

Sakai-Sugimoto model in QCD, five-dimensional Yang-Mills theory, and the Chern character appearing in the associated chiral anomaly

Tomohiro IWAMI (Kyushu Sangyo Univ.)

The Symposium of Algebraic Geometry, Kinosaki, Oct.20-Oct.24, 2014

1 Introduction

- D-branes probed systems [5] in the context of AdS/CFT correspondence:
 - the study of quiver moduli [2].
 - the Bogomolov-Gieseker type inequality on Calabi-Yau 3-folds [1].
- D4/D8/ $\overline{D8}$ probed system model (the so-called **Sakai-Sugimoto model** [4]):
 - mostly successful to describing Hadron models in QCD (quantum chromodynamics) without quark models.
 - to give the configurations of "skyrmions" (just like solitons) in five-dimensional Yang-Mills (YM) theory. (M. Atiyah et al, 1988).

In this note, based on the result [4], we discuss a chiral anomaly [4] associated to the WZW term of Sakai-Sugimoto model with regards to the Chern character appearing in the conjectural form of stability condition [1].

2 Sakai-Sugimoto D4/D8/ $\overline{D8}$ model and the associated five-dimensional Yang-Mills theory

Brief explanations [4] defines the chiral symmetry $U(N_f)_L \times U(N_f)_R$ gravitational model in 10-dimensional space, having the configurations of N_c D4 branes in (01234)-direction with compactification for (0123)-direction as S^1 by $\tau \sim \tau + 2\pi/U_{KK}$, and of N_f D8- $\overline{D8}$ branes ($\overline{D8}$ means having the inverse charge of D8) in (012356789)-direction, where N_c, N_f the numbers of colors and flavors, resp., U_{KK} the Kalza-Klein mass, and τ the angle coordinate in (4)-direction of S^1 -compactification.

- (1) Then, D4-brane solution is $ds^2 = \left(\frac{2U}{3}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2) + \left(\frac{3}{2U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$, where $f(U) := 1 - \frac{1}{U^3}$, $x^\mu (\mu = 0, 1, 2, 3)$ the 4-dimensional Minkowski coordinates, τ the above angle coordinate, U the radius coordinate in the 5-dimensional space orthogonal to D4-brane with $U \geq U_{KK}$, and $d\Omega_4^2$ the metric of 4-dimensional sphere S^4 surrounding D4-brane [4, (3.1)]. (Assume $U_{KK} = 1$ for simplicity in the sequel.)
- (2) Then, for introducing flavor-branes [3] of D8- $\overline{D8}$ as probes (assuming $N_f \ll N_c$), i.e. embedding D8 on D4-background as (1) with $U = U(\tau)$, the action of D8 is $S_{D8} \propto \int d^4x d\tau e^{-\phi} \sqrt{\det(-g_{D8})}$, where ϕ is dilaton, and g_{D8} is the metric induced on the world-sheet of D8.
- (3) Chiral fermions ϕ_L, ϕ_R are induced from open string between D4 and D8($\overline{D8}$).

- (4) Setting $(y, z) = (\sqrt{U^2 - 1}(\cos(U_{KK}\tau), \sin(U_{KK}\tau)))$, the configuration of continuing D8($\overline{D8}$) is given by $y = 0$. Then, world-volume of probe D8 is given by x^μ, z and S^4 -direction surrounding D4, i.e. (8+1)-dimensional.
- (5) **Assume that S^4 -dependency is ignored.** Then, the 5-dimensional YM theory is given as $S_{D8}^{YM} = \kappa \int d^4x dz \text{tr} \left\{ \frac{1}{2} K^{-1/3} F_{\mu\nu}^2 + K U_{KK}^2 F_{\mu\nu}^2 \right\}$, where $\kappa = \frac{N_c}{108\pi^3}$, $K(z) = 1 + z^2$, $\lambda = g_{YM}^2 N_c$, g_{YM} the YM coupling at U_{KK} , and $F_{\mu\nu}$ the field strengths. The associated Chern-Simons (CS) term is $S_{D8}^{CS} = \frac{N_c}{24\pi^2} \int_{M^4 \times \mathbb{R}} \omega_5(A)$, where $\omega_5(A) = \text{tr}(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5)$ the 5-form, $A = A(x^\mu, z)$ the gauge field on D8, and $F = (F_{\mu\nu})$.

3 Chiral anomaly on probe D8 and the associated Chern characters

Main Result By [4, (5.76), (5.77)], $S_{D8}^{CS} = \mu \int_{D8} C_3 \text{tr} F^3 = \mu \int_{D8} F_4 \omega_5(A)$, where $F_4 = dC_3$ the RR(Ramond-Ramond) 4-form field strength, $\mu = \frac{1}{48\pi^3}$ and $d\omega_5 = \text{tr} F^3$ holds. For the infinitesimal gauge transformation $\delta_\Lambda A = d\Lambda + [\Lambda, A]$, $\delta_\Lambda \omega_5(A) = d\omega_4^1(\Lambda, A)$, where $\omega_4^1(\Lambda, A) = \text{tr}(\Lambda d(\text{Ad}A + \frac{1}{2}A^3))$. Then, the gauge transformation of CS-term $\delta_\Lambda S_{D8}^{CS} = \frac{N_c}{24\pi^2} \int_{M^4 \times \mathbb{R}} d\omega_4^1(\Lambda, A)$ is given by taking $z \rightarrow \pm\infty$ for the gauge potentials $A_L(z, \Lambda)$ (resp. $A_R(z, \Lambda)$) of ϕ_L (resp. ϕ_R). So, $\delta_\Lambda S_{D8}^{CS}$ induces WZW terms. By considering $\text{tr} F^3$ as the third term of the Chern character $\text{ch}(F) = \text{tr} e^{\sqrt{-1}F/(2\pi)}$, the followings are settled: Let X be the smooth projective complex 3-fold as smooth compactification of $M^4 \times \mathbb{R} \times \mathbb{R} \ni (x^\mu, z, u)$ and let $(\tilde{x}^\mu) = (x^\mu, z, u)$ with $\tilde{x}^6 = u$. Let $B_1 = A_{\mu\nu}(\mu, \nu = 0, 1, \dots, 5)$ and $B_2 = A_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ the 2-forms in $H^2(X, \mathbb{C})$ such that B_2 is ample when X as the complex 3-fold. Define $\tilde{A} = \tilde{A}(x^\mu, z, u) = \tilde{A}(\tilde{x})$ the gauge field on X such that $\tilde{A}_u = 0$. Let the 2-forms $\tilde{F} = (\tilde{A}_{\mu\nu})(\mu, \nu = 0, 1, \dots, 5)$ on X and $\text{ch}_X(\tilde{F})$ the Chern character for $\tilde{F}$ on X , and define $Z_{B_1, B_2}(\tilde{F}) = \int e^{-(B_1 + \sqrt{-1}B_2)} \text{ch}_X(\tilde{F})$. Then;

Theorem As the chiral anomaly works even when $A_z = 0$ gauge [4, pp.869], $Z_{B_1, B_2}(\tilde{F})|_{M^4 \times \mathbb{R}} = \delta_\Lambda S_{D8}^{CS}$ is satisfied. Therefore, if [1, I, Conj.3.2.7] holds for $Z_{B_1, B_2}(\tilde{F})$, then $\delta_\Lambda S_{D8}^{CS}$ or $Z_{B_1, B_2}(\tilde{F})$ reproduces the chiral anomaly.

References

- [1] A.Bayer, A.Bertram, E.Macri, and Y.Toda: *Bogomolov-Gieseker type ...*, [math.AG]1103.5010, 1106.3430.
- [2] A.Ishii and K.Ueda: [math.AG]1012.5449v1, et al.
- [3] A.Karch, E.Katz: *Adding flavor ...*, hep-th/0205236.
- [4] T.Sakai and S.Sugimoto: *Low energy hadron physics in holographic QCD*, Prog.Theor.Phys., **113**(2005).
- [5] E.Witten: *Anti-de-Sitter space ...*, hep-th/9803131.