

Tight Graphs and Their Primitive Idempotents*

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Abstract

In this paper, we prove

Theorem 1. Let Γ denote a distance-regular graph with diameter $d \geq 3$. Suppose E and F are primitive idempotents of Γ , with cosine sequences $\sigma_0, \sigma_1, \dots, \sigma_d$ and $\rho_0, \rho_1, \dots, \rho_d$, respectively. Then the following are equivalent.

- i) The entry-wise product $E \circ F$ is a scalar multiple of a primitive idempotent of Γ .
- ii) There exists a real number ϵ such that

$$\sigma_i \rho_i - \sigma_{i-1} \rho_{i-1} = \epsilon (\sigma_{i-1} \rho_i - \sigma_i \rho_{i-1}) \quad (1 \leq i \leq d).$$

Let Γ denote a distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \dots > \theta_d$. In [1], Jurišić, Koolen and Terwilliger proved that the valency k and the intersection numbers a_1, b_1 satisfy

$$\left(\theta_1 + \frac{k}{a_1 + 1} \right) \left(\theta_d + \frac{k}{a_1 + 1} \right) \geq \frac{-ka_1 b_1}{(a_1 + 1)^2}.$$

They called the graph *tight* whenever Γ is not bipartite, and equality holds above. Combining Theorem 1 with some of their results, we obtain

Corollary 2. Let Γ denote a nonbipartite distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \dots > \theta_d$. The following are equivalent.

- i) There exist nontrivial primitive idempotents E, F of Γ such that (i), (ii) hold in Theorem 1.
- ii) Γ is tight.

Moreover, if (i), (ii) hold then the eigenvalues of Γ associated with E, F are a permutation of θ_1, θ_d .

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Reference

- [1] A. Jurišić, J. Koolen and P. Terwilliger, 1-Homogeneous Graphs (in preparation).

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