

Extension problems for spinors on S^4

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S^4 上のスピノールに対する延長問題

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1. The space of spinors on S^3

Here we shall explain the complex analytic point of view of Dirac operator on S^4 and discuss the eigenvalues of Hamiltonian acting on spinors on the equator $\simeq S^3$. These were obtained in [K].

a. Let us consider two copies of complex planes C_z^2 and $\hat{C}_w^2$ and a smooth bijection $v : C_z^2 \setminus \{0\} \rightarrow \hat{C}_w^2 \setminus \{0\}$ given by $w = v(z) = -\frac{\bar{z}}{|z|^2}$. We patch C_z^2 and C_w^2 by v to obtain a differentiable manifold $M = C^2 \bigsqcup_v \hat{C}^2$, which is homeomorphic to S^4 .

We endow M with a riemannian metric defined by

$$g = \begin{cases} (1 + |z|^2)^{-2} \sum_{i=1}^2 dz_i \otimes d\bar{z}_i & \text{on } C_z^2 \\ (1 + |w|^2)^{-2} \sum_{i=1}^2 dw_i \otimes d\bar{w}_i & \text{on } \hat{C}_w^2. \end{cases}$$

The Levi-Civita connection on M is given by gauge potentials

$$\begin{aligned} \Gamma(z) &= \frac{|z|^2}{1 + |z|^2} \sigma(z)^{-1} \cdot (d\sigma)_z & \text{for } z \in C_z^2 \\ \hat{\Gamma}(w) &= \frac{|w|^2}{1 + |w|^2} \sigma(w)^{-1} \cdot (d\sigma)_w & \text{for } w \in \hat{C}_w^2, \end{aligned}$$

where $\sigma(z) = |z|^2(v_*)_z$, v_* being the differential of v , and $\sigma(z)^{-1}(d\sigma)_z$ is a one-form valued in $\mathcal{G} = \{X \in gl(4, \mathbb{C}) : {}^tXK + KX = 0\} \simeq o(4, \mathbb{C})$, $K = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$.

On M there are a unique spin-structure $Spin(M)$ and the associated spinor bundle $S = Spin(M) \times_{Spin(4)} \Delta$. Δ is a basic representation space of $Spin(4)$ which is the direct sum of two irreducible representations of Δ^+ and Δ^- each of dimension 2. Let S^+ and S^- be the corresponding bundles whose cross sections are spinors of positive (respectively negative) chirality . We shall choose a frame of $S^\pm$ and denote the spinors in matrix form

$$\begin{pmatrix} \phi \\ \psi \end{pmatrix} \in \Gamma(S), \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \in \Gamma(S^+), \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \in \Gamma(S^-),$$

where Γ signifies the sections of a bundle. The inner product of two spinors $\phi, \varphi \in \Gamma(S^\pm)$ is defined by $\langle \phi(z), \varphi(z) \rangle = \phi_1(z)\bar{\varphi}_1(z) + \phi_2(z)\bar{\varphi}_2(z)$.

b The Dirac operator acting on the spinors is defined as the composition $\mathcal{D} = \mu \cdot \nabla$ where ∇ is the covariant derivative induced by the Levi-Civita connection and μ is Clifford multiplication . The Dirac operator switches S^+ and S^- and is of the form $\mathcal{D} = \begin{pmatrix} 0 & D^\dagger \\ D & 0 \end{pmatrix}$ where $D : \Gamma(S^+) \rightarrow \Gamma(S^-)$.

We gave in [K] the following matrix representation of the Dirac operator.

$$D = \begin{pmatrix} (1 + |z|^2)\frac{\partial}{\partial \bar{z}_1} - \frac{3}{2}\bar{z}_1 & -(1 + |z|^2)\frac{\partial}{\partial \bar{z}_2} + \frac{3}{2}z_2 \\ (1 + |z|^2)\frac{\partial}{\partial \bar{z}_2} - \frac{3}{2}\bar{z}_2 & (1 + |z|^2)\frac{\partial}{\partial \bar{z}_1} - \frac{3}{2}z_1 \end{pmatrix}$$

$$D^\dagger = \begin{pmatrix} (1 + |z|^2)\frac{\partial}{\partial \bar{z}_1} - \frac{3}{2}z_1 & (1 + |z|^2)\frac{\partial}{\partial \bar{z}_2} - \frac{3}{2}z_2 \\ -(1 + |z|^2)\frac{\partial}{\partial \bar{z}_2} + \frac{3}{2}\bar{z}_2 & (1 + |z|^2)\frac{\partial}{\partial \bar{z}_1} - \frac{3}{2}\bar{z}_1 \end{pmatrix}$$

We have a decomposition of D and $D^\dagger$ to their longitudinal parts and radial parts;

$$D = \gamma_0 (\mathbf{n} - \mathcal{P}) \quad D^\dagger = (\mathbf{n} + \mathcal{P})\gamma_0.$$

Here γ_0 signifies Clifford multiplication of the radial vector $\mathbf{n}$. We shall explain $\mathcal{P}$ soon after. First we introduce an orthonormal frame on M , but here we shall write down it only on the local coordinate $\mathbb{C}^2 \subset M$, the formulas

on $\widehat{C}^2 \subset M$ are easily obtained by the transition relation . This frame is important not only as it gives a neat expression of Dirac operators on M and on the equator $\simeq S^3$ but also as is associated to the Lie group structure of $S^3 \simeq SU(2)$ (see c). Let

$$\nu = \frac{1 + |z|^2}{|z|} \left(z_1 \frac{\partial}{\partial z_1} + z_2 \frac{\partial}{\partial z_2} \right) \quad \epsilon = \frac{1 + |z|^2}{|z|} \left(-\bar{z}_2 \frac{\partial}{\partial z_1} + \bar{z}_1 \frac{\partial}{\partial z_2} \right)$$

The radial vector field is given by

$$\mathbf{n} = \frac{1}{2}(\nu + \bar{\nu}).$$

Put

$$\theta_0 = \frac{1}{2\sqrt{-1}}(\nu - \bar{\nu}) \quad \theta_1 = \frac{1}{2}(\epsilon + \bar{\epsilon}) \quad \theta_2 = \frac{1}{2\sqrt{-1}}(\epsilon - \bar{\epsilon}).$$

Then $\sqrt{2}\mathbf{n}$, $\sqrt{2}\theta_0$, $\sqrt{2}\theta_1$, $\sqrt{2}\theta_2$ form an orthonormal frame on M and θ_0 , θ_1 , θ_2 are tangent to the constant altitude $\{|z| = \text{const}\}$.

$\mathcal{P} : S^+ \rightarrow S^+$ is given by $\mathcal{P} = -(\gamma_0 |S^-) \sum_{i=0}^2 \theta_i \nabla_{\theta_i}$ with γ_0 coming from Clifford multiplication of $\mathbf{n}$.

The matrix representation of $\mathcal{P}$ is written as

$$\mathcal{P} = \begin{pmatrix} -\sqrt{-1}\theta_0 & \bar{\epsilon} \\ -\epsilon & \sqrt{-1}\theta_0 \end{pmatrix} + \frac{3}{2}|z| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

Let $E = \{|z| = 1\}$ be the equator of M ; $E \simeq S^3$. E is endowed with the riemannian metric $g|_E$. Since $Spin(3)$ has the spinor representation on $\Delta^\pm$ the restrictions on E of bundle $S^\pm$ is the spinor bundle corresponding to the spin structure $Spin(E)$. γ_0 gives the isomorphism between $S^\pm$. The Dirac operator on E acting on spinors of positive chirality is given by $-\gamma_0 \mathcal{P}|_E$. The restriction $\mathcal{P}$ on E is called *Hamiltonian* on E .

c Here we shall discuss a little about infinitesimal representations of $SU(2)$ given by the vector fields $\sqrt{-1}\theta_i$, $i = 0, 1, 2$. First we note the commutation relations same as those of $sl(2)$;

$$[\sqrt{-1}\theta_0, \epsilon] = -2\epsilon, \quad [\sqrt{-1}\theta_0, \bar{\epsilon}] = 2\bar{\epsilon}, \quad [\epsilon, \bar{\epsilon}] = 4\sqrt{-1}\theta_0.$$

We now follow the isomorphism $B \simeq S^3 \simeq SU(2)$ and look the point $z \in B$ as $\tilde{z} = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix} \in SU(2)$. We shall then define the right action on E

of $g \in SU(2)$ by $z \cdot g =$ the first column of $\ddot{z} \cdot g$. Put $R_g f(z) = f(z \cdot g)$ for a continuous function f on E . Then the differentials become $dR(e_k) = -\theta_k$, $k = 0, 1, 2$, where

$$e_0 = \begin{pmatrix} \sqrt{-1} & 0 \\ 0 & -\sqrt{-1} \end{pmatrix}, \quad e_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & \sqrt{-1} \\ \sqrt{-1} & 0 \end{pmatrix}$$

are the basis of $su(2)$.

A polynomial that satisfies

$$P(az_1, bz_2, b\bar{z}_1, a\bar{z}_2) = a^k b^l P(z_1, z_2, \bar{z}_1, \bar{z}_2)$$

is called of class (k, l) . The set of polynomials of class (k, l) is denoted by $S_{k,l}$. Let $\mathcal{H}$ be the set of harmonic polynomials on $\mathbb{C}^2$ and put $\mathcal{H}_{k,l} = \mathcal{H} \cap S_{k,l}$. We have $S_{k,l} = \mathcal{H}_{k,l} \oplus |z|^2 S_{k-1,l-1}$, hence $\dim \mathcal{H}_{k,l} = k + l + 1$. It follows also that, on E , every polynomial is a sum of harmonic polynomials in $\mathcal{H}_{k,l}$'s. This ensures the fact that our family of eigenspinors of Hamiltonian on E obtained later is a complete system.

Put, for $r \geq 0$ and $0 \leq k, q \leq r$,

$$h_{k,r-k}^q(z) = \epsilon^q (z_1^k z_2^{r-k}).$$

For each pair r and $k \leq r$ the set $\{h_{k,r-k}^q; q = 0, \dots, r\}$ forms a basis of $\mathcal{H}_{k,r-k}$.

Proposition.

- (1) $\sqrt{-1}\theta_0 h_{k,r-k}^q = (r - 2q)h_{k,r-k}^q$
- (2) $\epsilon h_{k,r-k}^q = h_{k,r-k}^{q+1}$
- (3) $\bar{\epsilon} h_{k,r-k}^q = -4q(r - q + 1)h_{k,r-k}^{q-1}$

Hence the space of harmonic polynomials $\mathcal{H}$ (restricted on B) is decomposed by the right action R of $SU(2)$ into $\mathcal{H} = \sum_r \sum_{k=0}^r \mathcal{H}_{k,r-k}$. Each induced representation $R_{k,r-k} = (R, \mathcal{H}_{k,r-k})$ is an irreducible representation with the highest weight $\frac{r}{2}$.

d Put, for $r \leq 0$, $0 \leq k \leq r$, and $0 \leq q \leq r + 1$,

$$\phi_{k,r-k}^q = \begin{pmatrix} q2^{-q+1} h_{k,r-k}^{q-1} \\ -2^{-q} h_{k,r-k}^q \end{pmatrix}.$$

Then we have from the matrix representation of the Hamiltonian and the Proposition in $\mathfrak{c}$;

$$\mathcal{P}\phi_{k,r-k}^q = (r + \frac{3}{2})\phi_{k,r-k}^q.$$

Thus the positive eigenvalues and eigenfunctions of $\mathcal{P}$ are obtained. In particular the multiplicity of the eigenvalue r is $(r+1)(r+2)$.

The investigation of negative eigenspinors is related to the left action of $SU(2)$ on the harmonic polynomials. The left action of a $g \in SU(2)$ on E is defined by $g \cdot z =$ the first column of $g \cdot \ddot{z}$. Let $L_g f(z) = f(g^{-1} \cdot z)$ for a continuous function on E .

We introduce the following vector fields on $M - \{0, \hat{0}\}$, that have the following local expressions on $\mathbb{C}^2 - \{0\}$:

$$\mu = \frac{1 + |z|^2}{|z|} (z_2 \frac{\partial}{\partial z_2} + \bar{z}_1 \frac{\partial}{\partial \bar{z}_1}), \quad \delta = \frac{1 + |z|^2}{|z|} (\bar{z}_2 \frac{\partial}{\partial \bar{z}_1} - z_1 \frac{\partial}{\partial z_2}).$$

$$\tau_0 = \frac{1}{2\sqrt{-1}}(\mu - \bar{\mu}), \quad \tau_1 = \frac{1}{2}(\delta + \bar{\delta}), \quad \tau_2 = \frac{1}{2\sqrt{-1}}(\delta - \bar{\delta}).$$

We have $dL(e_i) = -\tau_i|E$; $i = 0, 1, 2$.

Let

$$\hat{h}_q^{r-k,k}(z) = \delta^q(\bar{z}_1^k z_2^{r-k}).$$

$\{\hat{h}_q^{l,k}; q = 0, \dots, r\}$ give a basis of $\hat{\mathcal{H}}^{l,k}$: the space of harmonic polynomials that satisfy the condition $P(az_1, az_2, b\bar{z}_1, b\bar{z}_2) = a^l b^k P(z_1, z_2, \bar{z}_1, \bar{z}_2)$. Put, for $r \geq 0$, $0 \leq k \leq r$, and $0 \leq q \leq r+1$,

$$\pi_q^{r-k,k} = \begin{pmatrix} 2^{-q} \hat{h}_q^{r-k+1,k} \\ 2^{-q} \hat{h}_q^{r-k,k+1} \end{pmatrix}.$$

By an easy calculus we have

$$\mathcal{P}\pi_q^{r-k,k} = -(r + \frac{3}{2})\pi_q^{r-k,k}.$$

Thus we have

Theorem 1 [K]. The eigenvalues of $\mathcal{P}$ are $\pm (\frac{3}{2} + r)$; $r = 0, 1, 2, \dots$ with multiplicity $(r+1)(r+2)$, in particular, there is no zero mode spinor of $\mathcal{P}$ and the spectrum are symmetric relative to 0.

e Here we note corresponding subjects on the other coordinate neighborhood $\hat{C}_w^2$. The transition function to describe the bundle $Spin(M)$ is $-{}^t(\gamma_0) = -\bar{\gamma}_0$ and a spinor on M is a pair of $\varphi(z) \in \Gamma(C_z^2 \times \Delta)$ and $\hat{\varphi}(w) \in \Gamma(\hat{C}^2 \times \Delta)$ that are patched by $\hat{\varphi}(v(z)) = -(\bar{\gamma}_0 \varphi)(z)$. The matrix representations of the Dirac operator on $\hat{C}^2 \subset M$ has the same form as those in (1-5) but the first and the second are changed since a section on $\hat{C}^2$ of the bundle S^+ (resp. S^-) is valued in Δ^- (resp. Δ^+). This is "CPT"-theorem. The counterpart of $\mathcal{P}$ is defined as $\mathcal{P} = (\gamma_0 | S^+) \sum \theta_1 \nabla_{\theta_i}$ acting on $\hat{\varphi} \in \Gamma(\hat{C}_w^2 \times \Delta^-)$. For a $\varphi \in \Gamma(C_z^2 \times \Delta^+)$, we have $D\hat{\varphi} = \bar{D}\varphi$ and $\widehat{\mathcal{P}\varphi} = \mathcal{P}\hat{\varphi}$.

2 Extension of spinors from the equator

a Let H be the space of square integrable spinors of positive chirality on E . Let $H_{\pm}$ be the closed subspace of H spanned by the eigenvectors ϕ_{λ} corresponding to the positive (resp. negative) eigenvalues λ of $\mathcal{P}$.

Put $c(r, q, k) = \left(\frac{q!k!(r-k)!}{(r+1-q)!} \right)^{-\frac{1}{2}}$. Then a complete orthonormal system of eigenspinors of $\mathcal{P}$ is given by

$$\left\{ c(r, q, k) \phi_{k, r-k}^q, c(r, q, k) \pi_q^{r-k, k}; r \geq 0, 0 \leq k \leq r, 0 \leq q \leq r+1 \right\}.$$

Take an eigenspinor φ_{λ} and extend it by $\Phi_{\lambda}(z) = r_{\lambda}(|z|) \varphi_{\lambda}(\frac{z}{|z|})$ to C^2 , where $r_{\lambda}(t) = t^{\lambda - \frac{3}{2}} (\frac{1+t^2}{2})^{\frac{3}{2}}$. Then $\Phi_{\lambda}(z)$ is a zero-mode spinor of D on C^2 . This is proved by the following calculus:

$$\begin{aligned} D\Phi_{\lambda}(z) &= \gamma_0(\mathbf{n} - \mathcal{P})(\Phi_{\lambda}(z)) \\ &= \gamma_0 \left((1 + |z|^2) r'_{\lambda}(|z|) - \left(\lambda - \frac{3}{2} \right) \frac{1 + |z|^2}{|z|} r_{\lambda}(|z|) - 3|z| r_{\lambda}(|z|) \right) \varphi_{\lambda}\left(\frac{z}{|z|}\right). \end{aligned}$$

But $r_{\lambda}(t)$ satisfies the equation

$$(1 + t^2) r'_{\lambda}(t) - \left(\lambda - \frac{3}{2} \right) \frac{1 + t^2}{t} r_{\lambda}(t) - 3t r_{\lambda}(t) = 0.$$

Therefore $D\Phi_{\lambda} = 0$.

Let $\mathcal{N}(U)$ (resp. $\mathcal{N}^{\dagger}(U)$) be the space of zero-mode spinors of Dirac operator D (resp. $D^{\dagger}$) on U that have L^2 -boundary values.

Theorem 2 [K]. Let $R = \{z \in \mathbb{C}^2; |z| < 1\}$ and $\hat{R} = \{w \in \hat{\mathbb{C}}^2; |w| < 1\}$.

- (1) H_+ is isomorphic to $\mathcal{N}(R)$,
- (2) H_- is isomorphic to $\mathcal{N}(\hat{R})$,
- (3) Every spinor in H is equal to the difference of the restrictions of zero mode spinors on R and on $\hat{R}$.

Proof: Let $\varphi \in H_+$ and expand it in $\varphi = \sum_{\lambda > 0} a_\lambda \phi_\lambda$. The spinor on R ; $\Phi(z) = \sum_{\lambda > 0} a_\lambda \Phi_\lambda(z)$ is well defined. In fact, consider the finite sum; $\Phi_m^n = \sum_{\lambda=m+\frac{3}{2}}^{n+\frac{3}{2}} a_\lambda \Phi_\lambda$. Then $\langle \Phi_m^n, \Phi_m^n \rangle(z)$ is subharmonic on R and is dominated by some constant multiple of its L^2 -norm on E , hence converges there to 0 compact uniformly as m, n tend to infinity. If we note the fact that each component of Φ is harmonic we see that it has L^2 -boundary value. Conversely let $\Phi \in \mathcal{N}(R)$ and let φ be its restriction to E . We can show that the eigenfunction expansion of φ by $\{\phi_\lambda\}$ can not contain the term with $\lambda < 0$ and $\varphi \in H_+$. As for (2) consider the function $r_{-\mu}(t) = t^{\mu-\frac{3}{2}}(\frac{1+t^2}{2})^{\frac{3}{2}}$, $t \geq 0$, where $-\mu = -r - \frac{3}{2}$, $r = 0, 1, \dots$ and do the same argument as in (1). Relations in e transform the result to that on $\hat{R}$.

b Let H^* be the space of square integrable spinors of negative chirality on E . γ_0 switches H and H^* ; $(\gamma_0|S^+)H = H^*$, $(\gamma_0|S^-)H^* = H$. We shall define $H_+^* = (\gamma_0|S^+)H_+$ and $H_-^* = (\gamma_0|S^+)H_-$.

Let $\psi^* \in H_-^*$ and suppose that $\psi = (\gamma_0|S^-)\psi^*$ is an eigenspinor belonging to a negative eigenvalue $\lambda = -(r + \frac{3}{2})$. Let $\Psi(z) = s_\lambda(|z|)\psi(\frac{z}{|z|})$, where $s_\lambda(t) = t^{-(\lambda-\frac{3}{2})}(\frac{2}{1+t^2})^{\frac{3}{2}}$. Then as before we can verify that $\Psi(z)$ extend ψ to $\mathbb{C}^2$, $\Psi(0) = 0$ and $D^\dagger \psi^* = (\mathbf{n} + \mathcal{P})\gamma_0 \psi^* = (\mathbf{n} + \mathcal{P})\psi = 0$.

Thus in the same manner as in Theorem 2 we have the following;

Theorem 3.

- (1) H_-^* is isomorphic to $\{\phi \in \mathcal{N}^\dagger(R); \phi(0) = 0\}$,
- (2) H_+^* is isomorphic to $\{\psi \in \mathcal{N}^\dagger(\hat{R}); \psi(\hat{0}) = 0\}$,
- (3) Every spinor in H^* is equal to the difference of the restrictions of zero mode spinors on R and on $\hat{R}$.

c From the definition $\langle \phi, \psi \rangle = 0$ for all $\phi \in H$ and $\psi \in H^*$.

Let ϕ and ψ be spinors on $R = \{|z| \leq 1\}$, Stokes' theorem states;

$$\int_R \frac{1}{(1+|z|^2)^4} (\langle D\phi, \psi \rangle + \langle \phi, D^\dagger \psi \rangle) dV = \frac{1}{8} \int_E \langle \phi, \gamma_0 \psi \rangle d\sigma.$$

Theorems 2,3 and Stokes' theorem yield immediately that

$$\int_E \langle \phi, \gamma_0 \psi \rangle d\sigma = 0 \quad \text{for } \phi \in H_+, \psi \in H_-^*.$$

Similarly

$$\int_E \langle \phi, \gamma_0 \psi \rangle d\sigma = 0 \quad \text{for } \phi \in H_-, \psi \in H_+^*.$$

The coupling between $H_\pm^*$ and $H_\pm$ does not vanish and is important to construct the geometric model of conformal field theory on S^4 which will be treated in the next paper.

d Actually eigenspinors $\phi_\lambda; \lambda > 0$ are extended to $\mathcal{N}(\mathbb{C}^2)$ and those for $\lambda < 0$ are extended to $\mathcal{N}(\widehat{\mathbb{C}}^2)$. We list here a table of expansion formula for $\phi_\lambda, \phi_\lambda^* = \gamma_0 \phi_\lambda$ for $\lambda > 0$ and $\pi_\lambda, \pi_\lambda^* = \gamma_0 \phi_\lambda$ for $\lambda < 0$.

- (1) $\Phi_\lambda(z) = |z|^{\lambda - \frac{3}{2}} \left(\frac{1+|z|^2}{2} \right)^{\frac{3}{2}} \phi_\lambda\left(\frac{z}{|z|}\right) \in \mathcal{N}(\mathbb{C}^2), \lambda > 0$ and $\Phi_\lambda(z) = \phi_\lambda(z)$ for $|z| = 1$.
- (2) $\widehat{\Phi}_\lambda^*(w) = |w|^{\lambda + \frac{3}{2}} \left(\frac{2}{1+|w|^2} \right)^{\frac{3}{2}} \widehat{\phi}_\lambda^*\left(\frac{w}{|w|}\right) \in \mathcal{N}^\dagger(\widehat{\mathbb{C}}^2)_0, \lambda > 0$ and $\widehat{\Phi}_\lambda^*(-\bar{z}) = -\overline{\gamma_0 \phi_\lambda^*}(z)$ for $|z| = 1$.
- (3) $\widehat{\Pi}_\lambda(w) = |w|^{-\lambda - \frac{3}{2}} \left(\frac{1+|w|^2}{2} \right)^{\frac{3}{2}} \widehat{\pi}_\lambda\left(\frac{w}{|w|}\right) \in \mathcal{N}(\widehat{\mathbb{C}}^2), \lambda < 0$ and $\widehat{\Pi}_\lambda(-\bar{z}) = -\overline{\gamma_0 \pi_\lambda}(z)$ for $|z| = 1$.
- (4) $\Pi_\lambda^*(z) = |z|^{-\lambda + \frac{3}{2}} \left(\frac{2}{1+|z|^2} \right)^{\frac{3}{2}} \pi_\lambda^*\left(\frac{z}{|z|}\right) \in \mathcal{N}^\dagger(\mathbb{C}^2)_0, \lambda < 0$ and $\Pi_\lambda^*(z) = \pi_\lambda^*(z)$ for $|z| = 1$.

References

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