

On the Possibility of Barkhausen-Kurz Oscillation. II

By

Isao Takahashi

(Received November 19, 1946)

1. Introduction

While in the previous paper¹ with the same title the author treated the case when no electrons are caught by the anode in Barkhausen-Kurz tube, in this paper, the interelectrode current of Barkhausen-Kurz tube is calculated in the case when the electrons of larger amplitude are captured by the anode and the occurrence of negative resistance is demonstrated in some numerical examples.

2. Currents between Electrodes, when the Electrons of larger Amplitude are captured by the Anode.

We consider this case for (a) and (b) with the anode d. c. voltage $V_a = 0$ in Fig. 1. The amplitude d_m of an electron from G to A can be obtained by means of u_2 in Part I:

$$\begin{aligned} d_m - d_2 &= \frac{\beta_2}{\omega^2} e^{j\omega(t_0+T_1)} (e^{j\omega T_2} - 1 - j\omega T_2) \\ &\quad + \frac{T_2}{T_1} \frac{\beta_1}{\omega^2} e^{j\omega t_0} (e^{j\omega T_1} - 1 - j\omega T_1 e^{j\omega T_1}) \\ &= \frac{\beta_2}{\omega^2} r e^{j(\omega t_0 - \varphi)}, \end{aligned} \quad (1)$$

where $\beta_1 = \frac{ev_1}{md_1}$ for (a) and (b), and

$$\beta_2 = \begin{cases} \frac{ev_1}{md_2} & \text{for (a)} \\ \frac{-ev_2}{md_2} & \text{for (b)} \end{cases}$$

and T_1 is the transit time from K to G , while T_2 is the transit time

¹ I. Takahashi: On the possibility of Bankhausen-Kurz oscillation, I. This Memoirs, A, Vol. 25, No 1, Article 3, 1947.

from G to the turning point. Thus the electrons leaving K at t_0 which makes the real part of (1) positive, that is, which satisfies

$$\left(2N - \frac{1}{2}\right)\pi < \omega t_0 + \varphi < \left(2N + \frac{1}{2}\right)\pi,$$

are destined to be caught by the anode, and the electrons starting at t_0 which makes the real part of (1) negative, that is, which satisfies

$$\left(2N + \frac{1}{2}\right)\pi < \omega t_0 + \varphi < \left(2N + \frac{3}{2}\right)\pi,$$

are to turn back in front of the anode.

Now, we know that the function $f(\omega t_0 + \varphi)$ such that

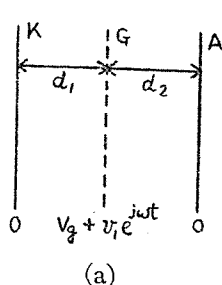
$$f(\omega t_0 + \varphi) = 0 \quad \text{for} \quad \left(2N - \frac{1}{2}\right)\pi < \omega t_0 + \varphi < \left(2N + \frac{1}{2}\right)\pi,$$

and

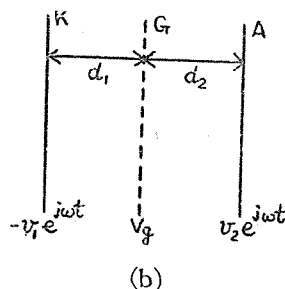
$$f(\omega t_0 + \varphi) = 1 \quad \text{for} \quad \left(2N + \frac{1}{2}\right)\pi < \omega t_0 + \varphi < \left(2N + \frac{3}{2}\right)\pi,$$

is expressed as:

$$\begin{aligned} f(\omega t_0 + \varphi) &= \frac{1}{2} + \frac{2}{\pi} \left(-\frac{\cos(\omega t_0 + \varphi)}{1} + \frac{\cos 3(\omega t_0 + \varphi)}{3} \dots \right) \\ &= \frac{1}{2} + F(t_0) \end{aligned} \quad (2)$$



(a)



(b)

A: anode
G: grid
K: cathode

V_g : positive grid voltage
 $v e^{j\omega t}$: oscillating voltage
 d : distance between electrodes

Fig. 1. Voltage distribution

We distinguish two classes of electrons. The first class consists of electrons which are not caught by the anode, thus being emitted by K in $n\{\frac{1}{2} + F(t_0)\}$ per unit time, and the second class consists of electrons which are caught by the anode, thus being emitted in $n\{\frac{1}{2} - F(t_0)\}$

per unit time. We proceed as in Part I. The current J_A flowing into G from outside on account of electron motions between G and A is

$$J_A = -\frac{ne}{d_2} \int_{t-T_1-T_2-\delta T_1''-\delta T_2''}^{t-T_1-\delta T_1'} \left(\frac{1}{2} + F(t_0) \right) u_2 dt_0 - \frac{ne}{d_2} \int_{t-T_1-T_2-\delta T_1'-\delta T_4'}^{t-T_1-\delta T_1'} \left(\frac{1}{2} - F(t_0) \right) u_2 dt_0,$$

where δT_4 is included in the instant $t = t_0 + T_1 + T_2 + \delta T_1(t_0) + \delta T_4(t_0)$ at which the second class electron reaches A and $\delta T_4'$ is equal to $\delta T_4(t - T_1 - T_2)$. Then, the d.c. part of J_A is

$$J_1' = -\frac{ne}{d_2} \left\{ \int_{t-T_1-T_2}^{t-T_1} \frac{1}{2} u_2' dt_0 + \int_{t-T_1-T_2}^{t-T_1} \frac{1}{2} u_2' dt_0 \right\} = -\frac{nea_2}{4d_2} T_2^2$$

and the a.c. part of frequency ω of J_A is

$$\begin{aligned} i_1' = -\frac{ne}{d_2} & \left\{ \int_{t-T_1-T_2-\delta T_1''-\delta T_2''}^{t-T_1-2T_2} \frac{1}{2} u_2' dt_0 + \int_{t-T_1-2T_2}^{t-T_1} \frac{1}{2} u_2'' dt_0 - \int_{t-T_1-2T_2}^{t-T_1} \frac{2}{\pi} \cos(\omega t_0 + \varphi) u_2' dt_0 \right. \\ & - \int_{t-T_1-\delta T_1'}^{t-T_1} \frac{1}{2} u_2' dt_0 + \int_{t-T_1-T_2-\delta T_1'-\delta T_4'}^{t-T_1-T_2} \frac{1}{2} u_2' dt_0 + \int_{t-T_1-T_2}^{t-T_1} \frac{1}{2} u_2' dt_0 \\ & \left. + \int_{t-T_1-T_2}^{t-T_1} \frac{2}{\pi} \cos(\omega t_0 + \varphi) u_2' dt_0 - \int_{t-T_1-\delta T_1'}^{t-T_1} \frac{1}{2} u_2' dt_0 \right\}. \end{aligned}$$

Thus

$$\begin{aligned} i_1' = & -\frac{ne}{d_2 \omega^2} \frac{2a_2}{\pi} e^{j\omega t} \left\{ e^{j\varphi} (e^{j\omega T_1} + 1 + j\omega T_2 e^{j\omega T_1}) \right\} \\ & + \frac{ne}{d_2 \omega^2} \beta_1 e^{j\omega t} \left\{ \left(\frac{2a_1 + a_2}{2a_1} \right) (1 - e^{j\omega T_1} - j\omega T_1 e^{-j\omega T_1}) e^{-j^2 \omega T_2} \right. \\ & - j\omega T_2 (1 - e^{-j\omega T_1}) e^{-j^2 \omega T_2} \\ & + (1 - e^{-j\omega T_1} - j\omega T_1 e^{-j\omega T_1}) \\ & - \frac{1}{2} (2 - e^{-j\omega T_1} - e^{-j^2 \omega T_2}) \\ & \times \left[\left(\frac{a_1 + a_2}{a_1} \right) \frac{(1 - e^{-j\omega T_1} - j\omega T_1 e^{-j\omega T_1})}{j\omega T_1} - (1 - e^{j\omega T_1}) \right] \\ & + \frac{1}{2} \frac{\beta_2}{\beta_1} (1 - e^{-j^2 \omega T_2} - j2\omega T_2 e^{-j^2 \omega T_2}) \\ & \left. + \frac{1}{2} \frac{\beta_2}{\beta_1} (2 - e^{-j\omega T_1} - e^{j^2 \omega T_2} - j3\omega T_2) \right\}, \quad (3) \end{aligned}$$

where $\alpha_1 = \frac{eV_g}{md_1}$, $\alpha_2 = \frac{eV_g}{md_2}$.

The current J_K flowing into G from outside on account of electron motions between K and G is

$$J_K = \frac{ne}{d_1} \int_{t-T_0-\delta T_0'}^{t-T_1-2T_2-\delta T_1''-\delta T_2''} \left(\frac{1}{2} + F(t_0) \right) u_3 dt_0.$$

Then, J_2 , the d. c. part and i_2' , a. c. part of frequency ω are respectively:

$$\begin{aligned} J_2 &= \frac{ne}{d_1} \left\{ \int_{t-T_0}^{t-T_1-2T_2} \frac{1}{2} u_3' dt_0 + \int_{t-T_1}^t u_1' dt_0 \right\} = \frac{ne\alpha_1 T_1^2}{4d_1}, \\ i_2' &= \frac{ne}{d_1} \left\{ \int_{t-T_0-\delta T_0'}^{t-T_0} \frac{1}{2} u_3' dt_0 + \int_{t-T_0}^{t-T_1-2T_2} \frac{1}{2} u_3'' dt_0 - \int_{t-T_0}^{t-T_1-2T_2} \frac{2}{\pi} \cos(\omega t_0 + \varphi) u_3' dt_0 \right. \\ &\quad \left. - \int_{t-T_1-2T_2-\delta T_1''-\delta T_2''}^{t-T_1-2T_2} \frac{1}{2} u_3' dt_0 + \int_{t-T_1-\delta T_1'}^{t-T_1} u_1' dt_0 + \int_{t-T_1}^t u_1'' dt_0 \right\}. \end{aligned}$$

Thus, we have

$$\begin{aligned} i_2' &= -\frac{ne}{d_1\omega^2} \frac{2d_1}{\pi} e^{j\omega t} \left\{ e^{j\varphi} (1 - e^{j\omega t} + j\omega T_1 e^{j\omega T_1}) \right\} \\ &\quad + \frac{ne}{d_1\omega^2} \beta_1 e^{j\omega t} \left\{ \left(\frac{2\alpha_1 + \alpha_2}{2\alpha_2} \right) (1 - e^{-j\omega T_1} - j\omega T_1 e^{-j\omega T_1}) e^{-j^2\omega T_2} \right. \\ &\quad \left. - j\omega T_2 (1 - e^{-j\omega T_1}) e^{-j^2\omega T_2} \right. \\ &\quad \left. + (1 - e^{-j\omega T_1} - j\omega T_1 e^{-j\omega T_1}) \right. \\ &\quad \left. + \frac{1}{2} \left[3(1 - e^{-j\omega T_1} - j\omega T_1) \right. \right. \\ &\quad \left. \left. - (1 - e^{-j\omega T_1})^2 e^{-j^2\omega T_2} \right. \right. \\ &\quad \left. \left. - \frac{\alpha_1 + \alpha_2}{\alpha_2} \frac{2(1 - e^{-j\omega T_1})(1 - e^{-j\omega T_1} - j\omega T_1) e^{-j^2\omega T_2}}{j\omega T_1} \right] \right. \\ &\quad \left. + \frac{1}{2} \frac{\beta_2}{\beta_1} (1 - e^{-j^2\omega T_2} - j2\omega T_2 e^{-j^2\omega T_2}) \right. \\ &\quad \left. + \frac{1}{2} \frac{\beta_2}{\beta_1} (1 - e^{-j\omega T_1}) \times \left[(1 - e^{-j^2\omega T_2}) \right. \right. \\ &\quad \left. \left. - \frac{\alpha_1 + \alpha_2}{\alpha_2} \frac{(1 - e^{-j^2\omega T_2} - j2\omega T_2 e^{-j^2\omega T_2})}{j\omega T_2} \right] \right\} \end{aligned}$$

Now, the total d. c. current becomes

$$J'_1 + J'_2 = \frac{ne}{4} \left(\frac{a_1}{d_1} T_1^2 - \frac{a_2}{d_2} T_2^2 \right) = 0, \quad (4)$$

because of $a_1 T_1 = a_2 T_2$.

3. Some Numerical Examples

i'_1 and i'_2 consist respectively of two components, one of which contains the factor α and the other the factor β .

We attach the suffixes a and b to i'_1 and i'_2 for the cases (a) and (b) in Fig. 1 respectively. For (b) $-i'_{1b}$ is chosen instead of i'_{1b} , on account of the sign of the oscillating voltage between G and A .

Hence

$$\begin{aligned} i'_{1a} &= \frac{ne}{d_2 \omega^2} \frac{2a_2}{\pi} e^{j\omega t} (A_1 + jB_1) + \frac{ne}{d_2 \omega^2} \beta_1 e^{j\omega t} (R_1 + jI_1), \\ i'_{2a} &= \frac{ne}{d_1 \omega^2} \frac{2a_1}{\pi} e^{j\omega t} (A_2 + jB_2) + \frac{ne}{d_1 \omega^2} \beta_1 e^{j\omega t} (R_2 + jI_2), \\ -i'_{1b} &= \frac{ne}{d_2 \omega^2} \frac{2a_2}{\pi} e^{j\omega t} (A_3 + jB_3) + \frac{ne}{d_2 \omega^2} \beta_1 e^{j\omega t} (R_3 + jI_3), \\ i'_{2b} &= \frac{ne}{d_1 \omega^2} \frac{2a_1}{\pi} e^{j\omega t} (A_4 + jB_4) + \frac{ne}{d_1 \omega^2} \beta_1 e^{j\omega t} (R_4 + jI_4), \end{aligned}$$

and we obtain the following table:

ωT_1	ωT_2	A_1	R_1	A_2	R_2	A_3	R_3	A_4	R_4
$\frac{\pi}{2}$	$\frac{\pi}{2}$	+0.57	+0.66	+0.57	+0.52	-0.30	+4.43	-1.11	+2.03
$\frac{\pi}{3}$	$\frac{2\pi}{3}$	-2.22	+3.16	+0.49	-0.32	-1.80	+2.65	-0.43	+0.00
$\frac{\pi}{4}$	$\frac{3\pi}{4}$	-0.97	+9.72	+0.30	-0.13	-2.36	+1.43	-0.12	-0.26

As the first terms in i'_1 and i'_2 are larger in magnitude than the second terms, corresponding to the ratio of α to β , so we expect the oscillation, when both A 's are negative, that is, for $-i'_{1b}$ and i'_{2b} in these examples. Here are presented only the possibility of oscillation but not the limit of oscillation.