

2-factorization theorem in finite groups

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1. 2-factorization in finite solvable groups

Let π be a set of primes which contains 2.

Hypothesis A. Let S be a non-identity finite 2-group and G a finite (solvable) group.

(A.1) S is a Sylow 2-subgroup of G .

(A.2) $O(G) = 1$.

(A.3) G is a π -group.

Question 1. Let S be a non-identity finite 2-group. Then does there exists a non-identity characteristic subgroup $W(S)$ of S which is normal in G , for any finite (solvable) group G which satisfies Hypothesis A ?

If π contains 3, there exists the following counter example group of Question 1 ; the semi-direct product group of a four group and $SL(2,2)$, i.e. a symmetric group of degree 4.

G. Glauberman proposed the question 1 as above, in the case $\pi = \{p ;$

prime $\neq 3$ }, in his paper [2]. In the case S is a p -group, p odd, this question had been solved by himself [1] so called ZJ-theorem, furthermore he showed a counter example group of ZJ-theorem in the case $p = 2$.

In the case $p = 2$, the almost only one result is one of J. G. Thompson [4], which is obtained until now. And he proposed an interesting conjecture [5]. After the congress in Kyoto, the autor may obtain the following result ;

Theorem [3]. In the case $\pi = \{2, p ; 2^m \not\equiv -1 \pmod{p}, m = 1, 2, \dots, p-1\}$, Question 1 is affirmative.

2. 2-factorization in finite non-solvable group

Hypothesis B. Let S be a finite 2-group and G a finite (non-solvable) group.

(B.1) S is a Sylow 2-subgroup of G .

(B.2) $C_G(O_2(G)) \subseteq O_2(G)$.

Question 2. Assume that S and G satisfy Hypothesis B. If $G \neq N_G(J(S))C_G(Z(S))$, we may say about G or $O_2(G)$?

If $G/O_2(G)$ is a simple group of Lie type of characteristic odd type, (in the half case) we have $G = N_G(J(S))C_G(Z(S))$.

Question 3. Assume that S and G satisfy Hypothesis B. Set $\bar{G} = G/\Phi(O_2(G))$ and $V = O_2(G)/\Phi(O_2(G)) = V_1 \times \dots \times V_r \times C_V(G)$, where V_i is an $\bar{G}$ -irreducible component of V on which $G/O_2(G)$ acts faithfully. (Note that we may not always set as this !) Suppose that $r \geq 2$. Then $G = N_G(J(S))C_G(Z(S))$?

References

- [1] G. Glauberman, A characteristic subgroup of a p -stable group, Can. J. Math. 20 (1968), 1101-1135.
- [2] G. Glauberman, Subgroups of finite groups, Bull. Amer. Math. Soc. 73 (1967), 1-12.
- [3] M. Hayashi, 2-factorization in finite solvable groups, to appear.
- [4] J. G. Thompson, Factorizations of p -solvable groups, Pacific J. Math. 16 (1966) 371-372.
- [5] J. G. Thompson, Replacement theorem for finite groups and a conjecture, J. Algebra 13 (1969), 149-151.