

Logic and memory devices of
nonlinear microelectromechanical resonator

Atsushi Yao

Abstract

This dissertation focuses on mechanical computation based on nonlinear microelectromechanical system (MEMS) resonators. In particular, this dissertation shows read, write, counter, and logic-memory operations in a single or coupled MEMS resonators. A MEMS resonator has two coexisting stable states when a nonlinear hysteretic response appears. The nonlinear MEMS resonator can be used as a mechanical 1-bit memory and/or logic devices.

We propose read and write operations of the memory device. The read and write operations imply a displacement measurement and a switching of two stable periodic vibrations, respectively. In this dissertation, we realize a displacement measurement along an approach avoiding supplemented sensors. In addition, we achieve the switching control between two coexisting periodic states by a displacement feedback control and a delayed feedback control.

This dissertation discusses a logical operation of multi-memories that consist of coupled nonlinear MEMS resonators. We develop a sequential logic device with coupled multi-resonators. From the viewpoint of application of nonlinear dynamics in coupled MEMS resonators, we show the experimental and numerical success of the controlling nonlinear behavior as a 2-bit binary counter.

This dissertation reports multifunctional operation based on the nonlinear dynamics in a single MEMS resonator. We address a logic-memory device that uses a closed loop control and a nonlinear MEMS resonator in which multiple states coexist. To obtain both logic and memory operations in a MEMS resonator, we examine the nonlinear dynamics with and without control input. Based on both experiments and numerical simulations, we develop a device that combines an OR gate and memory functions in a single MEMS resonator. In addition, a reprogrammable logic-memory device of the nonlinear MEMS resonator is numerically investigated. We numerically realize the reprogrammable logic function that consists of OR/AND gate by adjusting the excitation amplitude and the

memory function by storing logic information in the single nonlinear MEMS resonator.

Keywords: MEMS resonator, memory device, logic device, nonlinear dynamics, coupled resonators, logic-memory device, reprogrammable device

Acknowledgments

First and foremost, I would like to express my cordial gratitude to Professor Takashi Hikihara, Department of Electrical Engineering, Graduate School of Engineering, Kyoto University, for his continuous encouragement, in-depth discussions, insightful comments, constructive suggestions, and long-term support to accomplish this research.

I would like to appreciate Professor Masao Kitano, Department of Electronic Science and Engineering, Graduate School of Engineering, Kyoto University, and Associate Professor Hirofumi Yamada, Department of Electronic Science and Engineering, Graduate School of Engineering, Kyoto University, for providing helpful advice and taking part in my dissertation committee.

I would like to be grateful to Assistant Professor Suketu Naik, Weber State University, for his tremendous support in the design of MEMS resonators and useful advice.

I would like to acknowledge Associate Professor Nobuo Satoh, Department of Electrical, Electronics and Computer Engineering, Faculty of Engineering, Chiba Institute of Technology, for his technical help for experiments and moral support.

I would like to appreciate Dr. Yoshihiko Susuki and Dr. Ryo Takahashi, Department of Electrical Engineering, Graduate School of Engineering, Kyoto University, for their appropriate feedback, practical advice, and constant encouragement to achieve this study. I would like to thank Ms. Keiko Saito, who was an assistant professor in Department of Electrical Engineering, Graduate School of Engineering, Kyoto University, for constant encouragement.

I would like to be deeply grateful to Associate Professor Joachim Oberhammer, Micro and Nanosystems, School of Electrical Engineering, KTH Royal Institute of Technology, for giving me opportunity to be visiting student of KTH and to study many things.

I want to give special thanks to not only all present members but also past members of Professor Hikihara's laboratory. In particular, I would like to appreciate Mr. Yujirou Umezaki, Dr. Yuichi Yokoi, Dr. Tsuguhiro Takuno, Dr. Masataka Minami, Dr. Yama-

sue Kohei, Dr. Masayuki Kimura, Ms. Madoka Kubota, Mr. Hikaru Hoshino, Mr. Shinya Nawata, Mr. So Miyatake, Mr. Keiji Tashiro, Mr. Takashi Fumino, Mr. Takuya Kajiyama, Mr. Yoshihiko Yamaguchi, Mr. Alexandros Kordonis, Mr. Fredrik Raak, Dr. Nathabhat Phankong, Ms. Yanzi Zhou, and Mr. Takeya Matsumura for their supports to research environment, encouragement, and face-to-face discussion. I also want to thank Ms. Yoshiko Deguchi for her supports on research environment.

I would like to thank the members of Professor Kitano's research groups including the past members. Especially, I wish to thank Dr. Yosuke Nakata for his wide-ranging discussion and moral support.

I would like to express my gratitude to Dr. Quirin Unterreithmeier for fruitful comments through wire communication.

This work was partly supported by the Global COE of Kyoto University, Regional Innovation Cluster Program "Kyoto Environmental Nanotechnology Cluster", the JSPS KAKENHI (Grant-in-Aid for Exploratory Research) #21656074, and the Grant-in-Aid for JSPS Fellows #26462.

Finally, I want to thank my parents, Makoto and Tomoko Yao, from the bottom of my heart for their continuous supports and encouragement.

Contents

Abstract	iii
Acknowledgments	v
List of Acronyms	xi
1 Introduction	1
1.1 Mechanical computation	1
1.2 MEMS resonator	2
1.3 Nonlinear MEMS resonator	3
1.4 Purpose and outline of the dissertation	3
2 Nonlinear MEMS resonator and its application to memory and logic functions	7
2.1 Schematic of comb-drive resonator	7
2.2 MEMS resonator and its nonlinearity	8
2.2.1 Nonlinear restoring force	9
2.2.2 Hysteretic characteristics	9
2.2.3 Basins of attraction	10
2.3 Memory and logic functions	12
3 Read operation of memory device in MEMS resonator	15
3.1 Differential measurement	15
3.1.1 Excitation force	16
3.1.2 Current through capacitor	18
3.1.3 Experimental system	19
3.1.4 Experimental results and discussions	22

3.2	Numerical study on read operation	23
3.2.1	Parameters of model based on experimental results	25
3.2.2	Steady states	26
3.2.3	Transient states	29
3.3	Summary	31
4	Write operation of memory device in MEMS resonator	33
4.1	Experimental study on displacement feedback control	34
4.1.1	Experimental method and setup	34
4.1.2	Results and discussions	35
4.2	Numerical study on displacement feedback control	37
4.2.1	Numerical method	37
4.2.2	Results and discussions	37
4.2.3	Gain dependence	40
4.3	Numerical write operation without exact solutions	43
4.3.1	Numerical method	43
4.3.2	Numerical results and discussions	46
4.4	Summary	49
5	Counter operation in coupled MEMS resonators	53
5.1	Experimental study on 2-bit binary counter	53
5.1.1	Two MEMS resonators	53
5.1.2	Experimental switching method	54
5.1.3	Results and discussions	57
5.2	Numerical study on 2-bit binary counter	60
5.2.1	Dynamical model in two MEMS resonators	60
5.2.2	Numerical switching method	61
5.2.3	Numerical results and discussions	62
5.3	Summary	67
6	Logic-memory operation in a single MEMS resonator	69
6.1	Logic-memory device	69
6.1.1	Experimental and numerical systems	70
6.1.2	Logic operation (numerical results)	72

6.1.3	Memory operation (numerical results)	73
6.1.4	Experimental and numerical results and discussions	74
6.2	Reprogrammable logic-memory device	75
6.2.1	OR/AND gate and memory operations (numerical results)	76
6.3	Summary	79
7	Conclusions and future work	81
7.1	Conclusions	81
7.2	Future work	83
A	Classification of stability	85
B	Nonlinear analysis	87
C	RF pulse control	89
	Bibliography	93
	List of Publications	101

List of Acronyms

MEMS	Micro Electro Mechanical Systems
NEMS	Nano Electro Mechanical Systems
ac	Alternative Current
RF	Radio Frequency
dc	Direct Current
CMOS	Complementary Metal Oxide Semiconductor
DFC	Delayed Feedback Control
SOI	Silicon on Insulator
DRAM	Dynamic Random Access Memory

Chapter 1

Introduction

1.1 Mechanical computation

A mechanical computer consists of mechanical components such as gears, beams and so on. The history of mechanical computation began when the mechanical calculator was invented by Wilhelm Schickard in the 17th century. In 1644, Blaise Pascal designed and built a small and simple mechanical calculator (called Pascaline) that consisted of the fundamental operations of addition and subtraction. Some thirty years later, Gottfried Wilhelm Leibniz extended Pascal's invention by developing a mechanical calculator known as the Leibniz wheel that could perform not only addition and subtraction but also multiplication and division [1–3].

Many histories of the modern mainframe computer began with Charles Babbage's analytical engine. In the 19th century, Charles Babbage tried to build the analytical engine that had the concept of a programmable computer, the automatic storage and retrieval of information in coded form, the automatic execution of a sequence of operations, and so on [1, 2, 4–6]. In 1820, Charles Thomas de Colmar designed the Arithmometer that was mechanical calculator. The Arithmometer patented by Thomas de Colmar became the first commercially successful mechanical calculator [6, 7]. Thereafter, many mechanical calculators such as Tiger Calculator [8] shown in Fig. 1.1 continued to be utilized until the 1960's. However, mechanical calculators were replaced by electronic calculators in the early 1970's [3, 9]. Do the histories of mechanical computation finish?

This dissertation focuses on mechanical computation especially in micromechanical resonators. In addition, this dissertation opens the way for further investigations of controlling nonlinear behavior in the micromechanical resonator from the viewpoint of



Figure 1.1: Tiger Calculator.

application of nonlinear dynamics.

1.2 MEMS resonator

Microelectromechanical systems or nanoelectromechanical systems (MEMS or NEMS) devices have micro-scale or nano-scale dimensions and mainly contain both electrical and mechanical components. MEMS and NEMS devices have been extensively studied and used as novel functional devices such as accelerometers [10], ink jet nozzle printing arrays [11], radio-frequency (RF) MEMS [12–15], and so on [16, 17]. MEMS and NEMS devices will become a key technology component in the telecommunications and information technology, medical care, and chemical fields [16, 17].

In many MEMS devices, we focus on MEMS resonators. In the 1960's, Harvey C. Nathanson *et al.* produced the first MEMS resonator that was called resonant gate transistor [18, 19]. In recent years, MEMS and NEMS resonators have been used as frequency references, sensor elements, and filters due to high quality factor (Q factor) [17]. MEMS and NEMS resonators consist of common elements such as lumped masses, beams, plates, and so on [20].

Recently, many studies focus on mechanical computation based on MEMS or NEMS resonators [21–36]. In MEMS resonators, mechanical logic and memory devices have three

advantages. The first advantage of such devices is their possibility of lower power consumption [28, 37, 38]. In nano-scale dimensions, those devices will consume principally lower power than CMOS-based memories. The second advantage is their good environmental stability. Mechanical logic and memory devices can be used in outer space at high temperatures [37]. Finally, a single MEMS resonator allows active operations such as parallel logic operation [28], logic-memory operation [36, 39], reprogrammable logic operation [25], and so on.

Most modern computers use a binary representation that has two states “1” and “0”. In mechanical computation based on MEMS or NEMS resonators, some studies have addressed nonlinear dynamics in the MEMS or NEMS resonator that has two states [21, 22, 25–27, 30, 31, 34].

1.3 Nonlinear MEMS resonator

In 1987, M. V. Andres and colleagues showed that a MEMS resonator exhibited nonlinear response such as bistable and hysteretic characteristics [40]. The nonlinear dynamical responses are commonly observed in a microelectromechanical or nanoelectromechanical resonator [16, 17, 20, 41]. The nonlinear dynamics of the resonator is well known to be described by the Duffing equation [21, 27, 31, 42–44]. Such a nonlinear resonator has hysteretic characteristics with respect to the excitation frequency [16, 17] or excitation force [25, 34]. In the hysteretic region, the MEMS resonator exhibits two coexisting stable states that correspond to large and small amplitude vibrations [21, 25, 27, 31, 34, 42–44]. Thus, a nonlinear MEMS resonator can work as a 1-bit mechanical memory or logic device indicating “1” and “0” at large and small vibrations.

As a new logic element using vibrating states, Eiichi Goto invented the parametron that consisted of an LC resonant circuit in 1954 [45, 46]. In the LC resonant circuit, by using parameter excitation, it is possible to excite an oscillation. This oscillation has two phases (0 or π) that correspond to binary variables (“0” and “1”). The parametron computers shown in Fig. 1.2 were used until the early 1960’s [8].

1.4 Purpose and outline of the dissertation

This dissertation focuses on mechanical computation based on nonlinear microelectromechanical resonators. In particular, this dissertation shows 1) read, 2) write, 3)



Figure 1.2: Parametron Computer.

counter, and 4) logic-memory operations in a single or coupled resonators. In the following, the summarized contents of the dissertation are described.

Chapter 2 describes basics of comb-drive resonator and its nonlinearity. Furthermore, the basics of memory and logic functions of MEMS resonators are discussed. Firstly, a schematic diagram of MEMS resonator is explained. The electrostatically driven comb-drive resonator is fabricated with silicon on insulator (SOI) technology. Next, the nonlinearity of MEMS resonators is explained. The nonlinear dynamics of the fabricated MEMS resonator is described by the Duffing equation. Such a nonlinear resonator has hysteretic characteristics, which lead to two stable states and one unstable state, depending on the excitation frequency or excitation force. Finally, the basics of memory and logic functions are shown. In particular, memory, sequential logic, and logic-memory devices of the nonlinear MEMS resonators are discussed.

In chapter 3, we focus on a displacement measurement of a MEMS resonator as a read operation of a memory device. The displacement measurements of MEMS resonators, based on piezoelectric element [24], photodetector [47], electric circuit [42], and so on, have been recently studied. We propose the displacement measurement by using a self-sensing method for the combined structure of actuator and sensor in a MEMS resonator. That is, the MEMS resonator has a comb-drive as a forcing actuator, which is simultaneously used as a displacement sensor. The self-sensing can be a solution for the integration and the simplification of whole system such as measurement system, control system, and so

on. From this standpoint, we perform a self-sensing method, based on the measurement of the current through the capacitor of the MEMS resonator.

In chapter 4, we numerically and experimentally demonstrate the switching between two coexisting stable states by a displacement feedback control as a write operation of the memory device in the single MEMS resonator. Recently, Quirin P. Unterreithmeier and colleagues have shown a switching between two stable states in a nonlinear MEMS resonator at a constant excitation frequency [27]. Their results motivated us to find other appropriate continuous control methods for the practical use of MEMS resonators based on feedback control. Based on the displacement measurement shown in chapter 3, we discuss the switching between two stable periodic vibrations at a constant excitation frequency by the displacement feedback control. As a typical control method for nonlinear systems, we apply a delayed feedback control method (DFC) [48] to switching as a write operation without exact solutions by numerical simulations. DFC is applied to switching between stable periodic vibrations because DFC does not need any exact model except the periodic of the target solution. We numerically show the switching without any exact displacement amplitude.

In chapter 5, we investigate a binary counter that consists of a coupled system of MEMS resonators with nonlinear characteristics. Previous studies have addressed that a MEMS or NEMS resonator can be utilized as a mechanical 1-bit memory [21, 24, 26, 27, 30, 31, 34] or mechanical logic gates [23, 25, 28, 29]. The next phase is to develop a sequential logic operation from coupled multi-resonators. Based on both experiments and numerical simulations, we show the controlling nonlinear behavior as a 2-bit binary counter.

In chapter 6, we report multifunctional operation based on the nonlinear dynamics in a single MEMS resonator. Recently, Imran Mahboob and co-workers have demonstrated multifunctional operation in the form of a shift-register and a controlled NOT gate made from a single mechanical resonator [36]. We focus on a logic-memory device that uses a closed loop control and a nonlinear MEMS resonator in which multiple states coexist. The closed loop allows output and excitation signals to be fixed at a constant excitation frequency. To obtain both logic and memory operations in a MEMS resonator, we examine the nonlinear dynamics with and without control input. We discuss the experiments and numerical simulations that allow us to develop a device that combines multiple-input gate and memory functions in a single nonlinear MEMS resonator. In addition, we address a reprogrammable logic-memory device of a nonlinear MEMS resonator with multiple

states by numerical simulations. Recently, D. N. Guerra *et al.* have experimentally shown that the single nanomechanical resonator can work as a reprogrammable logic gate [25]. Based on their results and our study, we show the realization of the reprogrammable logic function and the memory function in the single nonlinear MEMS resonator.

In chapter 7, the conclusions of this dissertation are summarized. The course of the future work is also discussed.

Chapter 2

Nonlinear MEMS resonator and its application to memory and logic functions

This chapter discusses memory and logic functions of nonlinear MEMS resonator. The purpose of this chapter is to understand the fundamental matters of nonlinear MEMS resonator and to associate it with logic and memory devices. Firstly, the fabricated MEMS resonator is explained. Next, basics of the nonlinear characteristics in the fabricated MEMS resonator are discussed. Finally, basics of 1) read, 2) write, 3) counter, and 4) logic-memory operations in a single or coupled MEMS resonators are summarized.

2.1 Schematic of comb-drive resonator

Figure 2.1 shows a fabricated comb-drive MEMS resonator [43, 49–51]. Fig. 2.1(a) shows a top view of the resonator. The resonator consists of a perforated mass with a width, length, and thickness of 175, 575, and 25 μm , respectively. The perforated mass is supported by folded beams, which are designed to work as springs. They are connected to anchors. The MEMS resonator has two comb capacitors. The left and right capacitors are connected to the each side electrode, respectively. When the anchor is connected to ground and either of the right or the left electrode to ac voltage source with a dc bias voltage, the mass vibrates primarily in the lateral direction (X -direction) with weak link to the longitudinal and vertical directions.

Figure 2.1(b) illustrates a cross section at the dotted lines of Fig. 2.1(a). The MEMS resonator is fabricated using SOIMUMPs, which is a kind of silicon on insulator (SOI)

technology and is offered by Memscap, Inc [52]. In this process, the MEMS device consists of a $25\mu\text{m}$ thick Silicon layer as the structure layer, a $2\mu\text{m}$ thick Oxide layer as the insulating layer, and a $400\mu\text{m}$ thick Silicon layer as the substrate layer. The substrate and oxide layers underneath a movable structure are removed as shown in Fig. 2.1(b).

2.2 MEMS resonator and its nonlinearity

This section reports basics of the nonlinear characteristics in the fabricated MEMS resonator. A MEMS or NEMS resonator substantially shows nonlinear responses at large excitation force [16, 17, 21, 25, 25, 27, 31, 34, 34, 42–44, 49, 50]. In particular, it has been experimentally [30, 35, 39, 50, 51, 53] and numerically [31, 43, 49, 51, 54] confirmed that the fabricated MEMS resonator shown in Fig. 2.1 exhibits nonlinear responses.

In the following, Section 2.2.1 explains the nonlinear restoring force in the MEMS resonator. Next, Section 2.2.2 explains the hysteretic characteristics, which have two stable states and one unstable state, as a function of excitation frequency in the nonlinear MEMS resonator. Finally, the convergence conditions of two stable states are examined.

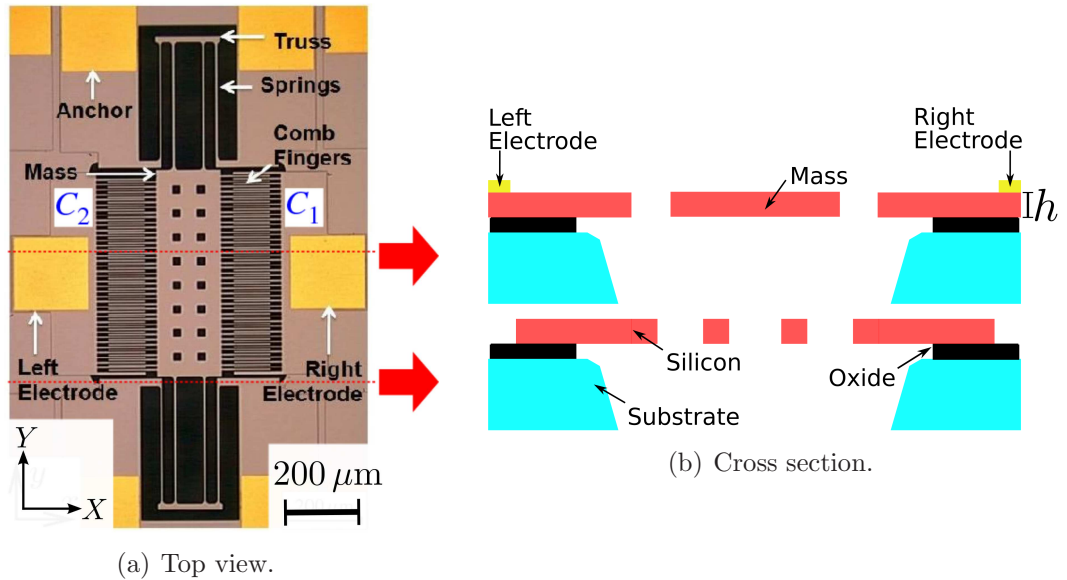


Figure 2.1: Schematic diagram of fabricated MEMS resonator.

2.2.1 Nonlinear restoring force

When the mass of the resonator vibrates in X -direction as mentioned previously, the linear dynamical model is given by [16, 17, 41]

$$m_e \frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + k_{s1}x = F \sin \omega t, \quad (2.2.1)$$

where x denotes the displacement, γ the damping constant, F the excitation amplitude, k_{s1} the spring constant, and m_e the effective mass. Here, it is assumed that the damping is proportional to the velocity. In Eq. (2.2.1), the spring is a linear spring and the total restoring force F_r follows Hooke's law,

$$F_r = -k_{s1}x. \quad (2.2.2)$$

MEMS resonators are known to produce nonlinear responses when the deflections become large [17]. As shown in Fig. 2.2, in the resonance of the MEMS resonator, the linear stiffness is augmented by a nonlinear restoring force of the form $k_{s3}x^3$ at large amplitude excitation [16, 17]. In this figure, the total restoring force F_r is described by [16]

$$F_r = -k_{s1}x - k_{s3}x^3, \quad (2.2.3)$$

where k_{s3} is the nonlinear mechanical spring constant. In Eq. (2.2.3), it should be noted that two types of behavior occur according to the sign of k_{s3} . If $k_{s3} > 0$, the spring is called a hard spring [16, 41, 55]. In contrast, for k_{s3} negative, the spring is a soft spring [16, 41, 55]. The fabricated comb-drive MEMS resonator has the hard spring [30, 31, 35, 39, 43, 49–51, 51, 53, 54]. Therefore, in the following study, we discuss the MEMS resonator with hard spring ($k_{s3} > 0$). As a result, the nonlinear dynamics of the MEMS resonator is described by Duffing's equation,

$$m_e \frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + k_{s1}x + k_{s3}x^3 = F \sin \omega t. \quad (2.2.4)$$

2.2.2 Hysteretic characteristics

At nonlinear responses, amplitude-frequency response curves bend toward higher frequencies owing to a hard spring effect [16, 41] as shown in Fig. 2.3. In Fig. 2.3, the red line corresponds to the responses at the up-sweep of frequency from f_L to f_H and the blue

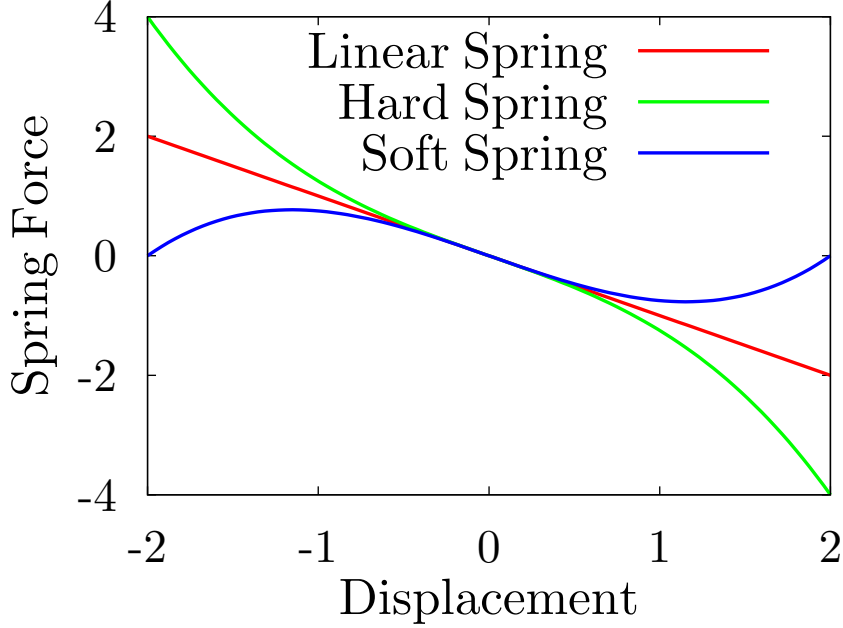


Figure 2.2: Linear and nonlinear spring force with respect to displacement.

line the responses at the downsweep of frequency from f_H to f_L . In the same figure, the coexisting unstable solution is shown by the green line.

At f_1 or f_2 , one stable solution and one unstable solution collide and then bifurcate. After the bifurcation, there is no solution originated by both solutions in the neighbourhood. Therefore, there occurs a jump to the other stable solution. The jump phenomenon is associated with the so-called saddle-node bifurcation [17, 55]. Notice that such a nonlinear resonator has hysteretic characteristics [17, 55] as shown by red and blue lines in Fig. 2.3. In the hysteresis region, two coexisting stable states and one unstable state appear. See Appendix A for the detailed classification of the stability of these periodic vibrations.

2.2.3 Basins of attraction

As mentioned above, the nonlinear dynamical model of MEMS resonator is described by Eq. (2.2.4) and has two stable solutions. In this study, conditions for existence and uniqueness of solutions are established. Here, the multiplicity of alternative stable attracting solutions depends on the initial conditions, which correspond to the displacement x

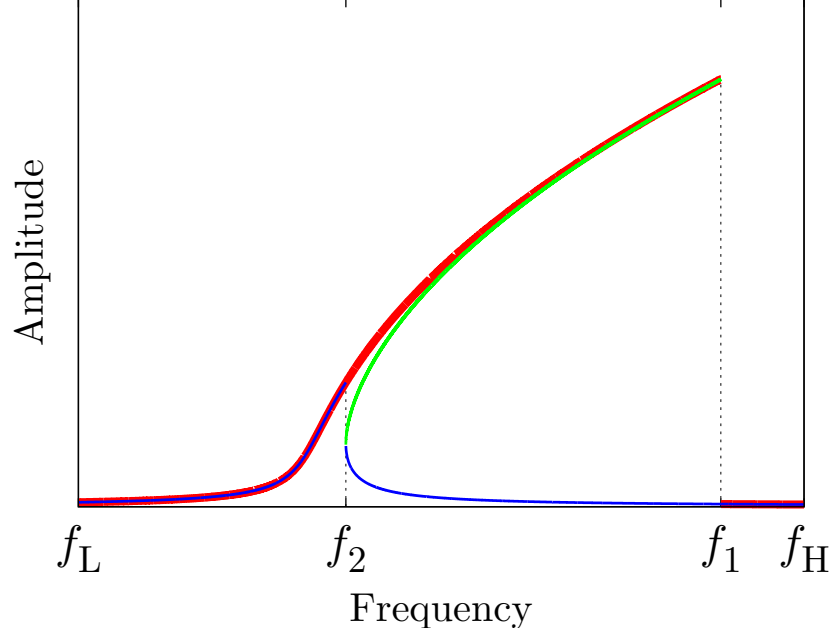


Figure 2.3: Amplitude frequency response of nonlinear MEME resonator.

and the velocity y (or the amplitude and the phase) [55]. A study of stroboscopic points of the xy plane onto itself serves to determine the basins of attraction [55, 56] of the original continuous differential equation (2.2.4). That is, as mentioned in Section A, a stroboscopic map [57] is considered.

Figure 2.4 shows one example of a set of initial conditions sampled by the period of the excitation frequency. The stable fixed point sampled by the period of the excitation force is called an attractor in the xy plane (See Appendix A for the stable fixed point.) [57, 58]. In Fig. 2.4, the green and blue points show two attractors. In the plane of the initial values of (x, y) at $t = 0$, there are basins of attraction such that the vibrations originating in white region converge to the green point (attractor) after the decay of transients, while vibrations starting in black region converge to the blue point (attractor). The aqua line between the basins of attraction is called a separatrix curve [55, 57, 58]. The basins of attraction for the two stable solutions are completely separated by stable manifolds of the unstable solution. In the nonlinear MEMS resonator, the basins of attraction tend to have a spiral form, which depends on both displacement and velocity. Here, two stable solutions have each basin of attraction around each solution. Therefore, the nonlinear

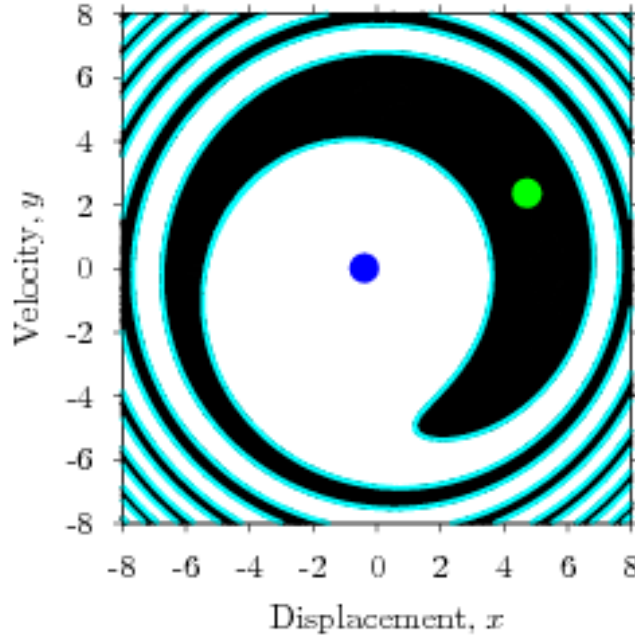


Figure 2.4: One example of basins of attraction. The white and black regions show the basins of two stable solutions. The green (blue) point shows the large (small) amplitude solution. Every initial state obviously corresponds to the convergence to small (white) and large (black) amplitude solutions. As shown by aqua line, the basins are completely separated by stable manifolds of an unstable solution.

MEMS resonator has the capability of staying at either of two stable solutions.

2.3 Memory and logic functions

In this section, the nonlinear MEMS resonator is associated with logic and memory devices. This dissertation focuses on the digital systems such as logic and memory devices using binary variables (“0” and “1”) that consist of vibrating states. As mentioned in previous sections, the nonlinear MEMS resonator has two stable states and an unstable state. Recently, some studies have shown that a nonlinear resonator can be utilized as a mechanical 1-bit memory [21, 22, 26, 27, 30, 31, 34] or as mechanical logic gates [25]. That is, as shown in Fig. 2.5, the nonlinear MEMS resonator has large and small vibrations designed “0” and “1”, respectively, for logic or memory output.

In the following, we explain basics of memory and logic devices. Generally, logic devices are classified in two categories: combinational logic devices and sequential logic

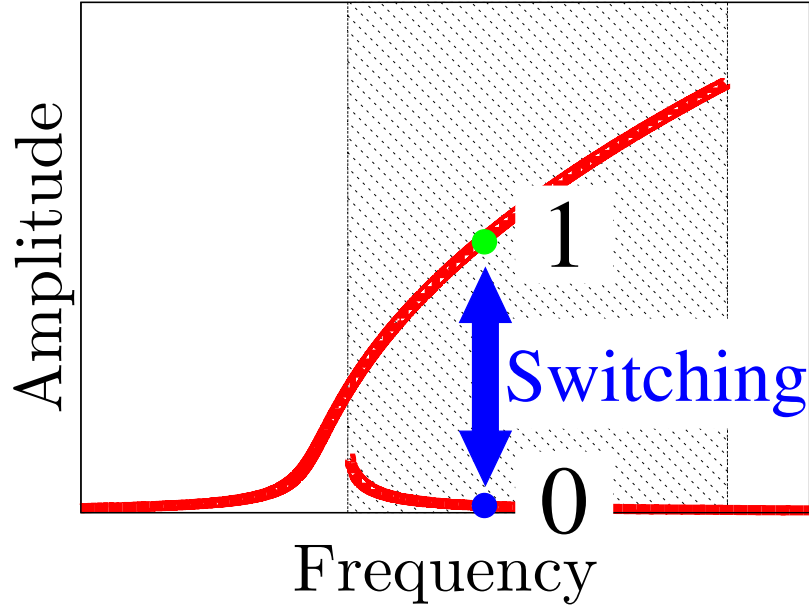


Figure 2.5: Nonlinear MEMS resonator: The large (small) amplitude vibration is regarded as a logical “1” (“0”).

devices [5, 59]. In addition, a logic-memory device is discussed. Finally, basics of four operations in logic and memory devices of a single or coupled MEMS resonators are summarized.

(a) Memory devices

Digital systems require memories to store the data used and generated by logic devices and so on. Memory devices have two states to store the binary variables [5]. For example, Dynamic Random-Access Memory (DRAM) stores a bit as the presence or absence of charge on a capacitor [5]. Detecting or recalling data from memory devices corresponds to the read or reading operation [59]. Storing data into a memory device corresponds to the write or writing operation [59]. In other words, the write operation implies switching between two states.

(b) Combinational logic devices

The outputs of a combinational logic device depend only on the current values of the inputs. That is, the device combines the current inputs to compute the output. Therefore, the device does not have memory. For instance, logic gates are combinational digital

devices. Logic gates take one or more binary inputs and produce a binary output [60].

(c) Sequential logic devices

The outputs of a sequential logic device depend on both current and previous values of the inputs. The device depends on the input sequence. Thus the device has memory. For example, a register and a counter correspond to sequential logic devices [60]. A binary counter is a sequential system that goes through a prescribed sequence of states upon the application of clock signals [61]

(d) Logic-memory device

As mentioned above, a combinational logic device such as a logic gate has no memory. Here, we focus on fabricating a multifunction device that offers logic gate and memory (called a “logic-memory device”) in the nonlinear MEMS resonator. The nonlinear MEMS resonator works as a logic gate and then a memory device by storing the logic. The output of the logic-memory device does not depend on the previous values of the inputs and memory but the current values of the inputs.

(e) Logic and memory devices of MEMS resonators

As explained in Section 1.4, this dissertation shows 1) read, 2) write, 3) counter, and 4) logic-memory operations in a single or coupled resonators. Based on the above discussions, in the memory and logic devices of MEMS resonators, the four operations are summarized as follows:

- 1) Read operation:** To measure two distinct stable vibrations
- 2) Write operation:** To switch between two stable periodic vibrations
- 3) Counter operation:** To perform the switching control sequence as a binary counter that consists of coupled nonlinear MEMS resonators
- 4) Logic-memory operation:** To generate multifunction devices that offer memory and multiple-input logic gates in a single MEMS resonator.

Chapter 3

Read operation of memory device in MEMS resonator

This chapter explains a read operation of the memory device in a MEMS resonator. As explained in Section 2.3, the read operation corresponds to a displacement measurement of a MEMS resonator. The comb-drive resonator shown in Fig. 2.1 is designed and fabricated to obtain hysteretic characteristics during upswing and downswing of frequency [43,50,51]. In Section 3.1, two stable vibrating states are measured in the hysteresis by using the differential measurement [62]. Section 3.2 shows the numerically calculated vibrating states.

3.1 Differential measurement

This section explains a displacement measurement of comb-drive resonator by using the differential measurement. The displacement measurements of MEMS resonators, based on piezoelectric element [24], photodetector [47], electric circuit [42], and so on, have been recently studied. In this dissertation, the displacement measurement by using a self-sensing method is proposed for the combined structure of actuator and sensor in a MEMS resonator. That is, the MEMS resonator has a comb-drive as a forcing actuator, which is simultaneously used as a displacement sensor. The self-sensing can be a solution for the integration and the simplification [63,64] of memory system. From this standpoint, we perform a self-sensing method, based on the measurement of the current through the capacitor of the MEMS resonator. Here, we show that the sum of the current through the capacitor only depends on the displacement in the differential measurement. This displacement measurement represents the read operation.

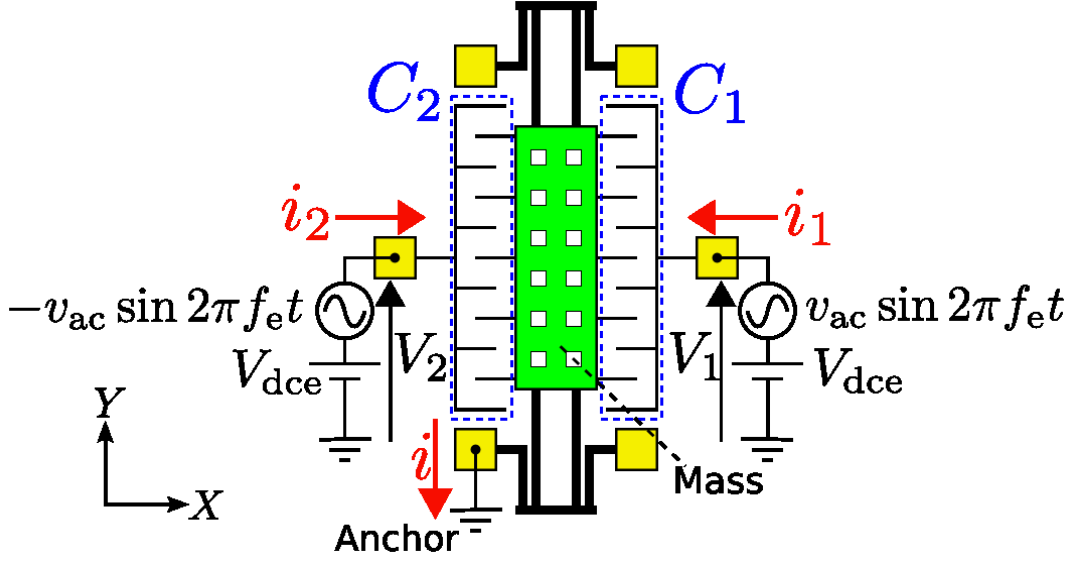


Figure 3.1: Input voltage and current through the capacitor in differential measurement. The nonlinear MEMS resonator, fabricated using silicon-on-insulator technology, is actuated by an ac excitation voltage v_{ac} with a dc bias voltage V_{dce} . When the MEMS resonator is excited, the mass vibrates in the X -direction.

In the following, the excitation force and the current are shown in the differential measurement. Next, the experimental system is explained. Finally, the experimental results are discussed.

3.1.1 Excitation force

This section explains the excitation force in the differential measurement shown in Fig. 3.1. That is, the voltage of right (left) electrode is excited by $V_1 = V_{dce} + v_{ac} \sin 2\pi f_e t$ ($V_2 = V_{dce} - v_{ac} \sin 2\pi f_e t$), where v_{ac} denotes the ac excitation voltage, V_{dce} the experimental dc bias voltage, and f_e the experimental excitation frequency. As explained in Section 2.1, when a MEMS resonator is actuated by applying an ac excitation voltage with a dc bias voltage between the mass and the electrodes, the mass vibrates in the X -direction.

When the mass vibrates, the right and left capacitances are given by

$$C_1 = 2\varepsilon N \frac{h(l+x)}{d} = 2\varepsilon N \frac{hl}{d} + 2\varepsilon N \frac{hx}{d} = C_0 + C(x), \quad (3.1.1)$$

$$C_2 = 2\varepsilon N \frac{h(l-x)}{d} = 2\varepsilon N \frac{hl}{d} - 2\varepsilon N \frac{hx}{d} = C_0 - C(x), \quad (3.1.2)$$

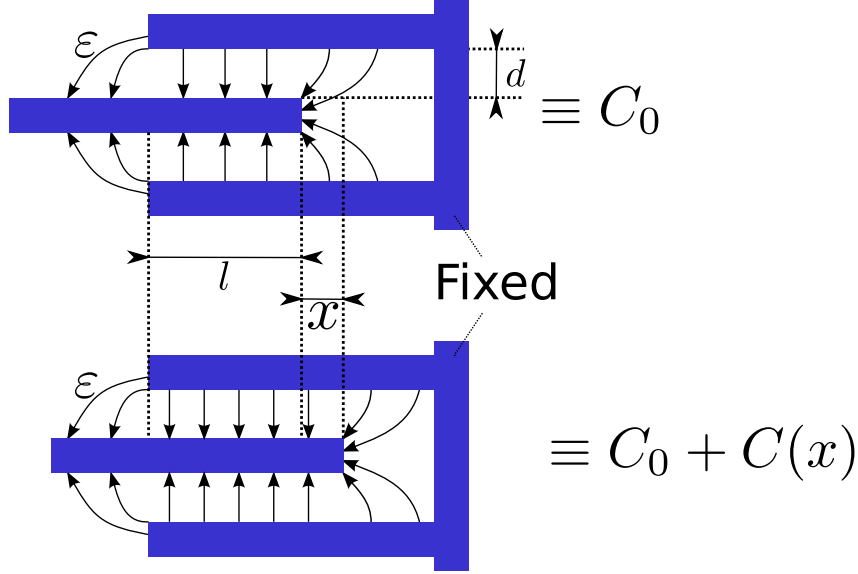


Figure 3.2: Schematic of comb capacitor at $N=1$.

Table 3.1: Device parameters in experiments.

Parameter		Value	Unit
N	comb number	39	
l	initial overlap between the fingers	100	μm
h	height of the finger	25	μm
d	gap between the fingers	3	μm
ε	permittivity	$8.85418782 \times 10^{-12}$	F/m

where C_1 denotes the capacitance of the right electrode, C_2 the capacitance of the left electrode, x the displacement, d ($= 3 \mu\text{m}$) the gap between the fingers, l ($= 100 \mu\text{m}$) the initial overlap between the fingers, ε ($= 8.85 \times 10^{-12} \text{ F/m}$) the permittivity, N ($= 39$) the comb number, and h ($= 25 \mu\text{m}$) the finger height. C_0 ($= 5.8 \times 10^{-13} \text{ F}$) is the initial capacitance and $C(x)$ is the change of capacitance as shown in Fig. 3.2. When the displacement x is set at $5 \mu\text{m}$, $C(x)$ becomes $2.9 \times 10^{-14} \text{ F}$. A parasitic capacitance between bond pad and substrate are estimated as $1.4 \times 10^{-12} \text{ F}$. Tab. 3.1 shows the device parameters.

The energy W_1 stored in the right capacitor is as follows:

$$W_1 = \frac{C_1 V_1^2}{2} = \frac{1}{2} (C_0 + C(x)) (V_{\text{dce}} + v_{\text{ac}} \sin 2\pi f_e t)^2. \quad (3.1.3)$$

In the same manner, the energy W_2 stored in the left capacitor is

$$W_2 = \frac{C_2 V_2^2}{2} = \frac{1}{2}(C_0 - C(x))(V_{\text{dce}} - v_{\text{ac}} \sin 2\pi f_e t)^2. \quad (3.1.4)$$

Therefore, the excitation force acting on the each capacitor is obtained as follows:

$$F_1 = \left(\frac{\partial W_1}{\partial x} \right)_v = \varepsilon N \frac{h}{d} (V_{\text{dce}}^2 + 2V_{\text{dce}} v_{\text{ac}} \sin 2\pi f_e t + v_{\text{ac}}^2 \sin^2 2\pi f_e t), \quad (3.1.5)$$

$$F_2 = \left(\frac{\partial W_2}{\partial x} \right)_v = -\varepsilon N \frac{h}{d} (V_{\text{dce}}^2 - 2V_{\text{dce}} v_{\text{ac}} \sin 2\pi f_e t + v_{\text{ac}}^2 \sin^2 2\pi f_e t), \quad (3.1.6)$$

where F_1 denotes the excitation force of the right electrode and F_2 the excitation force of the left electrode. The sum of these excitation is calculated as follows:

$$F_{\text{all}} = F_1 + F_2 = 4\varepsilon N \frac{h}{d} V_{\text{dce}} v_{\text{ac}} \sin 2\pi f_e t. \quad (3.1.7)$$

As shown in this equation, there exists the excitation force in the differential measurement.

3.1.2 Current through capacitor

This section addresses the current through the capacitor shown in Fig. 3.1. When the mass vibrates only in x -direction, a current of the anchor flows through the capacitor depending on the displacement of the comb-drive resonator [17]. However, the current is based not only on the displacement but also on the others such as the velocity when either of the electrodes is connected to the voltage source and the other to ground [17]. Therefore, the displacement can be extracted by the differential measurement of current.

As shown in Fig. 3.1, the current i_1 through the right capacitor and the current i_2 through the left capacitor are described by the following equations:

$$\begin{aligned} i_1 &= \frac{\partial(C_1 V_1)}{\partial t} \\ &= C_1 \frac{\partial V_1}{\partial t} + V_1 \frac{\partial C_1}{\partial t} \\ &= (C_0 + C(x))(2\pi f_e v_{\text{ac}} \cos 2\pi f_e t) + (V_{\text{dce}} + v_{\text{ac}} \sin 2\pi f_e t) 2\varepsilon N \frac{h}{d} \frac{\partial x}{\partial t}, \\ i_2 &= \frac{\partial(C_2 V_2)}{\partial t} \\ &= C_2 \frac{\partial V_2}{\partial t} + V_2 \frac{\partial C_2}{\partial t} \\ &= (C_0 - C(x))(-2\pi f_e v_{\text{ac}} \cos 2\pi f_e t) - (V_{\text{dce}} - v_{\text{ac}} \sin 2\pi f_e t) 2\varepsilon N \frac{h}{d} \frac{\partial x}{\partial t}. \end{aligned} \quad (3.1.8)$$

Therefor, the sum of the current i through the right and left capacitors are obtained as follows:

$$\begin{aligned}
i &= i_1 + i_2 \\
&= 4\pi C(x) f_e v_{ac} \cos 2\pi f_e t + 4v_{ac} \sin 2\pi f_e t \varepsilon N \frac{h}{d} \frac{\partial x}{\partial t} \\
&= (8\pi \varepsilon N \frac{h}{d} f_e v_{ac} \cos 2\pi f_e t) x + (4\varepsilon N \frac{h}{d} v_{ac} \sin 2\pi f_e t) \frac{\partial x}{\partial t}.
\end{aligned} \tag{3.1.9}$$

As shown in this equation, the sum of the current i through the right and left capacitors depends on the displacement x and velocity $\partial x / \partial t$ of the MEMS resonator.

When the displacement $x(t)$ is assumed as $A_0 \sin(2\pi f_e t + \phi)$, the current i is obtained as follows:

$$\begin{aligned}
i &= 8\pi \varepsilon N \frac{h}{d} f_e v_{ac} A_0 (\sin(2\pi f_e t + \phi) \cos 2\pi f_e t + \sin 2\pi f_e t \cos(2\pi f_e t + \phi)) \\
&= 8\pi \varepsilon N \frac{h}{d} f_e v_{ac} A_0 \sin(4\pi f_e t + \phi),
\end{aligned} \tag{3.1.10}$$

where A_0 and ϕ denote the amplitude and the phase of the displacement. Eq. (3.1.10) shows that the displacement can be extracted by the differential measurement of current. The frequency of current is twice the excitation frequency.

3.1.3 Experimental system

Here, we consider the experimental system to measure the sum of the current i through the right and left capacitors as mentioned in Section 3.1.2. Fig. 3.3 shows schematic diagram of measurement system to detect the current i that depends on the variation of displacement by differential measurement. The current i is converted to the output voltage V_{out} by two operational amplifiers (Burr-Brown; OPA627AP) as shown in Fig. 3.3. The first stage operational amplifier works as the current-to-voltage (I - V) converter and the second as the inverting amplifier. The output voltage V_{out} is obtained as

$$V_{out} = R \frac{R_2}{R_1} i = 8\pi f_e R \frac{R_2}{R_1} \varepsilon N \frac{h}{d} v_{ac} A_0 \sin(4\pi f_e t + \phi), \tag{3.1.11}$$

where R , R_1 , and R_2 denote three resistors. It can be confirmed that V_{out} depends on the amplitude A_0 and the phase ϕ of the displacement x .

Figure 3.4(a) shows two photographs of the experimental system. In this experimental system, the voltages (V_1 and V_2) of right and left electrodes are given by function generator

(Tektronix; AFG3022). The output voltage is measured by an oscilloscope (Tektronix; DPO4104). The supply voltage of two operational amplifiers is set at ± 15 V and supplied by stabilized power supply (TEXIO; PW36-1.5AD). The mechanical vibrations are verified by using the motion analysis microscope (KEYENCE; VW-6000).

Figure 3.4(b) shows a photograph of the internal vacuum chamber. The MEMS resonator is set in the vacuum chamber to reduce air resistance. In addition, in order to reduce an electrical noise due to a crosstalk, two operational amplifiers are also set in the vacuum chamber. The substrate and the vacuum chamber are grounded to reduce noise and parasitic capacitances.

In the following experiments in this chapter, the ac excitation amplitude v_{ac} is set at 0.6 V in vacuum at 10 Pa. As shown in Tab. 3.2, the resistors R , R_1 , and R_2 are set at 1 M Ω , 1 k Ω , and 100 k Ω , respectively. From the parameter settings, the excitation force F_{all} , the current i , and the output voltage V_{out} shown in Eq. (3.1.7), (3.1.10), and (3.1.11),

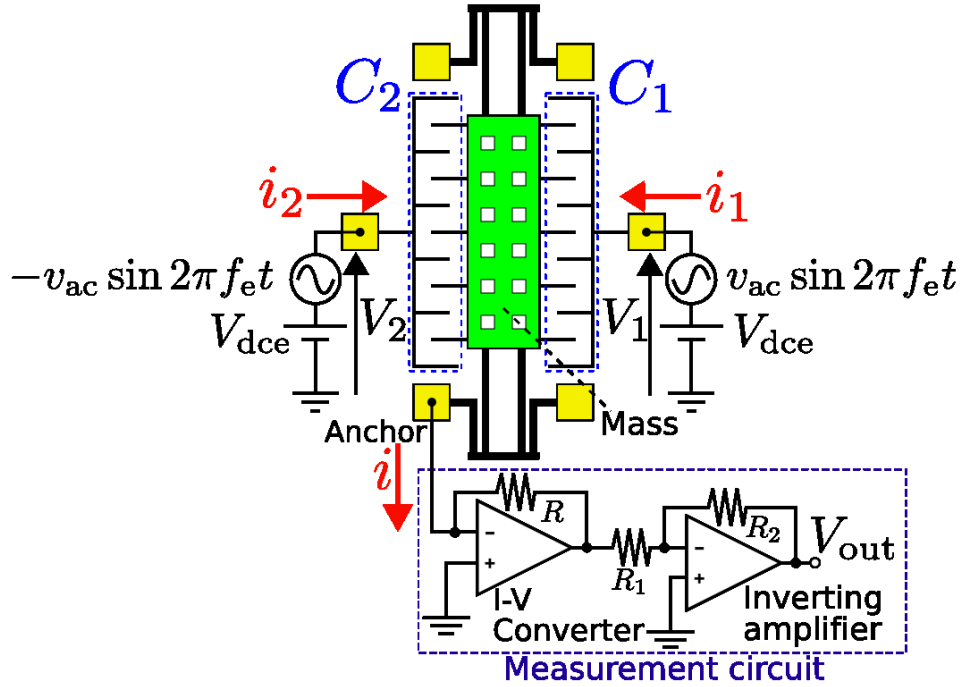
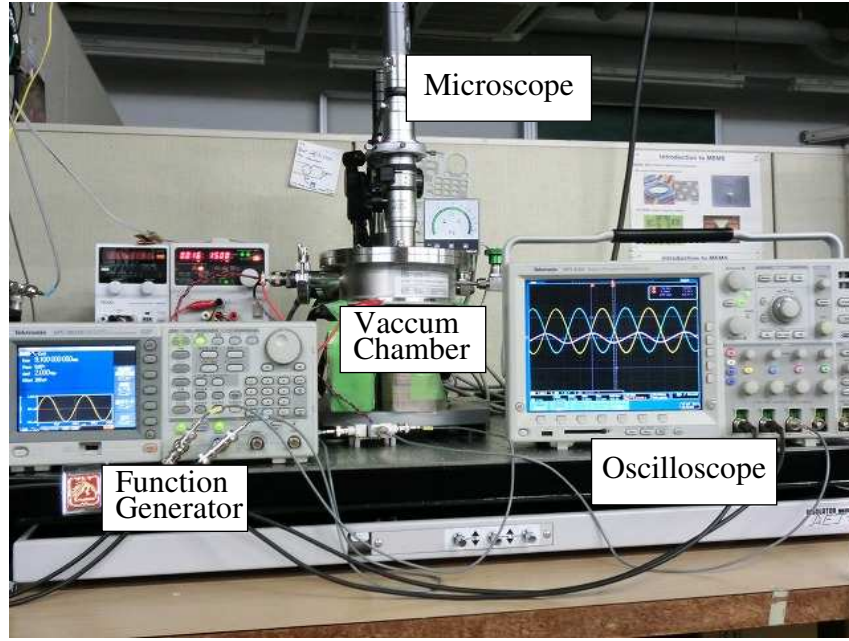
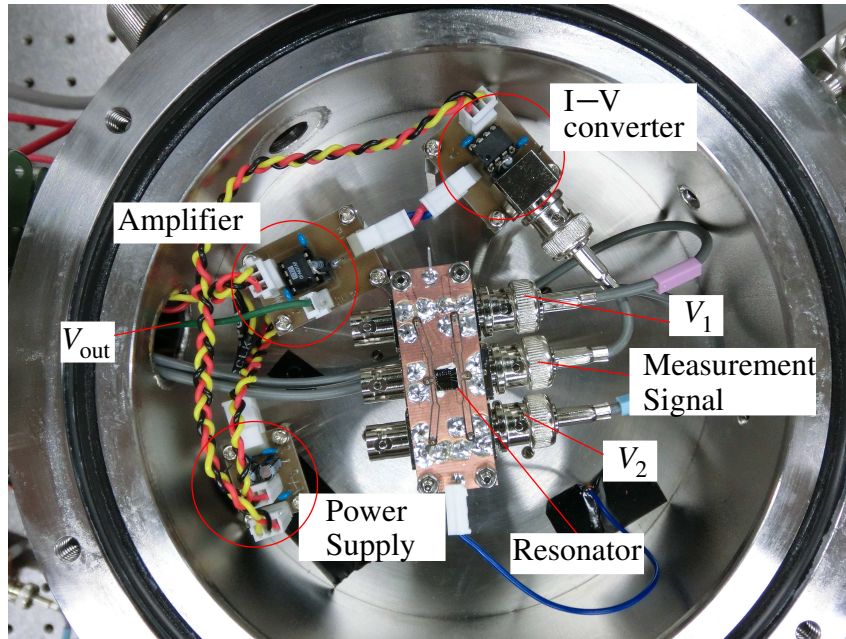


Figure 3.3: Schematic diagram of differential measurement. In the measurement system, the output voltage V_{out} depends on the amplitude and phase of the displacement in the nonlinear MEMS resonator.



(a) Photograph of whole experimental system.



(b) Photograph of internal vacuum chamber.

Figure 3.4: Photographs of the experimental system.

respectively, are calculated as follows:

$$F_{\text{all}} = 6.906 \times 10^{-9} \text{NV}^{-1} \times V_{\text{dcn}} \sin 2\pi f_e t, \quad (3.1.12)$$

$$i = 4.339 \times 10^{-8} \text{ As/mf}_e A_0 \sin(4\pi f_e t + \phi), \quad (3.1.13)$$

$$V_{\text{out}} = 4.339 \text{ V} / m f_e A_0 \sin(4\pi f_e t + \phi). \quad (3.1.14)$$

Table 3.2: Circuit parameters in experiments.

Parameter		Value	Unit
R	resistance	1	$\text{M}\Omega$
R_1	resistance	1	$\text{k}\Omega$
R_2	resistance	100	$\text{k}\Omega$

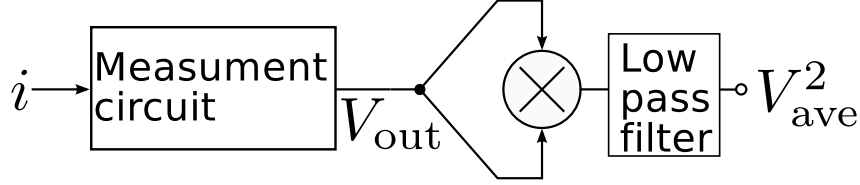


Figure 3.5: Analog multiplier and low-pass filter to convert the output ac voltage V_{out} to the slowly changing dc voltage V_{ave}^2 .

Figure 3.5 shows the circuit diagram to measure the amplitude of the output voltage V_{out} . The output ac voltage V_{out} is converted to the dc voltage V_{ave}^2 by an analog multiplier (ANALOG DEVICES; AD633) and a low-pass filter of the operational amplifier (Burr-Brown; OPA627AP). The time constant of the low-pass filter is set to about 0.47 s. Therefore, the dc voltage V_{ave}^2 is described as follows:

$$V_{\text{ave}}^2 = \frac{|V_{\text{out}}|^2}{2}. \quad (3.1.15)$$

From Eq. (3.1.14), the dc voltage V_{ave}^2 is calculated as follows:

$$V_{\text{ave}}^2 = \frac{(0.6906)^2 \text{Vs}^2/\text{m}^2 (2\pi f_e)^2 A_0^2}{2} \simeq 6.040 \times 10^{-3} \text{Vs}^2/\text{m}^2 f_e^2 A_0^2. \quad (3.1.16)$$

Here, it is assumed that the MEMS resonator has the vibrations at a single frequency (See Appendix B for this assumption.).

3.1.4 Experimental results and discussions

Figure 3.6 shows the experimentally obtained frequency response curves of the displacement at $V_{\text{dce}} = 150 \text{ mV}$. In Fig. 3.6, red and aqua lines show the response at upsweep and downsweep of frequency, respectively. We have confirmed that the MEMS resonator has hysteresis characteristics to frequency and two stable states coexist in the hysteresis region. Two stable states coexist at $8.6650 \text{ kHz} < f_e < 8.6666 \text{ kHz}$ in Fig. 3.6.

Figure 3.7 depicts the oscillogram of two stable periodic vibrations at 8.6654 kHz. The red and aqua lines are obtained by the average of 16 times measurements. It is found that these two stable periodic vibrations are obviously distinguished by the amplitude and the phase. In the following experiments in this chapter, the excitation frequency is fixed at 8.6654 kHz.

Figure 3.8 shows the experimentally determined hysteretic behavior as a function of dc bias voltage V_{dce} at 8.6654 kHz and $V_{\text{dce}} = 150$ mV. The hysteresis region exists at $95 \text{ mV} < V_{\text{dce}} < 275 \text{ mV}$. These stable regions, which correspond to large and small amplitude vibrations, define the two states of the single-output memory device in a single MEMS resonator. As a result, we realize the read operation by using the differential measurement.

3.2 Numerical study on read operation

This section focuses on the read operation of the nonlinear MEMS resonator by numerical simulations. The nonlinear dynamics of the MEMS resonator is discussed. Here, based on our reported parameter estimation method [65], the parameter settings are obtained. It is shown that the model of the MEMS resonator has two coexisting stable states

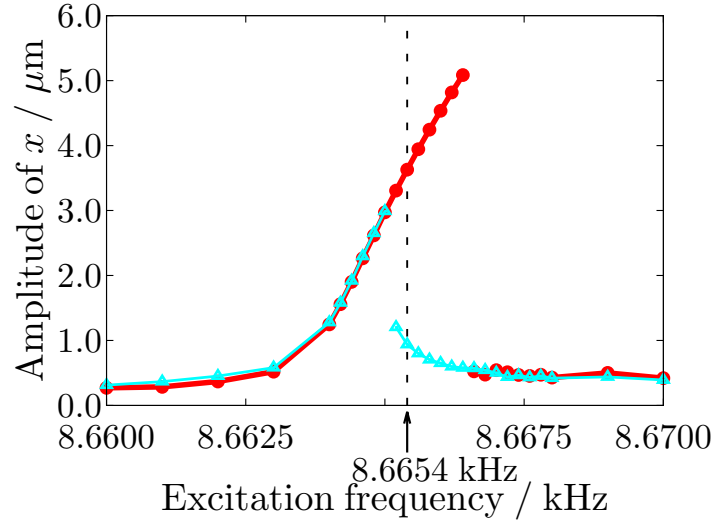


Figure 3.6: Experimentally obtained frequency response curves at $V_{\text{dce}} = 150$ mV. The dark (red) and thin (aqua) lines correspond to the responses to increasing and decreasing frequency sweeps, respectively. In the hysteresis region, two coexisting stable states appear.

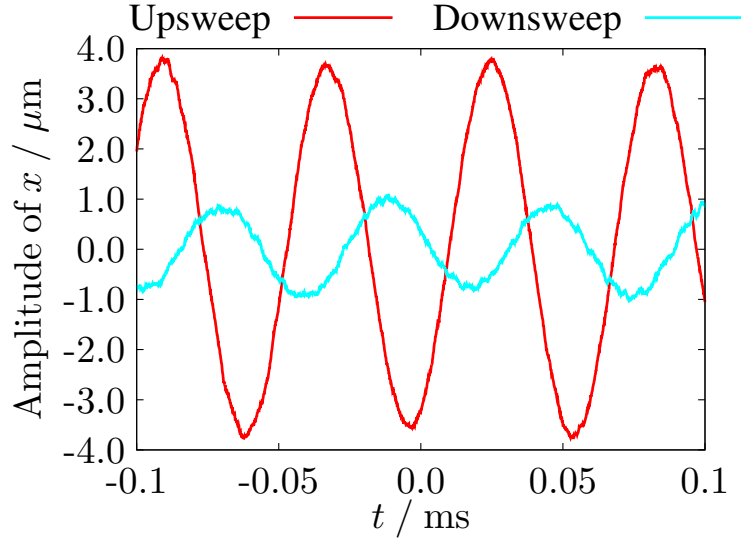


Figure 3.7: Two stable periodic vibrations at 8.6654 kHz and $V_{\text{dce}} = 150$ mV.

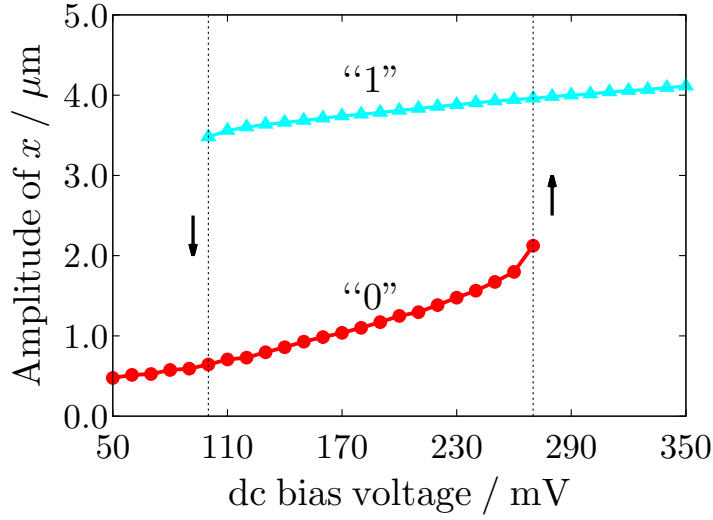


Figure 3.8: Measured hysteretic characteristics with respect to dc bias voltage V_{dce} at 8.6654 kHz. The excitation amplitude in the absence of the control input is proportional to the dc bias voltage V_{dce} . The experimentally obtained response shows the hysteretic behavior when the excitation amplitude is swept from left to right (thick red line) and right to left (thin aqua line). For memory output, the thin (dark) line is regarded as a logical “1” (logical “0”).

in the hysteretic region. In addition, in order to discuss one of the reasons for the difference between experimental and numerical results, the transient behavior is investigated by numerical simulations.

3.2.1 Parameters of model based on experimental results

As mentioned in Section 2.2, the nonlinear dynamics of the MEMS resonator is modeled as follows:

$$\frac{d^2x}{dt^2} + \frac{2\pi f_0}{Q} \frac{dx}{dt} + (2\pi f_0)^2 x + \alpha_3 x^3 = \frac{F_{\text{all}}}{m_e} \sin 2\pi f_n t, \quad (3.2.1)$$

where x denotes the displacement, f_0 ($= \sqrt{k_{s1}/m_e}/2\pi$) the resonance frequency, f_n the numerical excitation frequency, Q ($= \pi f_0 m_e / \gamma$) the quality factor, α_3 ($= k_{s3}/m_e$) the coefficient of cubic correction to the linear restoring force, F_{all} the excitation amplitude, and m_e the effective mass.

Using the experimental amplitude frequency response, the resonance frequency f_0 of the device is estimated as 8.6644 kHz. Fig. 3.9 shows the typical time evolution of the damped oscillation. The quality factor Q is approximated as around 25000 from

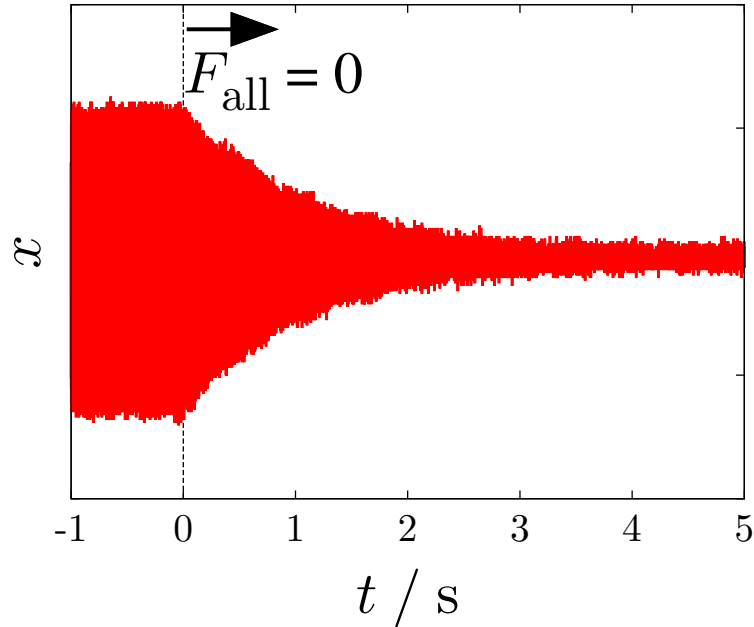


Figure 3.9: Time evolution of damped oscillation.

$Q = 2\pi f_0 \tau / 2$, where τ ($= 0.918$ s) denotes the time constant of the damped oscillation. The excitation force F_{all} is obtained by $6.906 \times 10^{-9} \text{ NV}^{-1} \times V_{\text{dcn}}$ as with Eq. (3.1.12), where V_{dcn} denotes the dc bias voltage. The value of the mass m_e is calculated from $m_e = F_{\text{all}} Q / 4\pi^2 f_0^2 A_{\text{max}} = 1.72 \times 10^{-9} \text{ kg}$, where A_{max} ($= 5.08 \mu\text{m}$) denotes the peak amplitude of the MEMS resonator. The numerical frequency response curves are fit to the measured data in Fig. 2(a), thereby obtaining $\alpha_3 = 7.06 \times 10^{16} (\text{sm})^{-2}$ as the numerically adjusted parameter.

3.2.2 Steady states

Figures 3.10 and 3.11 show numerical amplitude- and phase-frequency response curves generated from Eq. (3.2.1) at $V_{\text{dcn}} = 150 \text{ mV}$. The model of the MEMS resonator produces a hysteretic response. The red and aqua lines correspond to the responses at the upsweep and the downsweep of frequency, respectively. The solid (red and aqua) lines show two stable solutions and the dashed (green) line shows an unstable solution. At any given frequency in the hysteretic region, the MEMS resonator exhibits two coexisting stable states. These results show that the amplitude and phase of the displacement are obviously

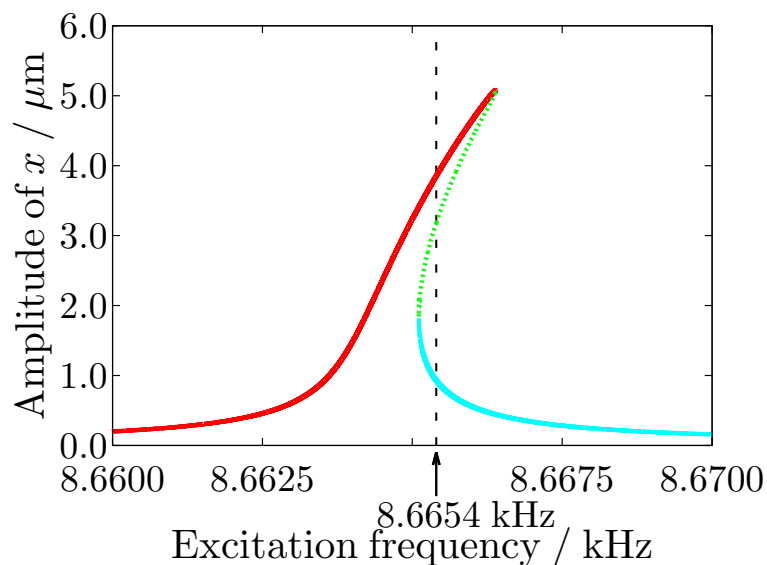


Figure 3.10: Numerical amplitude-frequency response curves generated from Eq. (3.2.1) for $V_{\text{dcn}} = 150 \text{ mV}$. The solid (red and aqua) lines show two stable solutions and the dashed (green) line shows an unstable solution.

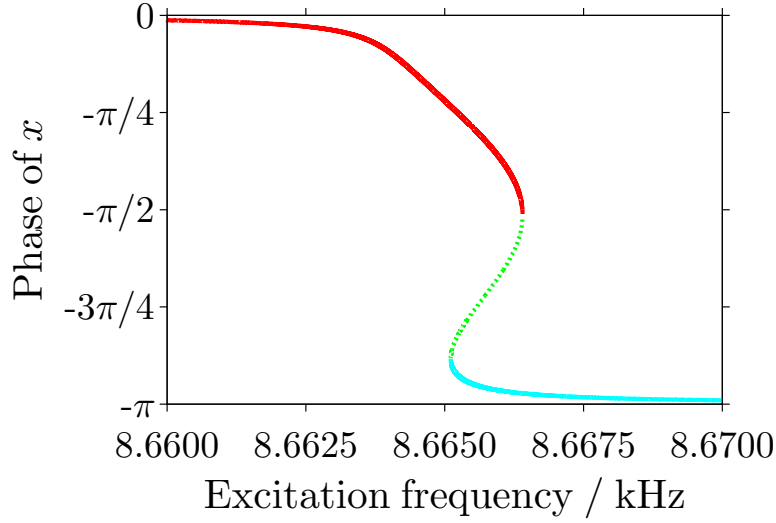


Figure 3.11: Numerical phase-frequency response curves generated from Eq. (3.2.1) for $V_{\text{dcn}} = 150 \text{ mV}$. The solid (red and aqua) lines show two stable solutions and the dashed (green) line shows an unstable solution.

distinguished. In addition, the numerically obtained results in Fig. 3.10 are qualitatively in agreement with experimental results in Fig. 3.6.

Figure 3.12 shows the time evolution of the two coexisting stable periodic vibrations at $f_n = 8.6654 \text{ kHz}$ and $V_{\text{dcn}} = 150 \text{ mV}$. In this figure, the red and aqua lines show the large and the small amplitude vibrations, respectively.

Figure 3.13 shows the numerically determined hysteretic behavior as a function of dc bias voltage V_{dcn} at 8.6654 kHz . The hysteresis region appears at $105 \text{ mV} < V_{\text{dcn}} < 245 \text{ mV}$ in Fig. 3.13. The nonlinear MEMS resonator has stable regions (solid line) that are completely separated by an unstable region (dashed line). These stable regions can be used as two states, corresponding to logical “0” and “1”, for memory function. In the numerical simulations, a displacement amplitude greater than $3.0 \mu\text{m}$ is regarded as a logical “1”; a value less than $3.0 \mu\text{m}$ is regarded as a logical “0” for memory output. As a result, we numerically demonstrate the read operation of memory device that consists of the MEMS resonator. The hysteresis region exists at $95 \text{ mV} < V_{\text{dce}} < 275 \text{ mV}$ in Fig. 3.8 and $105 \text{ mV} < V_{\text{dcn}} < 245 \text{ mV}$ in Fig. 3.13. In Section 3.2.3, the reason for the difference between experimental and numerical hysteresis results is discussed.

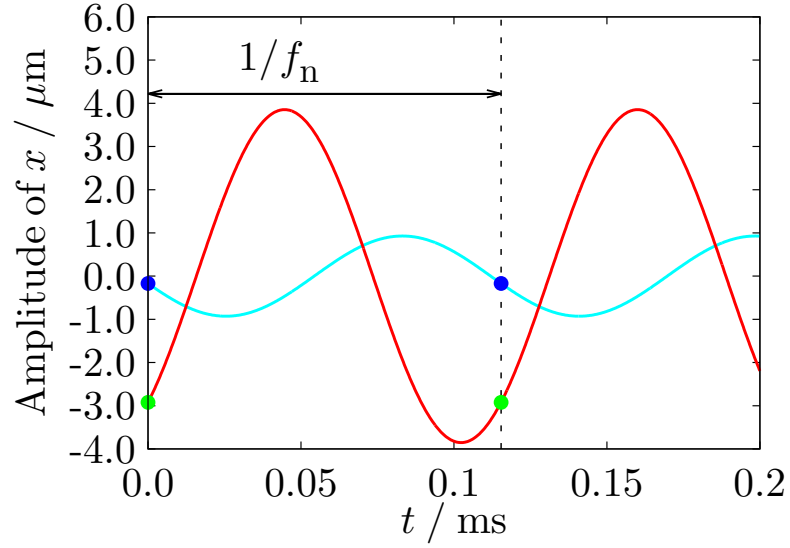


Figure 3.12: Two coexisting stable states at 8.6654 kHz.

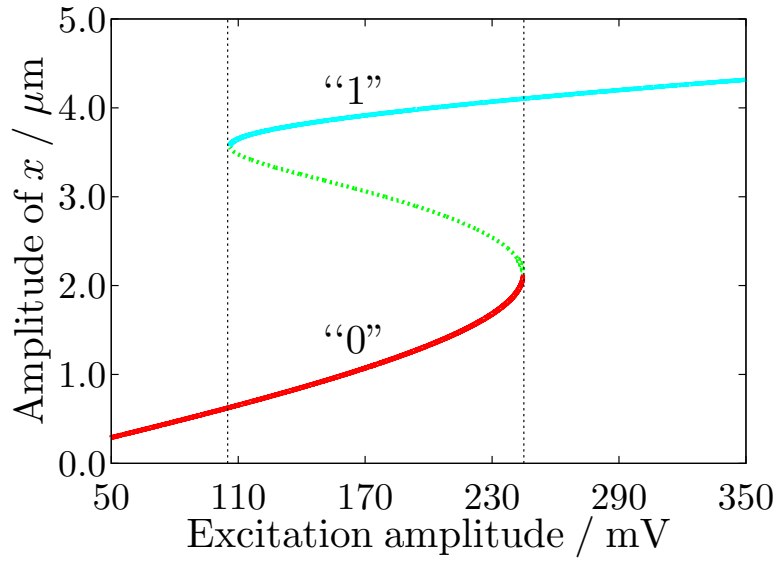


Figure 3.13: Numerical hysteretic characteristics as a function of dc bias voltage V_{dcn} at $f_n = 8.6654 \text{ kHz}$. The solid (red and aqua) lines show stable regions and the dashed (green) line shows an unstable region. These stable regions can be used as two states, corresponding to logical “0” and “1”, for memory function, as in Fig. 3.8.

3.2.3 Transient states

Here, in order to discuss the difference between experimental and numerical hysteresis results, the transient behavior is investigated by using numerical simulations under the low noise condition. The nonlinear dynamics of the MEMS resonator is described by the differential equations with the velocity y :

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -\frac{2\pi f_0}{Q} \frac{dx}{dt} - (2\pi f_0)^2 x - \alpha_3 x^3 + \frac{F_{\text{all}}}{m_e} \sin 2\pi f_n t. \end{cases} \quad (3.2.2)$$

As studied in Section 2.2.3, the set of initial conditions sampled by the period of the excitation frequency are considered as shown in Fig. 3.14. Here, the excitation frequency is fixed at $f_n = 8.6654 \text{ kHz}$. In the figure, the white and black regions show the basins of two stable solutions. The green and blue points correspond to the stroboscopic points at every period as shown by blue and green circles in Fig. 3.12. The green point shows the large amplitude solution and the blue point is the small amplitude solution. These two stable solutions possess their basins completely separated by stable manifolds of an unstable solution (aqua point). In white and black regions every initial state converges toward individual solutions. In Fig. 3.15, the red (aqua) line corresponds to the locus of

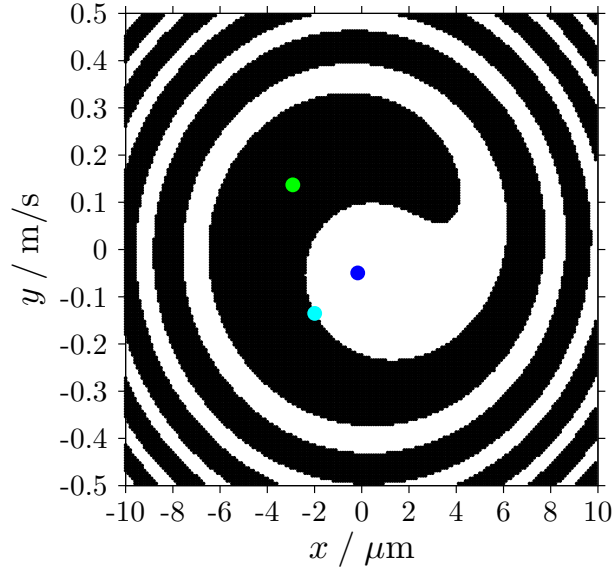


Figure 3.14: Basins of attraction at $f_n = 8.6654 \text{ kHz}$.

convergence to the large (small) amplitude solution in the xy plane.

Figures 3.16(a) and 3.16(b) show the set of initial conditions sampled by the period of each excitation frequency that corresponds to $f_n = 8.6652$ kHz and $f_n = 8.6656$ kHz, respectively. In the figures, the green, blue, and aqua points denote the stroboscopic

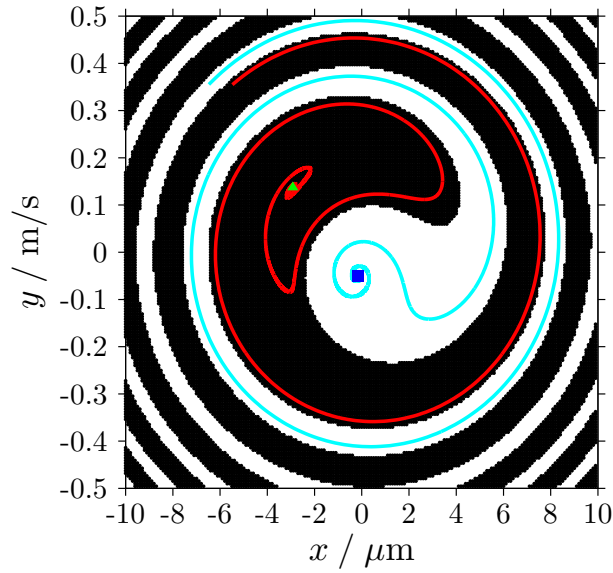


Figure 3.15: Basins of attraction at $f_n = 8.6654$ kHz. The red (aqua) line has the initial state in black (white) region.

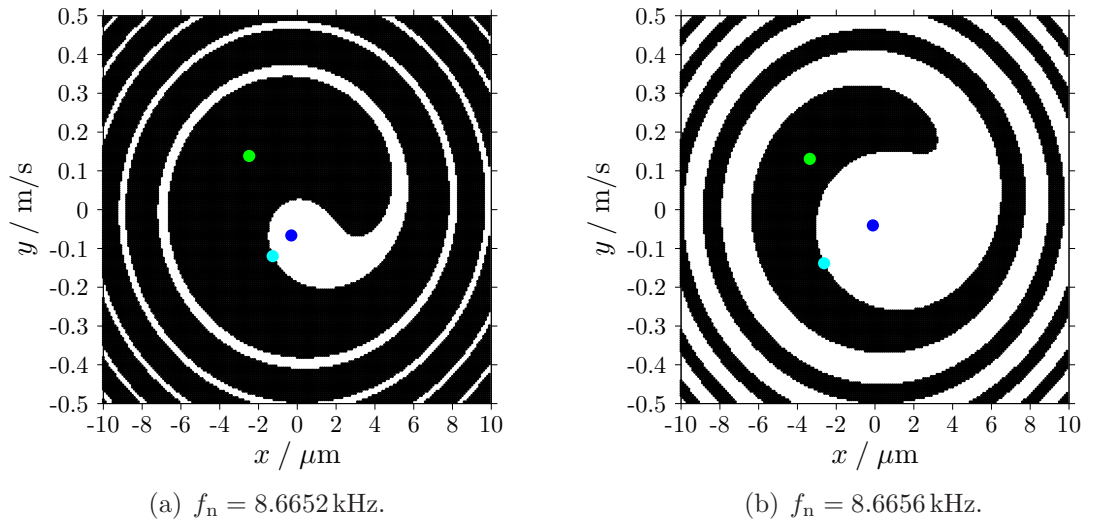


Figure 3.16: Basins of attraction.

points that correspond to the large amplitude, small amplitude, and unstable solutions at each excitation frequency, respectively. The white to black ratio depends on the excitation frequency as shown in Figs. 3.14 and 3.16. In other words, the white region expands at the upswing of frequency and vice versa. Here, as studied in Section 2.2.2, a stable state disappears through the saddle-node bifurcation in the quasi-static change. When the excitation frequency is shifted and then the initial state of resonator is perturbed by noise, it is anticipated that MEMS resonator does not maintain its original stable periodic vibration. Therefore, it is considered that the precise bifurcation points can not be measured in our experiments. It is considered that one of the causes of the different hysteresis regions is the difference of bifurcation points caused by noise. The precise bifurcation points in the experiments will be examined.

3.3 Summary

In this chapter, we numerically and experimentally demonstrate a read operation of a memory device that consists of a nonlinear MEMS resonator. It is theoretically shown that the excitation force exists at a single frequency and that the sum of the current through the capacitor depends on the displacement in the differential measurement. Based on these theoretical results, we have experimentally confirmed, without additional sensors except the current measurement, that two vibrations can be distinguished in the amplitude and the phase. The fabricated resonator has hysteretic characteristics depending on the excitation frequency and/or excitation force. As a result, the vibration displacement can be measured without additional sensors; therefore, the MEMS resonator is equipped with a comb drive that normally serves as a forcing actuator, but which simultaneously serves as a displacement sensor. Therefore, the read operation can be experimentally implemented by the self-sensing method. It is also cleared that the state information is kept through the measurement.

Next, through numerical simulations, we show the read operation in the MEMS resonator. Based on our experimental results, the parameters of the simulation model are obtained. It is shown by numerical method that the dynamical model of a single MEMS resonator exhibits hysteretic responses as a function of excitation frequency and/or excitation amplitude. By using transient analysis, it is also confirmed that the experimental noise is one of the causes of the difference between experimental and numerical results.

In the experiments, the precise bifurcation points will be examined.

Chapter 4

Write operation of memory device in MEMS resonator

Through the experiments and numerical simulations, this chapter focuses on the switching control between stable coexisting periodic vibrations in a MEMS resonator with hysteresis at a single excitation frequency. As mentioned in Section 2.3, the write operation implies a switching of two stable periodic vibrations in the nonlinear MEMS resonator.

Recently, Q. P. Unterreithmeier and co-workers realized a switching between two coexisting stable states in a nonlinear MEMS resonator at a constant excitation frequency [27]. They applied a radio-frequency (RF) pulse to control two stable states (See Appendix C for RF pulse control.). They clearly showed possibilities of switching states in their experiments. Their results motivated us to find other appropriate continuous control methods for the practical use of MEMS resonators based on feedback control.

In Section 4.1 and Section 4.2, the switching is experimentally and numerically performed between two coexisting stable states by a displacement feedback control as a write operation of the memory device in a MEMS resonator. Furthermore, in Section 4.3, we apply a delayed feedback control method (DFC) to switching as a write operation without exact solutions by numerical simulations.

4.1 Experimental study on displacement feedback control

4.1.1 Experimental method and setup

This section focuses on the switching control by a displacement feedback control at a constant excitation frequency as the write operation. It has already been explained in Section 3.1 that the output voltage V_{out} is returned as a proportional signal to the displacement by using the differential measurement. Therefore, based on the output voltage, we consider the switching by using the feedback control.

As explained in Section 3.1, the MEMS resonator has a comb-drive as a forcing actuator, which is simultaneously used as a displacement sensor. Here, we need to apply the control input to the MEMS resonator without any influence to measurement. Note that the excitation force F_{all} depends on both the ac excitation amplitude v_{ac} and the dc bias voltage V_{dce} , but the output voltage V_{out} only on the ac excitation amplitude v_{ac} , as given in Eqs. (3.1.7) and (3.1.11). Based on these features, the control input is applied as a slowly changed dc voltage to the MEMS resonator.

Figure 4.1 shows the switching control system based on displacement feedback control. The output ac voltage V_{out} is converted to the square average dc voltage V_{ave}^2 by an analog multiplier and a low pass filter of the operational amplifier as shown in Fig. 3.5. The control input u_{d} is given as a slowly changed dc voltage by

$$u_{\text{d}} = -V_{\text{ref}} + KV_{\text{ave}}^2, \quad (4.1.1)$$

where K ($= 5.1 \text{ V}^{-1}$) is the feedback gain and V_{ref} the external reference signal. The external reference signal V_{ref} is set at $K(V_{\text{ave}}^{\text{L}})^2$ ($K(V_{\text{ave}}^{\text{S}})^2$) when the state is requested to switch to a large (small) amplitude vibration. Here, $V_{\text{ave}}^{\text{L}}$ ($V_{\text{ave}}^{\text{S}}$) corresponds to the targeted average dc voltage for the large (small) amplitude vibration. As a result, the excitation force F_{all} with control input u_{d} is given by

$$F_{\text{all}} = 4\varepsilon N \frac{h}{d} (V_{\text{dc}} + u_{\text{d}}) v_{\text{ac}} \sin 2\pi f_{\text{e}} t, \quad (4.1.2)$$

$$= 4\varepsilon N \frac{h}{d} (V_{\text{dc}} - V_{\text{ref}} + KV_{\text{ave}}^2) v_{\text{ac}} \sin 2\pi f_{\text{e}} t. \quad (4.1.3)$$

Therefore, in this study, the switching speed depends not only on Q factor but also on the time constant of the low-pass filter.

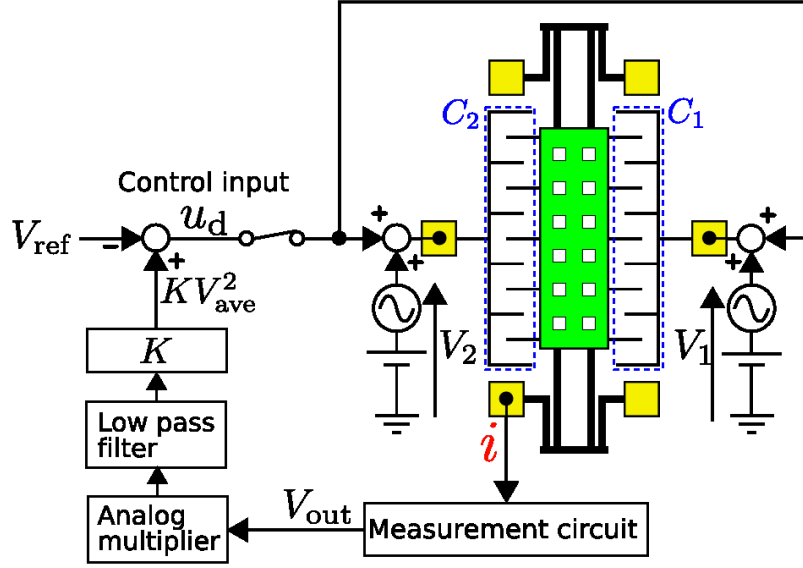


Figure 4.1: Control system for switching of two vibrations by feedback control in MEMS resonator.

Figure 4.2 shows the experimentally obtained frequency response curves of output voltage V_{out} . In Fig. 4.2, red and aqua lines show the response at upsweep and downsweep of frequency, respectively. In the following experiments in this section, the excitation frequency, the dc bias voltage, and the ac excitation amplitude are set at 9.0668 kHz, -0.15 V, and 0.7 V, respectively, in vacuum (≈ 10 Pa). Note that the MEMS resonators in Section 3.1 and this section are the same design on different chip dies.

4.1.2 Results and discussions

Figures 4.3(a) and 4.3(b) show the vibrations switched by the control input u_d . In these figures, aqua and purple lines correspond to the output voltage V_{out} and the control input u_d , respectively. The control input is applied at 1 s from the beginning of the oscillogram. Fig. 4.3(a) shows the result of the switching from small to large amplitude vibrations. It is found that the transition is slow from small to large amplitude vibrations. It takes around 5 s until the conversion. After the switching is completed, the control input u_d almost becomes null. Fig. 4.3(b) shows the result of the switching from large to small amplitude vibrations. It is observed that the converged state remains with a small

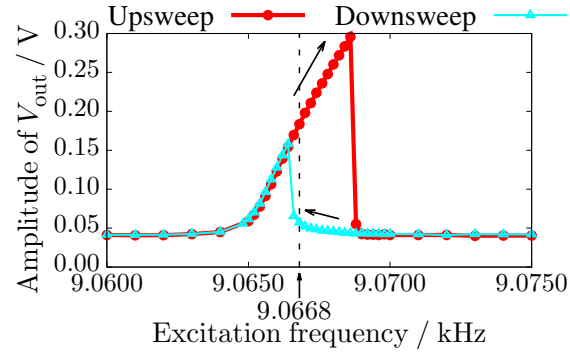
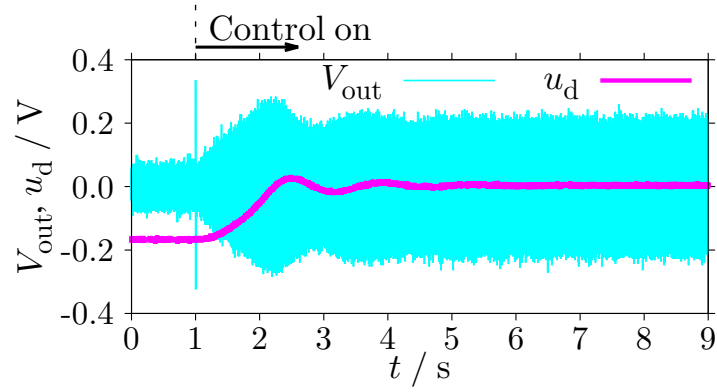
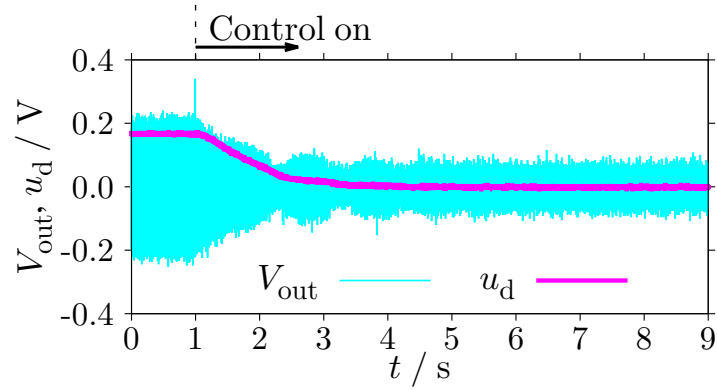


Figure 4.2: Displacement measurement by differential configuration.



(a) Switching from small to large amplitude vibration.



(b) Switching from large to small amplitude vibration.

Figure 4.3: Switching between two stable periodic vibrations.

amplitude periodic vibration in the comb-drive resonator. The switching is completed after 4 s and the control input u_d almost disappears. At the onset of the control (1 s), the output voltage V_{out} shows a surge each in Figs. 4.3(a) and 4.3(b). In addition, the electrical noise here appears. However, there does not happen any fault of switching operation. As a result, we surely realize the write operation from “0” to “1” and “1” to “0”.

4.2 Numerical study on displacement feedback control

Through the numerical simulations, this section reports the transient behavior in the switching as the write operation. When the control input is applied to the system, the basin, which is obtained by estimating the convergence without the control input, has no practical meaning for understanding their response. Therefore, the trajectories of the operating point under control are considered for the estimation of the ability of control.

4.2.1 Numerical method

Based on Eq. (3.2.2), the dynamical model of the MEMS resonator is given by the following non-dimensional differential equations with the displacement x , the velocity y , and the control input u :

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -\frac{y}{Q} - x - \alpha_3 x^3 + (k + u) \sin \omega t, \end{cases} \quad (4.2.1)$$

where ω ($= 1.02$) denotes the excitation frequency, Q ($= 282$) the quality factor, α_3 ($= 3.23$) the coefficient of cubic correction to the linear restoring force, and k ($= 0.001$) the amplitude of the excitation force. The parameter settings are due to Ref. [43] because the same device is assigned as shown in Fig. 2.1.

4.2.2 Results and discussions

Figure 4.4 shows the control system in numerical simulations. According to the experimental control method as shown in Section 4.1, the numerical control signal u is given

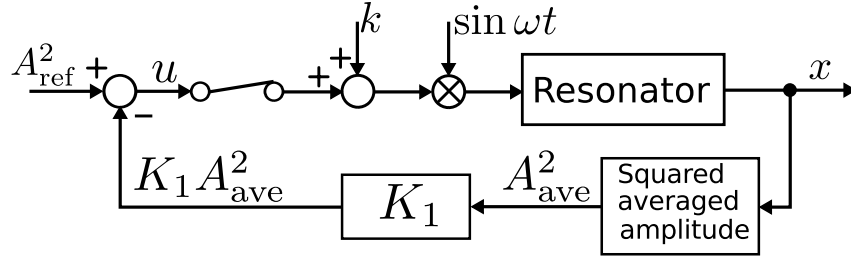


Figure 4.4: Control system in numerical simulations.

as the following equations:

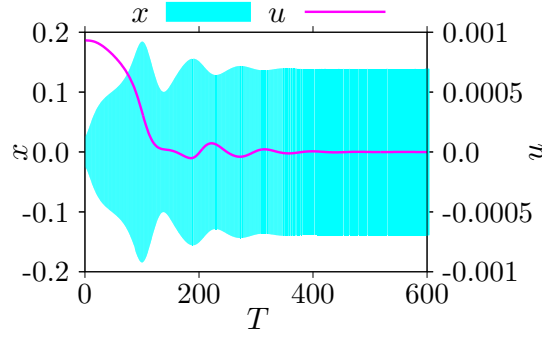
$$u = A_{\text{ref}}^2 - K_1 A_{\text{ave}}^2, \quad (4.2.2)$$

$$A_{\text{ave}}^2 = \frac{A_1^2 + A_2^2 + \cdots + A_m^2 + \cdots + A_M^2}{M}, \quad (4.2.3)$$

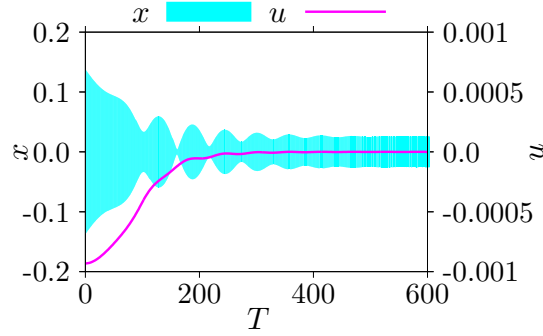
where K_1 denotes the feedback gain, A_{ref}^2 the external reference signal of squared amplitude, m natural number, M the average number, and A_m the displacement amplitude of the previous m periods within $1 \leq m \leq M$. Then, A_{ave}^2 is the average of A_m^2 . We set M at 100. When the state is switched to the large and small amplitude vibration, the external reference signal A_{ref}^2 is set at $K_1(A_{\text{ave}}^{\text{L}})^2$ and $K_1(A_{\text{ave}}^{\text{S}})^2$, respectively. Here, $A_{\text{ave}}^{\text{L}}$ and $A_{\text{ave}}^{\text{S}}$ show the target of amplitude to the large and small vibrations, respectively.

Figure 4.5(a) (4.5(b)) shows the time evolution from small (large) to large (small) vibration at $K_1 = 0.05$. In these figures, the purple and aqua plots correspond to the control signal and the displacement, respectively. These results show that we achieve the switching between two coexisting stable states. Note that the control input disappears after the switching is completed. We find close similarity of numerical results in Figs. 4.5(a) and 4.5(b) to experiments in Fig. 4.3.

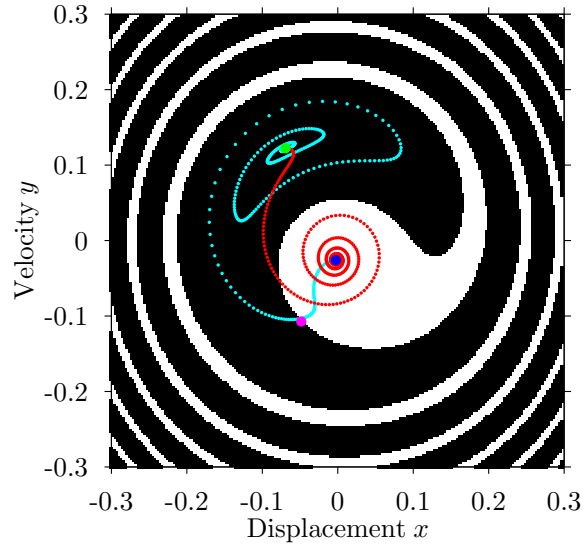
Figure 4.5(c) shows the obtained trajectories at $K_1 = 0.05$ in the xy plane. The aqua and red points imply the loci of stroboscopic states from small to large vibrations and vice versa. As mentioned in Section 3.2.3, in the figure, the white and black regions show the basins of two stable solutions without control signal. It should be noted that the states slowly transfer from small to large amplitude solutions and from large to small. These loci of stroboscopic states appear along unstable manifolds of an unstable saddle. Therefore, the feedback gain should be sufficiently high for the loci of stroboscopic states to pass across the unstable manifolds. Fig. 4.5(c) shows that there exist the points of intersection of these loci. The active MEMS memory may allow the rewrite operation when the vibrating state is the transient state near these points. It will be a new research



(a) Time evolution of switching from small to large amplitude vibration.



(b) Time evolution of switching from large to small amplitude vibration.



(c) Trajectories under control in xy plane. The green point shows the large amplitude solution and the blue the small solution without control signal.

Figure 4.5: Switching between two stable periodic vibrations at $K_1 = 0.05$ (numerical results).

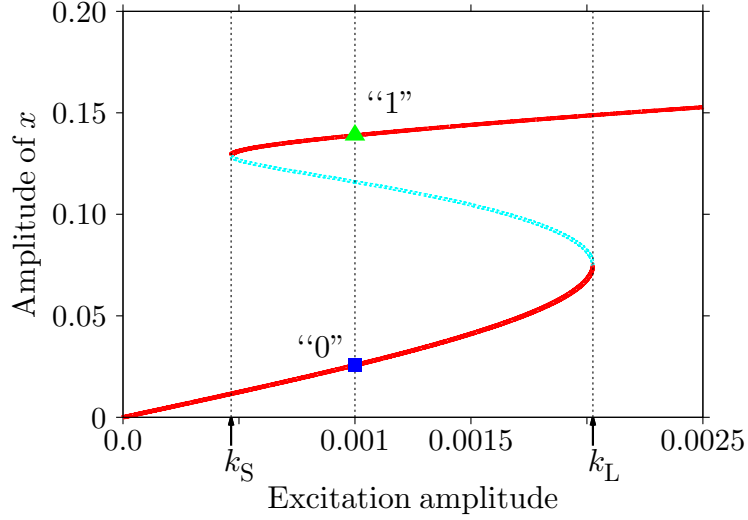


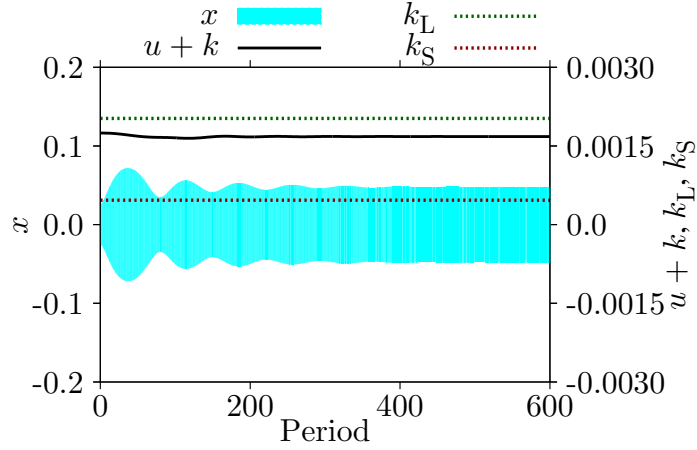
Figure 4.6: Hysteretic characteristics with respect to the excitation amplitude k at $\omega = 1.02$. Red lines show the stable states and aqua line the unstable state. Saddle-node bifurcations are denoted by k_S and k_L in the quasi-static change.

target. Consequently, we numerically demonstrate the write operation from “0” to “1” and from “1” to “0”.

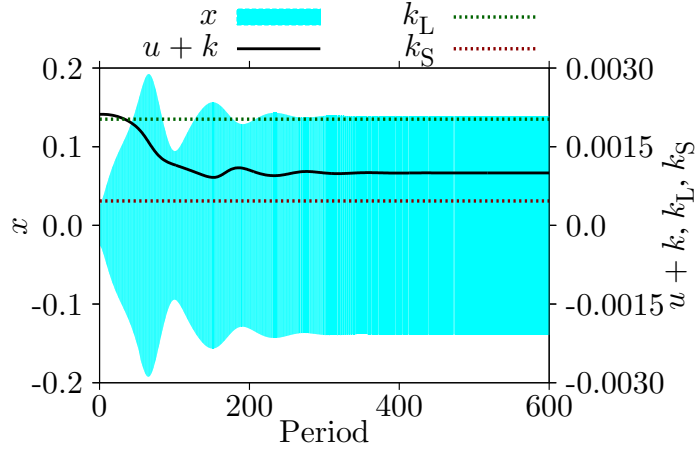
4.2.3 Gain dependence

This section focuses on the capability of the switching control between two stable periodic vibrations when the control input is applied to the MEMS resonator. Here, the gain dependence of the switching control is investigated. As shown in Fig. 4.6, a stable state disappears through the saddle-node bifurcation, denoted by k_S and k_L , in the quasi-static change.

Figures 4.7(a) and (b) show the switching from small amplitude solution at $K_1 = 0.04$ and at $K_1 = 0.06$, respectively. As shown in Fig. 4.7(a), the switching control does not work at $K_1 = 0.04$. At $K_1 = 0.04$, there still remains the control input when the vibration converges to the steady state. On the other hand, the switching control is achieved at $K_1 = 0.06$ as shown in Fig. 4.7(b). At $K_1 = 0.06$, the control input disappears after transient states is over. Note that the sum of the excitation force and the control input $u + k$ is less (more) than k_L at $K_1 = 0.04$ ($K_1 = 0.06$). It is considered that we could achieve the switching control from small to large amplitude states when $u + k$ exceeds the value around k_L , defined by k'_L , at the beginning of the control.



(a) $K_1 = 0.04$.

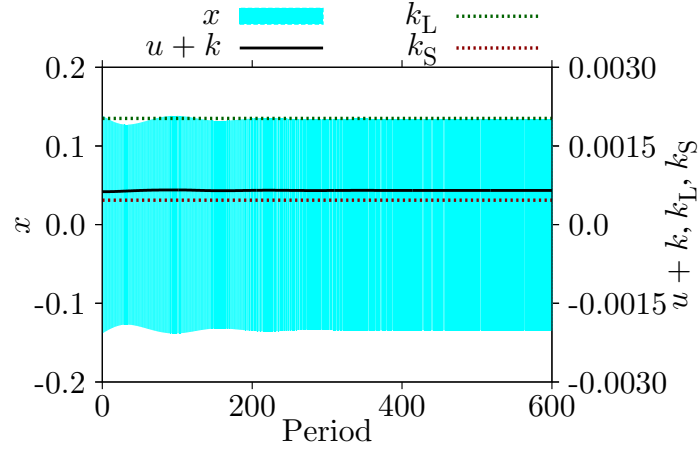


(b) $K_1 = 0.06$.

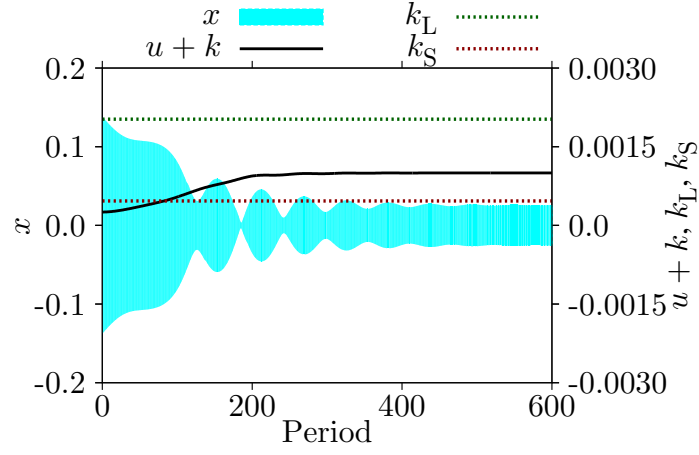
Figure 4.7: Switching control from small to large amplitude vibrations.

Figures 4.8(a) and (b) are obtained at the switching from large amplitude solution at $K_1 = 0.02$ and at $K_1 = 0.04$, respectively. At $K_1 = 0.02$, there exists the steady state near the initial state as shown in Fig. 4.8(a). Fig. 4.8(b) shows that the states change from large to small amplitude solutions. These results suggest that the switching control from large to small amplitude states can be realized when $u + k$ becomes less than the value around k_S , defined by k'_S , at the onset of the control.

In order to realize the switching between two states, we need to satisfy requirements as shown in Tab. 4.1. There possibly happens a fault of the switching control when the state of the resonator is perturbed by noises at the onset of the control.



(a) $K_1 = 0.02$.



(b) $K_1 = 0.04$.

Figure 4.8: Switching control from large to small amplitude vibrations.

Figure 4.9(a) (4.9(b)) illustrates the controlled result from small (large) solution at $K_1 = 0.02$. These results show that the switching control does not work satisfactorily at $K_1 = 0.02$. It is noted that there still remains the control input when the vibrations

Table 4.1: Requirements for the switching.

Switching results	Resonator $u + k$
"0" $\rightarrow$ "1"	$u + k > k'_L$
"1" $\rightarrow$ "0"	$u + k < k'_S$

converge to steady states. Figure 4.9(c) shows the controlled results at $K_1 = 0.02$ in the xy plane. Note that there exist the steady states near initial states when the control input is applied. When the control input is stopped in the MEMS resonator, the initial state cannot switch to the other stable state and converges to the original stable state due to the basins of attraction. As a result, when requirements shown in Tab. 4.1 are not satisfied, the states stay at “1” and “0” as shown in Fig. 4.9.

4.3 Numerical write operation without exact solutions

In this section, through numerical simulations, a write operation without exact solutions is discussed in the nonlinear MEMS resonator. As mentioned in previous sections, the displacement feedback control needs the external reference signal. Here, we apply a delayed feedback control method [48] to switching without any external reference as the write operation without exact solutions.

K. Pyragas proposed a delayed feedback control (DFC) that could be used for controlling chaos [66] in continuous-time system. It has been experimentally [67–69], numerically [70, 71], and theoretically [71] confirmed that DFC can stabilize unstable periodic orbits embedded in the chaotic attractor. Y. Yokoi and T. Hikihara have reported that DFC can be used as the start-up control of rotation in parametric pendulum [72, 73]. In particular, DFC confirmed successful experimental results in micro mechanical fields [68]. DFC uses time delay outputs.

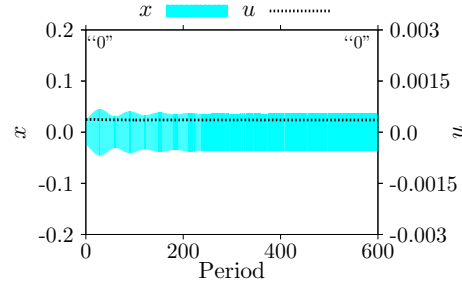
The switching between two stable states by DFC has three advantages. The first advantage of such a switching is a continuous closed loop control method. The second advantage is that the control method does not need external reference. Finally, the advantage is the disappearance of the control signal after the switching.

In the following, the idea of control is confirmed by numerical simulations.

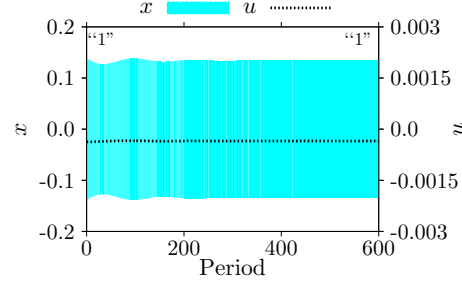
4.3.1 Numerical method

Based on Eq. (4.2.1), the system with control signal $u(t)$ is given by

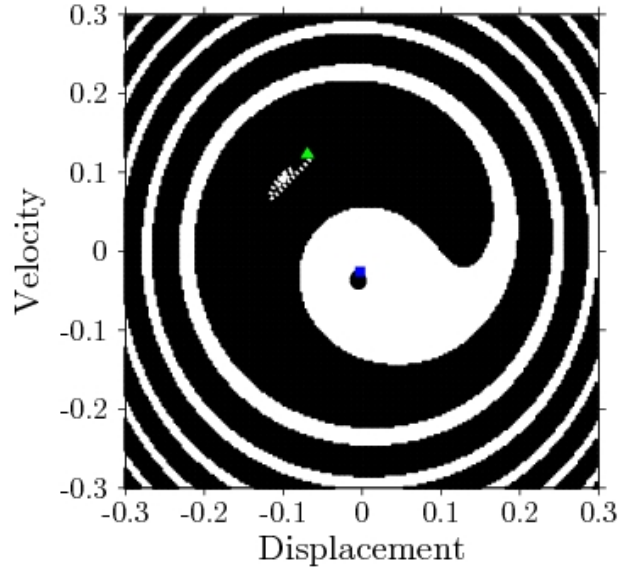
$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -\frac{y}{Q} - x - \alpha_3 x^3 + k \cos \omega t + u(t). \end{cases} \quad (4.3.1)$$



(a) Time evolution of switching from small amplitude vibration.



(b) Time evolution of switching from large amplitude vibration.



(c) Trajectories under control in xy plane. The solid and dotted lines show the loci of map from small and large amplitude vibrations, respectively.

Figure 4.9: Switching control between two stable periodic vibrations at $K_1 = 0.02$ (numerical results).

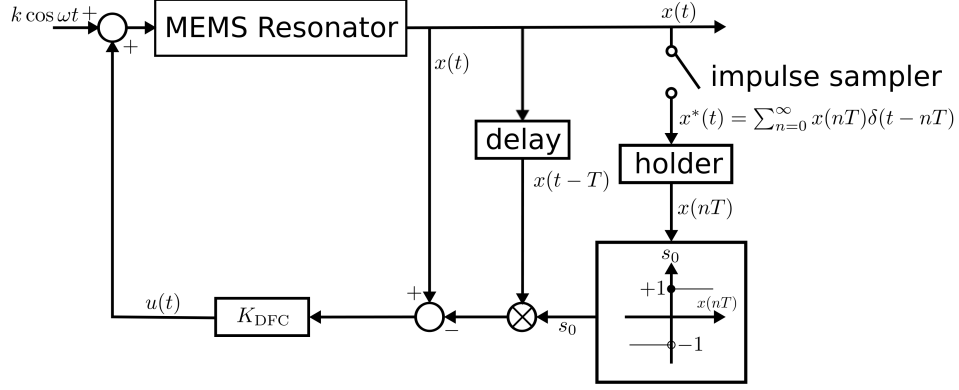


Figure 4.10: Switching system by using DFC from small to large amplitude vibrations.

Here the excitation frequency ω is set to 1.025. Note that the excitation force is given as the cosine wave. Note also that the control input is not added to the excitation amplitude. Therefore, we will examine the switching control method without any influence to measurement in our experimental system as explained in Section 4.1.

DFC employs the control signal [48] as

$$u(t) = K_{\text{DFC}}\{x(t) - x(t - T)\}, \quad (4.3.2)$$

where $T (= 2\pi/\omega)$ denotes the period and K_{DFC} is the feedback gain. As shown in Eq. (4.3.2), when the vibration in the MEMS resonator corresponds to one of two stable states, there does not appear the control input due to $x(t) = x(t - T)$. Therefore, the control input in Eq. (4.3.2) can not be applied to the switching between two states. Here, we give the direction to the control signal based on the difference between small and large amplitude vibrations.

In order to realize the switching by DFC, this study focuses on the phase difference. As shown in Fig. 3.11, the phase region of the small (large) amplitude vibration exists at $-\pi/2 > \phi > -\pi$ ($0 > \phi > -\pi/2$), where ϕ is the displacement phase. When the excitation force is given as the cosine wave, the stroboscopic point of the displacement has a positive (negative) value at $-\pi/2 > \phi > -\pi$ ($0 > \phi > -\pi/2$) [74].

Based on the above discussions, the direction to the control signal is given by s_g function. Fig. 4.10 (4.11) shows the switching system from small (large) to large (small) amplitude vibrations by using DFC. The control signal for the switching by DFC is

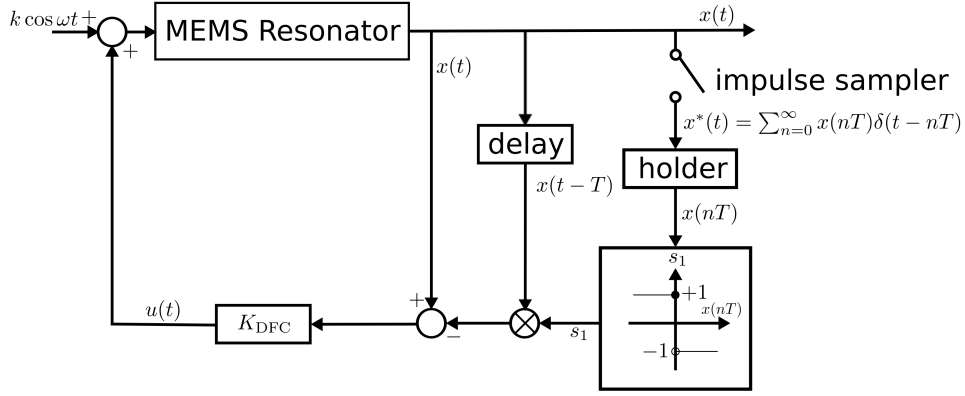


Figure 4.11: Switching system by using DFC from large to small amplitude vibrations.

obtained by

$$u(t) = K_{DFC}\{x(t) - s_g(x(nT)) \cdot x(t - T)\}, \quad (4.3.3)$$

where $g = 0, 1$. Here n is natural number at $t \in [nT, (n+1)T)$. $s_0(x(nT))$ is defined by

$$s_0(x(nT)) \equiv \begin{cases} +1 & (x(nT) \geq 0), \\ -1 & (x(nT) < 0), \end{cases} \quad (4.3.4)$$

when the states are requested to come towards the large amplitude vibration as shown in Fig. 4.10. When the state is requested to switch to the large amplitude vibration as shown in Fig. 4.11, $s_1(x(nT))$ is defined by the following equations:

$$s_1(x(nT)) \equiv \begin{cases} +1 & (x(nT) \leq 0), \\ -1 & (x(nT) > 0). \end{cases} \quad (4.3.5)$$

4.3.2 Numerical results and discussions

Figure 4.12 shows the results controlled by DFC in the xy plane. In this figure, the feedback gain K_{DFC} is set to -0.02 . The sequences of the red and aqua points show the loci of map with DFC from small to large and from large to small amplitude vibrations, respectively. It should be noted that the states slowly transfer from small to large or from large to small amplitude vibrations. It takes around 1500 (13800) periods before converging to the small (large) amplitude vibration.

DFC is a continuous closed loop control method. Here, we investigate the initial value dependence of the convergence. Fig. 4.13 shows the calculated results. In Fig. 4.13(a), yellow grid points indicate the initial states that converge to the large amplitude vibration.

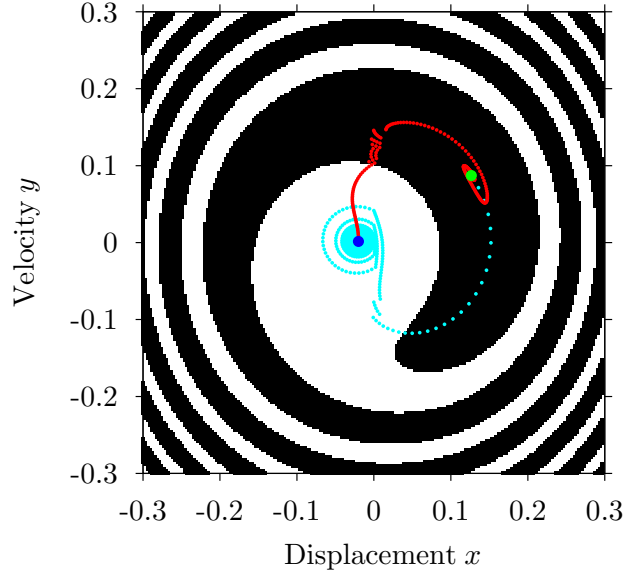


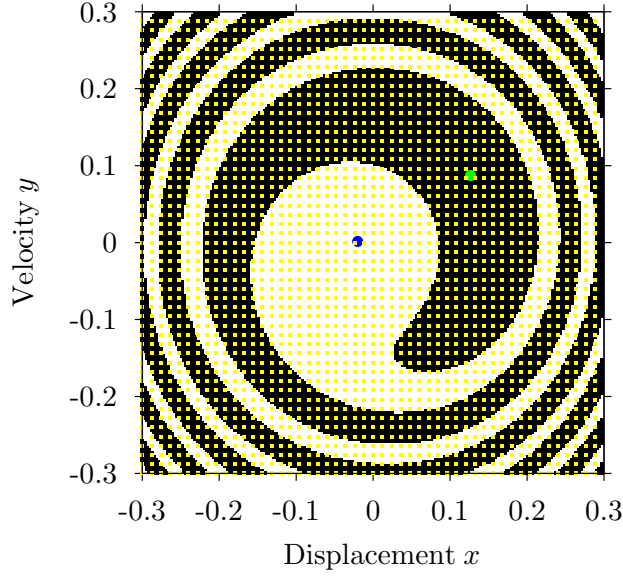
Figure 4.12: Switching between stable periodic vibrations at $\omega = 1.025$ and $K_{\text{DFC}} = -0.02$ by DFC: The basins of attraction corresponding to the two states are separated by the manifold of the unstable solutions. The red and aqua points show the switching from small to large and from large to small amplitude vibrations, respectively.

In the initial state plane, DFC can control the vibration to the large amplitude vibration without exception. In Fig. 4.13(b), red grid points indicate the initial states of convergence to the small amplitude vibration. There still remain the uncontrollable initial states.

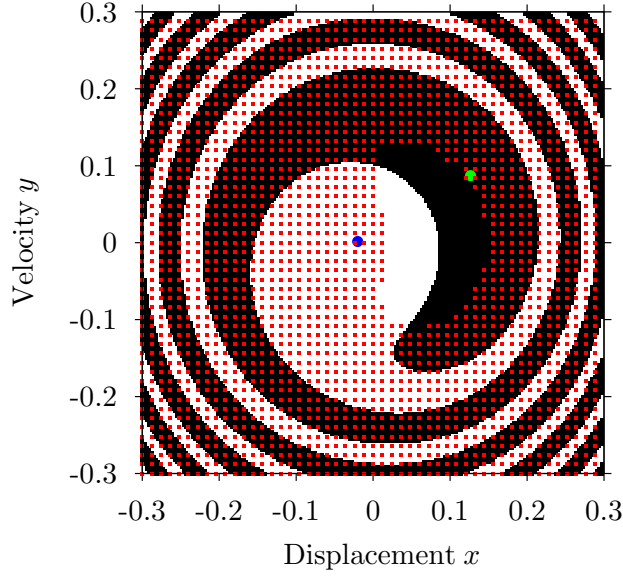
Figures 4.14 and 4.15 depict the time evolution of the control signal from small to large and large to small amplitude vibrations, respectively. These results show that the control signal of DFC disappears after the establishment of switching. It is confirmed that, based on the stroboscopic points depending on the displacement phase, the s_g function is changed. It is implied that the converged state is exactly one of the stable periodic vibrations in the resonator.

Figure 4.16 shows the gain dependence of the switching control by DFC. In Figs. 4.16(a), 4.16(b), and 4.16(c), each feedback gain is set to -0.03 , -0.01 , and 0.02 , respectively. In these figures, the red (aqua) points show the controlled result from small (large) amplitude solution. These results show that the switching control by DFC strongly depends on the feedback gain. Therefore, the gain dependence of the switching control by DFC will be investigated in more details.

The switching between two states is numerically achieved when the direction to the



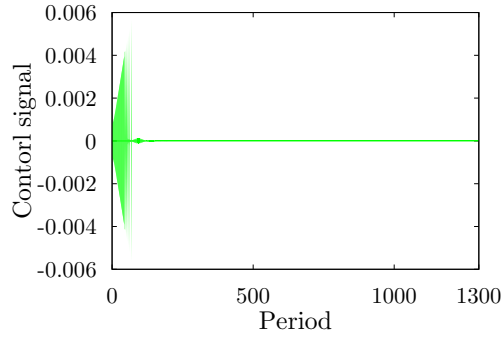
(a) Region of convergence to large amplitude vibration at $\omega = 1.025$ and $K_{\text{DFC}} = -0.02$.



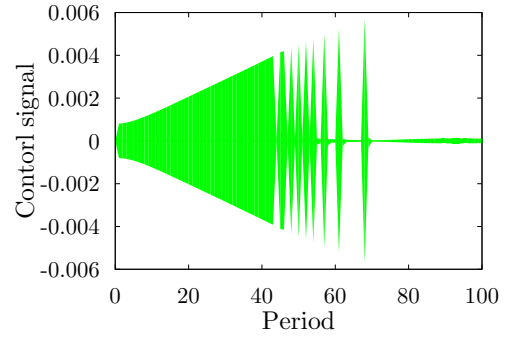
(b) Region of convergence to small amplitude vibration at $\omega = 1.025$ and $K_{\text{DFC}} = -0.02$.

Figure 4.13: Initial value dependence of convergence by DFC: Yellow and Red grid points show the region of convergence to large and small amplitude vibrations, respectively.

control signal of DFC is given by s_g function. The switching by DFC does not need the external reference signal. We numerically confirm the write operation without exact

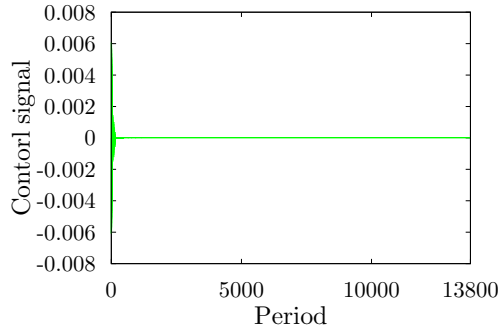


(a) Control signal.

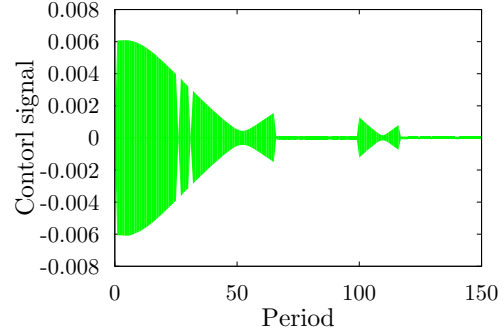


(b) Magnified view of Fig. 4.14(a).

Figure 4.14: Time evolution of control signal from small to large amplitude vibrations at $\omega = 1.025$ and $K_{DFC} = -0.02$.



(a) Control signal.



(b) Magnified view of Fig. 4.15(a).

Figure 4.15: Time evolution of control signal from small to large amplitude vibrations at $\omega = 1.025$ and $K_{DFC} = -0.02$.

solutions by using DFC. However, the switching system in DFC becomes complex in comparison with the displacement feedback control.

4.4 Summary

In this chapter, based on both experiments and numerical simulations, the switching between two coexisting stable states is done at a single excitation frequency as the write operation of memory device that consists of the nonlinear MEMS resonator.

Based on the displacement measurement as shown in Section 3.1, the switching between two stable periodic vibrations is achieved. The dc voltage bias is added to the

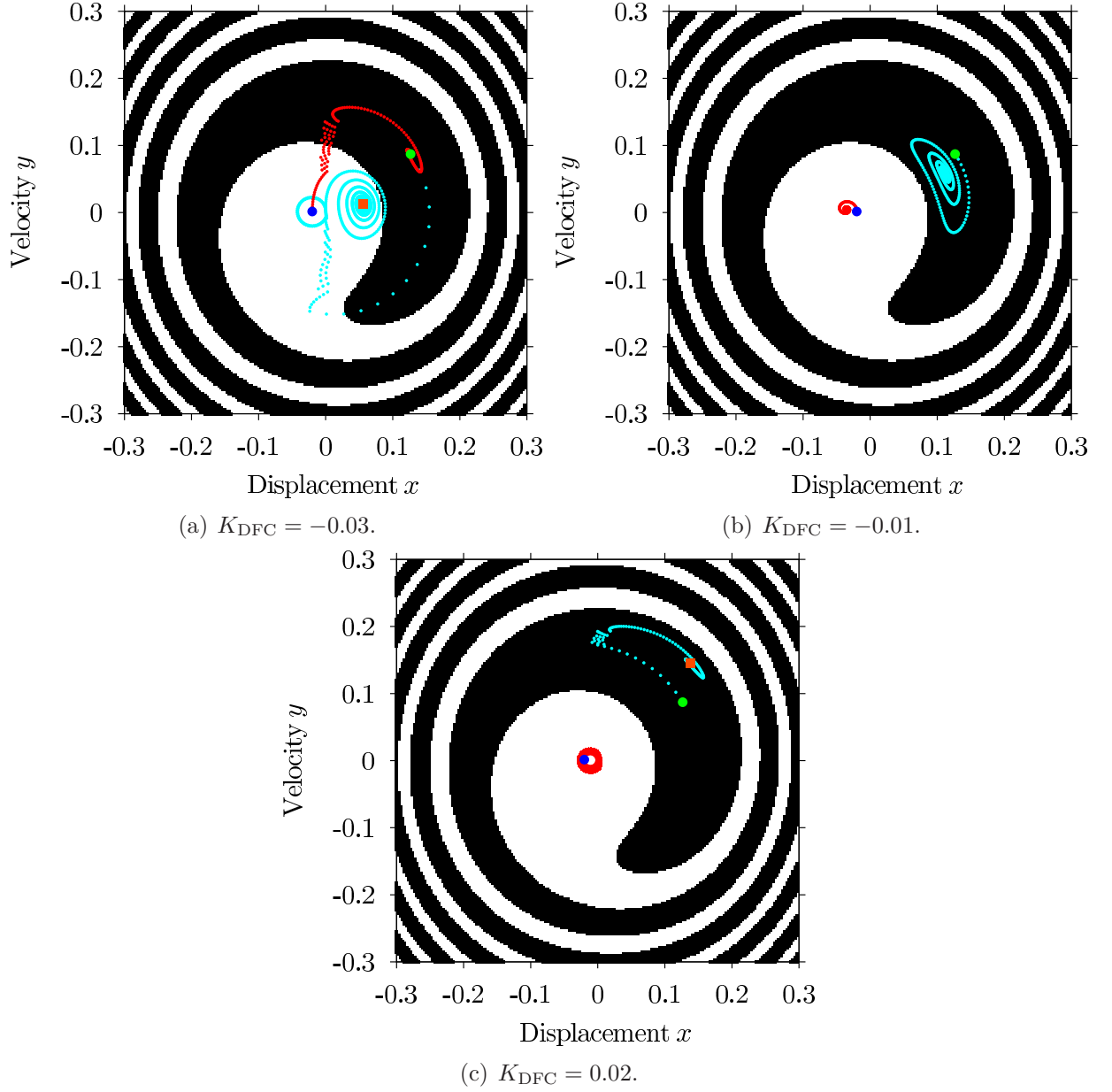


Figure 4.16: Switching between stable periodic vibrations with respect to gain at $\omega = 1.025$ by DFC.

excitation voltage for avoiding the influence to measurement. After the switching is completed, the control input becomes null. As a result, the switching operation is experimentally achieved between two coexisting stable states by the displacement feedback control at a constant excitation frequency in the hysteresis.

Next, the switching control between two states is numerically demonstrated. Further-

more, the transient behavior is numerically investigated in the write operation. When the transition of state is performed, we need to consider the system depending not only on the excitation force but also on the control input.

Finally, it is numerically confirmed that the switching control without any external reference by using DFC as the write operation without exact solutions in the nonlinear MEMS resonator. In a nonlinear MEMS resonator, when the direction to the control signal of DFC is given, the switching between two states is numerically demonstrated. The availability of the delayed feedback control is numerically confirmed. The control input disappears after the switching is completed. The dependence of initial conditions on the convergence is investigated. These results show that DFC is effective for application. However, in DFC and displacement feedback control, there still remains the precise gain dependence of the switching control for further investigation. The experimental success of the switching of two stable periodic vibrations will be realized by using DFC.

Chapter 5

Counter operation in coupled MEMS resonators

This chapter discusses a 2-bit binary counter [61] from the viewpoint of application of nonlinear dynamics in coupled MEMS memories. Previous studies address that a micro-electromechanical or nanoelectricalmechanical resonator can be utilized as a mechanical 1-bit memory [21, 24, 26, 27, 30, 31, 34] or mechanical logic gates [23, 25, 28, 29]. The next phase is the development of a sequential logic device such as a counter with coupled multi-resonators. From the viewpoint of application of nonlinear dynamics in coupled MEMS resonators, Section 5.1 and Section 5.2 show the experimental and numerical successes of the controlling nonlinear behavior as a 2-bit binary counter.

5.1 Experimental study on 2-bit binary counter

The overview of this section is organized as follows. Section 5.1.1 presents two fabricated MEMS resonators and its four coexisting stable states. Section 5.1.2 explains the switching control system in the coupled nonlinear MEMS resonators. In Section 5.1.3, the switching control sequence of counter operation is experimentally achieved and examined.

5.1.1 Two MEMS resonators

By using a differential measurement as shown in Section 3.1, the vibrations of two MEMS resonators are measured. In the differential measurement for a single MEMS resonator, excitation force $F_{\text{all}j}$ and output voltage $V_{\text{out}j}$ are obtained by the following

equations:

$$F_{\text{all}j} = 4\varepsilon N \frac{h}{d} V_{\text{dc}j} v_{\text{ac}j} \sin 2\pi f_j t, \quad (5.1.1)$$

$$V_{\text{out}j} = 8 \times 10^8 \pi f_j \varepsilon N \frac{h}{d} v_{\text{ac}j} A_j \sin(4\pi f_j t + \phi_j), \quad (5.1.2)$$

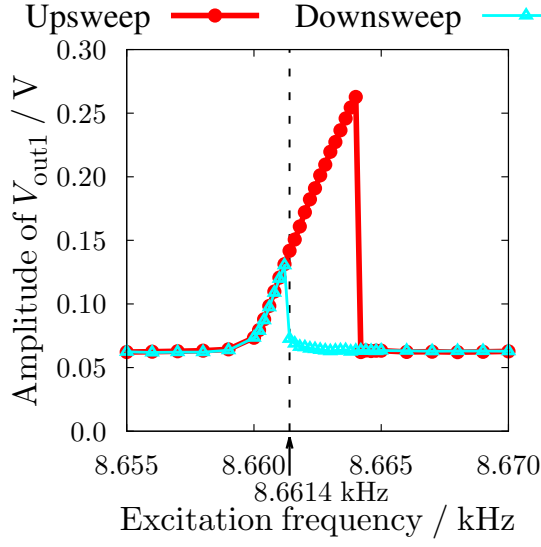
where $j = 1, 2$. Here f_j denotes the excitation frequency of the j -th MEMS resonator in the coupled system, A_j the amplitude of the displacement, and ϕ_j the phase. The device parameters have been shown in Tab. 3.1. In this section, the dc bias voltage $V_{\text{dc}j}$, the ac excitation amplitude $v_{\text{ac}j}$, and the pressure are set at -0.15 V, 0.6 V, and around 12 Pa at room temperature. The first (second) MEMS resonator is called Res. 1 (Res. 2) from here on.

Figure 5.1(a) (5.2(a)) shows the experimentally obtained frequency response curves in Res. 1 (Res. 2). The red and aqua lines correspond to the responses at the upswing and the downswing of frequency, respectively. The frequency response curves strongly depend on the sweep direction in the hysteresis region. We find that the behavior exhibited by the MEMS resonator qualitatively resembles to each other. Two stable states coexist at $8.6612 \text{ kHz} < f_1 < 8.6642 \text{ kHz}$ in Fig. 5.1(a) and at $8.6134 \text{ kHz} < f_2 < 8.6162 \text{ kHz}$ in Fig. 5.2(a). It is considered that the difference of hysteresis is caused by different doping angle, debris deposited during fabrication and die separation, and/or minute cracks [50].

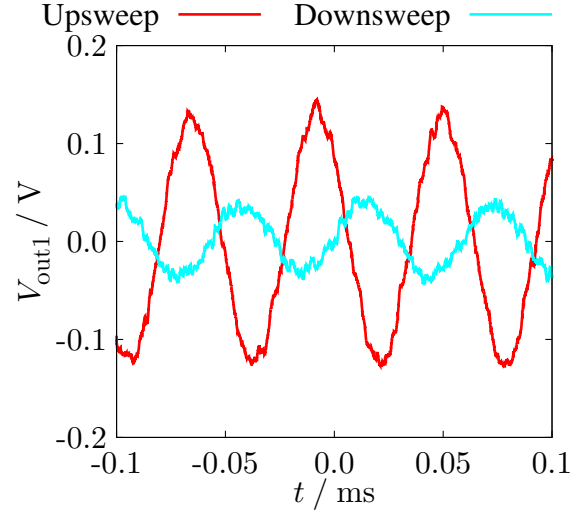
Figures 5.1(b) and 5.2(b) show the oscillogram of two stable periodic vibrations at 8.6614 kHz in Res. 1 and at 8.6136 kHz in Res. 2. The red and aqua lines are averaged out over 32 measurements. In the following experiments in this section, the excitation frequency is fixed at 8.6614 kHz in Res. 1 and at 8.6136 kHz in Res. 2. The large (small) amplitude vibration is regarded as the “1” (“0”) state for each resonator. In addition, Res. 1 holds the first bit and Res. 2 the second bit.

5.1.2 Experimental switching method

Figure 5.3 shows the switching control system. A binary counter is a sequential system that goes through a prescribed sequence of states upon the application of clock signals [61]. In a binary counter, the output transition of MEMS resonator (except the first) serves as a source for triggering other MEMS resonator. The 2-bit binary counter consists of a series connection of two MEMS resonators, with the output of the first resonator connected to the input of the second resonator. Here, two MEMS resonators are interconnected in a unidirectional coupled system. Although the electrical noise here appears due to electrical

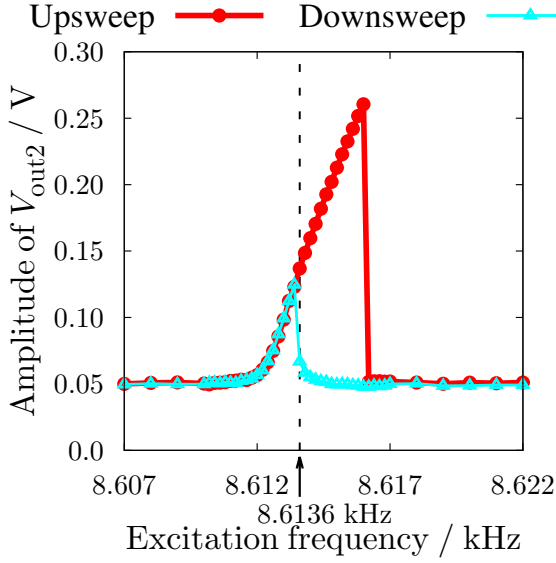


(a) Amplitude-frequency response curves of Res. 1.

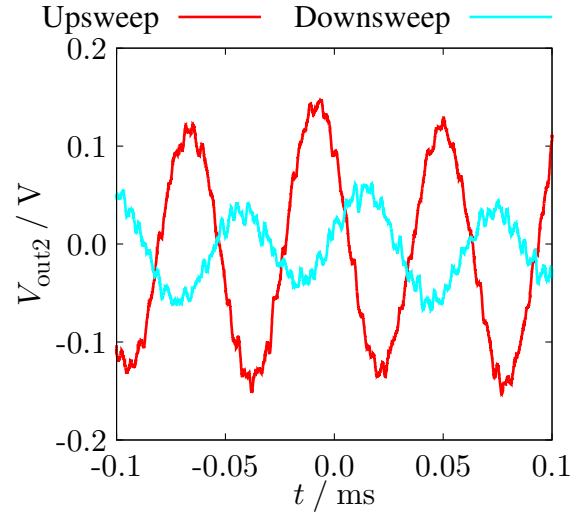


(b) Two coexisting stable states at $f_1 = 8.6614$ kHz of Res. 1.

Figure 5.1: Displacement measurement by differential configuration in Res. 1. At any given frequency in the hysteretic regime, the resonator can exist in two distinct amplitude states.



(a) Amplitude-frequency response curves of Res. 2.



(b) Two coexisting stable states at $f_2 = 8.6136$ kHz of Res. 2.

Figure 5.2: Displacement measurement by differential configuration in Res. 2. Two different vibrational states coexist.

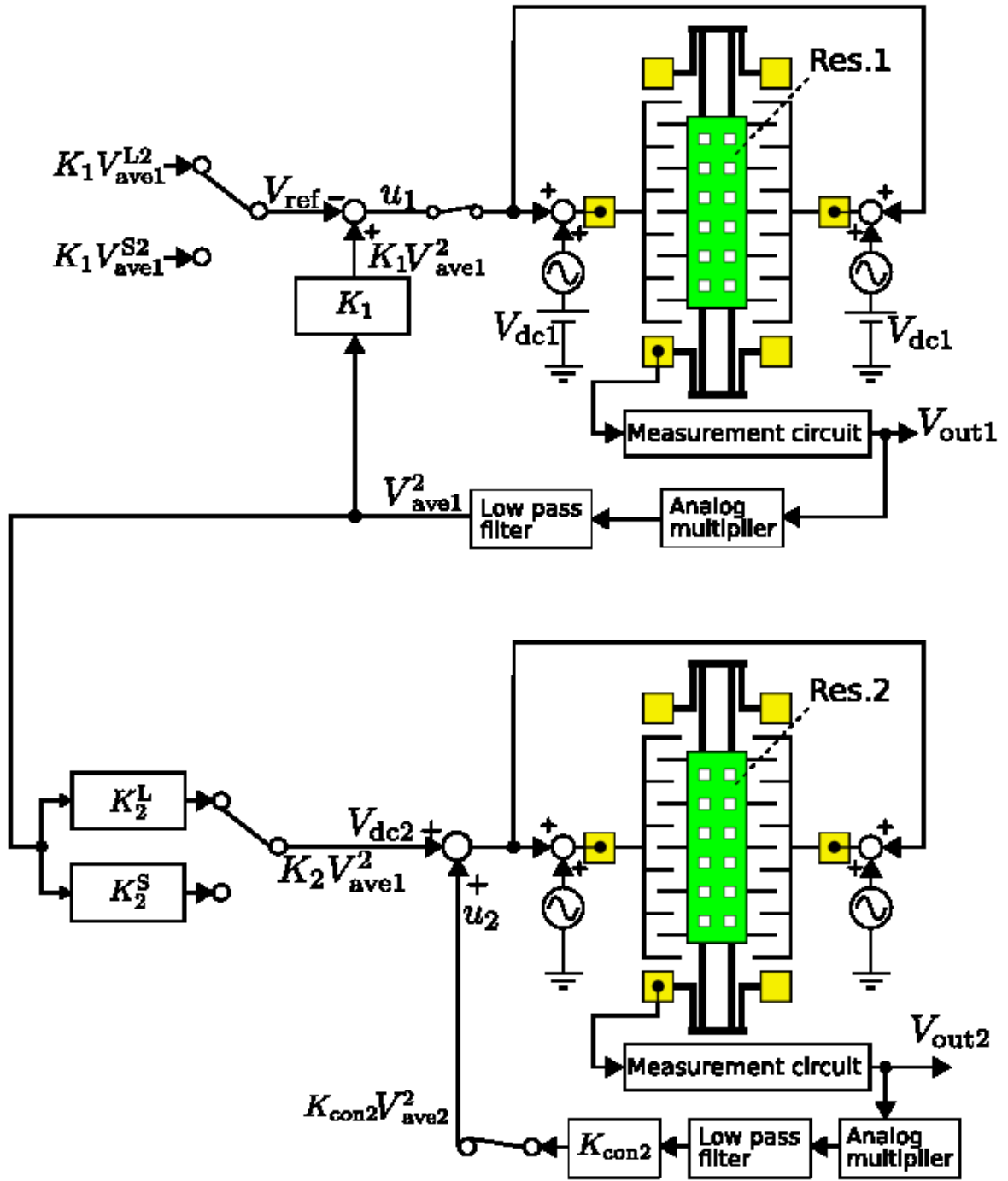


Figure 5.3: Switching control system.

coupling between two MEMS resonators, there does not happen any fault of switching control.

By using the same control method shown in Section 4.1, we construct a switching control system with the feedback control in Res. 1 and apply the control input as a slowly changing dc voltage to the MEMS resonator. In Res. 1, excitation force F_{all1} under control and the control input u_1 are defined by the following equations:

$$F_{\text{all1}} = 4\varepsilon N \frac{h}{d} (V_{\text{dc1}} + u_1) v_{\text{ac1}} \sin 2\pi f_1 t, \quad (5.1.3)$$

$$u_1 = -V_{\text{ref}} + K_1 V_{\text{ave1}}^2, \quad (5.1.4)$$

where K_1 ($= 9.1 \text{ V}^{-1}$) denotes the feedback gain, V_{ref} the external reference signal, and V_{ave1}^2 the square average dc voltage of Res. 1. The MEMS resonator exhibits the hysteretic behavior with respect to the excitation force at a fixed excitation frequency as shown in Fig. 3.13. We need to decrease (increase) the amplitude of the excitation force under control when the state is switched to the small (large) amplitude vibration.

The 2-bit binary counter consists of a series connection of two MEMS resonators, where the output of Res. 1 is connected to the input of Res. 2. As shown in Eq. (4.1.3), the excitation force depends on both dc bias voltage V_{dc1} and control input u_1 . Here, it is proposed that the dc bias voltage V_{dc2} of Res. 2 is given as the input depending on the square average dc voltage V_{ave1}^2 of Res. 1. In Res. 2, excitation force F_{all2} under control, dc bias voltage V_{dc2} , and control input u_2 are obtained by the following equations:

$$F_{\text{all2}} = 4\varepsilon N \frac{h}{d} (V_{\text{dc2}} + u_2) v_{\text{ac2}} \sin 2\pi f_2 t, \quad (5.1.5)$$

$$V_{\text{dc2}} = K_2 V_{\text{ave1}}^2, \quad (5.1.6)$$

$$u_2 = K_{\text{con2}} V_{\text{ave2}}^2, \quad (5.1.7)$$

where K_2 denotes the gain, K_{con2} ($= -3.9 \text{ V}^{-1}$) the control gain, and V_{ave2}^2 the square average dc voltage of Res. 2. When the state of Res. 1 is switched to the large (small) amplitude vibration, the external reference signal V_{ref} and the gain K_2 are set at $K_1(V_{\text{ave1}}^{\text{L}})^2$ ($K_1(V_{\text{ave1}}^{\text{S}})^2$) and K_2^{L} (K_2^{S}), respectively. Here, $K_2^{\text{L}}(V_{\text{ave1}}^{\text{L}})^2$ and $K_2^{\text{S}}(V_{\text{ave1}}^{\text{S}})^2$ are adjusted at -0.15 V . Furthermore, the feedback gain K_1 and the control gain K_{con2} are swept within the operating range and are adjusted at 9.1 V^{-1} and -3.9 V^{-1} .

5.1.3 Results and discussions

In order to realize the 2-bit binary counter as shown in Tab. 5.1, we apply the proposed switching control to two MEMS resonators. Fig. 5.4 shows the experimental results of the

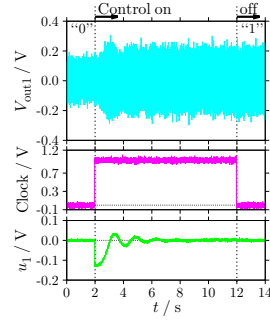
switching control. Figs. 5.4(a), (c), and (e) (Figs. 5.4(b), (d), and (f)) correspond to the switching control results in Res. 1 (Res. 2). When the clock signal is set at 0 V (1 V), the control input is off (on). The external reference signal V_{ref} and the dc bias voltage V_{dc2} of Res. 2 are switched by the rising edge of the clock signal.

The switching control from “00” to “01” is implemented as shown in Figs. 5.4(a) and (b). In these figures, Res. 2 stays at “0” and Res. 1 changes from “0” to “1”. Figs. 5.4(c) and (d) show the switching control from “01” to “10”. When Res. 1 changes from “1” to “0”, it triggers the switching control of Res. 2. Note that the absolute value of the excitation force (the sum of the dc bias voltage and the control input) in Fig. 5.4(d) exceeds that in Fig. 5.4(b) when the control input is applied. As a result, Res. 2 changes from “0” to “1”. Figs. 5.4(e) and (f) show that the present state is “10” and the next state becomes “11”. The transition is slow from “0” to “1” in Res. 1 and Res. 2 stays at “1”. After the switching control is completed, the control input u_1 of Res. 1 disappears due to the feedback control as shown in Figs. 5.4(a), (c), and (e). It is confirmed that the dc bias voltage V_{dc2} of Res. 2 becomes -0.15 V when the state of Res. 1 converges to a steady state. The switching control results are related to the initial states in two MEMS resonators.

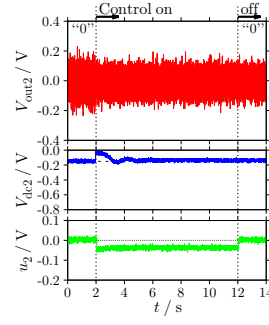
Figure 5.5 shows the oscillogram of the switching control sequence. The states in two MEMS resonators start from “00” and continues to “01”, “10”, and “11” at each clock signal. Here, the outputs must repeat the binary count sequence with a return to “00”. However, the switching from “11” to “00” is not realized in the proposed switching system. In order to implement the switching from “11” to “00”, it is proposed that a reset operation is applied to the coupled MEMS resonators. This reset operation is discussed in the next section.

Table 5.1: Count sequence for 2-bit binary counter.

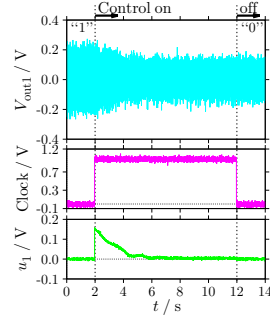
Clock	Res. 2 $V_{\text{out2}} (x_2)$	Res. 1 $V_{\text{out1}} (x_1)$
0	“0”	“0”
1	“0”	“1”
2	“1”	“0”
3	“1”	“1”



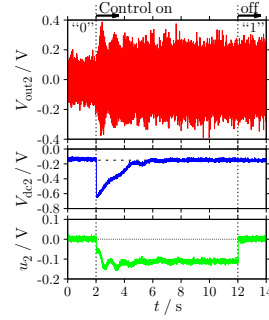
(a) Switching control from “0” to “1” in Res. 1 at $V_{\text{ref}} = K_1(V_{\text{ave1}}^L)^2$.



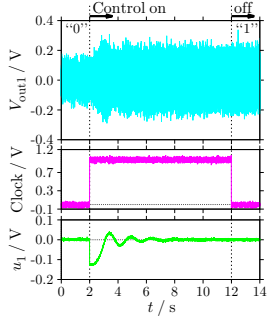
(b) Switching control from “0” to “0” in Res. 2 at $K_2 = K_2^L$.



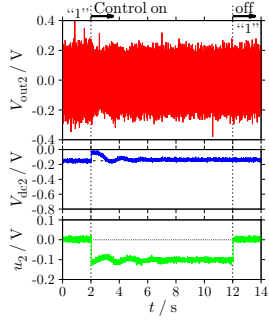
(c) Switching control from “1” to “0” in Res. 1 at $V_{\text{ref}} = K_1(V_{\text{ave1}}^S)^2$.



(d) Switching control from “0” to “1” in Res. 2 at $K_2 = K_2^S$.



(e) Switching control from “0” to “1” in Res. 1 at $V_{\text{ref}} = K_1(V_{\text{ave1}}^L)^2$.



(f) Switching control from “1” to “1” in Res. 2 at $K_2 = K_2^L$.

Figure 5.4: Switching control in coupled nonlinear MEMS resonator. Aqua and red lines correspond to output voltage V_{out1} in Res. 1 and V_{out2} in Res. 2. Purple, blue, and green lines show clock signal, dc bias voltage V_{dc2} of Res. 2, and control input. The control input is applied at 2s from the beginning of the oscillogram as shown in the first vertical dashed line. The second vertical dashed line at 12s represents the moment the control ends. External reference signal V_{ref} and gain K_2 are switched at 2s.

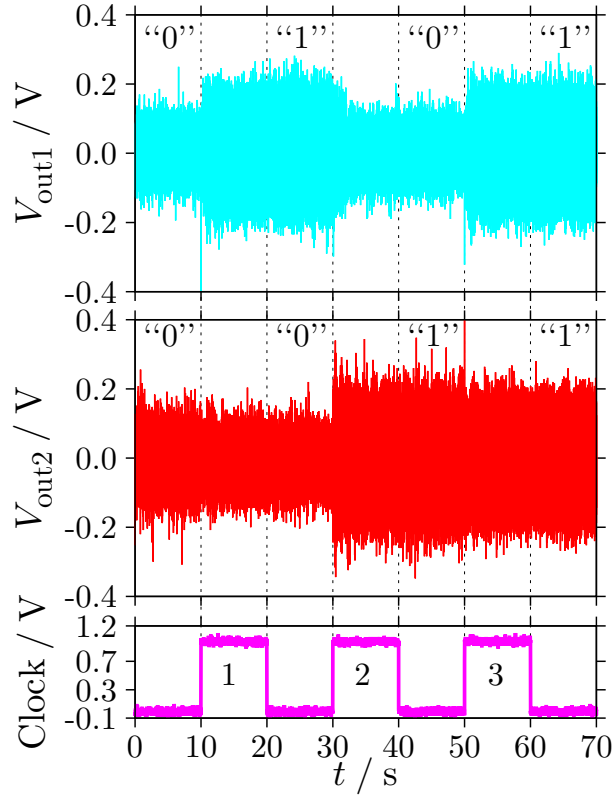


Figure 5.5: Switching control sequence ("00" $\rightarrow$ "01" $\rightarrow$ "10" $\rightarrow$ "11") as 2-bit binary counter in electrically coupled nonlinear MEMS resonators.

5.2 Numerical study on 2-bit binary counter

Based on our experimental results, the switching control is numerically confirmed in coupled nonlinear MEMS resonators as a 2-bit binary counter. In addition, the trajectories of the operating point are investigated for estimating the ability of switching control sequence and reset operation.

5.2.1 Dynamical model in two MEMS resonators

As in the above-mentioned experiments, two MEMS resonators are used. Based on the system described by Eq. (4.2.1), the dynamical model of a single MEMS resonator [43], which is a single element of the coupled MEMS resonators, is given by the following

non-dimensional differential equations:

$$\begin{cases} \frac{dx_j}{dt} = y_j, \\ \frac{dy_j}{dt} = -\frac{y_j}{Q} - x_j - \alpha_3 x_j^3 + (k_j + u_j) \sin \omega t, \end{cases} \quad (5.2.1)$$

for $j = 1, 2$. Here x_j denotes the displacement of the j -th MEMS resonator in the coupled system, y_j the displacement velocity of the j -th MEMS resonator, k_j the amplitude of the excitation force, and u_j the control input. Here, ω denotes the excitation frequency of the system, Q ($= 282$) the quality factor of both oscillators, and α_3 ($= 3.23$) the coefficient of cubic correction to each linear restoring force. k_1 is set at 0.001. As shown in Section 4.2, the parameter settings are due to Ref. [43], because the same device is assigned to the coupled system.

5.2.2 Numerical switching method

Figure 5.6 shows the numerical control system in two coupled MEMS resonators. According to the experimental method, the numerical control input u_1 of Res. 1, the numerical excitation force k_2 of Res. 2, and the numerical control input u_2 of Res. 2 are given by the following equations:

$$u_1 = A_{\text{ref}}^2 - K_1 A_{\text{ave1}}^2, \quad (5.2.2)$$

$$A_{\text{ave1}}^2 = \frac{A_1^2 + A_2^2 + \cdots + A_m^2 + \cdots + A_M^2}{M}, \quad (5.2.3)$$

$$k_2 = K_2 A_{\text{ave1}}^2, \quad (5.2.4)$$

$$u_2 = K_{\text{con2}} A_{\text{ave2}}^2, \quad (5.2.5)$$

$$A_{\text{ave2}}^2 = \frac{A_{21}^2 + A_{22}^2 + \cdots + A_{2m}^2 + \cdots + A_{2M}^2}{M}, \quad (5.2.6)$$

where K_1 denotes the feedback gain of Res. 1, A_{ref}^2 the external reference signal of squared amplitude, m natural number, M the average number, and K_{con2} the gain of Res. 2. Here, A_m (A_{2m}) is the displacement amplitude of Res. 1 (Res. 2) of the previous m periods within $1 \leq m \leq M$. Then, A_{ave1}^2 (A_{ave2}^2) is the average of A_m^2 (A_{2m}^2). M is set at 100. When the state of Res. 1 is switched to the large (small) amplitude vibration, the external reference signal A_{ref}^2 is set at $K_1(A_{\text{ave1}}^{\text{L}})^2$ ($K_1(A_{\text{ave1}}^{\text{S}})^2$) and the gain K_2 of the excitation force is set at K_2^{L} (K_2^{S}). Here $A_{\text{ave1}}^{\text{L}}$ ($A_{\text{ave1}}^{\text{S}}$) corresponds to the target of amplitude to the large (small) vibration. In addition, $K_2^{\text{L}}(A_{\text{ave2}}^{\text{L}})^2$ and $K_2^{\text{S}}(A_{\text{ave2}}^{\text{S}})^2$ are adjusted at 0.001.

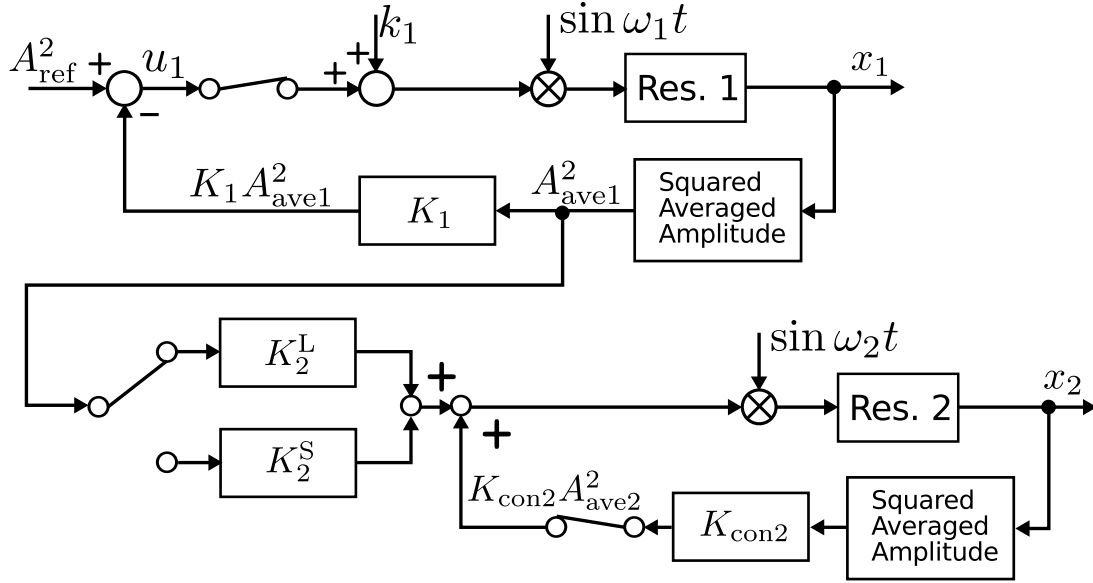


Figure 5.6: Numerical control system in coupled MEMS resonators.

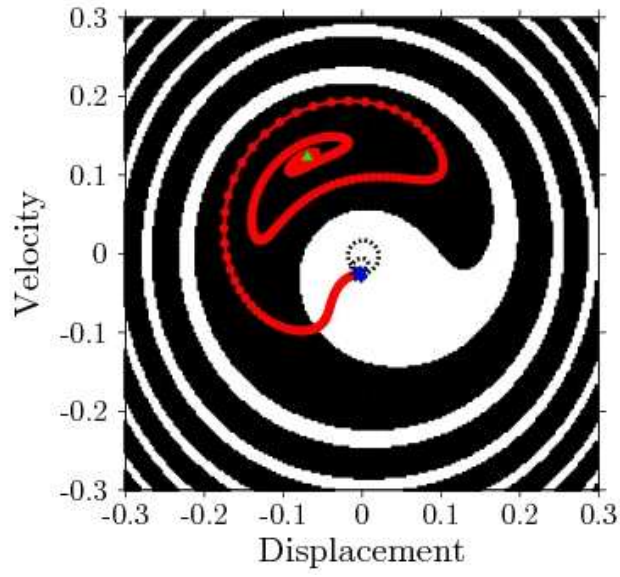
As mentioned in Section 4.2.3, the requirements of the switching control are obtained by using k'_L and k'_S . Therefore, the requirements for the 2-bit binary counter operation as shown in Tab. 5.2 are obtained from Tab. 4.1. In the following calculations, K_1 is set at 0.06 and K_{con2} at 0.03.

5.2.3 Numerical results and discussions

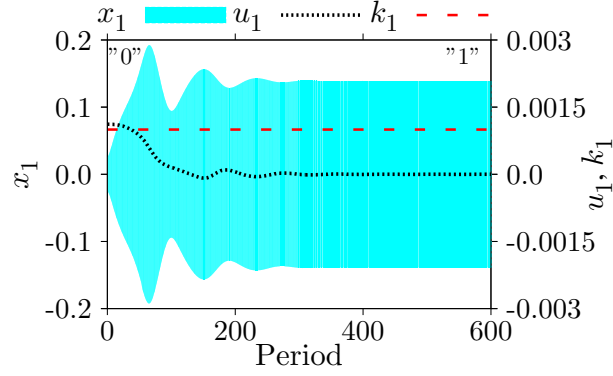
The proposed switching control is numerically applied to two MEMS resonators. In Figs. 5.7(a), 5.8(a), and 5.9(a), the solid and dotted lines show the loci of stroboscopic states of Res. 1 and Res. 2 in the phase space, respectively. Figs. 5.7(b), 5.8(b), and 5.9(b) display the time evolution of Res. 1. Figs. 5.7(c), 5.8(c), and 5.9(c) show the

Table 5.2: Requirements for 2-bit binary counter.

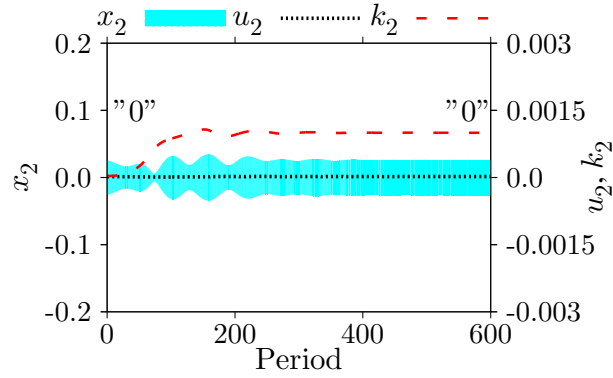
Count	Switching results	Res. 1 $u_1 + k_1$	Res. 2 $u_2 + k_2$
1	"00" $\rightarrow$ "01"	$u_1 + k_1 > k'_L$	$u_2 + k_2 < k'_L$
2	"01" $\rightarrow$ "10"	$u_1 + k_1 < k'_S$	$u_2 + k_2 > k'_L$
3	"10" $\rightarrow$ "11"	$u_1 + k_1 > k'_L$	$u_2 + k_2 > k'_S$
0	"11" $\rightarrow$ "00"	$u_1 + k_1 < k'_S$	$u_2 + k_2 < k'_S$



(a) Trajectories under control in Res. 1 (solid line) and Res. 2 (dotted line).

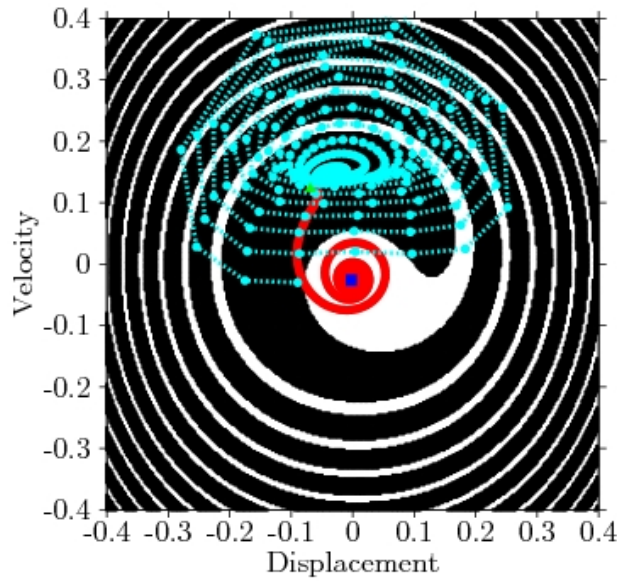


(b) Results of switching in Res. 1.

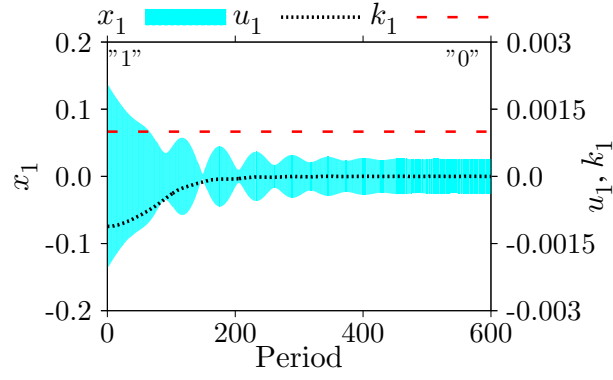


(c) Results of switching in Res. 2.

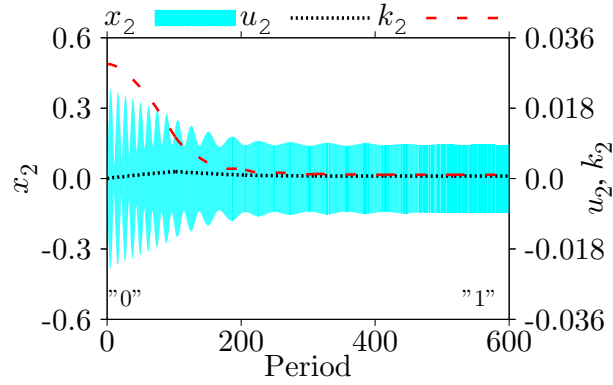
Figure 5.7: Switching from “00” to “01”.



(a) Trajectories under control in Res. 1 (solid line) and Res. 2 (dotted line).

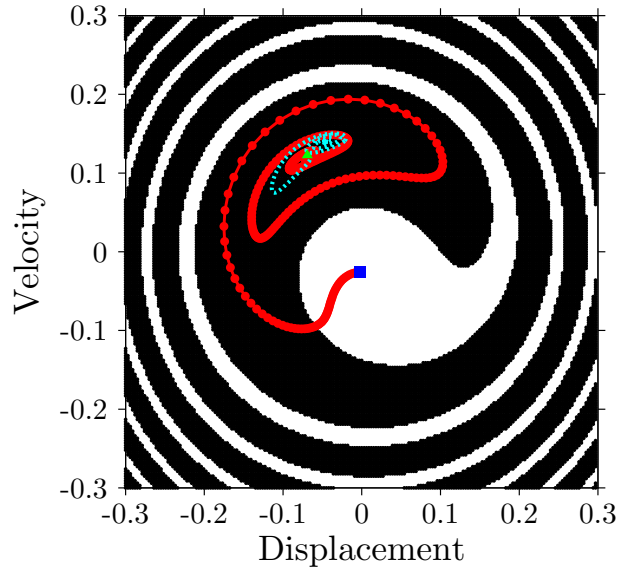


(b) Results of switching in Res. 1.

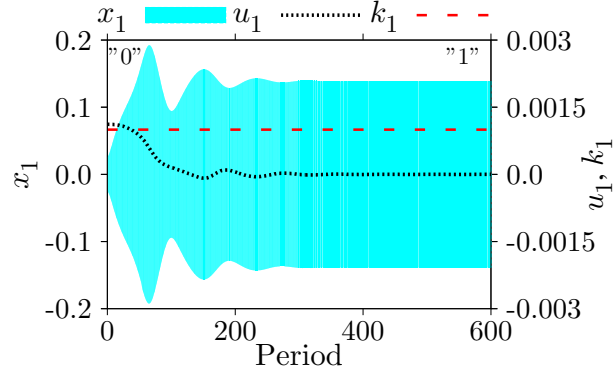


(c) Results of switching in Res. 2.

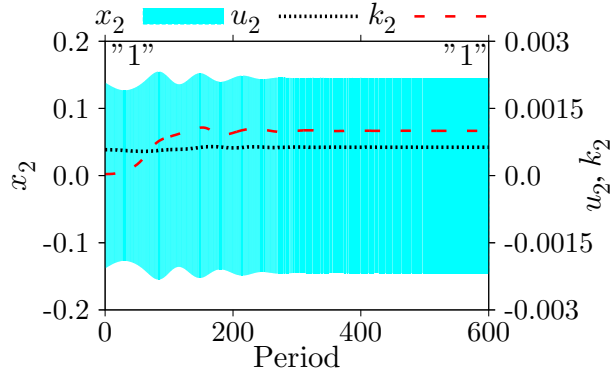
Figure 5.8: Switching from “01” to “10”.



(a) Trajectories under control in Res. 1 (solid line) and Res. 2 (dotted line).



(b) Results of switching in Res. 1.



(c) Results of switching in Res. 2.

Figure 5.9: Switching from “10” to “11”.

vibrating states of Res. 2. In Figs. 5.7(b), 5.8(b), 5.9(b), 5.7(c), 5.8(c), and 5.9(c), the dashed, solid, and dotted lines correspond to the excitation force, the displacement, and the control signal, respectively.

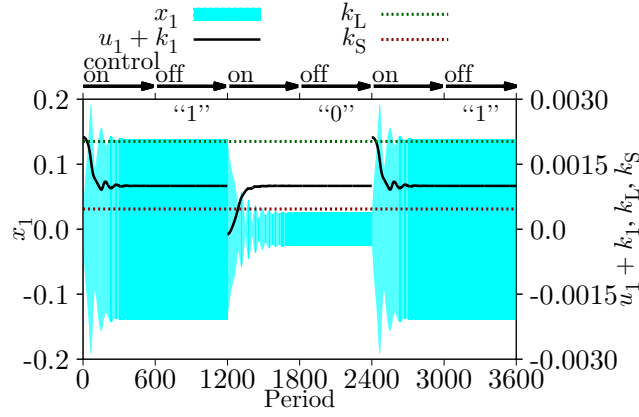
Figure 5.7 shows the switching from “00” to “01”. Here, while Res. 1 changes from “0” to “1”, Res. 2 keeps from “0” to “0” because the sum of the excitation force k_2 and the control input u_2 is equal to almost null.

In Fig. 5.8, the results show that the present state is “01” and the next state is “10”. When Res. 1 changes from “1” to “0”, it triggers Res. 2 and complements it. As a result, Res. 2 changes from “0” to “1”. Here, the stress for Res. 2 seems large for repeated use. The sum of the excitation force k_2 and the control input u_2 of Res. 2 is 30 times as large as the excitation force of Res. 1 at the onset of the control.

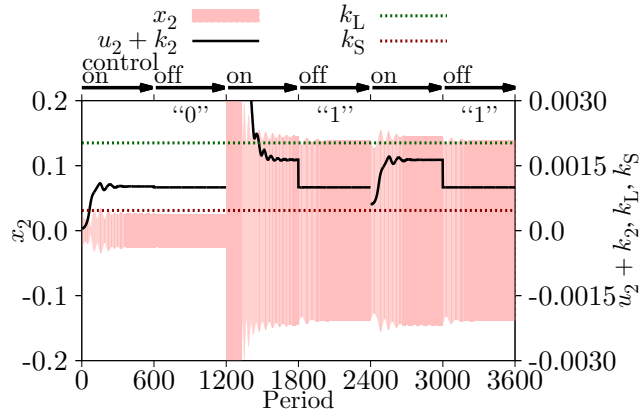
Figure 5.9 shows the switching from “10” to “11”. The transition is slow from “0” to “1” in Res. 1. Res. 2 changes from “1” to “1” because the control input u_2 of Res. 2 is kept around 0.0006.

Figures 5.10(a) and (b) show the switching control sequence (“00” $\rightarrow$ “01” $\rightarrow$ “10” $\rightarrow$ “11”) in two coupled MEMS resonators. In these figures, the control input is switched at intervals of 600 periods. Fig. 5.10(c) gives a partially magnified view of Fig. 5.10(b) and shows the switching control from “0” to “1” in Res. 2. The switching from large to small (small to large) amplitude vibrations is completed after around 300 periods (350 periods) in Res. 1. In Res. 2, the duration of the transient state is around 800 periods (700 periods) when the state stayed at the large (small) amplitude vibrations. It is found that the transition is slow from small to large amplitude vibrations in Res. 2. It takes around 900 periods until the conversion.

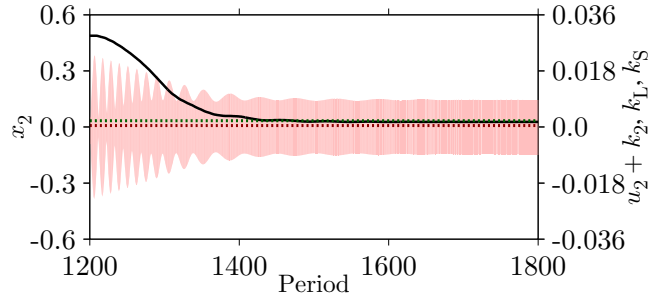
As mentioned above, the 2-bit counter must start from “00” and continues to counts “01”, “10”, “11” and then back to “00”. Therefore, for the switching control from “11” to “00”, the reset operation is applied to MEMS resonator. When the control input is not applied, every initial state converges toward either of two stable states (“0” and “1”) due to two basins of attraction. The small amplitude solution has the basin of attraction around the origin as shown in Figs. 5.7(a), 5.8(a), and 5.9(a). Based on these results, the reset operation can be realized. As a result, two coupled MEMS resonators can be used as a 2-bit binary counter by using the proposed switching control and the reset operation.



(a) Results of switching control in Res. 1.



(b) Results of switching control in Res. 2.



(c) Magnified view of Fig. 5.10(b)

Figure 5.10: Switching control from “00” to “11”.

5.3 Summary

Through the experiments and numerical simulations, this chapter reports a binary counter that consists of a coupled system of MEMS resonators with nonlinear characteristics. In order to realize a 2-bit binary counter, two MEMS resonators are interconnected

in a unidirectional coupled system. It is confirmed that the switching control results are related to the initial states in coupled resonators. Based on both experiments and numerical simulations, the novel switching control sequence (“00” $\rightarrow$ “01” $\rightarrow$ “10” $\rightarrow$ “11”) is realized as a 2-bit binary counter that consists of coupled nonlinear MEMS resonators. Based on the transient analysis, it is also proposed that a reset operation can be applied to MEMS resonators for the switching control from “11” to “00”.

Since the proposed switching control system is only a prototype, its performance will be improved by an optimized switching control method. Nevertheless, this study is the implementation of a sequential logic system consisting of electrically coupled nonlinear MEMS resonators. In addition, this study opens the way for further investigations of the switching control of coexisting states in coupled nonlinear dynamics. Based on our results, the coupled MEMS resonators may allow other sequential logic devices which are a shift register and so on.

Chapter 6

Logic-memory operation in a single MEMS resonator

This chapter reports multifunctional operation based on the nonlinear dynamics in a single MEMS resonator. This chapter focuses on fabricating a multifunction device that corresponds to a “logic-memory device” explained in Section 2.3. To obtain both logic and memory operations in the MEMS resonator, the nonlinear dynamics with and without control input is examined. Section 6.1 addresses the experiments and numerical simulations that allow us to develop a device that combines multiple-input gate and memory functions in a single nonlinear MEMS resonator. Section 6.2 focuses on a reprogrammable logic-memory device of a nonlinear MEMS resonator with multiple states by numerical simulations.

6.1 Logic-memory device

We experimentally and numerically study a device that combines an OR gate and memory functions in a single MEMS resonator. Recently, multifunctional operation has been demonstrated in the form of a shift-register and a controlled NOT gate made from a single mechanical resonator [36]. The next phase is to use a closed loop control to generate multifunction devices, which consist of memory and multiple-input gates, in a single device. The closed loop allows output and excitation signals to be fixed at a single frequency. The goal of the work presented in this section is to develop multifunction operation by applying the nonlinear MEMS resonator through coexistence of multi-states with closed loop control.

In this section, firstly, the experimental and numerical systems are shown to perform

the logic and memory operations. Next, in order to perform logic and memory operations, the nonlinear dynamics with and without control input are numerically examined. Finally, based on numerical results, the logic and memory operations are experimentally performed in a single MEMS resonator.

6.1.1 Experimental and numerical systems

A nonlinear MEMS resonator has hysteretic characteristics with respect to the numerical (experimental) dc bias voltage V_{dcn} (V_{dce}) as shown in Fig. 3.8 (3.13). In these figures, the stable regions, which correspond to large and small amplitude vibrations, define the two states of the single-output logic or memory device in a single MEMS resonator. In the numerical simulations and experiments, a displacement amplitude greater than $3.0 \mu\text{m}$ is regarded as a logical “1”; a value less than $3.0 \mu\text{m}$ is regarded as a logical “0” for logic and memory output. In experiments and simulations in this section, the excitation frequency and the ac excitation amplitude are set to 8.6654 kHz and 0.6 V in vacuum at 10 Pa . In Section 6.1, the numerical and experimental dc bias voltages (V_{dcn} and V_{dce}) are fixed at 150 mV . In the following, the experimental and numerical systems are explained.

(a) Experimental system

Figure 6.1 shows the control system to perform the logic operation. As studied in Section 4.1, the switching between two coexisting stable states was done by a displacement feedback control in the nonlinear MEMS resonator. Based on the results, the feedback control is performed. The logic inputs are applied to the MEMS resonator in the form of two dc voltages (L_{ine1} and L_{ine2}). In the experiments, the control input u_e is described as follows:

$$u_e = L_{\text{ine1}} + L_{\text{ine2}} - K_e V_{\text{ave}}^2, \quad (6.1.1)$$

where K_e denotes the feedback gain and V_{ave}^2 a slowly changing dc voltage that depends on the displacement [30]. Note that the external reference signal in Section 4.1 becomes two logic inputs in this section. Based on Eqs. (6.1.1) and (4.1.3), the excitation force under the control is obtained by

$$F_{\text{all}} = 4\varepsilon N \frac{h}{d} (V_{\text{dce}} + u_e) v_{\text{ac}} \sin 2\pi f t \quad (6.1.2)$$

$$= 4\varepsilon N \frac{h}{d} (V_{\text{dce}} + L_{\text{ine1}} + L_{\text{ine2}} - K_e V_{\text{ave}}^2) v_{\text{ac}} \sin 2\pi f t. \quad (6.1.3)$$

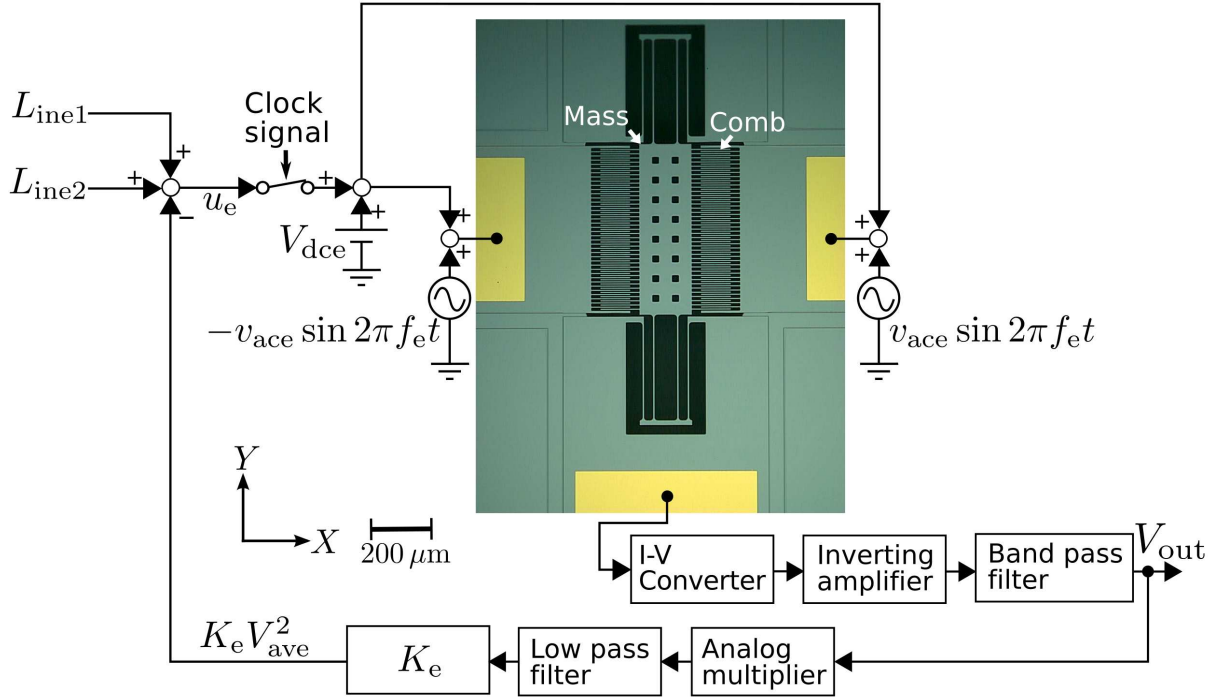


Figure 6.1: Schematic of MEMS resonator, measurement system, and control system that relates to logic inputs (Experimental system). The control system is implemented with a feedback input and logic inputs. The feedback input is given as the slowly changing dc voltage V_{ave}^2 , to which the output voltage V_{out} is converted by an analog multiplier and a low-pass filter as shown in Section 3.1. The logic inputs, represented by two dc voltages (L_{ine1} and L_{ine2}), are added to the dc bias voltage V_{dce} . As a result, the excitation force under control becomes proportional to $V_{\text{dce}} + u_e = V_{\text{dce}} + L_{\text{ine1}} + L_{\text{ine2}} - K_e V_{\text{ave}}^2$. Here L_{ine1} and L_{ine2} denote the input signals, which serve as the logic inputs, u_e is the control input, and K_e ($= 11 \text{ V}^{-1}$) is the feedback gain in the experiments.

(b) Numerical system

Figure 6.2 shows the numerical system. According to the experimental method as shown in Eq. (6.1.1), the corresponding numerical control input u_n is described as follows:

$$u_n = L_{\text{inn1}} + L_{\text{inn2}} - K_n A_{\text{nave}}^2, \quad (6.1.4)$$

$$A_{\text{nave}}^2 = \frac{A_{\text{n1}}^2 + A_{\text{n2}}^2 + \dots + A_{\text{nm}}^2 + \dots + A_{\text{nM}}^2}{M}, \quad (6.1.5)$$

where L_{inn1} and L_{inn2} denote the input signals that are the logic inputs. K_n denotes the feedback gain, m a natural number, M the average number, and A_{nm} the displacement amplitude of the previous m periods within $1 \leq m \leq M$ for the numerical simulations.

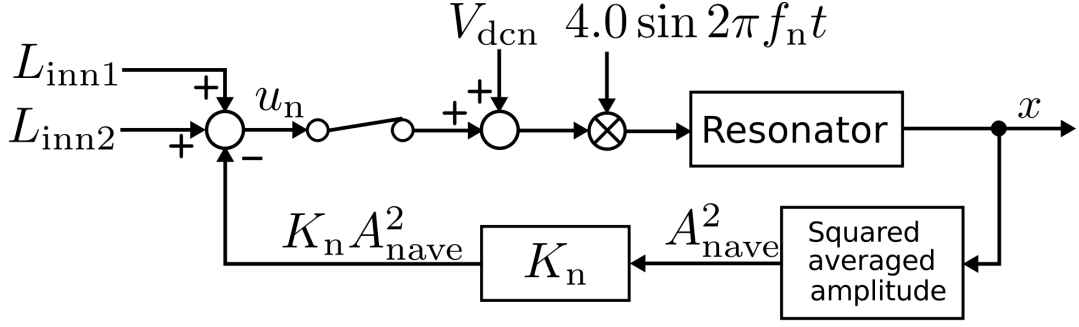


Figure 6.2: Numerical system.

In this case, A_{nave}^2 is the average of A_{nm}^2 . K_n is set to $1.42 \times 10^{10} \text{ Vm}^{-2}$ and M is set to 22000.

Based on Eqs. (3.2.1) and (6.1.3), the nonlinear dynamics of the MEMS resonator with the control input is obtained by

$$\begin{aligned} \frac{d^2x}{dt^2} + \frac{2\pi f_0}{Q} \frac{dx}{dt} + (2\pi f_0)^2 x + \alpha_3 x^3 &= 4.0 \text{ m(Vs}^2\text{)}^{-1} \times (V_{\text{dcn}} + u_n) \sin 2\pi f_n t \\ &= 4.0 \text{ m(Vs}^2\text{)}^{-1} \times (V_{\text{dcn}} + L_{\text{inn1}} + L_{\text{inn2}} - K_n A_{\text{nave}}^2) \sin 2\pi f_n t. \end{aligned} \quad (6.1.6)$$

6.1.2 Logic operation (numerical results)

In order to realize a logic operation in a MEMS resonator, we discuss the nonlinear dynamics with control input. Fig. 6.3 shows the numerically obtained steady states when the control input is applied to the MEMS resonator. The control input can induce a modulation of the resonator's amplitude and thus change the logical value of the output. In Fig. 6.3(a) (6.3(b)), the initial state of memory output is a logical “1” (logical “0”). In Figs. 6.3(a) and 6.3(b), there exist regions in which the displacement amplitude is the same because the control system depends on the feedback input. Assume that the control system receives only the input signals L_{inn1} and L_{inn2} . When the input signals $L_{\text{inn1}} = L_{\text{inn2}} = 0.0 \text{ mV}$ are sent to the MEMS resonator, the large stable state cannot switch to the small state. Therefore, a single MEMS resonator can be used as a logic gate because of the adjustment of the logic inputs and the feedback input.

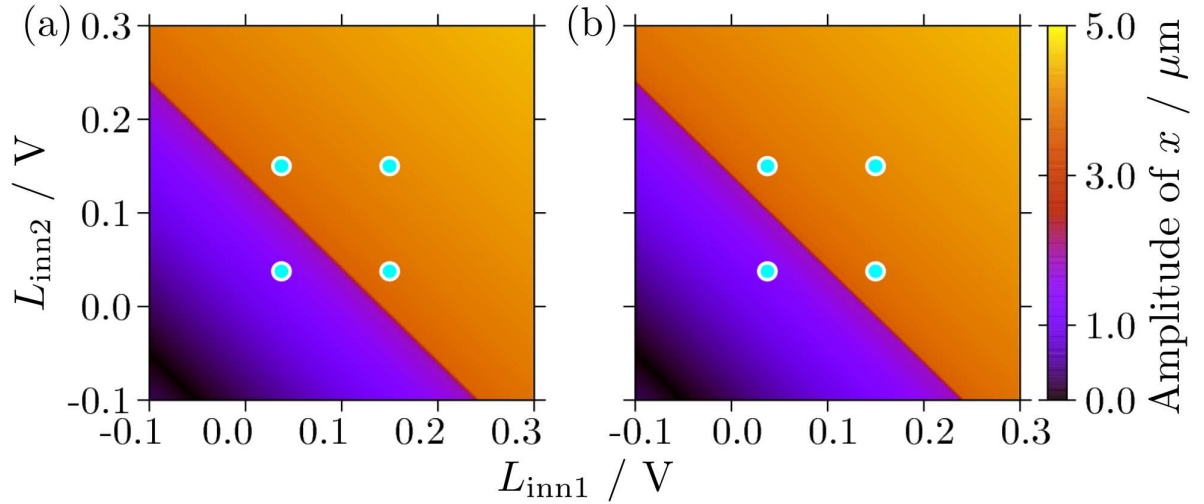


Figure 6.3: Amplitude modulation systematically varied in input signals L_{inn1} and L_{inn2} at $f_n = 8.6654 \text{ kHz}$ and $V_{\text{dcn}} = 150 \text{ mV}$ (numerical results). The light (dark) region corresponds to more than (less than) $3.0 \mu\text{m}$ in displacement amplitude, corresponding to a logical “1” (logical “0”) output. For an OR gate as shown in Tab. 6.1, input signals are set to 150.0 mV and 37.5 mV , as shown by the four circles with light (aqua) color: (a) Initial state is set to the large amplitude solution (logical “1” for memory output) (b) Initial state is the small amplitude solution (logical “0” for memory output).

6.1.3 Memory operation (numerical results)

To execute a memory operation in a MEMS resonator, we must consider the nonlinear dynamics without the control input. When the control input is not applied, every initial state corresponds to the convergence to either the small amplitude (black) or large amplitude (white) solutions, as shown in Fig. 6.4. As mentioned in Section 3.2, in the nonlinear MEMS resonator, the small and large amplitude solutions have each basin of

Table 6.1: Truth table of OR gate.

Input		Output
L_{inn1}	L_{inn2}	x
0	0	“0”
0	1	“1”
1	0	“1”
1	1	“1”

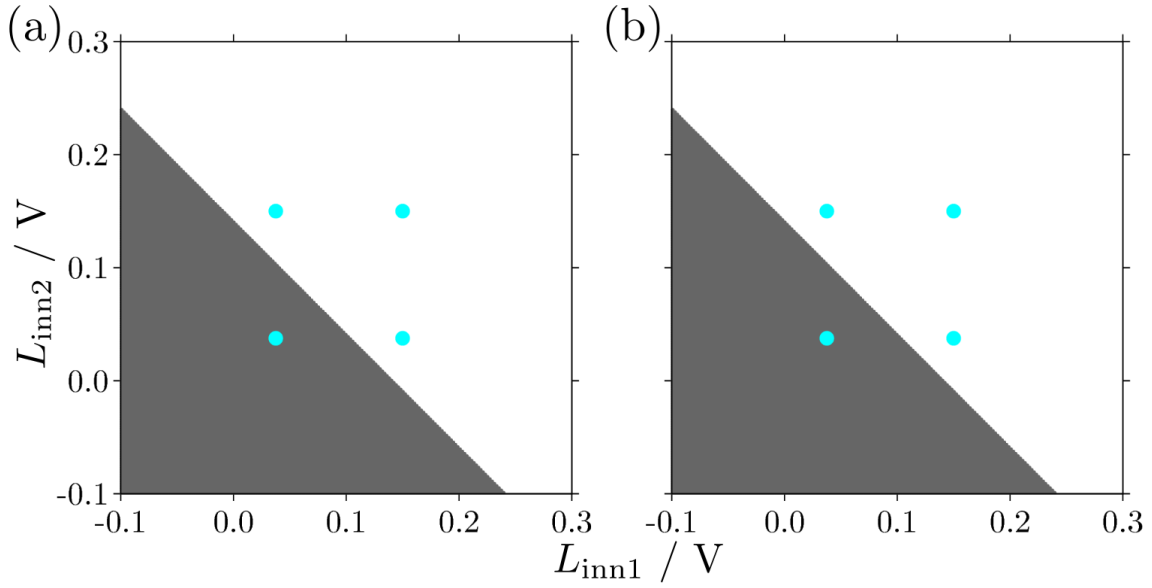


Figure 6.4: Calculated convergence conditions (final state) when the control input shown in Fig. 6.3 is off. The white (black) region corresponds to convergence to a logical “1” (logical “0”) for memory output. Light (aqua) circles show the logic input in our simulations, as in Fig. 6.3: (a) Numerical results corresponding to Fig. 6.3(a). (b) Numerical results corresponding to Fig. 6.3(b).

attraction around each solution. The convergence conditions depend on the two basins of attraction. The light region (displacement amplitude greater than $3.0\ \mu\text{m}$) in Fig. 6.3 corresponds to white region in Fig. 6.4 and vice versa. When the control input is not applied, the MEMS resonator maintains its original logic output. Thus, the nonlinear MEMS resonator works as the memory device by storing the logic information.

6.1.4 Experimental and numerical results and discussions

The results shown in Figs. 6.3 and 6.4 show that MEMS resonator works as a combined logic-memory device. The logic and memory operations can be programmed by adjusting the resonator’s operating parameters (input signals). In the numerical simulations, when the input signal (L_{inn1} or L_{inn2}) is set to $150.0\ \text{mV}$ ($37.5\ \text{mV}$), the logic input is regarded as logical 1 (logical 0). The logic inputs (0, 0) of input signals (L_{inn1} , L_{inn2}) have a value of $75.0\ \text{mV}$, (0, 1) and (1, 0) have a value of $187.5\ \text{mV}$, and finally (1, 1) have $300.0\ \text{mV}$, as shown by the light (aqua) circles in Figs. 6.3 and 6.4. The output of the device is

a logical “0”, when the logic inputs are (0, 0), which correspond to a value of 75.0 mV. However, when the logic inputs are set to (0, 1), (1, 0), or (1, 1), the output corresponds to a logical “1”. Therefore, the single MEMS resonator combines the function of an OR gate and memory.

These logic and memory operations can be demonstrated experimentally in a single MEMS resonator. The operations are confirmed for the behavior of device at clock evolution. The calculated time evolutions of the device are shown in Fig. 6.5(a) and the corresponding experimental time evolutions are shown in Fig. 6.5(b). The calculated results are consistent with the experimental results. When electrical noise and surges appear in Fig. 6.5(b), no logic faults occur and the memory operations are not perturbed. The experimental modulation of the amplitude and the convergence conditions will be examined in more detail in a future presentation. Nevertheless, this work demonstrates both experimentally and numerically a combined device of OR gate and memory functions in a single MEMS resonator.

Here, we estimate an instantaneous power of the MEMS resonator. In Fig. 6.1, when the voltage of right electrode is excited by $V_1 = V_{\text{dce}} + v_{\text{ac}} \sin 2\pi f_e t$ and the left by $V_2 = V_{\text{dce}} - v_{\text{ac}} \sin 2\pi f_e t$, the right and left comb capacitances (C_1 and C_2) are given by $C_1 = 5.75 \times 10^{-9} \text{ Fm}^{-1} \times (l + A_0 \sin(2\pi f_e t + \phi))$ and $C_2 = 5.75 \times 10^{-9} \text{ Fm}^{-1} \times (l - A_0 \sin(2\pi f_e t + \phi))$, where A_0 denotes the displacement amplitude and l ($= 100 \mu\text{m}$) is the initial overlap between the fingers as explained in Section 3.1. The power of the MEMS resonator p is described by $p = V_1 \partial(C_1 V_1) / \partial t + V_2 \partial(C_2 V_2) / \partial t$. In this study, lv_{ac}^2 is much greater than $A_0 V_{\text{dce}} v_{\text{ac}}$. Thus, the maximum power is estimated as $1.13 \times 10^{-8} \text{ W}$.

6.2 Reprogrammable logic-memory device

In this section, a reprogrammable logic-memory device is discussed by numerical simulations. In the previous section, a combination of an OR gate and a memory device is demonstrated in a single MEMS resonator. Recently, Guerra *et al.* have reported that the nonlinear mechanical resonator can be used as a reprogrammable logic gate [25]. This section focuses on the reprogrammable logic gate and memory devices in a single MEMS resonator.

In order to realize the reprogrammable logic-memory device, the nonlinear dynamics is examined at the change of excitation amplitude in the MEMS resonator. We confirm the

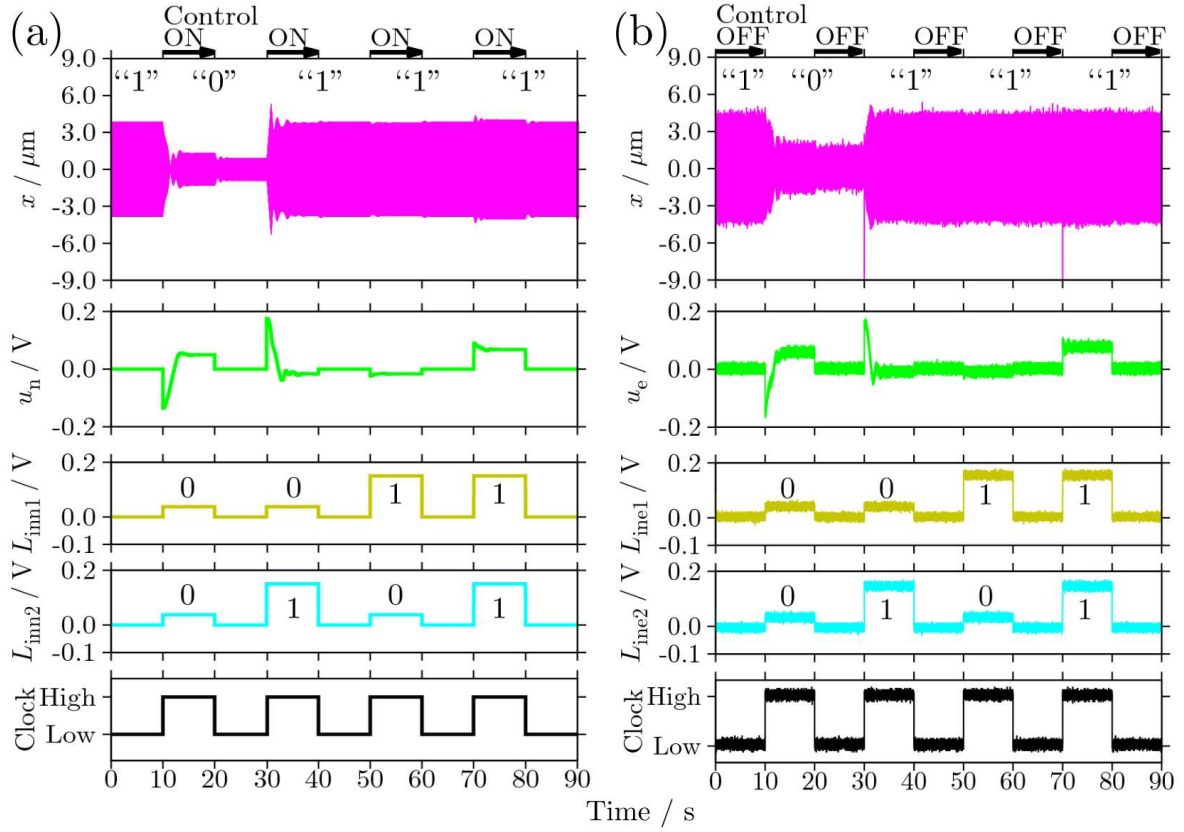
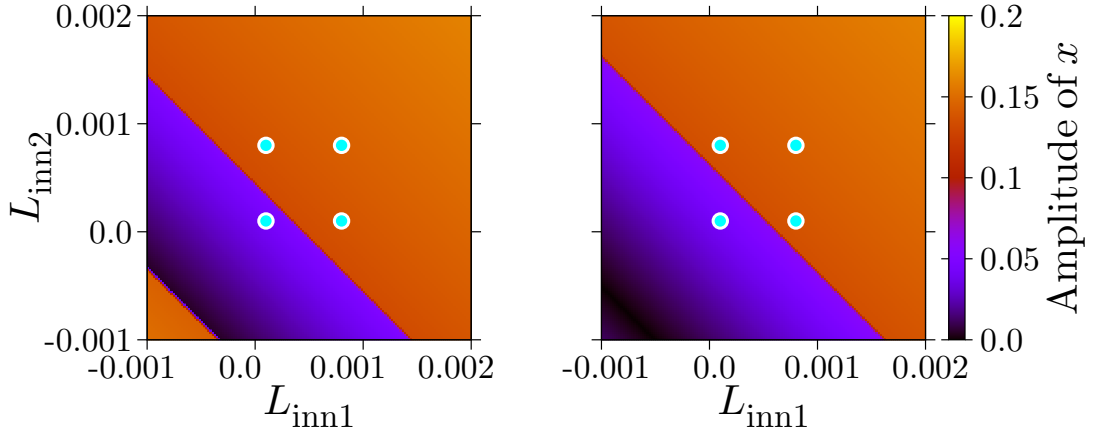


Figure 6.5: Time evolution of the combined device (OR gate and memory). The initial memory output is a logical “1”. When the clock signal is high (low), the control input is (is not) applied to the MEMS resonator. Note that the nonlinear MEMS resonator is used as a logic (memory) device at high (low) clock signal. The logic inputs start from (0, 0) and continue to (0, 1), (1, 0), and (1, 1). The logic output in a MEMS resonator changes to logical “0”, “1”, “1”, and “1” at each high clock signal: (a) Numerical results. (b) Experimental results. In our experiments, the logic input 1 (0) of experimental input signal (L_{ine1} or L_{ine2}) has a voltage of 150.0 mV (37.5 mV).

realization of the reprogrammable logic function by adjusting the excitation amplitude and the memory function by storing logic information in the single nonlinear MEMS resonator.

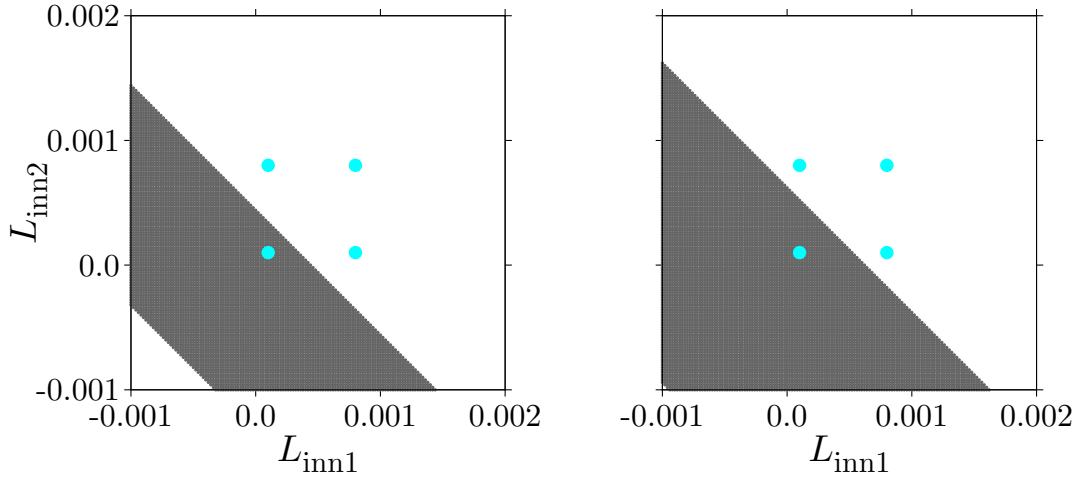
6.2.1 OR/AND gate and memory operations (numerical results)

By using the same method as shown in Section 6.1, we discuss both logic and memory operations in a MEMS resonator. In this section, the dynamical model of the MEMS



(a) Initial state is the logical "1" for memory output.

(b) Initial state corresponds to the logical "0" for memory output.



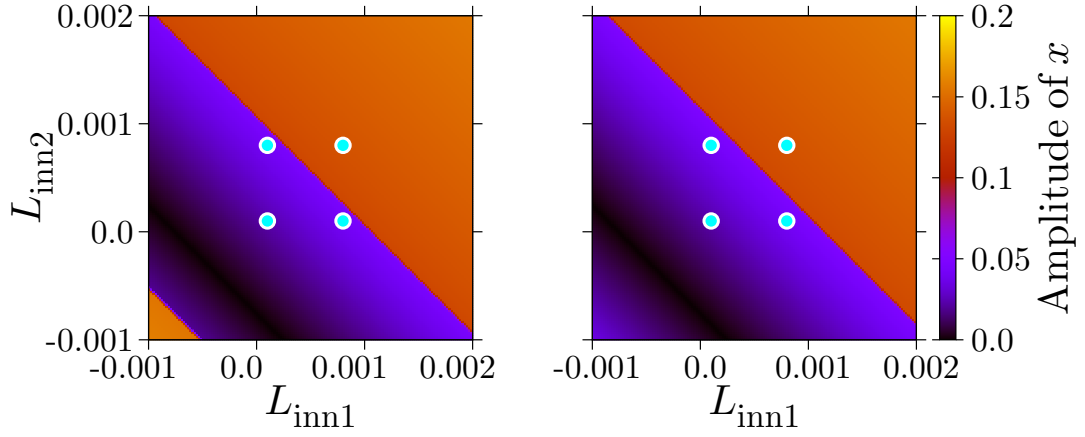
(c) Convergence conditions corresponding to Fig. 6.6(a).

(d) Convergence conditions corresponding to Fig. 6.6(b).

Figure 6.6: Displacement amplitude (logic output) with and without control input at $k = 0.00150$. (a) and (b) show the amplitude modulation systematically varied in input signals L_{inn1} and L_{inn2} . (c) and (d) show the final state when the control input is off.

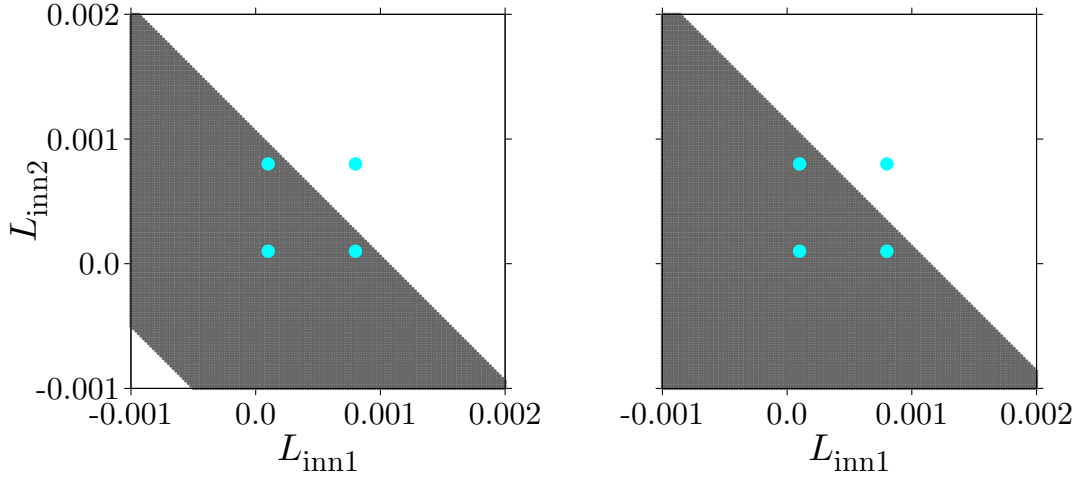
resonator is given by the non-dimensional differential equations (4.2.1) as explained in Section 4.2. Here, the excitation force k is set to either 0.00075 or 0.00150.

Figures 6.6(a) and (b) (Figures 6.7(a) and (b)) show the amplitude modulation systematically varied in input signals L_{inn1} and L_{inn2} when the control input is applied to the MEMS resonator at $k = 0.00150$ ($k = 0.00075$). In Figs. 6.6(a) and 6.7(a) (Figs. 6.6(b) and 6.7(b)), the initial state correspond to logical "1" ("0") for memory output. In these figures, the light (dark) region corresponds to more than (less than) 0.1 in displacement



(a) Initial state is set to the logical “1”.

(b) Initial state corresponds to the logical “0”.



(c) Convergence conditions corresponding to Fig. 6.7(a).

(d) Convergence conditions corresponding to Fig. 6.7(b).

Figure 6.7: Logic output with and without control input at $k = 0.00075$. (a) and (b) ((c) and (d)) show the amplitude modulation in the presence of the control input (the final state without the control input), as in Fig. 6.6.

amplitude, corresponding to a logical “1” (logical “0”) output. Note that the logic input is regarded as logical 1 (logical 0) when the input signal (L_{inn1} or L_{inn2}) is set to 0.0008 (0.0001), as shown by the four circles in Figs. 6.6 and 6.7. When the logic inputs are (0, 0), (0, 1), or (1, 0), the output of the device becomes a logical “0” at $k = 0.00075$. When the logic inputs are set to (1, 1), the output is a logical “1” at $k = 0.00075$. On the other hands, at $k = 0.00150$, when the logic inputs are set to (0, 1) or (1, 0), the output is a logical “0”. Therefore, the nonlinear MEMS resonator works as an OR (AND) gate at

Table 6.2: Truth table of AND gate.

Input		Output
L_{inn1}	L_{inn2}	x
0	0	“0”
0	1	“0”
1	0	“0”
1	1	“1”

$k = 0.00150$ ($k = 0.00075$) as shown in Tab. 6.1 (Tab. 6.2).

Next, the memory operation is considered in a MEMS resonator. Figs. 6.6(c) and 6.6(d) (Figs. 6.7(c) and 6.7(d)) show the calculated convergence conditions when the control inputs shown in Figs. 6.6(a) and 6.6(b) (Figs. 6.7(a) and 6.7(b)) are off. As shown in Figs. 6.6(a) and 6.6(b) (Fig. 6.7(c) and 6.7(d)), the nonlinear MEMS resonator can be used as the memory device by storing the output of OR (AND) gate. The single MEMS resonator combines the function of OR (AND) gate and memory at $k = 0.00150$ ($k = 0.00075$). As a result, we numerically demonstrate the reprogrammable logic function that consists of OR/AND gate and the memory function in the single nonlinear MEMS resonator due to the adjustment of the excitation amplitude.

6.3 Summary

In conclusion, a multifunctional device consisting of a nonlinear MEMS resonator is numerically and experimentally demonstrated. It is confirmed that when a control input is applied to a nonlinear MEMS resonator, two equal-amplitude regions exist because of the adjustment of the feedback input. Therefore, a single MEMS resonator can work as an OR gate. Numerical simulations are also used to show that in the absence of the control input, the nonlinear MEMS resonator maintains its original logical state. Thus, this resonator also serves as a memory device. Therefore, a combination of an OR gate and a memory device is demonstrated in a single MEMS resonator. I. Mahboob *et al.* have developed a device that combines a controlled NOT gate and memory functions in a single resonator at 2K [36]. In this study, a logic-memory device of high reliability operating at room temperature is realized with the logic inputs given as two dc voltages that do not depend on the phase.

Next, in numerical simulations, the reprogrammable logic gate and memory devices are demonstrated in a single MEMS resonator. It is numerically shown that when the excitation amplitude is adjusted, the logic function as the AND gate can be programmed. As a result, we numerically confirm the realization of the reprogrammable logic function that consists of OR/AND gate and the memory function in the single nonlinear MEMS resonator. By considering the closed loop and bias inputs, these results open the way to further research in high and multi functionality in single and coupled resonators, which may take the form of multiple-input gates such as three- or four-input logic gates and memory. In addition, in order to realize the arbitrary logic and memory devices, the system design based on the nonlinear dynamics will be discussed in the MEMS resonator.

Chapter 7

Conclusions and future work

This dissertation addressed four operations in mechanical computation based on a single or coupled nonlinear MEMS resonators. Firstly, the vibration displacement was measured without additional sensors as a read operation in a nonlinear MEMS resonator. Secondly, as a write operation, the switching control between two states was achieved by the feedback control. Thirdly, the switching control sequence of counter operation was achieved in the coupled nonlinear MEMS resonators. Finally, a logic-memory operation was realized in a single MEMS resonator. The conclusions of this study are presented and the future work is discussed as follows.

7.1 Conclusions

Chapter 2 focused on a nonlinear MEMS resonator and its application to memory and logic functions. This chapter showed a schematic of a fabricated comb-drive MEMS resonator. It was explained that the fabricated MEMS resonator has nonlinear responses, which lead to two stable states and one unstable state. Finally, this chapter explained memory, sequential logic, and logic-memory devices that consists of a single or coupled nonlinear MEMS resonators with hysteresis.

Chapter 3 described a displacement measurement of the comb-drive MEMS resonator as a read operation of the memory device. It was experimentally and theoretically confirmed that the MEMS resonator can be equipped with a comb drive that normally serves as a forcing actuator, but which simultaneously serves as a displacement sensor in the differential measurement. Therefore, it was confirmed that two vibrations can be detected, based on the measurement of the current through the capacitor without additional sen-

sors. These results showed that the read operation can be realized by the self-sensing method. It was also shown that the read operation does not destroy the bit value stored by the MEMS resonator. In addition, it was numerically shown that the dynamical model of the single MEMS resonator exhibits hysteretic responses.

Chapter 4 addressed the switching between two coexisting stable states as a write operation of the memory device in the single MEMS resonator. In particular, the switching was performed by using a displacement feedback control and a delayed feedback control. Based on the displacement measurement as studied in chapter 3, the switching between two stable periodic vibrations was experimentally and numerically demonstrated. The control input was applied to the MEMS resonator as the dc voltage bias without any influence to measurement. As a result, the write operation can be achieved by the displacement feedback control. In addition, the switching without any external reference was numerically realized by using a delayed feedback control method as a write operation. When the direction to the control signal of delayed feedback control was given, the switching between two states was realized by numerical simulations. These results showed that the delayed feedback control works as a write operation without exact solutions of the memory device in the resonator.

Chapter 5 proposed a binary counter that consists of a coupled system of MEMS resonators with nonlinear characteristics. An electronically coupled system of two MEMS resonators were investigated. It was confirmed that the switching control results depend on the initial states in coupled nonlinear MEMS resonators. The switching control sequence (“00” $\rightarrow$ “01” $\rightarrow$ “10” $\rightarrow$ “11”) were experimentally and numerically demonstrated as the counter operation of two coupled MEMS resonators. Based on the numerical transient behavior, it was also proposed that a reset operation is applied to MEMS resonators for the switching control from “11” to “00”. As a result, based on both experiments and numerical simulations, the controlling nonlinear behavior were realized as a 2-bit binary counter.

Chapter 6 reported multifunctional operation from the viewpoint of application of nonlinear dynamics in the single MEMS resonator. A logic-memory device, which consists of memory and multiple-input gates, was demonstrated in the nonlinear MEMS resonator by using a closed loop control. The nonlinear dynamics in the presence (absence) of control input was examined as the logic (memory) operation. It was confirmed that in the presence of the control input, two equal-amplitude regions exist because of the adjustment

of the feedback input. It was also shown that in the absence of the control input, the nonlinear MEMS resonator maintains its original logical state. Through the experiments and numerical simulations, a device that combines an OR gate and memory functions operating at room temperature was demonstrated in the single MEMS resonator. In addition, a reprogrammable logic-memory device of the nonlinear MEMS resonator was numerically investigated. This chapter numerically realized the reprogrammable logic function that consists of OR/AND gate by adjusting the excitation amplitude and the memory function by storing logic information in the single nonlinear MEMS resonator.

This dissertation realized read, write, counter, and logic-memory operations of the nonlinear MEMS resonator by the self-sensing method, the feedback control, the electronically coupled system, and the closed loop control. Chapters 4, 5, and 6 focused on the excitation force under control. In each chapter, the dc bias voltage and control input were given as shown in Tab. 7.1. By relatively easily changing the control system, memory and logic devices were realized. It is believed that four operations will be keys to develop histories of mechanical computation and new functions in the application of nonlinear MEMS.

7.2 Future work

In this study, a single MEMS resonator has two states and works as a mechanical 1-bit memory and/or logic devices. In the nonlinear dynamics described by Eq. (2.2.4), there appear other solutions owing to parameter changes [75]. Furthermore, due to nonlinear design, the nonlinear dynamics has other solutions [75] in the single MEMS resonator. Therefore, the single MEMS resonator may allow multiple-bit devices.

In a logic-memory device, the single nonlinear MEMS resonator has two functions that

Table 7.1: Control input and dc bias voltage. The excitation force under control becomes proportional to the sum of the dc bias voltage and the control input.

Chapter	Operation		Control input	dc bias voltage
4	Write operation		$-V_{\text{ref}} + KV_{\text{ave}}^2$	Constant
5	Counter operation	Res. 1	$-V_{\text{ref}} + K_1 V_{\text{ave1}}^2$	Constant
		Res. 2	$K_{\text{con2}} V_{\text{ave2}}^2$	$K_2 V_{\text{ave1}}^2$
6	Logic-memory operation		$L_{\text{in1}} + L_{\text{in2}} - KV_{\text{ave}}^2$	Constant

correspond to logic gate and memory. Furthermore, the nonlinear MEMS resonator allows a reprogrammable function. Based on these results, it is possible to include additional functions such as energy harvesting, sensing, learning and so on in the single MEMS resonator. In addition, the MEMS resonator may work as an on-demand device [76] that offers many functions. These studies open the way to research of the system of high reliability.

Appendix A

Classification of stability

This appendix shows the classification of the stability of vibrations as mentioned in Section 2.2.2 and Section 2.2.3. Let us rewrite Eq. (2.2.4) as a system of first-order equations,

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) = \mathbf{f}(\mathbf{x}, t + T), \quad (\text{A.0.1})$$

$$\mathbf{x} = (x, y)^\dagger \in \mathbb{R}^2, \quad (\text{A.0.2})$$

with a T -period orbit $\mathbf{x}_f(t) = \mathbf{x}_f(t + T)$.

When a stroboscopic map $\mathbf{P}$ for the system Eq. (A.0.1) can be constructed, Eq. (A.0.1) is described by

$$\mathbf{x}_{k+1} = \mathbf{P}(\mathbf{x}_k), \quad (\text{A.0.3})$$

where $(x_k, y_k) = (x(t; t_k, x_k, y_k), y(t; t_k, x_k, y_k))$ and $(x_{k+1}, y_{k+1}) = (x(t; t_k + T, x_{k+1}, y_{k+1}), y(t; t_k + T, x_{k+1}, y_{k+1}))$. (x_k, y_k) is a solution of Eq. (A.0.1) which is at the point $\mathbf{P}(\mathbf{x}_k)$ of the xy plane when $t = t_k$. Here k is an integer. $\mathbf{x}_k$ is called the stroboscopic point [57, 58]. For the stroboscopic map $\mathbf{P}$, the T -period orbit $\mathbf{x}_f(t) = \mathbf{x}_f(t + T)$ is given as

$$\mathbf{x}_f = \mathbf{P}(\mathbf{x}_f). \quad (\text{A.0.4})$$

Here $\mathbf{x}_f$ is called a fixed point. Therefore, the fabricated MEMS resonator has three fixed points.

In order to perform the classification of the stability, the neighboring behavior of the fixed point $\mathbf{x}_f$ is approximated by linearizing the map $\mathbf{P}$

$$\Delta\mathbf{x}_{k+1} = D\mathbf{P}(\mathbf{x}_f)\Delta\mathbf{x}_k, \quad (\text{A.0.5})$$

where $\mathbf{x}_{k+1} = \mathbf{x}_f + \Delta\mathbf{x}_{k+1}$, $\mathbf{x}_k = \mathbf{x}_f + \Delta\mathbf{x}_k$, and $D \equiv \partial/\partial\mathbf{x}$. The linear map $D\mathbf{P}(\mathbf{x}_f)$ is an 2×2 matrix. Two eigenvalues λ_1, λ_2 of the matrix determine the stability of the fixed point x_f . If both $|\lambda_1|$ and $|\lambda_2|$ are different from unity, simple fixed points are classified as follows [57, 58]:

- Completely stable if $|\lambda_1| < 1, |\lambda_2| < 1$,
- Completely unstable if $|\lambda_1| > 1, |\lambda_2| > 1$,
- Directly unstable if $0 < \lambda_1 < 1 < \lambda_2$,
- Inversely unstable if $\lambda_2 < -1 < \lambda_1 < 0$.

In this study, the stable fixed points correspond to the completely stable and the unstable fixed point is the directly unstable.

Appendix B

Nonlinear analysis

The following non-dimensionalized equation represents the nonlinear MEMS resonator by substituting $t_n = 2\pi f_0 t$ and $x = A_c x^*$ in Eq. (3.2.1):

$$\frac{d^2 x^*}{dt_n^2} + \frac{1}{Q} \frac{dx^*}{dt_n} + x^* + \alpha_3^* x^{*3} = B \sin \omega^* t_n \quad (\text{B.0.1})$$

where $\alpha_3^* = A_c^2 \alpha_3 / 4\pi^2 f_n^2$, $B = F_{\text{all}} / 4\pi^2 f_n^2 A_c m$, and $\omega^* = f_n / f_0$.

When the method of harmonic balance [75] is applied to the system (B.0.1), a periodic solution may be chosen as

$$x^* = x_\omega \sin \omega^* t_n + y_\omega \cos \omega^* t_n + x_{3\omega} \sin 3\omega^* t_n + y_{3\omega} \cos 3\omega^* t_n \quad (\text{B.0.2})$$

Substituting Eq. (B.0.2) into Eq. (3.2.1) and equating the coefficients of the terms containing $\sin \omega^* t$, $\cos \omega^* t$, $\sin 3\omega^* t$, $\cos 3\omega^* t$ separately to zero yields

$$\begin{aligned} & ((\omega^{*2} - 1) - 3\alpha_3^*(x_\omega^2 + y_\omega^2 + 2(y_{3\omega}^2 + x_{3\omega}^2)) / 4) x_\omega \\ & + (\omega^* / Q) y_\omega + 3\alpha_3^*(x_\omega^2 - y_\omega^2) x_{3\omega} / 4 + 3\alpha_3^* x_\omega y_\omega y_{3\omega} / 2 = B, \end{aligned} \quad (\text{B.0.3})$$

$$\begin{aligned} & (\omega^* / Q) x_\omega - ((\omega^{*2} - 1) - 3\alpha_3^*(x_\omega^2 + y_\omega^2 + 2(y_{3\omega}^2 + x_{3\omega}^2)) / 4) y_\omega \\ & - 3\alpha_3^*(x_\omega^2 - y_\omega^2) y_{3\omega} / 4 + 3\alpha_3^* x_\omega y_\omega x_{3\omega} / 2 = 0, \end{aligned} \quad (\text{B.0.4})$$

$$\begin{aligned} & ((9\omega^{*2} - 1) - 3\alpha_3^*(2(x_\omega^2 + y_\omega^2) + y_{3\omega}^2 + x_{3\omega}^2) / 4) x_{3\omega} + 3(\omega^* / Q) y_{3\omega} \\ & + x_\omega \alpha_3^*(x_\omega^2 - 3y_\omega^2) / 4 = 0, \end{aligned} \quad (\text{B.0.5})$$

$$\begin{aligned} & 3(\omega^* / Q) x_{3\omega} - ((9\omega^{*2} - 1) - 3\alpha_3^*(2(x_\omega^2 + y_\omega^2) + y_{3\omega}^2 + x_{3\omega}^2) / 4) y_{3\omega} \\ & - y_\omega \alpha_3^*(3x_\omega^2 - y_\omega^2) / 4 = 0 \end{aligned} \quad (\text{B.0.6})$$

$$r_1^2 = x_\omega^2 + y_\omega^2, \quad r_3^2 = x_{3\omega}^2 + y_{3\omega}^2 \quad (\text{B.0.7})$$

Figure B.1(a) (B.1(b)) shows the amplitude-frequency response curves for harmonic oscillations (higher-harmonic oscillations). Therefore, it is expected that the MEMS resonator has the vibrations at a single frequency.

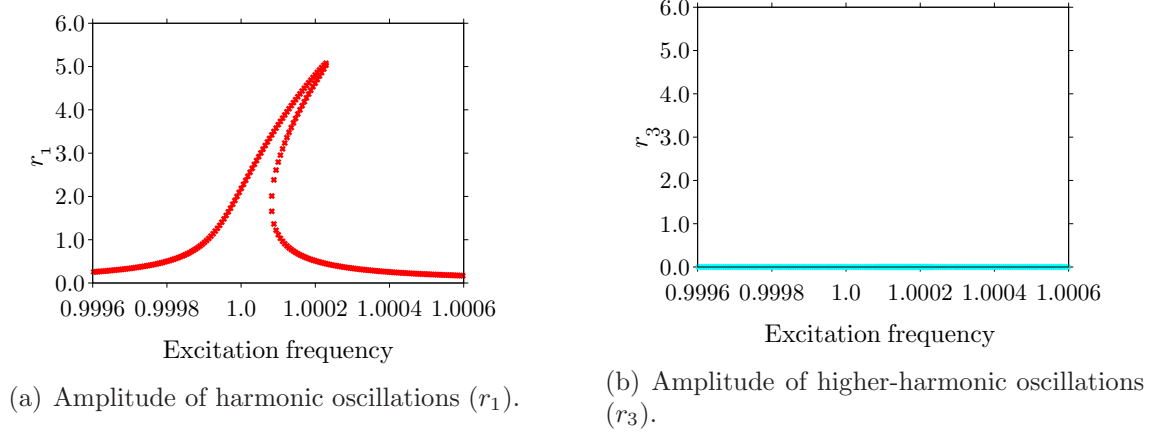


Figure B.1: Frequency response curves.

Appendix C

RF pulse control

In this appendix, we confirm the previously achieved results by Unterreithmeier *et al.* [27]. The system with control signal $u(t)$ is given by

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = -\frac{y}{Q} - x - \alpha_3 x^3 + k_1 \cos \omega t + u(t). \end{cases} \quad (\text{C.0.1})$$

Here the quality factor, the coefficient of cubic correction to the linear restoring force, the excitation amplitude, the excitation frequency are set to Q ($= 1.2 \times 10^5$), α_3 ($= 1.273 \times 10^{-5}$), k_1 ($= 9.818 \times 10^{-5}$), and ω ($= 1.0001245210$), respectively. The parameter settings are due to Ref. [27].

They applied a radio-frequency (RF) pulse to control signal $u(t)$ [27]:

$$u(t) = k_p \cos(\omega t + \varphi), \quad (\text{C.0.2})$$

where k_p ($= 1.767 \times 10^{-3}$) denotes the amplitude, and φ the phase of the RF pulse. The phase of the RF pulse φ is set at 77° (233°) for the control to the small (large) amplitude vibration. The RF pulse with adjustable duration t_p ($= 6470$) is used. The RF pulse control possibly switches the state between the stable vibrations when the amplitude, phase, and duration of the RF pulse are adjusted.

Figure C.1 shows the results controlled by the RF pulse. The red and aqua lines show the loci of map with the RF pulse from small to large and from large to small amplitude vibrations, respectively. Unterreithmeier *et al.* employ their extended knowledge to perform fast switching between the stable states no longer bound by relaxation times. The duration of the RF pulse was about 1030 periods.

Figure C.2 shows the initial value dependence by RF pulse control. In Fig. C.2(a) (C.2(b)), yellow (red) grid points indicate the initial states of convergence to the small (large) amplitude vibration. These results show that the RF pulse control can not insure the convergence exactly in the initial state plane for both directions. The complicated convergence area appears in the simple basin. The background mechanism is not clear at this moment. Therefore, it is considered the RF pulse control is weak for application from the standpoint of control engineering.

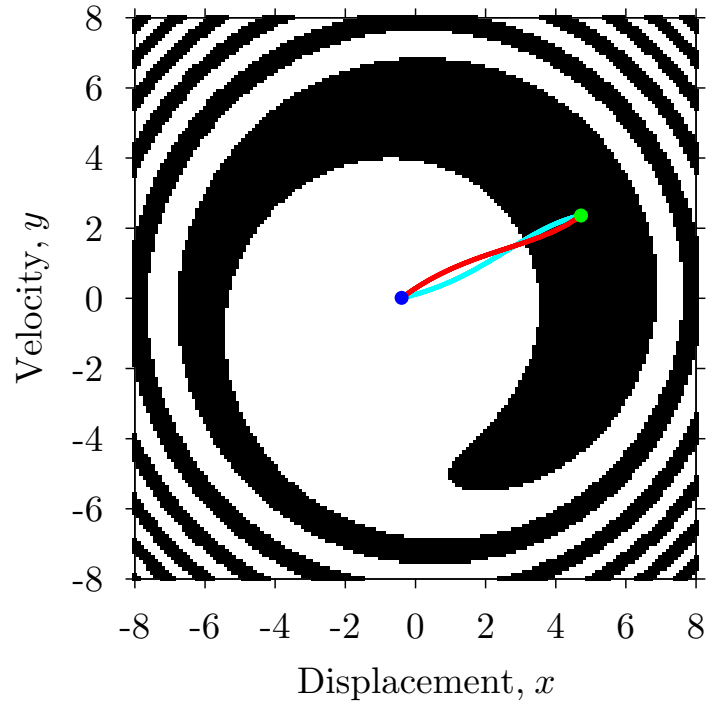
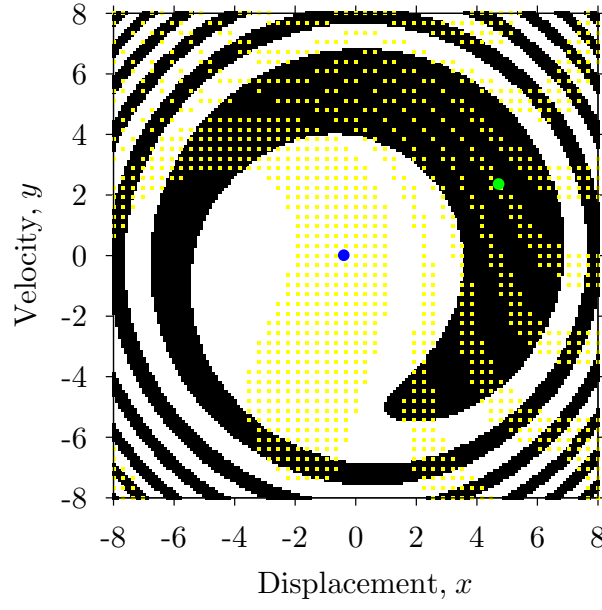
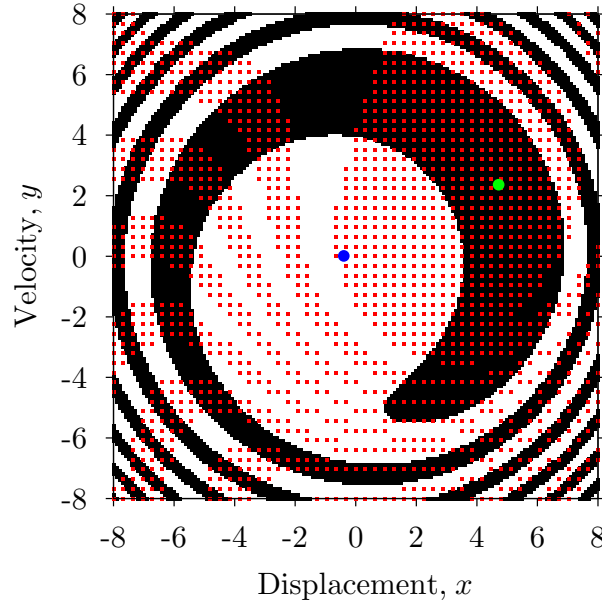


Figure C.1: Switching between stable periodic vibrations by RF pulse control: The basins of attraction corresponding to the two states are separated by the manifold of the unstable solutions. The red and aqua lines show the switching from small to large and from large to small amplitude vibrations, respectively.



(a) Region of convergence to large amplitude vibration.



(b) Region of convergence to small amplitude vibration.

Figure C.2: Initial value dependence of convergence by RF pulse control: Yellow and Red grid points show the region of convergence to large and small amplitude vibrations, respectively.

Bibliography

- [1] H. H. Goldstine, The computer from Pascal to von Neumann, (Princeton University Press, 1972).
- [2] P. E. Ceruzzi, Computing: A Concise History, (MIT Press, 2012).
- [3] J. Magnuson and P. C. Fu, Public Health Informatics and Information Systems, (Springer, 2014).
- [4] C. Eames and R. Eames, A Computer Perspective: Background to the Computer Age, (Harvard University Press, 1990).
- [5] D. M. Harris and S. L. Harris, Digital design and computer architecture, (Morgan Kaufmann, 2007).
- [6] B. Collier and J. MacLachlan, Charles Babbage: And the Engines of Perfection, (Oxford University Press, 2000).
- [7] J. W. Cortada, Before the computer: IBM, NCR, Burroughs, and Remington Rand and the industry they created, 1865-1956, (Princeton University Press, 2000).
- [8] IPSJ Computer Museum, <http://museum.ipsj.or.jp/en/index.html>.
- [9] G. O'Regan, A brief history of computing 2nd ed., (Springer, 2012).
- [10] L. M. Roylance and J. B. Angell, A batch-fabricated silicon accelerometer, *Electron Devices, IEEE Transactions on*, **26** (12), 1911–1917 (1979).
- [11] E. Bassous, H. H. Taub, and L. Kuhn, Ink jet printing nozzle arrays etched in silicon, *Applied Physics Letters*, **31** (2), 135–137 (1977).
- [12] U. Shah, M. Sterner, and J. Oberhammer, High-directivity MEMS-tunable directional couplers for 10–18-GHz broadband applications, (2013).

- [13] N. Somjit, G. Stemme, and J. Oberhammer, Power handling analysis of high-power-band all-silicon MEMS phase shifters, *Electron Devices, IEEE Transactions on*, **58** (5), 1548–1555 (2011).
- [14] U. Shah, M. Sterner, G. Stemme, and J. Oberhammer, Multi-position large tuning-range digitally tuneable capacitors embedded in 3D micromachined transmission lines, Micro Electro Mechanical Systems (MEMS), 2011 IEEE 24th International Conference on, 165–168 (2011).
- [15] M. Sterner, N. Roxhed, G. Stemme, and J. Oberhammer, Static zero-power-consumption coplanar waveguide embedded DC-to-RF metal-contact MEMS switches in two-port and three-port configuration, *Electron Devices, IEEE Transactions on*, **57** (7), 1659–1669 (2010).
- [16] J. A. Pelesko and D. H. Bernstein, Modeling MEMS and NEMS, (CHAMPMAN and HALL/CRC, 2003).
- [17] V. Kaajakari, Practical MEMS, (Small Gear Publishing, 2009).
- [18] H. C. Nathanson and R. A. Wickstrom, A resonant-gate silicon surface transistor with high-Q band-pass properties, *Applied physics letters*, **7** (4), 84–86 (1965).
- [19] H. C. Nathanson, W. E. Newell, R. A. Wickstrom, and J. R. Davis Jr, The resonant gate transistor, *Electron Devices, IEEE Transactions on*, **14** (3), 117–133 (1967).
- [20] J. F. Rhoads, S. W. Shaw, and K. L. Turner, Nonlinear dynamics and its applications in micro-and nanoresonators, *Journal of Dynamic Systems, Measurement, and Control*, **132** (3), 034001 (2010).
- [21] R. L. Badzey, G. Zolfagharkhani, A. Gaidarzhy, and P. Mohanty, A controllable nanomechanical memory element, *Applied Physics Letters*, **85** (16), 3587–3589 (2004).
- [22] R. L. Badzey and P. Mohanty, Coherent signal amplification in bistable nanomechanical oscillators by stochastic resonance, *Nature (London)*, **437** (7061), 995–998 (2005).

- [23] S. C. Masmanidis, R. B. Karabalin, I. De Vlaminck, G. Borghs, M. R. Freeman, and M. L. Roukes, Multifunctional nanomechanical systems via tunably coupled piezoelectric actuation, *Science*, **317** (5839), 780–783 (2007).
- [24] I. Mahboob and H. Yamaguchi, Bit storage and bit flip operations in an electromechanical oscillator, *Nature Nanotechnology*, **3** (5), 275–279 (2008).
- [25] D. N. Guerra, A. R. Bulsara, W. L. Ditto, S. Sinha, K. Murali, and P. Mohanty, A noise-assisted reprogrammable nanomechanical logic gate, *Nano Letters*, **10** (4), 1168–1171 (2010).
- [26] H. Noh, S.-B. Shim, M. Jung, Z. G. Khim, and J. Kim, A mechanical memory with a dc modulation of nonlinear resonance, *Applied Physics Letters*, **97** (3), 033116–1–033116–3 (2010).
- [27] Q. P. Unterreithmeier, T. Faust, and J. P. Kotthaus, Nonlinear switching dynamics in a nanomechanical resonator, *Physical Review B*, **81** (24), 241405–1–241405–4 (2010).
- [28] I. Mahboob, E. Flurin, K. Nishiguchi, A. Fujiwara, and H. Yamaguchi, Interconnect-free parallel logic circuits in a single mechanical resonator, *Nature Communications*, **2**, 198 (2011).
- [29] D. Hatanaka, I. Mahboob, H. Okamoto, K. Onomitsu, and H. Yamaguchi, An electromechanical membrane resonator, *Applied Physics Letters*, **101** (6), 063102–1–063102–5 (2012).
- [30] A. Yao and T. Hikihara, Reading and writing operations of memory device in micro-electromechanical resonator, *IEICE Electronics Express*, **9** (14), 1230–1236 (2012).
- [31] A. Yao and T. Hikihara, Read and Write Operations of Memory Device Consisting of Nonlinear MEMS Resonator, *Proc. of NOLTA2012*, 352–355 (2012).
- [32] N. A. Khovanova and J. Windelen, Minimal energy control of a nanoelectromechanical memory element, *Applied Physics Letters*, **101** (2), 024104–024104 (2012).
- [33] J.-S. Wenzler, T. Dunn, T. Toffoli, and P. Mohanty, A Nanomechanical Fredkin Gate, *Nano letters*, **14** (1), 89–93 (2013).

- [34] A. Uranga, J. Verd, E. Marigó, J. Giner, J. L. Muñoz-Gamarra, and N. Barniol, Exploitation of non-linearities in CMOS-NEMS electrostatic resonators for mechanical memories, *Sensors and Actuators A: Physical*, **197**, 88–95 (2013).
- [35] A. Yao and T. Hikihara, Counter operation in nonlinear micro-electro-mechanical resonators, *Physics Letters A*, **377** (38), 2551–2555 (2013).
- [36] I. Mahboob, M. Mounaix, K. Nishiguchi, A. Fujiwara, and H. Yamaguchi, A multimode electromechanical parametric resonator array, *Scientific reports*, **4** (4448), (2014).
- [37] I. Mahboob and H. Yamaguchi, Bit storage and bit flip operations in an electromechanical oscillator (Supplementary Information), *Nature nanotechnology*, **3** (5), 275–279 (2008).
- [38] M. L. Roukes, Mechanical computation, redux?, Electron Devices Meeting, 2004. IEDM Technical Digest. IEEE International, 539–542 (2004).
- [39] A. Yao and T. Hikihara, Logic-memory device of a mechanical resonator, *Applied Physics Letters*, **105** (12), 123104 (2014).
- [40] M. V. Andres, K. W. H. Foulds, and M. J. Tudor, Nonlinear vibrations and hysteresis of micromachined silicon resonators designed as frequency-out sensors, *Electronics Letters*, **23** (18), 952–954 (1987).
- [41] A. Frangi, Advances in multiphysics simulation and experimental testing of MEMS, Vol. 2, (World Scientific Publishing, 2008).
- [42] R. M. C. Mestrom, R. H. B. Fey, J. T. M. van Beek, K. L. Phan, and H. Nijmeijer, Modelling the dynamics of a MEMS resonator: Simulations and experiments, *Sensors and Actuators A: Physical*, **142** (1), 306–315 (2008).
- [43] S. Naik, T. Hikihara, H. Vu, A. Palacios, V. In, and P. Longhini, Local bifurcations of synchronization in self-excited and forced unidirectionally coupled micromechanical resonators, *Journal of Sound and Vibration*, **331** (5), 1127–1142 (2012).
- [44] D. Antonio, D. H. Zanette, and D. López, Frequency stabilization in nonlinear micromechanical oscillators, *Nature Communications*, **3**, 806 (2012).

- [45] E. Goto, The parametron, a digital computing element which utilizes parametric oscillation, *Proceedings of the IRE*, **47** (8), 1304–1316 (1959).
- [46] H. Takahasi and K. Zen'iti, Parametron, (Parametron institute, 1960).
- [47] Q. P. Unterreithmeier, E. M. Weig, and J. P. Kotthaus, Universal transduction scheme for nanomechanical systems based on dielectric forces, *Nature*, **458** (7241), 1001–1004 (2009).
- [48] K. Pyragas, Continuous control of chaos by self-controlling feedback, *Physics Letters A*, **170** (6), 421–428 (1992).
- [49] S. Naik and T. Hikiyara, Numerical study on the quasi-periodic behavior in coupled MEMS resonators, *NLP2010-10*, **110** (255), 25–30 (2010).
- [50] S. Naik and T. Hikiyara, Characterization of a MEMS resonator with extended hysteresis, *IEICE Electronics Express*, **8** (5), 291–298 (2011).
- [51] S. Naik, Investigation of synchronization in a ring of coupled MEMS resonators, Doctoral Dissertation, (Kyoto University, 2011).
- [52] A. Cowen, G. Hames, D. Monk, S. Wilcenski, and B. Hardy, SOIMUMPs design handbook, *MEMSCAP Inc*, (2011).
- [53] S. Naik, T. Hikiyara, A. Palacios, V. In, H. Vu, and P. Longhini, Characterization of synchronization in a unidirectionally coupled system of nonlinear micromechanical resonators, *Sensors and Actuators A: Physical*, **171** (2), 361–369 (2011).
- [54] A. Yao and T. Hikiyara, Logical Behavior in Memory Devices of Coupled Nonlinear MEMS Resonators, *Proc. of NOLTA2013*, 30–33 (2013).
- [55] H. B. Stewart and J. M. Thompson, Nonlinear Dynamics and Chaos, (John Wiley and Sons, 1986).
- [56] J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer, 1983) .
- [57] Y. Ueda, Theory of Chaotic Phenomena, (Corona publishing co., ltd., 2008 (in Japanese)).

- [58] Y. Ueda, R. H. Abraham, and H. B. Stewart, The road to chaos, (1992).
- [59] R. L. Tokheim, Schaum's outline of theory and problems of digital principles, (McGraw-Hill, 1994).
- [60] L. F. Lind and J. C. C. Nelson, Analysis and Design of Sequential Digital Systems, (Macmillan, 1977).
- [61] M. M. Mano, Digital Design (Prentice-Hall, 1991) Chap. 6.
- [62] V. Kempe, Inertial MEMS: Principles and Practice (Cambridge University Press, 2011) Chap. 6.
- [63] I. A. Ivan, M. Rakotondrabe, P. Lutz, and N. Chaillet, Quasistatic displacement self-sensing method for cantilevered piezoelectric actuators, *Review of Scientific Instruments*, **80** (6), 065102–1–065102–8 (2009).
- [64] K. Matsuda, Y. Okada, and J. Tani, Self-sensing magnetic bearing using the differential transformer principle, *Transactions of the Japan Society of Mechanical Engineers. C*, **63** (609), 1441–1447 (1997 (in Japanese)).
- [65] Y. Umezaki, A. Yao, and T. Hikiyara, Parameter Estimation of Nonlinear Micro Electromechanical Resonator, *Proc. of the 2014 IEICE General Conference*, **A-2-7**, 32 (2014 (in Japanese)).
- [66] E. Ott, C. Grebogi, and J. A. Yorke, Controlling chaos, *Physical review letters*, **64** (11), 1196 (1990).
- [67] T. Hikiyara and T. Kawagoshi, An experimental study on stabilization of unstable periodic motion in magneto-elastic chaos, *Physics Letters A*, **211** (1), 29–36 (1996).
- [68] K. Yamasue, K. Kobayashi, H. Yamada, K. Matsushige, and T. Hikiyara, Controlling chaos in dynamic-mode atomic force microscope, *Physics Letters A*, **373** (35), 3140–3144 (2009).
- [69] F. Alsaleem and M. I. Younis, Integrity analysis of electrically actuated resonators with delayed feedback controller, *Journal of Dynamic Systems, Measurement, and Control*, **133** (3), 031011 (2011).

- [70] K. Yamasue and T. Hikiyara, Control of microcantilevers in dynamic force microscopy using time delayed feedback, *Review of scientific instruments*, **77** (5), 053703 (2006).
- [71] T. Hikiyara and Y. Ueda, An expansion of system with time delayed feedback control into spatio-temporal state space, *Chaos: An Interdisciplinary Journal of Nonlinear Science*, **9** (4), 887–892 (1999).
- [72] Y. Yokoi and T. Hikiyara, Tolerance of start-up control of rotation in parametric pendulum by delayed feedback, *Physics Letters A*, **375** (17), 1779–1783 (2011).
- [73] Y. Yokoi and T. Hikiyara, Start Control of Parametric Pendulum into Periodic Rotation, *ISCIE Journal Systems, Control and Information*, **24** (3), 54–60 (2011 (in Japanese)).
- [74] A. Yao and T. Hikiyara, Switching Control between Stable Periodic Vibrations in a Nonlinear MEMS Resonator, *Proc. of NOLTA2011*, 709–712 (2011).
- [75] C. Hayashi, Nonlinear oscillations in physical systems, (Princeton Univ Press, 1964).
- [76] R. Yang, K. Terabe, G. Liu, T. Tsuruoka, T. Hasegawa, J. K. Gimzewski, and M. Aono, On-demand nanodevice with electrical and neuromorphic multifunction realized by local ion migration, *ACS nano*, **6** (11), 9515–9521 (2012).

List of Publications

Journal articles

- A. Yao and T. Hikihara, Reading and writing operations of memory device in micro-electromechanical resonator, *IEICE Electronics Express*, **9** (14), 1230–1236, (2012).
- A. Yao and T. Hikihara, Counter operation in nonlinear micro-electro-mechanical resonators, *Physics Letters A*, **377** (38), 2551–2555, (2013).
- A. Yao and T. Hikihara, Logic-memory device of a mechanical resonator, *Applied Physics Letters*, **105** (12), 123104-1–4, (2014).

International conference proceedings

- A. Yao and T. Hikihara, Switching Control between Stable Periodic Vibrations in a Nonlinear MEMS Resonator, 2011 International Symposium on Nonlinear Theory and its Applications (NOLTA2011), 709–712, Kobe Industrial Promotion Center, Japan, September 4–7, 2011.
- A. Yao and T. Hikihara, Read and Write Operations of Memory Device Consisting of Nonlinear MEMS Resonator, 2012 International Symposium on Nonlinear Theory and its Applications (NOLTA2012), 352–355, Gran Melia Victoria, Majorca, Spain, October 22–26, 2012
- A. Yao and T. Hikihara, Logical Behavior in Memory Devices of Coupled Nonlinear MEMS Resonators, 2013 International Symposium on Nonlinear Theory and its Applications (NOLTA2013), 30–33, Santa Fe Community Convention Center, USA, September 8–11, 2013.

- A. Yao and T. Hikihara, A Study on Reprogrammable Logic-Memory Device of a Nonlinear Microelectromechanical Resonator, ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, Boston, USA, August 2–5, 2015 (under review).

Conference presentations

- A. Yao and T. Hikihara, A Numerical Study on Switching between Stable Periodic Motions in a Nanomechanical Resonator, 2010 MicRO Alliance in Funai Hall, B-29, Kyoto, Japan, November 22, 2010.
- A. Yao and T. Hikihara, Switching and Measurement of Coexisting Stable Periodic Vibrations in MEMS Resonator: Towards Read and Write Operations as Memory Devices, 2011 Kyoto Workshop on NOLTA, NP10, Kyoto, Japan, November 30, 2011.
- A. Yao and T. Hikihara, A Numerical Study on Combined Device of Memory and Logic Gate in a MEMS Resonator, The 10th MicRO Alliance Meeting, Kyoto University Funai Tetsuro Auditorium, Japan, November 21, 2013.

Domestic conference proceedings

- A. Yao, S. Naik, and T. Hikihara, A Study on Switching Method for Coexisting Solutions in a Nonlinear MEMS Resonator, IEICE Technical Report, NLP2010-179, Tokyo University of Science, March 10, 2011 (in Japanese).
- A. Yao and T. Hikihara, A Numerical Study on Switching between Stable Periodic Vibrations in a Mechanical Resonator, The 2011 Annual Meeting of the Institute of Electrical Engineers of Japan. 3-139, Osaka University, March 17, 2011 (in Japanese).
- A. Yao and T. Hikihara, Displacement measurement of comb-drive resonator with differential measurement, The 2012 Annual Meeting of the Institute of Electrical Engineers of Japan, 3-123, Hiroshima Institute of Technology, March 21, 2012 (in Japanese).

- A. Yao and T. Hikihara, A Study on Read and Write Operations of Memory Device of Nonlinear MEMS Resonator, IEICE Technical Report, NLP2012-2, Ise City Plaza, April 19, 2012 (in Japanese).
- A. Yao and T. Hikihara, A Study on Displacement Measurement and its Control in Micro Electro Mechanical Resonator, The 56th Annual Conference of the Institute of Systems, Control and Information Engineers, W45-5, Kyoto Terrsa, May 23, 2012 (in Japanese).
- A. Yao and T. Hikihara, An Approach to Logical Behavior of Memory Devices in Coupled MEMS Resonators, IEICE Technical Report, NLP2012-102, Fukui City Communication Plaza, December 18, 2012.
- A. Yao and T. Hikihara, An Experimental Study of Counter Operation by Nonlinear Coupled MEMS-Resonator Memory, The 57th Annual Conference of the Institute of Systems, Control and Information Engineers, 216-4, Hyogo Prefecture Citizens' Hall, May 16, 2013 (in Japanese).
- A. Yao and T. Hikihara, A Study on Reprogrammable Logic-Memory Device of an Electro-Mechanical Resonator, The Papers of Technical Meeting on "Electronic Circuits", Tokyo Metropolitan University (Akihabara Satellite Campus), November 21, 2014 (in Japanese).