

Motion Characteristics Analysis of a Mecanum-Wheeled Omnidirectional Mobile Robot on a Slope

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Abstract. Mecanum-wheeled omnidirectional mobile robots (MOMRs) are widely used in the field of industry. However, since the mobility of the Mecanum wheel depends on the free rollers, limitations or undesirable behaviors may occur in the motions of the MOMR, especially when external forces are applied. Therefore, this study investigates the motion characteristics of the MOMR on a slope. First, we focus on the maximum possible acceleration considering the wheel skidding and clarify its relationship with the inclination angle of the slope and the structure of the MOMR. Then, the straight-traveling stability at the acceleration limit is analyzed to evaluate the effect of the resultant motion due to wheel skidding. The results show that the characteristics vary considerably depending on the inclination angle of the slope, and that directional dependences due to the free rollers are observed in the maximum acceleration limit and the straight-traveling stability, which should be considered in the operation of MOMRs.

Keywords: Omnidirectional Mobile Robot, Mecanum Wheel, Slope.

1 Introduction

The Mecanum wheel is an omnidirectional wheel mechanism that has several free rollers around its circumference mounted at a certain angle to the wheel plane. Due to the rotation of the free rollers, the Mecanum wheel can perform a passive diagonal movement in addition to the active forward/backward movement. Typically, four Mecanum wheels form an omnidirectional mobile robot. Such Mecanum-wheeled omnidirectional mobile robots (MOMRs) are widely used in factories and warehouses to transport parts and goods [1]–[3]. In recent years, the superb mobility of MOMRs has also attracted attention for their use as a mobile platform of a mobile manipulator [4]–[6]. To address the problem of low surmountability due to structural complexity, a new design of the Mecanum wheel has been proposed [7]. The idea of applying its working principle to a crawler mechanism has also been presented [8]. In addition, applications to the wheel-legged robot [9] and the reconfigurable robot [10] have been reported.

On the other hand, Mecanum wheels cannot provide a driving force in the direction in which the free rollers rotate. This may limit the possible motion of the robot or cause undesirable motion under conditions where external forces are acting. With this in

mind, this study considers a situation in which the MOMR moves on a flat slope and investigates its motion characteristics when gravity and inertial forces act on it.

2 Robot Modeling

2.1 Kinetic Model

First, we construct a kinetic model of the MOMR on a slope. Figure 1 illustrates the situation considered in this study. The MOMR moves on the flat surface of the slope, which has an inclination angle χ . The global reference frame $O-XYZ$ is fixed to the slope surface, where the direction of the Y -axis coincides with the slope gradient and the Z -axis is perpendicular to the slope surface. The local reference frame $P-xyz$ is fixed to the center of the MOMR. The x - and y -axes directions are rightward and forward, respectively. The center of gravity of the MOMR is assumed to be at a height h on the z -axis.

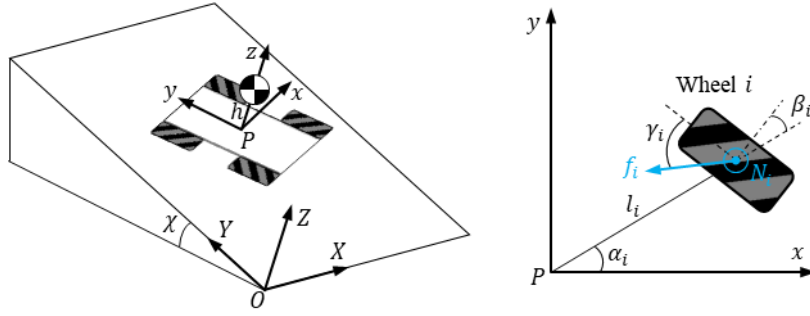


Fig. 1. Reference frames and wheel parameters in the local reference frame

The right figure in Fig. 1 shows the parameters for wheel i in the local reference frame. Here, the right front, left front, left rear, and right rear wheels of the MOMR are referred to as wheels 1, 2, 3, and 4, respectively. We represent the parameters belonging to wheel i by using the index i ($i = 1, 2, 3, 4$). Wheel i is located at a distance l_i and angle α_i from P and is oriented at an angle β_i . The setting angle between the axes of the free rollers and the wheel plane is γ_i . The driving force f_i and the normal force N_i are considered as the forces acting on wheel i .

From the above definitions, the equilibrium equations are given as follows:

$$\begin{cases} F_x = -\sum_i f_i \sin(\alpha_i + \beta_i + \gamma_i) \\ F_y = \sum_i f_i \cos(\alpha_i + \beta_i + \gamma_i) \\ F_z = \sum_i N_i \\ hF_y = -\sum_i l_i N_i \sin \alpha_i \\ hF_x = -\sum_i l_i N_i \cos \alpha_i \\ T = \sum_i l_i f_i \cos(\beta_i + \gamma_i) \end{cases} \quad (1)$$

where F_x , F_y , and F_z are the external forces acting on the center of gravity of the MOMR in the x -, y -, and z -axis directions, respectively, and T is the external torque around the z -axis.

This study deals with the MOMR that has coaxial left and right wheels and a point-symmetric structure. Namely, if $\alpha_1 = \alpha$ and $\gamma_1 = \gamma$, then $\alpha_2 = \pi - \alpha$, $\alpha_3 = -\pi + \alpha$, $\alpha_4 = -\alpha$, $\beta_i = -\alpha_i$, $\gamma_2 = \gamma_4 = -\gamma$, $\gamma_3 = \gamma$, and $l_1 = l_2 = l_3 = l_4 \equiv l$. Here, we set the half tread to $w = l \cos \alpha$ and the half wheelbase to $d = l \sin \alpha$. At this point, the driving forces f_i can be calculated from Eq. (1) as follows:

$$\begin{cases} f_1 = f + M_1 \\ f_2 = f + M_2 \\ f_3 = -f - M_3 \\ f_4 = -f - M_4 \end{cases} \quad (2)$$

where

$$\begin{cases} M_1 = -\frac{F_x}{4 \sin \gamma} + \frac{F_y}{4 \cos \gamma} + \frac{T}{4l \cos(\alpha - \gamma)} \\ M_2 = \frac{F_x}{4 \sin \gamma} + \frac{F_y}{4 \cos \gamma} - \frac{T}{4l \cos(\alpha - \gamma)} \\ M_3 = \frac{F_x}{4 \sin \gamma} - \frac{F_y}{4 \cos \gamma} + \frac{T}{4l \cos(\alpha - \gamma)} \\ M_4 = -\frac{F_x}{4 \sin \gamma} - \frac{F_y}{4 \cos \gamma} - \frac{T}{4l \cos(\alpha - \gamma)} \end{cases}$$

and f is an arbitrary value. Next, to determine the normal forces N_i , we assume that all four Mecanum wheels are always in contact with the slope surface at the same time. Then, N_i can be derived from Eq. (1) as follows:

$$\begin{cases} N_1 = \frac{1}{4} \left(-\frac{h}{w} F_x - \frac{h}{d} F_y + F_z \right) \\ N_2 = \frac{1}{4} \left(\frac{h}{w} F_x - \frac{h}{d} F_y + F_z \right) \\ N_3 = \frac{1}{4} \left(\frac{h}{w} F_x + \frac{h}{d} F_y + F_z \right) \\ N_4 = \frac{1}{4} \left(-\frac{h}{w} F_x + \frac{h}{d} F_y + F_z \right) \end{cases} \quad (3)$$

2.2 Non-skid Condition

Next, the condition for preventing the wheels from skidding is discussed. If we let μ be the coefficient of friction, the condition is given by

$$|f_i| \leq \mu N_i. \quad (4)$$

Substituting Eq. (2) into Eq. (4) yields

$$\max\{-\mu N_i - M_i\} \leq f \leq \min\{\mu N_i - M_i\}. \quad (5)$$

Therefore, the non-skid condition is that there exists f satisfying Eq. (5). When the equality is established, i.e., when the value of f is uniquely determined, this is the

condition for the limit of skidding. At this point, it is determined that skidding occurs at wheels i and j providing $\max\{-\mu N_i - M_i\}$ and $\min\{\mu N_j - M_j\}$.

2.3 Discussion on Redundancy

The MOMR has four driving wheels, while the required motion is three degrees of freedom (DOF). This means that there is one DOF of redundancy. In general, such redundant DOF can cause the coupled-drive configuration and the unnecessary internal forces to lead to component wear, wheel slippage, etc. One way to avoid these problems is to introduce constraints so that the internal force remains zero.

In fact, f in Eq. (2) can not be determined by the equilibrium equations. Referring to the kinematics and the principle of virtual work, $f = (f_1 + f_2 - f_3 - f_4)/4$ is derived and this force acts to deform the robot body. So the constraint $f = 0$ might be useful in practice. However, in this situation, only one wheel skidding will cause the equilibrium to be violated, resulting in poor performance at the skidding limit. Considering these circumstances, this study accepts the occurrence of the internal force and uses the condition with the largest skidding limit, i.e., the condition where at least two wheels reach the skidding limits simultaneously.

3 Analysis of Maximum Acceleration Limit

Based on the kinetic model constructed in the previous section, the motion characteristics of the MOMR on the slope are investigated. In this section, we focus on the maximum possible acceleration when the MOMR performs translational motion. The external forces are given as

$$\begin{cases} F_x = mg \sin \chi \sin \theta + ma \cos(\psi - \theta) \\ F_y = mg \sin \chi \cos \theta + ma \sin(\psi - \theta) \\ F_z = mg \cos \chi \\ T = 0 \end{cases} \quad (6)$$

where m is the weight of the MOMR, g is the gravitational acceleration, θ is the orientation of the MOMR, which is defined by the angle between the X - and x -axes, a is the translational acceleration of the MOMR, and ψ is the direction of acceleration from the X -axis. The maximum acceleration limit is calculated by solving $\max\{-\mu N_i - M_i\} = \min\{\mu N_i - M_i\}$ with respect to a as discussed in Subsection 2.2.

Figure 2 shows the calculation results of the maximum acceleration when $\gamma = 45^\circ$, $w = 0.24$ m, $d = 0.52$ m, $h = 0.12$ m, $m = 65$ kg, $\mu = 0.45$, and $g = 9.8$ m/s² and χ , ψ , and θ are varied. When the inclination angle of the slope $\chi = 0^\circ$ (indicated by the solid blue lines), the maximum acceleration is distributed in a squared shape. That is, the maximum acceleration is larger in the forward/backward and left/right directions of the robot and less in the diagonal directions. This is because the Mecanum wheels cannot exert any force in the diagonal direction due to the free rollers.

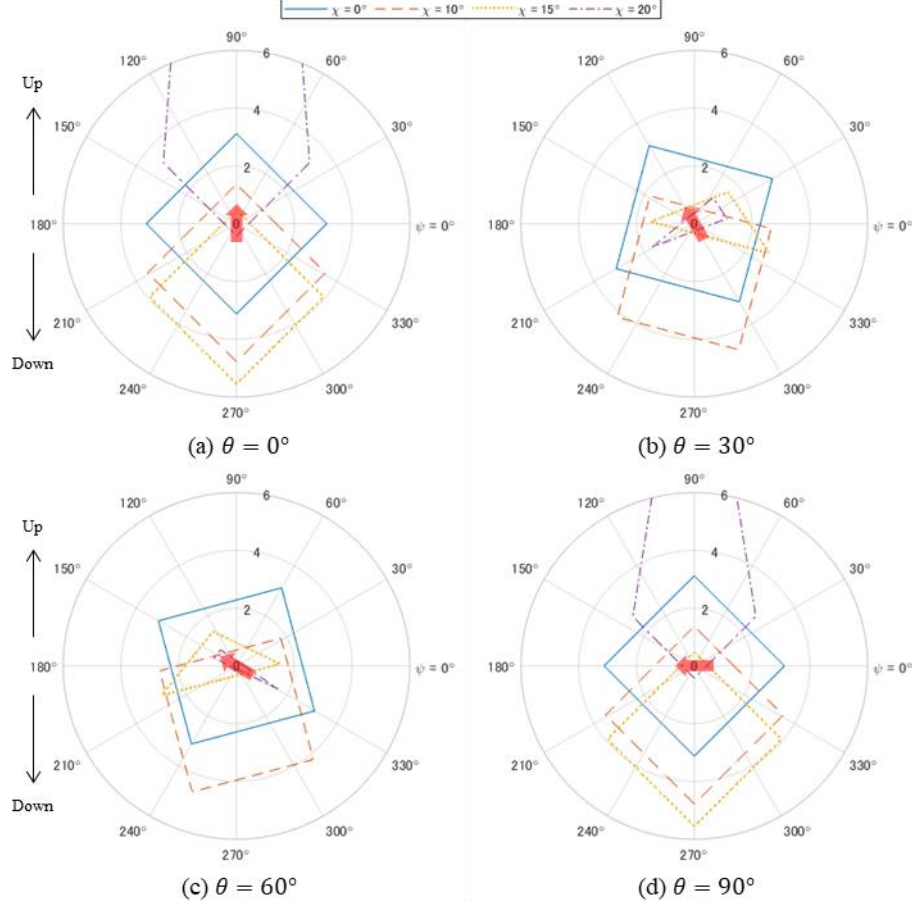


Fig. 2. Maximum acceleration limit versus acceleration direction ψ when robot orientation θ and inclination angle of the slope χ are varied. The red arrows indicate the forward direction of the robot corresponding to θ .

Focusing on the result for $\chi = 10^\circ$ (indicated by the orange dashed lines), Fig. 2 shows that the maximum acceleration distribution shifts in the downward direction, but the shape of the distribution is similar to the result for $\chi = 0^\circ$. Also, the distribution rotates according to the orientation of the robot θ , similar to the result for $\chi = 0^\circ$.

For $\chi = 15^\circ$ (indicated by the yellow dotted lines), the maximum acceleration distribution shows the same tendency as above in Fig. 2(a) and (d), while the distribution is quite different in Fig. 2(b) and (c). The maximum acceleration becomes larger partly in the upward direction but is strongly limited in the downward direction. This reflects the discrete change in the combination of the wheels that cause skidding as the inclination angle increases.

For $\chi = 20^\circ$ (indicated by the purple chain lines), the distribution changes significantly even in Fig. 2(a) and (d), where the downward acceleration is severely limited,

while the possible upward acceleration is extremely large. This is because the wheels must always exert a driving force in the upward direction to resist the gravitational force. When accelerating in the upward direction, the inertial force acts in the opposite direction, so that a large force can be used for acceleration. In addition, the grip of the rear wheels also increases, resulting in a synergistic effect. Note that there is no limit to the driving force that the robot can exert here. In the downward acceleration, the inertial force acts in the same direction as the driving forces, so the available extra force becomes small.

4 Straight-Traveling Stability at Acceleration Limit

The previous section shows that the behavior of the MOMR changes significantly depending on the inclination angle of the slope, especially that the maximum acceleration in the downward direction is limited when the inclination is large. Based on this result, we next investigate the straight-traveling stability of the MOMR by focusing on the torque generated at the skidding limit when the MOMR is moving down the slope.

The straight-traveling stability considered in this study is explained. As discussed in Subsection 2.3, in the MOMR here, two or more wheels skid simultaneously in the acceleration limit. If three or four wheels skid simultaneously, the motion of the robot is no longer determined and is considered to be unstable. On the other hand, if only two wheels skid, the unconstrained motion is limited to a certain direction. In addition, the driving force of the two remaining gripping wheels provides the torque T' to the robot.

$$T' = l(f_1 - f_2 - f_3 + f_4) \cos(\alpha - \gamma) \quad (7)$$

where f_i of the skidding wheels are set to zero. Figure 3 illustrates the resultant motions from the one-DOF unconstraint and the torque from the gripping wheels. As shown in the figure, the resultant motion will cause the robot to deviate to the left or right. Thus, if a robot moving to the left or right is further deviated to the left or right, the moving direction of the robot will not converge to the desired direction and the unstable motion will occur.

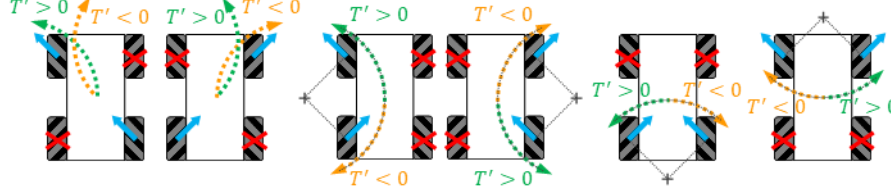


Fig. 3. Explanation of unstable motions when two wheels are skidding. The red cross marks indicate the skidding wheels. The blue arrows show the directions in which the motion of the gripping wheel is unconstrained. The orange and green dashed arrows represent the unstable resultant motions for $\psi \in (180^\circ, 270^\circ)$ and $\psi \in (270^\circ, 360^\circ)$, respectively.

We calculate the unstable regions by examining the combinations of the skidding wheels and the torque generated by the gripping wheels at the maximum acceleration limit when the MOMR is moving forward in the downward direction on the slope (i.e., the moving direction $\psi \in (180^\circ, 360^\circ)$ and the robot orientation $\theta = \psi - \pi/2$). Figure 4 shows the result of the calculation, where the yellow-highlighted regions indicate that the resultant motion can be unstable. In the case of the inclination angle $\chi = 0^\circ$, the conditions are the same for any θ , so the result is constant as shown in Fig. 4(a). Since the load is applied uniformly, all four wheels always reach the skidding limit at the same time. For $\chi = 10^\circ$, as shown in Fig. 4(b), the combinations of the skidding wheels change at the boundary of the gradient direction of the slope ($\theta = 180^\circ$). As a result, the resultant motion becomes unstable around the direction. However, since the maximum acceleration in this unstable region is relatively high, i.e., skidding is unlikely to occur, the robot can move, albeit slowly. Figure 4(c) shows that for $\chi = 15^\circ$, the combination of the skidding wheels changes differently and the proportion of the unstable regions increases. The unstable region around the gradient direction becomes smaller, but the maximum acceleration is zero at the boundaries of $\theta = 167.8^\circ$ and 192.2° , i.e., skidding inevitably occurs in these directions. Therefore, there is no problem when the MOMR accelerates exactly in the gradient direction, but when the MOMR is tilted at a certain angle, the robot may immediately fall into an unstable resultant motion. Figure 4(d) shows that the combination of the skidding wheels changes more complex for $\chi = 20^\circ$. Note that the region around the gradient direction becomes stable, considering the $\theta = 180^\circ$ direction is feasible because all four wheels skid simultaneously here. However, the maximum possible acceleration is relatively small.

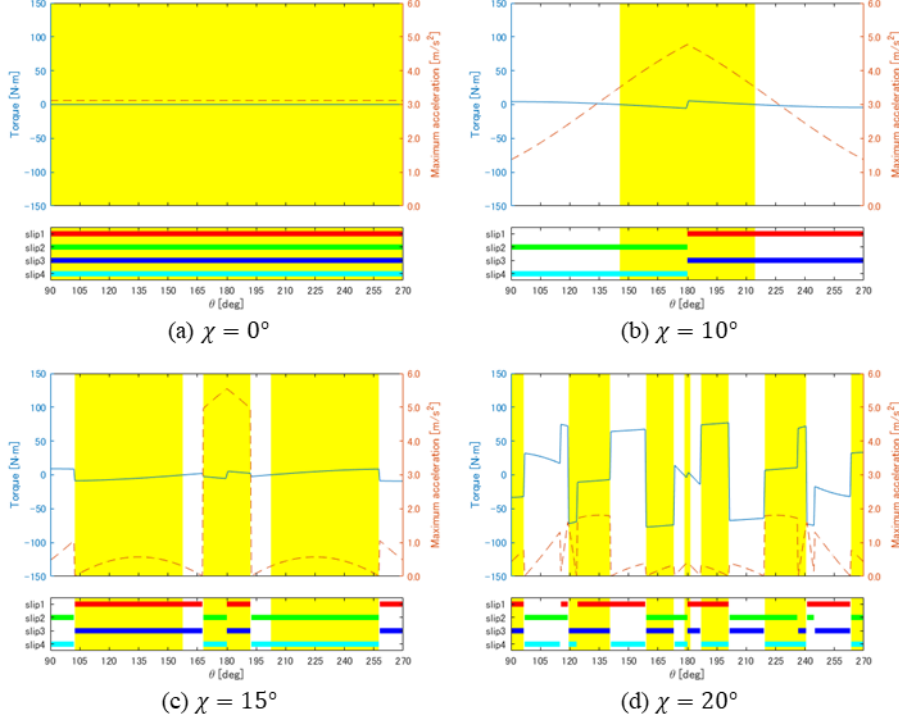


Fig. 4. Torque acting on the robot when the wheels start to skid and the maximum acceleration versus the robot orientation θ . Slip i in the lower graph means that wheel i is at the skidding limit. The regions highlighted in yellow indicate that the resultant motion may be unstable.

5 Conclusions

MOMRs are one of the most popular omnidirectional mobile robots, but the free rollers of the Mecanum wheel can negatively affect the kinetic performance. We then investigated the characteristics of the MOMR's motion on a slope under gravity and inertial forces.

The maximum acceleration limit, taking into account wheel skidding, was calculated. It was found that large acceleration can be achieved in the downward direction when the inclination angle of the slope is below a certain level, while the downward acceleration is limited and the upward acceleration becomes favorable when the inclination angle is above a certain level.

Based on the result, we also investigated the straight-traveling stability when the MOMR descends the slope at the acceleration limit. When the MOMR descends a slope with a slight angle of inclination, although the robot falls into unstable motion when the wheels start to skid, this can be avoided as long as the acceleration is kept at a moderate level. If the slope is slightly inclined, a certain deviation in the moving direction can cause the MOMR to immediately start skidding in an unstable state. If the slope

becomes steeper, the resultant motion becomes stable around the gradient direction, but skidding is more likely to occur.

In the future, we would like to investigate design conditions for the MOMR that are less likely to skid or more likely to become stable when skidding. In addition, we will conduct experiments using an actual MOMR to observe the skidding behavior when it is traveling on a slope, especially on a downward slope. In the experiment, a standard MOMR with four speed-controlled wheels can be used for basic validation. It is expected that approximately the same trends as in the simulation will be obtained, but there may be some differences due to the nonlinearity around the friction limits in the real world. In particular, some kind of suspension will be required on the actual machine, so it will be interesting to discuss whether its effect is either positive or negative in terms of skidding.

Acknowledgment

This work was supported by the JSPS KAKENHI under grant number JP21K14067.

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