

Spectrum of end of the world branes in holographic BCFTs

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ABSTRACT: We study overlaps between two regularized boundary states in conformal field theories. Regularized boundary states are dual to end of the world branes in an AdS black hole via the AdS/BCFT. Thus they can be regarded as microstates of a single sided black hole. Owing to the open-closed duality, such an overlap between two different regularized boundary states is exponentially suppressed as $\langle \psi_a | \psi_b \rangle \sim e^{-O(h_{ab}^{(\min)})}$, where $h_{ab}^{(\min)}$ is the lowest energy of open strings which connect two different boundaries a and b . Our gravity dual analysis leads to $h_{ab}^{(\min)} = c/24$ for a pure AdS₃ gravity. This shows that a holographic boundary state is a random vector among all left-right symmetric states, whose number is given by a square root of the number of all black hole microstates. We also perform a similar computation in higher dimensions, and find that $h_{ab}^{(\min)}$ depends on the tensions of the branes. In our analysis of holographic boundary states, the off diagonal elements of the inner products can be computed directly from on-shell gravity actions, as opposed to earlier calculations of inner products of microstates in two dimensional gravity.

KEYWORDS: AdS-CFT Correspondence, Boundary Quantum Field Theory, Conformal Field Theory

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1 Introduction

A conformal field theory (CFT) which is dual to classical gravity on an anti de-Sitter space (AdS) via the AdS/CFT [1], is expected to be strongly interacting and to have large degrees of freedom. Such CFTs, called holographic CFTs, have been considered to describe quantum systems with maximal quantum chaos [2]. Conformal bootstrap studies predict the special feature that a holographic CFT has a large spectrum gap and that its low energy spectrum is sparse [3–5]. Intuitively, such a large spectrum gap is believed to be produced due to strong interactions in the holographic CFT. This large spectrum gap is necessary to explain the black hole entropy expected from the AdS/CFT via a modular transformation. For example, if we assume a pure gravity theory on AdS_3 , the spectrum gap of the conformal dimension $\Delta = h + \bar{h}$ is expected to be $\Delta_{\text{gap}} = \frac{c}{12}$. Moreover, a basic property of quantum chaos so called eigenstate thermalization hypothesis (ETH) [6] has been derived from further studies of conformal bootstrap relations [7–9] (see also [10, 12] for earlier related works).

In this paper, we study an analogous spectrum property in holographic BCFTs and its physical implications. A boundary conformal field theory (BCFT) is defined as a CFT on

a manifold with boundaries where a part of conformal symmetry is preserved [13, 14]. The gravity duals of BCFTs (AdS/BCFT) can be obtained by inserting a class of end of the world branes in AdS backgrounds, which satisfy Neumann boundary conditions dual to the boundary conformal invariance [15–18]. End of the world branes play a crucial role in recent progress in understanding quantum aspects of black holes. Indeed, a class of microstates of a single sided AdS black hole with the fixed mass M can be constructed by the insertions of the end of the world branes to an eternal two sided black hole with the same mass M , [19–21]. Such a dictionary between black hole microstates and regularized boundary states plays key role in deriving the Page curve from the bulk perspective, therefore a resolution of black hole information paradox in the light of Island formula [22–27]. We also refer to [28, 29] for studies utilizing such branes to compute the entropy of Hawking radiation of higher dimensional black holes. End of the world branes also play a crucial role to find holographic duals of moving mirrors [30, 31], which mimic black hole evaporation processes.

Motivated by this, we will investigate a chaotic property of holographic boundary states (or so called Cardy states [13]), namely the inner products of two (regularized) boundary states. We will see that the off diagonal elements of them are exponentially suppressed in holographic BCFTs:

$$\langle \psi_a | \psi_b \rangle = \delta_{ab} + O(e^{-S_{BS}/2}), \quad (1.1)$$

where $|\psi_a\rangle$ denotes the regularized boundary states, labeled by a . The quantity S_{BS} estimates the entropy of microstates spanned by the boundary states. This behavior is directly related to the large spectrum gap in the open string between the two different boundaries a and b .

This paper is organized as follows. In section two, we give a very brief review of BCFT and boundary states and examine the inner products of boundary states. We will also present examples of inner products in a few solvable CFTs. In section three, starting with a short review of AdS/BCFT, we will study the relation between the inner products and open string spectra in two dimensional holographic CFTs, using a gravity dual. In section four, we extend the holographic analysis in section three to higher dimensional holographic CFTs. In section five, we will discuss our results, in particular by comparing them with the recent interpretation of a gravitational background as an ensemble of microstates. In appendix A, we give an argument which shows the absence of traversable wormholes in a class of Lorentzian AdS/BCFT setups.

2 BCFT and inner products of boundary states

A boundary conformal field theory (BCFT) is a CFT on a manifold with boundaries with a suitable boundary condition described below [14]. An Euclidean d dimensional CFT preserves a conformal symmetry $SO(d+1,1)$. We choose a boundary condition which preserves a subgroup $SO(d,1)$ for a BCFT. In two dimensions $d=2$, full conformal symmetry is enhanced to by a pair of infinite dimensional Virasoro algebras. In the presence of a conformal boundary, the chiral half of them is preserved as the symmetry of the system. A boundary state [13] is a useful description of a BCFT in terms of a quantum state. In this section, we start with a brief review of boundary states and study their inner products.

Although we focus on boundary states in two dimensional CFTs below, we can generalize their basic definition to higher dimensional CFTs in an obvious way.

2.1 Boundary states

A boundary state is a quantum state created by making a hole. Consider a two dimensional CFT on a cylinder as depicted in the left picture in figure 1. We describe this cylinder by the coordinate (τ, x) , imposing the periodicity $x \sim x + 2\pi$. The Hamiltonian in the Euclidean time τ direction is denoted by H_c (closed string Hamiltonian) and is written as

$$H_c = L_0 + \bar{L}_0 - \frac{c}{12}, \quad (2.1)$$

in terms of the Virasoro generators and the central charge c .

Adding a boundary along $\tau = 0$ is described by placing a boundary state (or Cardy state) $|B_a\rangle$, where a labels different boundary conditions. A Cardy state [13] is a linear combination of Ishibashi states $|I_k\rangle$ [32], where k labels all primary states in the CFT as

$$|B_a\rangle = \sum_k c_k^a |I_k\rangle. \quad (2.2)$$

The Ishibashi state $|I_k\rangle$ is a state constructed from a linear combination of descendant states on top of the primary state labeled by k and has maximal quantum entanglement between the left-moving and right-moving sectors. Ishibashi states are orthogonal to each other. The amplitude of the Euclidean time evolution by $\beta/2$ between two such states is computed as

$$\langle I_k | e^{-\frac{\beta}{2} H_c} | I_l \rangle = \delta_{kl} \chi_k(e^{-\frac{\beta}{2}}). \quad (2.3)$$

χ_k is the character for the primary k .

2.2 Open-closed duality and overlaps

On the other hand, the Cardy states are not orthogonal to each other but satisfy the open-closed duality relation as follow (refer to figure 1)

$$\langle B_a | e^{-\frac{\beta}{2} H_c} | B_b \rangle = \sum_k N_{a,b}^{(k)} \text{Tr}_k \left[e^{-2\pi t H_o} \right], \quad (2.4)$$

where

$$\beta = \frac{2\pi}{t}, \quad (2.5)$$

and $H_o = L_0 - \frac{c}{24}$ is the open string Hamiltonian. In the right hand side, $\text{Tr}_k[\dots]$ denotes we take the trace with respect to the primary k as well as its descendants. Moreover, $N_{a,b}^{(k)}$ counts the degeneracy of sectors which belong to the primary k in the open strings between the boundaries a and b .

Now if we take the limit $\beta \rightarrow 0$ (or equally $t \rightarrow \infty$), we find

$$\langle B_a | e^{-\frac{\beta}{2} H_c} | B_b \rangle \simeq N_{a,b}^{(k_m)} e^{-2\pi t \left(h_{a,b}^{(\min)} - \frac{c}{24} \right)}, \quad (2.6)$$

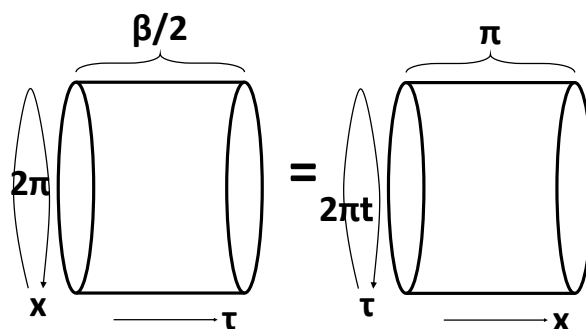


Figure 1. A sketch of cylinder amplitude from the viewpoint of closed string (left) and open string (right).

where k_m is the lightest primary among those satisfy $N_{a,b}^{(k_m)} \neq 0$, whose conformal dimension is denoted as $h_{a,b}^{(\min)}$.

Since we have $h_{a,a}^{(\min)} = 0$ when $a = b$, we can estimate the following ratio in the limit $\beta \rightarrow 0$ as

$$\frac{\langle B_a | e^{-\frac{\beta}{2} H_c} | B_b \rangle}{\sqrt{\langle B_a | e^{-\frac{\beta}{2} H_c} | B_a \rangle \cdot \langle B_b | e^{-\frac{\beta}{2} H_c} | B_b \rangle}} \simeq N_{a,b}^{(k_m)} e^{-4\pi^2 \frac{h_{a,b}^{(\min)}}{\beta}}. \quad (2.7)$$

Note that here we used the fact $N_{a,a}^{(0)} = 1$.

2.3 Thermal pure states

One of the reasons we are interested in boundary states is that, although they are pure states, in many respects they are indistinguishable from thermal mixed states under a suitable coarse-graining. This is in accord with the spirit of eigenstate thermalization hypothesis (ETH) [6], which claims many properties of typical pure states coincide with those of corresponding thermal mixed states. Thus, one anticipates that boundary states in CFT are indeed such typical states, therefore realize the ideas of ETH. To make this point explicit, now we consider a pure state given by a regularized boundary state

$$|\psi_a\rangle = \mathcal{N}_a \cdot e^{-\frac{\beta}{4} H_c} |B_a\rangle, \quad (2.8)$$

where we assume β is infinitesimally small and we choose the normalization $\mathcal{N}_a$ such that $\langle \psi_a | \psi_a \rangle = 1$.

This state is often employed as that just after a global quantum quench [33], which is an analytically tractable model of thermalization in an isolated quantum system. Thus, a late time limit of the time evolution of $|\psi_a\rangle$ can be regarded as a thermal pure state. Indeed, its expectation value of the energy is computed as

$$E_{th} = \frac{\langle \psi_a | H_c | \psi_a \rangle}{\langle \psi_a | \psi_a \rangle} = \frac{\pi^2 c}{3\beta^2}, \quad (2.9)$$

which agrees with the energy expectation value of the thermal ensemble with the temperature $1/\beta$. This indeed suggests the regularized boundary states are typical states (2.8) with the temperature β .

However, we can immediately see that not all typical states with the fixed energy (2.9) belong to the class of states (2.8), by counting the number of such regularized boundary states. The total number of the typical states can be read off from the thermodynamic entropy,

$$\#\text{Typical states} \sim e^{S_{th}}, \quad S_{th} = \frac{2\pi^2 c}{3\beta}. \quad (2.10)$$

Now, let us also estimate the number of such thermal pure states of the form (2.8), by counting the number of Cardy states. Since the label k of Ishibashi states coincides with that of primary states with the constraint that left and right are the same primary. Therefore, the number of different Cardy states at an effective temperature $1/\beta$ is estimated as

$$\#\text{Cardy states} \sim e^{S_{BS}} = e^{\frac{\pi^2 c}{3\beta}}, \quad (2.11)$$

where $S_{BS} = \frac{\pi^2 c}{3\beta}$ denotes the entropy naively associated with the Cardy states. Thus we see that the number of the regularized boundary states are too small to account all typical states.

It is also useful to note that in the AdS/CFT, this pure state (2.8) is dual to a microstate of a single sided BTZ black hole at inverse temperature β [19], or equally a three dimensional AdS spacetime with an end of the world-brane where boundary conformal invariance is preserved [15, 16]. This is indeed a half of the eternal BTZ black hole geometry which is dual to a thermal CFT.

2.4 Inner products

Now we would like to evaluate the inner products $\langle \psi_a | \psi_b \rangle$ of the pure states constructed from boundary states. In the high temperature limit $\beta \rightarrow 0$, we can estimate their inner products by using (2.7) as follows:

$$\langle \psi_a | \psi_b \rangle \simeq \delta_{ab} + N_{a,b}^{(k_m)} \cdot e^{-\frac{4\pi^2}{\beta} h_{a,b}^{(\min)}}. \quad (2.12)$$

In this way, the larger gap in the open string channel leads to a larger exponential suppression of off diagonal elements of inner products.

As we will explain in the next section, for an ideal holographic CFT which is dual to a pure gravity on AdS₃, we expect for $a \neq b$

$$h_{a,b}^{(\min)} = \frac{c}{24}. \quad (2.13)$$

This bound may be regarded as a chiral version of the well-known maximal gap $\Delta = \frac{c}{12}$ for a CFT dual of a pure gravity on AdS₃ [4, 34].

If we introduce a overall random phase for the state as $|\psi_a\rangle \rightarrow e^{i\theta_a} |\psi_a\rangle$, we find that the inner product takes the following behavior

$$\langle \psi_a | \psi_b \rangle \simeq \delta_{ab} + R_{ab} \cdot e^{-\frac{S_{BS}}{2}}, \quad (2.14)$$

where S_{BS} is the same as (2.11). Also R_{ab} is the random fluctuations such that $\langle R_{ab}^* R_{ab} \rangle = 1$. This ETH like property implies that such states are random states among $e^{S_{BS}}$ states and thus nicely agrees with the estimation (2.11).

This should be compared to the statistical fluctuations of overlaps between two typical states $|\phi_i\rangle$ [25],

$$\langle\phi_i|\phi_j\rangle = \delta_{ij} + e^{-S_{th}/2}R_{ij}, \quad (2.15)$$

with the random variable R_{ij} again satisfying $\langle R_{ij}^* R_{ij} \rangle = 1$. One can easily see the difference between the two, namely S_{BS} in the overlap between two regularized boundary states (2.14) is replaced to the thermodynamic entropy $S_{th} = 2S_{BS}$ in (2.15). This is again due to the fact that although such regularized boundary states are the frequently used model of thermal typical states, the number of such states are quite small compared to the total number of typical states.

2.5 Examples: free scalar and Liouville CFT

There are several boundary states in two dimensional conformal field theories, whose explicit forms are known. In this section, we first compute the overlaps of such boundary states in free boson theory and we compare the results with the general formula (2.7) presented above. For completeness, we also list several known results in Liouville theory, with caution that the general formula (2.7) cannot be applied directly to this theory due to its non-unitarity and continuous spectrum. Notice that these integrable boundary states have different properties than those of holographic boundary states.

2.5.1 Boundary states in free boson theory

Let us first consider the boundary states in free boson theory, whose action is given by

$$I = \frac{1}{2\pi} \int dz^2 \partial_z X \partial_{\bar{z}} X. \quad (2.16)$$

There are two types of such states. One is the Dirichlet state $|B_{D,x}\rangle$, which is labeled by a real parameter x of the localized point, and the other is the Neumann state $|B_N\rangle$.

The overlap between two Dirichlet boundary states $|B_{D,x_1}\rangle |B_{D,x_2}\rangle$ is estimated as

$$\frac{\langle B_{D,x_2} | e^{-\pi s H_c} | B_{D,x_1} \rangle}{\sqrt{\langle B_{D,x_1} | e^{-\pi s H_c} | B_{D,x_1} \rangle \cdot \langle B_{D,x_2} | e^{-\pi s H_c} | B_{D,x_2} \rangle}} = e^{-\frac{(x_1-x_2)^2}{4\pi s}} = e^{-\frac{(x_1-x_2)^2}{2\beta}}, \quad (2.17)$$

where we defined $s = \frac{\beta}{2\pi}$. The above exponential suppression agrees with the lowest open string energy $h_{DD}^{(\min)} = \frac{(x_1-x_2)^2}{8\pi^2}$ between the two boundaries, in accord with the general formula (2.7).

The overlap between the Neumann state and a Dirichlet state is,

$$\frac{\langle B_N | e^{-\pi s H_c} | B_{D,x_1} \rangle}{\sqrt{\langle B_N | e^{-\pi s H_c} | B_N \rangle \cdot \langle B_{D,x_1} | e^{-\pi s H_c} | B_{D,x_1} \rangle}} = \sqrt{\frac{4\pi\sqrt{s}\eta^3(is)}{\theta_2(is)}} \quad (2.18)$$

where the right hand side is given by where $\eta(\tau)$ is the eta function,

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n), \quad q = e^{2\pi i \tau} \quad \theta_2(\tau) = \prod_{n=1}^{\infty} (1 - q^n)(1 + q^n)^2. \quad (2.19)$$

In the $\beta \rightarrow 0$ limit, this ratio (2.18) behaves as $e^{-\frac{\pi^2}{4\beta}}$. This behavior agrees with the general formula (2.7), where the lowest open string energy is given by $h_{DN}^{(\min)} = \frac{1}{16}$ between the Dirichlet and Neumann boundary.

2.5.2 Boundary states in Liouville theory

Let us discuss a less trivial example, namely boundary states in Liouville theory. On a curved background its action reads,

$$I = \frac{1}{4\pi} \int dx^2 \sqrt{g} \left[g^{ab} \partial_a \phi \partial_b \phi + Q R \phi + 4\pi \mu e^{2b\phi} \right], \quad (2.20)$$

with $Q = b + \frac{1}{b}$, and R is the Ricci scalar. The central charge of this CFT is $c = 1 + 6Q^2$. In particular, we are interested in the large c limit $b \rightarrow 0$, where $c \simeq \frac{6}{b^2}$.

This theory has two types of boundary states, namely FZZT states [35, 36] and ZZ [37] states. Since the overlaps between two FZZT states are divergent even in the presence of the UV regulator, here we only discuss ZZ boundary states. ZZ states are characterized by two positive integers (m, n) , so we denote them $|B_{(m,n)}\rangle$. The amplitude between two such states was computed in [37]:

$$\langle B_{(m,n)} | e^{\pi i \tau H_{(m,n)(m',n')}} | B_{(m',n')} \rangle = \sum_{k=0}^{\min(m,m')-1} \sum_{l=0}^{\min(n,n')-1} \chi_{m+m'-2k-1, n+n'-2l-1}(\tilde{q}), \quad (2.21)$$

where

$$\tau = \frac{i\beta}{2\pi}, \quad \tilde{\tau} = -\frac{1}{\tau}, \quad \tilde{q} = e^{2\pi i \tilde{\tau}} = e^{-\frac{4\pi^2}{\beta}}, \quad (2.22)$$

with the caution that we are using an unusual convention for τ , in order to match the notation with (2.7). Also, we introduced the degenerate character,

$$\chi_{m,n}(q) = \frac{q^{-\frac{1}{4}(m/b+nb)^2} - q^{-\frac{1}{4}(m/b-nb)^2}}{\eta(\tau)} \quad (2.23)$$

For concreteness, let us focus on three simplest states in this class, ie $|B_{(1,1)}\rangle$, $|B_{(1,2)}\rangle$, and $|B_{(2,1)}\rangle$.

The overlap between $|B_{(1,1)}\rangle$ and $|B_{(1,2)}\rangle$ is

$$\frac{\langle B_{(1,1)} | e^{-\pi s H_c} | B_{(1,2)} \rangle}{\sqrt{\langle B_{(1,1)} | e^{-\pi s H_c} | B_{(1,1)} \rangle \langle B_{(1,2)} | e^{-\pi s H_c} | B_{(1,2)} \rangle}} = \frac{\chi_{1,2}}{\sqrt{\chi_{1,1}(\chi_{1,1} + \chi_{1,3})}}. \quad (2.24)$$

When we take $\beta \rightarrow 0$ limit and the large c limit $b \rightarrow 0$, we find

$$\langle \psi_{(1,1)} | \psi_{(1,2)} \rangle \simeq 1. \quad (2.25)$$

This means that $|B_{(1,1)}\rangle$ and $|B_{(1,2)}\rangle$ are indistinguishable in this limit.

On the other hand the overlap between $|B_{(1,1)}\rangle$ and $|B_{(2,1)}\rangle$ is

$$\frac{\langle B_{(1,1)} | e^{-\pi s H_c} | B_{(2,1)} \rangle}{\sqrt{\langle B_{(1,1)} | e^{-\pi s H_c} | B_{(1,1)} \rangle \langle B_{(2,1)} | e^{-\pi s H_c} | B_{(2,1)} \rangle}} = \frac{\chi_{2,1}}{\sqrt{\chi_{1,1}(\chi_{1,1} + \chi_{3,1})}}, \quad s = \frac{\beta}{2\pi}. \quad (2.26)$$

If we take the semi classical limit $b \rightarrow 0$ and the high temperature limit $\beta \rightarrow 0$ at the same time, we get,

$$\langle \psi_{(1,1)} | \psi_{(2,1)} \rangle \simeq e^{-\frac{\pi^2 c}{6\beta}}, \quad (2.27)$$

which interestingly coincides with the expectation in holographic CFTs.

More generally, in the large c limit $b \rightarrow 0$, we obtain

$$\langle \psi_{(n_1, m_1)} | \psi_{(n_2, m_2)} \rangle \simeq e^{-\frac{\pi^2 c}{6\beta} (n_1 - n_2)^2}. \quad (2.28)$$

Note that here we cannot apply directly the general formula (2.7) to Liouville theory because Liouville theory is non-unitarity and has continuous spectrum, as opposed to the usual CFTs which we assumed in the derivation of (2.7). Here we nevertheless mention these states because they are examples of boundary states whose explicit form are known.

3 Holographic analysis in $\text{AdS}_3/\text{BCFT}_2$

Now we move on to our main target: holographic BCFTs [16, 17, 38]. In this section, we focus on two dimensional BCFTs with classical gravity duals. As we explained in the previous section, the presence of a conformal boundary is described by a Cardy state, labeled by boundary conditions.

3.1 Lightning review of AdS/BCFT

We apply the AdS/BCFT duality [16–18] to study a gravity dual of a holographic CFT on a two-dimensional cylinder. The action of the gravitational system is given by

$$I = -\frac{1}{16\pi G_N} \int_N dx^3 \sqrt{g} (R - 2\Lambda) - \frac{1}{8\pi G_N} \int_Q \sqrt{h} (K - T). \quad (3.1)$$

In the above action, Q is the world volume of the end of the world brane in the bulk, which is anchored to the boundary of the BCFT region at the asymptotic boundary of the bulk spacetime. Also, in the action K is the trace of the extrinsic curvature of Q , and T denotes the tension of the brane. When we consider a gravity dual of a cylinder, there are two candidates of classical gravity solutions depending on whether the end of the world brane is connected or disconnected, as depicted in figure 2. We call these two a connected and disconnected solution, respectively.

First we consider a connected solution based on a thermal AdS_3 . We write the thermal AdS_3 metric as follows

$$ds^2 = R^2 \left(\frac{d\tau^2}{z^2} + \frac{dz^2}{h(z)z^2} + \frac{h(z)}{z^2} dx^2 \right), \quad (3.2)$$

where

$$h(z) = 1 - \left(\frac{z}{z_0} \right)^2. \quad (3.3)$$

To make the geometry smooth, the coordinate x is compactified as $x \sim x + 2\pi z_0$. We also compactify the Euclidean time τ such that $\tau \simeq \tau + 2\pi z_H$. A gravity dual of a CFT on a cylinder is given by the subregion [16, 17]

$$z_0 \arctan \left(\frac{RTz}{z_0 \sqrt{h(z) - R^2 T^2}} \right) \leq x(z) \leq \pi z_0 - z_0 \arctan \left(\frac{RTz}{z_0 \sqrt{h(z) - R^2 T^2}} \right). \quad (3.4)$$

Note that at the AdS boundary $z = 0$, this leads to $0 \leq x \leq \pi z_0$ and this interval is identified with the one where the BCFT is defined. The surface Q which is the boundary of the above region is a connected surface. By evaluating the gravity action on this background, we can holographically compute the BCFT partition function as follows

$$Z_{\text{con}} = e^{\frac{\pi c z_H}{12 z_0}}. \quad (3.5)$$

Note that the result is independent from the value of the tension T , which parameterizes different boundary conditions.

Next we consider the disconnected solution based on the BTZ black hole solution

$$ds^2 = R^2 \left(\frac{f(z) d\tau^2}{\tau^2} + \frac{dz^2}{f(z) z^2} + \frac{dx^2}{z^2} \right), \quad (3.6)$$

where

$$f(z) = 1 - \frac{z^2}{z_H^2}. \quad (3.7)$$

Now, in this solution, the Euclidean time like direction contractible, as opposed to thermal AdS.

A gravity dual of a CFT on a cylinder is given by the region [16, 17]

$$-z_H \cdot \text{arcsinh} \left(\frac{RT_a z}{z_H \sqrt{1 - R^2 T_a^2}} \right) \leq x(z) \leq \pi z_0 + z_H \cdot \text{arcsinh} \left(\frac{RT_b z}{z_H \sqrt{1 - R^2 T_b^2}} \right), \quad (3.8)$$

where $T_{a,b}$ is the tensions of each brane. By evaluating the gravity action on this background and we can again evaluate the BCFT partition function as follows

$$\begin{aligned} Z_{\text{dis}} &= e^{-I_{\text{dis}}}, \\ I_{\text{dis}} &= -\frac{\pi c z_0}{6 z_H} - S_{\text{bdy}}^{(a)} - S_{\text{bdy}}^{(b)}, \end{aligned} \quad (3.9)$$

where $S_{\text{bdy}}^{(i)}$, $i = a, b$ are the boundary entropies [39]

$$S_{\text{bdy}}^{(i)} = \frac{c}{3} \text{arctanh}(RT_i). \quad (3.10)$$

3.2 Inner products of holographic boundary states

When we consider the overlap $\langle B_a | e^{-\frac{\beta}{2} H_c} | B_a \rangle$ for an identical boundary condition a , then both the connected and disconnected solutions are allowed, which are depicted as the left and right picture in figure 2. The connected solution is favored in the limit $\beta \rightarrow 0$. By choosing $\pi z_0 = \frac{\beta}{2}$ and $z_H = 1$ in order to adjust the normalization, we find

$$\langle B_a | e^{-\frac{\beta}{2} H_c} | B_a \rangle \simeq e^{\frac{\pi^2 c}{6 \beta}}. \quad (3.11)$$

When we consider $\langle \psi_a | \psi_b \rangle$ for two different boundary conditions a and b , only the disconnected solution is allowed (i.e. the right picture in figure 2). Thus we can evaluate the overlap as follows

$$\langle B_a | e^{-\frac{\beta}{2} H_c} | B_b \rangle \simeq e^{\frac{c \beta}{12} + S_{\text{bdy}}^{(a)} + S_{\text{bdy}}^{(b)}}. \quad (a \neq b). \quad (3.12)$$

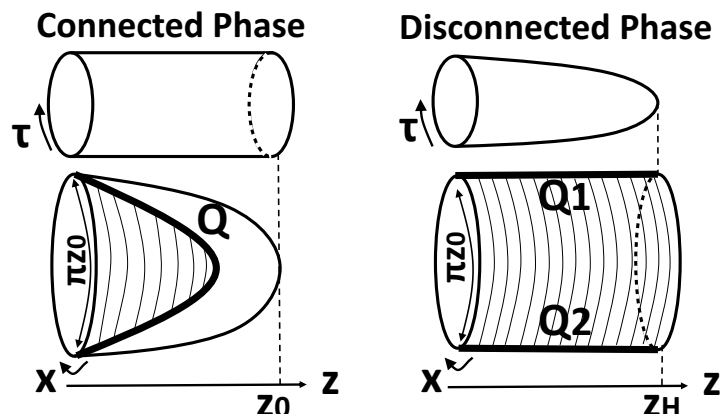


Figure 2. A sketch of gravity duals of a CFT on a cylinder in the connected phase (left) and disconnected phase (right) at $d = 2$.

Thus in the limit $\beta \rightarrow 0$ we find that the inner products of the normalized pure state $|\psi_a\rangle$ (2.8) are given by as follows (when $a \neq b$)

$$\langle \psi_a | \psi_b \rangle \simeq e^{-\frac{\pi^2 c}{6\beta} + S_{\text{bdy}}^{(a)} + S_{\text{bdy}}^{(b)}}. \quad (3.13)$$

This shows that the lowest dimensional state in the open string between a and b ($a \neq b$) is given by the previous expectation (2.13) i.e. $h_{a,b}^{(\min)} = \frac{c}{24}$, via (2.12), as promised.

4 Higher dimensional generalizations

The holographic approach based on the AdS/BCFT provides predictions also in higher dimensional BCFTs [17]. Notice that we can define a boundary state $|B_a\rangle$ as a state making a $d-1$ dimensional hole in d dimensional space. The inner product $\langle B_a|e^{-\frac{\beta}{2}H}|B_b\rangle$ can be computed as a partition function on a d dimensional open manifold $I_{\beta/2} \times T^{d-1}$, where $I_{\beta/2}$ is a length $\beta/2$ interval. As in the previous $d=2$ case, there are two different solutions, depending on whether the end of the world brane is connected or disconnected.

First we consider a connected solution based on a AdS_{d+1} soliton. We write the AdS_{d+1} soliton geometry as follows

$$ds^2 = R^2 \left(\frac{d\tau^2 + \sum_{i=1}^{d-2} dy_i^2}{z^2} + \frac{dz^2}{h(z)z^2} + \frac{h(z)}{z^2} dx^2 \right), \quad (4.1)$$

where

$$h(z) = 1 - \left(\frac{z}{z_0}\right)^d. \quad (4.2)$$

We compactify $x \sim x + \frac{4\pi}{d} z_0$ in order to have a smooth geometry. We also compactify the Euclidean time $\tau \sim \tau + \frac{4\pi}{d} z_H$ and other spacial coordinates $y_i \sim y_i + L_i$. The boundary CFT is defined in the bounded region

$$|x| < x(T). \quad (4.3)$$

The function $x(T)$ is introduced as

$$x(T) := \xi(T) \cdot z_0, \quad (4.4)$$

where $\xi(T)$ is defined when $T \leq 0$ as follows [17]:

$$\xi(T) \equiv \frac{\Gamma(1/d)\Gamma(1/2)}{\Gamma(1/2 + 1/d)} \frac{R|T|}{d(d-1)} \left(1 - \frac{R^2 T^2}{(d-1)^2}\right)^{1/d-1/2} F\left(1, \frac{1}{d}, \frac{1}{2} + \frac{1}{d}; 1 - \frac{R^2 T^2}{(d-1)^2}\right).$$

When $T > 0$, it is defined by $\xi(T) = \frac{2\pi}{d} - \xi(-T)$. Note that the tension takes values in the range $|T| < \frac{d-1}{R}$. As we see in the previous section, when we set $d = 2$ we have $\xi(T) = \frac{\pi}{2}$ for any T .

We would like to point out that in higher dimensions the behavior of $\xi(T)$ is different from that in $d = 2$. For $d > 2$, $\xi(T)$ non-trivially depends on T . We find $\xi(T)$ is a monotonically decreasing function of T such that $\xi(-\frac{d-1}{R}) = \infty$, $\xi(0) = \frac{\pi}{d}$ and $\xi(T_*) = 0$. Here $T_* > 0$ depends on the dimension. Since we have $\xi < 0$ for $T > T_*$, which looks unphysical, this implies that there is an upper bound of the tension $T < T_*$ for $d > 2$.

By evaluating the on-shell gravity action, the partition function is obtained as [17]

$$Z_{\text{con}} = e^{\frac{R^{d-1}(\prod_{i=1}^{d-2} L_i)z_H x(T)}{2dG_N z_0^d}}. \quad (4.5)$$

Next we consider the disconnected solution based on the AdS Schwartzschild black hole solution

$$ds^2 = R^2 \left(\frac{f(z)d\tau^2}{z^2} + \frac{dz^2}{f(z)z^2} + \frac{dx^2 + \sum_{i=1}^{d-1} dy_i^2}{z^2} \right), \quad (4.6)$$

where

$$f(z) = 1 - \left(\frac{z}{z_H}\right)^d. \quad (4.7)$$

We compactify $\tau \sim \tau + \frac{4\pi}{d} z_H$ in order to have a smooth geometry. We also compactify other spacial coordinates $y_i \sim y_i + L_i$, and the boundaries sit at $x = \pm x(T)$. For simplicity, we consider the case where the tension of the surface Q is vanishing $T = 0$. This is enough to extract the leading behavior in the limit $\beta \rightarrow 0$ as was true in $d = 2$. Then the disconnected solution is given by the region

$$|x| \leq x(T = 0) = \frac{\pi}{d} z_0. \quad (4.8)$$

By evaluating the gravity action on this background and we can estimate the partition function as follows

$$Z_{\text{dis}} = e^{-I_{\text{dis}}}, \quad (4.9)$$

$$I_{\text{dis}} = -\frac{R^{d-1}(\prod_{i=1}^{d-2} L_i)x(T = 0)}{dG_N z_H^{d-1}}. \quad (4.10)$$

Now let us evaluate the inner products of boundary states using the above holographic results. For an identical boundary condition a i.e. $\langle B_a | e^{-\frac{\beta}{2} H_c} | B_a \rangle$, both the connected and

disconnected solutions are allowed as in AdS_3 . The connected solution is favored in the limit $\beta \rightarrow 0$. We set $2x(T) = 2\xi(T)z_0 = \frac{\beta}{2}$ and take the periodicity in the τ and y_i direction to be $\frac{4\pi}{d}z_H = 2\pi$ and $L_i = L$, respectively. Then we find from (4.5)

$$\langle B_a | e^{-\frac{\beta}{2}H_c} | B_a \rangle_{\text{con}} \simeq e^{(4\xi(T))^d \pi c_{\text{eff}} \frac{L^{d-2}}{\beta^{d-1}}}. \quad (4.11)$$

Here we defined

$$c_{\text{eff}} := \frac{R^{d-1}}{16\pi G_N}. \quad (4.12)$$

Note that when $d = 2$, it is related to the central charge by $c_{\text{eff}} = \frac{c}{24\pi}$.

When we consider an inner product for two different boundary conditions a and b , only the disconnected solution is allowed (i.e. the right picture in figure 2). Thus we can evaluate the overlap for $T = 0$ from (4.9) as follows

$$\langle B_a | e^{-\frac{\beta}{2}H_c} | B_b \rangle \simeq e^{\frac{8\pi}{d^{d+1}} c_{\text{eff}} \beta L^{d-2}}. \quad (a \neq b). \quad (4.13)$$

Thus in the limit $\beta \rightarrow 0$ we find¹

$$\frac{\langle B_a | e^{-\frac{\beta}{2}H_c} | B_b \rangle}{\sqrt{\langle B_a | e^{-\frac{\beta}{2}H_c} | B_a \rangle \cdot \langle B_b | e^{-\frac{\beta}{2}H_c} | B_b \rangle}} \simeq e^{-\frac{\pi c_{\text{eff}}}{2} \frac{L^{d-2}}{\beta^{d-1}} ((4\xi(T_a))^d + (4\xi(T_b))^d)}, \quad (4.14)$$

where T_a and T_b are the tensions of surfaces Q_a and Q_b dual to the boundary states $|B_a\rangle$ and $|B_b\rangle$, respectively.

This predicts that the lowest energy among states in the open string between a and b ($a \neq b$) is given by

$$E_{a,b}^{(\text{min})} = \frac{\pi c_{\text{eff}}}{2} \left[\left(\frac{2\xi(T_a)}{\pi} \right)^d + \left(\frac{2\xi(T_b)}{\pi} \right)^d \right] V_{d-2}, \quad (4.15)$$

where $V_{d-2} = \left(\frac{2\pi L}{\beta} \right)^{d-2}$ is the volume of $d - 2$ dimensional torus in y^i direction. We took the length of x to be π and the periodicity of τ to be $2\pi t = \frac{4\pi^2}{\beta}$ as in the right of figure 1. Note also that the above energy gap decreasing as the tensions T_a and T_b gets larger and does vanish when $T_a = T_b = T_*$.

5 Discussions: JT gravity vs $\text{AdS}_3/\text{BCFT}_2$?

In this paper, we studied quantum states in CFTs $|\psi_a\rangle$, which are defined by regularized boundary states (Cardy states) for all possible conformal boundary conditions, labeled by a . This provides a class of microstates for single sided black holes in AdS. Since the left-right symmetric constraint is imposed for boundary states, the number of microstates spanned by boundary states, denoted by $e^{S_{BS}}$, is of the order of the square root of $e^{S_{th}}$ i.e. $S_{BS} \simeq \frac{S_{th}}{2}$. Here $e^{S_{th}}$ denotes the number of typical states $|\phi_i\rangle$ in a CFT which are indistinguishable from the corresponding thermal mixed state. S_{th} is also equal to the entropy of eternal black hole dual to the thermo field double of a holographic CFT.

¹Here we note that the T dependence for the disconnected solution can be negligible when we focus on the leading behavior of this ratio in the limit $\beta \rightarrow 0$.

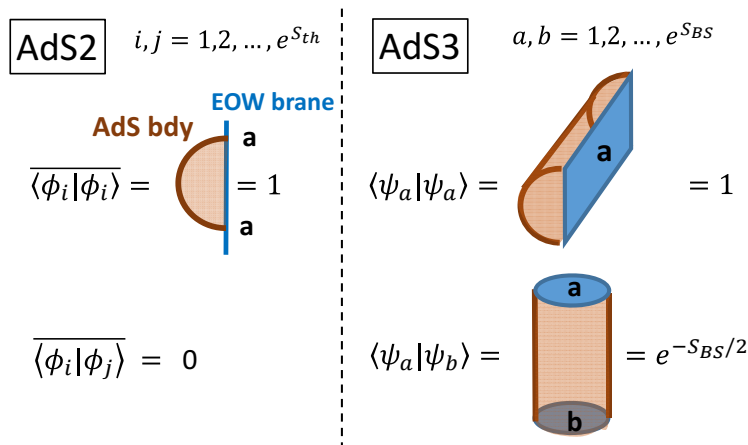


Figure 3. Gravity evaluations of the inner products of microstates $\langle \phi_i | \phi_j \rangle$ in AdS₂ gravity (left) and those of regularized boundary states $\langle \psi_a | \psi_b \rangle$ in AdS₃ gravity (right).

By using the open-closed duality we showed that the inner products $\langle \psi_a | \psi_b \rangle$ of boundary states for $a \neq b$ are exponentially small such that its exponent is proportional to the lowest energy in the open string spectrum between a and b as in (2.12). The holographic analysis of this inner products for a three dimensional AdS pure gravity shows that this lowest energy is $h_{ab}^{(\min)} = \frac{c}{24}$ in its dual two dimensional holographic CFT. Indeed, this value corresponds the estimation of inner products: $\langle \psi_a | \psi_b \rangle \sim e^{-S_{BS}/2}$. A most crucial fact behind this estimation is that there are two solutions in AdS/BCFT depending on whether the end of the world brane (EOW brane) is connected or the EOW branes are disconnected, as depicted as the top and bottom picture in figure 3, respectively. This result means that the quantum states $|\psi_a\rangle$ are random vectors in the full space spanned by the left-right symmetric states. Therefore we expect that this lowest energy $h_{ab}^{(\min)} = \frac{c}{24}$ is maximum possible values for any two dimensional BCFTs. This implies that such CFTs correspond to maximally chaotic BCFTs and that holographic BCFTs are the most chaotic BCFTs. We also generalized the above analysis to higher dimensional BCFTs, by applying the AdS/BCFT. An important new aspect in higher dimensions is that the lowest energy in open string turns out to depend on the value of the tension of the EOW branes. Therefore it is an intriguing future problem to understand the distributions of values of tension for the EOW branes which are dual to conformal boundary states.

Now we would like to compare our results with the analysis of microstates in two dimensional AdS gravity (JT gravity) performed in [25] to explain the physics behind the island formula in the black hole information loss problem. First of all, when we evaluate the inner products of microstates $\langle \phi_i | \phi_j \rangle$, they vanish except $a = b$ in the AdS₂ gravity as depicted in the left of figure 3. In [25], this is interpreted that the gravity calculation corresponds to taking a random ensemble average $\overline{\langle \phi_i | \phi_j \rangle}$ with respect to all possible microstates i, j . Even though the gravity analysis cannot directly compute the off diagonal element of $\langle \psi_i | \psi_j \rangle$, its square average $|\overline{\langle \phi_i | \phi_j \rangle}|^2$ is argued to be computable in the two dimensional gravity, by taking wormhole solutions (called replica wormholes) into account,

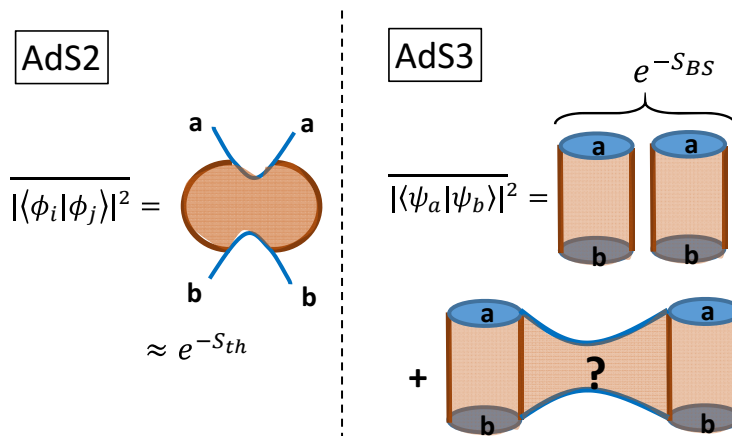


Figure 4. Gravity evaluations of the averages of squares of inner products in AdS_2 gravity (left) and AdS_3 gravity (right).

as depicted in the left of figure 4. This leads to the evaluation $|\overline{\langle\phi_a|\phi_b\rangle}|^2 \sim e^{-S_{th}}$ and this implies the behavior $\langle\phi_a|\phi_b\rangle \simeq \delta_{ij} + R_{ij}e^{-\frac{S_{th}}{2}}$ [25], where R_{ij} is a random matrix.

On the other hand, in the three dimensional AdS gravity we studied in this paper, the situation is a bit different. Since we can have the solution with two disconnected EOW branes in AdS_3 as depicted in the lower right picture of figure 3, the gravity prediction for the inner product $\langle\psi_a|\psi_b\rangle$ does not vanish even if $a \neq b$, as opposed to the AdS_2 case of [25]. Therefore we obtain the off diagonal value of $\langle\psi_a|\psi_b\rangle$ directly from a gravity calculation. This leads to the behavior $\langle\psi_a|\psi_b\rangle \simeq \delta_{ab} + O(e^{-\frac{S_{BS}}{2}})$, which leads to the lowest energy gap $h_{ab}^{(\min)} = \frac{c}{24}$ of open strings. These highlight the main difference between the microstate analysis in two dimensional JT gravity and that in three dimensional AdS gravity.

It may be plausible to think the ensemble average $|\overline{\langle\psi_a|\psi_b\rangle}|^2$ can be computed by the gravity partition function for a geometry with four boundaries i.e. two a s and two b s, which depicted in the right of figure 4. The first term is the direct product of the cylinder solutions. We expect that the random averages are responsible for the second term which corresponds to wormhole geometries which connect two cylinders. However, when we consider the construction of Euclidean AdS wormholes which connect two boundaries given in [40], each boundary has to be a surface with genus higher than one. In particular, we cannot connect two tori by an on shell AdS wormhole. Similarly, we might expect that it is not possible to connect two cylinders by an AdS wormhole with appropriate EOW branes.² This may suggest that in $\text{AdS}_3/\text{BCFT}_2$, we do not need to regard gravity path-integrals as averaged quantities of holographic CFTs. It would be an intriguing future problem to explore more on these aspects to understand precisely how gravity can describe CFT microstates.

²In the appendix A, we gave an argument that we cannot construct a traversable wormhole in the Lorentzian AdS/BCFT, by picking up a class of examples.

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A Averaged null energy condition and absence of traversal wormholes in AdS/BCFT

Here we will show that a construction of traversal wormhole with a locally Poincare AdS metric is not possible in AdS/BCFT when we impose the averaged energy condition (ANEC). An analogous statement of the absence of traversable wormholes in the standard AdS/CFT without end of the world branes was proven in [41, 42]. The ANEC for quantum field theories on flat spaces was derived in [43] using the AdS/CFT and in [44, 45] using field theoretic arguments. Recently, a necessary modification of ANEC for spaces with positive curvatures was proposed in [46] via holography.

In the Poincare AdS_{d+1}

$$ds^2 = R^2 \left(\frac{-dt^2 + dz^2 + dx^2 + \sum_{i=1}^{d-2} dy_i^2}{z^2} \right), \quad (\text{A.1})$$

we consider two BCFTs on regions confined by $x \leq -a$ or $x \geq a$, and ask whether we can have a static EOW brane that ends at these two boundaries. We write x coordinates of the brane at $x \leq 0$ by $x(z)$, in particular we have $x(0) = -a$ and $x'(z^*) = \infty$ where $z = z^*$ is the turning point. The null vector which generates null geodesic on the surface Q

$$(N^t, N^z, N^x) = Rz^2 \left(-1, \frac{1}{\sqrt{1 + x'(z)^2}}, \frac{x'(z)}{\sqrt{1 + x'(z)^2}} \right). \quad (\text{A.2})$$

We fixed the normalization of null vector such that we have $\frac{dX^\mu}{d\lambda} = N^\mu$ for an affine parameter λ , namely it satisfies $N^\mu \nabla_\mu N^\nu = 0$.

The averaged null energy can be decomposed into two parts: the one from $x = -a$ to $x = x(z_*)$ and the other one $x = x(z_*)$ to $x = a$. The averaged null energy condition tells

us that either of them is non-negative, which we can take to be the first one without losing any generality. Therefore we require

$$\int d\lambda (K_{\mu\nu} - K h_{\mu\nu}) N^\mu N^\nu = - \int_\epsilon^{z^*} dz \frac{zx''(z)}{(1+x'(z)^2)} \geq 0. \quad (\text{A.3})$$

However we can show that this integral should be negative by performing a partial integration:

$$\begin{aligned} & - \int_\epsilon^{z^*} dz \frac{zx''(z)}{(1+x'(z)^2)} \\ &= [-z \cdot \arctan(x')]_\epsilon^{z^*} + \int_0^{z_*} dz \arctan x' = -\frac{\pi}{2} z_* + \int_0^{z_*} dz \arctan x' < 0, \end{aligned} \quad (\text{A.4})$$

where we employed that the fact we have $x' = \infty$ at the turning point $z = z_*$ and the bound $|\arctan x'| < \frac{\pi}{2}$ for $\epsilon \leq z < z_*$. This clearly shows the wormhole solution in the geometry (A.1) is not possible.

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