

Constrution of algebraic vector bundles of rank 2
on non-singular algebraic varieties of arbitrary dimensions

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As for construction of algebraic vector bundles on non-singular algebraic varieties, the following methods are wellknown.

- (1) J.P.Serre : 2-bundles associated to closed subschemes of co-dimension 2, locally complete intersection.
- (2) R.Schwarzenberger :
 - (a) Ramified 2-coverings.
 - (b) Blowing-ups + Descents.
- (3) G.Horrocks : Monads.
- (4) M.Maruyama : Elementary transformations.

Using the methods (2), R.Schwarzenberger has constructed many indecomposable 2-bundles on algebraic surfaces. In this note, we shall report that we can generalize the method (2) (b) to algebraic varieties of arbitrary dimensions and obtain the following.

Theorem. Let X be a non-singular algebraic variety defined over an algebraically closed field k ($\text{char } k = p \geq 0$) and let $\{Y, Z\}$ be a pair of closed subschemes of X satisfying the following conditions :

- (a) D is a reduced divisor of X whose singular locus $\text{Sing}(D)$ is either empty or of $\text{codim}_X \text{Sing}(D) = 4$.
- (b) Z is a smooth closed subscheme of D with $\text{codim}_X Z = 2$ and it contains $\text{Sing}(D)$.
- (c) There is a rational map $f : D \longrightarrow \mathbb{P}^1$ such that the regular domain $D(f)$ of f contains $D - \text{Sing}(D)$ and $Z = f^{-1}(0)$ scheme theoretically.

Then there are an algebraic 2-bundle E on X and a section s of E satisfying the following properties :

- (1) Z coincides with $Z(s)$, the scheme of zeros of s .
- (2) $\mathcal{O}_X(D)$ is isomorphic to $\bigwedge^2 E$.

Moreover,

- (3) if $H^1(X, \mathcal{O}_X) = 0$, then there exists another section t of E

such that D coincides with $Z(s \wedge t)$, the scheme of zeros of $s \wedge t$.

(4) if X is projective and Z is connected and if $H^1(X, \mathcal{O}_X(-D)) = 0$, then E is determined uniquely up to isomorphisms.

As a corollary, we obtain the following.

Corollary. Let X be a non-singular projective variety defined over an algebraically closed field k ($\text{char } k = p \geq 0$), Y a non-singular divisor of X and let $\{Z_t\}_{t \in \mathbb{P}^1}$ be a Lefschetz pencil on Y with base locus W . Then there exist a reflexive sheaf E of rank 2 on X and a section s of E such that there is an exact sequence

$$0 \longrightarrow \mathcal{O}_X \longrightarrow E \longrightarrow \mathcal{O}_X(Y) \otimes \mathcal{I}_{Z_t} \longrightarrow 0$$

, where $\mathcal{I}_{Z_t}$ is the defining ideal sheaf of Z_t in X and $\text{Sing}(E)$ coincides with W .

The proof of the above theorem and corollary will be published elsewhere.