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Triple Systems and Applications to Gauge Theories

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1 Introduction

It has been expected that there exists M-theory, which unifies string theories. In M-theory, some structures of 3-algebras were found recently. First, it was found that field theories applied with $u(N)\oplus u(N)$ hermitian 3-algebras are the Chern-Simons gauge theories that describe effective actions of N coincident supermembranes [1–5], which are fundamental objects in M-theory. In a certain limit, a novel Higgs mechanism works, where the Chern-Simons gauge theories become the Yang-Mills theories that describe effective actions of D-branes in string theory. Second, 3-algebra models of M-theory themselves have been proposed and were studied in [6–13].

The hermitian 3-algebras [14–51] are special cases, where $<abc>=-<cba>$, of hermitian generalized Jordan triple systems $<abc>$ [52–74]. Therefore, it is natural to extend the $u(N)\oplus u(N)$ hermitian 3-algebras to more general hermitian generalized Jordan triple systems. Moreover, it is interesting to find a hermitian generalized Jordan triple system with which a Chern-Simons field theory reduces to a Yang-Mills theory in a certain limit.

In the following section, we review some results concerning with [75, 76].

2 Definitions

Let us start with a definition of a hermitian generalized Jordan triple systems.

Definition. A triple system U is said to be a hermitian generalized Jordan triple systems if relations (0)–(iv) satisfy;

0) U is a Banach space,

i) $[L(a,b),L(c,d)]=L(<abc>,d)-L(c,<bad>)$,

ii) $<xyz>$ is $\mathbf{C}$-linear operator on x,z and $\mathbf{C}$-anti-linear operator on y,

iii) $<abc>$ continuous with respect to a norm $||\ ||$ that is, there exists $K>0$ s.t.

$$||<xxx>||\leq K||x||^3 \text{ for all} x\in U.$$

iv)[1] There is a metric (x,y) that satisfies $(L(x,y)z,w)+(z,L(x,y)w)=0$ and $(x,y)=\overline{(y,x)}$.

[1]This definition is slightly different with that in [75, 76].

3 Generalization of the hermitian 3-algebra

In this section, we extend the $u(N) \oplus u(M)$ 3-algebras to a hermitian generalized Jordan triple system.

Let $D^*_{N,M}$ be the set of all $N \times M$ matrices with operation

$$< xyz >= x\bar{y}^T z - z\bar{y}^T x + zx^T\bar{y} - \bar{y}x^T z.$$

Then $D^*_{N,M}$ is a hermitian generalized Jordan triple system. In fact, it satisfies the conditions in the previous section with the metric $(x, y) := \mathrm{tr}(x\bar{y}^T)$. This is an extension of the $u(N) \oplus u(M)$ hermitian 3-algebras $< xyz >= x\bar{y}^T z - z\bar{y}^T x$.

4 Application to field theory

In this section, we apply the hermitian generalized Jordan triple system in the previous section to a field theory.

We start with

$$\begin{aligned} S = \int d^3x \mathrm{tr}(& -\mathbf{D}_\mu Z^A \overline{\mathbf{D}^\mu Z_A}^T \\ & + L\epsilon^{\mu\nu\lambda}(-A_{\mu\bar{b}c}\partial_\nu A_{\lambda\bar{d}a}\overline{T}^{T\bar{d}}[T^c, \overline{T}^{\bar{b}}, T^a] \\ & \qquad + \frac{2}{3}A_{\mu\bar{d}a}A_{\nu\bar{b}c}A_{\lambda\bar{f}e}[T^c, \overline{T}^{\bar{b}}, T^a]\overline{[T^f, \overline{T}^{\bar{e}}, T^d]})), \end{aligned}$$

where

$$\mathbf{D}_\mu Z^A = \partial_\mu Z^A - A_{\mu\bar{b}a}[T^a, \overline{T}^{\bar{b}}, Z^A].$$

Z^A and A_μ are matter and gauge fields, respectively. A runs from 1 to p, whereas μ runs from 0 to 2. This action is invariant under the transformations generated by the operator $L(x, y) - L(y, x)$. Here, we apply $[x, \overline{y}, z] :=< xyz >= (x\bar{y}^T - \bar{y}x^T)z - z(\bar{y}^T x - x^T\bar{y})$ to this action.

The covariant derivative is explicitly written down as

$$\mathbf{D}_\mu Z^A = \partial_\mu Z^A - iA^L_\mu Z^A + iZ^A A^R_\mu,$$

where $A^R_\mu := -iA_{\mu\bar{b}a}(\overline{T}^{T\bar{b}}T^a - T^{Ta}\overline{T}^{\bar{b}})$ and $A^L_\mu := -iA_{\mu\bar{b}a}(T^a\overline{T}^{T\bar{b}} - \overline{T}^{\bar{b}}T^{Ta})$ are real anti-symmetric matrices, which generate the $o(N)$ and $o(M)$ Lie algebras, respectively. The action can be rewritten in a covariant form with respect to $o(N)$ and $o(M)$ and we obtain a Chern-Simons gauge theory,

$$\begin{aligned} S &= \int d^3x \mathrm{tr}(-(\partial_\mu Z^A - iA^L_\mu Z^A + iZ^A A^R_\mu)\overline{(\partial_\mu Z_A - iA^L_\mu Z_A + iZ_A A^R_\mu)}^T \\ & \quad + L\epsilon^{\mu\nu\lambda}(\frac{1}{2}(A^L_\mu\partial_\nu A^L_\lambda - A^R_\mu\partial_\nu A^R_\lambda) + \frac{i}{3}(A^L_\mu A^L_\nu A^L_\lambda - A^R_\mu A^R_\nu A^R_\lambda))). \end{aligned}$$

In this action, A^L_μ and A^R_μ transform as adjoint representations of $o(N)$ and $o(M)$, respectively, whereas Z^A transforms as a bi-fundamental representation of $o(N) \oplus o(M)$;

$$\begin{aligned} \delta A^R_\mu &= [i\Lambda^R, A^R_\mu] \\ \delta A^L_\mu &= [i\Lambda^L, A^L_\mu] \\ \delta Z^A &= i\Lambda^L Z^A - Z^A(i\Lambda^R), \end{aligned}$$

where gauge parameters Λ^R and Λ^L are defined in the same way as A^R_μ and A^L_μ, respectively.

Next, let us examine whether the Novel Higgs mechanism works in this theory when M=N. By redefining the gauge fields as

$$\begin{aligned} A^L_\mu &= A_\mu + B_\mu \\ A^R_\mu &= A_\mu - B_\mu, \end{aligned}$$

we can separate a non-dynamical mode B_μ as

$$\begin{aligned} S &= \int d^3x \mathrm{tr}(-(D_\mu Z^A - i\{B_\mu, Z^A\})\overline{(D_\mu Z^A - i\{B_\mu, Z^A\})}^T \\ &\quad + L\epsilon^{\mu\nu\lambda}(B_\mu F_{\nu\lambda} + \frac{2i}{3}B_\mu B_\nu B_\lambda)), \end{aligned}$$

where

$$\begin{aligned} D_\mu Z^A &= \partial_\mu Z^A - i[A_\mu, Z^A], \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]. \end{aligned}$$

We divide Z^A into two real matrices as

$$Z^A = iX^A + X^{p+A},$$

and consider fluctuations around a background solution as $X^p = vI + \tilde{X}^p$. If we rescale L and B_μ as

$$\begin{aligned} L &= \mathcal{O}(v) \\ B_\mu &= \mathcal{O}(\frac{1}{v}), \end{aligned}$$

and use the equation of motion of B_μ,

$$B^\mu = \frac{L}{8v^2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - \frac{1}{2v}D^\mu X^{2p} + \mathcal{O}(\frac{1}{v^2}),$$

the action reduces to

$$S \to \int d^3x \mathrm{tr}(-g^2 F^2_{\mu\nu} - (D_\mu X^i)^2)$$

in $v \to \infty$, where $g = \frac{L}{v}$ and i runs from 1 to 2p-1. Therefore, we conclude that the Novel Higgs mechanism works in the Chern-Simons gauge theory with the hermitian generalized Jordan triple system in the previous section with M=N, and we obtain a Yang-Mills theory in this limit.

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