

Some remarks on Volume 17 of the *Taisei Sankei*

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Abstract

We shall see how the determinant was introduced in Volume 17 of the *Taisei Sankei* (1711), where the full expansion formulas of determinant of orders 2, 3, 4 and 5 were given in connection with the elimination theory of the system of algebraic equations with several unknowns. We shall examine, among others, the exactitude of the description of determinants.

§ 1. About the *Taisei Sankei*

The *Taisei Sankei* 大成算経 is an encyclopedic work of mathematics written by three mathematicians Seki Takakazu 関孝和 (1640s–1708), Takebe Kata'akira 建部賢明 (1661–1716) and Takebe Katahiro 建部賢弘 (1664–1739) during 1683–1711. Although many English translations of the title have been proposed, e. g., *Accomplished Classic of Mathematics* [2], *Complete Book of Mathematics* [5], *Great Accomplished Mathematical Treatise* [4], *Summa of Mathematical Canon* (SK), etc., we designate it in this paper by the romanised Japanese title *Taisei Sankei*.

The work is composed of twenty volumes preceded by an introductory volume.

Volumes 16 – 20 treat the theory of equations: that is,

Volume 16 題術辯 Discussions of problems, procedures and extractions,

Volume 17 全題解 Types of problems (direct calculations, implicit problems, concealed problems, submerged problems),

Volume 18 病題議 Restoration of defective problems,

Volume 19 演段例上 Operational examples (implicit problems, concealed problems),

Volume 20 演段例下 Operational examples (submerged problems).

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The theory of resultants and determinants, which is one of the most remarkable achievements of Seki, is expounded in Volume 17, while Volume 19 presents 15 examples to show how to apply this theory to concrete problems. Recently, the speaker translated these two volumes into English (see [12] and [13]).

Parts of this paper were presented orally at the "Mathematical Texts in East Asia Mathematical History" Workshop, March 11 – 15, 2016, Hainan, China and at the Bumwoo Conference on the History of Asian Mathematics, August 23 – 26, 2016, Seoul, Korea (see [10]).

§ 2. Manuscripts of the *Taisei Sankei*

There are more than 20 manuscripts of the *Taisei Sankei* (see Komatsu [1]). But in this paper, we work with the following four manuscripts.

1. MS University of Tokyo: This manuscript is called the Kashū edition 霞州本 and dates back to the Takebe Katahiro's time (ca. 1710),
2. MS Tokyo Science University: This manuscript dates back to ca. 1780,
3. MS Kyoto University B: This manuscript dates back to ca. 1850,
4. MS Kyoto University A: This is another manuscript conserved in the Library of Kyoto University (date unknown).

Note that the pages in MSS are numbered 1r (1 recto), 1v (1 verso), 2r (2 recto), 2v (2 verso), etc. In MS U. Tokyo, each page has 12 lines of 20 characters with no punctuation. In MS Tokyo Science U., each page has 10 lines of 20 characters with no punctuation. In MS Kyoto U. B, each page has 12 lines of 20 characters with vermilion punctuation. In MS Kyoto U. A, each page has 12 lines of 20 characters with vermilion punctuation.

§ 3. Three rules

Now we use the notation of modern mathematics. Let n be an integer.

§ 3.1. Rule of cross multiplications

Consider the simultaneous system of equations:

$$(3.1) \quad \begin{cases} P_{0,0} + P_{0,1}x + \cdots + P_{0,n-1}x^{n-1} = 0 \\ P_{1,0} + P_{1,1}x + \cdots + P_{1,n-1}x^{n-1} = 0 \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ P_{n-1,0} + P_{n-1,1}x + \cdots + P_{n-1,n-1}x^{n-1} = 0 \end{cases}$$

Then we have the equation

$$(3.2) \quad \det \begin{pmatrix} P_{0,0} & P_{0,1} & \cdots & P_{0,n-2} & P_{0,n-1} \\ P_{1,0} & P_{1,1} & \cdots & P_{1,n-2} & P_{1,n-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n-1,0} & P_{n-1,1} & \cdots & P_{n-1,n-2} & P_{n-1,n-1} \end{pmatrix} = 0$$

This theorem is called the rule of cross multiplication 交乘法 in the *Taisei Sanklei*, where the determinant is defined recursively in n by the Van der Monde formula.

§ 3.2. Rule of varied multiplications

The coefficient matrix in (3.2) is called symmetric (with respect to the anti-diagonal) if

$$\begin{aligned} P_{0,j} &= P_{n-1-j,n-1}, & j &= 0, 1, \dots, n-2 \\ P_{1,j} &= P_{n-1-j,n-2}, & j &= 0, 1, \dots, n-3 \\ &\dots \\ P_{n-2,0} &= P_{n-1,1}. \end{aligned}$$

The rule of cross multiplication for a symmetric coefficient matrix is called the rule of varied multiplication 変乘法 in the *Taisei Sankei*. Note that we transform a pair or two equations of degree n , then the transformed simultaneous system of equations is symmetric. Because of this fact, the symmetric case has a different name.

§ 3.3. Rule of deletion/generation

Problem: Let n be an integer. Consider the system of equations:

$$(3.3) \quad R_0 + R_1x + R_2x^2 + \dots + R_{n-1}x^{n-1} = 0, \quad x^n = Q^n$$

where $R_i, i = 0, 1, 2, \dots, n-1$ and Q are given constants. Eliminate the unknown x .

The equation (3.3) implies the simultaneous equation (3.4):

$$(3.4) \quad \begin{cases} R_0 + R_1x + R_2x^2 + \dots + R_{n-1}x^{n-1} = 0, \\ R_{n-1}Q^n + R_0x + R_1x^2 + \dots + R_{n-2}x^{n-1} = 0, \\ R_{n-2}Q^n + R_{n-1}Q^n + R_0x + \dots + R_{n-3}x^{n-1} = 0, \\ \dots \\ R_1Q^n + R_2Q^n x + R_3Q^n x^2 + \dots + R_0x^{n-1} = 0 \end{cases}$$

By the rule of cross multiplication, we can eliminate x as follows:

$$(3.5) \quad \det \begin{pmatrix} R_0 & R_1 & R_2 & \dots & R_{n-1} \\ R_{n-1}Q^n & R_0 & R_1 & \dots & R_{n-2} \\ R_{n-2}Q^n & R_{n-1}Q^n & R_0 & \dots & R_{n-3} \\ \dots & \dots & \dots & \dots & \dots \\ R_1Q^n & R_2Q^n & R_3Q^n & \dots & R_0 \end{pmatrix} = 0$$

In fact, we used the rule of varied multiplications because the coefficient matrix in (3.4) is symmetric. This is the modern interpretation of the rule of deletion/generation 消長法.

Notes: This problem was very popular in the early days of *wasan*. The cases $n = 2$ (quadratic) and $n = 3$ (cubic) were quite well-known in the beginning stage of *wasan*. In the *Hatsubi Sanpō* 発微算法 (1674), Seki Takakazu treated the problems of elimination only for these cases.

Later, the octic case ($n = 8$) was stated and published by Nakane Genkei 中根元圭 (1662 -- 1733) in the *Shichijōbeki Enshiki* 七乗幂演式 (Procedures for the Octic Power, 1691).

In the *Taisei Sankei* (1711) the exposition was only up to the cases $n = 4$ (quartic) and $n = 5$ (quintic) but the problem is clearly stated along with the theory of determinant (resultant).

§ 4. Statements in the *Taisei Sankei*

§ 4.1. Problem 17-56 (English translation)

Suppose given a pair of two quadratic formulas and transform it to obtain¹

$$(4.1) \quad \begin{array}{ll} \text{First formula} & \boxed{[b, a]} \quad [b + ax = 0] \\ \text{Second formula} & \boxed{[d, c]} \quad [d + cx = 0] \end{array}$$

(c is regarded as b in the symmetric case.)

Formula to be folded by the quadratic power

$$(4.2) \quad \boxed{[R_0, R_1]} \quad [R_0 + R_1x = 0]$$

Three rules for the quadratic case

平方交乘法 Quadratic rule of cross multiplications

$$\text{乙丙 (加)} \quad [+1bc] \quad \text{甲丁 (減)} \quad [-1ad]$$

変乘法 Rule of varied multiplications

$$\text{乙幂 (一段)(加)} \quad [+1b^2] \quad \text{甲丁 (一段)(減)} \quad [-1ad]$$

消長法 Rule of deletion/generation

$$\text{実幂 (一段)(長)} \quad [+1R_0^2] \quad \text{方幂商幂 (一段)(消)} \quad [-1R_1^2Q^2]$$

In the notation of modern mathematics, the three rules can be stated as follows:

Quadratic rule of cross multiplication: (4.1) implies

$$\det \begin{pmatrix} b & a \\ d & c \end{pmatrix} = bc - ad = 0.$$

¹To translate this text into English, we transcribe the ten branches by alphabet as follows:

$$\begin{array}{cccccccccc} \text{甲} & \text{乙} & \text{丙} & \text{丁} & \text{戊} & \text{己} & \text{庚} & \text{辛} & \text{壬} & \text{癸} \\ a & b & c & d & e & f & g & h & i & j \end{array}$$

In the translation a formula in the bracket [] is the transcription in modern mathematical notation.

Rule of varied multiplication: If $c = b$ (symmetric), then

$$\det \begin{pmatrix} b & a \\ d & b \end{pmatrix} = b^2 - ad = 0.$$

Rule of deletion/generation: If (4.2) and $x^2 = Q^2$, then

$$\det \begin{pmatrix} R_0 & R_1 \\ R_1 Q^2 & R_0 \end{pmatrix} = R_0^2 - R_1^2 Q^2 = 0.$$

§ 4.2. Problem 17--57 (English translation)

Suppose given a pair of two cubic formulas and transform it to obtain

First formula	$[c, b, a]$	$[c + bx + ax^2 = 0]$
Second formula	$[f, e, d]$	$[f + ex + dx^2 = 0]$
Third formula	$[i, h, g]$	$[i + hx + gx^2 = 0]$

(If these formulas are symmetric, d is regarded as b , g as c and h as f .)

Formula to be folded by the cubic power²

(4.4)	$[R_0, R_1, R_2]$	$[R_0 + R_1 x + R_2 x^2 = 0]$
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Three rules for the cubic case

立方交乘法 Cubic rule of cross multiplication

甲己辛 (加) $[+1afh]$ 乙丁壬 $[+1bdi]$ 丙戊庚 $[+ceg]$
 甲戊壬 (減) $[-1aei]$ 乙己庚 $[-1bfg]$ 丙丁辛 $[-1cdh]$

変乘法 Rule of varied multiplication

甲己冪 (一段)(加) $[+1af^2]$ 乙冪壬 (一段) $[+1b^2i]$
 丙冪戊 (一段) $[+1c^2e]$
 甲戊壬 (一段)(減) $[-1aei]$ 乙丙己 (二段) $[-2bcf]$

消長法 Rule of deletion/generation

実再 (一段)(長) $[+1R_0^3]$ 方再商再 (一段) $[+1R_1^3Q^3]$
 廉再商五 (一段) $[+1R_2^3Q^6]$
 実方廉商再 (三段)(消) $[-3R_0R_1R_2Q^3]$

²In this problem, we transcribe the Reality 実, the Square 方, the Side 廉 and the quotient 商 by R_0 , R_1 , R_2 and Q , respectively.

Restate the problem in modern mathematics. If we have (4.3), then

$$\begin{aligned} \det \begin{pmatrix} c & b & a \\ f & e & d \\ i & h & g \end{pmatrix} &= c \det \begin{pmatrix} e & d \\ h & g \end{pmatrix} - b \det \begin{pmatrix} f & d \\ i & g \end{pmatrix} + a \det \begin{pmatrix} f & e \\ i & h \end{pmatrix} \\ &= afh + bdi + ceg - aei - bfg - cdh = 0. \end{aligned}$$

This is called the rule of cross multiplications.

In the symmetric case, that is, $d = b$, $g = c$, $h = f$, the rule of cross multiplications is called the rule of varied multiplications:

$$\det \begin{pmatrix} c & b & a \\ f & e & b \\ i & f & c \end{pmatrix} = af^2 + b^2i + c^2e - aei - 2bcf = 0.$$

Suppose (4.4) and $-Q^3 + x^3 = 0$, where R_0 , R_1 , R_2 and Q^3 are known. Because we have

$$\begin{aligned} R_0 + R_1x + R_2x^2 &= 0, \\ R_2Q^3 + R_0x + R_1x^2 &= 0, \\ R_1Q^3 + R_2Q^3x + R_0x^2 &= 0, \end{aligned}$$

the rule of varied multiplications implies the rule of deletion/generation:

$$\det \begin{pmatrix} R_0 & R_1 & R_2 \\ R_2Q^3 & R_0 & R_1 \\ R_1Q^3 & R_2Q^3 & R_0 \end{pmatrix} = R_0^3 + R_1^3Q^3 - 3R_0R_1R_2Q^3 + R_2^3Q^6 = 0.$$

§ 4.3. Problem 17–58 (English translation)

Suppose given the pair of two formulas of degree 4 and transform it to obtain:³

(4.5)	First formula	$[\delta, \gamma, \beta, \alpha]$	$[\delta + \gamma x + \beta x^2 + \alpha x^3 = 0]$
	Second formula	$[\theta, \eta, \zeta, \epsilon]$	$[\theta + \eta x + \zeta x^2 + \epsilon x^3 = 0]$
	Third formula	$[\mu, \lambda, \kappa, \iota]$	$[\mu + \lambda x + \kappa x^2 + \iota x^3 = 0]$
	Fourth formula	$[\pi, \rho, \xi, \nu]$	$[\pi + \rho x + \xi x^2 + \nu x^3 = 0]$

³In order to translate the original text into English, we have to fix the rule of transcription of the twenty eight lodges by alphabets as follows:

角	亢	氏	房	心	尾	箕	斗	牛	女	虛	危	室	壁
α	β	γ	δ	ϵ	ζ	η	θ	ι	κ	λ	μ	ν	ξ
奎	婁	胃	昂	畢	觜	參	井	鬼	柳	星	張	翼	軫
ρ	π	ρ	σ	τ	υ	ϕ	χ	ψ	ω	Ω	張	翼	軫

Note that we need only $4^2 = 16$ ($5^2 = 25$) characters for a determinant of degree 4 (degree 5), respectively.

(In the symmetric case, we regard ϵ as β , ι as γ , κ as η , ν as δ , ξ as θ and o as μ .)

Formula to be folded by the (fourth) power: ⁴

$$(4.6) \quad \boxed{[R_0, R_1, R_2, R_3]} \quad [R_0 + R_1x + R_2x^2 + R_3x^3 = 0]$$

Three rules for the quartic case

三乘方交乘法 Quartic rule of cross multiplication

角尾虚婁 (加)	$[+1\alpha\zeta\lambda\pi]$	角箕危壁	$[+1\alpha\eta\mu\xi]$	角斗女奎	$[+1\alpha\theta\kappa o]$
亢心危奎	$[+1\beta\epsilon\mu o]$	亢箕牛婁	$[+1\beta\eta\iota\pi]$	亢斗虚室	$[+1\beta\theta\lambda\nu]$
氐心女婁	$[+1\gamma\epsilon\kappa\pi]$	氐尾危室	$[+1\gamma\zeta\mu\nu]$	氐斗牛壁	$[+1\gamma\theta\iota\xi]$
房心虚壁	$[+1\delta\epsilon\lambda\xi]$	房尾牛奎	$[+1\delta\zeta\iota o]$	房箕女室	$[+1\delta\eta\kappa\nu]$
角尾危奎 (減)	$[-1\alpha\zeta\mu o]$	角箕女婁	$[-1\alpha\eta\kappa\pi]$	角斗虚壁	$[-1\alpha\theta\lambda\xi]$
亢心虚婁	$[-1\beta\epsilon\lambda\pi]$	亢箕危室	$[-1\beta\eta\mu\nu]$	亢斗牛奎	$[-1\beta\theta\iota o]$
氐心危壁	$[-1\gamma\epsilon\mu\xi]$	氐尾牛婁	$[-1\gamma\zeta\iota\pi]$	氐斗女室	$[-1\gamma\theta\kappa\nu]$
房心女奎	$[-1\delta\epsilon\kappa o]$	房尾虚室	$[-1\delta\zeta\lambda\nu]$	房箕牛壁	$[-1\delta\eta\iota\xi]$

変乘法 Rule of varied multiplication

角尾虚婁 (一段) (加)	$[+1\alpha\zeta\lambda\pi]$	角箕斗危 (二段)	$[+2\alpha\eta\theta\mu]$	亢纂危纂 (一段)	$[+1\beta^2\mu^2]$
亢氏箕婁 (二段)	$[+2\beta\gamma\eta\pi]$	亢房斗虚 (二段)	$[+2\beta\delta\theta\lambda]$	氐纂斗纂 (一段)	$[+1\gamma^2\theta^2]$
氐房尾危 (二段)	$[+2\gamma\delta\zeta\mu]$	房纂箕纂 (一段)	$[+1\delta^2\eta^2]$		
角尾危纂 (一段) (減)	$[-1\alpha\zeta\mu^2]$	角箕纂婁 (一段)	$[-1\alpha\eta^2\pi]$	角斗纂虚 (一段)	$[-1\alpha\theta^2\lambda]$
亢纂虚婁 (一段)	$[-1\beta^2\lambda\pi]$	亢氏斗危 (二段)	$[-2\beta\gamma\theta\mu]$	亢房箕危 (二段)	$[-2\beta\delta\eta\mu]$
氐纂尾婁 (一段)	$[-1\gamma^2\zeta\pi]$	氐房箕斗 (二段)	$[-2\gamma\delta\eta\theta]$	房纂尾虚 (一段)	$[-1\delta^2\zeta\lambda]$

消長法 Rule of deletion/generation

実三 (一段) (長)	$[+1R_0^4]$	実方纂上商三 (四段)	$[+4R_0R_1^2R_2Q^4]$
実上下纂商七 (四段)	$[+4R_0R_2R_3^2Q^8]$	方纂下纂商七 (二段)	$[+2R_1^2R_3^2Q^8]$
上三商七 (一段)	$[+1R_2^4Q^8]$		
実纂方下商三 (四段) (消) *	$[-4R_0^2R_1R_3Q^4]$	実纂上纂商三 (二段)	$[-2R_0^2R_2^2Q^4]$
方三商三 (一段)	$[-1R_1^4Q^4]$	方上纂下商七 (四段)	$[-4R_1R_2^2R_3Q^8]$
下三商一十一 (一段)	$[-1R_3^4Q^{12}]$		

Notes on MSS (*) The character 消 (deletion, negative) is missing in the original text and is supplemented here.

We shall restate three rules in language of modern mathematics. If we have (4.5). then we

⁴In this problem, we transcribe the Reality 実, the Square 方, the Upper Side 上簾, the Lower Side 下簾 and the quotient 商 by R_0 , R_1 , R_2 , R_3 and Q , respectively.

have

$$\det \begin{pmatrix} \delta & \gamma & \beta & \alpha \\ \theta & \eta & \zeta & \epsilon \\ \mu & \lambda & \kappa & \iota \\ \pi & o & \xi & \nu \end{pmatrix} = 0.$$

The quartic rule of cross multiplications is given as the expansion formula of the determinant of a general matrix of degree 4:

In the symmetric case, we have the rule of varied multiplications:

$$\det \begin{pmatrix} \delta & \gamma & \beta & \alpha \\ \theta & \eta & \zeta & \beta \\ \mu & \lambda & \eta & \gamma \\ \pi & \mu & \theta & \delta \end{pmatrix} = 0.$$

If we have (4.6) and $Q^4 - x^4 = 0$, where R_0, R_1, R_2, R_3 and Q^4 are known, then we have

$$\det \begin{pmatrix} R_0 & R_1 & R_2 & R_3 \\ R_3 Q^4 & R_0 & R_1 & R_2 \\ R_2 Q^4 & R_3 Q^4 & R_0 & R_1 \\ R_1 Q^4 & R_2 Q^4 & R_3 Q^4 & R_0 \end{pmatrix} = 0$$

Expand this determinate and we have the rule of deletion/generation:

§ 4.4. Problem 17–59 (English translation)

Suppose given the pair of formula of degree 5. Transform it:

(4.7)	First formula	$[\epsilon, \delta, \gamma, \beta, \alpha]$	$[\epsilon + \delta x + \gamma x^2 + \beta x^3 + \alpha x^4 = 0]$
	Second formula	$[\kappa, \iota, \theta, \eta, \zeta]$	$[\kappa + \iota x + \theta x^2 + \eta x^3 + \zeta x^4 = 0]$
	Third formula	$[o, \xi, \nu, \mu, \lambda]$	$[o + \xi x + \nu x^2 + \mu x^3 + \lambda x^4 = 0]$
	Fourth formula	$[v, \tau, \sigma, \rho, \pi]$	$[v + \tau x + \sigma x^2 + \rho x^3 + \pi x^4 = 0]$
	Fifth formula	$[\Omega, \omega, \psi, \chi, \phi]$	$[\Omega + \omega x + \psi x^2 + \chi x^3 + \phi x^4 = 0]$

In the symmetric case, we regard ζ as β , λ as γ , μ as θ , π as δ , ρ as ι , σ as ξ , ϕ as ϵ , χ as κ , ψ as o and ω as v .

Formula to be folded by the (fifth) power:

$$(4.8) \quad [R_0, R_1, R_2, R_3, R_4] \quad [R_0 + R_1 x + R_2 x^2 + R_3 x^3 + R_4 x^4 = 0]$$

Three rules for the quintic case

四乗方交乘法 Rule of cross multiplication

角箕室畢星 (加)	[+1αηντΩ]	角箕壁觜鬼	[+1αηξυψ]	角箕奎昂柳	[+1αηοσω]
角斗危觜柳	[+1αθμνω]	角斗壁胃星	[+1αθξρΩ]	角斗奎畢井	[+1αθοτχ]
角牛危昂星	[+1αιμσΩ]	角牛室觜井	[+1αινυχ]	角牛奎胃鬼	[+1αιορψ]
角女危畢鬼	[+1ακμτψ]	角女室胃柳	[+1ακνρω]	角女壁昂井	[+1ακξσχ]
亢尾壁昂星	[+1βζξσΩ]	亢尾室觜柳	[+1βζνυω]	亢尾奎畢鬼	[+1βζοτψ]
亢斗虛畢星	[+1βθλτΩ]	亢斗壁觜參	[+1βθξυφ]	亢斗奎婁柳	[+1βθοπω]
亢牛虛觜鬼	[+1βιλλυψ]	亢牛室婁星	[+1βινπΩ]	亢牛奎昂參	[+1βιοσφ]
亢女虛昂柳	[+1βκλσω]	亢女室畢參	[+1βκντφ]	亢女壁婁鬼	[+1βκξπψ]
氐尾危畢星	[+1γζμτΩ]	氐尾壁觜井	[+1γζξυχ]	氐尾奎胃柳	[+1γζορω]
氐箕虛柳觜 (*)	[+1γηλωυ]	氐箕壁婁星	[+1γηξπΩ]	氐箕奎畢參	[+1γηοτφ]
氐牛虛胃星	[+1γιλρΩ]	氐牛危觜參	[+1γιμυφ]	氐牛奎婁井	[+1γιοπχ]
氐女虛畢井	[+1γκλτχ]	氐女危婁柳	[+1γκμπω]	氐女壁胃參	[+1γκξρφ]
房尾危觜鬼	[+1δζμυψ]	房尾室胃星	[+1δζνρΩ]	房尾奎昂井	[+1δζοσχ]
房箕虛昂星	[+1δηλσΩ]	房箕室觜參	[+1δηνυφ]	房箕奎婁鬼	[+1δηοπψ]
房斗虛觜井	[+1δθλνχ]	房斗危婁星	[+1δθμπΩ]	房斗奎胃參	[+1δθορφ]
房女虛胃鬼	[+1δκλρψ]	房女危昂參	[+1δκμσφ]	房女室婁井	[+1δκνπχ]
心尾危昂柳	[+1εζμσω]	心尾室畢井	[+1εζντχ]	心尾壁胃鬼	[+1εξξρψ]
心箕虛畢鬼	[+1εηλτψ]	心箕室婁柳	[+1εηνπω]	心箕壁昂參	[+1εηξσφ]
心斗虛胃柳	[+1εθλρω]	心斗危畢參	[+1εθμτφ]	心斗壁婁井	[+1εθξπχ]
心牛虛昂井	[+1ειλσχ]	心牛危婁鬼	[+1ειμπψ]	心牛室胃參	[+1εινρφ]
角箕室觜柳 (減)	[−1αηνυω]	角箕壁昂星	[−1αηξσΩ]	角箕奎畢鬼	[−1αηοτψ]
角斗危畢星	[−1αθμτΩ]	角斗壁觜井	[−1αθξυχ]	角斗奎胃柳	[−1αθορω]
角牛危觜鬼	[−1αιμνψ]	角牛室胃星	[−1αινρΩ]	角牛奎昂井	[−1αιοσχ]
角女危昂柳	[−1ακμσω]	角女室畢井	[−1ακντχ]	角女壁胃鬼	[−1ακξρψ]
亢尾室畢星	[−1βζντΩ]	亢尾壁觜鬼	[−1βζξυψ]	亢尾奎昂柳	[−1βζοσω]
亢斗虛觜柳	[−1βθλνω]	亢斗壁婁星	[−1βθξπΩ]	亢斗奎畢參	[−1βθοτφ]
亢牛虛昂星	[−1βιλσΩ]	亢牛室觜參	[−1βινυφ]	亢牛奎婁鬼	[−1βιοπψ]
亢女虛畢鬼	[−1βκλτψ]	亢女室婁柳	[−1βκνπω]	亢女壁昂參	[−1βκξσφ]
氐尾危觜柳	[−1γζμνω]	氐尾壁胃星	[−1γζξρΩ]	氐尾奎畢井	[−1γζοτχ]
氐箕虛畢星	[−1γηλτΩ]	氐箕壁觜參	[−1γηξυφ]	氐箕奎婁柳	[−1γηοπω]
氐牛虛觜井	[−1γιλνχ]	氐牛危婁星	[−1γιμπΩ]	氐牛奎胃參	[−1γιορφ]
氐女虛胃柳	[−1γκλρω]	氐女危畢參	[−1γκμτφ]	氐女壁婁井	[−1γκξπχ]
房尾危昂星	[−1δζμσΩ]	房尾室觜井	[−1δζνυχ]	房尾奎胃鬼	[−1δζορψ]
房箕虛觜鬼	[−1δηλνψ]	房箕室婁星	[−1δηνπΩ]	房箕奎昂參	[−1δηοσφ]
房斗虛胃星	[−1δθλρΩ]	房斗危觜參	[−1δθμνφ]	房斗奎婁井	[−1δθοπχ]
房女虛昂井	[−1δκλσχ]	房女危婁鬼	[−1δκμπψ]	房女室胃參	[−1δκνρφ]
心尾危畢鬼	[−1εζμτψ]	心尾室胃柳	[−1εζνρω]	心尾壁昂井	[−1εξξσχ]
心箕虛昂柳	[−1εηλσω]	心箕室畢參	[−1εηντφ]	心箕壁婁鬼	[−1εηξπψ]
心斗虛畢井	[−1εθλτχ]	心斗危婁柳	[−1εθμπω]	心斗壁胃參	[−1εθξρφ]
心牛虛胃鬼	[−1ειλρψ]	心牛危昂參	[−1ειμσφ]	心牛室婁井	[−1εινπχ]

Notes on MSS (*) 氐箕虛柳觜 should be 氐箕虛觜柳 if we follow the order of the 28 lodges. But this term is wrongly written in the all MSS. The last line (the 12th line) of 32v is missing in MS Kyoto U. A.

變乘法 Rule of varied multiplication

角箕室畢星 (一段) (加)	$[+1\alpha\eta\nu\tau\Omega]$	角箕壁奎觜 (二段)	$[+2\alpha\eta\xi\sigma\upsilon]$	角斗幕觜幕 (一段)	$[+1\alpha\theta^2\nu^2]$
角斗牛壁星 (二段)	$[+2\alpha\theta\iota\xi\Omega]$	角斗女奎畢 (二段)	$[+2\alpha\theta\kappa\sigma\tau]$	角牛幕奎幕 (一段)	$[+1\alpha\iota^2\sigma^2]$
角牛女室觜 (二段)	$[+2\alpha\iota\kappa\nu\upsilon]$	角女幕壁幕 (一段)	$[+\alpha\kappa^2\xi^2]$	亢幕室觜幕 (一段)	$[+1\beta^2\nu\nu^2]$
亢幕壁幕星 (一段)	$[+1\beta^2\xi^2\Omega]$	亢幕奎幕畢 (一段)	$[+1\beta^2\sigma^2\tau]$	亢氏斗畢星 (一段)	$[+1\beta\gamma\theta\tau\Omega]$
亢氏牛奎觜 (二段)	$[+2\beta\gamma\iota\sigma\upsilon]$	亢氏女壁觜 (二段)	$[+2\beta\gamma\kappa\xi\nu]$	亢房斗奎觜 (二段)	$[+2\beta\delta\theta\sigma\upsilon]$
亢房牛室星 (二段)	$[+2\beta\delta\iota\nu\Omega]$	亢房女壁奎 (二段)	$[+2\beta\delta\kappa\xi\sigma]$	亢心斗壁觜 (二段)	$[+2\beta\epsilon\theta\xi\nu]$
亢心牛壁奎 (二段)	$[+2\beta\epsilon\iota\xi\sigma]$	亢心女室畢 (二段)	$[+2\beta\epsilon\kappa\nu\tau]$	氐幕箕觜幕 (一段)	$[+1\gamma^2\eta\nu^2]$
氐幕牛幕星 (一段)	$[+1\gamma^2\iota^2\Omega]$	氐幕女幕畢 (一段)	$[+1\gamma^2\kappa^2\tau]$	氐房箕壁星 (二段)	$[+1\gamma\delta\eta\xi\Omega]$
氐房斗女觜 (二段)	$[+2\gamma\delta\theta\kappa\nu]$	氐房牛女奎 (二段)	$[+2\gamma\delta\iota\kappa\sigma]$	氐心箕奎畢 (二段)	$[+2\gamma\epsilon\eta\sigma\tau]$
氐心斗牛觜 (二段)	$[+2\gamma\epsilon\theta\iota\nu]$	氐心牛女壁 (二段)	$[+2\gamma\epsilon\iota\kappa\xi]$	房幕箕奎幕 (一段)	$[+2\delta^2\eta\sigma^2]$
房幕斗幕星 (一段)	$[+1\delta^2\theta^2\Omega]$	房幕女幕室 (一段)	$[+1\delta^2\kappa^2\nu]$	房心箕室觜 (二段)	$[+2\delta\epsilon\eta\nu\nu]$
房心斗牛奎 (二段)	$[+2\delta\epsilon\theta\iota\sigma]$	房心斗女壁 (二段)	$[+2\delta\epsilon\theta\kappa\xi]$	心幕箕壁幕 (一段)	$[+1\epsilon^2\eta\xi^2]$
心幕斗幕畢 (一段)	$[+1\epsilon^2\theta^2\tau]$	心幕牛幕室 (一段)	$[+1\epsilon^2\iota^2\nu]$		
角箕室觜幕 (一段) (減)	$[-1\alpha\eta\nu\nu^2]$	角箕壁幕星 (一段)	$[-1\alpha\eta\xi^2\Omega]$	角箕奎幕畢 (一段)	$[-1\alpha\eta\sigma^2\tau]$
角斗幕畢星 (一段)	$[-1\alpha\theta^2\tau\Omega]$	角斗牛奎觜 (二段)	$[-2\alpha\theta\iota\sigma\upsilon]$	角斗女壁觜 (二段)	$[-2\alpha\theta\kappa\xi\nu]$
角牛幕室星 (一段)	$[-1\alpha\iota^2\nu\Omega]$	角牛女壁奎 (二段)	$[-2\alpha\iota\kappa\xi\sigma]$	角女幕室畢 (一段)	$[-1\alpha\kappa^2\nu\tau]$
亢幕室畢星 (一段)	$[-1\beta^2\nu\tau\Omega]$	亢幕壁奎觜 (二段)	$[-2\beta^2\xi\sigma\upsilon]$	亢氏斗觜幕 (二段)	$[-2\beta\gamma\theta\nu^2]$
亢氏牛壁星 (二段)	$[-2\beta\gamma\iota\xi\Omega]$	亢氏女奎畢 (二段)	$[-2\beta\gamma\kappa\sigma\tau]$	亢房斗壁星 (二段)	$[-2\beta\delta\theta\xi\Omega]$
亢房牛奎幕 (二段)	$[-2\beta\delta\iota\sigma^2]$	亢房女室觜 (二段)	$[-2\beta\delta\kappa\nu\nu]$	亢心斗奎畢 (二段)	$[-2\beta\epsilon\theta\sigma\tau]$
亢心牛室觜 (二段)	$[-2\beta\epsilon\iota\nu\nu]$	亢心女壁幕 (二段)	$[-2\beta\epsilon\kappa\xi^2]$	氐幕箕畢星 (一段)	$[-1\gamma^2\eta\tau\Omega]$
氐幕牛女觜 (二段)	$[-2\gamma^2\iota\kappa\nu]$	氐房箕奎觜 (二段)	$[-2\gamma\delta\eta\sigma\upsilon]$	氐房斗牛星 (二段)	$[-2\gamma\delta\theta\iota\Omega]$
氐房女幕壁 (二段)	$[-2\gamma\delta\kappa^2\xi]$	氐心箕壁觜 (二段)	$[-2\gamma\epsilon\eta\xi\nu]$	氐心斗女畢 (二段)	$[-2\gamma\epsilon\theta\kappa\tau]$
氐心牛幕奎 (二段)	$[-2\gamma\epsilon\iota^2\sigma]$	房幕箕室星 (一段)	$[-1\delta^2\eta\nu\Omega]$	房幕斗女奎 (二段)	$[-2\delta^2\theta\kappa\sigma]$
房心箕壁奎 (二段)	$[-2\delta\epsilon\eta\xi\sigma]$	房心斗幕觜 (二段)	$[-2\delta\epsilon\theta^2\nu]$	房心牛女室 (二段)	$[-2\delta\epsilon\iota\kappa\nu]$
心幕箕室畢 (一段)	$[-1\epsilon^2\eta\nu\tau]$	心幕斗牛壁 (二段)	$[-2\epsilon^2\theta\iota\xi]$		

消長法 Rule of deletion/generation

実四 (一段) (長)	$[+1R_0^5]$	実幕方上幕商四 (五段) (*6)	$[+5R_0^2R_1R_2^2Q^5]$
実幕方幕中商四 (五段)	$[+5R_0^2R_1^2R_3Q^5]$	実幕上下幕商九 (五段)	$[+5R_0^2R_2R_4^2Q^{10}]$
実幕中幕下商九 (五段)	$[+5R_0^2R_3^2R_4Q^{10}]$	実方幕下幕商九 (五段)	$[+5R_0R_1^2R_4^2Q^{10}]$
実方上中下商九 (五段) (*1)	$[+5R_0R_1R_2R_3R_4Q^{10}]$	実上幕中幕商九 (五段)	$[+5R_0R_2^2R_3^2Q^{10}]$
方四商四 (一段)	$[+1R_1^5Q^5]$	方幕上幕下商九 (五段)	$[+5R_1^2R_2^2R_4Q^{10}]$
方幕上中幕商九 (四段) (*2)	$[+4R_1^2R_2R_3^2Q^{10}]$	方中幕下幕商一十四 (六段) (*3)	$[+6R_1R_3^2R_4^2Q^{15}]$
上四商九 (一段) (*5)	$[+1R_2^5Q^{10}]$	上幕中下幕商一十四 (五段)	$[+5R_2^2R_3R_4^2Q^{15}]$
中四商一十四 (一段)	$[+1R_3^5Q^{15}]$	下四商一十九 (一段)	$[+1R_4^5Q^{20}]$
実再方下商四 (五段) (消)	$[-5R_0^3R_1R_4Q^5]$	実再上中商四 (五段)	$[-5R_0^3R_2R_3Q^5]$
実方再上商四 (六段) (*4)	$[-6R_0R_1^3R_2Q^5]$	実方中再商九 (五段)	$[-5R_0R_1R_3^3Q^{10}]$
実上再下商九 (五段)	$[-5R_0R_2^3R_4Q^{10}]$	実中下再商一十四 (五段)	$[-5R_0R_3R_4^3Q^{15}]$
方再中下商九 (五段)	$[-5R_1^3R_3R_4Q^{10}]$	方上再中商九 (五段)	$[-5R_1R_2^3R_3Q^{10}]$
方上下再商一十四 (五段)	$[-5R_1R_2R_4^3Q^{15}]$	上中再下商一十四 (五段)	$[-5R_2R_3^3R_4Q^{15}]$

Notes on MSS (*1) This term is not "positive" (generation 長) but "negative" (deletion 消). It should be move to the "negative" part. This is a common error of all the MSS. (*2) The

coefficient 4 (四段) is an error. It should be 5 (五段). This is a common error of all the MSS. (*3 and *4) The coefficient 6 (六段) is an error. It should be 5 (五段). This is a common error of all the MSS. (*5) The coefficient 1 (一段) is correct in MSS U. Tokyo, Kyoto U. B and Kyoto U. A, but MS Tokyo Science U. is wrong here with the coefficient 2 (二段). (*6) The power 幂 is written with the formal style in MSS U. Tokyo and Tokyo Science U., while it is written with the simplified style 巾 in MSS Kyoto U. B and Kyoto U. A. This term is erroneously written as 実巾方上中商四 (五段) in MS Kyoto U. B. (The characters 巾 and 中 look similar.) MS Kyoto U. A is right at this term.

We shall restate three rules in the language of modern mathematics

Suppose (4.7). Then we have

$$\det \begin{pmatrix} \epsilon & \delta & \gamma & \beta & \alpha \\ \kappa & \iota & \theta & \eta & \zeta \\ o & \xi & \nu & \mu & \lambda \\ v & \tau & \sigma & \rho & \pi \\ \Omega & \omega & \psi & \chi & \phi \end{pmatrix} = 0.$$

This is called the quintic rule of cross multiplications.

The quintic rule of varied multiplications is the rule of cross multiplications for a "symmetric" coefficient matrix.

Suppose (4.8) and $Q^5 - x^5 = 0$, where R_0, R_1, R_2, R_3, R_4 and Q^5 are known. Then the rule of varied multiplications implies

$$\det \begin{pmatrix} R_0 & R_1 & R_2 & R_3 & R_4 \\ R_4 Q^5 & R_0 & R_1 & R_2 & R_3 \\ R_3 Q^5 & R_4 Q^5 & R_0 & R_1 & R_2 \\ R_2 Q^5 & R_3 Q^5 & R_4 Q^5 & R_0 & R_1 \\ R_1 Q^5 & R_2 Q^5 & R_3 Q^5 & R_4 Q^5 & R_0 \end{pmatrix} = 0$$

This is called the quintic rule of deletion/generation.

Around 2008, Komatsu Hikosaburo prepared a "critical" version of the *Taisei Sankei*. First four volumes were published by RIMS, Kyoto University but others are still circulating privately, for example., at the Nagoya seminar on the history of mathematics. In the Komatsu version, the rule of deletion/generation for the four multiplications is corrected accordingly.

§ 5. What is *shiki* 式 for Seki Takakazu?

Engaging ourselves in the mathematically faithful translation of Volume 17 of the *Taisei Sankei* 大成算経 into English, we want to transcribe a Japanese technical term into one specific English term. For example, *sū* 數 is transcribed into a "number". But sometimes a Japanese term has dual meanings and cannot be put into a single English term. This is the case for *shiki*

式. Although it was not clearly recognized by the authors of the *Taisei Sankei*, this term has two meanings: a ``formula" or an "equation".

tianyuanshu 天元術 (*tengenjutsu*)

Mathematics of Seki Takakazu and Takebe Katahiro stemmed from the Chinese mathematics. In fact, they studied Zhu Shijie's 朱世傑: the *Suanxue Qimeng* 算学啓蒙 (1299, Introduction to Computational Studies) and learned the *tianyuanshu* and extended this mathematical ideas into the *bōshohō* 傍書法, with which they could handle polynomials of several unknowns.

With the *tianyuanshu* we can handle polynomials of one unknown: the unknown x is represented by the column vector $\begin{bmatrix} 0 \\ +1 \end{bmatrix}$. A polynomial, for example, $+1 - 2x + 3x^2$ is represented by a column vector, in this case, $\begin{bmatrix} +1 \\ -2 \\ +3 \end{bmatrix}$.

In general a polynomial in x is represented by a *shiki* 式, which is a column vector displayed on the counting board 算盤 (*suanpan*) by means of two kinds of counting rods 算籌 (*suanchou*). (算木 (*sangi*) in Japanese). A red rod represents $+1$; a black rod -1 . We propose to use the term ``configuration" to mean the column vector on the counting board.

The Method for Solving Hidden Problems

Seki Takakazu 関孝和 wrote the *Kai Indai no Hō* 解隠題之法 (ca. 1685) (The Method for Solving Hidden Problems) and expounded the theory of the algebraic equation of one unknown. This book follows the *Suanxue Qimeng* but the exposition becomes general and abstract.

There are 5 sections in the book.

1. 立元 (Placement of the celestial unit)

$$\begin{bmatrix} 0 \\ +1 \end{bmatrix}$$

(Introduction of the unknown x)

2. 加減 (Addition and Subtraction)

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a + d \\ b + e \\ c + f \end{bmatrix}$$

(Addition and subtraction of polynomials in x .

$$(a + bx + cx^2) + (d + ex + fx^2) = (a + d) + (b + e)x + (c + f)x^2$$

The scalar multiplication is taken for granted.)

3. 相乗 (Multiplication)

$$\begin{bmatrix} 0 \\ +1 \end{bmatrix} \times \begin{bmatrix} 0 \\ +1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ +1 \end{bmatrix}$$

(Multiplication of two polynomials. Multiplication is assumed bi-linear.)

In Sections 1, 2 and 3 *shiki* 式 means a formula, a polynomial, or a configuration on the counting board. Note that a polynomial $P(x)$ is an element of the polynomial ring $\mathbf{Q}[x]$.

shiki 式 (a formula) is also called a virtual number 假數, while an ordinary numbers si called a true number (真數). This shows that the *shiki* is a mathematical object which can be added, subtracted and multiplied.

4. 相消 (Cancellation)

Place $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ on the counting board and move this configuration to the left. 寄左. Clear the board. Place $\begin{bmatrix} d \\ e \\ f \end{bmatrix}$ on the board and cancel it with the left to obtain the equation to be extracted 開方式

$$\boxed{\begin{bmatrix} d - a \\ e - b \\ f - c \end{bmatrix}} \text{ or } \boxed{\begin{bmatrix} a - d \\ b - e \\ c - f \end{bmatrix}}$$

(We find the equation $(a - d) + (b - e)x + (c - f)x^2 = 0$)

5. 求数 (Finding the numerical solution)

Apply the so called Horner's method to find a numerical solution.

(A positive solution must exit. If not, the problem is ill-posed.)

In sections 4 and 5 *shiki* 式 (more explicitly *kaihōshiki* 開方式) is an equation or an equation to be extracted. The equation $P(x) = 0$ can be considered as the *ideal* of the ring $\mathbf{Q}[x]$ which is generated by $P(x)$.

N.B. Let R be a ring. A subset I of R is a sub-ring if $f, g \in I$ implies $f + g, f - g, fg \in I$. A sub-ring I of R is an ideal if $f \in I$ and $g \in R$ implies $fg \in I$.

Did Seki Takakazu discover the determinant?

Proposition 5.1. *Let R be a ring, for example $\mathbf{Q}$, $\mathbf{Q}[y]$, or $\mathbf{Q}[y, z]$. Let $a, b, \dots$ be in the ring R . If we have*

$$\begin{cases} a + bx + cx^2 = 0 \\ d + ex + fx^2 = 0 \\ g + hx + ix^2 = 0 \end{cases}$$

then we can eliminate the unknown x :

$$(5.1) \quad aei + dhc + gb f - afh - bdi - ceg = 0.$$

In the *Kaifukudai no Hō* 解伏題之法 (ca. 1685) (Methods for Solving Concealed Problems) Seki Takakazu stated Proposition 5.1 and called it the cubic method of cross multiplications 交乘法 (*kōjōhō*).

The left hand side of (5.1) is, in modern language, the full expansion formula of the determinant of degree 3. The full expansion formula is given for the determinant of degree 2, 3, 4, and 5 in the *Taisei Sankei*, Vol. 17.

There was no notation of $\det$ in the *Taisei Sankei*. If we dare to use the notation $\det$, the determinant was defined in Volume 17 of the *Taisei Sankei* as follows

$$\begin{aligned} \det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} &= a \det \begin{pmatrix} e & f \\ h & i \end{pmatrix} - b \det \begin{pmatrix} b & c \\ h & i \end{pmatrix} + c \det \begin{pmatrix} b & c \\ e & f \end{pmatrix} \\ &= aei + dhc + gb f - afh - bdi - ceg. \end{aligned}$$

- There is no statement like

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -\det \begin{pmatrix} d & e & f \\ a & b & c \\ g & h & i \end{pmatrix}$$

- There is a statement like

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 0 \quad \text{if and only if} \quad \det \begin{pmatrix} ka & b & c \\ kd & e & f \\ kg & h & i \end{pmatrix} = 0$$

- There is no mention to the multi-linearity of $\det$.

§ 6. Concluding Remarks

In the *Taisei Sankei* the determinant is an *equation* and has not a specific name. It is not considered as a function of columns vectors (or row vectors) but rather as an equation.

In the *Taisei Sankei* (1711) the determinant 行列式 is defined inductively by the Van der Monde formula and the full expansion formulas of determinants of degree 2, 3, 4 and 5 are given correctly. But in the process of transcription, copiers did not recalculate the determinant of the fifth degree. One term in the full expansion formula of the determinant of degree 5 is in the wrong ordering. This error is conserved in all MSS which I examined.

According to the way of presentation of rules of cross multiplication, varied multiplication and deletion/generation, at least one of the authors realized the relation among them; that is, the rule of varied multiplications is a special case of the rule of cross multiplications and the rule of deletion/generation is a special case of the rule of varied multiplications. But the meaning of the rule of deletion/generation remained obscure to the author who made a final copy, Takebe Kata'akira. The given proof of the rule of deletion/generation is inconsistent of the arrangement of the three statements:

References

- [1] 小松彦三郎：「大成算経」校訂本作成の現状報告, 数理解析研究所講究録, 1444(2007), 140 – 156.
- [2] Annick Horiuchi: Japanese Mathematics in the Edo Period (1600 -- 1868), Birkhäuser, 2010. Translated from the French by Silke Wimmer-Zagier, (Annick Horiuchi: Les Mathématiques japonaises à l'époque d'Edo 1600 -- 1868, Librairie Philosophique J. Vrin, Paris, 1994)
- [3] 森本光生：和算における連立代数方程式を解くアルゴリズム, 数理解析研究所講究録 1787, 2012 年, 44--64
- [4] Morimoto, Mitsuo and Ogane, Tsukane: Mathematical Treatise on the Technique of Linckage: An Annotated English Translation of Takebe Katahiro's *Tetsujutsu Sankei* preserved in the National Archives of Japan, SCIAMVS, 13 (2012), 157 -- 286.
- [5] E. Knobloch, H. Komatsu, D.Liu (eds). Seki, founder of Modern Mathematics in Japan, A commemoration on His Tercentenary, Springer, 2013.
- [6] 森本光生 (Morimoto, Mitsuo): 『大成算経』における演段について (On the exhibition of steps in the *Taisei Sankei*), RIMS kôkyûroku, held in September, 2014.
- [7] 小川東・森本光生：江戸時代の数学最前線--和算から見た行列式---, 技術評論社, 2014 年
- [8] 森本光生：建部賢弘と『大成算経』 --- 生誕 350 周年を記念して, 数学文化 022, 日本評論社, 2014 年, 7--19.
- [9] 森本光生 (Morimoto, Mitsuo): 大成算経前集解題 (On Part A of the *Taisei Sankei*), RIMS Kôkyûroku Bessatsu B50 (2014), 41 -- 65.
- [10] Morimoto, Mitsuo: Theory of Equations in the *Taisei Sankei*, The Bumwoo Conference on the History of Asian Mathematics, Proceedings of the Korean Society for History of Mathematics, Vol. 26, No. 1 (Aug. 2016), 73-74.
- [11] Ogawa, Tsukane and Morimoto, Mitsuo: Methods for a Circle, Volume 12 of the *Taisei Sankei*, (in preparation), to appear in [14].
- [12] Morimoto, Mitsuo and Fujii, Yasuo: The Theory of Well-posed Equations, Volume 17 of the *Taisei Sankei*, (in preparation), to appear in [14].
- [13] Morimoto, Mitsuo and Fujii, Yasuo: The Fifteen Examples of Algebraic Equations, Volume 19 of the *Taisei Sankei*, (in preparation), to appear in [14]

- [14] Ogawa Tsukane and Morimoto Mitsuo (Eds): Mathematics of Takebe Katahiro and History of Mathematics in East Asia, Advanced Studies in Pure Mathematics, Mathematical Society of Japan (in preparation)