

STUDIES ON NONLINEAR WAVE PHENOMENA
IN A MODULATED ELECTRON
BEAM-PLASMA SYSTEM

Osamu FUKUMASA

April 1981



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4	4	13-15] which ----	[13-15] which ----
8	11	---- plasma [	---- plasma
8	12	46, 58] have ----	[46, 58] have ----
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30	F3	10^{-3} , ----	$= 2 \times 10^{-3}$, ----
43	14	---- in Fig.	--- in Fig. 2.12
43	15	2.12 have ----	have ----
43	F8	--- manner (see Fig.	--- manner. They
43	F7	2.14).	are shown in Fig. 2.14.
51	4	--- Figs. 2.18 and	--- Figs. 2.18 and 2.19.
51	5	2.19. Additional ----	Additional, ----
51	F6	--- collisions ($v/w_p \cong$	---- collisions (v/w_p
51	F5	10^{-3}). At the ----	$\cong 10^{-3}$). At the ----
65	3	----: $n_b/n_e =$	---- : n_b/n_e
65	4	10^{-2} , $v_b/v_t = 3$, ----	$= 10^{-2}$, $v_b/v_t = 3$, ----
71	12	---- in Fig.	---- in Fig. 3.5.
71	13	3.5. Dotted ----	Dotted ----
73	9	---- in Fig.	---- in Fig. 3.5
73	10	3.5 or experimental ----	or experimental ----
99	6	---, $(2n_e/n_b)^{1/3} =$	----, $(2n_e/n_b)^{1/3}$
99	7	5.85. Then, ----	$= 5.85$. Then, ----
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105	18	$1 \times 10^8 \text{ cm}^{-3}$, ----	$\cong 1 \times 10^8 \text{ cm}^{-3}$, ----
130	3	--- ratio $f_2/f_{pe} \cong$	--- ratio $f_2/f_{pe} \cong 0.38$,
130	4	0.38, where ----	where ----
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139	11	22] obtain ----	[22] obtain ----
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139	下5	$ekE(\tau)/(ms^2\omega_K^2), \dots$	$= ekE(\tau)/(ms^2\omega_K^2), \dots$
140	9	---- at $\tau =$	---- at $\tau = 14$.
140	10	14. The initial ----	The initial ----
140	下5	---- till $\tau =$	---- till $\tau = 14$.
140	下4	14. After this ----	After this ----
145	下8	---- of $v/\omega_{pe} \cong$	---- of v/ω_{pe}
145	下7	10^{-3} within the ----	$\cong 10^{-3}$ within the ----
145	下1	Finally, we think that ----	It should be noted that ----
151	下5	---, $n_e = 1 \times 10^8 - 6 \times$	---, $n_e = 1 \times 10^8 - 6 \times 10^9$
151	下4	10^9 cm^{-3} , and ----	cm^{-3} , and ----
158	3	--- in Figs. 5.3 and	--- in Figs. 5.3 and 5.4
158	4	5.4 were studied ----	were studied ----
159	1	--- and $\kappa T_e \cong$	--- and κT_e
159	2	2 eV. ii) Ratios ----	$\cong 2 \text{ eV}$. ii) Ratios ----
170	11	---- $(n_b/2n_e)^{1/3} (3^{1/2}/$	---- $(n_b/2n_e)^{1/3} (3^{1/2}/2)$
170	12	2) and from ----	and from ----
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178	下11	--- MHz. We think that	--- MHz. It is noted that
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191	下1	$n_b)^{1/3}$. As n_b/n_e ----	As n_b/n_e ----

ABSTRACT

This thesis is devoted to the experimental and theoretical studies on nonlinear wave phenomena, especially the nonlinear wave-particle interaction in a small cold beam-warm plasma system, and the suppression of beam-excited electron waves due to beam density modulation.

In Chapter 1, historical backgrounds concerning above-described subjects, and problems studied in this thesis are mentioned.

In Chapter 2, destruction of regular amplitude oscillations in a small cold beam-weakly collisional plasma system is discussed. Collisional effects in the plasma electrons, and competition between collisional and neighboring wave effects for the destruction of amplitude oscillations are studied numerically by using the extended single wave and many wave models. Weak collisions of plasma electrons, $\nu/\omega_{pe} \cong 10^{-3}$ where ν is the effective collision frequency and ω_{pe} is the electron plasma frequency, play an important role in the nonlinear stage and the persistent amplitude oscillations are destroyed by these weak collisions.

In Chapter 3, in order to study the control of wave spectrum or the suppression of plasma wave, the many wave model, i.e. the method of partial simulation, is applied to the modulated electron beam-plasma system. Numerical results are qualitatively in agreement with the experimental results.

In Chapter 4, suppression of beam-excited electron waves due

to beam density modulation is investigated in a bounded system. Measurements of beam distribution function are carried out, in relation to wave suppression. Experimental results indicate that suppression mechanism must be considered by taking into account nonlinear behaviors of beam electrons. These experimental results are discussed not by the so-called Van der Pol model but by the particle picture.

In Chapter 5, suppression of multimode electron waves and the ion-acoustic wave in the bounded beam-plasma system is studied experimentally. When a beam density is modulated at a frequency which is close to some frequency of electron waves, all of electron waves and the ion-acoustic wave are suppressed simultaneously. These experimental results are well explained from the viewpoint of quasilinear effects caused by the external wave.

In Chapter 6, the main results of this thesis are summarized. The problems left in this thesis are also mentioned.

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Chapter 1

GENERAL INTRODUCTION

It is a characteristic feature of a plasma that the plasma can carry collective oscillations and a rich variety of different waves [1]. For understanding of these collective processes in plasmas, the physics of the interaction of an electron beam with a plasma has been intensively studied theoretically and experimentally. Motz[2] notes the original proposal of Langmuir and gives references to the work of some of the pioneers in this field, including Pierce, Haeff, Twiss, Bohm and Gross, and Akhiezer and Fainberg. Briggs has written a monograph on this subject and also a more recent review[3].

Beam-plasma interactions have been topics of great interest to various fundamental physical problems, i.e. plasma production and heating, plasma diagnostics (observation of RF emission from laboratory plasma as well as from outer space), electron beam devices such as traveling wave tubes and klystrons, interaction between the solar wind and the magnetosphere, etc.. This is due to the fact that the beam-plasma interaction represents the simplest system where velocity-space instabilities occur in pure form[3]. Therefore, the beam-plasma interaction is useful for studying linear and nonlinear properties as well as for simulating the closely related instabilities of a hot plasma.

In the present thesis, nonlinear wave phenomena, i.e. the

nonlinear wave-particle interaction in a small cold beam-plasma system, and the suppression of beam-excited electron waves due to beam density modulation, have been studied experimentally, and theoretically and numerically. In the following sections, we describe the historical backgrounds concerning above-mentioned subjects, and problems studied in this thesis.

1.1 Nonlinear Wave-Particle Interaction in an Electron Beam-Plasma System

Two limiting cases for the linear growth of waves in the beam-plasma system are well known. One is the "monoenergetic beam" or "cold beam" case for which the beam and plasma temperatures are both so small that the motion is dominated by purely fluid effects. The other is the "gentle-bump" or "hot beam" case for which the beam density is small and beam temperature is large so that the beam may be considered to constitute a gentle bump on the electron velocity distribution function of the plasma.

A transition between these two cases is of interest not only from a theoretical point of view[4-8] but also from an experimental point of view, because in many experiments a monoenergetic beam is observed to be scattered into a smeared-out beam in velocity space [9-12]. The problem of the transition from cold to hot beam behaviors has been motivated by a study of the nonlinear beam-plasma interaction and has been discussed by several authors. For example, Shapiro[4] has argued that nonlinear evolution of beam-plasma system takes place in two stages: First, the interaction heats the

beam, transforming it from a delta function to a gentle bump. Then, the gentle bump is flattened in velocity space via quasilinear diffusion[13-15]. During the first stage of this process the roots of dispersion relation are like those for a cold beam case, and during the second stage they are like those for a gentle bump case. O'Neil and Malmberg[6] have shown that the transition, from cold to hot beams, occurs when $v_{tb} \cong v_b (n_b/2n_e)^{1/3}$, where n_b/n_e is the ratio of beam density to plasma density, v_{tb} is the thermal velocity of the beam, and v_b is the beam drift velocity. They also show that the transition is accompanied by a certain topological change in the roots of dispersion relation, namely a change from "beam-type" roots for the cold beam to "Landau-type" roots for the hot beam. Further discussion has been given by Self et al.[8]. They show that more significant is the change in the location of the phase velocities of the most unstable waves relative to the bump in the distribution function. Recently, similar discussion concerning whistler instabilities, i.e. transition between temperature anisotropy and beam type whistler instabilities, has been reported by Hashimoto and Matsumoto[16].

According to the above discussions, when $v_{tb}/v_b > (n_b/2n_e)^{1/3}$ the system includes hot electron beam and its theoretical treatment in the nonlinear stage is made within the framework of weak turbulence. In this system, there appear instabilities with large spread of phase velocity, i.e. $k_0 (\Delta(\omega/k))_s \gg \omega_b$, where $\omega_b^2 = (e/m)k_0^2 \phi_s$, $(\Delta(\omega/k))_s$ is the range of phase velocity of the spectrum at the saturation time, ϕ_s is the saturation potential amplitude and k_0 is a typical value of the spectrum k . It means that the time required for the randomization of relative phase of waves due

to the difference in phase velocity is much smaller than the time of the oscillation of the particles in the potential well. The wave-particle interaction is well described by quasilinear theory[13-15] which invokes random phase approximation. At that instance the resonant particles diffuse irreversibly until the plateau in velocity distribution is formed and hence the wave stops growing. Roberson et al.[17] have carried out experiments testing various aspects of the theory and verified the principal features of the theory.

While, the problems of cold beam-plasma system which induces narrow spectrum, i.e. $k_0(\Delta(\omega/k))_s \ll \omega_p$, lie outside the validity of weak turbulence theory[18,19]. In this limiting case, since the interaction of particles is essentially with a single wave, the effect of "particle trapping" is dominant. The effect has been seen in computer simulations[20,21], and was the basis for the trapping model("the single wave model") of the instability first proposed by Drummond et al.[22]. The analysis has been extended by several authors[23-28] to give a complete picture of the instability, its nonlinear limit and subsequent evolution. O'Neil et al.[23] have obtained the scaled equations which show the nonlinear evolutions of both beam electrons and unstable single sinusoidal wave. Further, they show that the solution obtained for temporally growing wave is equally valid for spatially growing one[24]. The essential feature of the model is that after many e foldings, the bandwidth of this instability is so narrow that the beam electrons interact with a wave which is nearly monochromatic. This wave grows until it traps the beam electrons, thereby saturating the instability. As the trapped particles slosh back and forth

in the wave potential, the amplitude oscillates as a result of energy conservation in the laboratory frame.

The single wave trapping model[23,24] has been successful in describing the initial stage of the nonlinear interaction between a small cold beam and a plasma. The predictions of the single wave model agree well with the experimental observations[29-35] through the initial trapping and up to the first amplitude oscillation. Beyond this point, however, the plasma experiments universally exhibit a rapid decay of the saturated wave rather than the persistent trapped particle oscillations predicted by the theory. For example, in experiment by Gentle and Lohr[31], during the first amplitude oscillation the wave spectrum is broadened by the growth of the neighboring waves and the single wave model is no longer applicable. Abe et al.[36] have shown, by a particle simulation of a spatial case, that the wave damps strongly to the first minimum and fails to regrow to the saturation level in the weakly collisional plasma. Phase-space evolution of beam electrons also become quite different from the theoretical and numerical results[23,24]. Dimonte and Malmberg[37] have shown experimentally that amplitude oscillations can be destroyed either by wave damping or by modulation of the main wave by unstable sidebands, where the background plasma is simulated by a traveling wave tube. Therefore, the initial nonlinear development of this instability is well understood by means of the single wave model, but the subsequent development remains to be explained.

Up to now, some mechanisms which destroy the regular amplitude oscillation have been proposed by some authors. These are the weak collisions[38], wave damping[37], the nonlinearities in

the background plasma[39,40], and the trapped particle instability [41-45] and the many wave effects associated with this instability [46,47]. Fukumasa et al.[38] have extended the single wave model to a more rigorous one including the collisional effects of plasma electrons. This extended model shows that the regular amplitude oscillations are destroyed by weak collisions which are too weak to change the linear property of the instability. Winfrey and Dunlop[47] have extended the single wave model to the model of fifteen waves and studied the role of neighboring waves. They show that, for the strong beam, the growth of neighboring waves occurs and a single wave no longer dominates the nonlinear evolution of the system. Destruction of amplitude oscillations due to neighboring waves are also reported by Matsiborko et al.[46]. In order to study competition of weakly collisional and neighboring wave effects for the destruction of amplitude oscillations, these two effects are treated together by using the many wave model including collisional effects of plasma electrons by Fukumasa et al.[48]. Simulation by Kainer et al.[39] has shown that nonlinearities in the background plasma occur for large beam strength, i.e. typically $n_b/n_e > 0.038$, and that plasma electrons can be trapped by the saturated wave when the trapping width is large. In this case, the background plasma is not a linear dielectric and the regular amplitude oscillations do not occur.

On the other hand, traveling wave tubes have been also investigated extensively[49]. The equations for the nonlinear interaction are first solved by Nordsieck[52]. The saturation mechanism was established experimentally as beam trapping by Cutler[51] with measurements of the phase-space distribution. However, the

investigation of the nonlinear interaction has been limited to the initial saturation region because the primary application for traveling wave tubes is as a broadband and/or high power amplifier. Consequently, the investigators had no compelling reasons to study the interaction much beyond saturation. Nordsieck[52] considered the large signal behavior of traveling wave amplifiers, following the analysis by Pierce[50] and others on the small signal behavior. He began with the equations of motion of the electron stream, and with a circuit equation for the electric field. From these equations, Nordsieck derived working equations. Tien[53] extends Nordsieck's analysis to include finite beam strength. In the small cold beam limit, the plasma acts essentially as a linear dielectric medium[23] capable of supporting slow space charge waves. The equations of the single wave model[23-25] are identical to these working equations for the nonlinear interaction in a traveling wave tube.

In order to avoid problems associated with the background plasma, i.e. the nonlinearities in the plasma and characteristics of laboratory plasma (density gradients and low-frequency potential fluctuations), Dimonte and Malmberg[37] have built a somewhat unconventional traveling wave tube to investigate the late-time development of the beam-plasma instability. By using this tube, they have shown that five trapped particle oscillations are occurred, and that these amplitude oscillations can be destroyed either by wave damping or by modulation of the main wave by unstable sidebands. They have also solved the working equations of Tien's model[53], which is essentially the single wave model with detuning, damping and finite-beam-strength corrections included.

As a whole, their experimental and numerical results are very similar to our numerical results[38,48].

Lastly, we note the followings: The essence of the single wave model consists of describing the behavior of the background plasma through a linear dielectric, and following the exact orbits of the beam particles. Therefore, I think that this technique is very useful to investigate the phenomena in which dynamics of beam particles or resonant particles predominate. By using this method, nonlinear state of the relativistic two-stream instability[54-56], effects of a DC electric field on beam trapping[57], and nonlinear interaction between modulated electron beam and plasma[46,58] have been studied by some authors. Further, simulation results on a Cerenkov interaction between obliquely propagating whistler waves and an electron beam have been discussed by the single wave model[59].

1.2 Suppression of Plasma Waves by an Externally Applied RF Signal

There has been considerable interest in the nonlinear mechanism which determines the saturation level of a self-excited oscillation(or instability) present in a plasma[60]. For example, harmonic effects, mode-mode coupling, wave-particle scattering, and particle trapping have been proposed as possible mechanism in various cases. The importance of instabilities in a plasma has been derived from not only basic plasma physics studies but also the practical side, i.e. the possible connection between the anom-

alously high diffusion of plasma across a containing magnetic field and the presence of these finite amplitude instabilities in the plasma. Therefore, any method which suppresses or controls these instabilities is a desirable feature in a plasma containment device. Plasma heating due to waves is another important aspect of the practical side. For these ends, it is necessary to understand excitation and suppression mechanisms of plasma waves. There have been a number of reports on the dynamic and the feedback stabilization methods, which are applicable to various plasma instabilities such as the electron waves, the ion-acoustic waves, and the drift waves. In this thesis, in order to study a mechanism of wave suppression in detail, particular attention is paid to the suppression of beam-excited electron waves by externally applied RF signal.

Controls of instability by beam modulation have been studied experimentally by Russian authors. For example, Kornilov et al. [61] found that a spontaneously excited high-frequency oscillation with frequency $2f_{ce}$ (f_{ce} : electron cyclotron frequency) and a low-frequency oscillation were suppressed by modulating a beam at a modulation frequency $f_m = 2f_{ce}$. Berezin et al. [62] reported that an oscillation having broad band spectrum whose peak was present at 880 MHz was controlled by an initial beam modulation. When a modulation frequency $f_m = 880$ MHz, the suppression was most effective and the power needed for the control was very low. Krivoruchko and Kornilov [63] studied the excitation of low-frequency oscillations associated with a two-stream instability, and showed the possibility of controlling their spectra by a high-frequency modulation of the electron beam.

On the other hand, Obiki et al.[64] also reported that ion-acoustic waves excited spontaneously in the electron beam-plasma system were suppressed by modulating a beam at a low-frequency nearly equal to the observed one.

So far, including the above-mentioned works, suppression of electron waves and ion-acoustic waves, by applying an external ac signal, have been reported by many authors[65-72]. Among them, Keen and Fletcher[65] reported that the ion-acoustic wave was suppressed by an external signal, and that this phenomena was phenomenologically explained by the Van der Pol equation with the external force term. Nakamura[66,67] reported that beam-excited electron oscillations were suppressed by beam density modulation. He also explained these phenomena by assuming that the oscillation obeyed the Van der Pol equation with the external force term. This idea was also applied to the suppression of drift waves by external ac signals[73] and the feedback stabilization of ion-acoustic waves[74,75].

Although Stix[76] has shown, by using the mode-mode coupling approach, that a finite-amplitude resistive drift wave saturates in a manner similar to the Van der Pol equations, the basic process of the suppression is still an open question. Because the physical meaning of the nonlinear terms, being essential to the Van der Pol equation, is not clarified. For example, the Van der Pol equation for ion-acoustic waves was derived by Keen and Fletcher[65] by means of the hydrodynamic equations for the ion and the equilibrium distribution for the electron. The nonlinear terms come from the source terms which are introduced supplementally in the continuity equation. Although these source terms

are interpreted as ionization etc. caused by large amplitude oscillations in the plasma, relationship between the coefficients of source terms and the plasma parameters is not clear. In the same way, the Van der Pol equation for an electron beam-plasma system was obtained by Nakamura[66]. In both cases, it is evident that the theoretical basis for the Van der Pol equation can not be easily furnished within the framework of the hydrodynamic approximation unless the source terms are intuitively supplemented. On the other hand, recently, it has been reported that in collisionless plasma where the fluid theory is not applicable and no source term is present, the Van der Pol equation is a phenomenological model for the ion-acoustic instability[72]. Furthermore, Kato et al.[77] show that the oscillations in a bounded electron beam-plasma system with density modulation of the electron beam are governed by the Mathieu equation instead of the Van der Pol equation. Therefore, it is an open question why the density fluctuations obey the Van der Pol equation.

For the first time, we discussed the suppression mechanism of beam-excited multimode electron waves from a viewpoint of particle picture[78]. Recently, we have observed the deformation of beam velocity distribution function in relation to the suppression of the electron waves in the bounded system[79,80]. Following this work, relationship between the wave suppression and velocity distribution function of plasma particles has been studied by many authors[72,81-88]. These experimental results suggest that suppression mechanism must be considered with including nonlinear behaviors of beam electrons. By using the method of partial simulation, we have studied numerically the interaction between the

modulated electron beam and the plasma, and obtained the qualitative agreement between the experimental and the numerical results [58,88]. According to these results, in the bounded system, feedback effects of reflected beams play an important role in suppression process and have close connection with whether deformation of beam distribution function occurs or not, in relation to wave suppression. The present consideration for electron waves may be applicable to the case of the ion-acoustic wave, because both the electron beam-plasma instability and the ion-acoustic instability are explained by the dispersion relation of almost the same form[89].

On the other hand, concerning nonlinear oscillations in an electron beam-plasma system, theoretical works have been reported by some authors[77,90,91]. Walsh and Hagelin[90] have obtained a kind of the Van der Pol equation by expanding the beam-plasma dispersion relation around the most unstable root and the perturbed beam electron orbit. Janssen and Rasmussen[91] have shown that the dynamical equation for the bump-on-tail instability is given by the nonlinear Landau equation. In both cases, it is considered that nonlinear terms being essential to the wave suppression are caused by the wave-particle interaction. We have also obtained the nonlinear rate equations of electron waves, by taking into account a perturbed orbit of a beam particle[88].

1.3 Objective and Contribution of the Present Works

In consideration of the above-mentioned historical background,

nonlinear wave phenomena are investigated in this thesis. It is organized into the following chapters.

In Chapters 2 and 3, nonlinear wave-particle interaction in a small cold beam - warm plasma system is investigated theoretically and numerically. In Chapter 2, particular attention is paid to the destruction mechanism of the regular amplitude oscillations of the electron wave[36,38,48]. It is found, by using the extended single wave model, that the persistent amplitude oscillations are destroyed by weak collisions of plasma electrons, i.e. $\nu/\omega_{pe} \cong 10^{-3}$ where ν is the effective collision frequency and ω_{pe} is the electron plasma frequency. Competition of weakly collisional and neighboring wave effects for the destruction of amplitude oscillations is also discussed, including the excitation mechanism of neighboring waves.

Chapter 3 describes an application of the many wave model, i.e. the method of partial simulation, to the phenomena in the modulated electron beam-plasma system[58]. Experimentally, controls of wave spectrum and wave amplitude are obtained by pre-modulation of the injected beams in both unbounded and bounded systems. These experimental results are discussed, including beam dynamics, by using the present model. Numerical results are qualitatively in agreement with experimental results.

In Chapters 4 and 5, we are concerned with the experimental studies on the suppression of beam-excited electron waves due to beam density modulation in a bounded system. Particular attention is paid to the suppression mechanism. These experimental results are discussed not by the Van der Pol model but by the particle picture.

In Chapter 4, experimental conditions are controlled so that a single mode of an electron wave is excited spontaneously. Measurements of beam distribution function are carried out in detail[79,80,82,84,86,88]. When injected beams are absorbed at the other end, deformation of the beam distribution is obtained in relation to the suppression of the electron wave. On the other hand, when beams are reflected, the externally applied wave suppresses the electron wave but distribution function shows no appreciable change. These experimental results are compared with the numerical ones obtained in Chapter 3, and are discussed by the particle picture.

Chapter 5 is devoted to simultaneous excitation and suppression of multimode beam-plasma instability in a bounded system[68, 78,92]. In this system, four or five electron waves are observed simultaneously, and are also accompanied with an ion-acoustic wave. When a beam density is modulated at a frequency which is close to some frequency of electron waves, all of electron waves and the ion-acoustic wave are suppressed simultaneously. These experimental results are discussed from the viewpoint of quasilinear effects of the external wave.

Chapter 6 summarizes the results which are obtained from the present works.

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Chapter 2

NUMERICAL STUDY OF NONLINEAR INTERACTION BETWEEN A SMALL COLD BEAM AND A WEAKLY COLLISIONAL PLASMA

2.1 Introduction

A nonlinear wave-particle interaction in a system of a small cold beam and a plasma has been studied by many authors[1-19]. In this system, since the interaction of particles is essentially with a single wave, the effect of "particle trapping" is dominant. Drummond et al.[1] and others[2-5] proposed a single wave model for such a nonlinear evolution of the system. According to the theory, the most unstable wave grows exponentially from the thermal fluctuation level, while the wave spectrum becomes progressively narrower. After several e foldings, the spectrum becomes so narrow that a beam electron can detect a nearly pure sinusoidal wave or a single wave, and the wave grows up to an amplitude sufficient to trap the beam. As the trapped particles slosh back and forth in the wave potential, the amplitude oscillates as a results of energy conservation. It is quite different from the warm beam case to which the quasi-linear theory[23] can be applied.

Predictions[1] or numerical results[3,4] based on the single wave model agree with the experimental results[14-19] through the initial trapping and up to the first amplitude oscillation. Beyond this point, the plasma experiments exhibit a rapid decay of

the wave amplitude rather than the regular amplitude oscillations predicted by the theory[1,3,4]. Therefore, the initial nonlinear development of this system is well understood, but the subsequent development remains to be explained.

Mechanisms which destroy the regular amplitude oscillation have been proposed by several authors. These are the wave damping[18], the weak collisions[11], the nonlinearities in the background plasma[12] and the many wave effects[8,10] associated with the sideband growth. However, there are few works which investigate the relationship between these destruction mechanisms.

In this chapter, we report the details of the numerical results on the destruction of amplitude oscillations in a small cold beam - a weakly collisional plasma system. Particular attention is paid to the competition of weakly collisional and neighboring wave effects for the destruction of amplitude oscillations. A novel feature of the excitation of neighboring waves is also discussed briefly.

2.2 Numerical Model and Basic Equations

2.2-1 Basic Equations

In order to study the competition between collisional and neighboring wave effects for the destruction of the amplitude oscillations, we consider the spatial evolution of a system of five modes or eleven modes. In this system, two or five modes are located on each side of the most unstable wave with frequency ω_0 . The intensity of the wave electric field, $E(x,t)$, with a stationary

amplitude $E(0)$ may be written as

$$E(x,t) = \sum_{\omega} E_{\omega}(x) \exp(-i\omega t) + \text{C.C.}, \quad (2-1)$$

where $E_{\omega}(x) = E_{\omega}(0) \exp(i \int_0^x k_{\omega}(x') dx')$. For the spatial evolution of the instability, temporal Fourier transformation of Poisson's equation is carried out by multiplying the equation by $e^{i\omega t}$ and integrating over a beat period T , i.e. $T = 2\pi/\Delta\omega = 2\pi l/\omega_0$, where $\Delta\omega$ is a frequency separation between modes and an integer l is $\omega_0/\Delta\omega$. Poisson's equation can be written for each Fourier component as

$$\frac{\partial}{\partial x} (E_{\omega}(x) - 4\pi \int \rho_{\omega} dx) = \frac{4\pi}{T} \int_0^T \rho_b(x,t) e^{i\omega t} dt, \quad (2-2)$$

where ρ_{ω} is the charge density of plasma electron and ρ_b for the beam density. Performing the integration of right hand side by parts, and using the continuity equation[9] for $\rho_b(x,t)$, eq.(2-2) becomes as follows:

$$\frac{\partial}{\partial x} (E_{\omega}(x) - 4\pi \int \rho_{\omega} dx) = - \frac{i4\pi}{T\omega} \int_0^T \frac{\partial}{\partial x} [v(x,t) \rho_b(x,t)] e^{i\omega t} dt.$$

This is simply integrated over x . Because plasma electrons are considered as linear dielectric medium[3], after integration, Poisson's equation may be rewritten as following form

$$\epsilon(\omega, k(x)) E_{\omega}(x) = - \frac{i4\pi}{T\omega} \int_0^T [v(x,t) \rho_b(x,t)] e^{i\omega t} dt, \quad (2-3)$$

where $\epsilon(\omega, k)$ is the plasma dielectric function. We assume that

N charge sheets pass the point $x = 0$ at times $t_j(x=0) = (j/N)T$. As these N sheets occupy a length in space $L = v_b T$, we have the following distribution function of the beam electrons

$$f_b(x, v, t) = \frac{n_b L}{N} \sum_j \delta(x - x_j(t)) \delta(v - v_j(t)) , \quad (2-4)$$

where $x_j(t)$ is the coordinate of the j -th beam particle and $v_j(t)$ is its velocity. By using this function, $v(x, t) \rho_b(x, t)$ is represented by

$$v(x, t) \rho_b(x, t) = - \int e v f_b dv = - \frac{e n_b L}{N} \sum_j v_j(t) \delta(x - x_j(t)) . \quad (2-5)$$

Substituting eq.(2-5) into eq.(2-3), Poisson's equation can be written as

$$\epsilon(\omega, k(x)) E_\omega(x) = i \frac{4\pi e n_b v_b}{N\omega} \sum_j \exp(i\omega t_j(x)) . \quad (2-6)$$

Here, $t_j(x)$ is the time when the j -th beam particle passes the point x . We adopt the following form of $\epsilon(\omega, k)$

$$\begin{aligned} \epsilon(\omega, k) &= 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)} \left(1 + \frac{3k^2 v_t^2}{\omega^2} \right) \\ &\approx 1 - \frac{\omega_p^2}{2} \left(1 + 3 \frac{k^2 v_t^2}{\omega^2} \right) + i \frac{\nu \omega_p^2}{3} \left(1 + 3 \frac{k^2 v_t^2}{\omega^2} \right) , \end{aligned} \quad (2-7)$$

where ν is the effective collision frequency between plasma electrons, ω_p is the electron plasma frequency, v_t is the thermal velocity of the plasma electron and k is the wavenumber. We set $k(x) = \omega/v_b + \delta k(x)$ and expand $\epsilon(\omega, k)$ up to the second order of

δk , namely

$$\varepsilon(\omega, k) = \varepsilon(\omega, \omega/v_b) + \left. \frac{\partial \varepsilon}{\partial k} \right|_{\omega, \omega/v_b} \delta k + \frac{1}{2} \left. \frac{\partial^2 \varepsilon}{\partial k^2} \right|_{\omega, \omega/v_b} (\delta k)^2, \quad (2-8)$$

where

$$\varepsilon(\omega, \omega/v_b) = 1 - \frac{\omega_0^2}{\omega^2} + i \frac{v \omega_0^2}{\omega^3}, \quad (2-9)$$

$$\left. \frac{\partial \varepsilon}{\partial k} \right|_{\omega, \omega/v_b} = \varepsilon' = -6 \frac{\omega_p^2}{\omega^2} \frac{v_t^2}{\omega v_b} \left(1 - i \frac{v}{\omega} \right), \quad (2-10)$$

$$\left. \frac{\partial^2 \varepsilon}{\partial k^2} \right|_{\omega, \omega/v_b} = \varepsilon'' = -6 \frac{\omega_p^2}{\omega^2} \frac{v_t^2}{\omega^2} \left(1 - i \frac{v}{\omega} \right), \quad (2-11)$$

$$\text{and} \quad \omega_0^2 = \omega_p^2 \left(1 + 3v_t^2/v_b^2 \right). \quad (2-12)$$

Imaginary parts of eqs. (2-10) and (2-11) are smaller than real parts by a factor v/ω . As we consider the effects of weak collisions, i.e. $v/\omega \cong 10^{-3}$, imaginary parts can be neglected. With the aid of eq. (2-8), we get the following equation from eq. (2-6),

$$\begin{aligned} & \left[\varepsilon(\omega, \omega/v_b) + \varepsilon' \delta k + \frac{\varepsilon''}{2} (\delta k)^2 \right] E_\omega(0) \exp\left(i \int_0^x \delta k(x') dx'\right) \\ & = i \frac{4\pi e n_b v_b}{N\omega} \sum_j \exp[i\omega(t_j(x) - x/v_b)]. \end{aligned} \quad (2-13)$$

The quantity δk can be replaced by a spatial derivative $-id/dx$. Therefore, we obtain

$$\left[\varepsilon(\omega, \omega/v_b) - i\varepsilon' \frac{d}{dx} - \frac{\varepsilon''}{2} \frac{d^2}{dx^2} \right] E_\omega(0) \exp\left(i \int_0^x \delta k(x') dx'\right)$$

$$= i \frac{4\pi en_b v_b}{N\omega} \sum_j \exp[i\omega \delta t_j(x)], \quad (2-14)$$

where $\delta t_j(x) = t_j(x) - x/v_b$. The time difference δt_j satisfies the equation of motion similar to Newton's law, i.e.

$$\frac{d\delta t_j}{dx} = \frac{1}{\dot{x}_j} - \frac{1}{v_b}, \quad \frac{d^2\delta t_j}{dx^2} = - \frac{1}{[\dot{x}_j]^2} \frac{d\dot{x}_j}{dx} = - \frac{1}{[\dot{x}_j]^3} \frac{d^2x_j}{dt^2}. \quad (2-15)$$

The equation of motion for the j -th beam particle is

$$\frac{d^2x_j}{dt^2} = \frac{e}{m} \left[\sum_{\omega} E_{\omega}(x_j) \exp(-i\omega t) + \text{C.C.} \right].$$

Then, eq. (2-15), i.e. $d^2\delta t_j/dx^2$, becomes

$$\frac{d^2\delta t_j}{dt^2} = \frac{1}{[\dot{x}_j]^3} \frac{e}{m} \left[\sum_{\omega} E_{\omega}(x_j) \exp(-i\omega t) + \text{C.C.} \right]. \quad (2-16)$$

We introduce dimensionless variables, i.e. the phase-space coordinate of the j -th beam electron $\xi_j = \omega_0 \delta t_j = \omega_0 [t_j(x) - x/v_b]$, the spatial coordinate $\eta = sx(\omega_0/v_b)$, the normalized electric field $E_{\omega}(\eta) = eE_{\omega}(x) \exp(-ix\omega/v_b) / (mv_b\omega_0 s^2)$. Here, the spatial scaling factor $s = [(n_b v_b^2) / (6n_e v_t^2)]^{1/3}$, and $\omega_0^2 = \omega_p^2 (1 + 3v_t^2/v_b^2)$. The spatial growth rate in the collisionless warm plasma is $-(3^{1/2}/2)sk_0 = -(3^{1/2}/2)(s\omega_0/v_b)$. We also call s beam strength [10] because the half-width of the growth rate function is proportional to s . In terms of the dimensionless variables, eqs. (2-14) and (2-16) becomes

$$(A + iB + iC \frac{d}{d\eta} + D \frac{d^2}{d\eta^2}) E_{\omega}(\eta) = \frac{i}{N} \sum_j \exp(i\omega \xi_j'/\omega_0), \quad (2-17)$$

$$\frac{d^2 \xi_j}{d\eta^2} = (1 + s \frac{d\xi_j}{d\eta})^3 [\sum_{\omega} E_{\omega}(\eta) \exp(-i\omega \xi_j / \omega_0) + \text{C.C.}], \quad (2-18)$$

where

$$A = s^2 \frac{\omega_0^2}{\omega_b^2} [(1 + \delta\omega/\omega_0) - (1 + \delta\omega/\omega_0)^{-1}],$$

$$B = s^2 \frac{\nu}{\omega_0} \frac{\omega_0^2}{\omega_b^2} (1 + \delta\omega/\omega_0)^{-2},$$

$$C = (1 + \delta\omega/\omega_0)^{-2} \quad \text{and} \quad D = \frac{s}{2} (1 + \delta\omega/\omega_0)^{-3}.$$

The value $\delta\omega = \omega - \omega_0 = n\Delta\omega$, where $n = 0, \pm 1$ and ± 2 in the case of five modes system, and $n = 0, \pm 1, \dots, \pm 5$ in the case of eleven modes system. We have retained the term $s d\xi_j/d\eta$ in the right hand side of eq.(2-18). Because this term becomes quite important when the beam strength s is not so small as one in other case [10]. Eqs.(2-17) and (2-18) make the self-consistent set of equations. In the present work, only collisional effects of plasma electrons are considered. As an effective collision frequency, we may adopt the reciprocal of the slowing down time associated with Coulomb collisions in a dense plasma.

Basic equations for the single wave model is included essentially in eqs.(2-17) and (2-18). For convenience, we describe the set of equations explicitly as follows:

$$(A' + iB' + i\frac{d}{d\eta} + \frac{s}{2} \frac{d^2}{d\eta^2}) E(\eta) = \frac{i}{N} \sum_j \exp(i\xi_j), \quad (2-19a)$$

$$\frac{d^2 \xi_j}{d\eta^2} = (1 + s \frac{d\xi_j}{d\eta})^3 E(\eta) \exp(-i\xi_j) + \text{C.C.}, \quad (2-19b)$$

where

$$A' = s^2 \frac{\omega_0^2}{\omega_b^2} \left[2 \frac{\delta\omega}{\omega_0} + \left(\frac{\delta\omega}{\omega_0} \right)^2 \right],$$

$$B' = s^2 \frac{v}{\omega_0} \frac{\omega_0^2}{\omega_b^2} \left(1 + \frac{\delta\omega}{\omega_0} \right)^{-1}$$

and $E(\eta) = eE_\omega(x) \exp(-ix\omega/v_b) / (mv_b \omega s^2)$. Here, the coefficient A' is caused by a difference between the frequency of the most unstable mode ω_0 and a given frequency ω , i.e. a detuning $\delta\omega = \omega - \omega_0$. This detuning is important to consider an external excitation of a wave due to pre-modulation of an electron beam[6,9].

2.2-2 Method of Numerical Calculation

In order to solve eqs.(2-17) and (2-18), we represent the beams by N charge sheets and solve for the exact nonlinear dynamics numerically. We typically employ a total of 200 charge sheets in the single wave model and 600 charge sheets in the many wave model.

The equations are integrated with the Runge-Kutta method with a step size of $\Delta\eta = 0.05$. The initial wave amplitudes are equal to 0.01. The entrance conditions of charge sheets for the numerical calculations are as follows: In the case of the single wave model,

$$\xi_j(0) = -2\pi(j/N) \quad \text{and} \quad \dot{\xi}_j(0) = 0, \quad (2-20)$$

and in the case of the many wave model,

$$\xi_j(0) = -2\pi l(j/N) \quad \text{and} \quad \dot{\xi}_j(0) = 0, \quad (2-21)$$

where l is the reciprocal of the mode spacing $\Delta\omega/\omega_0$.

The velocity, i.e. $v_j = \dot{x}_j$, in the laboratory frame is obtained from $\dot{\xi}_j = d\xi_j/d\eta$ by using the relation $\dot{x}_j = v_b/(1 + s\xi_j)$. A beam energy loss ΔW is defined by

$$\Delta W = - \left(\sum_j \frac{mv_j^2}{2} - W_0 \right) \quad \text{and} \quad W_0 = N \frac{mv_b^2}{2}, \quad (2-22)$$

where W_0 is the initial beam energy. Instantaneous wavenumber, i.e. $\Omega = \delta k/(sk_0)$, is calculated by the following relations

$$\Omega_r = (E_r dE_i/d\eta - E_i dE_r/d\eta)/|E|^2, \quad (2-23)$$

and

$$\Omega_i = -(E_r dE_r/d\eta + E_i dE_i/d\eta)/|E|^2. \quad (2-24)$$

2.3 Numerical Results

2.3-1 Destruction of Amplitude Oscillations due to

Weak Collisions between Plasma Electrons[11]

The spatial evolution of the single wave is shown in Fig.2.1. For numerical calculation, plasma parameters are chosen as follows: $n_b/n_p = 5 \times 10^{-4}$, $v_b/v_t = 9.9$ and $\delta\omega/\omega_0 = 0$. These parameters correspond to ones for the typical case of particle simulation[13], i.e. Run E3. The dashed line is $E^2(\eta)$ in the collisionless case. The solid line is $E^2(\eta)$ when $v/\omega_0 = 2 \times 10^{-3}$. The dotted line is $E^2(\eta)$ when $v/\omega_0 = 4 \times 10^{-3}$. The corresponding beam energy losses $\Delta W/W_0$ are shown in Fig.2.2.

As can be seen, in the collisionless case, the single wave grows exponentially and then saturates at $\eta = 10$. After this,

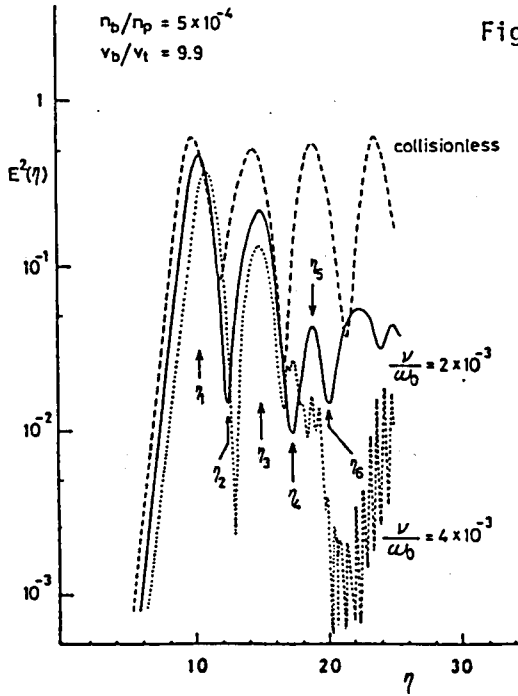


Fig.2.1 Wave power $E^2(\eta)$ vs η . Dashed, solid and dotted lines are computed with $\nu/\omega_0 = 0, 2 \times 10^{-3}$ and 4×10^{-3} , respectively. In all cases, frequency detuning $\delta\omega = 0$. Six arrows indicated by $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$ and η_6 show the characteristic positions from the first maximum to the third minimum in the collisional case, $\nu/\omega_0 = 2 \times 10^{-3}$.

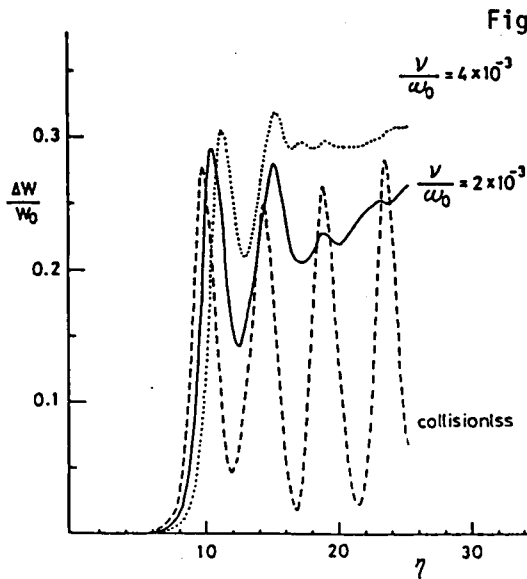


Fig.2.2 Beam energy loss $\Delta W/W_0$ vs η , corresponding to the evolution of $E^2(\eta)$ in Fig.2.1.

regular amplitude oscillations can occur satisfactorily. In the collisional case, up to the wave saturation, effects of weak collisions do not significantly alter the spatial evolution of the wave except that the wave amplitude saturates at a slightly lower level than in the collisionless case, and that saturation point shifts slightly to the posterior region. However, nonlinear evolution of the single wave becomes quite different. After the first maximum, the wave amplitude decreases to a level much smaller than that of the collisionless case and fails to regrow to the initial saturation level. Then, amplitude oscillations are destroyed in the posterior region.

Collisional effects on the spatial evolution of the single wave are studied for various plasma parameters. We adjusted the ratios n_b/n_p and v_b/v_t such that the spatial scaling factor s is 0.2, i.e. $n_b/n_p = 10^{-3}$ and $v_b/v_t = 7$; 5×10^{-4} and 9.9; and 2.5×10^{-4} and 14, respectively. These results are shown in Fig.2.3, where the first peak, the second peak and the first minimum are plotted vs B' . The second peak and the first minimum amplitudes are normalized by the amplitude of the first peak. The values of the first minimum are dependent strongly on the collisional effects. The arrows indicate the values of B' corresponding to two collision frequencies, respectively, when $n_b/n_p = 5 \times 10^{-4}$ and $v_b/v_t = 9.9$.

In order to consider the destruction mechanism of amplitude oscillations due to weak collisions, from various point of view, we compare the wave evolution of collisional case, i.e. $v/\omega_0 = 2 \times 10^{-3}$, with that of collisionless case in detail.

Figure 2.4 shows the spatial evolutions of the real and imaginary parts of the wavenumber, which is interpreted as the phase

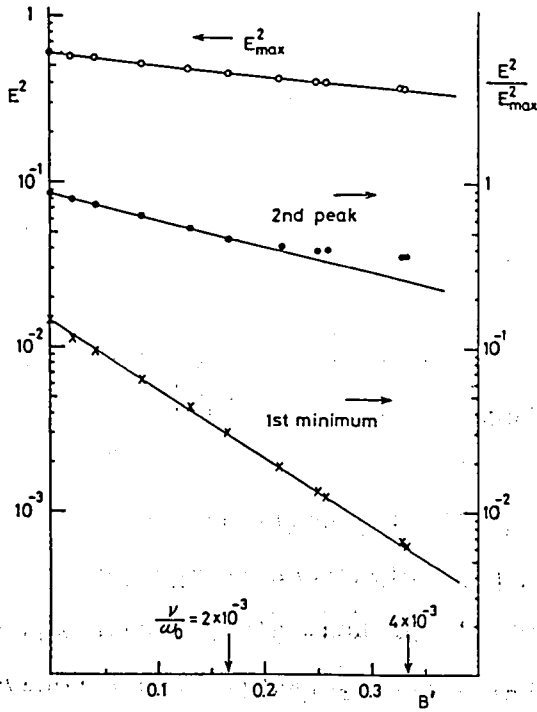


Fig.2.3 The first peak, the second peak and the first minimum vs B' . Plasma parameters n_b/n_p and v_b/v_t are chosen so that the spatial scaling factor s is equal to 0.2. In all cases, $\delta\omega = 0$.

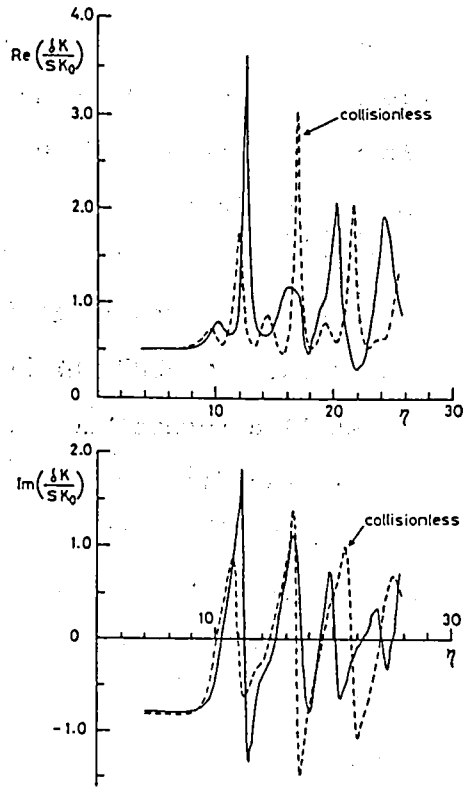


Fig.2.4 Instantaneous wavenumber $\delta k / (s k_0)$ vs η .

shift and growth rate, respectively. According to the linear theory, they are $(1/2)sk_0$ and $-(3^{1/2}/2)sk_0$, respectively. The values obtained from eqs.(2-23) and (2-24), before the wave saturation, correspond to the results of the linear theory. In this region, effects of weak collisions do not change the character of δk . After the wave saturation, two curves begin to depart from each other. Then, the wave undergoes a rapid phase shift as well as a strong damping of the amplitude near η_2 .

Figure 2.5 shows the phase-space evolution of the beam electrons in the collisional case. Each locus is composed of the phase points for the beam electrons at the particular positions denoted in Fig.2.1. In accordance with the amplitude oscillation, sloshing back and forth of trapped beam electrons in the wave trough appears. Though the motion of the beam electrons in phase space is still a reversible process, after saturation their behavior changes gradually from that of the collisionless case. Near η_3 , the locus splits into two parts. The beam electrons of the two parts begin to be smeared out and spill into adjacent wave troughs in an irreversible manner after η_4 .

On the other hand, the motion of the beam electrons in the collisionless case is still a reversible process after the second amplitude oscillation. Figure 2.6 shows the phase-space loci in the collisionless case at two positions, i.e. (a) the third maximum and (b) the third minimum of $E^2(\eta)$. Although some of the beam electrons spill into an adjacent wave trough, most of them do not spread irreversibly but continue their phase rotation in a reversible manner.

Furthermore, we compare phase-space trajectories of some test

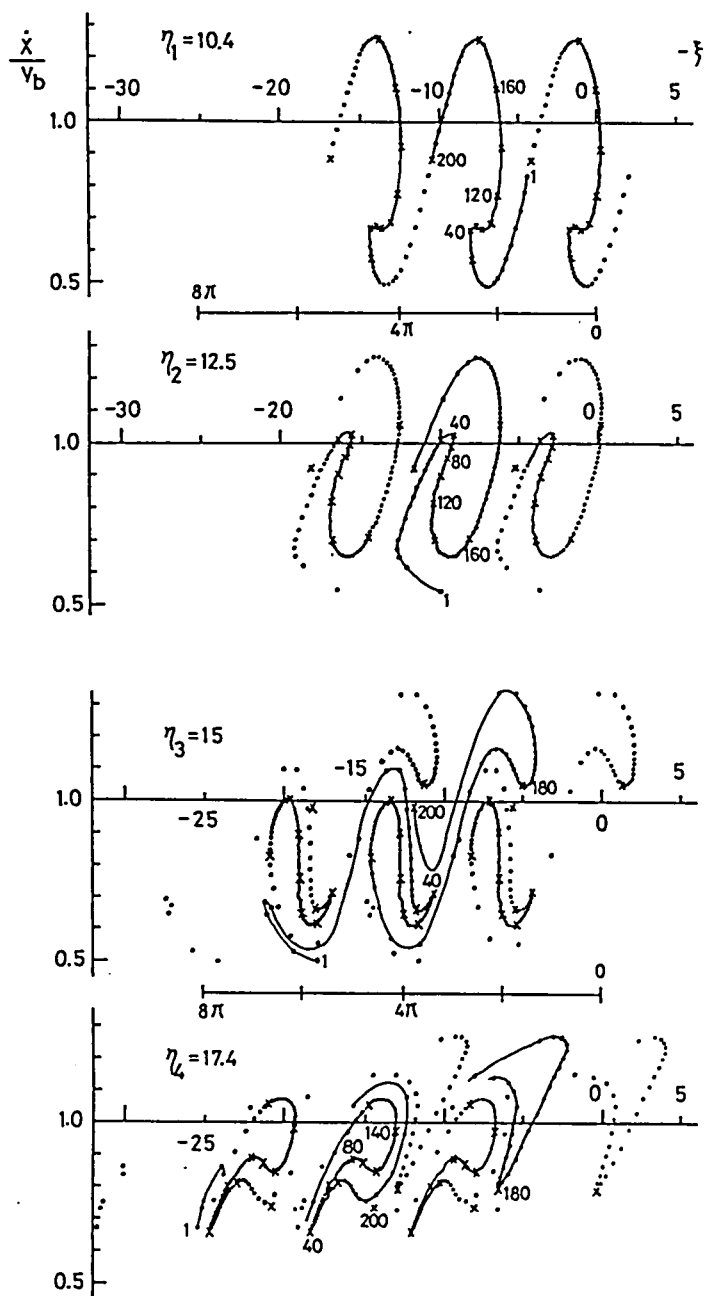


Fig.2.5

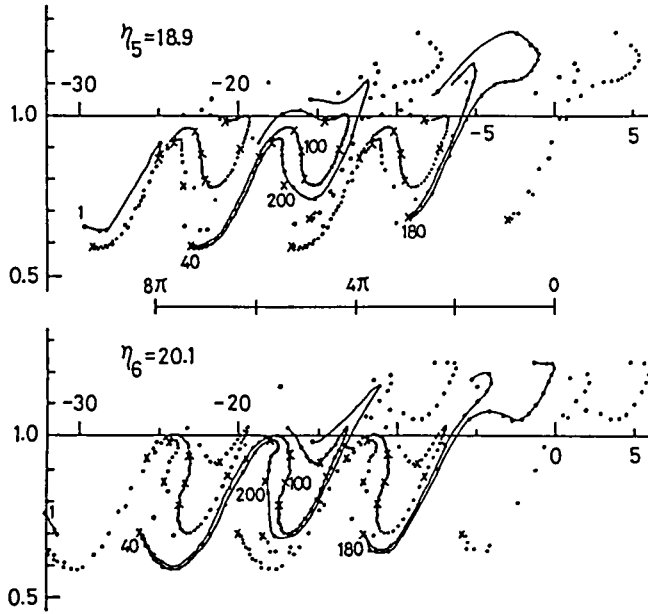
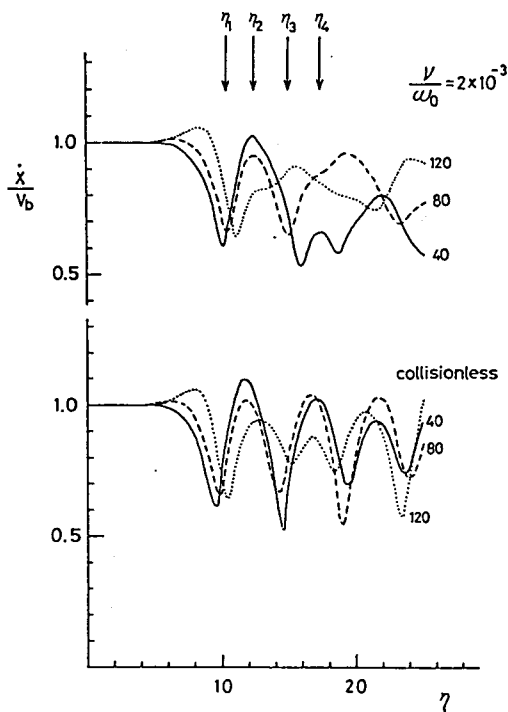
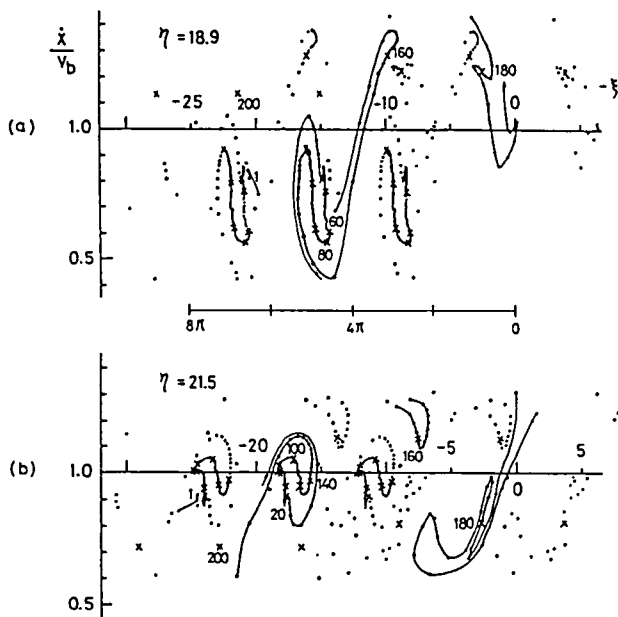


Fig.2.5(continue) Phase-space loci for the beam electrons in the collisional case, corresponding to six positions denoted in Fig.2.1. Each point gives the velocity $\dot{x}/v_b$ and the normalized spatial coordinate ξ for one beam electron. Since eqs.(2-19a) and (2-19b) are solved by using 200 beam electrons which satisfy the initial conditions in eq.(2-20), phase space is periodic in ξ with period 2π . In this figure, three groups of 200 electrons (the initial conditions $\xi_j(0)$ are $\xi_{-199 \leq j \leq 0} = -2\pi(j/N)$, $\xi_{1 \leq j \leq 200} = -2\pi(j/N)$ and $\xi_{201 \leq j \leq 400} = -2\pi - 2\pi(j-200)/N$) are displayed and every twentieth electron is marked. For reference, beam electrons with $\xi_{1 \leq j \leq 200} = -2\pi(j/N)$ are joined by a solid line.



particles in the $\dot{x}$ - η plane in the collisional case with ones in the collisionless case. This is shown in Fig.2.7. Motions of test particles in the collisional case become more irregular and their bounce periods also become longer because of the reduced amplitude of the wave. Consequently, net energy exchange between the wave and the beam electrons ceases after η_3 as shown in Fig.2.2, and the wave will remain at a relatively low power level. The phase average velocity distributions at various positions in Fig.2.1 are also shown in Fig.2.8.

Concerning spatial evolutions of the wave and the corresponding behaviors of beam electrons, features of the present numerical results in the collisional case resemble closely to those in particle simulation[13]. Here, we estimate the effective collision frequency ν in Run E3: The collision frequency ν of the one-dimensional particle simulation has the relationship with n_p and the Debye length λ_D [20,21], i.e. $\omega_p \nu^{-1} = \alpha (n_p \lambda_D)^2$, where α is a numerical factor and is about 0.1 in the model adopted there, and $n_p \lambda_D = 90$ in Run E3. Therefore, the calculated value of ν/ω_p is about 1.2×10^{-3} , which is comparable to the value $\nu/\omega_0 = 2 \times 10^{-3}$.

2.3-2 Competition of Weakly Collisional and Neighboring Wave

Effects for the Destruction of Amplitude Oscillations[22]

Next, we examine the effects of neighboring modes for the destruction of amplitude oscillation. For this end, we use the system of eleven modes with $\Delta\omega/\omega_0 = 0.02$. Plasma parameters are the same as those in Fig.2.1. Numerical results are shown in Fig.2.9. The spatial behaviors of the electric fields of three

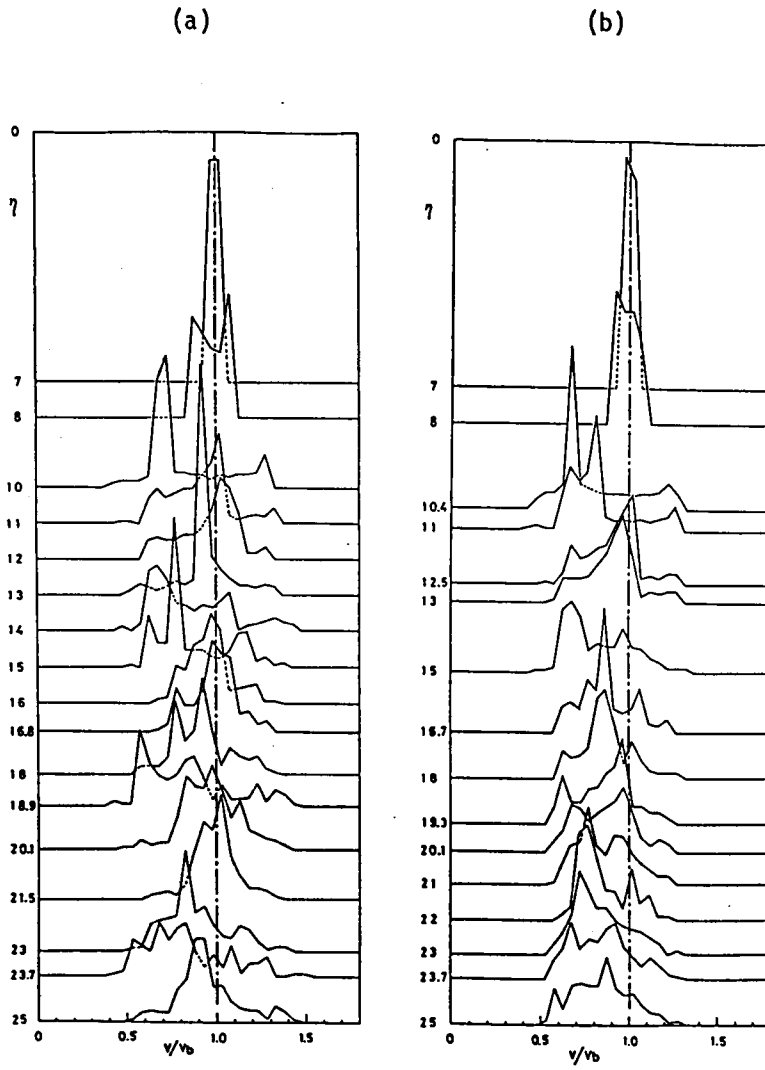


Fig.2.8 The phase average velocity distributions vs v_j/v_b :
 (a) in the collisionless case and (b) in the collisional case, where $v/\omega_0 = 2 \times 10^{-3}$.

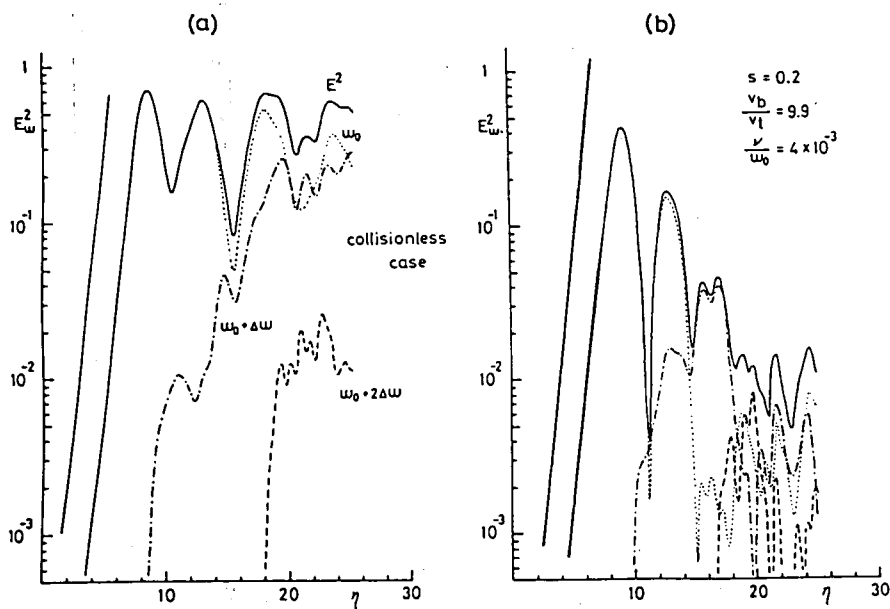


Fig.2.9 The total wave power $E^2(\eta)$ and the power in the spectral components in the collisionless case (a), and in the collisional case (b). These are computed with 600 beam electrons by using 11 modes system, where $\Delta\omega/\omega_0 = 0.02$. The solid straight lines in the figures correspond to the theoretical linear growth.

of eleven modes of the system, and the behavior of the total power E^2 are displayed. These are three most predominant modes in the system. In the collisionless case, in Fig.2.9(a), E^2 grows exponentially to saturation and then executes three amplitude oscillations. Up to the third peak of E^2 , the most unstable wave with frequency ω_0 dominates the nonlinear evolution of the system. However, in the nonlinear stage, excitation of the neighboring waves, whose frequencies are larger than ω_0 , occurs. Then, after the third peak, regular amplitude oscillations of E^2 are deformed due to the effects of neighboring waves because the wave with ω_0 no longer dominates the evolution of the system, and the wave spectrum is broadened[8,10,16,19]. In this plasma conditions, however, collisions are more effective than the excitation of neighboring waves for the destruction of the amplitude oscillations. This is clearly shown in Fig.2.9(b). The most unstable wave damps to the first minimum strongly and fails to regrow to the level of the first maximum. After this, the amplitude oscillations of E^2 is changed drastically and destroyed. This manner is quite similar to that in the single wave case shown in Fig.2.1.

In order to see the character of the excitation of the neighboring wave with frequency $\omega_0 + \Delta\omega$, we have obtained following two relations. Beam energy losses and mean beam velocities corresponding to Fig.2.9 are shown in Fig.2.10. Another example, i.e. $v/\omega_0 = 6 \times 10^{-3}$, is also plotted. Evolutions of the wavenumbers for two waves in the collisionless case are shown in Fig.2.11. From these results, it is found that the growth rate of the neighboring wave becomes large when the mean beam velocity is slowed down due to the growth of the most unstable wave. Therefore, for a certain

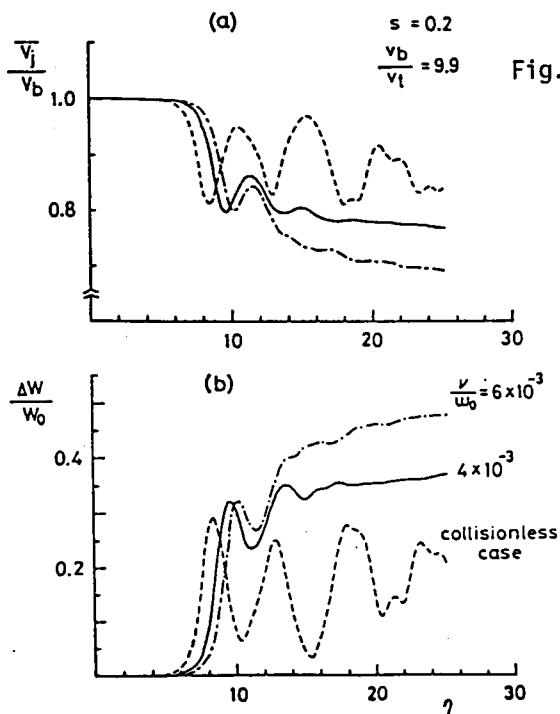


Fig.2.10 (a) Mean beam velocity V/v_b , and (b) beam energy loss $\Delta W/W_0$ for the calculation in Fig.2.9. The value of V is obtained by $\sum_j v_j/N$.

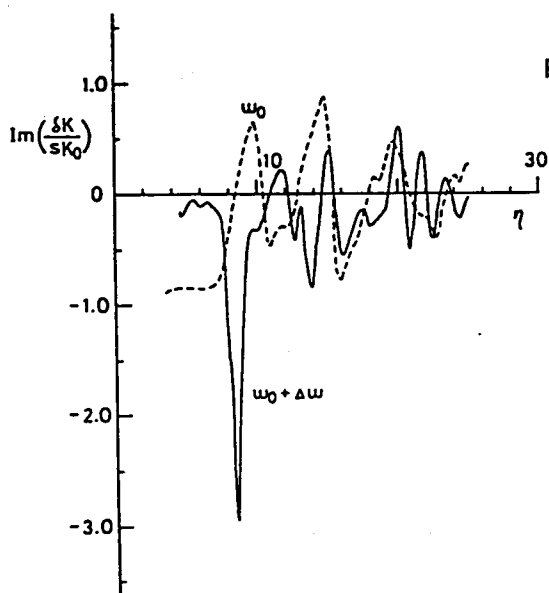


Fig.2.11 Imaginary parts of the wavenumber for two waves, i.e. the wave with frequency ω_0 and the wave with $\omega_0 + \Delta\omega$, respectively, corresponding to the wave evolutions in Fig.2.9(a).

plasma condition, an excitation mechanism of neighboring waves may be considered as the secondary beam-plasma interaction[10], where beam velocity is slowed down nearly equal to the phase velocity of the most unstable wave. This process is closely related to the recent works[24-27] on the mechanism of "sideband" or "trapped particle" instabilities.

The excitation of the neighboring waves depends strongly on plasma parameters, i.e. scaling factor s . In order to study competition of weakly collisional and neighboring wave effects for the destruction of amplitude oscillations in detail, eqs.(2-17) and (2-18) for five or eleven mode systems are solved for various plasma parameters. For calculation, 100, 200, 400 and 600 beam sheets are used, and there are no significant differences among four cases. Here, we show the results of five mode system with 200 beam sheets. Other conditions are as follows: $E_{\omega}(0) = 10^{-3}$ and $\Delta\omega/\omega_0 = 0.05$.

As is suggested in Ref.10, the most unstable wave no longer dominates the nonlinear evolutions of the system with increasing s . Excitation of neighboring waves, whose frequencies are larger than ω_0 , occur in the nonlinear stage, and deform the regular amplitude oscillations of the total wave power E^2 (see Fig.2.9).

On the excitation of neighboring waves, however, there is another feature. With increasing v_b/v_t under the constant s , it becomes difficult to excite neighboring waves. Therefore, it also causes the amplitude oscillation to continue persistently. This novel features is clearly shown in Fig.2.12. When $v_b/v_t = 5$, the excitation of the neighboring wave occurs in the early nonlinear stage and the amplitude oscillation of E^2 is destroyed after second

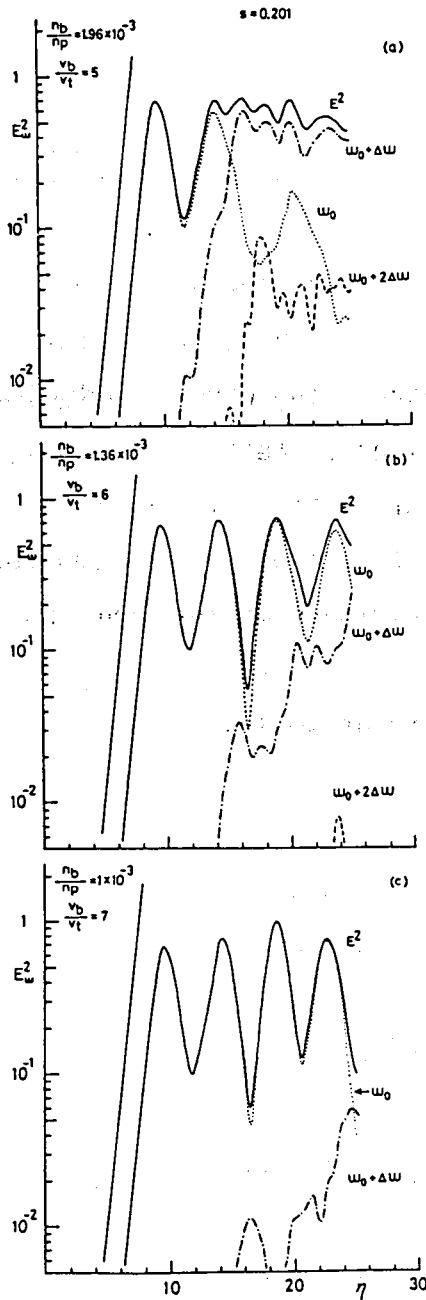


Fig.2.12 $E^2(\eta)$ and the power in the spectral components vs η in the collisionless case, where s is constant. These are computed with 200 beam electrons by using 5 modes system, where $\Delta\omega/\omega_0 = 0.05$.

maximum. On the other hand, when $v_b/v_t = 7$, the most unstable wave predominates in the nonlinear evolution of the system and the regular amplitude oscillation of E^2 is obtained.

These results are explained as follows: The initial beam velocity v_b is slowed down nearly equal to the phase velocity of the most unstable wave with ω_0 , i.e. the slowed velocity $v_b' \cong v_b(1-s)$. Then, the frequency of the most unstable wave ω_0' in the slowed beam-plasma system has the relationship, namely $\omega_0'^2 = \omega_p^2(1+3v_t^2/v_b'^2)$. The frequency ω_0' is larger than ω_0 because v_b' is smaller than v_b . By using these relations, it is found that a frequency shift $(\omega_0' - \omega_0)/\omega_p$ is proportional to $s(v_t^2/v_b^2)$. This shift becomes small with increasing v_b/v_t under the constant s . The smaller frequency shift can be considered to cause the less effects on the destruction of amplitude oscillation. Numerical results shown in Fig. 2.12 have the tendency like this.

Figures 2.13 and 2.14 show phase-space dynamics of beam electrons corresponding to the calculations in Fig. 2.12(a) and (c), respectively. The particle phase mixing is evident in the phase space plots in Fig. 2.13. However, when amplitude oscillations occur regularly, most of beam electrons do not spread irreversibly but continue their phase rotation in a reversible manner (see Fig. 2.14).

Taking into account the above-described features for the excitation of the neighboring waves, now, we consider the collisional effects in the many wave system. Figures 2.15 and 2.16 show spatial evolutions in the collisionless and in the collisional cases, respectively, when beam velocity is constant, i.e. $v_b/v_t = 4$. Collisional effects become remarkable as s becomes small. When

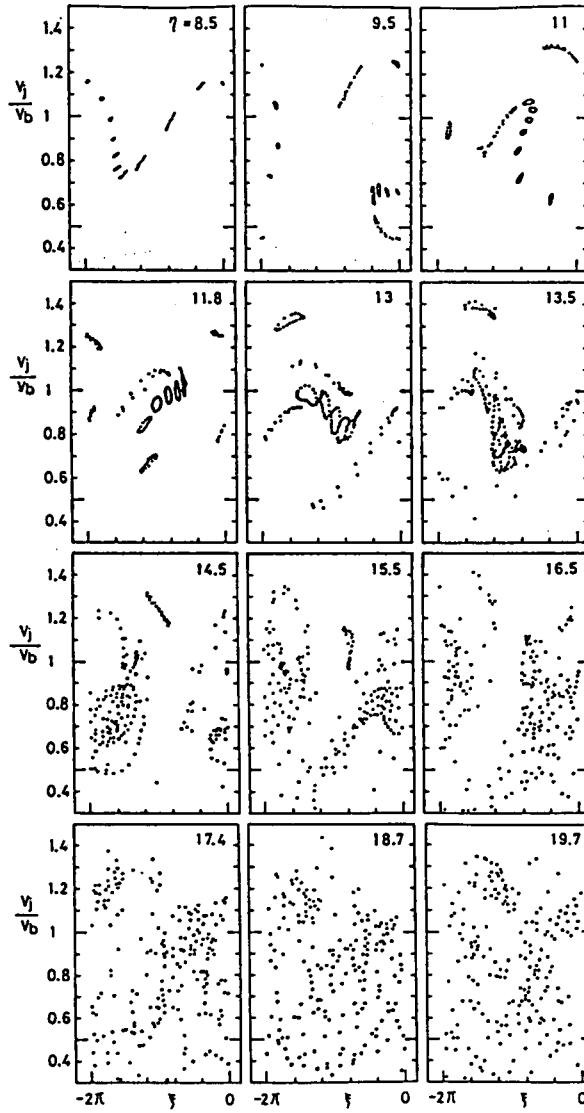


Fig.2.13 Phase-space dynamics at various positions η for computation in Fig.2.12(a). As $\omega_0/\Delta\omega = 20$ in this case, we need to follow the particles whose initial phases are in the interval $-40\pi \leq \xi_j(0) \leq 0$. In this figure, however, instantaneous particle phase $\xi_j(\eta)$ is transformed in the interval $-2\pi \leq \xi_j(\eta) \leq 0$.

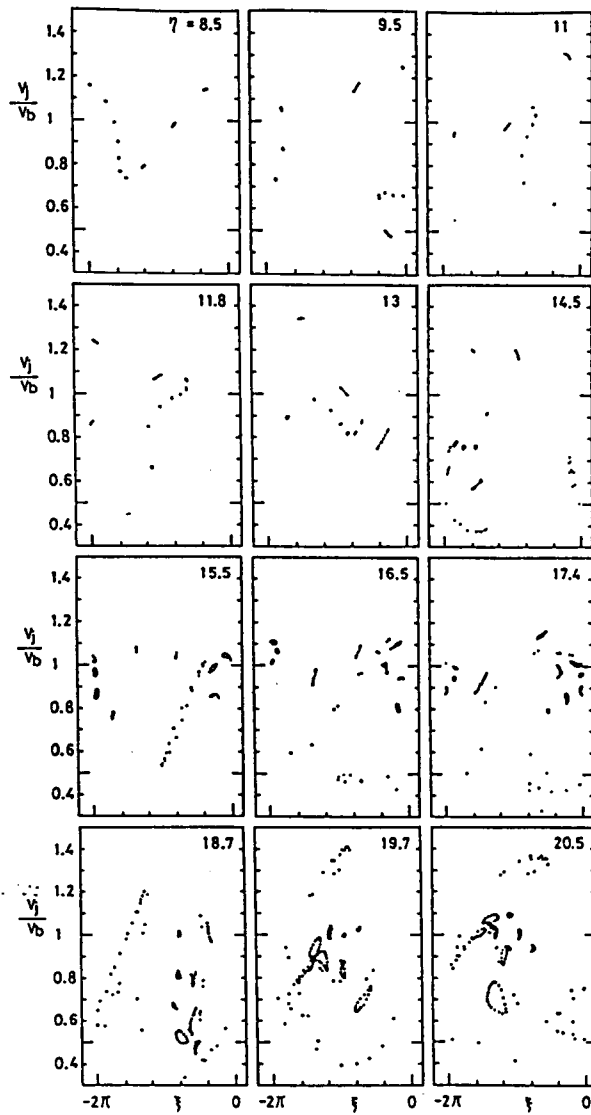


Fig.2.14 Phase-space dynamics at various positions η for computation in Fig.2.12(c). Conditions for plotting beam electrons are the same as in Fig.2.13.

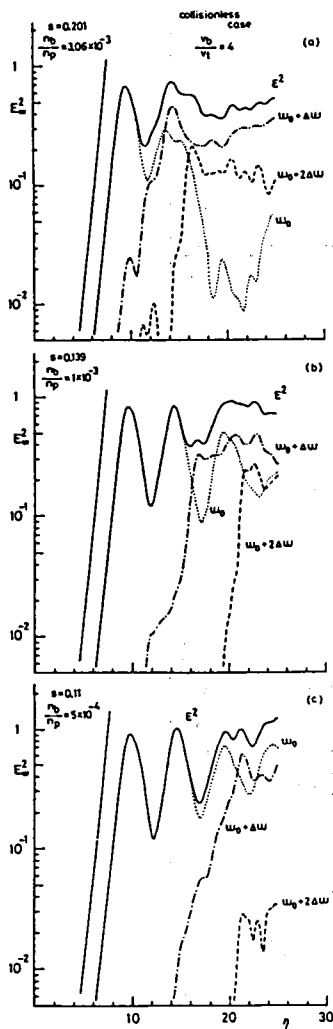


Fig.2.15 $E^2(\eta)$ and the power in the spectral components vs η in the collisionless case, where beam velocity is constant. These are computed with 200 beam electrons by using 5 modes system, where $\Delta\omega/\omega_0 = 0.05$.

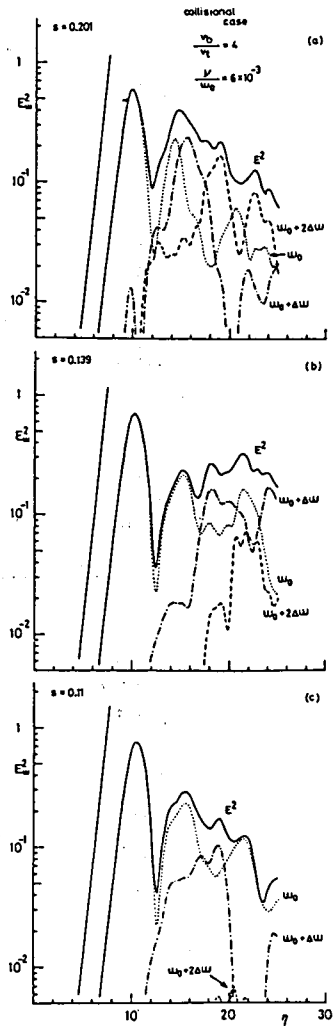


Fig.2.16 E^2 and the power in the spectral components vs η in the collisional case, where other conditions are the same as in Fig.2.15.

$s = 0.11$, through two amplitude oscillations of the total power E^2 in the collisionless case, the most unstable wave mainly dominates the nonlinear evolution of the system. In the collisional case, the most unstable wave damps to the first minimum strongly and fails to regrow to the level of the first maximum. Consequently, the amplitude oscillation of E^2 is changed drastically. Besides, amplitudes of the neighboring waves remain at a lower power level than in the collisionless case. When $s = 0.201$, however, the wave with frequency $\omega_0 + \Delta\omega$ grows to a large amplitude in the early nonlinear stage. After the saturation point, the evolution of E^2 cannot be determined by the most unstable wave alone. The amplitude oscillation of E^2 in the collisionless case is deformed due to the effects of the neighboring waves. Therefore, collisions do not modify the evolution of E^2 drastically as they do in the weak beam case ($s = 0.11$) although they make the additional change in the nonlinear behaviors.

Collisional effects also appear clearly in the spatial evolutions of the beam energy loss $\Delta W/W_0$. Figure 2.17 shows the evolutions of $\Delta W/W_0$, corresponding to the wave evolutions in Figs. 2.15 and 2.16. After the point, for example $\eta = 15$, where amplitude oscillations in the collisional case are destroyed, the evolution of $\Delta W/W_0$ for all beam strength do not oscillate because little net energy exchange occurs between the waves and beam particles. Before this point, however, the evolution of $\Delta W/W_0$ depends on s . The degree of the difference between $\Delta W/W_0$ in the collisionless case and one in the collisional case becomes remarkable with decreasing s . Therefore, collisions dominate the nonlinear evolutions in the weak beam system because the difference in $\Delta W/W_0$ is

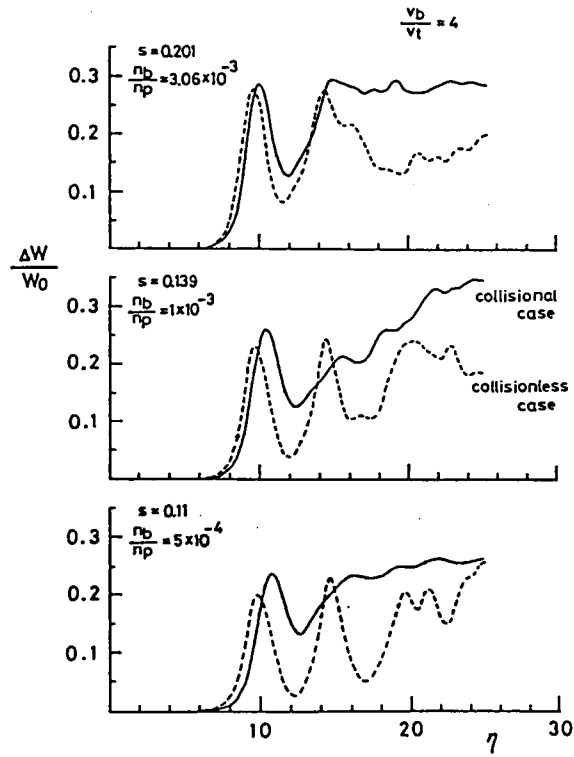


Fig.2.17 Beam energy loss $\Delta W/W_0$ vs η , corresponding to the wave evolutions shown in Figs.2.15 and 2.16.

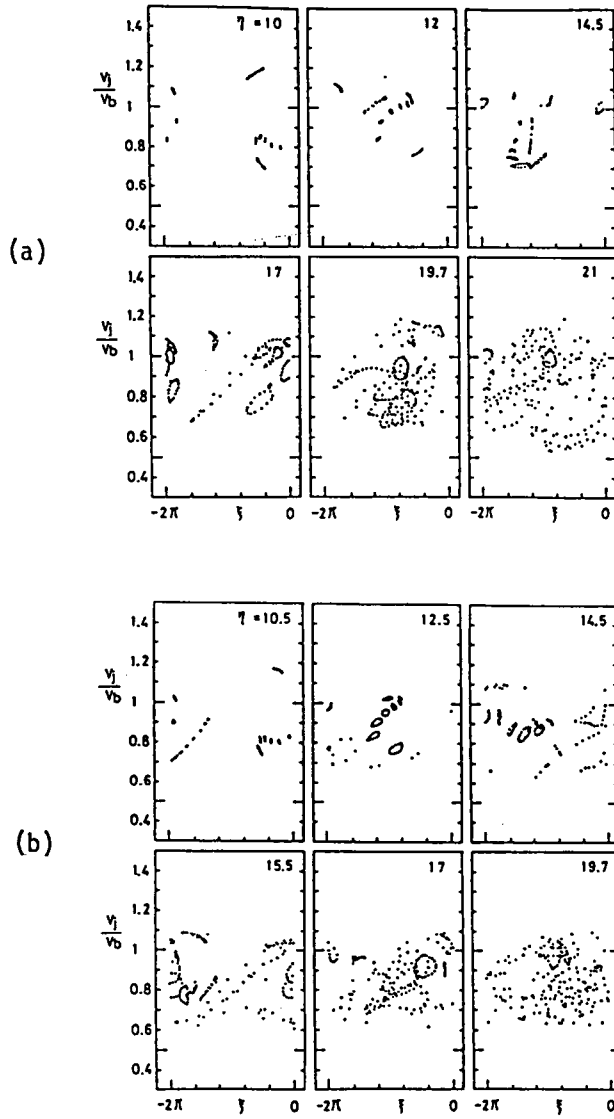


Fig.2.18 Phase-space dynamics, (a)in the collisionless case and (b)in the collisional case, at various positions η for computation in Figs.2.15 and 2.16. Plasma parameters are $s = 0.11$, $n_b/n_p = 5 \times 10^{-4}$, $v_b/v_t = 4$ and $v/\omega_0 = 6 \times 10^{-3}$. Conditions for plotting beam electrons are the same as in Fig.2.13.

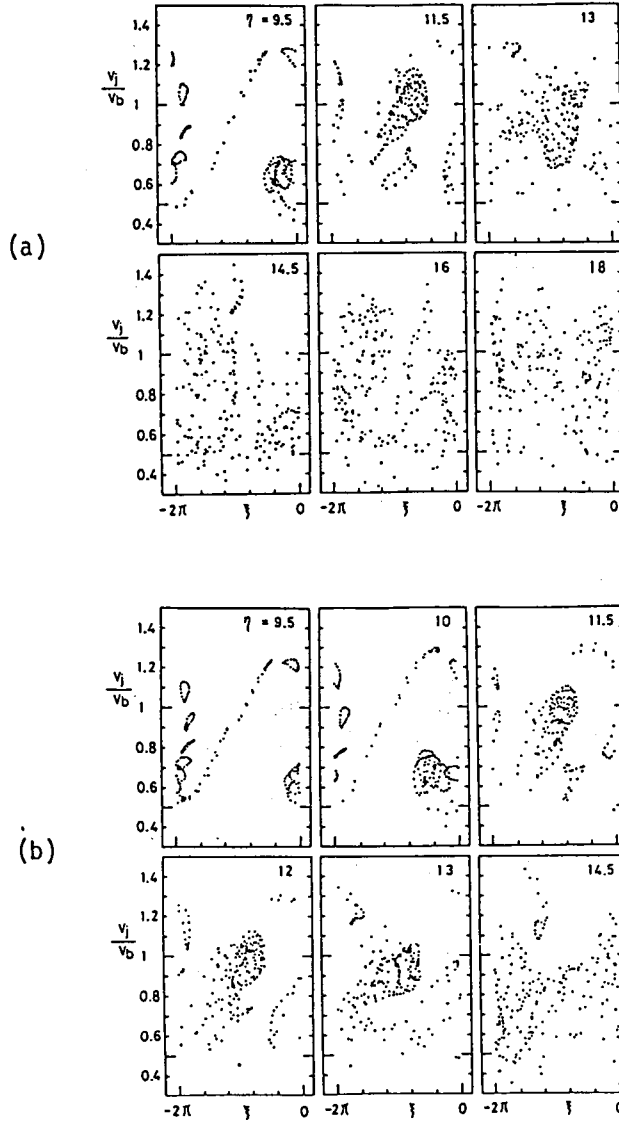


Fig.2.19 Phase-space dynamics, (a) in the collisionless case and (b) in the collisional case, at various positions η for computation in Figs.2.15 and 2.16. Plasma parameters are $s = 0.201$, $n_b/n_p = 3.06 \times 10^{-3}$, $v_b/v_t = 4$ and $\nu/\omega_0 = 6 \times 10^{-3}$. Other conditions are the same as in Fig.2.18.

due to the collisional effects for the interaction of the waves with the beam particles.

Phase-space plots of beam particles, corresponding to the wave evolutions in Figs.2.15 and 2.16, are shown in Figs.2.18 and 2.19. Additional phase mixing due to weak collisions is evident in the weak beam system(see Fig.2.18, $s = 0.11$). However, remarkable change does not appear in the strong beam system because phase mixing due to neighboring wave effects are occurred already(see Fig.2.19, $s = 0.201$).

2.4 Discussion

We have numerically observed that the regular amplitude oscillations can be destroyed by the weak collisions or by the growth of the neighboring waves. In both cases, particle phase mixing is evident in the phase space plots when the amplitude oscillations are destroyed.

Collisional destruction is as follows: After the first maximum point, the wave power falls drastically to the first minimum and fails to regrow to the initial saturation level. From the results shown in Figs.2.1 and 2.9, we can conclude that the origin of the overdamping of the first minimum is due to weak collisions($v/\omega_p \equiv 10^{-3}$). At the first minimum, the wave also undergoes a rapid phase shift. Following this point, the trapping oscillations are deformed strongly and destroyed. These numerical results, including phase-space loci of the beam electrons, are well in agreement with the results of our particle simulation[13].

Up to now, there have been some works concerning the destruction of amplitude oscillations. Here, we compare our results with other's.

Dimonte and Malmberg[18] studied the destruction of trapping oscillations both experimentally and numerically. In order to avoid the problems associated with the background plasma, the plasma is simulated by the slow wave structure of the traveling wave tube. They show that the amplitude oscillations can be destroyed either by the dissipation in the slow wave structure or by the modulation of the main wave by unstable sidebands. Characteristic features of the experimental results are also reproduced by the solutions of the working equations in a traveling wave tube. As a whole, our numerical results are well in agreement with their ones except that the trapping oscillations are deformed drastically following the second maximum, in the case of the traveling wave tube.

Winfrey and Dunlop[10] have extended the single wave model to the model of fifteen waves and studied the linear and nonlinear self-consistent evolutions with the beam and a non-resonant plasma. They showed that, for large values of the beam strength s , a single wave no longer dominated the nonlinear evolution but a broadband growth occurred, and concluded that the mechanism of the growth might be a secondary beam-plasma instability caused by the trapped electron distribution in the nonlinear beam.

We also obtain the same results as Winfrey and Dunlop[10] do in many wave system(see Fig.2.15). However, the effects of neighboring waves for the destruction of amplitude oscillations depend on not only s but also the beam velocity. As is shown in Fig.2.12,

under the constant s , regular amplitude oscillations appear with increasing v_b/v_t . As is mentioned in the previous section, a frequency shift $(\omega' - \omega_0)/\omega_p$ is proportional to $s(v_t/v_b)^2$. Then, it becomes small with increasing v_b/v_t under the constant s . Therefore, the instability of the monochromatic wave becomes significant and the persistent amplitude oscillation occurs. In a particle simulation[13], a broadband growth did not occur under the conditions: $s = 0.25$, $n_b/n_p = 10^{-3}$ and $v_b/v_t = 9.9$. From these results, in real experiments with a very weakly collisional plasma, i.e. $v/\omega_p \cong 10^{-4}$, persistent amplitude oscillations may be observed when v_b/v_t is considerably large.

Destruction due to many wave effects was also discussed by Matsiborko et al.[8]. They found that multiwave behavior depended on parameter δ , given by

$$\delta \equiv \frac{\omega - \omega_0}{\omega_0} \left(\frac{n_b}{n_p} \frac{v_g^2}{v_b^2} \right)^{-1/3} = \frac{2^{-1/3}}{3} \frac{\Delta\omega}{s\omega_0} \frac{v_b^2}{v_t^2},$$

where we use the relation that group velocity $v_g = 3v_t^2/v_b$. The last equation is the representation of δ in our notation. For constant $\Delta\omega/\omega_0$ and s , varying δ is equivalent to varying v_b/v_t . Figure 8 in their paper showed that a regular amplitude oscillation was destroyed with increasing δ , namely with increasing v_b/v_t . So, their numerical results have opposite features to our results. It is very difficult to understand the character of δ dependence. But, we cannot compare their results with ours further because there is no detailed description on the dependence of δ .

So far, we have shown that the weak collisions, the order of $v/\omega_0 \cong 10^{-3}$, cause overdamping of the amplitude at the first mini-

mum point, the associated destruction of the amplitude oscillation and modification of the phase-space loci of the beam electrons. As an effective collision frequency, we may adopt the reciprocal of the slowing down time associated with Coulomb collisions in a dense plasma. These collisions are too small to alter the properties in the linear stage of the beam-plasma interaction. Plasma electrons support the unstable waves in a linear dielectric medium. By analogy to an electric circuit, collisions between plasma electrons act as a phase shifter. According to the spatial evolution of $\text{Re}(\delta k/sk_0)$, the wave undergoes a rapid phase shift as well as a strong damping of the wave amplitude near the first minimum shown in Fig.2.4. Then, the behavior of the wave amplitude and beam electrons in the nonlinear stage can be changed strongly from ones expected by the collisionless single wave model[3,4].

In the present work, only collisional effects of plasma electrons are considered. Since the beam velocity is much greater than the plasma thermal velocity, i.e. $v_b/v_t = 9.9$ or so, collisional effects of beam electrons with plasma electrons are neglected. When plasma is generated by the injected beams, however, electron-neutral collisions of beam electrons are important in the nonlinear stage. For the destruction of amplitude oscillations, these collisional effects may also have other effects which are different from ones considered here. On the other hand, some weakly nonlinear processes (such as parametric instabilities) may result in an effective damping of the saturated wave even if collisions are negligibly small[9,18].

In the case of the interaction of a pre-modulated beam with

a plasma, waves can be excited with detuning[6,9]. Although our present model includes the effect of detuning, i.e. the coefficient A' in eq.(2-19a), numerical calculations on the detuning effect and the combination effect between detuning and weak collisions for the destruction of amplitude oscillations are not obtained systematically. We calculate only some cases of the wave evolutions with detuning by using the single wave model. The range of detuning is as follows: $|\delta\omega/\omega_0| < 0.01$. Generally speaking, our results have the same tendency as ones by Matsiborko et al.[6]. Saturation level of the wave amplitude depends on the sign of $\delta\omega$. For a negative $\delta\omega$, detuning leads to a decrease of the saturation amplitude comparing with the one without detuning. On the other hand, for a positive $\delta\omega$, detuning leads to an increase of the saturation amplitude. These trends are well explained by the slowing down of the injected beams. Namely, with decreasing v_b , the frequency of the most unstable mode shifts to the higher side, and the excitation condition of the wave with given frequency ω having a positive $\delta\omega$ becomes better.

2.5 Concluding Remarks

We have studied numerically the destruction of amplitude oscillations in a small cold beam-weakly collisional plasma system. Collisional effects are studied in detail by using the extended single wave model. Neighboring wave effects, and competition of collisional and neighboring wave effects are also studied by using the many wave model with collisions of plasma electrons. It is found

that the regular amplitude oscillations can be destroyed by the weak collisions or by the growth of the neighboring waves. In both cases, the particle phase mixing is evident in the phase-space plots when the amplitude oscillations are destroyed.

Collisions are too weak, namely the order of $\nu/\omega_0 = (1-6) \times 10^{-3}$, to change appreciably the linear stage of the interaction of beams with background plasma. After the wave saturation, however, the wave damps strongly to the first minimum and fails to regrow to the initial saturation level. At the same time, the wave undergoes the rapid phase shift at the first minimum. As a result, regular amplitude oscillations are destroyed and the wave power decreases to a lower level.

Nonlinear growth of the neighboring waves also leads to the destruction of amplitude oscillations due to phase modulation between the main wave and the trapped beam electrons. After the destruction of amplitude oscillations, total power does not decrease to a lower level as in the collisional case.

Destruction due to collisions involves the main wave only. Excitation of neighboring waves depends on plasma conditions, i.e. the beam strength s . We have also examined the competition between collisional and neighboring wave effects for the destruction of the amplitude oscillation. It is found that these two effects are competitive for the destruction. Either of them dominantly destroy the phase relation between the main wave and the beam electrons to cause the destruction of the regular amplitude oscillations. When the beam is weak, i.e. small s , collisions destroy the amplitude oscillation and suppress the growth of the neighboring waves. When the beam is strong, i.e. large s , neighboring

waves grow easily and destroy the amplitude oscillation. The salient features due to collisions are masked by the neighboring wave effects except that the total wave power decreases to a lower level in the posterior region.

We finally note that excitation of neighboring wave depends on not only s but also beam velocity. With increasing beam velocity under the constant s , the system becomes stable for the excitation of the neighboring waves. Therefore, collisional effects become important with increasing beam velocity, even if the beam is strong.

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Chapter 3

PARTIAL SIMULATION ON THE INTERACTION BETWEEN A MODULATED ELECTRON BEAM AND A PLASMA

3.1 Introduction

Nonlinear phenomena of plasma waves are of interest in plasma physics. One of them is the suppression or the spectrum control of plasma instabilities. Several suppression methods, applicable to a variety of instabilities, have been described in the literatures. Among them, for example, we refer to the spectrum control of beam excited instabilities by an initial beam modulation[1-3], the suppression of ion-acoustic waves by externally applied ac signals[4,5], and the suppression of beam-excited electron waves by beam density modulation[6-10].

Experimental results concerning wave suppressions were discussed phenomenologically by the solution of the Van der Pol equation with external force term[4-6,8-10]. This idea was also applied to the suppression of drift waves by external ac signals[11] and the feedback stabilization of ion-acoustic waves[12,13]. However, the basic process of the suppression is still an open question because the physical meaning of the nonlinear terms, being essential to the Van der Pol equation, is not clarified in the plasma. About this point, details are given in Chapter 4.

On the other hand, we discussed previously the suppression

mechanism of beam-excited multimode electron waves from a viewpoint of particle picture[14]. Recently, we have observed the deformation of beam distribution function in relation to the suppression of the electron waves in the bounded system[15]. Following this work, relationship between the wave suppression and velocity distribution function has been studied by several authors[5,16-20]. These experimental results show that nonlinear dynamics of beam particles play an important role for wave suppression.

In conjunction with the nonlinear waves, recent works on cold beam-plasma instabilities show that the trapping of beam particles in the wave potential is the dominant nonlinear saturation process[21,22]. The behavior of small wave propagating in the presence of a large amplitude plasma wave has been studied by many authors. Among them, Carr et al.[23] studied nonlinear mode competition in beam-plasma instability. They show that the saturation of small amplitude test wave is completely determined by the nonlinear behavior of the large amplitude wave, which saturates as a results of single-wave beam trapping[21,22]. DeNeef et al.[24] studied also the nonlinear coupling between two launched waves of nearly equal frequencies. The main wave modified the wave pattern of the test wave and slightly suppressed the test wave. It is considered that, in the context of the single-wave theory, the launched main wave has grown, trapped the beam, and consequently modified the dispersion relation of the other unstable wave at neighboring frequency.

In this chapter, suppression mechanism of beam-excited electron waves including nonlinear beam dynamics are studied numerically. For this purpose, the method of partial simulation[25-28]

for many wave system is used. In the case of the semi-infinite system, i.e. the unbounded system, spectrum control due to pre-modulation of injected beams or two wave interaction in an electron beam-plasma system were performed by some authors[24,25]. The instability is convective and the external wave is applied in the linear regime. However, most experiments on wave suppression have been performed in the bounded system[4-9,16-18,20]. The instability resembles an absolute instability and the amplitude of the wave is saturated before the application of the external wave. Therefore, it appears that the numerical model for the bounded case is different from the one for the unbounded case.

The objective of this chapter is to show numerical results concerning wave suppression in both cases. Particular attention is paid to the influence of the finite geometry for beam particles.

3.2 Spatially Growing Waves

3.2-1 Numerical Model and Basic Equations

In order to study the pre-modulated beam-plasma interaction in the semi-infinite space, we need to consider the spatially growing waves. The system of five modes is considered, where two modes are located on each side of the most unstable wave with frequency ω_0 . The intensity of the wave electric field, $E(x,t)$ with a stationary amplitude $E(0)$, may be written as

$$E(x,t) = \sum_{\omega} E_{\omega}(x) \exp(-i\omega t) + \text{C.C.} ,$$

where $E_\omega(x) = E_\omega(0) \exp(i \int_0^x k_\omega(x') dx')$. Basic equations are the same as ones derived in Chapter 2. Dimensionless variables are as follows: The phase-space coordinate of the j -th beam electron $\xi_j = \omega_0 \delta t_j = \omega_0 [t_j(x) - x/v_b]$, the spatial coordinate $\eta = sx(\omega_0/v_b)$ and the normalized electric field $E_\omega(\eta) = eE_\omega(x) \exp(-ix\omega/v_b) / (mv_b \omega_0 s^2)$, where the spatial scaling factor $s = [(n_b v_b^2) / (6n_e v_t^2)]^{1/3}$ and ω_0 is the frequency of the most unstable wave, i.e. $\omega_0^2 = \omega_p^2 (1 + 3v_t^2/v_b^2)$. In terms of these dimensionless variables, a set of equations becomes

$$(A + iB + iC \frac{d}{d\eta} + D \frac{d^2}{d\eta^2}) E_\omega(\eta) = \frac{i}{N} \sum_j \exp(i\omega \xi_j / \omega_0) , \quad (3-1)$$

$$\frac{d^2 \xi_j}{d\eta^2} = (1 + s \frac{d\xi_j}{d\eta})^3 (\sum_\omega E_\omega(\eta) \exp(-i\omega \xi_j / \omega_0) + C.C.) , \quad (3-2)$$

where

$$A = s^2 \frac{\omega_0^2}{\omega_b^2} [(1 + \frac{\delta\omega}{\omega_0}) - (1 + \frac{\delta\omega}{\omega_0})^{-1}] ,$$

$$B = s^2 \frac{v}{\omega_0} \frac{\omega_0^2}{\omega_b^2} (1 + \frac{\delta\omega}{\omega_0})^{-2} ,$$

$$C = (1 + \frac{\delta\omega}{\omega_0})^{-2} \quad \text{and} \quad D = \frac{s}{2} (1 + \frac{\delta\omega}{\omega_0})^{-3} .$$

The quantity $\delta\omega$ is the detuning, namely $\delta\omega = \omega - \omega_0 = n\Delta\omega$. The quantity $\Delta\omega$ is mode-spacing and the integer $n = 0, \pm 1$ and ± 2 in the present system.

3.2-2 Method of Numerical Calculation

The set of differential equations for $E_\omega(\eta)$ and $\xi_j(\eta)$ is solved for N beam electrons which satisfy the following initial conditions,

$$\xi_j(0) = -2\pi\lambda(j/N) \quad \text{and} \quad \dot{\xi}_j(0) = 0, \quad (3-3)$$

where λ is the reciprocal of the normalized mode-spacing, namely $\lambda = \omega_0/\Delta\omega$. In order to obtain the frequency dependence of beam modulation, we must change the value of the mode spacing $\Delta\omega/\omega_0$. As a closer mode spacing corresponds to large λ , a large number of beam electrons would be required. In the present case, we choose the mode-spacing as follows: $\Delta\omega/\omega_0 = 0.04, 0.05, 0.08$ and 0.1 . Various runs are made for 100, 200 and 600 beam electrons, and there are no significant differences among these three cases. In all cases, the initial values of wave fields $E_\omega(0)$ is kept constant value, i.e. $E_\omega(0) = 10^{-3}$.

Usually, effects of beam modulation are introduced by changing the initial amplitudes of a certain wave field $E_\omega(0)$ whose frequency is equal to a modulation frequency[24,25]. In the present simulation, however, we introduce the effects of the pre-modulation of beam electrons into $\xi_j(0)$ as follows:

$$\xi_j(0) = -2\pi\lambda(j/N) \left(1 + \sum_{\omega} MD(\omega) \sin[2\pi\lambda(j/N)(\omega/\omega_0)] \right), \quad (3-4)$$

where $MD(\omega)$ represents the modulation depth at frequency ω . While the initial values of all wave fields $E_\omega(0)$ are the same values, i.e. the noise level.

3.2-3 Numerical Results

A typical example of numerical solutions is shown in Fig.3.1. Plasma parameters and numerical conditions are as follows: $n_b/n_e = 10^{-2}$, $v_b/v_t = 3$, $N = 200$, $E_\omega(0) = 10^{-3}$ and $\Delta\omega/\omega_0 = 5 \times 10^{-2}$. The spatial evolution of the electric fields of three of five modes of the system is displayed. These are three most predominant modes. Total wave energy of the system is indicated by E^2 . In Fig.3.1(a), beam electrons are injected without pre-modulation. In Figs.3.1(b)-3.1(c), beam electrons are modulated by one frequency ω_0 . In Figs.3.1(d)-3.1(f), beam electrons are modulated by two frequencies, i.e. ω_0 and $\omega_0 + \Delta\omega$.

In Chapter 2, we give the details on the evolution of the many waves in the absence of beam modulation. Nonlinear evolution of the many wave system depends strongly on the parameter s and beam velocity [27,28].

For the strong beam considered here, i.e. large s , the most unstable wave $E(\omega_0)$ no longer dominates the evolution of the system during a linear growth region. Then, in the nonlinear stage, the regular amplitude oscillations of the total wave power is destroyed due to many wave effects (see Fig.3.1(a)) [25-28]. However, when beam electrons are modulated by ω_0 , $E(\omega_0)$ grows distinguishably and predominates the evolution of the system. The position, at which $E(\omega_0)$ starts to grow or the amplitude of $E(\omega_0)$ saturates, shifts towards the entrance of the system as the modulation depth $MD(\omega_0)$ is increased. Furthermore, one amplitude oscillation appears clearly. These trends show the possibility that wave spectrum can be controlled by the pre-modulation of the injected beam.

Figures 3.1(d)-3.1(f) show the interaction between two waves,

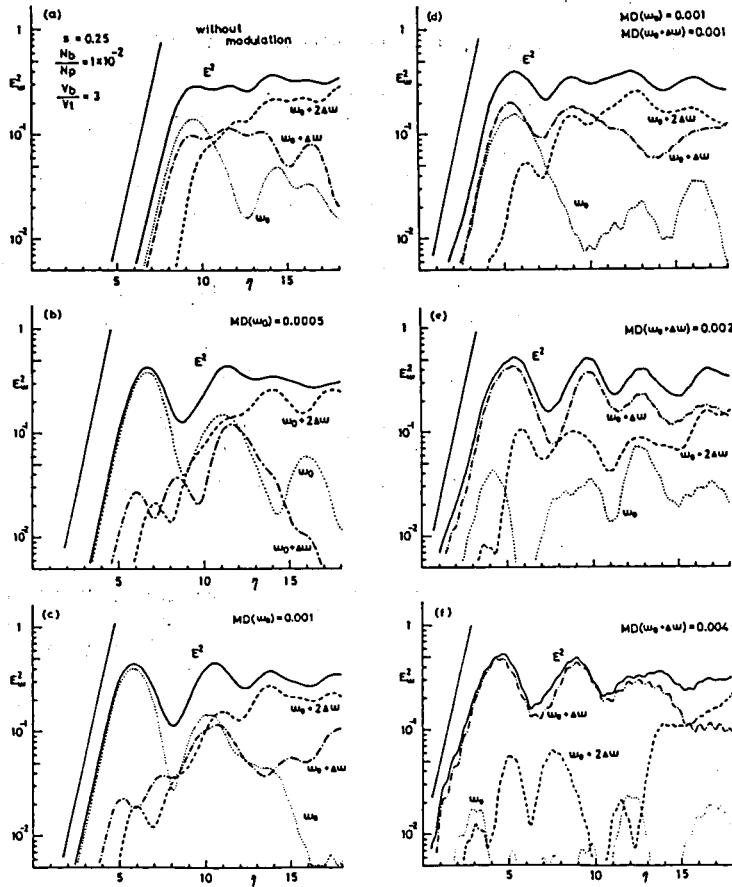


Fig.3.1 The total wave power $E^2(\eta)$ and the power in the spectral components vs η in the modulated beam-plasma system. The solid lines in the figure correspond to the theoretical linear growth. In (a), eqs. (3-1) and (3-2) are solved for 200 beam electrons which satisfy the initial conditions, i.e. eq.(3-3). A pre-modulation of the beam electrons are introduced into $\xi_j(0)$ as expressed in eq.(3-14). Beam electrons are modulated by one frequency ω_0 in (b) and (c). The modulation depth $MD(\omega_0)$ is 0.0005 in (b) and 0.001 in (c). In (d)-(f), beam electrons are modulated by two frequencies, i.e. ω_0 and $\omega_0 + \Delta\omega$. $MD(\omega_0 + \Delta\omega)$ is 0.001 in (d), 0.002 in (e) and 0.004 in (f), respectively, while $MD(\omega_0) = 0.001$ in all cases. Plasma parameters and other numerical conditions are $n_b/n_e = 0.01$, $v_b/v_{te} = 3$, $E_\omega(0) = 0.001$ and $\Delta\omega/\omega_0 = 0.05$.

i.e. $E(\omega_0)$ and $E(\omega_0 + \Delta\omega)$, where beam electrons are pre-modulated by two frequencies, ω_0 and $\omega_0 + \Delta\omega$. With increasing $MD(\omega_0 + \Delta\omega)$, $E(\omega_0 + \Delta\omega)$ increases its amplitude and suppresses $E(\omega_0)$. In this case, instead of $E(\omega_0)$, $E(\omega_0 + \Delta\omega)$ predominates the evolution of the system. Then, two amplitude oscillations of E^2 reappear when $MD(\omega_0 + \Delta\omega) = 0.004$.

In order to clarify the relationship between $E(\omega_0)$ and $E(\omega_0 + \Delta\omega)$, relative intensities at two different positions, i.e. $\eta = 4$ and 4.5, are plotted vs $MD(\omega_0 + \Delta\omega)$ in Fig.3.2. Wave amplitudes are normalized by the amplitude of $E(\omega_0)$ without modulation at each position. It is shown clearly that with increasing $MD(\omega_0 + \Delta\omega)$, $E(\omega_0)$ reduces its amplitude and $E(\omega_0 + \Delta\omega)$ becomes large. However, it is remarkable that $E(\omega_0)$ does not damp until the amplitude of $E(\omega_0 + \Delta\omega)$ becomes much larger than that of $E(\omega_0)$ in the case of no modulation.

Phase-space dynamics and phase averaged velocity distribution of beam electrons are shown in Figs.3.3 and 3.4, respectively. They correspond to the calculations in Figs.3.1(c)-(f). Dynamics of beam electrons are going to be modified when the amplitude of $E(\omega_0 + \Delta\omega)$ becomes comparable with that of $E(\omega_0)$ (see Fig.3.3(b)). With increasing the amplitude of $E(\omega_0 + \Delta\omega)$ further, the particle phase mixing becomes evident in the phase-space plot. Correspondingly, as is shown in Fig.3.4, the phase averaged beam distribution function is no longer peaked but broad.

From these results shown in Figs.3.2-3.4, we present a picture for the coupling process between two waves. The two waves do not interact directly, but through a wave-particle-wave process. One wave $E(\omega_0)$ does not damp easily until the other wave $E(\omega_0 + \Delta\omega)$ mod-

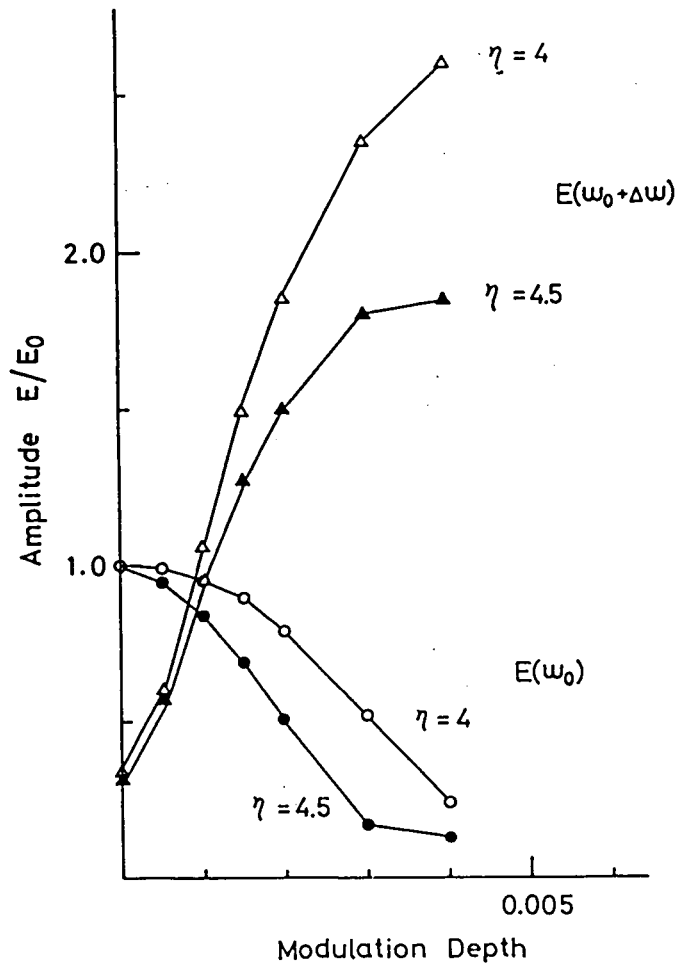


Fig.3.2 Relative intensities of two waves at different two fixed points vs $MD(\omega_0 + \Delta\omega)$. Wave amplitudes are normalized by the amplitude of $E(\omega_0)$ at the same point when $MD(\omega_0 + \Delta\omega) = 0$. Plasma parameters and conditions for calculations are the same as in Fig.3.1. Open and dotted symbols represent the amplitudes at $\eta = 4$ and $\eta = 4.5$, respectively.

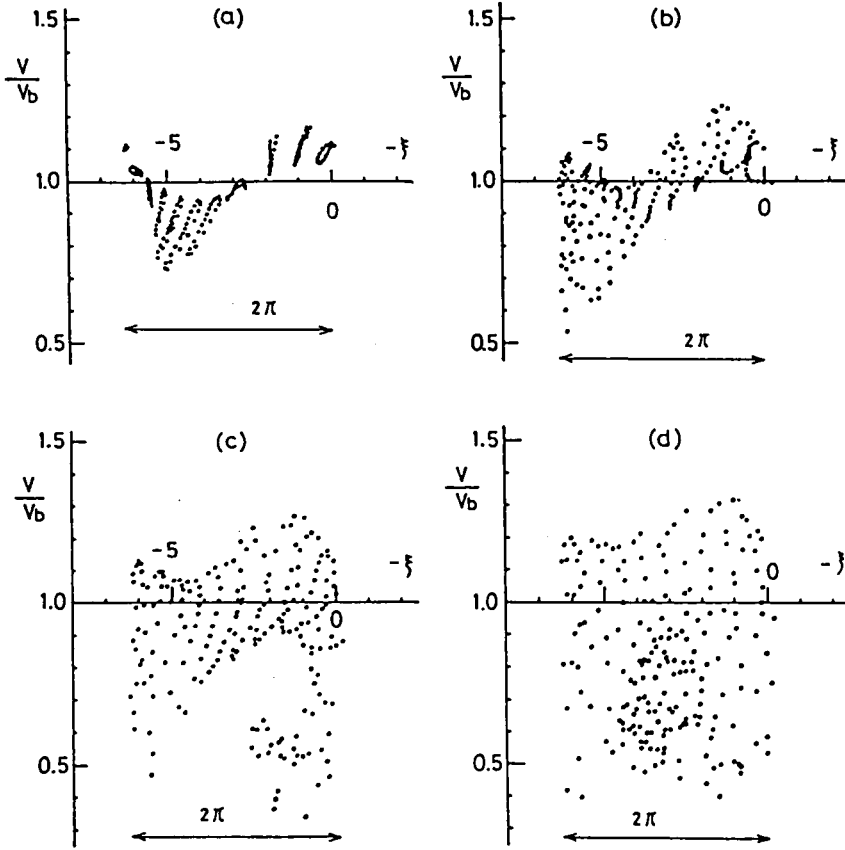


Fig.3.3 Phase-space behaviors of beam electrons at the point $\eta = 4$. These correspond to the calculations shown in Figs.1(c)-1(f). $MD(\omega_0) = 0.001$ in all cases. Parameters are the values of $MD(\omega_0 + \Delta\omega)$: (a) $MD(\omega_0 + \Delta\omega) = 0$, (b) 0.001, (c) 0.002 and (d) 0.004. As $\omega_0/\Delta\omega = 20$ in this case, we need to follow the particles whose initial phases are in the interval $-40\pi \leq \xi_j(0) \leq 0$. In this figure, however, instantaneous particle phase $\xi_j(\eta)$ is transformed in the interval $-2\pi \leq \xi_j(\eta) \leq 0$.

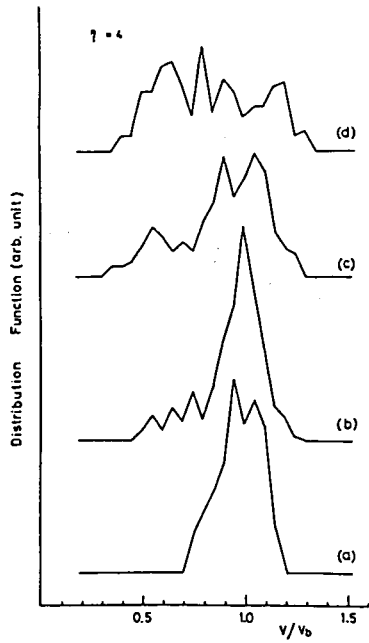


Fig.3.4 Phase averaged velocity distribution vs velocity normalized to the initial velocity v_b . They are corresponding to the phase space plots in Fig.3.3.

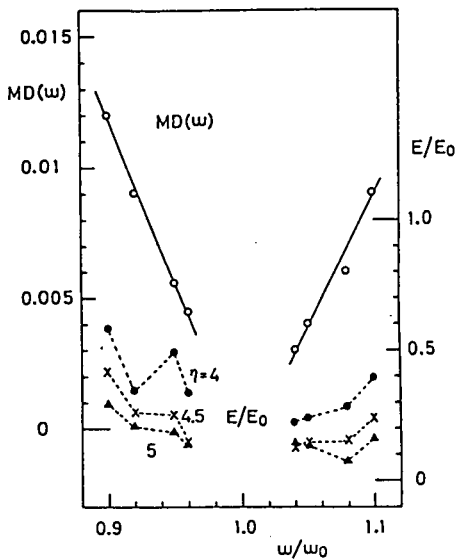


Fig.3.5 Relationship between the $MD(\omega)$ and the modulation frequency ω/ω_0 . Open circles are values of $MD(\omega)$ at each modulation frequency. Reduced amplitudes of the wave $E(\omega_0)$, at three different points, are indicated by the symbols, i.e. dotted circles ($\eta=4$), crosses ($\eta=4.5$) and dotted triangles ($\eta=5$), respectively. Amplitudes are normalized by $E(\omega_0)$ at the same point when $MD(\omega) = 0$.

ifies the beam distribution, although $E(\omega_0 + \Delta\omega)$ becomes comparable to $E(\omega_0)$ with increasing $MD(\omega_0 + \Delta\omega)$. The deformation process of beam distribution function is based on the behaviors of beam electrons in the presence of two waves. When $E(\omega_0 + \Delta\omega)$ becomes larger further, the main contribution to the beam dynamics is due to $E(\omega_0 + \Delta\omega)$. Then, $E(\omega_0)$ may not grow in this modified beam-plasma system and be suppressed.

Effects of $E(\omega_0 + \Delta\omega)$ for the suppression of $E(\omega_0)$ depend on not only the intensity of $MD(\omega_0 + \Delta\omega)$ but also the frequency difference between two waves, i.e. the magnitude of $\Delta\omega$. If we want to reduce the amplitude of $E(\omega_0)$ to a certain lower level at a fixed position, $MD(\omega_0 + \Delta\omega)$ becomes large with increasing $\Delta\omega$. This is shown in Fig. 3.5. Dotted circles, crosses and dotted triangles to the reduced amplitudes of $E(\omega_0)$ at each point when beams are pre-modulated at various frequencies $\omega_0 + \Delta\omega$ with the modulation depth indicated by open circles. According to this, in order to reduce $E(\omega_0)$ to a certain level, critical modulation depth becomes large linearly as $\Delta\omega$ increases. We do similar calculations by using the system of eleven modes. Numerical results have the same features as the above-described ones.

3.2-4 Discussion

Concerning the control of wave spectrum, numerical results show that broad band spectra of instabilities are controlled by a pre-modulation of the injected beams. On the other hand, about nonlinear interaction between two waves, especially suppression one wave due to the other wave, numerical results are well in a-

agreement with experimental ones in the unbounded system[23,24]. Deformation of the beam distribution function due to a large amplitude wave plays an important role for the evolution of the other wave. Two waves do not interact directly, but through a wave-particle-wave process. The above deformation process may be also important for the wave suppression in the bounded system[16,20].

Spectrum control or nonlinear interaction between two waves in a beam-plasma system was studied by some authors[24,25]. We discuss the differences between our results and their ones.

Matsiborko et al.[25] studied the interaction of the perturbations in the modulated beam-plasma system. They consider the nonlinear dynamics of these unstable perturbations, i.e. the fundamental wave with modulation frequency ω_0 and two test waves with frequencies $\omega_0 \pm \delta\omega$, where $\delta\omega$ is the frequency difference between the fundamental and the test waves. They assume that ω_0 coincides with the resonant plasma frequency ω_p , and that owing to the modulation, the initial amplitude of the fundamental wave substantially exceeds the initial amplitudes of the test waves. Under these conditions, the test perturbations are suppressed and a monochromatic wave of frequency ω_0 is excited at any value of the "detuning" δ . δ is a dimensionless frequency difference, i.e.

$$\delta = \frac{\delta\omega}{\omega_0} \left(\frac{n_e v_b^2}{n_b v_g^2} \right)^{1/3} = \frac{2^{-1/3}}{3} \frac{\delta\omega}{s\omega_0} \frac{v_b^2}{v_t^2},$$

where the group velocity $v_g = 3v_t^2/v_b$. For a larger difference between the initial amplitude of the fundamental and test waves, the distance over which the wave excited by the beam modulation remains monochromatic can be considerably increased. As a whole,

their results have the same feature as our results. But, we cannot understand the dependence of beam modulation on δ . They present the results that, with increasing the value of δ under the constant strength of modulation, the distance where the wave with ω_0 remains monochromatic is decreased. It means, according to our simulation model, that the effect of beam modulation becomes strong with increasing the frequency difference between the frequency of the electron wave and the modulation frequency. So, their results have opposite features to our results shown in Fig. 3.5 or experimental results[16,20].

DeNeef et al.[24] studied the nonlinear behavior of two launched waves of nearly equal frequencies on a beam-plasma system. They observed that the main wave with large amplitude modified the wave pattern of the test wave and slightly suppressed it. It was considered that the mechanism was based on a modification of dispersion relation by a change of beam distribution function. However, its measurement was not disclosed. In order to explain the experimental results, they calculate the behavior of the test wave by regarding it as a slow modulation of the main wave's amplitude and phase, i.e. a modulational calculation. This method assumes that the nonlinear behavior of one wave, launched alone, is known either from an experiment or a calculation. It means that the trapped particle distribution is not modified by other instabilities. In addition to this, there is a restriction on frequency difference $\Delta\omega$ between two waves. So, we cannot use this model to study the deformation of distribution function in the presence of two waves and/or to study the frequency dependence of beam modulation.

3.3 Temporally Growing Waves

3.3-1 Numerical Model and Basic Equations

In this section, we develop a simple model in order to study the wave suppression in a bounded electron beam-plasma system[20]. Experimentally, plasma parameters can be controlled so that a single mode with narrow frequency spectrum is excited spontaneously. The externally generated wave due to beam modulation also has narrow spectrum. It is also found experimentally that in the course of wave suppression only the electron wave and the external wave predominate the nonlinear beam-plasma interaction. For simplicity, only two waves are considered in the following discussions. Besides, the electron wave excited spontaneously is in the saturated state before the application of the external wave. Therefore, we develop a model in a spatially homogeneous beam-plasma system, where waves grow temporally and nonlinearities evolve with time.

The model simulates a one-dimensional fixed plasma with the system length L , through which a continuous stream of beam electrons(injected at the left end of the system, $x = 0$) flows. According to experimental conditions, two simplified boundary conditions at the right end of the system, $x = L$, are imposed on beam electrons. The one is the case of absorption(the Case I). Beam electrons which drift out the right end are removed without contributing to the electric field. The other is the case of reflection(the Case II). Beam particles which drift out the right end are reentered into the system with condition $x_j = 2L - x_j$, and the velocity $v_j = -v_j$ of the j -th beam particle. These reflected

beams are removed when they drift out the left end. In the model considered here, the ions are stationary and plasma electrons are described by linearized continuity and momentum equations, neglecting thermal motion. Trajectories of beam particles are calculated numerically. These beam particles must satisfy the continuity equation and are described in terms of its distribution function $f_b(x, v, t)$ as follows:

$$f_b(x, v, t) = \frac{n_b L}{N} \sum_j \delta(x - x_j) \delta(v - v_j) , \quad (3-5)$$

$$\rho_b(x, t) = -e \int f_b dv = - \frac{en_b L}{N} \sum_j \delta(x - x_j) , \quad (3-6)$$

and

$$\frac{\partial \rho_b}{\partial t} + \frac{\partial (\rho_b v)}{\partial x} = 0 . \quad (3-7)$$

Effect of beam modulation is introduced into Poisson's equation in the form of an externally oscillating density ρ_m as follows

$$\frac{\partial E}{\partial x} = -4\pi en + 4\pi \rho_b + 4\pi \rho_m , \quad (3-8)$$

$$\text{and } \rho_m = \rho_0 \cos(\omega_m t) [\delta(x - x_0 + \Delta X/2) - \delta(x - x_0 - \Delta X/2)] , \quad (3-9)$$

where n is the density perturbation of plasma electron, ρ_b is the charge density of the beam electron, ρ_0 is an external charge density, ω_m is a modulation frequency and a pair grid with spacing ΔX is located at x_0 which is the position near the left end.

Differentiating eq.(3-8) twice with respect to time and sub-

stituting the expression for n , ρ_b and ρ_m obtained from the linearized fluid equations, from the equation of continuity for the beam electrons and from eq.(3-9), eq.(3-8) becomes

$$\frac{\partial}{\partial x} \left(\frac{\partial^2 E}{\partial t^2} + \omega_p^2 E + 4\pi \frac{\partial J_b}{\partial t} \right) = -4\pi \omega_m^2 \rho_m, \quad (3-10)$$

where $\omega_p = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency and J_b is the beam electron current. After Fourier transformation with respect to the space coordinate, eq.(3-10) takes the form

$$\begin{aligned} \frac{d^2 E_k}{dt^2} + \omega_p^2 E_k = & -\frac{4\pi}{L} \frac{d}{dt} \left(\int_0^L J_b e^{-ikx} dx \right) \\ & - \frac{4\pi \omega_m^2}{kL} \rho_0 \exp(-ikx_0) [\exp(i\omega_m t) + \exp(-i\omega_m t)] \sin\left(\frac{k\Delta X}{2}\right), \end{aligned} \quad (3-11)$$

where

$$J_b = -e\rho_b v = -e \int v f_b dv = -\frac{en_b L}{N} \sum_j v_j \delta(x-x_j). \quad (3-12)$$

Substituting eq.(3-12) into J_b in eq.(3-11), we obtain

$$\begin{aligned} \frac{d^2 E_k}{dt^2} + \omega_p^2 E_k = & \frac{4\pi en_b}{N} \frac{d}{dt} \left[\sum_j v_j \exp(-ikx_j) \right] \\ & - \frac{4\pi \omega_m^2}{kL} \rho_0 \exp(-ikx_0) [\exp(i\omega_m t) + \exp(-i\omega_m t)] \sin\left(\frac{k\Delta X}{2}\right). \end{aligned} \quad (3-13)$$

More complete fluid models of the plasma electrons, including pressure forces and momentum transfer collisions, may be introduced by using the following equation of motion,

$$m \frac{\partial v}{\partial t} = -eE - mvv - \frac{3kT_e}{n_e} \frac{\partial n}{\partial x}, \quad (3-14)$$

where ν is the momentum transfer collision frequency, κ is the Boltzmann constant and T_e is the electron temperature. In this case, eq.(3-13) is rewritten as follows:

$$\begin{aligned} \frac{d^2 E_k}{dt^2} + \nu \frac{dE_k}{dt} + \omega_k^2 E_k &= \frac{4\pi en_b}{N} \frac{d}{dt} \left[\sum_j v_j \exp(-ikx_j) \right] \\ &- \frac{4\pi\omega_m^2}{kL} \rho_0 \exp(-ikx_0) [\exp(i\omega_m t) + \exp(-i\omega_m t)] \sin\left(\frac{k\Delta X}{2}\right), \quad (3-15) \end{aligned}$$

where $\omega_k^2 = \omega_k^2 + 3k^2 v_t^2$. The excited wave E_k , due to beam modulation is also governed by the similar equation.

$$\begin{aligned} \frac{d^2 E_{k'}}{dt^2} + \nu \frac{dE_{k'}}{dt} + \omega_{k'}^2 E_{k'} &= \frac{4\pi en_b}{N} \frac{d}{dt} \left[\sum_j v_j \exp(-ik'x_j) \right] \\ &- \frac{4\pi\omega_m^2}{k'L} \rho_0 \exp(-ik'x_0) [\exp(i\omega_m t) + \exp(-i\omega_m t)] \sin\left(\frac{k'\Delta X}{2}\right). \quad (3-16) \end{aligned}$$

We assume that electric field can be described as $E_k = E_k(t) \exp(-i\omega_k t)$ and $E_{k'} = F_{k'}(t) \exp(-i\omega_m t)$, where $E_k(t)$ and $F_{k'}(t)$ are the slowly varying parts. We consider the case that ω_m is nearly equal to ω_k , i.e. $\omega_m = \omega_k + \delta\omega$ ($\delta\omega \ll \omega_k, \omega_m$), and assume that the wavenumber of the excited wave $E_{k'}$ is the same as that of E_k (namely, $k' = k$). Substituting E_k and $E_{k'}$ into eqs.(3-15) and (3-16), respectively, following two equations for slowly varying parts $E_k(t)$ and $F_{k'}(t)$ are obtained,

$$\begin{aligned} \frac{d^2 E_k(t)}{dt^2} + (\nu - i2\omega_k) \frac{dE_k(t)}{dt} - i\nu\omega_k E_k(t) &= \\ &+ \frac{4\pi en_b}{N} \left\{ \frac{d}{dt} \left[\sum_j v_j \exp(-ikx_j) \right] \right\} \exp(i\omega_k t) \end{aligned}$$

$$- \frac{4\pi\omega_m^2}{kL} \rho_0 \exp(-ikx_0) \left\{ \exp[i(\omega_m + \omega_k)t] + \exp[-i(\omega_m - \omega_k)t] \right\} \sin\left(\frac{k\Delta X}{2}\right), \quad (3-17)$$

$$\begin{aligned} \frac{d^2 F_k(t)}{dt^2} + (\nu - i2\omega_m) \frac{dF_k(t)}{dt} + [(\omega_k^2 - \omega_m^2) - i\nu\omega_m] F_k(t) = \\ + \frac{4\pi en_b}{N} \left\{ \frac{d}{dt} [\sum_j v_j \exp(-ikx_j)] \right\} \exp(i\omega_m t) \\ - \frac{4\pi\omega_m^2}{kL} \rho_0 \exp(-ikx_0) [\exp(i2\omega_m t) + 1] \sin\left(\frac{k\Delta X}{2}\right). \end{aligned} \quad (3-18)$$

Corresponding to these two equations, the j -th beam particle is governed by the following equation of motion,

$$\begin{aligned} \frac{d^2 x_j}{dt^2} = - \frac{e}{m} E_k(t) \exp(-i\omega_k t) [\exp(ikx_j) + R \exp(-ikx_j)] \\ - \frac{e}{m} F_k(t) \exp(-i\omega_m t) [\exp(ikx_j) + R \exp(-ikx_j)] \\ + \text{C.C.} \end{aligned} \quad (3-19)$$

Here, we introduce the effects of reflected waves supplementally. R is the reflection coefficient of the wave. Usually, real experiments show that electron waves have a nature of a standing wave in the bounded system. As the standing wave is decomposed into two traveling waves, equation of motion for the beam particle may be described in the form of eq.(3-19). Before an application of beam modulation, beam particles are governed by the single wave $E_k(t)$ in eq.(3-19).

For convenience of numerical calculation, eqs.(3-17)-(3-19)

are described in terms of the following normalized variables: The temporal coordinate $\tau = s\omega_k t$, the position of the j -th beam $\xi_j = kx_j$, the normalized electric field $E(\tau) = ekE_k(t)/(ms^2\omega_k^2)$, and the temporal scaling factor $s = (n_b/2n_e)^{1/3}$. Then, the set of equations to be solved is as follows:

$$\frac{d^2 E}{d\tau^2} + \frac{1}{s} \left(\frac{\nu}{\omega_k} - i2 \right) \frac{dE}{d\tau} - i \frac{\nu}{s^2 \omega_k} E = \frac{\omega_b^2}{s^2 \omega_k^2} \frac{1}{N} \left(\frac{d}{d\tau} [\Sigma \dot{\xi}_j \exp(-i\xi_j)] \right) \exp(j \frac{\tau}{s}) - \frac{n_m \omega_m \omega_b^2}{n_b s^2 \omega_k^4} \exp(-ikx_0) \left(\exp[i(\frac{\omega_m}{\omega_k} + 1) \frac{\tau}{s}] + \exp[-i(\frac{\omega_m}{\omega_k} - 1) \frac{\tau}{s}] \right) \sin(\frac{k\Delta X}{2}), \quad (3-20)$$

$$\frac{d^2 F}{d\tau^2} + \frac{1}{s} \left(\frac{\nu}{\omega_k} - i2 \frac{\omega_m}{\omega_k} \right) \frac{dF}{d\tau} + \frac{1}{s^2} \left[\left(1 - \frac{\omega_m^2}{\omega_k^2} \right) - i \frac{\nu \omega_m}{\omega_k^2} \right] F = \frac{\omega_b^2}{s^2 \omega_k^2} \frac{1}{N} \left(\frac{d}{d\tau} [\Sigma \dot{\xi}_j \exp(-i\xi_j)] \right) \exp(i \frac{\omega_m \tau}{\omega_k s}) - \frac{n_m \omega_m \omega_b^2}{n_b s^2 \omega_k^4} \exp(-ikx_0) \left[\exp(i2 \frac{\omega_m \tau}{\omega_k s}) + 1 \right] \sin(\frac{k\Delta X}{2}), \quad (3-21)$$

and

$$\frac{d^2 \xi_j}{d\tau^2} = -[E \exp(-i \frac{\tau}{s}) + F \exp(-i \frac{\omega_m \tau}{\omega_k s})] [\exp(i\xi_j) + R \exp(-i\xi_j)] + \text{C.C.}, \quad (3-22)$$

where ω_b is the electron plasma frequency of the beam, n_m/n_b is the modulation depth and N is the total number of beam particles distributed in the system at the initial stage.

3.3-2 Method of Numerical Calculation

A set of differential equations for $E(\tau)$, $F(\tau)$ and $\xi_j(\tau)$ is solved for N beam electrons. Initial conditions of beam electrons are as follows: In the Case I where beam electrons injected at the

left end are absorbed at the right end, N beam electrons having velocity v_b are distributed uniformly in the system, i.e. initial position of the j -th beam $x_j(0) = L(j/N)$. In the Case II where beam electrons are reflected at the right end, the half of N beam electrons having velocity v_b and the other half having velocity $-v_b$ are distributed uniformly in the system.

The initial value of the wave amplitude $E(0)$ is usually set to be 10^{-3} . On the other hand, the initial value of the excited wave $F(\tau)$ is set to be $0.01E(\tau)$. Various runs are made for 100, 200, 400 and 600 beam electrons which are the total number of beams at the initial stage. There are no significant differences among four cases.

Mean beam velocity $V(\tau)$ and the beam velocity spread ΔV_b are obtained as a function of τ . These values are defined as follows:

$$V(\tau) = \sum_j v_j(\tau)/M \quad \text{and} \quad (\Delta V_b)^2 = \sum_j [v_j(\tau) - V(\tau)]^2/M, \quad (3-23)$$

where M is the total number of beam particles at each time step in the system, and is not necessarily equal to the total number N at the initial stage. Instantaneous wave frequency, i.e. $\Omega = \delta\omega/(s\omega_k)$, is also calculated by the following relation

$$\Omega_r = (E_r dE_i/d\tau - E_i dE_r/d\tau)/|E|^2 \quad (3-24)$$

and

$$\Omega_i = -(E_r dE_r/d\tau + E_i dE_i/d\tau)/|E|^2 \quad (3-25)$$

where Ω_r and Ω_i are real and imaginary parts of Ω , E_r and E_i are real and imaginary parts of E , and $|E|$ is the absolute value of E .

3.3-3 Numerical Results

(A) Single Wave Behavior

First of all, we present some results of numerical calculations in the case of no modulation. Figure 3.6 shows evolutions of wave energy, a mean beam velocity $V(\tau)$ and a beam velocity spread ΔV_b in the Case I, respectively. The total wave energy E^2 means the summation of the forward wave energy $E_k^2(\omega_0)$ and the backward wave energy $R^2 E_k^2(\omega_0)$, i.e. $(1+R^2)E_k^2(\omega_0)$. Here, we denote the mode frequency ω_k by ω_0 . Parameters used for calculation are $n_b/n_e = 10^{-2}$, $v_b/v_t = 4$, $v/\omega_0 = 0$, $R = 1.0$, $N = 200$ and mode number $m = 4$, where wavenumber $k = 2\pi m/L$. The wave grows exponentially and saturates its amplitude near $\tau = 10$. After this time, the wave keeps its amplitude nearly constant and no amplitude oscillation occurs. As is shown in Fig.3.6(b), after $\tau = 10$, both $V(\tau)$ and ΔV_b are nearly constant corresponding to the evolution of wave energy. Figure 3.7 shows instantaneous phase-space plots corresponding to the case shown in Fig.3.6.

Figure 3.8 shows evolutions of wave energy, $V(\tau)$ and ΔV_b in the Case II. Conditions of numerical calculation are the same as in Fig.3.6. In this case, the wave saturates its amplitude near $\tau = 14$. After this, the wave also keeps the amplitude constant. Temporal evolutions of injected beams of both $V(\tau)$ and ΔV_b correspond to the evolution of the wave. But, evolutions of the reflected beams reach the steady state at a slightly later time, i.e. $\tau = 18$, and the value of ΔV_b is much larger than that of injected beams.

Figure 3.9 shows instantaneous phase-space plots corresponding to the case shown in Fig.3.8. Evolutions of the injected beams

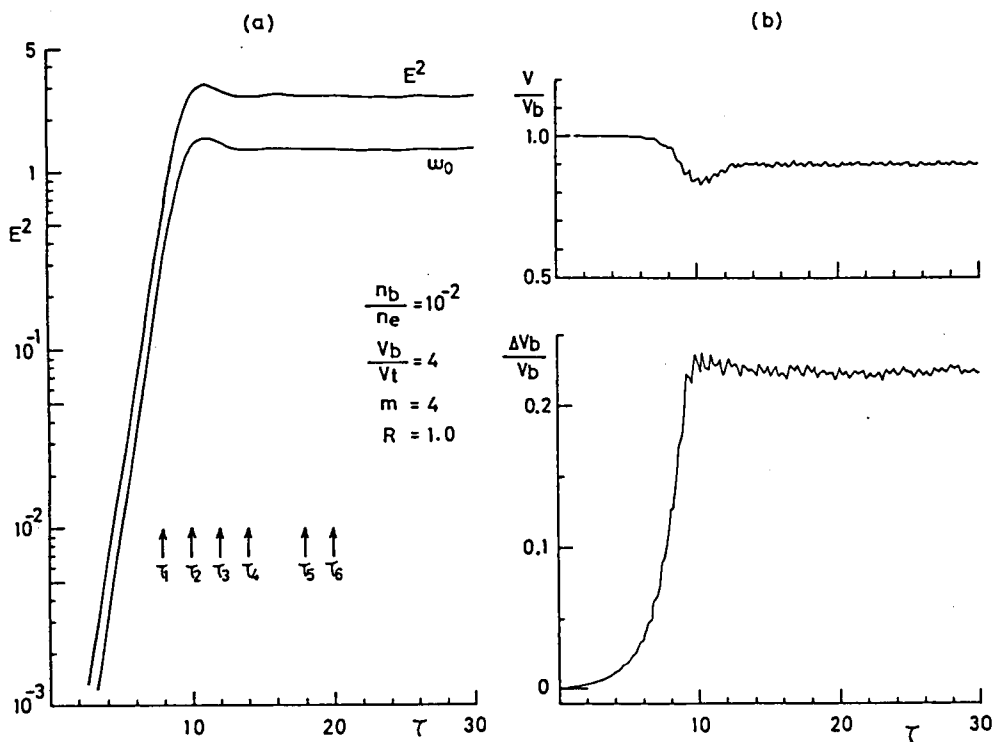


Fig.3.6 (a) Total wave power E^2 and the square of the wave amplitude ω_0 vs τ in the case that injected beams are absorbed at $x = L$.

(b) Instantaneous mean beam velocity V (upper) and the beam spread ΔV_b (lower) vs τ . They are normalized by the initial beam velocity v_b .

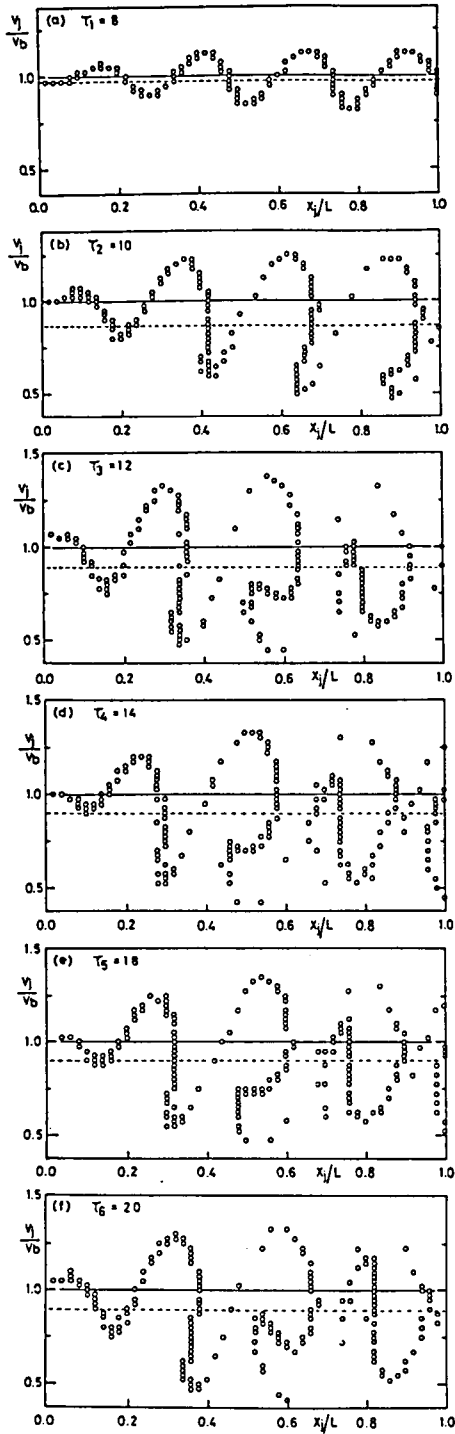


Fig.3.7 Instantaneous phase-space plots corresponding to six points denoted in Fig.3.6. Solid and dashed lines show the initial and the instantaneous mean beam velocities.

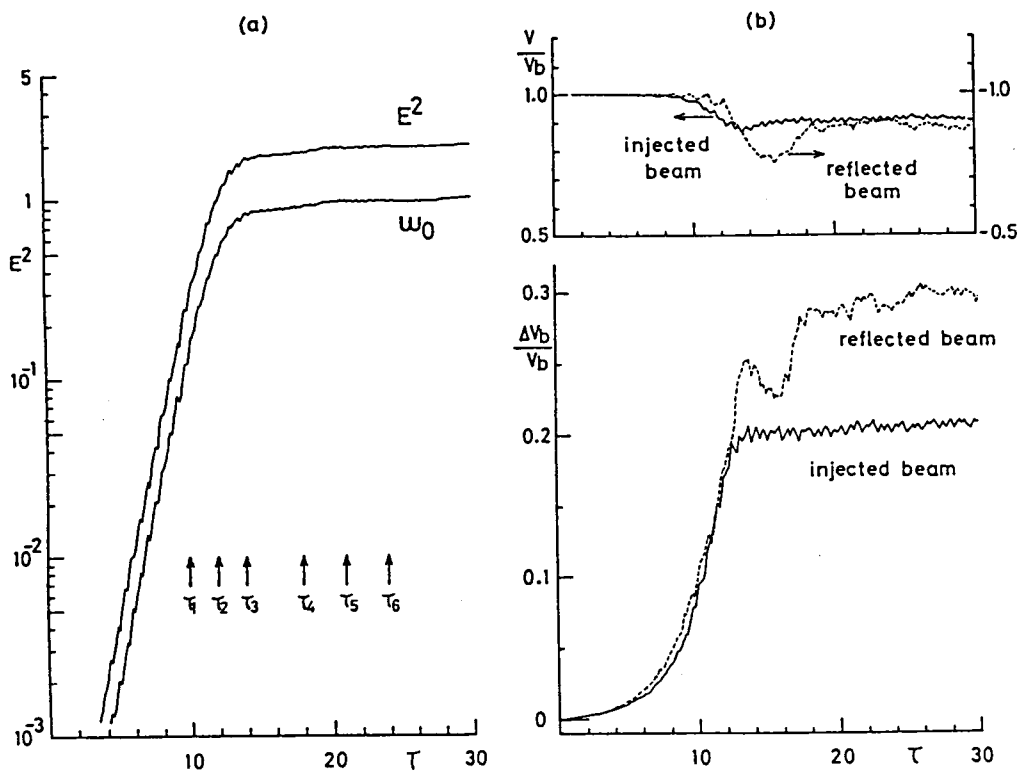


Fig.3.8 (a) Total wave power E^2 and the square of the wave amplitude ω_0 vs τ in the case that injected beams are reflected at $x = L$.

(b) Instantaneous mean beam velocity (upper) and the beam spread (lower) vs τ . Solid and dashed lines show the evolutions for injected beams and for reflected beams, respectively.

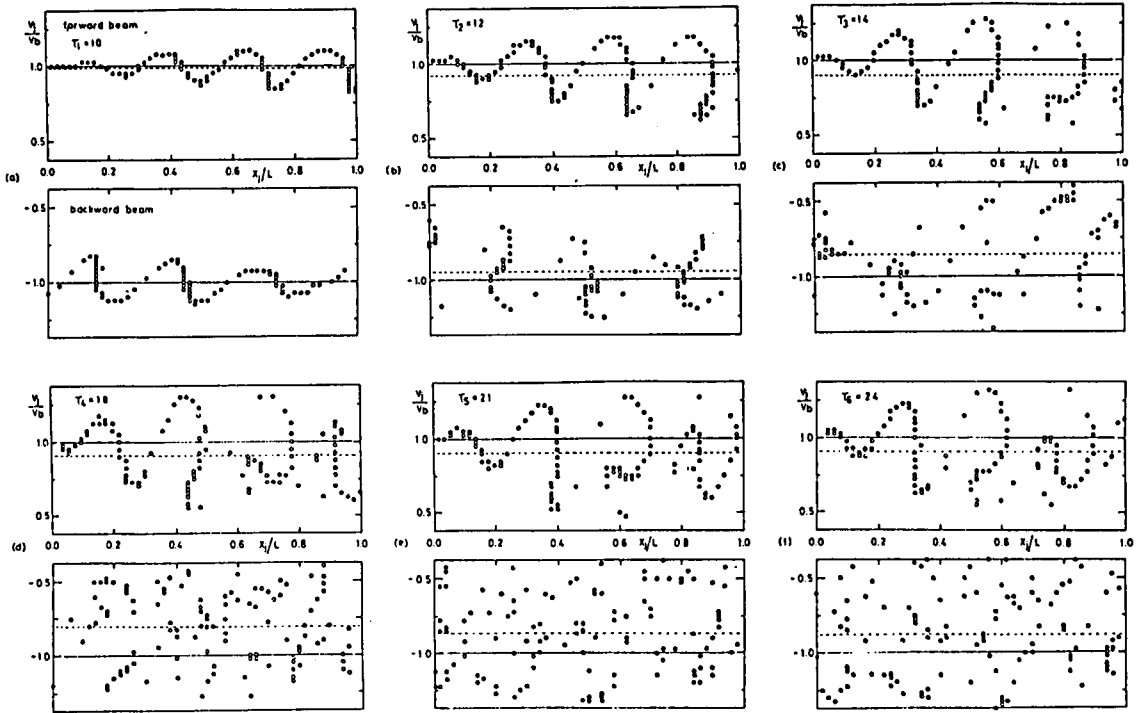


Fig.3.9 Instantaneous phase-space plots corresponding to six points denoted in Fig.3.8. The upper graph corresponds to the injected beams and the lower one does the reflected beams. Solid and dashed lines show the initial and the instantaneous mean beam velocities.

are very similar to those in the Case I. Behaviors of reflected beams in the phase-space are smeared out in the later time. However, mixing between the injected and the reflected beams does not occur.

Some runs are also made for different mode number m . With increasing the value of m , phase bunching becomes more indistinct.

(B) Interaction between Two Waves

Now, we present the results on wave suppression due to beam modulation. Numerical calculations are performed for various plasma conditions.

Figure 3.10 shows a typical example of the two wave interaction in the Case I, where plasma parameters and conditions for calculation are the same as in Fig.3.6. Then, evolutions of the system without modulation are the same as ones in Fig.3.6. Beam modulation is introduced at $\tau = 14$. Conditions are as follows: $n_m/n_b = 0.1$, the initial amplitude of $F_k(14) = 0.01E_k(14)$, the position of pair grid $x_G/L = 0.05$ and frequency difference between two waves $\delta\omega/\omega_0 = 0.02$. When the beam modulation is present, the total wave energy E^2 means the summation of the electron wave and the excited wave, i.e. $(1+R^2)[E_k^2(\omega_0)+F_k^2(\omega_0+\delta\omega)]$. With the growth of the excited wave, the electron wave decreases in amplitude up to $\tau = 20.3$. After this time, exchange of wave energy between the electron wave and the excited wave occurs. It is apparent that the additional broadening of the beam distribution occurs because of the excitation of another wave due to beam modulation.

Figure 3.11 shows the results in the Case II. Numerical con-

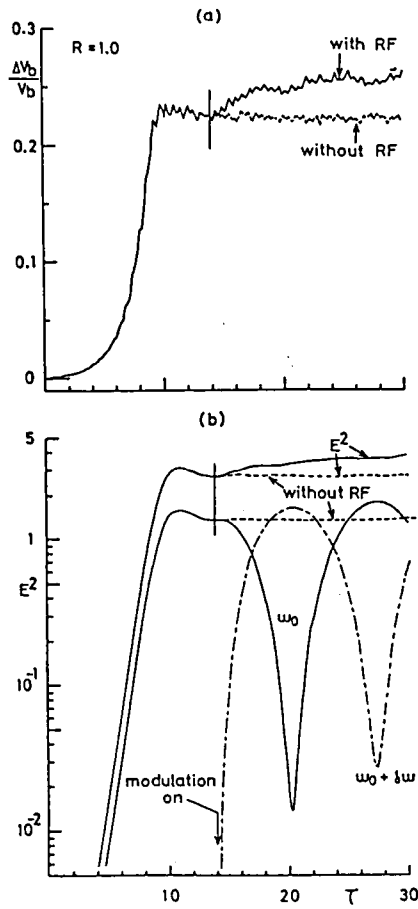


Fig.3.10 Evolutions of the beam spread $\Delta V_b/V_b$ and the wave power in the case that injected beams are absorbed at $x = L$. Beam modulation is introduced at $\tau = 14$. The orbits of beams are solved in the single wave approximation before the application of beam modulation. After this, the orbits are solved in the two wave approximation. Plasma conditions are $n_b/n_e = 0.01$, $v_b/v_{te} = 4$, $R = 1.0$ and $m = 4$.

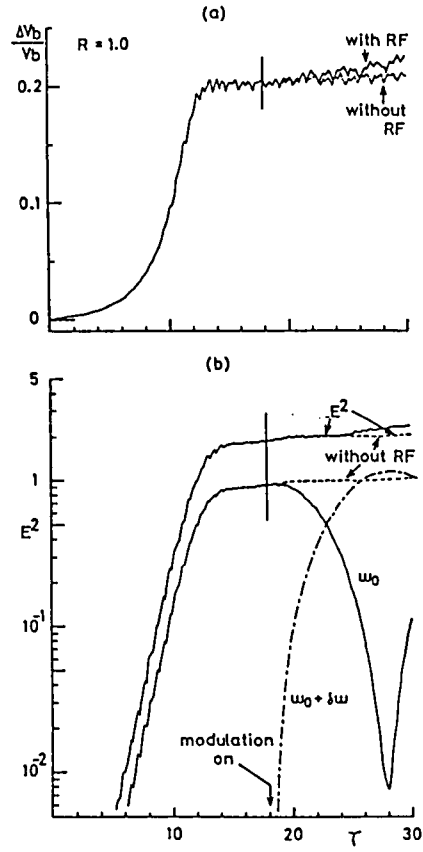


Fig.3.11 Evolutions of the beam spread $\Delta V_b/V_b$ (injected beams) and the wave power in the case that injected beams are reflected at $x = L$. Beam modulation is introduced at $\tau = 18$. Plasma conditions are the same as ones in Fig.3.10.

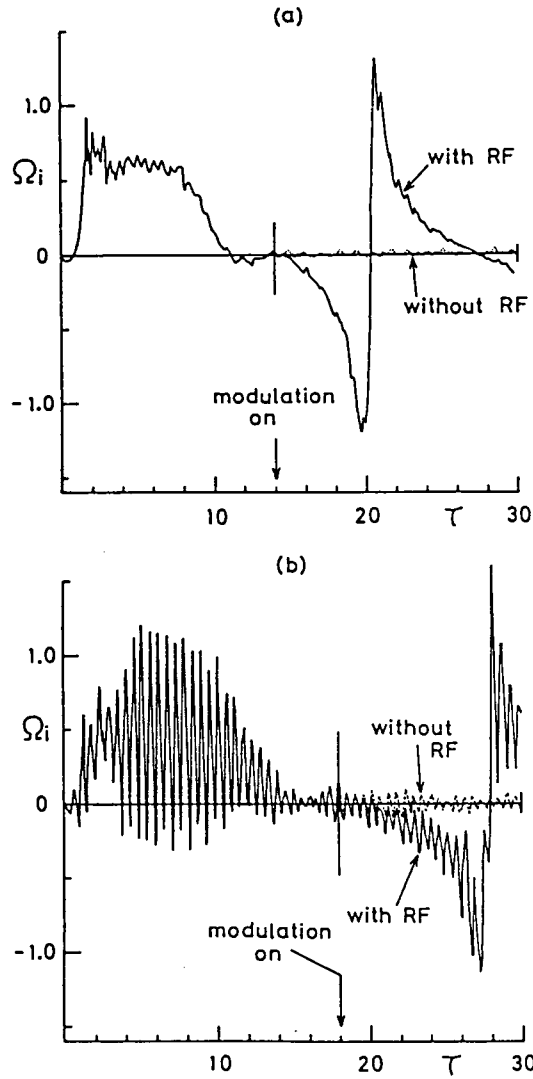


Fig.3.12 Evolutions of the growth rates corresponding to (a) the case in Fig.3.10, and (b) the case in Fig.3.11.

ditions for calculation are the same as ones in Fig.3.8(without modulation) and ones in Fig.3.10(with modulation), respectively. Beam modulation is introduced at $\tau = 18$ when the system reaches the steady state already. As the excited wave grows, the electron wave decreases in its amplitude. After the time, $\tau = 28$, exchange of wave energy between two waves also occurs as that does in the absence of reflected beams. However, there are some different features in this case from ones in the Case I. In spite of the excitation of the external wave, total wave energy has nearly the constant value. Corresponding to the evolution of total wave energy, the spread of beam distribution has nearly the constant value. Namely, the change of wave intensities occurs under the constant value of total wave energy, and the additional broadening of the beam distribution does not occur appreciably.

For reference, the evolutions of the growth rates for two cases in Figs.3.10 and 3.11, respectively, are shown in Fig.3.12.

In the present model, the backward wave is introduced supplementally as described in eq.(3-19). Therefore, between boundary condition of the beam particles and that of the waves including the reflection coefficient, there is no self-consistent relationship. They are assumed to have a certain condition independently each other. However, one of the advantages of the present model is that we can include or omit the backward wave at will and thus study its effect. In order to clarify the relation between the reflected beams and the backward wave, runs are made for various reflection coefficients with keeping the beam conditions same. Some results are shown in Figs.3.13 and 3.14. In the absence of reflected beams(the Case I), evolutions of both the wave and beam

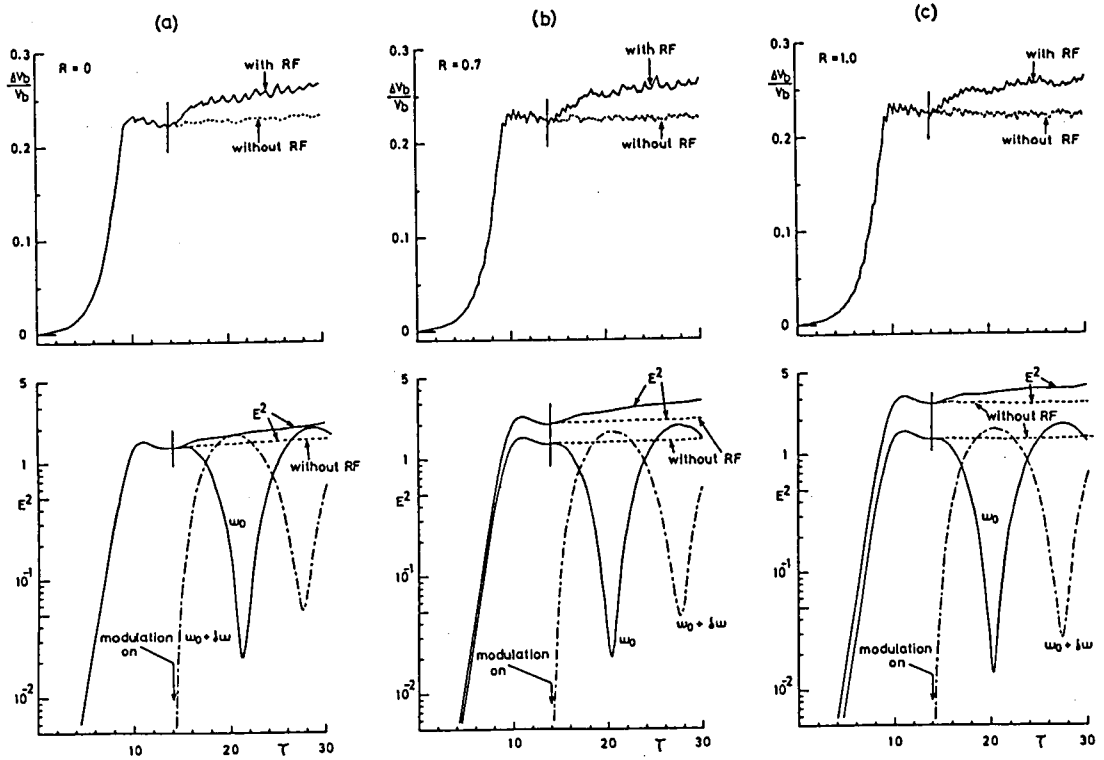


Fig.3.13 Evolutions of the beam spread $\Delta v_b/v_b$ and the wave power in the Case I. Conditions for calculation are the same as in Fig.3.10. Parameter is a magnitude of the reflection coefficient R :
(a) $R = 0$, (b) 0.7 and (c) 1.0 .

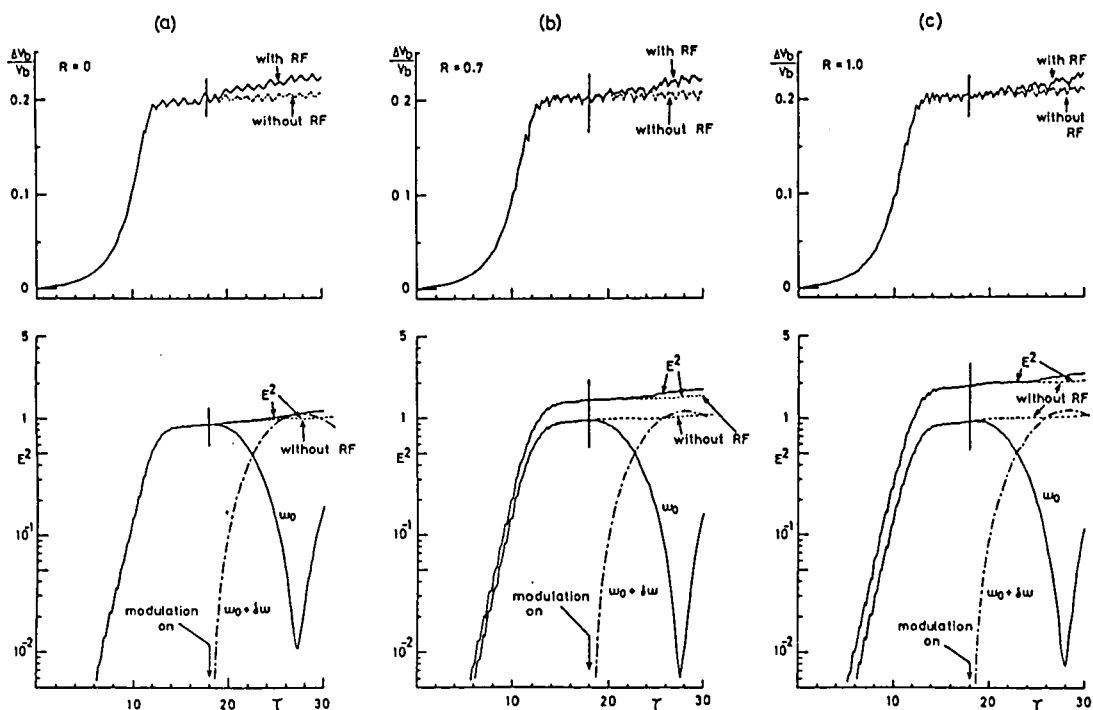


Fig.3.14 Evolutions of the beam spread $\Delta v_b/v_b$ (injected beams) and the wave power in the Case II. Conditions for calculation are the same as in Fig.3.11. Parameter is a magnitude of R : (a) $R = 0$, (b) 0.7 and (c) 1.0.

spread are not dependent on reflection coefficient so strongly. However, in the presence of reflected beams (the Case II), an additional broadening of beam spread due to the introduction of beam modulation becomes remarkable with decreasing reflection coefficient R .

In Chapter 2, we show [29] that in the semi-infinite system, nonlinear interaction between a small cold beam and a plasma can be changed drastically by introducing weak collisions within the background plasma, i.e. the ratio of collision frequency ν to plasma frequency ω_p is about 10^{-3} . We present some numerical results concerning collisional effects in the bounded system.

Figure 3.15 shows the evolution of the system where collisions between plasma electrons are included. In the absence of reflected beams, comparing the results in Fig. (a) with ones in Fig. 3.10, collisional effects does not change the evolution of both the waves and the beam spread appreciably except that the saturation amplitude and the beam spread decreases slightly at a lower level in the collisional case. However, in the presence of reflected beams, there is the appreciable difference between the results in Fig. (b) and ones in Fig. 3.11. Additional broadening of beam spread appears even if $R = 1.0$.

When plasma is generated by the injected beams, electron-neutral collisions of beam electrons are important in the nonlinear stage. Preliminary study of these collisional effects may be done by using the following equation of motion for the j -th beam electron, $x_j(t)$,

$$m \frac{d^2 x_j}{dt^2} = -e [E(x_j, t) + F(x_j, t)] - m\nu v_j ,$$

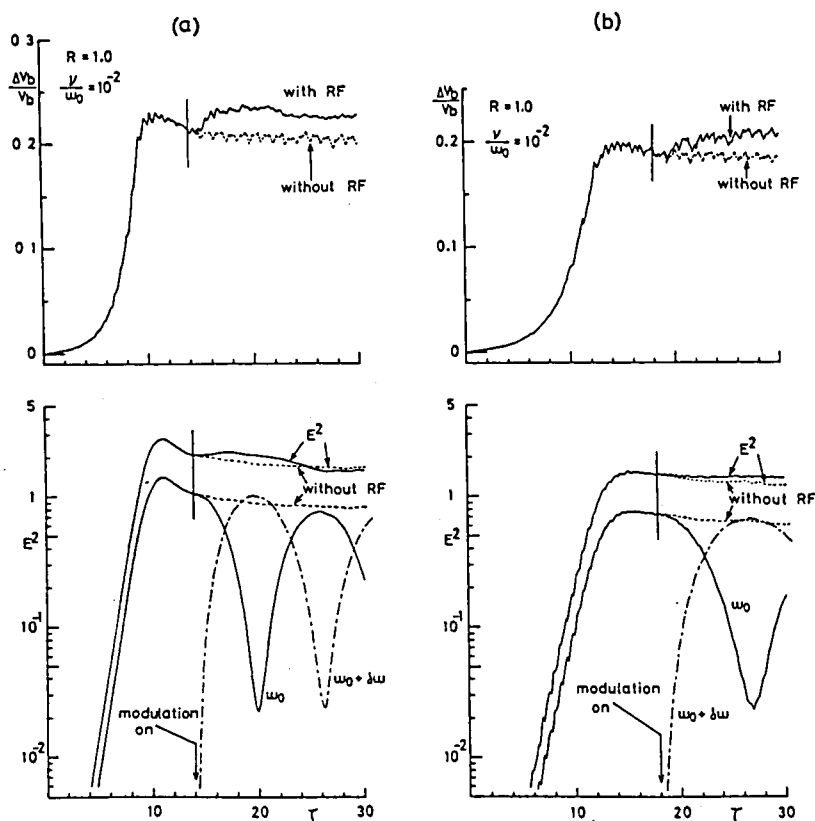


Fig.3.15 Evolutions of the beam spread $\Delta v_b/v_b$ and the wave power in the case that collisional effects between plasma electrons are considered, i.e. $\nu/\omega_0 = 0.01$: (a)The Case I, and (b)the Case II, respectively.

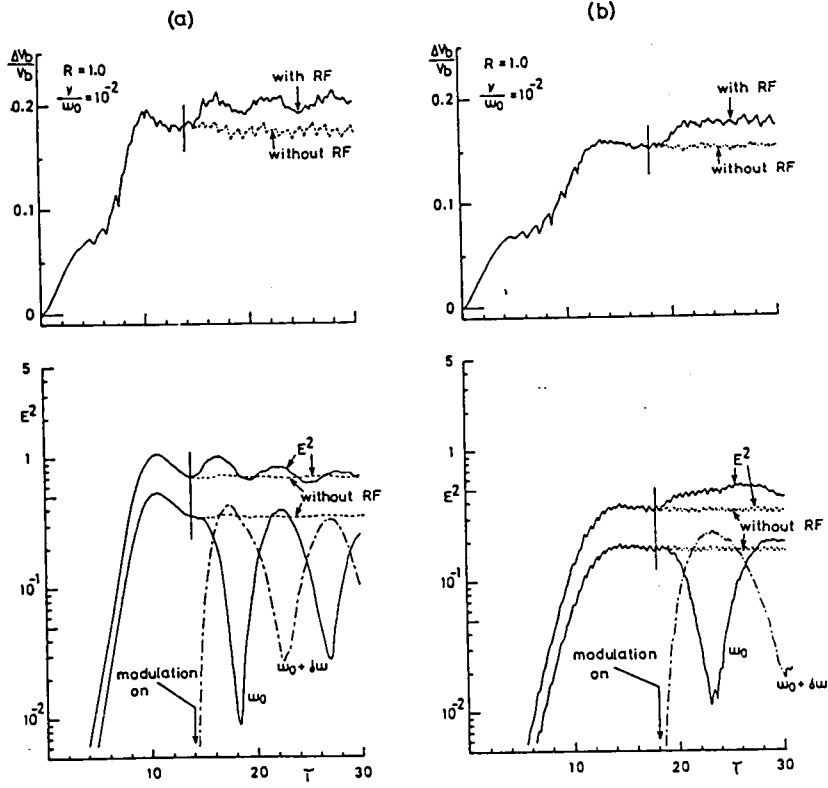


Fig.3.16 Evolutions of the beam spread $\Delta V_b/V_b$ and the wave power in the case that collisional effects both in plasma electrons and in the beam electrons are considered, i.e. $\nu/\omega_0 = 0.01$: (a)The Case I, and (b)the Case II, respectively.

where $E(x,t)$ and $F(x,t)$ are the electron and the excited waves, respectively, and v_j and x_j are the velocity and the position of the j -th beam electron. Figure 3.16 shows the evolution of the system where two collisional effects, i.e. collisions for both plasma electrons and beam electrons, are included. In both cases (with and without reflected beams), waves saturate their amplitudes at a pretty lower level as compared with the results in Figs. 3.10 and 3.11. In the Case II (shown in Fig. (b)), additional broadening of beam spread appears even if reflected beams are present and $R = 1.0$.

3.3-4 Discussion

In a bounded beam-plasma system, two different processes on the interaction between two waves are obtained. Beam electrons are injected at the end of the plasma column continuously, and is absorbed (the Case I), or reflected (the Case II) at the other end. In the Case I, the interaction of the electron wave with the excited wave is accompanied by the additional broadening of beam spread ΔV_b (see Fig. 3.10). In the Case II, however, ΔV_b shows no appreciable change in the course of two wave interaction (see Fig. 3.11).

Recently, suppression of beam-excited electron waves in a bounded system has been studied in connection with the beam distribution function [16-18,20]. These experimental results seem to support our numerical results at least in the early stage of the evolution of the excited wave, i.e. up to $\tau = 20.3$ in the Case I and up to $\tau = 28$ in the Case II.

For example, we have obtained the following experimental re-

sults[16,20]. In the absence of reflected beams(corresponding to the Case I in the present calculation), it looks that with increasing modulation voltage the excited wave increases its amplitude and the electron wave decreases independently each other. In particular, deformation of the beam distribution function, i.e. an additional broadening of the beam, is observed in relation to the suppression of the electron wave. However, in the presence of reflected beams(corresponding to the Case II), the rapid change in the ratio of wave intensities occur with increasing modulation voltage. This rapid change in amplitudes may be a consequence of the so-called mode competition[30]. During this process, total energy of two waves is kept nearly constant and beam distribution function has no appreciable change, if the frequency difference between the electron wave and the excited wave is within the spectral width at half-maximum of the electron wave.

Another example has been obtained by Amemiya[17]. In the single beam case(the Case I in our notation), a broadening of beam energy occurs associated with the suppression of the electron wave, while in the counter stream case(the Case II in our notation), a variation of beam energy distribution is little in relation to the wave suppression.

These two examples have the same features as the present numerical results have. We make a detailed comparison of our numerical and experimental results in Chapter 4.

We think that two different processes of wave suppression observed in experiments are caused by the feedback effects due to reflected beams[16,18,20] or the counter beams[17]. This point is also reproduced numerically. As is shown in Fig.3.14, the ad-

ditional broadening of the injected beams appears clearly with decreasing a magnitude of the reflection coefficient even if reflected beams are present. It suggests that the difference between the Case I and the Case II is caused by the feedback effects due to the interaction between the reflected beams and the backward wave.

Experimentally, the system has a steady state, whether beam modulation is present or not. In the present simulation, the steady state is also obtained when beam modulation is absent. However, in the presence of beam modulation, the simulation fails to develop a steady state. In the Case I, the electron wave and the excited wave oscillate their amplitudes due to energy exchange between them after $\tau = 20.3$. In the Case II, similar amplitude oscillations occur after $\tau = 28$. This may be due to the lack of support for the perfect phase lock in the code. In a steady state without modulation, fresh beams injected continuously are modulated by the electron wave and locked to a certain bounded state in the phase-space shown in Figs.3.7 and 3.9. It may be inferred that the suppression of the electron wave is due to a lock of beams to another bounded state governed by the excited wave. In the present simplified model, however, boundary conditions of beams and those of the waves including the reflection coefficients are not determined self-consistently. It is also assumed that two waves have the same wavenumbers because of small frequency difference between them.

Taking into account the present numerical results and the experimental results[16-18,20], we consider the suppression phenomena as follows: In the Case II(with feedback effects), the system behaves as a steady-state oscillator with a finite Q-value[5,18].

Suppression would be due to the mode competition and less related to the beam distribution function. On the other hand, in the Case I (without feedback effects), the system behaves as if it is unbounded, i.e. the case discussed in Section 3.3. Two waves do not interact directly, but through a wave-particle-wave process.

From numerical results which include collisional effects, we consider as follows: In general, collisions destroy the phase relation between the wave and the plasma particles. If there is a certain possibility that some of beam particles make a collision, phase bunching of beam particles are randomized due to collisions. Then, feedback effects due to reflected particles become weak. It may be reasonable that suppression processes in the Case II have the same features as ones in the Case I when collisional effects are introduced in the system (see Figs. 3.15 and 3.16).

As is shown in Fig. 3.9, for injected beams a cohesive beam is recognizable although particles are behaving nonlinearly. However, reflected beams become diffused strongly. From the viewpoint of feedback effects due to reflected beams, increasing a magnitude of mode number m causes feedback effects weak. Because, both injected and reflected beams become diffused more irregularly in the phase-space, and then phase relation between the wave and beam particles are destroyed considerably. As a numerical example, it is confirmed that an additional broadening of beam spread occurs in the Case II when $m = 8$.

In the present model, we assume that the motion of the plasma electrons can indeed be considered in the linear approximation. Finally, we check this assumption [31]. The velocity of the plasma electrons in the field of the wave can be estimated as $v_e = eE_{\max}/$

$(m\omega_p)$, where $E_{\max}$ is the maximum electric field of the wave at the moment of capture of the beam particles. The dominant nonlinear term in the equation describing the motion of the plasma electron is the term $v_e(\partial v_e/\partial x)$. Comparing it with the least linear term, γv_e , we find that even at the maximum amplitude of the wave field $\gamma v_e/(kv_e^2) \cong \omega_p/\gamma \cong (2n_e/n_b)^{1/3}$. When $n_b/n_e = 10^{-2}$, $(2n_e/n_b)^{1/3} = 5.85$. Then, $\gamma v_e/(kv_e^2) > 1$, and our approximation is valid.

3.4 Concluding Remarks

Concerning the control of wave spectrum and the suppression of the wave due to preliminary modulation of the beam, interaction between two waves in a modulated electron beam-plasma system is investigated by means of partial simulation. We have analyzed these problems in two different systems.

In the semi-infinite system (the unbounded case), the model of spatially growing waves is used. Spectrum control and wave suppression are obtained by pre-modulating injected beams. Deformation of beam distribution is occurred in relation to the suppression of one wave caused by the other wave. According to these numerical results, we think that two waves do not interact directly, but through the wave-particle-wave process.

On the other hand, the model of temporally growing waves is used in the bounded system. We have two different patterns for the wave suppression in this case. When injected beams are reflected at the other end, two wave interaction occurs under the constant wave energy. Besides, additional broadening of the beam

distribution does not occur. However, when injected beams are absorbed at the other end, suppression of one wave due to the other wave is accompanied with additional broadening of the beam distribution. These differences in suppression process are caused by whether feedback effects of reflected beams are present or not. So far, the steady state of the system is not obtained when the beam modulation is applied.

Present simulation results in both cases, i.e. the spatial and the temporal cases, are in agreement with experimental results for wave suppression, qualitatively.

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Chapter 4

SUPPRESSION OF BEAM-EXCITED ELECTRON WAVES

BY AN EXTERNALLY APPLIED RF SIGNAL

4.1 Introduction

Suppression of plasma waves such as electron waves and ion-acoustic waves, by applying an external ac signal, have been studied by many authors[1-12]. Among them, Keen and Fletcher[3] reported that the ion-acoustic instabilities excited in a plasma of arc discharge were suppressed by an external signal, and that these phenomena were explained by the solution of the Van der Pol equation with the external force term. Nakamura[4] reported that beam-excited longitudinal electron oscillations were suppressed by beam density modulation in an axially and transversely finite plasma system. He also explained these phenomena by assuming that the oscillations obeyed the Van der Pol equation with the external force term.

The above results do not, however, clearly delineate the physical processes involved in the wave suppression. In both cases[3,4], the Van der Pol equation was derived by means of the fluid equations. The nonlinear terms, being essential to the Van der Pol equation, come from the source terms introduced supplementally in the continuity equation. These source terms are interpreted as ionization etc. caused by large amplitude oscillations in the plasma[3]. However, in the collisionless plasma where the fluid theory is not applicable and no source term is present, it has been report-

ed recently that the Van der Pol equation is a phenomenological model for the ion-acoustic wave[13]. Furthermore, Kato et al.[14] have reported that the electron oscillations in a bounded electron beam-plasma system with density modulation of the electron beam are governed by the Mathieu equation instead of the Van der Pol equation. Therefore, the theoretical basis for the Van der Pol equation can not be easily furnished within the framework of the hydrodynamic approximation.

Recently, there have been some experiments[8-12], where relation between the wave suppression and the beam distribution function is studied in an electron beam-plasma system. These experimental results suggest that suppression mechanism must be considered by taking into account nonlinear behaviors of beam electrons.

This chapter gives the details of experimental measurements and a theoretical consideration for the suppression mechanism of beam-excited electron waves in a bounded system. Preliminary experiments have already been reported[8,9] in which deformation of the beam distribution function is observed in relation to the suppression of the electron waves. Further, this deformation depends strongly on whether the reflected beams are present or not[10]. In section 2, the experimental arrangement and procedure are described. Section 3 contains the experimental results. The discussion is presented in section 4. Finally, concluding remarks are presented in section 5.

4.2 Experimental Apparatus and Methods

The experimental apparatus is shown schematically in Fig.4.1. In a cylindrical stainless steel chamber, with 50 cm length and 10 cm diameter, there are an electron gun, an energy analyzer and two axially movable probes. The base pressure of the chamber is about 2×10^{-6} Torr. The experiments are performed under continuous pumping and flowing of argon gas. The working pressure is about 10^{-4} - 10^{-3} Torr.

The electron gun injects the electron beams continuously with an accelerating voltage V_B up to an energy of several tens of electron volts, and a beam current I_B to 10 mA. A beam diameter is about 20 mm. The value of I_B can be controlled by changing a voltage V_G of a grid G of the electron gun. In order to modulate the beam density, an external signal is applied to G at various frequencies. The argon plasma is generated by the injected beam. An axial magnetic field of about 100 G is applied to confine radially the plasma and to obtain uniform density along the axial direction. Plasma parameters are as follows: Electron density $n_e \cong 1 \times 10^8 \text{ cm}^{-3}$, electron temperature $kT_e \cong 3 \text{ eV}$ and the ratio of beam to plasma electron density $n_b/n_e \cong 1 \times 10^{-2}$.

Oscillation signals are picked up by a coaxial probe with a diameter of 0.3 mm and a length of 2.5 mm, and these signals are analyzed by means of a spectrum analyzer. The coaxial probe is movable axially and is at floating potential. This is also used as a Langmuir probe to measure plasma parameters.

The velocity distribution function is measured by both an energy analyzer and a plane probe with an area of $1 \times 2 \text{ mm}^2$ perpendicular to the magnetic field. After passing through the plasma column, the beam is collected in the analyzer. The analyzer is the

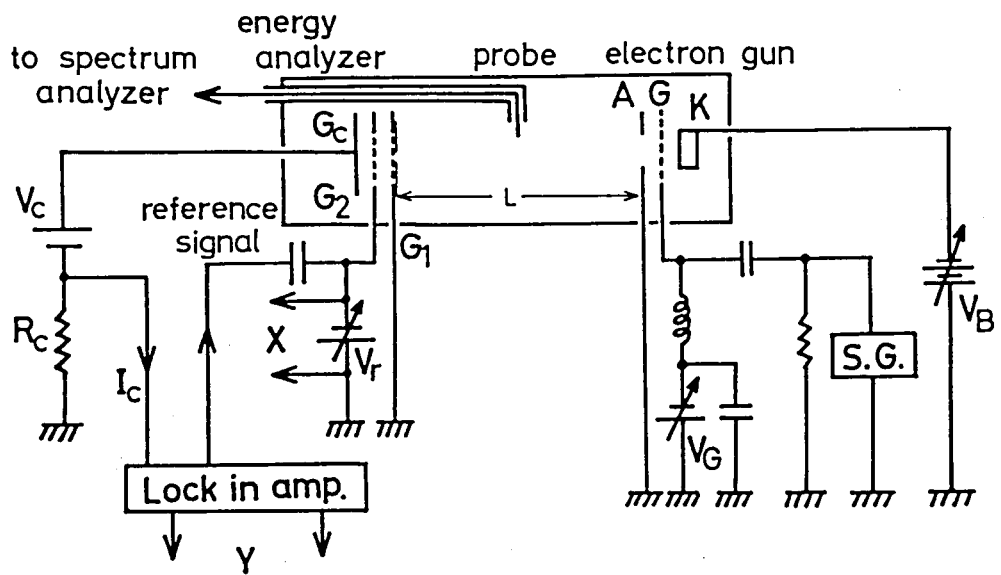


Fig.4.1 Schematic diagram of the beam-plasma system and the experimental set-up.

conventional multigrid type, having two mesh grids and a collector plate. Plasma terminates at the front grid G_1 . The second grid G_2 is used as the electron energy selector. The collector plate for the electron current I_c is kept at a positive potential of about 80 V. In order to obtain the electron velocity distribution function, it is necessary to take the first derivative of I_c with respect to the retarding voltage V_r [15]. For this purpose, is used a modulation method, where a small signal with voltage 100 mV_{p-p} and frequency 3.1 kHz is added to the dc retarding voltage. The ac component of I_c is proportional to the first derivative dI_c/dV_r . A schematic diagram of a system which accomplishes this measurement is also shown in Fig.4.1. Distribution function along the plasma column is obtained by the axially movable plane probe. Measurement method is the same as in the case of the analyzer described above.

In order to investigate the effects of reflected beams on wave suppression, experiments are performed under two different conditions. The one is with reflected beams, and the other is without reflected beams. These two conditions are arranged by keeping the voltage of G_1 floating or earth potential. Figure 4.2 shows current-voltage characteristics of the plane probe. The plane is directed towards the analyzer so as to collect the reflected beams. It is clearly shown that reflected beams are decreased to zero by increasing the voltage of G_1 .

4.3 Experimental Results

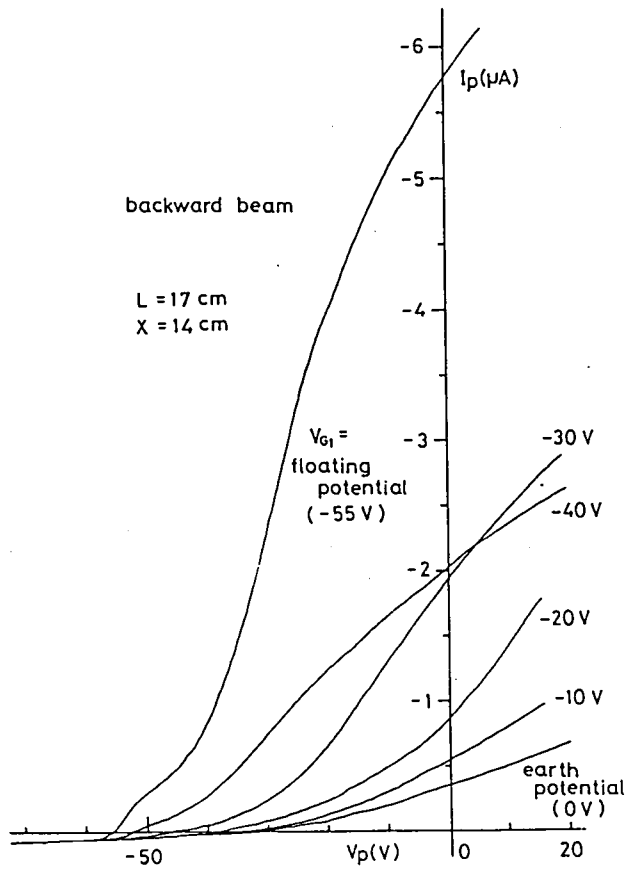


Fig.4.2 Current-voltage characteristics of the plane probe. A magnitude of the voltage G_1 is a parameter. The probe is 3 cm apart from the energy analyzer and its plane is set toward the analyzer so as to collect the reflected beams. Experimental parameters are as follows: $L = 17 \text{ cm}$, $V_B = 58 \text{ V}$, $I_B = 1.77 \text{ mA}$, $V_G = -30 \text{ V}$ and $p = 1 \times 10^{-5} \text{ Torr}$.

4.3-1 Beam-Excited Electron Waves

Under certain conditions, electron waves are excited spontaneously in both cases, i.e. with and without reflected beams. For unstable electron waves, discrete spectra are obtained, because injected beams and plasma produced by the beam are bounded by the anode A of the electron gun and G_1 . The wavelength λ_n and the frequency f_n are roughly determined by the following relations:

$$\lambda_n = \frac{2L}{n} \quad \text{and} \quad f_n = \frac{n}{2L} v_{pn}, \quad (4-1)$$

where a mode number n is an integer, L is the distance between A and G_1 , and v_{pn} is the phase velocity of the wave.

Figure 4.3 shows some examples of wave spectra and corresponding beam distribution function in the absence of reflected beams. In this case, V_B and L are kept constant and argon pressure p is a parameter. When $p = 2.1 \times 10^{-4}$ Torr, the electron wave E_3 is excited spontaneously. The mode number n is three and its frequency f_3 is 44 MHz. With increasing p , i.e. $p = (2.3-2.9) \times 10^{-4}$ Torr, the amplitude of E_3 becomes large and the width of the spectrum becomes broad. Wave frequency also shifts to higher side. With increasing p further, i.e. $p = (3.6-4.6) \times 10^{-4}$ Torr, the wave mode is changed one by one to next higher mode. In relation to these changes of wave spectra, beam distribution functions are also deformed. Not collisional effects but wave intensity and the shape of wave spectrum have relations closely with the deformation of the beam distribution. For collisions, electron-neutral collision is predominant under the present conditions, and is dependent on p . When $p = 3.8 \times 10^{-4}$ Torr and $V_B = 37$ V, the wave E_3 with frequency

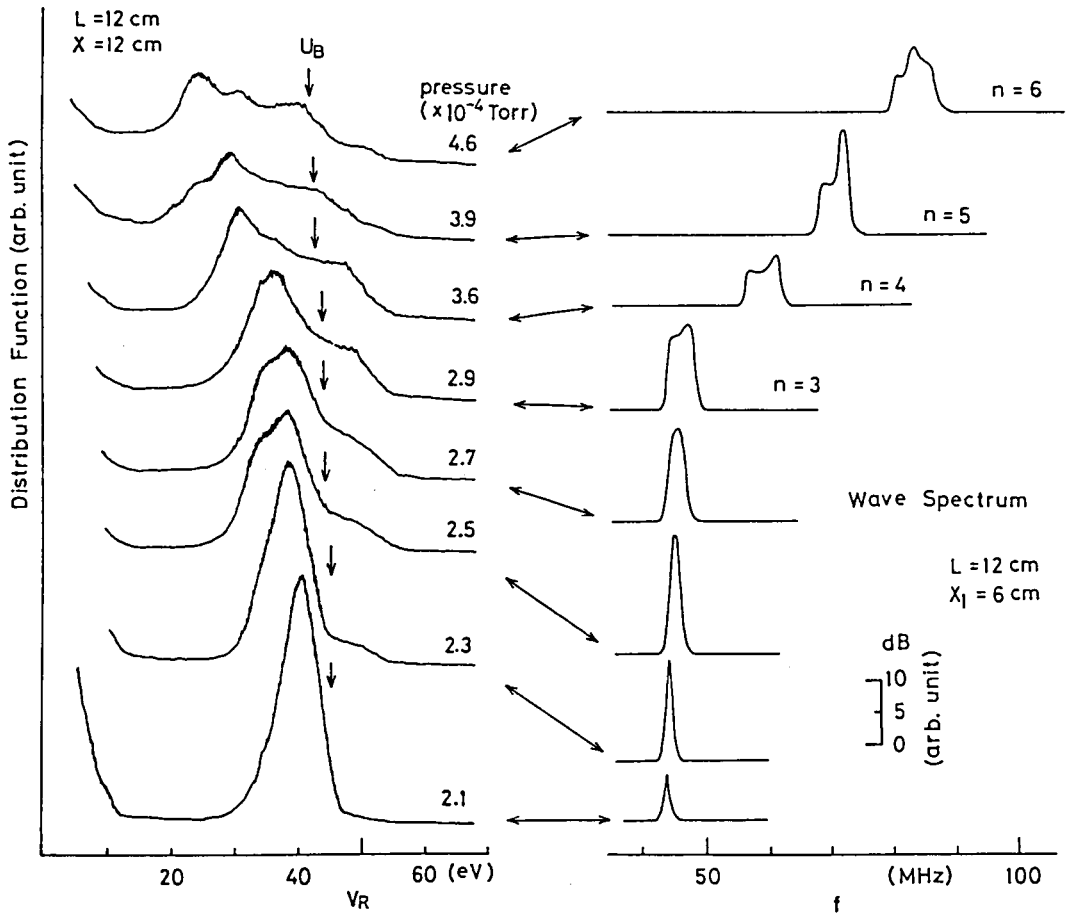


Fig.4.3 Relationship between the beam distribution function and the wave intensity. Gas pressure is a parameter. Distribution functions are measured by using the analyzer and waves are picked up by the coaxial probe. Probe position X_1 is 6 cm from A. n is the mode number. Throughout these measurements, V_B is almost kept 45 V which is indicated by arrows. Parameters: $L = 12$ cm, $V_G = -10$ V and $I_B = (4.2 - 4.7)$ mA.

53 MHz is excited and has a narrow spectrum as the wave has in the case shown in Fig.4.3, where $p = (2.1-2.3) \times 10^{-4}$ Torr and $V_B = 45$ V. Then, the resulting beam distribution keeps its shape narrow as one does in the case where $p = (2.1-2.3) \times 10^{-4}$ Torr.

Figure 4.4 shows some features of beam distribution function and wave spectra in the presence of reflected beams. In this case, V_B , L and p are kept constant. The voltage of the grid is a parameter. There is a best condition where a certain mode has its maximum amplitude. With increasing V_G , the amplitude of E_4 becomes large and E_4 has the maximum amplitude when $V_G = -32$ V. With increasing V_G further, the amplitude of E_4 becomes small and the next higher mode E_5 appears. A mode-jump is also observed by changing p as shown in Fig.4.3. In the course of the change of wave spectrum, there is no appreciable change in the width of the wave spectrum. The width at half-maximum is about 0.5 MHz throughout the present experiment. Beam distribution is deformed little by little as the wave amplitude becomes large and a multimode excitation is occurred.

In order to study the suppression mechanism in relation to the beam distribution, it is convenient to deal with the case where a single mode with a narrow frequency spectrum is excited. Because, in this case, we can consider the interaction between only two waves, i.e. the electron wave and the externally applied wave. Here after, plasma parameters are controlled so that a single mode with a narrow frequency spectrum is excited spontaneously.

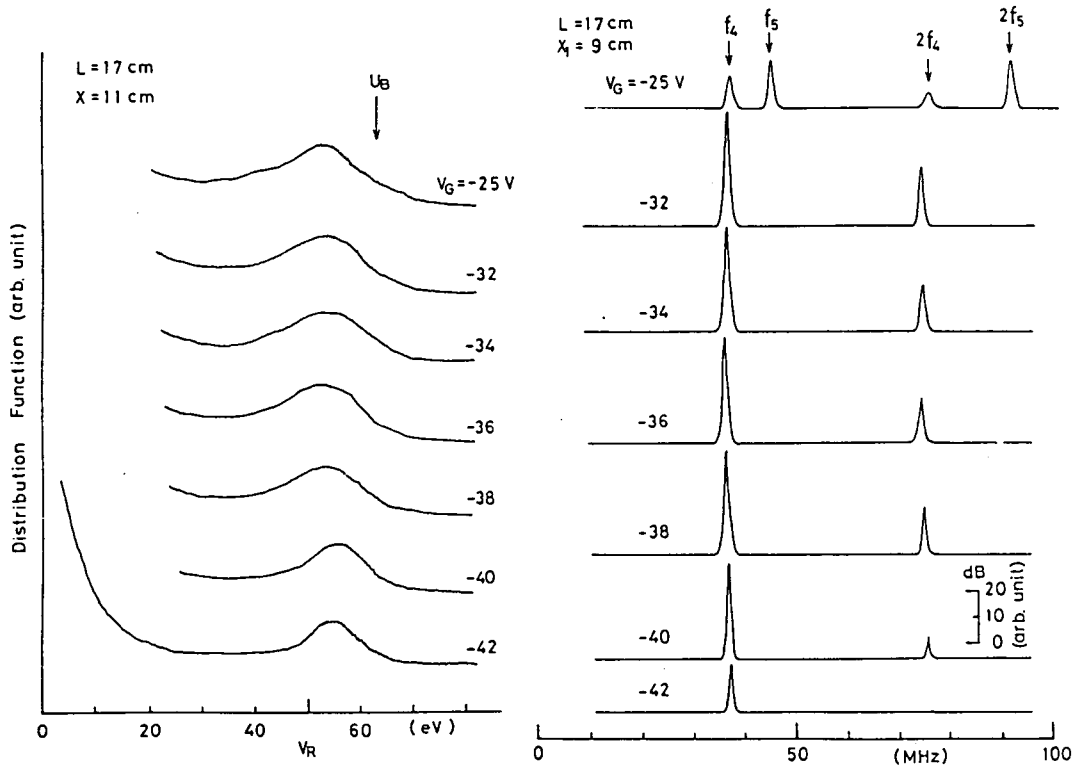


Fig.4.4 Relationship between the beam distribution function and the wave intensities in the presence of the reflected beams. V_G is a parameter. Distribution functions are measured by the plane probe whose position X is 11 cm from A. The plane is set toward the gun. Waves are picked up by the coaxial probe. Parameters: $L = 17 \text{ cm}$, $V_B = 62 \text{ V}$, $I_B = 3 \text{ mA}$ and $p = 3 \times 10^{-4} \text{ Torr}$.

4.3-2 Suppression of Electron Waves

in the Absence of Reflected beams

Before doing experiments of wave suppression, we show the dependence of wave frequency on the system length L . Typical raw data of wave spectrum and the relationship between the wave frequency and the reciprocal of L are shown in Fig.4.5. With decreasing L , wave frequency shifts to the higher side and a mode-jump occurs at about $L = 12.5$ cm. This frequency shift is proportional to L^{-1} . If we set $L = 11$ cm, only the wave E_2 is excited spontaneously. As is shown later, wave suppression is strongly dependent on modulation frequency f_m . In the case shown in Fig.4.5 (b), effects of beam modulation is very strong when f_m is close to the frequency of E_2 and in the shaded region. The shaded region is the frequency of E_3 , not excited spontaneously, neighboring with E_2 .

When the beam density is modulated by applying an external signal with frequency f_m to G, the excited wave E_m with the same frequency is observed. The amplitude of E_m becomes large as the modulation voltage V_m of the signal is increased. The amplitude of the electron wave can be controlled by the excited wave.

Two examples of wave suppression are shown in Fig.4.6. In the case (a), the electron wave E_1 with frequency 14.8 MHz is excited spontaneously and f_m is chosen to be the vicinity of f_1 . With increasing V_m , the amplitude of E_m increases and that of E_1 decreases. When $V_m = 0.8 V_{p-p}$, E_1 is suppressed and only E_m is observed. On the other hand, when f_m is chosen to be close to the frequency of the neighboring mode which is not excited spontaneously, similar phenomena is occurred. In the case (b), the electron

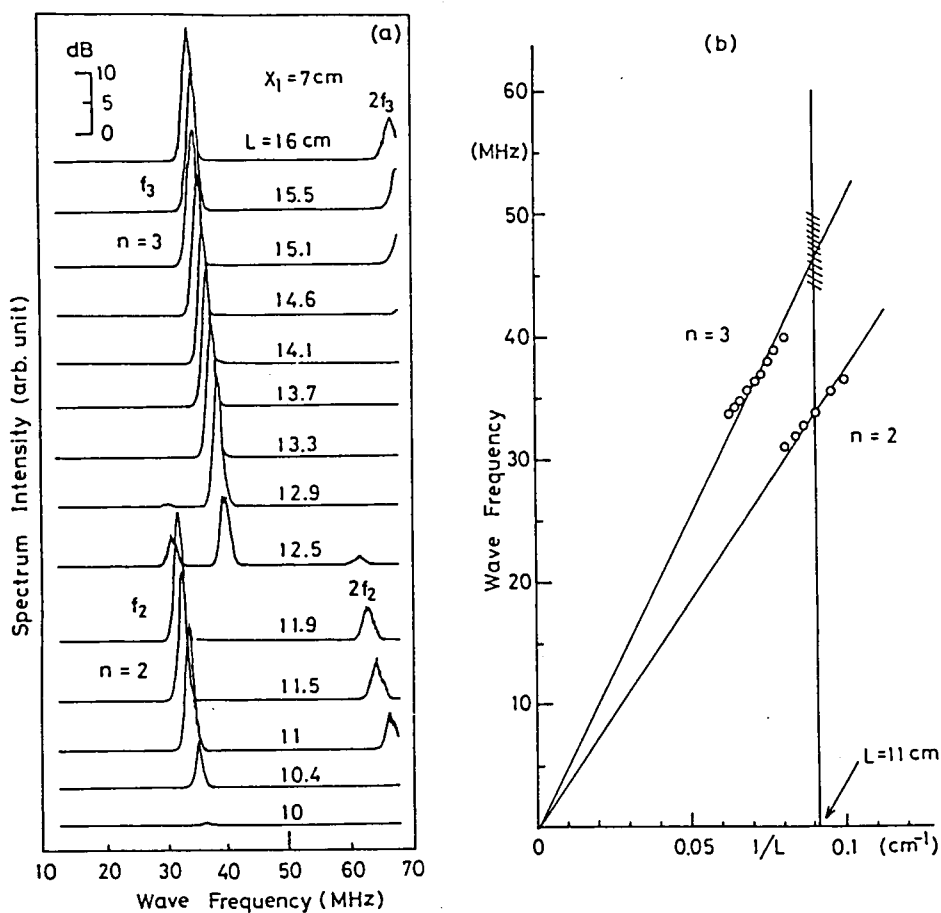


Fig.4.5 Characteristics of spontaneously excited electron waves.

(a) Frequency shift of the wave and its mode-jump. Plasma length which is changed by moving the energy analyzer is a parameter. The probe to pick up oscillation signals is kept 7 cm from A.

(b) Relationship between wave frequency and the reciprocal of L , i.e. dispersion relation.

Parameters: $V_B = 38$ V, $I_B = 4.5$ mA, $V_G = -10$ V and $p = 1.8 \times 10^{-4}$ Torr.

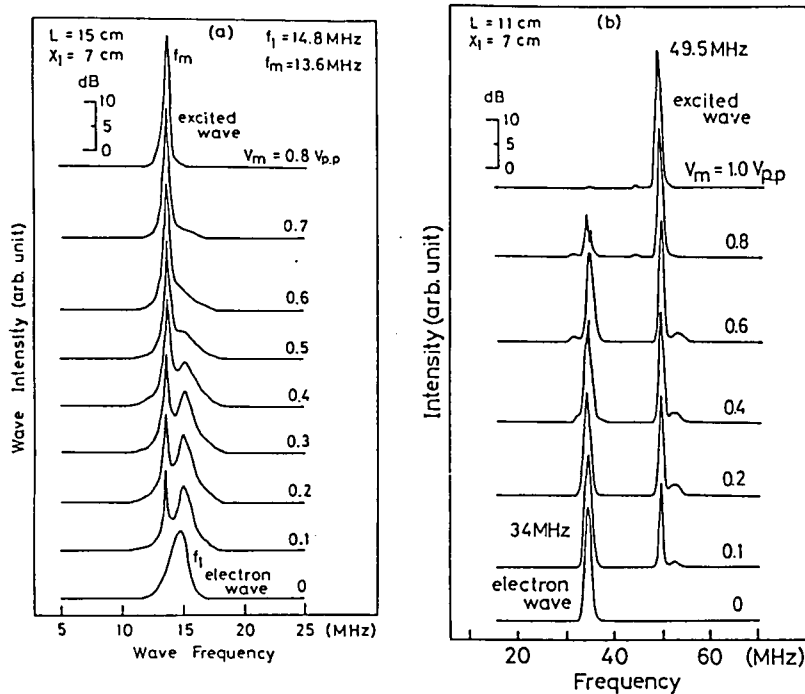


Fig.4.6 Two examples of wave suppression.

(a) Electron wave E_1 with frequency f_1 is excited spontaneously. f_m is set to be in the vicinity of f_1 . Parameters: $L = 15$ cm, $V_B = 37$ V, $I_B = 3.6$ mA, $V_G = -10$ V and $p = 1.1 \times 10^{-4}$ Torr.

(b) Electron wave E_2 with f_2 (34 MHz) is excited. In this case, f_m is set to be in the vicinity of f_3 which is the frequency of the mode neighboring with E_2 . Parameters: $L = 11$ cm, $V_B = 40$ V, $I_B = 4.6$ mA, $V_G = -10$ V and $p = 1.8 \times 10^{-4}$ Torr.

wave E_2 with frequency 34 MHz is excited spontaneously. f_m is set to 49.5 MHz which is close to that of the electron wave E_3 not excited spontaneously. When $V_m = 1 V_{p-p}$, E_2 is suppressed.

The critical modulation voltage V_c , above which the electron wave is suppressed by the beam modulation, is dependent on f_m . Relationship between V_c and f_m is shown in Fig.4.7(a), where the wave E_4 with frequency 51 MHz is excited. The value of V_c is nearly proportional to the frequency difference between f_m and the frequency of E_4 , i.e. $|f_m - f_4|$. The amplitude of the excited wave ϕ_c , which is obtained when $V_m = V_c$, is plotted as a function of f_m . For reference, the amplitude of E_4 without modulation is also plotted by the dotted circle. These wave amplitudes are measured by using the coaxial probe which is set to the fixed point. It is said that ϕ_c is larger than the amplitude of E_4 , and that the relation between ϕ_c and f_m is not so clear as the relation between V_c and f_m .

In order to understand the axial dependence of the excited wave, wave spectrum is measured by moving the coaxial probe along the plasma column. Three wave patterns of the excited waves are shown in Fig.4.7(b), where three values of V_m correspond to V_c at each f_m , respectively. It is found that throughout the plasma column every amplitude of the excited wave is larger than the amplitude of E_4 .

The change of the beam distribution function is observed in relation to the wave suppression. Figure 4.8 shows the distribution function $F(V)$, measured by the analyzer, corresponding to the case shown in Fig.4.7. The bottom trace of the dashed line is $F(V)$ without beam modulation. Comparing this with the deformed

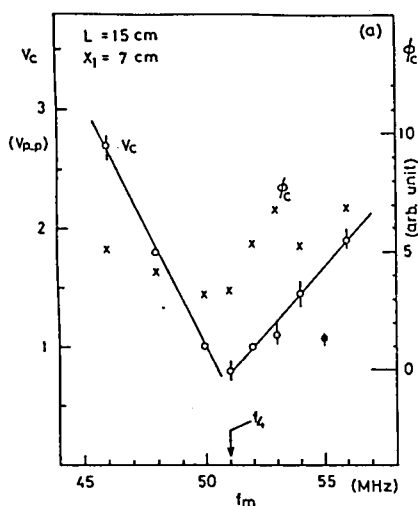
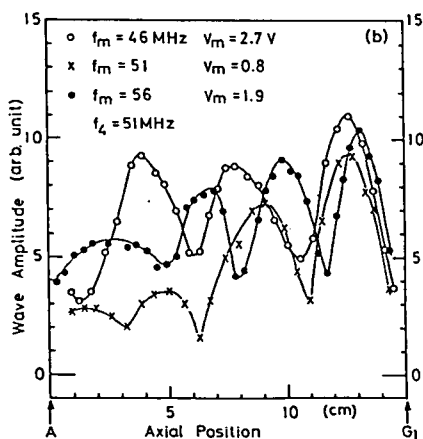


Fig.4.7 (a) Critical modulation voltage V_c and the resulting amplitude ϕ_c of the excited wave as a function of f_m . Electron wave E_4 with f_4 (51 MHz) is excited spontaneously. For reference, the amplitude of E_4 without modulation is indicated by the dotted circle. The probe is kept X_1 from A.



(b) Three amplitude patterns of the excited waves with different f_m . They are measured by moving the probe along the plasma column. In all cases, E_4 is suppressed almost completely because magnitudes of V_m correspond to those of V_c shown in Fig.(a).

Plasma parameters are as follows:
 $L = 15$ cm, $V_B = 42$ V, $I_B = 2.8$ mA,
 $V_G = -10$ V and $p = 1.4 \times 10^{-4}$ Torr.

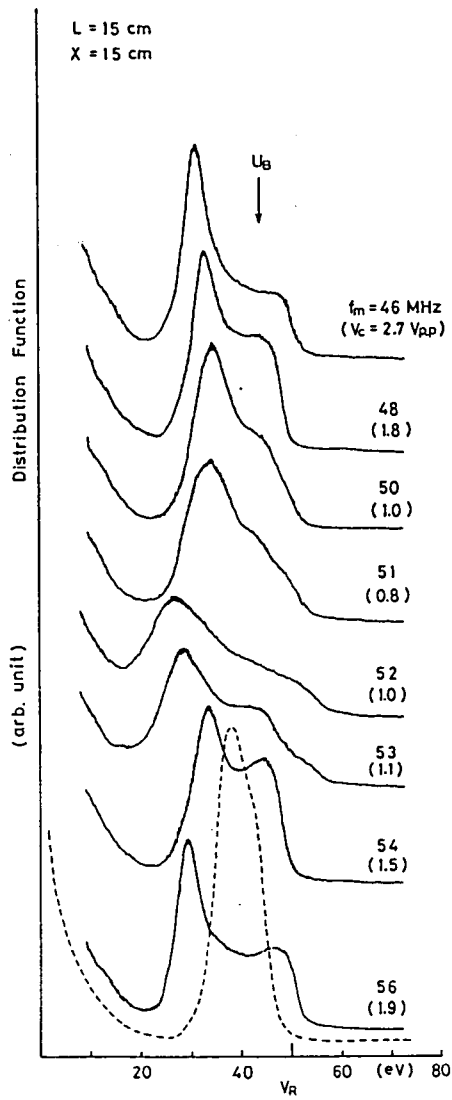


Fig.4.8 Beam distribution function in the fullfillment of wave suppression. These are measured by using the analyzer. Both f_m and V_c are parameters. Their values and experimental conditions are the same as in Fig.4.7.

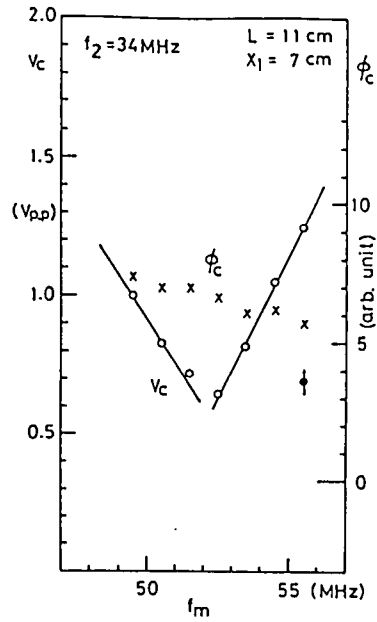


Fig.4.9 V_c and ϕ_c as a function of f_m .
 f_m is set to be in the vicinity
of f_3 which is the frequency of
the mode neighboring with E_2 .
Parameters: $L = 11$ cm, $V_B = 35$
V, $I_B = 4.5$ mA, $V_G = -10$ V and
 $p = 1.8 \times 10^{-4}$ Torr.

distribution, it is found that additional broadening or the degree of the deformation of $F(V)$ becomes remarkable as f_m approaches to f_4 . However, at the same time, the values of V_c become minimum when $f_m = f_4$.

These features shown in Figs.4.7 and 4.8 are also obtained in the case where f_m is chosen to be nearly equal to the frequency of the neighboring mode. For example, Fig.4.9 shows the relationship between V_c and f_m , and ϕ_c and f_m in this case, respectively.

In order to clear the relation between the wave suppression and deformation of $F(V)$, wave patterns and beam distributions are also measured simultaneously. A value of V_m is a parameter. A certain example of these simultaneous measurements is shown in Figs.4.10 and 4.11. In Fig.4.10, wave patterns of the electron wave E_2 and those of the excited wave E_m are shown for various modulation voltages. With increasing V_m , the amplitude of E_m becomes large and that of E_2 becomes small. It is found that no remarkable shift of wave pattern occurs in the axial direction, namely amplitudes of waves are varied by beam modulation without changing positions of their nodes and antinodes. Measurements of $F(V)$ corresponding to these wave amplitudes, are shown in Fig.4.11. The degree of the deformation becomes large as V_m is increased. It is important that a remarkable change of $F(V)$ is observed already before E_2 is suppressed. When $V_m = 2.1 V_{p-p}$, E_2 is almost suppressed. At the same time, $F(V)$ changes to be more broader and the shape of $F(V)$ is a double-humped distribution about the phase velocity of E_m .

It is not fruitful to discuss the fine structure of $F(V)$ further. Because, the final shape of $F(V)$ depends strongly on the

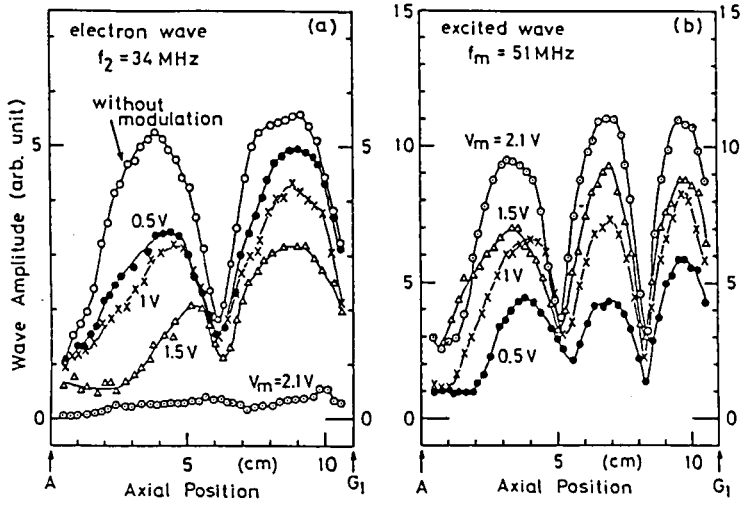


Fig.4.10 Amplitude patterns of the electron wave E_2 in (a) and the excited wave E_m in (b) for various modulation voltages. In this case, $V_c = 2.1 V_{p-p}$. Parameters: $L = 11 \text{ cm}$, $V_B = 38 \text{ V}$, $I_B = 4.6 \text{ mA}$, $V_G = -10 \text{ V}$ and $p = 1.7 \times 10^{-4} \text{ Torr}$.

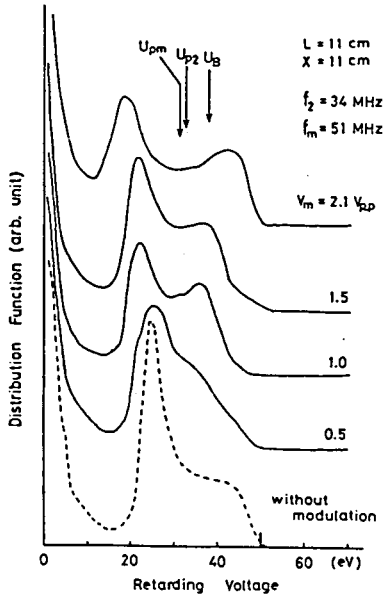


Fig.4.11 Deformation of the beam distribution due to the excited wave E_m , relating to the suppression of E_2 shown in Fig.4.10. The beam accelerating energy U_B , the phase velocity of the electron wave U_{p2} and that of the excited wave U_{pm} are indicated. The values of U_{p2} and U_{pm} are estimated from the wave patterns without modulation and that with $V_m = 2.1 V_{p-p}$, respectively.

shape of $F(V)$ without modulation although the change of $F(V)$ is caused by the presence of E_m . As is shown in Fig.4.3, the shape of $F(V)$ without modulation depends on some plasma parameters, i.e. V_B , p , L etc., which determine the character of the electron wave excited spontaneously in the plasma. We can only say that deformation of $F(V)$ occurs, and $F(V)$ becomes broader in relation to the suppression of the electron wave.

So far, only the deformation of $F(V)$, which are measured at the end of plasma column, is reported. Spatial dependence of the change of $F(V)$ is also measured. Figure 4.12(a) shows traces of $F(V)$ at various positions. As the amplitude of E_2 excited spontaneously is small in this case, suppression of E_2 occurs with weak modulation, i.e. $V_c = 0.5 V_{p-p}$. The energy at the peak of $F(V)$, obtained from Fig.4.12(a), and the amplitude of E_m are plotted versus axial position in Fig.4.12(b). On the change of $F(V)$ along the plasma column, there is a following tendency. The width of $F(V)$ becomes broader at a posterior region. Besides, $F(V)$ is velocity-modulated corresponding to the wave pattern of E_m . However, this type of deformation may be obtained clearly when suppression occurs with a small value of V_c and the mode number of the wave is small. Otherwise, broadening process becomes remarkable and the fine structure of velocity modulation is masked.

Figure 4.13 is another example concerning the spatial dependence of the change of $F(V)$. It shows deformation of $F(V)$ at three points, relating to the suppression of the electron wave E_3 . When $V_m = 1.1 V_{p-p}$, E_3 is almost suppressed. It is found that the degree of the change of $F(V)$ due to the presence of E_m becomes remarkable as the distance from A becomes large.

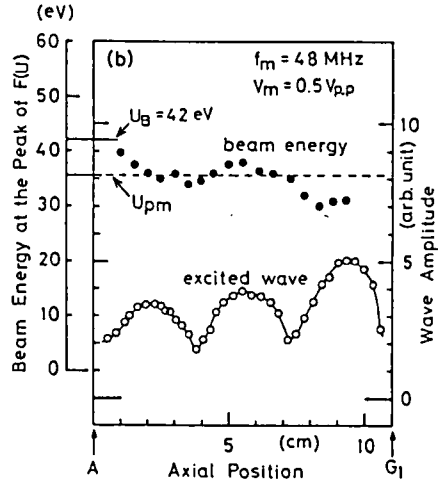
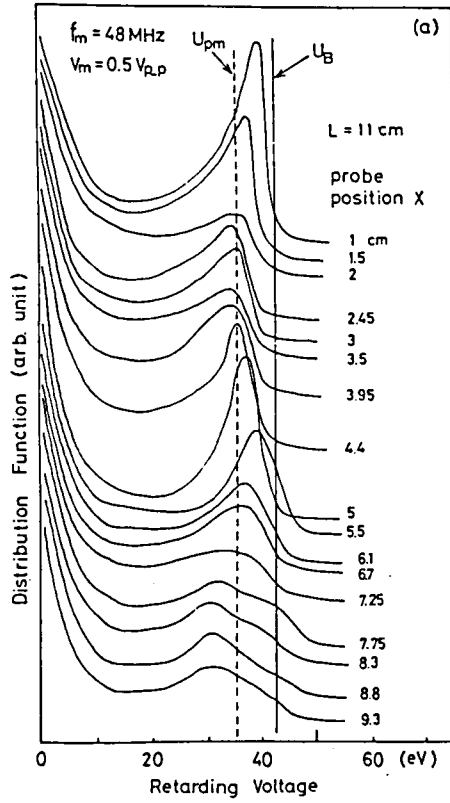


Fig.4.12 (a) Spatial variation of the beam distribution function in the presence of the excited wave. These traces are obtained by the axially movable plane probe. As the amplitude of E_2 with f_2 (37 MHz) is fairly small, suppression occurs with a weak modulation, i.e. $V_m = V_c = 0.5 V_{p-p}$. (b) Beam energy at the peak of the beam distribution function and the amplitude of E_m versus axial position, corresponding to the results shown in Fig.(a). U_B is the beam accelerating energy. U_{pm} is the phase velocity of E_m and is estimated from the wave pattern. Parameters: $L = 11$ cm, $V_B = 42$ V, $I_B = 5.1$ mA, $V_G = -10$ V and $p = 1.7 \times 10^{-4}$ Torr.

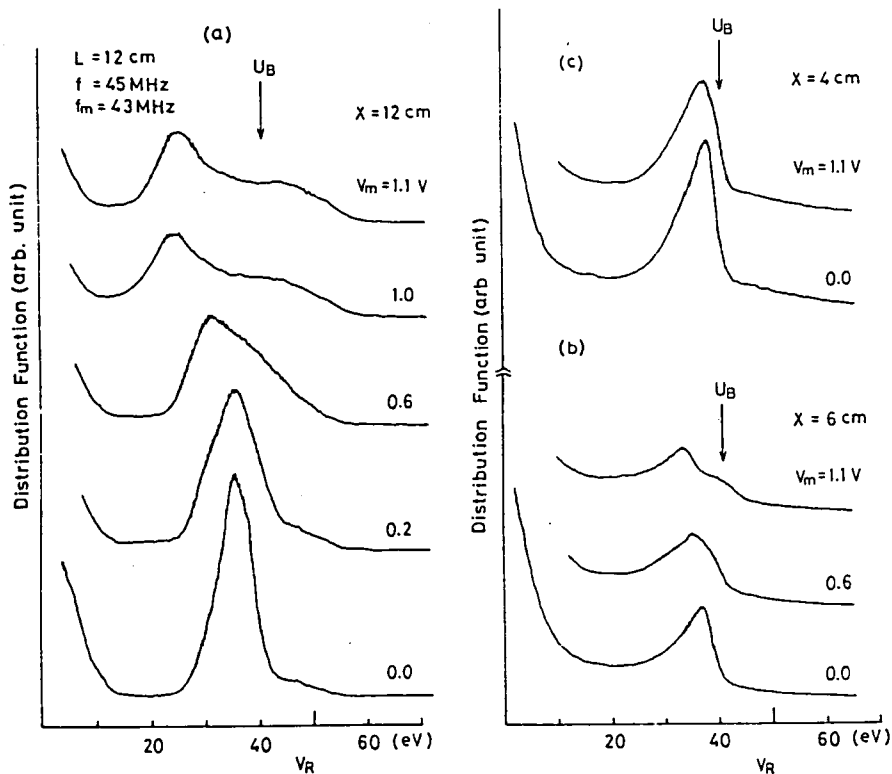


Fig.4.13 Deformation of beam distribution functions obtained at three different points, relating to the suppression of E_3 with f_3 (45 MHz). Distribution functions shown in (a) are obtained by using the analyzer, and those shown in (b) and (c) are by using the plane probe. Parameters: $L = 12$ cm, $V_B = 41$ V, $I_B = 4$ mA, $V_G = -10$ V and $p = 2.2 \times 10^{-4}$ Torr.

Finally, we note that the distribution function of the background plasma electrons shows no appreciable change throughout this experiment for wave suppression.

4.3-3 Suppression of Electron Waves

in the Presence of Reflected Beams

When the voltage of G_1 is kept at floating potential which is nearly equal to V_B , beam electrons passing through the plasma column are reflected at G_1 , and feedback effects of reflected beams are present.

Typical example of wave suppression at various modulation frequencies are shown in Fig.4.14. In the present experiment, f_m is chosen so that the frequency difference between f_m and f_4 is smaller than 1 MHz. Then, wave suppression occurs with a rather weak modulation. With increasing V_m , the amplitude of the electron wave E_4 decreases as that of the excited wave becomes large. Values of V_c decreases as f_m approaches to $f_4 = 38.5$ MHz. As a whole, these characteristics are the same as those obtained in the case, where reflected beams are absent.

There are, however, some different features from ones reported in section 4.3-2. The resulting amplitude of the excited wave is nearly equal to the amplitude of E_4 . Namely, wave suppression may occur under the constant wave energy. It is also found that in the course of wave suppression a sideband wave[16] with frequency $2f_4 - f_m$ or $2f_m - f_4$ appears more remarkably than one does in the absence of reflected beams.

Relationship between V_c and f_m is shown in Fig.4.15. The re-

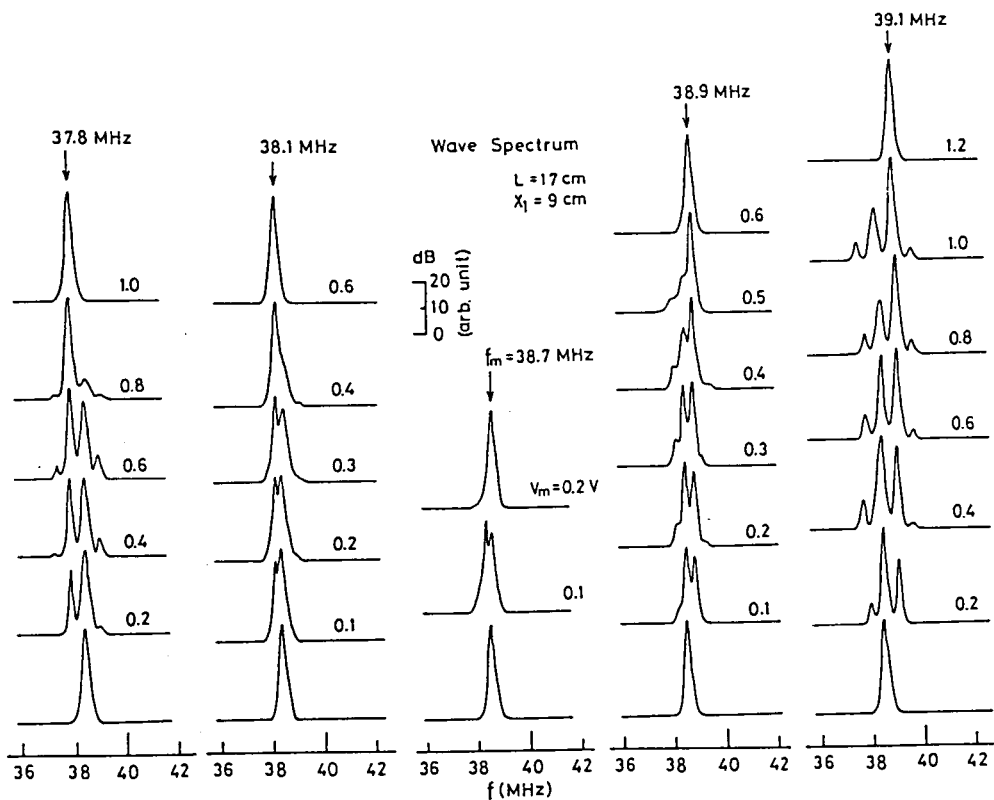


Fig.4.14 Typical examples of wave suppression in the presence of the reflected beams. f_m is a parameter. The electron wave E_4 with f_4 (38.5 MHz) is excited spontaneously. Waves are picked up by the coaxial probe which is 9 cm from A. The spectrum of E_m is indicated by the arrows. Parameters: $L = 17 \text{ cm}$, $V_B = 62 \text{ V}$, $I_B = 2.95 \text{ mA}$, $V_G = -37 \text{ V}$ and $p = 3 \times 10^{-4} \text{ Torr}$.

sulting amplitude of the excited wave ϕ_c is also plotted. The frequency width Δf at half-maximum of frequency spectrum of E_4 is about 0.5 MHz. When the frequency difference $|f_4 - f_m|$ is within Δf , the value of V_c is relatively small. The amplitude of the excited wave corresponding to V_c is also comparable to that of E_4 , namely total wave energy has constant value. As $|f_4 - f_m|$ exceeds Δf , however, the value of V_c increases sharply. Then, the resultant amplitude of E_m becomes larger than one of E_4 and the change of $F(V)$ is observed.

In relation to the wave suppression, measurement of $F(V)$ has been done and the results are shown in Fig.4.16. Experimental conditions are nearly the same as ones in Fig.4.15. Parameters are f_m and V_c . The bottom trace is the distribution function without modulation. Obviously, wave suppression occurs without appreciable change in $F(V)$, when f_m is nearly equal to f_4 . However, as $|f_4 - f_m|$ exceeds Δf , values of V_c becomes large abruptly. Then, wave suppression is accompanied with deformation of $F(V)$.

Lastly, we note the followings. As is mentioned in section 4.3-1, we restrict ourselves to the study of the case where a single mode is excited spontaneously. When some electron waves are excited simultaneously, however, suppression process is not so simple as one described above. In many wave case, wave suppression may be accompanied with the deformation of beam distribution even if the injected beams are reflected at the other end. Detailed experiments concerning this point are not performed in the present study.

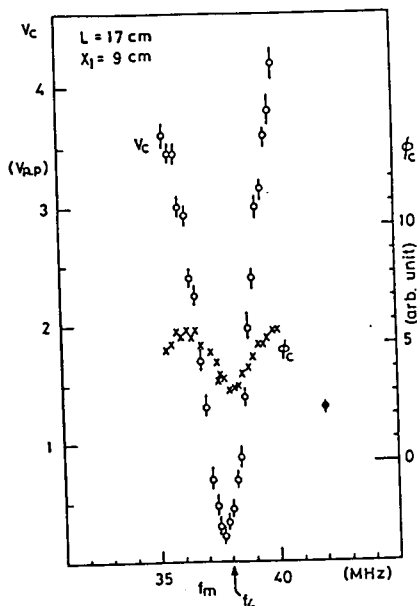


Fig.4.15 V_c and ϕ_c as a function of f_m . The amplitude of E_4 without modulation is indicated by the dotted circle. These wave amplitudes are obtained by the probe which is 9 cm from A. Parameters: $L = 17$ cm, $V_B = 58$ V, $I_B = 2.5$ mA, $V_G = -38$ V, $p = 3.3 \times 10^{-4}$ Torr, $f_4 = 38.2$ MHz and spectral width at half maximum $\Delta f \approx 0.4$ MHz.

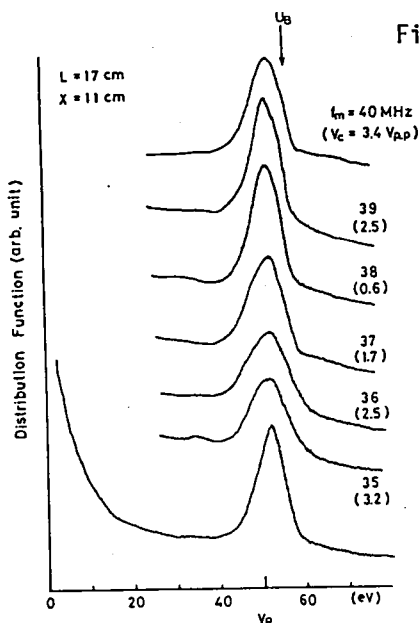


Fig.4.16 Beam distribution functions corresponding to the wave suppression. These are measured by the plane probe which is 11 cm from A. Bottom trace is the distribution function without modulation. Parameters: $L = 17$ cm, $V_B = 57$ V, $I_B = 2.6$ mA, $V_G = -37$ V, $p = 3.2 \times 10^{-4}$ Torr, $f_4 = 37.8$ MHz and $\Delta f \approx 0.4$ MHz.

4.4 Discussion

4.4-1 Beam-Excited Electron Waves

Observed electron waves are excited by the coupling of the slow space charge wave of the beam with the longitudinal electron wave of the plasma. When a beam-plasma system is bounded in both the longitudinal and transverse direction, a wave frequency ω can either be equal to the electron plasma frequency ω_{pe} or considerably lower than ω_{pe} depending on the relation between the wavelength λ and the transverse dimension, i.e. the beam radius a [17]. In the present system, λ is considerably larger than a . Dispersion relation can be written as follows:

$$\left[\frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pb}^2}{(\omega - kv_b)^2} \right] F^2 = 1, \quad (4-2)$$

and

$$F^2 = \frac{(ka)^2}{2} \ln\left(\frac{1}{ka}\right), \quad (ka \ll 1) \quad (4-3)$$

where ω_{pb} is the electron plasma frequency of the beam, v_b is the drift velocity of the beam, and k is the wavenumber. We assume that the motion of particles across the beam can be neglected. This is equivalent to the assumption that a rather strong magnetic field acts along the beam (electron cyclotron frequency $\omega_{ce} \gg \omega$). The ratio $\omega/\omega_{ce} \cong 1/7$ in our experiment, where $\omega/2\pi \cong 40$ MHz and $\omega_{ce}/2\pi \cong 280$ MHz. It is also assumed that both the beam and plasma are cold and that the radius of plasma is equal to that of beam. Then, the frequency of the electron oscillation is not ω_{pe} but $F\omega_{pe}$ in resonance. From electron density measured by the Langmuir

probe, the ratio of the observed oscillation frequency to ω_{pe} is determined. For example, in Fig.4.6(b) the electron wave E_2 with frequency $f_2 = 34$ MHz is excited and the measured ratio $f_2/f_{pe} \cong 0.38$, where $n_e \cong 1 \times 10^8 \text{ cm}^{-3}$. The evaluated ratio $f/f_{pe} = F$ from eq.(4-3) is about 0.32, where $a \cong 1 \text{ cm}$ and $\lambda \cong 10 \text{ cm}$.

4.4-2 Wave Suppression

(A) Interaction between Two Waves

In section 4.3, we have presented the experimental results on suppression of beam-excited electron waves due to beam density modulation. They are summarized as follows: In the absence of reflected beams, deformation of the beam distribution function is observed in relation to suppression of the electron wave. In the presence of reflected beams, however, the electron wave can be replaced by the excited wave under the constant total wave energy and distribution function of beam electrons shows no appreciable change.

Besides, the manner of interaction between the electron wave and the excited wave depends strongly on whether or not reflected beams are present. This is clearly shown in Figs.4.17 and 4.18. It looks that, with increasing V_m , the excited wave increases and the electron wave decreases independently each other when reflected beams are absent (see Fig.4.17). Taking both this results and the deformation of beam distribution function (see Fig.4.11) into account, we obtain the following picture for the interaction between two waves: The two waves do not interact directly, but through a wave-particle-wave process. However, when reflected beams are present, interaction between two waves is strong (see Fig.4.18).

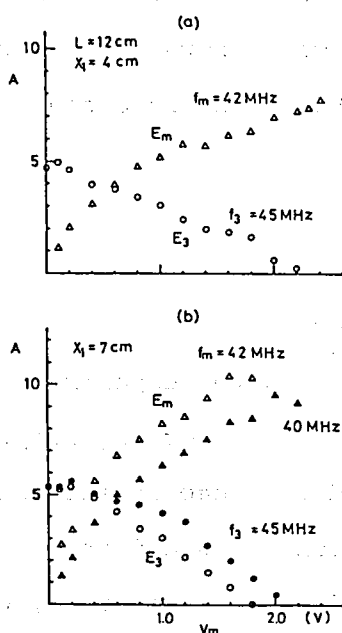


Fig.4.17 Wave amplitude vs V_m in the absence of the reflected beams. The electron wave E_3 with f_3 (45 MHz) is excited. The probe is 4 cm from A in (a), and 7 cm from A in (b). The amplitude of E_3 is represented by circles and that of E_m by triangles. Parameters: $L = 12$ cm, $V_B = 43$ V, $I_B = 4$ mA, $V_G = -10$ V and $p = 2.2 \times 10^{-4}$ Torr.

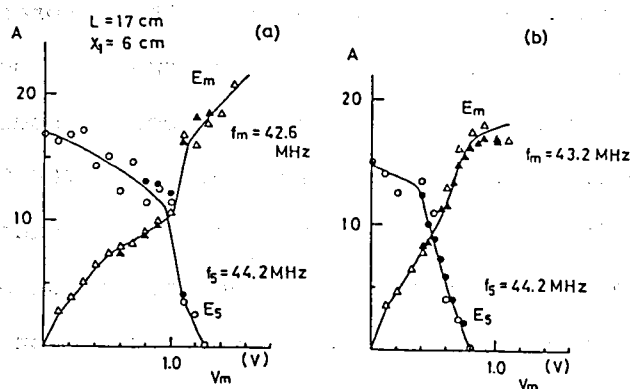


Fig.4.18 Wave amplitude vs V_m in the presence of the reflected beams. The electron wave E_5 with f_5 (44.2 MHz) is excited. The probe is 6 cm from A. f_m is 42.6 MHz in (a), and 43.2 MHz in (b), respectively. The amplitude of E_5 and that of E_m are represented by circles and triangles, respectively. Open symbols are measured by increasing V_m , and dotted ones are by decreasing V_m . Parameters: $L = 17$ cm, $V_B = 53.5$ V, $I_B = 3.4$ mA, $V_G = -28$ V and $p = 3 \times 10^{-4}$ Torr.

The most striking feature is that the rapid change in the ratio of wave intensities occurs when the excited wave grows up to a certain level with increasing V_m . This rapid change may be a consequence of the so-called mode competition[18].

(B) Comparison with Other Experimental Results

Here, we discuss the differences between our experimental results and some typical ones by other authors concerning suppression phenomena of the ion-acoustic wave[3] and the beam-excited electron waves[4,12]. As the beam-excited electron waves and the ion-acoustic waves have the dispersion relation of almost same form, we think that a certain suppression mechanism of the one wave may be applicable to the other wave.

Keen and Fletcher[3] have reported the suppression of the ion-acoustic wave. The experiment is performed in the positive column of a neon arc discharge with a mercury-pool cathode. The plasma was contained in an axial magnetic field $B_0 \cong 200$ G. The instability has predominantly a single frequency, independent of B_0 , of $\cong 7.5$ kHz with a zero azimuthal mode number, and an axial wavelength $\lambda_z = 80$ cm. By applying the density perturbation externally, the ion-acoustic wave is quenched. Experimental results, i.e. behavior of the instability amplitude, critical amplitude of the external force vs modulation frequency etc., are explained by the solutions of the Van der Pol equation.

Nakamura has reported the suppression of two-stream instability by beam modulation[4]. The plasma is generated by the injected beam. At an end of the beam is provided a movable repeller. As

the voltage of the repeller is set equal to the beam acceleration one, this beam-plasma system is quite similar to ours of the case described in section 4.3-3. He also discussed the experimental results by using the model of the Van der Pol oscillator.

Although the suppression phenomena were shown to fit well to the Van der Pol equation, the mechanism is not clarified. The one reason is due to the derivation of the Van der Pol equation. The other reason is due to a lack of discussion concerning the relationship between wave suppression and particle dynamics. Usually, the equation is derived from the framework of the hydrodynamic approximation[3,4]. The nonlinear terms which are essential to the Van der Pol equation come from the source terms introduced supplementally in the continuity equation. This is given by

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = S ,$$

where n_0 is the zero-order density, n_1 is the perturbed density, v_1 is the perturbed velocity and S is a source term. The source term is taken to be of the following form,

$$S = \alpha n_1 - \beta n_1^2 - \gamma n_1^3 ,$$

where α is the linear growth rate, and β and γ are the nonlinear saturation coefficients. In Ref.3, the source term is interpreted as plasma heating, ionization, etc., caused by the large amplitude fluctuation in a plasma. But, this interpretation is not clear to us. Namely, relationship between the coefficients of the Van der Pol equation and plasma parameters or basic process of the

wave suppression is not clarified.

Our experimental results in the presence of reflected beams may be explained by the Van der Pol model. However, this model cannot describe the beam dynamics and the results in the absence of reflected beams must be considered by using another model. The deformation process of the beam distribution function is essentially based on the nonlinear beam-plasma interaction. Besides, experimental results suggest that feedback effects of reflected beams are quite important for wave suppression. Therefore, by means of not the Van der Pol model but a particle picture, we discuss our experimental results in both cases systematically.

On the other hand, Amemiya has studied the suppression phenomena of electron waves in connection with the measurement of beam distribution function[12]. He does experiment in two different beam-plasma systems. The one is the case of single beam, and the other is the case of two beams, i.e. a counter-stream type. In the case of single-stream type, when an external wave suppresses an electron wave, the beam distribution function is altered appreciably. In the counter-stream type, however, the beam distribution function shows no appreciable change. These two cases may correspond to our two experimental conditions, namely without and with reflected beams. The characters of two different processes in his experiments are the same as our results qualitatively. However, plasma conditions are quite different from each other. In his experiment, the ratio of collision frequency ν to the angular plasma frequency ω_{pe} is of the order of $2\pi\nu/\omega_{pe} = 0.2-0.4$ and the mean free path of the beam along the plasma column is about 3 cm. Because of the collision dominancy and the absence of a mag-

netic field, the relaxation of the beam has a strong dependence on axial distance and one dimensionality of the system is not strict. Contrary to his beam-plasma system, our system has a magnetic field and is considered as collisionless system because $2\pi v/\omega_{pe} = 5 \times 10^{-3}$. Then, suppression process must be discussed by a nonlinear beam-plasma interaction in collisionless regime.

4.4-3 Theoretical and Numerical Consideration for Suppression Mechanism

Throughout the present experiments, we control the plasma parameters so that the electron wave of single mode with narrow frequency spectrum is excited spontaneously. The excited wave due to beam modulation also has a narrow spectrum. In the course of wave suppression, it is also found that the electron wave and the excited wave predominate the nonlinear beam-plasma interaction. The higher harmonics of these two waves or the sideband modes are much smaller than the electron or the excited wave. Therefore, we shall consider that only two waves, namely the electron wave and the excited wave, determine the nonlinear behaviors of beam electrons in the bounded system.

In order to investigate the experimental results including nonlinear beam dynamics in the presence of two waves, the method of partial simulation is used[19,20]. This technique assumes that the plasma response remains linear and can be described through its dielectric properties, while the beam trajectories are calculated exactly. In the present analysis, the plasma is described through the linearized fluid equations. The details of the model

is described in Chapter 3.

(A) Numerical Model and Basic Equations

The present model simulates a one-dimensional beam-plasma system with the length L , where beam electrons are injected continuously at the left end of the system, $x = 0$. According to experimental conditions, two different boundary conditions for the beam particles are imposed at the right end of the system, $x = L$. The one is the case of absorption (the Case I). Beam particles which drift out the right end are removed. The other is the case of reflection (the Case II). Beam particles which drift out the right end are reentered into the system with conditions, i.e. position $x_j = 2L - x_j$ and velocity $v_j = -v_j$ of the j -th beam particle. These reflected beams which drift out the left end are removed. As the electron wave excited spontaneously is in the saturated state before the application of the beam modulation, we develop a model in a spatially homogeneous beam-plasma system, where waves grow temporally and nonlinearities evolve with time.

Periodic boundary conditions are imposed on waves. Then, we assume the following form of the electron wave including the effect of backward wave,

$$E(x,t) = E_k(t) \exp(-i\omega_k t) (e^{ikx} + R e^{-ikx}) + \text{C.C.} , \quad (4-4)$$

where $E_k(t)$ is a slowly varying function, $k = 2\pi m/L$, m is an integer, ω_k is the characteristic frequency of the mode k , and R is the reflection coefficient. Effect of beam modulation is introduced

into Poisson's equation in the form of an external oscillating charge density, ρ_m ,

$$\rho_m = \rho_0 \cos(\omega_m t) [\delta(x-x_0+\Delta X/2) - \delta(x-x_0-\Delta X/2)] , \quad (4-5)$$

where ρ_0 is external charge density, ω_m is a modulation frequency, x_0 is the position of near the left end, and a pair grid with spacing ΔX is located at x_0 . Here, we consider the case that ω_m is nearly equal to ω_k , i.e. $\omega_m = \omega_k + \delta\omega$ and $\delta\omega \ll \omega_k, \omega_m$. We also assume the following form of the excited wave, i.e.

$$F(x,t) = F_k(t) \exp(-i\omega_m t) (e^{ikx} + \text{Re}^{-ikx}) + \text{C.C.} , \quad (4-6)$$

where $F_k(t)$ is a slowly varying function. Time evolution of $E_k(t)$, $F_k(t)$, and the position $x_j(t)$ and the velocity $v_j(t)$ of the j -th beam particle are solved numerically. The orbits of the beams are calculated in the single wave approximation before the application of beam modulation. After applying modulation, they are solved in the two wave approximation. The set of equations to be solved is as follows:

$$\begin{aligned} \frac{d^2 E_k}{dt^2} - i2\omega_k \frac{dE_k}{dt} &= \frac{4\pi en_b}{N} \left(\frac{d}{dt} [\sum_j v_j \exp(-ikx_j)] \right) \exp(i\omega_k t) \\ &- \frac{4\pi\omega_m^2}{kL} \rho_0 \exp(-ikx_0) \left(\exp[i(\omega_m + \omega_k)t] + \exp[-i(\omega_m - \omega_k)t] \right) \sin\left(\frac{k\Delta X}{2}\right), \end{aligned} \quad (4-7)$$

$$\begin{aligned} \frac{d^2 F_k}{dt^2} - i2\omega_m \frac{dF_k}{dt} - (\omega_m^2 - \omega_k^2) F_k &= \frac{4\pi en_b}{N} \left(\frac{d}{dt} [\sum_j v_j \exp(-ikx_j)] \right) \exp(i\omega_m t) \\ &- \frac{4\pi\omega_m^2}{kL} \rho_0 \exp(-ikx_0) [\exp(i2\omega_m t) + 1] \sin\left(\frac{k\Delta X}{2}\right), \end{aligned} \quad (4-8)$$

and

$$\frac{d^2 x_j}{dt^2} = -\frac{e}{m} E_k [\exp(ikx_j) + \text{Rexp}(-ikx_j)] \exp(-i\omega_k t) + \text{C.C.}$$

$$- \frac{e}{m} F_k [\exp(ikx_j) + \text{Rexp}(-ikx_j)] \exp(-i\omega_m t) + \text{C.C.}, \quad (4-9)$$

where n_b is the beam density and N is the total number of beam particles in the system at the initial stage.

Before presenting the numerical results, we consider further the terms of the right hand sides in eqs.(4-7) and (4-8), which represent the effects of beam electrons. By taking into account perturbed orbits of beam particles, the right hand sides can be represented in terms of $E_k(t)$ and $F_k(t)$. Then, nonlinear rate equations for $E_k(t)$ and $F_k(t)$ are obtained and the role of reflected beam electrons is also clarified. After some algebra(described briefly in Appendix), we obtain the nonlinear rate equations of the following forms:

$$\frac{dE_k}{dt} = \alpha_1 E_k - \beta_1 E_k^3 - \theta_{12} F_k^2 E_k, \quad (4-10)$$

$$\frac{dF_k}{dt} = \alpha_2 F_k - \beta_2 F_k^3 - \theta_{21} E_k^2 F_k, \quad (4-11)$$

where α_1 and α_2 are the linear growth rates, and β_1 , β_2 , θ_{12} and θ_{21} are the nonlinear saturation coefficients, respectively. In the two wave system described above, the mode competition arises from the θ terms[18,21]. It is well known that the system is weak coupling when $\beta_1 \beta_2 > \theta_{12} \theta_{21}$, and that the system is strong coupling when $\beta_1 \beta_2 < \theta_{12} \theta_{21}$. The Van der Pol oscillator prefers operation at a single frequency and exhibits hysteresis phenomena as in the

case of strong coupling[21]. It is not easy to judge whether the system is weak or strong coupling without a detailed calculation. However, when the reflected beams are present, the θ terms become large due to the effects of interaction between the reflected beams and the backward waves. Consequently, the mode coupling may become strong. It is reasonable that the rapid change in the ratio of wave intensities occurs in the system where reflected beams are present (see Figs. 4.17 and 4.18).

Nonlinear rate equations similar to our results described above have also been derived by some authors. Walsh and Hagelin[22] obtain a kind of the Van der Pol equation by expanding the beam-plasma dispersion relation around the most unstable root and the perturbed orbit of the beam electron. Janssen and Rasmussen[23] show that the dynamical equation for the bump-on-tail instability is given by the well-known nonlinear Landau equation. In both cases, wave-particle interaction may be considered as a candidate for the origin of the nonlinear terms.

(B) Numerical Results

Now, we are in the position to report the results of simulation. Dimensionless variables are introduced as follows: The temporal coordinate $\tau = s\omega_k t$, the normalized electric field $E(\tau) = ekE(t)/(ms^2\omega_k^2)$, and the temporal scaling factor $s = (n_b/2n_e)^{1/3}$. Mean beam velocity $V(\tau)$ and the beam velocity spread ΔV_b are also obtained as a function of τ , i.e. eq.(3-23).

Numerical calculations are performed under various plasma conditions. Here, typical examples are presented and others are in

Chapter 3. Figure 4.19 shows the results in the Case I. At the initial stage, 200 beam electrons with velocity v_b are distributed uniformly in the system. The unstable electron wave E_k^2 grows exponentially and saturates its amplitude at $\tau = 10$. After this point, the wave has constant amplitude in the case of no modulation (shown by dashed line). Corresponding to the evolution of the wave amplitude, the beam spread ΔV_b also becomes large up to the saturation point of the wave amplitude and has nearly the constant spread in the later region. Beam modulation is introduced at $\tau = 14$. The initial value of $F_k(14)$ is $0.01E_k(14)$. Then, with the growth of the excited wave F_k^2 , the electron wave decreases in amplitude up to $\tau = 20.3$. After this time, energy exchange between the electron wave and the external wave occurs. The additional broadening of the beam distribution also occurs with the introduction of beam modulation. The temporal evolution of the beam spread corresponds to that of the total wave energy. The magnitude of the additional broadening is about 20-25 percent of the beam spread without modulation.

Figure 4.20 shows the result in the Case II. At the initial stage, 100 beam electrons with velocity v_b and 100 with $-v_b$ are distributed uniformly in the system. The electron wave grows up and saturates its amplitude at $\tau = 14$. The saturation level is slightly lower than that in the Case I. Correspondingly to the wave growth, the spread of the forward beam becomes large till $\tau = 14$. After this time, beam distribution keeps its spread nearly constant as the wave has nearly the constant amplitude. Beam modulation is applied at $\tau = 18$ when the wave amplitude is saturated already. The electron wave decreases in amplitude up to $\tau = 28$,

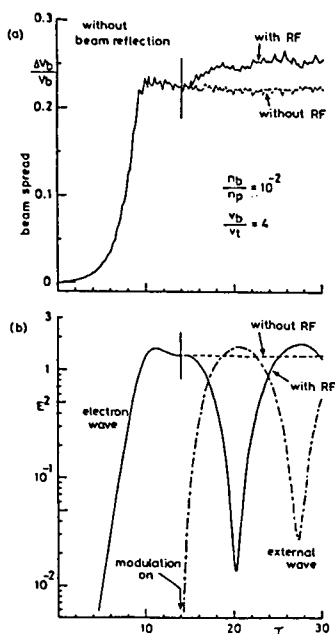


Fig.4.19 Evolution of wave energy and beam spread in a modulated beam-plasma system, where reflected beams are absent. Beam modulation is introduced at $\tau = 14$. The orbits of the beams are solved in the single wave approximation before the application of beam modulation. After this, the orbits are solved in the two wave approximation. Parameters are as follows: $n_b/n_e = 10^{-2}$, $v_b/v_{te} = 4$, $R = 1.0$ and $m = 4$.

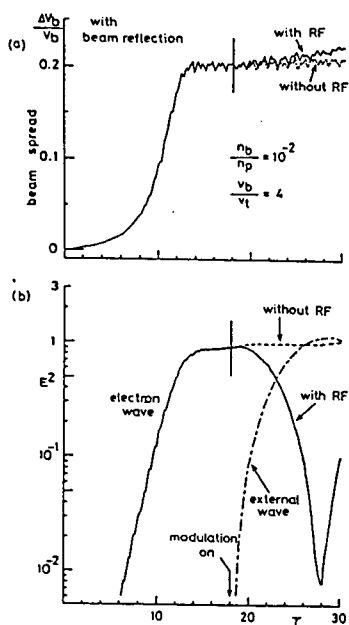


Fig.4.20 Evolution of wave energy and beam spread of the injected beams in a modulated beam-plasma system, where reflected beams are present. Beam modulation is introduced at $\tau = 18$. Parameters are the same as ones in Fig.4.19.

while the excited wave grows larger and larger. After this time, energy exchange between two waves also occurs as that does in the Case I. However, there are some different features in the Case II. In spite of the excitation of the external wave, total wave energy has nearly the constant value in the early stage although it increases gradually after the time $\tau = 25$. Corresponding to the evolution of total wave energy, the spread of beam distribution has nearly the constant value. Namely, up to the time $\tau = 28$, the change of wave intensities between the electron wave and the excited wave occurs under nearly the constant value of the total wave energy and the additional broadening of the beam distribution does not occur appreciably.

(C) Comparison between Experimental and Numerical Results

Experimentally, whether or not beam modulation is applied, the system has a steady state. Measurements of wave amplitudes and beam distribution function are carried out under the steady state, where the electron wave and the excited wave with constant amplitudes corresponding to various strength of beam modulations, respectively, are present. A certain steady state is determined by the nonlinear behaviors of beam electrons injected continuously in the presence of the two wave fields. When the suppression of the electron wave occurs, the beam electrons are locked to the bounded state caused by the excited wave.

In the simulation, the steady state is observed when beam modulation is absent, but it is not observed when beam modulation is present. As is discussed in Chapter 3, this may be due to the

lack of support for the perfect phase lock in the code. In the present simplified model, boundary conditions of the beam and those of the waves including the reflection coefficients are not determined self-consistently. It is also assumed that two waves have the same wavenumber.

Although there are some faults in this model, the numerical results in the early stage of the excited wave, i.e. up to $\tau = 20.3$ in the Case I and up to $\tau = 28$ in the Case II, reproduce the salient features of the experimental results. They are as follows: (1) When the reflected beams are absent (Case I), additional broadening of the beam distribution occurs in relation to the suppression of the electron wave. (2) When the reflected beams are present (Case II), suppression of the electron wave occurs under the constant width of the beam distribution. (3) The saturation level of the electron wave in the Case I is larger than that in the Case II. These features are the same as those of experimental results.

Concerning the additional broadening of the beam distribution, we have obtained the qualitative agreement between experimental and numerical results. But, there is a difference between them quantitatively. In simulation, the ratio of additional broadening of beam spread to the spread in the absence of modulation is about 20-25 percent. On the other hand, in experiments, this ratio is about 40-70 percent, where the data in Fig.4.8 are used and the magnitude is estimated from the width at half-maximum in the beam distribution function. This difference may be caused by the followings. The numerical result is the mean value in the system, which is defined by eq.(3-23). According to the results shown in Fig.4.13, the degree of the deformation becomes large with increas-

ing the distance from the anode A. Therefore, from the viewpoint of mean value in the system, the obtained value from the data in Fig.4.8 is overestimated because beam distribution functions are measured at the end of the plasma column. Another cause may be as follows: In experiment, the electron wave is suppressed almost completely. Namely, the ratio of the reduced amplitude to the initial one is about 0.01. On the other hand, in simulation, the ratio is from about 0.1 to 0.05. So, deformation and additional broadening may be smaller than ones in experiment. At any rate, even if we take these situations into account, the experimental value of additional broadening is larger than the numerical one.

Next, we estimate the reflection coefficient R in the present experiment. The value of R is obtained from the voltage standing wave ratio R_{vswr} of the wave pattern. For this end, amplitudes at the anti-node and the node nearest to the grid G_1 are used because the effects of reflection appear most remarkably there. When the beam electrons are absorbed at G_1 (Case I), R_{vswr} is from 2.2 to 3.1. Then, R is from 0.37 to 0.51. When the beam electrons are reflected at G_1 (Case II), R_{vswr} is from 5.8 to 7. Then, R is from 0.7 to 0.75. Numerical results in Figs.4.19 and 4.20 are calculated when $R = 1$. Runs are made for several values of R with keeping the beam conditions same. In the Case I, evolutions of both the wave and beam spread are not dependent on the magnitude of R . Principal features are the same as those in Fig.4.19. In the Case II, however, the additional broadening of beam spread due to the excited wave becomes remarkable with decreasing the magnitude of R . These results suggest that two different processes in the course of wave suppression, shown in Figs.4.19 and 4.20, are

caused by feedback effects due to the interaction between the reflected beams and the backward wave.

In many situations, collisions between plasma electrons and neutral atoms or ions play an important role in the development of a beam-plasma instability. They can become the deciding factor in limiting the temporal or spatial growth and determining its magnitude. In cases where the beam has a finite velocity spread, collisions drastically reduce the growth rate, and the delicate interplay of collision frequency and velocity spread of the beam can lead to a quenching of the instability[24].

In the present system, electron-neutral collision is predominant. We estimate the collision frequency ν_{e-n} under the following conditions: $kT_e \cong 3$ eV, $p = 2 \times 10^{-4}$ Torr and $n_e \cong 1 \times 10^8$ cm $^{-3}$. Thermal velocity of the plasma electron v_{te} is about 1.03×10^8 cm/sec. Mean free path of plasma electron λ_{mfp} is about 250 cm. Then, $\nu_{e-n} = v_{te}/\lambda_{mfp} = 0.43$ MHz and $2\pi\nu_{e-n}/\omega_{pe} = 4.6 \times 10^{-3}$. So, this system is considered as the collisionless system and the collisional effects is not so important as ones in other system[12,24]. However, we have shown that, in the semi-infinite space, nonlinear interaction between a small cold beam and a plasma can be changed drastically by introducing weak collisions of the order of $\nu/\omega_{pe} \cong 10^{-3}$ within the background plasma[25]. In the present model, numerical calculation with collisional effects has also been performed when $\nu/\omega_{pe} = 5 \times 10^{-3}$. But, in the bounded system, anomalous effects due to weak collisions do not occur except that the saturation amplitude decreases to a slightly lower level than the saturation level in the collisionless case.

Finally, we think that present consideration may be applicable

to the suppression phenomena of the ion-acoustic wave which has the same dispersion relation as the beam-excited electron wave.

4.5 Concluding Remarks

We have investigated the suppression phenomena of beam-excited electron waves in the bounded beam-plasma system. Experimental conditions have been controlled carefully so that a single mode of the electron wave with narrow frequency spectrum is excited spontaneously. Two types of wave suppression processes are observed. They depend on whether feedback effects of reflected beams are present or not.

In the absence of reflected beam electrons, deformation of the beam distribution function is observed in relation to the suppression of the electron wave. When beam electrons are reflected, however, the excited wave suppresses the electron wave but beam distribution function shows no appreciable change. In these two cases, distribution function of background plasma electrons shows no appreciable change. It is also found experimentally that only the electron wave and the excited wave predominate in the process of the wave suppression.

By using the method of partial simulation, interactions between two waves including nonlinear motions of beam electrons is studied. Salient features of the experimental results in two cases, namely the case with and the case without reflected beams, are corroborated by the numerical solutions.

These experimental and numerical results show that nonlinear

behaviors of beam electrons are quite important for deciding the suppression process of the wave.

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Chapter 5

EXCITATION AND SUPPRESSION OF A MULTIMODE BEAM-PLASMA INSTABILITY IN A BOUNDED SYSTEM

5.1 Introduction

Interaction between a modulated electron beam and a plasma have been investigated experimentally by Russian authors[1-3] and Obiki et al.[4]. These experimental results show that wave amplitudes and wave spectra can be controlled by pre-modulation of injected beams. Nakamura[5,6] for beam excited electron waves, and Keen and Fletcher[7] for ion-acoustic waves show, respectively, that suppression phenomena are interpreted phenomenologically by the model of the classical Van der Pol oscillator.

So far, suppression phenomena have been studied by many authors, and experimental results have been discussed based on the model of the Van der Pol oscillator[5-15]. However, the reason why the self-excited oscillations in plasmas obey the Van der Pol equation has not been discussed rigorously and not clarified. In order to investigate the suppression mechanism, quite recently, measurements of beam distribution function have been made by some authors[16-18], in relation to the suppression of beam-excited electron waves. These results suggest that nonlinear behaviors of beam electrons play an important role for wave suppression.

In Chapter 4, we have investigated the suppression mechanism

of beam-excited electron waves in connection with the beam distribution function, and shown that it is necessary to consider these phenomena in terms of particle picture. However, we study only the case where a single mode is excited spontaneously. There are few experimental studies on the suppression of a multimode beam-excited instability. Then, in this chapter, we investigate suppression phenomena of multimode instability.

On the other hand, the interaction between a high-frequency wave and a low-frequency one has also been studied by several authors. Jones and Emeleus[19] pointed out the possibilities of the interaction of an electron wave with an ion-acoustic wave. Fedorchenko et al.[20] observed that an electron plasma wave decayed into an electron plasma wave of another mode and an ion-acoustic wave of axial mode. This was the first experiment suggesting the possibility of the theory of decay instability[21]. Amagishi reported some observations of the high-frequency oscillation excited by the external low-frequency fields[22]. Itatani et al.[23-25] reported that an ion-acoustic wave could be excited through electron waves of beam modes by nonlinear wave-particle interaction, i.e. nonlinear Landau damping.

We have observed an excitation of multimode beam-plasma instability in a bounded system. At least, four or five electron waves of axial beam modes are observed simultaneously. They are also accompanied with an ion-acoustic wave of an axial mode under a certain plasma condition. Here, simultaneous suppression of electron waves and the ion-acoustic wave have been investigated experimentally[26]. Interaction between electron waves and ion-acoustic waves is also studied. We attempt to interpret the

present experimental results in terms of wave-particle interaction [27]. In section 5.2, the experimental apparatus and the procedure are described. Experimental results are given in section 5.3. Section 5.4 is devoted to discussions of experimental results. For the suppression of electron waves, a semi-quantitative explanation is introduced. Comparisons between the model and the experimental results are demonstrated. The results are summarized in the final section 5.5.

5.2 Experimental Set-up and Procedure

The experiment has been performed in a cylindrical discharge tube (made of glass) of the hot-cathode type with an electron gun. A conceptual drawing of the discharge tube is shown in Fig.5.1. The accelerating electrode E_3 of the gun works as an anode for the main discharge in pair with the hot-cathode K. A plasma is produced by means of a mercury vapor discharge and the background pressure is controlled by changing a temperature T_r of a mercury pool. In order to measure both an electron density n_e and an electron temperature T_e , we use a Langmuir probe (0.2 mm in diameter and 2 mm in length) inserted between a grid G and E_3 . Plasma parameters are as follows: Discharge current $I_D = 10\text{--}20$ mA, working pressure $p = 5 \times 10^{-4} - 1 \times 10^{-3}$ Torr ($T_r = 10\text{--}25$ °C), $n_e = 1 \times 10^8 - 6 \times 10^9$ cm $^{-3}$, and $kT_e \cong 2$ eV.

The electron beam is injected into the plasma along the axis through a hole of the anode, whose diameter is about 5 mm. Accelerating voltage V_B of the beam is from 10 to 50 V. Beam density

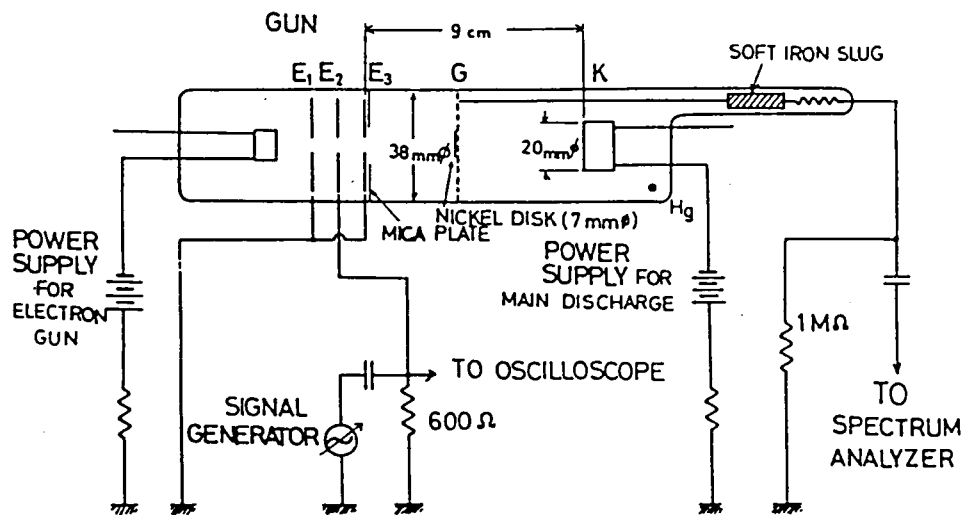


Fig.5.1 Schematic diagram of the experimental apparatus

n_b and n_e depend weakly on V_B . Three examples of current-voltage characteristics of the probe are shown in Fig.5.2. When the range of beam current I_B is controlled from 10 to 20 mA, n_b is changed from 10^8 to 10^9 cm⁻³. Present experiment is performed under the condition $I_B < 10$ mA. Then, the ratio n_b/n_e is from 0.01 to 0.1.

The movable mesh grid G is inserted between the cathode K and the anode E_3 in order to fix the boundary conditions of oscillations excited by beam-plasma instability. For the purpose of stopping the injected beam, a small nickel disk (about 10 mm in diameter) is mounted at the center of the mesh. This grid is also used as the probe. Beam velocity v_b and beam temperature T_b are estimated from the current-voltage characteristics of G. When the range of V_B is from 20 to 30 V, $kT_b = 1-2$ eV. On the other hand, beam energy is lower than the accelerating energy by about 6 eV. This may be due to the plasma production. As the present experiment is performed under this condition, here after we use the revised value for V_B , which is lower than the accelerating energy by amount of 6 V.

The first and the third electrodes, i.e. E_1 and E_3 , are grounded lest an applied external signal should affect the plasma directly. The external sinusoidal signal is applied to the electron gun by the second accelerating electrode E_2 so as to modulate the beam density.

Oscillation signals in the plasma are detected by both G and the Langmuir probe. Low-frequency oscillations are detected by G. Signals are led to both an oscilloscope and a spectrum analyzer. High-frequency oscillations are detected by the probe which is connected with a field-intensity meter. There is no external mag-

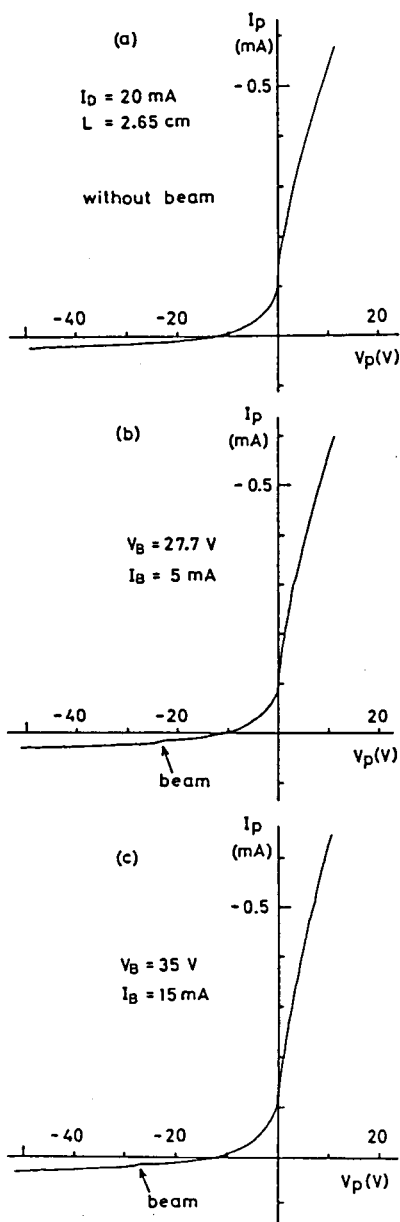


Fig.5.2 Current-voltage characteristics of the Langmuir probe for three different plasma conditions. Calculated values of electron density n_e and electron temperature T_e are as follows:
 (a) $n_e = 5.18 \times 10^9 \text{ cm}^{-3}$; $kT_e = 1.8 \text{ eV}$, (b) $5.68 \times 10^9 \text{ cm}^{-3}$; 2.2 eV , (c) $6.25 \times 10^9 \text{ cm}^{-3}$; 2.3 eV .

netic field.

5.3 Experimental Results

5.3-1 Spontaneous Oscillations

We have observed both high-frequency oscillations (550-700 MHz) and low-frequency oscillations (10-30 kHz) simultaneously under appropriate plasma conditions. When the electron beam is injected into the plasma, the floating potential of G becomes nearly equal to the beam accelerating voltage V_B . Therefore, the mounted nickel disk of G might play the same role as the repeller used in Ref.5. Reflection of the beam electrons at G would result in a closed amplifying feedback loop [28] and the system works as a self-sustained high-frequency oscillator.

(A) High-Frequency Oscillations

In Fig.5.3, the observed frequencies of high-frequency oscillations and the amplitude of the low-frequency oscillation whose frequency is 19 kHz are shown as a function of v_b . The magnitude of v_b is estimated by the relation $v_b = (2eV_B/m)^{1/2}$, where e and m are an electron charge and an electron mass, respectively. With increasing v_b , frequencies shift higher side.

To see the dependence of oscillations on the length L of the beam-plasma system, i.e. the distance between E_3 and G, we observe the oscillation spectra by changing L with keeping v_b and I_D constant. Detected oscillation frequencies are shown in Fig.5.4 as a function of L^{-1} . The oscillation has its maximum amplitude in

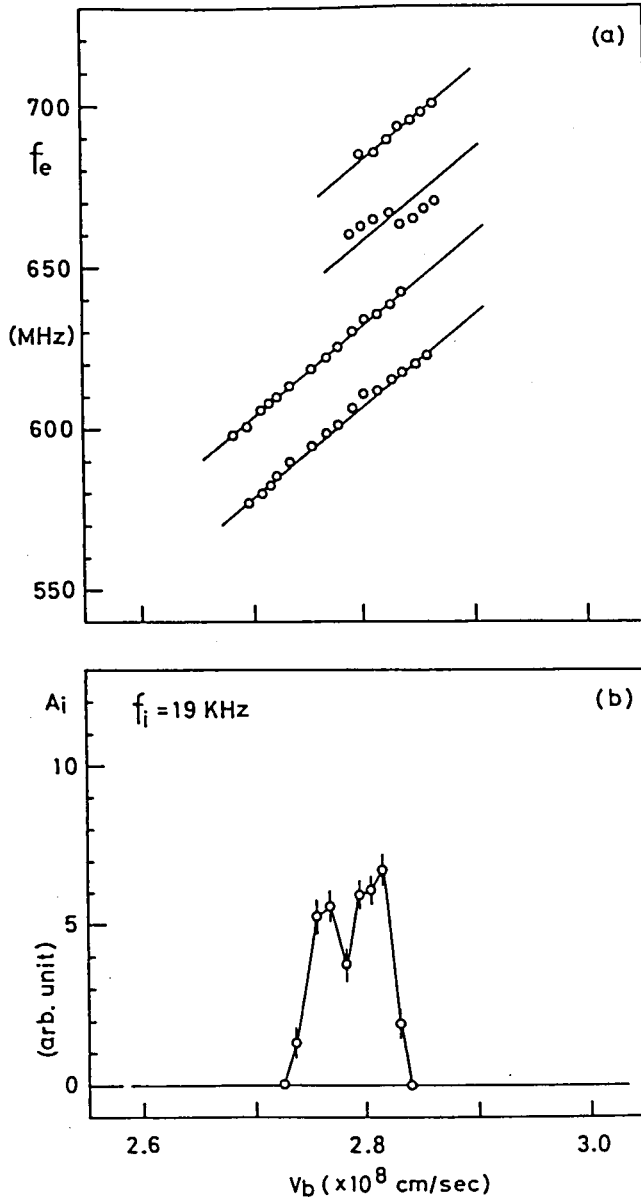


Fig.5.3 Observed frequencies of (a)high-frequency oscillations and (b)the amplitude of the low-frequency oscillation with frequency 19 KHz as a function of v_b . Plasma parameters are as follows: $I_D = 17$ mA, $L = 3.15$ cm and $T_r = 18.5$ °C.

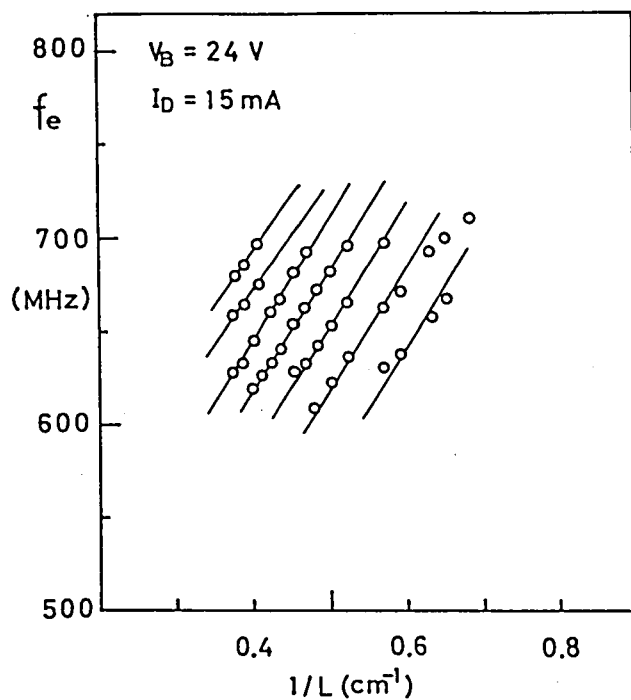


Fig.5.4 Observed frequencies of high-frequency oscillations as a function of L^{-1} . Plasma parameters are as follows: $I_D = 15 \text{ mA}$, $V_B = 24 \text{ V}$ and $T_r = 16.5 \text{ }^\circ\text{C}$.

the middle of the region where it is observed. In this case, frequencies shift higher side as L^{-1} becomes large.

A mode-spacing and a frequency-shift shown in Figs.5.3 and 5.4 were studied by Looney and Brown[29], Kofoed[28], experimentally, and explained theoretically by Sumi[30], considering the complex frequency derived from the dispersion equation with real wave-number. In a bounded system, a wavenumber is determined from the boundary conditions, i.e. L in our case. If a standing wave is excited, both a wavelength and a frequency should take the following discrete values, respectively,

$$n \frac{\lambda_n}{2} = L \quad \text{and} \quad f_n = v_{ph} \frac{n}{2L}, \quad (5-1)$$

where λ_n and f_n are the wavelength and the frequency of the n -th mode, respectively, v_{ph} is the phase velocity, and the mode number n is an integer. According to eq.(5-1), an each mode-spacing δf_n is nearly equal to the fundamental frequency f_1 and is proportional to L^{-1} , i.e.

$$\delta f_n = f_{n+1} - f_n = \frac{v_{ph}}{2L} = f_1. \quad (5-2)$$

Figure 5.3(a) shows that δf_n is equal to each other. Figure 5.4 shows that δf_n has a tendency to become large with the increase of L^{-1} . Therefore, the observed δf_n can be explained by the standing wave model.

Features of the experimental results on high-frequency oscillations are summarized as follows: i) Oscillations are observed when a ratio of v_b to the thermal velocity of the plasma electron v_{te}

lies in the range from 3 to 3.5, where $v_{te} = (2\kappa T_e/m)^{1/2}$ and $\kappa T_e \cong 2$ eV. ii) Ratios of the observed frequencies to the plasma frequency f_{pe} are from 0.8 to 0.95, where $f_{pe} \cong 700$ MHz ($n_e \cong 6 \times 10^9$ cm⁻³). iii) The frequency-shift is proportional to the product of v_b and L^{-1} . iv) The mode-spacing δf is from 25 to 35 MHz. v) Mode numbers are from 20 to 27 ($k\lambda_e$ is from 0.25 to 0.3, where λ_e is the Debye length). vi) Phase velocities of oscillations are nearly equal to each other and a ratio of v_{ph} to v_b is about 0.7 in the present case. vii) A ratio of velocity difference Δv between v_{ph} and v_b to the thermal velocity of beam electrons v_{tb} is from 1.0 to 1.2, where $v_b/v_{te} = 3-3.5$, $v_{tb} = (2\kappa T_b/m)^{1/2}$, $\kappa T_e \cong 2$ eV and $\kappa T_b \cong 1.5$ eV. These results are well explained by the solutions of dispersion equation of electron waves in the electron beam-plasma system (see eq. (5-3)).

(B) Low-Frequency Oscillations

As is shown in Fig. 5.3, the high-frequency oscillations, i.e. electron waves of beam mode in the electron beam-plasma system, are accompanied with a low-frequency oscillation. Figure 5.5 shows the observed frequency f_L as a function of L^{-1} . The frequency-shift of the low-frequency oscillation is observed only when L is changed. Beam velocity does not change the frequency of the oscillation. The wavelength λ_L is measured by means of phase-sensitive detection [31], and it is found that the low-frequency oscillation is a half-wavelength standing wave between E_3 and G. The components of higher harmonics, not shown in Fig. 5.5, are also observed. These low-frequency oscillations are identified with the

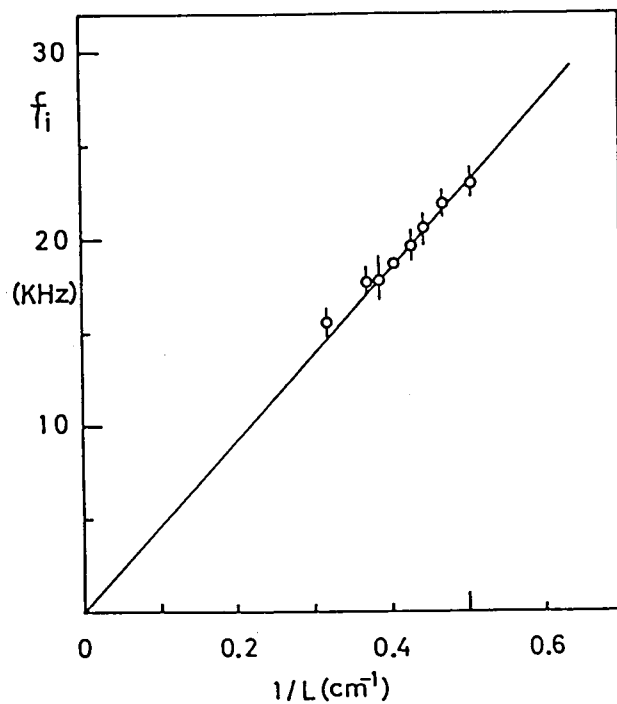


Fig.5.5 An observed frequency of a low-frequency oscillation as a function of L^{-1} . Plasma parameters are the same as in Fig.5.4.

ion-acoustic wave in the following ways. First, frequencies are proportional to L^{-1} , i.e. the wavenumber. Second, the phase velocity $f_L \lambda_L$ obtained from the slope of the solid line in Fig.5.5 coincides with the sound velocity C_s calculated from $\kappa T_e \cong 2$ eV. They are as follows:

$$f_L \lambda_L \cong 9.3 \times 10^4 \text{ cm/sec} \quad \text{and} \quad C_s = \left(\frac{\kappa T_e}{M} \right)^{1/2} \cong 9.7 \times 10^4 \text{ cm/sec},$$

where M is the ion mass.

Concerning the excitation of the ion-acoustic wave, the most remarkable point is as follows. The ion-acoustic wave can be observed only when electron waves are present. At the same time, v_b is about three times as large as v_{te} . Under these conditions, the linear dispersion relation of ion waves shows that ion-acoustic wave is never excited directly by the electron beam-plasma interaction. Excitation mechanism of the ion-acoustic wave has been studied in detail by Obiki[25]. Concerning this point, we will discuss briefly in the section 5.4-4.

5.3-2 Suppression of Waves

Under the conditions: $V_B = 23.5$ V, $I_B = 6.7$ mA, $I_D = 15$ mA, $L = 3.17$ cm and $T_r = 18$ °C, four electron waves and an ion-acoustic wave are observed simultaneously. This is shown in Fig.5.6. Frequencies of these observed electron waves are 587, 608, 636 and 668 MHz, respectively, and that of the ion-acoustic wave is 21 kHz. By applying a sinusoidal signal to E_2 , the beam density is modulated. A modulation frequency f_m is varied continuously from 500

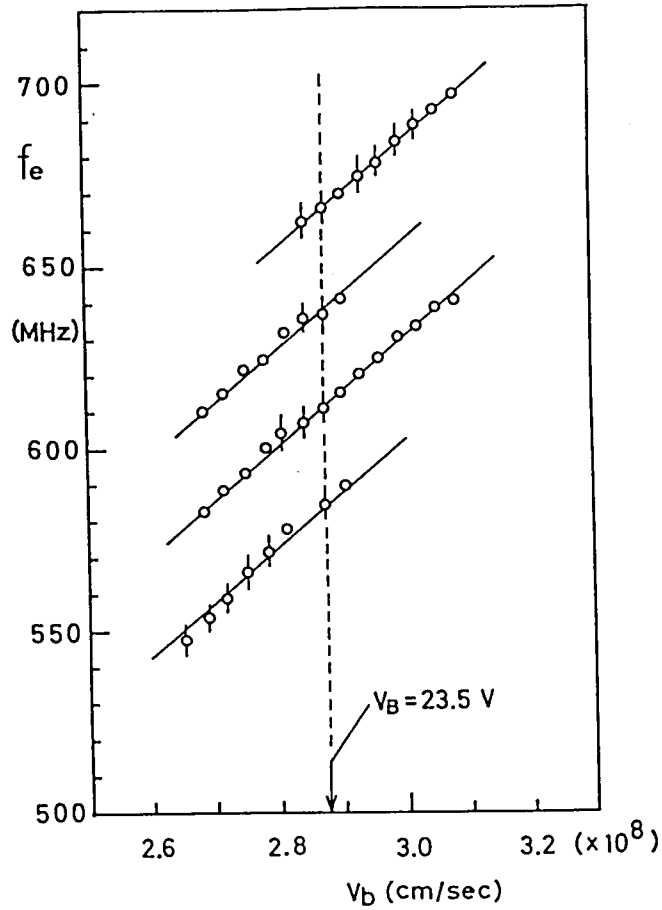


Fig.5.6 Frequency spectrum of the electron waves as a function of v_b . Plasma parameters are as follows: $I_D = 15$ mA, $L = 3.17$ cm and $T_r = 18$ °C.

to 700 MHz, while a modulation voltage V_m is kept constant.

All of the observed waves are reduced in their amplitudes by the beam density modulation when f_m is close to some frequency f_n of the observed electron waves. A typical example is shown in Fig.5.7. Throughout this experiment, there is no frequency-shift of electron waves. Figures 5.7(a) and (b) show the output signals of the field intensity meter as a function of f_m . Tuning frequencies are 587 MHz in (a) and 608 MHz in (b), respectively. Other choice of tuning frequency, i.e. 636 or 668 MHz, also behave like as ones shown in Fig.5.7. Figure 5.7(c) is the output signal of the spectrum analyzer tuned at 21 kHz. In this case, the variation of the amplitude of the ion-acoustic wave follows that of the electron wave with frequency 587 MHz.

As well as the amplitude of the electron waves, the amplitude of the excited wave generated by the beam modulation with f_m is also dependent on f_m . This is demonstrated clearly in Fig.5.8, where the amplitude of the electron wave A_e with frequency 608 MHz and that of the excited wave A_m are plotted, respectively. V_m is kept at a constant value, i.e. $0.05 V_{p-p}$. If f_m approaches to 608 MHz, the electron wave is reduced in its amplitude and at the same time the excited wave increases. When f_m is between 603 and 610 MHz, the electron wave is suppressed completely.

Figure 5.9 shows the output signals of the field intensity meter as a function of f_m . Tuning frequency is 595 MHz. Modulation voltage V_m is a parameter. If f_m is inside the frequency range indicated by two arrows, the electron wave is suppressed completely. But, when f_m is outside of this range, the wave is suppressed partially. The frequency range becomes wide with increas-

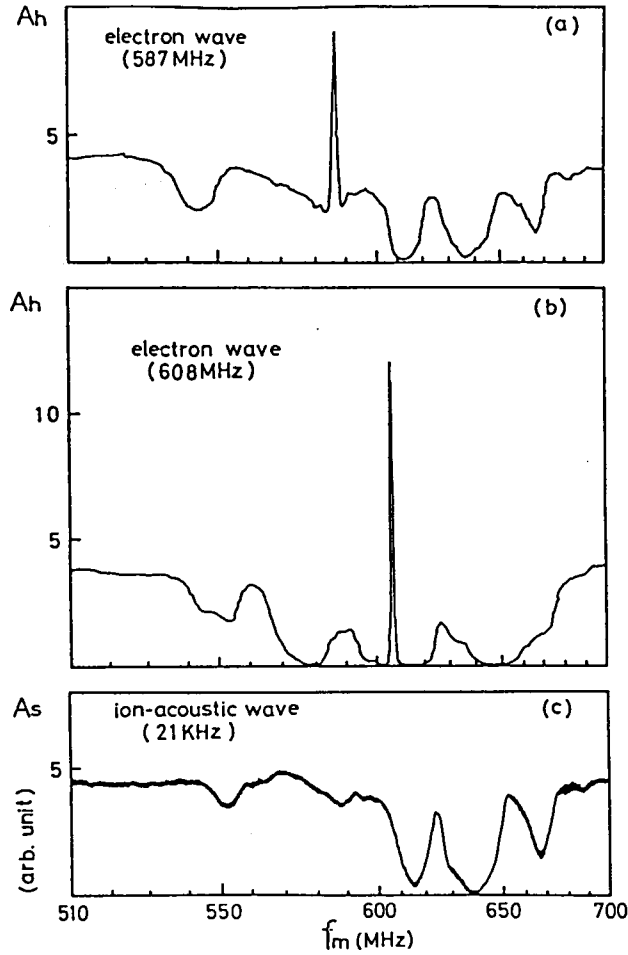


Fig.5.7 Behaviors of (a)the electron wave amplitude A_e (587 MHz), (b)the electron wave amplitude A_e (608 MHz) and (c)the amplitude of the ion-acoustic wave A_s (21 kHz) against f_m with a constant modulation voltage $V_m = 0.06 V_{p-p}$. Peak values near the tuning frequencies in (a) and (b) are the amplitudes of the excited wave generated by the beam modulation. Plasma parameters are as follows: $I_D = 15$ mA, $V_B = 23.5$ V, $I_B = 6.7$ mA, $L = 3.17$ cm and $T_r = 18$ °C.

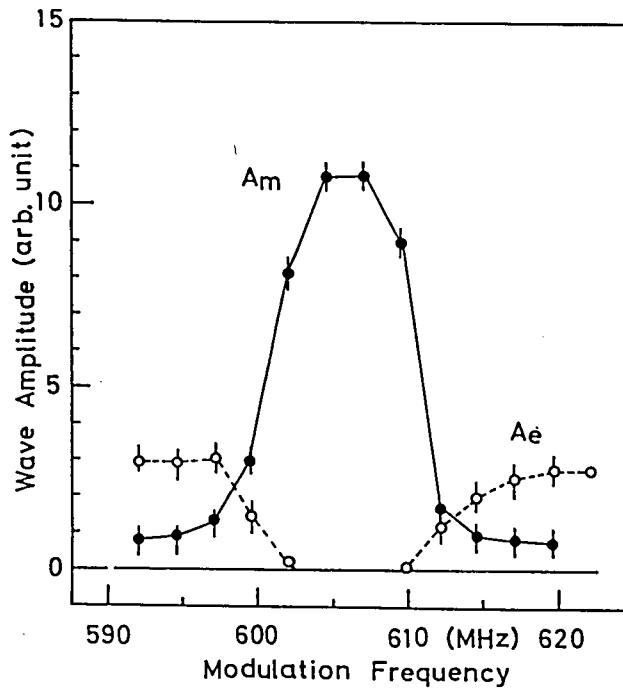


Fig.5.8 Dependences of the amplitude of the electron wave A_e (608 MHz) and that of the excited wave A_m on f_m with a modulation voltage $V_m = 0.05 V_{p-p}$. Plasma parameters are as follows: $I_D = 12$ mA, $V_B = 23$ V, $I_B = 6.7$ mA, $L = 3.1$ cm and $T_r = 19$ °C.

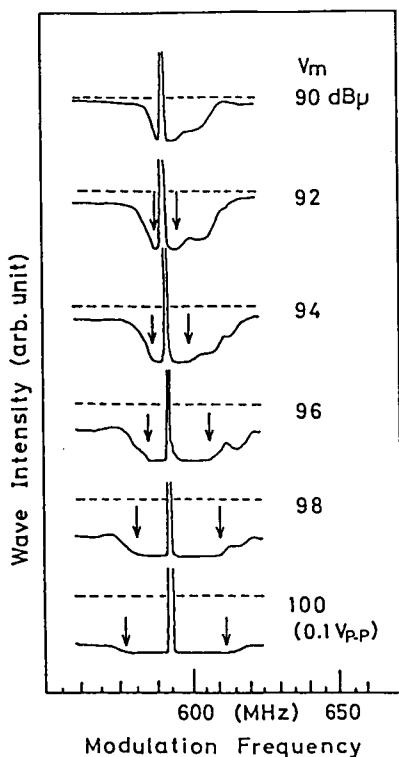


Fig.5.9 Behaviors of the electron wave amplitude A_e (594 MHz) against f_m for various modulation voltages.

Plasma parameters are as follows:

$I_D = 10.4$ mA, $V_B = 22.2$ V, $I_B = 5.3$ mA, $L = 3.1$ cm and $T_r = 18$ °C.

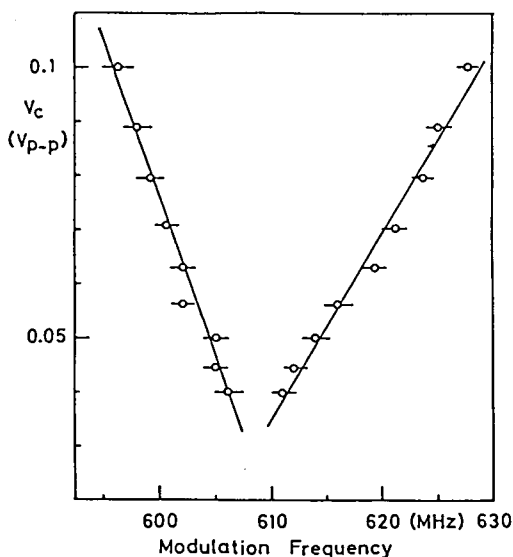


Fig.5.10 Critical modulation voltage V_c for the suppression of the electron wave (608 MHz) as a function of f_m .

Plasma parameters are as follows: $I_D = 15$ mA, $V_B = 23.5$ V, $I_B = 6.7$ mA, $L = 3.17$ cm and $T_r = 18$ °C.

ing V_m . From these measurements, we can obtain the relationship between the critical modulation voltage V_c and f_m . If V_m is superior to V_c , the electron wave is suppressed. Figure 5.10 shows V_c vs f_m . The magnitude of V_c becomes small as f_m is close to the frequency of the electron wave f_n , i.e. 608 MHz in this case. Experimentally, V_c is nearly proportional to a frequency difference between f_m and f_n , i.e. $|f_m - f_n|$.

5.4 Discussions

5.4-1 Electron Waves of Beam Modes in an Electron Beam-Plasma System

When an electron beam of density n_b , with a shifted Maxwellian distribution (drift velocity v_b and temperature T_b), is injected into a stationary plasma (density n_e and temperature T_e), the dispersion equation can be written as follows:

$$\epsilon(\omega, k) = 1 - \frac{k_e^2}{k^2} \frac{Z'(\alpha_e)}{2} - \frac{k_b^2}{k^2} \frac{Z'(\alpha_b)}{2} = 0, \quad (5-3)$$

where ω and k are an angular frequency and a wavenumber, $k_e^2 = 4\pi n_e e^2 / (\kappa T_e)$, $k_b^2 = 4\pi n_b e^2 / (\kappa T_b)$, $\alpha_e = \omega / (k v_{te})$, $\alpha_b = (\omega/k - v_b) / v_{te}$ and $Z'(\alpha)$ is the derivative of the plasma dispersion function [32]. In our case, eq. (5-3) can be solved with a complex angular frequency $\omega = \omega_r + i\omega_i$ and a real wavenumber k . The normalized growth rate ω_i / ω_{pe} , the wave frequency ω_r / ω_{pe} and the phase velocity $v_{ph} / v_b = \omega_r / (k v_b)$ are obtained as a function of the wavenumber $k \lambda_e$.

Figure 5.11 shows a linear dispersion relation of a beam mode

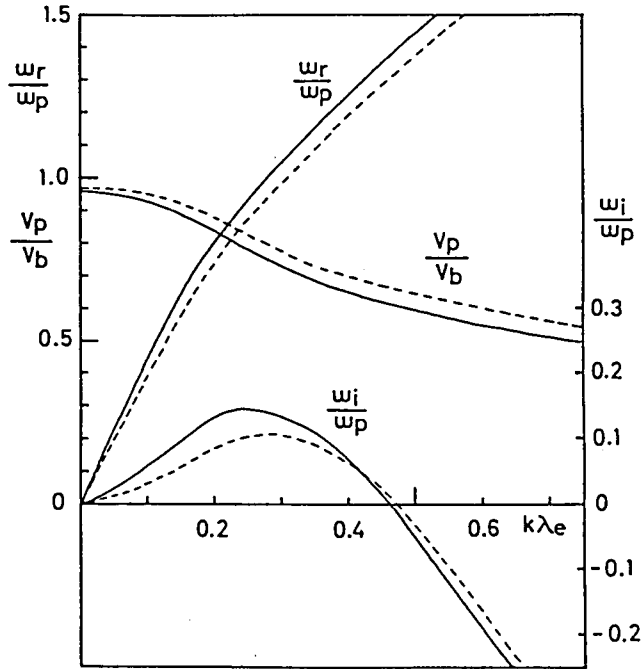


Fig.5.11 Dispersion relation of the electron beam-plasma system calculated from eq.(5-3). Wave frequency ω_r/ω_{pe} , growth rate ω_i/ω_{pe} and phase velocity v_{ph}/v_b are plotted as a function of $k\lambda_e$. The typical experimental parameters $n_b/n_e = 0.08$ and $T_e/T_b = 1$ are used in the calculation. The beam velocity v_b/v_{te} is chosen to be 3.4 in the solid line and 3.0 in the dashed line, respectively.

obtained from eq.(5-3), where parameter is a beam velocity v_b/v_{te} . Plasma parameters for the case shown in Fig.5.3, i.e. $n_e = 6 \times 10^9 \text{ cm}^{-3}$, $n_b/n_e = 0.08$, $\kappa T_e = 2 \text{ eV}$ and $T_e/T_b = 1.0$, are inserted. ω_i/ω_{pe} has a maximum value in a range of wavenumber $k\lambda_e = 0.2-0.3$. At this time, $\omega_r/\omega_{pe} = 0.8-1.0$ and $v_{ph}/v_b = 0.7-0.85$. Characteristics of these numerical results are well in agreement with the experimental results summarized in section 5.3-1.

It is also resonable that electron waves are observed in the region of parameter $k\lambda_e$ where the growth rate becomes maximum. In a real experiment, effective loss rate δ due to collisions, reflection and diffusion of particles etc., is present. δ should be subtracted from the growth rate. Therefore, waves having a net positive growth rate and satisfying the longitudinal boundary condition, i.e. eq.(5-1), are excited spontaneously.

5.4-2 Nonlinearity of Beam-Excited Electron Waves

Saturation of amplitudes and suppression of multimode electron waves described in section 5.3 are caused essentially by nonlinear interaction between the electron beam and the plasma.

Generally, beam-plasma systems are classified into two limiting cases, i.e. a hot beam and a cold beam cases, based on a criterion $v_{tb}/v_b > (n_b/2n_e)^{1/3}$ or vice versa[33,34]. When $v_{tb}/v_b > (n_b/2n_e)^{1/3}$, the nonlinear wave-particle interaction is well described by quasilinear theory[35-37]. On the other hand, when $v_{tb}/v_b < (n_b/2n_e)^{1/3}$, the interaction of particles is essentially with a single wave and the effect of particle trapping is dominant. Drummond et al.[38] and others[39-41] proposed a single wave model

for such a nonlinear evolution of a weak cold beam-plasma system.

According to the criterion described above, our experiments have been done near the transition region, namely transition from a hot beam to a cold beam. By using the full dispersion equation, i.e. eq.(5-3), we study in detail the change of dispersion roots numerically.

Figure 5.12(a) shows growth rates for various density ratio n_b/n_e covered in the present experiments. Other parameters are $T_e/T_b = 1$ and $v_b/v_{te} = 3.2$. Figure 5.12(b) shows the maximum growth rate, corresponding to Fig.5.12(a), as a function of n_b/n_e . Cold beam and hot beam limits are estimated from $(n_b/2n_e)^{1/3}(3^{1/2}/2)$ and from $(\pi/2e)^{1/2}(n_b/2n_e)(v_b/v_{tb})$, respectively[34].

Strictly speaking, according to the numerical solutions of dispersion equation shown in Fig.5.12, whether our beam-plasma system is fit for a hot beam limit or a cold beam limit is not so clear when n_b/n_e is nearly equal to or larger than 0.1. However, we think that a quasilinear treatment may be applicable to the present system. There are several arguments which support this assumption. First, the present beam-plasma interaction is resonant because the velocity distribution of beam particles is spread out over a velocity region of the order $\Delta v = v_b - v_{ph}$. Experimentally, $\Delta v/v_{tb} = 0.9 - 1.2$. Second, the spectra show no side-band caused by the trapped-particle instability. Third, many electron waves, i.e. at least four modes, are excited and are interacting simultaneously. The region of the resonance particles for any one of electron waves is given, in order of magnitude, by the following inequality

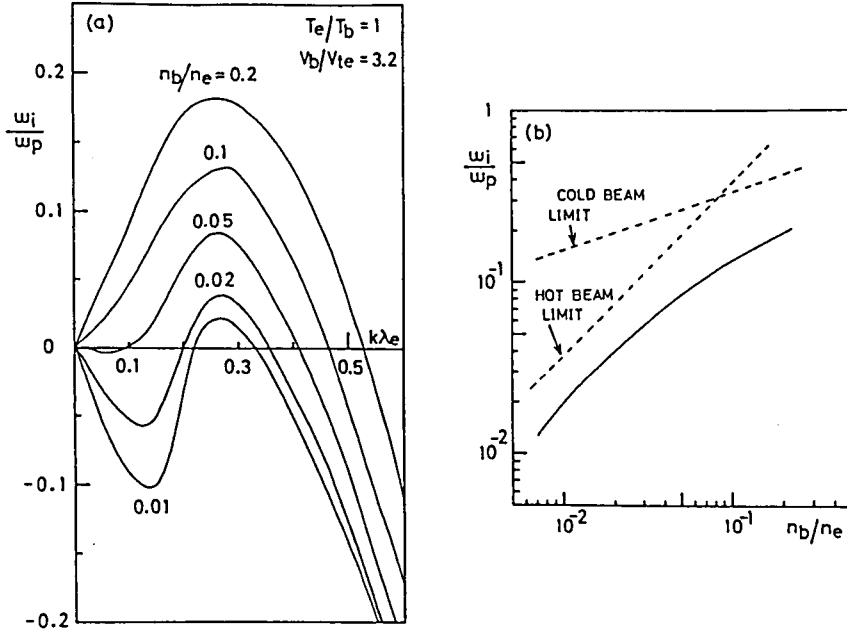


Fig.5.12 (a)Growth rate vs $k\lambda_e$. Density ratio n_b/n_e is a parameter. In the present experiment, these values of n_b/n_e are controlled by changing the combination of I_D and I_B . Other parameters are chosen as follows: $v_b/v_{te} = 3.2$ and $T_e/T_b = 1.0$.

(b)Maximum temporal growth rate corresponding to Fig.(a) as a function of density ratio. Two dashed lines are obtained by maximum temporal growth rates in the cold and in the hot beam limits, respectively.

$$v_{pn} - \Delta v_n \leq v \leq v_{pn} + \Delta v_n \quad \text{and} \quad \Delta v_n = (4e\phi_{en}/m)^{1/2}, \quad (5-4)$$

where $v_{pn} = 2\pi f_n/k_n$ is the phase velocity of the electron wave of n -th mode, Δv_n is the half resonance width and ϕ_{en} is the potential amplitude of the electron wave. As the observed electron waves are excited by the electron beam, their phase velocities are nearly equal to each other. Then, overlapping of the resonance regions within electron waves is easily occurred. This supports the random phase approach, i.e. a quasilinear treatment.

5.4-3 Suppression of Electron Waves

On suppression phenomena for the electron waves, there are following two features:

- (i) When a modulation frequency f_m is close to some frequency f_n of the electron waves, effects of the beam modulation become remarkable and all of the electron waves are simultaneously reduced in their amplitudes (see Figs. 5.7 and 5.8).
- (ii) The critical modulation voltage V_c above which the electron waves are suppressed is nearly proportional to the frequency difference between f_m and f_n , i.e. $V_c \propto |f_m - f_n|$ (see Fig. 5.10).

The feature (i) is explained as follows. In the bounded system, a wavenumber k_n has a discrete value and an integral multiple of the fundamental wavenumber k_1 determined by the boundary, i.e. $k_1 = \pi/L$ and $k_n = nk_1 = n\pi/L$. Allowed excitation of electron waves is that waves satisfy both the linear dispersion, i.e. eq.

(5-3), and the boundary condition, i.e. eq.(5-1). As the observed electron waves are excited by the electron beam, their phase velocities are nearly equal to each other. On the other hand, the excited wave generated by the beam modulation is the response of the plasma for an external force with a frequency f_m . It may be inferred that the excited wave as well as the electron wave has discrete values of wavenumber. When f_m is close to f_n , the excitation condition and the phase velocity of the excited wave become the same as those of the electron waves. Therefore, if $f_m \approx f_n$, the excited wave interacts strongly with the electron waves.

Next, we consider the suppression mechanism of the multimode electron waves due to the excited wave and the frequency dependence of V_c , i.e. the feature (ii).

Although numerous works of wave suppression have been made in the case that a single unstable mode is present, only a few experimental studies have been reported on suppression of a multimode beam-plasma instability[6,26,42].

Nakamura[6] observed that spontaneous oscillation was suppressed by beam density modulation, where modulation frequency was set nearly equal to that of the mode neighboring with the mode appearing spontaneously. He explained the results by using the set of the Van der Pol equations with mode-mode coupling terms. However, the physical meaning of the nonlinear terms including mode-mode coupling terms is not clear theoretically and experimentally. As is discussed in Chapter 4, according to Refs.5 and 7, the nonlinear terms come from the source terms which are introduced supplementally in the continuity equation of the fluid model. Therefore, relationship between the coefficients of the Van der Pol e-

quation and the physical process of wave suppression is not clear.

On the other hand, Amemiya[42] observed the wave suppression in the counter-stream system, under a nearly constant form of the beam distribution function and a constant wave energy. Although he proposed the model for the wave suppression, the form of a monotonic increasing function G , being essential to his model, was not specified. So, physical meaning of the wave suppression is indistinct. Besides, his beam-plasma interaction is non-resonant.

Consequently, in order to explain our experimental results, we cannot apply these models described above. Now, taking into account the discussion in section 5.4-2, we consider the suppression mechanism of multimode instability due to the excited wave.

Under the action of a wave, as is predicted by the quasilinear theory, the distribution function becomes distorted in the resonance particle region and a plateau forms on the distribution function. However, Coulomb collisions of resonance particles with the remaining particles in the plasma gradually smooth out the plateau and establish a stationary state[37,43]. Since the action of the wave for the distribution function has the effect of changing not the distribution function itself but mainly its derivative at the phase velocity of the wave, the effective growth rate γ has the form

$$\gamma = \frac{\gamma_L}{1 + \tau_1/\tau_2} \quad , \quad (5-5)$$

where γ_L is the linear growth rate, τ_1 is a characteristic time associated with the setting up of a localized Maxwellian distribution and τ_2 is a characteristic time associated with the distortion of the distribution function under the action of the field of the

wave packet[37]. Such an interpretation enables us to make an estimate of the growth rate for the electron waves in the presence of the excited wave. Then, the slope of the velocity distribution function of electrons at the phase velocity of the electron wave is ruled by the counterbalance between the trapping by the excited wave and the detrapping by collisions.

Let the amplitude of the potential in the excited wave be ϕ_m . The region of the resonance particles for the excited wave is given as follows:

$$v_f - \Delta v_1 \leq v \leq v_f + \Delta v_1 \quad \text{and} \quad \Delta v_1 = \left(\frac{4e\phi_m}{m} \right)^{1/2}, \quad (5-6)$$

where $v_f = 2\pi f_m / k_n$ is the phase velocity of the excited wave.

Due to small-angle Coulomb collisions, localized equilibrium will be reestablished in the region after a time $\tau_1 \cong (\Delta v_1)^2 / D \cong 4e\phi_m / (v\kappa T_e)$, where D is the diffusion coefficient in the velocity space, $D \cong v\nu_{te}^2 = v\kappa T_e / m$, v is an electron-electron collision frequency and κ is the Boltzmann constant. The equivalent time for the non-linear distortion due to the interaction with the excited wave is of the order $\tau_2 = \lambda_m / (e\phi_m / m)^{1/2}$, where λ_m is the wavelength of the excited wave. Therefore, the effective growth rate γ in this case is expressed as follows:

$$\gamma = \frac{\gamma_L}{1 + 4(e\phi_m)^{3/2} / (\kappa T_e \lambda_m v m^{1/2})} \quad (5-7)$$

Strictly speaking, eq. (5-7) is valid when the phase velocity of the electron wave is equal to that of the excited wave. In the present experiment, there is a difference in a phase velocity between

the excited wave and the electron wave when f_m is not equal to f_n . In this case, a phase velocity difference Δv_2 is given by

$$\Delta v_2 = \frac{2\pi}{k_n} |f_m - f_n| = \frac{2\pi \Delta f}{k_n} \quad (5-8)$$

Under the experimental conditions where suppression takes place, however, Δv_1 is greater than Δv_2 . Then, the reduced growth rate of the electron wave can be estimated by eq. (5-7).

Electron waves are suppressed when

$$\gamma = \nu_d \quad (5-9)$$

where ν_d is the magnitude of the damping coefficient and is equivalent to effective loss rate δ in section 5.4-1. So, the critical amplitude of the excited wave ϕ_{mc} can be estimated as follows:

$$\phi_{mc} = [(\gamma_L/\nu_d - 1) \kappa T_e \lambda_m \nu_m^{1/2}/4]^{2/3}/e \equiv C \quad (5-10)$$

The magnitude of ϕ_{mc} is independent of the frequency difference between f_m and f_n . On the other hand, ϕ_m can be understood as the response of the plasma for the external force, the absolute value of ϕ_m is given by

$$\frac{e |\phi_m|}{\kappa T_e} = \frac{(n_b/n_e) (n_m/n_b)}{2k_n^2 \lambda_e^2 |\epsilon(k_n, \omega_m)|} \quad (5-11)$$

where $\epsilon(k_n, \omega_m)$ is a dielectric function of the plasma, $\omega_m = 2\pi f_m$ and n_m is the perturbed number density of the modulated electrons. As the modulation depth of the beam density n_m/n_b may be propor-

tional to V_m , frequency dependence of V_c can be obtained by solving the following equation,

$$\frac{e|\phi_{mc}|}{\kappa T_e} = \frac{(n_b/n_e)\beta V_c}{2k_n^2 \lambda_e^2 |\epsilon(k_n, \omega_m)|} \quad (5-12)$$

where β is a coupling coefficient between the electron beam and the accelerating electrode E_2 . As $|f_m - f_n| \ll f_m$ and f_n , dielectric function can be approximated by the following form

$$|\epsilon(k_n, \omega_m)| = |\epsilon'| [(\omega_m - \omega_n)^2 + (\gamma_m - \gamma_n)^2]^{1/2} \quad (5-13)$$

where $\epsilon' = \partial\epsilon/\partial\omega|_{\omega=\omega(k)}$, ω_n and γ_n are the real and imaginary parts of $\omega(k_n)$, respectively, and γ_m is the imaginary part of $\omega(k_n)$ when $\omega_r(k_n) = \omega_m = 2\pi f_m$. Substituting eq.(5-13) into eq.(5-12), we obtain the relation

$$(\omega_m - \omega_n)^2 + (\gamma_m - \gamma_n)^2 = \frac{(n_b/n_e)^2 \beta^2 V_c^2}{(2k_n^2 \lambda_e^2)^2 |\epsilon'|^2} \left(\frac{\kappa T_e}{e|\phi_{mc}|} \right)^2 \quad (5-14)$$

In the region of $k\lambda_e$, where growth rate becomes maximum, imaginary part is not so dependent on $k\lambda_e$ as real part is (see Fig. 5.11). Then, if we may assume $|\omega_m - \omega_n| > |\gamma_m - \gamma_n|$, eq.(5-14) is written as follows:

$$V_c \cong \frac{4\pi k_n^2 \lambda_e^2 |\epsilon'|}{(n_b/n_e)\beta} \left(\frac{e|\phi_{mc}|}{\kappa T_e} \right) |f_m - f_n| \quad (5-15)$$

This relation, i.e. eq.(5-15), is in good agreement with the observed relationship between V_c and f_m , shown in Fig.5.10. It is also noted that features of these relations, i.e. eqs.(5-14) and

and (5-15), are the same as those deduced from the solution of the Van der Pol equation with a periodic perturbation term[5,7].

In addition to the above qualitative explanation, now, we have a quantitative consideration about this model. According to the experimental conditions where the results shown in Figs.5.8 and 5.10 are observed, plasma parameters are as follows: $\kappa T_e \cong 2$ eV, $L \cong 3 \times 10^{-2}$ m, $f_{pe} \cong 700$ MHz, $f_n = 608$ MHz, $T_e/T_b \cong 1$, $n_b/n_e \cong 0.05$, $p = 9 \times 10^{-4}$ Torr and mode number of the electron wave $n = 22$.

First, we evaluate the magnitude of ϕ_{mc} from eq.(5-10). When the values of $\kappa T_e \cong 2$ eV, $\lambda_m \cong 2L/n \cong 2.73 \times 10^{-3}$ m, $\nu \cong \nu_{e-e} = 1$ MHz and $\gamma_L/\nu_d = 28$ are put into eq.(5-10), $\phi_{mc} = C = 0.2$ V. In this calculation, γ_L is not the experimental value but the solution of the dispersion equation, where $T_e/T_b = 1$ and $n_b/n_e = 0.05$. The magnitude of ν_d is assumed to be the same as that of an electron-neutral collision frequency ν_{e-n} . Therefore, ν_d is under-estimated. Another estimation of the magnitude of ν_d is obtained from the resonance curve for the excited wave shown in Fig.5.8, i.e. the full width at the half-maximum $\Delta f_0 \cong 10$ MHz. We think that the latter case is more realistic concerning the estimation of ν_d . In this case $\gamma_L/\nu_d = \gamma_L/(2\pi\Delta f_0) \cong 5.6$, and $\phi_{mc} \cong 6 \times 10^{-2}$ V.

On the other hand, experimentally, the potential amplitude of the electron wave ϕ_e is estimated from the fluctuation of floating potential of the probe, and $|e\phi_e/\kappa T_e|$ may be from 10^{-3} to 10^{-2} . Therefore, the magnitude of ϕ_{mc} , i.e. 6×10^{-2} V, is rather large compared with that of the electron wave. However, this beam-plasma system has a pretty high quality factor, i.e. $Q = f_0/\Delta f_0 \cong 60$, where $f_0 = 608$ MHz is the resonance frequency in Fig.5.8. The magnitude of V_c is from 0.04 to 0.1 V in Fig.5.10. So, the esti-

mated value of ϕ_{mc} is not unrealistic.

Next, we compare the phase velocity difference Δv_2 with the width of the resonance region Δv_1 . For example, in Fig.5.10, $\Delta v_2 = 2.73 \times 10^4$ m/sec, where $\Delta f = 10$ MHz and $k_n = \pi n/L \cong 2.3 \times 10^3$ m⁻¹. According to eq.(5-6), $\Delta v_1 = \Delta v_2$ when $\phi_{mc} \cong 1.1 \times 10^{-3}$ V. So, we can say that Δv_1 is greater than Δv_2 whenever the electron waves are suppressed, and that the reduced growth rate γ is well estimated from eq.(5-7).

5.4-4 Suppression of the Ion-Acoustic Wave

The amplitude of the ion-acoustic wave varies with frequency change of the beam modulation as shown in Fig.5.7. Among four electron waves excited simultaneously, the one wave of frequency 587 MHz has strong correlation with the ion-acoustic wave in its intensity.

On the other hand, as is shown in Fig.5.3, the ion-acoustic wave can be excited spontaneously only when the electron waves are present. According to the linear theory, the ion-acoustic wave can not be excited directly by the beam-plasma interaction under the present experimental conditions[25]. So, the simultaneous suppression of both the electron waves and the ion-acoustic wave in our experiment suggests that the ion-acoustic wave is generated from the electron wave by some nonlinear processes[44].

Experimentally, we have obtained the growth rate of ion-acoustic waves, γ_1 , as a function of the amplitude of the electron wave, A_e [23]. When the beam density is modulated by low-voltage rectangular pulse signal applied to E_2 , the ion-acoustic wave is ex-

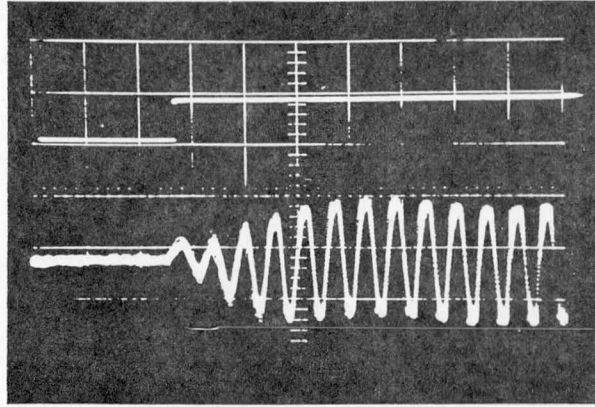


Fig.5.13 Oscillation traces. The upper trace represents the applied low-voltage pulse. The lower trace shows the temporal growth of the ion-acoustic wave. The time scale is 0.1 msec/div. Plasma parameters are as follows: $I_D = 15$ mA, $V_B = 20.7$ V, $L = 3.1$ cm and $T_r = 17$ °C.

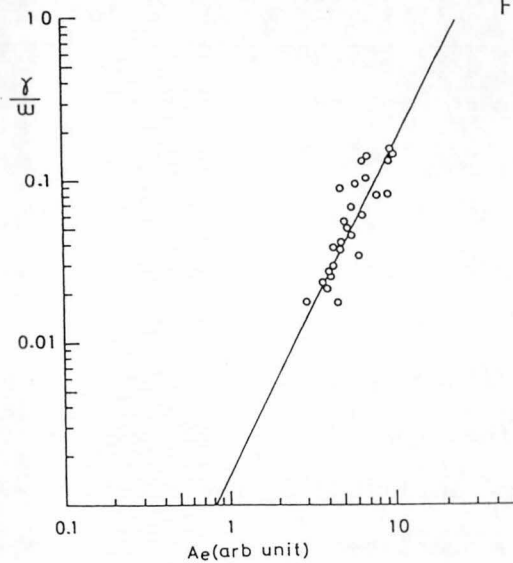


Fig.5.14 Relationship between γ_L of the ion-acoustic wave (20 kHz) and A_e (400 MHz). Plasma parameters are as follows: $I_D = 18.9$ mA, $V_B = 26$ V, $I_B = 6.25$ mA, $L = 3.1$ cm and $T_r = 17$ °C.

cited during the application of the pulse voltage. The temporal growth of the ion-acoustic wave is directly observed by an oscilloscope (see Fig.5.13). In this experiment, the grid G is connected with a variable condenser. By varying the capacitance, we can control A_e without changing the DC components of the plasma parameters. Therefore, we can obtain γ_l as a function of A_e by varying the capacitance. The experimental results are shown in Fig.5.14. The solid line shows a relationship $\gamma_l \propto A_e^2$. The results agree well with this line.

As the coupling mechanism between high-frequency waves and low-frequency ones, the wave-wave interaction and wave-particle interaction are well known[45]. Obiki[25] have verified that the coupling mechanism between the electron wave and the ion-acoustic wave in our system is due to the nonlinear wave-particle interaction, i.e. the nonlinear Landau damping.

Then, according to these experimental results and discussions, we can conclude that the suppression of the ion-acoustic wave is not due to the direct effect of the beam modulation but the damping of the electron wave which constitutes an energy source of the ion-acoustic wave.

5.5 Concluding Remarks

In a bounded electron beam-plasma system, we have observed that electron waves excited by the beam-plasma interaction are accompanied with the ion-acoustic wave. Both electron waves and the ion-acoustic wave are suppressed simultaneously by the beam

density modulation when the modulation frequency is close to the frequency of any one of electron waves. The critical amplitude of the modulation voltage is approximately proportional to the frequency difference between the modulation frequency and that of the electron wave. Characteristic features of these experimental results are well explained by the quasilinear effects of the excited wave.

The ion-acoustic wave observed in this experiment is excited by the electron wave through the nonlinear Landau damping. By applying the beam modulation, the ion-acoustic wave is also shown to be suppressed through the damping of the electron wave which constitutes an energy source of the ion-acoustic wave.

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Chapter 6

CONCLUSIONS

In this thesis, nonlinear wave-particle interaction and suppression of beam-excited electron waves are investigated theoretically and numerically, and experimentally. We present a summarization of each chapter as follows:

CHAPTER 2: The destruction mechanism of the amplitude oscillations in the system of a small cold beam and a weakly collisional plasma is examined by means of partial simulation. The regular amplitude oscillations can be destroyed either by weak collisions within the background plasma or by nonlinear growth of neighboring waves of the most unstable mode.

Collisions, the order of magnitude $\nu/\omega_{pe} \cong 10^{-3}$, are too weak to change the linear stage. However, they alter the phase relation between the most unstable mode, namely the single wave, and the trapped electrons in the nonlinear stage. Therefore, after the wave saturation, the amplitude of the wave at the first minimum decreases anomalously and the wave cannot regrow to the first maximum. As a result, the persistent trapped-particle oscillations in the collisionless case are destroyed.

On the other hand, nonlinear growth of the neighboring waves also leads to the destruction of the amplitude oscillations due to phase-mixing between beam electrons and the single wave.

Either of the collisional and neighboring wave effects competes with another in playing a decisive role for the destruction of the amplitude oscillations. Effects of neighboring waves for the destruction depend strongly on plasma conditions. Because, the system becomes stable for the excitation of the neighboring waves when the beam velocity increases under the constant expansion parameter s , where s is given by the relation, $s = [(n_b v_b^2) / (6n_e v_t^2)]^{1/3}$. In this case, collisional effects become predominant for the destruction of the amplitude oscillations.

CHAPTER 3: Concerning the control of wave spectrum and the suppression of the wave due to pre-modulation of the beam, interaction between two waves in a modulated electron beam-plasma system is investigated by means of partial simulation. We have analyzed those problems in the following two different cases.

(a) The unbounded case(semi-infinite system)

In this case, the model of spatially growing waves is used. Spectrum control and wave suppression are obtained by pre-modulating the injected beams. Deformation of beam distribution function is occurred in relation to the suppression of one wave caused by the other wave. According to these numerical results, two waves do not interact directly, but through a wave-particle-wave processes.

(b) The bounded case(finite system)

In this case, the model of temporally growing waves is used, and we have obtained two different patterns for the wave suppression. When injected beams are reflected at the other end, two wave interaction occurs under the constant total energy of the

waves. Besides, additional broadening of the beam distribution does not occur.

On the other hand, when injected beams are absorbed at the other end, suppression of the one wave due to the other wave is accompanied with additional broadening of the beam distribution function. Differences between these two processes are caused by whether feedback effects of reflected beams are present or not.

CHAPTER 4: Suppression of the beam-excited electron waves caused by beam density modulation in a bounded system is investigated experimentally, in connection with the beam distribution function. Plasma parameters are controlled so that one mode with narrow frequency spectrum is excited spontaneously.

Wave suppression has two different processes depending on whether injected beams are reflected at the other end or absorbed. In the absence of reflected beams, deformation of the beam distribution function is observed in relation to the suppression of the electron wave. However, when beam electrons are reflected, the excited wave generated by beam modulation suppresses the electron wave but the beam distribution function shows no appreciable change.

These experimental results show that nonlinear behaviors of beam electrons, namely behaviors of reflected beams, are quite important for wave suppression. Interpretations of experimental results in both cases are made systematically without using the phenomenological model of the classical Van der Pol oscillator. By using the method of partial simulation described in Chapter 3, interaction between two waves in the bounded system, including nonlinear motions of beam electrons, is studied numerically.

Qualitative agreement between experimental and numerical results is obtained.

CHAPTER 5: Suppression of multimode beam-plasma instability is studied experimentally. Four or five longitudinal electron waves of axial beam modes are observed simultaneously in a bounded electron beam-plasma system. They are accompanied with an ion-acoustic wave of axial mode under appropriate plasma conditions. According to the linear theory, however, the ion-acoustic wave cannot be excited directly by beam-plasma interaction in the present case, where $v_b/v_{te} > 3$.

When a beam density is modulated by an external RF signal whose frequency is close to some frequency of electron waves excited spontaneously, all of electron waves and the ion-acoustic wave are suppressed simultaneously. It is discussed that the suppression of electron waves is attributed to quasilinear effects of the excited wave caused by beam modulation. It is also clarified experimentally that suppression of the ion-acoustic wave is not brought about directly by the beam modulation but by the damping of the electron wave which constitutes the energy source of the ion-acoustic wave.

So far, we have investigated several nonlinear wave phenomena in the modulated electron beam-plasma system, and have delineated the physical processes involved in them. However, this does not mean that all problems have been thoroughly understood. Next, we mention the problems left in this thesis in the followings:

In Chapter 2, we have studied competition between collisional effects and many wave effects for the destruction of the persistent amplitude oscillations. It is shown that in both cases the destruction is caused by phase randomization between the most unstable mode and the trapped beam electrons. Further, we have also clarified the excitation mechanism of the neighboring waves, and its dependence on plasma parameters in detail. So, concerning competition between two effects for the destruction, thorough results are obtained qualitatively. However, we have not yet derived the theoretical criterion that determines whichever of these two effects is dominant for the destruction of the amplitude oscillations.

In Chapter 3, concerning the study for the bounded case, we have used a very simplified model where the two waves are assumed to have the same linear properties, i.e. the same wavenumber. So, we cannot choose freely the magnitude of the frequency difference $\delta\omega$ between two waves, but restrict $\delta\omega$ to a small value. Consequently, relationship between the modulation depth and the modulation frequency is not clarified although salient features of the experimental results are corroborated by the present model. However, extension of this model to the more appropriate one is easily obtained. Our basic consideration and techniques would be applicable to even the more complex case.

In Chapters 4 and 5, suppression mechanisms of beam-excited electron waves in the bounded system are discussed from a viewpoint of the wave-particle interaction. In the case where a single

mode with narrow frequency spectrum is excited spontaneously, relationship between the wave suppression and the beam distribution function is clarified experimentally and theoretically. On the other hand, in the case where a multimode instability is excited spontaneously, relationship between the wave suppression and the beam distribution function is not so clear as in the single mode case although experimental results are well explained qualitatively by the particle picture. To study this point further in detail, we must have a more subtilized measurement which could reveal microscopic movement of beam particles relative to the wave phase, because not only the slowly varying part as reported in this thesis but also the quickly varying part of the distribution function may play an important role.

However, so far as we know, we have discussed the suppression phenomena from a viewpoint of particle picture for the first time. Following this work, recently, suppression of beam-excited electron waves and ion-acoustic waves have been studied and discussed in connection with the velocity distribution function of plasma particles by many authors.

Appendix

THE DERIVATION OF Eqs. (4-10) AND (4-11)

In this appendix, we show the outline of derivation of eqs. (4-10) and (4-11). In order to obtain the rate equations, it is sufficient to examine the following approximate equations which are derived from eqs. (4-7)-(4-9),

$$\begin{aligned} \frac{dE_k}{dt} = & - \frac{2\pi en_b}{\omega_k N} \sum_j \frac{dv_j}{dt} \sin(\omega_k t - kx_j) \\ & + \frac{2\pi en_b k}{\omega_k N} \sum_j v_j^2 \cos(\omega_k t - kx_j) , \end{aligned} \quad (A-1)$$

$$\begin{aligned} \frac{dF_k}{dt} = & - \frac{2\pi en_b}{\omega_m N} \sum_j \frac{dv_j}{dt} \sin(\omega_m t - kx_j) \\ & + \frac{2\pi en_b k}{\omega_m N} \sum_j v_j^2 \cos(\omega_m t - kx_j) , \end{aligned} \quad (A-2)$$

and

$$\begin{aligned} \frac{d^2 x_j}{dt^2} = \frac{dv_j}{dt} = & -2 \frac{e}{m} [E_k \cos(\omega_k t - kx_j) + RE_k \cos(\omega_k t + kx_j) \\ & + F_k \cos(\omega_m t - kx_j) + RF_k \cos(\omega_m t + kx_j)] . \end{aligned} \quad (A-3)$$

As the terms $E_k(t)$ and $F_k(t)$ vary slowly compared to the high frequency ω_k and ω_m , we may neglect the second order derivative. For example, assuming that $E_k(t) = E_k(0)e^{\gamma t}$, where γ is the linear growth rate, the ratio $2\omega_k (dE_k/dt)/(d^2 E_k/dt^2) = 2\omega_k/\gamma \cong 2(2n_e/n_b)^{1/3}$. As $n_b/n_e \cong 10^{-2}$ in the present experiments, the above

ratio is about 12. The terms due to beam modulation are also neglected, because they are the external source terms and not important for the derivation of the nonlinear rate equations. Lastly, we assume that E_k and F_k are real quantities. Strictly speaking, they are complex quantities because system has a nonlinear phase shift. But, in order to discuss and describe the suppression of instability by an external wave, only the amplitude variation is of importance.

Next, we consider the perturbation of the orbit of the j -th beam particle. Let the forward beam, i.e. the injected beam

$$x_j(t) = x_j(0) + v_b t + \delta x_j, \quad (\text{A-4})$$

then,

$$\begin{aligned} \frac{d^2 x_j}{dt^2} = \frac{d^2 \delta x_j}{dt^2} = & - \frac{2eE_0}{m} \left(e^{\gamma t} \cos[k\delta x_j + \eta_0 t + kx_j(0)] \right. \\ & \left. + \operatorname{Re} \gamma^t \cos[k\delta x_j + \eta_1 t + kx_j(0)] \right) \\ & - \frac{2eF_0}{m} \left(e^{\gamma' t} \cos[k\delta x_j + \eta_0' t + kx_j(0)] \right. \\ & \left. + \operatorname{Re} \gamma'^t \cos[k\delta x_j + \eta_1' t + kx_j(0)] \right). \end{aligned} \quad (\text{A-5})$$

Let the backward beam, i.e. the reflected beam,

$$x_j(t) = x_j(0) - v_b t + \delta x_j, \quad (\text{A-6})$$

then,

$$\frac{d^2 \delta x_j}{dt^2} = - \frac{2eE_0}{m} \left(e^{\gamma t} \cos[\eta_1 t - k\delta x_j - kx_j(0)] \right.$$

$$\begin{aligned}
& + \operatorname{Re} \gamma^t \cos[\eta_0 t - k \delta x_j - k x_j(0)] \Big] \\
& - \frac{2eF_0}{m} \left(e^{\gamma' t} \cos[\eta_1' t - k \delta x_j - k x_j(0)] \right. \\
& \left. + \operatorname{Re} \gamma'^t \cos[\eta_0' t - k \delta x_j - k x_j(0)] \right) , \tag{A-7}
\end{aligned}$$

where $\eta_0 = kv_b - \omega_k$, $\eta_1 = kv_b + \omega_k = \eta_0 + 2\omega_k$, $\eta_0' = kv_b - \omega_m = \eta_0 - \delta\omega$, $\eta_1' = kv_b + \omega_m = \eta_0 + 2\omega_k + \delta\omega$, and γ and γ' are the linear growth rates of the electron and the excited waves, respectively. A value of $\delta\omega$ is the frequency difference between ω_k and ω_m . Linearizing the eqs.(A-5) and (A-7), we can obtain the slowly time varying part of the orbit perturbation as follows: The forward beam has

$$\begin{aligned}
\delta v_j = & - 2 \frac{e}{m} \frac{E_k(t)}{a_0} \left[c_1 \cos[kx_j(0)] - s_1 \sin[kx_j(0)] \right] \\
& - 2 \frac{e}{m} \frac{F_k(t)}{a_1} \left[c_2 \cos[kx_j(0)] - s_2 \sin[kx_j(0)] \right] , \tag{A-8}
\end{aligned}$$

and

$$\begin{aligned}
\delta x_j = & - 2 \frac{e}{m} \frac{E_k(t)}{a_0^2} \left[A_1 \cos[kx_j(0)] - B_1 \sin[kx_j(0)] \right] \\
& - 2 \frac{e}{m} \frac{F_k(t)}{a_1^2} \left[A_2 \cos[kx_j(0)] - B_2 \sin[kx_j(0)] \right] . \tag{A-9}
\end{aligned}$$

The backward beam has

$$\begin{aligned}
\delta v_j = & - 2 \frac{e}{m} \frac{RE_k(t)}{a_0} \left[c_1 \cos[kx_j(0)] + s_1 \sin[kx_j(0)] \right] \\
& - 2 \frac{e}{m} \frac{RF_k(t)}{a_1} \left[c_2 \cos[kx_j(0)] + s_2 \sin[kx_j(0)] \right] , \tag{A-10}
\end{aligned}$$

and

$$\delta x_j = - 2 \frac{e}{m} \frac{RE_k(t)}{a_0^2} \left[A_1 \cos[kx_j(0)] + B_1 \sin[kx_j(0)] \right]$$

$$- 2 \frac{e}{m} \frac{RF_k(t)}{a_1^2} \left(A_2 \cos[kx_j(0)] + B_2 \sin[kx_j(0)] \right), \quad (A-11)$$

where $a_0 = \gamma^2 + \eta_0^2$, $a_1 = \gamma'^2 + \eta_0'^2$,

$$c_1 = \gamma \cos \eta_0 t + \eta_0 \sin \eta_0 t - \gamma \exp(-\gamma t),$$

$$c_2 = \gamma' \cos \eta_0' t + \eta_0' \sin \eta_0' t - \gamma' \exp(-\gamma' t),$$

$$s_1 = \gamma \sin \eta_0 t - \eta_0 \cos \eta_0 t + \eta_0 \exp(-\gamma t)$$

$$s_2 = \gamma' \sin \eta_0' t - \eta_0' \cos \eta_0' t + \eta_0' \exp(-\gamma' t),$$

$$A_1 = \gamma c_1 + \eta_0 s_1 - \gamma a_0 \exp(-\gamma t),$$

$$A_2 = \gamma' c_2 + \eta_0' s_2 - \gamma' a_1 \exp(-\gamma' t),$$

$$B_1 = \gamma s_1 - \eta_0 c_1 + \eta_0 a_0 \exp(-\gamma t)$$

and $B_2 = \gamma' s_2 - \eta_0' c_2 + \eta_0' a_1 \exp(-\gamma' t).$

After substituting eqs.(A-5) and (A-7) into dv_j/dt in eqs. (A-1) and (A-2), we expand the right hand sides of eqs.(A-1) and (A-2) up to the third order of perturbations. Then, replacing these δx_j and δv_j with the expressions in eqs.(A-8)-(A-11), assuming that the initial positions $x_j(0)$ are distributed uniformly and using the following relation, i.e.

$$\sum_j \sin^2 kx_j(0) = \sum_j \cos^2 kx_j(0) = \frac{N}{2}, \quad (A-12)$$

and

$$\sum_j \sin kx_j(0) \cos kx_j(0) = 0, \quad (A-13)$$

we find the following set of equations:

$$\frac{dE_k}{dt} = \alpha_1 E_k - \beta_1 E_k^3 - \theta_{12} F_k^2 E_k, \quad (A-14)$$

$$\frac{dF_k}{dt} = \alpha_2 F_k - \beta_2 F_k^3 - \theta_{21} E_k^2 F_k. \quad (A-15)$$

These two equations are the same as the eqs. (4-10) and (4-11) in the text, respectively. Nonlinear saturation coefficients, i.e. β_1 , β_2 , θ_{12} and θ_{21} , have the following forms in the absence of reflected beams:

$$\beta_1 = \frac{\omega_p^2}{\omega_k} \left(\frac{e}{m} \right)^2 \frac{k^2}{a_0^4} \left\{ [B_1(c_1^2 + s_1^2) + 2A_1c_1s_1] \cos \eta_0 t + 2A_1B_1 \cos 2\eta_0 t - [A_1(c_1^2 + s_1^2) + 2B_1c_1s_1] \sin \eta_0 t \right\}, \quad (A-16)$$

$$\beta_2 = \frac{\omega_p^2}{\omega_m} \left(\frac{e}{m} \right)^2 \frac{k^2}{a_1^4} \left\{ [B_2(c_2^2 + s_2^2) + 2A_2c_2s_2] \cos \eta_0' t + 2A_2B_2 \cos 2\eta_0' t - [A_2(c_2^2 + s_2^2) + 2B_2c_2s_2] \sin \eta_0' t \right\}, \quad (A-17)$$

$$\begin{aligned} \theta_{12} = & \frac{\omega_p^2}{\omega_k} \left(\frac{e}{m} \right)^2 k^2 \left\{ \frac{2A_2B_2}{a_1^4} \cos 2\eta_0 t + \frac{2(A_1B_2 + A_2B_1)}{a_0^2 a_1^2} \cos(\eta_0 + \eta_0') t \right. \\ & + \frac{2(A_1A_2 + B_1B_2)}{a_0^2 a_1^2} \sin(\eta_0 - \eta_0') t + \frac{1}{a_0^2 a_1^3} [a_1B_1(c_2^2 + s_2^2) \\ & + 2a_0B_2(c_1c_2 + s_1s_2) + 2a_1A_1c_2s_2 + 2a_0A_2(c_1s_2 + c_2s_1)] \cos \eta_0 t \\ & - \frac{1}{a_0^2 a_1^3} [a_1A_1(c_2^2 + s_2^2) + 2a_0A_2(c_1c_2 + s_1s_2) + 2a_1B_1c_2s_2 \\ & \left. + 2a_0B_2(c_1s_2 + c_2s_1)] \sin \eta_0 t \right\}, \quad (A-18) \end{aligned}$$

$$\begin{aligned} \theta_{21} = & \frac{\omega_p^2}{\omega_m} \left(\frac{e}{m} \right)^2 k^2 \left\{ \frac{2A_1B_1}{a_0^4} \cos 2\eta_0' t + \frac{2(A_1B_2 + A_2B_1)}{a_0^2 a_1^2} \cos(\eta_0 + \eta_0') t \right. \\ & + \frac{2(A_1A_2 + B_1B_2)}{a_0^2 a_1^2} \sin(\eta_0' - \eta_0) t + \frac{1}{a_1^2 a_0^3} [a_0B_2(c_1^2 + s_1^2) \\ & + 2a_1B_1(c_1c_2 + s_1s_2) + 2a_0A_2c_1s_1 + 2a_1A_1(c_1s_2 + c_2s_1)] \cos \eta_0' t \\ & \left. - \frac{1}{a_1^2 a_0^3} [a_0A_2(c_1^2 + s_1^2) + 2a_1A_1(c_1c_2 + s_1s_2) + 2a_0B_2c_1s_1] \sin \eta_0' t \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{1}{a_1^2 a_0^3} [a_0 A_2 (c_1^2 + s_1^2) + 2a_1 A_1 (c_1 c_2 + s_1 s_2) + 2a_0 B_2 c_1 s_1 \\
& + 2a_1 B_1 (c_1 s_2 + c_2 s_1)] \sin \eta_0 t \Big] . \quad (A-19)
\end{aligned}$$

When the reflected beams are present, the mode-mode coupling terms, i.e. θ_{12} and θ_{21} , are modified due to the interaction between the reflected beams and the backward wave. The magnitudes of them are increased by θ'_{12} ($\omega_k \theta'_{12} = \omega_m \theta'_{21}$) which is expressed as follows:

$$\theta'_{12} = \frac{\omega_{pb}^2}{\omega_k} \left(\frac{e}{m}\right)^2 \frac{2k^2}{a_0^2 a_1^2} R^2 (A_1 A_2 + B_1 B_2) \sin(\eta_1' - \eta_1) t . \quad (A-20)$$