

**Dynamic soil-structure decoupling impact on
serviceability of underpass structures installed in
road embankment**

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ABSTRACT

Recent earthquakes, including the 2004 Mid Niigata Prefecture Earthquake, the 2011 Great East Japan Earthquake, and the 2016 Kumamoto Earthquake, highlighted the vulnerability of RBCs to joint openings in regions with sandy alluvial deposits, including liquefiable soils. Damage surveys conducted by Nakashima et al., (2021) documented numerous cases of joint openings and associated sediment outflows in expressway embankments.

To address these gaps, Nanazawa et al., (2022) and Miyazaki et al., (2024) conducted experimental investigations using centrifuge testing to explore the behavior of rigid box culverts during seismic events. Their studies identified a critical phenomenon: the preferential progression of liquefaction beneath culverts, which leads to stress imbalances and joint openings. These experiments provided valuable insights into the deformation mechanisms at play and marked a significant step forward in understanding the factors influencing RBC performance.

Building on the findings of damage surveys and experimental studies, this current study employs advanced numerical analysis to replicate and extend the observed phenomena in sandy alluvial deposits, particularly in liquefiable soils. By integrating hydrological, geological, and structural factors, this research aims to elucidate the mechanisms of joint opening and provide actionable insights for improving the seismic resilience of RBCs. This included reproduction and validation of the damage survey findings, validation of experimental observations and quantification of the joint opening and influencing factors. Finally, a mitigation measure based on the research findings is proposed.

Initially, the simulator validated the experimental results in a better way, and the simulator parameters adopted for the current numerical works. In 2D analysis, the undrained boundary between the culvert and foundation ground alleviated the liquefaction levels beneath the culvert, resulting in high EPWP at the joint section that the mouth sections. 2 culvert models produced significant joint openings, but this was not the case for the 4 culvert element models. The joint model could precisely produce the joint opening. Concave and convex soil deformation which resulted to V shaped and inverted V shaped joint openings respectively were confirmed for shallow and deep liquefiable soils, and this also reflected the experimental centrifuge test findings. A relationship between the culvert bump and joint opening index was developed which can be used in real site situation in prediction occurred culvert joint opening in case an earthquake occurs and the culvert bump forms.

For the 3D analysis, localized liquefaction occurred beneath the culverts as compared to the embankment side. This also reflected the order of liquefaction; whereby higher liquefaction occurs first beneath the culverts then beneath the embankments. The culvert joint opening was like the 2D results. All these results agreed with the experimental centrifuge results from past researchers.

The research proposes a mitigation measure of sheet pile extending the whole depth of liquefiable ground and rigidly fixed at the culvert mouth. This was influenced by the analytical results whereby, the ground beneath culvert mouth would initially undergo deformation causing initial inclinations and subsequent joint openings. Further studies are proposed on this mitigation measure, alongside variation of the input wave and structural properties of the culvert.

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CHAPTER ONE

INTRODUCTION

1.1 Background and context

Over the past decade, rigid box culverts (RBCs) have consistently suffered joint opening damage during seismic events, particularly in regions with liquefiable soils and sandy alluvial deposits. This recurrent issue has led to significant disruptions in transportation infrastructure, raising concerns about the resilience of essential lifelines. Despite numerous studies aimed at addressing this problem, the fundamental mechanisms driving joint opening remain unresolved, and effective mitigation strategies have yet to be established.

The seismic performance of underground structures, including RBCs, is distinct from that of surface structures due to their interaction with surrounding soil. Past research has identified key factors influencing the seismic response of RBCs, such as the flexibility ratio, racking deformation, and soil-structure interaction effects. Studies by Wang (1993) and Penzien (2000) have provided foundational insights into these phenomena, proposing simplified models to evaluate structural distortions. However, these models often assume uniform soil conditions and do not account for the complex behaviors observed during real seismic events.

Recent earthquakes, including the 2004 Mid Niigata Prefecture Earthquake, the 2011 Great East Japan Earthquake, and the 2016 Kumamoto Earthquake, highlighted the vulnerability of RBCs to joint openings in regions with sandy alluvial deposits, including liquefiable soils. Damage surveys conducted by Nakashima et al., (2021) documented numerous cases of joint openings and associated sediment outflows in expressway embankments. These findings suggest that liquefaction-induced deformation plays a critical role in amplifying damage. However, the mechanisms driving joint openings in these conditions remain poorly understood due to the lack of experimental data.

To address these gaps, Nanazawa et al., (2022) and Miyazaki et al., (2024) conducted experimental investigations using centrifuge testing to explore the behavior of rigid box culverts during seismic events. Their studies identified a critical phenomenon: the preferential progression of liquefaction beneath culverts, which leads to stress imbalances and joint openings. These experiments provided valuable insights into the deformation mechanisms at play and marked a significant step forward in understanding the factors influencing RBC performance. However, the experimental findings also highlighted the complexity of the

problem, necessitating further study through advanced numerical simulations to generalize and expand upon their results.

Building on the findings of damage surveys and experimental studies, this dissertation employs advanced numerical analysis to replicate and extend the observed phenomena in sandy alluvial deposits, particularly in liquefiable soils. By integrating hydrological, geological, and structural factors, this research aims to elucidate the mechanisms of joint opening and provide actionable insights for improving the seismic resilience of RBCs.

1.2 Research objectives

1.3 Scope and methodology

This study employs a combination of two-dimensional (2D) and three-dimensional (3D) elastoplastic finite element analysis to investigate the interaction between RBCs and different soil types. The research methodology is structured as follows, corresponding to the chapters of this dissertation:

1. **Literature Review (Chapter 2):** A detailed review of past studies on the seismic performance of underground structures, with a focus on mechanisms of joint opening and the role of soil-structure interaction. This chapter also emphasizes the gaps in existing research, particularly in sandy alluvial deposits, that this dissertation aims to address.
2. **Analytical Framework (Chapter 3):** Development of 2D and 3D finite element models, incorporating advanced soil constitutive models and structural parameters. This chapter provides the theoretical and methodological foundation for the analysis.
3. **2D Analysis (Chapter 4):** Examination of hydrological and culvert dimension factors affecting joint openings in liquefiable soils using 2D models, focusing on drainage conditions, soil depth, and structural configurations. Results from this chapter offer initial insights into the critical factors influencing joint openings.
4. **3D Analysis (Chapter 5):** Validation and extension of 2D findings using 3D models to simulate realistic in situ conditions and capture complex interactions. This chapter explores how three-dimensional effects influence ground deformation and joint opening mechanisms by liquefaction.
5. **Discussion of Findings (Chapter 6):** Analysis of the implications of numerical results, including relationships between liquefaction progression, joint opening indices, and structural performance. This chapter integrates findings from the previous chapters to derive practical recommendations.
6. **Conclusions and Recommendations (Chapter 7):** Summary of key findings, proposed mitigation strategies, and recommendations for future research. This chapter consolidates the contributions of the study to geotechnical engineering and infrastructure resilience.

Through these efforts, this dissertation seeks to advance the field of geotechnical engineering, offering innovative solutions to a problem that has persisted for over a decade.

References

- Miyazaki, Y., Shizuka, E., Mutua, B. M., Toshinori, Y., Satoshi, T., Yu, F., Hirofumi, H. and Hiroaki, H. (2024). Traffic Disruptions in Box-Type Embankment-Underpasses Due to Past Earthquakes in Japan, *Proceedings of the 5th International Conference on Transportation Geotechnics (ICTG)*, vol. 2, pp. 45-53. Lecture Notes in Civil Engineering, vol 403. Springer, Singapore. https://doi.org/10.1007/978-981-97-8217-8_6
- Nakashima, K., Kusaka, H., Saeki, M., Mano, M. and Ro, H. (2021). Analysis of damage to expressway embankments and culverts from earthquake on soft ground, *Journal of Japan Society of Civil Engineers, Ser. A1 (Structural Engineering & Earthquake Engineering (SE/EE))*, vol. 77, Issue 3, pp. 384-394.
- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki, M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management, Ministry of Land, Infrastructure, Transport and Tourism, Japan*, No. 1219.
- Penzien, J. (2000). Seismically induced racking of tunnel linings, *Earthquake Engineering Structural Dynamics*, vol. 29, pp. 683–691. [https://onlinelibrary.wiley.com/doi/10.1002/\(SICI\)1096-9845\(200005\)29:5%3C683::AID-EQE932%3E3.0.CO;2-1](https://onlinelibrary.wiley.com/doi/10.1002/(SICI)1096-9845(200005)29:5%3C683::AID-EQE932%3E3.0.CO;2-1)
- Wang, J. N. (1993). Seismic design of tunnels: A state-of-the-art approach. Monograph 7. New York: Parsons Brinckerhoff Quade & Douglas.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the damage caused by past earthquake occurrences on rigid box culverts (RBCs). This involves various studies by researchers in trying to understand the joint opening phenomena in rigid box culverts. Most of the studies involved investigating the seismic response of the culverts in transverse or longitudinal direction hence this classification is adopted.

2.2 General performance of underground structures during seismic activities

Rigid box culverts (RBCs) are indispensable components of urban transportation infrastructure, particularly in highways, where their damage during earthquakes can disrupt emergency vehicle access due to surface displacements (Tokida et al., 2007), posing a significant threat to human lives. Being an example of a buried structure, they represent a complex soil-structure interaction whereby the relative stiffness between the backfill soil and culvert materials is a critical factor in their performance.

During an earthquake occurrence, the effects appear in the form of seismic waves that are generated at bedrock level and can be transferred through a soil medium to the ground surface. The seismic behavior of underground features is distinct from most surface structures, mostly due to (1) their complete enclosure in soil or rock, and (2) their significant length. Hence, the design of underground facilities to withstand seismic loading has aspects that are very different from the seismic design of surface structures.

When underground structures are subjected to seismic activities, the effects can be grouped into two categories: 1) ground shaking; and 2) ground failure such as liquefaction, fault displacement, and slope instability (Anderson et al., 2008). The major factors influencing shaking damage include: 1) the shape, dimensions and depth of the structure; 2) the properties of the surrounding soil or rock; 3) the properties of the structure; and 4) the severity of the ground shaking (Dowding and Rozen, 1978; St. John and Zahrah, 1987).

A collapse caused by shear failure of supporting columns damaged the shinkansen tunnel through Rokko mountain, Southeastern Hyogo prefecture, according to report by Asakura et al., (1996). Since the collapse of several underground structures in the past, various seismic design methods have been applied to rectangular underground structures, e.g., cut and- cover tunnels, which are structurally similar to RBCs, and various researchers have developed ways to evaluate their seismic behavior and seismic design (Abuhajar et al., 2015; Debiasi et al., 2013; Huo et al., 2006; Hushmand et al., 2016; Penzien, 2000;

Tsinidis, 2017; Tsinidis et al., 2016; Wang, 1993). The study results were presumed to be applicable to the seismic design of RBCs because the cross-section of RBCs and the material of members of RBCs are similar to the underground structures described above. However, careful considerations should be made when dealing with RBCs due to the existence of haunches which might affect their seismic response.

Conventional culverts satisfying the Road Earthwork and Culvert Construction Guiding Principle (Japan Road Associations, 2010), exhibited good seismic performance, even without a seismic design. This was because RBCs were thought to behave in unison with the surrounding ground, and therefore, the lining was not subjected to large seismic forces during earthquake occurrences. Though serious damage to geotechnical structures occurred, (e.g., the Niigata Prefecture Chuetsu-oki Earthquake in 2007, the Great East Japan Earthquake in 2011, and the Kumamoto Earthquake in 2016), no significant damage to RBCs was reported (EJECI, 2016; Murakami, 2011; NILIM and PWRI, 2004; Toyota, 2007).

2.3 Seismic performance of culverts considering transverse direction

Various researchers have reported the existence of the relationship between the surrounding ground strain and shear deformation angle, namely, the racking ratio–flexibility ratio relationship, (Abuhajar et al., 2015; Debiassi et al., 2013; Huo et al., 2006; Hushmand et al., 2016; Penzien, 2000; Tsinidis et al., 2016; Tsinidis, 2017; Wang, 1993). Racking ratio can be expressed as the structure shear strain over the ground shear strain. Yatsumoto et al., (2019) confirmed this relationship through numerical studies whereby the rigid box culverts were found to develop rocking rotations when the ground strains reached 0.08%. In the same research, a relationship between the shear deformation angle (α) and the rocking rotation (RR) angle (β) was derived as shown in Eq. (2.1) and Eq. (2.2) based on the coordinates of the four corners of the RBC shown in Figure 2.1. $x_1 - x_4$ are the x-coordinates of the RBC corners, $y_1 - y_4$ are the y-coordinates of the RBC corners, W is the RBC width and H is the RBC height. Also, in addition to failure mode in Figure 2.1, Miyazaki, (2014) suggested the various modes of culvert failure during seismic activity as shown in Figure 2.2.

$$\tan \beta = \frac{(y_4 - y_3)}{W} \quad 2.1$$

$$\tan (\alpha + \beta) = \frac{(x_1 - x_4)}{H} \quad 2.2$$

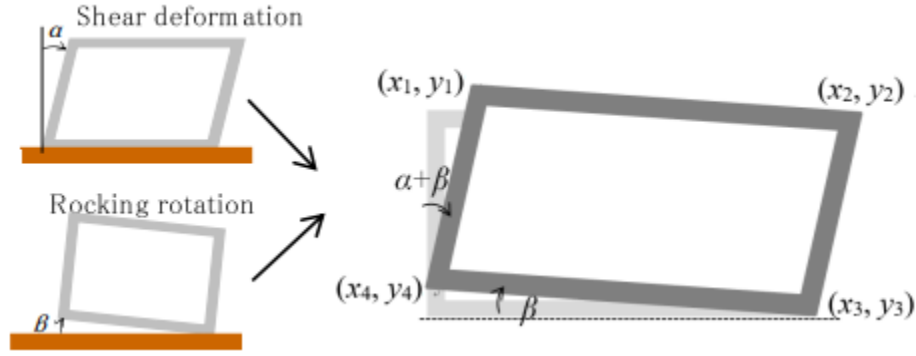


Figure 2.1. Relationship between rocking rotation and shear deformation of RBCs (Yatsumoto et al., 2019).

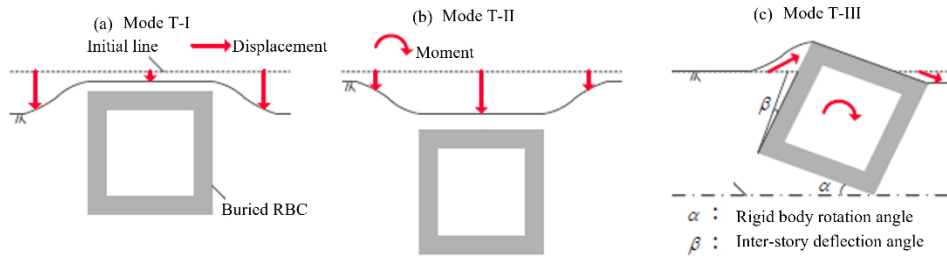


Figure 2.2. Culverts displacement modes in transverse directions; (a) ground subsidence on culverts sides (b) ground subsidence below the culvert and (c) culvert rotation (modified after Miyazaki et al., 2024).

Studies by Pitilakis et al., (2014), with simplified response in Figure 2.3; suggested that the inertia of the surrounding soils is much higher than the inertia of the structure itself, consequently the response of the embedded structure is dominated by the response of the surrounding soil and the transient ground deformations. This characteristic of underground structures makes their seismic behavior very distinct from the typical above ground structures, where the response is mainly related to the structure's inertia.

Soil deformation in the proximity of the structure, which in general is not elastic, imposes displacement constraint on the structure's cross-section. Since there exists stiffness difference between the two media, the structure does not follow the imposed ground deformation. The relatively rigid structure will resist seismic ground deformations, while a flexible structure will follow the ground distortions. For the very flexible structure, the structural distortion may be even higher than the free field ground deformation. The overall seismic behavior of the structure depends on the properties of the surrounding soil, which are not constant during strong excitation, and the inertial properties of the structure itself (Tsinidis, 2017).

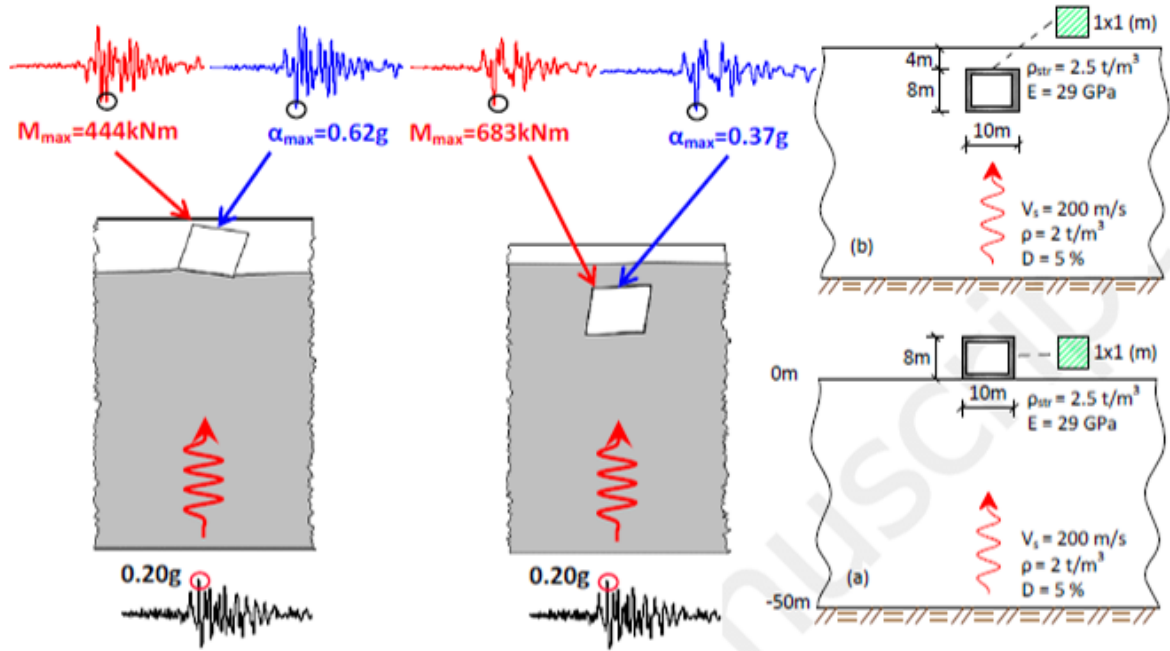


Figure 2.3. Response of a simple rectangular structure embedded or simply founded on the ground (Pitilakis et al., 2014).

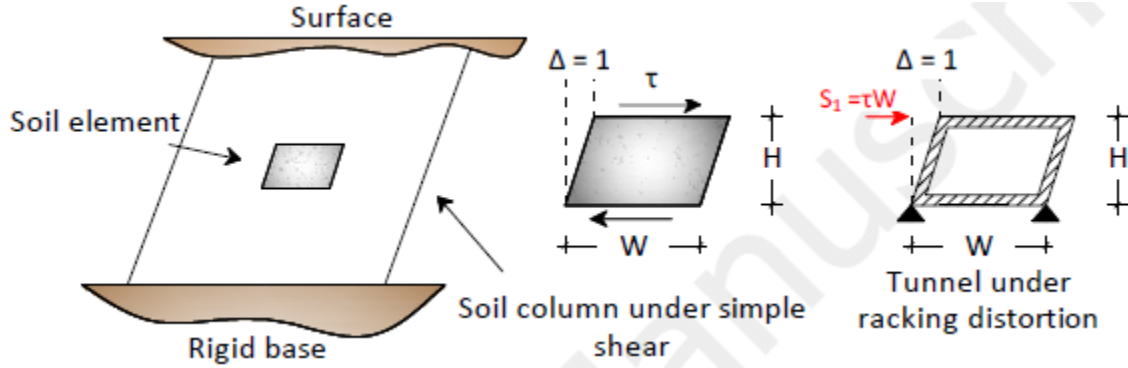
Also, Tsinidis (2017) showed that the relationship between the culvert surrounding ground strain (γ_{ff}) and the RR angle (θ) of its walls known as flexibility ratio (F) is given by θ/γ_{ff} . The flexibility ratio relationship is obtained from numerical analysis. The flexibility ratio value is closely related to the expected stress level on the structure, whereby, if: -

1. $F \rightarrow 0$: the structure is rigid and will not display any type of deformation.
2. $F < 1$: the structure is stiffer than the surrounding soil, thus the structural deformation level will be smaller than the free-field deformation level.
3. $F = 1$: the structure and the surrounding soil share the same level of stiffness, so the tunnel will follow the free-field deformation.
4. $F > 1$: the racking deformation of the structure is amplified compared to the free-field deformations.

For the case of rectangular structures, the flexibility ratio is estimated according to Wang (1993) as shown in Eq. (2.3) whereby, G_m is the soil shear modulus, W is the width of the structure, H is the height of the structure and S is the required force to cause a unit racking deflection of the structure, which is estimated through a simple static elastic frame analysis, as shown in Figure 2.4. The procedure does not include rocking but assumes pure racking deformation as the prevailing deformation mode, but it is not always the case.

$$F = \frac{(G_m \times W)}{S \times M}$$

2.3



$$\text{Soil: } \gamma = \frac{\Delta}{H} = \frac{\tau}{G_m} \rightarrow \frac{\tau}{\gamma} = \frac{\tau}{\Delta/H} = G_m; \text{ Structure: } \gamma = \frac{\Delta}{H} = \frac{P}{HS_1} = \frac{\tau W}{HS_1} \rightarrow \frac{\tau}{\gamma} = \frac{\tau}{\Delta/H} = \frac{HS_1}{W};$$

$$\text{Flexibility ratio: } F = \frac{G_m W}{HS_1}$$

Figure 2.4. Simplified model to evaluate flexibility ratio for rectangular structures (modified after Wang, 1993).

According to (Tsinidis et al., 2013b, c, d), experimental centrifuge results on rectangular model structures showed that the settlements above the tunnel are smaller than those recorded far from the tunnel and at the same time the shear modulus is much higher in the soil mass above the tunnel as shown in Figure 2.5. Pitilakis et al., (2014) emphasized that it is common in engineering practice to evaluate the static and the seismic behavior separately, then finally, the final detailing is performed as a combination of both the static and the seismic loads.

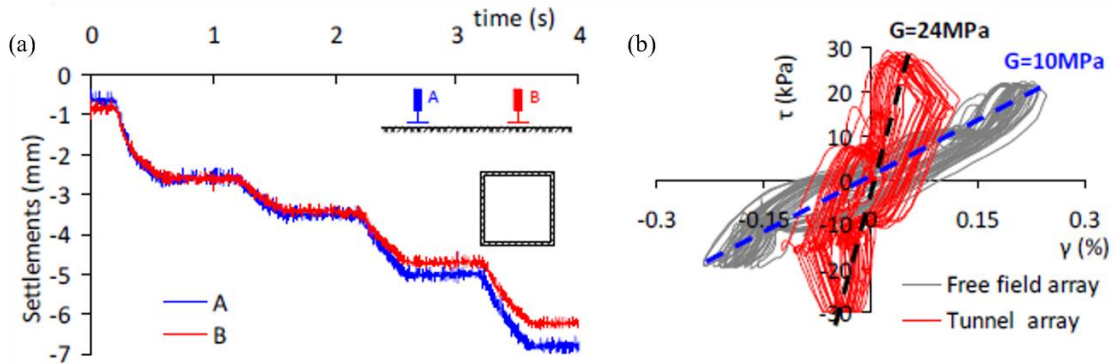


Figure 2.5. Soil response during experimental centrifuge tests (a) dynamic settlements as recorded during a centrifuge test at two points of the soil surface, (b) Stress–strain loops computed by the recorded accelerations at free field and near the tunnel (Tsinidis et al., 2013b, c, d).

The analysis and design of underground structures consists of four major steps: -

1. Seismic hazard assessment and selection of the design earthquake.
2. Evaluation of transient ground response and induced phenomena i.e. liquefaction, ground failure.
3. Evaluation of structure seismic response.
4. Synthesis of static and seismic loads.

Soil structure interaction effects are more significant in the case of rectangular structures, as they usually have larger and thicker dimensions, to withstand the static loads. This unique characteristic of rectangular structures may lead to a strong modification of the structural racking deformation with respect to the ground racking distortion. Wang (1993) and Penzien (2000) proposed simplified approaches accounting to this modification, through the racking ratio (structural/ground distortion) and showed that the racking ratio is related to the flexibility ratio. Wang (1993) relates the racking ratio (R) to the flexibility ratio (F) using the full dynamic analyses results of soil-tunnel systems, while Penzien (2000) correlated the two ratios based on a pseudo-static approach. Indicative racking ratio (R) to the flexibility ratio (F) relations are presented in Figure 2.6. According to study by Wang (1993), for a given flexibility ratio, the normalized structural distortion (racking ratio) of a rectangular structure tends to be reduced by 10 % with respect to a circular tunnel.

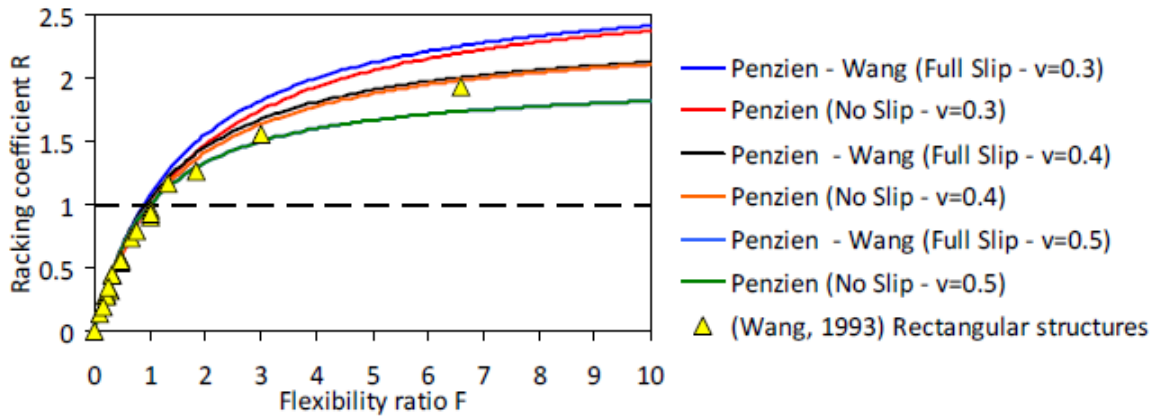


Figure 2.6. Racking ratio (R) to the flexibility ratio (F) correlation for circular and rectangular structures (modified after Wang, 1993 and Penzien, 2000).

$$\Delta_{structure} = R \times \Delta_{ff} \quad 2.4$$

Thus, the conclusion can be made that the results for circular tunnels are proposed to be used as an upper limit. If the estimation of the racking ratio R can be done, for instance, using diagrams like those in Figure 2.6, the structural deflection can be computed as shown in Eq. (2.4) where Δ_{ff} is free soil distortion. Similar to structural deflection, the force can be estimated and imposed on the structure, as a concentrated

force or as distributed force (Figure 2.7) in a simple elastic static frame analysis. These internal forces computed by this procedure are combined with the static internal forces, for the final structure detailing.

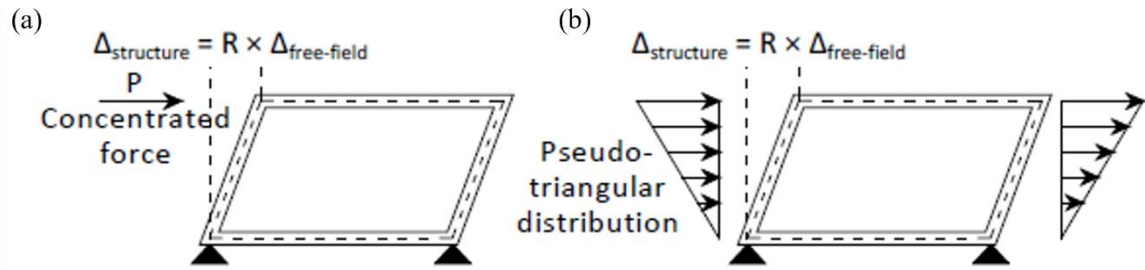


Figure 2.7. Simplified frame analysis models (a) concentrated load at the corner, (b) distributed load on the side walls (modified after Wang, 1993).

The discussed methodology is based on the hypothesis of pure racking deformation of the structure, both for the flexibility ratio determination and for the final analysis procedure. As mentioned, for rigid structures, rocking movement may be observed, whereas for relatively flexible structures inward deformations like for slabs and the side walls may occur. The exclusion of these deformation patterns may alter considerably the design internal loads.

Abuhajar et al., (2015) used centrifuge tests and numerical analysis to check the to check the effect of the soil fill height to culvert width ratio (H/B_c) on the kinematic soil culvert interaction. From his conclusions, as the amplitude of an earthquake increases, the reduction in peak ground acceleration (PGA) increases. The results from the 1995 Kobe earthquake at $PGA = 0.3 \text{ g}$ showed that the reduction in the PGA values can reach 50%. Also, it was shown that the kinematic interaction effect appears as the amplitude of the earthquake exceeds 0.1 g . Hence, the presence of buried underground structures will have minimal kinematic interaction effects if the earthquake has a PGA less than 0.1 g . The rocking angle for the foundation and box culvert were found to be very small; and changed with the PGA of the ground input motion. Even though these angles were very small the values for the foundation were higher than those of the box culverts. This behavior happened due to the confinement of the surrounding soil around the box culvert which was not the case for the foundations on the soil surface. Another important aspect was the lateral movement of the surface foundations. The results showed that the peak ground displacement (PGD) and the PGA at the top of the foundations increased as the PGA of the ground input motion at the base of the model increased. From the numerical analysis results, it was found that the PGA values for both free field and structural field decrease as the H/B_c ratio increases; and this happened for all the H/B_c ratios less than 3.07. For larger values, there was not much difference between them. The conclusion could be made

that the effect of kinematic interaction appears to be more significant when the height of the soil fill above the culvert is less than three times the width of the culvert.

2.4 Seismic performance of culverts considering longitudinal direction

The behavior of culverts connectivity in the longitudinal direction, during seismic activities portrays very well the joint opening mechanism. The failure mechanisms can result in the joint opening in any of the following forms shown in Figure 2.8, according to Yamaki et al., (2011): -

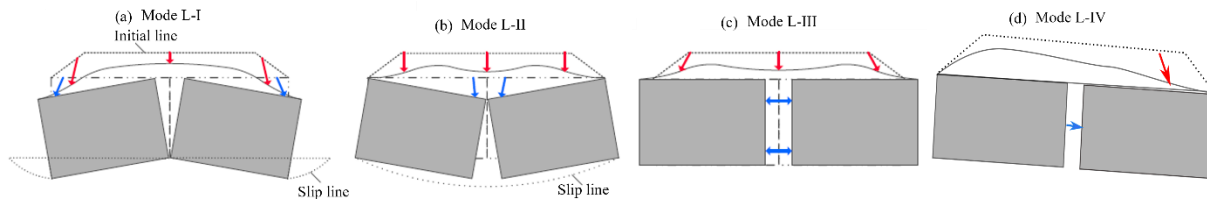


Figure 2.8. Culverts displacement modes in longitudinal directions (a) upward joint opening (b) downward joint opening (c) lateral joint separation and (d) inclined joint separation.

After the 2004 Mid Niigata prefecture Earthquake, Yasuda and Fujioka (2009) reported that the most serious damage occurred in places where: -

- (a) The embankments were constructed on slopes of hills mainly by cutting and filling and sliding of the filled embankments occurred at several sites during this earthquake.
- (b) The embankments were constructed on level grounds by filling soil and large settlement of the embankments occurred in this section.

Based on the mechanism of failure of the embankments, the damage of Kan-etsu expressway could be classified into three types as shown in Figure 2.9. From this figure, it's evident that the longitudinal direction effect of the earthquake on the culverts is well portrayed. Type II damage causes a large difference in level of the road surface and the box culvert but the creation of spacing of the joints of the box culvert and the difference in level of the crossway are small. For Type III, the damage creates a separation of the joints of the box culvert, and a large difference in level of the crossway and the box culvert. From the reproduction numerical analysis by Ishikawa et al., (2019); in Type II (Figure 2.10), the amount of difference in levels of a box culvert and service road is defined by the penetrated settlement of a box culvert, and the amount of difference in levels was 17.5 cm. Also, from the reproduction numerical analysis results of Type III (Figure 2.11), the amount of difference in levels of a box culvert and a service road is defined by the penetrated settlement of a box culvert, and the amount of difference in levels is 42.2 cm.

During the 2016 Kumamoto Earthquake, a field survey was done to ascertain the damage caused by the earthquake. Nanazawa et al., (2022) conducted study on 30 facilities mainly culverts. It was confirmed that there was no noticeable damage to the main body of the culvert, and that the main deformation was the opening of the joints. These structural soundness of the culverts walls during earthquakes were supported by earlier researchers including Youd and Beckman, (1996); Nishioka and Unjoh, (2003) and Wood (2005). During the joint repair process, depending on the size of the joint opening, traffic disruptions occurred, and Figure 2.12 shows the repair methods and how the traffic disruptions were managed. For this reason, to minimize the impact on road functions, it is desirable to limit the opening of joints to less than 20 cm, which allows repair by steel plate blockages that do not require upper road restrictions. Also, for the repair method, the smaller the joint opening during an earthquake, the faster and easier the repair method, and vice versa as portrayed by Figure 2.13.

Nanazawa et al., (2022) classified the culverts displacement modes as either convex- for shallow liquefiable grounds; and concave- for deep liquefiable grounds through a series of dynamic centrifuge tests. Depending on the thickness of the liquefiable layers, the convex displacement mode produces V-shaped joint openings, (Figure 2.14 a), on shallow liquefiable depth of 3.0 m in real dimension, while the concave displacement mode produces inverted V-shaped joint openings, (Figure 2.14 b), on deep liquefiable depth of 10.0 m in real dimension. Not only the effect of the liquefiable layer on the deformation mode but also the effect of the overburden ratio, the aspect ratio of RBC, the casting type and so on were investigated.

From the generalized failure modes, the net liquefiable area is smaller in shallow than deep liquefiable ground. For the shallow liquefiable ground, the net liquefied area is small, and the embankment flow leads the culverts to tilt to the outside mouths, leading to the deformed modes shown in Figure 2.14 (a). The high ground deformation below the culverts in deep liquefiable ground leads to culverts sinking and taking the deformation shown in Figure 2.14 (b).

From centrifuge tests conducted by Miyazaki, (2024), there was development of sand boil at the joint beneath the culverts, (Figure 2.15 a), which was caused by the development of excess pore water pressure during liquefaction. This caused the culverts to develop V-shaped joint opening which was caused by sand boil development due to liquefaction of the deep liquefiable foundation ground. The wing shape effect strongly contributes to V-shaped joint opening than the thickness of the liquefiable layer.

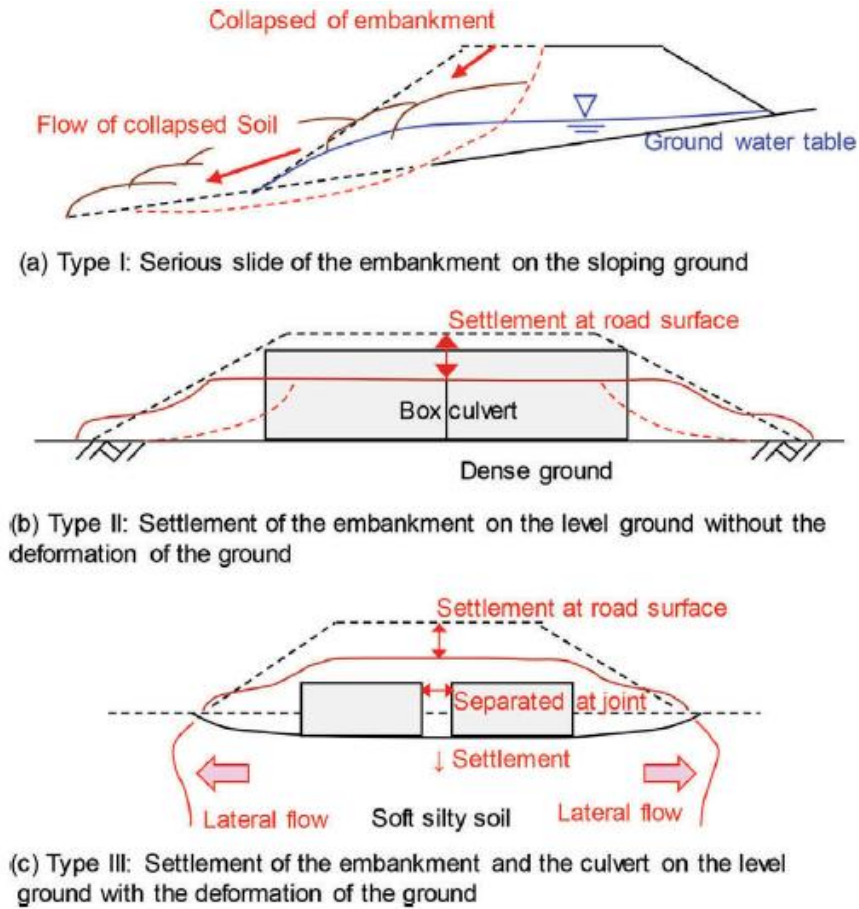


Figure 2.9. Classification of the damage to the embankment of Kan-etsu expressway according to the mechanism of failure (Ishikawa et al., 2019).

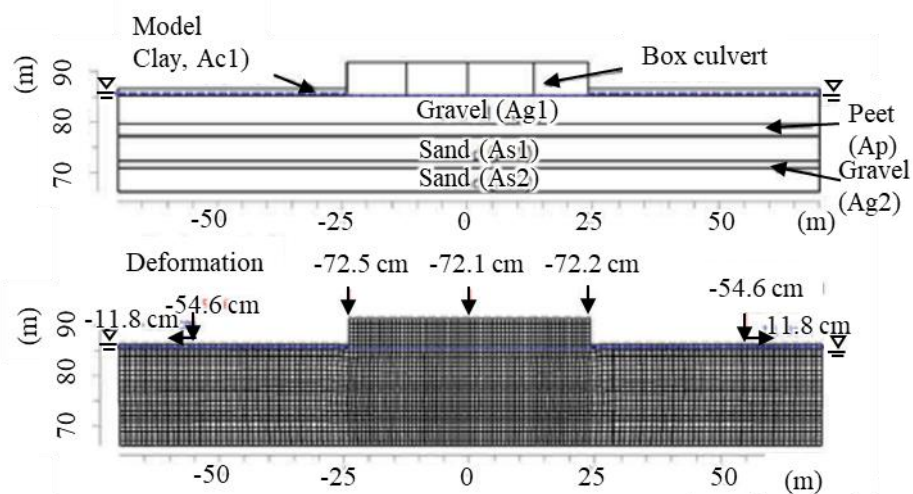


Figure 2.10. Residual deformation analysis results for disaster Type II (modified after Ishikawa et al., 2019).

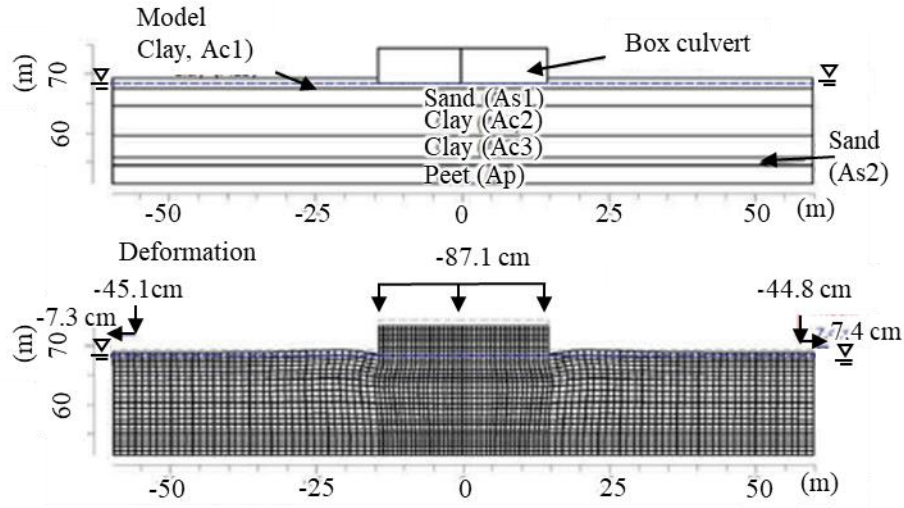


Figure 2.11. Residual deformation analysis results for disaster Type III (modified after Ishikawa et al., 2019).

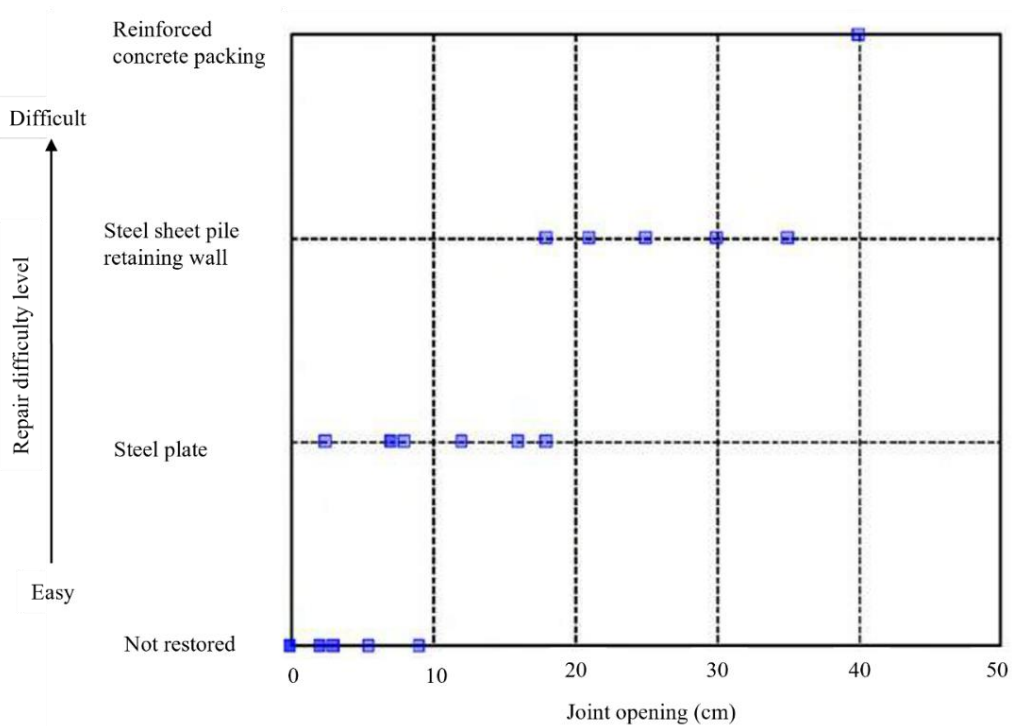


Figure 2.12. Joint opening and repair methods for the 2016 Kumamoto Earthquake (modified after Nanazawa et al., 2022).

The sand boil was suspected to be caused by the unbalanced drainage boundary behavior beneath the culverts and the embankment, which amplified the EPWP at the joint under culverts (Figure 2.15 b). The

order of liquefaction development was from the bottom of the culvert and later to the embankment and this agreed to the discussion by Nanazawa et al., (2022) in Figure 2.16. The joint opening was evident in Miyazaki, (2024) experimental studies and from Nanazawa et al., (2022) report on the aftermath of the 2016 Kumamoto Earthquake.

The excess pore water pressure was found to increase sharply after the initial tremor after the shaking and reached maximum at maximum shaking (Figure 2.16). This caused the longitudinal elongation of the culverts due to joint opening and reached climax at maximum shaking. Similarly, the order of liquefaction was found to start from the bottom of the culvert then beneath the embankment (Figure 2.16).

	Joint opening amount δ		
	$7 \text{ cm} \leq \delta < 20 \text{ cm}$	$20 \text{ cm} \leq \delta < 40 \text{ cm}$	$40 \text{ cm} \leq \delta$
Repair difficulty level	Relatively easy	Somewhat difficult	Difficult
Repair method	Construction can be done from the interior road. No traffic restrictions are required on the upper road.	Construction will begin from the upper road. Traffic restrictions will be in place on the upper road.	Construction will be carried out with complete closure of the upper and inner road.
	Repair by blocking with steel plates.	Repair using steel sheet piles.	Repair using reinforced concrete filling.
Condition at the time of or after repair			
			 

Figure 2.13. Joint repair process and traffic management after the 2016 Kumamoto Earthquake (modified after Nanazawa et al., 2022).

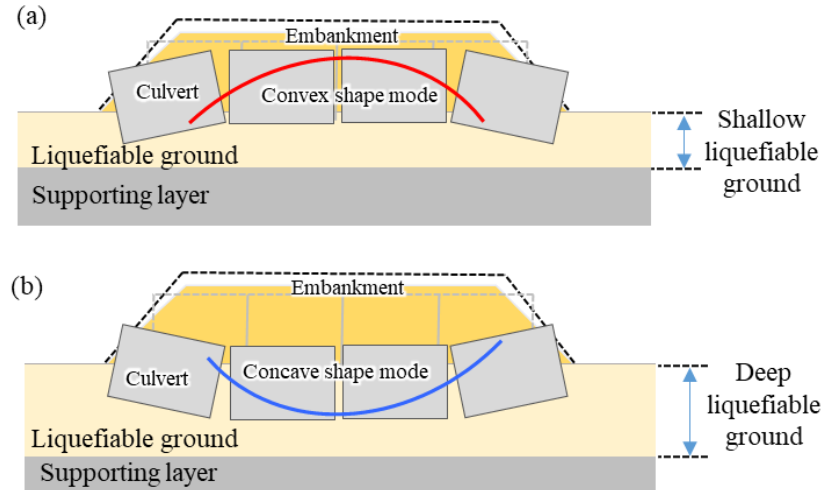


Figure 2.14. Generalized culverts deformation modes for (a) shallow and (b) deep liquefiable grounds (modified after Nanazawa et al., 2022).

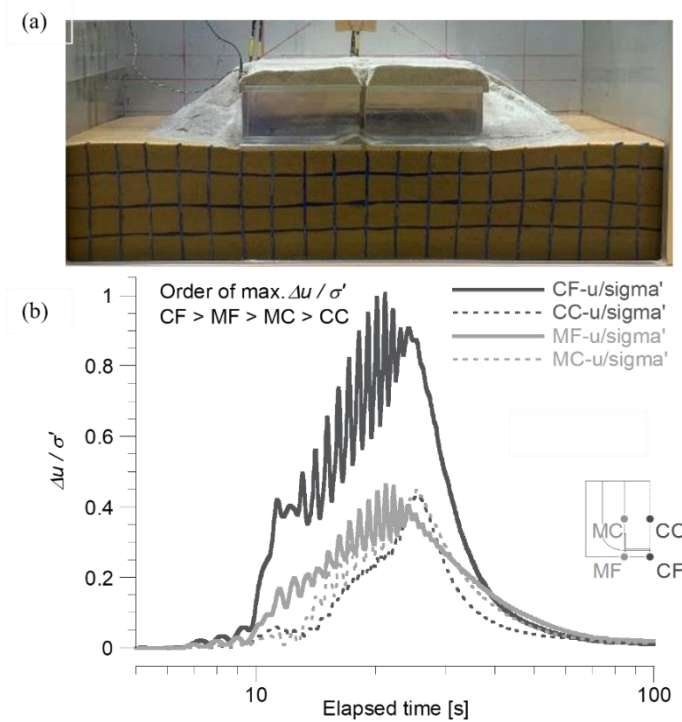


Figure 2.15. Development of sand boil during centrifuge test (a) model deformed state after final shaking and (b) Time histories of excess porewater pressure ratios beneath underpass and embankment during the first and the second shaking (MC and CC are directly under the embankment, MF and CF are directly under the underpass. C means center, M means mouth of the underpass, and F means front of the soil chamber) (Miyazaki et al., 2024).

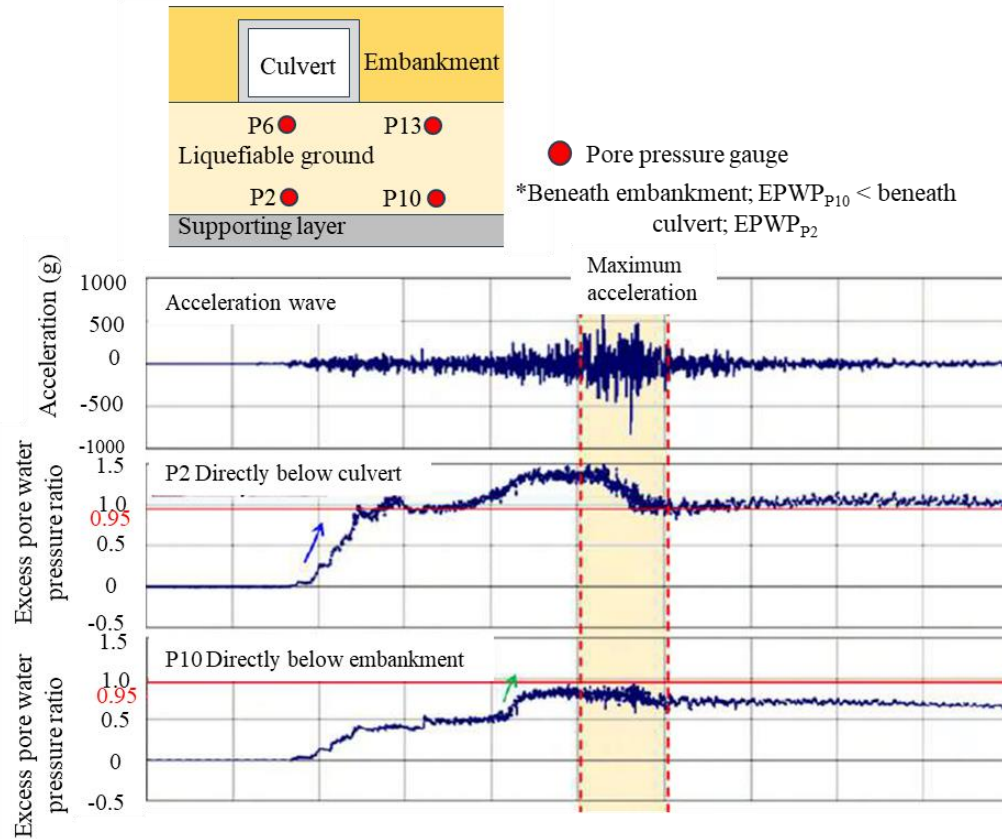


Figure 2.16. Order of excess pore water pressure (EPWP) development beneath the culvert and embankment during centrifuge experiment tests (modified after Nanazawa et al., 2022).

2.5 Experimental model adopted for the current reproduction analytical study and procedure

Nanazawa et al., (2022) conducted centrifuge experimental tests to verify culverts behavior due to liquefaction of foundation soils during earthquakes using large dynamic centrifugal loading test device (Figure 2.17) owned by the Public Works Research Institute; a national research and development agency. The experiment model was geometrically reduced to 1/75 scale of the actual ground model, which was constructed to simulate a support layer, a liquefaction layer, a culvert and an embankment in a soil tank measuring 150 cm wide, 50 cm high, and 50 cm deep. For the testing, the soil tank was attached to a rotating arm, and a constant-speed centrifugal acceleration was applied to the experimental device to vibrate in the centrifugal force field to investigate the behavior of the ground and the model. The culvert model was made of aluminum (A5052), and the dimensions were based on road earthwork - culvert work guidelines and the actual ground model is shown in Figure 2.18. The dimensions were set in consideration of the actual structural dimensions of the cast-in-place box culvert determined because of the design based on the constant load specifications.



Figure 2.17. Large dynamic centrifuge loading equipment (Nanazawa et al., 2022).

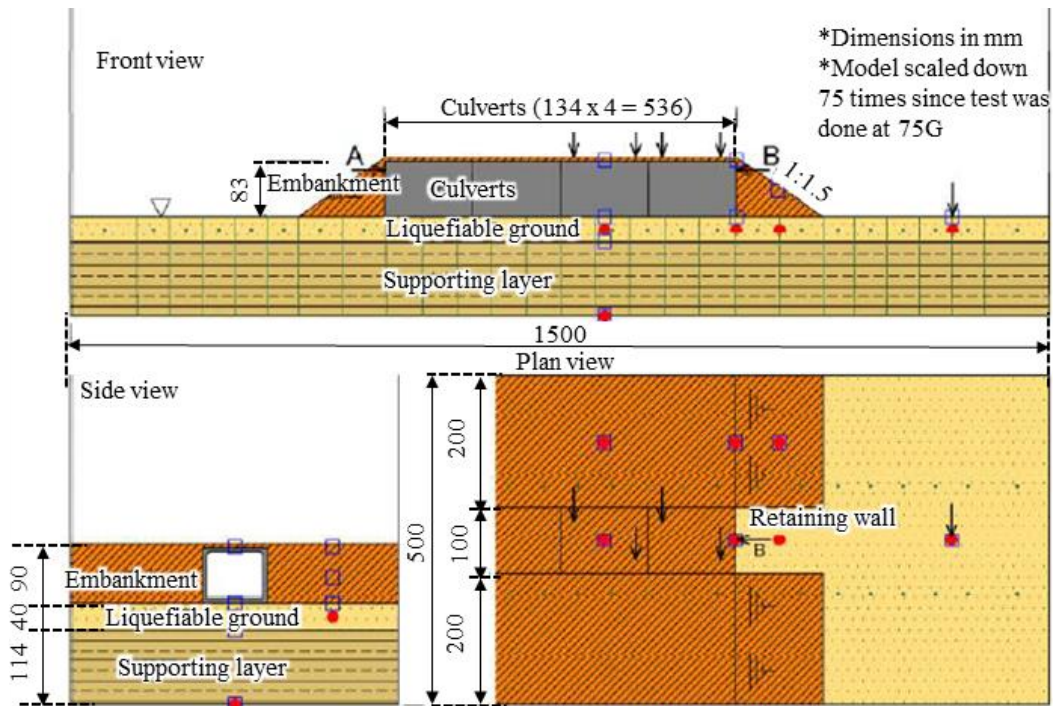


Figure 2.18. Experimental model adopted for the current analytical study (modified after Nanazawa et al., 2022).

For the supporting layer, air-dried Tohoku Silica Sand No. 7 was used for the support layer with production pitch of 20 mm. The liquefied layer was produced by the air-drop method targeting Dr of 60%, in which the air-dried Tohoku Silica Sand No. 7 was naturally dropped using a sand hopper. Saturation of the liquefied layer was performed by a negative pressure method using a vacuum chamber. As the ground pore fluid, we used a Metolose aqueous solution was adjusted to have a viscosity 75 times that of water against the centrifugal acceleration (75G) during the experiment. The embankment was made by compacting wet *Edosaki* sand adjusted to the optimum water content and the targeted degree of compaction (D_c) was 90%. For the measurements, displacement gauges, accelerometers, pore water pressure gauges, and displacement markers were installed on the embankment, the liquefaction layer, and the bottom of the soil layer. For the dynamic analysis simulation, a standard acceleration waveform was used.

2.6 Summary of earthquake damage to RBCs in the past

From the various studies discussed in this chapter and by Hashash et al., (2001); the following general observations can be made on how earthquakes have damaged underground structures in the past, especially RBCs: -

1. Underground structures are found to suffer less damage than surface structures.
2. Reported damage cases decreased with an increase in overburden, whereby deep structures are safer than surface structures.
3. Underground structures constructed in soils can be expected to suffer more damage compared to openings constructed in competent rock.
4. Damage caused by earthquake shaking can be reduced by stabilizing the ground around the tunnel and by improving the contact between the lining and the surrounding ground through grouting.
5. Damage can be related to peak ground acceleration and velocity based on the magnitude and epicentral distance of the earthquake.
6. During earthquakes, the duration of strong-motion shaking is of utmost importance because it may cause fatigue failure and therefore large deformations.
7. During shaking, high frequency motions may explain the local spalling of concrete along planes of weakness.

2.7 Research problem of culverts joint opening during seismic activities and subsequent loss of serviceability of entire road embankment

During the past earthquake occurrences in Japan in the past, as discussed in the literature review, the damage mainly reported was the joint openings between the culvert units along the culvert longitudinal direction, as shown in Figure 2.19. This joint opening results in: -

- Traffic flow interruptions leading to partial or total closure of the road usage until repairs are done.
- Time wastage and resulting economic losses related to the traffic interruptions in transportation of people and goods.
- High expenses incurred in the repair of the damaged sections of the road depending on the magnitude of the damage on the road.

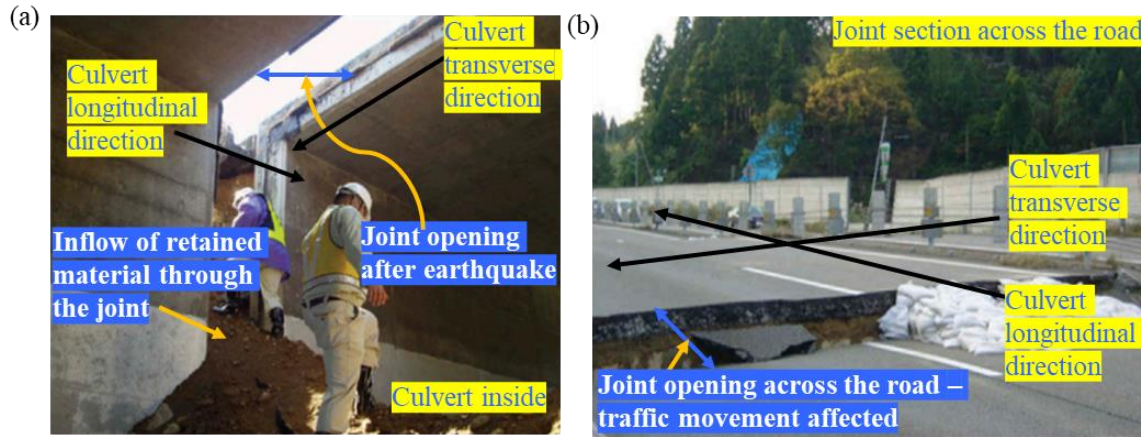


Figure 2.19. Problem visualization after the Mid Niigata Prefecture Earthquake in 2004 (a) Joint opening and inflow of retained soil material into the inside of the culvert and (b) separation of road section affecting traffic flow; loss of serviceability problem (modified after Ishikawa et al., 2019).

From Figure 2.19, a pictorial presentation of the research problem was developed, as shown in Figure 2.20. Before the earthquake occurrence, the underground structure, RBCs, being hollow and lighter as compared to the embankment soil, exerts lower confining stress to the foundation ground than the embankment soil, (exerts higher confining stresses to the foundation ground). During seismic activity, the lighter hollow underpass structure will undergo higher shaking and stress imbalances occur between the culvert-foundation and embankment. This stress interaction behavior and the permeability differences of the underground structure (RBCs), foundation ground and the embankment are suspected to accelerate the joint opening, if the soil beneath undergoes liquefaction.

In Figure 2.21, the joint opening after seismic activity, and the experimental centrifuge results by (Nanazawa et al., 2022) modified by Miyazaki et al., (2024), the joint opening is predicted to be caused by destabilization of stresses due to localized liquefaction of foundation ground beneath the culverts during seismic activities. Hence, the joint opening, the link to localized liquefaction beneath the culverts and the destabilization of the stresses during the earthquake is the main focus for the current study through numerical analysis.

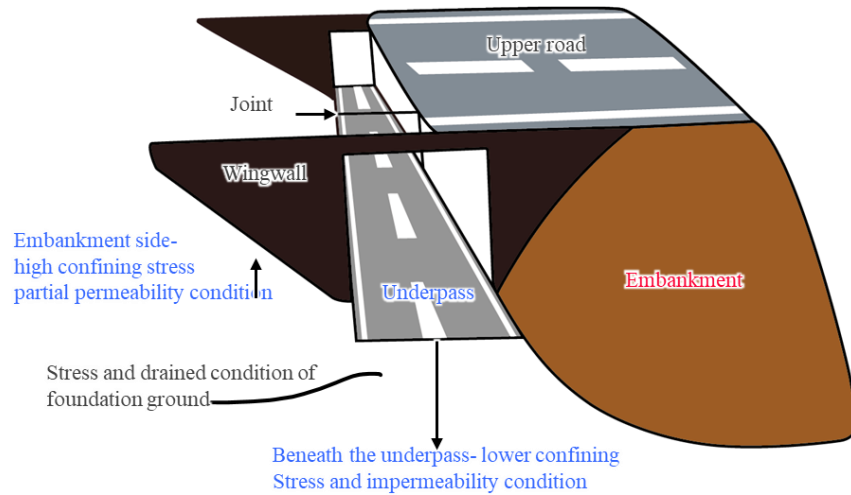


Figure 2.20. Stress and drainage distributions across the underpass, foundation and embankment.

	Underpass section	Underpass cross section	Embankment section
Stress boundary	Semi - shallow foundation	Lower confining stress beneath underpass	Trapezoidal distributed load
Drainage boundary	Impermeable drainage condition	Locally impermeable drainage beneath underpass	Partially permeable drainage condition beneath embankment
Mechanical behavior	Contact- impact due to multi rigid structures	Composite behavior of contact-impact and elasto-plastic deformation	Elasto-plastic deformation of soil
C.P. Confining pressure L.L. Liquefaction layer L.R. Liquefaction range			

Figure 2.21. Soil-multiple structures-soil interactions (Miyazaki et al., 2024; modified after Nanazawa et al., 2022).

In order to achieve the study objectives stipulated in Chapter One in understanding the culvert displacements behavior during earthquakes, the study will be done as per the chapters presented in Figure 2.22: -

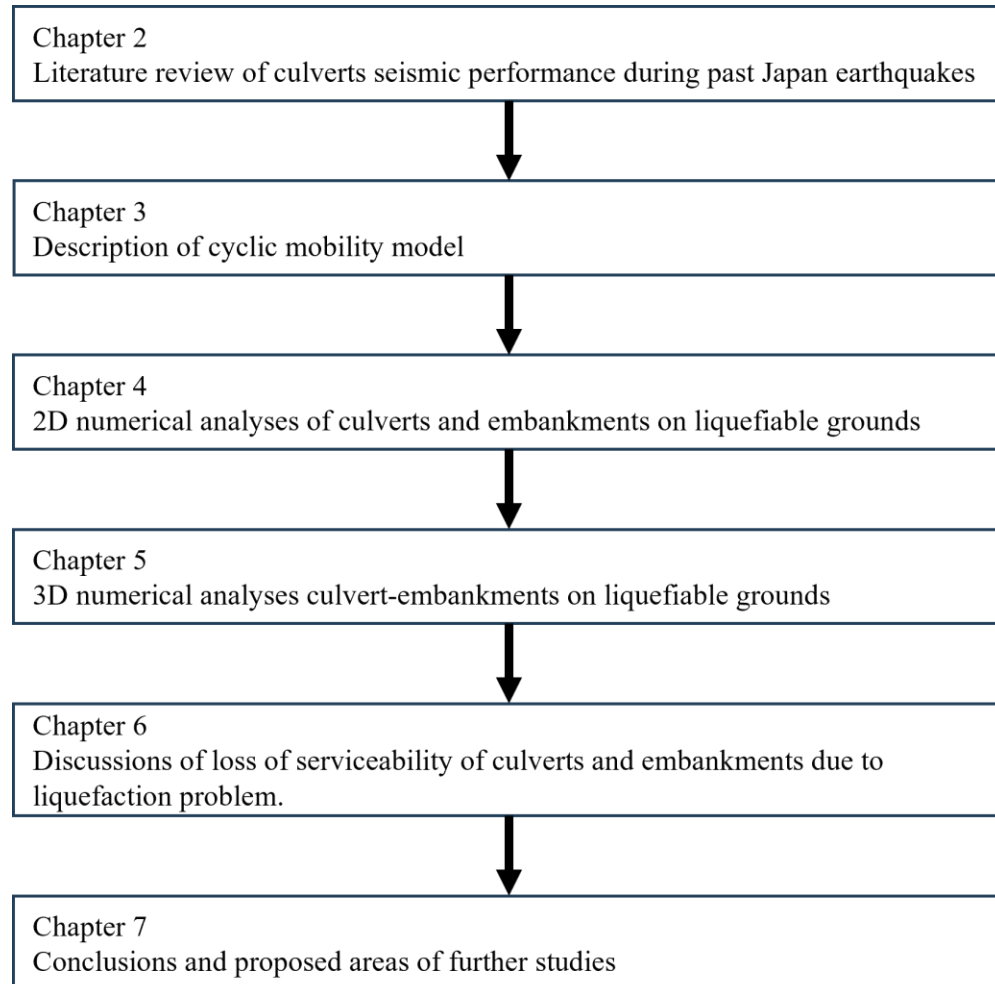


Figure 2.22. Onward thesis structure.

References

- Abuhajar, O., El Naggar, H. and Newson, T. (2015). Experimental and numerical investigations of the effect of buried box culverts on earthquake excitation, *Soil Dynamics and Earthquake Engineering*, vol. 79, pp. 130–148.
- Anderson, D.G., Martin, G.R., Lam, I. and Wang, J.N. (2008). Seismic analysis and design of retaining walls, buried structures, slopes and embankment, *National Cooperative Highway Research Program Report No. 611*, Transportation Research Board, National Research Council, Washington, D.C.

- Debiasi, E., Gajo, A. and Zonta, D. (2013). Seismic response of shallow-buried rectangular structures, *Tunnelling and Underground Space Technology*, vol. 38, pp. 99–113.
- Dowding, C.H. and Rozen, A. (1978). Damage to rock tunnels from earthquake shaking, *Journal of Geotechnical Engineering Division, American Society of Civil Engineers*, vol. 104, No. 2, pp. 175–191.
- Eight-Japan Engineering Consultants Inc (EJECI) (2016). Report on the 2016 Kumamoto Earthquake. Retrieved September 18, 2018, from <http://www.ejec.ej-hds.co.jp/center/pdf/160712.pdf>.
- Hashash, Y.M.A. and Park, D. (2001). Seismic design and analysis of underground structures, *Tunneling and Underground Space Technology*, vol. 16, pp. 247–293. doi:10.1016/S0886-7798(01)00051-7.
- Huo, H., Bobet, A., Fernández, G. and Ramírez, J. (2006). Analytical solution for deep rectangular structures subjected to far-field shear stresses, *Tunneling and Underground Space Technology*, vol. 21, pp. 613–625. <https://doi.org/10.1016/j.tust.2005.12.135>.
- Hushmand, A., Dashti, S., Davis, C., McCartney, J.S. and Hushmand, B. (2016). A centrifuge study of the influence of site response, relative stiffness, and kinematic constraints on the seismic performance of buried reservoir structures, *Soil Dynamics and Earthquake Engineering*, vol. 88, pp. 427–438.
- Ishikawa, K., Yasuda, S., Nakamura, H. and Fujioka, K. (2019). Damage to Expressway Embankment Caused by the Niigata Prefecture Chuetsu Earthquake- Evaluation of applicability of residual deformation analysis, *15th Japan Earthquake Engineering Symposium*.
- Ito, H., Atsuro, S., Masakazu, S. and Toshiaki, N. (2021). Study on behavior of culvert due to liquefaction during earthquake, *Journal of Structural Engineering*, vol. 67A, pp. 161–173. (in Japanese)
- Japan Road Associations (2010). Design and Construction guiding principle for road earth works and culverts. Maruzen Print Co. Ltd (in Japanese).
- Kawashima, K. (2000). Seismic design of underground structures in soft ground: a review in: Kusakabe, Fujita, Miyazaki (eds) *Geotechnical aspects of underground construction in soft ground*. Balkema, Rotterdam. ISBM 90 5809 1 066.
- Miyazaki, Y., Shizuka, E., Mutua, B. M., Toshinori, Y., Satoshi, T., Yu, F., Hirofumi, H. and Hiroaki, H. (2024). Traffic Disruptions in Box-Type Embankment-Underpasses Due to Past Earthquakes in Japan, *Proceedings of the 5th International Conference on Transportation Geotechnics (ICTG)*, vol. 2, pp. 45-53. Lecture Notes in Civil Engineering, vol 403. Springer, Singapore. https://doi.org/10.1007/978-981-97-8217-8_6
- Murakami, S. (2011). Report of the 2011 Great East Japan Earthquake, retrieved September 18, 2018, from www.geo.civil.ibaraki.ac.jp/murakami/GIS/topics/TohokuTaiheiyo2011/IbadaiGeo201104N08-2.pdf.
- Nakamura, S., Yoshida, N. and Iwatate, T. (1996). Damage to Daikai Subway Station During the 1995

- Hyogoken-Nambu Earthquake and Its Investigation, *Japan Society of Civil Engineers, Committee of Earthquake Engineering*, pp. 287–295.
- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki, M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.
- National Institute for Land and Infrastructure Management, NILIM and Ministry of Land and Public Works Research Institute- PWRI (2006). Report on damage to infrastructures by the 2004 Mid Niigata Prefecture Earthquake. In Research report of National Institute for Land and Infrastructure Management, No. 27, January 2006. (in Japanese) Retrieved September 18, 2018, from <http://www.nilim.go.jp/lab/bcg/siryou/rpn/rpn0027.htm>.
- Nishioka, T. and Unjoh, S. (2003). A simplified evaluation method for the seismic performance of underground common utility boxes.
- Penzien, J. (2000). Seismically induced racking of tunnel linings, *Earthquake Engineering Structural Dynamics*, vol. 29, pp. 683–691.
- Pitilakis, K. and Tsinidis, G. (2014). Performance and Seismic Design of Underground Structures. In: Maugeri, M., Soccodato, C. (eds) *Earthquake Geotechnical Engineering Design, Geotechnical, Geological and Earthquake Engineering*, vol 28. Springer, Cham. https://doi.org/10.1007/978-3-319-03182-8_11
- St. John, C.M. and Zahrah, T.F. (1987). Aseismic design of underground structures, *Tunneling Underground Space Technology*, vol. 2, No. 2, pp. 165–197.
- Tokida, K., Oda, K., Nakahara, A., Hayashi, K. and Yorifuji, M. (2007). Performance-based Road management against bump damage induced by earthquake based on driving test, *Proceedings of the JSCE Earthquake Engineering Symposium*, vol. 29, pp. 596-604. (in Japanese)
- Toyota, H. (2018). Report of the 2007 Niigata-ken Chuetsu-oki Earthquake. Retrieved September 18, 2018, from <http://133.44.114.42/HTML/shear/>.
- Tsinidis, G., Heron, C., Pitilakis, K. and Madabhushi, S.P.G. (2013c). Physical modeling for the evaluation of the seismic behavior of square tunnels. In: Ilki A, Fardis MN (eds) *Seismic evaluation and rehabilitation of structures. Geotechnical, geological and earthquake engineering*, vol. 26, pp 389–406. doi:10.1007/978-3-319-00458-7_22.
- Tsinidis, G., Pitilakis, K., Anastasiadis, A., Pitilakis, D., Heron, C., Madabhushi, S.P.G., Stringer, M. and Paolucci, R. (2013d). Investigation of several aspects affecting the seismic behavior of shallow rectangular underground structures in soft soils, *Seismic engineering research infrastructures for European synergies*. <http://www.series.upatras.gr/TUNNELSEIS>.
- Tsinidis, G., Pitilakis, K., Heron, C. and Madabhushi, S.P.G. (2013b). Experimental and numerical

- investigation of the seismic behavior of rectangular tunnels in soft soils, *Computational methods in structural dynamics and earthquake engineering conference (COMPDYN 2013)*, Kos, Greece.
- Tsinidis, G. (2017). Response characteristics of rectangular tunnels in soft soil subjected to transversal ground shaking, *Tunnelling and Underground Space Technology*, vol. 62, pp. 1-22.
- Tsinidis, G., Rovithis, E., Pitilakis, K. and Chazelas, J. L. (2016). Seismic response of box-type tunnels in soft soil: Experimental and numerical investigation, *Tunnelling and Underground Space Technology*, vol. 59, pp. 199–214.
- Wang, J. N. (1993). Seismic design of tunnels: A state-of-the-art approach. Monograph 7. New York: Parsons Brinckerhoff Quade & Douglas.
- Wood, J.H. (2005). Earthquake design of rectangular underground structures. Wellington: *New Zealand Society for Earthquake Engineering*.
- Yamaki, M., Tanimoto, S. and Sasaki, T. (2011). Dynamic centrifuge model tests on dynamic behavior of box culvert in large earthquake, *Proceedings of 56th Symposium of Geotechnical Engineering*, pp. 37-44. (in Japanese)
- Yasuda, S. and Fujioka, K. (2009). Study on the method for the seismic design of expressway embankments, *Earthquake Geotechnical Engineering Satellite Conference XVIIth international Conference on Soil Mechanics & Geotechnical Engineering*, pp. 1–22.
- Yatsumoto, H., Mitsuyoshi, Y., Sawamura, Y. and Kimura, M. (2019). Evaluation of seismic behavior of box culvert buried in the ground through centrifuge model tests and numerical analysis, *Underground Space*, Volume 4, Issue 2, pp. 147-167.
- Youd, T.L. and Beckman, C.J. (1996). Highway culvert performance during past earthquakes. Buffalo, N.Y.: *National Center for Earthquake Engineering Research*.

CHAPTER THREE

FORMULATION OF FINITE ELEMENT ANALYSIS

3.1 Overview

This chapter first describes the constitutive model used and the governing equations. The joint model is also described. Additionally, experimental methods used, the analytical method, and their respective results are presented. The validation of the experimental results with the analytical results is done. Then finally, the summary of the respective soil parameters for use in the current study is presented.

3.2 Yield functions for analytical model

3.2.1 Soil water coupled theory (u - p) equations based on Biot's theory (Liquid phase equations)

When the soils of saturated foundation grounds are subjected to dynamic load, the interaction between solid skeleton and water can be described in u - p equations based on Biot's theory. In this theory, saturated porous media is treated as consisting of solid and liquid phases. The u - p formulation used in this study (Oka et al.,1994) treats displacement of solid phase, u and pore water pressure, p as unknowns. It is used as the field equation for the soil-water coupled problem. If the pore water pressure defined by the center of gravity of the element (Christian and Boehmar, 1968) the number of degrees of freedom to be solved can be reduced, and this reduces greatly the computational cost.

The field equations are spatially discretized using finite difference and finite element methods. The finite difference method is used for spatial discretization of the pore water pressure term in the continuity equation while the finite element method is used for spatial discretization of the balance equation (Akai and Tamura, 1978). Pore water pressure is defined as a constant value at the element center of gravity. Also, stress and strain are defined at the element center of gravity by reduction of integration. This method reduces the degree of freedom of the calculation and avoids shear locking under undrained conditions (Hisada and Noguchi, 1995). The accuracy of these analysis methods has been verified by Oka et al., (1994) and in the following derivations, the following assumptions are made: -

1. Infinitesimal strain is used.
2. Porosity distribution in the soil is sufficiently smooth.
3. Relative acceleration of the fluid phase to that of the solid phase is much smaller than the acceleration of the solid phase.

4. Soil grain particles are incompressible.

Based on the above assumptions and basic equations, both the balance and continuity equations are derived. The detailed derivations and symbol meanings are presented in the Appendix, but in this section, the main equations are presented.

The solid and liquid phase expressions for the law of conservation of momentum are defined in Eqs. (3.1) and (3.2) respectively. Here, $\bar{\rho}^s$ and $\bar{\rho}^f$ are the apparent densities of the solid and liquid phase respectively, σ_{ij}^s and σ_{ij}^f are the corresponding solid phase and liquid phase stress tensors. u_i^s and u_i^f are the displacement vector of the solid phase and liquid phase. b_i is the body force, and R_i is the term that represents the interaction between the solid and liquid phases. R_i is defined in Eq. (3.3).

$$\bar{\rho}^s \ddot{u}_i^s = \frac{\partial \sigma_{ij}^s}{\partial x_j} + \bar{\rho}^s b_i - R_i \quad (3.1)$$

$$\bar{\rho}^f \ddot{u}_i^f = \frac{\partial \sigma_{ij}^f}{\partial x_j} + \bar{\rho}^f b_i + R_i \quad (3.2)$$

$$R_i = -n \frac{\gamma_w}{k} \dot{w}_i \quad (3.3)$$

Summing up Eqs. (3.1) and (3.2), we obtain the momentum conservation law for the mixture, Eq. (3.4): -

$$\bar{\rho}^s \ddot{u}_i^s + \bar{\rho}^f \ddot{u}_i^f = \frac{\partial \sigma_{ij}^s}{\partial x_j} + \frac{\partial \sigma_{ij}^f}{\partial x_j} + \bar{\rho}^s b_i + \bar{\rho}^f b_i \quad (3.4)$$

By substituting Eqs. (A3), (A5), and (A13) into Eq. (3.4) and rearranging, we obtain Eq. (3.5).

$$\rho \ddot{u}_i^s + \rho^f \ddot{w}_i = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \quad (3.5)$$

From assumption 3, if we set $\ddot{w}_i \cong 0$; the **balance equation**; Eq. (3.5) can be expressed as follows: -

$$\rho \ddot{u}_i^s = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \quad (3.6)$$

Where ρ is bulk density of soil; σ_{ij} is the total stress tensor, u_i^s is the displacement vector of the solid phase and b_i is the component of body force vector.

When the compressibility of the solid phase is assumed, and substituting Eq. (A29) into Eq. (A28), the resulting equation becomes: -

$$\frac{\partial \dot{w}_l}{\partial x_i} + \dot{\varepsilon}_{ii}^s + \frac{n}{\rho^f} \dot{\rho}^f = 0 \quad (3.7)$$

In Eq. (3.7), the third term on the left side is a term that indicates the compressibility of the pore water, but this term can further be modified to make it easier to handle. So, by substituting Eq. (A2) into Eq. (A17), which is the law of conservation of mass in the liquid phase, and rearranging it, we obtain: -

$$\dot{\rho}^f = -\rho^f \frac{\partial \dot{u}_i^f}{\partial x_i} \quad (3.8)$$

Expressing the term on the right-hand side of Eq. (A9) in terms of strain, we obtain: -

$$\dot{\rho}^f = -\rho^f \dot{\varepsilon}_{ii}^f \quad (3.9)$$

Substituting the liquid phase constitutive law Eq. (A11), we obtain: -

$$\dot{\rho}^f = -\frac{\rho^f}{K^f} \dot{p} \quad (3.10)$$

Also, considering that the time change of the hydrostatic pressure is 0 from Eq. (A14), using the excess pore water pressure (p_d), Eq. (3.10) becomes: -

$$\dot{\rho}^f = -\frac{\rho^f}{K^f} \dot{p}_d \quad (3.11)$$

If Eq. (3.11) is substituted into Eq. (3.7), the continuity equation becomes: -

$$\frac{\partial \dot{w}_l}{\partial x_i} + \dot{\varepsilon}_{ii}^s - \frac{n}{K^f} \dot{p}_d = 0 \quad (3.12)$$

It is noted that the equation still contains the relative displacement w_l of the liquid phase with respect to the solid phase. To eliminate this relative displacement, the equation is transformed as follows: -

$$\bar{\rho}^f \ddot{u}_i^s + \frac{\bar{\rho}^f}{n} \ddot{w}_i = \frac{\partial \sigma_{ij}^f}{\partial x_j} \bar{\rho}^f b_i + R_i \quad (3.13)$$

Based on assumption 3, the following equation is considered: -

$$\ddot{u}_i^s \gg \ddot{w}_i \quad (3.14)$$

Considering Eq. (3.14) in Eq. (3.13), it reduces to: -

$$\bar{\rho}^f \ddot{u}_i^s = \frac{\partial \sigma_{ij}^f}{\partial x_j} \bar{\rho}^f b_i + R_i \quad (3.15)$$

Also, substituting Eqs. (A2), (A7) and (A20) into Eq. (3.15), it results to: -

$$n \rho^f \ddot{u}_i^s = \frac{\partial n p}{\partial x_i} + n \rho^f b_i - n \frac{\gamma_w}{k} \dot{w}_i \quad (3.16)$$

If it is assumed that the porosity is smooth in space based on assumption 2, then: -

$$\frac{\partial n}{\partial x_i} = 0 \quad (3.17)$$

If Eq. (3.17) is substituted into Eq. (3.16), it results in: -

$$\rho^f \ddot{u}_i^s = \frac{\partial p}{\partial x_i} + \rho^f b_i - \frac{\gamma_w}{k} \dot{w}_i \quad (3.18)$$

If the second term on the right-hand side in Eq. (3.18) is the body force due to gravity, Eq. (3.19) holds if we use the hydrostatic pressure p_s and ignore the dynamic terms.

$$\frac{\partial p_s}{\partial x_i} + \rho^f b_i = 0 \quad (3.19)$$

Substituting Eq. (3.19) into Eq. (3.18) and considering Eq. (3.14), we obtain: -

$$\rho^f \ddot{u}_i^s = \frac{\partial p_s}{\partial x_i} - \frac{\gamma_w}{k} \dot{w}_i \quad (3.20)$$

Solving for w_i results into the following equation:

$$\dot{w}_i = \frac{k}{\gamma_w} (\rho^f \ddot{u}_i^s - \frac{\partial p_d}{\partial x_i}) \quad (3.21)$$

w_i can be expressed in terms of the solid phase displacement and excess pore water pressure. Substituting Eq. (3.21) into Eq. (3.12), it results in Eq. (3.22). However, here we assume that the permeability coefficient k is sufficiently smooth in space.

$$\rho^f \frac{\partial \ddot{u}_i^s}{\partial S_i} - \frac{\partial^2 p_d}{\partial x_i^2} - \frac{\gamma_w}{k} \left(\dot{\varepsilon}_{ii}^s - \frac{n}{K_f} \dot{p}_d \right) = 0 \quad (3.22)$$

In addition, taking Eq. (A8) into consideration, and factoring it in Eq. (3.22), the final **continuity equation** becomes: -

$$\rho^f \dot{\varepsilon}_{ii}^s - \frac{\partial^2 p_d}{\partial x_i^2} - \frac{\gamma_w}{k} \left(\dot{\varepsilon}_{ii}^s - \frac{n}{K^f} \dot{p}_d \right) = 0 \quad (3.23)$$

Where ρ^f is fluid density; ε_{ii}^s is the volumetric strain of solid skeleton; n is the soil porosity; k is the permeability coefficient of soil; K^f is the bulk modulus of fluid; γ_w is the unit volume weight of pore water; p_d is the excess pore water pressure and b_i is the component of body force vector.

3.2.2 Solid phase equations

For the foundation ground soils, a rotating hardening elastoplastic model, Cyclic Mobility Model, developed by Zhang et al., (2007) was used. This model systematically combines the evolution equations for the structure of soil as proposed by Asaoka et al., (2002) for the development of stress induced anisotropy and the changes in the over-consolidation of soils. The advantage of the proposed model is that it can express the behavior of soils under cyclic loading with different drained conditions and by uniformly considering their densities, over-consolidation ratio, naturally accumulated structure of the soil and stress-induced anisotropy of the soil.

According to Asaoka et al., (2002), the similarity ratio of the superloading surface to normal yield surface R^* and the similarity ratio of the superloading surface to subloading surface R are the same (Figure 3.1), as shown in Eqs. (3.24) to (3.26) where (p, q) , $(\tilde{p}, \tilde{q})$, and $(\bar{p}, \bar{q})$ represent the present stress state, the corresponding normally consolidated state, and the structured stress state at the p-q plane, respectively.

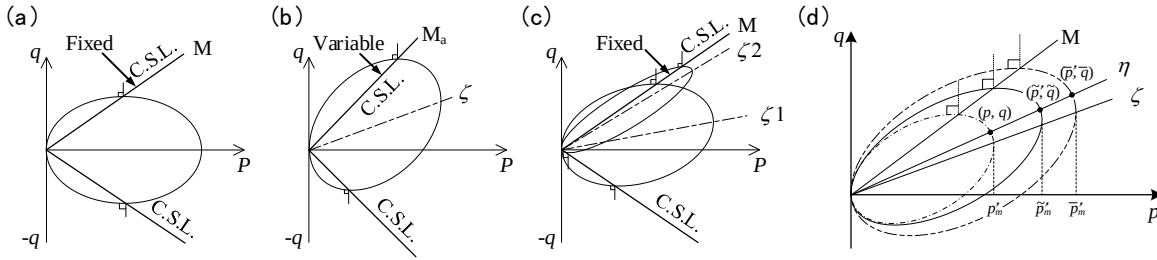


Figure 3.1. Development of yielding surfaces in cyclic mobility model (Zhang et al., 2007) with changes in anisotropy ζ in (a) modified Cam-clay model (Schofield and Wroth, 1968), (b) SYS Cam-clay model (Asaoka et al., 2002), (c) cyclic mobility model, and (d) subloading, normal, and superloading yield surfaces in p-q plane adopted in cyclic mobility model (Miyazaki, 2019).

$$R^* = \frac{\tilde{p}}{\bar{p}} = \frac{\tilde{q}}{\bar{q}}, \quad 0 < R^* \leq 1 \quad (3.24)$$

$$R = \frac{p}{\bar{p}} = \frac{q}{\bar{q}}, \quad 0 < R \leq 1, \quad R = \frac{1}{OCR} \quad (3.25)$$

$$\frac{\bar{q}}{\bar{p}} = \frac{\tilde{q}}{\tilde{p}} = \frac{q}{p} \quad (3.26)$$

The critical state line (C.S.L.) is the border line distinguishing dilation and compression defined by M_a in the relationship ($M_a^2 = M^2 + \zeta^2$) and ζ is the magnitude of the stress induced anisotropy, hence the larger the stress induced anisotropy, ζ , the larger the ellipse formed, and Eqs. (3.27) to (3.30) shows the necessary relationships whereby S_{ij} is the deviatoric stress tensor, β_{ij} is the anisotropic stress tensor and σ_{ij} is the component of Cauchy effective stress tensor assumed to be positive in compression. The final consistency equation can be presented in the form of Eq. (3.31). The detailed derivation of the equation is shown in Appendix Eq. (A47) to (Eq. (A56)).

$$f = \ln \frac{\tilde{p}}{p} + \ln \frac{M^2 - \zeta^2 + \eta^{*2}}{M^2 - \zeta^2} - \frac{\varepsilon_v^p}{C_p} = 0 \quad (3.27)$$

$$\eta^* = \sqrt{\frac{3}{2} \hat{\eta}_{ij} \hat{\eta}_{ij}}, \quad \hat{\eta}_{ij} = \eta_{ij} - \beta_{ij}, \quad \eta_{ij} = \frac{S_{ij}}{p}, \quad S_{ij} = \sigma_{ij} - p \cdot \delta_{ij}, \quad p = \frac{\sigma_{ij}}{3} \quad (3.28)$$

$$\eta = \sqrt{\frac{3}{2} \eta_{ij} \eta_{ij}}, \quad \varsigma = \sqrt{\frac{3}{2} \beta_{ij} \beta_{ij}} \quad (3.29)$$

$$C_p = \frac{\lambda - \kappa}{1 + e_0} \quad (3.30)$$

Based on Eq. (3.27), after the derivatives, the consistency equation can be expressed as,

$$df = 0 \Rightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \beta_{ij}} d\beta_{ij} + \frac{1}{R^*} dR^* - \frac{1}{R} dR - \frac{1}{C_p} \varepsilon_v^p = 0 \quad (3.31)$$

The Cyclic Mobility Model requires eight parameters, out of which, five (R_{cs} , λ , κ , N , ν_e) can be determined by triaxial compression tests. The remaining three parameters (a , m , b_r) are obtained through trial and error by fitting the results obtained from repeated triaxial test results. The parameters' respective meanings are as shown below: -

R_{cs} : Principal stress ratio at failure under triaxial compression condition given in Eq. (3.32)

$$R_{CS} = \left(\frac{\sigma_1}{\sigma_3} \right)_{CS(comp.)} \quad (3.32)$$

λ : Compression index - The parameter is determined by isotropic consolidation and an unloading test; the gradient is $e-\ln p'$ in the loading process of the normal consolidation.

κ : Swelling index - The parameter is determined by isotropic consolidation and an unloading test; the gradient is $e-\ln p'$ in the loading process of the normal consolidation.

N : Void ratio at air pressure ($p' = 98$ kPa, $q = 0$ kPa).

ν_e : Poisson's ratio.

a : Degradation parameter of structure.

m : Degradation parameter of overconsolidation state.

b_r : Evolution parameter of anisotropy.

3.3 Multi linear spring element modelling for RBCs joints, parameters and related joint opening equations

In the development of the joint model, culvert element is explained first. The experimental culvert model was made of aluminum (A5052), and the parameters are shown in Table 3.1. Since the culvert is a hollow structure, the effect of the varied volume was considered in the two-dimensional (2D) analysis as shown in Figure 3.2. The top and bottom continuous culvert plates in y direction were assigned the whole unit weight (γ) is 2.71 g/cm^3 (Table 3.1); ($\alpha \times \gamma$). For the thin-walled middle section (discontinuous in y direction) between the top and bottom, a unit weight of 0.361 g/cm^3 ; ($\beta \times \gamma$) was assigned. This proportion was based on the section volume to the whole volume of the entire cross section in 3D extension. Note that the haunch was not considered to simplify the 3-dimensional density effect. These proportioned unit weights were used in the spring element coefficients computations.

Table 3.1: RBC model parameters

	Units	Value
Young's modulus E	kPa	7.00×10^7
Poisson's ratio ν	-	0.334
Unit weight γ	g/cm^3	2.71, 0.361
Damping ratio h	-	0.02

For the contact-impact problem by a grid scheme and a particle method, such as the finite element method and the moving particle simulation, the solutions are based on Lagrange's method of undetermined multipliers (Hughes et al., 1976) or the penalty method (e.g., Suzuki and Toi, 1987). The penalty method is relatively simple to implement in the simulation since it involves inserting a spring into each of the contact points. The springs interaction model is shown in (Figure 3.3 (a)). For this study, multilinear spring elements were inserted into each contact node (in x, y and rotational direction for the 2D analysis; and x, y and z in 3D analysis) between the culvert elements (Figure 3.3 (b), in 3D k_θ replaced by k_z) along with the original penalty method. Their behavior was used to predict the culvert displacements and accelerations, and their coefficients (Table 3.2) were based on the force transfer between the culverts.

In this analysis, 1D spring elements were employed to model the contact and separation behavior between RBCs. For the contact direction, a high stiffness was assigned based on the rigidity of culverts, whereby the section proportioned density, and the spring influence area were considered (Figure 3.2).

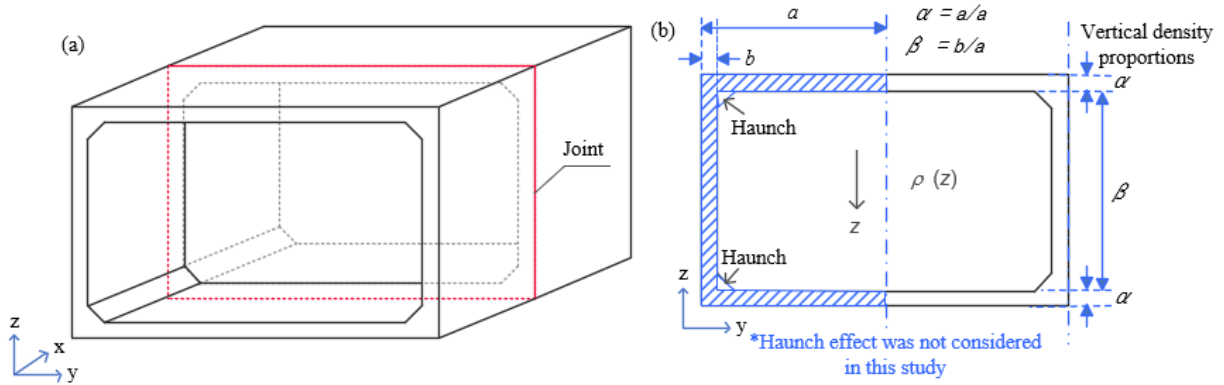


Figure 3.2. RBCs with (a) two units connected at the joint section and (b) cross section at joint section showing density distribution (haunch was not considered).

For the tangential direction, stiffness was reduced using the friction coefficient of aluminum alloys as a reference. Additionally, the non-contact threshold was set to 1 mm, based on the measurement accuracy of joint opening widths reported in the damage investigation by Nakashima et al., (2021). The spring coefficients are presented in Table 3.2.

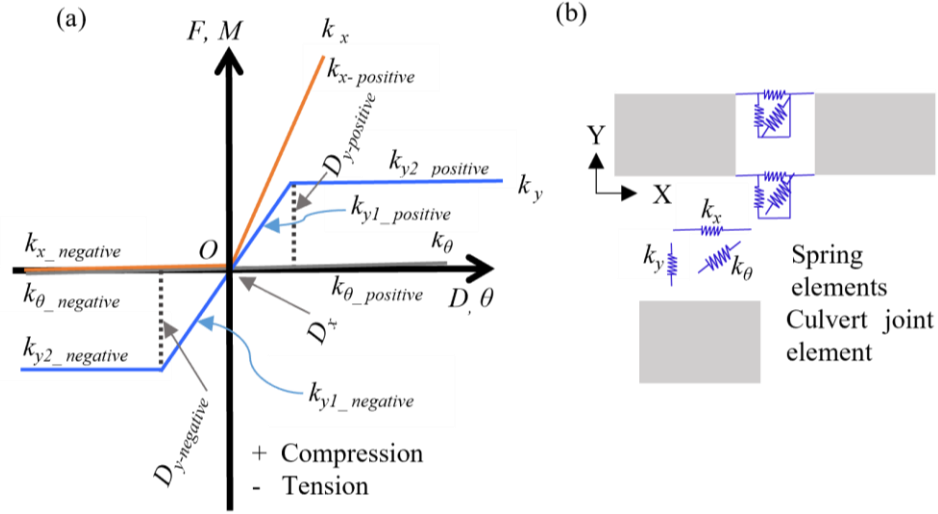


Figure 3.3. 2D spring model with (a) coefficients and (b) an example of inter - culvert elements-springs

Table 3.2: Spring coefficients

Spring coefficients	Units	Value
$k_{x- \text{ positive}}$	kN/m	2.6×10^8
$k_{x- \text{ negative}}$	kN/m	1.0×10^{-9}
$k_{y1- \text{ positive}}$	kN/m	7.9×10^7
$k_{y2- \text{ positive}}$	kN/m	1.0×10^{-7}
$k_{y1- \text{ negative}}$	kN/m	7.9×10^7
$k_{y2- \text{ negative}}$	kN/m	1.0×10^{-7}
$k_{\theta- \text{ positive}}$	kN*m/rad	1.0×10^{-9}
$k_{\theta- \text{ negative}}$	kN*m/rad	1.0×10^{-9}
$D_{y- \text{ positive}}$	m	0.001
$D_{y- \text{ negative}}$	m	0.001
D_x	m	1.0×10^{-9}

In the analytical results, various components to express the rigid box culverts joint openings are explained.

- (a) Culvert bump (B_c)- Once the settlement of the foundation ground occurs, depending on the differences between the various positions beneath the culvert, the culvert will rotate and the central joint section will be left protruding upwards, hence bump is created, Figure 3.4. The resulting effect of culvert inclination towards their mouths results in joint opening.
- (b) Joint opening index (I_o)- is the rate of joint opening per unit height of the culvert (Figure 3.4). The culvert bump and joint opening index is based on two culverts next to each other in any model.

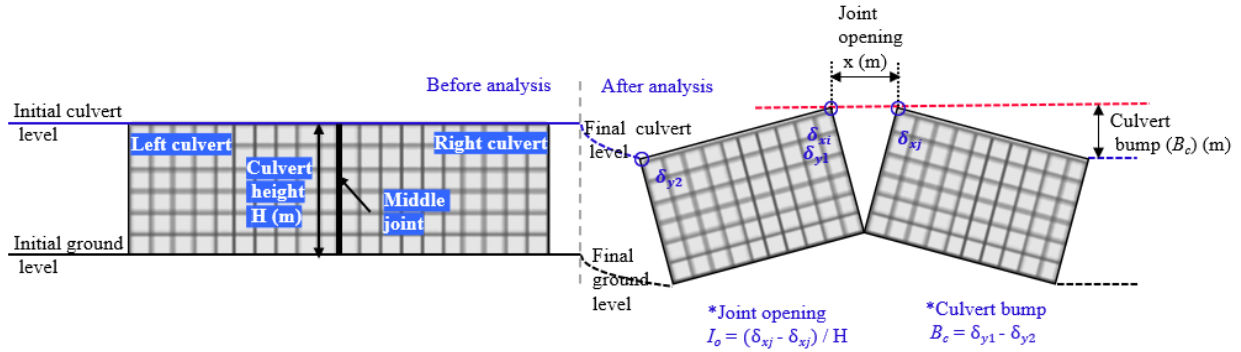


Figure 3.4. Culvert bump and joint opening phenomena.

- (c) Net settlement (S) - is the difference in settlement of the mouth and the joint of the culvert, which causes the culvert to rotate (Figure 3.5). It should be considered with respect to y-direction for 2D and z-direction for 3D model.
- (d) Settlement gradient (S_{gradient}) - is the net settlement per unit length of the culvert, which is referred to as the culvert rotation angle in the direction of consideration (Figure 3.5). Effective stress decreasing ratio, $ESDR$ is the liquefaction index used in this study.
- (e) Effective stress decreasing ratio ($ESDR$) - is the liquefaction index proposed by Oka et al. (2015) based on the rate of the rate of reduction of mean effective stress from the initial mean effective stress (Eq. 3.37).
- (f) Liquefaction gradient ($ESDR_{\text{gradient}}$) - is the difference between the maximum $ESDR$ at the joint section and the maximum $ESDR$ at the mouth of the culvert, divided by the length of the culvert under consideration (Figure 3.5). This consideration can be extended to any number of culvert units depending on the section of consideration of the researcher.

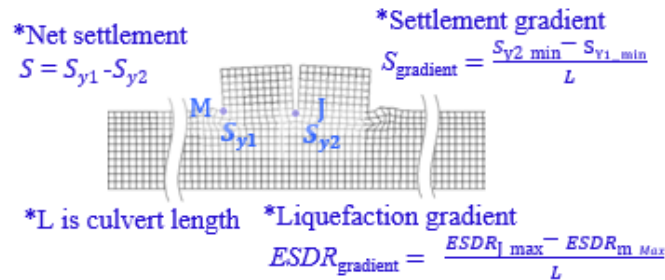


Figure 3.5. Settlement gradient and $ESDR$ gradient

The equations summary is presented below: -

$$\text{Culvert bump, } B_c \quad B_c = \delta_{y1} - \delta_{y2} \quad (3.33)$$

$$\text{Joint opening index, } I_0 \quad I_0 = \frac{\delta_{xj} - \delta_{xi}}{H} \quad (3.34)$$

$$\text{Net settlement, } S \quad S = S_{y1} - S_{y2} \quad (3.35)$$

$$\text{Settlement gradient, } S_{\text{gradient}} \quad S_{\text{gradient}} = \frac{S_{y2_min} - S_{y1_min}}{L} \quad (3.36)$$

$$\text{Effective stress decreasing ratio, } ESDR \quad ESDR = 1 - \frac{\sigma'_m}{\sigma'_{m0}} \quad (3.37)$$

$$\text{Liquefaction gradient, } ESDR_{\text{gradient}} \quad ESDR_{\text{gradient}} = \frac{ESDR_{J_max} - ESDR_{M_max}}{L} \quad (3.38)$$

3.4 Joint model simulation

The joint element simulation was conducted using 2 units of culverts subjected to a simple sine wave of 1 hertz. Figure 3.6 shows the model for the joint simulation. The bottom edges of the culverts are fixed in x, y and free in Θ ; to maintain the model in position but still able to rotate, while all the edges are free in all directions (x, y and Θ). To simulate the joint opening and to study the difference when the joint element is inserted or not inserted, one case was conducted without inserting the spring element while the second case is conducted by inserting the spring element at the middle joint as shown in Figure 3.6 (b). The input wave is shown in Figure 3.7. The results are shown in Figure 3.8.

The joint opening was influenced by the input wave behavior. Hence, for the culvert with joint model, the trend of joint opening reflected the fluctuations of the input wave. Because of inertia forces of the culvert at rest during start of simulation, the input wave and the joint opening were in opposite directions. For the culvert without joint model, the central joint opened continuously until the shaking of the wave ended. This did not reflect the influencing factor, that is the fluctuations of the input wave.

The advantages of using the joint model are: -

- (a) The joint opening will reflect the factor influencing the two culverts to move, since it is elastically modelled. Hence for the above case, the trend of the input wave and the joint opening were in harmony with the case with joint model but not reflected in the case without joint model.

- (b) Joint opening was modeled with maximum (1 m) and minimum (0.001 m) values; hence the joint cannot open infinitely or close completely to cause collision between the culverts but opening will be determined by the influencing factor. This was reflected well in the case with the joint model but the model without the joint model, opening stopped when shaking ended.
- (c) The joint model ensures safe force transfer between the culverts and at the same time controlling collision of culvert elements. For the culverts without joint model, collision of elements is bound to occur and at the same time the culverts are acting independently.

Hence, based on the joint model performance in the simulation, the spring coefficients in Table 3.2 were adopted for the current numerical analysis.

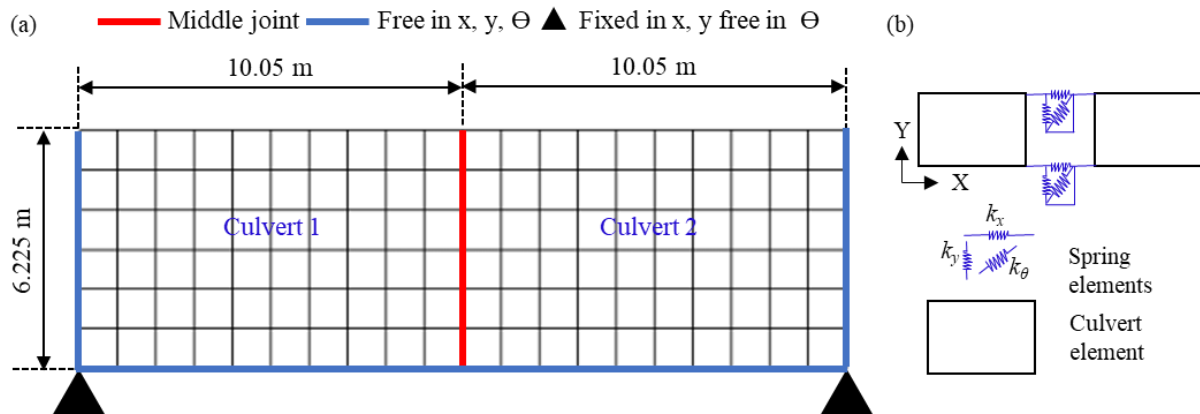


Figure 3.6. Joint simulation model (a) culverts with boundary conditions (b) culvert element to element connection using spring element.

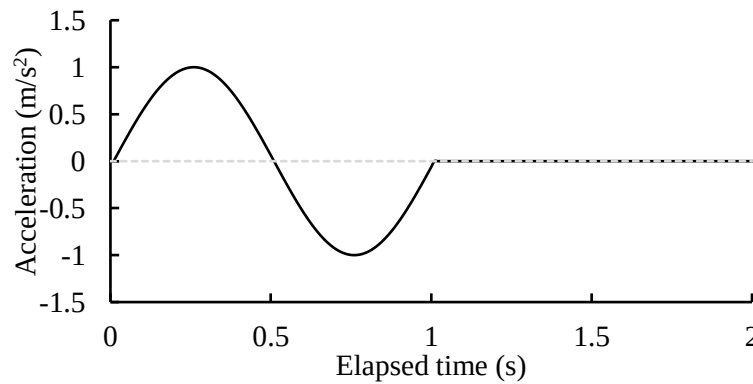


Figure 3.7. Input wave: sine wave with 1 Hz, 1 cycle and magnitude of 1 m/s²

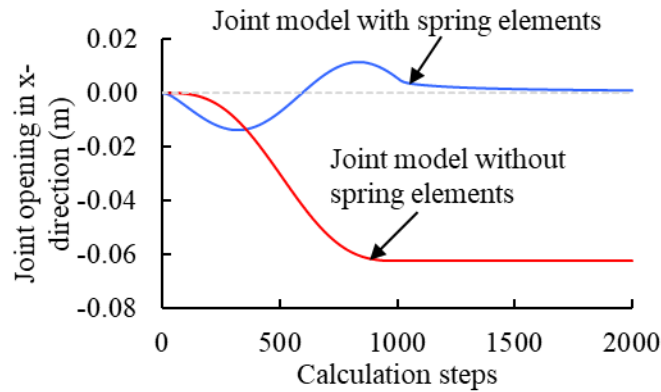


Figure 3.8. Culvert middle joint opening simulation

3.5 Experimental results for undrained cyclic triaxial test results for foundation grounds soils.

In the experimental centrifuge tests, the supporting layer and the liquefiable ground were made using air-dried Tohoku Silica Sand No. 7 while the embankments were made using *Edosaki* sand. To simulate liquefaction, the liquefied layer was produced by the air-drop method targeting Dr of 60%, in which the air-dried Tohoku Silica Sand No. 7 was naturally dropped using a sand hopper. Nanazawa et al., (2022) conducted an undrained cyclic triaxial test to generate the liquefaction strength properties of the foundation soils, (whose parameters are shown in Table 3.3).

In his report, general Tohoku Silica Sand No. 7 liquefaction properties were conducted without separating the supporting layer and liquefiable ground. The report does not give detailed information on the experimental conditions, setups and results. *Edosaki* sand experimental and analytical results comparisons are discussed in Chapter 3.7.2 due to absence of comparable experimental results of the embankment soil. The cyclic stress ratio against the number of cycles for various axial strains and EPWP were developed as shown in Figure 3.9.

Figure 3.9 represents the Cyclic Stress Ratio (CSR) versus the Cyclic Number (N_c) for liquefaction resistance of Tohoku Silica Sand No. 7 under different conditions, categorized by "Exp" (Experiment). The results are plotted from (using trial and error method), from the reported results in Nanazawa et al., (2022) report. From the results, the markers for different axial strains; DA (1%, 2%, 5%, and 10%) and excess pore water pressure (EPWP) show a systematic progression in resistance.

Table 3.3: Experimental foundation ground soil parameters (Nanazawa et al., 2022)

	Tohoku Silica Sand No. 7 (Liquefiable ground and supporting layer)	Edosaki sand (Embankment)
Density ρ_s (g/cm ³)	2.638	2.71
Sand content (%)	96.3	89.2
Silt content (%)	3.7	6.3
Clay content (%)	0	4.5
Uniformity factor U_c	1.43	4.67
Average particle size D50 (mm)	0.161	0.2641
Maximum dry density ρ_{d-max} (g/cm ³)	1.614	-
Minimum dry density ρ_{d-min} (g/cm ³)	1.272	-
Maximum compacted density (g/cm ³)	-	1.674
Optimum water content (%)	-	16.9

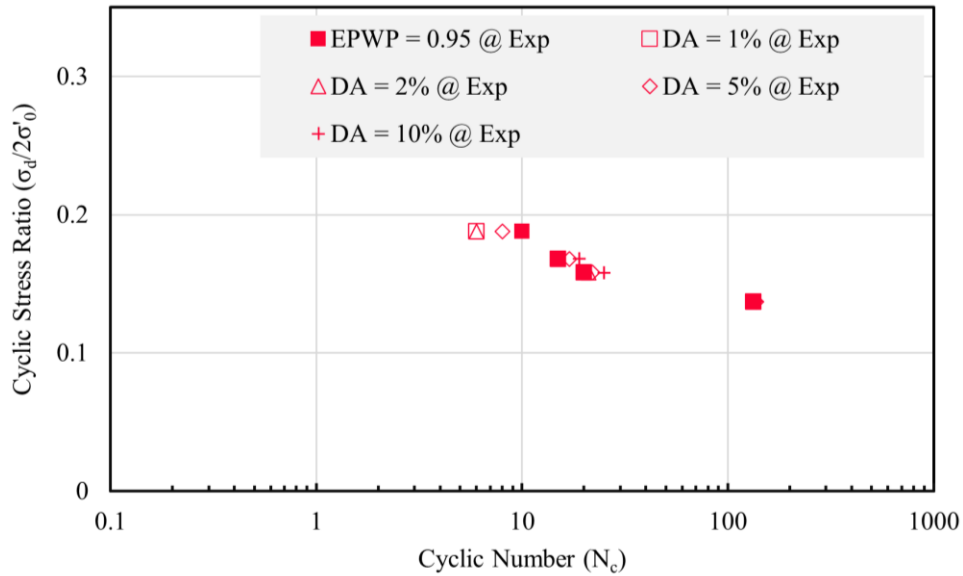


Figure 3.9. Liquefaction strength characteristics for liquefiable layer Tohoku Silica sand No. 7 used in centrifuge experimental test (modified after Nanazawa et al., 2022).

3.6 Analytical results for undrained cyclic triaxial test for foundation ground soils.

For the analytical cases presented in this study, Toyoura sand with D_r of 55% was used as the liquefiable ground while for the supporting layer, Toyoura sand with D_r of 95% was used. This soil has been widely researched and most of its information is available. Also, the grain size distribution of Tohoku Silica Sand No. 7 and Toyoura sand are relatively consistent (Hirado and Nitta, 2010). The stress paths for the foundation soils are shown in Figure 3.10. The foundation soil parameters (shown in Table 3.4) were obtained from the undrained cyclic triaxial test conducted by Mommae et al., (2019). Similarly, the embankment soil parameters were obtained by Miyazaki (2019).

Table 3.4: Analytical foundation ground soils parameters (Mommae et al., 2019; Miyazaki, 2019)

	Toyoura sand (Liquefiable)	Toyoura sand (Non- liquefiable)	<i>Edosaki</i> sand (Embankment)
Fully saturated	Yes	Yes	No
Internal friction angle ϕ	37.5	42.1	38.25
Earth pressure coefficient at rest k_0	0.391	0.330	0.381
Wet density ρ_s (g/cm ³)	1.929	2.012	1.761
Relative density D_r (%)	55	95	-
Degree of compaction D_c (%)	-	-	90
Moisture content (%)	29.2	23.7	16.9
Maximum dry density ρ_d (g/cm ³)	-	-	1.674
Particle density G_s (g/cm ³)	2.646	2.646	2.71
Initial void ratio e_0	0.772	0.626	0.799
Poisson ratio ν	0.3	0.3	0.276

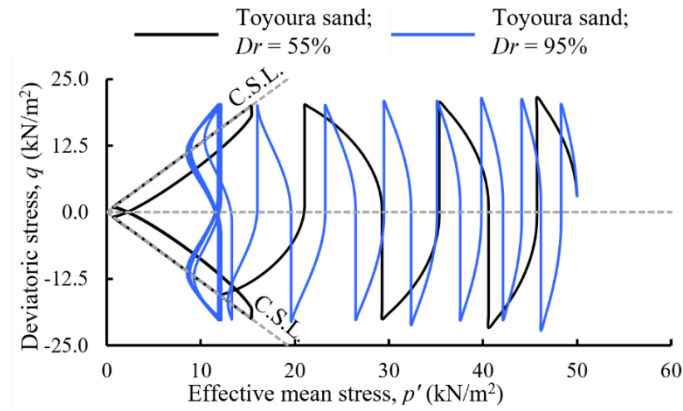


Figure 3.10. Stress paths for foundation soils ($p'_0 = 50$ kPa).

From Figure 3.10, the supporting layer ($D_r = 95\%$) has longer stress paths compared to the liquefiable ground ($D_r = 55\%$) and different behavior towards end of analysis. The shear stress-strain relationships for the two foundation grounds indicate that, for the liquefiable ground, Toyoura sand with D_r of 55% (Figure 3.11) is expected to have larger shear strains as compared to the supporting layer, Toyoura sand with D_r of 95% (Figure 3.12).

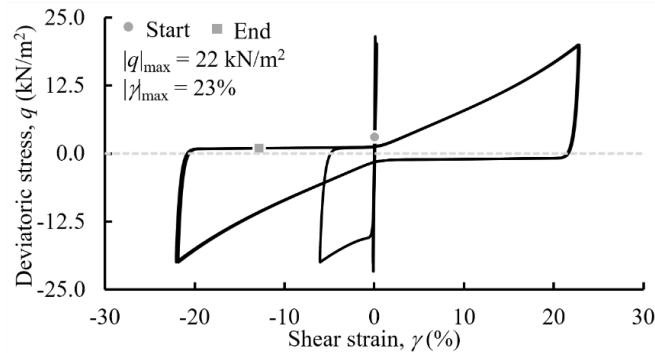


Figure 3.11. Stress-strain relationship for Toyoura sand ($D_r = 55\%$).

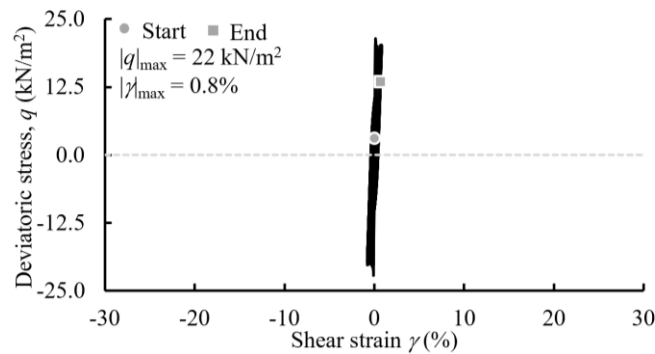


Figure 3.12. Stress-strain relationship for Toyoura sand ($D_r = 95\%$).

3.7 Comparisons between experimental and analytical test results

3.7.1 Foundation ground soils comparisons

Figure 3.13 represents the cyclic stress ratio (CSR) versus the cyclic number (N_c) for liquefaction resistance under different conditions, categorized as analytical "Sim" (Simulation) and "Exp" (Experiment). DA means different axial strains used for simulation.

The key observations and characteristics of both simulation and experimental results are as follows:

-

Simulation results: The CSR generally decreases as the cyclic number increases, following a logarithmic trend. This is consistent with the typical behavior of soils under cyclic loading. Also, the markers for different axial strains (1%, 2%, 5%, and 10%) show a systematic progression in resistance.

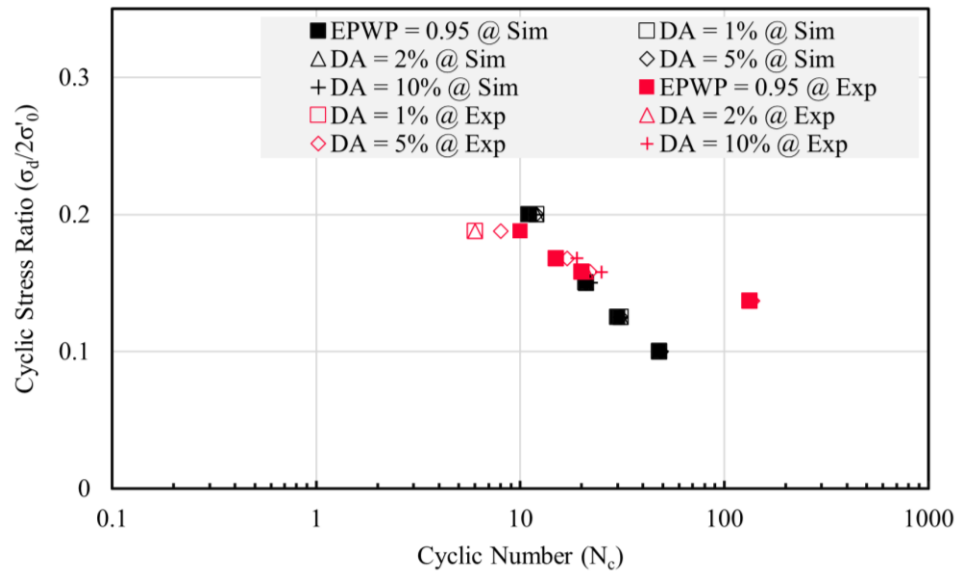


Figure 3.13. Liquefaction strength characteristics for experimental and analytical foundation soils.

In addition, the simulation results exhibit a relatively smooth and consistent decline in CSR. The CSR values for specific strain levels (e.g., DA 1%, DA 2%) are slightly higher compared to experimental results, indicating a more optimistic resistance prediction in simulations. The simulation results provide a clear distinction between different strain levels, which might reflect idealized assumptions in numerical modeling.

The key significance of the simulation results is that, the simulated data likely assumes uniform material properties and controlled boundary conditions, leading to a less scattered response. This data can serve as a benchmark for validating experimental results.

Experiment results: The results also show a decrease in CSR with increasing N_c but exhibit more variability compared to simulations. The curve for each strain level overlaps more significantly, indicating a greater degree of scatter in the data. The experimental results for higher axial strains (e.g., DA 10%) tend to deviate more from the simulated results, particularly at lower cyclic numbers, suggesting non-ideal or localized failure mechanisms in the real test. The variability in CSR reflects the influence of natural soil heterogeneity, equipment limitations, and potential sample disturbances.

The key significance of the experimental is that the data provides a realistic measure of liquefaction resistance but may underestimate or overestimate resistance due to inherent uncertainties in testing conditions. The results are critical for validating and calibrating simulation models to bridge the gap between idealized and real-world conditions.

Consistency and deviations of simulation and experimental results

Consistency: In both simulation and experimental data show a general agreement in trends, confirming the reliability of the numerical model in predicting liquefaction resistance.

Deviations: Simulation tends to overestimate CSR, especially at higher strain levels, due to the absence of real-world imperfections in the numerical model. Experimental results show greater scatter, reflecting the complexity of real soil behavior under cyclic loading. Hence, both sets of results (experimental and simulation) agree within a particular range, but generally acceptable, based on the general trend.

Application: The simulation results are useful for predictive modeling and parametric studies while the experimental results are essential for validating simulations and ensuring safety margins in practical design.

In conclusion, the graph highlights the complementary roles of simulation and experimental data in understanding liquefaction resistance. Simulations provide idealized predictions with clear trends, while experimental results reflect real-world uncertainties and complexities. A combined approach leveraging both datasets can lead to more robust predictions and safer geotechnical designs.

3.7.2 Embankment soils comparisons

For the embankment soil, *Edosaki* sand, from Table 3.3 and 3.4 results, Miyazaki (2019) conducted comparative analysis, and the experimental and simulation results were comparatively presented in Figure A1 in the Appendix. From the simulation results, which reproduced the experimental results relatively well,

the parameters for the Cyclic Mobility Model and the state variables of R^*_0 and ζ_0 were determined. Also, from the results, there is better agreement between the experimental and analytical results showing that the proposed soil material would replicate well the behavior observed in the experimental model.

3.8 Summary of formulation of finite element and foundation soils parameters adopted for current analytical study

In this chapter, the relevant finite element equations for Cyclic Mobily Model used for numerical analysis studies have been presented. Also, the joint spring element model and the analytical equations for characterization of the joint opening have been derived. The spring parameters for determination of the joint displacements in numerical studies have been formulated.

Undrained cyclic triaxial tests were done by past researchers to determine the experimental soil (Tohoku Silica Sand No. 7) and simulation soil (Toyoura Sand) parameters for the foundation ground. The numerical analysis and results of the foundation soils were found to reflect the experimental results in a better way, even though the soils used for the separate cases were different. Hence, based on the comparative analysis of both experimental and simulation results, Toyoura Sand could be used for the current numerical studies in investigating the culvert joint opening behavior during earthquakes, since its performance compare in a better way with the experimental soil, Tohoku Silica Sand No.7. In the current numerical studies, the foundation ground used Toyoura sand; Ts-L with Dr of 55% and Toyoura sand; Ts-NL with Dr of 95% for liquefiable ground and supporting layer respectively. The soil numerical parameters proposed by Mommae et al., (2019) are presented in Table 3.5.

Also, for the embankment soil, *Edosaki* sand, the simulation results matched very well with the experimental results, since the soil used in both cases was the same, hence it could be used in the current numerical works. The numerical parameters proposed by Miyazaki (2019) are presented in Table 3.6.

In general, experimental and simulation performance of both the foundation ground and embankment soil for use in the current numerical studies gives the researcher confidence in using the given soils to conduct the current numerical studies using the proposed analytical code, DBLEAVES developed by Ye et al., (2007), in explaining the culvert joint opening behavior due to liquefaction induced by earthquake. The code has been verified as an important tool in analyzing liquefaction induced problems (contact problems and soil-water coupled behavior) for underground structures by various researchers. These studies include Kheradi et al., (2018), Miyazaki et al., (2021), Bao et al., (2017) and Yatsumoto et al., (2019), hence it was deemed adequate for use in conducting the current numerical research.

Table 3.5: Foundation ground soils parameters (Mommae et al., 2019)

	Ts-L	Ts-NL
	Liquefiable soil	Non-liquefiable soil
Constitutive model	CMM	CMM
Compression index λ	0.05	0.05
Swelling index κ	0.0064	0.0064
Critical stress ratio M	1.30	1.30
Void ratio $p' = 98 \text{ kPa}$ N	0.87	0.87
Poisson's ratio ν	0.30	0.30
Degradation parameter of overconsolidation state m	0.01	0.01
Degradation parameter of structure a	0.50	0.50
Evolution parameter of anisotropy b_r	1.50	1.50
Initial void ratio e_0	0.772	0.626
Reference degree of structure R^*_r	0.80	0.80
Reference anisotropy x_r	0.0	0.0
Permeability k	2.35	1.72
Damping ratio h	0.05	0.05

Table 3.6: Embankment soil parameters (Miyazaki, 2019)

Principal stress ratio at critical state $R_{CS} = (\sigma_1/\sigma_3)_{CS(comp.)}$	4.0
Compression index λ	0.082
Swelling index κ	0.101
$N = e_{NC}$ at $p = 98$ kPa & $q = 0$ kPa	1.06
Poisson's ratio ν_e	0.276
Degradation parameter of overconsolidation state m	0.020
Degradation parameter of structure a	0.65
Evolution parameter of anisotropy b_r	0.4
Wet unit weight (kN/m ³) γ_t	17.738
Initial degree of structure R^*_0	0.1491 ~ 0.1584
Initial anisotropy ζ_0	0.5

References

- Akai K. and Tamura, T. (1978). Numerical analysis of multidimensional consolidation using elastic-plastic constitutive equations, *Proceedings of the Japan Society of Civil Engineers*, No. 269, pp. 98-104.
- Asaoka, A., Noda, T., Yamada, E., Kaneda, K. and Nakano, M. (2002). An elasto-plastic description of two distinct volume change mechanisms of soils, *Soils and Foundations*, 42 (5), pp. 47–57. [dx.doi.org/10.3208/sandf.42.5_47](https://doi.org/10.3208/sandf.42.5_47)
- Bao, X., Xia, Z., Ye, G., Fu, Y. and Su, D. (2017). Numerical analysis on the seismic behavior of a large metro subway tunnel in liquefiable ground, *Tunnelling and Underground Space Technology*, vol. 66, pp. 91-106.
- Christian, J. T. and Boehmar, J. W. (1968). Plane Strain Consolidation by Finite Elements, *Journal of Soil Mechanics, American Society of Civil Engineers*, vol. 96, No.SM4, pp. 1435-1457.

- Hirado, T. and Nitta, H., 2010. Study on Evaluation Method for Stripping Resistance of Polymer Modified Asphalt Using Silica Sand. *65th Annual Conference of the Japan Society of Civil Engineers*, pp. 245-246. (in Japanese)
- Hisada, T. and Noguchi, H. (1995). Fundamentals and Applications of Nonlinear Finite Element Method, Maruzen, pp. 161-175.
- Hughes, T. J. R., Taylor, R. L., Sackman, J. L., Curnier, A. and Kanoknukulchai, W. (1976). A Finite Element for a Class of Contact-Impact Problems, *Computer Methods in Applied Mechanics and Engineering*, vol. 8, pp. 249-276.
- Kheradi, H., Nagano, K., Nishi, H. and Zhang, F. (2018). 1-g shaking table tests on seismic enhancement of existing box culvert with atrial ground-improvement method and its 2D dynamic simulation, *Soils and Foundations*, vol. 58, pp. 563-581.
- Miyazaki, Y. (2019). Fundamental study on seismic behavior of hinge types of precast arch culverts in culvert longitudinal direction, Kyoto University, 2019, Ph. D. (Engineering).
- Miyazaki, Y., Sawamura, Y., Kishida, K. and Kimura, M. (2021). Influence of longitudinal structural connectivity on seismic performance of three-hinged precast arch culverts, *Soils and Foundations*, vol. 61, Issue 6, pp. 1681-1698.
- Mommae, F., Isobe, K. and Egawa, T. (2019). Numerical simulation on liquefied behavior of volcanic ash soil in intricate deposit situation, *54th JGS annual conference*, pp. 2161–2162. (in Japanese).
- Nakai, T. and Hinokio, M. (2004). A Simple Elastoplastic Model for Normally and over Consolidated Soils with Unified Material Parameters, *Soils and Foundations*, vol. 44, no.2, pp. 53-70.
- Nakashima, K., Kusaka, H., Saeki, M., Mano, M. and Ro, H. (2021). Analysis of damage to expressway embankments and culverts from earthquake on soft ground, *Journal of Japan Society of Civil Engineers, Ser. A1 (Structural Engineering & Earthquake Engineering (SE/EE))*, vol. 77, Issue 3, pp. 384-394.
- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.
- Oka, F., Hirokazu, Y., Sayuri, K. and Teppei, K. (2015). New Evaluation Method for Liquefaction of Ground Using Dynamic Liquefaction Analysis Method and Its Application, *International Journal of Geomechanics, American Society of Civil Engineers*, vol. 16, No. 5.
- Oka, F., Yashima, A., Shibata, T., Kato, M. and Uzuoka, R. (1994). FEM-FEM Coupled Liquefaction Analysis of a Porous Soil Using an Elasto-Plastic Model, *Applied Scientific Research*, vol. 52, pp. 209-245.
- Schofield, A. N. and Wroth, C. P. (1968). Critical State Soil Mechanics, McGraw-Hill, London.
- Suzuki, N. and Toi, Y. (1987). Finite Element Analysis of Dynamic Frictional Contact Problems by Using the Penalty Function Method -Consideration on the Penalty Numbers and its Application to the

Friction-Excited Vibration Problem, *Journal of the Society of Naval Architects of Japan*, no. 162, pp. 364-373. (in Japanese)

- Yatsumoto, H., Mitsuyoshi, Y., Sawamura, Y. and Kimura, M. (2019). Evaluation of seismic behavior of box culvert buried in the ground through centrifuge model tests and numerical analysis, *Underground Space*, vol. 4, Issue 2, pp. 147-167.
- Ye, B., Zhang, F. and Yashima, A. (2007). Experiment and numerical simulation of repeated liquefaction-consolidation of sand, *Soils and Foundations*, vol. 47, No. 3, pp. 547–558.
- Zhang, F., Ye, B., Noda, T., Nakano, M. and Nakai, K. (2007). Explanation of cyclic mobility of soils, Approach by stress-induced anisotropy, *Soil and Foundations*, vol. 47, No. 4, pp. 635-648. dx.doi.org/10.3208/sandf.47.635

CHAPTER FOUR

SEISMIC RESPONSE OF RIGID BOX CULVERTS ON LIQUEFIABLE GROUND THROUGH 2D ELASTOPLASTIC ANALYSIS

4.1 Introduction

In this chapter, dynamic responses of rigid box culverts (RBCs), often referred to as culverts in this chapter, upon foundation ground liquefaction are discussed. For cases built up, two and four culvert units are initially considered and under each case, the effect of variation of the depth of liquefiable ground is analyzed. Also, the hydrological role of the bottom plate of the culverts as a boundary between the foundation ground and the culverts is also considered. The drainage condition of culverts bottom plates is the ability to dissipate excess pore water pressure which builds up during liquefaction due to the confinement by the bottom plates, due to existence of two different surfaces, soil and concrete. This is suspected to bring stress imbalances between the drained and undrained surfaces. This destabilization might result in increased joint openings; hence it is necessary to consider the variation of this condition and check its effects on the culverts damage during seismic activities.

Considering traffic loading, the minimum overburden is designed as 0.5 m, according to Japan Road Association (2010), which has very little effect on the culvert. Nakashima et al., (2021) investigated 333 damaged RBCs on soft and liquefiable grounds to determine the relationships between the magnitude of joint openings, design considerations and earthquake characteristics during the 2004 Niigata-Chuetsu earthquake, the 2011 off the Pacific Coast of Tohoku earthquake and the 2016 Kumamoto earthquake. According to the survey, the joint opening of the RBCs concentrated on the specific overburden thickness of 0.0-2.0 m. Based on the culvert design criteria and the damaged investigation, embankment-culvert-foundation dynamic interactions should be studied for joint-opening mechanism. Embankment-culvert-foundation ground interactions are further investigated with 3D analytical model in Chapter Five. Note that, considering traffic loading, the minimum overburden is designed as 0.5 m, according to Japan Road Association, (2010), which has very little effect on the culvert.

4.2 Seismic induced liquefaction damage of culvert units considering varied liquefiable depth and drainage conditions

4.2 Model descriptions and analytical conditions

4.2.1 Analysis models

The two culvert units models (for shallow and deep liquefiable grounds) shown in Figures 4.1 and 4.2, comprised of culvert elements and two foundation layers, the liquefiable ground resting on a firm supporting layer. For the four culverts units' models shown in Figures 4.3 and 4.4, Figures 4.1 and 4.2 were

modified by adding an extra identical structural unit on each side of the RBCs maintaining the longitudinal connectivity and drainage conditions. The current research was a reproduction analysis to verify the 2016 Kumamoto earthquake results by Nanazawa et al., (2022) hence their experimental model dimensions were adopted (Figure 2.21 in Chapter Two). The seismic behavior of rigid structure without any deep foundation support was the issue to be solved in this study, hence, it was deemed necessary to firstly focus on the culverts response to seismicity without considering the embankment. Two depths of liquefiable layer, 3 m (shallow) and 10 m (deep) depths were chosen to explain the effect of the depth of the liquefiable layer, and 2 and 4 RBCs were chosen to explain the effect of the number of structural units on the liquefiable ground.

The numerical work was conducted using two-dimensional finite element elasto-plastic analysis operated in DBLEAVES (Ye et al., 2007). The RBCs were elastically modeled based on experimental studies by Ito et al., (2021), on liquefiable ground. Due to the hollow nature of the RBCs (in 3D extension)-transverse direction, the full volume of the top and the bottom section, and the discontinuous volume of the middle section were proportioned to the full unit weight of the culvert as shown in Figure 4.5. Since the culvert is a hollow structure, the effect of the varied volume was considered in the two-dimensional (2D) analysis. The top and bottom continuous culvert plates in y direction were assigned the whole unit weight (γ) is 2.71 g/cm^3 ; ($\alpha \times \gamma$). For the thin-walled middle section (discontinuous in y direction) between the top and bottom, a unit weight of 0.361 g/cm^3 ; ($\beta \times \gamma$) was assigned. This proportion was based on the section volume to the whole volume of the entire cross section in 3D extension. Note that the haunch was not considered to simplify the 3-dimensional density effect.

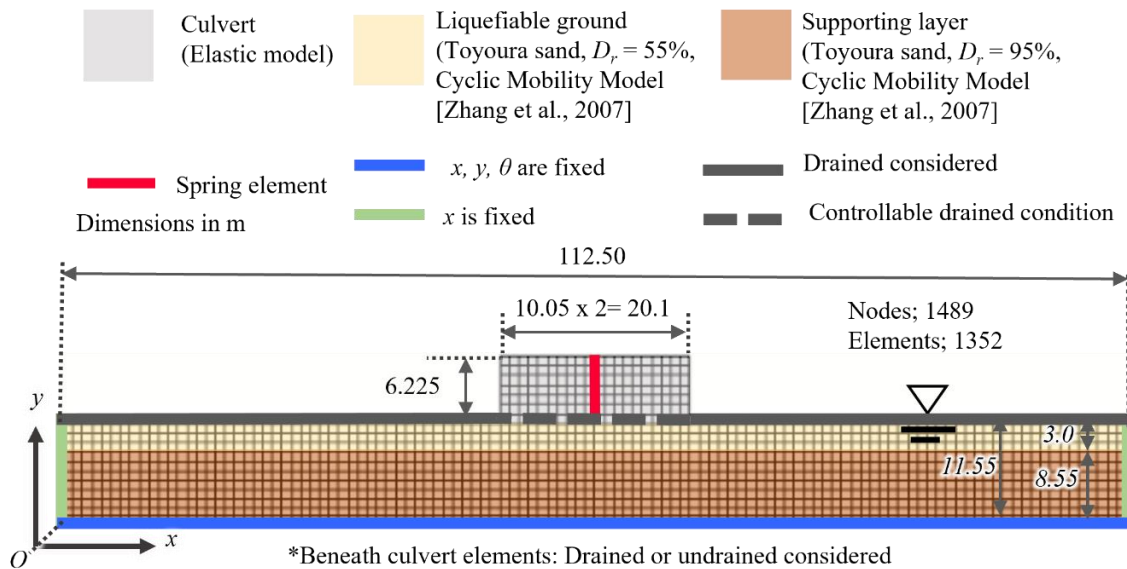
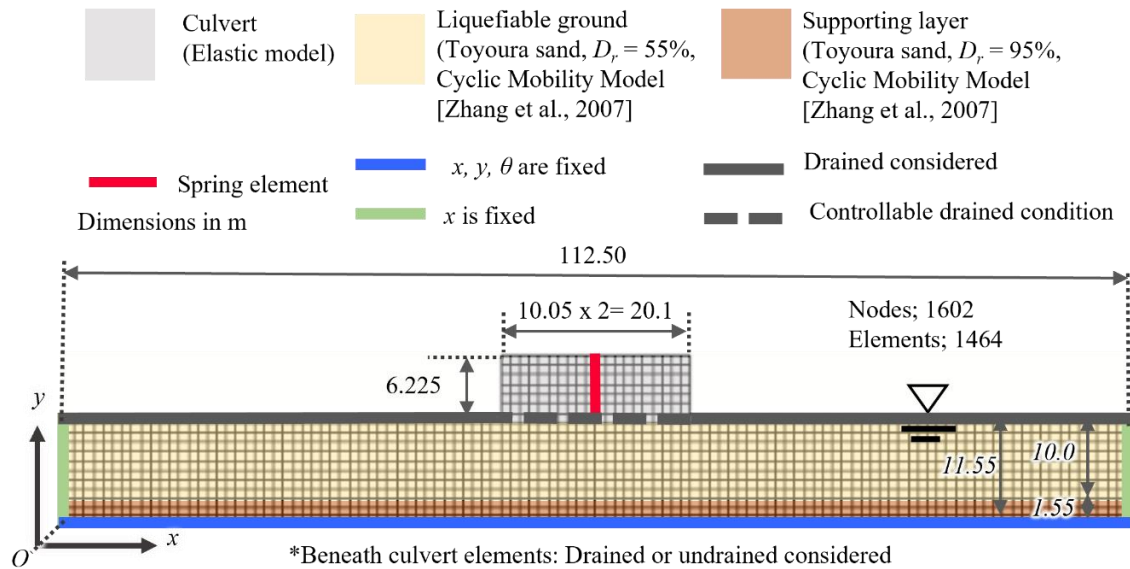


Figure 4.1. Model of two culvert units on shallow liquefiable ground.



4.2.2 Boundary conditions

The drainage variation was applied only at the boundary between the culvert units and the liquefiable ground, by considering drained (impervious) and undrained (pervious) nature of the bottom slabs of the RBCs. For the boundary condition on the sides of the foundation ground model, the bottom was rigidly fixed in x, y, and rotation θ . The sides were fixed in x direction only.

4.2.3 Soil and culvert parameters

The foundation ground soil parameters and cyclic mobility model are discussed in Chapter Three.

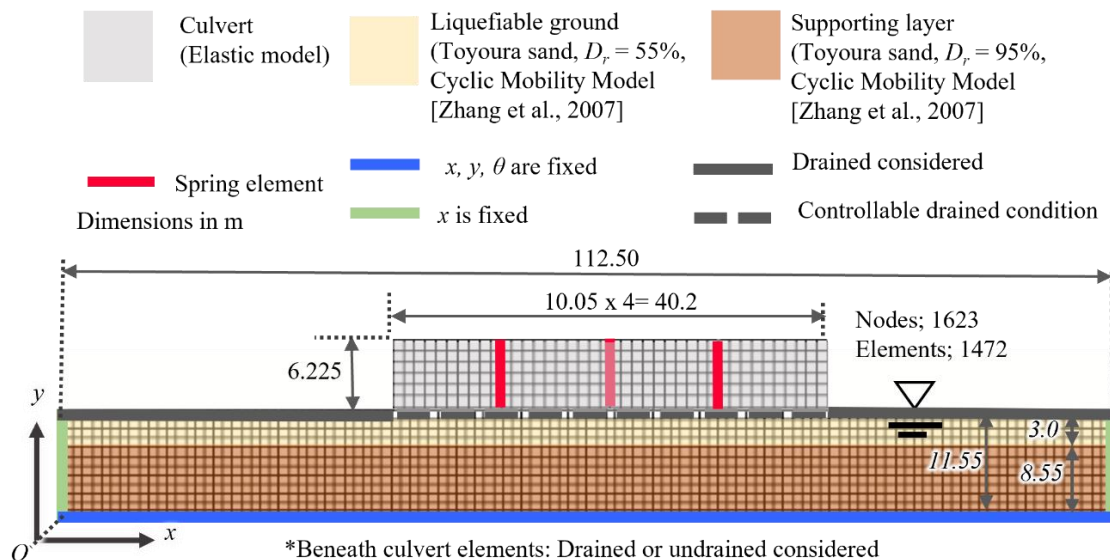
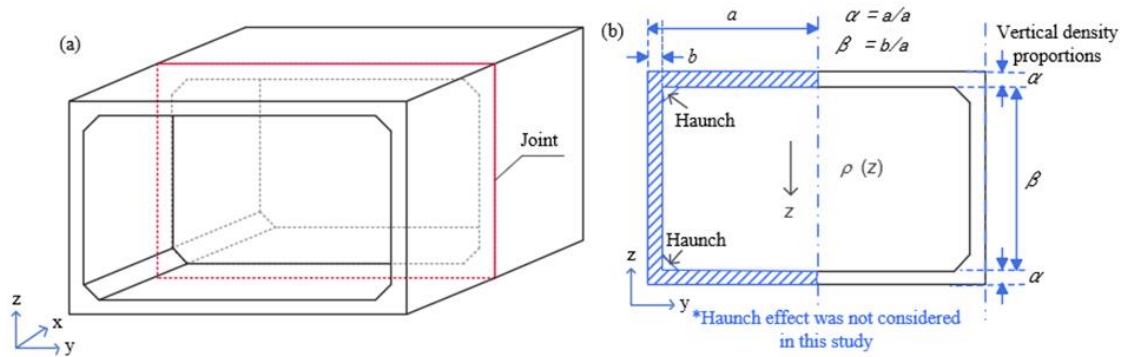
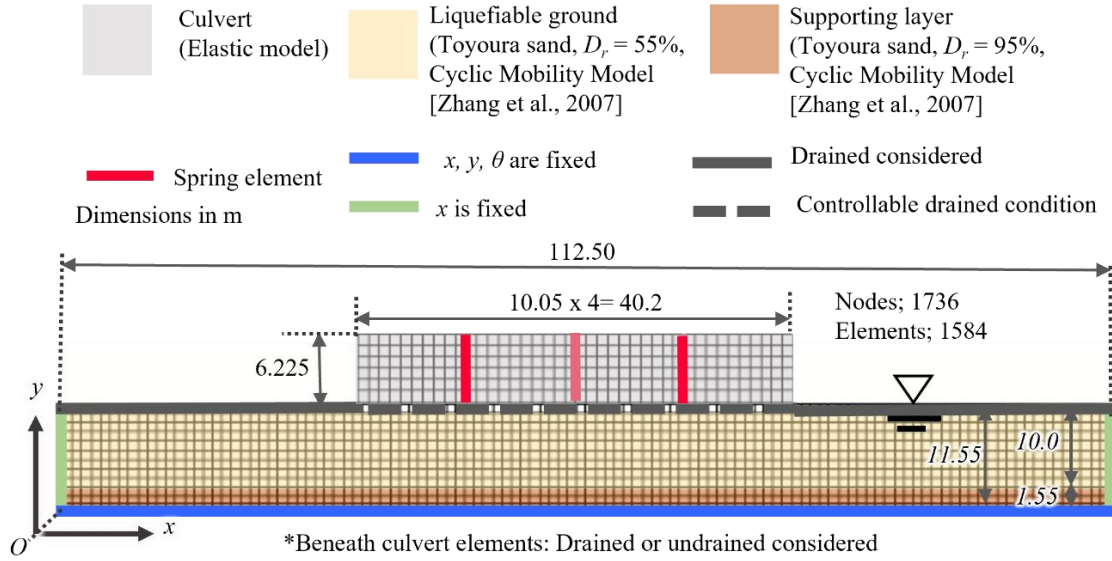


Figure 4.3. Model of four culvert units on shallow liquefiable ground.



4.2.4 Input wave, self-weight analysis and analysis cases

The inputted seismic wave (Figure 4.6) at the bottom of the foundation ground in the longitudinal; was the TR component on the ground around the Tsugaru Ohashi Bridge, 1983 Central Japan Sea Earthquake and was classified as Level 1 earthquake according to Japan Road Association, (2012). Its time interval of calculation was 0.001 seconds, and the time integration was based on a Newmark- β method ($\beta = 1/4$, $\gamma = 1/2$).

To accurately analyze the localized liquefaction beneath RBCs examined in this study, the initial stress state of the analytical model is of critical importance. In this analysis, a self-weight analysis was conducted using an elastic model with deformation parameters derived from undrained triaxial simulations based on the Cyclic Mobility Model parameters (Chapter Three).

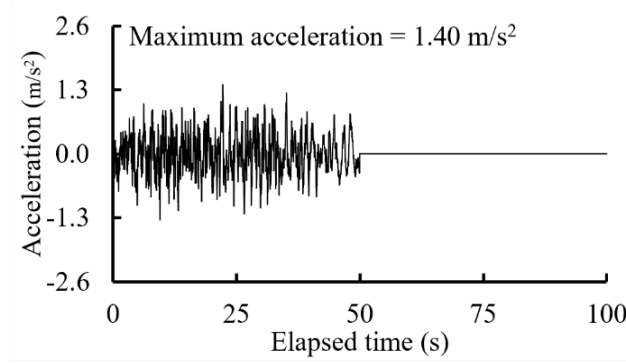


Figure 4.6. Acceleration-time profile of input wave recorded around Tsugaru Ohashi Bridge during 1983 Central Japan Sea Earthquake

The deformation parameters were adjusted by multiplying their values by a factor of four, referencing the Specifications for Highway Bridges by the Japan Road Association (2012). The groundwater table was assumed to coincide with the ground surface (refer to Figures 4.1 to 4.4). After confirming the dissipation of excess pore water pressure, the initial stress state was modified under the assumption that the maximum principal stress direction aligns with σ_y and the minimum principal stress direction aligns with σ_x . This modified initial stress state was subsequently employed in dynamic analysis. Earth pressure coefficient K_0 was calculated using Jaky's formula in Eq. (4.1), where φ is the internal friction angle and Poisson's ratio ν determined from Eq. (4.2).

$$K_0 = 1 - \sin \varphi \quad (4.1)$$

$$\nu = \frac{K_0}{1 - K_0} \quad (4.2)$$

For the analysis, the cases presented in Table 4.1 were investigated. These cases represented the analytical conditions to be investigated including the effects of the geological condition (depth of liquefiable ground), structural effect (number of culvert units) and the hydrological effect (drainage condition) at the boundary between the culverts and foundation ground.

Table 4.1: 2D analytical cases

	2 culvert units model		4 culvert units model	
	Drained	Undrained	Drained	Undrained
3 m deep liquefiable ground	Case-2D3	Case-2U3	Case-4D3	Case-4U3
10 m deep liquefiable ground	Case-2D10	Case-2U10	Case-4D10	Case-4U10

4.3 Case study for verification of simulator parameters performance in current research

A case study was conducted to verify the performance of adopted parameters in the current research. The target element selected (at mid liquefiable layer beneath the joint section, Figure 4.8) was the one whose initial mean effective stress was close to 50kPa, like the simulator. For the verification study, analytical Case-2U3 was selected and the results comparisons with the simulator are shown in Figure 4.7.

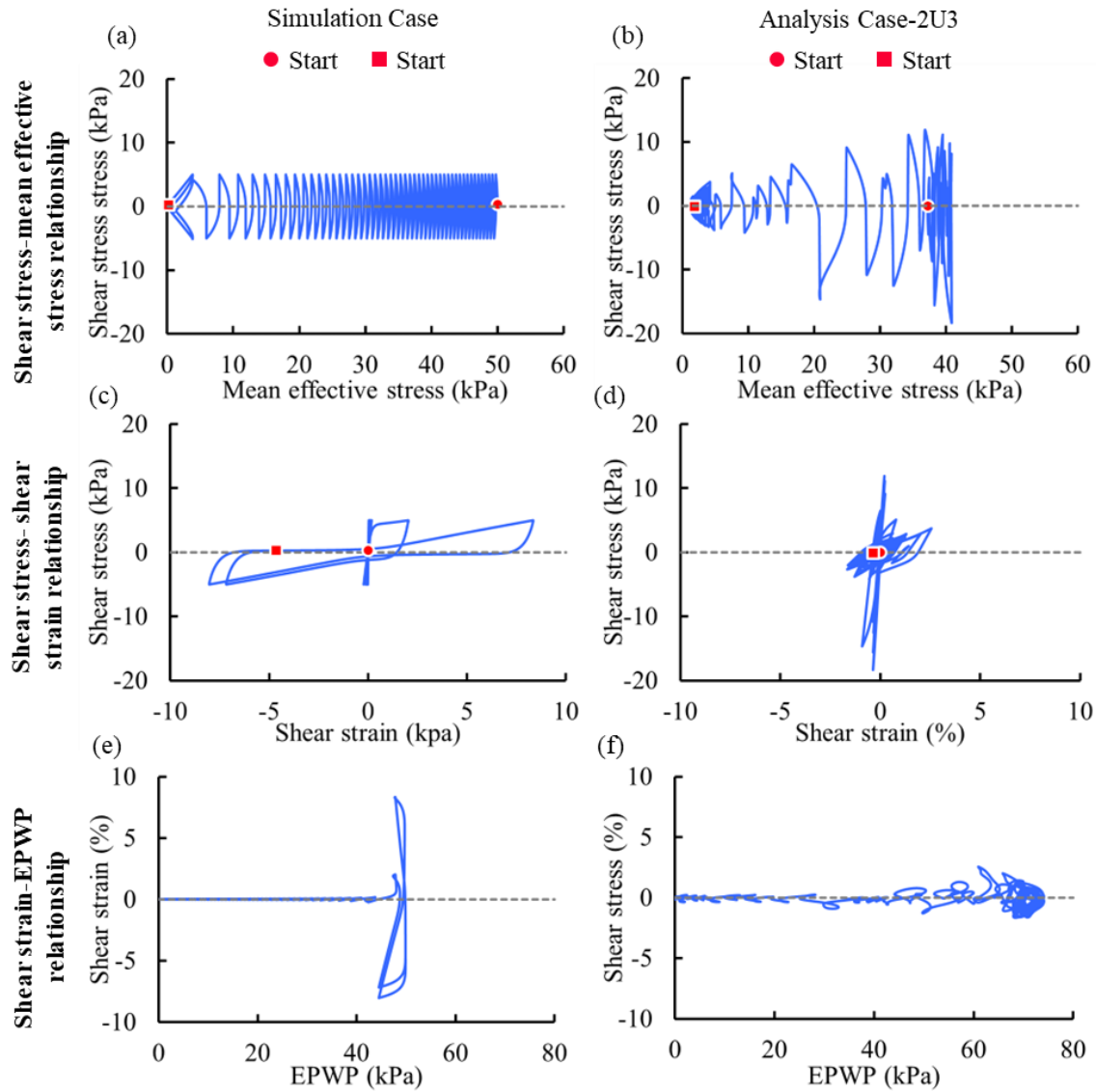


Figure 4.7 Comparisons of simulator and analysis case performances

From the results, the stress paths relationships show similar stress paths between the two cases, whereby the mean effective stress is decreasing as the analysis progresses. The performance of the simulator is controlled hence uniform stress path as compared to the analysis case where the performance is due to

the input earthquake wave. This makes the simulator have longer stress paths as compared to the analytical case presented. As a result of long stress paths, higher shear strains are developed in the simulator as compared to the analytical case. For the Shear strain-excess pore water pressure (EPWP) relationship, in both cases, maximum excess pore water pressure occurs at maximum strains.

In general, the adopted parameters could produce similar result trends with the simulator hence, the researcher can confidently apply the said parameters in the current research.

4.4 Foundation ground and culverts response to input wave acceleration

For this section, Figure 4.8 is also used to express the details of nodal pick-up points for accelerations analysis. To understand how the culverts responded to the input wave (Figure 4.6) with maximum acceleration of 1.40 m/s^2 , the culverts top left corner of left culvert (LC) and top right corner of right culvert (RC) response to the input acceleration wave were evaluated as shown in Figure 4.9. It can be noted that there was an amplification of the input wave acceleration by the culverts in all cases. This is because the culverts movements were not restricted (the culverts were freely standing), hence small ground shaking caused larger culverts shaking, leading to higher accelerations. For the 4 culvert model cases; Figures 4.9 (e), (f), (g) and (h), there was very abnormal acceleration at 1 second, before the results stabilized, hence these results up to 1 second were not considered for the comparisons. They were attributed to the culvert - ground synchronization during the initial shaking.

For the depth of liquefiable layer effect, 2 culvert units cases had no difference but for the 4 culvert units cases, the acceleration was much reduced from shallow to deep liquefiable layer cases. This reduction was attributed to the effect of the higher self-weight effect of the 4 culvert units as compared to the 2 culvert units on the liquefied foundation ground causing the culverts to sink and subsequently, their movements being reduced.

To understand the accelerations distributions in terms of their frequency and comparison with the input wave, Fourier spectral amplitudes for representative cases were computed as shown in Figure 4.10. These cases represented drainage, liquefiable depth and number of units variations. There was symmetry in the response of the culverts as seen for the maximum left and right culvert accelerations in the selected cases. From the Fourier spectral amplitude analysis, and in all cases, it can be confirmed that the highest accelerations were concentrated within frequency of 1 Hertz. Even though the input and the culverts accelerations response did not match very well as explained earlier, the symmetry in their magnitudes for left and right culverts confirmed that they responded well to the input wave acceleration.

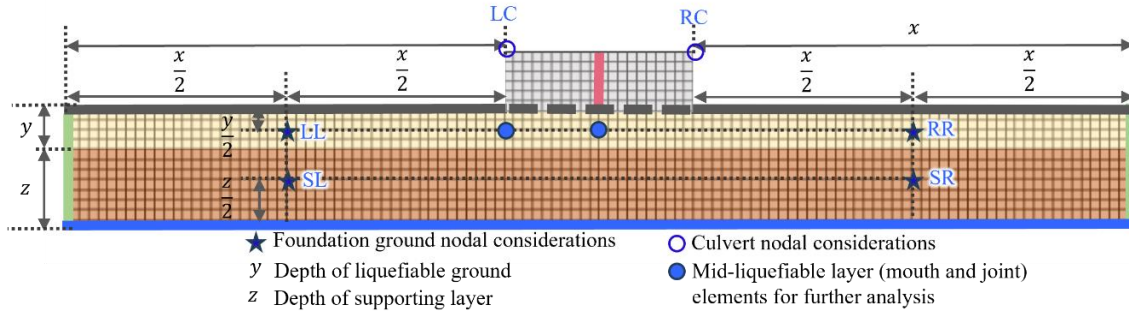


Figure 4.8. Critical analysis points for culverts and foundation ground (same demonstration model was adopted for the 4 culverts model).

Similarly, the foundation ground response to the input acceleration wave was also evaluated. For the foundation ground consideration, the midpoints of the free ground beyond the left and the right culvert mouths were considered. Mid liquefiable ground on left and right sides were denoted as LL and LR respectively while mid supporting layer on left and right sides were demoted as SL and SR respectively, as shown in Figure 4.8 and the results are as shown in Figure 4.11. The drainage effect was not considered because only varied drainage conditions below the culverts existed, and separately it was confirmed that the picked areas were not affected by this condition. From the time history analysis of the acceleration responses, all the cases except the shallow liquefiable ground (Figures 4.11 (a) and (e)), there was reduction effect response to the input acceleration wave. This was because of the restraint boundary conditions to movement in x-direction.

For the shallow liquefiable ground (Figures 4.11 (a) and (e)), the selected element being closer to the free zone of the top ground surface than the other cases; resulted in the exceptional behavior of amplification of the input wave acceleration as seen for the cases of the culverts responses.

Also, to check the symmetry in ground response to input wave and frequency concentrations of the accelerations, the Fourier spectral amplitude response was conducted on selected cases representing the liquefiable ground, supporting layer, liquefiable depth and the number of culvert units effect, as shown in Figure 4.12. Similar trend as discussed in the culverts was also noticed with the ground response. Symmetry in ground response to the input wave acceleration occurred between the left and the right side of the ground in the foundation layers, but lower than the input wave acceleration. Also, the concentrations of the accelerations were centered around 1 Hertz frequency.

In summary, the culverts being free on the ground without any movement restrictions as the foundation ground layers had amplification effect to the input acceleration wave. In all the responses, for

culverts and the foundation ground layers, all the highest responses were centered around frequency of 1 Hertz, and there was symmetry between the left and the right elements of the analysis model. Hence, this concludes that the input wave was well manifested in the model elements.

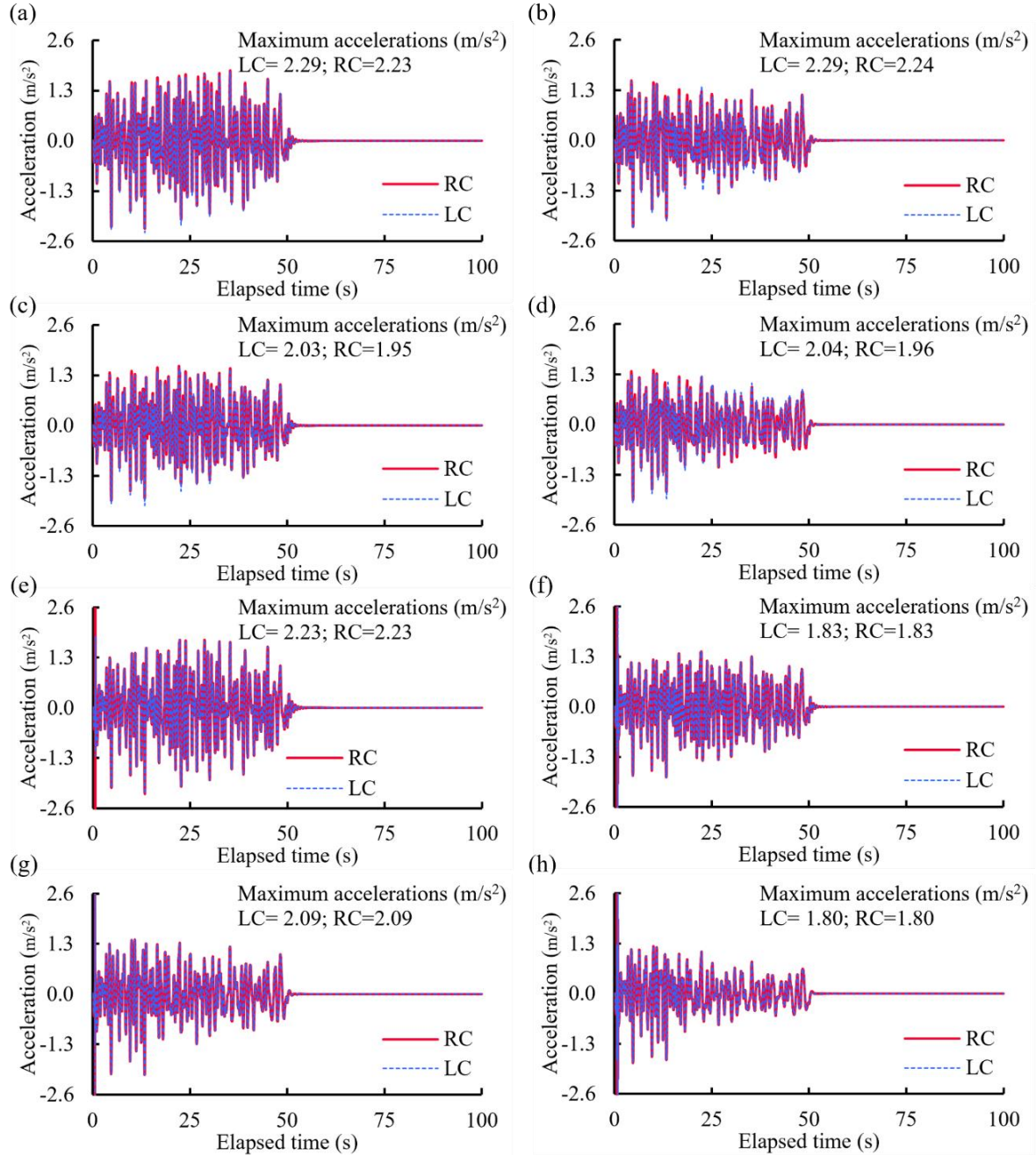


Figure 4.9. Culverts response to input wave for 2 culvert units (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 (d) Case-2U10 and 4 culvert units (e) Case-4D3 (f) Case-4U3 (g) Case-4D10 (h) Case-4U10.

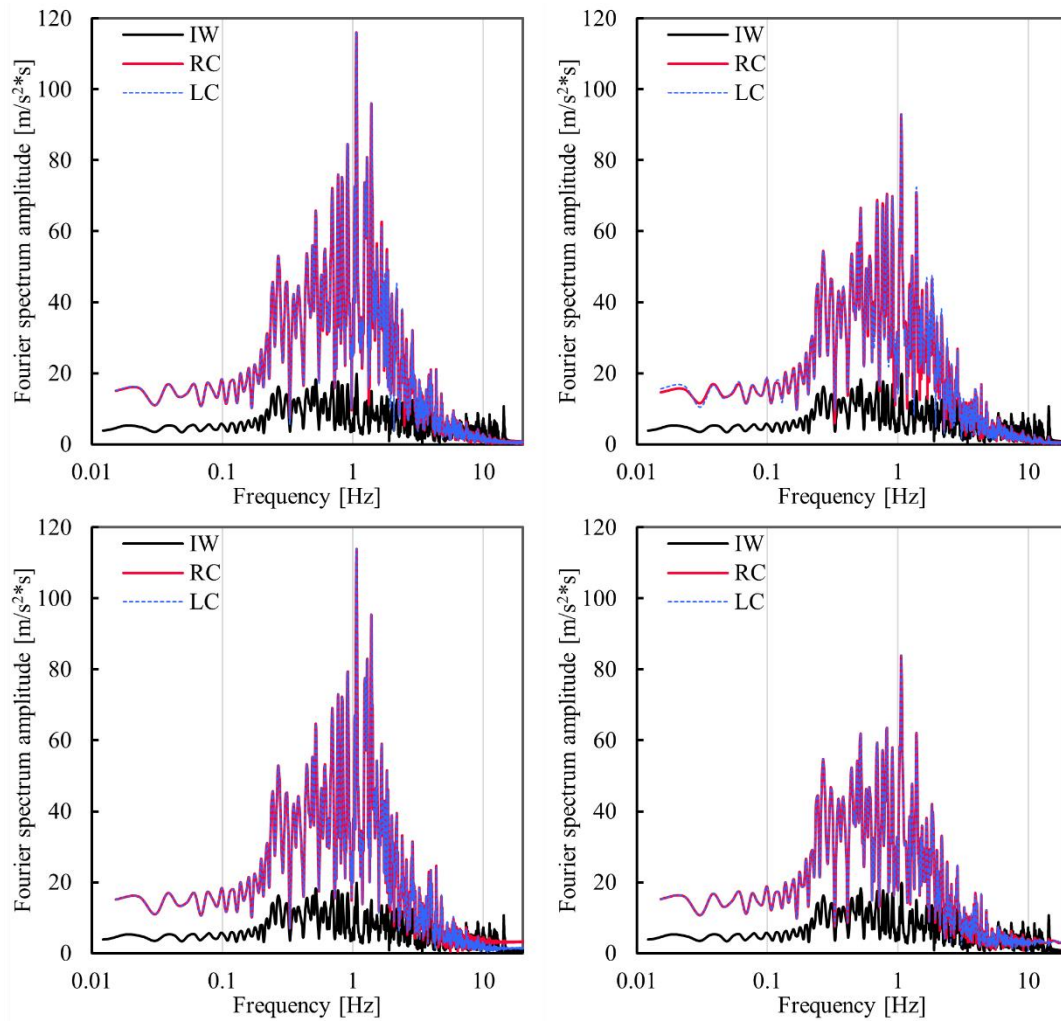


Figure 4.10. Fourier spectral analysis of input wave - culverts acceleration response for 2 culverts units (a) Case 2D3, (b) Case 2U10 and 4 culvert units (c) Case 4D3 and (d) Case 4U10.

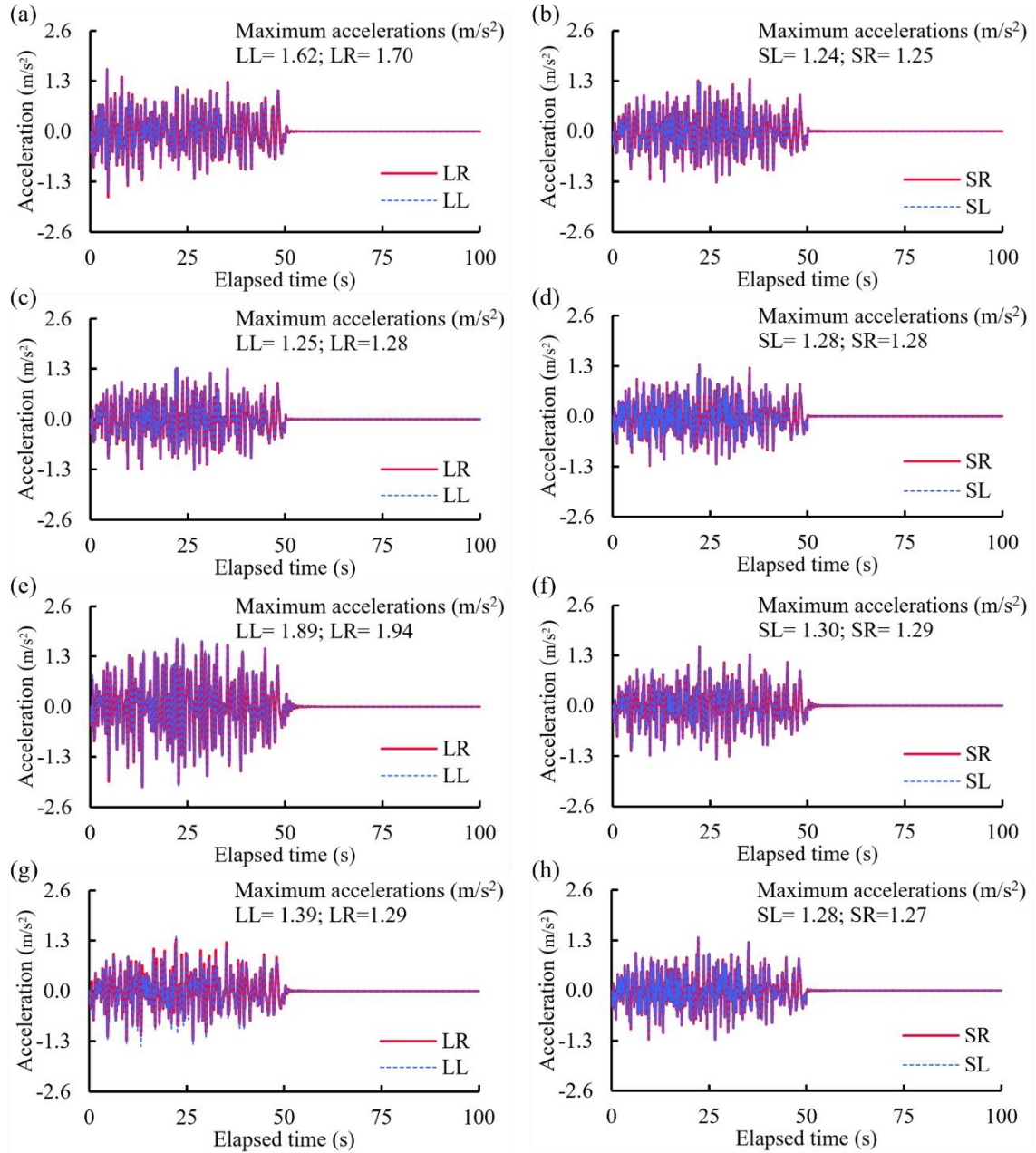


Figure 4.11. Foundation ground response to input wave for 2 culvert units for Case-2D3 in (a) and (b), Case-2D10 in (c) and (d), 4 culvert units for Case-4D3 in (e) and (f) and for Case 4D10 in (g) and (h).

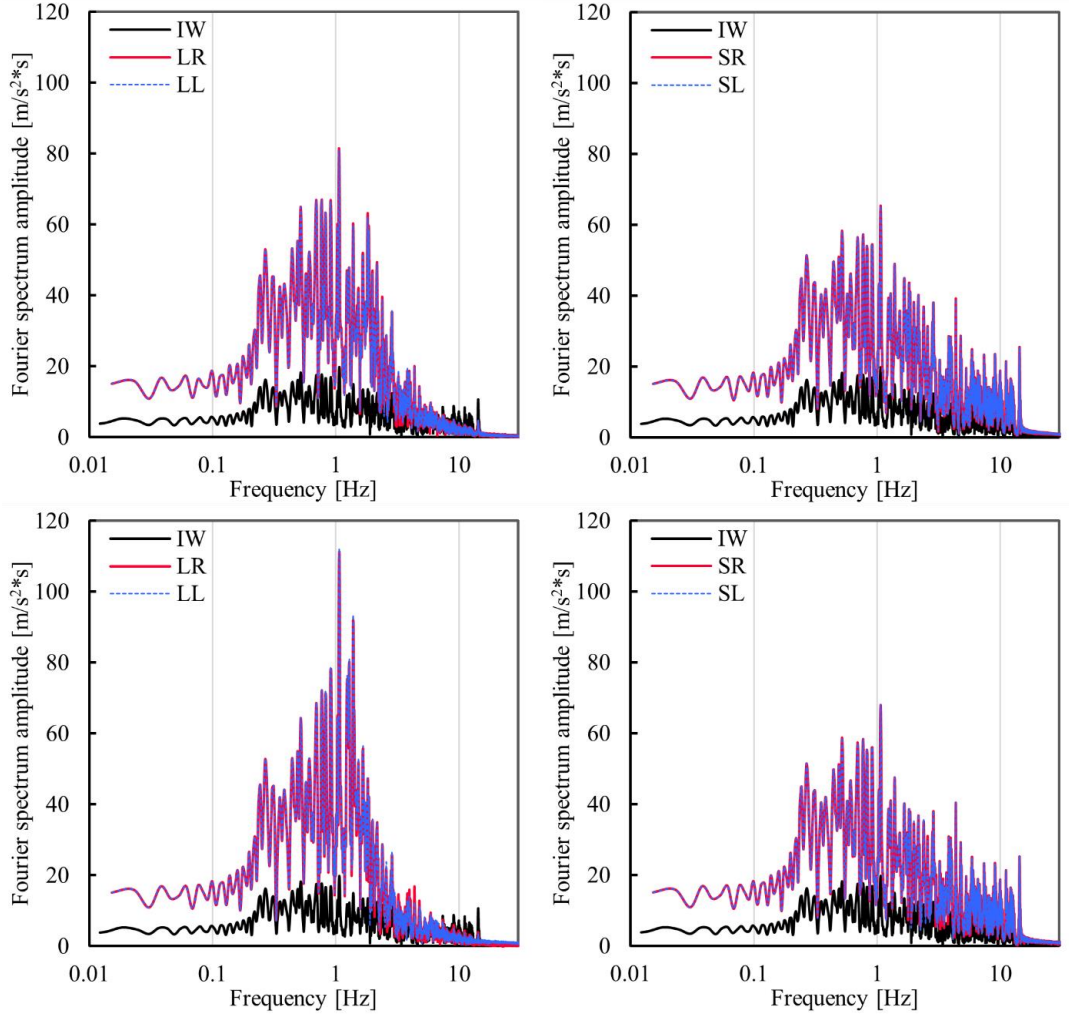


Figure 4.12. Fourier spectral analysis of input wave - culverts acceleration response for 2 culvert units for (a) Case-2D3, (b) Case-2D10, 4 culvert units for (c) Case-4D3 and (d) Case-4D10.

4.5 Culverts and ground response to earthquake induced liquefaction

Effective stress decreasing ratio (*ESDR*) in Eq. (4.3), is the liquefaction index proposed by Oka et al. (2015); whereby σ'_m is the current mean effective stress while σ'_{m0} is the initial mean effective stress. At the onset of the analysis, σ'_m and σ'_{m0} are supposed to be equal hence *ESDR* is zero. Theoretically, upon full liquefaction, all mean effective stress is converted to excess pore water pressure (EPWP) hence σ'_m becomes zero, and at that time, *ESDR* becomes one. In some instances, the *ESDR* may be reduced to negative value. This means the current mean effective stress becomes more than the initial mean effective stress. This might happen due to ground stiffening due to recovery of mean effective stress through dissipation of EPWP.

4.5.1 Liquefaction (*ESDR*) and EPWP variations

Figure 4.13 shows the contour heatmaps of the final *ESDR* and the corresponding foundation grounds settlements at the end of the analysis for both drainage cases. Figure 4.14 shows the element location for EPWP analysis to explain Figure 4.13.

$$ESDR = 1 - \frac{\sigma'_m}{\sigma'_{m0}} \quad (4.3)$$

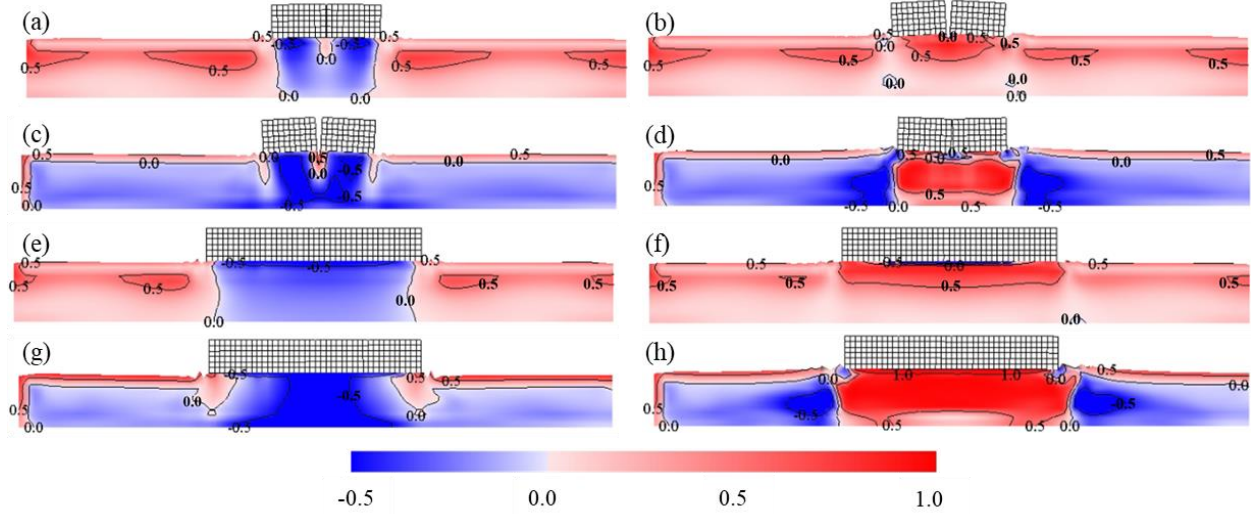


Figure 4.13. Final *ESDR* heatmaps at 100 seconds for 2 culvert units (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 (d) Case-2U10 and 4 culvert units (e) Case-4D3 (f) Case-4U3 (g) Case-4D10 (h) Case-4U10 at the end of analysis.

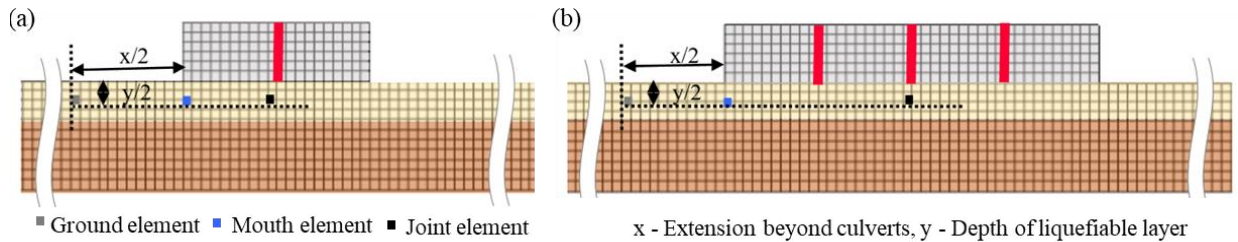


Figure 4.14. EPWP analysis elements' location for (a) 2 culvert units and (b) 4 culvert units.

From the *ESDR* heatmaps generated at the end of the analysis, higher liquefaction levels were experienced beneath the culverts whereby undrained boundary condition beneath the culverts alleviated these levels. In the shallow liquefiable ground, the net liquefied area was much lower compared to the deep liquefiable ground, and this was clearer from the undrained cases. The negative *ESDR* was due to the recovered and enhanced mean effective stress against the initial mean effective stress (current mean effective stress being higher than initial mean effective stress). This implies that the current mean effective

stress is higher than the initial mean effective stress, hence Eq. (4.3) becomes negative. From the analytical results in Figure 4.15, the cause of these negative *ESDR* in the drained condition was due to the dissipation of the liquefaction (decrease in EPWP after maximum liquefaction levels were reached) which had occurred and the improved confining stress after the foundation deformed, hence the culvert displacement changed (contacting pressure mode change). In this sense, in drained conditions (for both 2 and 4-culvert elements cases), faster dissipation of excess pore water pressure occurred as compared to the undrained condition. This resulted in higher liquefaction levels being favored by the undrained condition beneath the 2 and 4-culvert elements on foundation ground at the end of analysis. The dissipation of EPWP outside the culvert region is slower in all the drained cases, since little impact of culvert action is influencing this region. Also, there is evident higher peak EPWP levels with undrained condition as compared to drained. This is brought about by the high liquefaction levels beneath the culvert due to the undrained conditions, as seen in Figure 4.13. These elevated EPWP levels have a counteracting effect of reducing the EPWP outside the culvert region.

Generally, the undrained condition has an effect of amplifying the liquefaction levels beneath the culverts, resulting in high EPWP levels as compared to the drained condition. Similarity in EPWP behavior is noted for both 2 and 4 culvert in all cases, for instance, there is similarity in behavior between shallow drained cases; Case-2D3 and Case-4D3, deep drained cases; Case-2D10 and Case-4D10, shallow undrained cases; Case-2U3 and Case-4U3; and deep undrained cases; Case-2D10 and Case-4D10. Since Figure 4.13 was based on the final analysis results at the end of analysis at 100 seconds, it might not have been the true picture of how high the ground liquefied, since dissipation of EPWP occurred, after peak liquefaction. Hence, it was deemed necessary to check the maximum liquefaction levels of the foundation grounds, hence the maximum liquefaction profiles across the foundation grounds were analyzed as shown in Figure 4.16.

From the foundation ground liquefaction profiles in Figure 4.16, there was minimal liquefaction to the supporting layer as compared to liquefiable ground occurred, and that's how the ground was expected to perform during the analysis. This is because the ground relative density of the supporting layer was very high, 95%, as compared to the liquefiable ground, 55%. These liquefaction profiles show where and how high the ground is liquefied. From the profiles, it can be demonstrated that the presence of culverts on the foundation ground affects how the ground behaves during seismic activities. The ground outside the culverts region generally behaved in a similar manner in all cases with higher liquefaction on the liquefiable ground and lower liquefaction levels in the supporting layer. For the drained condition, the liquefaction levels were very low as compared to the undrained condition.

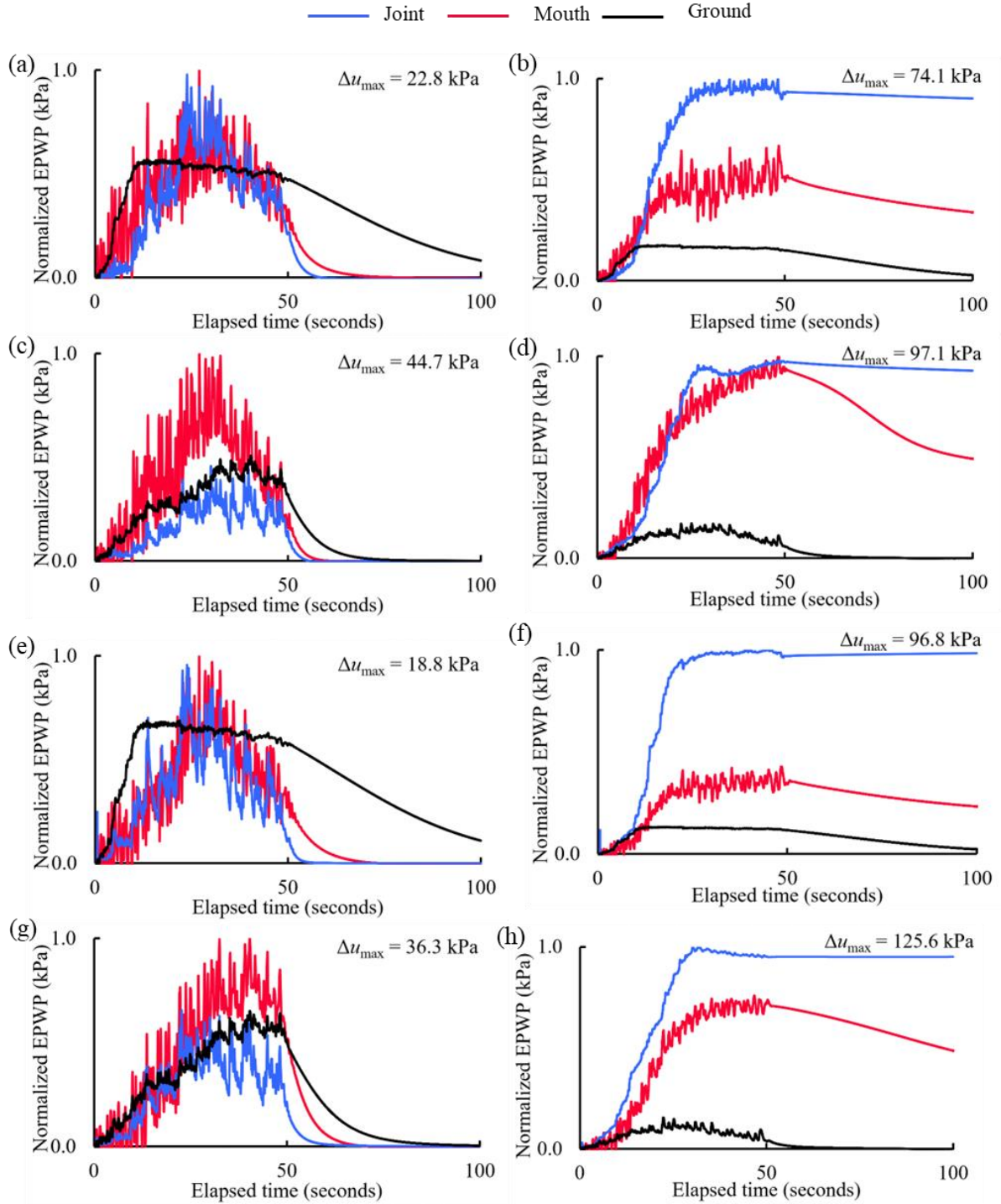


Figure 4.15. Time history of normalized excess pore water pressure (EPWP) variation below the culvert mouth, joint and outside the culverts, for 2 culvert units (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 (d) Case-2U10 and 4 culvert units (e) Case-4D3 (f) Case-4U3 (g) Case-4D10 (h) Case-4U10 during analysis. The EPWP values are normalized with maximum values in each case.

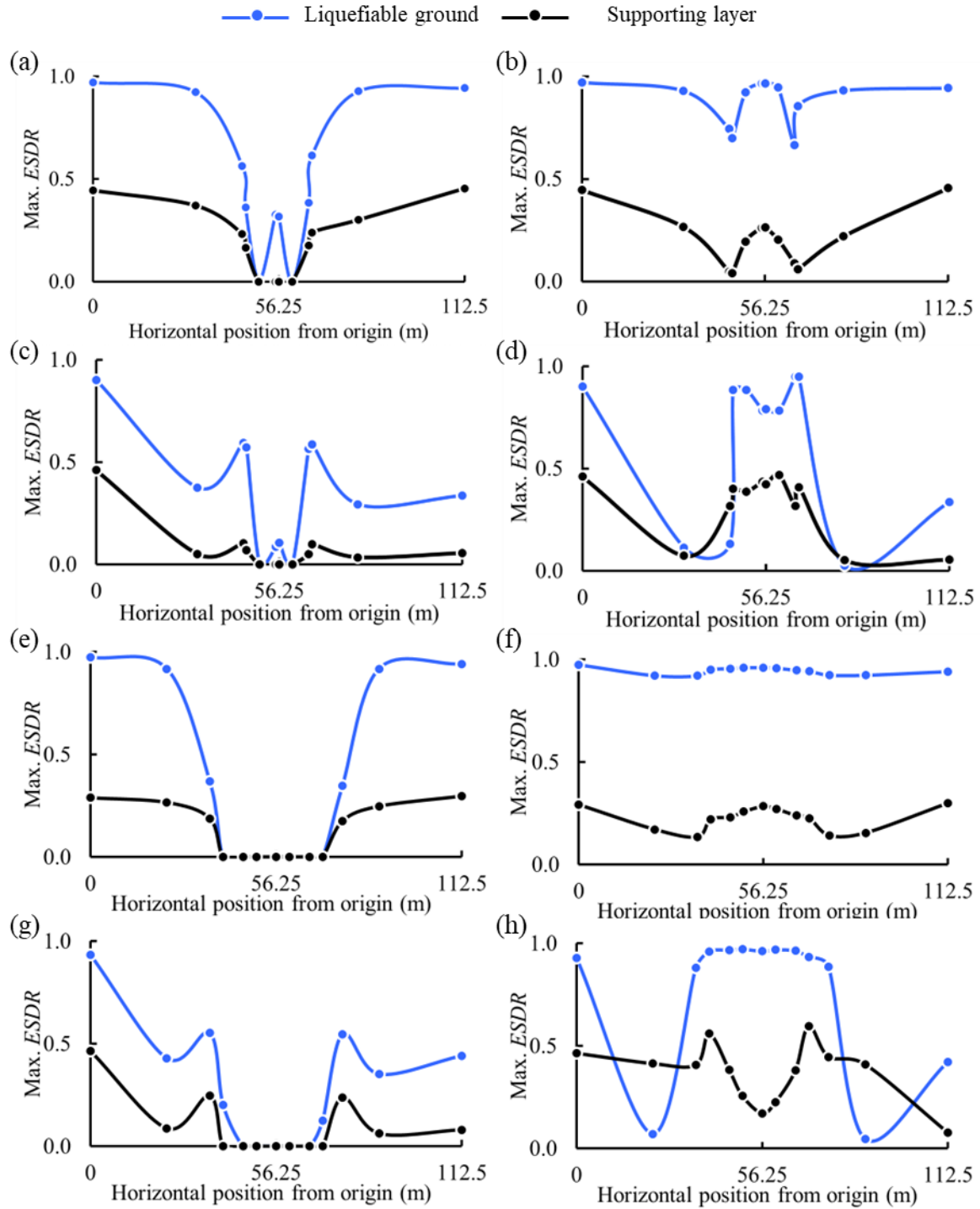


Figure 4.16. Maximum liquefaction ($ESDR$) profiles of foundation ground for, for 2 culvert units (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 (d) Case-2U10 and 4 culvert units (e) Case-4D3 (f) Case-4U3 (g) Case-4D10 (h) Case-4U10.

The presence of undrained boundary condition beneath the culverts (porous culvert bottom plate) provided room for the accumulation of higher EPWP beneath the culverts, despite the self-weight action on the foundation ground, resulting in escalation of the liquefaction levels as seen in Figure 4.13. On contrary, the non-porosity effect of the bottom plates (drained condition), and the self-weight action of the culverts on the foundation ground, suppressed the creation of more EPWP beneath the culverts, a situation which led to low buildup of EPWP due to liquefaction. As seen in the pattern of the EPWP in Figure 4.15, a similar pattern was replicated by the maximum liquefaction profiles in the foundation grounds.

4.5.2 Culvert joint opening analysis

Due to these liquefaction levels variation, the culverts joint opening occurred, and the results are as shown in Figure 4.17 for the 2 culvert units cases and Figures 4.18 and 4.19 for the 4 culvert unit cases. From Figure 4.17, at the end of analysis, the joints between the culverts opened at varying magnitudes and mechanisms. The joint openings were calculated using the spring displacements in the x-directions. The spring displacements were normalized using the maximum displacement of each joint and negative signs indicate that the springs experienced tensional forces from the opposite culverts.

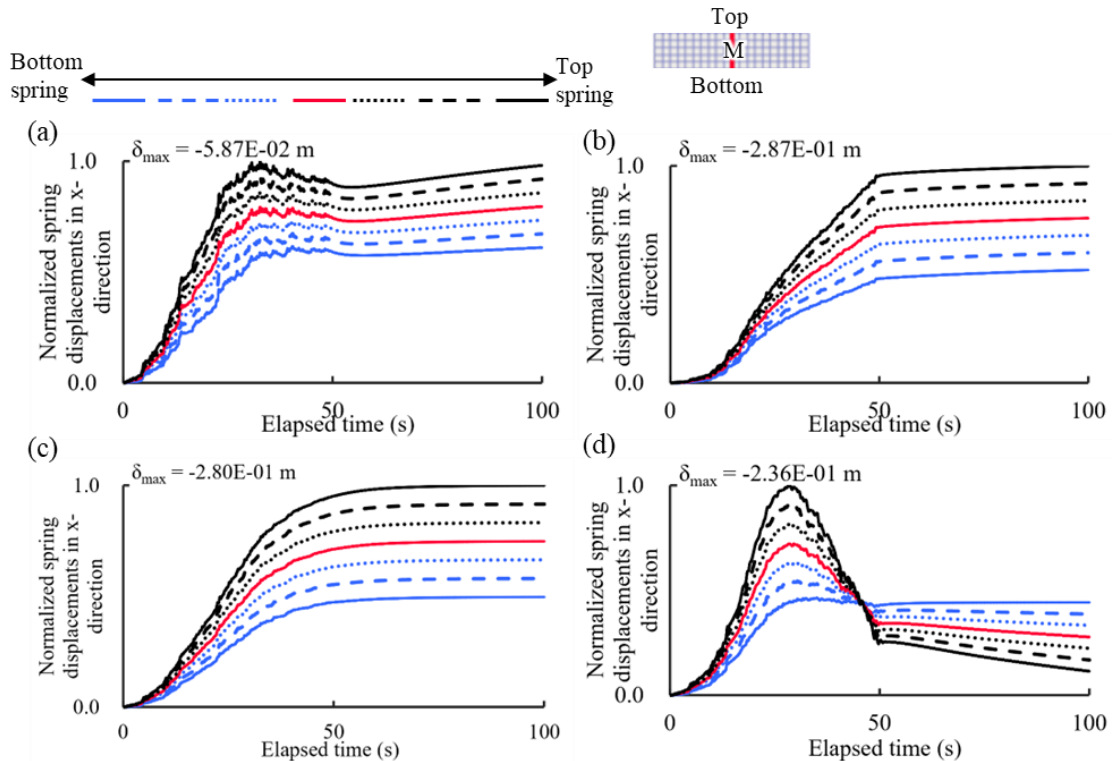


Figure 4.17. Spring element displacements for middle (M) joint for 2 culvert units (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 and (d) Case-2U10. Spring displacements in x-direction are normalized with maximum values in elapsed time in each analysis case.

From the results, the joint openings were increasing from bottom to top as the analysis progressed except for the deep liquefiable layer with undrained condition (unique case). For the unique case, the joint opening increased up to a maximum value (0.236 m), then underwent closure of the joint and eventually opened towards the bottom direction. All these joint openings of the culverts were caused by the culvert movements due to liquefaction of the soil layers beneath them, a condition which seems to have been increased by the undrained boundary condition between the culverts and the soil layers, in the four cases. The joint openings were in V-shaped form except for the special case of deep undrained condition. These were due to the development of EPWP during liquefaction, and differential ground deformations at the mouth and the joint sections beneath the culverts. Figures 4.18 and 4.19 show the time history of spring displacement behavior for the 4-culvert cases under drained and undrained conditions for the left (L), middle (M) and right (R) joints. The spring displacements were insignificant compared to the 2-culvert cases. This was attributed majorly to the combined increased self-weight of the four culverts on the liquefiable ground suppressing the ground vibration hence the limited culvert displacements despite the development of higher excess pore water pressures at the middle joint in undrained condition. The determination of the order of springs displacements from bottom to top of the joint was not possible due to the very small springs displacements. In summary, comparing the outputs of this numerical study (undrained cases selected due to unique behavior in culverts displacements in deep liquefiable ground) and the experimental study results of Nanazawa et al., (2022) in terms of culverts joint openings and final culverts inclination modes, the following observations are made:- (a) Both studies are in agreement with the final ground deformation modes, whereby shallow liquefiable ground took a convex deformation mode, (hence pushing the joint opening upwards) while the deep liquefiable ground took a concave deformation mode (hence pushing the joint opening downwards). (b) For the numerical study, the maximum culvert joint opening for shallow liquefiable ground was 0.29 m while the deep liquefiable ground was 0.24 m (before joint closing started), whereas in the experimental study, the shallow liquefiable ground was 0.63 m while the deep liquefiable ground was 0.70 m. The difference can be attributed to i) difference in number culverts units used (experimental work involved four culverts while numerical work involved two culverts). ii) Experimental case had effect of overburden of 0.5 m on culverts while numerical case did not have overburden and finally, iii) the numerical case explains the complete path of opening and closure to downward opening of the joint while experimental case presents the final joint opening position, hence not possible to tell the complete process of joint opening. According to Nanazawa et al., (2022), it was recommended that the joint openings should be maintained below 200 mm to facilitate quick repairs with minimum impact on the road functionality.

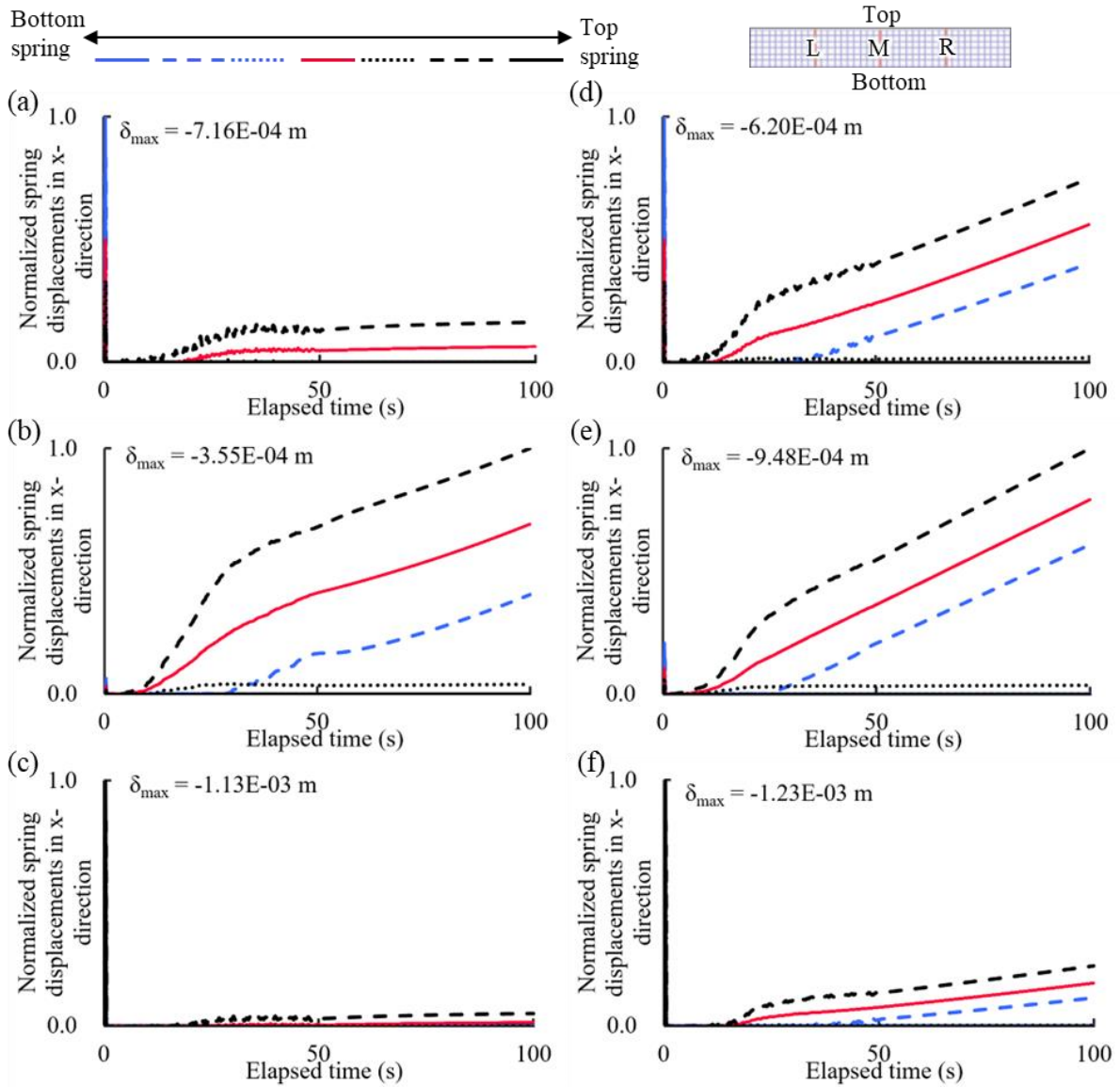


Figure 4.18. Spring element displacements for 4 culvert units in 3 m liquefiable ground for Case-4D3 (a) left (L) (b) middle (M) and (c) right (R) joints and Case-4U3 (d) left (L) (e) middle (M) and (f) right (R) joints. Spring displacements in x-direction are normalized with maximum values in elapsed time in each analysis case.

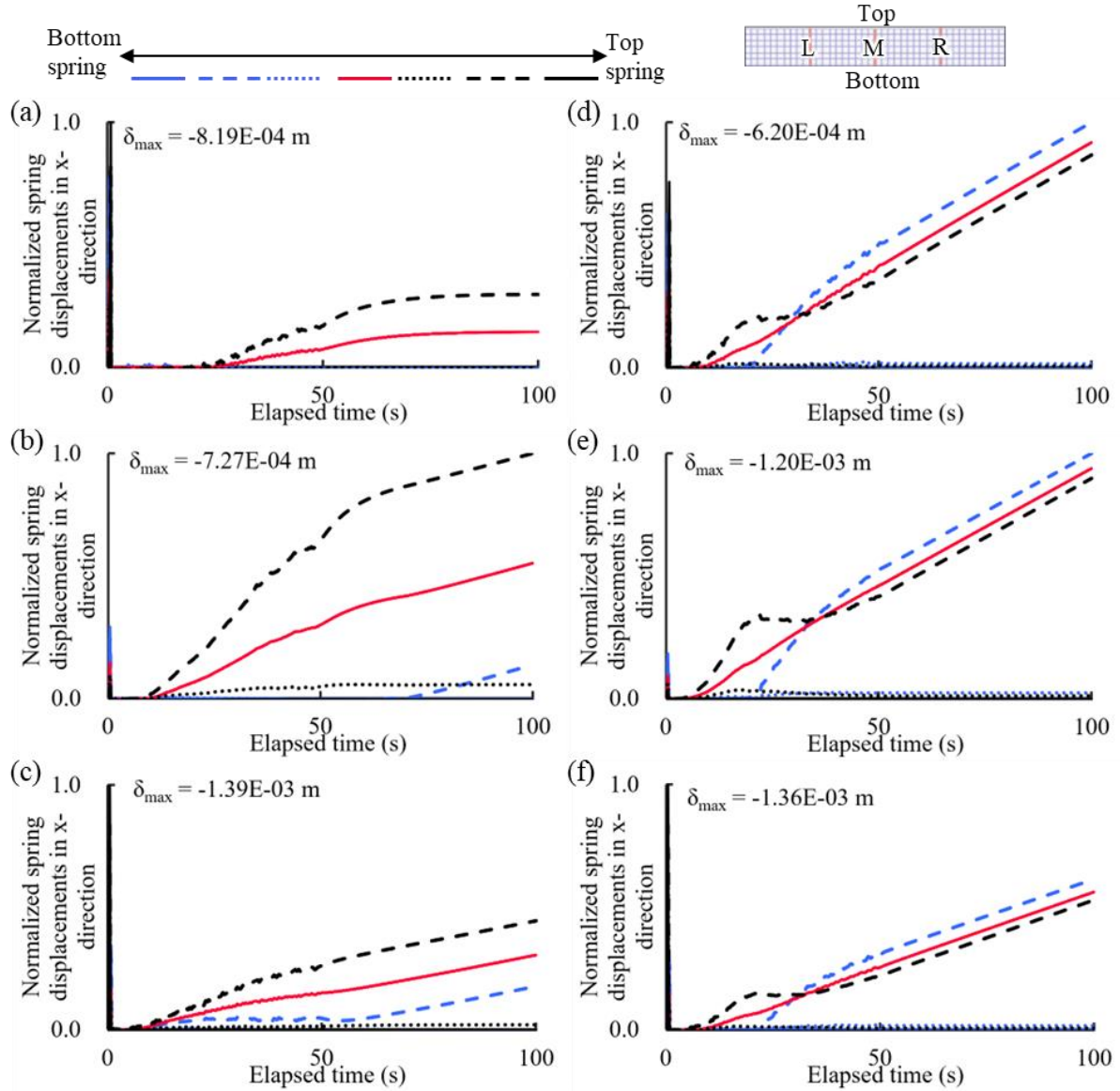


Figure 4.19. Spring element displacements for 4 culvert units in 10 m liquefiable ground for Case-4D10 (a) left (L) (b) middle (M) and (c) right (R) joints and Case-4U10 (d) left (L) (e) middle (M) and (f) right (R) joints. Spring displacements in x-direction are normalized with maximum values in elapsed time in each analysis case.

4.5.3 Relationship between liquefaction (*ESDR*) and culverts joint opening index (*I_O*)

After careful consideration of all the analytical cases, detailed analysis on the relationships between the joint openings, liquefactions and ground performances were conducted. To achieve this, only two culvert unit cases were considered since they had considerable joint openings. Representative elements for further analysis were picked at the liquefiable layers only as the third and the fourth elements from the top for 3 m

(shallow) and 10 m (deep) liquefiable grounds respectively in the ground beneath the culverts mouth and joint section (Figure 4.20). The chosen elements were at the mid-section of the liquefiable ground in each case, since they had high liquefaction levels. The high liquefaction levels within the liquefiable layer were the cause of (a) ground deformation and subsequent loss of their shear strength leading to (b) the culverts movements. These elements were analyzed further in the next subtopics.

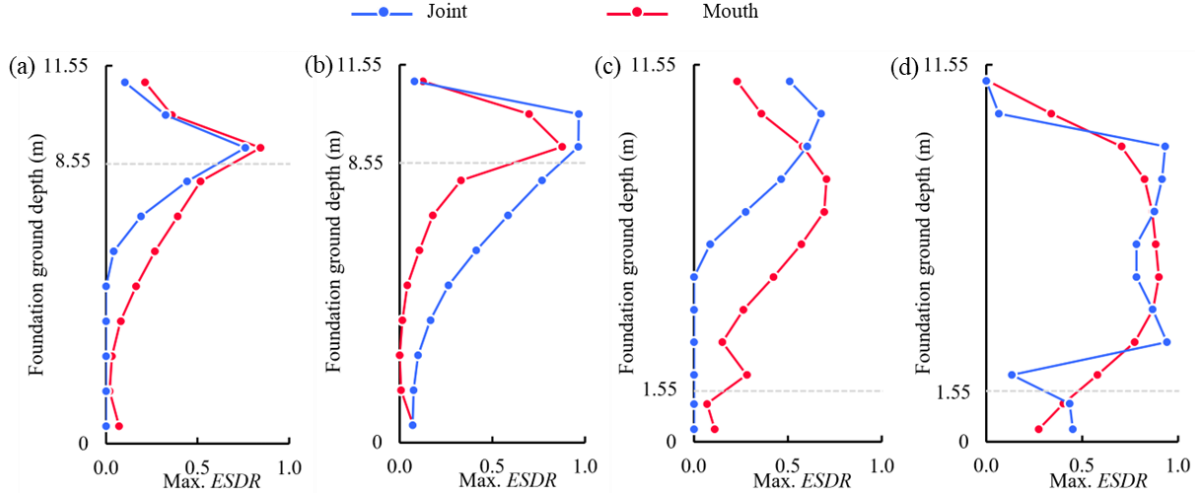


Figure 4.20. Vertical maximum *ESDR* variations beneath the mouth and joint in the foundation ground for (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 and (d) Case-2U10.

To correlate the ground deformation and culverts movements, the relationships between the opening index (I_o) at the top of the culvert joint, ground deformations and the *ESDR* of the selected elements beneath the mouth and the joint of the culverts; were analyzed as shown in Figure(s) 4.21 and 4.22.

The joint opening index, I_o , in Eq. (4.4), is the rate of change of joint opening per unit height of the culvert whereby δ_{xi} and δ_{xj} are the displacements in x-direction of the opposite nodes at the point of consideration along the joint while H is the total height of the culvert.

$$I_o = \frac{\delta_{xj} - \delta_{xi}}{H} \quad (4.4)$$

Ground deformation took place faster at the mouth section as compared to the joint section in all cases (Figure 4.21) at the initial stage of joint opening for all cases, and this controlled the culvert tilting on the foundation ground. For the deep undrained liquefiable ground, after the maximum joint opening was attained, faster deformation of the ground beneath the joint controlled the joint closure as compared to minimal gain in deformation beneath the mouth of the culvert. This resulted in the collapse of the joint opening which had occurred. From Figure 4.22., at the start of the analysis, it can be confirmed that the

initial stresses were captured well since the current and the initial stresses were equal, hence all the *ESDR* at start of joint opening, for the four cases, were zero.

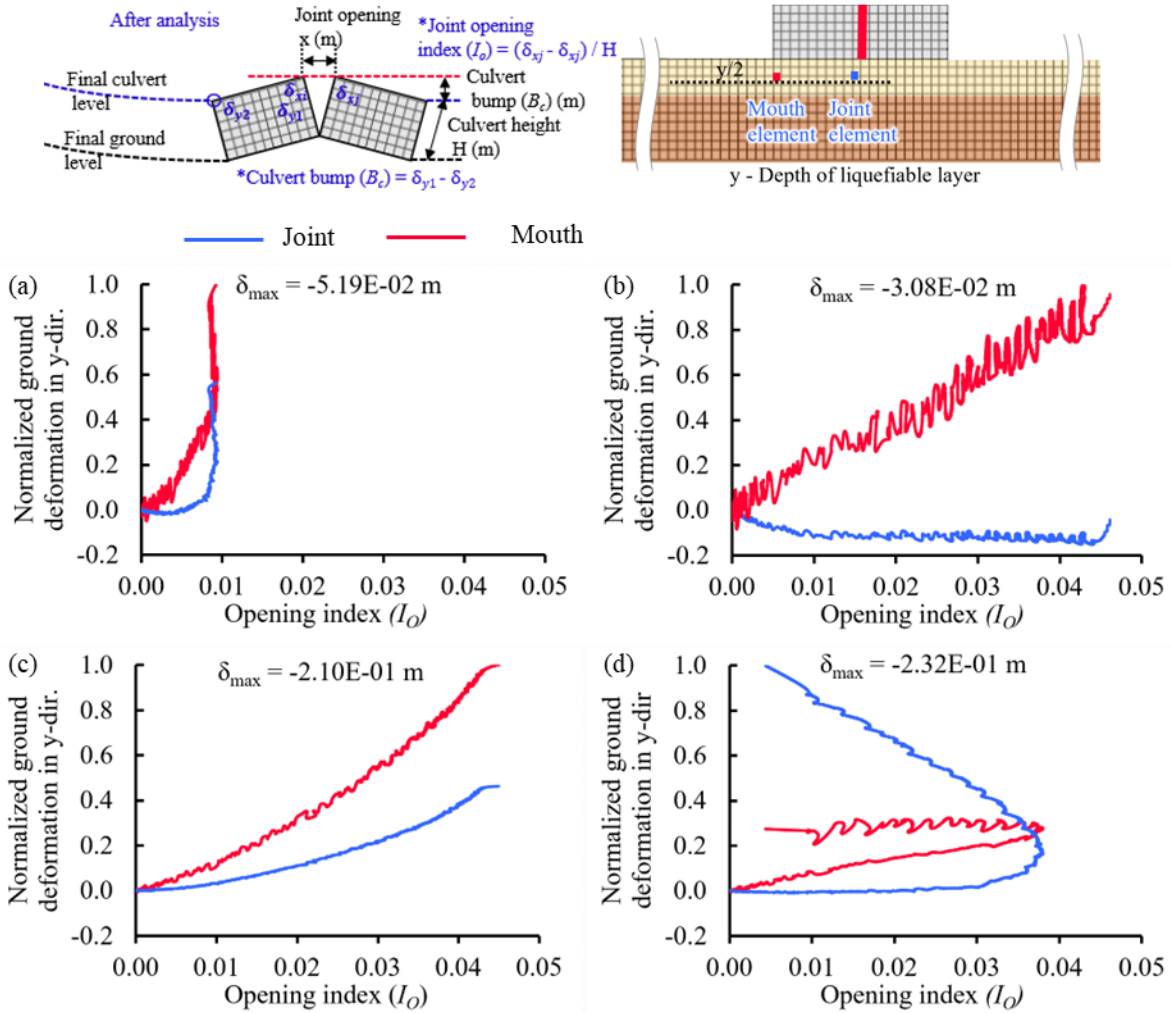


Figure 4.21. Ground deformation (in y - direction) - Joint opening index (I_O) relationships for (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 and (d) Case-2U10.

From all the four cases, all the relationships have three phases of *ESDR* variations with increase or decrease in I_O , (a) initial *ESDR* decrease (pre-liquefaction stage), (b) steady *ESDR* increase to maximum level (liquefaction stage) then (c) final *ESDR* changes (post liquefaction). The *first phase* (a) is characterized by an increase in mean effective stress of the supporting soil layer due to the culvert's initial shaking and loading of its dead weight on the foundation ground. This increase in mean effective stress results in negative *ESDR* as per Eq. (4.3).

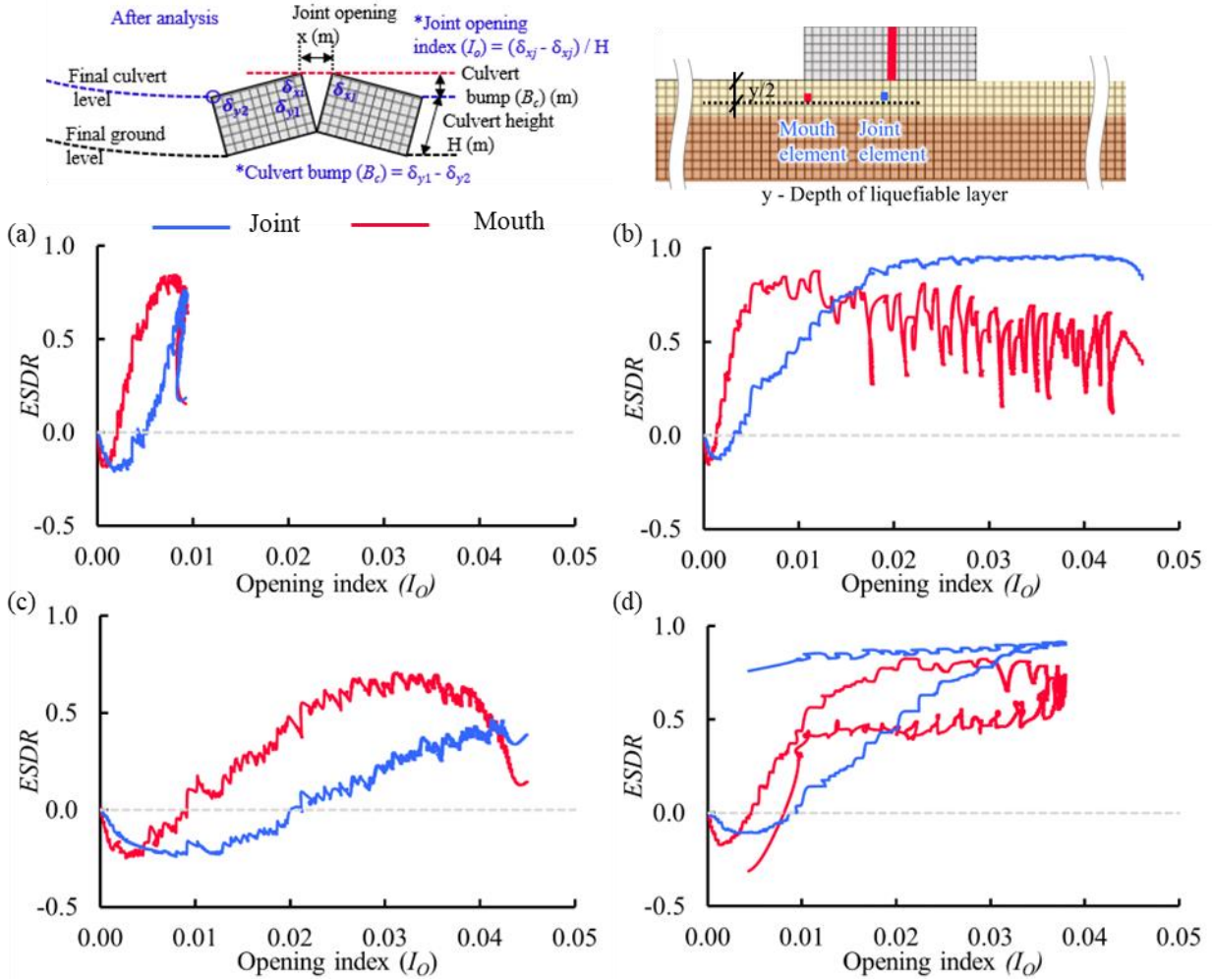


Figure 4.22. Effective stress decreasing ratio (ESDR) - Joint opening index (I_o) relationships for (a) Case-2D3 (b) Case-2U3 (c) Case-2D10 and (d) Case-2U10.

Once the foundation ground and the culvert loads are synchronized, then the analysis enters the *second phase (b)* of rapid joint opening due to liquefaction increase. At this stage, mean effective stress is converted to excess pore water pressure, liquefiable ground loses its strength becoming flowable hence culvert movement increases resulting in an increase in the culvert's joint opening. The joint opening is a function of higher liquefaction rate at the mouth as compared to the joint section. The ground beneath the culvert mouth loses its strength at a higher rate as compared to the mouth section resulting to the subsequent culvert inclination towards the mouth section. This scenario occurs because the culverts dead weight effect on the ground to suppress the shear strength reduction during liquefaction is lower as compared to the area beneath the culvert including the joint section. Finally, after the maximum liquefaction levels are reached, the analysis enters the *third phase (c)*.

This is categorized by ground effective mean stress recovery at various levels at the mouth and joint part. This *ESDR* recovery difference results in the final positioning or inclination of the culvert.

Phase one does not contribute significantly to the joint opening. From phase two, the joint opening is caused by the higher liquefaction at the mouth as compared to the joint. This rate of change of I_o with *ESDR* change is higher in deep liquefiable ground as compared to the shallow ground. The influence of undrained boundary condition between the culverts and foundation ground has an amplifying effect to the I_o . In the last phase, the final position of the culvert inclination is determined, and the following cases are discussed.

Case 1: For the drained shallow and deep liquefiable ground, once maximum *ESDR* is reached, the final joint opening and culvert inclination position are achieved, despite slight recovery of mean effective stress recovery at the joint and the mouth which take place at the same magnitude at the post liquefaction stage. The final joint opening was high for the undrained conditions, except for the deep liquefiable ground where the joint opening was lost due to enhanced liquefaction beneath the culverts. For the drained condition, deep liquefiable ground favored the joint opening, due to higher net liquefied ground beneath the culverts.

Case 2: For the undrained shallow and deep liquefiable ground, maximum *ESDR* does not define the final joint opening and culvert inclination position. For the shallow liquefiable ground, maximum *ESDR* is reached when the I_o is still low, then rapid increase in I_o to maximum levels takes place with minimal mean effective stress recovery. For the deep liquefiable ground, once the maximum *ESDR* and I_o are reached, rapid decrease in I_o follows. In terms of effective mean stress recovery rate, a very high rate occurs at the mouth section and almost zero recovery at the joint section. This stress imbalance occasioned by ground regaining its strength at mouth section and still low strengths at the joint section results in the loss of the joint opening and inclination of the culvert which had taken place initially, hence the inverted V shape joint opening as seen in Figure 4.17 (d). The deep liquefiable ground case with undrained boundary condition results in a unique final culvert opening behavior (inverted V- Δ) as opposed to the normal V shaped joint opening and normal culverts inclinations to their mouths. These joint displacement results in Figure 4.17 confirm the discussion of the effective stress decreasing ratio (*ESDR*)-joint opening index (I_o) relationships, on the culverts joint opening modes during analysis and final joint opening magnitudes at the end of the analysis.

4.5.4 Foundation ground stress paths and shear stress - shear strain relationships

The elastoplastic model used was based on the concept of subloading and superloading, maintaining the critical stress ratio ($M=1.3$; Table 3.5); the gradient of critical state line (C.S.L.); constant. Since the critical

areas for culvert inclination were found to be beneath the mouth and the joint section, as shown in Figure 4.13 and Figure(s) 4.21 and 4.22, the foundation grounds stress paths and the stress-strain behavior, Figure 4.23 and Figure 4.24 were analyzed. The mid-liquefiable layer elements beneath the culvert mouth and joint used in developing Figure(s) 4.21 and 4.22, were selected for consistency in the discussions, since they had a lot of influence on the liquefaction and resulting deformation of the soils in the foundation ground and existed within middle section of the liquefiable ground.

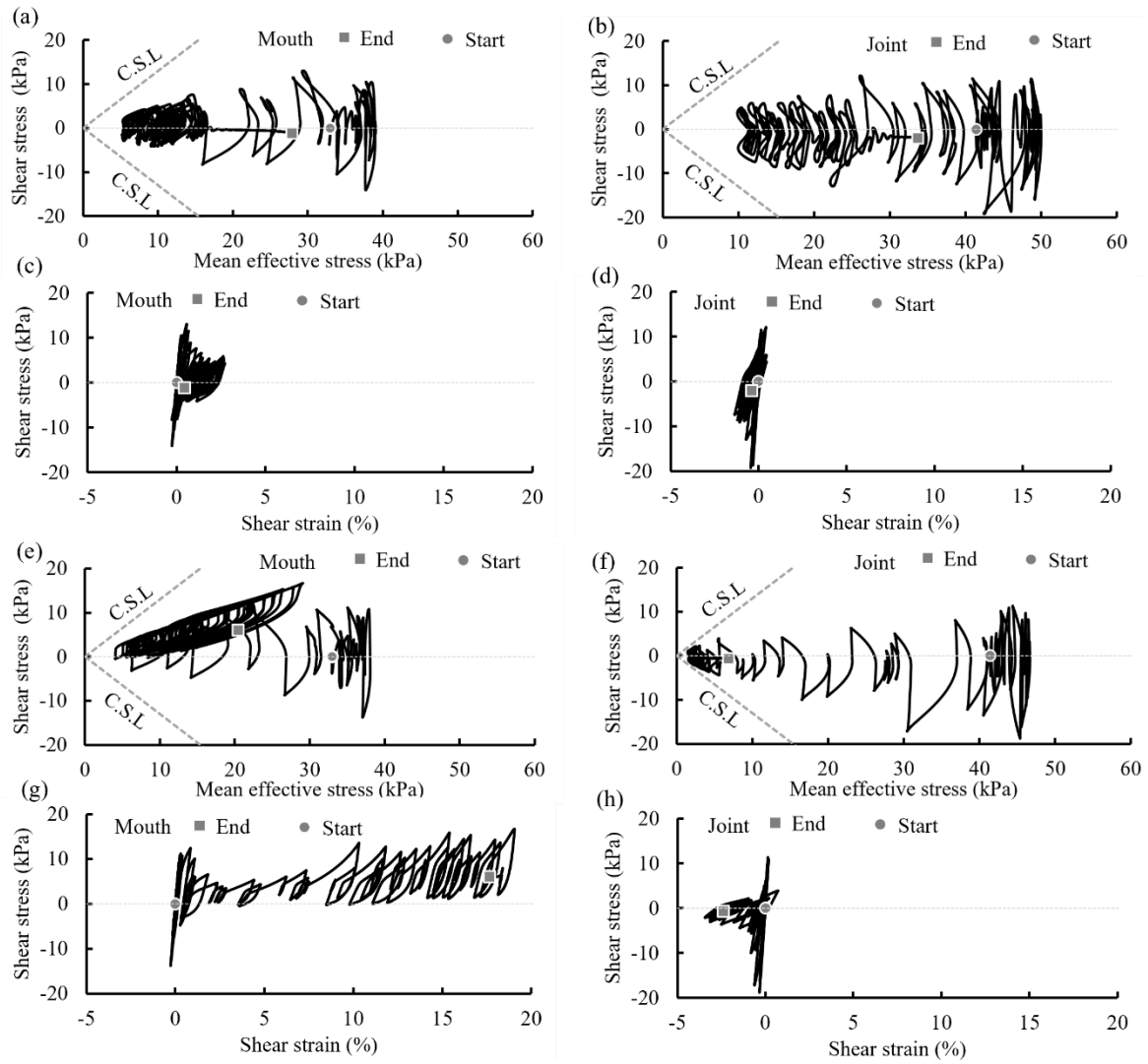


Figure 4.23. Case-2D3 stress paths beneath culvert at (a) mouth and (b) joint and corresponding shear strains at (c) mouth and (d) joint. Case-2U3 stress paths beneath culvert at (e) mouth and (f) joint and corresponding shear strains at (g) mouth and (h) joint.

Similar to the explanation of Figure 4.22, the initial mean effective stress increase is observed in all cases. The cyclic behavior of the foundation soils was clearly replicated. There was a commonality of stress

paths for undrained condition at mouth and joint, whereby, mean effective stress (*MES*) recovery after final *ESDR* had been reached was witnessed at the mouth section but very little at joint section. This was thought to be the result of the large shear strains at the mouth section as compared to the joint section. The undrained cases (mouth and joint) shear stress behavior is thought to have resulted in the culvert inclination due to high deformations at the mouth section as compared to the joint section, and residual high excess pore water pressures at the joint sections (Figure 4.15).

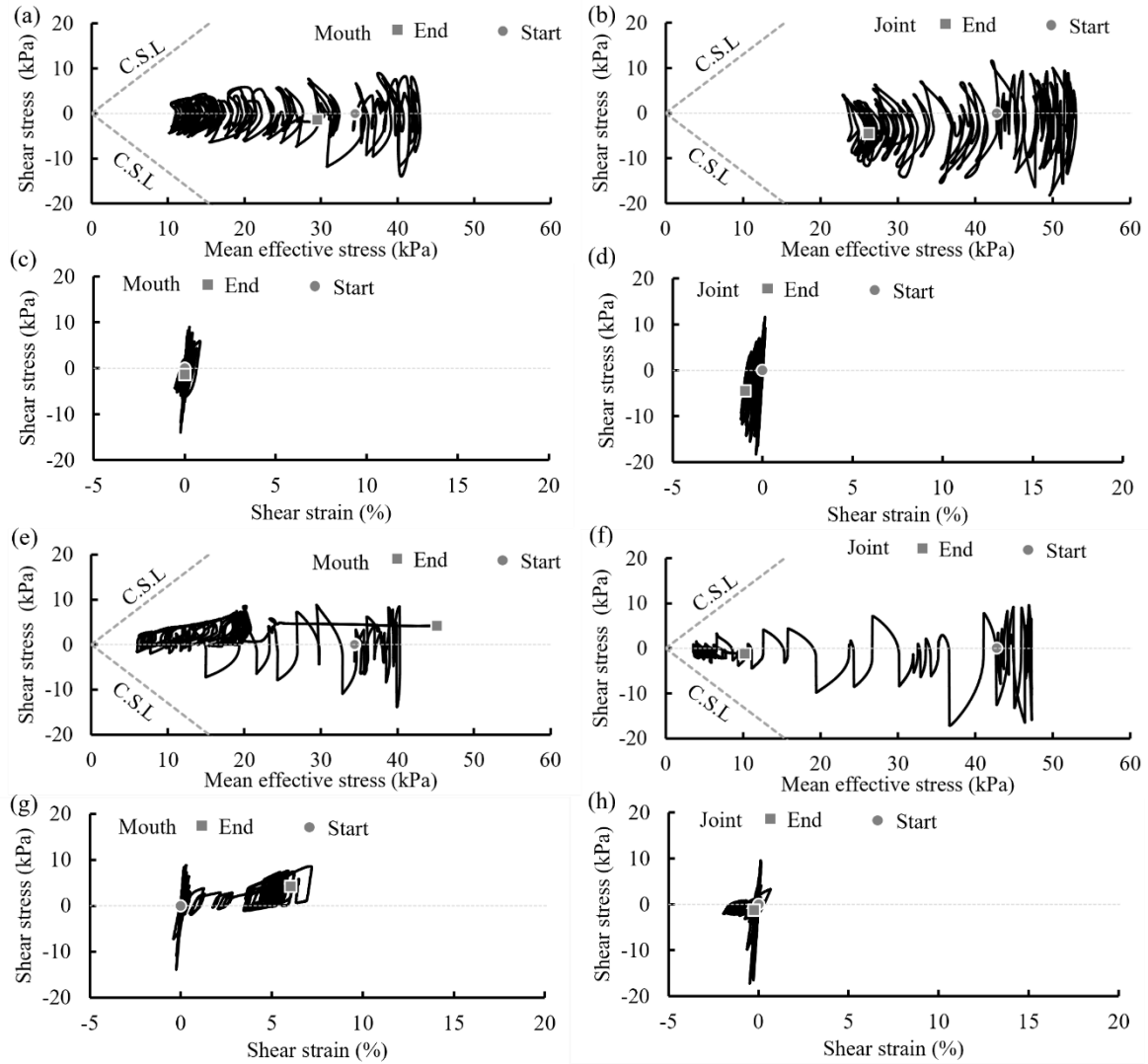


Figure 4.24. Case-2D10 stress paths beneath culvert at (a) mouth and (b) joint and corresponding shear strains at (c) mouth and (d) joint. Case-2U10 stress paths beneath culvert at (e) mouth and (f) joint and corresponding shear strains at (g) mouth and (h) joint.

It can be concluded that the undrained cases explained the culverts inclination behavior based on the residual EPWP, *ESDR* and *MES* at the joints and the mouths. Also, the maximum shear strains were within the experimental results limit of 23% (Figure 3.8) considering the analysis was done for the liquefiable ground elements.

The initial shear stress at K_0 is confirmed to be zero for all the cases and at that time no shear strains were generated hence the analysis was believed to have captured the initial set parameters well.

4.5.5 Culvert bump (B_c) development during liquefaction, and (I_o - B_c) relationships

During liquefaction, apart from lateral movement, culverts experience vertical movement resulting in the creation of a bump. This also affects the traffic movement on the road surface, since if the value is not controlled, the bumps may lead to traffic restrictions being implemented on the road as discussed by Tokita et al., (2007) during the Northern Miyagi Prefecture Earthquake in 2003 and the Mid Niigata Prefecture Earthquake in 2004. Culvert bump, for this study, is defined as the vertical displacement difference between the top opposite nodes of the culvert (at the joint- δ_{y1} and at the mouth - δ_{y2}) under consideration and is denoted by B_c (Eq. 4.5).

$$B_c = \delta_{y1} - \delta_{y2} \quad (4.5)$$

The time history series of these bumps during liquefaction were evaluated as portrayed in Figure 4.24.

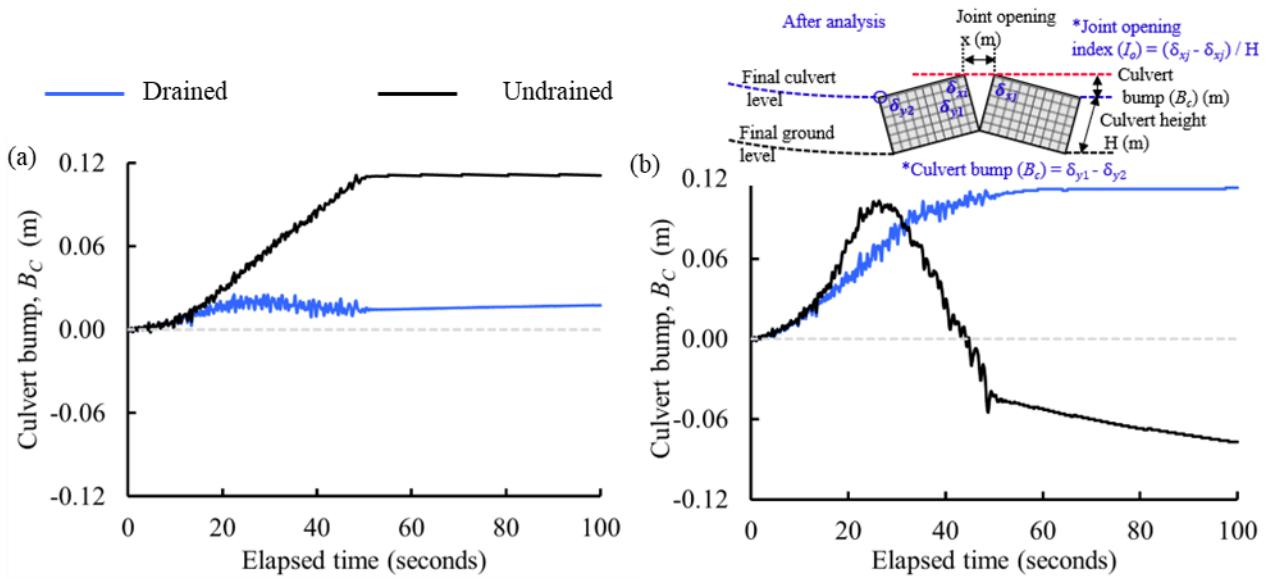


Figure 4.25. Culvert bump (B_c) time histories for (a) Case-2D3 and Case-2U3 and (b) Case-2D10 and Case-2U10.

From the bump behavior, normal upward bump creation, resulting from joint separation (Figure 4.17) during liquefaction, occurred in shallow liquefiable case with higher bump created in undrained boundary condition case due to higher joint separation as compared to the drained boundary case. These bumps were retained at the end of the liquefaction, similar explanation as given to joint opening index (I_o). Similarly, like in the undrained shallow case, normal bump was created and retained at the end of liquefaction in drained deep liquefiable layer case, but the case was different in undrained deep case. The bump was created, lost and then negative bump (depression was created). This was in response to the initial upward joint opening (V), closing and final downward opening of the joint (Λ) as explained in section 4.5.3. Thus, a correlation was believed to exist between the joint opening index (I_o) and culvert bump (B_c) hence, Figure 4.26 was generated. It should be noted that the joint opening index was developed using the top nodes of the joint. Hence, from Figure 4.26, a residual separation of the culverts remains at top. A reverse behavior is expected to occur for the bottom nodes due to linearity nature of joint opening mechanisms (Figure 4.17).

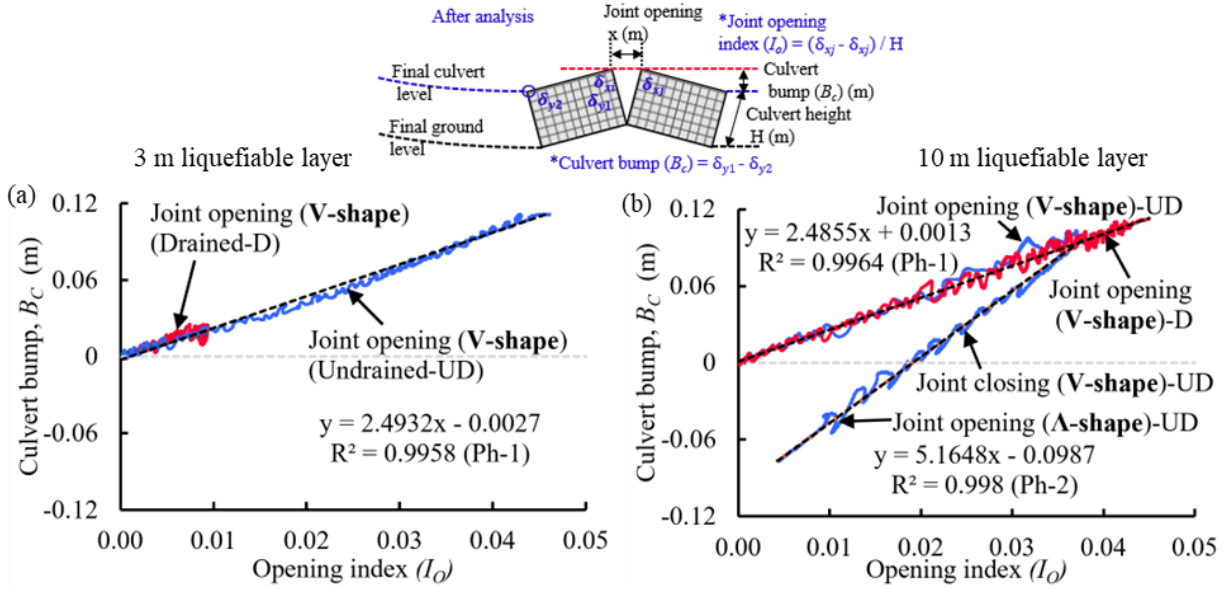


Figure 4.26. Culvert bump (B_c)-joint opening index (I_o) relationships for (a) Case-2D3 and Case-2U3 and (b) Case-2D10 and Case-2U10.

From Figure 4.26, a new relationship for approximating the bump development due to culvert joint opening is discovered {with very good coefficient of determination, R^2 , of 0.996 in phase 1 (Ph-1)}. This was deduced after the relationships between upward (V) joint opening index (I_o) and culvert bump (B_c) for 3 m and 10 m shallow and deep liquefiable layer (4 cases; Case-2D3, Case-2U3, Case-2D10 and Case-2U10) gave similar correlation which can be approximated linearly (as in form of $y=mx+c$). The term c can be ignored since its value is insignificant in the expression (1-2 %), hence the final relationship is given as

(Eq. 4.6); where y is the formed bump in meters, due to x joint opening index, calculated based on the maximum opening of the culvert top part of the joint and culvert height, while m is the correlation value given as 2.5 meters. For the downward (Λ) joint opening relationship in phase 2 (Ph-2), further studies with different cases are required to arrive at good correlations.

$$y = mx \quad (4.6)$$

It is worth noting that from Figure 4.26, the following can be evaluated, (a) magnitude of culvert joint opening (b) amount of the bump created (c) joint opening shape; if the gradient of the line is positive, opening of joint is upwards, if gradient is negative, the opening is inverted, and if the gradient is zero, the joint is closed -Eq(s). 4.7 and (d) reading with Figure 4.25, the time for various changes in the joint opening shape can also be determined.

$$\text{If } \frac{\delta B_c}{\delta I_o} > 0, V - \text{shape}; \frac{\delta B_c}{\delta I_o} = 0, \text{joint closed and } \frac{\delta B_c}{\delta I_o} < 0; \Lambda - \text{shape} \quad (4.7)$$

4.6 Summary of 2D analysis

The general trend of the simulator results and current numerical case study results were similar. The presence of culverts on the foundation ground influenced the liquefaction behavior of the foundation ground. The undrained boundary beneath the culverts alleviated the liquefaction levels due to high EPWP levels at the culvert joint and reducing towards the culvert mouths. In the joint opening behavior, the spring model produced the culvert displacements, and this was confirmed by the actual analysis of the joints (joint opening index) based on the culvert's nodes alone. V shaped joint opening for the shallow liquefiable ground and inverted V shaped joint opening for the deep liquefiable ground occurred like the experimental centrifuge test results for the 2 culvert models. No significant joint opening occurred for the 4 culvert unit models. Also, a relationship between the culvert bump and joint opening index was developed which can be used in real site situations to know the amount of opened joint based on the bump formed.

References

- Hughes, T. J. R., Taylor, R. L., Sackman, J. L., Curnier, A. and Kanoknukulchai, W. (1976). A Finite Element for a Class of Contact-Impact Problems, *Computer Methods in Applied Mechanics and Engineering*, vol. 8, pp. 249-276.
- Ito, H., Atsuro, S., Masakazu, S. and Toshiaki, N. (2021). Study on behavior of culvert due to liquefaction during earthquake, *Journal of Structural Engineering*, vol. 67A, pp. 161–173. (in Japanese)

- Japan Road Association (2010). *Road Earthwork Guidelines - Guidelines for Culvert Work* (in Japanese).
- Japan Road Association (2012). *Specifications for Highway Bridges Vol. V: Seismic Design*.
- Kim, S. H., Shah, S. H. A., Woo, S. K., Chu, I. and Sim, C. (2022). Probability-Based Crack Width Estimation Model for Flexural Members of Underground RC Box Culverts, *Applied Sciences* (Switzerland), vol. 12(4), Article 2063. <https://doi.org/10.3390/app12042063>
- Nakashima, K., Kusaka, H., Saeki, M., Mano, M. and Ro, H. (2021). Analysis of damage to expressway embankments and culverts from earthquake on soft ground, *Journal of Japan Society of Civil Engineers, Ser. A1 (Structural Engineering & Earthquake Engineering (SE/EE))*, vol. 77, Issue 3, pp. 384-394. https://www.jstage.jst.go.jp/article/jscejseee/77/3/77_384/pdf/-char/ja
- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki, M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.
- Oka, F., Hirokazu, Y., Sayuri, K. and Teppei, K. (2015). New Evaluation Method for Liquefaction of Ground Using Dynamic Liquefaction Analysis Method and Its Application, *International Journal of Geomechanics*, American Society of Civil Engineers, vol. 16, No. 5.
- Suzuki, N. and Toi, Y. (1987). Finite Element Analysis of Dynamic Frictional Contact Problems by Using the Penalty Function Method -Consideration on the Penalty Numbers and its Application to the Friction-Excited Vibration Problem, *Journal of the Society of Naval Architects of Japan*, No. 162, pp. 364-373. (in Japanese)
- Tokida, K., Oda, K., Nakahara, A., Hayashi, K. and Yorifuji, M. (2007). Performance-based Road management against bump damage induced by earthquake based on driving test, *Proceedings of the JSCE Earthquake Engineering Symposium*, vol. 29, pp. 596-604. (in Japanese)
- Zhang, F., Ye, B., Noda, T., Nakano, M., and Nakai, K. (2007). Explanation of cyclic mobility of soils, Approach by stress-induced anisotropy, *Soil and Foundations*, vol. 47, No. 4, pp. 635–648.

CHAPTER FIVE

3D ELASTOPLASTIC ANALYSIS OF CULVERT-EMBANKMENT ON LIQUAFIABLE GROUND

5.1 Introduction

This chapter investigates the critical 2D analytical cases combining various components. This is because, in 2D analysis, the 2 culvert unit models had significant joint opening upon liquefaction of the foundation ground. For the 4 culvert unit models, though the liquefaction and EPWP variations were found to take similar trends as the 2 culvert unit models, insignificant joint openings occurred.

Since the 3D analysis is a confirmation of the analytical 2D selected cases, the analysis narrows down to the culvert joint opening and the localized liquefaction of the foundation ground. Stability analysis of the model, in terms of ground and culvert wave responses, is similar to 2D analysis, hence this section will be omitted in this chapter.

In the 2D analysis, the overburden effect was not considered as explained earlier in chapter Four. In this chapter, the effect of overburden and embankment are considered. The same seismic wave is used to bring the excitation of the model and induce liquefaction. In combination of all these components, the study narrows down to two analytical cases of 2 culvert unit models with shallow (3 m) and deep (10 m) liquefiable layers. Undrained boundary condition beneath the culverts is considered, whereas the open ground, without embankment and culvert, drained condition is considered.

5.2 Model descriptions

The two analytical models (for shallow and deep liquefiable grounds) shown in Figures 5.1 and 5.2, comprised of two culvert units, embankments on top and either side of the culverts (considering the transverse direction); modeled on two foundation layers, the liquefiable ground resting on a firm supporting layer. The dimensions were based on the experimental model proposed by Nanazawa et al., (2022), like the 2D analysis.

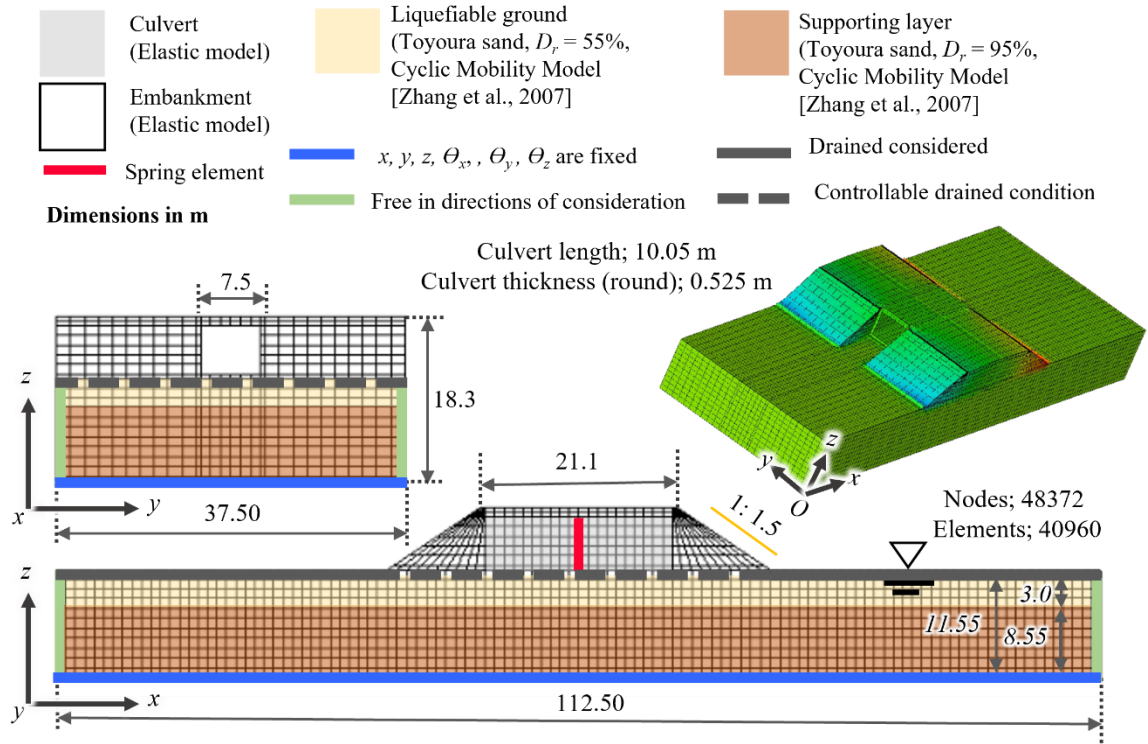


Figure 5.1. 3D model of two culverts units on shallow liquefiable ground.

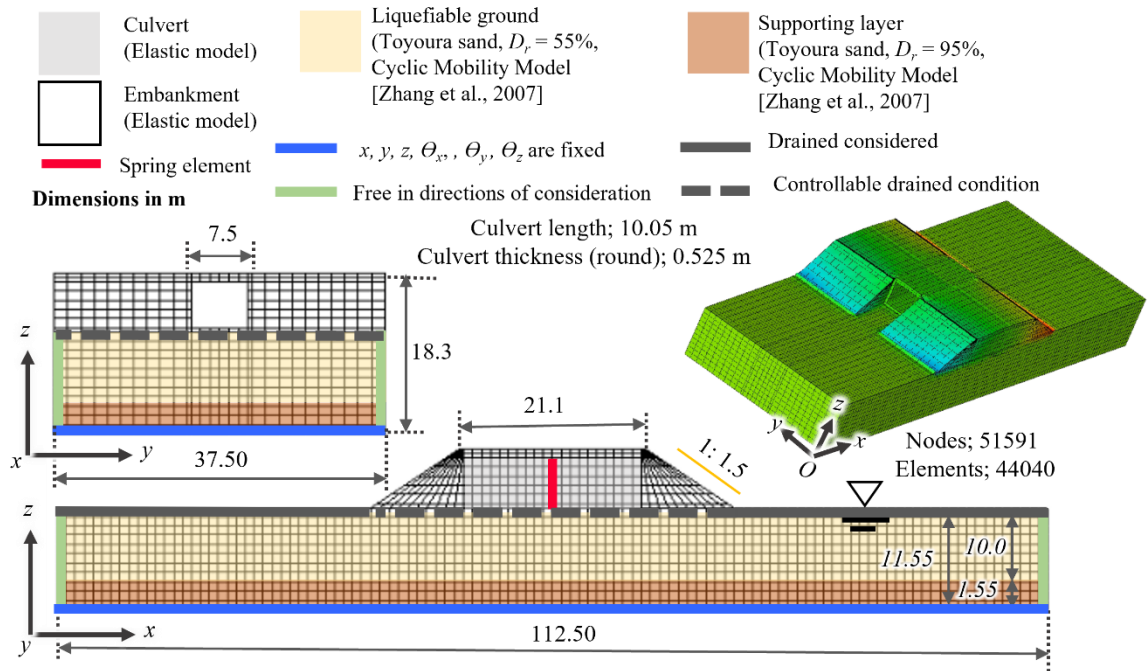


Figure 5.2. 3D model of two culverts units on deep liquefiable ground.

5.3 Parameters formulations, initial stresses, input wave and boundary conditions

The foundation ground, embankment soil, and cyclic mobility model are discussed in Chapter Three. The culvert parameters are shown in Table 3.1, in Chapter Four. For the spring elements parameters, the formulation principle was similar to the 2D analysis, but for the 3D, the rotation component (in 2d analysis) was replaced with z component. The undrained boundary condition was applied only at the contact point between the culvert units and the liquefiable ground. Beneath the embankments, natural phenomena was assumed to exist hence no drainage condition was considered. On top of the foundation ground (section extending past the culverts and embankment), drained condition was considered since there was no structural unit or embankment was resting on the ground. The bottom of the foundation ground was rigidly fixed in x, y, and z and their respective rotations. The vertical sides were free to rotate in all directions and move in their directions of their orientation. The initial stresses were calculated in a similar way to the 2D analysis. The current results are based on initial stress of the model components determined as the K_0 state of the soil, considering the stability of the calculation. Separately, initial stresses of the soils of the foundation ground and the culverts were determined through a self-weight analysis of the elastic model. In the elastic model, the Young's modulus of the embankment was four times that of the initial Young's modulus obtained by the triaxial simulation of Edosaki sand (Japan Road Association, 2012).

5.4 Input wave and analytical cases

The input seismic wave (Figure 4.6 in Chapter Four) at the bottom of the foundation ground in the culvert longitudinal direction, was same as for the 2D analysis. For the analysis, the cases presented in Table 5.1 were investigated.

Table 5.1: 3D analytical cases

	2 culvert units model
3 m deep liquefiable ground	Case 1-3m
10 m deep liquefiable ground	Case 2-10m

5.5 Results for 3D elasto-plastic FEM analysis

The analytical results were mainly for comparisons with the 2D analytical cases, hence, to be able to capture the main comparison areas, more focus was done at the mid liquefiable layers for the two cases. The localization of liquefaction beneath the culvert was the critical area of focus for the 3D analysis. To bring more clarity to the results, Figure 5.3 was used to show the analytical elements for further analysis and

discussions. The elements along the culvert mouth were referred to as (a1, a2 and a3) and the ones along the culvert joint were referred to as (b1, b2 and b3). 1, 2 and 3 means elements beneath embankment, culvert edge and midpoint of the culvert along the y-direction.

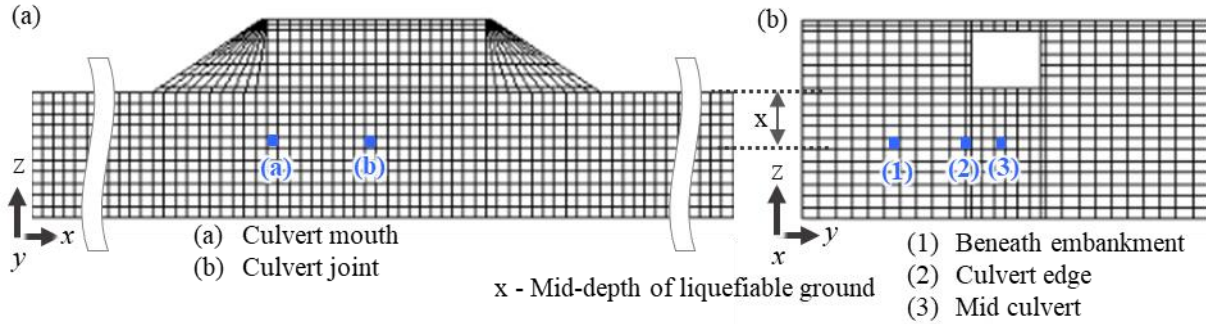


Figure 5.3. Analysis element's locations (a) beneath culvert mouth and joint for (b) beneath embankment, culvert edge and mid culvert points.

5.5.1 Liquefaction analysis and ground settlement

For the liquefaction analysis, the concept of effective stress decreasing ratio (*ESDR*) was adopted, similar to 2D analysis, in Chapter Four. The heatmaps for the liquefaction results are as shown in Figure 5.4. The shown results were up to the end of the seismic wave which was 50 seconds.

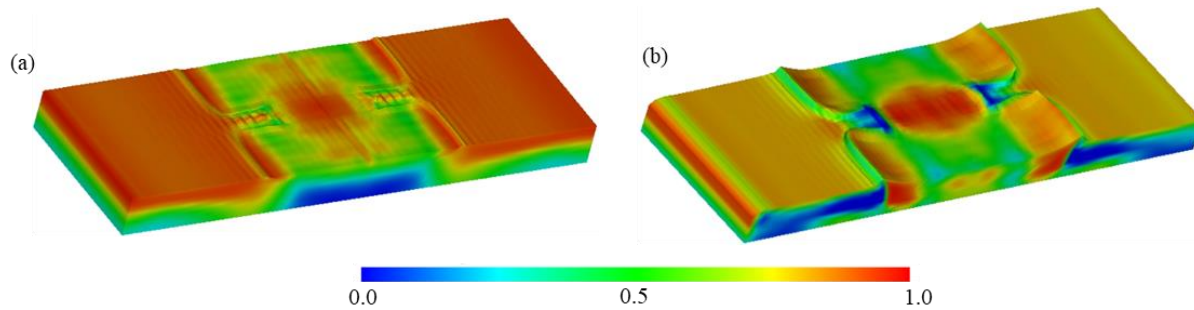


Figure 5.4. *ESDR* heatmaps for the foundation ground (at the end of analysis at 50 seconds) for Case 1-3m and Case 2-10m.

From the heatmaps, the model with deeper liquefiable ground generated higher net liquefaction area beneath the culverts and embankments as compared to the shallow liquefiable ground. This was contributed by the large soft ground at the end of liquefaction for the deep liquefiable ground as compared to the shallow liquefiable ground.

The influence of culvert on ground liquefaction is clearly portrayed in both cases. Beneath the culvert higher liquefaction occurred as compared to the section beneath the embankment. This clearly shows that

the presence of culvert on foundation ground accelerates the liquefaction due to its light weight and less confining stress to the foundation ground. This causes the culvert shaking higher than the embankment which exerts higher confining stress on the foundation ground. This increased shaking accelerates the liquefaction beneath the culvert elements, and the effect is increased by the increased depth of the liquefiable ground. Further analysis of the liquefaction propagation beneath the culvert and embankment was carried out as shown in Figure 5.5.

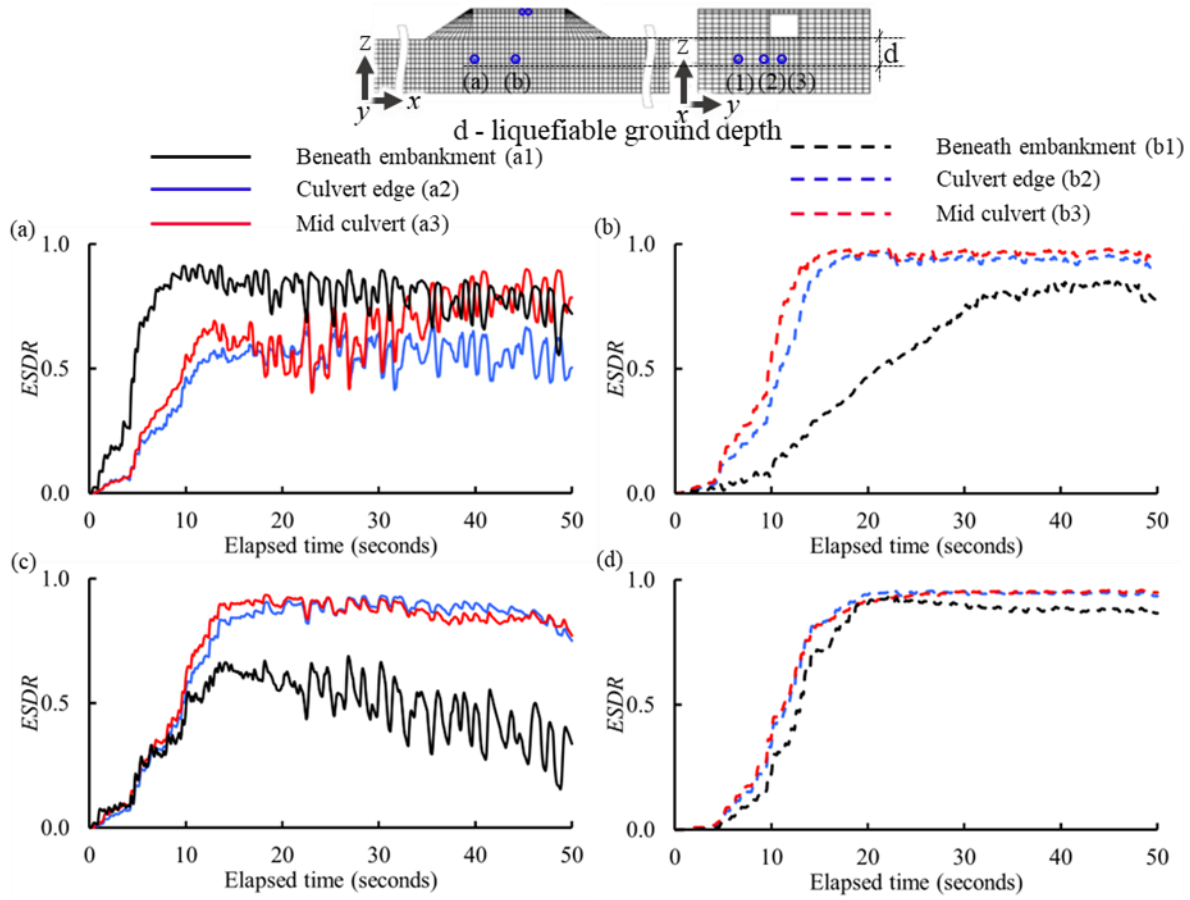


Figure 5.5. *ESDR* variations along Case 1-3 m (a) culvert mouth; (b) culvert joint and Case 2-10 m (c) culvert mouth; (d) culvert joint.

From Figure 5.5 (b) and (d), along the joint section, the order of liquefaction propagation was from the middle of the culvert, then to the culvert edge and finally to the embankment. This order was a confirmation of the experimental results by Nanazawa et al., (2022) and Miyazaki et al., (2024), where liquefaction increased rapidly beneath the culvert then beneath the embankment.

Similarly, along the mouth section, deep liquefiable ground (Figure 5.5 (c)) also confirmed the experimental results by Nanazawa et al., (2022) and Miyazaki et al., (2024), but for the shallow liquefiable

ground, rapid liquefaction took place first beneath the embankment then at the middle of the culvert and finally at the culvert edge.

Notably, also, is that, beneath the culvert, at the middle section and edge, the liquefaction behavior was not so much different, portraying the uniformity in the influence of the culvert on the liquefaction of the foundation ground as compared to the influence of the embankment. The liquefaction of the foundation ground was brought about by the conversion of mean effective stress to excess pore water pressure as seen in Figure 5.6.

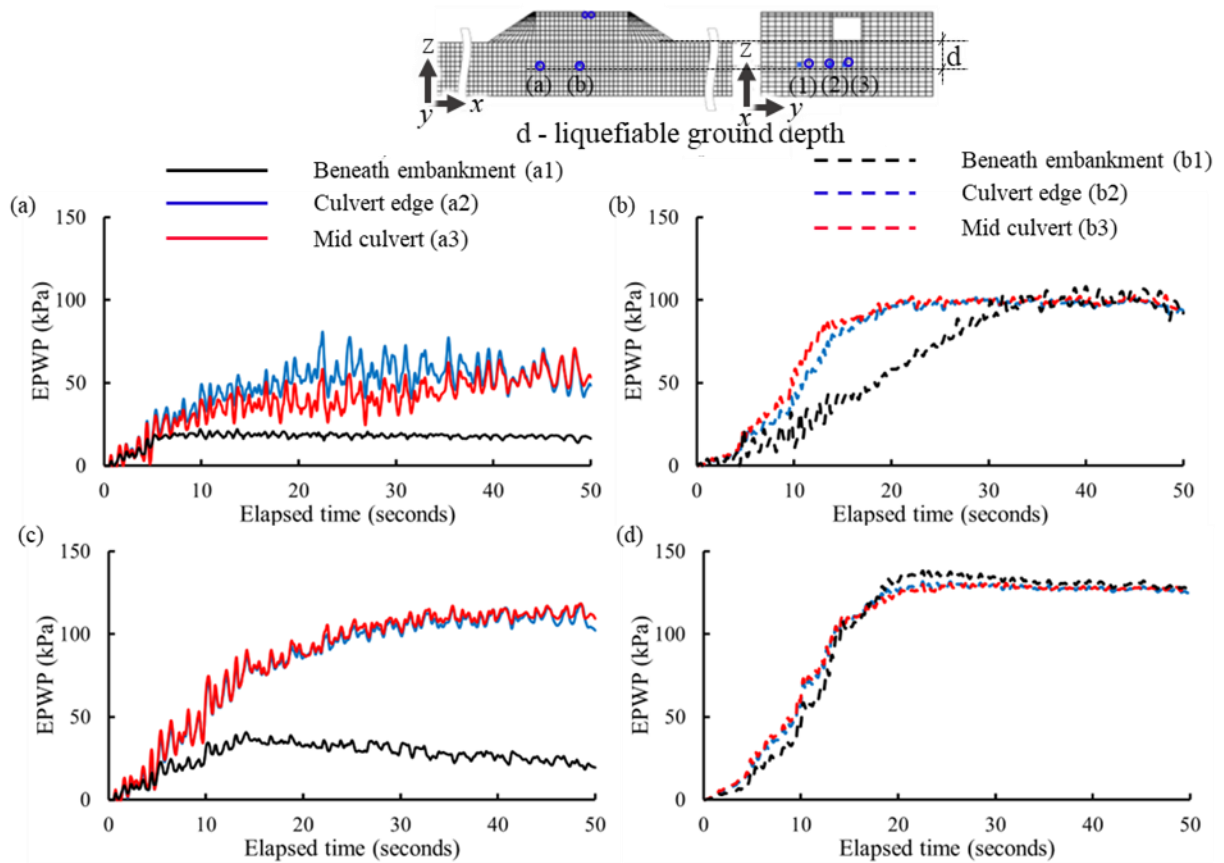


Figure 5.6. EPWP variations along Case 1-3 m (a) culvert mouth; (b) culvert joint; and Case 2-10 m (c) culvert mouth; (d) culvert joint.

The joint section achieved higher EPWP levels as compared to the mouth section. During the culvert shaking, the combined effect of liquefaction beneath each culvert contributes to the high liquefaction levels. This behavior is like the 2D analysis in all analytical cases where high EPWP leading to higher liquefaction levels at the culvert middle joints occurred. Clearly, the same performance of the EPWP reflects the exact performance of the liquefaction, in Figure 5.5. This confirms that EPWP variation during analysis was the

main contributor to the liquefaction of the foundation ground. To generalize the liquefaction behavior beneath the culvert mouths and joints, Figure 5.7 was developed. From the summary, there was an increase in liquefaction from the mouth to the joint section. Also, the effect of deep liquefiable ground on increased liquefaction was confirmed. These results were a confirmation of liquefaction performance for the 2D analytical cases, where joint sections had higher liquefaction than mouth section in most of the analytical cases. Also, the results confirmed Nanazawa et al., (2022) and Miyazaki et al., (2024) experimental results where localized and progressive liquefaction directly under the box culvert had occurred and progressively reduced towards the embankment side.

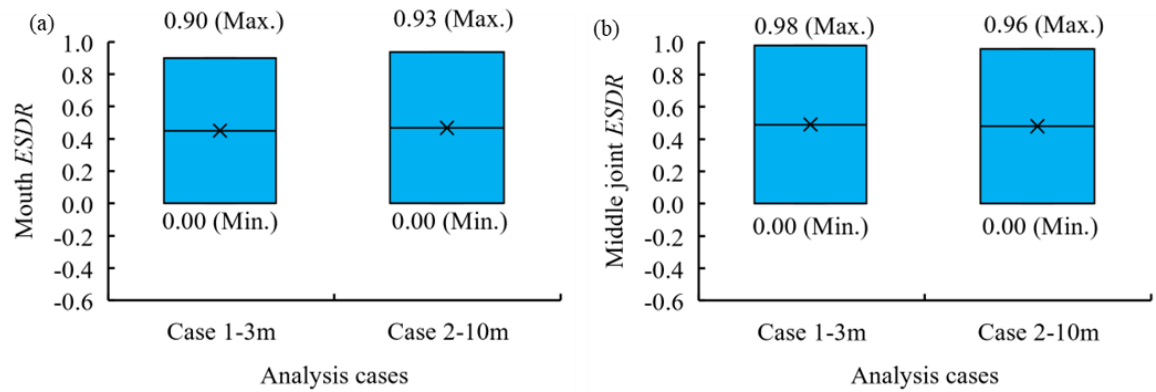


Figure 5.7. Summary of liquefaction variations between the mouth and joint section for each analysis case.

Due to the ground softening due to liquefaction, model displacements occurred as shown in Figure 5.8. It can be confirmed that the deep liquefiable ground had greater ground displacements in both z and x directions as compared to the shallow foundation. The embankment flow towards the foundation ground is higher for the deep than the shallow liquefiable ground. This is attributed to the high liquefaction beneath the culverts softening the ground more hence the sinking of the culverts and the residual displacement of the embankments. At the toes of the embankment, positive displacement in z-direction occurred. This was due the scooping out effect (displacement) of the adjacent foundation ground by the sinking embankment toes, hence pushing the adjacent ground upwards. The effect varies with the depth of liquefiable ground, since the softer the ground, the higher the displacement effect.

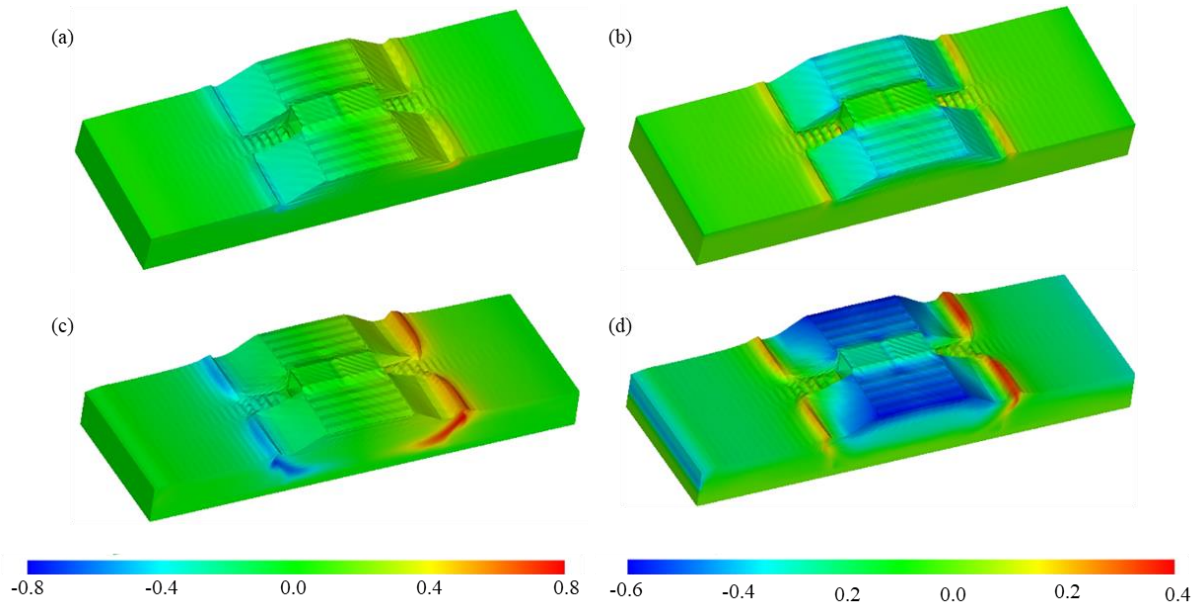


Figure 5.8. Displacements heatmaps for Case 1-3 m in (a) x-direction (b) in z-direction and for Case 2-10 m in (c) x-direction (d) in z-direction.

5.5.2 Culvert joint displacements

The culverts displacements were determined from the spring elements as shown in Figure 5.9. The 3D spring displacement behavior confirms the 2D analytical cases for shallow and deep undrained boundary conditions of 2 culvert units models.

For the shallow, liquefiable ground, at the end of the earthquake (at 50 seconds), joint displacement was increasing with time and increasing from bottom to top of the culvert. This confirms the V-shaped joint opening behavior for similar 2D analytical cases.

For the deep liquefiable ground, joint opening increased, reached maximum and started closing, by the end of the earthquake. Notably is that faster joint opening was occurring in deep liquefiable ground than shallow liquefiable ground, hence the deep liquefiable ground reached peak joint displacement faster than shallow liquefiable ground. The shape of the joint opening as portrayed by the order of the spring elements displacements shows a V-shaped joint opening.

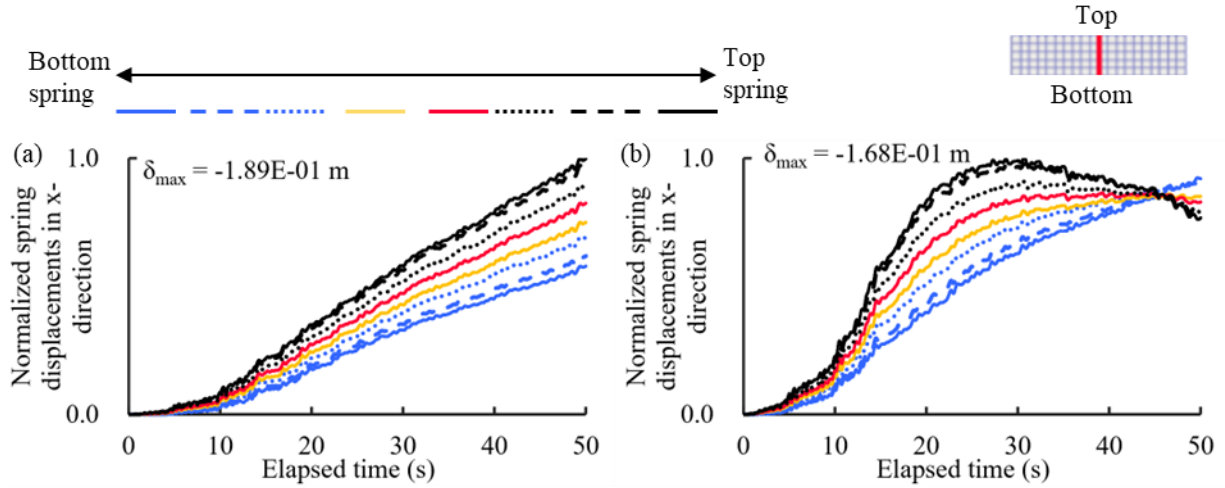


Figure 5.9. Spring element displacements for middle joint for (a) Case 1-3m and (b) Case-2. Spring displacements in x-direction are normalized with maximum values in elapsed time in each analysis case.

Figure 5.10 gives the variation of the joint opening summary for each analysis case. Though the maximum joint opening seems to be less deep as compared to the shallow liquefiable ground, from Figure 5.9, it was confirmed that joint collapse process took place after maximum attainment of the joint opening. It is also noted that the minimum joint opening for the deep liquefiable ground was higher than the shallow liquefiable ground, which was directly related to the higher liquefaction beneath the culverts. Deep liquefiable ground resulted in faster joint opening in both cases.

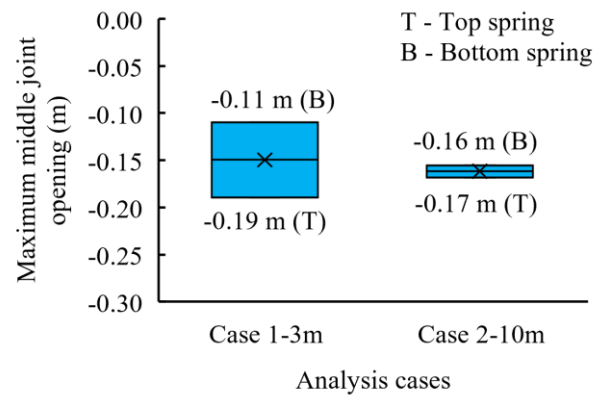


Figure 5.10. Spring element displacements for middle joint for (a) Case 1-3m and (b) Case-2. Spring displacements in x-direction are normalized with maximum values in elapsed time in each analysis case.

Analysis of ground settlement was done to check the behavior of the culvert rotation with respect to the mouth and joint part of the culvert, and the results are shown in Figure 5.11.

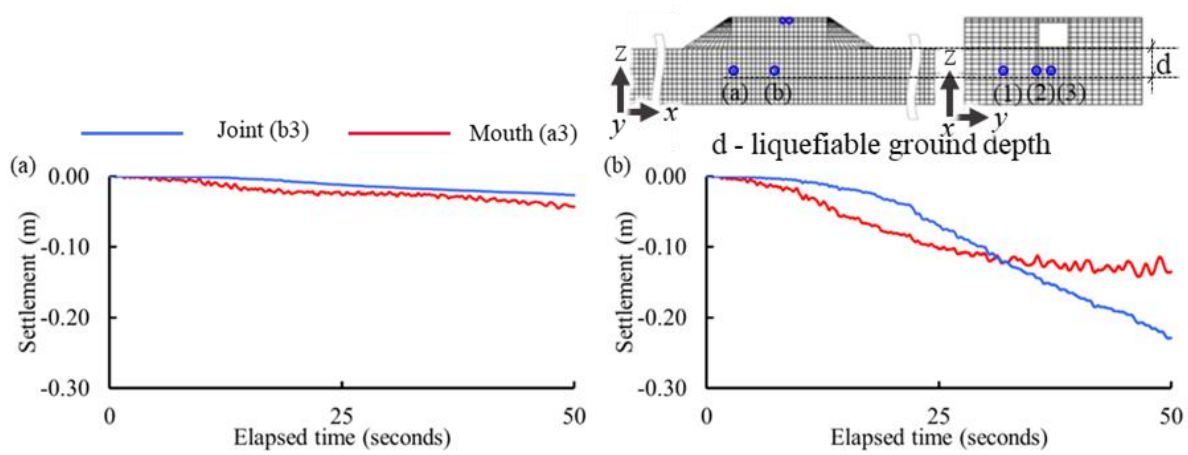


Figure 5.11. Ground settlement at the center of the culvert for (a) Case 1-3m and (b) Case 2-10m.

In both cases, the ground settlement took place faster at the mouth as compared to the joint section at the initial stages. For the shallow liquefiable ground, this process continued to the end of analysis which resulted to the culvert rotation towards the mouth due to the differential ground settlement between the joint and mouth section. For the deep liquefiable ground, initially the mouth section had faster settlement then the joint section took over. This scenario might have been caused by the higher liquefied net area beneath the joint section after the earthquake had stopped. At that time, the ground hardening at the mouth section seems to have taken place, hence reversing the culvert rotation at this point. These scenarios led to the joint opening behavior shown in Figure 5.9. The relationship between the joint opening and ground settlement was evaluated as shown in Figure 5.12.

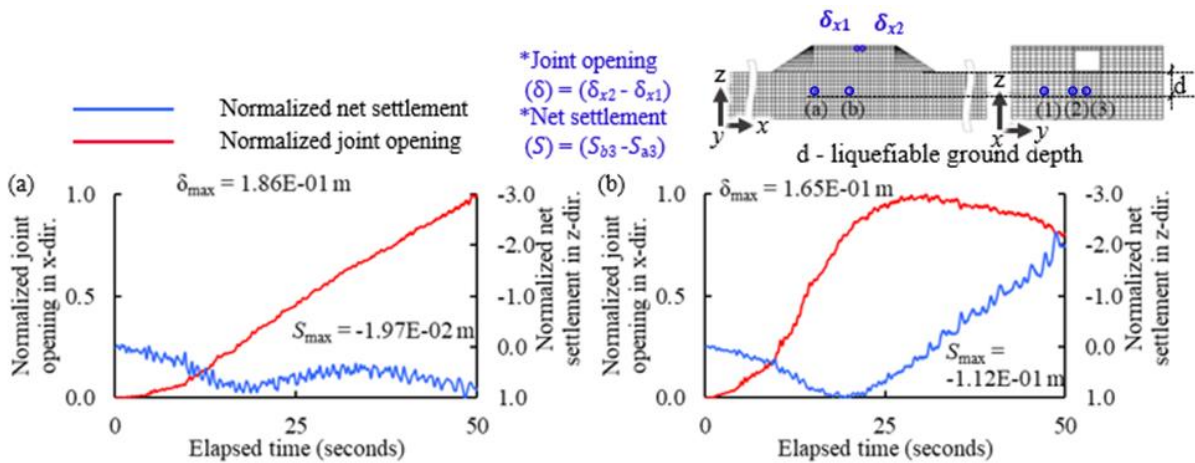


Figure 5.12. Relationship between joint opening and ground settlement (a) Case 1-3m and (b) Case 2-10m.

This relationship summarizes the behavior of joint displacement and ground settlement. The net settlement for the undrained condition changed upon reaching maximum, due to the ground hardening, upon end of the earthquake, at the mouth section whereas the joint section, the ground had not hardened. This was due to the high liquefaction at the joint section as compared to the mouth section. For the shallow liquefiable ground, the net settlement was not very much as compared to the deep liquefiable ground. From the same figure, the maximum joint opening can also be shown.

5.5.3 Stress paths and shear stress-strain relationships

The stress paths for the foundation ground elements were computed (Figures 5.13 and 5.14) to check how the ground responded to the seismic wave. For consistency, the same elements used for Figure 5.5 (*ESDR* analysis) and Figure 5.6 (EPWP analysis) were used.

For the shallow liquefiable depth case, larger stress paths occurred at the joint section as compared to the mouth section. Also, large shear strains occurred at the mouth section as compared to the joint section. The shear strains were increasing from the embankment side towards the middle of the culvert. The larger shear strains along the mouth section are contributed by the increased displacements of the embankment material towards the foundation ground. The larger stress paths along the joint section are because of increased shaking of the culvert structure due to its light weight as compared to the higher weight of the embankment material along the mouth section. Also, increased EPWP along the joint section is also a contributing factor to the large shear strain along the section.

For the deep liquefiable ground, along the joint section, there is minimal variations in terms of the stress paths and shear strains. Along the mouth section, the shear strains increased towards the middle of the culvert. From the culvert edge to the middle of the culvert the ground behavior is the same, a situation comparable with the liquefaction (*ESDR*) and EPWP variations.

The stress paths were within the Critical State Line (C.S.L.). The strains were within the maximum range of 23% as expected from the performance of the liquefiable soil in Figure 3.7 in Chapter 3.

5.6 Summary of 3D analysis

The 3D analysis replicated well the 2D analytical cases which were extended to the 3D analysis in terms of joint opening mechanism and liquefaction behavior of the foundation ground. For the 3D analysis, undrained boundary condition beneath the culverts was applied and this resulted to similar boundary conditions with 2D cases which, represented the realistic situation in the actual field.

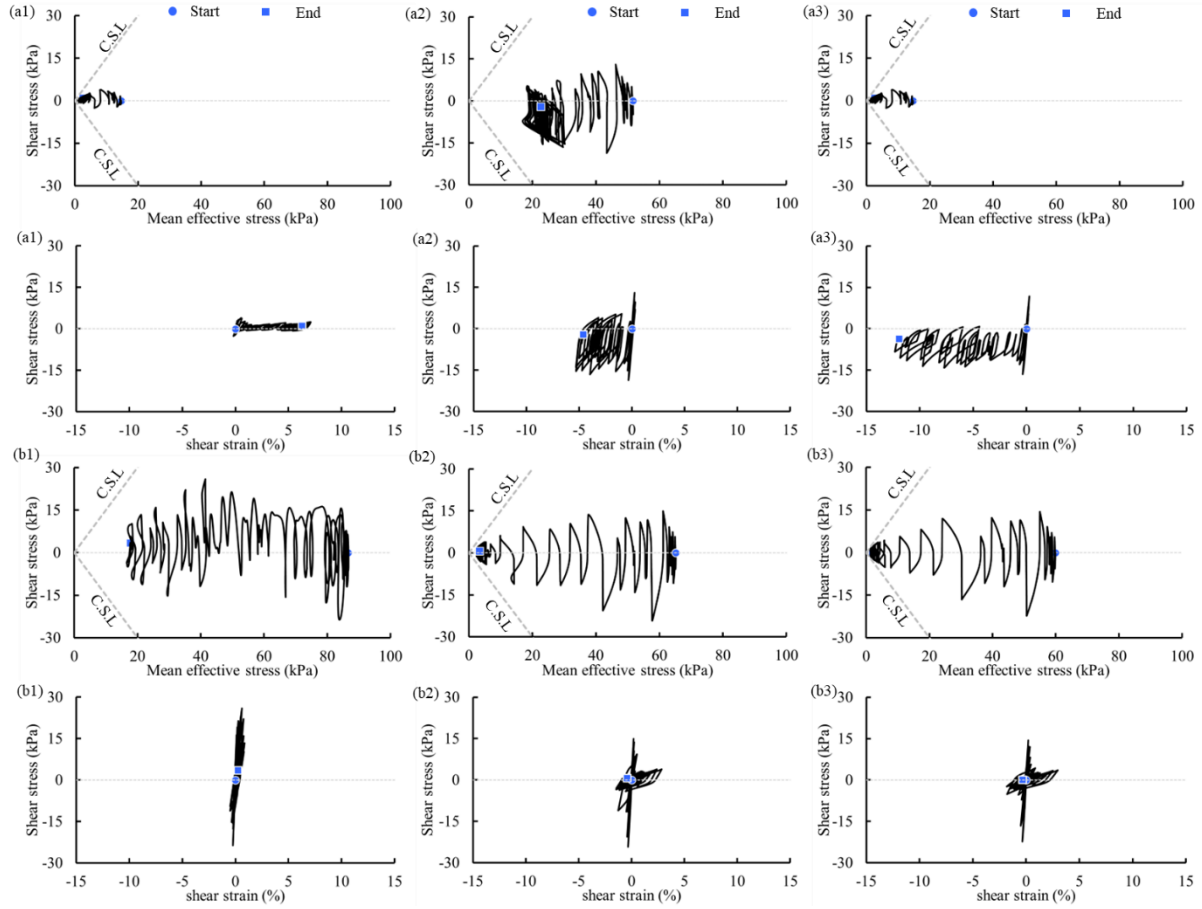


Figure 5.13. Case 1-3m: mouth elements stress paths for (a1), (a2) and (a3) and their corresponding stress - strain relationships; and joint elements stress paths for (b1), (b2) and (b3) and their corresponding stress - strain relationships.

From the analysis, the following can be deduced: -

1. Despite incorporation of overburden and embankment into the 3D model, it reproduced very well the 2D analytical results.
2. According to Nanazawa et. al., (2022) and Miyazaki et al., (2024) experimental results, localization of liquefaction occurred beneath the culverts. Liquefaction took place first beneath the culverts then progressed to the embankment side. This was confirmed by the 3D analytical results.

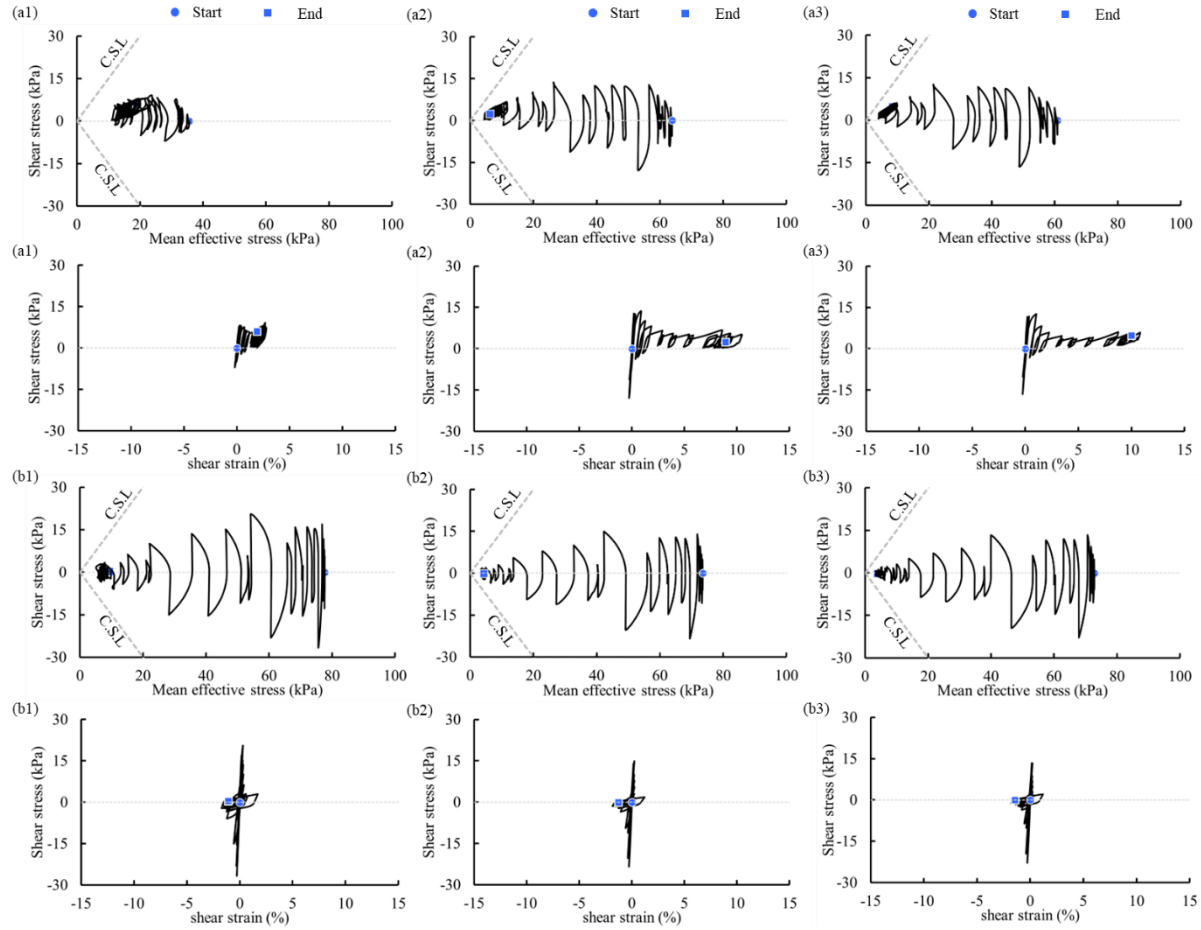


Figure 5.14. Case 2-10m: mouth elements stress paths for (a1), (a2) and (a3) and their corresponding stress - strain relationships; and joint elements stress paths for (b1), (b2) and (b3) and their corresponding stress - strain relationships.

3. High liquefaction (*ESDR*) and *EPWP* concentrations at the joint section were noted. This was comparable with the 2D analytical results.
4. Longer stress paths in deep liquefiable ground produced higher stress paths in deep liquefiable ground than shallow liquefiable ground. The shear stress paths were within the expected range.
5. Joint opening for the shallow liquefiable ground was convex in nature while for the deep liquefiable ground, it was concave in nature. This scenario was portrayed in the experimental results by Nanazawa et. al., (2022).

References

Japan Road Association (2012). *Specifications for Highway Bridges Vol. V: Seismic Design*.

- Miyazaki, Y., Shizuka, E., Mutua, B. M., Toshinori, Y., Satoshi, T., Yu, F., Hirofumi, H. and Hiroaki, H. (2024). Traffic Disruptions in Box-Type Embankment-Underpasses Due to Past Earthquakes in Japan, *Proceedings of the 5th International Conference on Transportation Geotechnics (ICTG)*, vol. 2, pp. 45-53. Lecture Notes in Civil Engineering, vol 403. https://doi.org/10.1007/978-981-97-8217-8_6
- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.
- Zhang, F., Ye, B., Noda, T., Nakano, M. and Nakai, K. (2007). Explanation of cyclic mobility of soils, Approach by stress-induced anisotropy, *Soil and Foundations*, vol. 47, No. 4, pp. 635–648.

CHAPTER SIX

DISCUSSIONS OF LOSS OF SERVICEABILITY OF CULVERTS DUE TO LIQUEFACTION PROBLEM

6.1 Introduction

Clarification of liquefaction seismic induced damage to culverts was investigated through both 2D and 3D elastoplastic finite element analysis. The initial assumption was that presence of culverts on the liquefiable ground accelerates the liquefaction of the foundation ground, and subsequently, this brings about increased culvert displacements and damage to the joints. This causes the whole embankment system to lose its serviceability. The presence of culverts on the foundation ground brings about amplified liquefaction, a localization problem, which tends to reduce from the culvert middle joint towards the mouth and embankment side.

6.2 Liquefaction behavior of foundation grounds beneath culverts

The presence of culverts on the liquefiable ground was found to increase the liquefaction. A situation which was amplified by the porous nature of the bottom plates of the culverts (undrained condition). The existence of undrained condition beneath the culverts causes the destabilization of the stresses due to liquefaction, (EPWP) development. Hence increasing these EPWP at the bottom of the culverts as compared to the free ground outside the culvert regions. This scenario of amplification of liquefaction beneath the culverts was similar for all the 2 and 4 culvert units models. Generally, higher stress paths were also developed by these high liquefaction levels and this led to higher shear strains in the soils. From the deep undrained case of 2 culvert units, the post liquefaction scenario led to collapse of the soil, due to higher mean stress recovery at the mouth than the joint section.

In summary, for the 2D analysis, from Figure 6.1, undrained condition effect on amplification of liquefaction in both mouth and joint sections occurred regardless of the number of culvert units. Also, drained condition had more recovery of mean effective stress as compared to undrained condition (portrayed by the negative ESDR) for both 2 and 4 culvert units at mouth and joint sections. From Figure 20, the higher mean effective stress recovery in drained condition is believed to have resulted in less residual shear strains as compared to the undrained condition which resulted in higher residual strains in the foundation ground in each case except the 4 units drained case. This might have been because of the contact – discontact behavior due to multiple culvert units on the foundation ground during the liquefaction process. Deep undrained liquefiable ground of 2-unit cases had exceptional behavior due to the joint closure and downward opening at the end of the earthquake.

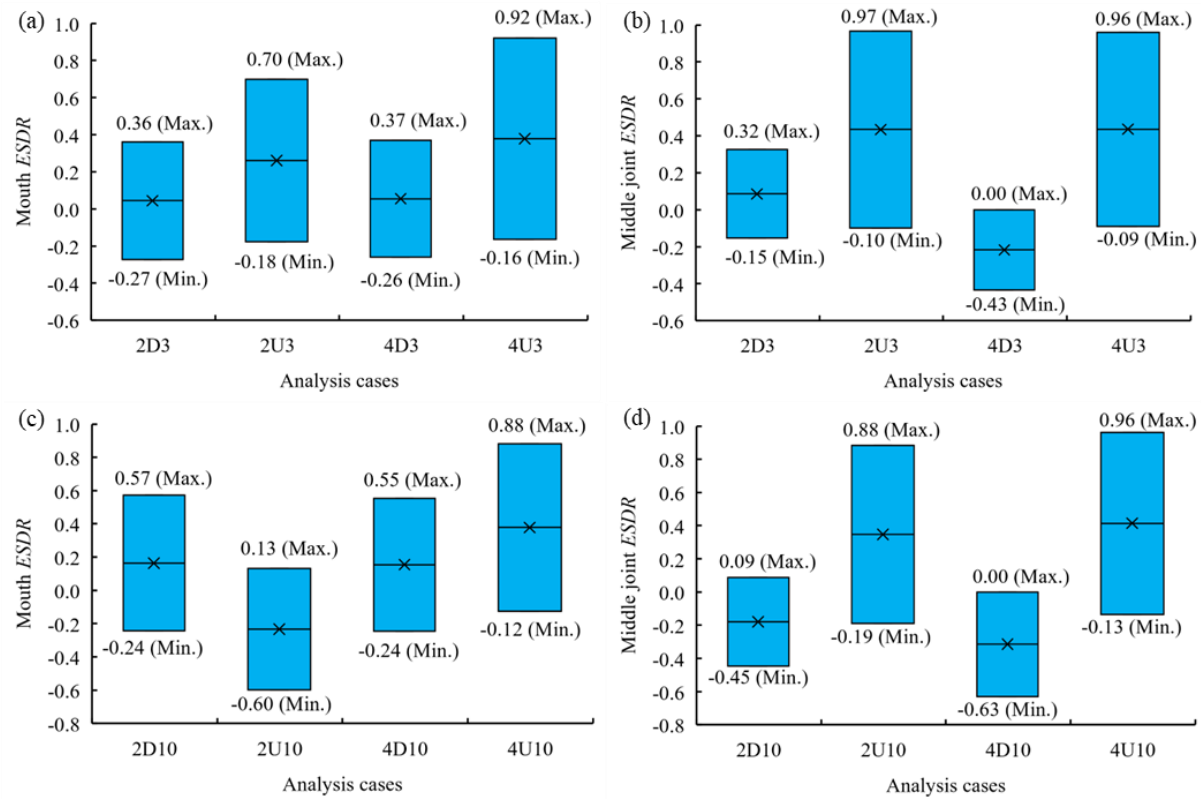


Figure 6.1. 2D liquefaction generalization at the mouth and joint sections for shallow liquefiable ground (a) mouth and (b) joint sections and deep liquefiable ground (c) mouth and (d) joint sections. Analysis cases are defined in Chapter Four (for instance 2U3 means 2 culvert units, undrained case, 3 m deep liquefiable layer).

For the 3D analysis, the same behavior was noted where the influence of deep liquefiable ground increased the liquefaction level. Also, higher liquefaction at the joint as compared to the mouth of the culvert was noted.

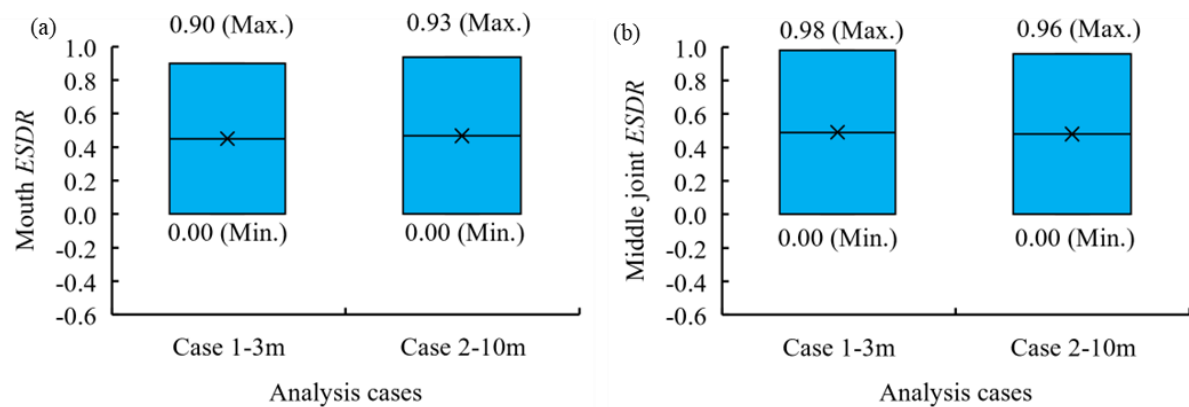


Figure 6.2. 3D liquefaction generalization at (a) mouth and (b) joint sections.

In summary, except for the case 2U10 mouth section, from Figure 6.1, undrained condition effect on amplification of liquefaction in both mouth and joint sections occurred regardless of the number of culvert units. Also, drained condition had more recovery of mean effective stress as compared to undrained condition (portrayed by the negative ESDR) for both 2 and 4 culvert units at mouth and joint sections. The exceptional behavior was due to the joint opening, closure and inverted joint opening transition and the contributing factors discussed in Chapter Four.

6.3 Culvert displacement modes and ground settlements

During liquefaction, the ground lost its strength, and this led to the settlement of the ground at varying rates at the mouth and the joint of the culverts. As discussed in Chapter Four, insignificant joint opening occurred for the four culvert unit cases. For the 2 culvert unit cases, the relationships between the joint opening and net settlements are shown in Figure 6.3 for 2D analysis and Figure 6.4 for 3D analysis.

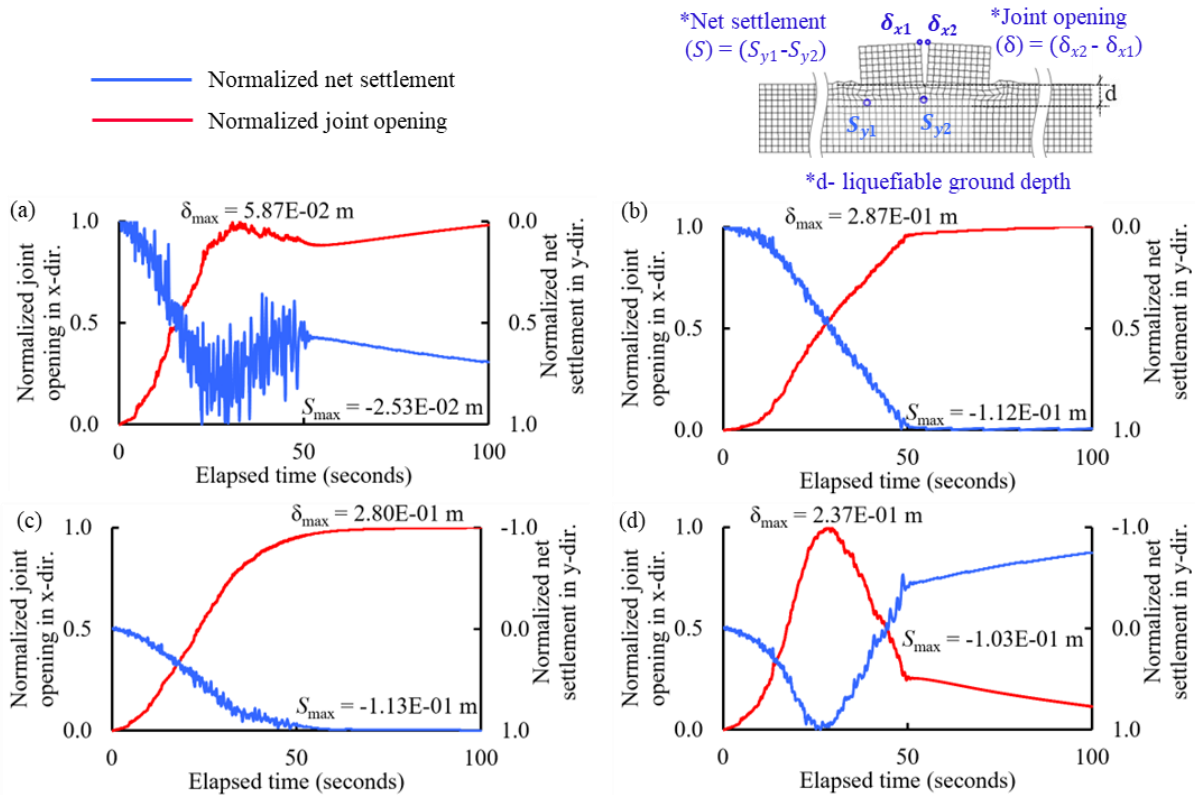


Figure 6.3. Relationship between joint opening and ground settlement for 2D analysis for 3 m liquefiable depth (a) drained and (b) undrained conditions and for 10 m liquefiable depth (c) drained and (d) undrained conditions.

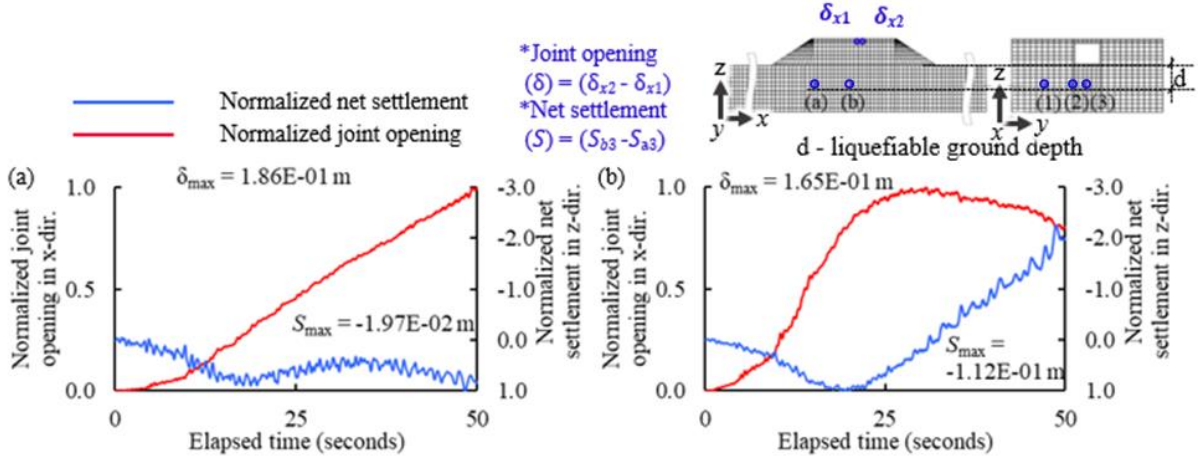


Figure 6.4. Relationship between joint opening and ground settlement for 3D analysis (a) Case 1-3m and (b) Case 2-10m

From the relationships, joint openings occurred because of differential ground settlement between the mouth and joint of the culverts. The higher the difference the higher the joint opening. For the deep liquefiable ground, the joint opened, closed and started opening in the opposite direction. This also correlated well with the joint opening behavior.

Figure 6.5 and 6.6 summarizes the number of culvert units influence on the joint opening behavior. The negative signs indicates that the joint opening was caused by tensional forces experienced by the springs. In general, the 2-culvert model had the highest joint openings as compared to the 4 culvert units model, meaning that the higher the number of culvert units above the foundation ground, the lower the joint opening despite the liquefaction behavior appearing the same for both 2 and 4 culvert units model, drained and undrained cases. Also notable is that the undrained condition, for the shallow liquefiable ground, amplified the joint opening approximately five times as compared to the drained condition, as evidently seen in the 2-culvert model. The situation was different for the deep liquefiable condition for the 2 culvert unit cases due to the upward joint opening, closure and downward joint opening transitions and the contributing factors discussed in Chapter 4. This scenario was same for the deep liquefiable ground in 3D case. The results for the 2D analytical case for undrained 2 culvert unit cases are comparable to 3D cases, though with some differences due to the additional effects of overburden and embankment for the 3D analysis.

From Nanazawa et al., (2022), joint opening for shallow liquefiable ground was in V-shape while in deep liquefiable ground it was inverted V shape due to ground deformations. These V shaped modes were confirmed in all the shallow liquefiable grounds, both 2D and 3D cases. For deep liquefiable grounds with undrained conditions, the inverted V shape mode was confirmed. The magnitudes of the final joint

separation are summarized in Figure 6.7 for both 2D and 3D cases. From the figure, it can be shown how various final joint separation modes resulted into. The final V shaped and inverted V shaped final culvert displacement modes can be deduced from Figure 6.7. The effect of drainage boundary condition and depth of liquefiable layer brought about the differences in these joint openings. Majorly, the impact of differential deformations of the ground at the culvert mouth and at the joints and mean effective stress recovery caused the culvert inclination and the final displacement modes.

6.4 Liquefaction gradient and settlement gradient relationships

From the joint opening index and *ESDR*, it could be observed that initial rapid deformation due to liquefaction of the culvert mouth caused initial joint opening. Mean effective stress recovery caused the final culvert inclination position. After liquefaction, free ground outside the culvert region did not experience any EPWP confinement as for the case of the beneath the culvert, hence regain of mean effective stress and dissipation of EPWP occurred. These scenarios dictated the culvert joint opening position during and after liquefaction of the foundation ground.

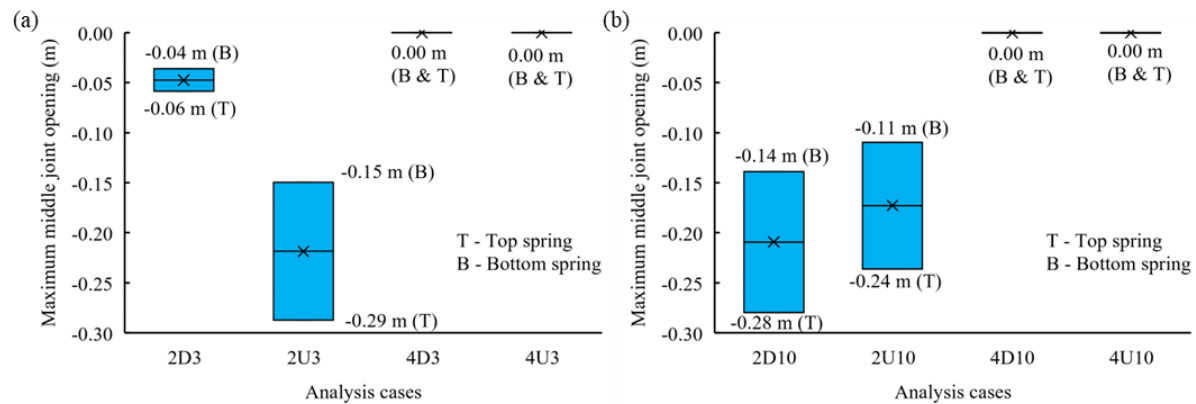


Figure 6.5. Generalization of joint opening for the 2D analytical cases.

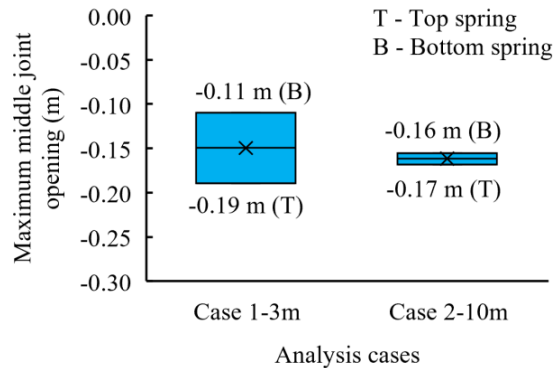


Figure 6.6. Generalization of joint opening for the 3D analytical cases.

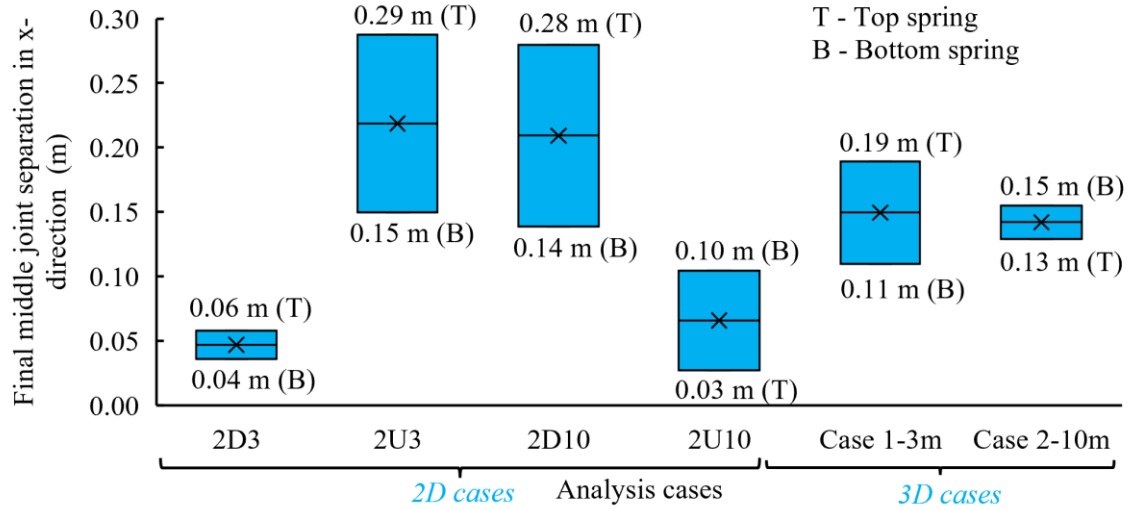


Figure 6.7. Final middle joint separation in x-direction for all 2 culvert units, 2D and 3D, analytical cases.

As the dynamic analysis progressed, the deformed foundation ground caused the culverts to rotate along the settled ground profile. This was caused by the ground net (differential) settlements between the mouth and joint. To analyze the impact of liquefaction and the culvert rotations, liquefaction gradient, $ESDR_{\text{gradient}}$, and settlement gradient, S_{gradient} , were computed, as shown in Figure 6.8. $ESDR_{\text{gradient}}$ is the ratio of the difference between the $ESDR$ beneath the culvert middle joint and mouth to the length of the culvert. S_{gradient} , which is taken as the culvert rotation angle, is the ratio of the net settlement beneath the culvert middle joint and mouth to the length of the culvert. For the 4 culvert units model, beneath the first joint and the left mouth positions were considered, since the liquefaction beneath the culverts was same, as discussed in Chapter Four.

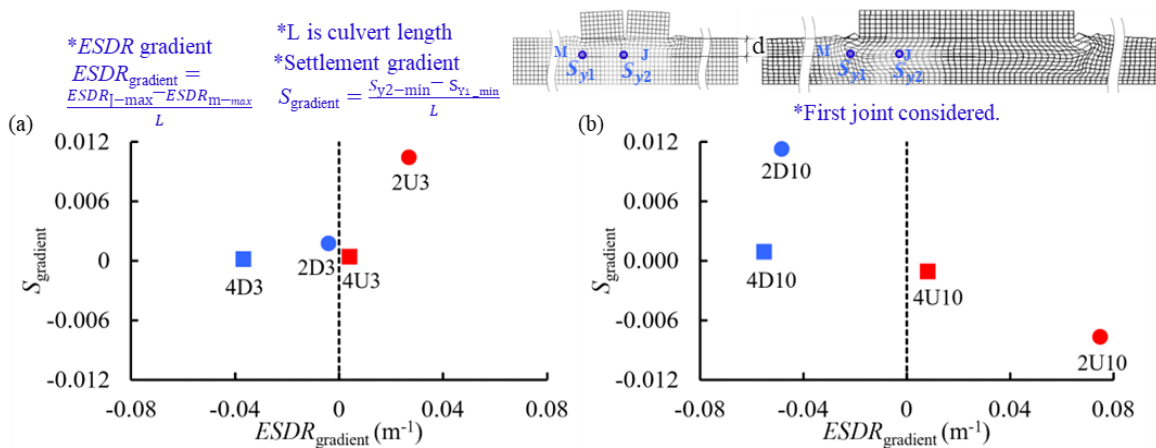


Figure 6.8. Generalization of settlements variations with $ESDR$ for 2D analytical cases.

From Figure 6.8, there were insignificant culvert rotations for the 4 culvert units, which is contributed by very minute net differential ground settlements beneath the culverts middle joint and mouth. This resulted in insignificant joint openings. For the 2 culvert units, significant culvert rotations occurred, and clearly, this was influenced by the liquefaction.

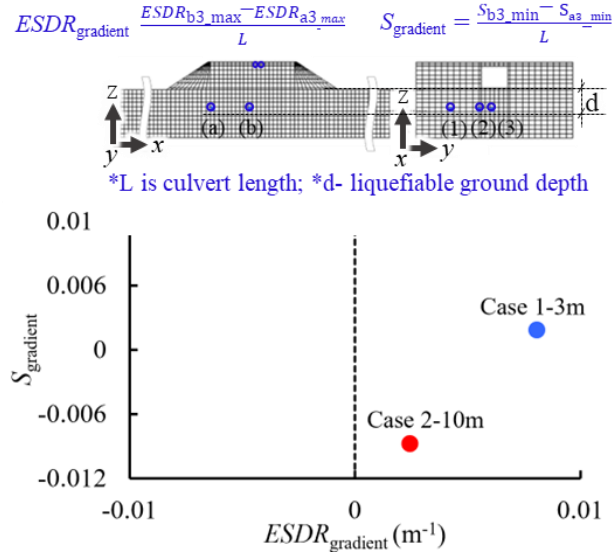


Figure 6.9. 3D variations of settlements with $ESDR$.

This as a result caused the joint opening differences in Figure 4.16 (Chapter Four) for the drained and undrained condition. For the 3D analysis, undrained boundary condition was applied beneath the culverts. In Figure 6.8 and 6.9, drained condition causes negative liquefaction gradient, while undrained condition causes positive liquefaction gradient despite the number of culvert units. The negative liquefaction gradient is due to the recovery of mean effective stress (MES) after maximum liquefaction is attained in each analytical case. This MES recovery is not observed in the undrained condition hence the positive liquefaction gradient.

6.5 Proposal for mitigation measures against liquefaction damage to culverts and embankments

From the analysis, referring to Figure 4.13 and Figure 4.15, the most critical liquefaction areas in the foundation which had impact on the culvert's displacements were the joint and mouth section. This is numerically confirmed from the relationships between the bump development (mouth-joint displacements relationship) and the joint opening index. Hence, if the culvert mouth displacement is contained, Figure 4.26 graphs will flatten (no bump formation will be formed, since B_c becomes zero), hence there will be no joint opening and finally the liquefaction of the foundation ground will have very minimal impact on the culvert joint opening.

To achieve that, the research proposes a sheet pile rigidly connected to the culvert mouth and extending the whole of the liquefiable ground. This idea is thought to achieve: - (a) restrain the vertical displacement of the culvert at its mouth hence reducing the culvert bump (B_c) formation to almost zero and (b) restrain the stresses (excess pore water pressure) spread past the culvert mouth from beneath the culvert. This will help in controlling the stresses imbalances between the region with culvert and without culvert (free ground). Once these two objectives are achieved, from the study outcomes, the culvert destruction by liquefaction will be reduced to minimum and non-destructive levels. The details of the sheet pile proposed and the study to determine its effectiveness in containing the liquefaction on the culvert destruction during seismic activities is proposed as area of further research for this study.

References

Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki, M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.

CHAPTER SEVEN

CONCLUSIONS AND PROPOSED AREAS OF FURTHER STUDIES

7.1 Conclusions of research findings

The study effectively determined the effect of deep and shallow foundations on culvert destruction due to liquefaction of the foundation ground by application of a real earthquake wave at the bottom of the foundation ground in the longitudinal; to the two-culvert model on varying liquefiable depths and drainage conditions. The main objective was to clarify the joint opening mechanisms by studying the soil-culvert interactions. From the study conducted, the following conclusions are made: -

1. Final culvert position is not always determined by the highest liquefaction level the soil experiences since once maximum *ESDR* is attained, the soils have been found to experience some considerable mean effective stresses recovery in some cases as portrayed by the stress paths and residual shear strain formations. Careful study of the changes in mean effective stress with time is considered appropriate in understanding soil-structure response to seismic activities. More emphasis should be placed on the liquefaction and deformation behavior of the soil supporting the culverts mouth and joint, since change of boundary and load conditions at these ends causes destabilization of stresses leading to varied strain levels and subsequent culvert inclinations and joint openings.
2. The undrained boundary between the culvert and foundation ground has a great effect of increasing the liquefaction levels beneath the culvert, hence more culvert joint openings as compared to the drained boundary condition. Emphasis should be placed on making the boundary between the bottom plates of the rigid box culverts and the foundation ground watertight to reduce this effect during seismic activities.
3. New parameters for assessing the culvert displacements, namely joint opening index (I_o) and culvert bump (B_c) have been developed and a relationship between them confirmed, which can help road engineers in determining the level of bump formation based on the maximum joint opening at the top of the culvert. Also, from the relationship between the joint opening index (I_o) and culvert bump (B_c), the shape of the joint opening can be determined from relationships given in Eqs. 14, since from the study, the shape can either upward (V) or downward (Λ) joint opening.
4. Finally, from the research outcomes, a proposal of sheet pile rigidly fixed to the culvert mouth extending to the whole depth of the liquefiable ground, which is subject to further studies has been proposed. This will considerably reduce effects of the culverts inclinations and resulting formation of culvert bumps and joint opening which greatly affect the traffic flow on the supported road. Since the joint openings were found to exceed the recommended maximum values in several cases,

Nanazawa et al., (2022) and Tokida et al., (2007) proposed that 200 mm joint openings and 200 mm bumps on culvert joints respectively, are the maximum limits allowable, which would not affect traffic flows on the roads during quick repairs after an earthquake.

7.2 Recommended areas of further studies

From the research outcomes, the following areas are proposed for further studies: -

1. The analysis was based on one type of input seismic wave. The results can be extended to check the effect of various long and short waves.
2. The culvert structural dimensions, for instance the uniform thickness against the width or height can be varied to check the differences with the current research.
3. Only one culvert in transverse direction was considered. Depending on the nature where the culverts will be installed, multiple culverts in transverse direction might be required, hence, in addition to varying the structural units in longitudinal direction, a comparison in varying the units in transverse direction can be conducted.
4. Also, from the proposed mitigation measure, analysis incorporating sheet pile, extending full depth of liquefiable layer can be conducted and the impacts verified if the mitigation to liquefaction is achieved.

References

- Nanazawa, T., Ito, H., Tani, T., Sasaki, A. and Sasaki, M. (2022). Study on behavior of culvert due to liquefaction during earthquake, *Technical Note of National Institute for Land and Infrastructure Management*, Ministry of Land, Infrastructure, Transport and Tourism, Japan, No. 1219.
- Tokida, K., Oda, K., Nakahara, A., Hayashi, K. and Yorifuji, M. (2007). Performance-based Road management against bump damage induced by earthquake based on driving test, *Proceedings of the JSCE Earthquake Engineering Symposium*, vol. 29, pp. 596-604. (in Japanese)

APPENDIX

1. Comparison of experimental and simulation results of *Edosaki* sand for embankments

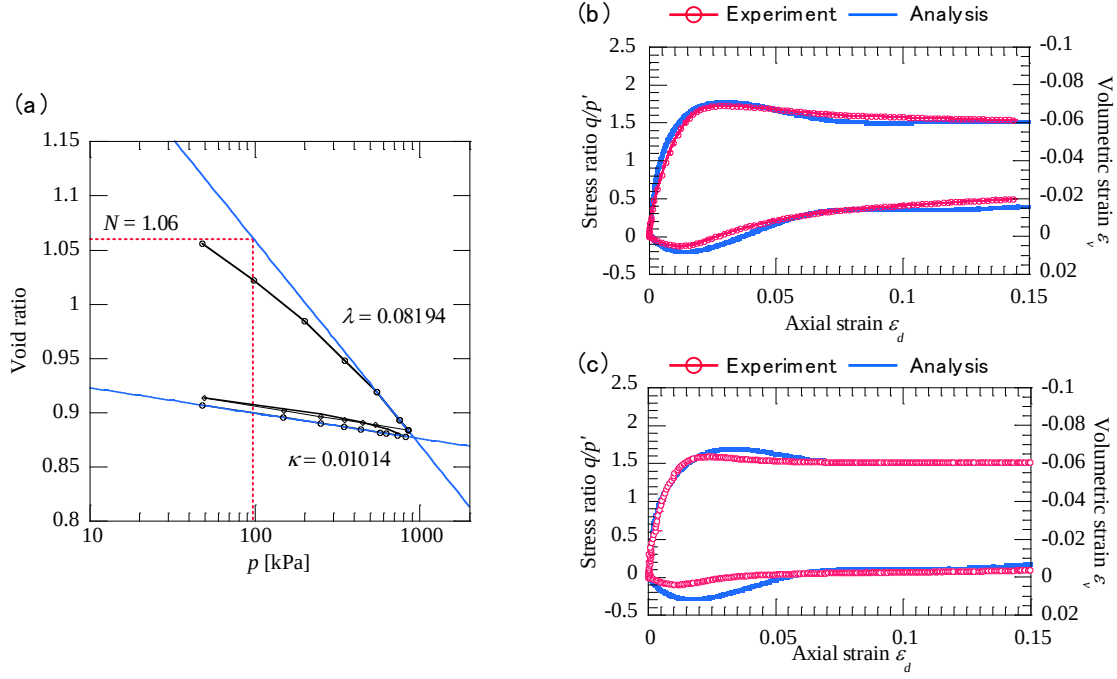


Figure A1. Results for (a) isotropic consolidation tests and triaxial compression tests and their simulations in (b) 50 kPa and (c) 100 kPa of confining stress for *Edosaki* sand (Miyazaki, 2019).

2. Step by step derivation of yield functions (Oka et al., 1994)

(a) Density of two-phase mixture

Expressing the apparent density $\bar{\rho}^s$ of the solid phase and the apparent density $\bar{\rho}^f$ of the liquid phase per unit volume of a two-phase mixture using the porosity n , where ρ^s is solid phase density and ρ^f is liquid phase density respectively.

$$\bar{\rho}^s = (1 - n)\rho^s \quad (\text{A1})$$

$$\bar{\rho}^f = n\rho^f \quad (\text{A2})$$

Combining the two equations, Eqs. (A1) and (A2), the density ρ of the mixture becomes: -

$$\rho = \bar{\rho}^s + \bar{\rho}^f = (1 - n)\rho^s + n\rho^f \quad (\text{A3})$$

(b) Two-phase mixtures stresses

Using pore water pressure, the relationship between total stress and effective stress can be expressed as Eq. (A4) where σ_{ij} is the total stress tensor, σ'_{ij} is the effective stress tensor, p is the pore water pressure and δ_{ij} is the Kronecker delta. Total stress is now expressed as the sum of the partial stresses acting on the solid and liquid phases as shown in Eq. (A5).

$$\sigma_{ij} = \sigma'_{ij} + p\delta_{ij} \quad (\text{A4})$$

$$\sigma_{ij} = \sigma^s_{ij} + \sigma^f_{ij} \quad (\text{A5})$$

Where σ^s_{ij} is the solid phase stress tensor, and σ^f_{ij} is the liquid phase stress tensor. The solid phase stress can now be expressed as the sum of the effective stress and the pore water pressure acting on the solid phase as shown: -

$$\sigma^s_{ij} = \sigma'_{ij} + (1 - n)p\delta_{ij} \quad (\text{A6})$$

Also, the partial stress of the liquid phase can be expressed as the pore water pressure acting on the liquid phase as shown in Eq. (A7). It should be noted that the signs of stress and pore water pressure are positive for the tensile component.

$$\sigma^f_{ij} = np\delta_{ij} \quad (\text{A7})$$

(c) Displacement - strain relationship

Based on assumption 1 at the start of the derivations, the relationship between strain and displacement for the solid and liquid phases is expressed in Eq. (A8) and Eq. (A9). Where ε^s_{ij} is the strain tensor of the solid phase, u^s_i is the displacement vector of the solid phase, ε^f_{ij} is the strain tensor of the liquid phase, and u^f_i is the displacement vector of the liquid phase.

$$\varepsilon_{ij}^s = \frac{1}{2} \left(\frac{\partial u_i^s}{\partial x_j} + \frac{\partial u_j^s}{\partial x_i} \right) \quad (\text{A8})$$

$$\varepsilon_{ij}^f = \frac{1}{2} \left(\frac{\partial u_i^f}{\partial x_j} + \frac{\partial u_j^f}{\partial x_i} \right) \quad (\text{A9})$$

(d) Constitutive formula

The stress-strain relationship in the solid phase can be defined as the effective stress increment tensor-strain increment tensor relationship in the same solid phase by the Eq. (A10), where $\Delta\sigma'_{ij}$ is the effective stress increment tensor, D_{ijkl} is the D matrix, $\Delta\varepsilon_{kl}^s$ is the strain increment tensor of the solid phase. If it is assumed that the liquid phase is treated as a material that does not resist shear deformation and that viscous resistance is not considered, the constitutive equation for the liquid phase can be expressed as Eq. (A11), where k^f is the bulk modulus of the liquid phase.

$$\Delta\sigma'_{ij} = D_{ijkl} \Delta\varepsilon_{kl}^s \quad (\text{A10})$$

$$p = k^f \varepsilon_{ij}^f \quad (\text{A11})$$

(e) Darcy rule

If it's assumed that the relative velocity of the liquid phase flowing into the gap with respect to the solid phase is proportional to the hydraulic gradient, and applying Darcy's law, Eq. (A12) is obtained. $\dot{w}_i$ is the relative displacement vector of the liquid phase with respect to the solid phase, which is defined by Eq. (A13).

$$\dot{w}_i = -k \frac{\partial h}{\partial x_i} \quad (\text{A12})$$

$$w_i = n(u_i^f - u_i^s) \quad (\text{A13})$$

If h is taken as the total water head, Eq. (A14) is also defined. h_z is the position head, γ_w is the unit volume weight of pore water, p_s is the hydrostatic pressure, and p_d is the excess pore volume as an increment from the hydrostatic pressure. Also, in addition, the unit volume weight of pore water γ_w has a relationship with the density of the liquid phase ρ^f as shown in Eq. (A15). g is taken as gravitational acceleration.

$$h = h_z - \frac{p}{\gamma_w} = h_z - \left(\frac{p_s}{\gamma_w} + \frac{p_d}{\gamma_w} \right) \quad (\text{A14})$$

$$\gamma_w = \rho^f g \quad (\text{A15})$$

(f) Law of conservation of mass

The solid and liquid phase expressions for law of conservation of mass are defined in Eqs. (A16) and (17) respectively. $\frac{D}{Dt}$ is an operator that represents real differentiation.

$$\frac{D\bar{\rho}^s}{Dt} + \bar{\rho}^s \frac{\partial u_i^s}{\partial x_i} = 0 \quad (\text{A16})$$

$$\frac{D\bar{\rho}^f}{Dt} + \bar{\rho}^f \frac{\partial u_i^f}{\partial x_i} = 0 \quad (\text{A17})$$

(g) Law of conservation of momentum

The solid and liquid phase expressions for law of conservation of momentum are defined in Eqs. (A18) and (19) respectively. Here, b_i is the body force, and R_i is the term that represents the interaction between the solid and liquid phases. R_i is defined in Eq. (A20).

$$\bar{\rho}^s \ddot{u}_i^s = \frac{\partial \sigma_{ij}^s}{\partial x_j} + \bar{\rho}^s b_i - R_i \quad (\text{A18})$$

$$\bar{\rho}^f \ddot{u}_i^f = \frac{\partial \sigma_{ij}^f}{\partial x_j} + \bar{\rho}^f b_i + R_i \quad (\text{A19})$$

$$R_i = -n \frac{\gamma_w}{k} \dot{w}_i \quad (\text{A20})$$

Based on the above basic equations for deriving the field equations. The balance equation and the continuity equation can be derived below.

(h) Balance equation

Summing up Eqs. (A18) and (19) to obtain the momentum conservation law for the mixture, Eq. (A21): -

$$\bar{\rho}^s \ddot{u}_i^s + \bar{\rho}^f \ddot{u}_i^f = \frac{\partial \sigma_{ij}^s}{\partial x_j} + \frac{\partial \sigma_{ij}^f}{\partial x_j} + \bar{\rho}^s b_i + \bar{\rho}^f b_i \quad (\text{A21})$$

By substituting Eqs. (A3), (A5), and (A13), into Eq. (A21) and rearranging, we obtain the Eq. (A22).

$$\rho \ddot{u}_i^s + \rho^f \ddot{w}_i = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \quad (\text{A22})$$

From assumption 3, if we set $\ddot{w}_i \cong 0$; the **balance equation**; Eq. (A22) can be expressed as follows: -

$$\rho \ddot{u}_i^s = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i \quad (\text{A23})$$

(i) Continuity equation

Substituting Eq. (A1) into the law of conservation of mass of the solid phase Eq. (A16) and expanding it, we obtain Eqs. (A24) and (A25): -

$$(1-n) \frac{\partial \rho^s}{\partial t} + \rho^s \frac{\partial (1-n)}{\partial t} + \rho^s \frac{\partial \{(1-n)\dot{u}_i^s\}}{\partial x_i} + (1-n)\dot{u}_i^s \frac{\partial \rho^s}{\partial x_i} = 0 \quad (\text{A24})$$

Similarly, for the liquid phase and expanding it, we obtain the following equation.

$$n \frac{\partial \rho^f}{\partial t} + \rho^f \frac{\partial n}{\partial t} + \rho^f \frac{\partial \{n\dot{u}_i^f\}}{\partial x_i} + n\dot{u}_i^f \frac{\partial \rho^f}{\partial x_i} = 0 \quad (\text{A25})$$

Multiplying Eq. (A24) by ρ^f / ρ^s and adding both sides to Eq. (A25) results to: -

$$\begin{aligned} & \rho^f \left(\frac{\partial (1-n)}{\partial t} + \frac{\partial n}{\partial t} \right) + \rho^f \frac{\partial \{n(\dot{u}_i^f - \dot{u}_i^s)\}}{\partial x_i} + \rho^f \frac{\partial \dot{u}_i^s}{\partial x_i} + n \left(\frac{\partial \rho^f}{\partial t} + \dot{u}_i^f \frac{\partial \rho^f}{\partial x_i} \right) + (1 \\ & - n) \frac{\rho^f}{\rho^s} \left(\frac{\partial \rho^s}{\partial t} + \dot{u}_i^s \frac{\partial \rho^s}{\partial x_i} \right) = 0 \end{aligned} \quad (\text{A26})$$

Eq. (A18) has the first term as 0. Furthermore, by substituting Eqs. (A8) and (A13) and dividing both sides by ρ^f we obtain: -

$$\frac{\partial \dot{w}_i^l}{\partial x_i} + \dot{\varepsilon}_{ii}^s + \frac{n}{\rho^f} \left(\frac{\partial \rho^f}{\partial t} + \dot{u}_i^f \frac{\partial \rho^f}{\partial x_i} \right) + \frac{(1-n)}{\rho^s} \left(\frac{\partial \rho^s}{\partial t} + \dot{u}_i^s \frac{\partial \rho^s}{\partial x_i} \right) = 0 \quad (\text{A27})$$

$\dot{\varepsilon}_{ii}^s$ is the volumetric strain of the solid phase. Also, by the definition of the real derivative, Eq. (A27) can be expressed as: -

$$\frac{\partial \dot{w}^l}{\partial x_i} + \dot{\varepsilon}_{ii}^s + \frac{n}{\rho^f} \dot{\rho}^f + \frac{(1-n)}{\rho^s} \dot{\rho}^s = 0 \quad (\text{A28})$$

Based on assumption 4, the following equation is valid: -

$$\dot{\rho}^s = 0 \quad (\text{A29})$$

Substituting Eq. (A29) into Eq. (A28), the resulting equation becomes the continuity equation when the compressibility of the solid phase is assumed.

$$\frac{\partial \dot{w}^l}{\partial x_i} + \dot{\varepsilon}_{ii}^s + \frac{n}{\rho^f} \dot{\rho}^f = 0 \quad (\text{A30})$$

In Eq. (A30), the third term on the left side is a term that indicates the compressibility of the pore water, but this term can further be modified to make it easier to handle. Substituting Eq. (A2) into Eq. (A17), which is the law of conservation of mass in the liquid phase, and rearranging it, we obtain: -

$$\dot{\rho}^f = -\rho^f \frac{\partial \dot{u}_i^f}{\partial x_i} \quad (\text{A31})$$

Expressing the term on the right-hand side of Eq. (A9) in terms of strain, we obtain: -

$$\dot{\rho}^f = -\rho^f \dot{\varepsilon}_{ii}^f \quad (\text{A32})$$

Substituting the liquid phase constitutive law Eq. (A11), we obtain: -

$$\dot{\rho}^f = -\frac{\rho^f}{K^f} \dot{p} \quad (\text{A33})$$

Also, considering that the time change of the hydrostatic pressure is 0 from Eq. (A14), Eq. (A33) becomes as follows using the excess pore water pressure (p_d).

$$\dot{\rho}^f = -\frac{\rho^f}{K^f} \dot{p}_d \quad (\text{A34})$$

If Eq. (A34) is substituted into Eq. (A30), the continuity equation becomes: -

$$\frac{\partial \dot{w}_l}{\partial x_i} + \dot{\varepsilon}_{ii}^s - \frac{n}{K^f} \dot{p}_d = 0 \quad (\text{A35})$$

It is noted that the equation still contains the relative displacement w_l of the liquid phase with respect to the solid phase. To eliminate this relative displacement, the equation is transformed as follows: -

$$\bar{\rho}^f \ddot{u}_i^s + \frac{\bar{\rho}^f}{n} \ddot{w}_i = \frac{\partial \sigma_{ij}^f}{\partial x_j} \bar{\rho}^f b_i + R_i \quad (\text{A36})$$

Based on assumption 3, the following equation is considered: -

$$\ddot{u}_i^s \gg \ddot{w}_i \quad (\text{A37})$$

Considering Eq. (A37) in Eq. (A36), it reduces to: -

$$\bar{\rho}^f \ddot{u}_i^s = \frac{\partial \sigma_{ij}^f}{\partial x_j} \bar{\rho}^f b_i + R_i \quad (\text{A38})$$

Also, substituting Eqs. (A2), (A7) and (A20) into Eq. (A38), it gives the following: -

$$n \rho^f \ddot{u}_i^s = \frac{\partial n p}{\partial x_i} + n \rho^f b_i - n \frac{\gamma_w}{k} \dot{w}_i \quad (\text{A39})$$

If it is assumed that the porosity is smooth in space based on assumption 2, the following: -

$$\frac{\partial n}{\partial x_i} = 0 \quad (\text{A40})$$

If Eq. (A40) is substituted into Eq. (A39), it results into: -

$$\rho^f \ddot{u}_i^s = \frac{\partial p}{\partial x_i} + \rho^f b_i - \frac{\gamma_w}{k} \dot{w}_i \quad (\text{A41})$$

If the second term on the right-hand side in Eq. (A41) is the body force due to gravity, Eq. (A42) holds if we use the hydrostatic pressure p_s and ignore the dynamic terms.

$$\frac{\partial p_s}{\partial x_i} + \rho^f b_i = 0 \quad (\text{A42})$$

Substituting Eq. (A42) into Eq. (A41) and considering Eq. (A14), we obtain: -

$$\rho^f \ddot{u}_i^s = \frac{\partial p_s}{\partial x_i} - \frac{\gamma_w}{k} \dot{w}_i \quad (\text{A43})$$

Solving for w_i gives us the following equation:

$$\dot{w}_i = \frac{k}{\gamma_w} \left(\rho^f \ddot{u}_i^s - \frac{\partial p_d}{\partial x_i} \right) \quad (\text{A44})$$

Using the Eq. (A44) derived above, w_i can be expressed in terms of the solid phase displacement and excess pore water pressure. Substituting Eq. (A44) into Eq. (A35) gives Eq. (A45). However, here we assume that the permeability coefficient k is sufficiently smooth in space.

$$\rho^f \frac{\partial \ddot{u}_i^s}{\partial S_i} - \frac{\partial^2 p_d}{\partial x_i^2} - \frac{\gamma_w}{k} \left(\dot{\varepsilon}_{ii}^s - \frac{n}{K^f} \dot{p}_d \right) = 0 \quad (\text{A45})$$

In addition, taking Eq. (A8) into consideration, the final **continuity equation** to be solved is obtained as follows:

$$\rho^f \dot{\varepsilon}_{ii}^s - \frac{\partial^2 p_d}{\partial x_i^2} - \frac{\gamma_w}{k} \left(\dot{\varepsilon}_{ii}^s - \frac{n}{K^f} \dot{p}_d \right) = 0 \quad (\text{A46})$$

3. Cyclic Mobility Model derivations (Asaoka et al., 2002)

The following equations are detailed equations for the cyclic mobility model operations, as an extension in Chapter Three.

$$f = \ln \frac{\tilde{p}}{p} + \ln \frac{M^2 - \zeta^2 + \eta^{*2}}{M^2 - \zeta^2} - \frac{\varepsilon_v^p}{C_p} = 0 \quad (\text{A47})$$

The first term in Eq. (A47) is given by: -

$$\ln \frac{\tilde{p}}{\tilde{p}_0} = \ln \frac{\tilde{p}}{\bar{p}} - \ln \frac{\bar{p}}{\tilde{p}_0} + \ln \frac{p}{\tilde{p}_0} = \ln R^* - \ln R + \ln \frac{p}{\tilde{p}_0} \quad (\text{A48})$$

Substituting Eq. (A48) to Eq. (A47), it can be rewritten as: -

$$f = \ln \frac{p}{\tilde{p}_0} + \ln \frac{M^2 - \zeta^2 + \eta^{*2}}{M^2 - \zeta^2} + \ln R^* - \ln R - \frac{\varepsilon_v^p}{C_p} = 0 \quad (\text{A49})$$

Further, the partial differentials of Eq. (A49) can be formulated as follows. When $\partial f / \partial p = 0$, the C.S.L. defined by the above conditions always satisfies $\eta = M$, which proves that constant C.S.L. is independent of the evolution of the stress-induced anisotropy.

$$\frac{\partial f}{\partial p} = \frac{M^2 - \eta^2}{(M^2 - \zeta^2 + \eta^{*2})p} \quad (\text{A50})$$

$$\frac{\partial f}{\partial S_{ij}} = \frac{3\hat{\eta}_{ij}}{(M^2 - \zeta^2 + \eta^{*2})p} \quad (\text{A51})$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{3\hat{\eta}_{ij}}{(M^2 - \zeta^2 + \eta^{*2})p} + \frac{M^2 - \zeta^2}{(M^2 - \zeta^2 + \eta^{*2})p} \cdot \frac{\delta_{ij}}{3} \quad (\text{A52})$$

$$\frac{\partial f}{\partial \beta_{ij}} = \frac{3(-M^2 \hat{\eta}_{ij} + \eta^{*2} \beta_{ij} + \zeta^2 \hat{\eta}_{ij})}{(M^2 - \zeta^2 + \eta^{*2})(M^2 - \zeta^2)} \quad (\text{A53})$$

The plastic volumetric strain is given in the following equation and definitions where $\mathbf{J}$ is the Jacobian determination of deformation gradient tensor $\mathbf{F}$. The term $\mathbf{D}^p$ is the plastic component of stretching $\mathbf{D}$ which can be expressed by $\tilde{\lambda}$ and $\tilde{\kappa}$, the compression and swelling indexes, respectively.

$$\varepsilon_v^p = -\int_0^t J \text{tr} \mathbf{D}^p d\tau \quad (\text{A54})$$

$$J = \det \mathbf{F} = \frac{v}{v_0} = \frac{1+e}{1+e_0} \quad (\text{A55})$$

$$\mathbf{D} = \frac{\tilde{\lambda} - \tilde{\kappa}}{M(1+e_0)} = \frac{\tilde{\lambda} - \tilde{\kappa}}{Mv_0} \quad (\text{A56})$$