

Comparing Expressiveness of First-Order Modal μ -calculus and First-Order CTL*

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Abstract

In this paper we introduce a first-order extension of propositional modal μ -calculus (first-order modal μ -calculus) and a first-order extension of CTL* (first-order CTL*), and then compare their expressiveness. More precisely we show that first-order CTL* is strictly less expressive than first-order modal μ -calculus. It is known that CTL* is strictly less expressive than propositional modal μ -calculus, hence our result shows that our two first-order extensions of CTL* and $\text{PM}\mu$ preserve an expressiveness relation.

1 Introduction

Formal verification is an important research area in computer science, and many people verify safety critical systems with formal verification. In formal verification, 1) a verification target is formalized as a theory and a model of a logic, 2) a verification item is formalized as a formula of the logic, 3) and then a verifier checks whether the formalized verification target satisfies the formalized verification item with theorem proving and model checking, which is based on the logic. [Propositional] temporal logics[3, 9] and propositional modal μ -calculus $\text{PM}\mu$ [7] have been often used for this purpose. As verification targets and verification items are becoming complicated, extensions, in particular first-order extensions, of propositional temporal logics have been introduced [2, 4, 5, 10]. The author and Kashima introduced a first-order extension of $\text{PM}\mu$ in [6, 8].

There are many propositional temporal logics, linear temporal logic LTL, computation tree logic CTL and CTL* etc., thus their expressiveness results have been proved. For example, it is known that CTL is strictly less expressive than CTL* (see [1]), and that CTL* is strictly less expressive than $\text{PM}\mu$ (see [3]). $\text{PM}\mu$ is a yardstick for comparing expressiveness of propositional temporal logics. Then the author thinks that our first-order extension of $\text{PM}\mu$ can be also a yardstick for comparing expressiveness of first-order extensions of propositional temporal logics.

In this paper we compare expressiveness of a first-order extension of $\text{PM}\mu$ and a first-order extension of CTL*, more precisely show that the first-order extension of CTL* is strictly less expressive than the first-order extension of $\text{PM}\mu$.

2 First-Order Modal μ -calculus

In this section, we define a first-order extension of propositional modal μ -calculus and call the logic *first-order modal μ -calculus* ($FOM\mu$).

2.1 Syntax of $FOM\mu$

Definition 1 A signature σ for first-order modal μ -calculus consists of two infinite sets $IVar$ and $PVar$, and a set $Pred_n$ for each natural number $n \geq 0$.

We call elements of $IVar$ *individual variables* and write $x, y, \dots$ for them, and also call those of $PVar$ *propositional variables* and write $X, Y, \dots$ for them. We call elements of $Pred_n$ *n -ary predicate symbols* and write $P, Q, \dots$ for them.

Definition 2 Let σ be a signature of $FOM\mu$. Then the σ -formulas of $FOM\mu$ are the following:

1. $X \in PVar \Rightarrow X$ is a σ -formula,
2. $P \in Pred_n$ and $\bar{x} \in IVar \Rightarrow P(\bar{x})$ is a σ -formula,
3. φ and ψ are σ -formulas $\Rightarrow \neg\varphi, \varphi \vee \psi, \Box\varphi$ are σ -formulas.
4. $x \in IVar$ and φ is a σ -formula $\Rightarrow \forall x.\varphi$ is a σ -formula,
5. If $X \in PVar$, φ is a σ -formula and no free occurrence of X in φ is negative $\Rightarrow \mu X.\varphi$ is a σ -formula.

We use usual abbreviations $\varphi \wedge \psi, \varphi \supset \psi, \exists x.\varphi$ and two abbreviations

- $\Diamond\varphi \stackrel{\text{def}}{=} \neg\Box\neg\varphi,$
- $\nu X.\varphi \stackrel{\text{def}}{=} \neg\mu X.\neg\varphi[(\neg X)/X]$

where $\varphi[(\neg X)/X]$ denotes the syntactic substitution of a formula $\neg X$ for free occurrences of a propositional variable X in φ .

2.2 Semantics of $FOM\mu$

We define a structure of $FOM\mu$, and it is the same as structure of first-order modal logic.

Definition 3 Let σ be a signature of $FOM\mu$. A σ -structure is the quadruple $\langle S, R, D, I \rangle$ where

1. S is a non-empty set (of states),
2. D is a non-empty set,
3. R is a binary relation on S and

4. I is a function of the type $\text{Pred}_n \times S \rightarrow \wp(D^n)$ for every $n \in \mathbb{N}$.

A structure $\langle S, R, D, I \rangle$ of $\text{FOM}\mu$ is an extension of a Kripke frame $\langle S, R \rangle$ of propositional modal μ -calculus, in which a state s represents a first-order structure $\langle D, I(-, s) \rangle$.

$\text{FOM}\mu$ has two kinds of variables, namely individual variables and propositional variables, hence we define a valuation of $\text{FOM}\mu$ as the mixture of those of first-order logic and modal μ -calculus.

Definition 4 Let $\mathcal{A} = \langle S, R, D, I \rangle$ be a σ -structure. A valuation in $\mathcal{A}$ is a function V which takes an element $x \in \text{IVar}$ to an element of D , and an element $X \in \text{PVar}$ to an element of $\wp(S)$.

In the sequel, we fix a σ -structure $\mathcal{A} = \langle S, R, D, I \rangle$ and a valuation V in $\mathcal{A}$ unless stated otherwise.

Now we prepare functions to define denotations of formulas. Let $x, y \in \text{IVar}$, $d \in D$, $X, Y \in \text{PVar}$ and $T \in \wp(S)$ of S . Then we define functions $V[d/x]$ and $V[T/X]$ as follows.

$$V[d/x](Y) = V(Y), \quad V[d/x](y) = \begin{cases} d, & \text{if } y = x \\ V(y), & \text{if } y \neq x \end{cases}$$

$$V[T/X](y) = V(y), \quad V[T/X](Y) = \begin{cases} T, & \text{if } Y = X \\ V(Y), & \text{if } Y \neq X \end{cases}$$

Finally we define denotations of formulas. In modal μ -calculus, the denotation of a formula is defined as the set of states at which the formula is true. Therefore we extend that idea to $\text{FOM}\mu$.

Definition 5 Let φ be a formula of $\text{FOM}\mu$, then we define the denotation $\llbracket \varphi \rrbracket_V^{\mathcal{A}} (\in \wp(S))$ of φ as follows:

1. $\llbracket X \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} V(X)$ for $X \in \text{PVar}$,
2. $\llbracket P(\bar{x}) \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} \{ s \in S \mid V(\bar{x}) \in I(P, s) \}$. where $P \in \text{Pred}_n$ and $\bar{x} \in \text{IVar}$,
3. $\llbracket \neg \varphi \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} S \setminus \llbracket \varphi \rrbracket_V^{\mathcal{A}}$,
4. $\llbracket \varphi \vee \psi \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} \llbracket \varphi \rrbracket_V^{\mathcal{A}} \cup \llbracket \psi \rrbracket_V^{\mathcal{A}}$,
5. $\llbracket \forall x. \varphi \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} \bigcap_{d \in D} \llbracket \varphi \rrbracket_{V[d/x]}^{\mathcal{A}}$,
6. $\llbracket \Box \varphi \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} \{ s \in S \mid \forall t \in S, R(s, t) \Rightarrow t \in \llbracket \varphi \rrbracket_V^{\mathcal{A}} \}$,
7. $\llbracket \mu X. \varphi \rrbracket_V^{\mathcal{A}} \stackrel{\text{def}}{=} \bigcap \{ T \subseteq S \mid \llbracket \varphi \rrbracket_{V[T/X]}^{\mathcal{A}} \subseteq T \}$.

Let $\mathcal{A} = \langle S, R, D, I \rangle$ be a structure, (v, V) a valuation in $\mathcal{A}$ and $s \in S$. Then we define a satisfaction relation $\models$ as follows:

$$\mathcal{A}, V, s \models \varphi \stackrel{\text{def}}{\iff} s \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$$

Remark 6 Let φ be a formula, $\mathcal{A}$ a structure and V a valuation in $\mathcal{A}$.

1. By the definition of $\llbracket \mu X.\varphi \rrbracket_V^{\mathcal{A}}$, the denotation $\llbracket \mu X.\varphi \rrbracket_V^{\mathcal{A}}$ is the least fixed-point of the function which takes an element T of $\wp(S)$ to $\llbracket \varphi \rrbracket_{V[T/X]}^{\mathcal{A}}$.
2. By the definition of the formula $\nu X.\varphi$,

$$\llbracket \nu X.\varphi \rrbracket_V^{\mathcal{A}} = \bigcup \{T \subseteq S \mid \llbracket \varphi \rrbracket_{V[T/X]}^{\mathcal{A}} \supseteq T\},$$

hence $\llbracket \nu X.\varphi \rrbracket_V^{\mathcal{A}}$ is the greatest fixed-point of the function which takes an element T of $\wp(S)$ to $\llbracket \varphi \rrbracket_{V[T/X]}^{\mathcal{A}}$.

3 First-Order CTL*

In this section, we define a logic FOCTL* which is a first-order extension of CTL*, i.e., we define syntax and semantics of FOCTL*.

3.1 Syntax of FOCTL*

A signature of FOCTL* is the same as that of FOM μ , and let σ be a signature of FOCTL*.

Definition 7 We mutually define the state σ -formulas and the path σ -formulas of FOCTL* as follows:

1. $X \in PVar \Rightarrow X$ is a state σ -formula,
2. $P \in Pred_n$ and $\bar{x} \in IVar \Rightarrow P(\bar{x})$ is a state σ -formula,
3. φ, φ' are state σ -formulas $\Rightarrow \neg\varphi, \varphi \vee \varphi'$ are state σ -formulas,
4. φ is a state σ -formula and $x \in IVar \Rightarrow \forall x.\varphi$ is a state σ -formula,
5. ψ is a path σ -formula $\Rightarrow \mathbf{E}\psi$ is a state σ -formula,
6. Every state σ -formula is a path σ -formula,
7. ψ, ψ' are path σ -formulas $\Rightarrow \neg\psi$ and $\psi \vee \psi'$ are path σ -formulas.
8. ψ is a path σ -formula and $x \in IVar \Rightarrow \forall x.\psi$ is a path σ -formula.
9. ψ, ψ' are path σ -formulas $\Rightarrow \mathbf{X}\psi$ and $\psi \mathbf{U} \psi'$ are path σ -formulas.

We use an abbreviation $\mathbf{A}\psi \stackrel{\text{def}}{=} \neg\mathbf{E}\neg\psi$ for a path formula ψ . When a signature σ is not important or clear from context, we often write a state (path) formula for state (respectively path) σ -formula. We say that θ is a formula of FOCTL* if it is a state formula or a path formula of FOCTL*.

3.2 Semantics of FOCTL*

In this subsection we define semantics of FOCTL*, i.e., we define structures and a satisfaction relation for FOCTL*. A definition of structure of FOCTL* is the same as that of FOM μ . But a definition of the satisfaction relation is different from that of FOM μ .

Definition 8 1. A structure of FOCTL* is defined in the same way as FOM μ , namely it is a quadruple $\langle S, R, D, I \rangle$ satisfying the conditions in Definition 3.

2. A valuation of FOCTL* is defined in the same way as FOM μ , namely it is a function V satisfying the conditions in Definition 4.

3. For a structure $\langle S, R, D, I \rangle$ of FOCTL*, a full path in S is a maximal sequence $s_0, s_1, \dots$, of elements of S such that $(s_i, s_{i+1}) \in R$.

For a full path $\pi = s_0, s_1, \dots$ and a natural number i , we write π^i for $s_i, s_{i+1}, \dots$.

Since there are two kinds of formulas in FOCTL*, namely state formulas and path formulas, a definition of a satisfaction relation for state formulas is different from that for path formulas. Thus we mutually define the satisfaction relation $\models$ for state formulas and path formulas.

Definition 9 Let $\mathcal{A} = \langle S, R, D, I \rangle$ be a structure of FOCTL*, V a valuation in $\mathcal{A}$, $s \in S$ and π a full path in S . Let X be a propositional variable, P an n -ary predicate symbol and $\bar{x}$ individual variables. Let φ, φ' be state formulas, and ψ, ψ' path formulas. Put Π is the set of full paths in S and $\pi(0)$ denotes the first element of π . Then we define a satisfaction relation $\models$ as follows:

1. $\mathcal{A}, V, s \models X \stackrel{\text{def}}{\iff} s \in V(X),$
2. $\mathcal{A}, V, s \models P(\bar{x}) \stackrel{\text{def}}{\iff} V(\bar{x}) \in I(P, s),$
3. $\mathcal{A}, V, s \models \neg\varphi \stackrel{\text{def}}{\iff} \mathcal{A}, V, s \models \varphi$ does not hold.
4. $\mathcal{A}, V, s \models \varphi \vee \varphi' \stackrel{\text{def}}{\iff} \mathcal{A}, V, s \models \varphi$ or $\mathcal{A}, V, s \models \varphi',$
5. $\mathcal{A}, V, s \models \forall x.\varphi \stackrel{\text{def}}{\iff} \mathcal{A}, V[d/x], s \models \varphi$ for any $d \in D,$
6. $\mathcal{A}, V, s \models \mathbf{E}\psi \stackrel{\text{def}}{\iff} \exists \pi \in \Pi$ such that $\pi(0) = s$ and $\mathcal{A}, V, \pi \models \psi,$
7. $\mathcal{A}, V, \pi \models \psi \stackrel{\text{def}}{\iff} \mathcal{A}, V, \pi(0) \models \psi$ for a state formula $\psi,$
8. $\mathcal{A}, V, \pi \models \neg\psi \stackrel{\text{def}}{\iff} \mathcal{A}, V, \pi \models \psi$ does not hold,
9. $\mathcal{A}, V, \pi \models \psi \vee \psi' \stackrel{\text{def}}{\iff} \mathcal{A}, V, \pi \models \psi$ or $\mathcal{A}, V, \pi \models \psi'.$
10. $\mathcal{A}, V, \pi \models \forall x.\psi \stackrel{\text{def}}{\iff} \mathcal{A}, V[d/x], \pi \models \psi$ for any $d \in D,$

11. $\mathcal{A}, V, \pi \Vdash \mathbf{X}\psi \stackrel{\text{def}}{\iff} \mathcal{A}, V, \pi^1 \Vdash \psi,$
12. $\mathcal{A}, V, \pi \Vdash \psi \mathbf{U} \psi' \stackrel{\text{def}}{\iff} \exists j \geq 0. \mathcal{A}, V, \pi^j \Vdash \psi' \text{ and } 0 \leq \forall k < j. \mathcal{A}, V, \pi^k \Vdash \psi$

4 Comparison of Expressiveness of $\text{FOM}\mu$ and FOCTL^*

In this section we compare expressiveness of $\text{FOM}\mu$ and FOCTL^* . More precisely we show that FOCTL^* is strictly less expressive than $\text{FOM}\mu$. On the other hand $\text{FOM}\mu$ (FOCTL^*) is a first-order extension of $\text{PM}\mu$ (respectively CTL^*), and it is known that CTL^* is strictly less expressive than $\text{PM}\mu$ (See Theorem 4.1.4 in [3] which is a linear temporal version of the statement.) Thus $\text{FOM}\mu$ and FOCTL^* preserve a expressiveness relation.

4.1 Definitions of Comparing Expressiveness

For a signature σ of FOCTL^* , hence of $\text{FOM}\mu$, $\text{FOCTL}^*[\sigma]$ ($\text{FOM}\mu[\sigma]$) denotes the set of σ -formulas of FOCTL^* (respectively $\text{FOM}\mu$).

Definition 10 *Let σ be a signature.*

1. *We say that a state formula φ of $\text{FOCTL}^*[\sigma]$ can be expressed in the set $\text{FOM}\mu[\sigma]$ if there is a formula θ of $\text{FOM}\mu[\sigma]$ such that $\mathcal{A}', V', s' \Vdash \varphi \iff \mathcal{A}', V', s' \Vdash \theta$ for any structure $\mathcal{A} = \langle S', R', D', I' \rangle$, valuation V' in $\mathcal{A}'$ and an element $s' \in S'$.*
2. *We say that a path formula ψ of $\text{FOCTL}^*[\sigma]$ can be expressed in the set $\text{FOM}\mu[\sigma]$ if there is a formula θ of $\text{FOM}\mu[\sigma]$ such that $\mathcal{A}', V', s' \Vdash \mathbf{A}\psi \iff \mathcal{A}', V', s' \Vdash \theta$ for any structure $\mathcal{A} = \langle S', R', D', I' \rangle$, valuation V' in $\mathcal{A}'$ and an element $s' \in S'$.*
3. *We say that the set $\text{FOCTL}^*[\sigma]$ is less expressive than $\text{FOM}\mu[\sigma]$ if any formula of $\text{FOCTL}^*[\sigma]$ can be expressed in $\text{FOM}\mu[\sigma]$. When $\text{FOCTL}^*[\sigma]$ is less expressive than $\text{FOM}\mu[\sigma]$, we write $\text{FOCTL}^*[\sigma] \leq \text{FOM}\mu[\sigma]$.*
4. *We say that FOCTL^* is less expressive than $\text{FOM}\mu$ if $\text{FOCTL}^*[\sigma] \leq \text{FOM}\mu[\sigma]$ for any signature σ . When FOCTL^* is less expressive than $\text{FOM}\mu$, we write $\text{FOCTL}^* \leq \text{FOM}\mu$.*

Proposition 11 *FOCTL^* is less expressive than $\text{FOM}\mu$.*

Proof We can prove the proposition in a similar way to the fact that CTL^* is less expressive than $\text{PM}\mu$. (See pp. 367 in [3] for example.) $\square$

Definition 12 *Let σ be a signature.*

1. We say that a formula θ of the set $FOM\mu[\sigma]$ cannot be expressed in the set $FOCTL^*[\sigma]$ if, for each state formula φ of $FOCTL^*[\sigma]$, there is a structure $\mathcal{A}' = \langle S', R', D', I' \rangle$, a valuation V' in $\mathcal{A}'$ and an element $s' \in S'$ such that $\mathcal{A}', V', s' \models \theta \not\models \mathcal{A}', V', s' \models \varphi$.
2. We say that $FOCTL^*[\sigma]$ is strictly less expressive than $FOM\mu[\sigma]$ (or $FOM\mu[\sigma]$ is strictly more expressive than $FOCTL^*[\sigma]$) if $FOCTL^*[\sigma] \leq FOM\mu[\sigma]$, and there is a formula φ of $FOM\mu[\sigma]$ which cannot be expressed in $FOCTL^*$. We write $FOCTL^*[\sigma] \leq FOM\mu[\sigma]$ when $FOCTL^*[\sigma]$ is strictly less expressive than $FOM\mu[\sigma]$.
3. We say that $FOCTL^*$ is strictly less expressive than $FOM\mu$ (or $FOM\mu$ is strictly more expressive than $FOCTL^*$) if $FOCTL^* \leq FOM\mu$, and $FOCTL^*[\sigma] \leq FOM\mu[\sigma]$ for some signature σ . We write $FOCTL^* \leq FOM\mu$ when $FOCTL^*$ is strictly less expressive than $FOM\mu$.

4.2 Expressiveness of $FOM\mu$

In this subsection we give an expressiveness result of $FOM\mu$, in particular that of a formula $\nu X.\varphi \wedge \Box\Box X$. In Lemma 14, we introduce a structure $\mathcal{A}_k$ so that we will compare expressiveness of $FOM\mu$ and $FOCTL^*$ in the next subsection.

Lemma 13 Let $R = \{(i, i+1) \mid i \in \mathbb{N}\}$, D be a non-empty set, I an interpretation. Let V be a valuation in the structure $\mathcal{A} = \langle \mathbb{N}, R, D, I \rangle$ and φ a formula which does not contain a free propositional variable X . Then

$$\llbracket \nu X.\varphi \wedge \Box\Box X \rrbracket_V^{\mathcal{A}} = \{i \in \mathbb{N} \mid \mathcal{A}, V, i + 2k \models \varphi \text{ for any } k \in \mathbb{N}\}.$$

Proof ($\subseteq$) We show that $i \in \llbracket \nu X.\varphi \wedge \Box\Box X \rrbracket_V^{\mathcal{A}} \Rightarrow i + 2k \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$ for all $k \in \mathbb{N}$.

Put $\llbracket \nu X.\varphi \wedge \Box\Box X \rrbracket_V^{\mathcal{A}} = M$. Since M is the [greatest] fixed-point of the function which takes an element $T \in \wp(\mathbb{N})$ to the element $\llbracket \varphi \wedge \Box\Box X \rrbracket_{V[T/X]}^{\mathcal{A}}$, We have the following equality:

$$M = \llbracket \varphi \wedge \Box\Box X \rrbracket_{V[M/X]}^{\mathcal{A}} (= \llbracket \varphi \rrbracket_{V[M/X]}^{\mathcal{A}} \cap \llbracket \Box\Box X \rrbracket_{V[M/X]}^{\mathcal{A}}). \quad (*)$$

Assume that $i \in M$. Then $i \in \llbracket \varphi \rrbracket_{V[M/X]}^{\mathcal{A}}$ by (*), hence $i \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$. Again by (*), $i \in \llbracket \Box\Box X \rrbracket_{V[M/X]}^{\mathcal{A}}$, this implies that $i + 2 \in \llbracket X \rrbracket_{V[M/X]}^{\mathcal{A}}$. Thus we have that $i + 2 \in M$. By repeating this argument, we have that $i + 2k \in M$ (in particular $i + 2k \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$) for any $k \in \mathbb{N}$.

($\supseteq$) Put $M' = \{i \in \mathbb{N} \mid \mathcal{A}, V, i + 2k \models \varphi \text{ for all } k \in \mathbb{N}\}$. Since $\llbracket \nu X.\varphi \wedge \Box\Box X \rrbracket_V^{\mathcal{A}}$ is the [greatest] fixed-point of the function which takes an element $T \in \wp(\mathbb{N})$ to the element $\llbracket \varphi \wedge \Box\Box X \rrbracket_{V[T/X]}^{\mathcal{A}}$, it is enough to show that $M' \subseteq \llbracket \varphi \wedge \Box\Box X \rrbracket_{V[M'/X]}^{\mathcal{A}}$.

Assume that $i \in M'$. Then $i + 2k \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$ for any $k \in \mathbb{N}$, in particular $i \in \llbracket \varphi \rrbracket_V^{\mathcal{A}}$ when $k = 0$. Hence $i \in \llbracket \varphi \rrbracket_{V[M'/X]}^{\mathcal{A}}$. On the other hand,

$$i \in M' \Rightarrow i + 2 \in M' \Leftrightarrow i \in \llbracket \Box\Box X \rrbracket_{V[M'/X]}^{\mathcal{A}}.$$

Thus $i \in \llbracket \varphi \wedge \Box\Box X \rrbracket_{V[M'/X]}^{\mathcal{A}} (= \llbracket \varphi \rrbracket_{V[M'/X]}^{\mathcal{A}} \cap \llbracket \Box\Box X \rrbracket_{V[M'/X]}^{\mathcal{A}})$. $\square$

In the following, we consider a signature which consists of a single unary predicate symbol P .

Lemma 14 *Let $R = \{(i, i+1) \mid i \in \mathbb{N}\}$ and D a non-empty set, and let I_k be an interpretation such that*

$$I_k(P, i) = \begin{cases} \emptyset, & \text{if } k = i, \\ D, & \text{otherwise.} \end{cases}$$

Put $\mathcal{A}_k = \langle \mathbb{N}, R, D, I_k \rangle$ and let V be a valuation in $\mathcal{A}$ such that $V(X) = \mathbb{N}$ for any propositional variable X . Then $\mathcal{A}_k, V, 0 \Vdash \nu X. P(x) \wedge \Box\Box X \Leftrightarrow k$ is odd.

Proof

$$\begin{aligned} & \mathcal{A}_k, V, 0 \Vdash \nu X. P(x) \wedge \Box\Box X \\ \Leftrightarrow & 0 \in \llbracket \nu X. P(x) \wedge \Box\Box X \rrbracket_V^{\mathcal{A}_k} \\ \Leftrightarrow & 0 \in \{i \in \mathbb{N} \mid \mathcal{A}_k, V, i + 2j \Vdash P(x) \text{ for any } x \in \mathbb{N}\} \quad (\text{Lemma 13}) \\ \Leftrightarrow & k \text{ is odd} \end{aligned}$$

$\square$

We remark that a valuation in a structure $\mathcal{A}_k$ can be a valuation in a structure $\mathcal{A}_l$ for any natural numbers k, l , hence a valuation in $\mathcal{A}_k$'s makes sense.

4.3 FOCTL* is strictly less expressive than FOM μ

In this subsection we show that FOCTL* is strictly less expressive than FOM μ . Similarly it is known that CTL* is strictly less expressive than PM μ .

Our proof of Theorem 18 is a first-order (and branching) extension of Theorem 4.1.4 in [3], hence we must also consider formulas of the form $\forall x.\varphi$ and a valuation V . This requires modification of induction statements in proofs, thus we introduce a binary relation $\sim$ on the set of valuations in a structure.

Definition 15 *Let $\langle S', R', D', I' \rangle$ be a structure and $\mathcal{V}$ the set of valuations in it and $V, V' \in \mathcal{V}$. We define a binary relation $\sim$ on $\mathcal{V}$ as follows: $V' \sim V''$ if there is a finite subset $I\text{Var}_0$ of $I\text{Var}$ such that*

1. $V'(x) = V''(x)$ for any $x \in I\text{Var} \setminus I\text{Var}_0$ and
2. $V'(X) = V''(X)$ for any $X \in P\text{Var}$.

Lemma 16 *Let $\mathcal{A}_j = \langle \mathbb{N}, R, D, I_j \rangle$ be the structure for each natural number j and the valuation V defined in Proposition 14.*

1. *For any state formula φ of FOCTL*[{ P }], natural numbers $k, i \geq 0$ and valuation $V' \sim V$, $\mathcal{A}_k, V', i \Vdash \varphi \Leftrightarrow \mathcal{A}_{k+1}, V', i+1 \Vdash \varphi$.*

2. For any path formula ψ of $\text{FOCTL}^*[\{P\}]$, natural numbers $k, i \geq 0$ and valuation $V' \sim V$, $\mathcal{A}_k, V', \pi^i \Vdash \psi \Leftrightarrow \mathcal{A}_{k+1}, V', \pi^{i+1} \Vdash \psi$ where $\pi = 0, 1, 2, \dots$

Proof Let φ be a state formula and ψ a path formula. We prove the lemma by mutual induction on the construction of φ and ψ . Let V' be a valuation with $V' \sim V$.

Case 1 $\varphi \equiv X \in PVar$: By the definition of V' , $\mathcal{A}_k, V', i \Vdash X$ for any $k, i \in \mathbb{N}$. In particular $\mathcal{A}_k, V', i \Vdash X \Leftrightarrow \mathcal{A}_{k+1}, V', i+1 \Vdash X$.

Case 2 $\varphi \equiv P(x)$: (Recall that P is the unique predicate symbol.) By the definition of I_k , $\mathcal{A}_k, V', i \Vdash P(x) \Leftrightarrow k \neq i$ for any $k, i \in \mathbb{N}$. Then the following holds for any $k, i \in \mathbb{N}$.

$$\mathcal{A}_k, V', i \Vdash P(x) \Leftrightarrow k \neq i \Leftrightarrow k+1 \neq i+1 \Leftrightarrow \mathcal{A}_{k+1}, V', i+1 \Vdash P(x)$$

Case 3 $\varphi \equiv \neg\varphi'$ or $\varphi' \vee \varphi''$ for some state formulas φ' and φ'' : We skip proofs for these cases.

Case 4 $\varphi \equiv \forall x.\varphi'$ for some state formula φ' :

$$\begin{aligned} & \mathcal{A}_k, V', i \Vdash \forall x.\varphi' \\ \Leftrightarrow & \mathcal{A}_k, V'[d/x], i \Vdash \varphi' \quad \text{for any } d \in D \\ \Leftrightarrow & \mathcal{A}_{k+1}, V'[d/x], i+1 \Vdash \varphi' \quad \text{for any } d \in D \quad (V'[d/x] \sim V, \text{IH}) \\ \Leftrightarrow & \mathcal{A}_{k+1}, V', i+1 \Vdash \forall x.\varphi' \end{aligned}$$

Since the number of the logical symbol $\forall$ in φ is finite, valuations occurring in the induction must be the form $V'[d_1/x_1] \dots [d_n/x_n] (\sim V)$ for some elements $d_1, \dots, d_n$ of D and individual variables $x_1, \dots, x_n$. Thus our induction works.

Case 5 $\varphi \equiv \mathbf{E}\psi$ for some path formula ψ :

$$\begin{aligned} \mathcal{A}_k, V', i \Vdash \mathbf{E}\psi' & \Leftrightarrow \mathcal{A}_k, V', \pi^i \Vdash \psi' \\ & \Leftrightarrow \mathcal{A}_{k+1}, V', \pi^{i+1} \Vdash \psi' \quad (\text{IH}) \\ & \Leftrightarrow \mathcal{A}_{k+1}, V', i+1 \Vdash \mathbf{E}\psi' \end{aligned}$$

Case 6 ψ is a path formula which is also a state formula: Proofs for these cases are similar to those of the cases 1, 2, 3, 4 and 5.

Case 7 $\psi \equiv \neg\psi'$ or $\psi' \vee \psi''$ for some path formulas ψ' and ψ'' : We skip proofs for these cases.

Case 8 $\psi \equiv \forall x.\psi'$ for some path formula ψ' : A proof for this case is similar to that of the case 4.

Case 9 $\psi \equiv \mathbf{X}\psi'$ for some path formula ψ' :

$$\begin{aligned} \mathcal{A}_k, V', \pi^i \Vdash \mathbf{X}\psi' & \Leftrightarrow \mathcal{A}_k, V', \pi^{i+1} \Vdash \psi' \\ & \Leftrightarrow \mathcal{A}_{k+1}, V', \pi^{i+2} \Vdash \psi' \quad (\text{IH}) \\ & \Leftrightarrow \mathcal{A}_{k+1}, V', \pi^{i+1} \Vdash \mathbf{X}\psi' \end{aligned}$$

Case 10 $\psi \equiv \psi_1 \mathbf{U} \psi_2$ for some path formulas ψ_1, ψ_2 :

$$\begin{aligned}
& \mathcal{A}_k, V', \pi^i \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \exists i' > i. [\mathcal{A}_k, V', \pi^{i'} \Vdash \psi_2 \ \& \ (i < \forall j < i'. \mathcal{A}_k, V', \pi^j \Vdash \psi_1)] \\
\Leftrightarrow & \exists i' > i. [\mathcal{A}_{k+1}, V', \pi^{i'+1} \Vdash \psi_2 \ \& \ (i < \forall j < i'. \mathcal{A}_{k+1}, V', \pi^{j+1} \Vdash \psi_1)] \quad (\text{IH}) \\
\Leftrightarrow & \exists i'' > i+1. [\mathcal{A}_{k+1}, V', \pi^{i''} \Vdash \psi_2 \ \& \ (i+1 < \forall j < i''. \mathcal{A}_{k+1}, V', \pi^j \Vdash \psi_1)] \\
\Leftrightarrow & \mathcal{A}_{k+1}, V', \pi^{i+1} \Vdash \psi_1 \mathbf{U} \psi_2
\end{aligned}$$

□

Lemma 17 shows limitations of expressiveness of FOCTL* with respect to the structure $\mathcal{A}_k$.

Lemma 17 *Let $\mathcal{A}_j = \langle \mathbb{N}, R, D, I_j \rangle$ be the structure for each natural number j and the valuation V defined in Proposition 14.*

1. *For every state formula φ of FOCTL* $\{\{P\}\}$, there is a natural number l such that $\mathcal{A}_k, V', 0 \Vdash \varphi \Leftrightarrow \mathcal{A}_l, V', 0 \Vdash \varphi$ for any natural number $k \geq l$ and valuation $V' \sim V$.*
2. *We write $\pi^i = i, i+1, i+2, \dots$ for the path $\pi = 0, 1, 2, \dots$. For every path formula ψ of FOCTL* $\{\{P\}\}$, there is a natural number l such that $\mathcal{A}_k, V', \pi^0 \Vdash \psi \Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi$ for any natural number $k \geq l$ and valuation $V' \sim V$.*

Proof We prove the lemma by mutual induction on the construction of the formula φ and ψ .

Case 1 $\varphi \equiv X$ for some propositional variable X : By the definition of V' , we have that $\mathcal{A}_k, V', 0 \Vdash X$ for any $k \geq 0$. Thus put $l = 0$.

Case 2 $\varphi \equiv P(x)$ for some individual variable x : (Recall that P is the unique predicate symbol.) By the definition of I_k , we have that $\mathcal{A}_k, V', 0 \Vdash P(x) \Leftrightarrow k \neq 0$. Thus put $l = 1$.

Case 3 $\varphi \equiv \neg\varphi'$ or $\varphi' \vee \varphi''$ for some state formulas φ' and φ'' : We skip proofs for these cases.

Case 4 $\varphi \equiv \forall x. \varphi'$ for some state formula φ' : By induction hypothesis, for the formula φ' , there is a natural number $l_{\varphi'}$ such that $\mathcal{A}_k, V', 0 \Vdash \varphi' \Leftrightarrow \mathcal{A}_{l_{\varphi'}}, V', 0 \Vdash \varphi'$ for any valuation $V' \sim V$ and natural number $k \geq l_{\varphi'}$. Put $l = l_{\varphi'}$ and $k' \geq l$.

$$\begin{aligned}
& \mathcal{A}_{k'}, V', 0 \Vdash \forall x. \varphi' \\
\Leftrightarrow & \mathcal{A}_{k'}, V'[d/x], 0 \Vdash \varphi' \text{ for all } d \in D \\
\Leftrightarrow & \mathcal{A}_l, V'[d/x], 0 \Vdash \varphi' \text{ for all } d \in D \quad (k', l \geq l_{\varphi'}, V'[d/x] \sim V) \\
\Leftrightarrow & \mathcal{A}_l, V', 0 \Vdash \forall x. \varphi'
\end{aligned}$$

Case 5 $\varphi \equiv \mathbf{E}\psi$ for some path formula ψ : By induction hypothesis, for the path formula ψ , there is a natural number l_ψ such that $\mathcal{A}_k, V', \pi^0 \Vdash \psi \Leftrightarrow$

$\mathcal{A}_{l_\psi}, V', \pi^0 \Vdash \psi$ for any valuation $V' \sim V$ and natural number $k \geq l_\psi$. Put $l = l_\psi$ and $k' \geq l$.

$$\begin{aligned} \mathcal{A}_{k'}, V', 0 \Vdash \mathbf{E}\psi &\Leftrightarrow \mathcal{A}_{k'}, V', \pi^0 \Vdash \psi \\ &\Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi \quad (k', l \geq l_\psi) \\ &\Leftrightarrow \mathcal{A}_l, V', 0 \Vdash \mathbf{E}\psi \end{aligned}$$

Case 6 ψ is a path formula which is also a state formula: Proofs for these cases are similar to those of the cases 1, 2, 3, 4 and 5.

Case 7 $\psi \equiv \neg\psi'$ or $\psi' \vee \psi''$ for some path formulas ψ' and ψ'' : We skip proofs for these cases.

Case 8 $\psi \equiv \forall x.\psi'$ for some path formula ψ' : A proof for this case is similar to that of the case 4.

Case 9 $\psi \equiv \mathbf{X}\psi'$ for some path formula ψ' : By induction hypothesis, for the path formula ψ' , there is a natural number $l_{\psi'}$ such that $\mathcal{A}_k, V', \pi^0 \Vdash \psi' \Leftrightarrow \mathcal{A}_{l_{\psi'}}, V', \pi^0 \Vdash \psi'$ for any valuation $V' \sim V$ and natural number $k \geq l_{\psi'}$. Put $l = l_{\psi'} + 1$ and $k \geq l$.

$$\begin{aligned} \mathcal{A}_k, V', \pi^0 \Vdash \mathbf{X}\psi' &\Leftrightarrow \mathcal{A}_k, V', \pi^1 \Vdash \psi' \\ &\Leftrightarrow \mathcal{A}_{k-1}, V', \pi^0 \Vdash \psi' \quad (k \geq 1, \text{Lemma 16}) \\ &\Leftrightarrow \mathcal{A}_{l-1}, V', \pi^0 \Vdash \psi' \quad (k-1, l-1 \geq l_{\psi'}) \\ &\Leftrightarrow \mathcal{A}_l, V', \pi^1 \Vdash \psi' \quad (\text{Lemma 16}) \\ &\Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \mathbf{X}\psi' \end{aligned}$$

Case 10 $\psi \equiv \psi_1 \mathbf{U} \psi_2$ for some path formulas ψ_1, ψ_2 : By induction hypothesis, we can choose a natural number l_{ψ_1} (l_{ψ_2}) for ψ_1 (respectively ψ_2) such that, for every path formula ψ of $\text{FOCTL}^*[\{P\}]$, there is a natural number l such that $\mathcal{A}_k, V', \pi^0 \Vdash \psi_1 \Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi_1$ (respectively $\mathcal{A}_k, V', \pi^0 \Vdash \psi_2 \Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi_2$) for any natural number $k \geq l$ and valuation $V' \sim V$. Similarly, without referring a natural number $l_{\psi_1 \mathbf{U} \psi_2}$, we can choose a natural number $l_{\mathbf{X}\psi_1}$ ($l_{\mathbf{X}\psi_2}$) for the formula $\mathbf{X}\psi_1$ (respectively $\mathbf{X}\psi_2$). Then we put $l = \max\{l_{\mathbf{X}\psi_1}, l_{\mathbf{X}\psi_2}\}$.

By induction on k , we prove that

$$\mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \Leftrightarrow \mathcal{A}_k, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \text{ for any natural number } k \geq l.$$

Base Case: We prove that $\mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \Leftrightarrow \mathcal{A}_{l+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2$.

$$\begin{aligned}
& \mathcal{A}_{l+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
\Leftrightarrow & \mathcal{A}_{l+1}, V', \pi^0 \Vdash \mathbf{X}\psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
\Leftrightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \mathbf{X}\psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \quad (l \geq l_{\mathbf{X}\psi_2}) \\
\Leftrightarrow & \mathcal{A}_l, V', \pi^1 \Vdash \psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
\Rightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
\Rightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \text{ or } \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \quad (\text{Lemma 16}) \\
\Leftrightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\\
& \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_l, V', \pi^i \Vdash \psi_2 \text{ and } 0 < \forall j < i. (\mathcal{A}_l, V', \pi^j \Vdash \psi_1)] \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and} \\
& \quad 0 < \forall j < i. (\mathcal{A}_l, V', \pi^j \Vdash \psi_1)] \quad (\text{Lemma 16}) \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and} \\
& \quad \mathcal{A}_l, V', \pi^1 \Vdash \psi_1 \text{ and } 0 < \forall j < i. (\mathcal{A}_l, V', \pi^j \Vdash \psi_1)] \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and} \\
& \quad \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \text{ and } 0 < \forall j < i. (\mathcal{A}_l, V', \pi^j \Vdash \psi_1)] \quad (l \geq l_{\mathbf{X}\psi_1}) \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and} \\
& \quad \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \text{ and } 0 < \forall j < i. (\mathcal{A}_{l+1}, V', \pi^{j+1} \Vdash \psi_1)] \quad (\text{Lemma 16}) \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and} \\
& \quad \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \text{ and } 1 < \forall j' < i+1. (\mathcal{A}_{l+1}, V', \pi^{j'} \Vdash \psi_1)] \\
\Leftrightarrow & \exists i > 0. [\mathcal{A}_{l+1}, V', \pi^{i+1} \Vdash \psi_2 \text{ and } 0 < \forall j' < i+1. (\mathcal{A}_{l+1}, V', \pi^{j'} \Vdash \psi_1)] \\
\Rightarrow & \mathcal{A}_{l+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2
\end{aligned}$$

Induction Step: Assume that $\mathcal{A}_{k'}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2$ for any natural number k' with $l \leq k' \leq k$. Then we prove that $\mathcal{A}_{k+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \Leftrightarrow \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2$.

$$\begin{aligned}
& \mathcal{A}_{k+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \mathcal{A}_{k+1}, V', \pi^1 \Vdash \psi_2 \text{ or } \mathcal{A}_{k+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
\Leftrightarrow & \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_2 \text{ or } \mathcal{A}_{k+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \quad (k+1, l+1 \geq l_{\mathbf{X}\psi_2}) \\
\Leftrightarrow & \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_2 \text{ or } \mathcal{A}_{l+1}, V', \pi^1 \Vdash \psi_1 \wedge (\psi_1 \mathbf{U} \psi_2) \\
& \quad (k+1, l+1 \geq l_{\mathbf{X}\psi_1}, \text{ Lemma 16 and I.H.}) \\
\Leftrightarrow & \mathcal{A}_{l+1}, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \\
\Leftrightarrow & \mathcal{A}_l, V', \pi^0 \Vdash \psi_1 \mathbf{U} \psi_2 \quad (\text{Base Case})
\end{aligned}$$

□

Theorem 18 *For a unary predicate symbol P , $\text{FOCTL}^*[\{P\}]$ is strictly less expressive than $\text{FOM}\mu[\{P\}]$. In particular the formula $\nu X.P(x) \wedge \Box\Box X$ of $\text{FOM}\mu[\{P\}]$ cannot be expressed in $\text{FOCTL}^*[\{P\}]$, namely, for each state formula φ of $\text{FOCTL}^*[\sigma]$, there is a structure $\mathcal{A}' = \langle S', R', D', I' \rangle$, a valuation V' in $\mathcal{A}'$ and an element $s' \in S'$ such that $\mathcal{A}', V', s' \models \nu X.P(x) \wedge \Box\Box X \not\models \varphi$.*

Proof Assume that the formula $\nu X.P(x) \wedge \Box\Box X$ can be expressed as a state formula φ of $\text{FOCTL}^*[\{P\}]$, namely that $\mathcal{A}', V', s' \models \varphi \Leftrightarrow \mathcal{A}', V', s' \models \nu X.P(x) \wedge \Box\Box X$ for any structure $\mathcal{A}' = \langle S', R', D', I' \rangle$, valuation V' in $\mathcal{A}'$ and element $s' \in S'$.

Recall that we have introduced the structure $\mathcal{A}_k = \langle \mathbb{N}, R, D, I_k \rangle$ and the valuation V in Lemma 14. By Lemma 17, there is a natural number l such that, for any valuation $V' \sim V$ and natural numbers $k, k' \geq l$,

$$\begin{aligned} \mathcal{A}_k, V', 0 \models \nu X.P(x) \wedge \Box\Box X &\Leftrightarrow \mathcal{A}_k, V', 0 \models \varphi \\ &\Leftrightarrow \mathcal{A}_l, V', 0 \models \varphi \quad (k \geq l, \text{Lemma 17}) \\ &\Leftrightarrow \mathcal{A}_{k'}, V', 0 \models \varphi \quad (k' \geq l, \text{Lemma 17}) \\ &\Leftrightarrow \mathcal{A}_{k'}, V', 0 \models \nu X.P(x) \wedge \Box\Box X \end{aligned}$$

But this contradicts to the fact (Lemma 14) that

$$\mathcal{A}_k, V', 0 \models \nu X.P(x) \wedge \Box\Box X \Leftrightarrow k \text{ is odd.}$$

Thus the formula $\nu X.P(x) \wedge \Box\Box X$ cannot be expressed in $\text{FOCTL}^*[\{P\}]$, hence $\text{FOCTL}^*[\{P\}]$ is strictly less expressive than $\text{FOM}\mu[\{P\}]$. $\square$

Corollary 19 *FOCTL^* is strictly less expressive than $\text{FOM}\mu$.*

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