364 Lib would not be necessarily low mass transfer rate. We suspect its long orbital period is possibly the source of infrequent outbursts. This is because it would take much time to form a dense disk enough to trigger outbursts in V364 Lib due to its long orbital period which means a large value of R_{L1} . The relation between long outburst intervals and long orbital periods in several DNe has been pointed out in Bruch (2017), and we show it in Figure 4.18. The values of outburst intervals in the long-period objects were derived from Menzies et al. (1986); Shears, Poyner (2009); Sekiguchi (1992); Bruch (2017) and see also Figure 6.2 in Appendix A. V1017 Sgr also has a very long orbital period, but we excluded it from the regression shown in Figure 4.18 because it is clearly out of the trend.

For instance, compared with an typical dwarf nova having an orbital period of 4 hours, a mass ratio of 0.45 and outburst intervals of 30–100 days, V364 Lib has an R_{L1} -value about 2.6 times larger. If the transfer rate is almost equal to that in a typical dwarf nova, then the accretion disk in V364 Lib would accumulate ~ 2.5 – 7 -times more mass than that in a typical dwarf nova until an outburst is triggered. In other words, based on mass transfer in ordinary DNe, we would expect V364 Lib to have an outburst interval of about 70–700 days, but this value seems to be shorter than the observational outburst intervals in V364 Lib. Actually, V364 Lib may not follow the linear relation between the orbital period and the logarithm of the outburst intervals in these systems, although all outbursts in this object might not be detected due to their small amplitudes.

Since the above discussion is a simple approach, our estimation has some uncertainty. For example, the mass transfer rate is likely larger in the systems having longer orbital periods so that the rate in this object might be about 4 times larger than that in an typical dwarf nova according to equations (38) and (39) in Hameury et al. (1998). It is, however, possibly similar to the typical one, because this system enters inside-out outbursts. In addition, the structure and density profile in the disk would also affect the outburst intervals, and the expected intervals may

become longer. We thus could not reject either of the long orbital period or the low mass-transfer rate as the origin of the infrequent outbursts in V364 Lib. The value of $\dot{M}$ in Table 4.2 represents not the mass-transfer rate but the accretion rate in outburst. The actual mass transfer rate should be smaller than that value.

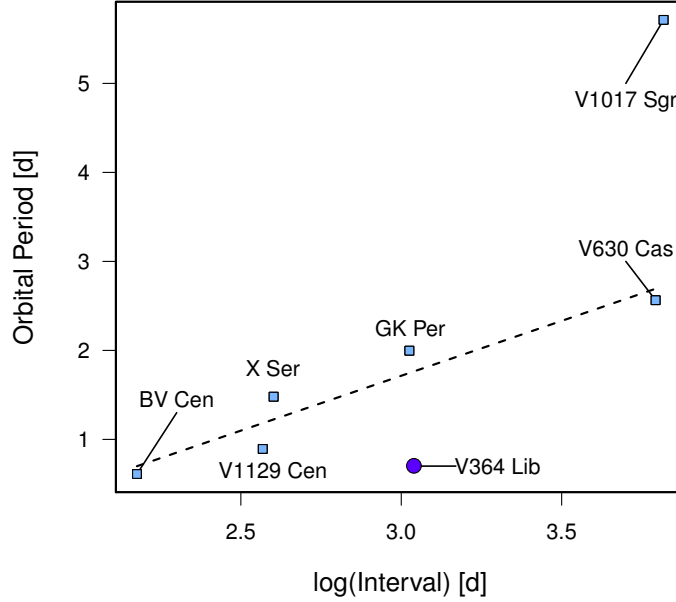


Figure 4.18: Linear relation between the orbital periods and the logarithm of the outburst intervals in several DNe having very long orbital periods. The dashed line represents the regression formula $y = 1.2x - 2.0$ estimated by the least-squares method with the data of BV Cen, V1129 Cen, GK Per, X Ser and V630 Cas. Here, x and y represent the orbital period and the interval between outbursts.

The inherent low-mass transfer rates and/or long orbital periods would naturally explain the characteristic long outburst duration and inside-out outbursts in the three objects within the context of the disk instability model. This is because a lot of mass is expected to accumulate in the disk and because the mass would be easily diffused inward on viscous timescales in these conditions. Actually, the numerical simulations of the disk instability model showed that, in the dwarf nova GK Per having a very long orbital period, most of its outbursts are easily triggered at the inner disk, and that this behavior cannot be reproduced by mass-transfer bursts (Kim et al., 1992). It is also consistent to the idea of the disk-instability model that the large stored disk mass before the onset of an outburst can be related to the long outburst durations. The slow rises of these outbursts, in addition to their small amplitudes, may also make it difficult to detect them. Long-lasting inside-out outbursts were commonly observed in V1129 Cen and HS 0218, which are similar to V364 Lib and 1SWASP J1621, respectively (Bruch, 2017; Golysheva

et al., 2012); Golysheva et al. (2012) have already pointed out that an inside-out outburst is expected from systems with low mass-transfer rates, which is considered to be associated with the almost symmetrical phase profiles, as in HS 0218.

The mechanism that causes lower mass-transfer rates in the three objects than those in many other dwarf novae above the period gap may be unclear. One plausible explanation is that the mass-transfer rate varies in the same system on much longer timescales than the interval between outbursts for some reason. For example, Shara et al. (1986) and Kovetz et al. (1988) proposed hibernation scenario. According to their proposition, during about a century after a nova eruption, the mass-transfer rate remains high due to the strong irradiation from the white dwarf, and thereafter, it decreases. V1213 Cen, a classical nova, revealed dwarf-nova outbursts 6 years before its eruption (Mróz et al., 2016), and this phenomenon is understood as evidence of hibernation. The hibernation is, however, believed to occur only when the binary separation expands just after a nova eruption. On the basis of equations (14) and (21), which represent the relation between the variation of binary separation and the mass ratio in Shara et al. (1986), it is doubtful whether the binary separation expands after a nova eruption in V364 Lib. 1SWASP J1621, BD Pav may be in hibernation, but V364 Lib may not.

4.6.4 Appearance of highly ionized emission lines

The He II 4686 emission line was observed in 1SWASP J1621, BD Pav, and V364 Lib during the outburst state (Scaringi et al. (2016); Barwig, Schoembs (1987); Sec. 4.4). In addition, the C III/N III emission line was also observed in the outburst of V364 Lib. These highly ionized lines are barely detected in the outbursts of many DNe. In the case of V364 Lib, a massive white dwarf is a natural consequence on the basis of its massive companion star (see Sec. 4.4). Since the primary star has degenerated earlier, the progenitor of the primary white dwarf would be more massive than a F-type companion star as for V364 Lib. For example, the relation between the final white-dwarf mass and the initial main-sequence mass given in Fig. 2 in Salaris et al. (2009) shows the white-dwarf mass can exceed $1M_{\odot}$ if the initial mass exceeds $\sim 6M_{\odot}$, and our observations do not deny the possibility that the initial mass of the primary exceeds $\sim 6M_{\odot}$. It is thus suggested that the primary white dwarf in this system is likely heavier than those in the CVs which do not have massive donor stars. In spite of the large uncertainty, our estimation in Sec. 4.4 implies the possibility that this object has a massive (more than $1M_{\odot}$) white dwarf. If the white dwarf is massive, the inner disk has a relatively high temperature. The massive white dwarf may be, therefore, the origin of the highly ionized emission lines in this system. We note that some DNe having massive white

dwarfs showed highly-ionized emission lines in outburst (e.g., Wilber et al., 2015; Wargau, 1988; Morales-Rueda, Marsh, 2002). In addition, this type of line was detected in a recurrent nova having a massive white dwarf (Diaz et al., 2010).

On the other hand, the source to produce the He II emission line in outburst in normal DNe having $\sim 0.8-M_{\odot}$ white dwarfs (Zorotovic et al., 2011) is unclear, but such a highly ionized emission line is sometimes observed in high inclination systems (e.g., Baba et al., 2002; Harlaftis et al., 1992). The He II emission line in 1SWASP J1621 and BD Pav may arise due to their high inclinations, although their relatively massive white dwarfs, heavier than $\sim 0.8-M_{\odot}$, may contribute to the line as in V364 Lib. This possibility is also notified by Zola et al. (2017). In the high-inclination systems, the accretion disk is optically thick. Additionally, the He II emission line in the 1985 outburst of BD Pav showed a single-peaked profile according to Figure 1 in Barwig, Schoembs (1987), although our modeling and other works suggest the inclination of this system is ~ 75 deg. The emission line from the accretion disk will have a clear double-peaked shape due to its high inclination (Horne, Marsh, 1986). Thus the radiation is not likely to originate from a disk. Such a single-peaked He II emission line was also observed in outburst in HT Cas, another dwarf nova, and a disk wind is proposed as its possible origin (Murray, Chiang, 1997). In the outbursts of DNe, the disk temperature becomes high as in nova-like systems. The single-peaked emission lines whose origin is interpreted as the disk wind are often observed in nova-like systems (e.g., Baptista et al., 2002), and hence, it would not be strange that the strong single-peaked emission lines are detected in some dwarf-nova outbursts. Additionally, an optically-thin disk wind may contribute to the phase profile in the B band.

4.6.5 Evolutionary stage of V364 Lib

It seems to be strange at first glance that V364 Lib, whose mass ratio is likely more than 1 has low mass transfer rate. In a subclass of CVs which have a companion more massive than the primary white dwarf, if the companion star completely fills with its Roche lobe, the mass transfer becomes easily unstable since the mass loss from the companion star induces the binary separation to shrink. We call this subclass “supersoft X-ray sources”. In these systems, the mass transfer rate is as high as $1-4 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ (see Kahabka, van den Heuvel, 1997, for a review and references therein). This value is more than 100 times higher than the typical mass transfer rate in DNe (Shafter et al., 1986). On the contrary, if we consider the whole amount of the stellar wind flows via the L_1 point under the condition that the radius of the companion star has not yet filled with its Roche lobe, the mass transfer rate is too low to trigger dwarf-novae type outbursts. Under the

assumption that the wind velocity is the escape velocity, the mass transfer rate estimated to be $\sim 10^{-11-12} M_{\odot}/\text{yr}$ by using the equation $\dot{M} = 4\pi R_2^2 \rho v_{\text{wind}}$. We finally suggest that in V364 Lib, the thin outer layer of the companion fills with its Roche lobe. Since the estimated value of the Roche radius on the companion side is about twice of the radius of a typical main-sequence F-type star, the companion is likely a sub-giant if our suggestion is correct.

Although the mass transfer rate in V364 Lib seems to be low at present, it may increase as the companion evolves and/or the orbital angular momentum decreases. Then this system may show nova eruptions repeatedly since the white dwarf is likely massive (Fujimoto, 1982). In addition, if the mass transfer becomes unstable (i.e., the transfer rate becomes close to $\sim 10^{-7} M_{\odot}/\text{yr}$), the system behaves as a super soft source. V364 Lib might eventually become a supernova via the super-soft-source phase. The duration of the post-common-envelope phase, before the system evolves to a completely semi-detached binary, is likely long, since the companion star evolves on timescales of $\sim 10^9$ yr and the orbital angular momentum is decreasing on timescales of $\sim 10^{8-9}$ yr (e.g., Figure 12 in Knigge et al., 2011). It would not be strange that some CV-like systems having massive companions show dwarf-nova-like outbursts. Recent numerical simulations on the CV evolution suggest that some CVs having F-type donor stars enter dwarf-nova type outbursts (Figures 4 and 16 in Kalomeni et al., 2016).

4.7 Summary

We presented the optical photometric observations of 1SWASP J1621, BD Pav, and V364 Lib and the optical spectroscopy of V364 Lib, and investigated their properties by using the numerical modeling. Our main findings and discussion are summarized in the following descriptions.

- There are many common features in the outburst behavior and the orbital variability in 1SWASP J1621, BD Pav, and V364 Lib. They show infrequent and small-amplitude outbursts. The outburst durations are long (about a few tens of days), and slow rises in brightness are observed at an early stage. In addition, all of the three systems show prominent ellipsoidal variations in quiescence.
- In spite of the similarities in the outburst behavior, some properties in each system are somewhat different from the others. Our observations and numerical modeling suggest that 1SWASP J1621 has a very high inclination (~ 90

deg) plus a faint hot spot, and these properties are similar to those in BD Pav, though it has a little lower inclination (~ 75 deg). On the other hand, V364 Lib likely has an F-type bright companion star, a massive white dwarf, and a low inclination angle.

- The characteristic low-amplitude outbursts in the three objects can be explained by a high inclination or a hot companion and inside-out outbursts.
- The almost symmetrical phase profiles with respect to the primary minima, which arise due to faint hot spots, and long intervals between outbursts, would suggest a low mass-transfer rate in 1SWASP J1621 and BD Pav. The long orbital period implies the large disk for V364 Lib. The low mass-transfer rate and/or the large disk may be related to long outburst intervals, long outburst durations, and inside-out outbursts in these three objects.
- The commonly observed, highly ionized emission lines in their outbursts would originate from the high inclinations, for 1SWASP J1621 and BD Pav, and from the massive white dwarf for V364 Lib. A disk wind may be related to the profile of these highly ionized emission lines.
- In spite of a donor star more massive than the primary white dwarf, V364 Lib shows dwarf-nova like outbursts. This implies only the thin outer layer of the donor star fills its Roche lobe in this system. This object may evolve into a recurrent nova or a supersoft X-ray source.

Although the properties of the three objects are somewhat different from normal DNe, (in particular, V364 Lib is different from many other DNe having cool companions), our results show that the peculiar outburst behavior and the changing orbital profiles between outburst and quiescence in the three objects can be explained in terms of the disk instability model in normal DNe. It is not necessary to consider strong magnetic activity and mass transfer bursts. The estimation of the mass ratio in V364 Lib has a large error, and hence, high-dispersion spectroscopy in the outburst state is important to determine it with high accuracy. We consider that the three systems recently reported by Bruch, Monard (2017); Bruch (2017); Golysheva et al. (2013) have properties similar to those of 1SWASP J1621, BD Pav, or V364 Lib, and our discussion is applicable to their peculiar outburst behavior. Although the total number of this kind of object may be large, it is difficult to find them due to their low-amplitude and rare outbursts.

Thermal-viscous instability in tilted accretion disks: a possible application to IW And-type dwarf novae

5.1 Introduction

Recently, a newly identified group of anomalous DNe attract many researchers. Simonsen (2011) noticed the presence of two unusual DNe which show repeated outbursts to standstills terminated by brightening (instead of fading as in ordinary Z Cam stars which we introduce in Sec. 1.7). After that, Szkody et al. (2013) called these objects “anomalous Z Cam stars” and discussed the potential relation with small outbursts in NLs. Recently, Kato (2019) found three more objects showing the same behavior as that reported by these authors and named these objects “IW And-type stars”. He has pointed out that more or less regular repetition of light curve patterns, i.e., “quasi-standstills” (i.e., state in intermediate brightness with (damping) oscillatory variations) terminated by brightening are common to these objects. He further suggested the presence of a previously unknown type of limit-cycle oscillation in IW And-type stars.

Hameury, Lasota (2014) explored for the first time the cause of “anomalous Z Cam phenomenon” and proposed a model in which variation in mass transfer rates from the secondary star is responsible for the IW And-type phenomenon. However, there is no positive evidence for triggering modulations of transfer rates; the brightening of bright spots, which should occur simultaneously with the sudden increase of transfer rates, remains observationally undetected (Honeycutt, 2001; Schlegel, Honeycutt, 2019). Hameury, Lasota (2014) sought the cause of the IW And-type

phenomenon in the outside of the disk, i.e., enhanced mass transfer from the secondary star. However, it would be more preferable if a previously unknown type of limit-cycle oscillation is found within the disk, as suggested by Kato (2019). Since the standard thermal-viscous instability model is very unlikely to produce the IW And-type phenomenon, for instance, standstill terminated by brightening, we need to seek some new aspects not considered in the standard disk instability model. To this end, recently, time-resolved optical photometry gives us a clue, the detections of negative superhumps in some of IW And stars.

Gies et al. (2013) and Armstrong et al. (2013) observed negative superhumps in some DNe that were later identified to be IW And-type stars (e.g., Kato, 2019). The negative superhumps are photometric light modulations with periods slightly shorter than the orbital period, i.e., they show a negative excess to the orbital period (Harvey et al., 1995), and they are interpreted as the transit of the bright spot on a tilted and/or warped disk misaligned to the orbital plane, which experiences retrograde nodal precession (Wood et al., 2000; Murray et al., 2002; Wood, Burke, 2007). Importantly, this interpretation means that the gas stream often flows into the inner part in tilted disks, while it always collides with the outer edge of the disk in non-tilted disks. In fact, Kato (2019) suggested that quasi-standstills in IW And-type DNe may somehow be maintained as the inner part of the disk stays in the hot state, while the outer part is in the cool state, and that thermal instability starting from the outer part of the disk terminates quasi-standstills. The mass input in a tilted disk might achieve such a new limit cycle by keeping the inner disk hot during quasi-standstills.

Motivated by this suggestion, we study the disk instability model in the case of tilted accretion disks. Our aim is to investigate how the thermal instability works in tilted accretion disks and to see if it could explain the essential feature of the IW And-type phenomenon. In Sec. 5.2, we introduce the observational light variations in IW And-type stars and specify their essential features, which we aim at reproducing. In Sec. 5.3, we give our assumptions and the method of our simulations in time evolution of the one-dimensional disk. In Sec. 5.4, we calculate the patterns of mass input in tilted disks, a key point of our simulations, and list the calculated models. In Sec. 5.5, we present the numerical simulations. We discuss our results in Sec. 5.6, comparing them with observations. Finally, we give our summary in Sec. 5.7.

5.2 Observational light variations in IW And-type stars

In the last couple of years we have obtained much more knowledge about the light variations in IW And stars than we did when the first example of the IW And-type DNe was recognized. Here we briefly introduce the essential features of the IW And-type light variations on the basis of the latest observations.

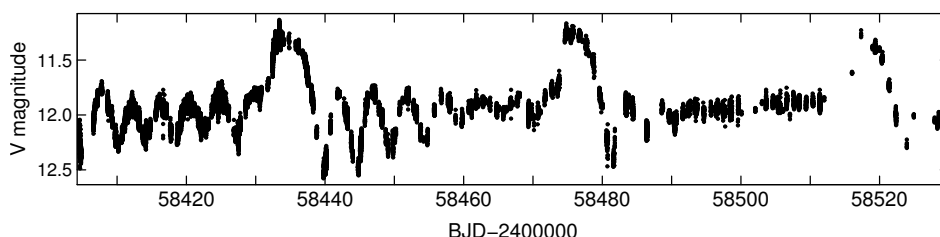


Figure 5.1: An example of the IW And-type phenomenon from a part of the 2018 light curves of IM Eri, which are obtained by a campaign led by Variable Star Network (VSNET). All of the IW And-type phenomenon of this object in 2018 is presented in Kato et al. (2019a). Here, BJD is barycentric Julian date.

IW And-type stars commonly show repetitive light variations as pointed out by Kato (2019). Figure 5.1 displays the typical IW And-type light variations of IM Eri*, which was identified to belong to IW And-type stars by Kato (2019). The most characteristic feature of the typical IW And-type light variations is “quasi-standstills” terminated by brightening, and they are one of the essential features of light variations in IW And stars. Here quasi-standstills are states of intermediate brightness with (damping) oscillatory variations, which are different from standstills with almost constant luminosity in normal Z Cam stars. We call this the IW And-type phenomenon hereafter. Deep luminosity dips occasionally follow brightening (see also Figure 5.2). The amplitudes of brightening are typically less than 1 mag. The average interval between brightening is ~ 50 days, but its length never remains in one constant value within one object.

Moreover, long-term photometric surveys for optical transients revealed that diverse light variations can be observed on long timescales even within one IW And star. Figure 5.2 illustrates long-term light curves of two IW And stars, FY Vul and HO Pup, which were recognized as IW And-type DNe by one of the authors (T.K. 2018, vsnet-chat 8101[†], 8162[‡]). It is another essential feature of the light

*Kato et al. (2019a) report in detail this kind of light variations of this object, which was observed in 2018.

[†]<<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-chat/8101>>

[‡]<<http://ooruri.kusastro.kyoto-u.ac.jp/mailarchive/vsnet-chat/8162>>

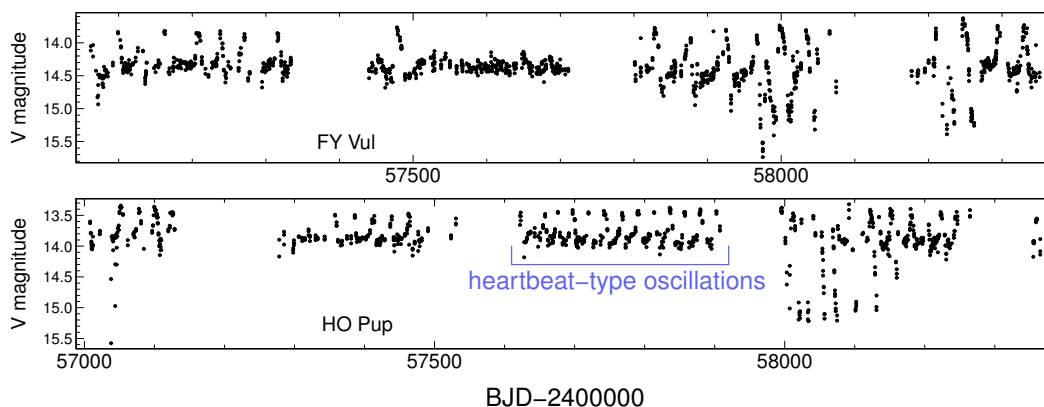


Figure 5.2: A wide variety in long-term light curves of FY Vul and HO Pup, IW And-type stars. We have obtained the data from ASAS-SN data archive (Davis et al., 2015).

variations in IW And stars that they alternate within one object between the IW And-type phenomenon, Z Cam-type standstills, normal dwarf-nova outbursts, and heartbeat-type oscillations on timescales of ~ 100 -1000 d. The averaged optical luminosities among those different types of light variations are almost constant and thus the time variations in mass transfer rates are unlikely to be the cause of their diversity. Here the heartbeat-type oscillations indicated in the lower panel of Figure 5.2 seem to be a new type of light variability, but may be common in IW And stars, because similar behavior has also been detected in other objects (e.g., Ramsay et al., 2016).

5.3 Method of numerical calculations for time-dependent disks

5.3.1 Basic assumptions of our model

In this study, we consider how the problem of thermal-viscous disk instability in CVs could be applied to tilted disks. Generally speaking, if the disk is tilted, it would have warped structures and it would experience retrograde nodal precessions. In the binary frame of reference, the disk structure would vary with the period of negative superhumps, i.e., the synodic period between the orbital period and the precession period. To study such complicated problem, we have to perform a full 3-dimensional hydrodynamical simulations of a non-axisymmetric disk whose structures change with time, which is not an easy task.

However, we think that one of the most important effects of the disk tilt on the problem of thermal-viscous disk instability is that the gas stream from the secondary star will penetrate deeply into the inner part of disk, in contrast to the standard case of a non-tilted disk where the gas stream arrives mostly in the outer disk edge. In this paper, we concentrate on studying this particular effect of the tilted disk. For this purpose, we adopt various simplifying assumptions as far as possible. We first assume that the tilted disk is not warped and rigidly tilted and we take this tilted plane misaligned to the orbital plane as our frame of reference. We further assume that the disk is axisymmetric in this frame of reference and we choose the cylindrical coordinates r and z in this plane, where r is the distance from the central white dwarf and z is the distance perpendicular to this plane. By these simplified assumptions our problem reduces to the same problem as the standard case of a non-tilted disk, and the effects of the tilt enter only through the mass supply pattern being different to that in the standard case. In fact, the standard case of a non-tilted disk is regarded as one of our special cases with the zero tilt angle.

Furthermore, we do not discuss any problems of time variations shorter than the negative-superhump period, and we average the mass supply pattern over that period (see also Sec. 5.4.1). This is because we focus on the light variations on timescales of days, much longer than the period of negative superhumps.

5.3.2 Basic equations for a viscous disk

We calculate the time evolution of a geometrically thin and axisymmetric accretion disk with the assumption of a coplanar but tilted disk, basically in the same way as in Ichikawa, Osaki (1992). It is noted here that since the original computer code of Ichikawa, Osaki (1992) does not exist any more, we have newly written our simulation code from the beginning in this study, following that paper.

We assume that the disk under consideration is geometrically thin and that it is in hydrostatic equilibrium in the vertical z -direction. We adopt one-zone model in the z -direction. Here, we use the cylindrical coordinates, (r, ϕ, z) , with the z -axis being the disk rotation axis. The surface density, Σ , is then given in one-zone model by

$$\Sigma = \int_{-\infty}^{\infty} \rho \, dz = 2\rho_c H, \quad (5.1)$$

where ρ is the density, H is the scale height, i.e., half the thickness of the disk which is given later by equation (5.6), and ρ_c is the density at the disk mid-plane, respectively. Hereafter we express the variables at the disk mid-plane with the subscript “c”. We use vertically integrated basic equations for a viscous disk: the

equations for conservation of mass, angular momentum, and energy, by adopting vertically one-zone model.

The equation for mass conservation is

$$\frac{\partial(2\pi r\Sigma)}{\partial t} = \frac{\partial\dot{M}}{\partial r} + s, \quad (5.2)$$

where $\Sigma(r, t)$ is the surface density at radial position, r , from the central white dwarf and time, t , in units of g cm^{-2} , and $\dot{M}(r, t)$ is the mass accretion rate in units of g s^{-1} , which is defined by $-2\pi r\Sigma v_r$, where v_r is the radial velocity of the gas flow in the disk, and the source term, $s(r)$, is the mass supply rate per unit distance via the stream from the secondary star at a given radius of the accretion disk, respectively. We assume the source term to be time independent in the present study, and give our formulation for $s(r)$ in the case of tilted disks in Sec. 5.4.1.

The equation for angular momentum conservation is

$$\frac{\partial(2\pi r\Sigma h)}{\partial t} = \frac{\partial(\dot{M}h)}{\partial r} - \frac{\partial}{\partial r}(2\pi r^2 W) - D + h_{\text{LS}} s, \quad (5.3)$$

where W is the vertically integrated viscous stress, D is the tidal torque exerted by the secondary star, and $h = \sqrt{GM_1 r}$ and $h_{\text{LS}} = \sqrt{GM_1 r_{\text{LS}}}$ are the specific angular momentum of the disk matter and that of the gas stream from the secondary star at a given radius, respectively. Here M_1 is the mass of the primary star, and G is the gravitational constant, respectively. We assume that the specific angular momentum of the gas stream is conserved to be that at the Lubow-Shu radius, r_{LS} (Lubow, Shu, 1975). We consider only the $r\phi$ -component of the viscous stress tensor, expressed by $w_{r\phi}$, and hence, $W \equiv \int -w_{r\phi} dz$. We take ϕ as the azimuthal angle in the cylindrical coordinates. Here $w_{r\phi}$ includes the shear viscosity coefficients, and is expressed as $-3\rho\nu\Omega/2$, using the kinematic viscosity ν . By adopting the α -prescription proposed by Shakura, Sunyaev (1973), $w_{r\phi}$ is represented as $-\alpha P$, where P is the pressure and α is the viscosity parameter formulated in Sec. 5.3.4, respectively. The vertically integrated viscous stress, W , is then expressed under one-zone approximation as

$$W = \frac{3}{2}\nu\Sigma\Omega = 2H\alpha P_c = \alpha \frac{\mathcal{R}}{\mu_c} \Sigma T_c, \quad (5.4)$$

where T is the temperature, $\Omega = \sqrt{GM_1/r^3}$ is the Keplerian angular velocity, $\mathcal{R}$ is the gas constant, and μ is the mean molecular weight, respectively. We do not consider the radiation pressure, and P represents the gas pressure defined as

$$P = \frac{\mathcal{R}}{\mu} \rho T. \quad (5.5)$$

The scale height of the disk, H , is defined as

$$H = \frac{1}{\Omega} \sqrt{\frac{\mathcal{R}}{\mu_c}} T_c, \quad (5.6)$$

using $\frac{P_c}{\rho_c} = (H\Omega)^2$ on the basis of one-zone approximation.

As for the tidal torque, we adopt the following expression by Smak (1984):

$$D = c\omega r \Sigma \nu \left(\frac{r}{a}\right)^5, \quad (5.7)$$

where a is the binary separation. By substituting equation (5.2) into equation (5.3), we obtain the following relation:

$$\dot{M} \frac{\partial h}{\partial r} = \frac{\partial}{\partial r} (2\pi r^2 W) + D + (h - h_{\text{LS}})s. \quad (5.8)$$

The equation for energy conservation is

$$C_P \left[\frac{\partial}{\partial t} (2\pi r \Sigma T_c) - \frac{\partial}{\partial r} (\dot{M} T_c) - 2\pi \Sigma \nu_{\text{th}} \frac{\partial(r F_r)}{\partial r} - s T_c \right] = 2\pi r (Q^+ - Q^-), \quad (5.9)$$

where $T_c(r, t)$ is the temperature at the mid-plane of the disk in units of K. Also, Q^+ and Q^- represent the heat generation and the radiative loss from the disk surface per unit surface area, respectively. The thermodynamic quantities of μ in equation (5.4) and the specific heat at constant pressure, C_P , are evaluated at the mid-plane disk. We adopt the chemical composition of population I stars: $X = 0.70$, $Y = 0.27$, and $Z = 0.03$, where X , Y , and Z are the hydrogen content, the helium content, and the metallicity, respectively. In calculating the thermodynamic quantities, we take into account the ionization of hydrogen, and the first and second ionizations of helium, and the dissociation of the H_2 molecule in the same way as described by Paczyński (1969). The second, third, and fourth terms on the left-hand side of equation (5.9) represent those of the advection, the thermal diffusion, and the input energy of the gas stream from the secondary star, respectively. In the thermal diffusion term, F_r is the temperature gradient expressed as $\partial T_c / \partial r$, and ν_{th} signifies the thermal diffusivity. Here we consider the dynamical diffusivity only, and ν_{th} is given by $2W / \Omega \Sigma$ (Meyer, 1984; Mineshige, 1986). In the fourth term, we assume the gas stream has the same temperature as the pre-existing disk matter for simplicity. We note here that all variables appearing in these equations are now calculated once the two quantities, Σ and T_c , at a given r are known.

5.3.3 Heating and cooling functions

To solve the energy conservation denoted by equation (5.9), we need to express Q^+ and Q^- as functions of T_c and Σ . In this study, we consider the following three

terms as the heating source Q^+ :

$$Q_1^+ = \frac{3}{2}W\Omega, \quad (5.10)$$

$$Q_2^+ = D \frac{\Omega - \omega}{2\pi r}, \quad (5.11)$$

$$Q_3^+ = \frac{\beta}{2} \frac{GM_1}{r} \frac{s}{2\pi r}. \quad (5.12)$$

Here, Q_1^+ , Q_2^+ , and Q_3^+ represent the shear viscous heating, the tidal dissipation, and the energy dissipation of the gas stream from the secondary star, respectively. The fraction of energy dissipation β depends on the radius, and is calculated in Sec. 5.4.1. The cooling rate is expressed as $Q^- = 2F$. The radiative loss function F is obtained by integrating the vertical structure of the convective accretion disk as calculated by e.g. Mineshige, Osaki (1983); Hameury et al. (1998). We, however, use simplified interpolation formulae below in this study. The thermal equilibrium curve of the disk is composed of three branches: a hot branch where hydrogen is fully ionized, an intermediate branch where hydrogen is partially ionized, and an optically thin cool branch where hydrogen is neutral.

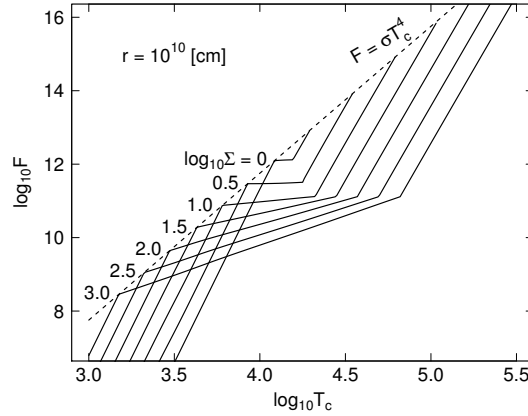


Figure 5.3: The relation between the temperature and the radiative flux as for 6 different values of Σ at $r = 10^{10}$ cm, calculated by equations (5.15), (5.18), and (5.20). The dashed line represents $F = \sigma T_c^4$.

In the one-zone model, the radiative flux from the optically thick disk is given as

$$F = \frac{16\sigma T_c^4}{3\kappa_c \rho_c H}, \quad (5.13)$$

where σ is the Stefan-Boltzmann constant, and $\tau = \kappa_c \rho_c H$ is the optical depth of the disk, respectively. As for the hot branch, the opacity (κ) is given by Kramers' law of ionized gas (equation (3.14) in Cannizzo et al., 1988) as follows:

$$\kappa = 2.8 \times 10^{24} \rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1}. \quad (5.14)$$

By combining equations (5.6), (5.13), and (5.14), the radiative flux at the hot branch is obtained as

$$\log F_{\text{hot}} = 8 \log T_c - \log \Omega - 2 \log \Sigma - 0.5 \log \mu_c - 23.405. \quad (5.15)$$

By contrast, we adopt the opacity of the negative hydrogen given by the following interpolation formula in the optically thin region on the basis of the Cox and Stewart opacity (Cox, Stewart, 1969) as shown in Fig. 2 in Cannizzo, Wheeler (1984):

$$\kappa = 5.13 \times 10^{-19} \rho^{0.62} T^{5.8} \text{ cm}^2 \text{ g}^{-1}, \quad (5.16)$$

where we do not consider the molecular opacity. The radiative flux in the optically thin case is then as follows:

$$F = \tau \sigma T_c^4. \quad (5.17)$$

From equations (5.6), (5.16), and (5.17), the radiative flux at the cool branch is obtained as

$$\log F_{\text{cool}} = 9.49 \log T_c + 0.62 \log \Omega + 1.62 \log \Sigma + 0.31 \log \mu_c - 25.48. \quad (5.18)$$

As for the intermediate branch, we assume that the cool branch extends to the critical temperature T_A at which $F = \sigma T_A^4 = F_A$, and that the hot branch extends to T_B at which the radiative flux is approximately represented by

$$\log F_B = 11 + 0.4 \log \left(\frac{2.0 \times 10^{10}}{r} \right). \quad (5.19)$$

Here, F_B is equal to F_A if $F_B < F_A$. The radiative flux at the intermediate branch is as follows:

$$\log F_{\text{int}} = (\log F_A - \log F_B) \log \frac{T_c}{T_B} / \log \frac{T_A}{T_B} + \log F_B. \quad (5.20)$$

Some examples of the relation between the temperature and the radiative flux are given in Figure 5.3.

5.3.4 Radial dependence of the viscosity parameter

The past studies suggest the necessity of r dependence of α in equation (5.4) in quiescence and/or a difference in α between the hot and cool branches in order to reproduce the observational amplitudes of outbursts and frequent outside-in outbursts (Mineshige, Wood, 1989; Hameury et al., 1998; Cannizzo, 1993). We therefore adopt the following formulation:

$$\log \alpha = \frac{1}{2} (\log \alpha_{\text{hot}} - \log \alpha_{\text{cool}}) \left[1 - \tanh \left(\frac{4 - \log T_c}{0.4} \right) \right] + \log \alpha_{\text{cool}}, \quad (5.21)$$

$$\alpha_{\text{cool}} = 0.03 \left(\frac{r}{r_{\text{tidal}}} \right)^{0.5}, \quad (5.22)$$

$$\alpha_{\text{hot}} = 0.3, \quad (5.23)$$

where α is a function of r and T_c . The α value becomes close to α_{cool} in the cool state, and becomes close to α_{hot} in the hot state. We now have the definitions of the variables necessary for our simulations except for those of $s(r)$ and β . We also obtain the relation between the surface density and the effective temperature, which is called the thermal equilibrium curve as in Figure 5.4. In this figure, we do not consider Q_3^+ because $s(r)$ depends on the mass input pattern described in Sec. 5.4.1.

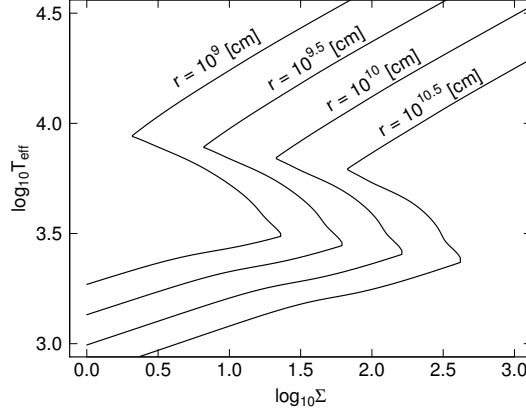


Figure 5.4: The thermal equilibrium curves at $r = 10^9$, $10^{9.5}$, 10^{10} , and $10^{10.5}$ cm, calculated by $2F = Q_1^+ + Q_2^+$. The vertical axis represents the effective temperature. Here we use the binary parameters of U Gem, given in Sec. 5.4.2.

5.3.5 Finite-difference scheme

The finite-difference scheme used in this paper is the same as that described by Ichikawa, Osaki (1992). We treat the conservation of the total angular momentum of a disk in the scheme, by letting the outer disk edge variable. We divide the accretion disk into N concentric annuli. We define the interface between i -th and $(i + 1)$ -th annuli by r_i . The number of interfaces is then $N + 1$ ($i = 0, 1, 2, \dots, N$). The inner boundary of the disk is now given by r_0 and the outer boundary is given by r_N . In this study, we choose the inner boundary at the surface of the primary white dwarf. We give Σ and T_c at the center of each annulus and these variables are expressed with the half-integer subscript $i - 1/2$, while the mass accretion rate, $\dot{M}$, is defined at the interface of each annulus with the integer subscript i .

As for the time integration, we adopt a hybrid method of explicit integration for mass conservation and implicit integration for the energy equation. The mass of the i -th annulus, $\Delta M_{i-1/2}$ is then given by $\Delta M_{i-1/2} = \pi(r_i^2 - r_{i-1}^2)\Sigma_{i-1/2}$ and the equation of mass conservation (i.e., equation (5.2)) is now written in our finite-

difference scheme as

$$\Delta M_{i-1/2}^{\text{new}} = \Delta M_{i-1/2} + (\dot{M}_i - \dot{M}_{i-1}) dt + \dot{M}_{s,i-1/2} dt, \quad (5.24)$$

(for $i = 1, 2, 3, \dots, N$),

where dt is the time step, and $M_{s,i-1/2} = \int_{r_{i-1}}^{r_i} s(r) dr$ is mass supply rate to the i -th annulus. The variable with the subscript “new” means that at the new time step. On the other hand, as for the conservation of angular momentum, we use equation (5.8). By integrating equation (5.8) over $r_{i-1/2}$ and $r_{i+1/2}$, we obtain

$$\begin{aligned} \dot{M}_i(h_{i+1/2} - h_{i-1/2}) = & (2\pi r^2 W)_{i+1/2} - (2\pi r^2 W)_{i-1/2} \\ & + [D + (h - h_{\text{LS}})s]_i(r_{i+1/2} - r_{i-1/2}) \\ & \text{(for } i = 0, 1, 2, \dots, N-1), \end{aligned} \quad (5.25)$$

where the last term is defined in equation (10) of Ichikawa, Osaki (1992). The finite difference equation for the energy equation is written as follows:

$$\begin{aligned} C_{P,i-1/2}^{\text{new}} [\Delta M_{i-1/2}^{\text{new}} T_{c,i-1/2}^{\text{new}} - \Delta M_{i-1/2} T_{c,i-1/2}] = & (5.26) \\ C_{P,i-1/2}^{\text{new}} [(\dot{M} T_c)_i - (\dot{M} T_c)_{i-1} + \dot{M}_{s,i-1/2} T_{c,i-1/2}^{\text{new}}] dt + \\ \frac{4\pi C_{P,i-1/2}^{\text{new}} W_{i-1/2}^{\text{new}}}{\Omega_{i-1/2}} \left[r_i \frac{T_{c,i+1/2} - T_{c,i-1/2}}{r_{i+1/2} - r_{i-1/2}} - r_{i-1} \frac{T_{c,i-1/2} - T_{c,i-3/2}}{r_{i-1/2} - r_{i-3/2}} \right] dt + \\ \pi(r_i^2 - r_{i-1}^2)(Q^+ - Q^-)_{i-1/2}^{\text{new}} dt \\ \text{(for } i = 1, 2, 3, \dots, N), \end{aligned}$$

where T_c at the interfaces of annuli is chosen to be the value at the center of annuli at the upstream side of the flow.

The most important point in this scheme is that the width of the outermost annulus is variable, while those of other annuli are fixed. Some special treatment is necessary to handle the outermost annulus and in our formulation we vary r_N at each time step in such a way as to conserve the total angular momentum of the disk, and its detailed description is given in Ichikawa, Osaki (1992) and that paper should be consulted (see Appendix B for details). The number of annuli N is also variable as a mesh is either added or deleted if the width of the outermost annulus, Δr_N , exceeds some prespecified size or shrinks below another prespecified size when the disk expands or contracts. The way to increase or decrease the number of meshes is described in Sec. 5.2.3 of Ichikawa, Osaki (1992).

As for the inner boundary condition, we adopt $r_{-1/2} = r_0$ and $W_{-1/2} = 0$, which means stress free at the inner edge of the disk. Here the mass accretion rate to the white dwarf, $\dot{M}_0$, is approximately calculated from equation (5.25) as

$$\dot{M}_0 = (2\pi r^2 W)_{1/2} / (h_{1/2} - h_0), \quad (5.27)$$

where we assume the tidal torque is minute at the innermost annulus, and $s_{1/2} = 0$.

For the outer boundary condition, we adopt $W_N = 0$ and $\dot{M}_N = 0$, which means no material outside the disk and that the mass does not escape from the outer disk edge. In this formulation, the outer disk absorbs the angular momentum transferred from the inside by its expansion when the outer disk enters the hot state, having high viscosity. When the disk reaches the tidal truncation radius r_{tidal} and tries to expand beyond it, we fix the disk radius at that radius by removing the extra angular momentum from the disk. The extra angular momentum to be removed is calculated by using equation (15) of Ichikawa, Osaki (1992).

The total mass and angular momentum of the disk are now written as

$$M_{\text{disk}} = \int_{r_0}^{r_N} 2\pi r \Sigma dr = \Sigma_{i=1}^N \Delta M_{i-1/2}, \quad (5.28)$$

$$J_{\text{disk}} = \int_{r_0}^{r_N} 2\pi r \Sigma h dr = \Sigma_{i=1}^N h_{i-1/2} \Delta M_{i-1/2}. \quad (5.29)$$

It has been demonstrated (see, Sec. 5.2.3 of Ichikawa, Osaki (1992)) that the equations of mass and angular momentum conservation of the disk are written in our finite-difference scheme as

$$M_{\text{disk}}^{\text{new}} - M_{\text{disk}} = (\dot{M}_{\text{tr}} - \dot{M}_0) dt \quad (5.30)$$

$$J_{\text{disk}}^{\text{new}} - J_{\text{disk}} = (h_{\text{LS}} \dot{M}_{\text{tr}} - h_0 \dot{M}_0 - D_{\text{total}}) dt, \quad (5.31)$$

where $\dot{M}_{\text{tr}}$ is the total mass transfer rate to the disk from the secondary star and it is given by

$$\dot{M}_{\text{tr}} = \int_{r_0}^{r_N} s(r) dr. \quad (5.32)$$

Here, D_{total} is the total tidal torque exerted on the accretion disk,

$$D_{\text{total}} = \int_{r_0}^{r_N} D dr. \quad (5.33)$$

Equation (5.30) means that the variation of the total disk mass is determined by the mass supply rate from the secondary minus the mass accretion rate onto the white dwarf, while the variation in the total angular momentum of the disk is determined by the angular momentum supply via the gas stream minus the angular momentum loss carried with the accreted matter minus the tidal removal of angular momentum from the disk (equation (5.31)). That is, the exact conservation of mass and angular momentum in the disk is preserved during the time evolution in our numerical scheme.

The calculations at each time step proceed as follows. The necessary variables at the old time step are N , r_N , $\Sigma_{i-1/2}$, and $T_{c,i-1/2}$ ($i = 1, 2, \dots, N$). First, we calculate $\dot{M}_i$ ($i = 0, 1, 2, \dots, N-1$) from the variables such as W and D by equation

(5.25). Next, we compute $\Delta M_{i-1/2}^{\text{new}}$ by equation (5.24), and derive r_N at the new time step from equation (15) of Ichikawa, Osaki (1992). Finally we calculate $\Sigma_{i-1/2}^{\text{new}}$ by using r_N at the new time step, and compute $T_{c,i-1/2}^{\text{new}}$ from equation (5.26).

We determine each time step Δt to satisfy the following conditions in all of the meshes.

$$\Delta t \leq \min \left(\frac{0.3(\Delta r)^2}{\nu}, \frac{0.3\Delta r(2\pi r\Sigma)}{|\dot{M}|} \right), \quad (5.34)$$

$$|T_{c, \text{new}} - T_c| \leq 0.03T, \quad (5.35)$$

$$|\Sigma_{\text{new}} - \Sigma| \leq 0.03\Sigma. \quad (5.36)$$

The first one is the Courant condition to secure numerical stability and means that each time step should be smaller than the diffusion timescale. Here, $T_{c, \text{new}}$ and Σ_{new} are the temperature at the mid-plane of the disk and the surface density after the new time step, respectively.

In our calculations in this paper, we set the first 90 annuli to have the same width in logarithmic scales, and that the others do in linear scales.

For $r \geq 0.1a$ ($i \geq 111$),

$$dr = \frac{r_{\text{tidal}} - 0.1a}{90}, \quad (5.37)$$

and for $r_{\text{input, min}} \leq r < 0.1a$ ($91 \leq i \leq 110$),

$$dr = \frac{0.1a - r_{\text{input, min}}}{20}, \quad (5.38)$$

and for $r < r_{\text{input, min}}$ ($i \leq 90$),

$$\log(r_i) - \log(r_{i-1}) = \frac{\log(r_{\text{input, min}}) - \log(r_0)}{90}. \quad (5.39)$$

Here, $r_{\text{input, min}}$ signifies the innermost radius where the gas stream from the secondary star reaches. The reason for dividing the region where $r_{\text{input, min}} \leq r < 0.1a$ into finer meshes than those in the nearby region is to smooth the mass input in this region (see also Sec. 5.6.3).

5.4 Mass input from the secondary star to a tilted disk

5.4.1 Mass input from the secondary star to a tilted disk

As described in Sec. 5.3.1, the key part of our simulations is to consider how outburst behavior changes when we vary the mass supply pattern, $s(r)$, in tilted

disks where the gas stream enters not only the disk outer edge but also the inner disk. The location of a bright spot on the disk surface varies depending on the position of the secondary star against the tilted disk. To treat this problem, we have estimated which radius of the tilted disk the gas-stream trajectory first collides with, while the tilted disk rotates half way round against the secondary during the half period of negative superhumps. We then obtain the time-averaged mass input pattern in the r -direction, $s(r)$. Finally we have formulated three representative mass input patterns in the low-tilt case, the moderate-tilt case, and the high-tilt case, respectively.

We have computed the ballistic trajectory of a particle at first by solving the restricted three-body problem with the binary parameters of U Gem. We have used equations (1) and (2) in Flannery (1975), which represent the equation of motion in a co-rotating frame with the binary. Here we take x - y plane as the orbital plane of the binary, and the z' -direction perpendicular to the orbital plane. We set the primary and the secondary on the x -axis as point masses. The gas stream comes into the primary Roche lobe via the Lagrangian point (the L1 point), and the movement of a particle is governed by the gravitational fields of the primary and the secondary, and the Coriolis force. We assume the initial velocity of a particle toward the x -direction is 0.03, which is normalized by the orbital velocity of the binary. This value is consistent with the sound speed of the atmosphere of the secondary star (Lubow, Shu, 1975). The resultant gas-stream trajectory is shown as the thick black line in Figure 5.5.

We next determine the geometry of the disk. Here we assume that the tilted disk has the thickness expected from the steady standard disk for convenience (Shakura, Sunyaev, 1973) as follows:

$$\frac{H}{r} = 1.72 \times 10^{-2} \alpha^{-1/10} \dot{M}_{16}^{3/20} \left(\frac{M_1}{M_\odot} \right)^{3/8} r_{10}^{1/8} \left[1 - \left(\frac{r_{\text{in}}}{r} \right)^{1/2} \right]^{3/5}, \quad (5.40)$$

where r_{10} is r in units of 10^{10} cm, and $\dot{M}_{16}$ is $\dot{M}_{\text{tr}}$ in units of 10^{16} g s^{-1} , respectively. Here $\dot{M}_{\text{tr}}$ is the mass transfer rate from the secondary star. We set $\dot{M}_{16}$ and α in equation (5.40) to be 10 and 0.3, respectively, and assume that the outermost radius of the disk is the tidal truncation radius. In reality, the thickness and the size of the disk vary with time but we do not consider such effects here for simplicity.

By using the trajectory and the disk geometry that we have prepared, we have calculated the first-crossing points between the gas stream and the tilted disk, once we fix a tilt angle ($\theta = 7$ deg in Figure 5.5). The tilted accretion disk rotates clockwise around the white dwarf (i.e., the z' -axis) in the co-rotating frame with the binary, and it takes one period of negative superhumps for one rotation. The

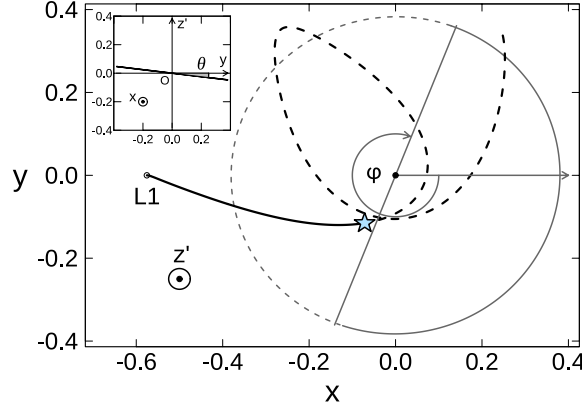


Figure 5.5: Trajectory of the gas stream from the secondary star, which moves on the x - y plane. The grids are normalized by the binary separation. The L1 point is located at $(x, y) = (0, -0.575)$. The white dwarf and the secondary are located at $(x, y) = (0, 0)$ and $(-1, 0)$, respectively. The black point represents the center of the white dwarf. We adopt the tilt angle, $\theta = 7$ deg, and we show the particular case of $\varphi = 289.6$ deg, where φ is the angle by which the nodal line (shown by the diametric line) makes with the x -axis and it is counted clockwise. The solid thick line represents the trajectory, and the mark ‘star’ is the first crossing point of the gas-stream trajectory against the surface of the tilted disk. The thick dashed line represents the trajectory of the gas stream after that, if no collision had occurred. The gray thin line represents the contour of the tilted mid-plane disk. The solid gray line means that mid-plane is above the x - y plane, and the dashed gray line means that it is below the x - y plane. In the small insert, we indicate the y - z' plane and the tilt angle $\theta = 7$ deg when $\varphi = 0$.

rotational angle is denoted as φ and is defined as the angle from the positive x -axis to the nodal line of the tilted disk, measured clockwise. It varies according to $\varphi = \omega_{\text{NSH}} t + \varphi_0$, where ω_{NSH} is the angular velocity of the tilted disk, and φ_0 is the initial value which we choose when the gas stream collides with the nodal line of the tilted disk at the disk edge. We show the definitions of φ and θ in Figure 5.5.

The gas stream moves on the x - y plane. For example, the gas stream collides with the tilted disk surface at the mark ‘star’ in Figure 5.5 when $\varphi = 289.6$ deg and $\theta = 7$ deg. When the tilt angle is small, the crossing point is slightly deviated from the nodal line since we assume the disk has a finite thickness. We have calculated and recorded the radial distance of the crossing points from the white dwarf during one rotation of the tilted disk, by incrementing φ by 1 deg from φ_0 to $\varphi_0 + 2\pi$. The results for $\theta = 7$ deg are shown in Figure 5.6. Here we show the results as the dashed line if the gas stream collides on the back face of the disk, and as the solid line if the gas stream collides on the front face where we assume that the observer looks at the disk from above, for instance, with an inclination angle $i =$

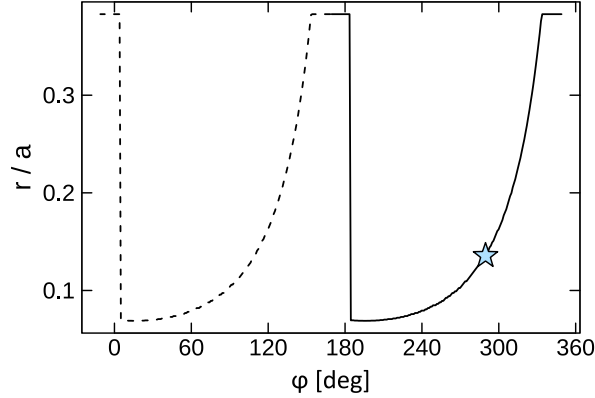


Figure 5.6: Radial coordinates of the first crossing points of the gas stream from the secondary star when a tilted disk rotates around the z' -axis during one period of negative superhumps in the case of $\theta = 7$ deg. The horizontal axis represents the rotational angle φ of the accretion disk in the co-rotating frame with the binary. The vertical axis represents the radial distance of the crossing points from the white dwarf, which is normalized by the binary separation. The mark ‘star’ corresponds to the one in Figure 5.5. The dashed line means the gas stream enters the back face of the tilted disk, and the solid line means it enters the front face, respectively.

45 deg. Since these two cases (i.e., the back face and the front face) are the same with respect to the radial distribution, we have calculated the crossing points only when the gas stream collides with the disk on the front face during a half period of negative superhumps.

We have repeated these kinds of calculations over the range of θ from 1 to 30 deg, every 1 deg. We have then estimated how frequently each region of the accretion disk in the r -direction receives the gas stream as shown in Figure 5.7. Here we prepare three examples with three different tilt angles. These three panels show the time-averaged mass input patterns themselves. It turns out that the patterns hardly depend on the tilt angle above 15 deg. The corresponding tilt angles to these three panels are 3, 7, and 15 deg in Figure 5.7 when we assume the standard disk geometry in the hot disk for convenience. However, the actual disk does not always stay in the hot state and the disk is mostly thinner than in the case we calculated. Therefore these tilt angles do not represent real ones and are very much overestimated. In what follows, we refer to these three mass input patterns as those in the slightly tilted disk, in the moderately tilted disk, and in the highly tilted disk, respectively, and we do not use these tilt angles any more. The mechanism of the disk tilt is still unknown (see also Sec. 5.6.2) and the tilt angle of the accretion disk is hard to measure. There is no negative observational evidence about high tilt angles. That is the reason why we consider the high-tilt case as well. We see the gas stream often reaches the vicinity of $r_{\text{input, min}}$ in the high-tilt case, while it

is mostly intercepted at the outer edge in the low-tilt case.

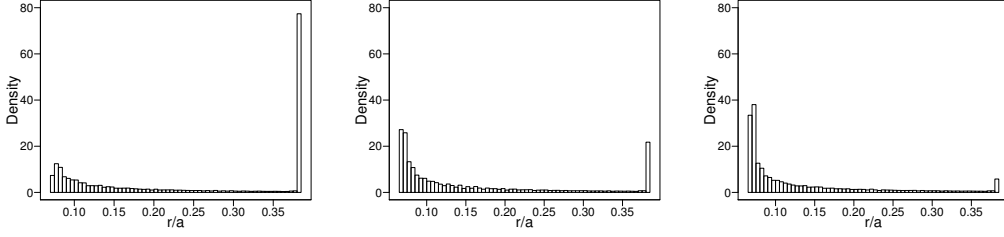


Figure 5.7: These panels show how frequent the gas stream enters each annulus during the period of negative superhumps, i.e., while the tilted disk rotates once against the secondary star. (Left) The low-tilt case with $\theta = 3$ deg. (Middle) The moderate-tilt case with $\theta = 7$ deg. (Right) The high-tilt case with $\theta = 15$ deg.

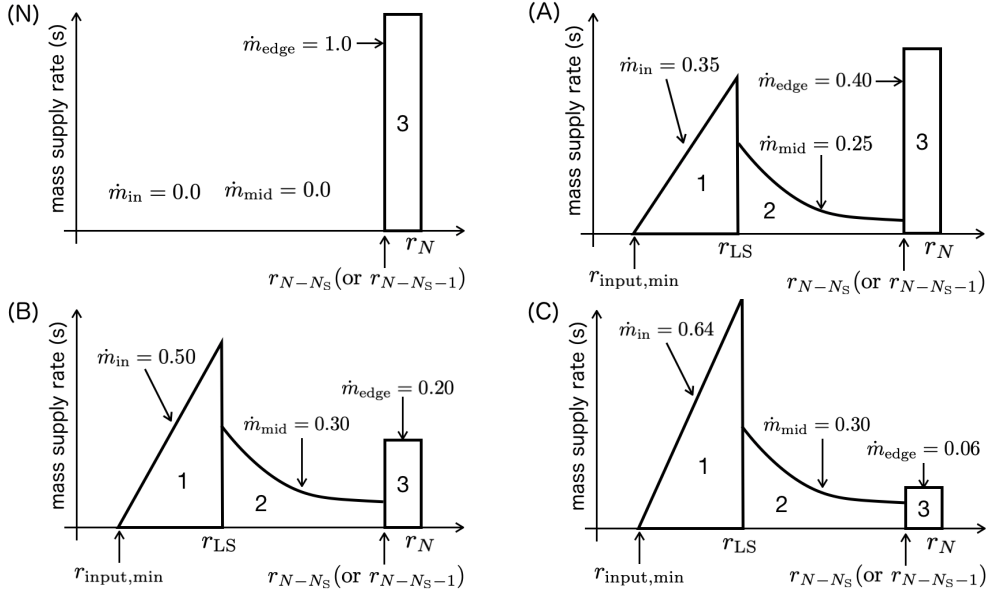


Figure 5.8: Schematic pictures of the mass input patterns that we use in our simulations on the basis of the results shown in Figure 5.7. The regions 1, 2, and 3 are the annulus between $r_{\text{input, min}}$ and r_{LS} , that between r_{LS} and r_{N-N_S} (or r_{N-N_S-1}), and that between r_{N-N_S} (or r_{N-N_S-1}) and r_N , respectively. Pattern (N): The non-tilted standard case where the gas stream always enters the outer edge of the disk. Pattern (A): The low-tilt case corresponding to the left panel of Figure 5.7, Pattern (B): The moderate-tilt case corresponding to the middle panel of Figure 5.7, Pattern (C): The high-tilt case corresponding to the right panel of Figure 5.7.

To implement the three mass input patterns shown in Figure 5.7 in our simulations, we have formulated them by dividing the accretion disk into the three regions shown in the panels of Figure 5.8. We set the three values of $\dot{m}_{\text{in}} = \dot{M}_{\text{in}}/\dot{M}_{\text{tr}}$, $\dot{m}_{\text{mid}} = \dot{M}_{\text{mid}}/\dot{M}_{\text{tr}}$, and $\dot{m}_{\text{edge}} = \dot{M}_{\text{edge}}/\dot{M}_{\text{tr}}$, respectively. Here, $\dot{M}_{\text{tr}}$ is the mass transfer rate from the secondary star, and $\dot{M}_{\text{in}}$, $\dot{M}_{\text{mid}}$, and $\dot{M}_{\text{edge}}$ are the mass

input rates into regions 1, 2, and 3, respectively. We have calculated these values according to the results shown in Figure 5.7, and show them in Figure 5.8. Here we also display the mass input pattern of the non-tilted standard case in its upperleft panel. In this case, both of $\dot{m}_{\text{in}}$ and $\dot{m}_{\text{mid}}$ are zero. We thus see that the difference in our simulations between the non-tilted case and the tilted cases is only the mass supply pattern. Next, we have approximately expressed the source term $s(r)$ as follows:

$$s(r) = \frac{2\dot{M}_{\text{in}}}{(r_{\text{LS}} - r_{\text{input, min}})^2} (r - r_{\text{input, min}}) \quad (r_{\text{input, min}} \leq r \leq r_{\text{LS}}), \quad (5.41)$$

$$s(r) = \frac{\dot{M}_{\text{mid}}}{\log \frac{r_{N-N_S}}{r_{\text{LS}}}} \frac{1}{r} \quad (r_{\text{LS}} \leq r \leq r_{N-N_S}), \quad (5.42)$$

$$s(r) = \frac{\dot{M}_{\text{edge}}}{N_S \text{ dr}} \quad (r_{N-N_S} \leq r \leq r_N), \quad (5.43)$$

for $\text{dr}_N \geq \text{dr}$. Here N_S is 10, and dr_N is the width of the outermost radius, respectively. If $r < r_{\text{input, min}}$, $s(r)$ is 0. When dr_N is less than dr as defined in equation (5.37), the boundary between regions 2 and 3 becomes r_{N-N_S-1} instead of r_{N-N_S} in equations (26) and (27). We have chosen the triangular distribution as for the mass supply rate at region 1 in Figure 5.8 to avoid numerical difficulties (see the discussion in Sec. 5.6.3).

We have also estimated the value of β in equation (12), which depends on the radius where the gas stream collides with the tilted disk, by calculating the relative speed between the gas stream and the disk matter rotating around the central white dwarf with Keplerian velocity. If we write the relative velocity as v_{rel} , the β value is defined as

$$\beta = 0.5 \frac{v_{\text{rel}}^2/2}{GM_1/2r}. \quad (5.44)$$

Here, we assume that half of the energy thermally dissipated by the gas stream is radiated locally as a bright spot, and the other half is used to heat the disk. The resultant β value is given in Figure 5.9.

5.4.2 Model parameters and lists of calculated models

We first specify the binary parameters for our simulations. Since the aims of the present study are to examine how outburst properties depend on the mass input pattern and how the tilted disk could reproduce the essential features of the light variations in IW And-type stars, it is preferable to adopt the typical binary parameters of IW And-type DNe in our simulations. Unfortunately, the binary parameters of IW And stars are not well known, but we have some knowledge that

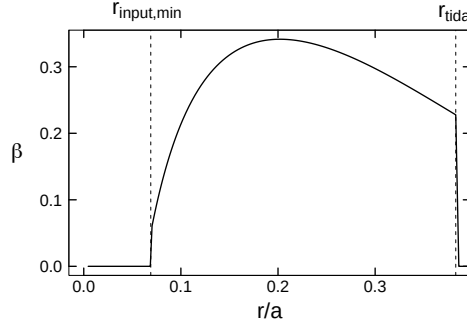


Figure 5.9: Fraction of the energy thermally dissipated by the gas stream with respect to $GM_1/2r$ to be used to heat the disk, β , depending on the radius where the gas stream collides with the tilted disk. We estimate this value between $r_{\text{input, min}}$ and r_{tidal} .

their orbital periods fall within the wide range of 0.14–0.48 d above the period gap (Rodríguez-Gil et al., 2007; Armstrong et al., 2013; Szkody et al., 2013; Echevarría, Michel, 2007; Ringwald, 1994; Cieslinski et al., 1998; Gies et al., 2013; Thorstensen et al., 2004). We can thus presume that IW And stars would have similar binary parameters to those of U Gem stars, with a similar range of orbital periods except for the mass transfer rate. Under such circumstances, we basically adopt the binary parameters appropriate for U Gem in the following calculations, because the period of U Gem is within the period range of IW And stars mentioned above and because the binary parameters of U Gem are the best known among DNe. Besides, we have made additional simulations with the binary parameters of KIC 9406652 in order to confirm that our main results do not depend very much on the binary parameters used. KIC 9406652 is only one object whose mass ratio was measured among IW And-type stars (Gies et al., 2013).

The binary parameters of U Gem adopted in this study are as follows: the orbital period (P_{orb}) is 0.176906 d, the white dwarf mass (M_1) is $1.18M_{\odot}$, the mass of the secondary (M_2) is $0.55M_{\odot}$, the binary separation (a) is 1.115×10^{11} cm, the tidal truncation radius (r_{tidal}) is $0.383a$, the Lubow-Shu radius (r_{LS}) is $0.117a$, and the inner edge of the disk (r_0) is 5×10^8 cm (i.e., at the surface of the primary white dwarf), respectively (Anderson, 1988). Additionally, we set $c\omega$ in equation (5.7) to be 0.4 as a realistic value according to Ichikawa, Osaki (1994). According to Lubow, Shu (1975), $r_{\text{input, min}}$ is estimated to be $0.069a$ when the binary mass ratio (q), which is defined as the mass ratio of the secondary with respect to the primary, is 0.47.

We next need to specify the mass transfer rate, $\dot{M}_{\text{tr}}$. Since one of our aims in this paper is to try to understand the outbursts of IW And-type DNe, the mass transfer

rates, discussed here are rather high.

We have shown four mass input patterns including the non-tilted, slightly tilted, moderately tilted, and highly tilted cases in the previous subsection (see also Figure 5.8), and examine them with a constant mass transfer rate. The models and parameters that we deal with are summarized in Table 5.1. The mass input patterns and the mass transfer rates do not change with time within one model.

models	mass input pattern*	degree of tilt	$\dot{M}_{\text{tr}}^\dagger$	figure(s)
N1	N	zero	$10^{16.75}$	5.10, 5.11
A1	A	low	$10^{16.75}$	5.11
B1	B	moderate	$10^{16.75}$	5.11–5.14
C1	C	high	$10^{16.75}$	5.11
N2	N	zero	10^{17}	5.15
A2	A	low	10^{17}	5.15
B2	B	moderate	10^{17}	5.15
C2	C	high	10^{17}	5.15
N3	N	zero	$10^{16.5}$	5.16
A3	A	low	$10^{16.5}$	5.16
B3	B	moderate	$10^{16.5}$	5.16
C3	C	high	$10^{16.5}$	5.16
N4	N	zero	$10^{17.25}$	5.17
A4	A	low	$10^{17.25}$	5.17
B4	B	moderate	$10^{17.25}$	5.17
C4	C	high	$10^{17.25}$	5.17

*See Figure 5.8 for the definition of the mass input patterns.

† In units of g s^{-1} .

Table 5.1: Summary of the models and parameter sets in our simulations.

5.5 Results of numerical simulations

5.5.1 Case of a non-tilted disk (Model N1)

As mentioned in Sec. 5.3.1, the non-tilted standard case is a special case of our model with a zero tilt angle (see also Figure 5.8). It is very convenient to use this case as a test of our numerical code. We have first calculated this case with $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$ (i.e., Model N1) by our code. The resultant time evolution of the disk is shown in Figure 5.10. In calculating the V -band absolute magnitude, we consider the disk luminosity and the bright spot, and use the method described by Dubus et al. (2018). We set the inclination angle to be 45 deg. Here the inclination angle is not the inclination of the binary system but the inclination of the disk

to the observer. We assume that the disk emission is multi-color blackbody, and that the bright spot emits single-temperature blackbody. We assume the size of the bright spot is 2% of the disk, and its luminosity is simply approximated to be $0.25GM\dot{M}_{\text{tr}}/r_N$.

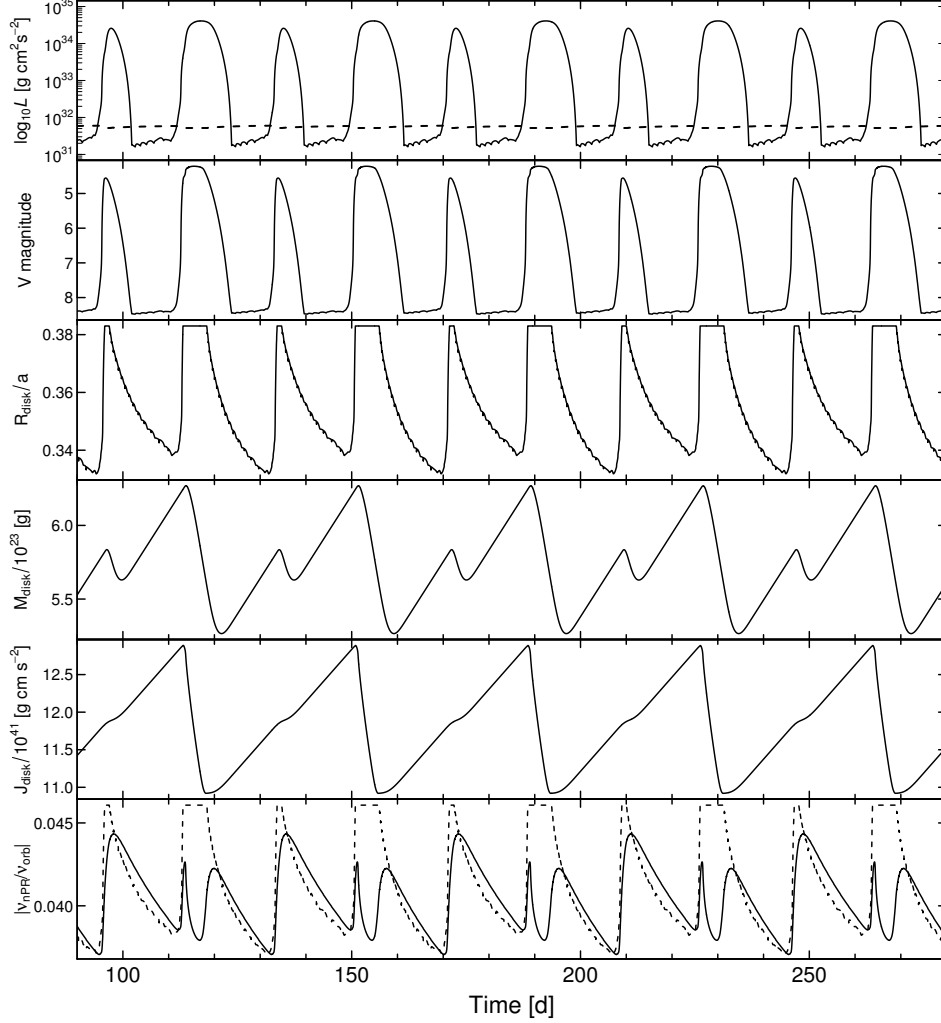


Figure 5.10: Time evolution of the non-tilted accretion disk in the case of $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$ (Model N1 in Table 5.1). From top to bottom: the luminosity of the disk, the absolute V -band magnitude, the disk radius in units of the binary separation, the total disk mass, the total angular momentum, and the absolute value of the normalized nodal precession rate of the disk. The dashed line in the top panel represents the luminosity of the bright spot ($= 0.25GM\dot{M}_{\text{tr}}/r_N$). The observed luminosity in quiescence is expected not to be lower than this line. The dashed line in the bottom panel represents the nodal precession rate calculated by equation (5.46) with $\eta = 1.17$.

As seen in Figure 5.10, our results reproduce outside-in outbursts which are typical of dwarf-nova outbursts. The variations of disk luminosity, disk radius, disk mass,

and disk angular momentum are similar to those in the previous simulation works (e.g., Ichikawa, Osaki, 1992; Hameury et al., 2000). We see that the resultant light variations regularly repeat long and short outbursts. This is not common in all DNe, but is observed in SS Cyg at least in some epochs (e.g., during JD 2450100–2450450 in Fig. 1 in McGowan et al. (2004)). The disk radius suddenly increases at the onset of outbursts because of a sudden increase in the angular momentum transport from the inner disk to the outer disk, while the disk slowly shrinks after the luminosity maximum because of the addition of mass having low specific angular momentum at the outer edge in the cool disk. These reproduce the typical observational disk-radius variations (e.g., Smak, 1984). The disk radius becomes constant around the luminosity maxima because of the tidal truncation (see also Sec. 5.3.5).

The bottom panel of Figure 5.10 represents the time variations of the nodal precession rate (ν_{nSH}) of a tilted rigid disk. If the tilt angle of the disk is very low, the gas stream would almost always enter around the disk edge. Then the time evolution of the disk would be almost the same one as that in the non-tilted case.

The precession rate normalized by the orbital frequency is estimated by the following equation (Papaloizou, Terquem, 1995; Larwood, 1998):

$$|\epsilon_-^*| = |\nu_{\text{nPR}}/\nu_{\text{orb}}| = \frac{3}{4} \frac{GM_2}{a^3} \frac{\int \Sigma r^3 dr}{\int \Sigma \Omega r^3 dr} \cos \theta, \quad (5.45)$$

where ν_{nPR} is the nodal precession rate of a tilted disk and ν_{orb} is the orbital frequency, respectively. Here, we set θ to be zero. Also, ϵ_-^* is always negative, which means retrograde precession. We also calculate $|\epsilon_-^*|$ depending only on the disk radius by assuming $\Sigma(r) \propto r^{-3/4}$, which comes from the standard disk (Shakura, Sunyaev, 1973), via the following equation introduced by Osaki, Kato (2013b):

$$|\epsilon_-^*| = |\nu_{\text{nPR}}/\nu_{\text{orb}}| = \eta \frac{3}{7} \frac{q}{\sqrt{1+q}} (r_N/a)^{3/2} \cos \theta, \quad (5.46)$$

where η is a correction factor, which typically ranges from 0.8 to 1.2 (see Appendix 1 in Osaki, Kato, 2013b, for details). The values estimated by equation (5.45) are displayed as solid lines and those by equation (5.46) are given as dashed lines in the bottom panels of Figures 5.10 and 5.12.

We see the precession rate estimated by equation (5.45) (the solid line) traces the expansion/contraction of the disk radius (the dashed line) well in the short outbursts, but not in the long outbursts, because $\Sigma(r)$ is drastically altered. At the onset of long outbursts, the whole disk enters the hot state and a large amount of mass accumulated at the outer disk is transported to the inner disk soon after the disk radius expands. The disk stays in the hot state for a while, and finally the

cooling wave develops from the outer edge. It propagates inwards, and simultaneously sweeps up disk matter outward across the cooling front. Thus the complex changes in the nodal precession rate, i.e., the consecutive sets of the rapid increase and decrease and then again increase, reflect the three phenomena superimposed: the disk radius expansion, the inward transport of a large amount of mass, and the redistribution of the disk mass by the cooling wave.

5.5.2 How do the light variations change with the disk tilt ?

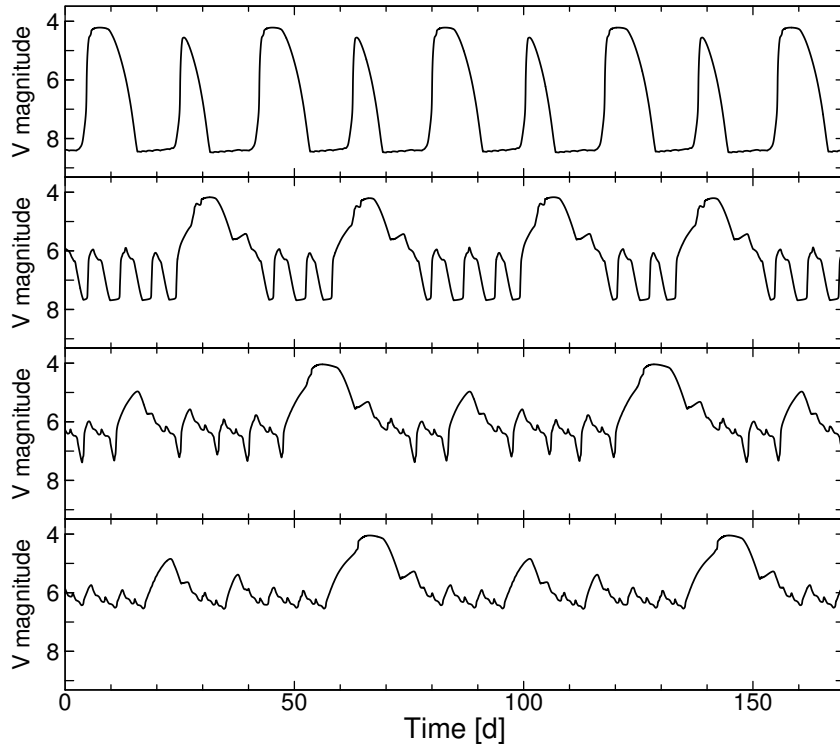


Figure 5.11: Time evolution of the V -band magnitude of the tilted accretion disk in the case of $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$. The contribution of the bright spot is included in our simulations. From top to bottom: the non-tilted standard case (Model N1), the low-tilt case with mass input pattern (A) (Model A1), the moderate-tilt case with mass input pattern (B) (Model B1), and the high-tilt case with mass input pattern (C) (Model C1).

Now we are ready to investigate how the light curves change from those in the non-tilted standard case when the disk is tilted. We show the resultant V -band light curves of the simulations with four mass input patterns in the case of the same mass transfer rate as the previous subsection $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$ (Models N1, A1, B1, and C1) in Figure 5.11. Here we show the later parts of simulations in which the effects of the initial conditions have already faded, and some arbitrary

offsets of time are made for visibility. The luminosity of the bright spot is included in the light curves in the tilted cases (Models A1, B1, and C1) as in the non-tilted case (Model N1), and it is assumed to be $0.25GM\dot{M}_{\text{tr}}/r_{\text{LS}}$ for simplicity, since the gas stream from the secondary arrives between $r_{\text{input, min}}$ and r_N . Since the gas stream reaches deeper in the potential well in the tilted cases as compared with the non-tilted case, the level of minima in V-band luminosity is thus higher in the tilted disk than in the non-tilted disk (see the top and 2nd panels of Figure 5.11).

In Figure 5.11, we can see that diverse light variations appear in the tilted disk depending on the mass input patterns, for the same mass transfer rate. As described in the preceding subsection, the disk alternately experiences long and short outbursts in the non-tilted disk (the top panel). In Model A1, the disk alternates between a large outburst and a few small outbursts (the 2nd panel). In Model B1, large outbursts sandwich the mid-brightness interval with repetitive dips (the 3rd panel). In Model C1, we see repetition of oscillations and brightening (the bottom panel). The intervals between brightening become longer and the amplitudes of outbursts become smaller as the tilt becomes larger. In the tilted cases, we always see a slow rise at the onset of outbursts (or brightening), which represent inside-out outbursts (Mineshige, Osaki, 1985). Since the fraction of gas stream arriving in the inner part of the disk becomes larger in the tilted disk, the outbursts are easily triggered in the inner disk.

The light curve in Model B1 shown in the 3rd panel of Figure 5.11 is most interesting, since it shows a large outburst and a middle level brightness oscillation accompanied with a dip. We therefore examine the results in this case in detail.

5.5.3 Time evolution of the disk in Model B1

We show the time-dependent properties of an accretion disk in Model B1 in Figure 5.12. The interval covering days 221–293 in this figure corresponds to one cycle, consisting of a large outburst, and oscillatory light variations in mid-brightness interval occasionally accompanied by dips.

To help understanding this behavior, we show two more figures, Figures 5.13 and 5.14. In Figure 5.13, we show three snapshots of the distributions of temperature (the left column) and surface density (the right column), where the times shown as three circles in Figure 5.12 (numbered as 1, 2, and 3 in its top panel) correspond to those at a large outburst, the mid-level brightness interval, and a dip, respectively. The dashed line in each panel in the left column represents the minimum temperature for achieving the hot state. The dashed line in the 2nd and 3rd panels in the

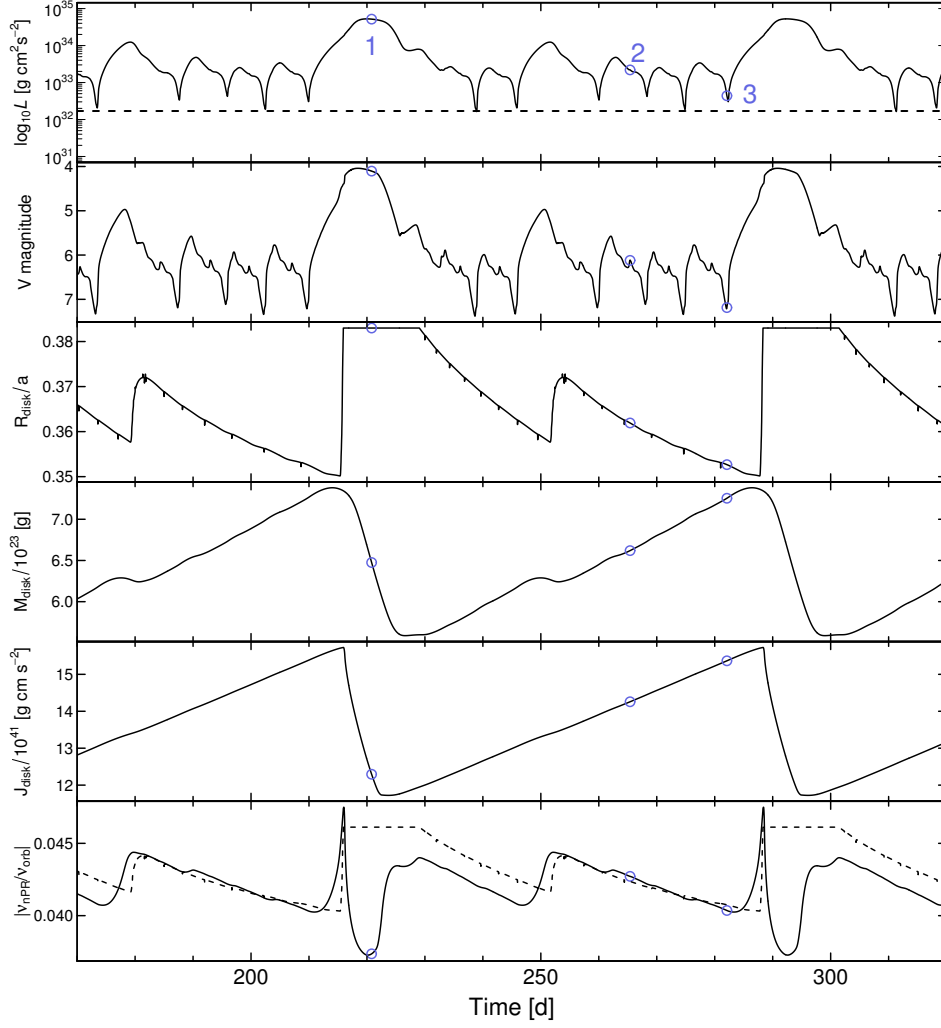


Figure 5.12: Time evolution of the tilted accretion disk in the case of $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$ with mass input pattern (B) displayed in the lower-left panel of Figure 5.8 (Model B1 in Table 5.1). From top to bottom: as in Figure 5.10. The circles correspond to the time picked up at each panel in each column of Figure 5.13. We assume that the luminosity of the bright spot is approximately represented as $0.25GM\dot{M}_{\text{tr}}/r_{\text{LS}}$. Also, we use 1.18 as the η value in calculating the dashed line in the bottom panel.

right column represents the maximum surface density for keeping the entire disk cool.

Figure 5.14 shows time variations of temperature in one cycle at three representative radii of the outer, the middle, and the inner parts of the disk ($\log_{10} r = 10.43, 10.16$, and 9.75) from the top to the bottom, respectively. Although the light curve itself is very complicated in one cycle, the time variations of the total angular momentum in the disk, J_{disk} , in the 5th panel of Figure 5.12, simply show

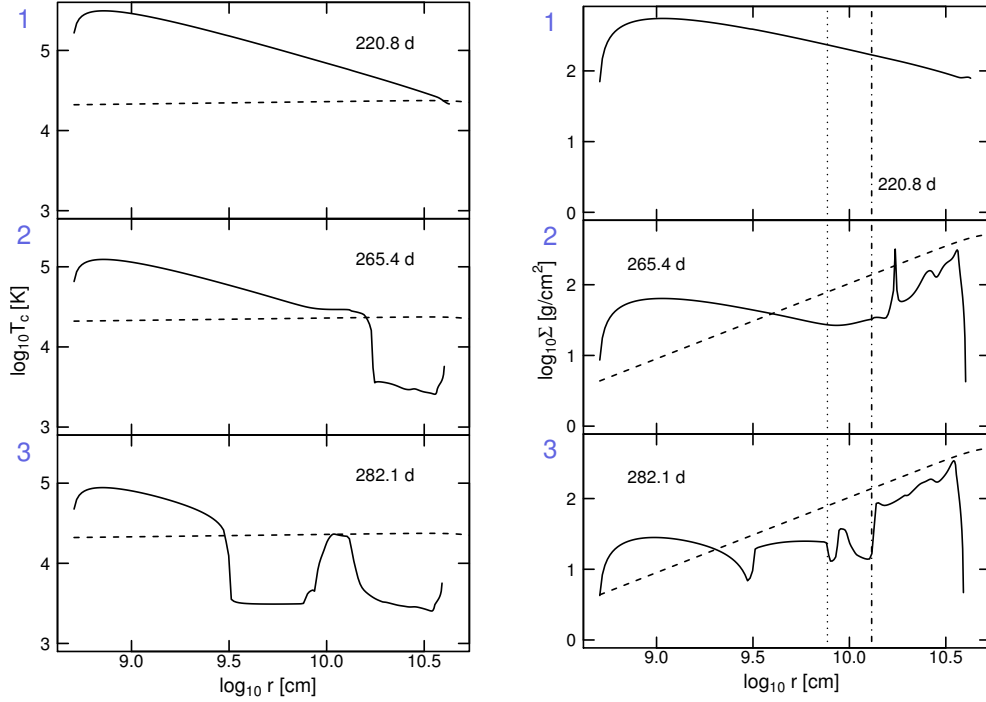


Figure 5.13: Part of the time evolution of the temperature (left) and the surface density (right) of the disk in the case of Figure 5.12. The corresponding time (date) is written in each panel. They are also marked at the top panel of Figure 5.12. The dashed line in the left panel represents the minima of temperature ($T_{\text{hot, min}}$) for achieving the hot state as calculated in equation (38) of Ichikawa, Osaki (1992). The dashed line in the right panel represents the maxima of surface density ($\Sigma_{\text{cool, max}}$) for keeping the disk cool as calculated in equation (35) of Ichikawa, Osaki (1992). The dotted and dash-dotted lines in the right plane represent $r_{\text{input, min}}$ and r_{LS} , respectively.

a slow monotonic increase and then rapid monotonic decrease after its maximum in one cycle. We thus use J_{disk} as a good indicator of the phase of one cycle. The evolutionary tracks for two cycles in Figure 5.14 completely overlap and the arrows in the top panel represent the direction of time evolution.

We can roughly understand this behavior by dividing the disk into three parts, the inner, the middle, and the outer parts. The inner part spends most of the time in the hot state (see, the bottom panel of Figure 5.14), and the outer part stays in the cool state for most of the time during one cycle (its top panel), and in the middle part between the hot inner part and the cool outer part, transition waves (i.e., heating waves and cooling waves) frequently go back and forth there (its middle panel). Thus different regions of the disk behave differently.

At the beginning of one cycle (at brightening), the whole disk stays in the hot

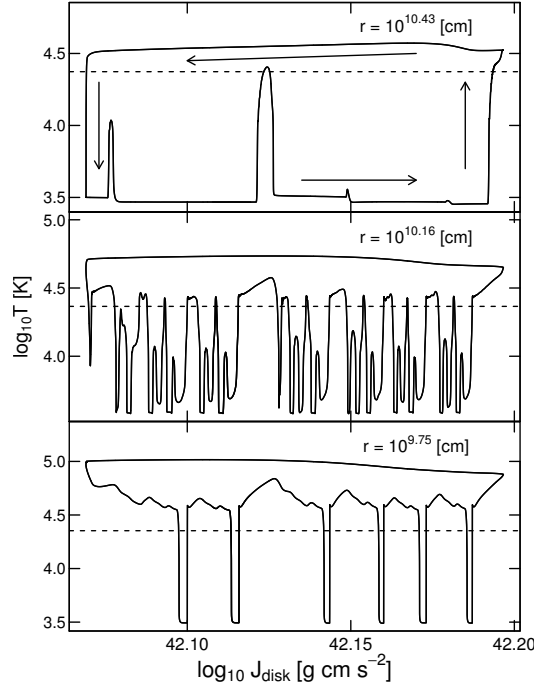


Figure 5.14: T - J_{disk} planes at three representative points in the disk, i.e., $\log_{10} r = 10.43, 10.16$, and 9.75 , corresponding to the outer, the middle, and the inner parts of the disk, respectively, during 170–320 days. The arrows in the top panel represent the direction of time evolution. The dashed line denotes $T_{\text{hot, min}}$ defined in Figure 5.13.

quasi-steady state (see the 1st panel of the left column in Figure 5.13). After the end of a large outburst, the cooling wave starting from the outer edge propagates inward and it transforms the outer part of the disk into the cool state. While the outer part gradually accumulates mass in the cool state (see, the 4th panel of Figure 5.12, and the 2nd and 3rd panels of the right column in Figure 5.13), the inner part usually stays in the hot state (see the 2nd panel of the left column of Figure 5.13), making an intermediate-brightness state. Cooling and heating waves alternately develop in the middle region (see the middle panel of Figure 5.14), causing oscillatory variations in the disk luminosity. Occasionally a cooling wave propagating to the inner part causes luminosity drop, i.e., 6 dips in the light curve correspond to 6 drops in temperature in the inner disk per cycle (see the bottom panel of Figure 5.14 and the 3rd panels of Figure 5.13). Finally, a heating wave developed at the middle part propagates outward all the way and reaches the outer edge of the disk, turning the entire disk into the hot state (see the top panels of Figure 5.13). At that time, brightening is regenerated. The drain of a large amount of mass onto the central white dwarf during the large inside-out outburst returns the disk to the starting point of one cycle. The time necessary to accumulate a large

amount of mass in the outer disk determines the duration of one cycle. The disk thus experiences a new type of accretion cycle if the gas stream from the secondary penetrates to the inner disk.

In the bottom panel of Figure 5.12, we also estimate the variations of the nodal precession rates as we do in Model N1. They show cyclic variations, and large and rapid variations occur around the large outburst. This kind of violent variation is very similar to those in the long outbursts in Model N1, but there exists one difference between these two cases, that is, the delay of the expansion of the disk to the increase of the nodal precession rate at the onset of the large outburst (see also the bottom panel of Figure 5.10). This is because the outward propagation of the heating wave precedes the disk-radius expansion in inside-out outbursts, i.e., the mass redistribution by the heating wave occurs before the disk expansion, while in long outside-in outbursts in the case of non-tilted disks, the heating wave propagation from the outer edge occurs almost simultaneously with the disk radius expansion. Interestingly, the changes in the surface density distribution near the outer edge of the disk govern the nodal precession rate rather than the disk radius variations. On the other hand, the precession rate decreases during small outbursts in parallel with the disk radius variations, unless the heating wave reaches the outermost region.

5.5.4 Brief explanations of the light variations in Models A1 and C1

We explore the light curves in the other tilt cases for $\dot{M}_{\text{tr}} = 10^{16.75} \text{ g s}^{-1}$ at first on the basis of our investigation in Sec. 5.5.3. In Model A1, there are clear quiescent states between outbursts and more frequent large outbursts as compared with those in Model B1 (see, the 2nd and 3rd panels of Figure 5.11). This is because the inner disk often drops to the cool state and keeps cool for a longer time due to the smaller mass input to the inner disk, and because the outer disk often enters the hot state due to the larger mass input to the outer edge of the disk. On the other hand, in Model C1, the dips seen in Model B1 disappear, because the inner disk always stays in the hot state due to the sufficient mass supply inside r_{LS} (see the 4th panel of Figure 5.11). The outer disk stays longer in the cool state because of the reduced mass supply to it, and hence, the duration of one cycle becomes longer as compared with that in Model B1.

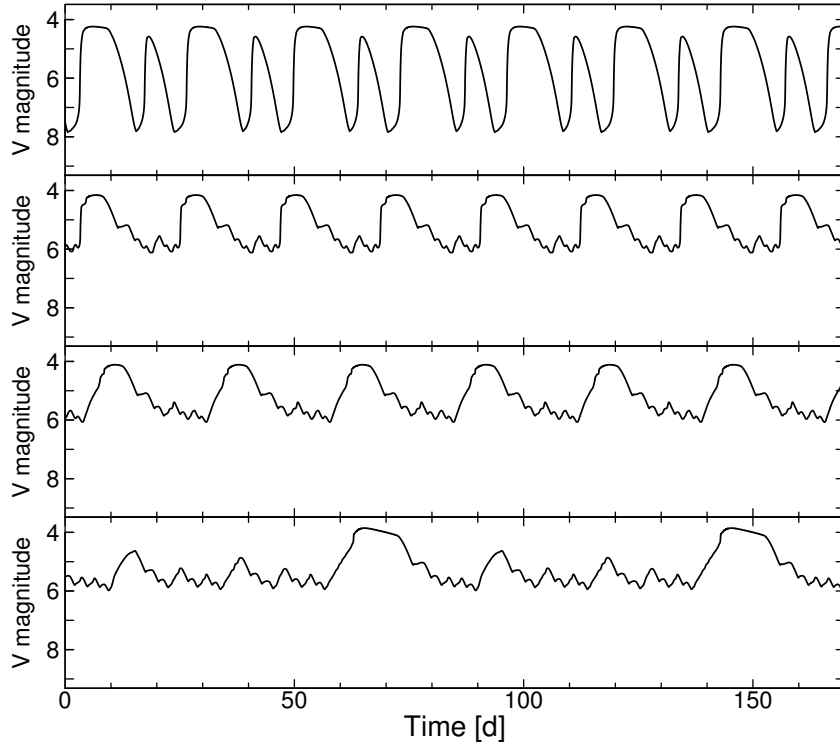


Figure 5.15: Same as Figure 5.11 but for $\dot{M}_{\text{tr}} = 10^{17} \text{ g s}^{-1}$ (Models N2, A2, B2, and C2).

5.5.5 Brief explanations of the light variations in the case of other mass transfer rates

We next examine the light variations for different mass transfer rates. The resultant V-band light curves for 4 different mass input patterns with $\dot{M}_{\text{tr}} = 10^{17} \text{ g s}^{-1}$ (Models N2, A2, B2, and C2) in Figure 5.15. We see that the quiescent state completely disappears in the tilt cases (Models A2, B2, and C2). Under this high mass transfer rate, the mass input rate to the inner disk is high enough to keep it hot even in Model A2. Small-amplitude brightening is repeated in Models A2 and B2, since the interval between brightening becomes shorter and the mid-brightness interval disappears because of the large amount of mass supply to the outer disk edge. The behavior in Model C2 is similar to Model C1.

In the case of $\dot{M}_{\text{tr}} = 10^{16.5} \text{ g s}^{-1}$, the light curves alternate between several small outbursts and a large outburst even in Model C3 just as in Model A1 when the disk is tilted. This is because the cyclic accretion explained in Sec. 5.5.3 also occurs and because the mass supply rate in the inner disk is not enough to keep it hot even in the highly tilted disk. In the case of $\dot{M}_{\text{tr}} = 10^{17.25}$, the disk in the standard non-tilted case settles down to the hot steady disk, corresponding to NL stars. In

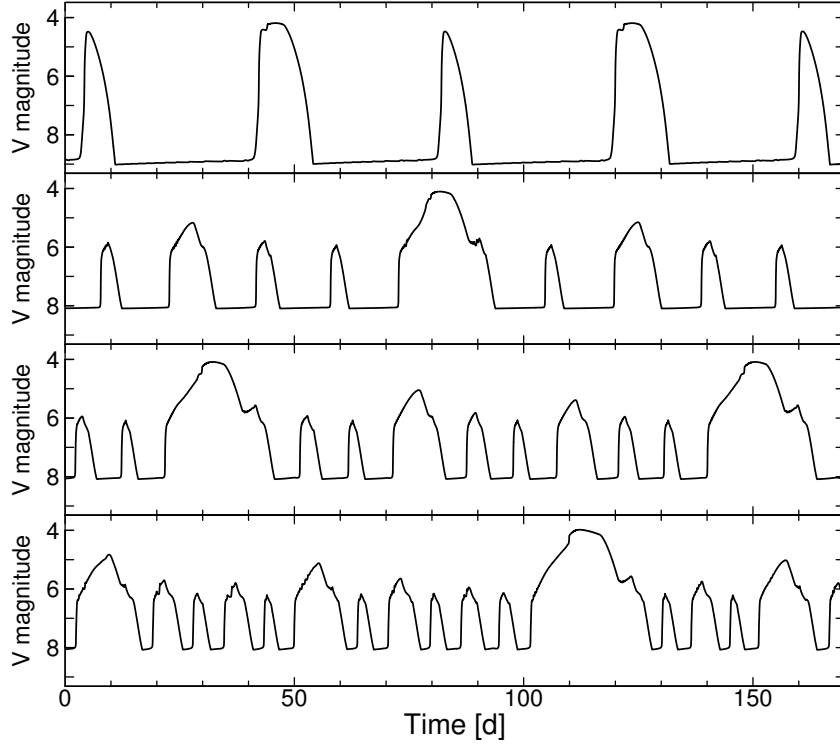


Figure 5.16: Same as Figure 5.11 but for $\dot{M}_{\text{tr}} = 10^{16.5} \text{ g s}^{-1}$ (Models N3, A3, B3, and C3).

tilted disks, the reduced mass supply in the outer edge cannot keep the outer disk hot continuously. The outer disk thus occasionally drops from the hot steady state, and it produces a light curve with small-amplitude variations.

5.5.6 Test simulations with another set of binary parameters

We have also checked whether our main results are sensitive to the binary parameters or not, by performing some simulations with the binary parameters of KIC 9406652 (Gies et al., 2013). According to Gies et al. (2013), the orbital period (P_{orb}) is 0.2545 d, the white dwarf mass (M_1) is $0.9M_{\odot}$, the mass of the secondary (M_2) is $0.75M_{\odot}$, the binary separation (a) is $1.39 \times 10^{11} \text{ cm}$, the tidal truncation radius (r_{tidal}) is $0.328a$, and the Lubow-Shu radius (r_{LS}) is $0.093a$, respectively. The inner edge of the disk (r_0) is the same as that in our previous calculations, and $c\omega$ in equation (5.7) is 4. We also changed the number of meshes from 200 to 190, and used 80, 30, and 80 meshes in the three regions defined by equations (5.37), (5.38), and (5.39) in Sec. 5.3.5. Then $r_{\text{input, min}}$ is estimated to be $0.053a$ (Lubow, Shu, 1975).

We have tried simulations of the non-tilted standard case and the three tilted cases

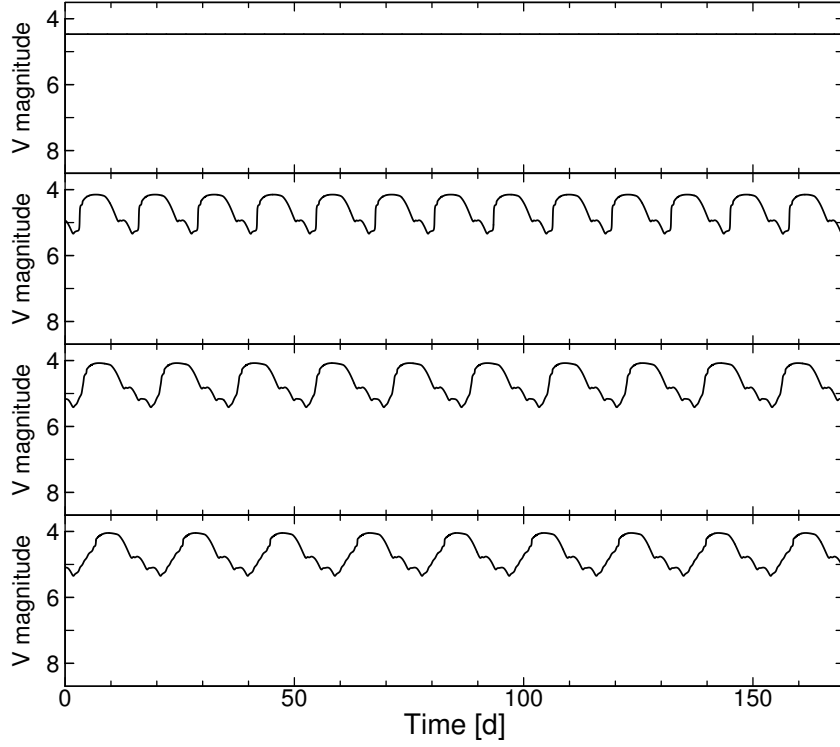


Figure 5.17: Same as Figure 5.11 but for $\dot{M}_{\text{tr}} = 10^{16.5} \text{ g s}^{-1}$ (Models N4, A4, B4, and C4).

in the case of $\dot{M}_{\text{tr}} = 10^{16.9} \text{ g s}^{-1}$ by using the same mass input patterns as in the case of U Gem. The resultant *V*-band light curves are exhibited in Figure 5.18. The results are similar to those in Figure 5.11, and the major features in the simulations with the binary parameters of U Gem: frequent outside-in outbursts in the non-tilted case, repetition of small inside-out outbursts and a large inside-out outburst in the low-tilt case, repetition of oscillatory state with dips terminated by brightening in the moderate-tilt case, and similar repetition without dips in the high-tilt case are reproduced. We have thus confirmed that a new accretion cycle also occurs if the system has a tilted disk and a relatively high mass transfer rate.

5.6 Discussion

5.6.1 Comparison of our simulations with observations

As described in Sec. 5.2, there are two essential features of the light variations in IW And-type DNe: cyclic light variations with a characteristic pattern, which we call the IW And-type phenomenon, and a wide variety in the long-term light curves

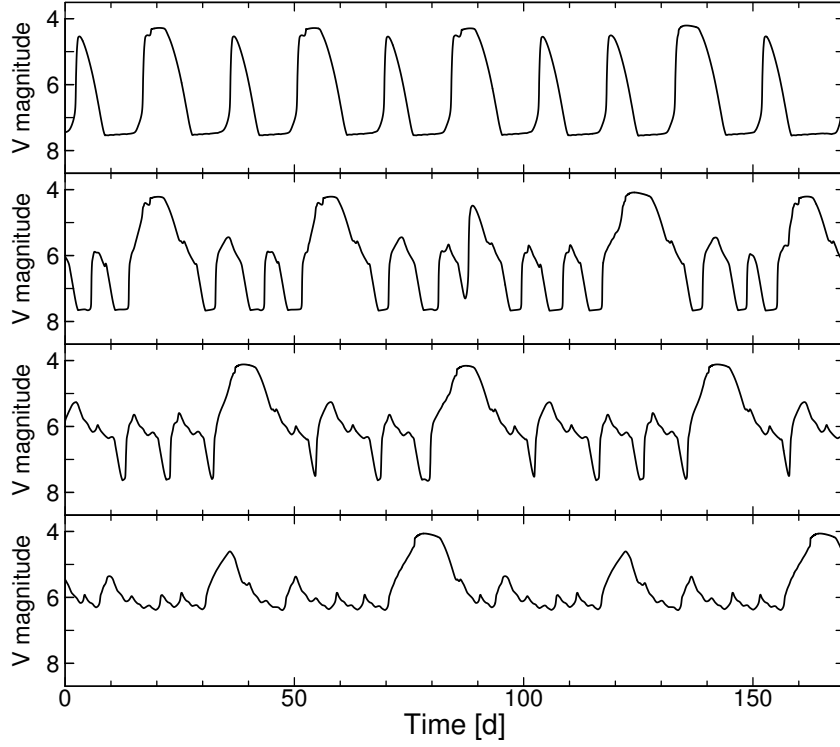


Figure 5.18: Same as Figure 5.11 but for $\dot{M}_{\text{tr}} = 10^{16.9} \text{ g s}^{-1}$ and with the binary parameters of KIC 9406652.

even within one object. Our simulations for the tilted disk give the following main results with respect to these features of IW And stars.

- Some of our simulations of the tilted disk can produce light curves reminiscent of the IW And-type phenomenon: regular repetition of the mid-brightness interval with oscillations and sometimes dips followed by an outburst (brightening) (see, the third and the bottom panels of Figure 5.11, the bottom panel of Figure 5.15, and the third and the bottom panel of Figure 5.18). This is produced because the inner, the middle, and the outer regions behave differently in the tilted disk where the gas stream reaches not only the outer disk edge but also the inner disk. The inner disk mostly stays in the hot state and it helps to sustain the mid-brightness interval. The outer disk spends a long time in the cool state, and once it accumulates enough mass, an inside-out outburst occurs which terminates the mid-brightness interval. Cooling and heating waves frequently go back and forth in the middle region between the inner hot disk and the outer cool disk, which produces oscillatory variations in the mid-brightness interval. A cooling wave occasionally reaches the inner disk, and a dip (a quiescence) then appears in the light curve. These phe-

nomena combined can produce a kind of new accretion cycle in tilted disks, as suggested by Kato (2019).

- Our simulations have demonstrated that the thermal instability working on the tilted disk can produce a wide variety of light curves depending on mass input patterns, i.e., the tilt angles and/or the disk geometry, even with constant mass transfer rates (see Figures 5.11, 5.15, and 5.18). That is, different mass input patterns make different lengths of intervals between brightening and different excursion times to the cool state (i.e., the difference between clear quiescence, a short dip, and no quiescence in the mid-brightness interval). For example, the systems can alternate between dwarf-nova outbursts, repetition of the intermediate-brightness state terminated by brightening, which resembles the IW And-type phenomenon, and repetitive brightening similar to heartbeat-type oscillations observed in HO Pup (see the lower panel of Figure 5.2 and Figure 5.15) if the mass transfer rate is relatively high and if the mass input pattern varies on long timescales.

On the other hand, we have found it difficult to explain by our model the details of the observational IW And-type light variations. For example, luminosity dips sometimes appear just after brightening in observations (see also Figures 5.1 and 5.2), but in our simulations, dips appear during the mid-brightness interval instead of after brightening (see the 3rd panel of Figure 5.11). Also, the amplitudes of brightening are typically less than 1 mag in observations, but they are ~ 1 -mag larger in our simulations. Although oscillations in quasi-standstills are sometimes moderate in the observational light curves (see also Figure 5.1), our simulations always show large-amplitude oscillations in the mid-brightness interval. Z Cam-type standstills are sometimes observed in long-term light curves as well (see the upper panel of Figure 5.2), but our simulations do not reproduce them. This is because the inner hot and the outer cool disks coexist when the mid-brightness interval is generated by our models. The luminosity never settles down in this case, since cooling and heating waves are always triggered between the hot and cool regions. Since we have performed our numerical simulations with a certain set of assumptions, and since we have not explored other possible assumptions, it is not expected to reproduce all of observational details of IW And-type stars in this work. Other possible assumptions to be explored are, for instance, a more realistic thermal equilibrium curve instead of the simple ‘S’-shaped thermal equilibrium curve, other combinations of the viscosity parameters, α_{hot} and α_{cool} , other forms of mass input patterns, and so on. Those attempts are beyond the scope of this paper.

5.6.2 Do IW And-type dwarf novae have tilted disks ?

Our simulations suggest light variations similar to the IW And-type phenomenon appear in tilted disks with relatively high mass transfer rates which are close to those of Z Cam-type DNe (see, the third and the bottom panels of Figure 5.11, and the bottom panel of Figure 5.15). Although the mechanisms for inducing tilted or warped structures are controversial, Montgomery, Martin (2010) proposed that lift may force a disk to be tilted. The lift is the same force as exerted on the surface area of the wings of airplanes. They showed that tilt by lift happens more easily in the objects having higher mass transfer rates. Their and our results are consistent with the recent observations in that the IW And-type phenomenon is prevalent in Z Cam-type DNe (e.g., Kato, 2019). Therefore the disk tilt is regarded as a promising assumption to explain the IW And-type phenomenon, and we could naturally interpret that SS Cyg stars having lower mass transfer rates never show this kind of phenomenon.

As displayed in the bottom panels of Figures 5.10 and 5.12, we can now calculate the frequency variations of negative superhumps. The comparison between our calculations and observations might give more evidence of the disk tilt and our model. Since the time variations in the precession rate reflect the changes in mass distribution within the disk, we can also discuss the mass distribution in the disk through the variations in the negative-superhump periods with high time-cadence light curves. We will report on this in our next paper.

5.6.3 Possibility of gap formation in the disk

Let us come back to a problem of the mass input pattern discussed in Sec. 5.4.1, particularly concerning a triangular profile in region 1 (see also Figure 5.8), and we now discuss why we have used such a profile in this work. We started our simulations by adopting a step function in region 1 for the source function, $s(r)$, in the same way as in region 3. However, we had to stop our computation because we encountered some numerical difficulty around $r_{\text{input, min}}$ when the inner part of the disk enters the cool state. What happened was that a vacuum (or a gap in the disk) seemed to be formed around $r_{\text{input, min}}$, although our code could not handle such a hole.

If the inner disk enters the cool state, the disk matter hardly moves there because of the low viscosity. Since the inner part of the disk below $r_{\text{input, min}}$ has no mass supply from the secondary star, the disk matter sits in its place in such a case. On the other hand, the disk matter between $r_{\text{input, min}}$ and r_{LS} receives mass from

the secondary star with a specific angular momentum ($\sqrt{GM r_{\text{LS}}}$) larger than that of the disk matter, and is forced to move rapidly towards the Lubow-Shu radius. This is because the third term on the right-hand side of equation (5.8) takes a large negative value which dominates over the viscous diffusion term (i.e., the first term). In this case, the disk may split into two parts, the inner cool disk and the outer disk having an inner boundary around the Lubow-Shu radius.

To avoid such a difficulty, we have used a mass input profile in the region 1 such that the source term, $s(r)$, tapers down to zero at $r_{\text{input, min}}$. In our problem with our interest, the cool state in the inner part of the disk is rather short and we have been able to avoid that numerical difficulty by this treatment. In fact, even in our treatment we can see such a tendency of thinning disk matter just above $r_{\text{input, min}}$ as seen in the third panel of the right column in Figure 5.13 when the inner disk stays in a cool state. Although full three dimensional hydrodynamic simulations may be necessary to confirm this, the formation of a gap in the disk seems very likely to occur if the gas stream from the secondary penetrates deeply in the cool disk. We need to consider such a case in future.

5.7 Summary

We have performed numerical simulations of the disk instability of dwarf novae in the case of tilted disks by taking into account the various mass supply patterns when the gas stream penetrates deeply in the disk under some simplifying assumptions. We have found that a new kind of accretion cycle can occur in tilted accretion disks, as suggested by Kato (2019). This is made possible because the different parts of the disk (i.e., the inner, middle, and outer parts of the disk) can stay in different thermal states in tilted disks. The inner disk mostly stays in the hot state while the outer disk spends most of the time in the cool state but eventually makes an outburst when enough mass is accumulated there. As a result, alternating mid-brightness interval (repeating small outbursts) and brightening (a large outburst) are reproduced. The oscillatory light variations in the mid-brightness interval are inevitable because cooling and heating waves are always triggered in the middle part between the hot inner disk and the cool outer disk.

By this cyclic accretion, light variations reminiscent of the IW And-type phenomenon appear when the mass transfer rate is relatively high. Thermal instability in the tilted disk would be a plausible model for explaining the most characteristic feature of the IW And-type phenomenon. Furthermore, we have found that a quite different variety in light curve patterns can be produced depending on the mass supply patterns in tilted disks even within a given mass transfer rate, which could

be a key for to understanding the wide variety in long-term light curves of IW And stars. Successful reproduction of the essential features of IW And-type light curves is possible by considering the thermal-viscous instability in the tilted disk, although we could not reproduce the details such as the mid-brightness interval with low-amplitude oscillations and Z Cam-type standstills by our model.

General discussion

6.1 Test of the disk-instability model

This section is devoted to answer the question: “Can the disk instability model explain a rich variety in dwarf-nova outbursts ?” throughout these studies, which is posed in Sec. 1.10. The first subsection summarizes the standard properties of dwarf-nova outbursts. The second subsection poses some questions about some kinds of light variations deviated from the classical picture. The third, fourth, fifth, and sixth subsections describe how each chapter in this dissertation addresses these problems. The final subsection draws the conclusions.

6.1.1 Classical picture of dwarf-nova outbursts

The orbital period and the mass transfer rate from the secondary classify DNe into several classes (see also Figure 1.22). Except for post-bounce objects, the longer the orbital period is, the larger the binary mass ratio is, and the period gap between ~ 2 – 3 hrs is the border below which the systems experience superhumps and superoutbursts. The orbital period of DNe is usually less than ~ 12 hrs (see also Sec. 1.3.3 and Figure 1.8). Some of subclasses of DNe and their light variations are summarized in Figure 6.1. If the mass transfer rate is above $\dot{M}_{\text{crit}}$, the disk is thermally stable and the luminosity is almost constant.

U Gem-type stars above the period gap have mass transfer rates below $\dot{M}_{\text{crit}}$, and repeat only normal outbursts with amplitudes of ~ 2 – 6 mag and with short duration of ~ 10 days (see Figure 1.10). The larger the amplitude of normal outbursts is, the longer their outburst interval is (van Paradijs, 1985), and the outburst interval is typically less than 1 yr. The outbursts are usually triggered at the outer part of the disk, i.e., outside-in outbursts. If mass transfer rates are around $\dot{M}_{\text{crit}}$, the systems alternate between frequent outbursts and standstills (Z Cam-type stars).

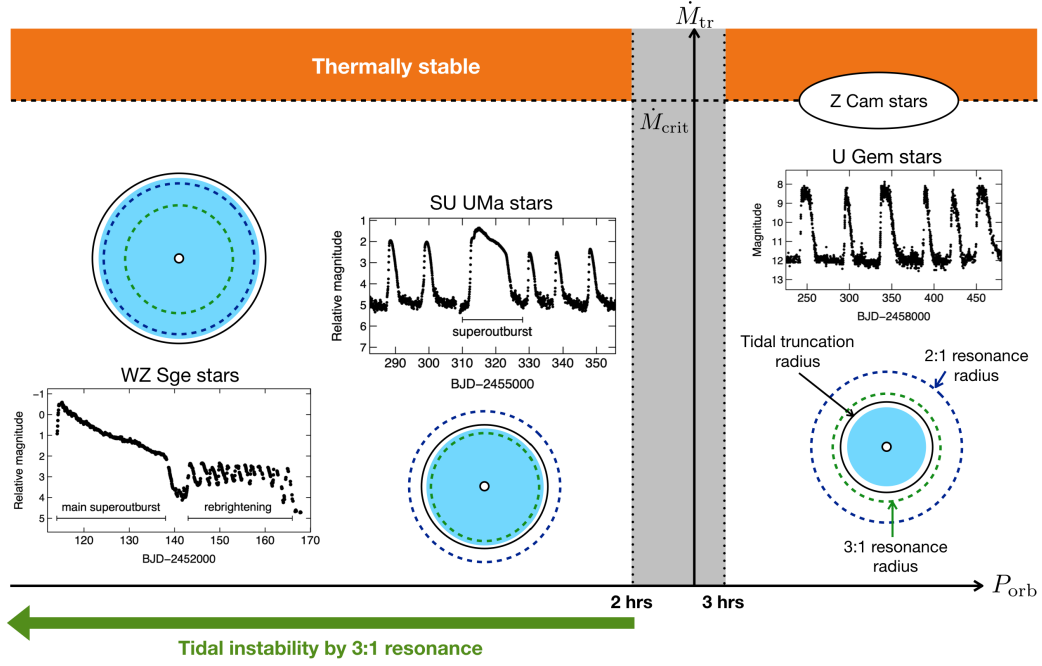


Figure 6.1: Classification of dwarf novae and the light curves and the size of the accretion disk at the beginning of outbursts.

SU UMa-type stars below the period gap repeat several normal outbursts and a superoutburst with superhumps. The amplitude of superoutbursts is around 6–8 mag, and they last for about two weeks. WZ Sge-type stars are an extreme subclass of SU UMa stars, and show only superoutbursts with very long intervals of decades. The amplitude of their superoutbursts is typically ~ 8 mag, and greater than that of SU UMa-type superoutbursts. The duration is also longer than that of SU UMa-type superoutbursts, and typically around three weeks. In the early stage of superoutbursts, early superhumps are observed instead of ordinary superhumps. Many of WZ Sge stars enter rebrightening just after the main superoutburst (Kato, 2015).

Dwarf-nova outbursts are roughly classified into normal outbursts and superoutbursts. Normal outbursts can be explained by the thermal-viscous instability triggered by partial ionization of hydrogen (see Sec. 1.5.2). Superoutbursts are believed to be caused by the TTI model (the combination of the thermal instability and the tidal instability) (Osaki, 1989). The tidal instability develops when the disk radius exceeds the 3:1 resonance radius in the systems whose binary mass ratios are smaller than 0.25 (Whitehurst, 1988). The disk becomes eccentric and undergoes prograde precessions. If the system enters an outburst and the disk radius exceeds the 3:1 resonance radius, the tidal dissipation is strongly enhanced

and the outburst becomes a superoutburst (see Sec. 1.6.2). If the binary mass ratio is smaller than 0.08, the disk can expand beyond the 2:1 resonance radius at the beginning of superoutbursts. The 2:1 resonance forms two-armed spiral density structures in the disk, which is regarded as the origin of early superhumps. The tidal dissipation becomes efficient. Although the 2:1 resonance and the 3:1 resonance have different modes, their effects are the same one: enhanced energy dissipation by tidal forces exerted by the secondary, which makes the long-lasting plateau characteristic of superoutbursts. When the 2:1 resonance is fading, the 3:1 resonance begins to develop. The mass transfer rate from the secondary star is constant in both of the thermal-instability model and the TTI model. As for Z Cam stars, it is suggested fluctuations in mass transfer rates makes the alternation between normal outbursts and standstills (Meyer, Meyer-Hofmeister, 1983), but this idea is not confirmed by observations.

6.1.2 Atypical light variations in dwarf novae

There are some kinds of light variations deviated from the classical picture explained in the preceding subsection. The discovery of these light variations is accelerated these days thanks to the development of optical observational networks and wide-field surveys. Some people still argue that other physical mechanisms except for the disk instability model are required for dwarf-nova outbursts.

There is a little population in WZ Sge stars, which shows double plateau stages during superoutburst, while most of them exhibit a single plateau in their main superoutbursts. It is unclear how the dip of luminosity in the middle of superoutbursts is generated. The possibility that double superoutbursts are connected to extremely low mass ratios was pointed out by Kato et al. (2013b), but that was not confirmed.

Although the small outbursts frequently occur in DNe, some DNe above the period gap exhibit small-amplitude but infrequent outbursts. Some people doubt even that these systems have the same nature as DNe. For example, the merger of two MSs in V1309 Sco, which was a W UMa-type star, is invoked (Scaringi et al., 2016). It is suggested that they are possibly classified into β Lyr systems composed of heavy and extended stars (Bruch, 2017). Qian et al. (2017) stress that mass transfer from the secondary ceases because of a star spot staying at the L_1 point in quiescence.

Additionally, some of postnovae (old novae) experience low-amplitude outbursts with long outburst intervals (see the left column of Figure 6.2). The outburst shape is asymmetric and indicates inside-out outbursts. Since the disk in postnovae is regarded to be thermally stable because of the high mass transfer rate from the ir-

radiated secondary, this behavior seems weird. Their very long orbital periods more than 1 day are also exceptional. GK Per which is also known as an intermediate polar in which the WD contains strong magnetic fields, has been well investigated, and for example, Bianchini et al. (1986) considered that the transient events in this object are caused by periodic modulations of the mass transfer rate, which are employed by oscillations of the K-type secondary star. The outburst interval is not stable in this object, and Bianchini (1990) proposed that the cyclic variations in outburst intervals in postnovae and NLs are produced by the several-years cycle of activity in solar-type secondaries. However, Kim et al. (1992) performed numerical simulations as for GK Per without considering enhanced mass transfer by taking into account the truncated inner disk by magnetic fields, and reproduced inside-out outbursts with long intervals, which are consistent with observations. The mass transfer rate that they assumed is consistent with that derived from observational data by Smak (1982), and their simulations do not need large-amplitude changes in mass transfer rates in the MTB model.

On the other hand, Honeycutt et al. (1998) defined the small-amplitude outbursts sometimes accompanied by deep dips in postnovae and NLs as “stunted outbursts”. They seem repetitive small bursts from standstills, which are considered to be generated when the disk is thermally stable. They suggested that mass transfer events are required to cause this phenomenon, and that the thermal instability is hard to explain some observational features in stunted outbursts. However, the origin of this phenomenon is inconclusive, and no evidence for enhanced mass transfer was detected from the recent high time-cadence optical light curves of AC Cnc (Schlegel, Honeycutt, 2019).

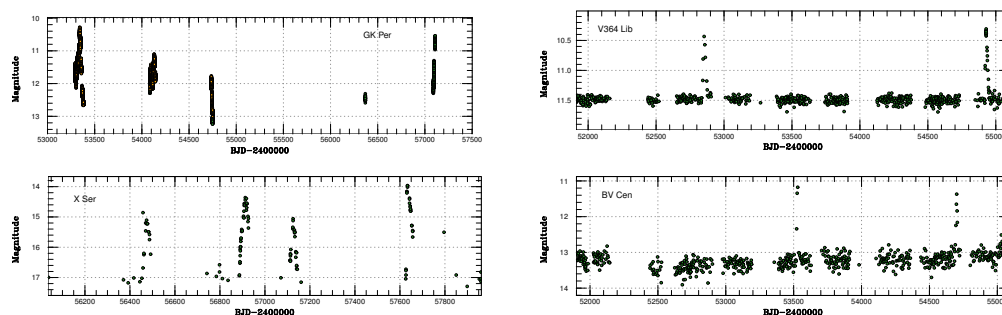


Figure 6.2: Long-term light curves in long-period objects. The light curves of V364 Lib and BV Cen are derived from the ASAS-3 data archive. The light curves of X Ser are derived from the ASAS-SN data archive. The light curves of GK Per were gathered by the VSNET collaboration. Green and orange points represent the observations in the V band and no (clear) filter, respectively.

IW And-type stars which have been recognized as anomalous Z Cam-type stars because they show brightening from a standstill instead of fading in normal Z

Cam-type stars (Simonsen, 2011; Szkody et al., 2013). Some people think that the IW And-type phenomenon resembles stunted outbursts (e.g., Gies et al., 2013; Schlegel, Honeycutt, 2019). Hameury, Lasota (2014) argue that variations in mass transfer rates reproduce this peculiar behavior, since the past numerical simulations cannot reproduce the Z Cam-type behavior without oscillations in mass transfer rates (e.g., Buat-Ménard et al., 2001).

In addition, the physical mechanism of rebrightening in WZ Sge-type DNe has been puzzling for a long time (see also Sec. 1.6.4). Some people think that this phenomenon is the evidence of enhanced mass transfer because of strong irradiation after outbursts, which would be employed by high-temperature WD surface (Hameury et al., 2000).

Chapters 2–5 deal with rebrightening in WZ Sge stars, double plateaus in superoutbursts, low-amplitude, infrequent, and inside-out outbursts, and the IW And-type phenomenon, respectively. Examples of these unexpected light variations are displayed in Figure 6.3.

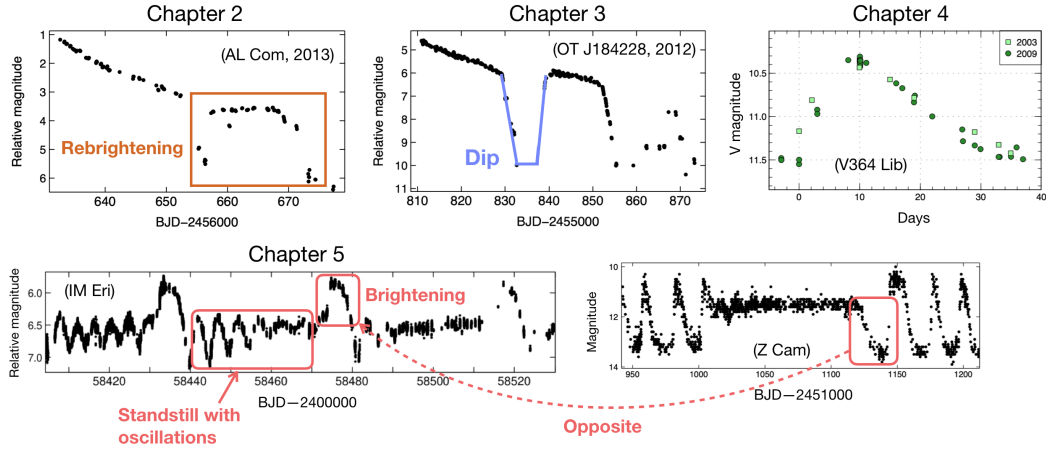


Figure 6.3: Unexpected light variations dealt with in this dissertation. In each panel, the abbreviation of the object name is given. The data are taken by the VSNET collaboration and the ASAS-3 data archive.

6.1.3 Luminosity dip in the plateau stage of superoutbursts

Chapter 3 reports on three WZ Sge stars showing a luminosity dip in a superoutburst. The time evolution of the disk luminosity and superhumps suggest that the three objects probably have extremely low mass ratios. Actually, the binary mass ratio is directly estimated to be much smaller than those of ordinary WZ Sge stars in one of them. These results furnish the classification of the plateau

of superoutbursts: single plateau in ordinary WZ Sge stars and double plateaus in WZ Sge stars with extremely low mass ratios, i.e., possible candidates for the period bouncer (see also Figure 3.12).

The tidal force becomes naturally weaker in the systems with tinier secondaries, and the growth of the tidal instability by the 3:1 resonance becomes slower. In ordinary WZ Sge stars, the 3:1 resonance matures before the tidal dissipation by the 2:1 resonance completely ceases, but the development of eccentricity delays against the end of the 2:1 resonance in the period-bouncer candidates. The outer edge of the disk cannot maintain the high-viscosity state because of no enhancement of removal of angular momenta, and drops to the cool state, which triggers propagation of a cooling wave. Rapid decay of the disk luminosity is observed. However, the disk radius is still beyond the 3:1 resonance radius, and the eccentricity gradually evolves. The tidal dissipation becomes efficient again, and the disk jumps up to the hot state, and the luminosity rises again.

This interpretation successfully adds one new type (superoutburst with a luminosity dip in its plateau stage) to the variety in the light curve of the beginning of superoutbursts, which are proposed in Osaki, Meyer (2003) for the first time (see also Figure 1.19), by considering extremely slow development of the tidal instability in the objects with extremely low mass ratios. Since there is no object showing double superoutbursts until Osaki (2005) was published, they did not think about this case. Here Figure 6.4 exhibits the schematic picture of the new type.

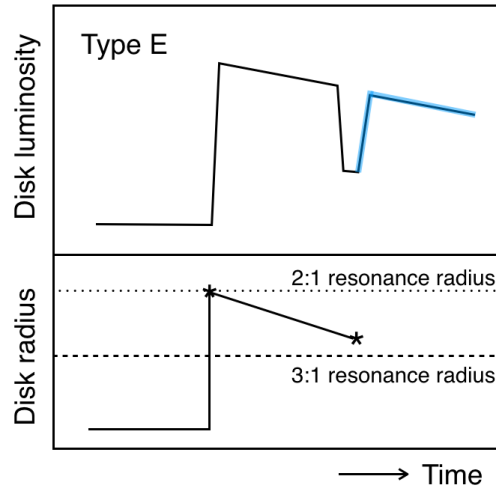


Figure 6.4: New type in the variety of the beginning of superoutbursts. I made this on the basis of the classification in Osaki (2005). The 2nd * mark indicates the start of the tidal instability.

The study in chapter 3 not only defines a new type of superoutbursts but also

recently gave some hints in thinking about the mechanism for a rare superoutburst in CS Ind having larger mass ratio than the condition for triggering superoutbursts (Kato et al., 2019b). Kato et al. (2019b) suggested that period-bouncer candidates and CS Ind-like objects could show the luminosity dip in their very long outbursts before ordinary superhumps emerge, since the eccentric disk very slowly develops. In CS Ind, the 2:1 resonance radius would be replaced by the tidal truncation radius in Figure 6.4.

6.1.4 Low-amplitude, infrequent, and slow-rise outbursts

Chapter 4 treats three systems showing small-amplitude and rare inside-out outbursts. The modeling of orbital and/or eclipsing modulations and spectral analyses revealed that these three systems are DNe but abnormal properties. Two of them have high inclination angles and low mass-transfer rates for some reasons. One of them contains a hot companion star (an F-type star), very long orbital period (~ 16.9 hrs), and a massive WD.

As for the former two objects, the outburst amplitude is tend to be smaller in higher inclination systems. The modeling of orbital modulations in quiescence proved that the disk was already large before their outbursts. This rules out the cessation of mass transfer by a star spot in quiescence, which is proposed in Qian et al. (2017). In addition, there might be a possibility that these two undergo long-lasting hibernation a few centuries after nova eruptions (Shara et al., 1986). If the mass-transfer rate is low, inevitable diffusion of the accumulated mass in long quiescence conveys a part of it inwards. Inside-out outbursts would be easily triggered under this condition because $\Sigma_{\text{cool, max}} \propto r$ (Hameury et al., 1998).

In contrast, the system having a very long orbital period and an F-type companion star would enter low-amplitude and rare outbursts, because of a big disk and a large contribution of the companion star in its luminosity. In a big disk, it takes long time to stock mass enough to trigger outbursts. In addition, inside-out outbursts easily occur because of $\Sigma_{\text{cool, max}} \propto r$. The behavior in V364 Lib is consistent with the scenario proposed in Kim et al. (1992) as for GK Per. Figure 4.18 investigates the correlation between the outburst interval and the orbital period in very long period objects including some of postnovae. If their low-amplitude outbursts are hard to be detected, all of them follow the linear correlation. Kim et al. (1992) treated only GK Per, but this investigation shows that the disk instability model may be widely applicable for atypical outbursts in long-period CVs. It might be general that some of postnovae enter dwarf-nova-type outbursts.

It might be doubtful that low-amplitude and infrequent outbursts are caused by the disk instability model as described in the preceding subsection. However, the observational works in chapter 3 prove that no other mechanisms except for the disk instability are necessary to explain rare outbursts in peculiar DNe.

6.1.5 Anomalous Z Cam-type light variations

Chapter 5 presents the model of thermal instability in tilted accretion disks. The motivation of this study is the discovery of negative superhumps in some IW And-type DNe (e.g., Gies et al., 2013). Time evolution of the thermal instability in tilted accretion disks can be simulated by changing mass input patterns from the secondary basically with the same method of non-tilted disks under some simplified assumptions. Some of numerical simulations successfully reproduce repetitive cycles of a mid-brightness interval brightening, which invoke the IW And-type phenomenon.

The inner disk almost always in the hot state and forms the mid-brightness interval. The cool outer disk jumps up to the hot state once per cycle and generates brightening. This behavior was already predicted in Kato (2019), even though he did not refer the possibility of tilted disks. However, the simulation results provided new knowledge that the middle region between the inner and the outer disks frequently jumps between the cool state and the hot state. In this model, oscillatory variations in the mid-brightness interval could not be suppressed because of fluctuations in the mass flow rate.

In addition, the resultant light variations differ if the mass input pattern is different even without variations in mass transfer rates. Recent observations show IW And stars show a wide variety in their long-term light curves. Some of the results in chapter 5 suggest that this kind of rich variety is possibly reproduced by the thermal instability in tilted accretion disks if the tilt angle and/or the disk geometry vary on long timescales.

Thus the study in chapter 5 provides a new idea that anomalous light variations in IW And-type DNe would originate from the thermal instability in tilted accretion disks. The similarity with stunted outbursts is pointed out by some authors (see Sec. 6.1.2), but mass transfer events would not be responsible for the IW And-type phenomenon. There might be a possibility that some of stunted outbursts are associated with tilted disks. However, the numerical simulations in chapter 5 cannot reproduce detailed light variations in IW And-type phenomenon like small-amplitude brightening, a deep dip soon after brightening, and Z Cam-like standstills

and the model proposed in this chapter should be tested by observations and needs to be improved.

6.1.6 Rebrightening in WZ Sge-type stars

Although the physical mechanism of rebrightening is still unclear, the study in chapter 2 proceeded that research and by obtaining key information about the rebrightening origin. Unexpected superoutbursts and rebrightening in 2015 in AL Com, one of famous WZ Sge stars, suggest that rebrightening types are highly likely inherent to each object. It naturally results in the interpretation that the properties inherent to each system like the mass ratio, the masses of the primary and the secondary, the viscosity, the constant mass transfer rate and so on, are somehow associated with the mechanism of rebrightenings. Prior to this study, theoretical discussion about the origin of rebrightening was stuck (see Sec. 1.6.4). The results in chapter 2 bring some new ideas: for example, the way how the tidal instability develops, which is related to mass ratios, and the mass distribution before outbursts, which depend on the mass transfer rate and the viscosity, in the cool state determine the rebrightening type.

6.1.7 Towards deeper understanding of the disk instability

The studies in this dissertation propose the possible mechanisms of some kinds of light variations, which are deviated from the classical picture of dwarf-nova outbursts, by using the disk instability model (especially in the studies in chapters 3 and 5), and contribute to the testing and the expanding this framework. The important job carried out in these studies is to add new aspects to the disk instability model. Chapter 3 considered extremely slow development of the tidal instability. Chapter 4 applied the disk instability to very long-period rare objects. Chapter 5 took into account specific mass input patterns from the secondary in the tilted disk. It is unnecessary to introduce other models like variants of the MTB model (e.g., Smak, 2000; Hameury, Lasota, 2014; Qian et al., 2017).

Lasota (2001) also points out that additional physical mechanisms should be taken into account in addition to the simple disk instability model. In this review, he mainly discusses about the irradiation on the surface of the secondary star by the high-temperature WD and the inner part of the disk during outburst as the refinements of the disk instability model for dwarf-nova outbursts on the basis of the numerical simulations carried out by Hameury (2000). This kind of irradiation may be the cause of enhanced mass transfer in DNe. For example, Hameury (2000)

considered a hybrid model of the MTB model and the disk instability model can reproduce frequent superoutbursts in ER UMa stars. In contrast, what I assert in the previous paragraph is that the instability within the disk has potential for explaining all kinds of dwarf-nova outbursts systematically. The lack of understanding about the disk instability seems to have led to wrong conclusions for long time.

6.2 Test of the standard scenario in the binary evolution

As mentioned in Sec. 1.10, the study of dwarf-nova outbursts give some information about binary parameters of DNe, which can proceed the study of the binary evolution of low-mass stars. For example, there are many difficulties in CV evolution like the period minimum problem, the missing population problem, and rare population of CVs having very long orbital periods more than 12 hrs. In this dissertation, the studies in chapters 3 and 4 treat post-bounce objects and outlier DNe containing massive secondaries.

6.2.1 Identification of possible candidates for the period bouncer

The study in chapter 3 report on three promising candidates for period bouncers, which were recently detected, and simultaneously summarizes the outburst properties of period-bounce objects among WZ Sge stars. This will advance the study for resolving the gap in the observational and the theoretically predicted populations of period-bounce objects. Period bouncers are considered to be faint because of their dim secondary stars (brown dwarfs), and it seems hard to observe them in quiescence. It is technically difficult to estimate the binary mass ratio via spectroscopic observations in these objects. However, they brighten by more than ~ 6 mag due to accretion of a huge amount of mass stored in the disk during quiescence. The bright outbursts are helpful to search period-bouncer candidates only from their outburst behavior.

In the missing population problem, the final goal would be to know the true population for post-bounce objects by observations in order to examine whether it is comparable to the theoretically predicted population. The bright outbursts are easily observed, and hence, even amateur astronomers can contribute this kind of studies. Moreover, if early superhumps and stage A superhumps are observed during superoutburst, the dynamical estimation of the binary mass ratio is possible

(see also Sec. 1.9.3). This constrains the evolutionary track of each system as shown in Figure 3.11.

6.2.2 Evolutionary paths of long-period dwarf novae with hot companions

The study for the CV evolution is currently limited to the CVs having low-mass MSs (e.g., Kolb, 1993; Knigge et al., 2011), but a part of the works in chapter 4 found that V364 Lib contains an F-type companion star. I notice that other two CVs resemble V364 Lib. One of them is V1129 Cen whose orbital period is 21.4 hrs. Its companion star is probably slightly evolved F2 star (Bruch, 2017). The other one is BV Cen. Its orbital period is 14.7 hrs, and the primary mass and the secondary mass are $1.18^{+0.28}_{-0.16} M_{\odot}$ and $(0.9 \pm 0.3) M_{\odot}$, respectively (Hollander et al., 1993; Watson et al., 2007). Both of them show outbursts with intervals of a few yrs. In general, DNe having very long orbital periods exceeding 12 hrs are deviated from the standard scenario of CV evolution (see also Sec. 1.3.3). However, there is a possibility that optical monitoring surveys miss their outbursts because of their small amplitudes. Although the number of the known candidates is a few, there may be a hidden population of the DNe having F-type stars.

The evolutionary path of V364 Lib-like objects is puzzling. In these objects, the secondary star is likely more massive than the primary WD, and the mass transfer should be unstable (see also equation (1.15)), while the candidates behave like objects with low mass transfer rates. The secondary star may be slightly evolved stars only thin outer layer of which fills their Roche lobe by considering the condition of stable mass transfer and that the radius of the star is $\sim 40\%$ smaller than the Roche lobe if dwarf stars are assumed. Kimura et al. (2018b) proposed one possible scenario about its evolutionary path in which the binary will evolve to an SSS. I now notice another possibility that the system has experienced an SSS and survived. This seems consistent with the massive WD. If either of these proposed scenario is correct, V364 Lib-like objects might evolve to type-Ia supernovae. In the former case, the mass of their massive WDs may exceed the Chandrasekhar mass because of rapid mass accretion (Hachisu et al., 1999). In the latter case, the system possibly repeat nova eruptions, and the primary mass may increase (Kato, Hachisu, 2012).

6.3 Implications for other kinds of systems with accretion disks

This dissertation is devoted to only dwarf-nova outbursts, but all kinds of objects having accretion disks seem to share common accretion physics, since transient accretion events are ubiquitous. The knowledge gained in this dissertation study will be applicable for other kinds of objects. In particular, accretion disks in LMXBs and AGNs are in the same range of temperature around several thousands of Kelvin. Therefore they could enter the thermal instability. Actually, the thermal instability is believed as the cause of outbursts in LMXBs.

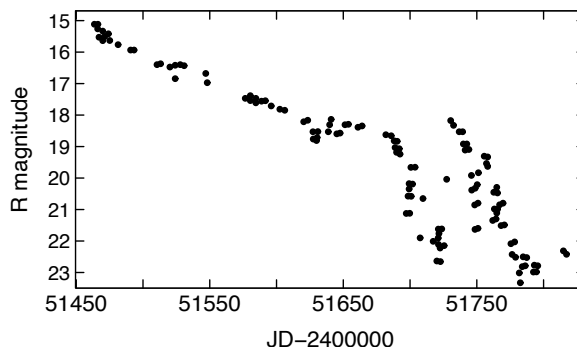


Figure 6.5: *R*-band light curves of the 1999–2000 outburst in XTE J1859+226. The data are taken from Zurita et al. (2002).

If LMXBs have black holes as the primary star, the binary mass ratio tends to be as small as those in WZ Sge stars, because the minimum value of stellar black-hole masses is $3 M_{\odot}$. Actually, superhump candidates have been detected in some black-hole LMXBs (Kato et al., 1995; O’Donoghue, Charles, 1996; Uemura et al., 2002; Zurita et al., 2006). Kimura, Done (2019) revisited multi-wavelength spectra of XTE J1859+226, one of the LMXBs which showed superhumps in the past, and investigated the irradiation effect at the outer part of the accretion disk during its 1999–2000 outburst (see also Figure 6.5). The orbital period of this system is 6.6 hrs and more than 4-times longer than that of WZ Sge-type stars (Corral-Santana et al., 2011). It seems reasonable that the outburst in this system has ~ 10 -times longer duration in comparison with that in WZ Sge stars. In LMXBs, the inner disk emits a lot of hard and soft X-rays, so that the accretion at the outer disk is thought to be governed by X-ray reprocessing. A part of X-rays radiated from the inner disk are absorbed in the outer disk and optical photons are re-emitted. The irradiation keeps the outer disk in the hot state for long time, and prevents it from dropping to the cool state (Dubus et al., 2001). The analyses at the early stage of

a superoutburst are consistent with this picture, but the optical/UV excess cannot be explained only by X-ray irradiation. Thus Kimura, Done (2019) proposed that the tidal instability may affect this outburst and that its effect overcomes the irradiation effect at the late stage of the outburst.

As described above, the tidal instability could significantly affect the outer-disk structure even in LMXBs, but this effect has not been well investigated, because there were only several known objects which brighten during outburst at both of X-ray and optical wavelengths. However, the recent development of transient surveys will give us a lot of opportunities for observing outbursts in LMXBs in the near future. It must be important to adopt accretion physics at the outer disk which has been well studied in the field of DNe for LMXBs.

I also notice that rebrightening is common in outbursts of DNe and LMXBs (see also around JD 51750 in Figure 6.5), which was already pointed out by Kuulkers et al. (1996); Kuulkers (2000), and hence, rebrightening is associated to the pure disk activity regardless of strong gravitational field. There is no collective view about the mechanism of rebrightening, but one of key information obtained via the study in chapter 2 may be apprecable. The study of rebrightening would be connected to seeking the basic accretion physics generating rebrightening, which is common in both of DNe and LMXBs.

Additionally, I would like to mention the similarities in accretion physics between LMXBs and AGNs. Changing-look AGNs which are challenging objects against the conventional picture of AGNs draw attention these days. They seem to experience big changes in accretion rates onto supermassive black holes within 10 yrs. Noda, Done (2018) reported that the time evolution of multi-wavelength spectra in one changing-look AGN resembles the spectral transition in LMXBs, which implies that the thermal-viscous instability triggered by partial ionization of hydrogen, may work in AGNs. However, there is a big difficulty for explaining short timescales of light variations in changing-look AGNs by the disk-instability model. The predicted timescale of outbursts is around 10^{4-5} yrs (Mineshige, Shields, 1990; Hameury et al., 2009), which is much longer than the observational timescale.

6.4 Future perspective

One of the final goals in the study of dwarf-nova outbursts is to explain all kinds of their light variations systematically. I think that the disk instability model itself could be the most plausible as concluded in Sec. 6.1.7, and hence, it is important to understand deeper the disk instability via both of observations and numerical simu-

lations. Ideally, it is desirable to discover new types of phenomena by observations and to reproduce them by numerical simulations.

Chapter 3 suggests that the luminosity dip in the middle of superoutbursts is formed by slow development of the the eccentric disk. The next step is to confirm this by numerical simulations. Additionally, it seems difficult to reproduce rebrightening, but I have some ideas. For instance, it would be possible to take into account the growth of eccentricity in the disk which depends on each system in simulations on the basis of many observational works (e.g., Kato et al., 2009; Nakata et al., 2014).

One of the conclusions in chapter 4 is that the accretion disk can enter the thermal instability in V364 Lib with a hot companion. It would be worth investigating how the thermal instability works in long-period CVs with hot companions in details. I think the important point is the temperature of the disk is low enough to trigger the thermal instability even if the system harbors a high-temperature companion. It is necessary to verify that by seeing whether S-shaped region appears or not in the thermal equilibrium curve through numerical calculations.

Chapter 5 gives some implications for the IW And-type phenomenon, but there were some gaps between the observational and theoretical light curves. In order to test the model proposed in chapter 5, it may be useful to compare the predictions of frequency variations of negative superhumps by numerical simulations and the observational frequency variations. The radial mass distribution of the disk will be discussed by that comparison as well. The study in chapter 5 also reminds that there are only several simulation works except that which treat correctly the conservation of angular momentum (e.g., Ichikawa, Osaki, 1992; Hameury et al., 1998). Past simulation works might leave some important problems unresolved, and now I have opportunities for solving them.

In addition, Sec. 6.2.2 points out that there may be a hidden population of V364 Lib-like objects. This launches a new project as one of the studies on binary evolution. To seek the hidden population of DNe with F-type companion stars, optical photometry and spectroscopy are good enough. They show ellipsoidal variations in quiescence due to their hot donors, and become double-spectroscopic binaries during outburst, which enable to measure the masses of their components. Infrequent outbursts, ellipsoidal variations, and massive white dwarfs and companion stars are the conditions for identifying V364 Lib-like objects. The rapid development of optical transient surveys will provide big data enough for the systematic search, and this would open a new window in the field of binary evolution.

Conclusions

This research mainly aimed to investigate whether the disk instability can explain some of recently detected light variations which seemed hard to be understood from the classical picture about dwarf-nova outbursts through optical observations and numerical simulations. My interest is time evolution of physical processes in accretion disks, and optical observations continuously performed during outburst are suitable to explore it in DNe. Numerical simulations for the time-dependent viscous disk can reproduce the time-varying disk luminosity, which enables a direct comparison with observational light curves. The deficiency in most of the past numerical simulations of dwarf-nova outbursts is the conservation of total angular momentum of the disk. This is the reason why I built the numerical code in which the disk radius varies with time.

Throughout the research in this dissertation, it can be concluded that the disk instability is potentially the source of all kinds of dwarf-nova outbursts. The main results gained from each of the studies are summarized as follows:

- The luminosity dip in the middle stage of superoutbursts would be generated by extremely slow growth of eccentricity in the disk. If the growth of eccentricity is significantly delayed against the fading of strong tidal dissipation triggered by the 2:1 resonance, the disk edge would drop to the cool state because of the cessation of efficient removal of its angular momentum (chapter 3).
- High-inclination systems with low mass transfer rates and long-period objects with massive WDs and high-temperature companions can show low-amplitude, infrequent, and inside-out outbursts without any fluctuations in mass transfer rates. The massive WD in the latter group might explode as a type Ia supernova (chapter 4).

- The gas stream can reach the inner part of the disk if the disk is tilted. Some of numerical simulations including this effect reproduce the light modulations similar to the cyclic light variations composed of a standstill with oscillatory variations and small brightening in IW And stars. This suggests that the thermal instability in tilted accretion disks becomes possibly a good model for the IW And-type phenomenon (chapter 5).
- The 2015 superoutburst in AL Com showed the same type of rebrightening as the previous superoutburst, although its plateau stage was much shorter than the previous one. The rebrightening type in WZ Sge stars may be intrinsic to each system (chapter 2).

Thus these works provided feasible physical interpretations by using the disk instability model and/or gained new information as for four different kinds of peculiar light variations. I figured out that the disk instability model has a wide range of its application, i.e., is versatile, if some new aspects are taken into account. Any instabilities outside of the disk like enhanced mass transfer are not required at least within this investigation.

The ultimate goal in the study of dwarf-nova outbursts would be to explain dwarf-nova outbursts systematically. In my opinion, it is indispensable to understand the essence of instability within the disk in order to achieve this goal, but it was not straightforward before, and the current understanding of the disk instability would be not adequate.

The viscous timescale of the disk in CVs is only ~ 1 day. It is desirable to continue observations every day during outburst in order not to miss any information about mass-accreting processes, but it was difficult to secure plenty of observational time with limited resources of optical telescopes. In addition, the most interesting and important physical process triggered by the thermal instability and/or the tidal instability in the disk seems to be that the local instability propagates globally. Each local region in the disk thus interacts with each other. It implies that multi-wavelength observations are necessary to know the activities in the entire disk, but the knowledge on the accretion disk in CVs basically relies on optical observations. Soft X-rays and extreme UV observations as for the inner disk are lacking.

In numerical simulations, the angular-momentum conservation of the disk was not correctly treated as mentioned in the first paragraph. In other words, the specific angular momentum of the gas stream was neglected, and the disk radius was fixed in most of past works. However, the disk-radius variations can determine the overall behavior of outbursts, since the disk outer edge is sensitive to cooling waves, and since they affect the mass distribution in the disk during quiescence. The

classical picture of dwarf-nova outbursts seem to have been constructed without this important effect.

These difficulties possibly keep us away from the full understanding of the disk instability, and many people seem to leap to conclusions. However, nowadays the big time-series data are provided by several optical transient surveys, and the international collaboration of many small ground-based telescopes are solving the difficulty in observations as the VSNET team performs. Some of multi-wavelength observational networks are also to be organized. In addition, I have devoted to taking into account the realistic disk-radius variations in my numerical code. I hope that these developments in observational networks and numerical methods will greatly advance the study for dwarf-nova outbursts. It is the time to unveil the essence of the disk instability.

Supplementary materials about the observations and analyses

Here I put the tables for supplementary issues in Kimura et al. (2016a,b, 2018a,b) like the observational logs and the $O - C$ values.

Table A1: Log of observations of AL Com in 2015.

Start*	End*	Mag [†]	Error [‡]	$N^§$	Obs	Band [#]	exp[s]
86.4595	86.6849	14.253	0.002	307	RPc	<i>V</i>	60
87.8162	87.9366	14.116	0.011	72	AAVSO	<i>V</i>	120
89.4328	89.6030	14.326	0.004	214	DPV	<i>C</i>	60
90.3395	90.4716	14.437	0.004	170	DPV	<i>C</i>	60
90.4260	90.6889	14.261	0.003	285	RPc	<i>V</i>	60
90.4903	90.5754	14.225	0.006	120	AAVSO	<i>V</i>	60
91.3727	91.6040	14.572	0.002	285	DPV	<i>C</i>	60
91.5368	91.7189	14.356	0.003	248	RPc	<i>V</i>	60
91.8336	91.9516	14.418	0.004	100	AAVSO	<i>V</i>	120
91.8388	91.9311	14.376	0.004	92	DKS	<i>C</i>	60
92.0640	92.2760	14.215	0.004	541	Kis	<i>Ic</i>	30
92.4319	92.7412	14.498	0.003	267	RPc	<i>V</i>	60
92.7250	92.7544	14.454	0.009	29	DKS	<i>C</i>	60
93.0458	93.2710	14.588	0.002	511	Kis	<i>C</i>	30
94.0966	94.2286	14.718	0.003	347	Kis	<i>C</i>	30
95.1349	95.1804	14.844	0.005	121	Kis	<i>C</i>	30
98.3414	98.5572	16.985	0.006	140	DPV	<i>C</i>	60
99.3920	99.6271	15.643	0.003	302	DPV	<i>C</i>	60
102.3807	102.6172	15.993	0.005	159	DPV	<i>C</i>	60
103.5077	103.5213	15.927	0.015	20	AAVSO	<i>V</i>	60
105.3546	105.5033	15.439	0.002	191	DPV	<i>C</i>	60
106.3781	106.4277	15.466	0.003	65	DPV	<i>C</i>	60
107.0658	107.2406	15.276	0.004	457	Kis	<i>C</i>	30
108.0868	108.2301	15.326	0.006	247	Kis	<i>C</i>	30
108.2852	108.5968	14.823	0.002	273	CRI	<i>C</i>	90
108.4375	108.6461	15.331	0.002	290	RPc	<i>C</i>	60
108.3752	108.5311	15.313	0.002	194	IMi	<i>C</i>	60
113.3872	113.6240	16.518	0.006	257	IMi	<i>C</i>	60

*BJD – 2457000.0.

[†]Mean magnitude.

[‡] 1σ of mean magnitude.

[§]Number of observations.

^{||}Observer’s code: RPc (Roger D. Pickard),

AAVSO (AAVSO observers: Boardman James and Rodda Anthony),

DPV (Pavol A. Dubovsky), DKS (Shawn Dvorak), Kis (Seiichiro Kiyota),

CRI (Elena P. Pavlenko and Aleksei A. Sosonovskij), and IMi (Ian Miller)

[#]Filter. “*V*” means *V* filter, “*C*” means no filter (clear),

and “*Ic*” means *I* filter with unfiltered zeropoint.

Table A2: Log of observations of ASASSN-15jd in 2015.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]	exp[s]
58.3835	58.5979	14.300	0.002	173	Van	<i>C</i>	60
59.4360	59.5155	14.386	0.002	93	Van	<i>C</i>	60
60.3841	60.5916	14.484	0.001	227	Van	<i>C</i>	60
61.3155	61.5612	14.523	0.001	306	DPV	<i>C</i>	60
61.3888	61.5035	14.517	0.001	153	Trt	<i>C</i>	60
61.5031	61.5974	14.540	0.001	95	Van	<i>C</i>	60
62.0047	62.1933	14.530	0.001	492	KU1	<i>C</i>	30
62.1640	62.1899	14.500	0.001	70	OKU	<i>C</i>	30
62.4018	62.5906	14.571	0.001	192	Van	<i>C</i>	60
62.4563	62.5377	14.576	0.001	150	IMi	<i>CV</i>	30
63.2956	63.4021	14.674	0.002	100	CRI	<i>C</i>	120
63.3849	63.5911	14.707	0.001	231	Van	<i>C</i>	60
64.1458	64.2850	14.706	0.002	275	KU1	<i>C</i>	30
64.1566	64.2693	14.732	0.001	156	OKU	<i>C</i>	30
65.2691	65.3961	15.423	0.009	61	CRI	<i>C</i>	120
65.3866	65.4792	15.476	0.003	107	Van	<i>C</i>	60
66.3356	66.5482	14.906	0.006	145	CRI	<i>C</i>	120
66.3865	66.5916	14.844	0.003	195	Van	<i>C</i>	60
66.4446	66.5118	14.843	0.002	126	IMi	<i>CV</i>	30
66.5903	66.8343	14.787	0.003	268	DKS	<i>C</i>	60
67.1612	67.2776	14.661	0.004	149	OKU	<i>C</i>	30
67.2943	67.5335	14.655	0.004	110	CRI	<i>C</i>	120
67.3753	67.5206	14.643	0.002	359	Ter	<i>C</i>	30
67.9861	68.2617	14.690	0.001	719	KU1	<i>C</i>	30
68.1661	68.2851	14.695	0.002	318	OKU	<i>C</i>	30
68.2725	68.3749	14.696	0.003	73	CRI	<i>C</i>	120
68.6699	68.8851	14.700	0.002	272	SWI	<i>C</i>	60
69.0767	69.2842	14.960	0.002	288	Ioh	<i>C</i>	60
69.1712	69.2695	14.722	0.001	246	OKU	<i>C</i>	30
69.4363	69.5539	14.777	0.002	228	IMi	<i>CV</i>	30
69.5811	69.6179	14.812	0.006	52	RPc	<i>V</i>	60
70.1466	70.2738	14.856	0.002	336	OKU	<i>C</i>	30
70.6038	70.8384	14.938	0.002	279	DKS	<i>C</i>	60

Table A2: Log of observations of ASASSN-15jd in 2015 (continued).

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]	exp[s]
71.3427	71.4869	15.007	0.002	186	DPV	<i>C</i>	60
71.3710	71.4926	15.002	0.003	169	Trt	<i>C</i>	60
71.5756	71.7965	15.039	0.002	256	DKS	<i>C</i>	60
72.2365	72.3010	15.061	0.005	141	KU1	<i>C</i>	30
74.4216	74.5255	16.666	0.010	114	RPc	<i>CV</i>	60
75.5129	75.5338	18.999	0.159	28	DPV	<i>C</i>	60
77.1767	77.1926	18.298	0.036	9	OKU	<i>C</i>	30
86.1662	86.2473	15.812	0.019	40	OKU	<i>C</i>	30
86.4214	86.5462	15.364	0.001	128	Van	<i>C</i>	60
86.6490	86.8169	15.360	0.002	215	SWI	<i>C</i>	120
87.4142	87.5649	16.265	0.006	128	Van	<i>C</i>	60

*BJD – 2457100.0.

[†]Mean magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer's code: Van (Tonny Vanmunster), DPV (Pavol A. Dubovsky), Trt (Tamás Tordai), KU1 (Kyoto Univ. Team), OKU (Osaka Kyoiku Univ. Team), IMi (Ian Miller), CRI (Crimean Observatory Team), DKS (Shawn Dvorak), Ter (Terskol Observatory), SWI (William L. Stein) Ioh (Hiroshi Itoh), and RPc (Roger D. Pickard)

[#]Filter. “V” means *V* filter, “*C*” and *CV* mean no filter (clear).

Table A3: Times of superhump maxima in ASASSN-15jd.

E	Max [†]	Error	$O - C^{\ddagger}$	$N^{\S}$
0	57166.4601	0.0008	0.0009	136
1	57166.5191	0.0007	-0.0051	84
2	57166.5840	0.0011	-0.0051	56
3	57166.6536	0.0011	-0.0006	64
4	57166.7186	0.0007	-0.0006	60
5	57166.7833	0.0004	-0.0010	63
11	57167.1774	0.0004	0.0030	59
12	57167.2401	0.0003	0.0007	61
13	57167.3122	0.0009	0.0077	21
14	57167.3727	0.0005	0.0033	87
15	57167.4349	0.0003	0.0004	154
16	57167.4968	0.0003	-0.0027	136
24	57168.0244	0.0005	0.0047	137
25	57168.0914	0.0007	0.0066	131
26	57168.1518	0.0004	0.0020	169
27	57168.2164	0.0003	0.0016	274
28	57168.2821	0.0007	0.0022	121
19	57168.3470	0.0011	0.0021	36
35	57168.7369	0.0004	0.0019	63
36	57168.7997	0.0004	-0.0003	68
41	57169.1251	0.0009	-0.0001	72
42	57169.1894	0.0004	-0.0008	177
43	57169.2517	0.0006	-0.0035	189
46	57169.4495	0.0004	-0.0008	77
47	57169.5165	0.0005	0.0012	99
57	57170.1691	0.0004	0.0035	126
58	57170.2299	0.0003	-0.0007	139
64	57170.6177	0.0016	-0.0030	44
65	57170.6834	0.0017	-0.0024	64
76	57171.3983	0.0005	-0.0028	140
77	57171.4634	0.0010	-0.0027	133
79	57171.5994	0.0019	0.0033	60
80	57171.6622	0.0009	0.0010	65
81	57171.7251	0.0011	-0.0011	56
82	57171.7902	0.0016	-0.0011	38
89	57172.2470	0.0011	0.0006	79

*Cycle counts.

[†]BJD-2400000.0.

[‡] $C = 2457166.459117 + 0.0650258 E$.

[§]Number of points used for determining the maximum.

Table A4: Log of observations of the 2016 outburst in ASASSN-16dt.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs
57482.1006	57482.2068	13.199	0.019	84	SPE
57483.5701	57483.7663	13.417	0.020	110	HMB
57484.5278	57484.7410	13.501	0.029	124	HMB
57485.0301	57485.2374	13.491	0.025	116	SPE
57485.2343	57485.4249	-0.018	0.018	549	MLF
57485.5292	57485.7608	13.563	0.021	102	HaC
57486.2857	57486.4086	0.064	0.014	351	MLF
57486.5348	57486.7581	13.623	0.015	106	HMB
57487.2795	57487.3615	0.122	0.012	237	MLF
57487.5282	57487.7552	13.710	0.020	108	HaC
57488.5268	57488.7504	13.793	0.016	106	HaC
57489.2968	57489.4274	0.282	0.014	376	MLF
57489.5268	57489.7497	13.868	0.017	106	HaC
57490.0690	57490.1896	13.874	0.012	94	SPE
57490.5248	57490.7470	13.925	0.019	96	HMB
57491.7255	57491.7457	14.000	0.017	11	HaC
57492.5232	57492.7410	14.046	0.024	95	HaC
57493.2464	57493.4544	0.561	0.018	598	MLF
57493.5602	57493.7401	14.145	0.019	64	HaC
57494.5236	57494.7371	14.204	0.015	137	HaC
57495.5177	57495.7343	14.262	0.032	120	HMB
57495.7321	57495.8591	13.944	0.030	180	SGE
57496.8774	57497.0056	14.283	0.010	162	MGW
57499.5158	57499.7530	16.774	0.352	124	HMB
57500.5370	57500.7504	16.749	0.701	37	HaC
57500.8773	57501.0057	18.604	0.357	156	MGW
57502.8776	57503.0052	17.742	0.129	161	MGW
57504.2149	57504.4214	0.978	0.040	567	MLF
57504.3673	57504.5505	14.572	0.043	207	deM
57504.8605	57505.1721	14.203	0.033	389	MGW
57504.9694	57505.0386	-0.567	0.054	195	KU1
57505.2143	57505.5014	0.639	0.039	823	MLF
57505.2921	57505.4930	14.223	0.046	193	BSM
57505.3857	57505.5202	14.266	0.037	150	deM
57505.5493	57505.7691	14.150	0.040	294	DKS
57505.9719	57506.1719	14.111	0.039	248	MGW
57506.5123	57506.7358	14.223	0.036	130	HMB
57506.8985	57507.1090	14.092	0.034	263	MGW
57507.5114	57507.7671	14.230	0.036	103	HaC
57508.5115	57508.7649	14.285	0.036	121	HMB
57508.9882	57509.1964	14.282	0.168	305	Ioh
57509.5110	57509.7620	14.341	0.033	119	HaC
57510.2135	57510.4897	0.769	0.037	784	MLF
57510.5097	57510.7592	14.418	0.029	136	HaC

Table A4: Log of observations of the 2016 outburst in ASASSN-16dt (continued).

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs
57511.0128	57511.0548	-0.425	0.045	118	KU1
57511.2339	57511.4076	0.894	0.030	497	MLF
57511.5121	57511.7565	14.487	0.028	132	HMB
57512.0048	57512.0523	14.489	0.024	30	SPE
57512.2163	57512.4337	0.897	0.027	619	MLF
57512.5091	57512.7530	14.549	0.029	125	HaC
57512.8567	57513.1718	14.462	0.028	387	MGW
57512.9488	57513.1390	-0.302	0.045	522	KU1
57513.1065	57513.1898	14.591	0.025	64	SPE
57513.2467	57513.4953	1.045	0.032	717	MLF
57513.3047	57513.4691	14.573	0.041	166	BSM
57513.5085	57513.7281	14.622	0.069	103	HaC
57513.6198	57513.7287	14.668	0.021	85	BJA
57513.8567	57514.1253	14.539	0.021	336	MGW
57514.2284	57514.4699	1.118	0.036	587	MLF
57514.5454	57514.7371	14.595	0.031	212	DKS
57514.5834	57514.7549	14.493	0.040	170	UJH
57514.8542	57515.1715	14.629	0.027	397	MGW
57515.5091	57515.7304	14.798	0.062	113	HaC
57515.5454	57515.7477	14.692	0.037	272	DKS
57515.5852	57515.6621	14.534	0.046	35	UJH
57516.0414	57516.1724	14.740	0.087	45	Ioh
57516.5506	57516.7373	14.896	0.026	78	HaC
57516.5512	57516.7362	14.781	0.040	187	DKS
57518.5068	57518.7364	16.521	0.168	117	HaC
57519.8462	57520.0213	18.005	0.134	177	MGW
57520.0184	57520.0209	3.267	0.278	3	KU1
57520.5076	57520.7301	18.318	0.384	100	HaC
57521.5070	57521.7289	18.539	0.376	98	HaC
57521.8505	57522.0214	18.451	0.152	213	MGW
57522.8520	57523.1295	18.614	0.113	186	MGW
57530.5033	57530.5044	18.526	0.002	2	HaC
57531.5030	57531.5041	18.530	0.040	2	HMB
57531.8460	57532.1077	18.158	0.222	172	MGW
57532.5027	57532.5038	15.157	0.003	2	HMB
57532.9298	57533.1074	14.994	0.061	119	MGW
57533.5023	57533.5034	15.437	0.019	2	HaC
57534.5026	57534.5037	16.479	0.046	2	HaC
57535.5020	57535.5031	18.445	0.302	2	HaC
57537.8864	57537.8879	18.433	0.115	2	MGW
57538.8439	57538.8455	18.442	0.188	2	MGW
57539.8442	57539.8457	18.512	0.283	2	MGW

Table A4: Log of observations of the 2016 outburst in ASASSN-16dt (continued).

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs
57542.4941	57542.4951	19.788	0.152	2	HaC
57543.4998	57543.5009	19.343	0.325	2	HaC
57544.4993	57544.5004	19.203	0.249	2	HaC
57547.4998	57547.5009	19.261	0.115	2	HaC
57548.5008	57548.5019	19.607	0.607	2	HaC
57551.4941	57551.4952	19.230	0.112	2	HaC

*BJD – 2400000.0.

[†]Mean magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer's code: SPE (Peter Starr), HaC (Franz-Josef Hambsch), MLF (Berto Monard), SGE (Geoff Stone), MGW (Gordon Myers), deM (Enrique de Miguel), KU1 (Kyoto Univ. Team), BSM (Stephen M. Brincat), DKS (Shawn Dvorak), Ioh (Hiroshi Itoh), BJA (Boardman James) and UJH (Joseph Ulowetz)

Table A5: Log of observations of the 2016 outburst in ASASSN-16hg.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs
57482.1006	57586.9298	14.450	0.026	126	HaC
57587.5914	57587.9316	14.573	0.033	124	HaC
57588.1796	57588.3012	15.827	0.042	65	KU1
57588.5700	57588.9314	14.673	0.042	147	HaC
57589.1455	57589.2971	15.919	0.055	86	KU1
57589.6285	57589.9299	14.838	0.038	106	HaC
57590.6264	57590.9293	15.367	0.083	101	HaC
57591.6232	57591.9304	14.866	0.081	118	HaC
57592.6389	57592.9300	14.733	0.050	112	HaC
57593.6371	57593.9296	14.839	0.056	113	HaC
57594.7362	57594.9297	14.967	0.041	65	HaC
57595.6319	57595.9309	15.075	0.047	95	HaC
57596.6855	57596.9303	15.185	0.031	121	HaC
57597.6264	57597.9300	15.324	0.037	137	HaC
57598.6237	57598.9295	16.262	0.144	138	HaC
57599.6210	57599.9281	18.176	0.392	126	HaC
57600.7491	57600.9295	18.736	0.441	89	HaC
57603.7192	57603.7192	18.403	–	1	HaC
57604.7336	57604.7336	15.578	–	1	HaC
57605.7310	57605.7310	16.637	–	1	HaC
57606.7283	57606.7283	18.031	–	1	HaC
57608.7227	57608.7227	15.622	–	1	HaC
57609.7199	57609.7209	16.703	0.012	2	HaC
57610.7175	57610.9250	18.200	0.250	189	HaC

*BJD – 2400000.0.

[†]Mean magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer's code: see the annotation in Table A8.

Table A6: Times of superhump maxima in ASASSN-16dt.

E	Max [†]	Error	$O - C^{\ddagger}$	$N^{\S}$
0	57502.9004	0.0012	-0.0247	65
1	57502.9716	0.0008	-0.0181	64
21	57504.2745	0.0005	-0.0070	149
22	57504.3356	0.0007	-0.0104	148
23	57504.4028	0.0007	-0.0079	156
25	57504.5329	0.0007	-0.0069	50
31	57504.9243	0.0002	-0.0031	65
32	57504.9887	0.0002	-0.0033	63
33	57505.0547	0.0002	-0.0019	65
34	57505.1197	0.0002	-0.0015	65
36	57505.2494	0.0004	-0.0010	148
37	57505.3147	0.0004	-0.0002	200
38	57505.3793	0.0003	-0.0002	215
39	57505.4454	0.0003	0.0013	262
40	57505.5101	0.0004	0.0013	108
41	57505.5745	0.0004	0.0011	68
42	57505.6390	0.0003	0.0011	69
43	57505.7049	0.0003	0.0024	70
48	57506.0299	0.0003	0.0045	63
49	57506.0938	0.0006	0.0038	65
50	57506.1587	0.0004	0.0041	47
56	57506.5464	0.0006	0.0042	31
57	57506.6115	0.0005	0.0047	29
58	57506.6779	0.0009	0.0065	29
62	57506.9337	0.0003	0.0039	65
63	57506.9993	0.0003	0.0049	63
64	57507.0630	0.0003	0.0041	64
71	57507.5157	0.0008	0.0046	19
72	57507.5827	0.0005	0.0070	30
73	57507.6449	0.0005	0.0046	14
74	57507.7117	0.0009	0.0068	14
87	57508.5507	0.0005	0.0061	31
88	57508.6136	0.0006	0.0044	22
89	57508.6807	0.0007	0.0069	21
90	57508.7405	0.0010	0.0022	21
94	57509.0044	0.0010	0.0077	60
95	57509.0661	0.0007	0.0048	90
96	57509.1266	0.0007	0.0007	95

Table A6: Times of superhump maxima in ASASSN-16dt (continued).

E	Max [†]	Error	$O - C^{\ddagger}$	$N^{\S}$
102	57509.5199	0.0015	0.0064	23
103	57509.5855	0.0007	0.0075	28
104	57509.6483	0.0012	0.0057	20
105	57509.7150	0.0010	0.0078	21
113	57510.2308	0.0003	0.0069	127
114	57510.2936	0.0003	0.0050	149
116	57510.4223	0.0003	0.0045	148
118	57510.5519	0.0005	0.0050	42
119	57510.6181	0.0009	0.0066	21
120	57510.6829	0.0008	0.0068	21
121	57510.7510	0.0016	0.0102	17
129	57511.2585	0.0005	0.0010	145
130	57511.3257	0.0005	0.0036	149
131	57511.3881	0.0006	0.0015	128
134	57511.5819	0.0007	0.0015	37
135	57511.6469	0.0016	0.0019	21
136	57511.7104	0.0013	0.0008	21
141	57512.0316	0.0013	-0.0010	30
144	57512.2265	0.0009	0.0002	120
145	57512.2919	0.0004	0.0010	147
146	57512.3573	0.0004	0.0017	149
147	57512.4256	0.0006	0.0055	106
149	57512.5517	0.0006	0.0024	42
150	57512.6152	0.0022	0.0013	19
151	57512.6799	0.0010	0.0014	19
152	57512.7418	0.0020	-0.0013	15
154	57512.8690	0.0003	-0.0033	48
155	57512.9360	0.0004	-0.0009	65
156	57513.0018	0.0004	0.0003	62
157	57513.0665	0.0005	0.0004	65
158	57513.1325	0.0004	0.0019	103
163	57513.4509	0.0005	-0.0027	199
166	57513.6463	0.0006	-0.0011	55
167	57513.7092	0.0007	-0.0028	38
170	57513.9048	0.0004	-0.0010	65

Table A6: Times of superhump maxima in ASASSN-16dt (continued).

E	Max [†]	Error	$O - C^{\ddagger}$	$N^{\S}$
171	57513.9709	0.0004	0.0005	63
172	57514.0354	0.0005	0.0005	65
173	57514.0967	0.0005	-0.0028	65
175	57514.2292	0.0040	0.0005	69
176	57514.2910	0.0008	-0.0023	123
177	57514.3561	0.0013	-0.0018	115
178	57514.4225	0.0008	0.0000	149
181	57514.6159	0.0010	-0.0004	176
182	57514.6785	0.0012	-0.0023	136
183	57514.7412	0.0012	-0.0043	116
185	57514.8640	0.0011	-0.0107	47
186	57514.9381	0.0006	-0.0011	65
187	57515.0012	0.0004	-0.0026	63
188	57515.0648	0.0006	-0.0036	65
189	57515.1308	0.0006	-0.0022	65
196	57515.5853	0.0025	0.0001	117
197	57515.6400	0.0012	-0.0098	97
198	57515.7086	0.0015	-0.0057	88
211	57516.5537	0.0020	-0.0003	53
212	57516.6127	0.0025	-0.0059	122
213	57516.6737	0.0011	-0.0095	124
214	57516.7433	0.0043	-0.0045	92
263	57519.9120	0.0008	-0.0008	64
264	57519.9789	0.0021	0.0015	44
293	57521.8583	0.0007	0.0077	44
294	57521.9259	0.0014	0.0107	65
295	57521.9903	0.0009	0.0105	64

*Cycle counts.

[†]BJD−2400000.0.

[‡] $C = 2457502.925071 + 0.0653055 E$.

[§]Number of points used for determining the maximum.

Table A7: Times of superhump maxima in ASASSN-16hg.

E	Max [†]	Error	$O - C^{\ddagger}$	$N^{\S}$
0	57591.6600	0.0065	-0.0010	18
1	57591.7237	0.0037	0.0003	18
2	57591.7863	0.0013	0.0006	19
3	57591.8442	0.0015	-0.0038	21
4	57591.9102	0.0015	-0.0002	20
16	57592.6570	0.0012	-0.0015	16
17	57592.7208	0.0014	-0.0001	18
18	57592.7796	0.0016	-0.0037	19
19	57592.8439	0.0017	-0.0017	21
20	57592.9060	0.0016	-0.0020	21
32	57593.6548	0.0022	-0.0013	15
33	57593.7179	0.0011	-0.0006	18
34	57593.7767	0.0012	-0.0041	19
35	57593.8421	0.0017	-0.0011	20
36	57593.9059	0.0016	0.0004	20
50	57594.7769	0.0021	-0.0015	16
51	57594.8361	0.0018	-0.0046	17
52	57594.9042	0.0015	0.0011	17
64	57595.6516	0.0017	0.0003	14
65	57595.7146	0.0048	0.0010	15
66	57595.7729	0.0016	-0.0030	16
67	57595.8356	0.0015	-0.0027	16
68	57595.9068	0.0015	0.0061	15
81	57596.7117	0.0018	0.0006	15
82	57596.7738	0.0032	0.0003	27
83	57596.8389	0.0030	0.0030	28
84	57596.8956	0.0022	-0.0026	28
96	57597.6508	0.0027	0.0044	14
97	57597.7086	0.0029	-0.0001	15
98	57597.7678	0.0033	-0.0032	25
99	57597.8382	0.0059	0.0048	28
100	57597.8903	0.0047	-0.0055	28
114	57598.7596	0.0037	-0.0090	23

*Cycle counts.

[†]BJD-2400000.0.

[‡] $C = 2457591.6610 + 0.0623475 E$.

[§]Number of points used for determining the maximum.

Table A8: Log of observations of the 2016 outburst of 1SWASP J1621.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]
466.9381	467.0381	14.705	0.015	47	Mic	R_C
466.9387	467.0365	15.341	0.017	46	Mic	V
466.9395	467.0373	16.254	0.026	46	Mic	B
468.9367	468.9931	14.845	0.027	26	Mic	R_C
468.9373	468.9937	15.472	0.029	26	Mic	V
468.9382	468.9922	16.348	0.029	25	Mic	B
505.7052	506.0115	16.268	0.016	137	Mic	B
505.7060	506.0101	14.750	0.013	136	Mic	R_C
505.7066	506.0107	15.379	0.014	136	Mic	V
542.8993	542.9831	13.053	0.006	38	Mic	R_C
542.8999	542.9837	13.382	0.007	38	Mic	V
542.9007	542.9846	13.818	0.009	38	Mic	B
543.6787	543.9624	13.501	0.014	812	SGE	CV
543.8746	543.9054	14.243	0.048	34	HBB	V
544.6798	544.9507	13.804	0.020	400	SGE	CV
544.7206	544.8464	13.786	0.016	313	COO	CV
544.8811	544.9792	13.519	0.052	44	Mic	R_C
544.8817	544.9776	13.925	0.062	43	Mic	V
544.8826	544.9784	14.449	0.079	43	Mic	B
545.3104	545.5287	1.759	0.013	485	CRI	CV
545.4019	545.5645	13.891	0.026	219	RPc	V
545.4241	545.5385	13.875	0.007	66	SHU	CV
545.6793	545.9420	13.885	0.016	300	SRI	CV
545.7179	545.9854	14.102	0.022	362	SGE	CV
545.7365	545.9182	13.489	0.010	877	COO	R_C
546.3322	546.4039	14.259	0.041	101	Trt	V
546.4068	546.6473	13.621	0.023	150	Mic	R_C
546.4071	546.6477	14.056	0.027	150	Mic	V
546.4078	546.6483	14.624	0.038	150	Mic	B
546.4359	546.5066	13.781	0.009	47	SHU	R_C
546.4375	546.4540	14.009	0.008	2	SHU	CV
546.4691	546.4796	14.396	0.013	14	JSJ	B
546.6157	546.7485	13.847	0.004	242	LCO	CV
546.6840	546.9420	14.179	0.017	270	SRI	CV

Table A8: Log of observations of the 2016 outburst of 1SWASP J1621 (continued).

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]
547.2813	547.4122	2.134	0.025	31	CRI	R_C
547.2853	547.4100	1.592	0.019	31	CRI	B
547.2940	547.4115	1.980	0.023	30	CRI	V
547.2953	547.4128	2.229	0.025	30	CRI	I_C
547.3109	547.5321	2.162	0.013	500	CRI	CV
547.3536	547.4979	14.165	0.028	134	SHU	R_C
547.3596	547.4988	14.220	0.019	6	SHU	CV
547.4130	547.6069	14.341	0.017	351	IMi	V
547.4426	547.5714	14.179	0.005	181	Trt	V
547.5081	547.6157	14.139	0.007	125	BSM	CV
547.6819	547.9724	14.355	0.012	648	SGE	CV
548.2983	548.5226	2.155	0.056	52	CRI	B
548.2998	548.5200	2.439	0.040	52	CRI	V
548.3005	548.5207	2.526	0.032	52	CRI	R_C
548.3011	548.5213	2.557	0.025	52	CRI	I_C
548.3439	548.4874	14.802	0.027	93	SHU, NKa	R_C
548.3584	548.5426	2.473	0.012	416	CRI	CV
548.3680	548.4865	14.411	0.043	4	SHU	CV
548.3890	548.6118	14.598	0.014	266	BPO	CV
548.4251	548.5936	14.716	0.016	314	IMi	V
548.5078	548.6157	14.456	0.010	119	BSM	CV
548.6831	548.9865	14.714	0.012	541	SGE	CV
549.3060	549.4802	2.406	0.016	44	CRI	B
549.3074	549.4776	2.668	0.016	43	CRI	V
549.3082	549.4783	2.728	0.017	43	CRI	R_C
549.3087	549.4789	2.699	0.017	43	CRI	I_C
549.6008	549.8552	15.795	0.031	95	CMJ	B
549.6020	549.8564	15.010	0.020	96	CMJ	V
549.6277	549.8538	14.966	0.014	176	RIT	CV
549.6900	549.9682	14.985	0.010	548	SGE	CV
549.6976	549.9180	15.741	0.038	89	GFB	B
549.6985	549.9190	14.985	0.023	90	GFB	V

Table A8: Log of observations of the 2016 outburst of 1SWASP J1621 (continued).

Start*	End*	Mag [†]	Error [‡]	$N^{\S}$	Obs	Band [#]
550.0408	550.1700	14.838	0.010	338	OKU	<i>CV</i>
550.0501	550.2506	14.832	0.012	308	Kis	<i>CV</i>
550.0555	550.2780	14.625	0.010	284	Ioh	<i>R_C</i>
550.2851	550.5242	2.734	0.030	60	CRI	<i>B</i>
550.2866	550.5216	2.921	0.023	59	CRI	<i>V</i>
550.2873	550.5223	2.936	0.021	59	CRI	<i>R_C</i>
550.2879	550.5229	2.860	0.018	59	CRI	<i>I_C</i>
550.3179	550.6242	14.940	0.012	183	BSM	<i>CV</i>
550.3451	550.5412	15.016	0.082	6	SHU	<i>CV</i>
550.6839	550.9838	15.127	0.009	541	SGE	<i>CV</i>
550.6942	550.9266	15.103	0.015	136	SRI	<i>CV</i>
550.6974	550.9731	15.972	0.020	230	GFB	<i>B</i>
551.2732	551.5204	2.999	0.028	62	CRI	<i>B</i>
551.2747	551.5218	3.096	0.025	62	CRI	<i>V</i>
551.2754	551.5226	3.061	0.022	62	CRI	<i>R_C</i>
551.2760	551.5232	2.963	0.019	62	CRI	<i>I_C</i>
552.4002	552.4594	15.087	0.021	32	SHU, Nka	<i>CV</i>
552.6653	552.8563	16.368	0.024	72	CMJ	<i>B</i>
552.6666	552.8550	15.385	0.023	70	CMJ	<i>V</i>
553.5893	553.8566	16.317	0.024	103	CMJ	<i>B</i>
553.5905	553.8553	15.325	0.017	102	CMJ	<i>V</i>
553.6977	553.9739	16.306	0.014	230	GFB	<i>B</i>
553.7135	553.8156	15.181	0.012	26	SGE	<i>V</i>
554.0120	554.1428	15.103	0.007	289	OKU	<i>CV</i>
554.3890	554.5240	15.312	0.018	71	PVE	<i>V</i>
554.4558	554.5183	15.092	0.012	67	SHU, Nka	<i>CV</i>
554.6905	554.9057	15.333	0.024	72	SGE	<i>V</i>
554.6935	554.9087	16.363	0.036	72	SGE	<i>B</i>
555.3449	555.5094	14.169	0.066	9	PVE	<i>I_C</i>
555.3524	555.5439	15.359	0.025	55	PVE	<i>V</i>
555.3917	555.5158	16.326	0.085	8	PVE	<i>B</i>
555.6884	555.9814	15.211	0.023	49	SGE	<i>V</i>
555.6913	555.9845	16.242	0.028	53	SGE	<i>B</i>

Table A8: Log of observations of the 2016 outburst of 1SWASP J1621 (continued).

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]
556.3195	556.5463	14.141	0.019	61	PVE	I_C
556.3574	556.5290	16.347	0.085	8	PVE	B
556.3631	556.5347	15.311	0.050	8	PVE	V
556.6001	556.8552	16.352	0.021	99	CMJ	B
556.6014	556.8539	15.352	0.018	97	CMJ	V
556.6899	556.9798	15.297	0.020	94	SGE	V
556.6930	556.9829	16.310	0.026	94	SGE	B
557.3278	557.4053	14.093	0.034	14	PVE	I_C
557.3304	557.4133	15.278	0.045	15	PVE	V
558.3199	558.5466	15.373	0.019	99	PVE	V
563.0314	563.0755	14.638	0.010	62	OKU	R_C
566.0187	566.1714	14.774	0.013	160	OKU	R_C
617.3656	617.5738	15.229	0.009	288	RPc	CV
630.3350	630.5050	15.238	0.009	236	RPc	CV
665.2928	665.3731	15.202	0.012	106	RPc	CV
823.4359	823.5977	14.696	0.143	30	WTH	R_C

*BJD–2457000.0.

[†]Mean magnitude. Here, CRI reports the relative magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer’s code: GFB (William Goff), Kis (Seiichiro Kiyota), COO (Lewis M. Cook), Ioh (Hiroshi Itoh), CRI (Crimean Observatory), OKU (Osaka Kyoiku Univ. team), Trt (Tamás Tordai), RPc (Roger D. Pickard), IMi (Ian Miller), RIT (Michael Richmond), SGE (Geoff Stone), SHU (S. Shugarov team), SRI (Richard Sabo), LCO (Colin Littlefield), NKa (Natalia Katysheva & Sergei Yu. Shugarov), PVE (Velimir Popov), HBB (Barbara Harris), CMJ (Michael Cook), WTH (Wikander, Thomas), Mic (Raúl Michel), BSM (Stephen M. Brincat), JSJ (Steve Johnson).

[#]Filter. “ CV ” means no (clear) filter.

Table A9: Log of observations of the 2006 outburst and the 2013 quiescence of BD Pav.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]
979.2944	79.4928	12.481	0.003	539	MLF	<i>CV</i>
980.1966	80.5440	12.540	0.002	903	MLF	<i>CV</i>
982.1992	82.5138	12.635	0.003	851	MLF	<i>CV</i>
983.2130	83.5227	12.699	0.003	880	MLF	<i>CV</i>
984.1914	84.5195	12.776	0.003	930	MLF	<i>CV</i>
985.1892	85.5080	12.965	0.003	858	MLF	<i>CV</i>
3454.7143	2554.8467	15.152	0.114	150	OAR	<i>V</i>
3455.6887	2555.9171	15.157	0.144	220	OAR	<i>V</i>
3457.7425	2557.9143	15.101	0.109	167	OAR	<i>V</i>
3460.7053	2560.8474	15.039	0.030	9	OAR	<i>V</i>
3461.7647	2561.7685	14.956	0.015	5	OAR	<i>V</i>
3462.7649	2562.7688	15.077	0.033	5	OAR	<i>V</i>
3463.7621	2563.7657	15.156	0.016	5	OAR	<i>V</i>

*BJD–2453000.0.

[†]Mean magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer’s code: MLF (Berto Monard).

[#]Filter. “*CV*” means no (clear) filter.

Table A10: Log of observations of the 2009 outburst of V364 Lib.

Start*	End*	Mag [†]	Error [‡]	N [§]	Obs	Band [#]
928.0757	28.2011	10.412	0.001	405	Mhh	V
928.1202	28.2177	11.478	0.003	223	Njh	CV
928.2010	28.2883	1.822	0.001	481	OUS	CV
928.2017	28.2036	10.569	0.006	6	Mhh	B
929.1080	29.1733	10.711	0.001	89	Kis	CV
929.1197	29.2284	1.893	0.002	351	OUS	CV
930.0970	30.2722	11.460	0.001	412	Njh	CV
930.1155	30.2758	1.880	0.001	885	OUS	CV
930.1287	30.1968	10.732	0.002	96	Kis	CV
930.1431	30.2359	10.436	0.002	138	Mhh	V
930.1434	30.2355	10.628	0.002	136	Mhh	B
931.0911	31.3198	11.493	0.001	537	Njh	CV
931.0944	31.1718	10.710	0.003	104	Kis	CV
932.0938	32.2945	11.552	0.001	465	Njh	CV
932.1091	32.1847	10.770	0.003	102	Kis	CV
932.1812	32.2890	10.729	0.001	410	Mhh	B
937.1064	37.1435	11.750	0.002	88	Njh	CV
938.0842	38.2074	11.906	0.003	237	Njh	CV
939.2176	39.3233	11.199	0.001	591	Mhh	CV
945.1195	45.2613	12.179	0.001	335	Njh	CV
947.1338	47.1456	12.247	0.005	29	Njh	CV
949.0912	49.2735	12.347	0.002	422	Njh	CV
950.1414	50.2637	12.363	0.002	288	Njh	CV
951.0445	51.2736	12.407	0.002	540	Njh	CV
951.3060	51.6199	0.042	0.001	1623	MLF	V
952.0562	52.1361	12.431	0.002	189	Njh	CV
953.0907	53.2577	12.433	0.001	392	Njh	CV
965.0846	65.2432	12.440	0.001	370	Njh	CV

*BJD-2454000.0.

[†]Mean magnitude. Here, OUS reports the relative magnitude.

[‡]1 σ of mean magnitude.

[§]Number of observations.

^{||}Observer's code: Mhh (Hiroyuki Maehara), Njh (Kazuhiro Nakajima), OUS (Okayama U. of Science), Kis (Seiichiro Kiyota).

[#]Filter. "CV" means no (clear) filter.

Table A11: Log of spectroscopic observations of V364 Lib.

Date*	Name [†]	Start Time [‡]	End Time [§]	Exp	Number [#]	Grating [¶]	Site**
2009-04-07	V364 Lib	54929.1854	54929.2062	60	11	Grism	G
2009-04-09	V364 Lib	54931.1862	54931.2022	60	15	Grism	G
2009-04-10	V364 Lib	54932.1536	54932.1703	60	12	Grism	G
–	V364 Lib	54932.1953	54932.2043	60	9	Grism	G
2009-04-15	V364 Lib	54937.0754	54937.0764	30	3	I_C	G
–	V364 Lib	54937.0816	54937.0948	60	11	Grism	G
2009-04-18	V364 Lib	54940.1137	54940.1151	60	2	Grism	G
2009-04-19	V364 Lib	54941.0797	54941.0901	60	10	Grism	G
2009-04-22	V364 Lib	54944.0798	54944.0902	60	10	Grism	G
2009-04-23	V364 Lib	54945.1173	5494.1257	60	5	Grism	G
2009-04-26	V364 Lib	54948.1084	54948.1209	120	5	Grism	G
2009-04-27	V364 Lib	54949.0751	54949.0820	60	6	Grism	G
2009-04-28	V364 Lib	54950.1300	54950.1473	60	12	Grism	G
2009-05-02	V364 Lib	54954.0627	54954.0738	60	10	Grism	G
2009-05-05	V364 Lib	54961.8932	54961.9392	900	5	std-Yd	S
–	V364 Lib	54961.9619	54961.0183	900	6	std-Bc	S
–	V364 Lib	54962.0348	54962.0573	900	3	std-Yd	S
2009-05-08	V364 Lib	54960.1634	54960.0759	60	10	Grism	G
2009-05-13	V364 Lib	54965.1412	54965.0419	20	3	R	G
–	V364 Lib	54965.1510	54965.1579	60	7	Grism	G
2009-05-20	V364 Lib	54972.1113	54972.1183	60	6	Grism	G

*Observational date (Japan Standard Time).

[†]Name of star.

[‡]Start time of observations in the unit of BJD–2400000.

[§]End time of observations in the unit of BJD–2400000.

^{||}Exposure time of each observation in unit of seconds.

[#]Number of observations.

[¶]Diffraction or echelle gratings.

**Name of observatory: S (Subaru Observatory), G (Gunma Observatory).

Table A12: Radial velocity measured in the 2009 quiescence of V364 Lib.

Time*	Phase [†]	Radial Velocity	Error
54929.1934	0.7912	–107.8	10.8
54931.1895	0.6329	–50.3	11.3
54932.1952	0.0648	73.3	13.7
54937.0838	0.0243	42.0	22.2
54941.0801	0.7135	–99.8	37.9
54944.1177	0.0379	–5.6	46.4
54945.1577	0.5186	–12.3	52.5

*Time of observations in the unit of BJD–2400000.

[†]Orbital phase under the assumption that the orbital period is 0.7024293 d.

Table A13: Radial velocity measured in the 2009 outburst of V364 Lib.

Time*	Phase [†]	Radial Velocity	Error
54961.3873	0.3746	−49.7	4.3
54961.3994	0.3917	−38.1	3.3
54961.4107	0.4078	−34.3	3.7
54961.4220	0.4239	−28.4	3.7
54961.4334	0.4401	−21.1	4.1
54961.4561	0.4724	−6.0	5.3
54961.4676	0.4888	2.4	4.3
54961.4788	0.5048	8.5	4.2
54961.4901	0.5208	13.4	3.1
54961.5013	0.5367	22.5	3.1
54961.5125	0.5527	30.8	2.2
54961.5289	0.5762	40.5	6.7
54961.5403	0.5923	49.5	5.5
54961.5515	0.6083	51.4	4.5

*Time of observations in the unit of BJD−2400000.

[†]Orbital phase under the assumption
that the orbital period is 0.7024293 d.

Details of the numerical methods for a time-dependent viscous disk

B.1 Conservation of the total angular momentum of the disk

This section describes the way how to conserve the total angular momentum of the disk, i.e., the way how to change the disk radius in the simulations in chapter 5. As described in Sec. 5.3.5, this is the same way as that used in Ichikawa, Osaki (1992). The disk radius is determined by the angular momentum conservation. The condition is derived from equation (5.8) as follows:

$$\frac{(h\Delta M)_{N-1/2}^{\text{new}} - (h\Delta M)_{N-1/2}}{\Delta t} = (2\pi r^2 W)_{N-1} - D_{N-1/2} \Delta r_N - \dot{M}_{N-1} h_{N-1} + \dot{M}_{s,N-1/2} h_{\text{LS}}, \quad (\text{B.1})$$

where Δr_N represents the width of the outermost radius. Since W_{N-1} and h_{N-1} are not defined at the interface, equation (5.8) is applied to eliminate these quantities. Here the angular momentum in the inner half of the outermost annulus is conserved as follows:

$$\begin{aligned} \dot{M}_{N-1} (h_{N-1/2} - h_{N-1}) &= (2\pi r^2 W)_{N-1/2} - (2\pi r^2 W)_{N-1} \\ &+ D_{N-1/2} (r_{N-1/2} - r_{N-1}) - \frac{\dot{M}_{s,N-1}}{2} (h_{\text{LS}} - h_{N-1/2}). \end{aligned} \quad (\text{B.2})$$

By combining equations (5.24), (B.2), and (B.3), the following relation is obtained as

$$\begin{aligned} \Delta M_{N-1/2}^{\text{new}} \frac{h_{N-1/2}^{\text{new}} - h_{N-1/2}}{\Delta t} &= (2\pi r^2 W)_{N-1/2} - D_{N-1/2} (r_{N-1/2} - r_{N-1}) \\ &- \frac{\dot{M}_{s,N-1/2}}{2} (h_{N-1/2} - h_{\text{LS}}), \end{aligned} \quad (\text{B.3})$$

which represents the time variation of $r_{N-1/2}$, and hence, the disk radius r_N can be estimated.

B.2 Treatment for the tidal truncation

If the disk radius expands beyond the tidal truncation radius, the extra angular momentum is removed from the outer edge of the disk in our numerical simulations. When the extra tidal torque that exerts on the outermost annulus during the tidal truncation is defined as D_{ext} , the total angular momentum of the disk at a new time step is expressed as

$$J_{\text{disk}}^{\text{new}2} - J_{\text{disk}}^{\text{old}} = (h_{\text{LS}}\dot{M}_{\text{tr}} - h_0\dot{M}_0 - D_{\text{total}} - D_{\text{ext}})dt. \quad (\text{B.4})$$

If the tidal truncation is not triggered, the total angular momentum of the disk at a new time step should be

$$\begin{aligned} J_{\text{disk}}^{\text{new}} - J_{\text{disk}}^{\text{old}} &= (h_{\text{LS}}\dot{M}_{\text{tr}} - h_0\dot{M}_0 - D_{\text{total}})dt \\ &= \sum_{i=1}^{N-1} h_{i-1/2}^{\text{new}} \Delta M_{i-1/2}^{\text{new}} + h_{N-1/2}^{\text{new}} \Delta M_{N-1/2}^{\text{new}}, \end{aligned} \quad (\text{B.5})$$

where $h_{N-1/2}^{\text{new}} = \sqrt{GM_1(r_{N-1} + R_d)/2}$. After the tidal truncation, the disk radius is fixed to r_{tidal} , and

$$J_{\text{disk}}^{\text{new}2} = \sum_{i=1}^{N-1} h_{i-1/2}^{\text{new}} \Delta M_{i-1/2}^{\text{new}} + h_{N-1/2}^{\text{new}2} \Delta M_{N-1/2}^{\text{new}}, \quad (\text{B.6})$$

where $h_{N-1/2}^{\text{new}2} = \sqrt{GM_1(r_{N-1} + r_{\text{tidal}})/2}$. Finally, D_{ext} is obtained as $(J_{\text{disk}}^{\text{new}} - J_{\text{disk}}^{\text{new}2})/dt = ((h_{N-1/2}^{\text{new}} - h_{N-1/2}^{\text{new}2})\Delta M_{N-1/2}^{\text{new}})/dt$.

B.3 Splitting and merging processes of the outermost annulus

As described in Sec. 5.3.5, the mesh number changes in accordance with the variation of the disk radius. If $0.5dr \leq \Delta r_N \leq 1.5dr$, we do not change the mesh number. If $\Delta r_N < 0.5dr$, we merge the N -th and $(N-1)$ -th annuli. Then the mesh number decreases by 1. If $\Delta r_N > 1.5dr$, we split the N -th annulus into two adjacent annuli. Then the mesh number increases by 1.

In changing the mesh number, the total mass, the total angular momentum, and $\sum_1^N T_c \Delta M$ of the disk are conserved. In merging the two annuli, the following

conditions are imposed as

$$N^{\text{new}} = N - 1, \quad (\text{B.7})$$

$$\Delta M_N^{\text{new}} = \Delta M_{N-1} + \Delta M_N \quad (\text{B.8})$$

$$\Delta M_N^{\text{new}} \Delta T_{c,N}^{\text{new}} = \Delta M_{N-1} \Delta T_{c,N-1} + \Delta M_N \Delta T_{c,N}. \quad (\text{B.9})$$

In splitting the outermost annulus, the following conditions are imposed as

$$N^{\text{new}} = N + 1, \quad (\text{B.10})$$

$$\Delta M_N^{\text{new}} + \Delta M_{N-1}^{\text{new}} = \Delta M_N, \quad (\text{B.11})$$

$$T_{c,N}^{\text{new}} = T_{c,N-1}^{\text{new}} = T_{c,N}. \quad (\text{B.12})$$

In order to conserve the angular momentum, it is necessary to recalculate the disk radius apart from the calculation for determining the condition of the merge or split as $J_{\text{disk}}^{\text{new}} = \sum_1^{N^{\text{new}}} h_{i-1/2}^{\text{new}} \Delta M_{i-1/2}^{\text{new}}$. In addition, we have to recalculate the density because μ_c also changes.

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Colophon

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