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Procurement Auctions with General Price-Quality Evaluation^{*}

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Abstract

We offer a general framework to study procurement auctions when quality matters. In this environment, sellers compete for a project by bidding a price-quality pair, and the winning bidder is determined by the score assigned to each bid. In contrast to the existing study in which only the quasilinear scoring rule is considered, our analysis allows a broad class of scoring rules including many other realistic ones. We focus on the analyses of the equilibrium bidding behavior of first-score (FS) and second-score (SS) auctions. We find that FS or SS auctions can be transformed into equivalent, single-dimensional score-bid auctions where the bidder's utility (payoff upon winning) is non-linear in the score-bid. Our analysis demonstrates that the ranking of the two auction formats, in terms of expected scores, depends on the scoring rule and that the equivalence fails unless scoring rules are quasilinear. FS auctions induce less aggressive bidding than SS auctions if, for example, the scoring rule is price-quality ratio (PQR).

Key words: scoring auctions, non-quasilinear scoring rules, procurement

JEL classification: D44, H57, L13

1 Introduction

Public sector spending amounts, on average, to 15 percent of the GDP in OECD member countries (OECD (2007)). To meet the needs of the public without increasing government liability, the call for the efficient and effective use of public funds is greater than ever.

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Accordingly, public procurement mechanisms must play a substantial role in saving public funds. As a competitive, transparent, and accountable allocation mechanism, low-price auctions have been widely used. However, because of the growing pressure to seek *value for money*, more and more procurement buyers have introduced mechanisms by which relevant prices and *qualities* of proposals in the entire procurement cycle are assessed. *Scoring auctions*, or equivalently multi-parameter bidding, are actually the prevailing mechanism that aims at competitiveness in price and value for money at the same time.

In this paper, we offer a general framework to study scoring auctions when quality matters, where sellers compete for a project by bidding a price-quality pair, and the winning bidder is determined by the score assigned to each bid. In contrast to existing studies in which only the quasilinear scoring rule is considered, our analysis allows for a broad class of scoring rules, for which price is non-linear in the score. We focus on the characterization and comparisons of equilibrium bidding behavior of first-score (FS) and second-score (SS) auctions. This study contributes to our understanding of strategic behaviors of bidders and the resulting performance evaluation for the procurement buyers in scoring auctions with non-quasilinear scoring rules.

A wide variety of scoring rules that are not quasilinear in price have been adopted in many countries. For example, many state departments of transportation (DOTs) in the U.S., including those in Alaska, Michigan, North Carolina, and South Dakota, have adopted the “adjusted bid,” under which the price bid is adjusted by being divided by the quality bid (Molenaar and Yakowenko (2007)). In Japan, most public procurement contracts are allocated to the bidder with the highest price-to-quality bid ratio.¹ In Sweden, scoring rules in public procurement are frequently additively separable but non-linear in price (Bergman and Lundberg (2011)). To examine the properties of scoring auctions with these scoring rules, the existing studies in which quasilinear rules are allowed do not apply. It should be emphasized here that any monotonic function cannot transform such a non-quasilinear scoring rule into a quasilinear (QL) form.²

We establish a model of scoring auctions in which general *independent* scoring rules are accepted. A scoring rule is independent if the score depends only on the associated bidder’s price and quality.³ Following the literature, we consider ex ante symmetric, risk-

¹The department of Health and Aging in Australia also employs a price-quality-ratio awarding rule for contracts that need to achieve better returns on public investment (The Department of Health and Ageing, Australia (2011)).

²For instance, taking a logarithm of a price-quality-ratio scoring rule only gives an additively separable scoring rule with price being non-linear in the score; a necessary condition for quasilinearity is thus violated.

³Our theory does not apply to *interdependent* scoring rules in which the bidder’s score is determined not only by his or her bids but also those of other bidders. For instance, the price score is given by the minimum or average price bid divided by the bidder’s price bid. See Albano et al. (2009) for the classification of scoring rules. More detailed arguments for the scoring rule are given in Dimitri et al., eds (2006).

neutral bidders that have a convex cost function which is parameterized via a single-dimensional private signal.⁴ We analyze and compare two standard auction formats: FS and SS auctions. In a FS auction, the bidder with the lowest score wins and follows the contract as specified in his winning bid. In a SS auction, the bidder with the lowest score wins and is free to choose the price and quality of the finalized contract as long as the score value based on the price and quality matches the minimal rival score.⁵

We approach the scoring auction with a general independent scoring rule by showing that a multi-dimensional auction can be transformed into an equivalent, single-dimensional score-bid auction from the following observations. First, instead of taking a price-quality pair as a bid, we substitute it with the associated score-quality pair. This is without loss of generality for the scoring rule that is monotonic and thus invertible in price given a quality. Second, the quality component in the analysis is endogenized given a score based on the fact that the equilibrium quality is consistent with the winner's profit maximization. In a FS auction, the quality component of a bid must be profit maximizing ex post, because the bidder can adjust the price-quality pair without affecting the score that has solely determined the probability of winning. In a SS auction, the quality component is re-selected to attain the second-lowest score. Given the score, the quality component in a bid is irrelevant to the bidder's ex post profit and the probability of winning. Hence, in either auction format, the only relevant component of a bid is the score. We can therefore suppress the quality component of a bid and focus only on the score for the analysis of strategic bidding behaviors in FS and SS auctions.

The above observations suggest that the analysis of FS and SS auctions might be analogous to that of first- and second-price auctions by just replacing price with score. Nonetheless, there is an important difference between scoring and price-only auctions; in scoring auctions, each bidder's utility upon winning cannot be exogenously given but is derived consistently with the scoring rule. More precisely, each bidder's utility function upon winning is the function of its cost parameter and the exercised score (that is, the winning score bid in a FS auction, or the second-lowest score bid in a SS auction) derived from profit maximization. This fact makes it difficult to analyze strategic bidding because the induced utility function is generally implicit without specifying functional forms of scoring rules and cost functions. To maintain the generality of our analysis, we avoid specific functional forms but instead impose a mild restriction on the induced utility, specifically, the

⁴With some assumptions, our model can relax this restriction and be applied to the multi-dimensional signal environment.

⁵There is a difference between the SS auction and the so-called *second-preferred* score auction, in which the lowest-scoring bidder wins but must adopt the price and quality of the second-lowest bidder. The second-preferred score auction generates an ex post loss and is subject to renegotiation.

log-supermodularity of score and cost parameter. This condition holds for a broad class of scoring rules, including quasilinear (QL) and price-quality-ratio (PQR) rules.

Our first result shows that there is a weakly dominant strategy equilibrium in a SS auction, in which each bidder bids the score for which the utility upon winning is equal to zero. We call such a score the break-even score. The proof is similar to that of a second-price auction. If the bidder submits a score lower than the break-even score and wins, there is a possibility that the second-lowest score is below the break-even score, thereby incurring a loss. The bidder could have avoided this loss by bidding the break-even score. Analogously, it is suboptimal for the bidder to submit a score higher than the break-even score. Note that this result requires only that the break-even score be well-defined. Therefore, the result can be generalized even to multidimensional types.

Our second result characterizes a symmetric Bayesian Nash equilibrium for a FS auction under the log-supermodularity assumption. In a FS auction, a higher score bid implies a tradeoff between a higher utility upon winning and a lower probability of winning. Assuming the equilibrium is symmetric and sorting (i.e., a strictly increasing bidding strategy in cost type), we derive the equilibrium condition dictating that the marginal benefit equals the marginal loss. Although the existence of a solution to the associated differential equation is immediate, the characterization is not straightforward. Under log-supermodularity, we can indeed find a “change of variables for types” that induces the de facto type space for bidding in the following sense:⁶ the equilibrium condition of the original auction replicates that of a hypothetical first-price auction in which each bidder’s procurement cost is drawn from the de facto type space with the associated distribution. Since solving the latter equilibrium condition is routine, we can characterize the original equilibrium accordingly.⁷ It should be noted here that the change of variables generally involves the equilibrium bidding function, and therefore the equilibrium characterization is implicit.⁸

We then rank the expected scores to be exercised in FS and SS auctions. We follow the approach by Maskin and Riley (1984) and show that bidding behavior in a FS auction is more (respectively, less) aggressive than in the associated SS auction if the bidder’s utility function exhibits concavity (respectively, convexity) in score, and therefore the expected exercised score is smaller (respectively, larger) in the FS auction. Intuitively, the curvature of the induced utility affects the marginal utility upon winning but not the probability of winning directly, and hence concavity results in a more aggressive, lower score bid in a FS

⁶We name the de facto type as the *effective cost measured by score*.

⁷We follow Maskin and Riley (1984) to adopt this logic.

⁸For a quasilinear scoring rule, the de facto type is derived without the equilibrium bidding. This corresponds to the productive potential in Che and the pseudotype in Asker and Cantillon.

auction.⁹ By contrast, the equilibrium bidding behavior in a SS auction is qualitatively unaffected: each bidder bids the break-even score. Our result includes the equivalence result by Che (1993) and Asker and Cantillon (2008) for the case of QL scoring rules, in which the induced utility is linear in score.

Two factors determine the curvature of the induced utility function. First, the shape of the scoring rule in price partly affects the curvature of utility. If the scoring rule is concave in price, the inverse with respect to price is convex in score, and therefore the bidder's utility upon winning is more convex in score. Second and more importantly, a score change is, generally, associated with a change in the optimal quality provided. Because of this flexibility in quality, the marginal utility of score tends to increase as the score increases (becomes more convex). Hence, if the scoring rule is concave in price, the curvature of the induced utility is definitely convex, whereas if the scoring rule is convex in price, the curvature depends on the relative effects of the two factors.

To illustrate the above results, we select two important classes of scoring rules: QL rules (score $s = \text{total payment } p - \text{quality } q$) and PQR rules ($s = p/q$). It is indeed instructive to think of scoring auctions as *auctions of contracts*; given a scoring rule, each bidder submits a particular form of cost reimbursement contract for quality, and a quality level is chosen upon winning. In a QL-scoring auction, the contract is $p = s + q$; the score specifies a fixed payment upon winning, and any additional payment is proportional to q with the price-per-quality of one. In a PQR-scoring auction, the contract is $p = sq$; the score is *price per quality*. In QL scoring auctions, the optimal quality is determined regardless of the level of score because the quality price is fixed at one. The induced utility function is the fixed payment (score) plus constant (maximum of quality value - cost of quality) and thus is linear in score. In PQR scoring auctions, the optimal quality increases in score simply because the score is the quality price. The induced utility function thus increases in score faster than linearly; i.e., it is convex in score.

By our ranking result, the expected exercised score in a FS auction is the same as in the associated SS auction for QL rules, while it is higher for PQR rules. The ranking result of PQR rules can also be shown through a simple argument of the complementarity of the score and the optimal quality. A higher score in FS auctions (i.e., higher quality price) induces a higher quality level, reaping a higher profit upon winning. By contrast, this complementarity effect is absent in the SS auction since the optimal quality upon winning depends only on the second-lowest score, which is why bidders are less aggressive (i.e.,

⁹Note that our auction is a low-score bid auction while Maskin and Riley's is a high-price bid auction, and therefore the implication of the curvature on aggressiveness is opposite. Note also that our result has nothing to do with the risk attitude of bidders. Bidders are assumed to be risk-neutral in our setup.

higher bidding) in PQR scoring FS auctions.

We consider an extension to the model with a multi-dimensional type space. Except for QL rules (e.g., Asker and Cantillon (2008)), it is not easy to apply our analysis to a full-fledged multi-dimensional type space.¹⁰ To proceed, we impose a restriction on the induced utility so that it is decomposed: the equilibrium score bid depends only on a dimension in the multi-dimensional type space. This decomposition applies to a PQR scoring rule, for instance, if cost functions are homothetic among bidders whose signals are identical in the relevant dimension. Given the restriction, we predict that the contracted price and quality are scattered in the price-quality space, which would better fit the real-world scoring auction data. This extension to the multi-dimensional signal environment will be attractive in the structural econometrics of FS auctions with a non-quasilinear scoring rule.

Related Literature The theoretical study of scoring auctions with quasilinear scoring rules was pioneered by Che (1993), who found that equilibria of multi-dimensional auctions can be characterized with the standard methodology to analyze price-only auctions. Branco (1997) extended the results to the case in which bidders' signals are correlated. More recently, a thorough analysis by Asker and Cantillon (2008) showed that these properties are maintained even if a player's signal is multi-dimensional.¹¹

For auctions with non-quasilinear scoring rules, theoretical research in economics on the scoring auction with PQR rules was conducted by Hanazono (2010), who examined an example in which n risk-neutral bidders receive single-dimensional private signals independently from a common uniform distribution. Having derived an equilibrium bidding strategy, he verified that there exists a symmetric Bayesian Nash equilibrium in a FS auction with a PQR scoring rule. For empirical research, Nakabayashi (2009) constructed a model of scoring auctions with PQR rules in which he assumes that the bidder's cost function is an inverse L shape so that a bidder's optimal quality choice is uniquely determined by the bidder's signal even under PQR scoring rules. Because of the restriction on either the specific distribution or shape of the cost function, both analyses are silent on the general properties of scoring auctions examined in this paper.

The remainder of this paper is organized as follows. In Section 2, the model of scoring

¹⁰This is because the translation of the original type space (multi-dimensional) to the de facto type space (one-dimensional) requires partitions of types by which the associated equilibrium score bids are classified. However, finding such partitions that are consistent with equilibrium is not at all straightforward and is beyond the scope of our current analysis. Note that this remark is only relevant for FS auctions, whereas for SS auctions, the same analysis applies to the case of a multi-dimensional type space as mentioned earlier.

¹¹Bajari et al. (2006) empirically examines the procurement auction with unit price bids. In this auction, bidders submit multiple prices and the winner is the bidder whose weighted sum of all multiple bids is the lowest. The theoretical model is based on Che (1993) and Asker and Cantillon (2008).

auctions is described. In Section 3, symmetric equilibria in FS and SS auctions are analyzed, and expected winning scores in FS and SS auctions are compared. In Section 4, the two commonly used scoring functions, QL and PQR, are discussed. In Section 5, an extension of the analysis to a multi-dimensional signal environment is delivered. The final section is the conclusion.

2 The model

Consider a procurement buyer who auctions off a project contract to n risk-neutral bidders. All bidders are ex ante symmetric. At the bid preparation stage, each bidder obtains a signal $\theta \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$ distributed following the publicly known cumulative distribution $F(\theta)$. Let $q \in [q, \bar{q}] \subset \mathbb{R}_+$ denote the non-monetary attribute (quality) with which the bidder performs the contract.¹² We shall suppose that the bidder's cost schedule satisfies the following conditions

Assumption 1 (Cost function).

1. $C(q|\theta) \geq 0$,
2. $C_q(q|\theta) \geq 0$,
3. $C_{qq}(q|\theta) > 0$,
4. $C_\theta(q|\theta) > 0$.

That is, the cost function is increasing and convex in q . The fourth item implies that the more efficient bidder produces the same q at a lower cost.

In the scoring auction, the bidder submits a price bid $p \in \mathbb{R}_+$ and quality q . The publicly known independent scoring rule $S(p, q) : \mathbb{R}^2 \rightarrow \mathbb{R}$ maps these multi-dimensional bids into the score, s .

We assume that the scoring rule corresponds to the buyer's utility function. The buyer is indifferent to the exchange of any amount of the numeraire p and the quality level q so long as the score value is unchanged. Although out of the scope of our analysis, procurement buyers may use a scoring rule that differs from their true preference in practice. In this case, the quality price is still specified by the slope of the iso-score curve. Che (1993) and Asker and Cantillon (2010) show that buyers may be better off if setting the quality price lower than his true MRS to limit information rents obtained by the winning bidder.

We impose the following mild assumption on the scoring function:

¹²A single-dimensional quality is easily extended to the multi-dimensional quality as far as the quality component in the score is summarized to a single index. See Appendix A for more details.

Assumption 2 (Scoring function).

1. $S_p(p, q) > 0$,
2. $S_q(p, q) < 0$.

Note that a monotonic transformation of a scoring rule does not affect the bidder's behavior in auctions. However, such a transformation certainly changes the buyer's evaluation of outcomes in terms of score and may not be valid for a true representation of preferences.

The lowest-scoring bidder wins the contract in both FS and SS auctions. The winner receives the payment p and fulfills the contract providing the quality level q in a FS auction. In a SS auction, the winning bidder is free to choose p and q finalized in the contract as long as $S(p, q)$ is equal to the second-lowest score in the auction. In both auctions, the winner's finalized p and q , as well as the associated score value s , satisfy $s = S(p, q)$. In other words, the winner's payment p can be rewritten as a function of q and s . Therefore, there is no loss of generality in considering that each bidder chooses a scoring bid s and a quality bid q . The winner receives a payment $p(s, q)$, where the payment function satisfies $S(p(s, q), q) \equiv s$.

Now, let s^e denote the *exercised score by the winner*, i.e., the winning bidder's scoring bid s in FS auctions or the rivals' minimum scoring bid $\hat{s}$ in SS auctions. The bidder's payoff upon winning is then given by

$$p(s^e, q) - C(q|\theta).$$

Given the exercised score s^e , each bidder's choice of quality q must be consistent with ex post profit maximization. Otherwise, the bidder can increase profits without changing the winning probability, which is inconsistent with equilibrium bidding behavior. This observation suggests that each bidder cares only about the exercised score upon winning. Therefore, each bidder's ex post utility function can be written based on the score and private information only.

The following assumption is imposed on both the score function and the cost function to ensure the existence of a unique solution $q \in [\underline{q}, \bar{q}]$ for the maximization of payoff upon winning: $p(s^e, q) - C(q, \theta)$:

Assumption 3. For all $q \in [\underline{q}, \bar{q}]$,

$$p_{qq}(s, q) - C_{qq}(q, \theta) < 0.$$

Let $q(s^e, \theta)$ be the optimal q , which satisfies

$$q(s^e, \theta) = \arg \max_q p(s^e, q) - C(q, \theta),$$

for any s^e . For notational convenience, let us also define

$$u(s^e, \theta) = p(s^e, q(s^e, \theta)) - C(q(s^e, \theta), \theta),$$

as a bidder's ex post utility upon winning. The optimal quality choice $q(s^e, \theta)$ satisfies $p_q(s^e, q(s^e, \theta)) - C_q(q(s^e, \theta)|\theta) = 0$ if the optimal q has an interior solution. An important observation here is that the contractor in a scoring auction chooses q so that the marginal revenue matches the marginal cost. In a scoring auction, the winning bidder's revenue function is the payment function $p(s^e, q)$. Therefore, $q(s^e, \theta)$ is an analogue to the profit-maximizing quantity in producer theory.

An interior solution for the optimal q is not necessary in our analysis. If the optimal q is a corner solution, then the solution is generically insensitive to s , i.e., $q_s(s^e, \theta) = 0$. Therefore, regardless of whether $q(s^e, \theta)$ is an interior or corner solution, we have $[p_q(s^e, q) - C_q(q(s^e, \theta)|\theta)]q_s(s^e, \theta) = 0$. Applying the envelope theorem to $u(s, \theta)$, we have the derivatives of $u(s, \theta)$ with respect to s and θ as

$$\begin{aligned} u_1(s^e, \theta) &= p_s(s^e, q(s^e, \theta)), \\ u_2(s^e, \theta) &= -C_\theta(q(s^e, \theta)|\theta), \end{aligned}$$

respectively. Note that $u_1(s, \theta) \equiv p_s(s, q(s, \theta))$ is strictly positive by Assumption 2-1 and that $u_2(\cdot)$ is strictly negative by Assumption 1-4.

When $q(s^e, \theta)$ is an interior solution, the marginal cost of providing an additional quality unit matches the buyer's marginal rate of substitution (MRS), or equivalently, the marginal revenue to the winning bidder.¹³ In short, the choice of quality is taken as endogenous, and the scoring bid s is a sufficient statistic to examine the bidder's problem in scoring auctions. Thus, we henceforth restrict our attention to the reduced auction game in which bidders choose their scoring bids. The bidder's problem in the reduced auction game is given by

$$\max_s u(s^e, \theta) \Pr(\text{win}|s). \tag{1}$$

¹³In fact, the derivative of $C(q|\theta)$ with respect to q is the marginal rate of technical substitution (MRTS) between the numeraire and quality. Hence, as seen in a general equilibrium model, the buyer's MRS equals the producer's MRTS in scoring auctions.

3 Equilibrium analysis

The equilibrium bidding strategy in SS auctions is straightforward. In a SS auction, the bidder's payoff upon winning:

$$u(\hat{s}, \theta),$$

is independent of his own scoring bid. Furthermore, the winning bidder has a non-negative payoff. Therefore, as in a second-price auction, where bidders submit their willingness to pay, bidding the break-even score is a dominant strategy in a SS auction.

Define $k^-(\theta)$ as the minimum possible score the bidder with type θ makes with a non-negative utility. That is, for all θ , $k^-(\theta)$ satisfies

$$u(k^-(\theta), \theta) \equiv 0.$$

Since $u_1(s, \theta)$ is strictly positive, there is a unique value of $k^-(\theta)$ for all θ . Furthermore, the bidder has a strictly positive payoff upon winning for any $s > k^-(\theta)$. In addition, $k^-(\theta)$ is strictly increasing in θ .¹⁴

The following theorem summarizes this result.

Theorem 1. *In a SS auction, there exists a dominant strategy equilibrium, $s^{SS}(\theta) = k^-(\theta)$, in which bidders report their break-even score as their optimal scoring bids, i.e.,*

$$u(s^{SS}(\theta), \theta) = 0.$$

Proof. Let $G(\hat{s})$ be an arbitrary distribution of the lowest rival's scoring bid for the bidder with $G(\bar{s}) = 1$. Then, the bidder's optimization problem is given by

$$\max_s \int_s^{\bar{s}} u(\tau, \theta) dG(\tau).$$

The first-order condition gives

$$-u(s, \theta) = 0.$$

Since $u_1(s, \theta)$ is strictly positive, the objective function is maximized at $\{s : u(s, \theta) = 0\}$. □

Note that the dominant strategy equilibrium of a SS auction exists in a fairly general environment. For instance, the type space is extended to be generally multi-dimensional.

¹⁴Taking the derivative of $u(k^-(\theta), \theta) \equiv 0$ on both sides with respect to θ gives $u_1(\cdot)(k^-)'(\theta) + u_2(\cdot) = 0$. By Assumption 1-4 and Assumption 2-1, $u_1(\cdot) > 0$ and $u_2(\cdot) < 0$.

Furthermore, the signals across bidders can be correlated. As long as every bidder finds a unique, efficient scale quality level, the equilibrium is characterized in a SS auction.

We now characterize the equilibrium bidding strategy in FS scoring auctions. The following condition guarantees the existence of a symmetric increasing Bayesian Nash equilibrium in the FS auction.

Assumption 4 (Single-crossing condition).

$$\frac{d^2}{dsd\theta} \log u(s, \theta) > 0.$$

The log-supermodularity of the bidder's utility function is sufficient to guarantee the existence of a Bayesian Nash equilibrium shown by Athey (2001). Let $s(\theta)$ be the strictly increasing equilibrium bidding function in a FS auction. The equilibrium scoring bid in a FS auction is then characterized as follows:

Theorem 2. *The equilibrium strategy in a FS auction is given by*

$$p(s(\theta), q(s(\theta), \theta)) = C(q(s(\theta), \theta) | \theta) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(s(\tau), \tau) | \tau) \left[\frac{1 - F(\tau)}{1 - F(\theta)} \right]^{n-1} d\tau. \quad (2)$$

Proof. The bidder's problem (1) is rewritten by

$$\max_s u(s, \theta) [1 - F(s^{-1}(s))]$$

in equilibrium. By imposing the symmetric condition, the first-order condition is given by

$$u_1(s(\theta), \theta) s'(\theta) [1 - F(\theta)]^{n-1} = u(s(\theta), \theta) (n-1) f(\theta) [1 - F(\theta)]^{n-2}.$$

It follows that

$$\begin{aligned} [u_1(s(\theta), \theta) s'(\theta) + u_2(s(\theta), \theta)] [1 - F(\theta)]^{n-1} \\ - u(s(\theta), \theta) (n-1) f(\theta) [1 - F(\theta)]^{n-2} = u_2(s(\theta), \theta) [1 - F(\theta)]^{n-1}. \end{aligned}$$

Taking the integral on both sides from θ through $\bar{\theta}$ yields

$$u(s(\theta), \theta) [1 - F(\theta)]^{n-1} = \int_{\theta}^{\bar{\theta}} -u_2(s(\tau), \tau) [1 - F(\tau)]^{n-1} d\tau. \quad (3)$$

Therefore, the equilibrium strategy in a FS auction, $s(\theta)$, is characterized by

$$p(s(\theta), q(s(\theta), \theta)) = C(q(s(\theta), \theta)|\theta) + \int_{\theta}^{\bar{\theta}} C_{\theta}(q(s(\tau), \tau)|\tau) \left[\frac{1 - F(\tau)}{1 - F(\theta)} \right]^{n-1} d\tau.$$

□

Several observations can be made. First, as shown in Maskin and Riley (1984), the first-order condition is sufficient for optimality as well when the bidder's objective function satisfies the single-crossing condition.¹⁵ Second, (2) is an extension of the bidding function in a FS auction presented by Che (1993) to the non-quasilinear scoring function environment. Unlike the case of a QL scoring rule, the choice of q is endogenous in s under the general scoring function. Thus, the explicit solution for the equilibrium strategy $s(\theta)$ is not obtained.

Finally, just as in the first-price auction, the equilibrium score bid can be expressed by the conditional expectation of the the second-lowest bidder's cost-related variable. To see this, we define a bidder's *effective cost measured by score* as

$$k(s(\theta), \theta) = s(\theta) - \frac{u(s(\theta), \theta)}{u_1(s(\theta), \theta)}.$$

This definition can be interpreted as a cost in terms of score since u/u_1 represents the "utility" in terms of score (evaluated at the marginal utility of score) while the score itself can be interpreted as the "gross utility." Then, the increasing strategy $s(\theta)$ is characterized by the order statistic of $k(\cdot)$, as shown in the following corollary.

Corollary 1. *In a FS auction, the equilibrium scoring bid is the conditional expectation*

¹⁵Suppose the bidder with type θ reports an arbitrary value $\tilde{\theta}$ and his payment is given by $p(s(\tilde{\theta}), q(s(\tilde{\theta}), \theta))$. Then, the bidder's expected profit is given by

$$u(s(\tilde{\theta}), \theta)[1 - F(\tilde{\theta})]^{n-1}.$$

Taking the derivative with respect to $\tilde{\theta}$ gives

$$\begin{aligned} & u_1(s(\tilde{\theta}), \theta) s'(\tilde{\theta}) [1 - F(\tilde{\theta})]^{n-1} - (n-1) f(\tilde{\theta}) u(s(\tilde{\theta}), \theta) [1 - F(\tilde{\theta})]^{n-2} \\ &= u_1(s(\tilde{\theta}), \theta) \left\{ s'(\tilde{\theta}) - (n-1) \frac{f(\tilde{\theta})}{1 - F(\tilde{\theta})} \frac{u(s(\tilde{\theta}), \theta)}{u_1(s(\tilde{\theta}), \theta)} \right\} \end{aligned}$$

Note that $u(\cdot)/u_1(\cdot)$ is decreasing in θ (log-supermodular). Therefore, if $s'(\tilde{\theta}) - (n-1) \frac{f(\tilde{\theta})}{1 - F(\tilde{\theta})} \frac{u(s(\tilde{\theta}), \theta)}{u_1(s(\tilde{\theta}), \theta)} = 0$, and $s(\tilde{\theta}) = k^-(\tilde{\theta})$, then

$$\frac{d}{d\tilde{\theta}} u(s(\tilde{\theta}), \theta) [1 - F(\tilde{\theta})]^{n-1} \begin{cases} > 0 \text{ if } \tilde{\theta} < \theta, \\ = 0 \text{ if } \tilde{\theta} = \theta, \\ < 0 \text{ if } \tilde{\theta} > \theta. \end{cases}$$

of the second lowest bidder's effective cost k :

$$s(\theta) = \int_{\theta}^{\bar{\theta}} \frac{(n-1)f(\tau) [1 - F(\tau)]^{n-2}}{[1 - F(\theta)]^{n-1}} k(s(\tau), \tau) d\tau. \quad (4)$$

Proof. The first-order condition of the bidder's maximization problem regarding s was

$$u_1(s(\theta), \theta)[1 - F(\theta)]^{n-1} + u(s(\theta), \theta)(n-1)f(\theta)[1 - F(\theta)]^{n-2} \frac{1}{s'(\theta)} = 0.$$

Since $u_1 > 0$, dividing both sides by u_1 yields

$$[1 - F(\theta)]^{n-1} + (s(\theta) - k(s(\theta), \theta))(n-1)f(\theta)[1 - F(\theta)]^{n-2} \frac{1}{s'(\theta)} = 0.$$

Solving the differential equation gives (4). □

Literature has shown that, when a scoring auction uses QL scoring functions, each bidder's private information can be rewritten as *productive potential* (in Che (1993)) or *pseudo-type* (in Asker and Cantillon (2008)). The effective cost defined above is a generalization of this concept. With the QL scoring function $S(p, q) = p - q$, $k(s(\theta), \theta)$ is given by

$$k(s(\theta), \theta) = C(q(s(\theta), \theta) | \theta) - q(s(\theta), \theta),$$

which is the break-even score for quality $q(s(\theta), \theta)$. Note that the optimal quality $q(s(\theta), \theta)$ maximizes the ex post profit upon a given winning score, i.e.,

$$p(s, q) - C(q | \theta) = s + q - C(q | \theta),$$

and thus only depends on θ for a QL scoring function. Hence, $k(s(\theta), \theta)$ equals $\min_q \{q - C(q | \theta)\} = k^-(\theta)$, which in turn equals $(-1) \times$ the productive potential.¹⁶ With a general scoring function, however, k hinges on s and is, hence, endogenous in the equilibrium strategy in a FS auction. Thus, $k(s(\theta), \theta)$ is generally not a sufficient statistic to summarize the bidder's effective type in a FS auction.

An advantage of characterizing k in a scoring auction may lie in the structural estimation of scoring auctions. Since $s(\theta)$ is the conditional second-lowest order statistic of $k(\cdot)$ in a FS auction or the minimum effective cost $k^-(\theta)$ in a SS auction, applying the structural estimation technique of price-only auctions (e.g., Guerre et al. (2000)) allows us to identify

¹⁶The negative sign in our model is an artifact of our modeling in which the lowest score bidder wins, whereas in Che or Asker-Cantillon, the highest score bidder wins.

the latent variable $k(\cdot)$ from both an observable s and its distribution. Note that, given θ , $k(s(\theta), \theta)$ is monotone in the bidder's effective production cost $C(q(s(\theta), \theta)|\theta)$. Hence, the bidder's equilibrium $C(q(s(\theta), \theta)|\theta)$ is identified.

Now, we proceed to analyze the revenue ranking in scoring auctions with a generalized scoring function. The revenue equivalence theorem (Myerson (1981); Riley and Samuelson (1981)) has been extended to multi-dimensional auction games by Che (1993), showing that if a QL scoring rule is used, the expected exercised scores in both FS and SS auctions are equivalent. Furthermore, Asker and Cantillon (2008) extend Che (1993)'s result to the multidimensional private information environment. We show, however, that QL is a special case in which the equivalence holds.

The following theorem illustrates that when the scoring function is a general form, the expected score ranking between FS and SS auctions depends on the scoring function.

Theorem 3 (expected scores). *The expected exercised score in a FS auction is greater (smaller) than that in a SS auction if $u_{11}(s, \theta)$ is positive (negative), i.e.,*

$$\begin{aligned} E[s^{FS}(\theta_{(1)})] &> E[s^{SS}(\theta_{(2)})] \text{ if } u_{11}(s, \theta) > 0, \\ E[s^{FS}(\theta_{(1)})] &= E[s^{SS}(\theta_{(2)})] \text{ if } u_{11}(s, \theta) = 0, \\ E[s^{FS}(\theta_{(1)})] &< E[s^{SS}(\theta_{(2)})] \text{ if } u_{11}(s, \theta) < 0. \end{aligned}$$

Proof. Let $s^{SQ}(\theta)$ be the conditional expectation of the second-lowest bidder's break-even score such that

$$\begin{aligned} s^{SQ}(\theta) &= E[k^-(\theta_{(2)})|\theta_{(2)} > \theta], \\ &= \int_{\theta}^{\bar{\theta}} \frac{(n-1)[1-F(\tau)]^{n-2}f(\tau)}{[1-F(\theta)]^{n-1}} k^-(\tau) d\tau, \end{aligned}$$

where $s^{SQ}(\bar{\theta}) = k^-(\bar{\theta})$. Taking the derivative with respect to θ gives

$$s_{\theta}^{SQ}(\theta) = [s^{SQ}(\theta) - k^-(\theta)] \frac{(n-1)f(\theta)}{1-F(\theta)}.$$

On the other hand, the derivative of the equilibrium bidding function of a FS auction is

$$s_{\theta}^{FS}(\theta) = \frac{u(s^{FS}(\theta), \theta)}{u_1(s^{FS}(\theta), \theta)} \frac{(n-1)f(\theta)}{1-F(\theta)}. \quad (5)$$

We know that $u(k^-(\theta), \theta) = 0$ and that $u_{11}(s, \theta) \geq 0$ for all $s \geq k^-(\theta)$. Thus, we have

$u_1(s, \theta) \cdot [s - k^-(\theta)] \geq u(s, \theta)$. Given the fact that $u_1(s, \theta) > 0$, it follows that

$$s - k^-(\theta) \geq \frac{u(s, \theta)}{u_1(s, \theta)},$$

for all $s \geq k^-(\theta)$. Therefore, (5) becomes

$$s_\theta^{FS}(\theta) = \frac{u(s^{FS}(\theta), \theta)}{u_1(s^{FS}(\theta), \theta)} \frac{(n-1)f(\theta)}{1-F(\theta)} \leq [s^{FS}(\theta) - k^-(\theta)] \frac{(n-1)f(\theta)}{1-F(\theta)}. \quad (6)$$

If $s^{FS}(\theta) < s^{SQ}(\theta)$, then $[s^{FS}(\theta) - k^-(\theta)] \frac{(n-1)f(\theta)}{1-F(\theta)} < [s^{SQ}(\theta) - k^-(\theta)] \frac{(n-1)f(\theta)}{1-F(\theta)}$. Therefore, by (6), $s_\theta^{FS}(\theta) < s_\theta^{SQ}(\theta)$. In addition, $s^{FS}(\bar{\theta}) = s^{SQ}(\bar{\theta}) = k^-(\bar{\theta})$. Hence, for any $\theta < \bar{\theta}$, $s^{FS}(\theta) > s^{SQ}(\theta)$. $\square$

Several points are worth noting here. First, as seen in (1), a multi-dimensional auction is, in general, reduced into a one-dimensional auction in which bidders with a non-linear utility function submit scores. Bidders with a convex (concave) utility function bid less (more) aggressively in a FS auction than in a SS auction. This is analogous to the comparison of the bidding behavior between first- and second-price auctions with non-risk-neutral bidders. Consequently, the proof of Theorem 3 directly follows from the standard proof of the revenue ranking between first- and second-price auctions, as demonstrated by Maskin and Riley (1984).

Second, two factors affect the expected score ranking between FS and SS auctions: i) the degree of cross effect of score and quality and ii) the convexity or concavity of the score function in price. Specifically, we have

$$u_{11}(s, \theta) = p_{sq}(s, q(s, \theta))q_s(s, \theta) + p_{ss}(s, q(s, \theta)).$$

Factors i) and ii) are associated with the first and the second terms on the right-hand side of the equation, respectively. Regarding factor i), it is easily seen that the cross effect $p_{sq}(s, q(s, \theta))q_s(s, \theta)$ is always non-negative and is strictly positive if q_s is non-zero.¹⁷ The intuition behind this is straightforward: the awarded bidder has the freedom to choose the

¹⁷We have

$$u_{11}(s, \theta) = p_{ss}(s, \theta) + p_{sq}(s, \theta)q_s(s, \theta).$$

On the other hand, for all s , $q(s, \theta)$ satisfies

$$p_q(s, q(s, \theta)) \equiv C_q(q(s, \theta)|\theta).$$

Taking the derivative on both sides with respect to s yields

$$p_{sq}(s, q(s, \theta)) \equiv [C_{qq}(q(s, \theta)|\theta) - p_{qq}(s, q(s, \theta))] q_s(s, \theta).$$

By Assumption 3, $C_{qq} - p_{qq}$ is strictly positive. Hence, $p_{sq}(s, q(s, \theta))q_s(s, \theta)$ is non-negative.

profit-maximizing quality level. A higher s raises the bidder's payment upon winning no slower than linearly. As this effect increases ceteris paribus, the bidder's ex post utility function becomes more convex in score, which induces bidders to bid less aggressively in a FS auction. Notice that under a QL scoring function, the bidder's profit-maximizing quality is independent of s ; thereby, $p_{sq}q_s = 0$. Combining the fact that $p_{ss} = 0$, we can see that a scoring auction with a QL scoring function is an analogy to a risk-neutral bidder in a price-only auction.

The effect of varying non-price attributes on a bidder's aggressiveness is analyzed analogously by Hansen (1988) in a different context. He analyzes a homogeneous multiple-object procurement auction in which risk-neutral bidders submit a unit price of the item, and it is the auctioneer who chooses the quantity purchased ex post. The non-price attribute thus represents *quantity* instead of *quality*. Given that the auctioneer chooses a quantity from a downward sloping demand, a higher unit price raises the winning bidder's payment no faster than linearly. As a result, risk-neutral bidders bid more aggressively in a first-price multi-unit auction with homogeneous items.

Regarding factor ii), suppose a scoring function is weakly concave in p for any q , i.e., S_{pp} is non-positive, or equivalently, p_{ss} is non-negative. Then u_{11} is necessarily non-negative, and therefore the bidders bid less aggressively in a FS auction than in the associated SS auction. In particular, if the score function is either quasilinear or price-quality-ratio in the form of $p/V(q)$, the score function is linear in price. Later we will see that u_{11} is strictly positive in a PQR case. In order to make u_{11} strictly negative, p_{ss} needs to be sufficiently negative. Only in such a case do the bidders bid more aggressively in a FS auction, and thus a lower expected score results than in a SS auction.

Finally, the non-linearity of the bidder's utility function partially hinges on the smoothness of the cost function around the optimal quality level. Suppose, as an extreme counter-example, that the cost function is an inverse L shape such that $C(q|\theta) = C(\theta)$ if $q \leq q(\theta)$ and $C(q|\theta) = \infty$ if $q > q(\theta)$. Then, the optimal q is constant for all $s \geq k^-(\theta)$ such that $q(k^-(\theta), \theta) = q(\theta)$. The payoff upon winning is thus given by

$$u(s, \theta) = u_1(s, \theta)[s - k^-(\theta)],$$

with $u_1(s, \theta) = p_s(s, q(k(\theta), \theta))$. If, in addition, $p_{ss}(\cdot) = 0$, $u_1(s, \theta)$ is constant for all s and thus u is linear in s . As happens under a QL scoring rule, the expected score equivalence holds between FS and SS auctions even with a general scoring function.

4 Comparing QL and PQR scoring functions

In this section, we restrict our attention to QL and PQR scoring functions, the most commonly used awarding rules in real-world procurement auctions.

Definition 1 (QL and PQR).

1. $S(p, q) = p/q$: *Price-quality-ratio (PQR)*
2. $S(p, q) = p - q$: *Quasi-linear (QL)*.

The reduced scoring auction game can be interpreted as the following supplier's competition. Bidders are asked to bid s , which is *the price of quality* under the PQR or the amount of a lump-sum subsidy under the QL scoring function. The lowest bidder wins. The payment rule is linear under the PQR and non-linear under the QL scoring function. Under the PQR scoring function, a unit of quality is reimbursed at the price equal to either the lowest scoring bid in FS or the second-lowest scoring bid in SS auctions. Under the QL function, a unit of quality is reimbursed linearly at the quality price equal to one, and a lump-sum subsidy is made which is equal to the lowest scoring bid in a FS or the second-lowest scoring bid in a SS auction.

Given these two different competitive environments, the bidder's optimal quality choices in PQR and QL functions are given, respectively, as

1. $C_q(q(s^e, \theta)|\theta) = s^e$ (PQR),
2. $C_q(q(s^e, \theta)|\theta) = 1$ (QL).

The optimal quality $q(s, \theta)$ is the one at which the marginal cost equals the quality price. Under the QL rule, the quality price is given as a constant. Therefore, bidders are "price takers" under the QL rule. Hence, the bidder's quality choice is $q(k^-(\theta), \theta)$, independent of s .

With (4), we showed that the bidder's equilibrium scoring bid in a FS auction is the expected value of $k(s(\theta_{(2)})|\theta_{(2)})$, i.e., the expectation of the second-order statistic of the bidder's effective cost measured by score. With the QL and PQR scoring functions, $k(s, \theta)$ is given by

1. $k(s, \theta) \equiv C(q(s, \theta)|\theta)/q(s, \theta)$ (PQR),
2. $k(s, \theta) \equiv k^-(\theta) \equiv C(q(k^-(\theta), \theta)|\theta) - q(k^-(\theta), \theta)$ (QL).

That is, $k(\cdot)$ is i) the average cost (AC) curve under the PQR and ii) the bidder's net cost, i.e., the gross production cost C subtracted by the cost reimbursement of the increased quality level under the QL function.

Since k^- is unique for any θ , a unique quality level exists at which either AC is minimized (*efficient scale*) under the PQR or the net cost is minimized under the QL scoring function. A SS auction with the PQR scoring function is the competition with respect to the minimum AC. The lowest minimum AC bidder wins and freely chooses the quality level given the quality price equal to the second-lowest bidder's minimum AC.

Notice that the winning bidder provides an excessive quality level (above the efficient scale) under the PQR rule. Because of the winning bidder's informational rents, the quality price is above the minimum AC. Hence, just as a profit-maximizing firm produces beyond the efficient scale in a short run if the market price is above the break-even price, the winning bidder chooses a larger quality level than the efficient scale. Under the QL scoring rule, the quality price is exogenous. Hence, the contracted quality may be greater or smaller than the efficient scale depending on MRS of the procurement buyer's utility function.

In a FS auction, bidders commit the *ex post* optimal quality level when submitting the scoring bid s . Under the PQR scoring rule, informational rents accrue to bidders by submitting s strictly greater than their minimum AC, and thus, the lowest losing bidder's AC, $k(s, \theta)$, is above his or her minimum AC. Recall that the equilibrium bid in a FS is the average k of the lowest losing bid. In this sense, the winning bidder takes advantage of the losing bidder's informational rents to obtain a larger payoff upon winning in a FS auction. This is another explanation of a higher expected exercised score in a FS than in a SS auction with the PQR scoring function.

5 Extension to multiple-dimensional type space

Our theoretical model has so far assumed that the bidder's type space is one-dimensional. In this section, we relax this assumption. As mentioned before, the equilibrium analysis in a SS auction is possible under the general multi-dimensional type space without any change. We now demonstrate that the equilibrium characterization in a FS auction is also possible under the multi-dimensional type space if the bidder's cost function satisfies an assumption.

To simplify the analysis, we suppose that the bidder's signal θ is two-dimensional such that $\theta = (\theta^0, \theta^1)$, where $\theta^0 \in [\underline{\theta}^0, \bar{\theta}^0]$ and $\theta^1 \in [\underline{\theta}^1, \bar{\theta}^1]$. Let $\underline{u}(s, \theta^0)$ be the bidder's payoff upon winning such that $\underline{u}(s, \theta^0) = u(s, \theta^0, \underline{\theta}^1)$. Then, we assume that there exists a

monotonic function $h(\theta) \geq 1$ such that, for all θ^0 , $dh(\theta)/d\theta^1 > 0$ and for all s and θ^0 ,

$$u(s, \theta) = h(\theta) \underline{u}(s, \theta^0). \quad (7)$$

For example, suppose the scoring function is PQR such that $S(p, q) = p/q$. Then u can be decomposed as above if and only if $C(q|\theta)$ is a homothetic function of $C(q|\theta^0, \underline{\theta}^1)$. The monotonic function $h(\theta) = h(\theta^1)$ is a multiplier such that $C(q|\theta) = h(\theta^1) \cdot C(q \cdot h(\theta^1) | \theta^0, \underline{\theta}^1)$.

With this assumption, we show that the equilibrium bidding strategy $s(\theta)$ is independent of θ^1 . Suppose that there are two bidders whose private signals are (θ^0, θ^1) and $(\theta^0, \tilde{\theta}^1)$. That is, two bidders have the same θ^0 but different θ^1 signals.

The equilibrium bid strategy $s(\theta)$ maximizes the bidder's expected payoff. The bidders' objective functions are given by

$$\begin{aligned} \max_s h(\theta^0, \theta^1) \underline{u}(s, \theta^0) \Pr\{win|s\}, \\ \max_s h(\theta^0, \tilde{\theta}^1) \underline{u}(s, \theta^0) \Pr\{win|s\}. \end{aligned}$$

Since the two maximization problems are monotonic transformations of each other, the two objective functions are maximized at the same s . This fact implies that the equilibrium bid strategy $s(\theta)$ is dependent only on θ^0 and is independent of θ^1 .

Taking the derivative of (7) with respect to s gives

$$u_1(s, \theta) = h(\theta) \underline{u}_1(s, \theta^0).$$

Therefore,

$$\frac{u(s, \theta)}{u_1(s, \theta)} = \frac{h(\theta) \underline{u}(s, \theta^0)}{h(\theta) \underline{u}_1(s, \theta^0)} = s - k(s, \theta^0, \underline{\theta}^1).$$

This result suggests that $k(s, \theta)$ is independent of θ^1 for all s . Thus, taking $\underline{u}(s, \theta^0)$ as $u(s, \theta)$ in the model discussed in the previous sections, the multi-dimensional signal case can be analyzed in our framework so that there are n risk-neutral bidders whose utility function is $h(\theta) \underline{u}(s, \theta^0)$. The log-supermodularity of $\underline{u}(s, \theta^0)$ is a sufficient condition for the existence of an increasing equilibrium in a FS auction. Due to the assumption on the utility function, the winning bidder is unchanged in FS and SS auctions.

By classifying the bidders with an identical θ^0 as in a group, a single dimension of θ , such as θ^0 , ends up governing the differentiation of a bidder's equilibrium behavior with respect to s , yet the bidders in the same group choose different q according to the other dimensions of θ . It follows that the equilibrium-exercised score and quality can be scattered in the price quality space.

6 Concluding remarks

In this paper, we have established a model of scoring auctions in which the assumption that scoring rules are QL is relaxed. We have demonstrated the existence of equilibria and characterized the equilibrium bidding strategies in FS and SS auctions. We have further found that equivalence fails between FS and SS mechanisms under general scoring rules.

The complexity associated with a general scoring function lies in the fact that the bidder's optimal quality depends on the score, whereas it does not with QL scoring functions. Hence, our equilibrium characterization must rely on the summary statistic that incorporates varying quality choice. In a SS auction, the winning bidder chooses ex post an optimal quality for the second-lowest score. Therefore, as in the case of second-price auctions, it is weakly dominant that each bidder truthfully reports the break-even score, which gives zero ex post profit for the optimal quality.

In a FS auction, the winning bidder chooses an optimal quality for his or her own score. Plugging the optimal quality for the score bid, we can transform the multi-dimensional auction into an auction of single-dimensional scores. Since a higher score is associated with a higher payment, each bidder faces a tradeoff between a higher profit upon winning and a lower probability of winning, which is analogous to the tradeoff observed in a first-price auction (FPA).

Our analysis departs from the standard price-only auctions because the bidder's ex post utility function is derived by optimal quality choice and may not be an explicit function. We have shown that, if the bidder's ex post utility function satisfies the log-supermodularity between score and private cost information, the existence and characterization of a symmetric equilibrium in a FS auction are guaranteed. Moreover, the ranking of the expected exercised score between FS and SS auctions depends on the second-order derivative of ex post utility with respect to score, which is reminiscent of the analysis of auctions with risk-averse bidders.

Our model has assumed that the buyer's preference is represented by the given scoring rule. In practice, however, a procurement buyer may use a scoring rule which differs from his or her true utility. As Che (1993) argues, buyers can benefit from not using the scoring rule that represents the true utility. Our theorem suggests that if the procurement buyer's true preference is PQR, a FS auction never implements the optimal mechanism since a SS auction always dominates the FS counterpart in terms of the expected exercised scores.

The issue of the awarding procedure between PQR and QL has also been discussed in the context of cost-benefit analysis (see, for instance, Stiglitz (2000)), with the conclusion that the advantage of either procedure depends exclusively on the buyer's preference. Our

results, however, suggest that when the scoring rule is PQR, a FS auction tends to leave more rents to bidders. Thus, if the preference of a risk-neutral procurement buyer is based on PQR, the buyer should use a SS auction.

In this study, we have restricted our attention to independent scoring rules by which each bidder's score depends only on his or her price and quality. In practice, interdependent scoring rules are also used. For instance, the quality measurement is an index based on the highest, lowest, or average quality bid submitted. Another example is that the lowest price bidder receives 100 points and a higher price bidder receives a scaled point accordingly. That is, the score depends not only on the bidder's own price and quality bid, but also on some or all competitors' price and quality bids. An important extension would make a theoretical consideration of the scoring auction with such a *interdependent scoring rule*.

Appendix A Multi-dimensional quality

Consider the cost function $C(q|\theta)$ where the quality bid is multi-dimensional such that $q \in \mathbb{R}_+^L$ with $L > 1$. Let $V(q)$ be a quality index where $V_q(q) > 0$ and $V_{qq}(q)$ is negative semi-definite. Suppose that the scoring rule $S(p, q)$ is given by $S(p, q) = p/V(q)$ for PQR and $S(p, q) = p - V(q)$ for QL scoring rules. Then, without loss of generality, V can be substituted with $v \in \mathbb{R}_+^L$ as follows: Define

$$C(v, \theta) = \min C(q|\theta) \text{ s.t. } V(q) = v,$$

for $v \in \text{Range of } V(q)$. If FOC is necessary for the minimization problem, we have

$$C(v, \theta) = C(q(v|\theta), \theta)$$

where $(q(v|\theta), \lambda(v|\theta))$ is a solution to

$$\begin{cases} C_q(q|\theta) - \lambda V_q(q) = 0 \in \mathbb{R}^L \\ V(q) - v = 0 \in \mathbb{R}. \end{cases}$$

Suppose C is strictly convex in q and $V(q)$ is weakly concave. Then

$$C_v = \lambda(v|\theta) > 0$$

$$C_{vv} = \lambda_v(v|\theta) > 0.$$

To see the first condition, note that from the second equation for optimality,

$$V_q(q(v|\theta)) q_v(v|\theta) = 1$$

which implies, by the first equation,

$$C_q(q(v|\theta), \theta) q_v(v|\theta) = \lambda.$$

This shows $C_v = \lambda(v|\theta)$. For the second condition, note that from the implicit function theorem

$$\begin{pmatrix} C_{qq} - \lambda V_{qq} - V_q^T \\ V_q & 0 \end{pmatrix} \begin{pmatrix} q_v \\ \lambda_v \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

This implies

$$\begin{aligned} V_q^T \lambda_v &= (C_{qq} - \lambda V_{qq}) q_v \\ \Rightarrow \underbrace{q_v^T V_q^T}_1 \lambda_v &= q_v^T \underbrace{(C_{qq} - \lambda V_{qq})}_{\text{positive definite}} q_v \\ \Rightarrow \lambda_v &> 0 \end{aligned}$$

This shows that there is no loss of generality to restrict attention to one-dimensional quality as long as $V(q)$ and C satisfy certain regularity conditions.

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