

Homogeneous isoparametric hypersurfaces with four distinct principal curvatures and moment maps

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Main Results

Homog. Herm. isoparametric hypersurf. ($\subsetneq \mathbb{S}^n$, $g = 4$) is obtained from some moment maps!!

Our Goal

Isoparametric hypersurf. $\subsetneq \mathbb{S}^n$, $g = 4$



Is there any relation?

Moment maps

Isotropy representations

G/K : Herm. symm. sp. of rank 2,

$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$: the Cartan decomp. of $\mathfrak{g} = \text{Lie}(G)$,

1. the principal orbit of $K \curvearrowright^{\text{Ad}} \mathfrak{p}$

= homog. isoparametric hypersurf.

$\Rightarrow \exists \varphi : \mathfrak{p} \rightarrow \mathbb{R}$: isoparametric,

$\Rightarrow \varphi$: K -invariant,

2. $K \curvearrowright^{\text{Ad}} \mathfrak{p}$: Hamiltonian K -action,

$\Rightarrow \exists \mu : \mathfrak{p} \rightarrow \mathfrak{k}^*$: the moment map,

$\Rightarrow \mathfrak{p} \xrightarrow[\text{K-equivariant}]{\text{moment map } \mu} \mathfrak{k}^* \xrightarrow[\text{K-invariant norm } \|\cdot\|]{\text{moment map } \mu} \mathbb{R}$

What is an isoparametric hypersurface?

$M \subsetneq \mathbb{S}^n$: a hypersurf. in sphere,

• M : isoparametric

$\stackrel{\text{def}}{\iff} \exists \varphi : \mathbb{S}^n \rightarrow \mathbb{R}$: isoparametric & $\exists c \in \mathbb{R}$ s.t. $M = \varphi^{-1}(c)$.

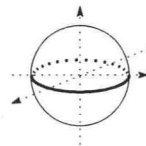
◦ $\varphi : \mathbb{S}^n \rightarrow \mathbb{R}$: isoparametric

$\stackrel{\text{def}}{\iff} \exists A(t), \exists B(t) \in C^\infty(\mathbb{R})$ s.t.
 $\begin{cases} \|\text{grad } \varphi\|^2 = A \circ \varphi, \\ \Delta \varphi = B \circ \varphi. \end{cases}$

Example of isoparametric hypersurf.

$\mathbb{S}^2 \supsetneq M = \mathbb{S}^1(r)$, $0 < r \leq 1$.

$\implies M$: isoparametric.



$\varphi : \mathbb{S}^2 \rightarrow \mathbb{R}$,
 $\varphi(x, y, z) := z$.

Facts

$M \subsetneq \mathbb{S}^n$: hypersurf.

• M : homog. $\Rightarrow M$ has constant principal curvatures,
 $\iff M$: isoparametric,

• M : homog. $\iff M$: the principal orbit of the isotropy rep. of $\exists G/K$: symmetric sp. of rank 2.

• M : isoparametric,

$g := \#$ (distinct principal curvatures of M),

◦ g must be 1, 2, 3, 4, or 6,

◦ $g \leq 3 \Rightarrow "M$: homog. $\iff M$: isoparametric",

◦ $g = 4 \Rightarrow$ "non-homog., classification is still open."