

KYOTO UNIVERSITY

DOCTORAL THESIS

**Analysis of $\Lambda(1405)$ based on
chiral SU(3) dynamics**

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Abstract

In recent hadron physics, one of the most active fields is the search for exotic hadrons with different structure from the ordinary meson (quark-antiquark) or baryon (three-quark) states. The $\Lambda(1405)$ baryon resonance is a promising candidates, expected to be the molecular state of an antikaon $\bar{K}$ and a nucleon N decaying to the channel with a pion π and a Sigma baryon Σ . This picture leads to the prediction of related interesting states, $\bar{K}$ nuclei, consisting of an antikaon and nuclei. Thanks to the recent precise experiment by the SIDDHARTA Collaboration, we are at the stage of quantitative discussion to elucidate the nature of these states. However, the two-body potential, which is the fundamental building block for the study of the $\Lambda(1405)$ and $\bar{K}$ nuclei, has not been constructed in a realistic form. In this thesis, we construct reliable potentials between the meson and baryon which compose the $\Lambda(1405)$.

In the former part of this thesis, we establish the potential construction procedure and apply it to the $\bar{K}N$ single-channel and $\bar{K}N$ - $\pi\Sigma$ coupled-channel potentials. First, to theoretically extrapolate the experimental data to the $\Lambda(1405)$ energy region directly inaccessible by the K^-p scattering experiments, we employ the chiral unitary approach which respects the chiral $SU(3)$ symmetry in quantum chromodynamics (QCD) and the unitarity of S matrix of the two-body scattering. Next, we construct the local potential used in the Schrödinger equation which equivalently reproduces the results of chiral unitary approach. Here, the quantitative reproducibility is necessary not only on the real energy axis but also in the complex energy plane, because the essential information of the $\Lambda(1405)$ is included in the complex plane as a resonance pole. With the newly developed procedure, we obtain the energy-dependent complex and real potentials in the single and coupled-channel systems, respectively.

As the first application of the simpler single-channel potential, we calculate the $\bar{K}$ nuclei up to seven-body systems with the accurate few-body calculation method, the stochastic variational method with correlated Gaussian basis. In the $\bar{K}NN$ system, relatively small binding energy is obtained, and in seven-body system, the possibility of the inversion of the level ordering is predicted. For more detailed discussion of some states near the $\pi\Sigma$ threshold, the explicit treatment of lower channels like the $\pi\Sigma$ would be necessary. Next, we theoretically predict the K^-p correlation function in heavy-ion collisions. We find the unique isospin combination in the correlation function, which would lead to complementary information of the $\bar{K}N$ interaction to the ordinary K^-p scattering amplitudes.

In the latter half of this thesis, we discuss how to analyze the wave function from the resulting potential. The difficulty stems from the energy dependence of the potential, which breaks the essential requirements of the quantum mechanics, the norm conservation and the orthogonality relation. To recover these requirements, we derive the new normalization scheme in the non-Hermitian system with the energy-dependent potential, respecting the continuity equation. Furthermore, interpreting the origin of the energy dependence in terms of the channel elimination, we reveal the meaning of the new normalization scheme in association with the compositeness and the elementariness. With this new formulation, we analyze the compositeness and the spatial structure of the $\Lambda(1405)$ to confirm the $\bar{K}N$ molecular picture.

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Contents

Acknowledgements	ii
1 Introduction	1
1.1 Overview of $\Lambda(1405)$	1
1.1.1 Theoretical studies	1
1.1.2 Experiments	4
1.2 Overview of $\bar{K}$ -nuclei	10
1.2.1 Theoretical studies	10
1.2.2 Experiments	13
1.3 Purpose of this work	15
2 Meson-Baryon local potential of $\Lambda(1405)$ system	17
2.1 Formulation	17
2.1.1 Meson-baryon amplitudes from chiral unitary approach	17
2.1.2 Construction of equivalent potential	23
2.2 $\bar{K}N$ single-channel potential	29
2.2.1 Reanalysis of previous work	29
2.2.2 Precision near the real energy axis	32
2.2.3 Precision in large $ \text{Im}\sqrt{s} $ region	34
2.2.4 Realistic potential with SIDDHARTA constraint	35
2.3 Application of $\bar{K}N$ single-channel potential	40
2.3.1 $\bar{K}$ -nuclei	40
2.3.2 K^-p correlation	47
2.4 $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ coupled-channel potential	53
2.4.1 Determination of range parameter	53
2.4.2 Adjustment ΔV	57
2.4.3 Parametrization of potential	59
2.4.4 $I = 1$ channel	61
3 Analysis of $\Lambda(1405)$ wave function	66
3.1 Formulation	66
3.1.1 Normalization of wave function for E -dep. complex potential	66
3.1.2 Relation between new normalization and elementariness	70
3.2 Analysis of $\Lambda(1405)$ wave function	75
3.2.1 $\bar{K}N$ single-channel system	75
3.2.2 $\bar{K}N$ - $\pi\Sigma$ coupled-channel system	80
4 Summary	85
Bibliography	88

Chapter 1

Introduction

In this chapter, we first review the long-standing theoretical and experimental works of the $\Lambda(1405)$ in Sec. 1.1 and a related topic, $\bar{K}$ nuclei in Sec. 1.2. Based on the current status, we show the motivation and purpose of this work in Sec. 1.3.

1.1 Overview of $\Lambda(1405)$

1.1.1 Theoretical studies

The $\Lambda(1405)$ is an excited state of the Λ baryon, with the strangeness $S = -1$, spin-parity $J^P = \frac{1}{2}^-$, and isospin $I = 0$, and the mass is slightly lighter than the $\bar{K}N$ threshold energy (~ 1435 MeV¹) [1–3]. Prior to experimental measurements, the existence is theoretically predicted in 1959 in Refs. [4, 5]. In these works, the possible existence of a resonance in the $\pi\Sigma$ scattering amplitude is pointed out analyzing the $\bar{K}N$ scattering length from low-energy bubble-chamber data with the unitary condition in the $\bar{K}N$ - $\pi\Sigma$ system. As we will show later, the existence and the spin-parity of the $\Lambda(1405)$ are experimentally established, and this state is catalogued by the Particle Data Group (PDG) with four star status [1]. However, there are two unsolved issues, the composite nature and the pole structure of the $\Lambda(1405)$.

The former is related to the too light mass of the $\Lambda(1405)$ which makes it difficult to describe the state by the ordinary three-quark picture in constituent quark models [6–9]. From the simple three-quark model, the $\Lambda(1405)$, the negative parity excited state of the Λ , is expected to be at least heavier than the corresponding excited state of the nucleon, the $N(1535)$. Furthermore, the approximate degeneracy of the spin-orbit partner, the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ states, is also expected as in the excited states of the nucleon, $N(1535)$ and $N(1520)$. Actually, regarding the Σ baryon which contains the strange quark as in the Λ , it is experimentally established that the excited state with $J^P = \frac{1}{2}^-$, the $\Sigma(1620)$, is heavier than the $N(1535)$, and close to the $J^P = \frac{3}{2}^-$ state, the $\Sigma(1580)$, within the 40 MeV difference. In contrast, regarding the $\Lambda(1405)$, the mass is unexpectedly much smaller than the $N(1535)$ by more than 100 MeV, though it includes the heavier strange quark. Furthermore, there is more than 100 MeV difference from the spin-orbit partner, the $\Lambda(1520)$, which is definitely different from the Σ case (~ 40 MeV difference) as well as the nucleon case (~ 15 MeV difference). We illustrate the situation in Fig. 1.1. Due to this unnaturally small mass, the $\Lambda(1405)$ has been paid a lot of attention, and considered to be an "exotic hadron", which has a different structure from ordinary three-quark states. To reveal the inner structure, the concept of compositeness [10, 11], which represents the

¹In the charge basis, the K^-p and $\bar{K}^0n$ thresholds are 1432 MeV and 1437 MeV, respectively. In this thesis, unless otherwise specifically noted, we use the isospin averaged value as the $\bar{K}N$ threshold.

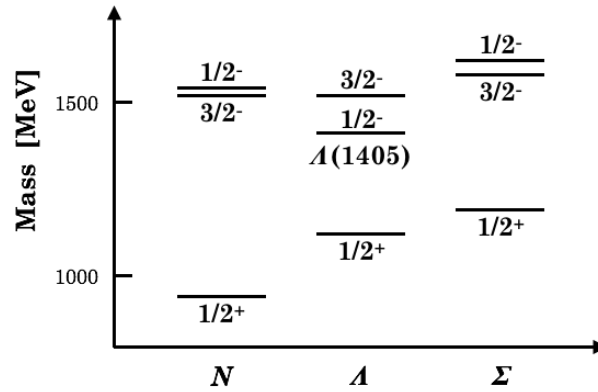


FIGURE 1.1: Mass spectrum of negative parity baryons. The $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ states are shown for the nucleon, Λ , and Σ . For comparison, we show the ground states.

norm of a two-body state, has been recently applied to the $\Lambda(1405)$, and it has been suggested that the $\Lambda(1405)$ seems mainly consist of the $\bar{K}N$ quasibound state [12].

Apart from the composite nature, the resonance pole structure is also controversial problem for the $\Lambda(1405)$. Generally, a bound or resonance state is represented by a pole of the scattering amplitude in the complex energy plane, one-to-one. On the other hand, in some studies, it is suggested that the $\Lambda(1405)$ is a superposition of two resonance poles, which is known as the double pole structure [2, 3, 13]. However, other models only with single resonance pole can also reproduce present experimental data, and we have not reached a conclusion for the pole structure yet. Not only to reveal the $\Lambda(1405)$ nature but also to give some predictions of other systems like $\bar{K}$ nuclei, it is necessary to clarify the pole structure, too. Theoretically, various approaches have been adopted to describe this interesting $\Lambda(1405)$. As typical examples, we review the quark model, chiral unitary approach, and lattice QCD calculation, here.

In quark models, the simplest one describes a baryon as a three-body state of the constituent quarks. Here, between quarks, confinement interactions and hyperfine interactions are employed. As confinement interactions, harmonic oscillator type or linear type ones are often used. In some cases, hyperfine interactions are constructed from one-gluon exchange and/or meson exchange such as scalar, pseudoscalar, and vector mesons. This simple picture has achieved great success for various baryons including hyperons and excited states [6–9]. In these works, to describe the negative parity of the $\Lambda(1405)$, the quarks are excited in the P -wave state. However, these works failed to reproduce the light $\Lambda(1405)$ mass and the predicted one exceeded the experimental value by around 100 MeV. This result indicates the difficulty to describe the $\Lambda(1405)$ with the simple quark models. An alternative approach has been proposed by considering a $qqqq\bar{q}$ state. This expectation is based on the idea that the existence of an antiquark is responsible for the negative parity without the P -wave excitation in contrast to the three-quark state where the P -wave excitation leads to the higher mass. The five-quark picture for the $\Lambda(1405)$ has been studied with the MIT bag model [14] and the diquark model [15]. The latter is based on the preceding work to describe the Θ^+ state as a compound of two diquarks and one antiquark [16]. Though these models well describe the light nature of the $\Lambda(1405)$, there appeared some extra excited states which have not been experimentally observed.

Next, we briefly show the presently most successful model to describe the $\Lambda(1405)$ as the meson-baryon resonance or quasibound state, chiral unitary approach. This approach is based on the meson-baryon molecular picture of the $\Lambda(1405)$, which has been implied by the difficulty in the quark models. This picture had already been proposed in Ref. [17], and the spectrum of the $\Lambda(1405)$ was well reproduced by the meson-baryon interaction via the vector meson exchange. Concerning the meson-baryon interactions, recent chiral perturbation theory based on the essential feature of QCD, the chiral symmetry, has achieved great success in various systems. Chiral unitary approach is an extended method of chiral perturbation theory to nonperturbative systems like resonant states. For more reliable discussion of the meson-baryon picture, this approach has been adopted to the $\Lambda(1405)$ in many works [18–20], and succeeded in reproducing its spectrum. One of interesting results in chiral unitary approach is the appearance of two resonance poles for the $\Lambda(1405)$, namely, the above mentioned double pole structure [20]². In this approach, the origin of each pole has been analyzed and turned out to be two attractive interactions in the SU(3) singlet and octet channels [13], or the $\bar{K}N$ and $\pi\Sigma$ channels [22]. Recent works, taking into account precise experimental data [23–26], achieve consensus on the qualitative results of the double pole structure and the quantitative results of one of the pole positions which is close to the $\bar{K}N$ threshold with narrow decay width. On the other hand, concerning the other pole, it is impossible to add quantitative constraint at present, while it tends to lie at relatively lower energy with large decay width [3]. We should note that the double pole structure itself is model dependent and still controversial. Thus, further studies are demanded including experimental ones. In Sec. 2.1.1, we will explain chiral unitary approach in more detail.

Finally, we show the application to the $\Lambda(1405)$ of the most reliable first principle calculation of QCD, lattice QCD. Lattice QCD enables the nonperturbative numerical calculation of QCD by discretizing the space-time which is originally continuous in QCD, respecting the gauge symmetry. Lattice QCD has succeeded in reproducing masses of various hadrons, though there is difficulty for light hadrons like the pion because of heavy computational cost. Concerning the $\Lambda(1405)$, in Ref. [27], the mass was extracted with quenched approximation from the baryon correlator, where the $\Lambda(1405)$ was expressed with three-quark operators. The result was approximately 300 MeV higher than the experimental value, which would indicate that the $\Lambda(1405)$ is not a simple three-quark state, and source operator or the quenched approximation should be improved. The quenched calculation with five-quark source operators was conducted in Ref. [28], but it also failed to reproduce the $\Lambda(1405)$ mass. On the other hand, a full-QCD calculation with three-quark operators succeeded in reproducing the $\Lambda(1405)$ below the $\bar{K}N$ threshold [29]. Here the pion mass ranged from 702 down to 156 MeV. The same conclusion was confirmed in Ref. [30]. Recently, from the lattice calculation, it has been indicated that the $\Lambda(1405)$ is the meson-baryon state, specifically, the $\bar{K}N$ state [31]. In this work, the strange quark contribution to the magnetic form factor of the $\Lambda(1404)$ was calculated and revealed to vanish near the physical quark masses, which can be naturally explained by the $\bar{K}N$ picture where the strange quark is confined in the spin-0 $\bar{K}$ [31]. The above works extracted the effective hadron mass from the hadron two-point correlation function. On the other hand, recently, it has been considered that the hadron scattering should be analyzed from the four-point correlation function to describe unstable resonances like the $\Lambda(1405)$, and such methods have become main stream. Representative approaches are the Lüscher’s finite volume method [32–34]

²The double pole structure was first pointed out in Ref. [21] with the cloudy bag model.

	Facility	Reaction	Peak position [MeV]
1961 [43]	Lawrence Radiation Lab.	$K^- p \rightarrow \pi\pi\pi\Sigma$	1405 MeV
1965 [44]	BNL	$\pi^+ d \rightarrow KN\pi\Sigma$	1382 ± 8 MeV
1973 [45]	BNL	$\pi^- p \rightarrow K^+\pi\Sigma$	1405 MeV
1977 [46]	CERN	$K^- d \rightarrow \pi^+\Sigma^-n$	1420 MeV
1985 [47]	CERN	$K^- p \rightarrow \pi^-\pi^+(\pi\Sigma)$	1406.5 ± 4.0 MeV

TABLE 1.1: Bubble-chamber experiments for the $\Lambda(1405)$.

and the HAL QCD method [35–37], and both have been developed. In the former approach, the hadron scattering amplitude in the infinite volume, or the phase shift, is extracted from the discrete energy spectra in the finite volume in the lattice calculation. As the application, some resonances such as the ρ and $a_0(980)$ have been calculated in Refs. [38, 39]. Another promising method, HAL QCD method [35–37], can extract a hadron-hadron potential from Nambu-Bethe-Salpeter wave functions in lattice QCD. This method has been applied to various systems including the ones related to the basic nuclear force [35, 40] and the $Z_c(3900)$ which is a candidate of exotic hadron with charm quarks [41]. For the application to the $\Lambda(1405)$ system, there are several problems such as the necessity of the channel coupling, light quark mass, and the quark-antiquark annihilation³. For instance, the annihilation has not been taken into account in HAL QCD method, because it needs so-called all-to-all propagators which cause huge computational cost. However, this problem is being solved by introducing smeared sink or source operators which are favorable for the computational cost. In Ref. [42], the validity of the smeared-sink operator was shown comparing with the well established point-sink operator⁴. Such development to treat the all-to-all propagators would enable us to calculate the $\bar{K}N$ potential by lattice QCD, which would lead to the elucidation of the nature of the $\Lambda(1405)$.

1.1.2 Experiments

The existence of the $\Lambda(1405)$ pioneered in Refs. [4, 5] is experimentally indicated in Ref. [43] in 1961 for the first time, where a peak appears around 1405 MeV in the invariant mass distribution of the $\pi\Sigma$ channel in the $K^- p \rightarrow \pi\pi\pi\Sigma$ reaction. Thereafter, the resonance had been confirmed by a lot of bubble-chamber experiments, and the mass is specified as 1382–1420 MeV. We summarize the results of these experiments in Table 1.1.

Aside from the old bubble-chamber experiments, relatively recently, various experiments have been conducted to analyze the $\pi\Sigma$ invariant mass distribution. In 2003, the first photoproduction experiment for the $\Lambda(1405)$ analysis was carried out by LEPS Collaboration [48], and the differential cross section was obtained for the first time from the photoproduction in 2008 by the same collaboration [49]. In both works, the mass spectra of charged $\pi\Sigma$ states, $\pi^\pm\Sigma^\mp$, are measured in the reaction, $\gamma p \rightarrow K^+\pi\Sigma$. Theoretically, the spectra of the $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, and $\pi^+\Sigma^-$ channels had been predicted to be different from each other due to the interference of isospin

³The quark-antiquark annihilation is important for most exotic candidates or resonant states, except for "manifestly exotic hadrons" which do not include $q\bar{q}$ pairs like $u\bar{u}$ and $c\bar{c}$, for example, the Θ^+ ($uudd\bar{s}$) and H -dibaryon ($uuddss$) states.

⁴It was also pointed out that the non-locality of the potential should be carefully taken into account for the smeared operator [42].

channels [50]. This can be understood by considering the isospin decomposition of the physical states⁵,

$$|\pi^0\Sigma^0\rangle = -\sqrt{\frac{1}{3}}|I=0, I_z=0\rangle + \sqrt{\frac{2}{3}}|2,0\rangle, \quad (1.2)$$

$$|\pi^-\Sigma^+\rangle = -\sqrt{\frac{1}{3}}|0,0\rangle + \sqrt{\frac{1}{2}}|1,0\rangle - \sqrt{\frac{1}{6}}|2,0\rangle, \quad (1.3)$$

$$|\pi^+\Sigma^-\rangle = -\sqrt{\frac{1}{3}}|0,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle - \sqrt{\frac{1}{6}}|2,0\rangle, \quad (1.4)$$

which leads to the cross sections,

$$\frac{d\sigma(\pi^0\Sigma^0)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2, \quad (1.5)$$

$$\frac{d\sigma(\pi^-\Sigma^+)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2 + \frac{1}{2}|T^{(1)}|^2 - \frac{2}{\sqrt{6}}\text{Re}[T^{(0)}T^{(1)*}], \quad (1.6)$$

$$\frac{d\sigma(\pi^+\Sigma^-)}{dM_I} \propto \frac{1}{3}|T^{(0)}|^2 + \frac{1}{2}|T^{(1)}|^2 + \frac{2}{\sqrt{6}}\text{Re}[T^{(0)}T^{(1)*}] \quad (1.7)$$

with the invariant mass M_I and the $\pi\Sigma$ scattering amplitude in the isospin basis $T^{(I)}$. Here, the $I = 2$ component is neglected⁶. In the charged channels, the $\Lambda(1405)$ contribution (i.e. the $I = 0$ contribution) is contaminated by the $I = 1$ channel and the interference of $I = 0$ and $I = 1$. From the opposite sign of the interference term, the two charged channels are expected to show the different mass spectrum. In the experiments by LEPS [48, 49], the difference was seen and the interference effect was verified experimentally. On the other hand, for the aim to directly analyze the $\Lambda(1405)$ in the $I = 0$ channel, the $\pi^0\Sigma^0$ channel turns out to be the most adequate. In 2013, CLAS Collaboration measured all the three $\pi\Sigma$ channels with higher precision [51–53]. One of the important achievements by CLAS Collaboration is the definite determination of the spin-parity of the $\Lambda(1405)$ [53], which had been assumed to be $1/2^-$ for a long time. In Ref. [53], they determined the spin as $\frac{1}{2}$ from the decay angular distribution in the $\gamma p \rightarrow K^+\pi^-\Sigma^+$ reaction⁷. Producing the spin-polarized $\Lambda(1405)$, the parity was directly measured as negative through the unchanging direction of the Σ^+ polarization against the $\Lambda(1405)$ polarization direction. Because the CLAS Collaboration [51–53] produces the most precise data of the $\pi\Sigma$ spectra at various initial photon energies, many theoretical works have employed the data to analyze the $\Lambda(1405)$ [26, 54, 55].

However, we note here that the analysis of such $\pi\Sigma$ spectra has some difficulty for the extraction of the $\Lambda(1405)$ information. The $\pi\Sigma$ invariant mass distribution is considered to be determined from the combination of two-body scattering amplitudes $T_{MB,\pi\Sigma}$ from a meson-baryon state, MB , to the $\pi\Sigma$ state, which represent the

⁵The following phase convention in Ref. [19] is adopted,

$$\begin{aligned} |\pi^+\rangle &= -|I=1, I_z=+1\rangle, \quad |K^-\rangle = -|1/2, -1/2\rangle, \\ |\Sigma^+\rangle &= -|1, +1\rangle, \quad |\Xi^-\rangle = -|1/2, -1/2\rangle. \end{aligned} \quad (1.1)$$

⁶Generally, the interaction in an "exotic channel" like $\pi\Sigma$ ($I = 2$) where more than three valence quarks are include ($u\bar{d}uus$ in the $\pi\Sigma$ ($I = 2, I_z = +2$) case) is considered to be negligible, because any resonant states in such channel have not been observed at present.

⁷In regard to the spin, previous works [44, 45, 47] also showed that the data were consistent with the spin $\frac{1}{2}$, though the statistics were insufficient to draw a definite conclusion.

	Collaboration (Facility)	Reaction	Note
2003 [48]	LEPS (SPring-8)	$\gamma p \rightarrow K^+ \pi^\pm \Sigma^\mp$	difference between $\pi^+ \Sigma^-$ and $\pi^- \Sigma^+$ spectra
2004 [58]	Crystal Ball (BNL)	$K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$	relatively higher peak position around 1420 MeV ⁸
2008 [49]	LEPS (SPring-8)	$\gamma p \rightarrow K^+ \pi^\pm \Sigma^\mp$	first differential cross section for photo production
2008 [59]	ANKE (COSY-Jülich)	$pp \rightarrow pK^+ \pi^0 \Sigma^0$	(M, Γ) = (1400 MeV, 60 MeV)
2013 [51, 52]	CLAS (J-lab.)	$\gamma p \rightarrow K^+ \pi \Sigma$	simultaneous measurements of all $\pi \Sigma$ channels
2013 [60]	HADES (GSI)	$pp \rightarrow pK^+ \pi^\pm \Sigma^\mp$	relatively lower peak position around 1385 MeV
2014 [53]	CLAS (J-lab.)	$\gamma p \rightarrow K^+ (\pi \Sigma)$	determination of the spin-parity as $J^P = \frac{1}{2}^-$

TABLE 1.2: Summary of the recent experiments about the $\pi \Sigma$ spectra. M and Γ respectively represent the mass and decay width of the $\Lambda(1405)$ obtained from the experimental data.

final-state interactions, as

$$\frac{d\sigma}{dM_{\text{inv}}} \propto \left| \sum_{MB} c_{MB} G_{MB}(M_{\text{inv}}) T_{MB, \pi \Sigma}(M_{\text{inv}}) \right|^2, \quad (1.8)$$

where G_{MB} denotes the meson-baryon loop function at the $\pi \Sigma$ invariant mass M_{inv} . The difficulty for the $\Lambda(1405)$ analysis, that is, the extraction of $T_{MB, \pi \Sigma}$, is attributed to the ambiguity of the coefficient c_{MB} , which could generally depend on various elements such as the total energy of the system or the scattering angle of the final $\pi \Sigma$ cluster. The coefficient is related to the production mechanism of MB before the final-state interaction, and is obtained only under some theoretical assumption. The difficulty to determine c_{MB} is seen from the complicated mechanism as in Refs [56, 57]. Furthermore, Eq. (1.8) is also obtained under an assumption. Generally, many-body interactions contribute to the whole reaction, which are neglected in Eq. (1.8). The reason for such ambiguity is that the $\pi \Sigma$ spectra cannot be measured from an elastic scattering experiment and some production process becomes necessary. Thus, regardless of the experimental precision, the $\pi \Sigma$ analysis is accompanied by some ambiguity unless the reaction mechanism is perfectly handled, which is not the case at least at present.

To manage the situation, it would be desirable to analyze various types of $\pi \Sigma$ experiments, especially the ones which are theoretically handled easily, as well as to clarify the photoproduction mechanism. In Table 1.2, we summarize the recent $\pi \Sigma$ experiments. As an experiment other than the photoproduction, the $K^- p \rightarrow \pi^0 (\pi \Sigma)$ measurement by Crystal Ball Collaboration [58] can be given, where the neutral $\pi^0 \Sigma^0$ channel was measured. The proton-proton collision, $pp \rightarrow pK^+ (\pi \Sigma)$, was also carried out for the $\Lambda(1405)$ analysis by ANKE Collaboration [59] and HADES Collaboration [60]. Recently, a new type of analysis with a weak decay of a heavy hadron has found to be a good tool to reveal the hadron interaction [62], thanks to

⁸The pole position was analyzed in the different theoretical work [61].

experimental developments in LHCb, Belle, BES III, and other facilities. One of the most striking achievements is the unexpected finding of two pentaquark states in the $J/\psi p$ channel in the $\Lambda_b \rightarrow J/\psi K^- p$ reaction by LHCb Collaboration [63, 64], which prompted active theoretical studies. Concerning the $\Lambda(1405)$ analysis, the usefulness of the similar reaction, $\Lambda_b \rightarrow J/\psi \Lambda(1405)$, was theoretically proposed in Ref. [65], where the $\bar{K}N$ and $\pi\Sigma$ spectra were expected to have an advantage in extracting the $I = 0$ state. Actually, in the LHCb measurements [63, 64], the tail of the $\Lambda(1405)$ signal was clearly visible in the $K^- p$ channel, which is consistent with the theoretical prediction [65]. In our work, we analyzed the similar reaction from the Λ_c whose production rate is much larger than the Λ_b , and found that the $\Lambda_c \rightarrow \pi^+ \Lambda(1405)$ reaction will benefit the extraction of the $I = 0$ amplitude including the $\Lambda(1405)$ and $\Lambda(1670)$ [66]. We should note that the same reaction was suggested to be useful to obtain the $\pi\Sigma$ scattering length in Ref. [67]. Aside from the above nonleptonic decays, the semileptonic decay, $\Lambda_c \rightarrow \nu e^+ \Lambda(1405)$, was also studied with the same theoretical framework aiming at a clearer signal without the contamination from some complicated mechanisms [68].

Concerning with the pole structure of the $\Lambda(1405)$, some discussion has been expanded with the $\pi\Sigma$ amplitudes. In the double pole structure indicated in many chiral unitary approaches, the higher (lower) pole is considered to be strongly coupled to the $\bar{K}N$ ($\pi\Sigma$) channel [13]. Thus, a peak in the $\pi\Sigma$ spectra is expected to lie at higher (lower) energy than 1405 MeV if the fraction of the $\bar{K}N$ ($\pi\Sigma$) production before the final-state interaction is dominant, that is, the coefficient $c_{\bar{K}N}$ ($c_{\pi\Sigma}$) in Eq. (1.8) is much larger than the others. The shift of the peak position to the higher energy can be seen in the $K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$ reaction by Crystal Ball Collaboration [58], which was theoretically analyzed in Ref. [61] and the mass M and the decay width Γ of the $\Lambda(1405)$ was found to be $(M, \Gamma) = (1420, 38)$ MeV. On the other hand, in Ref. [69], it was shown that the shift is caused by the interference of one pole and the continuum amplitude and the single pole model can explain it. Similar discussion was expanded in Ref. [70] about the old bubble-chamber measurement, $K^- d \rightarrow \pi^+ \Sigma^- n$, by Ref. [46], where the peak appears around 1420 MeV. From the theoretical studies, the same feature is expected in some heavy baryon decays such as the Λ_b and Λ_c decays [65, 66, 68]. Contrarily, the possibility of the downward shift of the peak position due to the lower pole of the $\Lambda(1405)$ was suggested in the $pp \rightarrow pK^+(\pi^\pm \Sigma^\mp)$ analysis by HADES Collaboration [60], where the $\Lambda(1405)$ pole was estimated as $(M, \Gamma) \sim (1385, 50)$ MeV. This possibility was also supported in Ref. [71], where a good description of the HADES data was achieved with a relatively large contribution of the lower pole. Up to the present, the qualitative pole structure as well as the pole positions has not been settled. One of the main reasons would be the difficulty to directly derive the two-body amplitudes from the spectra of the reactions, that is, the ambiguity of c_{MB} in Eq. (1.8). Another reason may be the large decay width of the controversial lower pole, the effect of which is not clear in the mass distribution. For further study, the $K^- d \rightarrow \pi\Sigma n$ reaction was proposed because the behavior of the $\pi\Sigma$ amplitudes was found to be related to the pole structure with the full three-body calculation [72]. As another candidate, we propose the Ξ_b decay as the lower pole dominant reaction [73]. It is the first prediction of the lower pole dominant mechanism.

Apart from the $\pi\Sigma$ spectra, the $\bar{K}N$ data are also essential for the $\Lambda(1405)$ analysis. In contrast to the $\pi\Sigma$ analysis, the $\Lambda(1405)$ signal cannot be observed as a peak in the $\bar{K}N$ spectra, because the $\Lambda(1405)$ lies below the $\bar{K}N$ threshold. However, because the mass is just below the threshold, the data in the vicinity of the $\bar{K}N$ threshold give crucial constraint on the $\Lambda(1405)$. Besides, there is no ambiguity related to c_{MB} in

Eq. (1.8) because the K^-p scattering is possible in experiments, and the two-body amplitudes $T_{K^-p,MB}$ can be directly obtained. Basic data are the low-energy elastic and inelastic total cross sections accumulated from 1960s to 1980s [74–81]. Other ones are threshold branching ratios which represent the ratios of the decay branches from the K^-p channel to other channels at the K^-p threshold. The ratios were quantitatively well determined from K^- capture experiments by hydrogens as [82, 83]

$$\begin{aligned}\gamma &= \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.04, \\ R_c &= \frac{\Gamma(K^-p \rightarrow \text{charged particles})}{\Gamma(K^-p \rightarrow \text{all})} = 0.664 \pm 0.011, \\ R_n &= \frac{\Gamma(K^-p \rightarrow \pi^0\Lambda)}{\Gamma(K^-p \rightarrow \text{neutral particles})} = 0.189 \pm 0.015.\end{aligned}\tag{1.9}$$

As another information on the threshold, the $\bar{K}N$ scattering length should be taken into account. Theoretically, in 1981, the scattering length $a^{(I)}$ of each isospin channel ($I = 0, 1$) was derived by extrapolating the $\bar{K}N$ scattering amplitudes above the threshold onto the threshold as $a^{(0)} = -1.70 + 0.65i$ fm, $a^{(1)} = 0.37 + 0.60i$ fm [84]. Measurements of the kaonic hydrogen are the experimental approaches to extract the K^-p scattering length a_{K^-p} , and have recently been refined. Though the energy level of the kaonic hydrogen is mainly determined from the electromagnetic interaction, there appears some deviation from the quantum electrodynamics (QED) results due to the strong interaction, which is called the energy shift. Considering the short-range nature of the strong interaction, it turns out that the energy shift mainly occurs for the ground 1S state. In the similar K^-d system, it is theoretically ascertained with the accurate three-body calculations that the shift in the excited 2S state is an order of magnitude smaller than in the 1S state and the one in the 2P state is negligible in eV order [85]. Thus, the QCD effect in the K^-p system can be extracted from the measurement of the X ray emitted in the transition from the 2P to the 1S state, which is related to the energy shift of the 1S state. The connection of the energy shift to the scattering length is obtained from the improved Deser-Trueman formula [86, 87] including the isospin breaking correction in Ref. [88],

$$E_{\text{exp.}} - E_{\text{EM}} = -\Delta E - i\Gamma/2 = -2\alpha^3\mu_r^2 a_{K^-p} [1 + 2\alpha\mu_r(1 - \ln \alpha)a_{K^-p}], \tag{1.10}$$

with the fine-structure constant α and the K^-p reduced mass μ_r . ΔE and Γ respectively represent the energy shift and the width, which are defined as the difference between the X-ray energies measured in the experiment $E_{\text{exp.}}$ and theoretically calculated only with the electromagnetic interaction E_{EM} . Results from various kaonic hydrogen measurements are shown in Table . For comparison, we show the result from the extrapolation of the low-energy scattering data [84], $a_{K^-p} = -0.67 + 0.63i$ fm, too. While a_{K^-p} from the scattering data [84] is "repulsive"¹⁰($\text{Re}[a_{K^-p}] < 0$), the early measurements [89–91] resulted in the "attractive" scattering lengths. They are incompatible with the quasibound state picture of the $\Lambda(1405)$ because the existence of a (quasi)bound state leads to the "repulsive" scattering length. This contradiction had been unsolved for a long time. However, in 1997, the "repulsive" result consistent with the scattering data [84] was obtained with precise measurements

⁹The scattering length with the SIDDHARTA data was calculated in Ref. [24].

¹⁰We call the results with $\text{Re}[a_{K^-p}] < 0$ ($\text{Re}[a_{K^-p}] > 0$) or $\Delta E < 0$ ($\Delta E > 0$) "repulsive" ("attractive"). It does not necessarily mean that the interaction is repulsive (attractive). In fact, in the K^-p system, the interaction is strongly attractive in contrast to the "repulsive" scattering length.

	Collaboration (Facility)	$(\Delta E, \Gamma)$ [eV]	a_{K^-p} [fm]
1979[89]	NIMROD	$(+40 \pm 60, 0_{-0}^{+230})$	$+0.10 + i0$
1980[90]	CERN	$(+270 \pm 80, 560 \pm 260)$	—
1983[91]	CERN	$(+197 \pm 60, 80_{-80}^{+220})$	—
1997[92]	KEK-PS K3	$(-323 \pm 63 \pm 11, 407 \pm 208 \pm 100)$	$-0.78 + i0.49$
2005[93]	DEAR (DAΦNE)	$(-193 \pm 37 \pm 6, 249 \pm 111 \pm 30)$	$-0.468 + i0.302$
2011[94, 95]	SIDDHARTA (DAΦNE)	$(-283 \pm 36 \pm 6, 541 \pm 89 \pm 22)$	$-0.65 + i0.81^9$
1981 [84]	extrapolation of scattering data :		$-0.67 + 0.63i$

TABLE 1.3: Energy shift ΔE , width Γ , and K^-p scattering length a_{K^-p} from kaonic hydrogen measurements. For comparison, we attach the result from the extrapolation of the scattering data [84].

by KEK [92]. Still, quantitative deviation had remained and the KEK result contradicted with the following DEAR result [93]. To break through this situation, much more precise experiment was carried out by SIDDHARTA Collaboration [94, 95]. In this measurement, the error was significantly reduced and the result was found to be consistent with the KEK one [92]. The SIDDHARTA data is the most precise at present, and we have to take into account this constraint for quantitative discussion.

Though the highly precise result about a_{K^-p} was obtained by SIDDHARTA Collaboration, there is still room for improvement related to the isospin decomposition. The K^-p scattering length can be decomposed into isospin basis as $a_{K^-p} = (a^{(0)} + a^{(1)})/2$. Together with the properties of the $\Lambda(1405)$, the $I = 0$ component has been well constrained. On the other hand, in the $I = 1$ channel, there remains some uncertainty. Therefore, to add stronger constraint on theories, the measurement of the kaonic deuterium, which is sensitive to the $I = 1$ component, has been recommended [24, 85], and is planned at J-PARC E57 [96] and SIDDHARTA-2 [97, 98]. In the similar direction, we propose to measure the two hadron intensity correlation in high-energy heavy-ion collisions [99, 100]. We find that it provides complementary information of the isospin decomposition with the K^-p elastic scattering amplitude. The results of this work is discussed in Sec. 2.3.2.

	Approach ($\pi\Sigma N$ channel)	$\bar{K}N$ interaction	$(B_{\bar{K}NN}, \Gamma_{\bar{K}NN})$ [MeV]
2007 SGMR [109, 110]	Faddeev (explicit)	Pheno. (E -indep.)	(50-70, 90-110)
2007 YA [104]	Variational (effective)	Pheno. (E -indep.)	(48, 60)
2007 IS [111]	Faddeev (explicit)	Chiral (E -indep.)	(60-95, 45-80)
2009 DHW [105, 106]	Variational (effective)	Chiral (E -dep.)	(17-23, 40-70)
2010 IKS [112]	Faddeev (explicit)	Chiral (E -dep.)	(9-16, 34-46)
			(67-89, 224-320) ¹¹
2012 BGL [107]	Variational (effective)	Chiral (E -dep.)	(16, 41)
2014 RS [113]	Faddeev (explicit)	Pheno. (E -indep.)	(47-53, 50-65)
		Chiral (E -dep.)	(32, 49)
2015 DIM [114]	Variational (effective)	Chiral (E -dep.)	(20-28, 23-47)
2016 OHMH [108]	Variational (effective)	Chiral (E -dep.)	(26-28, 31-59)
2017 DIM (full) [115]	Variational (explicit)	Chiral (E -dep.)	(14-38, 16-36)

TABLE 1.4: Methods and results of the theoretical $\bar{K}NN$ calculation which do not take into account the non-mesonic decay. $B_{\bar{K}NN}$ and $\Gamma_{\bar{K}NN}$ are the binding energy and decay width, respectively. The terms, "Pheno." and "Chiral", represent the phenomenological interactions and the ones motivated by chiral effective theory, respectively. In the item of the few-body approach, we show how these theories treat the $\pi\Sigma N$ channel.

1.2 Overview of $\bar{K}$ -nuclei

1.2.1 Theoretical studies

The picture of the $\Lambda(1405)$ as the quasibound state of the antikaon and nucleon means that the $\bar{K}N$ interaction is strongly attractive. Based on the strong attraction, the existence of nuclei including an antikaon ($\bar{K}$ nuclei) has been predicted [101]. In particular, in Refs. [102, 103], drastic predictions were made by g -matrix calculations of some $\bar{K}$ nuclei, where optical potentials for the nuclei were obtained from phenomenological meson-baryon two-body interactions. For instance, it was predicted for the $K^- + {}^3\text{H}$ system that the binding energy is so large and exceeds the lower threshold ($\pi\Sigma NN$ threshold), which prohibits the main decay mode and the decay width becomes drastically small. Furthermore, in the $K^- + {}^8\text{Be}$ system, it was anticipated that the distances between nucleons shrink due to the strong $\bar{K}N$ attraction, and quite high density approximately five times larger than the ordinary nuclear density would be realized. Inspired by these interesting predictions, the studies about the $\bar{K}$ nuclei have been developed in both the theoretical and experimental sides. However, when we treat such high density states, the two-nucleon absorption like $\bar{K}NN \rightarrow \Lambda N$ should be taken into account. It has been also pointed out that the model with the hadron degrees of freedom would break down in too high density and the quark degrees of freedom should be explicitly considered. Furthermore, we should note that the results were obtained with the optical potentials which are applicable to nuclear matter but are not adequate in few-body systems.

In such situation, the $\bar{K}NN$ system has gained prominence. This system is the simplest one in the $\bar{K}$ nuclei, and can be rigorously calculated with the variational approach [104–108] or Faddeev approach [109–113] which are accurate methods for few-body systems with the interactions. In Table 1.4, we summarize the results of the $\bar{K}NN$ binding energy and decay width with various calculation methods. In

these theoretical calculations, the qualitative result that the $\bar{K}NN$ system has a resonance between $\bar{K}NN$ and $\pi\Sigma N$ thresholds has been agreed with each other. On the other hand, quantitatively, both the binding energy and the decay width heavily depend on the models, ranging 16-95 and 23-110 MeV, respectively. Possible main causes are the approximations of the few-body calculations in practical applications and the difference of the $\bar{K}N$ interactions.

Concerning the few-body calculation approach, it was confirmed by a benchmark test in Ref. [116] that the Faddeev and variational approaches lead to the same result if they are applied strictly with same interactions. However, in practical calculation of the $\bar{K}NN$ system, some approximation is used for simplicity. For example, in the Faddeev calculations in Refs. [109–113], only s -wave interactions were taken into account, and furthermore the separable two-body interactions were assumed, $\hat{V} = |g\rangle\gamma\langle g|$, with the coupling constant γ and the vertex form factor $|g\rangle$. Considering the introduced form factor $\langle p|g\rangle = \Lambda^2/(p^2 + \Lambda^2)$, there is an artificial divergence when extrapolating p below the threshold down to $p^2 = -\Lambda^2$. On the other hand, in regard to the variational approach, energy dependence of the $\bar{K}N$ potential becomes an origin of ambiguity. As explained in Sec. 2.1.1, the energy dependence naturally comes from the chiral symmetry¹². However, because the two-body energy in many-body systems cannot be uniquely determined, we have to use some approximate prescription in the variational approach. Furthermore, in most of the present variational methods, $\bar{K}N$ single channel potentials have been employed where the effect of other channels like $\pi\Sigma$ channel is effectively incorporated. In Refs. [110, 117], the importance of the explicit treatment of other channels was discussed, comparing the $\bar{K}NN$ - $\pi\Sigma N$ full calculation and the effective calculation only with the $\bar{K}NN$ subspace where the employed $\bar{K}N$ effective interaction was constructed to reproduce the full channel two-body scattering amplitudes. In these works, it was estimated that the effective calculations result in roughly 10-25 MeV smaller binding energy than the full calculations. In some variational works, another approximation was used related to the determination of the "minimum value" of a complex eigenenergy. In Refs. [105–107], they diagonalized only the real part of the Hamiltonian and the imaginary contribution was perturbatively taken into account. In recent our work [108], we have succeeded in determining the complex eigenenergy nonperturbatively (See Sec. 2.3.1). As another prescription, the scheme with the complex scaling method (CSM) has been developed in Refs. [114, 115, 118]. CSM also enables the nonperturbative determination of the complex eigenenergy. However, other approximations in the variational approaches have also been adopted here. Very recently, the coupled-channel calculation has been carried out in Ref. [115].

The other reason for the uncertainty of the $\bar{K}NN$ calculation, that is, the uncertainty of the $\bar{K}N$ interaction, seems to be more important. For example, we compare the results in Refs. [111, 112]. Between these works, only difference is the employed two-body interaction, and the same few-body calculation approach is adopted. However, as seen from Table 1.4, the $\bar{K}NN$ results are quite different from each other, and the difference of the binding energy amounts to around 50 MeV. The origin of such uncertainty is the extrapolation of the $\bar{K}N$ interaction to below the $\bar{K}N$ threshold. While the observable scattering data above the threshold is well reproduced, the $\bar{K}N$ information below the threshold cannot be directly observed by the $\bar{K}N$ scattering experiments, and we have to conduct the extrapolation with some model. At this stage, the uncertainty comes up, because the interaction below the

¹¹In Ref. [112], two poles for the $\bar{K}NN$ system were simultaneously obtained.

¹²At the same time, when we effectively reduce the number of channels with the Feshbach projection, additional energy dependence appears.

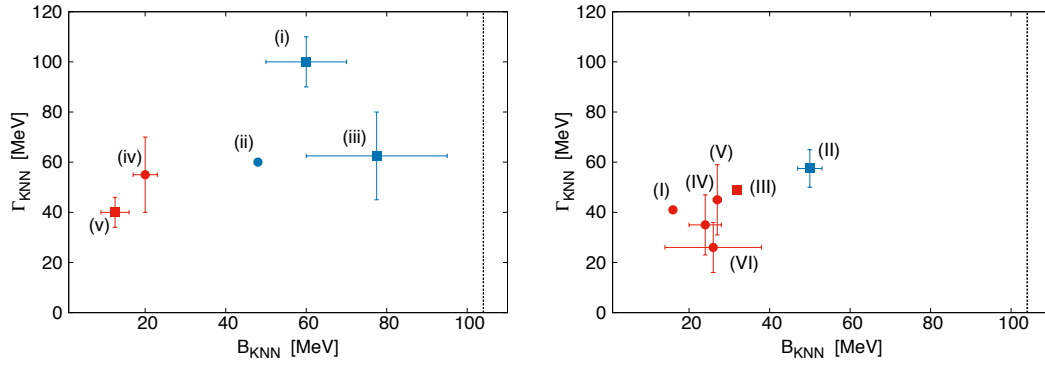


FIGURE 1.2: $\bar{K}NN$ binding energies and decay widths of several works. Left (right) panel is the results without (with) the SIDDHARTA constraint. Circle and square points respectively represent the variational and Faddeev variational approaches. Red and blue colors represent the E -dep. and E -indep. interactions, respectively. (i) stands for SGMR [109, 110], (ii) for YA [104], (iii) for IS [111], (iv) for DHW [105, 106], (v) for IKS [112], (I) for BGL [107], (II) for RS (with phenomenological interaction) [113], (III) for RS (with chirally motivated interaction) [113], (IV) for DIM (with effective single-channel) [114], (V) for OHMH [108], and (VI) for DIM (with coupled-channel treatment) [115]. The vertical dashed line represents the $\pi\Sigma N$ threshold.

$\bar{K}N$ threshold is essential to the calculation of the quasibound states, or the $\bar{K}$ nuclei. Roughly speaking, the model of interactions can be divided into two types, the E -dependent and E -independent types. Generally, the E -dep. potentials are based on the chiral effective theory, and E -indep. potentials are phenomenological ones¹³.

From Table 1.4, we find that the binding energies with E -dep. interactions (~ 10 – 40 MeV) tend to be much smaller than the ones with E -indep. interactions (~ 50 – 95 MeV). However, still, large uncertainty remains even in the each type of the interaction. This is because the extrapolation to below the $\bar{K}N$ threshold had not been well restricted by previous experimental data. This situation is changed; recently, thanks to the precise SIDDHARTA data (see Sec. 1.1.2), the uncertainty of the $\bar{K}N$ interaction has been significantly reduced, which enables us to proceed the next stage about $\bar{K}$ nuclei for quantitative discussion. Motivated by the recent development, we constructed the realistic $\bar{K}N$ potential with the SIDDHARTA constraint (See Sec. 2.2 or Refs. [119, 120]), and calculated several $\bar{K}$ nuclei with the variational approach (See Sec. 2.3 or Ref. [108]). Together with other recent works considering the SIDDHARTA constraint [107, 113–115], we show our results in Table 1.4¹⁴. In Fig. 1.2, we compare the $\bar{K}NN$ results with and without the SIDDHARTA constraint. It turns out that the SIDDHARTA data well constrains the $\bar{K}NN$ results.

We should note that the above results have not taken into account the effect of the multinucleon absorption like $\bar{K}NN \rightarrow \Lambda N, \Sigma N$. The effect was perturbatively estimated in Ref. [106], and an increase of the decay width by an amount of order 10 MeV was obtained. In Ref. [121], the effect was incorporated in meson-baryon

¹³The interaction in Ref. [111] was motivated by the chiral effective theory but the energy was fixed at threshold energies.

¹⁴Among the theoretical works of the $\bar{K}NN$ system, there are only two works [108, 113] which take into account both the K^-p scattering data and the SIDDHARTA data.

loop function, and it was nonperturbatively estimated that the decay width would increase by about 30 MeV. Thus, after establishing the quantitative calculation without the multinucleon absorption, for more elaborated discussion, this should be also taken into account.

1.2.2 Experiments

After the drastic prediction in Refs. [102, 103], several experimental works attempted to measure the $\bar{K}$ nuclei. In particular, the $\bar{K}NN$ has gained prominence because this is the simplest system of the $\bar{K}$ nuclei and expected to put strong constraint on the $\bar{K}N$ interaction. The first measurement was carried out in 2005 by FINUDA Collaboration [122]. In this measurement, K^- was stopped and absorbed in some nuclear targets like ${}^6\text{Li}$, ${}^7\text{Li}$, and ${}^{12}\text{C}$ to form the $\bar{K}NN$ quasibound state. As a decay channel of the $\bar{K}NN$ state, they analyzed the Λp invariant mass distribution and found a signal of this quasibound state with $(B_{\bar{K}NN}, \Gamma_{\bar{K}NN}) = (115, 67)$ MeV. This binding energy is too large and exceeds the lower $\pi\Sigma N$ threshold, which disagrees with all theoretical predictions in Table 1.4. Here, we should note that the interpretation of the results is not straightforward and it cannot be necessarily concluded that the measured signal represents the $\bar{K}NN$ state. For instance, in Ref. [123], it was pointed out that the signal interpreted as the $\bar{K}NN$ quasibound state is naturally explained by the simple two-nucleon absorption, $K^- pN \rightarrow \Lambda N$, and the final-state interaction of the Λ or N with the daughter nuclei.

Another famous experimental result was reported in 2010 by DISTO Collaboration [124], where the K^+ missing mass and Λp invariant mass spectra were measured in the $pp \rightarrow K^+ \Lambda p$ reaction and the broad peak corresponding to $(B_{\bar{K}NN}, \Gamma_{\bar{K}NN}) = (103, 118)$ MeV was found. However, to this interpretation, several problems have also been pointed out. For instance, other reactions like $pp \rightarrow N^* p \rightarrow K^+ \Lambda p$ would strongly affect the Λp spectra, or the measurement missed some effects like $\pi\Sigma p$ threshold effect or quasi-free $K^- pp$ effect which should appear naturally [125]. In fact, a recent reanalysis of the DISTO data contradicted the finding of the $\bar{K}NN$ quasibound state [126]. In 2014, J-PARC E27 Collaboration reported a $\bar{K}NN$ -like structure at $(B_{\bar{K}NN}, \Gamma_{\bar{K}NN}) = (95, 162)$ MeV in missing mass spectrum of the $\pi^+ d \rightarrow K^+ X$ reaction [127]. Still, the problems in the DISTO measurement has remained in the J-PARC E27 one, too. On the other hand, any signal for the $\bar{K}NN$ quasibound state has not been seen in some recent works [128–130], and the existence of such quasibound state itself has not been experimentally confirmed yet.

In this line, the latest work in 2016 by J-PARC E15 Collaboration has observed a clear peak structure in the Λp invariant mass distribution of the $K^- {}^3\text{He} \rightarrow \Lambda pn$ reaction, which has been fitted by a Breit-Wigner formula with a Gaussian form factor to result in $(B_{\bar{K}NN}, \Gamma_{\bar{K}NN}) = (15, 110)$ MeV [131]. The likelihood of the $\bar{K}NN$ quasibound state has been also suggested in a theoretical work [132]. It has been considered that this experiment has several advantages. One is that the momentum transfer to the Λp state is estimated to be the smallest of all experiments so far, which would enhance the probability of the $\bar{K}$ sticking to the NN system. Another merit is the identification of the final state (Λp). Furthermore, it is also expected that the contamination from background by other mechanism would be less than others. Comparing with the theoretical works, it turns out that the J-PARC E15 results ($B_{\bar{K}NN} \sim 15$ MeV) are closer to the $\bar{K}NN$ predictions based on the chiral effective theory, that is, the ones with E -dependent interactions ($B_{\bar{K}NN} \sim 10\text{--}40$ MeV), than the previous experimental results ($B_{\bar{K}NN} \sim 100$ MeV). For more precise discussion,

	Collaboration	Facility	Reaction	$(B_{\bar{K}NN}, \Gamma_{\bar{K}NN})$ [MeV]
2005 [122]	FINUDA	DAΦNE	$K^- + A \rightarrow \Lambda p + A'$	(115, 67)
2007 [133]	OBELIX	CERN	$\bar{p} + {}^4\text{He} \rightarrow \Lambda p + X$	(160.9, < 24.4)
2010 [124]	DISTO	Saclay	$pp \rightarrow K^+ \Lambda p$	(103, 118)
2014 [127]	J-PARC E27	J-PARC	$\pi^+ d \rightarrow K^+ X$	(95, 162)
2014 [128]	LEPS	SPring-8	$\gamma d \rightarrow K^+ \pi^- X$	no signal
2014 [129]	J-PARC E15	J-PARC	$K^- + {}^3\text{He} \rightarrow X + n$	no signal
2014 [130]	HADES	GSI	$pp \rightarrow p K^+ \Lambda$	no signal
2015 [131]	J-PARC E15	J-PARC	$K^- + {}^3\text{He} \rightarrow \Lambda pn$	(15, 110)

TABLE 1.5: Experimental results of the $\bar{K}NN$ quasibound state. $B_{\bar{K}NN}$ and $\Gamma_{\bar{K}NN}$ are the binding energy and decay width, respectively. A and A' in FINUDA measurement represent nuclei like ${}^6\text{Li}$, ${}^7\text{Li}$, and ${}^{12}\text{C}$.

J-PARC E15 Collaboration has proceeded to further analysis. In Table 1.5, we summarize the experimental situation so far.

1.3 Purpose of this work

The $\Lambda(1405)$ has attracted a lot of attention due to the exotic nature. Because of the difficulty to describe the state in quark models, it has been considered as an exotic baryon which has different inner structure from ordinary three-quark states. The most successful picture is the one to describe the $\Lambda(1405)$ as the $\bar{K}N$ quasibound state (see Sec. 1.1). This picture indicates that the $\bar{K}N$ interaction is strongly attractive, which leads to the predictions of quasibound states in the systems of nucleons and an antikaon, namely, the $\bar{K}$ nuclei. Among others, the drastic predictions in Refs. [102, 103], where extremely high density states (or compact states) with small decay widths were obtained, has prompted vigorous theoretical and experimental works (See Sec. 1.2). The base of studies about the $\Lambda(1405)$ and $\bar{K}$ nuclei is the $\bar{K}N$ interaction. However, because previous experimental data had not well constrained the interaction, there had been large uncertainty in particular below the $\bar{K}N$ threshold which is the essential region for quasibound states (See for example Sec. 1.2.1). This situation has been recently changed thanks to the precise data related to the $\bar{K}N$ scattering length by SIDDHARTA Collaboration [94, 95] (see Sec. 1.1.2). Hence, in our work, aiming to establish a base to reveal the nature of the $\Lambda(1405)$ and $\bar{K}$ nuclei through quantitative discussion, we construct $\bar{K}N$ potentials with the SIDDHARTA constraint which are reliable even below the $\bar{K}N$ threshold.

Similarly to the studies of the nuclear force, three approaches to the construction of the $\bar{K}N$ interaction can be considered, that is, the phenomenological one, the one based on the chiral effective theory, and lattice QCD. As phenomenological interactions, the Akaishi-Yamazaki potential constructed to reproduce the scattering lengths and the $\Lambda(1405)$ energy [102, 103] and Jülich potential based on boson exchanges [134] were proposed. However, we have to note that the reliability of these potentials is lost below the threshold because the data in this region cannot be directly obtained by experiments. Besides, the recent SIDDHARTA data were not considered there. Regarding lattice QCD, though the methods to extract the hadron potentials have been developing recently, the $\bar{K}N$ potential has not been calculated yet for several difficulties (see Sec. 1.1.1). In contrast, chiral unitary approach, corresponding to chiral effective field theory for the nuclear force, precisely reproduces the experimental data, and furthermore it can give reliable results even in the experimentally inaccessible region below the threshold because it takes into account the essential feature of QCD, the chiral symmetry, in constructing the Lagrangian. Besides, the interactions can be systematically improved by taking higher order terms in chiral perturbation theory. For all of these reasons, in our work, we construct the hadron potentials for the $\Lambda(1405)$ system based on chiral unitary approach. In particular, local type potentials, applicable to the quantum mechanics, are useful for the calculation of the $\bar{K}$ nuclei and the analysis of the spatial structure of the $\Lambda(1405)$. Because the main results of chiral unitary approach are the scattering amplitudes not the local potentials, it is necessary to establish the method to construct the local potential based on chiral unitary approach.

Motivated by the above situation, the former part of this work is organized as follows: in Sec. 2.1, we will show the procedure to construct the local potential. As a first step, we briefly explain the formulation of chiral unitary approach with the great success in reproducing existing data including the SIDDHARTA data. Subsequently, we will explain how to connect chiral unitary approach to the local potential in general coupled-channel case, which is an extended version of previous works for single-channel case [22, 119, 120]. Based on the procedure, in subsequent sections, we will establish the way to construct quantitatively reliable potentials. The point

here is how to make some adjustment of the local potential to reproduce the original amplitudes from chiral unitary approach, especially paying attention to the complex energy plane. This is because information of the mass and decay width of a resonant state is represented by the resonance pole position in the complex plane, and other nature would be also decisively influenced by the pole position. Thus, in Sec. 2.2, we will establish the method in the single-channel case to precisely reproduce the original amplitudes based on the uniqueness of the analytic continuation. Employing this method, we will construct the reliable $\bar{K}N$ local potential with the precise SIDDHARTA constraint (Sec. 2.2.4). Applications of the realistic $\bar{K}N$ potential will be shown in Sec. 2.3. First one is the search for $\bar{K}$ nuclear states, where an accurate few-body calculation method, the stochastic variational method with correlated Gaussian basis, is employed (Sec. 2.3.1). Another application is to the K^-p correlation function in heavy-ion collisions, which would give complementary information of the $\bar{K}N$ interaction to the ordinary K^-p scattering experiments (Sec. 2.3.2).

In the latter part of Chapter 2, we will extend the construction of the single-channel potential to the coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential, which is demanded for more elaborated studies. This is because a substantial contribution of the $\pi\Sigma$ channel to the $\Lambda(1405)$ resonance poles has been pointed out in several works. Furthermore, as explained in Sec. 1.2.1, the $\pi\Sigma$ elimination, though the effect is effectively taken into account in the two-body system, would cause some ambiguity in few-body calculations [110, 117]. Hence, in Sec. 2.4, we will construct the realistic $\bar{K}N$ - $\pi\Sigma$ potential with the similar manner to the single-channel case. As expected, the coupled-channel potential can reproduce the original pole structure more naturally than the single-channel one, and the energy dependence of the potential is simplified. The simplification may give advantage to the few-body calculations. As explained in Sec. 1.2.1 and pointed out in Ref. [113], the treatment of the energy dependence of the potential has been a troublesome problem in the variational method to cause ambiguity in the few-body calculation. However, in Refs [135–137], it was indicated that the energy dependence can be eliminated by renormalizing the wave function if the dependence is linear. And in Ref. [137], it was suggested to apply the renormalized potential to few-body calculations. Thus, in Sec 2.4.3, we will also construct a potential linearly depending on the energy.

In Chapter 3, to analyze the wave function of the $\Lambda(1405)$, we will establish how to treat a general energy-dependent potential in the non-Hermitian system. For energy-dependent potential in the Hermitian system, it is known that some careful treatment is necessary to preserve essential properties, the norm conservation and the orthogonality relation. We will derive a new treatment for the energy-dependent potential in the non-Hermitian systems such as the $\Lambda(1405)$ in Sec. 3.1.1. We find that the new formulation is related to the compositeness and elementariness, considering that the energy dependence of the potential comes from the elimination of some channels. In Sec 3.1.2, we will summarize the relation with the compositeness introducing the Feshbach projection [138, 139]. In Sec. 3.2, applying the new formulation, we will analyze the compositeness and the spatial structure of the $\Lambda(1405)$ with the single and coupled-channel potentials developed in Secs. 2.2 and 2.4.

Chapter 2

Meson-Baryon local potential of $\Lambda(1405)$ system

2.1 Formulation

In quantum chromodynamics (QCD) describing the strong interaction, the analysis in the low energy region is difficult due to the large coupling constant which disables the perturbative treatment. In such nonperturbative region, some effective theory with the hadron degrees of freedom is useful. Chiral unitary approach is one of the most successful effective theories in particular to analyze hadron resonances. In Sec. 2.1.1, we will briefly show the formulation of chiral unitary approach. Based on the approach, we will construct an equivalent local potential applied to the quantum mechanics, which is useful to study the spatial structure of the $\Lambda(1405)$ or the properties of $\bar{K}$ nuclei. In Sec. 2.1.2, we will establish a basic method to connect chiral unitary approach to the equivalent potential in a coupled-channel system.

2.1.1 Meson-baryon amplitudes from chiral unitary approach

In QCD, the Lagrangian is expressed with the quark q_f and gluon $A_{a\mu}$ degrees of freedom as¹

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G_{a\mu\nu} G_a^{\mu\nu}, \quad (2.1)$$

$$G_{a\mu\nu} \equiv \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - gf_{abc} A_{b\mu} A_{c\nu}, \quad D_\mu \equiv \partial_\mu + igA_\mu, \quad A_\mu \equiv A_{a\mu} \frac{\lambda_a^c}{2}, \quad (2.2)$$

where f ($= 1, \dots, N_f$), μ ($= 0, 1, 2, 3$), and a ($= 1, \dots, 8$) respectively denote the flavor, Lorentz, and color indices. g , γ_μ , and λ_a^c are respectively the gauge coupling constant in QCD, the gamma matrix, and the Gell-Mann matrix in the color space. In the chiral limit, $m_u = m_d = m_s = 0$, the QCD Lagrangian is invariant under the chiral transformation, $SU(3)_L \times SU(3)_R$,

$$q_L \mapsto Lq_L = \exp\left(-i \sum_{a=1}^8 \Theta_{La} \frac{\lambda_a}{2}\right) q_L, \quad (2.3)$$

$$q_R \mapsto Rq_R = \exp\left(-i \sum_{a=1}^8 \Theta_{Ra} \frac{\lambda_a}{2}\right) q_R, \quad (2.4)$$

where $q_L = \frac{1-\gamma_5}{2} q$ and $q_R = \frac{1+\gamma_5}{2} q$ respectively represent the left and right-handed quarks. L and R represent the elements of $SU(3)_L$ and $SU(3)_R$, respectively. $\Theta_{L(R)a}$

¹ The color and spinor indices in the quark field q_f are implicit in Eq. (2.1).

is the corresponding real parameter. λ_a is the Gell-Mann matrix in the flavor space. For a while, we assume the exact chiral symmetry neglecting the explicit breaking by the quark mass term.

In the low energy region, due to the confinement of the quark, only the hadron degrees of freedom appear in the system. At the same time, original global symmetry $G = SU(3)_L \times SU(3)_R$ is spontaneously broken to the subgroup $H = SU(3)_V$, which is called the spontaneous breaking of the chiral $SU(3)$ symmetry. In this stage, following the Nambu-Goldstone theorem [140–143], there appears the pseudoscalar meson octet as the Nambu-Goldstone bosons (NG bosons) corresponding to the number of generators of the broken symmetries. In Refs. [144, 145], based on the chiral $SU(3)$ symmetry and its spontaneous breaking, the effective theory with the hadron degrees of freedom was constructed, which is well known as chiral perturbation theory. In chiral perturbation theory, the Lagrangian can be systematically expanded with respect to the power of the momentum and the quark mass which are much smaller in the low energy region than the scale of the symmetry breaking characterized by the pion decay constant f_π ($4\pi f_\pi \approx 1170 \text{ MeV} \approx 1 \text{ GeV}$), and the renormalizability is ensured in each order [144–147]. Thanks to the systematical treatment, chiral perturbation theory has been greatly successful in various systems including not only mesons but also baryons.

The Lagrangian is constructed in the nonlinear realization of the chiral symmetry. The chiral field is constructed as

$$u(x) = \exp [i\phi(x)/2f_0], \quad (2.5)$$

with the field of the pseudoscalar meson octet,

$$\phi = \sum_{a=1}^8 \phi_a \lambda_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \quad (2.6)$$

The constant f_0 is identified as the meson decay constant in the chiral limit [147]. Then, the chiral transformation of u , which means the transformation of the meson octet field, is

$$u(x) \mapsto Ru(x)K^\dagger = KuL^\dagger, \quad (2.7)$$

where $K(L, R, \phi) \in SU(3)_V$. On the other hand, the baryon octet field B with the spin-parity $J^P = 1/2^+$ is represented as

$$B = \sum_{a=1}^8 B_a \frac{\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (2.8)$$

The transformation of the baryon field is governed by the element of H as

$$B \mapsto B' = K(L, R, \phi)BK^\dagger(L, R, \phi). \quad (2.9)$$

Because $K(L, R, \phi)$ is related to $\phi(x)$ which is an element of the local $SU(3)_V$ transformation, a covariant derivative should be introduced for the baryon field. Defining

the covariant derivative as

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B], \quad (2.10)$$

with the so-called chiral connection,

$$\Gamma_\mu = \frac{1}{2}[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger], \quad (2.11)$$

it turns out that $D_\mu B$ transforms in the same way as B ,

$$D_\mu B \mapsto [D_\mu B]' = K(L, R, \phi) [D_\mu B] K^\dagger(L, R, \phi). \quad (2.12)$$

Γ_μ is related to the vector current. On the other hand, as the one related to the axial-vector current, the so-called chiral vielbein is introduced,

$$u_\mu = i [u^\dagger \partial_\mu u - u \partial_\mu u^\dagger], \quad (2.13)$$

which meets the following transformation,

$$u_\mu \mapsto K(L, R, \phi) u_\mu K^\dagger(L, R, \phi). \quad (2.14)$$

Considering these transformation related to the meson and baryon fields, the effective Lagrangian is constructed to be invariant under $SU(3)_L \times SU(3)_R$. The contribution of the quark mass term in Eq. (2.1), which explicitly breaks the chiral symmetry, can be taken into account with the assumption that the mass $\mathcal{M} \equiv \text{diag}(m_u, m_d, m_s)$ transforms as

$$\mathcal{M} \mapsto R \mathcal{M} L^\dagger, \quad (2.15)$$

though it is just a constant in fact. Under this assumption, the original QCD Lagrangian becomes invariant under the chiral transformation. Therefore, the effective Lagrangian is also constructed to be invariant with this assumption.

In the meson-baryon sector which is the object of our thesis, the leading Lagrangian in the order $\mathcal{O}(p)$ is

$$\mathcal{L}_{MB}^{(1)} = \text{Tr} [\bar{B}(i\not{D} - M_0)B] + \frac{D}{2} \text{Tr} (\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}) + \frac{F}{2} \text{Tr} (\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]), \quad (2.16)$$

where M_0 is the baryon mass in the chiral limit. The axial vector couplings D and F can be determined from experimental data of the semi-leptonic decays of several baryons [148]. The next-to-leading order Lagrangian in $\mathcal{O}(p^2)$ relevant to the meson-baryon scattering is

$$\begin{aligned} \mathcal{L}_{MB}^{(2)} = & b_D \text{Tr} (\bar{B} \{\chi_+, B\}) + b_F \text{Tr} (\bar{B} [\chi_+, B]) + b_0 \text{Tr} (\bar{B} B) \text{Tr} (\chi_+) \\ & + d_1 \text{Tr} (\bar{B} \{u^\mu, [u_\mu, B]\}) + d_2 \text{Tr} (\bar{B} [u^\mu, [u_\mu, B]]) \\ & + d_3 \text{Tr} (\bar{B} u_\mu) \text{Tr} (u^\mu B) + d_4 \text{Tr} (\bar{B} B) \text{Tr} (u^\mu u_\mu), \end{aligned} \quad (2.17)$$

where

$$\chi_+ = u^\dagger \mathcal{M} u^\dagger + u \mathcal{M}^\dagger u. \quad (2.18)$$

b_D, b_F, b_0 , and d_0 - d_4 are the low energy constants.

In the low energy region, dominant contribution to the scattering amplitude

comes from the s -wave component. Thus, in our work, we only consider the meson-baryon two-body scattering amplitude in s wave. The lowest order one can be calculated from the first term in Eq. (2.16) as

$$V_{ij}^{\text{WT}}(\sqrt{s}) = -\frac{C_{ij}}{4f_i f_j} (2\sqrt{s} - M_i - M_j) \sqrt{\frac{E_i + M_i}{2M_i}} \sqrt{\frac{E_j + M_j}{2M_j}}, \quad (2.19)$$

which is known as the Weinberg-Tomozawa (WT) term. Here, $\sqrt{s}$ is the total center-of-mass energy. M_i and E_i denote the mass and energy of the baryon in channel i . The meson decay constant f_i of the meson in channel i is determined from the experimental data of the leptonic decays of mesons [1]. The symmetric coefficient C_{ij} is obtained from the expansion of Eq. (2.16). For instance, in the strangeness $S = -1$ sector and in the isospin basis, C_{ij} is determined as ²,

$$C_{ij}^{I=0} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} & \frac{3}{\sqrt{2}} & 0 \\ & 4 & 0 & \sqrt{\frac{3}{2}} \\ & & 0 & -\frac{3}{\sqrt{2}} \\ & & & 3 \end{pmatrix}, \quad C_{ij}^{I=1} = \begin{pmatrix} 1 & -1 & -\sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} & 0 \\ & 2 & 0 & 0 & 1 \\ & & 0 & 0 & -\sqrt{\frac{3}{2}} \\ & & & 0 & -\sqrt{\frac{3}{2}} \\ & & & & 1 \end{pmatrix}, \quad (2.21)$$

where each channel represents the $\bar{K}N$ ($i = 1$), $\pi\Sigma$ ($i = 2$), $\eta\Lambda$ ($i = 3$), and $K\Xi$ ($i = 4$) channels for $I = 0$, and the $\bar{K}N$ ($i = 1$), $\pi\Sigma$ ($i = 2$), $\pi\Lambda$ ($i = 3$), $\eta\Sigma$ ($i = 4$), and $K\Xi$ ($i = 5$) for $I = 1$. The important point to note is that this WT term is model-independently derived from the low energy theorem, too [151, 152]. In fact, Eq. (2.19) does not include any low energy constants, and uniquely determined from the group theory. Another important point is the energy dependence of the WT term. As discussed in Sec. 2.1.2, as a result of this energy dependence, the local potential to be used in the Schrödinger equation, which is our objective in this thesis, also becomes energy dependent. This requires a careful treatment in analysis of the wave function (see Chapter 3).

As the next-to-leading order amplitudes, there are two types. One is calculated from a diagram using two three-point vertices in the second or third term of the lowest order Lagrangian in Eq. (2.16), which is called the Born term V^{Born} ³. The other is from a diagram with one four-point vertex in the second lowest order Lagrangian in Eq. (2.17), which we call V^{NLO} . In V^{Born} , only the low energy constants are the axial couplings, D and F , which are well determined from the experimental data of the hyperon semi-leptonic decay. On the other hand, in V^{NLO} , the low energy constants, b_D , b_F , b_0 , and d_1 - d_4 should be fitted by the meson-baryon scattering data.

²In chiral unitary approach discussed later, isospin states are naively assigned to the meson and baryon states except for the following states [19],

$$\begin{aligned} |\pi^+\rangle &= -|I=1, I_z=+1\rangle, & |K^-\rangle &= -|\frac{1}{2}, -\frac{1}{2}\rangle, \\ |\Sigma^+\rangle &= -|1, +1\rangle, & |\Xi^-\rangle &= -|\frac{1}{2}, -\frac{1}{2}\rangle. \end{aligned} \quad (2.20)$$

The theoretical base of this phase convention is the Clebsch-Gordan coefficients for the octet states [149]. In Ref. [150], a related topic is discussed in appendix.

³ While the Born term is generally in $\mathcal{O}(p)$, it is in higher order in the s wave with the non-relativistic expansion.

The explicit forms of V^{Born} and V^{NLO} are shown in Refs. [2, 24].

In chiral perturbation theory, the scattering amplitude can be calculated from diagrams up to required order with the expanded Lagrangian. However, due to the perturbative treatment, it cannot be applied to systems with bound or resonant states. Chiral unitary approach is a model developed for the application to such non-perturbative phenomena [18–20, 153]. In this approach, some tree-level amplitudes from chiral perturbation theory V such as V^{WT} , V^{Born} , and V^{NLO} are resummed, as in the similar strategy to the one in the nuclear force [154, 155]. The Bethe-Salpeter equation for the two-body T-matrix T is

$$T_{ij}(\sqrt{s}; k_i, k_j) = V_{ij}(\sqrt{s}; k_i, k_j) + \sum_k i \int \frac{d^4 q_k}{(2\pi)^4} V_{ik}(\sqrt{s}; k_i, q_k) \tilde{G}_k(\sqrt{s}; q_k) T_{kj}(\sqrt{s}; q_k, k_j), \quad (2.22)$$

with

$$\tilde{G}_k(\sqrt{s}; q_k) = \frac{2M_k}{(P - q_k)^2 - M_k^2 + i\epsilon} \frac{1}{q_k^2 - m_k^2 + i\epsilon}, \quad (2.23)$$

where M_k and m_k respectively represent the baryon and meson masses in channel k . k_i is the on-shell relative four-momentum of an initial or final state, and q_i is the meson four-momentum of the intermediate state. In stead of directly calculating the Bethe-Salpeter equation, it is convenient to adopt the approximation which is called on-shell factorization [156]. The interaction kernel is separated to the on-shell and off-shell parts as

$$V_{ik}(\sqrt{s}; k_i, q_k) \simeq V_{ik}(\sqrt{s}; k_i, q_k^2 = m_k^2) + \beta_{ik}(\sqrt{s}; k_i, q_k)(q_k^2 - m_k^2), \quad (2.24)$$

and inserted in the second term in Eq. (2.22), which results in

$$\begin{aligned} & \sum_k i \int \frac{d^4 q_k}{(2\pi)^4} V_{ik}(\sqrt{s}; k_i, q_k) \tilde{G}_k(\sqrt{s}; q_k) \\ &= \sum_k i V_{ik}(\sqrt{s}; k_i, q_k^2 = m_k^2) \int \frac{d^4 q_k}{(2\pi)^4} \frac{2M_k}{(P - q_k)^2 - M_k^2 + i\epsilon} \frac{1}{q_k^2 - m_k^2 + i\epsilon} \\ &+ \sum_k i \int \frac{d^4 q_k}{(2\pi)^4} \frac{2\beta_{ik}(\sqrt{s}; k_i, q_k) M_k}{(P - q_k)^2 - M_k^2 + i\epsilon}. \end{aligned} \quad (2.25)$$

The second term in the right-hand side, that is, the off-shell contribution, just corresponds to a tadpole diagram where a baryon loop is attached to the meson-baryon four-point vertex. This contribution can be absorbed in the renormalization constant of the lowest order four-point vertex. Hence, only the remaining term should be explicitly calculated, which reduces original integral equation to a simpler algebraic equation,

$$\begin{aligned} T_{ij}(\sqrt{s}) &= V_{ij}(\sqrt{s}) + \sum_k V_{ik}(\sqrt{s}) G_k(\sqrt{s}) T_{kj}(\sqrt{s}) \\ &= \left(\left[V^{-1}(\sqrt{s}) - G(\sqrt{s}) \right]^{-1} \right)_{ij}, \end{aligned} \quad (2.26)$$

with the two-body loop function,

$$G_k(\sqrt{s}) = i \int \frac{d^4 q_k}{(2\pi)^4} \frac{2M_k}{(P - q_k)^2 - M_k^2 + i\epsilon} \frac{1}{q_k^2 - m_k^2 + i\epsilon} \quad (2.27)$$

In most works, this simpler equation is solved to calculate meson-baryon amplitudes. The divergent loop function is regularized with the dimensional regularization scheme, and the finite part becomes

$$\begin{aligned} G_k(\sqrt{s}; a_k(\mu)) = & \frac{1}{(4\pi)^2} \left\{ a_k(\mu) + \ln \frac{M_k^2}{\mu^2} + \frac{m_k^2 - M_k^2 + s}{2W^2} \ln \frac{m_k^2}{M_k^2} \right. \\ & + \frac{\bar{q}_k}{\sqrt{s}} \left[\ln(s - (M_k^2 - m_k^2) + 2W\bar{q}_k) + \ln(s + (M_k^2 - m_k^2) + 2\sqrt{s}\bar{q}_k) \right. \\ & \left. \left. - \ln(-s + (M_k^2 - m_k^2) + 2\sqrt{s}\bar{q}_k) - \ln(-s - (M_k^2 - m_k^2) + 2\sqrt{s}\bar{q}_k) \right] \right\}, \end{aligned} \quad (2.28)$$

where $\bar{q}_k$ stands for the three-momentum in the center-of-mass frame,

$$\bar{q}_k = \frac{\sqrt{[s - (M_i - m_i)^2][s - (M_i + m_i)^2]}}{2\sqrt{s}}. \quad (2.29)$$

Here, μ is the renormalization scale and $a_k(\mu)$ is the subtraction constant at μ . In chiral unitary approach, the subtraction constant a_k is also a free parameter to fit the experimental data.

Chiral unitary approach has been applied to the $\Lambda(1405)$ system in many works [23–26, 157–163]. Among them, the Ikeda-Hyodo-Weise (IHW) model [23, 24] has succeeded in well reproducing experimental data including the recent precise SIDDHARTA data [94, 95] (See Sec. 1.1) with the interaction kernels up to next-to-leading order ($V = V^{\text{WT}} + V^{\text{Born}} + V^{\text{NLO}}$). In the IHW model, the free parameters are the low-energy constants in the NLO Lagrangian, b_D , b_F , b_0 , and d_1 – d_4 in Eq. (2.17), and the subtraction constants a_i in the loop function in Eq. (2.28). With these parameters, the following experimental data were fitted;

- $K^- p$ elastic and inelastic total cross sections [74–81],
- branching ratios at the $K^- p$ threshold [23, 24]:

$$\begin{aligned} \gamma &= \frac{\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)}{\Gamma(K^- p \rightarrow \pi^- \Sigma^+)} = 2.36 \pm 0.04, \\ R_c &= \frac{\Gamma(K^- p \rightarrow \text{charged particles})}{\Gamma(K^- p \rightarrow \text{all})} = 0.664 \pm 0.011, \\ R_n &= \frac{\Gamma(K^- p \rightarrow \pi^0 \Lambda)}{\Gamma(K^- p \rightarrow \text{neutral particles})} = 0.189 \pm 0.015. \end{aligned} \quad (2.30)$$

- the energy shift ΔE and the width Γ of the kaonic hydrogen in the SIDDHARTA measurements [94, 95] which are related to the scattering length with the improved Deser-Trueman formula [88]:

$$\Delta E = -283 \pm 36 \text{ (stat.)} \pm 6 \text{ (sys.) eV}, \quad \Gamma = 541 \pm 89 \text{ (stat.)} \pm 22 \text{ (sys.) eV}. \quad (2.31)$$

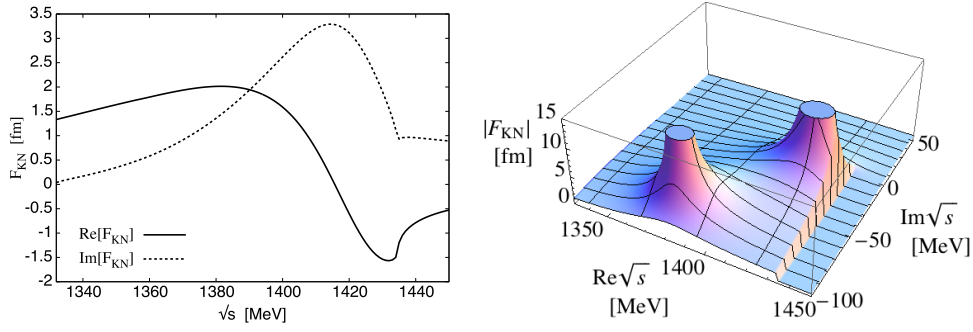


FIGURE 2.1: $\bar{K}N$ diagonal scattering amplitude $F_{\bar{K}N}$ in the IHW model. In the left panel, the real and imaginary parts of $F_{\bar{K}N}$ are respectively represented by the solid and dashed lines. In the right panel, the absolute value of $F_{\bar{K}N}$ is depicted.

The result of the χ^2 fit is $\chi^2/\text{d.o.f} = 0.96$. Thus, as the base of quantitative discussion, we respect the IHW model in the following sections. As pointed out in most chiral unitary approaches [13], the IHW model also shows the peculiar pole structure where two resonance poles describe one peak in a spectrum corresponding to the $\Lambda(1405)$. In Fig. 2.1, we show the $\bar{K}N$ ($I = 0$) diagonal amplitude $F_{\bar{K}N} = -\frac{M_N}{4\pi\sqrt{s}} T_{\bar{K}N,KN}$ on the real energy axis and in the complex energy plane⁴. In Ref. [22], it is indicated that the pole lying at the higher (lower) $\text{Re}[\sqrt{s}]$ region seems to be generated from the attraction in the $\bar{K}N$ ($\pi\Sigma$) channel.

2.1.2 Construction of equivalent potential

In anticipation of few-body calculations or spatial-structure analysis of the $\Lambda(1405)$, we construct meson-baryon local potentials, which we call equivalent potentials, based on the amplitudes derived from chiral unitary approach. In this section, we establish the framework to construct a general coupled-channel equivalent potential, based on the single-channel case in Refs. [18, 22].

The equivalent potential V^{equiv} is the one used in the Schrödinger equation,

$$\left[-\frac{\nabla^2}{2\mu_i} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}^{\text{equiv}}(\mathbf{r}, E) \right] \psi_j(\mathbf{r}) = E \psi_i(\mathbf{r}), \quad (2.32)$$

where E is the two-body energy. μ_i , ΔM_i , and ψ_i are the reduced mass, the mass difference from a reference energy, and the two-body wave function in channel i , respectively. It is expected that V^{equiv} is related to the interaction term in chiral unitary approach. However, because the two frameworks, the quantum mechanics and chiral unitary approach, are different from each other, the interactions used in the two models should not be directly connected. Here, as a reference, we first regard the Fourier transform of V^{Ch} as V^{equiv} . As in Eq. (2.19), the lowest order WT term is momentum independent, we obtain a delta-type potential as

$$V_{ij}^{\text{equiv}}(\mathbf{r}, E) = \delta^3(\mathbf{r}) N_{ij}(\sqrt{s}) V_{ij}^{\text{Ch}}(\sqrt{s}), \quad (2.33)$$

⁴ $F_{\bar{K}N}$ in Fig. 2.1 is the simplified one of the IHW model neglecting the isospin-breaking mass difference as explained in Sec. 2.2.4.

where $\sqrt{s} = E + M_i + m_i - \Delta M_i$ is the total center-of-mass energy. N is the kinematic factor to compensate the model difference between chiral unitary approach and the quantum mechanics. Following Ref. [18], we determine this kinematic factor in the Born approximation. In chiral unitary approach, scattering amplitudes F_{ij}^{Ch} is

$$F_{ij}^{\text{Ch}}(\sqrt{s}) = -\frac{\sqrt{M_i M_j}}{4\pi\sqrt{s}} T_{ij}^{\text{Ch}}(\sqrt{s}) \stackrel{\text{Born}}{\approx} -\frac{\sqrt{M_i M_j}}{4\pi\sqrt{s}} V_{ij}^{\text{Ch}}(\sqrt{s}). \quad (2.34)$$

M_i represents the baryon mass in channel i . On the other hand, by using the equivalent potential in Eq. (2.33), the amplitudes with the Schrödinger equation F_{ij}^{Sch} is

$$\begin{aligned} F_{ij}^{\text{Sch}}(\sqrt{s}) &= -(2\pi)^2 \sqrt{\mu_i \mu_j} T_{ij}^{\text{Sch}}(E) \\ &\stackrel{\text{Born}}{\approx} -(2\pi)^2 \sqrt{\mu_i \mu_j} \langle \mathbf{k}_i | \hat{V} | \mathbf{k}_j \rangle \\ &= -\frac{\sqrt{\mu_i \mu_j}}{2\pi} \int d\mathbf{r} e^{-i(\mathbf{k}_i - \mathbf{k}_j) \cdot \mathbf{r}} V_{ij}^{\text{equiv}}(\mathbf{r}, E) \\ &= -\frac{\sqrt{\mu_i \mu_j}}{2\pi} N_{ij}(\sqrt{s}) V_{ij}^{\text{Ch}}(\sqrt{s}) \int d\mathbf{r} e^{-i(\mathbf{k}_i - \mathbf{k}_j) \cdot \mathbf{r}} \delta^3(\mathbf{r}) \\ &= -\frac{\sqrt{\mu_i \mu_j}}{2\pi} N_{ij}(\sqrt{s}) V_{ij}^{\text{Ch}}(\sqrt{s}), \end{aligned} \quad (2.35)$$

where μ_i and k_i respectively represents the reduced mass and the meson momentum in the center-of-mass frame. Comparing Eqs. (2.34) and (2.35), N can be determined as⁵

$$N_{ij}(\sqrt{s}) = \frac{1}{2} \sqrt{\frac{M_i M_j}{s \mu_i \mu_j}}. \quad (2.38)$$

Because the delta potential appears from the Fourier transform of V^{Ch} , some regularization is necessary to solve the Schrödinger equation. In a similar sense to the momentum cutoff regularization or the dimensional regularization in chiral unitary approach, we smear the delta-type potential with finite range parameters to

⁵ In the previous works [22, 119, 120], respecting the pioneering work [18] for the $\Lambda(1405)$ system, the flux factor N is derived from the semi-relativistic form of the scattering amplitude,

$$F_{ij}^{\text{Sch}}(\sqrt{s}) = -(2\pi)^2 \sqrt{\tilde{\omega}_i \tilde{\omega}_j} T_{ij}^{\text{Sch}}(\sqrt{s}). \quad (2.36)$$

where $\tilde{\omega}_i = \omega_i E_i / (\omega_i + E_i)$ stands for the reduced energy. $\omega_i = (s - M_i^2 + m_i^2) / 2\sqrt{s}$ and $E_i = (s - m_i^2 + M_i^2) / 2\sqrt{s}$ are the meson energy and baryon energy, respectively, with the meson mass m_i and baryon mass M_i . Thus, the resultant flux factor is

$$N_{ij}(\sqrt{s}) = \frac{1}{2} \sqrt{\frac{M_i M_j}{s \tilde{\omega}_i \tilde{\omega}_j}}. \quad (2.37)$$

Considering that the Schrödinger equation is non-relativistic, we should use the factor in Eq. (2.38) obtained by the non-relativistic relation in our present work about the coupled-channel potential. Nevertheless, the resulting potentials in the previous works are still reliable. The determination of the range parameter in the single-channel case does not depend on the difference of N , because both the factors give the same value on the threshold. Furthermore, the difference of N in the previous works can be absorbed in the adjustment term ΔV . Therefore, the final potential after adding ΔV is equivalent to the one from the correct flux factor.

On the other hand, in the coupled-channel case, the treatment of N affects the determination of the range parameters. This is because even on the threshold of one channel, there appears the effect of the normalization factor of the other channels. Thus, we have to use the correct factor in Eq. (2.38) in the coupled-channel case to be consistent with the nonrelativistic Schrödinger equation (2.32).

construct a coordinate-space potential. The potential range would be related to the σ meson exchange (or the two-pion exchange) or the vector-meson exchange, which are not explicitly taken into account. For a while, we consider a general function $g_{ij}(\mathbf{r})$ as the spatial distribution of the potential,

$$V_{ij}^{\text{equiv}}(\mathbf{r}, \sqrt{s}) = g_{ij}(\mathbf{r}) N_{ij}(\sqrt{s}) V_{ij}^{\text{Ch}}(\sqrt{s}). \quad (2.39)$$

Considering that there are only the diagonal subtraction constants in chiral unitary approach with the dimensional regularization, we determine the range parameters of the local potential only for the diagonal parts and the off-diagonal part is determined from the diagonal parts. Requirements for the diagonal parts of g_{ij} are given by the prescription that the equivalent potential reproduces the amplitudes from chiral unitary approach

- in diagonal channels
- on threshold of each channel
- in the Born approximation,

that is, to meet

$$F_{ii}^{\text{Sch}}(\sqrt{s} = M_i + m_i) \Big|_{\text{Born}} = F_{ii}^{\text{Ch}}(\sqrt{s} = M_i + m_i) \Big|_{\text{Born}}. \quad (2.40)$$

Once again, we consider the amplitude from the equivalent potential,

$$\begin{aligned} F_{ij}^{\text{Sch}}(\sqrt{s}) &\stackrel{\text{Born}}{\approx} -\frac{\sqrt{\mu_i \mu_j}}{2\pi} \int d\mathbf{r} e^{-i(\mathbf{k}_j - \mathbf{k}_i) \cdot \mathbf{r}} V_{ij}^{\text{equiv}}(\mathbf{r}, \sqrt{s}) \\ &= -\frac{\sqrt{M_i M_j}}{4\pi\sqrt{s}} V_{ij}^{\text{Ch}}(\sqrt{s}) \int d\mathbf{r} e^{-i(\mathbf{k}_j - \mathbf{k}_i) \cdot \mathbf{r}} g_{ij}(\mathbf{r}) \end{aligned} \quad (2.41)$$

Here, if we set $i = j$ and $\sqrt{s} = M_i + m_i$, $\mathbf{k}_i = \mathbf{k}_j = 0$ is satisfied and the requirement for g_{ij} becomes

$$\int d^3r g_{ii}(\mathbf{r}) = 1. \quad (2.42)$$

Regarding the off-diagonal parts, we take something like the average of the spatial distribution of the diagonal parts as

$$g_{ij}(\mathbf{r}) = (g_{ii}(\mathbf{r}) g_{jj}(\mathbf{r}))^{-1/2}. \quad (2.43)$$

The validity of the above strategy will be confirmed in Sec. 2.4.1⁶.

Up to here, we have considered the general function g_{ij} . In the following, we assume a central force, especially a Gaussian type potential as the smeared delta function, which is convenient in few-body calculations. Because we consider only

⁶We have confirmed that our prescription regarding the off-diagonal part of the equivalent potential in Eq. (2.43) results in almost the same range parameters b_i and the scattering amplitudes $F_{ij}^{\text{equiv, eff}}$ as the ones from another prescription with the averaged off-diagonal range $b_{ij} = (b_i + b_j)/2$ and $g_{ij} = e^{-r^2/b_{ij}^2} / (\pi^{3/2} b_{ij}^3)$ derived from the normalization $\int d^3r g_{ij} = 1$, which is the same as the diagonal part in Eq. (2.42).

the s wave state for the $\Lambda(1405)$, the Schrödinger equation (2.32) is reduced to

$$\left[-\frac{1}{2\mu_i} \frac{d^2}{dr^2} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}^{\text{equiv}}(r, E) \right] u_j(r) = E u_j(r), \quad (2.44)$$

where $u_i(r)$ represents the s -wave part of the two-body radial wave function in channel i . Considering the above requirements (Eqs. (2.42) and (2.43)), an explicit form of the spatial distribution is

$$g_{ij}(r) = \frac{e^{-r^2(1/2b_i^2 + 1/2b_j^2)}}{\pi^{3/2} b_i^{3/2} b_j^{3/2}}, \quad (2.45)$$

where b_i represents the diagonal-potential range in channel i .

As the next step, we determine the range parameter b_i . The prescription is as follows: the equivalent potential reproduces the amplitude F^{Ch}

- in diagonal channels
- on each threshold
- with full calculation (not in the Born approximation).

that is,

$$F_{ii}^{\text{Sch}}(\sqrt{s} = M_i + m_i; b_i) = F_{ii}^{\text{Ch}}(\sqrt{s} = M_i + m_i). \quad (2.46)$$

Generally, even on thresholds, F^{Sch} depends on the choice of the range parameter in the full calculation. Thus, we regard b_i as the parameter to compensate the deviation on the thresholds.

For practical application of the potential to few-body calculations, we shall reduce the number of coupling channels from the full channels in chiral unitary approach. In our work, $\bar{K}N$ single or $\bar{K}N-\pi\Sigma(-\pi\Lambda)$ ⁷ coupled channel is chosen respecting the importance in the $\Lambda(1405)$ energy region. The reduction is conducted adopting the Feshbach projection to V_{ij}^{Ch} of the right-hand side in Eq. (2.39). When we reduce full N channels to n channels (leave $i = 1, 2, \dots, n$ channels and eliminate $i = n+1, n+2, \dots, N$ channels), the effective n -channel interaction $V_{ij}^{\text{Ch}(n)}$ can be obtained as⁸

$$\begin{aligned} V_{ij}^{(n)} &= V_{ij} + \tilde{V}_{ij}, \\ \tilde{V}_{ij} &= \sum_{\alpha=n+1}^N V_{i\alpha} G_{\alpha} V_{\alpha j} + \sum_{\alpha, \beta=n+1}^N V_{i\alpha} G_{\alpha} \tilde{T}_{\alpha\beta}^{(N-n)} G_{\beta} V_{\beta j}, \\ \tilde{T}_{\alpha\beta}^{(N-n)} &= V_{\alpha\beta}^{(N-n)} + \sum_{\gamma=n+1}^N V_{\alpha\gamma}^{(N-n)} G_{\gamma}^{(N-n)} \tilde{T}_{\gamma\beta}^{(N-n)} \\ &= \left(\left[\left\{ V^{(N-n)} \right\}^{-1} - G^{(N-n)} \right]^{-1} \right)_{\alpha\beta}, \end{aligned} \quad (2.47)$$

where G_i stands for the loop function in chiral unitary approach (Eq. (2.28)). The superscript $(N-n)$ means a matrix just eliminating $i = n+1, \dots, N$ channels. Though

⁷ In the $I = 0$ channel, we will construct the $\bar{K}N-\pi\Sigma$ potential, and the $\pi\Lambda$ channel will be taken into account in the $I = 1$ channel, assuming the isospin symmetry.

⁸For simplicity, we omit the superscript, "Ch", from the following equations.

we call $V^{(n)}$ the effective interaction, it exactly reproduces the full channel amplitude T_{ij} with the following equation,

$$\begin{aligned} T_{ij}^{(n)} &= V_{ij}^{(n)} + \sum_{k=1}^n V_{ik}^{(n)} G_k^{(n)} T_{kj}^{(n)} \\ &= \left(\left[\left\{ V^{(n)} \right\}^{-1} - G^{(n)} \right]^{-1} \right)_{ij} \\ &= T_{ij}, \end{aligned} \quad (2.48)$$

where the quantities with the superscript (n) are $n \times n$ matrices. With this effective interaction, the equivalent potential is constructed as

$$V_{ij}^{\text{equiv}}(r, \sqrt{s}) = \frac{e^{-r^2/(1/2b_i^2+1/2b_j^2)}}{2(\pi b_i b_j)^{3/2}} \sqrt{\frac{M_i M_j}{s \mu_i \mu_j}} V_{ij}^{\text{Ch, eff}}(\sqrt{s}), \quad (2.49)$$

where $V_{ij}^{(n)}$ is rewritten to $V_{ij}^{\text{Ch, eff}}$.

Here, we have to note that the effective interaction $V_{ij}^{\text{Ch, eff}}$ can be a complex value. It can be seen from Eq. (2.47). All the interaction terms V are real values (for example, see Eqs. (2.19)), and the only origin of the imaginary part is G_α ($\alpha = n+1, \dots, N$). The loop function G_α has the imaginary part only when the energy is above the threshold of the channel α . Hence, if we consider the energy region below all the eliminated channels, the effective interaction is a real value. On the other hand, if the energy exceeds one of the eliminated threshold, $V_{ij}^{\text{Ch, eff}}$ should be a complex value. In actual application, we consider the energy region around the $\Lambda(1405)$, which is above the $\pi\Sigma$ threshold and below the $\bar{K}N$ threshold. Thus, in the $\bar{K}N$ single-channel case, the effective interaction becomes complex, and results in a complex equivalent potential. In contrast, in the $\bar{K}N$ - $\pi\Sigma$ coupled channel case, the effective interaction and the equivalent potential remain real values.

In addition, the Feshbach projection is related to energy-dependence of the effective interaction or the equivalent potential. This is because G_α in Eq. (2.47) depends on the energy. Hence, even if the original interaction V_{ij}^{Ch} is energy-independent, $V_{ij}^{\text{Ch, eff}}$ becomes energy-dependent. Actually, as shown in previous section (see Eq. (2.19)), V_{ij}^{Ch} is originally energy-dependent. The Feshbach projection further includes the energy dependence.

With the equivalent potential in Eq. (2.49), the original amplitude F_{ij}^{Ch} would be well reproduced around the thresholds, considering the prescription to determine b_i on the thresholds (see Eq. (2.46)). However, the reproducibility would become worse far from the thresholds, because chiral unitary approach and the standard quantum mechanics are originally different frameworks. At least, the relativistic effect included in chiral unitary approach is not taken into account in the Schrödinger equation, and the regularization is conducted by the dimensional regularization scheme in chiral unitary approach but by the smearing of the delta-function type potential in the equivalent potential. To compensate the deviation, we add a term for adjustment ΔV_{ij} to Eq. (2.49) in each energy,

$$V_{ij}^{\text{equiv}}(r, \sqrt{s}) = \frac{e^{-r^2/(1/2b_i^2+1/2b_j^2)}}{2(\pi b_i b_j)^{3/2}} \sqrt{\frac{M_i M_j}{s \mu_i \mu_j}} \left(V_{ij}^{\text{Ch, eff}}(\sqrt{s}) + \Delta V_{ij}(\sqrt{s}) \right). \quad (2.50)$$

Thanks to this adjustment, we can apply the equivalent potential to a wider energy region. Detailed procedure to add ΔV_{ij} is explained in the following sections.

Due to the artificial adjustment at each energy, the potential cannot be analytically continued to the complex energy plane. To make it possible, we parametrize the strength of the potential (2.50) by a polynomial in the energy,

$$V_{ij}^{\text{equiv}}(r, E) = e^{-r^2(1/2b_i^2 + 1/2b_j^2)} \sum_{\alpha=0}^{\alpha_{\text{max}}} K_{\alpha,ij} E^{\alpha}, \quad (2.51)$$

$$(E = \sqrt{s} - M_N - m_K)$$

where the degree of a polynomial α_{max} is determined adequately as explained in the following sections.

Model	$b_{\bar{K}N}$ [fm]	Poles in $F_{\bar{K}N}^{\text{equiv}}$ [MeV]	Poles in $F_{\bar{K}N}^{\text{Ch}}$ [MeV]
ORB	0.52	$1419 - 42i$	$1427 - 17i, 1389 - 64i$
HNJH	0.47	$1421 - 35i$	$1428 - 17i, 1400 - 76i$
BNW	0.51	$1404 - 46i$	$1434 - 18i, 1388 - 49i$
BMN	0.41	$1416 - 27i$	$1421 - 20i, 1440 - 76i^{11}$

TABLE 2.1: Range parameter $b_{\bar{K}N}$ and $\Lambda(1405)$ pole positions from the previous equivalent potential [22]. The pole positions are obtained by analytically continuing $F_{\bar{K}N}^{\text{equiv}}$ to the complex energy plane [119]. For comparison, we show the original pole positions of the original amplitude $F_{\bar{K}N}^{\text{Ch}}$.

2.2 $\bar{K}N$ single-channel potential

Aiming at the quantitative discussion of the $\Lambda(1405)$ and the $\bar{K}$ nuclei, we reanalyze the previous $\bar{K}N$ potential in Ref. [22], which qualitatively reproduces the original scattering amplitude from chiral unitary approach, F^{Ch} , on the real energy axis. In this work, we establish a new method to quantitatively reproduce F^{Ch} even in the complex plane, where essential information of a resonant state is included. With this method, we construct the realistic $\bar{K}N$ potential with the recent SIDDHARTA constraint. This work is summarized in Refs. [119, 120].

2.2.1 Reanalysis of previous work

A similar formulation to the one in Sec. 2.1 is introduced in the previous work [22]. In that work, the equivalent local potentials are constructed to reproduce F^{Ch} in some chiral unitary approaches before the recent SIDDHARTA experiment [94, 95], the Oset-Ramos-Bennhold (ORB) [157], Hyodo-Nam-Jido-Hosaka (HNJH) [158, 159], Borasoy-Niβler-Weise (BNW) [160, 161], and Borasoy-Meißner-Niβler (BMN) [162] models. The form of the equivalent potential is the one in Eq. (2.50)⁹, where the range parameter $b_{\bar{K}N}$ is determined to make $\text{Re}[F_{\bar{K}N}^{\text{equiv}}] = 0$ at $\sqrt{s} \sim 1420$ MeV as in $F_{\bar{K}N}^{\text{Ch}}$. This prescription respects the binding energy of the quasibound state where the real part of the amplitude becomes close to zero¹⁰. The results of the ranges are summarized in Table 2.1.

Without the adjustment ΔV , the local potential fails to reproduce F^{Ch} especially in the $\sqrt{s} < 1420$ MeV region. This is because the connection of chiral unitary approach to the equivalent potential is conducted under the requirements: in the Born approximation and on the threshold. Thus, to broaden the applicable energy region from the near threshold region, the adjustment ΔV is added in $\sqrt{s} < 1420$ MeV. The resulting equivalent potentials are parametrized by a third order polynomial in energy as in Eq. (2.51).

⁹We note that the semi-relativistic flux factor (Eq. (2.37)) is used in the previous work, which is corrected to the non-relativistic one in our coupled-channel work (Sec. 2.4). However, because the effect of the different flux factor can be absorbed by the adjustment term ΔV , the final results are not affected by the choice of the flux factor.

¹⁰On the other hand, we determine the range on the $\bar{K}N$ threshold, respecting the whole flow of the potential construction (see Sec. 2.1).

¹¹One of the poles in the BMN model ($1440 - 76i$ MeV) lies above the $\bar{K}N$ threshold in the unphysical sheet.

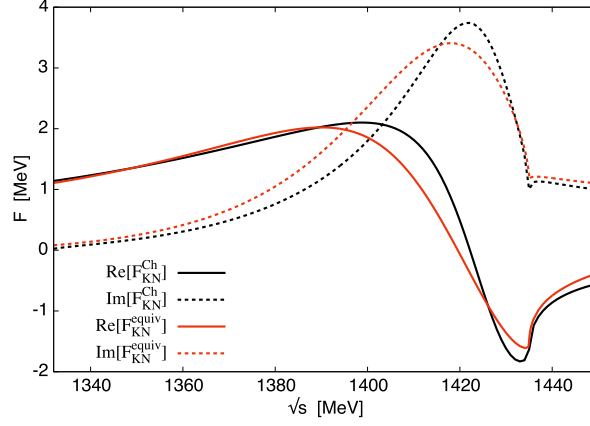


FIGURE 2.2: Scattering amplitude of the $\bar{K}N$ ($I = 0$) channel on the real energy axis in the previous work [22] (HNJH model [158, 159]). The real (imaginary) parts of the amplitudes are shown by the solid (dashed) lines. The black and red lines denote $F_{\bar{K}N}^{\text{Ch}}$ and $F_{\bar{K}N}^{\text{equiv}}$, respectively.

The results of the $\bar{K}N$ ($I = 0$) scattering amplitude from the equivalent potential, $F_{\bar{K}N}^{\text{equiv}}$, is shown in Fig. 2.2, compared with the one from chiral unitary approach (HNJH model), $F_{\bar{K}N}^{\text{Ch}}$. As seen from this figure, the previous equivalent potential almost reproduces the qualitative behavior of $F_{\bar{K}N}^{\text{Ch}}$ on the real energy axis.

As a first step of our work, we reanalyze the equivalent potential focusing on the amplitude in the complex energy plane. In general, the eigenenergy of a resonant state, including a quasibound state, is represented by a divergent pole of the amplitude in the complex plane. Thus, the nature of the resonance strongly depends on the pole structure, that is, the amplitude in the complex plane. Furthermore, the pole structure in a two-body system is considered to affect many-body systems like $\bar{K}$ nuclei [112]. Hence, the equivalent potential should reproduce $F_{\bar{K}N}^{\text{Ch}}$ in the complex plane, or the pole structure. To check it, we analytically continue $F_{\bar{K}N}^{\text{equiv}}$ to the complex energy plane, and search for the $\Lambda(1405)$ poles. The behavior of $F_{\bar{K}N}^{\text{equiv}}$ in the complex plane is shown in Fig. 2.3, and the pole positions are summarized in Table 2.1. It turns out from these figure and table that $F_{\bar{K}N}^{\text{equiv}}$ in the complex energy plane is different from $F_{\bar{K}N}^{\text{Ch}}$ qualitatively as well as quantitatively. In contrast to the double pole structure in chiral unitary approaches, the equivalent potentials reproduce only one resonance pole. Moreover, the pole position corresponds to neither of two poles from chiral unitary approach. Hence, to obtain a more reliable equivalent potential even in the complex plane, we improve the potential construction procedure.

To estimate the degree of the reliability of the potential, we define several quantities. Because we mainly focus on the $\Lambda(1405)$, we consider the following energy region for the definition,

$$\begin{aligned} 1332 \text{ MeV} &\leq \text{Re}[\sqrt{s}] \leq 1450 \text{ MeV}, \\ -100 \text{ MeV} &\leq \text{Im}[\sqrt{s}] \leq 50 \text{ MeV}. \end{aligned} \quad (2.52)$$

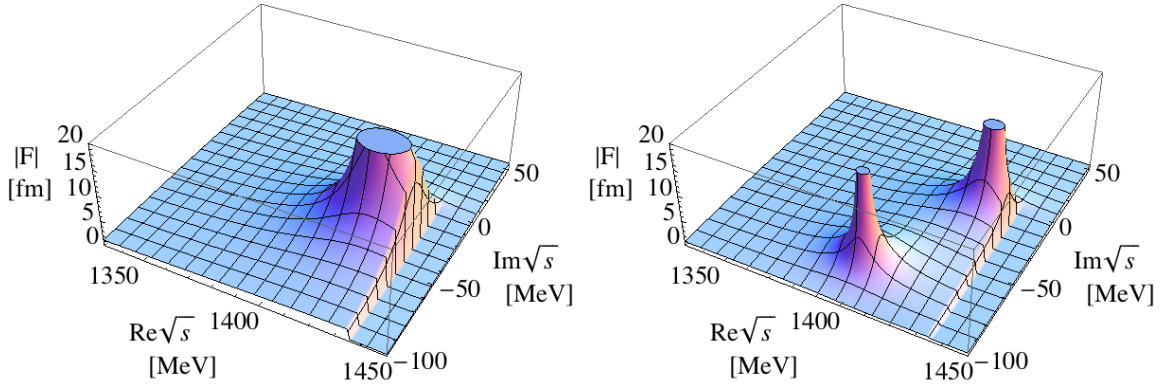


FIGURE 2.3: Magnitude of the $\bar{K}N$ ($I = 0$) scattering amplitudes with the HNJH model [158, 159] in the complex energy plane. The left panel shows $|F_{\bar{K}N}^{\text{equiv}}|$ obtained by analytically continuing the previous work [22]. The right panel shows $|F_{\bar{K}N}^{\text{Ch}}|$ for comparison.

Among this region, we first introduce the average deviation on the real axis ΔF_{real} between $F_{\bar{K}N}^{\text{equiv}}$ and $F_{\bar{K}N}^{\text{Ch}}$ as

$$\Delta F_{\text{real}} = \frac{\int d(\text{Re}\sqrt{s}) |F_{\bar{K}N}^{\text{Ch}}(\sqrt{s}) - F_{\bar{K}N}^{\text{equiv}}(\sqrt{s})|}{\int d(\text{Re}\sqrt{s}) |F_{\bar{K}N}^{\text{Ch}}(\sqrt{s})|}, \quad (2.53)$$

where the integration range is the one in Eq. (2.52). ΔF_{real} means the averaged deviation on the real axis normalized by the average of $|F_{\bar{K}N}^{\text{Ch}}|$. In the previous work [22] with the HNJH model¹², the deviation is $\Delta F_{\text{real}} = 0.14$.

Next, to estimate the reliability in the complex energy plane, we define the deviation of the amplitude in the complex $\sqrt{s}$,

$$\Delta F(\sqrt{s}) = \left| \frac{F_{\bar{K}N}^{\text{Ch}}(\sqrt{s}) - F_{\bar{K}N}^{\text{equiv}}(\sqrt{s})}{F_{\bar{K}N}^{\text{Ch}}(\sqrt{s})} \right|. \quad (2.54)$$

Here, considering the error bands with the SIDDHARTA constraint (see, for example, Fig. 4 in Ref. [24]), we name the energy region "precise region" where the deviation is less than 20 %,

$$\Delta F(\sqrt{s}) < 0.2. \quad (2.55)$$

The percentage of this precise region P_{comp} within the region of interest in Eq. (2.52) is calculated as

$$P_{\text{comp}} = \frac{\int \int d(\text{Re}\sqrt{s}) d(\text{Im}\sqrt{s}) \Theta(0.2 - \Delta F(\sqrt{s}))}{\int \int d(\text{Re}\sqrt{s}) d(\text{Im}\sqrt{s})} \times 100. \quad (2.56)$$

In our work, we aim to maximize P_{comp} .

Concerning the previous equivalent potential, we show the contour plot of ΔF in Fig. 2.4¹³. While the deviation on the real axis is relatively small ($\lesssim 20\%$), the one

¹²Hereafter, we take the equivalent potential with the HNJH model [22] as an example of the previous potentials.

¹³We choose the most adjacent Riemann sheet to the real energy axis.

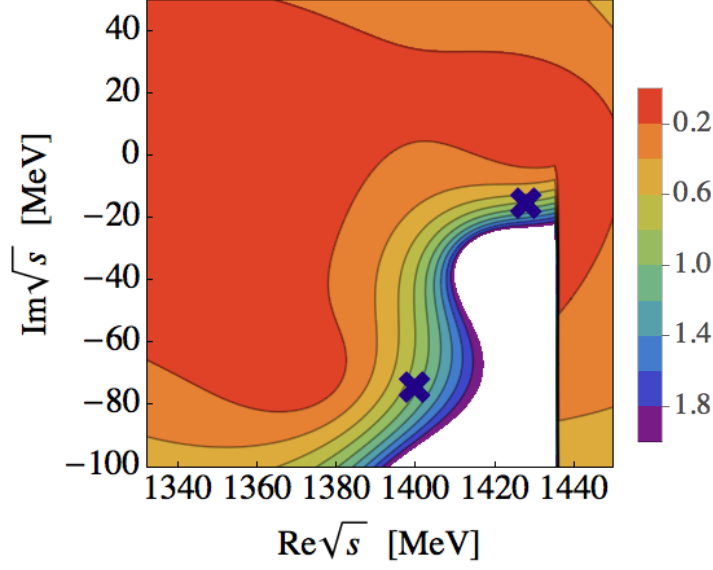


FIGURE 2.4: Contour plot of ΔF from the previous equivalent potential with the HNJH model [22]. The red region meeting $\Delta F < 0.2$ represents the "precise region". The pole positions of the $\Lambda(1405)$ in $F_{\bar{K}N}^{\text{Ch}}$ are represented by the crosses.

around the $\Lambda(1405)$ pole positions is much larger ($\gg 20\%$), related to the failure to reproduce the original pole structure. The percentage of the precise region, which corresponds to the red region in Fig. 2.4, is only 50 % ($P_{\text{comp}} = 50$). This result indicates the insufficient precision in the complex plane.

2.2.2 Precision near the real energy axis

For a while, we use the HNJH model to examine how the reproducibility is improved comparing with the previous work [22]. After establishing the method to precisely construct the potential, we impose the recent SIDDHARTA constraint.

First, we focus on the energy region near the real axis, expecting to reproduce the higher pole of the $\Lambda(1405)$ with relatively small decay width. In such region, the reproducibility on the real energy axis should be essential. From Figs. 2.2 and 2.4, we find that there remains some deviation around $\sqrt{s} \sim 1400 - 0i$ MeV. This is because the adjustment ΔV in Eq. (2.50) is chosen to be a real value in the previous work [22]. On the other hand, because the $\pi\Sigma$ channel, which is the decay channel of the $\Lambda(1405)$, is eliminated to obtain the effective interaction $V^{\text{Ch, eff}}$ in Eq. (2.50), the $\bar{K}N$ equivalent potential is a complex quantity to account for the decay effect. Thus, there is no reason to restrict ΔV to be real as in the previous work. Furthermore, this seems to be related to the inadequate region to add ΔV below 1400 MeV, which does not cover the higher pole position. Actually, when we try to add the real ΔV above 1400 MeV, an unphysically large ΔV is necessary.

Taking into account the above problem, in this work, we add ΔV

- as a complex value,
- to the relevant energy region for the higher pole, 1332-1450 MeV¹⁴.

¹⁴The lower limit, 1332 MeV, is set slightly above the $\pi\Sigma$ threshold. In our $\bar{K}N$ single-channel case, the non-analytic behavior on the $\pi\Sigma$ threshold cannot be reproduced by the $\bar{K}N$ analytic potential.

	previous potential [22]	Potential I	Potential II
$b_{\bar{K}N}$ [fm]	0.47	0.46	0.46
ΔV	Real	Complex	Complex
Polynomial type	Third order	Third order	Tenth order
Adjustment range	1300-1400	1332-1450	1332-1521
Parametrized range	1300 1450	1332-1450	1332-1521
ΔF_{real}	1.4×10^{-1}	4.8×10^{-3}	4.0×10^{-4}
P_{comp}	50	68	85
Pole positions [MeV]	1421 – 35 <i>i</i>	1427 – 17 <i>i</i>	1428 – 17 <i>i</i> , 1400 – 77 <i>i</i>

TABLE 2.2: Properties of the previous potential [22], Potential I, and Potential II. The original chiral unitary approach for all the potentials is the HNJH model. "Adjustment range" and "Parametrized range" respectively represent the energy range where ΔV is added and the one where the potential is parametrized by a polynomial. We should compare the pole positions to the original one from chiral unitary approach, 1428 – 17*i* MeV and 1400 – 76*i* MeV.

$\Delta V(\sqrt{s})$ is determined to minimize the deviation between $F_{\bar{K}N}^{\text{equiv}}$ and $F_{\bar{K}N}^{\text{Ch}}$ on the real energy axis,

$$\left| F_{\bar{K}N}^{\text{equiv}}(\sqrt{s}) - F_{\bar{K}N}^{\text{Ch}}(\sqrt{s}) \right| \quad (\sqrt{s} \in \mathbb{R}), \quad (2.57)$$

at each energy. We parametrize the resulting equivalent potential by a polynomial in the energy¹⁵,

$$V_{\bar{K}N}^{\text{equiv}}(r, \sqrt{s}) = \frac{e^{-r^2/b_{\bar{K}N}^2}}{\pi^{3/2}b_{\bar{K}N}^3} \frac{M_N}{2\tilde{\omega}_{\bar{K}N}\sqrt{s}} \sum_{\alpha=0}^{\alpha_{\text{max}}} K_{\alpha}(\sqrt{s} - M_N - m_{\bar{K}})^{\alpha}. \quad (2.58)$$

Here, the degree of the polynomial is set to be third order, to directly compare with the previous work. We name the first improved potential Potential I, and summarize the properties in Table 2.2¹⁶. Here, "Adjustment range" and "Parametrized range" respectively represent the energy range where ΔV is added and the one where the potential is parametrized by a polynomial. In Fig. 2.5, we show the contour plot of ΔF in Eq. (2.54).

Thanks to the improvements, the average deviation on the real axis ΔF_{real} of Potential I is reduced by two orders of magnitude from the previous one. The precision on the real energy axis leads to the better description in the complex energy plane as seen from the comparison of Fig. 2.5 with Fig. 2.4. It turns out that the precise region, the red area in Fig. 2.5 where $\Delta F < 0.2$ is met, is enlarged near the real axis, and covers the higher pole position. The enlargement is also confirmed from the increase of the percentage of the precise region, P_{comp} , from 50 to 68 %. As a result, we succeed in reproducing the higher pole almost at the original position from chiral unitary approach. In this way, it is shown that the complex ΔV considering the decay effect and the relevant adjustment range for a resonance pole are necessary to reproduce the original amplitude in the small $\text{Im}\sqrt{s}$ region as well as on the real energy axis.

¹⁵We note that the parametrizing function here is different from the one in Eq. (2.51)

¹⁶The range parameter $b_{\bar{K}N}$ is determined as 0.46 fm to minimize the quantity $|F_{\bar{K}N}^{\text{Ch}} - F_{\bar{K}N}^{\text{equiv}}|$ on the $\bar{K}N$ threshold before adding ΔV (see Sec. 2.1.2).

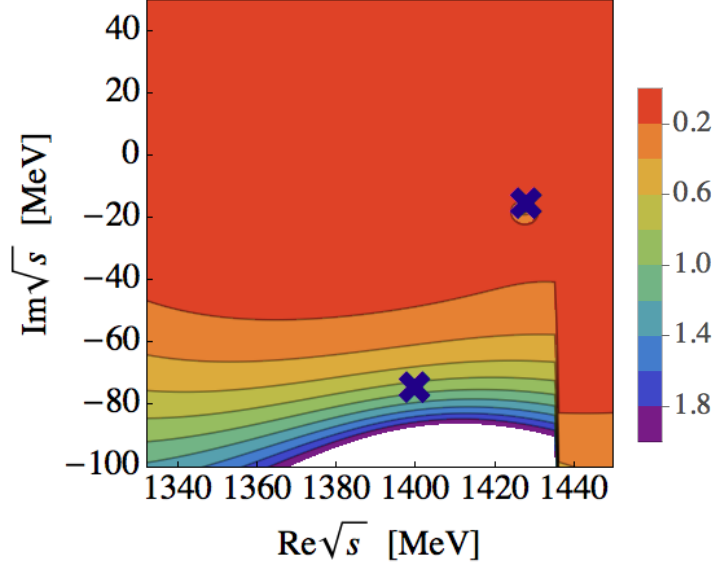


FIGURE 2.5: Contour plot of ΔF from Potential I with the HNJH model [22]. The red region meeting $\Delta F < 0.2$ represents the "precision region". The pole positions of the $\Lambda(1405)$ in $F_{\bar{K}N}^{\text{Ch}}$ are represented by the crosses.

2.2.3 Precision in large $|\text{Im}\sqrt{s}|$ region

Though Potential I reproduces $F_{\bar{K}N}^{\text{Ch}}$ near the real energy axis, $F_{\bar{K}N}^{\text{equiv}}$ starts to deviate from $F_{\bar{K}N}^{\text{Ch}}$ as the energy goes far from the real axis to $\text{Im}\sqrt{s} \lesssim -50$ MeV (see Fig. 2.5). The lower pole of the $\Lambda(1405)$ at $1400 - 76i$ MeV lies in such a large $|\text{Im}\sqrt{s}|$ region, and Potential I fails to generate it. In this section, we further improve the potential to reproduce $F_{\bar{K}N}^{\text{Ch}}$ in the large $|\text{Im}\sqrt{s}|$ region.

Here, we respect the uniqueness of the analytic continuation, which means if $F_{\bar{K}N}^{\text{equiv}}$ is completely same as $F_{\bar{K}N}^{\text{Ch}}$ on the whole real energy axis, $F_{\bar{K}N}^{\text{equiv}}$ in the complex energy plane is also same as $F_{\bar{K}N}^{\text{Ch}}$. This indicates that the increase of the adjustment range and the parametrized range¹⁷ will improve the reproducibility in the large $|\text{Im}\sqrt{s}|$ region. However, we should note the limitation of the parametrized range. In general, there appears a non-analytic behavior of the scattering amplitude at two-body thresholds. With the polynomial energy dependence of the strength of the single-channel $\bar{K}N$ potential, we cannot reproduce the non-analyticity at thresholds other than the $\bar{K}N$ channel, while the $\bar{K}N$ threshold effect can be naturally obtained. Therefore, the extension of the parametrized range should be limited between the nearest thresholds, the $\pi\Sigma$ threshold (1331 MeV) and the $\eta\Lambda$ threshold (1664 MeV) in this case. There is another note that the precision on the real energy axis, that is, the smallness of ΔF_{real} , should be kept while broadening the parametrized range. For this reason, we change the degree of the polynomial in Eq. (2.58) from the third order to tenth order.

Based on the above strategy, we vary the parametrized range by 1 MeV step. In Table 2.3, we show some of the results about ΔF_{real} , P_{comp} , and the pole positions. We find that both the $\Lambda(1405)$ poles appear here. The higher one is stable and the lower is sensitive against the change of the parametrized range. Generally, more poles tend

¹⁷Hereafter, we set the parametrized range equal to the adjustment range, and just call it "Parametrized range".

upper boundary [MeV]	ΔF_{real}	P_{comp}	pole positions [MeV]
1450	1.8×10^{-4}	59	1428 – 17i, 1388 – 60i, unphysical poles
1500	2.6×10^{-4}	71	1428 – 17i, 1404 – 70i, unphysical poles
1521	4.0×10^{-4}	85	1428 – 17i, 1400 – 77i
1550	5.9×10^{-4}	79	1428 – 17i, 1392 – 82i
1600	6.8×10^{-4}	77	1428 – 17i, 1389 – 83i
1650	8.8×10^{-4}	77	1428 – 17i, 1389 – 87i

TABLE 2.3: Results of the reproducibility of the HNJH model with equivalent potentials against the change of the parametrized range. The lower boundary is fixed just above the $\pi\Sigma$ threshold (1332 MeV), and the higher one is variously changed. The original pole positions of $F_{\bar{K}N}^{\text{Ch}}$ are 1428 – 17i MeV and 1400 – 76i MeV.

to appear as we increase a degree of polynomial. The appearance of the lower pole would be related to the increase of the degree of the polynomial. However, in such case, we have to note the existence of unphysical poles. Actually, in our case some unphysical poles, which do not exist in the original amplitude F^{Ch} , appear around the $\Lambda(1405)$ energy region for relatively narrow parametrized range. However, as expected from the above-mentioned uniqueness of the analytic continuation, as we broaden the parametrized range, the unphysical poles disappear and only the physical $\Lambda(1405)$ poles remain. What is different from the expectation is that the precise region P_{comp} and the reproducibility of the pole positions do not monotonically become better, but they have an optimal parametrized range halfway. One of the reasons would be the limitation of the lower boundary of the parametrized range at the $\pi\Sigma$ threshold, which makes it difficult to enlarge the precise region in the lower energy region. It may be another reason that the average deviation on the real energy axis, ΔF_{real} , becomes slightly larger as the parametrized range is broadened, because a pole position with a large decay width is quite sensitive to the precision on the real energy axis.

Anyway, with a larger parametrized range (1332-1521 MeV) than Potential I, we obtain an optimal potential, which is named "Potential II". The properties are summarized in Table 2.2, and the deviation in the complex energy plane, ΔF , is shown in Fig. 2.6. The precise region is further extended near the lower pole as well as the higher pole, resulting in the larger percentage, $P_{\text{comp}} = 85$. Consequently, both the $\Lambda(1405)$ poles are reproduced at the correct positions. Thanks to this success, it becomes possible to discuss the nature or the effect of the lower pole, too¹⁸.

2.2.4 Realistic potential with SIDDHARTA constraint

In the previous section, we confirm with the HNJH model that the new procedure can reproduce the original amplitude broadly in the complex energy plane. Applying the procedure, we construct a realistic $\bar{K}N$ potential with the recent experimental constraint from the SIDDHARTA data [94, 95], which is crucial for quantitative discussion of the $\Lambda(1405)$ and $\bar{K}$ nuclei. Keeping the application to the $\bar{K}$ nuclei in mind, we construct equivalent potentials both in $I = 0$ and $I = 1$ channels.

¹⁸There is an unphysical pole around 1440 – 100i MeV. If we employ such potential, we should be careful of the corresponding energy region.

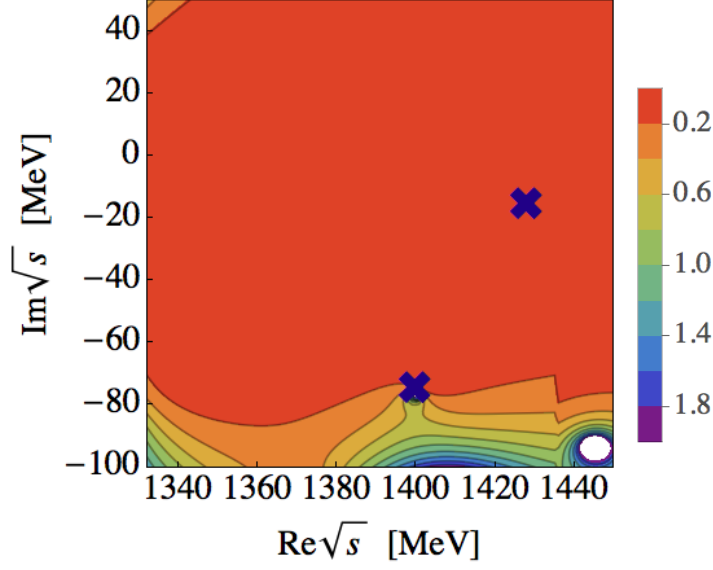


FIGURE 2.6: Contour plot of ΔF from Potential II with the HNJH model [22]. The red region meeting $\Delta F < 0.2$ represents the "precise region". The pole positions of the $\Lambda(1405)$ in F_{KN}^{Ch} are represented by the crosses.

As an original amplitude, we employ the Ikeda-Hyodo-Weise (IHW) model [23, 24], which is a chiral unitary approach utilizing the interactions of chiral perturbation theory up to next-to-leading order terms to fit the energy shift of the SID-DHARTA measurements (see Sec. 2.1.1). We note that the particle basis is adopted in the IHW model with the isospin-breaking mass difference. In applications, the isospin basis without such isospin breaking is more convenient. Furthermore, in the potential construction, the multiple thresholds in the particle basis prevent us from broadening the parametrized range for the single-channel potential. Therefore, we first reconstruct an isospin symmetric $\bar{K}N$ amplitude based on the IHW model. The process is just replacing the physical hadron masses with the isospin averaged ones, fixing all other parameters same as the ones in the IHW model. In Fig. 2.7, we compare the isospin symmetric $I = 0$ $\bar{K}N$ amplitude to the original one with the particle basis¹⁹. It turns out that both the amplitudes well accord with each other except for the $\bar{K}N$ threshold region, and the pole positions are also same as the original ones within 1 MeV accuracy. Because the deviated region is tiny and negligible for the analysis of the $\Lambda(1405)$ and $\bar{K}$ nuclei, we employ the isospin symmetric amplitude as the basis of the equivalent potential.

The range parameter b_{KN} in Eq. (2.50) is determined as $b_{KN} = 0.38$ fm for the $I = 0$ channel, following the procedure in Sec. 2.1.2. Applying the procedure in Secs. 2.2.2 and 2.2.3, the adjustment ΔV in Eq. (2.50) is chosen to be complex, and the upper boundary of the parametrized range is variously changed by 1 MeV fixing the lower boundary just above the $\pi\Sigma$ threshold. The optimal parametrized range is found to be 1332-1657 MeV, and we name it "Kyoto $\bar{K}N$ potential ($I = 0$)". We summarize the properties of the potential in Table 2.4 and the deviation of the amplitudes in the complex energy plane, ΔF , in Fig. 2.8. The average deviation on the real energy axis, ΔF_{real} , is kept quite small over a wide energy range almost from

¹⁹To obtain the corresponding amplitude to the $I = 0$, we combine the original particle-basis amplitudes as $(F_{K^-p, K^-p} + 2F_{K^-p, \bar{K}^0n} + F_{\bar{K}^0n, \bar{K}^0n})/2$.

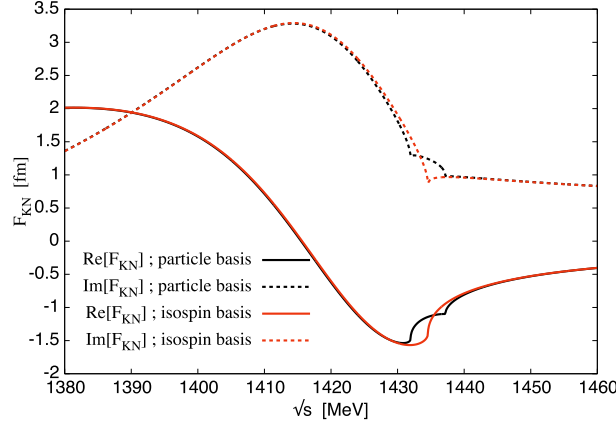


FIGURE 2.7: Isospin symmetric $I = 0$ $\bar{K}N$ amplitude (red lines) and the corresponding combination of the original IHW model with isospin breaking [23, 24], $(F_{K^-p, K^-p} + 2F_{K^-p, \bar{K}^0n} + F_{\bar{K}^0n, \bar{K}^0n})/2$ (black lines). The real (imaginary) parts of the amplitudes are shown by the solid (dashed) lines.

	Kyoto $\bar{K}N$ potential ($I = 0$)
b [fm]	0.38
ΔV	complex
polynomial type	tenth order
parameterized range [MeV]	1332-1657
ΔF_{real}	5.4×10^{-3}
P_{comp}	96
pole positions [MeV]	$1424 - 26i, 1381 - 81i$

TABLE 2.4: Properties of Kyoto $\bar{K}N$ potential ($I = 0$) based on the IHW model. The original pole positions of the IHW model are $1424 - 26i$ MeV and $1381 - 81i$ MeV.

the $\pi\Sigma$ to $\eta\Lambda$ threshold. As a result, the precise region extends over the lower pole position as well as the higher one, and the percentage amounts to 96 %. Regarding the pole positions, Kyoto $\bar{K}N$ potential completely reproduces both ones in the IHW model within 1 MeV accuracy.

In exactly the same way, we construct the potential for the $I = 1$ channel²⁰, and name it "Kyoto $\bar{K}N$ potential ($I = 1$)", again. $b_{\bar{K}N}$ is determined to be 0.37 fm, and ΔV is chosen to be a complex value. However, in this case, we cannot determine the best parametrized range comparing $F_{\bar{K}N}^{\text{equiv}}$ with $F_{\bar{K}N}^{\text{Ch}}$ in the complex energy plane. This is because in the $I = 1$ channel, the simple analytic continuation to the complex plane is impossible for the prescription to avoid the unphysical cut in $F_{\bar{K}N}^{\text{Ch}}$ [161]. Thus, we adopt the same parametrized range as in the $I = 0$ case, 1332-1657 MeV. Such determination would be sufficient, considering that the strength of the $I = 1$ potential is weaker than the $I = 0$ one and main contribution to the $\bar{K}$ nuclei is expected to come from the $I = 0$ channel.

Finally, we show the coefficients in Eq. (2.58) for Kyoto $\bar{K}N$ potentials in Tables 2.5 and 2.6, and the energy dependence of the potentials at $r = 0$ in Fig. 2.9.

²⁰The combination of the particle basis corresponding to the $I = 1$ channel is $(F_{K^-p, K^-p} - 2F_{K^-p, \bar{K}^0n} + F_{\bar{K}^0n, \bar{K}^0n})/2$.

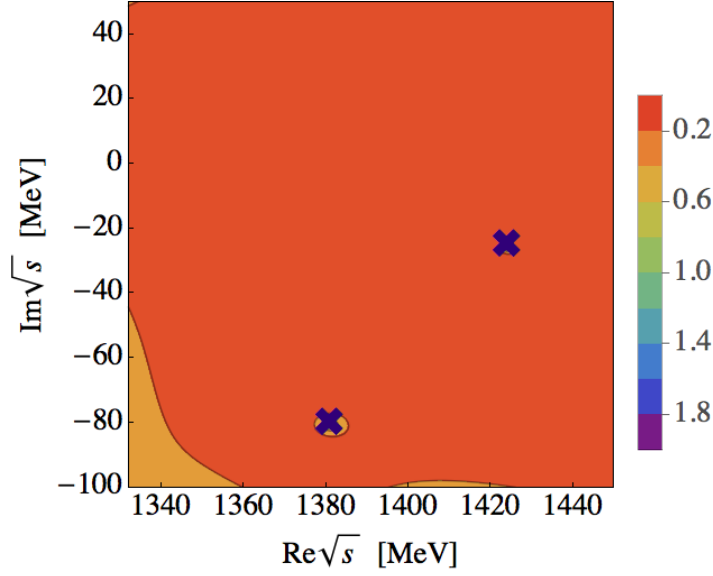


FIGURE 2.8: Contour plot of ΔF from Kyoto $\bar{K}N$ potential ($I = 0$) based on the IHW model. The red region meeting $\Delta F < 0.2$ represents the "precise region". The pole positions of the $\Lambda(1405)$ in original $F_{\bar{K}N}^{\text{Ch}}$ are represented by the crosses.

K_0 [fm]		K_1 [fm]		K_2 [fm]		K_3 [fm]	
Re	Im	Re	Im	Re	Im	Re	Im
-10.833	-1.8149	-2.7962	-0.64315	-0.47980	0.88991	-0.64480	-0.55225
K_4 [fm]		K_5 [fm]		K_6 [fm]		K_7 [fm]	
Re	Im	Re	Im	Re	Im	Re	Im
0.44645	0.0048399	0.089658	0.47326	-0.23222	-0.38284	0.027650	-0.072843
K_8 [fm]		K_9 [fm]		K_{10} [fm]			
Re	Im	Re	Im	Re	Im		
0.059123	0.22152	-0.024071	-0.099375	0.0022208	0.014415		

TABLE 2.5: Coefficients K_α in Eq. (2.58) of the strength of Kyoto $\bar{K}N$ potential ($I = 0$).

From the figure, we find that the strength of the $I = 0$ real part is approximately twice stronger than the one of the $I = 1$, which indicates the much stronger attraction in the $I = 0$ channel. On the other hand, the imaginary parts have an almost same strength for the $I = 0$ and $I = 1$ especially around the $\Lambda(1405)$ energy region. This suggests that the decay to the $\pi\Sigma$ and $\pi\Lambda$ channels occurs equally in the $I = 0$ and $I = 1$ channels.

We can say that Kyoto $\bar{K}N$ potentials are realistic in the sense that they succeed in reproducing the scattering amplitudes in the IHW model which fits the experimental data with $\chi^2/\text{d.o.f} \simeq 1$ (See Sec. 2.1.1), and the pole positions listed in PDG [1]. These realistic $\bar{K}N$ potentials enable us to quantitatively discuss various systems including the antikaons and nucleons, such as the $\bar{K}$ nuclei. In the following sections, we will show some applications of our potentials.

K_0 [fm]		K_1 [fm]		K_2 [fm]		K_3 [fm]	
Re	Im	Re	Im	Re	Im	Re	Im
-6.2261	-1.7382	-2.1909	-0.63300	-0.37668	-0.082052	-0.14782	-0.18206
K_4 [fm]		K_5 [fm]		K_6 [fm]		K_7 [fm]	
Re	Im	Re	Im	Re	Im	Re	Im
2.9791	0.52183	-0.53283	0.29745	-3.0760	-0.59389	1.5430	-0.062035
K_8 [fm]		K_9 [fm]		K_{10} [fm]			
Re	Im	Re	Im	Re	Im		
0.64668	0.36800	-0.59141	-0.17606	0.10746	0.026128		

TABLE 2.6: Coefficients K_α in Eq. (2.58) of the strength of Kyoto $\bar{K}N$ potential ($I = 1$).

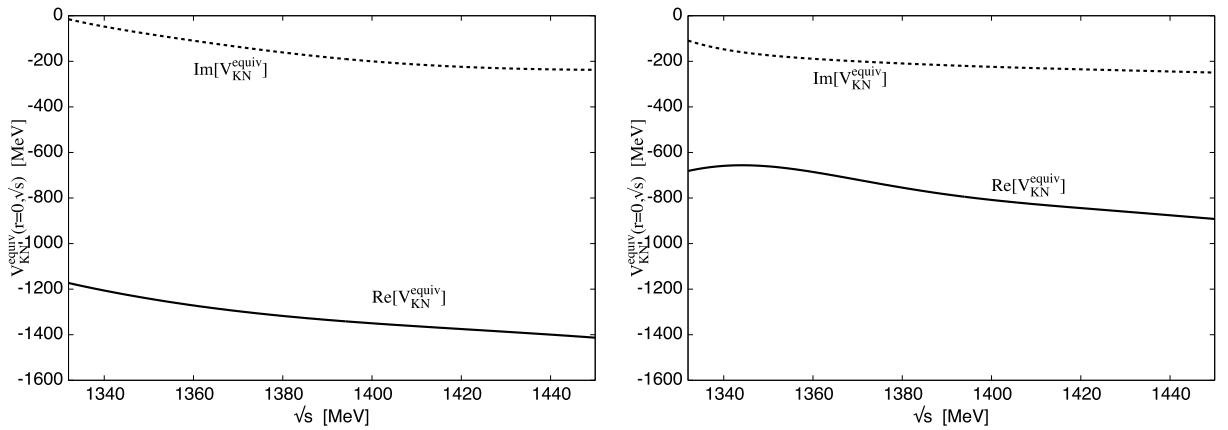


FIGURE 2.9: Energy dependence of Kyoto $\bar{K}N$ potentials at $r = 0$, $V_{\bar{K}N}^{\text{equiv}}(r = 0, \sqrt{s})$. The $I = 0$ and $I = 1$ potentials are shown in the left and right panels, respectively. The solid (dashed) lines represents the real (imaginary) part of the potentials.

2.3 Application of $\bar{K}N$ single-channel potential

Before constructing a coupled-channel potential, we apply the realistic single-channel potential to some systems. The first example is the $\bar{K}$ nuclei, which consist of some nucleons and one anti-kaon, to theoretically predict new states. The second one is the hadron-hadron correlation in heavy-ion collisions, which is expected to lead to the information of a hadron interaction.

2.3.1 $\bar{K}$ -nuclei

Recently, with progressing experimental studies about the $\bar{K}$ nuclei [122, 124, 127–131, 133], more precise theoretical studies have become needed to confirm their existence and clarify the properties. In this work, we employ the realistic $\bar{K}N$ potential obtained in Sec. 2.2, and the accurate few-body calculation method, the stochastic variational method (SVM) with correlated Gaussian basis (CG) [164–166], aiming at quantitative discussion of the $\bar{K}$ nuclei. This work is summarized in Ref. [108].

For a system with $(\mathcal{N} - 1)$ nucleons and one antikaon, the Hamiltonian is

$$H = \sum_{i=1}^{\mathcal{N}} T_i - T_{\text{cm}} + \sum_{i<j}^{\mathcal{N}-1} V_{ij}^{(NN)} + \sum_{i=1}^{\mathcal{N}-1} V_{i\mathcal{N}}^{(\bar{K}N)} + \sum_{i<j}^{\mathcal{N}} V_{ij}^{\text{Coul}}, \quad (2.59)$$

where T_i denotes the kinetic energy of the i -th particle and T_{cm} is the center-of-mass energy. $V_{ij}^{(NN)}$, $V_{i\mathcal{N}}^{(\bar{K}N)}$, and V_{ij}^{Coul} represent the NN strong interaction, the $\bar{K}N$ strong interaction, and the Coulomb interaction, respectively. For the nuclear force, we utilize the Argonne V4' potential (AV4') [167], which is obtained by deleting the electromagnetic, spin-orbit, and tensor terms from the realistic Argonne V18 potential [168], while the central potential is readjusted to obtain the deuteron binding. To light nuclei, the AV4' is reasonably applicable.

As the $\bar{K}N$ potential, we employ our realistic single-channel potential (see Sec. 2.2 and our previous work [119, 120]). Here we have to note that the potential is complex and energy dependent. In particular, it is important that, because of the energy dependence, there appears ambiguity in few-body calculations. The energy in the $\bar{K}N$ potential is the $\bar{K}N$ two-body energy, and we cannot uniquely determine it in the few-body systems. As a prescription, following the previous works [105, 106], we relate the two-body energy $\sqrt{s}$ to the calculable quantities in the variational method,

$$\sqrt{s} = M_N + m_{\bar{K}} + \delta\sqrt{s} \quad (2.60)$$

$$\text{Type I: } \delta\sqrt{s} = \langle H \rangle_G - \langle H_N \rangle_G, \quad (2.61)$$

$$\text{Type II: } \delta\sqrt{s} = \frac{\langle H \rangle_G - \langle H_N \rangle_G}{\mathcal{N} - 1}, \quad (2.62)$$

where the subscript G denotes the expectation value with the Gamow vector (see Sec. 3.1.1). H_N is the Hamiltonian only for the nucleons,

$$H_N = \sum_{i=1}^{\mathcal{N}-1} T_i - T_{\text{cm}}^N + \sum_{i<j}^{\mathcal{N}-1} V_{ij}^{(NN)} + \sum_{i<j}^{\mathcal{N}-1} V_{ij}^{\text{Coul}}, \quad (2.63)$$

with the center-of-mass energy of the $\mathcal{N} - 1$ nucleons T_{cm}^N subtracted. Here, $\langle H_N \rangle_G - \langle H \rangle_G$ can be interpreted as an "antikaon binding energy". Type I is based on the picture that the antikaon field collectively surrounds all nucleons. And Type II is

based on the one that the antikaon energy is split among all nucleons (same explanation as Ref. [105]). In numerical calculation, we require the self-consistency for $\sqrt{s}$. That is, we first assume some $\sqrt{s}^{(0)}$ as an input, and calculate with the variational method to obtain an optimized wave function. With the wave function, we calculate the tow-body energy $\sqrt{s}^{(1)}$ by Eq. (2.60), and compare with the input value $\sqrt{s}^{(0)}$. If $\sqrt{s}^{(0)} \neq \sqrt{s}^{(1)}$, then we use $\sqrt{s}^{(1)}$ as a next input. Until the energy $\sqrt{s}^{(n)}$ converges to $\simeq \sqrt{s}^{(n-1)}$, we iterate the above procedure.

In this work, we consider only systems with zero total orbital momentum, $L = 0$, which means that the total angular momentum J becomes the total spin S , $J = S$. Because the NN and $\bar{K}N$ interactions consist of only central forces, the channel coupling between different L vanishes and we do not have to consider states with $L > 0$. The $\mathcal{N}$ -body wave function is expanded by the basis functions,

$$\Psi_{JMM_T}(\mathbf{x}) = \Psi_{SM_S M_T}(\mathbf{x}) = \sum_{k=1}^K c_k \Phi_{SM_S M_T}(\mathbf{x}, A_k), \quad (2.64)$$

where M , M_S , and M_T respectively represent the z component of the total angular momentum, the total spin, and the total isospin. $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ denotes the Jacobi coordinate with the $\mathcal{N} - 1$ components. The basis function is formed as

$$\Phi_{SM_S M_T} = \mathcal{A} \{ \exp(-\tilde{x} A x) \chi_{SM_S} \eta_{M_T} \}, \quad (2.65)$$

where the operator $\mathcal{A}$ is an antisymmetrizer for nucleons. χ_{SM_S} and η_{M_T} represent the spin and isospin wave functions, respectively. $\tilde{x}$ denotes a transpose of x . Here, we should note that the effect of the higher partial waves for each coordinate can be considered thanks to the cross term, $x_i \cdot x_j$, of the CG basis in Eq. (2.65). A $(\mathcal{N} - 1) \times (\mathcal{N} - 1)$ matrix A is symmetric and positive definite, and its elements are the variational parameters, as well as the parameters c_k in Eq. (2.64). To optimize the parameters, we apply the efficient and economical method, SVM, where the basis size is increased one-by-one to avoid the costly repetition of the full diagonalization [164–166].

The variational method is usually applied to Hermitian systems, where eigenenergies are always real values, and the variational parameters are determined to minimize the value. On the other hand, when we deal with a complex eigenenergy as in the Non-Hermitian system, the "minimum value" becomes meaningless. Our target is also such Non-Hermitian system, where the $\bar{K}N$ potential is complex reflecting the reduction of the $\pi\Sigma$ channel, and some ingenuity is necessary to obtain a complex eigenenergy. In the previous works [105, 106], the variational method is conducted only with the real part of the potential to obtain a real energy interpreted as the real part of the genuine resonance energy. And the decay width is estimated perturbatively as $\Gamma = -2\langle\Psi|\text{Im}V^{(\bar{K}N)}|\Psi\rangle$. In other works [114, 118], the resonance energy is determined with the complex-scaling method (see Sec. (3.2.2)). A θ -independent energy among energies obtained by diagonalizing the Hamiltonian is judged as the resonance energy, which is based on the so-called ABC theorem [169, 170] about the θ dependence of resonances and scattering states. In our work [108], we propose a new strategy to obtain the complex eigenenergy nonperturbatively. The scheme is as follows:

- i). apply the SVM only for the real part of the Hamiltonian;
- ii). obtain the minimum value of the real eigenenergy and the corresponding basis functions;

- iii). with the basis functions, we diagonalize the full Hamiltonian to obtain the complex eigenenergy.

The validity of this strategy is checked in the $\bar{K}N$ two-body and few-body systems. We confirm that the eigenenergies of the full Hamiltonian converges with the basis functions to achieve the convergence of the eigenenergies of the real par of the Hamiltonian. In the two-body system where an exact eigenenergy can be obtained from the analysis of the scattering amplitude, our method reproduces the exact value. In the few-body systems, using the phenomenological AY potential [102, 104] for the $\bar{K}N$, we confirm the consistency of our results with the ones from other calculations [102–104].

To analyze the spatial structure of few-body systems, we define the N mean-squared radii $\langle r_N^2 \rangle$ and the $\bar{K}$ mean-squared radii $\langle r_{\bar{K}}^2 \rangle$ as

$$\langle r_N^2 \rangle = \int d^3r \left\{ \sum_i^{\mathcal{N}-1} \frac{|\mathbf{r}_i - \mathbf{x}_N|}{\mathcal{N}-1} \right\} |\Psi_{SM_S M_T}(\mathbf{x})|^2, \quad (2.66)$$

$$\langle r_{\bar{K}}^2 \rangle = \int d^3r \{ |\mathbf{r}_{\bar{K}} - \mathbf{x}_N|^2 \} |\Psi_{SM_S M_T}(\mathbf{x})|^2, \quad (2.67)$$

where $\mathbf{r}_i$ and $\mathbf{r}_{\bar{K}}$ respectively denote the single-particle coordinate of the i th nucleon and the antikaon. $\mathbf{x}_N$ stands for the center-of-mass coordinate. These quantities mean the averaged distance of the nucleon or the antikaon from the center-of-mass of the whole system. Here, we note that the expectation values are calculated with the absolute value of $\Psi_{SM_S M_T}$, though strictly, the Gamow vector should be used for the non-Hermitian system and the E -derivative of the potential should appear in the normalization condition for E -dependent potential (see Sec. 3.2.1 and Sec. 3.1.1). However, as explained in Sec. 3.2.1, the dumping of the wave function can be extracted from the standard normalization scheme, where the absolute value of the wave function is used, for the quasibound state, whose wave function converges to zero at sufficiently large distance. Respecting the statement, we use the standard scheme to extract the spatial structure.

We summarize the results of the three-body ($\bar{K}NN$) to seven-body ($\bar{K}NNNNNN$) systems in Table 2.7. The binding energy B , decay width Γ , $\bar{K}N$ two-body energy from the $\bar{K}N$ threshold $\delta\sqrt{s}$, and the root-mean-squared radii (rms) for the nucleon $\sqrt{\langle r_N^2 \rangle}$ and antikaon $\sqrt{\langle r_{\bar{K}}^2 \rangle}$ are shown here.

We first focus on the three-body systems ($\bar{K}NN$). Here, we calculate both spin-parity states, $J^\pi = 0^-$ and 1^- , and find a quasibound state only for $J^\pi = 0^-$ below the $\Lambda(1405) + N$ threshold. On the other hand, we could not find such states for $J^\pi = 1^-$, which corresponds to the system of the deuteron with an antikaon. This is the inverse case to the two-nucleon system, where only the $J^\pi = 1^+$ deuteron is obtained as a bound state. The reason for this inversion is that $J^\pi = 0^-$ states contain a larger fraction of the $I = 0$ $\bar{K}N$ components, which is more attractive than the $I = 1$ $\bar{K}N$ potential. This result is consistent with the previous works [104–107, 109–112, 114, 117, 171]. For the quantitative discussion, we compare the results from our $\bar{K}N$ potential named Kyoto $\bar{K}N$ potential in Table 2.7. Regarding the binding energy, the two results from Type I and Type II are close to each other and it seems insensitive to the choice of the two-body energy, $\delta\sqrt{s}$. On the other hand, the decay width is sensitive to the choice of $\delta\sqrt{s}$ and the result from Type II (~ 60 MeV) is almost twice as large as the one from Type I (~ 30 MeV). This ambiguity comes from the large difference of $\delta\sqrt{s}$.

	$\bar{K}N$ potential	(B, Γ) [MeV]	$\delta\sqrt{s}$ [MeV]	$\sqrt{\langle r_N^2 \rangle}$ [fm]	$\sqrt{\langle r_{\bar{K}}^2 \rangle}$ [fm]
$K^- pp-$ $\bar{K}^0 pn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(27.9, 30.9)	$-61.0 - i25.0$	1.12	1.14
	(Type II)	(26.1, 59.3)	$-30.2 - i23.7$	1.08	1.10
	AY	(48.7, 61.9)	—	0.958	0.988
$K^- pn-$ $\bar{K}^0 nn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(27.6, 31.6)	$-60.2 - i25.6$	1.13	1.15
	(Type II)	(25.3, 59.4)	$-29.4 - i23.8$	1.09	1.11
	AY	(48.1, 61.6)	—	0.963	0.993
$K^- ppn-$ $\bar{K}^0 pnn$ ($J^\pi = 1/2^-$)	Kyoto (Type I)	(45.3, 25.5)	$-70.4 - i20.7$	1.17	1.17
	(Type II)	(49.7, 69.4)	$-26.1 - i18.6$	1.11	1.08
	AY	(72.6, 78.6)	—	1.09	1.03
$K^- pppn-$ $\bar{K}^0 pppn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(67.9, 28.3)	$-67.6 - i23.0$	1.22	1.22
	(Type II)	(72.7, 74.1)	$-18.4 - i15.0$	1.18	1.12
	AY	(85.2, 86.5)	—	1.27	1.14
$K^- ppnn-$ $\bar{K}^0 pnnn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(69.6, 28.0)	$-68.7 - i22.4$	1.21	1.21
	(Type II)	(75.5, 74.5)	$-19.1 - i14.9$	1.17	1.11
	AY	(87.4, 87.2)	—	1.26	1.13
$\bar{K}NNNNN$ ($L = 0$)	Kyoto (Type I) (Type II) AY	no state (see the text)			
$K^- ppppn-$ $\bar{K}^0 ppppn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(69.8, 23.7)	$-76.2 - i18.1$	1.82	1.61
	(Type II)	(79.7, 75.6)	$-15.1 - i10.2$	1.78	1.49
	AY	(103, 88.0)	—	1.66	1.43
$K^- pppnn-$ $\bar{K}^0 pppnn$ ($J^\pi = 0^-$)	Kyoto (Type I)	(70.6, 23.9)	$-75.6 - i18.3$	1.81	1.61
	(Type II)	(80.0, 75.5)	$-14.9 - i10.2$	1.77	1.49
	AY	(103, 88.0)	—	1.66	1.43
$K^- ppppn-$ $\bar{K}^0 ppppn$ ($J^\pi = 1^-$)	Kyoto (Type I)	(70.8, 26.4)	$-70.2 - i21.5$	1.91	1.54
	(Type II)	(77.5, 75.2)	$-13.2 - i10.4$	1.88	1.45
	AY	(92.9, 88.0)	—	1.83	1.40
$K^- pppnn-$ $\bar{K}^0 pppnn$ ($J^\pi = 1^-$)	Kyoto (Type I)	(72.8, 26.0)	$-71.6 - i20.8$	1.91	1.53
	(Type II)	(80.7, 75.6)	$-13.8 - i10.4$	1.87	1.44
	AY	(95.6, 88.5)	—	1.82	1.39

TABLE 2.7: Results of $\bar{K}$ nuclei. The binding energy B , decay width Γ , $\bar{K}N$ two-body energy $\delta\sqrt{s}$ (Type I or Type II), and the root-mean-squared radii of the nucleon $\sqrt{\langle r_N^2 \rangle}$ and antikaon $\sqrt{\langle r_{\bar{K}}^2 \rangle}$ are shown. The spin-parity state of each $\bar{K}$ nucleus is denoted by J^π . As the $\bar{K}N$ potential, we employ our single channel potential, Kyoto $\bar{K}N$ potential (see Sec. 2.2 and Ref. [119]), and the phenomenological one, AY potential [102, 104]. For the energy-dependent Kyoto $\bar{K}N$ potential, we employ the two choices of the $\bar{K}N$ two-body energy, Type I in Eq. (2.61) and Type II in Eq. (2.62).

Comparing with other theoretical works, our results of the binding energy (~ 20 -30 MeV) is much smaller than the ones from E -independent potential (~ 50 -100 MeV) [104, 109–111, 113], and closer to the ones from E -dependent potential based on chiral SU(3) dynamics (~ 10 -40 MeV) [105–107, 112–115]. The resulting binding energy is consistent with the recent experimental data by J-PARC E15 [131] ($B_{\bar{K}NN} = 15_{-6}^{+8}$ (stat.) ± 12 (syst.) MeV). In contrast, the decay width from Kyoto $\bar{K}N$ potentials (~ 30 -60 MeV) is smaller than the experimental result ($\Gamma_{\bar{K}NN} = 110_{-17}^{+19}$ (stat.) $\pm$

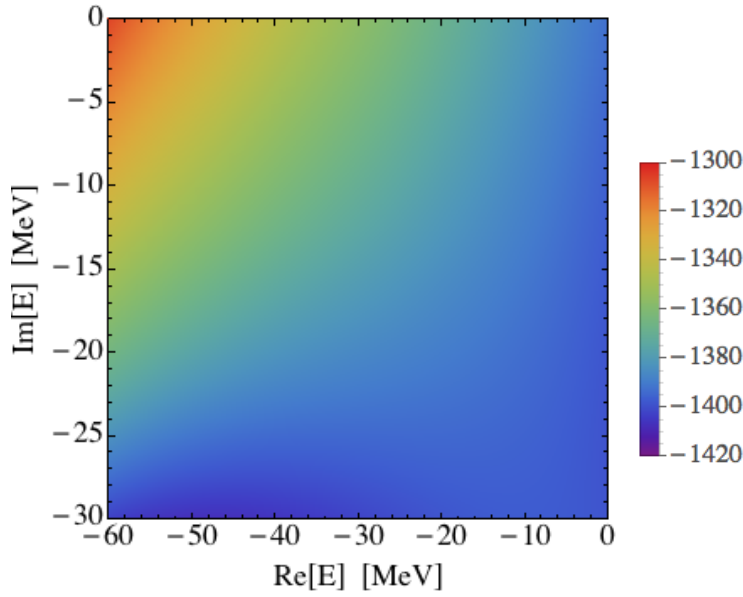


FIGURE 2.10: Strength of $\text{Re}[V_{\bar{K}N}(r=0, E)]$ in Sec. 2.2 in the complex energy plane.

27 (syst.) MeV). We note that we neglects the nonmesonic decays with multinucleon absorptions like $\bar{K}NN \rightarrow \Lambda N$. In some studies [106, 172–174], the nonmesonic decays are expected to increase the decay width by several tens MeV. Besides, in the experimental aspect, the $\bar{K}NN$ pole position depends on how to analyze the results. In Ref. [131] with $\Gamma \sim 110$ MeV, the simple Breit-Wigner formula is applied. But it is suggested in Ref. [132] that the model with much smaller width (~ 72 MeV) can reproduce the experimental results. Thus, for further discussion of the decay width, refinement both from the theoretical and experimental sides is necessary.

Here, we discuss the dependence of the $\bar{K}NN$ binding energy on the treatment of $\text{Im}[V_{\bar{K}N}]$. In the similar theoretical works [105–107] with the E -dependent complex $\bar{K}N$ potential [22] and the variational method, only the real part of the potential is used to determine the binding energy²¹. We discuss the difference from our full calculation including the imaginary part, following the two steps;

- i). switching off the E -dependence of the potential, estimate the effect of $\text{Im}[V_{\bar{K}N}]$,
- ii). switching on the E -dependence, as in our case, do the same thing.

In the case i), the binding energy from the calculation including $\text{Im}[V_{\bar{K}N}]$ is smaller than the one only from $\text{Re}[V_{\bar{K}N}]$. It may mean that the inclusion of $\text{Im}[V_{\bar{K}N}]$ for an E -independent potential tends to decrease the binding energy. On the other hand, in the case ii), the binding energy of the full calculation is larger than the one only from $\text{Re}[V_{\bar{K}N}]$. The reason can be deduced from the energy-dependence of $\text{Re}[V_{\bar{K}N}(r=0, E)]$ in the complex energy plane (Fig. 2.10). With the E -dependent and complex potential, the two-body energy E in the three-body system becomes complex. As seen from Fig. 2.10, the $\bar{K}N$ potential is more attractive for the complex E than for $\text{Re}[E]$, $|\text{Re}[V_{\bar{K}N}(E)]| > |\text{Re}[V_{\bar{K}N}(\text{Re}E)]|$. Though the tendencies in the case i) and ii) are opposite, in our case, it seems that the attraction in the complex two-body energy surpasses the other effect. As a result of this difference of the few-body calculation

²¹As mentioned above, the imaginary part is employed to estimate the decay width perturbatively.

as well as the different $V_{\bar{K}N}$, our binding energy ($\sim 20\text{--}30$ MeV) becomes larger than the previous ones ($\sim 5\text{--}10$ MeV) [105–107].

In Table 2.7, we also show the results from the previous E -independent phenomenological potential (AY potential) [102, 104] with our accurate few-body calculation method. AY potential is more attractive than Kyoto $\bar{K}N$ potential and generates larger binding energy in the $\Lambda(1405)$ system. As expected, from this E -independent potential, we obtain larger binding energy (~ 50 MeV) in the $\bar{K}NN$ system, which is almost twice that from the E -dependent Kyoto $\bar{K}N$ potential. And the shrinkage effect is also stronger, which can be seen from the smaller the rms radii with AY potential.

The results of more than four-body systems are shown in Table 2.7, too. The rough tendency is similar to the three-body case. The binding energies are not so sensitive to the choice of the two-body energy, while the decay width greatly depends on the choice. Furthermore, we find that the root-mean-squared radii of the nucleon are smaller than the corresponding nuclei (see Table I in Ref. [108]). For example, in the $K^-ppnn-\bar{K}^0pnnn$ system, the nucleon rms radius $\sqrt{\langle r_N^2 \rangle}$ (~ 1.2 fm) is relatively smaller than the one in the corresponding ${}^4\text{He}$ (~ 1.4 fm) which is known to be still compact comparing with other nuclei. This means that the system shrinks for the insertion of the antikaon, which is expected from the strong $\bar{K}N$ attraction and no repulsive core between $\bar{K}N$. The values from AY potential reveal more attractive results, the larger binding energies and smaller rms radii. Here, we note that the drastic results in Refs. [103, 175], where they indicate that the binding energies exceed 100 MeV in some systems, are not realized even with the stronger AY potential when the few-body system is rigorously solved. Such drastic results would be the artifacts caused by the approximated calculation of the few-body systems. Furthermore, calculating the density distribution of the nucleons and antikaon, we find that the surprisingly high central densities in Ref. [175] (for example, 8.2 times the normal density in the $\bar{K}NNN$ system) are not realized as well. In our results, the central densities reach at most the twice those of the corresponding nuclei.

In the six-body systems, we do not find any quasibound states. This would be because we only consider the total orbital momentum $L = 0$ case. In terms of the original nucleus, ${}^5\text{He}$, before injecting the antikaon, the ground state is considered to be the resonant state with $J^\pi = 3/2^-$. Hence, if we consider $L > 0$, we may find some states with the antikaon.

In the seven-body systems, the interesting results are obtained, related to the inversion of the ground state. It is the similar situation as the three-body $\bar{K}NN$ states, where the ground state is inverted from the $J^\pi = 1^+$ state, or the deuteron, to the $J^\pi = 0^-$ states after adding the antikaon. In the seven-body systems, the situation is more complicated. In the normal nuclei without an antikaon, the ground state of the six-nucleon system is a $J^\pi = 1^+$ state not $J^\pi = 0^+$ states. As the quasibound states of $\bar{K}$ nuclei, we obtain both the $J^\pi = 0^-$ and $J^\pi = 1^-$ states. In Type I, the binding energy is 1 MeV (2.2 MeV) larger for the $J^\pi = 1^-$ state, $K^-ppppnn-\bar{K}^0ppppnn$ ($K^-ppppnn-\bar{K}^0ppppnn$), than for the $J^\pi = 0^-$ states, which means that no ground-state inversion takes place here. Conversely, in Type II, the inversion occur for the $K^-ppppnn-\bar{K}^0ppppnn$ state, and the binding energy is 2.2 MeV smaller for the $J^\pi = 1^-$ state than for the $J^\pi = 0^-$ states. Still, for the $K^-ppppnn-\bar{K}^0ppppnn$ state, the ordering of the ground state is kept. The reason for the inversion in Type II is again related to the strong $I = 0$ $\bar{K}N$ potential (refer to the three-body case). In the $J^\pi = 0^-$ states, a larger fraction of the $I = 0$ $\bar{K}N$ components are contained than in the $J^\pi = 1^-$ states. Thus, if the $I = 0$ $\bar{K}N$ potential is strongly attractive

enough, the ground state inversion can occur. In Type II, the $\bar{K}N$ two-body energy $\delta\sqrt{s}$ is much smaller than in Type I (see Table 2.7), which results in the more attractive potential as shown in Fig. 2.10. Similarly, the inversion takes place with the stronger ΛY potential. In this case, the effect appears more evidently, and in both the states, $K^- ppppnn-\bar{K}^0 pppnnn$ and $K^- pppnnn-\bar{K}^0 pppnnn$, the $J^\pi = 0^-$ states become ground states, whose binding energies are roughly 10 MeV larger than the ones in the $J^\pi = 1^-$ states. This difference of the inversion situation against the potential models would give a new constraint on the $\bar{K}N$ system.

2.3.2 K^-p correlation

To elucidate the nature of the $\Lambda(1405)$ and $\bar{K}$ nuclei, it is essential to precisely determine the $\bar{K}N$ interaction. In the low energy region, the K^-p scattering data of old bubble chamber experiments are employed, and relatively large uncertainty remains. On and below the threshold, while the $I = 0$ $\bar{K}N$ amplitude has been theoretically well constrained [23, 24] due to the recent SIDDHARTA experiment [94, 95], there are some ambiguity in the $I = 1$ channel. Concerning the latter problem, the kaonic-deuterium measurement is expected to be promising because the $I = 1$ $\bar{K}N$ component is three times as large as $I = 0$ in this system, and planned by J-PARC E57 [96] and SIDDHARTA-2 [97, 98]. The energy level shift is theoretically studied in Ref. [85], where our precise $\bar{K}N$ potential is employed. As another source to constrain the $I = 1$ amplitude, we propose to analyze the two hadron intensity correlation [176–184] in high-energy heavy-ion collisions. As it turns out, the correlation can provide a complementary information with the K^-p elastic scattering amplitude, related to the two $\bar{K}N$ components, that is, $I = 0$ and $I = 1$ in the isospin basis. This work is summarized in Ref. [99] only with the $\bar{K}N$ strong interaction, and in Ref. [100] including the Coulomb interaction, too.

The correlation function is mainly determined from quantum statistics and the final-state interaction [176–182]. The former is known as Hanbury-Brown and Twiss (HBT) [185] or Goldhaber-Goldhaber-Lee-Pais (GGLP) [186] effect. The quantum statistical effect can be used to extract the source size in the heavy-ion collisions [187, 188]. The final-state interaction effect is expected to be significant when the interaction of the particle pairs is sufficiently strong in the range comparable to the effective source size [178, 179]. Hence, the measurement of the correlation function might provide the information of the pairwise interaction as in the scattering experiments. Its advantage against the scattering experiments is the possibility to extract the information of any measurable channels which are difficult in the scattering experiments. $\Lambda\Lambda$ system is an interesting example of such systems, and its correlation in the heavy-ion collisions has been measured at the Relativistic-Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory [189], which has been theoretically analyzed in Refs. [99, 189, 190]. As another application, we theoretically predict the K^-p correlation ahead of future experiments. In this system, the correlation function is directly related to the pairwise final-state interaction, because the system is free from the quantum statistical effect for identical particles [178].

The two-particle correlation function from a chaotic source is given by the Koonin-Pratt (KP) formula as [176–178],

$$C(\mathbf{q}, \mathbf{P}) = \frac{\int d^4x_1 d^4x_2 S_1(x_1, \mathbf{p}_1) S_2(x_2, \mathbf{p}_2) |\psi^{(-)}(\mathbf{r}, \mathbf{q})|^2}{\int d^4x_1 S_1(x_1, \mathbf{p}_1) \int d^4x_2 S_2(x_2, \mathbf{p}_2)} \quad (2.68)$$

$$\simeq \int d^3r S_{12}(\mathbf{r}) |\psi^{(-)}(\mathbf{r}, \mathbf{q})|^2, \quad (2.69)$$

with the source function $S_i(x_i, \mathbf{p}_i)$ of the hadron i of momentum $\mathbf{p}_i$. $\mathbf{q} = (m_2 \mathbf{p}_1 - m_1 \mathbf{p}_2)/(m_1 + m_2)$ and $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ respectively represent the relative and center-of-mass momenta, with the mass of hadron i , m_i . $\mathbf{r}$ is the relative coordinate with time difference correction. $\psi^{(-)}$ denotes the relative wave function in the two-body outgoing state with the asymptotic relative momentum $\mathbf{q}$. When the time difference of the emission and the momentum dependence of the source are negligible, we can integrate out the center-of-mass coordinate, which results in Eq. (2.69) with the normalized source function in the relative coordinate $S_{12}(\mathbf{r})$. As a first theoretical

prediction of the K^-p correlation, we employ a simple static and spherical source,

$$S_i(x, \mathbf{p}) \propto \exp \left[-\frac{x^2}{2R_i^2} \right] \delta(t - t_0) \quad (2.70)$$

which leads to

$$S_{12}(\mathbf{r}) = \frac{e^{-r^2/(4R^2)}}{(4\pi R^2)^{3/2}} \left(R = \sqrt{(R_1^2 + R_2^2)/2} \right). \quad (2.71)$$

For a whole, we consider the K^-p system only with the strong interaction, neglecting the Coulomb interaction. In this system, because K^- and p are distinguishable and K^- has no spin, the relative wave function is expressed with the partial wave decomposition as

$$\begin{aligned} \psi^{(-)}(\mathbf{r}, \mathbf{q}) &= \sum_{l=0}^{\infty} i^l (2l+1) P_l(\cos \theta) \psi_q^l(r) \\ &= e^{i\mathbf{q} \cdot \mathbf{r}} - e^{i\mathbf{q} \cdot \mathbf{r}} + \sum_{l=0}^{\infty} i^l (2l+1) P_l(\cos \theta) \psi_q^l(r) \\ &= e^{i\mathbf{q} \cdot \mathbf{r}} + \sum_{l=0}^{\infty} i^l (2l+1) P_l(\cos \theta) \left[\psi_q^l(r) - j_l(qr) \right], \end{aligned} \quad (2.72)$$

where P_l is the Legendre polynomials with $\cos \theta$ representing the angle between $\mathbf{r}$ and $\mathbf{q}$ as the argument. $\psi_q^l(r)$ and $j_l(qr)$ are the wave function and the spherical Bessel function with angular momentum l , respectively. ψ_q^l meets the outgoing boundary condition. If we assume that the interaction in $l \geq 1$ is negligible and the full wave function is not very much modified from the plane wave, we can approximate the wave function as²²

$$\psi(\mathbf{r}, \mathbf{q}) \simeq e^{i\mathbf{q} \cdot \mathbf{r}} + \psi_{K^-p}(r) - j_0(pr), \quad (2.73)$$

where, for simplicity, the s -wave K^-p wave function $\psi_p^{l=0}(r)$ is denoted as $\psi_{K^-p}(r)$. Inserting Eq. (2.73) into Eq. (2.69), the form of the correlation function results in

$$C(q) \simeq 1 + \int d^3r S_{12}(r) \left[|\psi_{K^-p}(r)|^2 - |j_0(qr)|^2 \right]. \quad (2.74)$$

The second term shows the interaction effects; when the wave function is enhanced due to the attraction, the correlation is enhanced accordingly.

A characteristic point of the K^-p system is the channel coupling to $\bar{K}^0 n$, which has not been paid attention to in previous calculations of the correlations. In such coupled-channel system, the correlation function gives a somewhat different information from the usual scattering amplitude. The point is the difference of the boundary condition of the wave function. In the correlation case, the outgoing boundary condition is required,

$$\psi_{K^-p}(r) \xrightarrow{r \rightarrow \infty} \frac{1}{2iqr} \left[e^{iqr} - \tilde{S}_{K^-p}^{-1} e^{-iqr} \right]. \quad (2.75)$$

²²We have to note that the $\Lambda(1520)$ effect, which comes from the d -wave component, is omitted here. Therefore, our predictive correlation function would deviate from the future experimental data at around $\sqrt{s} = 1520$ MeV ($q = 0.24$ GeV/ c in the momentum).

$\tilde{S}_{K^-p}$ is different from the usual S-matrix obtained from the condition of the plane wave as the incident state,

$$\psi_{K^-p}(r) \xrightarrow{r \rightarrow \infty} \frac{i}{2} \left[e^{-iqr} - S_{K^-p} e^{iqr} \right]. \quad (2.76)$$

$\tilde{S}_{K^-p}$ and S_{K^-p} can be related considering the isospin basis, which is connected to the physical basis as²³

$$\begin{pmatrix} |\bar{K}N^{I=0}\rangle \\ |\bar{K}N^{I=1}\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} |K^-p\rangle \\ |\bar{K}^0n\rangle \end{pmatrix}. \quad (2.77)$$

A general form of the $\bar{K}N$ wave function $\psi_{\bar{K}N, I=0}^{(-)}$ can be written as the superposition of the isospin wave function $\psi_I(r)$ with the outgoing boundary condition,

$$\begin{aligned} \psi_{\bar{K}N, I=0}^{(-)} &= C_0 \frac{\chi(K^-p) + \chi(\bar{K}^0n)}{\sqrt{2}} \psi_0(r) + C_1 \frac{-\chi(K^-p) + \chi(\bar{K}^0n)}{\sqrt{2}} \psi_1(r) \\ &= \chi(K^-p) \frac{C_0 \psi_0(r) - C_1 \psi_1(r)}{\sqrt{2}} + \chi(\bar{K}^0n) \frac{C_0 \psi_0(r) + C_1 \psi_1(r)}{\sqrt{2}}, \end{aligned} \quad (2.78)$$

with the isospin wave functions of the physical states, $\chi(K^-p)$ and $\chi(\bar{K}^0n)$. Thus, the spatial wave function in the physical basis can be expressed by the one in isospin basis,

$$\begin{aligned} \psi_{K^-p} &= \frac{C_0 \psi_0(r) - C_1 \psi_1(r)}{\sqrt{2}}, \\ \psi_{\bar{K}^0n} &= \frac{C_0 \psi_0(r) + C_1 \psi_1(r)}{\sqrt{2}}. \end{aligned} \quad (2.79)$$

Here, we consider the asymptotic behavior of the isospin wave function. As in Eq. (2.76), the S-matrix in the isospin basis S_I is defined from the following asymptotic form,

$$\psi_I(r) \xrightarrow{r \rightarrow \infty} \frac{i}{2} \left[e^{-iqr} - S_I e^{iqr} \right]. \quad (2.80)$$

Because each isospin channel can be decomposed assuming the isospin symmetry, the outgoing wave function is simply expressed with the S-matrix as

$$\psi_I(r) \xrightarrow{r \rightarrow \infty} \frac{1}{2iqr} \left[e^{iqr} - S_I^{-1} e^{-iqr} \right]. \quad (2.81)$$

Hence, the asymptotic behavior of the physical wave functions become

$$\begin{aligned} \psi_{K^-p} &\xrightarrow{r \rightarrow \infty} \frac{1}{2iqr} \left[\frac{C_0 - C_1}{\sqrt{2}} e^{iqr} - \frac{C_0 S_0^{-1} - C_1 S_1^{-1}}{\sqrt{2}} e^{-iqr} \right], \\ \psi_{\bar{K}^0n} &\xrightarrow{r \rightarrow \infty} \frac{1}{2iqr} \left[\frac{C_0 + C_1}{\sqrt{2}} e^{iqr} - \frac{C_0 S_0^{-1} + C_1 S_1^{-1}}{\sqrt{2}} e^{-iqr} \right]. \end{aligned} \quad (2.82)$$

²³We use the phase convention in Eq. (2.20) for each hadron.

The original outgoing boundary condition in Eq. (2.75) means the purely K^-p outgoing wave without the $\bar{K}^0n$ one, that is, the coefficient of e^{iqr} in the K^-p channel should be $1/2iqr$ and the one in the $\bar{K}^0n$ channel should disappear. These conditions result in $C_0 = C_1 = 1/\sqrt{2}$, and the asymptotic K^-p wave function is turned to be

$$\psi_{K^-p}(r) \xrightarrow{r \rightarrow \infty} \frac{1}{2iqr} \left[e^{iqr} - \frac{S_0^{-1} + S_1^{-1}}{2} e^{-iqr} \right], \quad (2.83)$$

which means

$$\tilde{S}_{K^-p} = \frac{2}{S_0^{-1} + S_1^{-1}}. \quad (2.84)$$

Because $\tilde{S}_{K^-p}$ has the different combination of the isospin components from the usual S-matrix, $S_{K^-p} = (S_0 + S_1)/2$, it would complement the existing data of the scattering experiments.

To calculate the s-wave K^-p wave function, which is equivalent to obtain the correlation function from Eq. (2.74), we solve the following coupled-channel Schrödinger equation,

$$\begin{pmatrix} -\frac{1}{2\mu} \frac{d^2}{dr^2} + V_{K^-p, K^-p}^{\text{strong}} & V_{K^-p, \bar{K}^0n}^{\text{strong}} \\ V_{\bar{K}^0n, K^-p}^{\text{strong}} & -\frac{1}{2\mu} \frac{d^2}{dr^2} + V_{\bar{K}^0n, \bar{K}^0n}^{\text{strong}} \end{pmatrix} \begin{pmatrix} \psi_{K^-p} \\ \psi_{\bar{K}^0n} \end{pmatrix} = E \begin{pmatrix} \psi_{K^-p} \\ \psi_{\bar{K}^0n} \end{pmatrix}, \quad (2.85)$$

where a common reduced mass μ is used to both the K^-p and $\bar{K}^0n$ channels, and the isospin breaking in the kinematics is neglected. The $\bar{K}N$ strong interaction in the physical basis, V^{strong} , is derived from our isospin basis potentials (see Sec. 2.2), $V^{I=0}$ and $V^{I=1}$, as

$$\begin{pmatrix} V_{K^-p, K^-p}^{\text{strong}} & V_{K^-p, \bar{K}^0n}^{\text{strong}} \\ V_{\bar{K}^0n, K^-p}^{\text{strong}} & V_{\bar{K}^0n, \bar{K}^0n}^{\text{strong}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} V^{I=0} & 0 \\ 0 & V^{I=1} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad (2.86)$$

$$= \frac{1}{2} \begin{pmatrix} V^{I=0} + V^{I=1} & V^{I=0} - V^{I=1} \\ V^{I=0} - V^{I=1} & V^{I=0} + V^{I=1} \end{pmatrix}. \quad (2.87)$$

We show the result of the K^-p correlation function in Fig. 2.11. The source size $R = \sqrt{(R_K^2 + R_p^2)/2}$ in Eq. (2.71) is set to be 3 fm, because the kaon source size is estimated as 2-5 fm from Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV [191, 192], and the proton is expected to have the similar source size. Comparing with the source size, the range parameters of the $\bar{K}N$ potential around 0.4 fm are relatively small because of the absence of the pion exchange. Thus, the correlation function is steady against the change of the range parameter. We confirm the fact varying the range parameters from 0.2 to 0.8 fm, keeping the scattering amplitude unchanged. Such short range systems can be well described by the Lednický and Lyuboshits model (LL model) [179], where the wave function is replaced by its asymptotic form, that is, the zero-range potential is assumed. We overlay the K^-p correlation with the LL model in Fig. 2.11, and confirm the great reproducibility of the model.

An interesting feature of the correlation function is the bump and dip structure in $q \sim 0.05\text{-}0.15$ GeV/c. The characteristic structure seems to be caused by the unique isospin combination in Eq. (2.84). For comparison, we show the $\bar{K}N$ ($I = 0$) and $\bar{K}N$ ($I = 1$) correlation functions in Fig. 2.11. In particular, in the energy region of the dip structure at $q \sim 0.15$ GeV/c, both the $I = 0$ and $I = 1$ correlations exceed

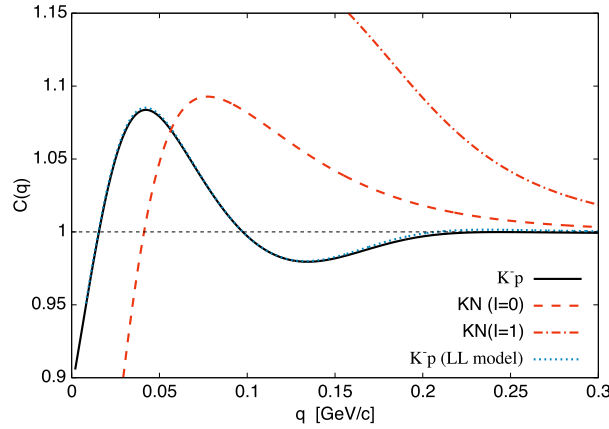


FIGURE 2.11: K^-p correlation function (black line) without the Coulomb interaction. A static source with $R = 3$ fm is employed. For comparison, we show the $\bar{K}N$ correlations in the $I = 0$ (red dashed line) and the $I = 1$ (red dash-dotted line) channels. The K^-p correlation by the LL model is described by the blue dotted line.

unity, reflecting the attractive interactions in both the channels, and the dip seems to be generated from the unique interference of the isospin components. Here, we show the dependence of the correlation function on the source size R in Fig. 2.12. R

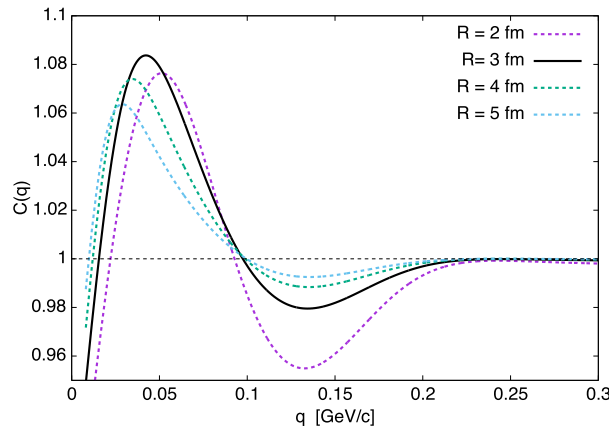


FIGURE 2.12: K^-p correlation function with several source size R . The results from $R = 2, 3, 4$, and 5 fm are respectively described with the purple, black, green, and blue lines. All correlations do not include the Coulomb effect.

is varied among 2-5 fm, taking into account of the uncertainty of R . We find that the qualitative behavior is not changed by the source size. Such characteristic behavior does not appear in the usual K^-p scattering amplitudes, and the analysis of the K^-p correlation would lead to information complementary to the one from the scattering experiments.

Finally, to directly compare with future experiments, we include the Coulomb effect referring to the work for the $p\Omega$ correlation [193]. The difference appears in the K^-p diagonal component of the Schrödinger equation (2.85). Accordingly, the asymptotic behavior of the K^-p wave function is modified, and the outgoing boundary condition (Eq. (2.75)) and Eqs. (2.72-2.74) are also altered. In Fig. 2.13, we show the K^-p correlation function including the Coulomb interaction. Resulting from the long range Coulomb attraction, there appears strong enhancement at low q region

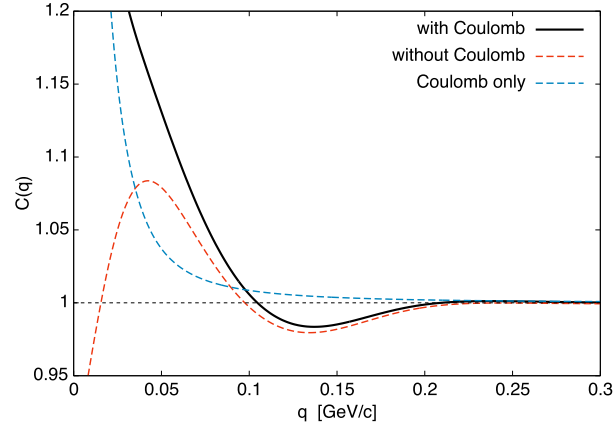


FIGURE 2.13: K^-p correlation function (black line) with the Coulomb interaction. A static source with $R = 3$ fm is employed. For comparison, we show the K^p correlations without the Coulomb interaction (red dashed line) and only with the Coulomb interaction, that is, without the strong interaction (blue dotted line).

($q \lesssim 0.1$ GeV/ c), which is seen in the $p\Omega$ case, too [193]. On the other hand, because the intermediate momentum ($q \sim 0.15$ GeV/ c) is dominated by the strong interaction, the characteristic dip structure remains. We should note that the $\Lambda(1520)$ contribution related to the d -wave $\bar{K}N$ wave function is neglected here. However, considering the higher q region of the state ($q \sim 0.24$ GeV/ c) and the relatively small decay width (~ 15 MeV), the contamination to the dip structure would not be so large, and the structure is expected in actual experiments.

2.4 $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ coupled-channel potential

In the previous sections, we have explicitly treated only the $\bar{K}N$ channel respecting the picture of the $\Lambda(1405)$ as the $\bar{K}N$ quasibound state. However, the importance of the $\pi\Sigma$ channel to the $\Lambda(1405)$ system has also been pointed out. In particular in chiral unitary approach which generates the two resonance poles, the relatively large contribution of the $\pi\Sigma$ channel to the lower pole has been expected, based on the large coupling constant [13], the formation of the corresponding pole only from the $\pi\Sigma$ single-channel interaction [22], or the relatively large compositeness [194]. In addition, in the calculation of the $\bar{K}$ nuclei, it is indicated that the effective treatment of the $\bar{K}NN$ system, eliminating the $\pi\Sigma N$ channel, would cause deviation from the full coupled-channel treatment [110, 117]. In Sec. 2.3.1, we predicted some $\bar{K}$ nuclei near the corresponding $\pi\Sigma$ threshold; for example, the seven-body system with $B \sim 70$ -80 MeV lies closer to the lower $\pi\Sigma NNNNN$ threshold. In such systems, the kinematics of the lower channel corresponding to the $\pi\Sigma$ would be more important. Hence, in this section, we construct a $\bar{K}N$ - $\pi\Sigma$ coupled-channel potential, extending the single-channel case in the previous section. Concentrating on the $\Lambda(1405)$ system, we first construct the $\bar{K}N$ - $\pi\Sigma$ potential in the $I = 0$ channel, where the $\pi\Lambda$ in $I = 1$ does not contribute assuming the isospin symmetry. The results of this work are summarized in Ref. [195].

2.4.1 Determination of range parameter

We consider the Schrödinger equation for the coupled-channel radial wave function u_i in the s -wave,

$$\begin{pmatrix} -\frac{1}{2\mu_{\pi\Sigma}} \frac{d^2}{dr^2} - \Delta M + V_{\pi\Sigma, \pi\Sigma}^{\text{equiv}} & V_{\pi\Sigma, \bar{K}N}^{\text{equiv}} \\ V_{\bar{K}N, \pi\Sigma}^{\text{equiv}} & -\frac{1}{2\mu_{\bar{K}N}} \frac{d^2}{dr^2} + V_{\bar{K}N, \bar{K}N}^{\text{equiv}} \end{pmatrix} \begin{pmatrix} u_{\pi\Sigma} \\ u_{\bar{K}N} \end{pmatrix} = E \begin{pmatrix} u_{\pi\Sigma} \\ u_{\bar{K}N} \end{pmatrix}, \quad (2.88)$$

where E and μ_i are the energy from the $\bar{K}N$ threshold and the reduced mass of channel i , respectively. ΔM represents the mass difference of the two channels,

$$\Delta M \equiv (m_{\bar{K}} + M_N) - (m_{\pi} + M_{\Sigma}). \quad (2.89)$$

The explicit form of the equivalent potential is shown in Eq. (2.50).

We first determine the range parameter b_i in V^{equiv} following Sec. 2.1.2. In this step, adjustment ΔV_{ij} is not necessary, and we temporarily set

$$\Delta V_{ij} = 0 \quad (2.90)$$

in this subsection. In this coupled-channel system, there are two thresholds and we have to reproduce F_{ii}^{Ch} on i -th threshold using two free parameters, $b_{\pi\Sigma}$ and $b_{\bar{K}N}$. Because each F_{ii} is affected by both the range parameters, we cannot determine $b_{\bar{K}N}$ and $b_{\pi\Sigma}$ separately. Thus, we try to determine both b_i to minimize the sum of the deviations between F^{Ch} and F^{equiv} at thresholds in the $\bar{K}N$ and $\pi\Sigma$ channels,

$$\begin{aligned} \Delta F_{\text{threshold}} \equiv & |F_{\pi\Sigma, \pi\Sigma}^{\text{Ch}}(\sqrt{s} = m_{\pi} + M_{\Sigma}) - F_{\pi\Sigma, \pi\Sigma}^{\text{equiv}}(\sqrt{s} = m_{\pi} + M_{\Sigma})| \\ & + |F_{\bar{K}N, \bar{K}N}^{\text{Ch}}(\sqrt{s} = m_{\bar{K}} + M_N) - F_{\bar{K}N, \bar{K}N}^{\text{equiv}}(\sqrt{s} = m_{\bar{K}} + M_N)|. \end{aligned} \quad (2.91)$$

The behavior of $\Delta F_{\text{threshold}}$ against $b_{\pi\Sigma}$ and $b_{\bar{K}N}$ is shown in Fig. 2.14. The optimized

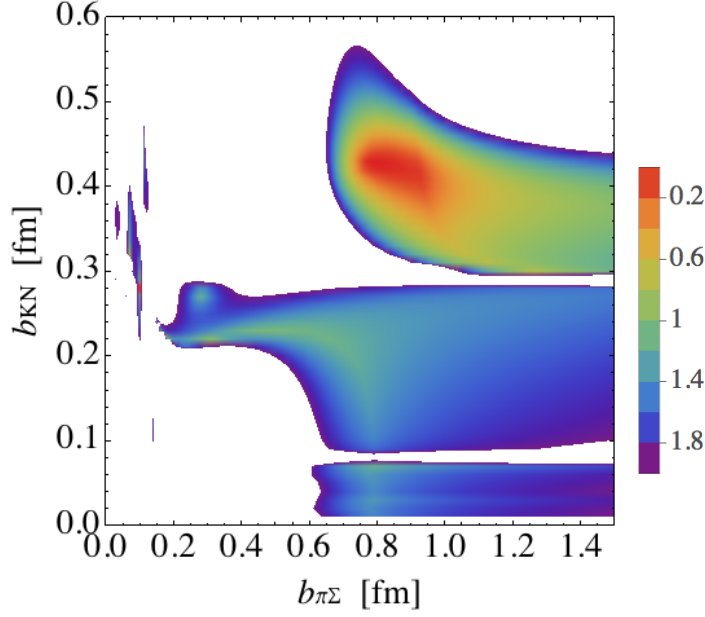


FIGURE 2.14: $\Delta F_{\text{threshold}}$ against $b_{\pi\Sigma}$ and $b_{\bar{K}N}$ in the IHW model. The horizontal and vertical axes respectively represent $b_{\pi\Sigma}$ and $b_{\bar{K}N}$.

ranges are

$$b_{\pi\Sigma} = 0.80 \text{ fm}, \quad b_{\bar{K}N} = 0.43 \text{ fm}. \quad (2.92)$$

Here, we consider the validity of the above prescription to determine the range parameters. The physical mechanism to give the longest range of the potential would be the σ -meson exchange (or two-pion exchange). Thus, to directly confirm whether the above prescription reflects this microscopic mechanism, we prepare a Yukawa type potential assuming the one-boson exchange picture. Determination procedure of the range parameters is just the same as in the above Gaussian case. The Yukawa potential which meets the requirements in the previous section, Eq. (2.42) and (2.43), is the following one,

$$V_{ij}^{\text{equiv}}(r, \sqrt{s}) = \frac{1}{4\pi b_i^{\text{Yukawa}} b_j^{\text{Yukawa}}} \frac{e^{-r/(1/2b_i^{\text{Yukawa}} + 1/2b_j^{\text{Yukawa}})}}{r} \sqrt{\frac{M_i M_j}{s \mu_i \mu_j}} V_{ij}^{\text{Ch, eff}}(\sqrt{s}). \quad (2.93)$$

With this potential, we optimize the ranges to minimize $\Delta F_{\text{threshold}}$ in Eq. (2.91), which results in

$$b_{\pi\Sigma}^{\text{Yukawa}} = 0.66 \text{ fm}, \quad b_{\bar{K}N}^{\text{Yukawa}} = 0.34 \text{ fm}. \quad (2.94)$$

This result would indicate that the range parameter of the Gaussian potential appears larger than the one estimated from the meson exchange. As in the Gaussian potential case, $b_{\pi\Sigma}^{\text{Yukawa}}$ is larger than $b_{\bar{K}N}^{\text{Yukawa}}$. One of the reasons may be the Pauli blocking in the $\pi^0 \Sigma^0$ ($\sim [u\bar{u} - d\bar{d}][uds]$) channel, which does not appear in the other channels of the $\bar{K}N$ and $\pi\Sigma$. Actually, in terms of the subtraction constants, the larger result of the $\pi\Sigma$ channel ($a_{\pi\Sigma}(\mu = 1 \text{ GeV}) = +4.35 \times 10^{-3}$) than the $\bar{K}N$ channel ($a_{\bar{K}N}(\mu = 1 \text{ GeV}) = -2.38 \times 10^{-3}$) in the IHW model [24] indicates the smaller

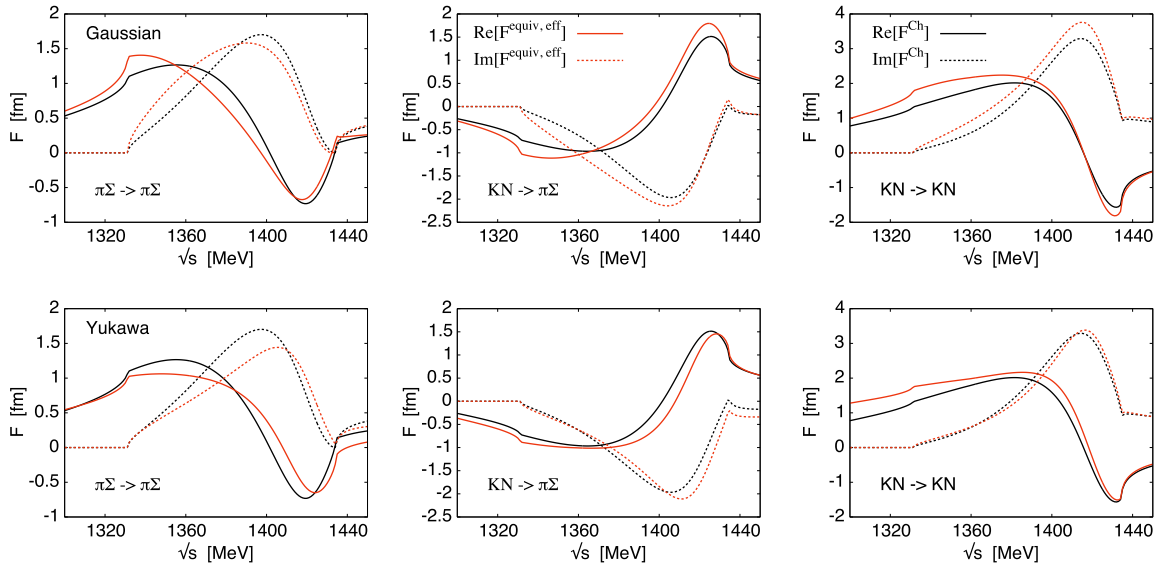


FIGURE 2.15: Scattering amplitudes $F^{\text{equiv, eff}}$ with Gaussian potential (upper figures) and Yukawa-type potential (lower ones) in the IHW model. Black lines (red lines) represent the ones from chiral unitary approach (the equivalent potential). Solid lines and dashed lines are real and imaginary part, respectively. Each figure represents $F_{\pi\Sigma, \pi\Sigma}$, $F_{\pi\Sigma, \bar{K}N}$, and $F_{\bar{K}N, \bar{K}N}$ from left to right.

absolute value of the loop function in Eq. (2.28), which leads to the smaller effect of the attractive interaction via the Lippmann-Schwinger equation in Eq. (2.26)²⁴. Because our Gaussian and Yukawa potentials do not explicitly include a repulsive component, the $\pi\Sigma$ attractive potential may be somewhat weakened via the range parameter²⁵. From b^{Yukawa} , the masses of some exchanged mesons can be expected as

$$\pi\Sigma \text{ channel : } \sim 300 \text{ MeV}, \quad (2.95)$$

$$\bar{K}N \text{ channel : } \sim 580 \text{ MeV}. \quad (2.96)$$

These values are consistent with the mass of the σ (~ 500 MeV) considering the large decay width (~ 400 - 700 MeV) [1]. It would indicate that our prescription to determine the range parameters is reasonable.

In Fig. 2.15, we show how the potential only from $V^{\text{Ch, eff}}$ can reproduce the original amplitudes F^{Ch} . The amplitudes from $V^{\text{Ch, eff}}$ is denoted as $F^{\text{equiv, eff}}$. Though we connect the different models, chiral unitary approach and the quantum mechanics, it turns out that the potential astonishingly reproduces the original amplitudes in all channels. It would also support the validity of our prescription. In fact, when we use different range parameters, for example, $b_{\pi\Sigma} = b_{\bar{K}N} = 0.2, 0.6$, and 1.0 fm with the Gaussian potential, the reproducibility of F^{Ch} becomes decisively worse (see Fig. 2.16). In Fig. 2.17, we show $F^{\text{equiv, eff}}$ in the $\bar{K}N$ single-channel case for comparison. While the single-channel potential roughly reproduces $F^{\text{Ch}}_{\bar{K}N}$, the de-

²⁴The convention of the subtraction constant is different from the one in Sec. 2.1.1. The relation is shown in Ref. [22].

²⁵We numerically check that the effect of the attractive potential becomes weaker as the range parameter increases keeping the volume integral fixed. This tendency can be seen in the behavior of the binding energy of a bound or resonant state.

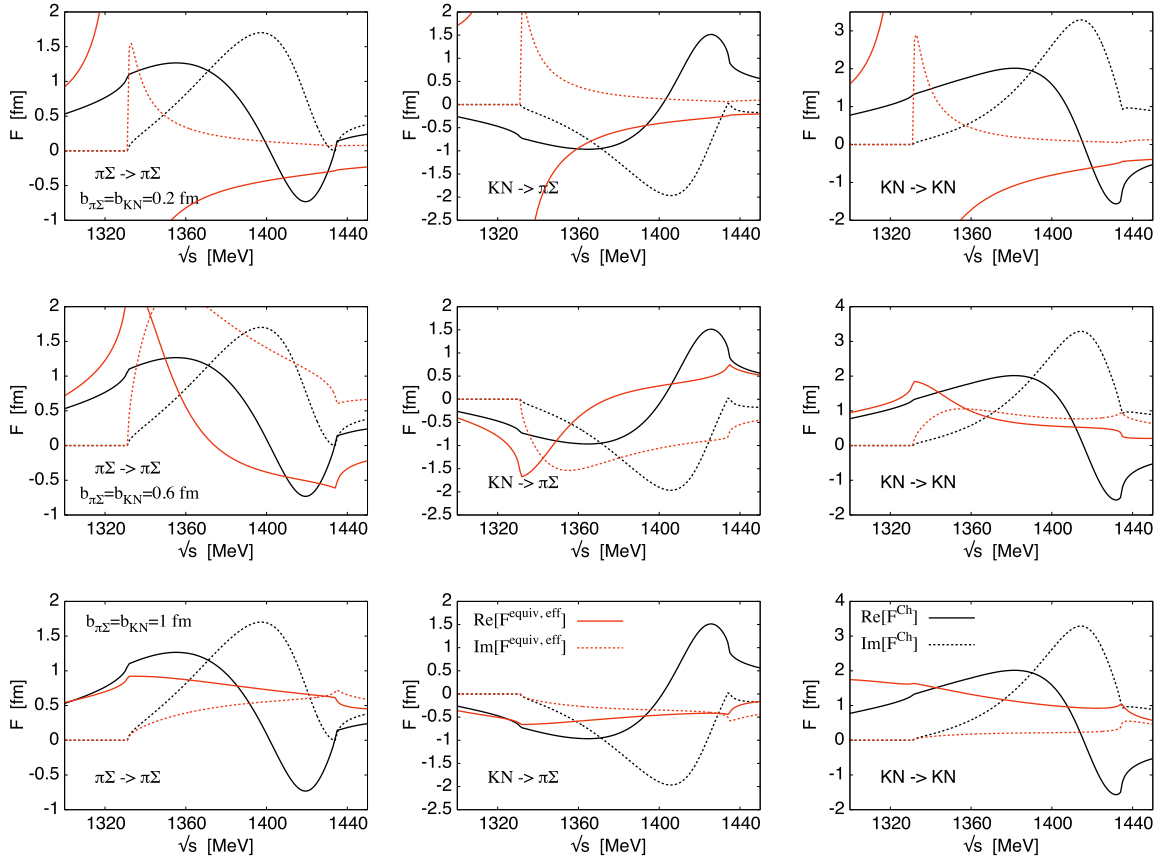


FIGURE 2.16: Scattering amplitudes $F^{\text{equiv, eff}}$ with $b_{\pi\Sigma} = b_{\bar{K}N} = 0.2$ fm (upper figures), 0.6 fm (middle figures), and 1.0 fm (lower ones). The local potentials are constructed with the Gaussian based on the IHW model. Black lines (red lines) represent the ones from chiral unitary approach (the equivalent potential). Solid lines and dashed lines are real and imaginary part, respectively. Each figure represents $F_{\pi\Sigma, \pi\Sigma}$, $F_{\pi\Sigma, \bar{K}N}$, and $F_{\bar{K}N, \bar{K}N}$ from left to right.

viation becomes larger than Fig. 2.15 in the lower energy region. It would indicate the importance of explicitly treating the $\pi\Sigma$ channel which is considered to largely contribute to the $\Lambda(1405)$.

We show the pole positions of $F_{ij}^{\text{equiv, eff}}$ in the coupled-channel system in Table 2.8. It turns out that both the Gaussian and Yukawa potentials relatively well reproduce the higher pole position with a few MeV precision while the lower position is deviated from the original one beyond the theoretical uncertainty in Ref. [24] in the Gaussian case and do not appear in the Yukawa case. Because the pole positions are essential for the analysis of the $\Lambda(1405)$ and $\bar{K}$ nuclei, they should be modified for quantitative discussion. Thus, in the next section, we add ΔV as in the single-channel case.

²⁶Here, the flux factor in Eq. (2.38) is adopted.

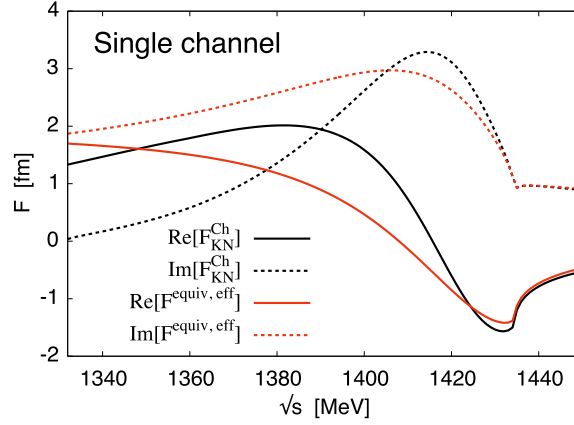


FIGURE 2.17: Scattering amplitudes $F_{\bar{K}N}^{\text{equiv, eff}}$ in the single-channel system (see Sec. 2.2) with the Gaussian potential in the IHW model²⁶. Black lines (red lines) represent the ones from chiral unitary approach (the equivalent potential). Solid lines and dashed lines are real and imaginary part, respectively.

potential type	higher pole [MeV]	lower pole [MeV]
Gaussian	$1425 - 23i$	$1336 - 69i$
Yukawa	$1423 - 24i$	—
original pole	$1424 - 26i$	$1381 - 81i$

TABLE 2.8: Pole positions of $F^{\text{equiv, eff}}$ before adding the adjustment ΔV with Gaussian and Yukawa-type potentials in the IHW model. For comparison, we also show the original pole positions from chiral unitary approach.

2.4.2 Adjustment ΔV

We add adjustment term ΔV_{ij} to the strength of the equivalent potential. Here, we note that the effective interaction V_{ij}^{eff} in Eq. (2.50) is real in the $\Lambda(1405)$ energy region (below $\eta\Lambda$ threshold), in contrast to the complex $V_{\bar{K}N}^{\text{eff}}$ in the single-channel case (see Sec. 2.2). Hence, ΔV_{ij} should be real in this coupled-channel case. With the three parameters, $\Delta V_{\pi\Sigma, \pi\Sigma}$, $\Delta V_{\pi\Sigma, \bar{K}N}$, and $\Delta V_{\bar{K}N, \bar{K}N}$, the real and imaginary parts of the original amplitudes, $F_{\pi\Sigma, \pi\Sigma}^{\text{Ch}}$, $F_{\pi\Sigma, \bar{K}N}^{\text{Ch}}$, and $F_{\bar{K}N, \bar{K}N}^{\text{Ch}}$, should be reproduced in each energy. Because all V_{ij} are entangled for the calculation of each F_{ij}^{Ch} , we minimize the following deviation,

$$\Delta F_{\text{real}}(E) \equiv \sum_{i \leq j} |F_{ij}^{\text{Ch}}(E) - F_{ij}^{\text{equiv, } \Delta V}(E)|, \quad (2.97)$$

simultaneously using all ΔV_{ij} . The amplitudes with ΔV_{ij} are represented by $F_{ij}^{\text{equiv, } \Delta V}$ and summarized in Fig. 2.18.

It turns out from Fig. 2.18 that the original amplitudes are quite well reproduced in the $\bar{K}N$ -diagonal and off-diagonal channels. Relatively visible deviation around $\sqrt{s} \sim 1300\text{--}1400$ MeV in the $\pi\Sigma$ -diagonal channel is still within the theoretical uncertainty of the original amplitude (see, for example, Fig. 6 in Ref. [24]).

In Fig. 2.19, we compare the energy dependence of the equivalent potential with

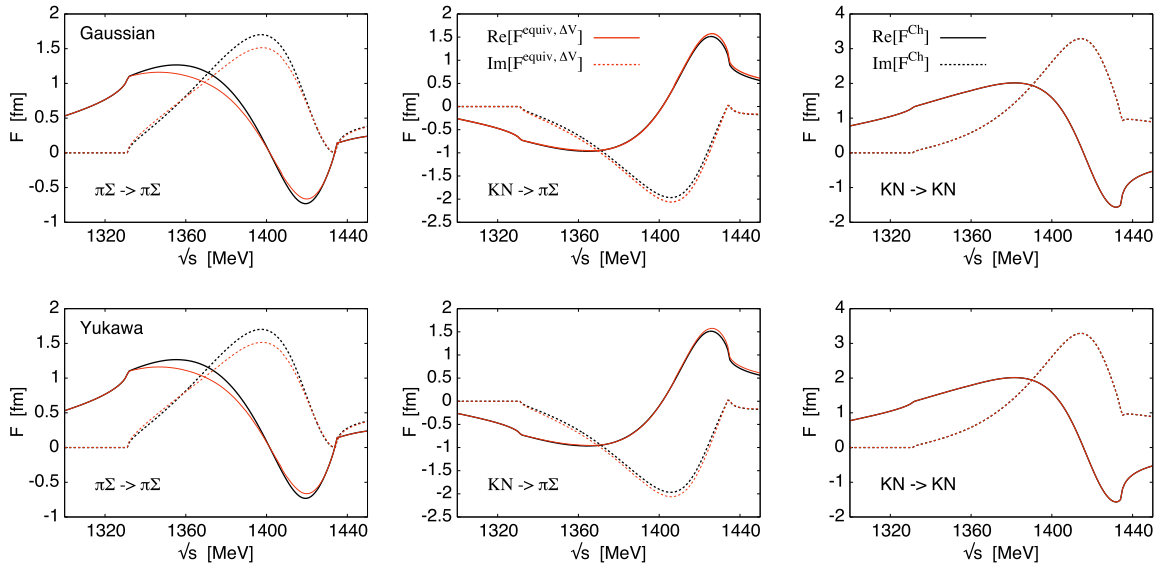


FIGURE 2.18: $F_{ij}^{\text{equiv}, \Delta V}$ in the IHW model with the Gaussian (upper figures) and Yukawa-type (lower ones) potentials. Black lines represent F_{ij}^{Ch} . Solid lines and dashed lines are real and imaginary part, respectively. From the left, each figure represents $F_{\pi\Sigma, \pi\Sigma}$, $F_{\pi\Sigma, \bar{K}N}$, and $F_{\bar{K}N, \bar{K}N}$.

ΔV_{ij} , $V_{ij}^{\text{equiv}, \Delta V}(r=0, \sqrt{s})$, and the one without ΔV_{ij} , $V_{ij}^{\text{equiv}, \text{eff}}(r=0, \sqrt{s})$. For both the Gaussian and Yukawa potentials, the leading dependence is determined from $V_{ij}^{\text{equiv}, \text{eff}}$. In particular, in the $\Lambda(1405)$ region between the $\pi\Sigma$ and $\bar{K}N$ thresholds, ΔV is quite small and the deviation between $V_{ij}^{\text{equiv}, \text{eff}}$ and $V_{ij}^{\text{equiv}, \Delta V}$ is less than 15 %. Further favorable feature is the linearity against the energy in this region. In our previous work of the single channel $\bar{K}N$ potential [119], the 10th order polynomial for the parametrizing function was necessary to reproduce the pole positions. On the other hand, in this coupled-channel case, the parametrizing function is expected to become much simpler because of the smoother energy dependence of the potential strength.

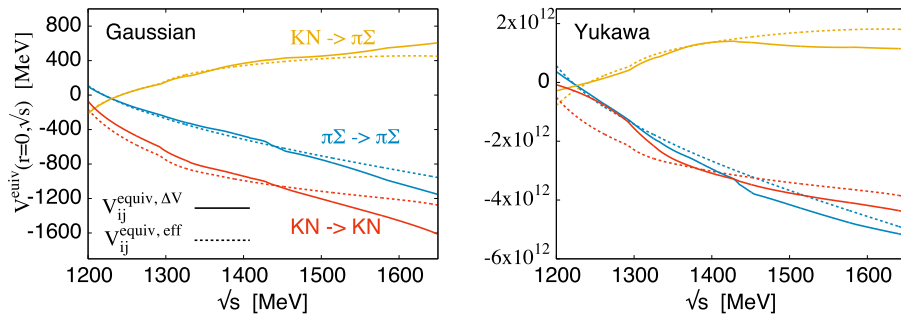


FIGURE 2.19: $V_{ij}^{\text{equiv}, \text{eff}}(r \rightarrow 0, \sqrt{s})$ (dashed lines) and $V_{ij}^{\text{equiv}, \Delta V}(r \rightarrow 0, \sqrt{s})$ (solid lines) with the Gaussian (left figure) and Yukawa-type (right figure) potentials²⁷. The red, yellow, and blue lines respectively represents the $\bar{K}N$ -diagonal, off-diagonal, and $\pi\Sigma$ -diagonal potential.

²⁷For Yukawa-type potential, we slightly shift $r = 0$ to $r = 10^{-10}$ fm to avoid the divergence.

2.4.3 Parametrization of potential

To parametrize the resultant equivalent potential with ΔV_{ij} , we use the polynomial in Eq. (2.51). As explained in Secs. 2.2.2 and 2.2.3, the important points are the parametrized range and parametrizing function. Because in Fig. 2.19 the energy dependence of $V^{\text{equiv}, \Delta V}$ looks almost linear around the $\Lambda(1405)$ region, we use first and second order polynomials. To discuss the reproducibility of F_{ij}^{Ch} , we define the deviation of the amplitudes in the complex energy plane,

$$\Delta F_{ij}(E) \equiv \left| \frac{F_{ij}^{\text{Ch}}(E) - F_{ij}^{\text{equiv, par}}(E)}{F_{ij}^{\text{Ch}}(E)} \right|, \quad (2.98)$$

where $F_{ij}^{\text{equiv, par}}$ denotes an amplitude with a parametrized equivalent potential. In Sec. (2.2.1), we defined the "precise region" to satisfy $\Delta F(E) < 0.2$. In the coupled-channel case, we define the precise region by the average of $\Delta F_{ij}(E)$,

$$\Delta F_{\text{avg}} \equiv \frac{\Delta F_{\pi\Sigma, \pi\Sigma}(E) + \Delta F_{\pi\Sigma, \bar{K}N}(E) + \Delta F_{\bar{K}N, \bar{K}N}(E)}{3} < 0.2. \quad (2.99)$$

Following Sec. 2.2.1 the target-energy region, where the percentage of the precise region is discussed, is set to

$$\begin{aligned} 1332 \text{ MeV} &\leq \text{Re}[\sqrt{s}] \leq 1450 \text{ MeV} \\ -100 \text{ MeV} &\leq \text{Im}[\sqrt{s}] \leq 50 \text{ MeV}, \end{aligned} \quad (2.100)$$

and we judge the best potential by this percentage,

$$P_{\text{comp}} = \frac{\int \int d(\text{Re}\sqrt{s}) d(\text{Im}\sqrt{s}) \Theta(0.2 - \Delta F_{\text{avg}}(\sqrt{s}))}{\int \int d(\text{Re}\sqrt{s}) d(\text{Im}\sqrt{s})} \times 100. \quad (2.101)$$

The parametrized range is variously changed within 1200-1660²⁸ MeV with 1 MeV stride to find the optimized value,

$$\begin{aligned} 1200 &\leq \sqrt{s}_{\text{low}} \leq 1450 \text{ MeV}, \\ \sqrt{s}_{\text{low}} &< \sqrt{s}_{\text{up}} \leq 1660 \text{ MeV}, \end{aligned} \quad (2.102)$$

where $\sqrt{s}_{\text{low (up)}}$ stands for the lower (upper) boundary of the parametrized range.

The results of the best potentials against the parametrized range are summarized in Table 2.9. The amplitudes on the real energy axis, $\Delta F_{ij}(E)$ in the complex energy plane, and the parametrized equivalent potentials are shown in Figs. 2.20, 2.21 and 2.22, respectively. From Table 2.9, we see that the optimized parametrized range of the second order potential covers the $\Lambda(1405)$ region. On the other hand, the one of the first order potential seems relatively narrow, and especially the lower boundary is far above the $\pi\Sigma$ threshold (~ 1330 MeV). However, as seen from the large P_{comp} , the first order potential is well extrapolated to much wider energy region. This fact can be confirmed from Fig. 2.21. The precise region in the first order case go beyond the $\pi\Sigma$ threshold on the real energy axis. Even in the complex energy plane, the

²⁸The threshold of the $\eta\Lambda$ channel lies at around 1664 MeV. Because we do not explicitly treat the channel, the non-analytic behavior at the threshold cannot be reproduced by our analytic potential. Therefore, we restrict the parametrized range below the threshold.

potential type	polynomial in E	$\sqrt{s}_{\text{low}} - \sqrt{s}_{\text{up}}$ [MeV]	P_{comp} [%]	pole position [MeV]
Gaussian	1st	1403-1440	84	$1424 - 26i, 1377 - 80i$
	2nd	1362-1511	99	$1424 - 27i, 1381 - 81i$
original pole	—	—	—	$1424 - 26i, 1381 - 81i$

TABLE 2.9: Results of the best equivalent potential after parametrization. Shown are the polynomial type of the parametrizing function, the best parametrized range to maximize the precise region ($\sqrt{s}_{\text{low}} - \sqrt{s}_{\text{up}}$), the percentage of the precise region P_{comp} , and pole positions.

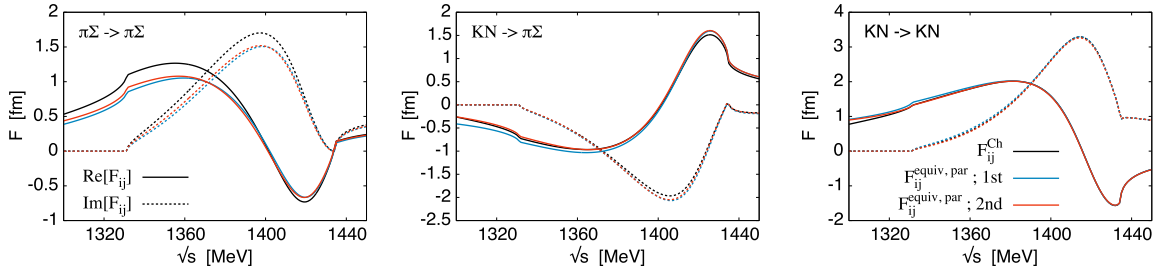


FIGURE 2.20: $F^{\text{equiv,fit}}$ is compared with F^{Ch} (black lines) with the Gaussian potential in the IHW model. The blue (red) lines represent $F^{\text{equiv,par}}$ with first (second) order polynomial and 1403-1440 MeV (1362-1511 MeV) parametrized range. Solid lines and dashed lines are real and imaginary part, respectively. From the left, each figure represents $F_{\pi\Sigma,\pi\Sigma}$, $F_{\pi\Sigma,\bar{K}N}$, and $F_{\bar{K}N,\bar{K}N}$.

potential succeeds in reproducing F^{Ch} precisely and largely. The precise region covers the higher pole and extends near to the lower one. In the second order case, the region completely covers both the poles, and the percentage of the region amounts to 99 %. As a result, the first order potential reproduces the higher pole perfectly and the lower one within 10 MeV difference. And the second order one completely reproduces both the poles. Hence, the employment of the 1st order potential would be reasonable for the quantitative analysis of systems like $\bar{K}$ nuclei focusing only on the higher pole. Of course, the study related to the lower pole would be fruitful enough. When we discuss the effect of both the poles more precisely, the second order potential would be useful.

Here, we remember the single-channel case in Sec. 2.2, where the lower pole appears with a higher-order polynomial like the 10th one. The success in this coupled-channel case with the much lower polynomials would be due to the explicit treatment of the $\pi\Sigma$ channel. In the original chiral unitary approach, the main interaction comes from the WT term, which linearly depends on the energy. Though it cannot be directly connected to the quantum mechanics, we naively expect the local potential in the Schrödinger equation has also the linear dependence. However, when we apply the Feshbach projection to effectively reduce some channel (see Eq. (2.47)), there appears additional energy dependence. In particular, because the $\pi\Sigma$ channel has large contribution to the $\Lambda(1405)$, and moreover, the threshold lies below the $\Lambda(1405)$ mass region, the reduction of the $\pi\Sigma$ channel would generate complicated energy dependence, which cannot be parametrized with a linear potential. On the other hand, in the $\bar{K}N$ - $\pi\Sigma$ coupled-channel case, we only reduce the $\eta\Lambda$ and $K\Xi$ channels, whose contribution is relatively smaller [22, 24]. Moreover, because the thresholds of these channels lie much above the $\Lambda(1405)$ region, the loop function

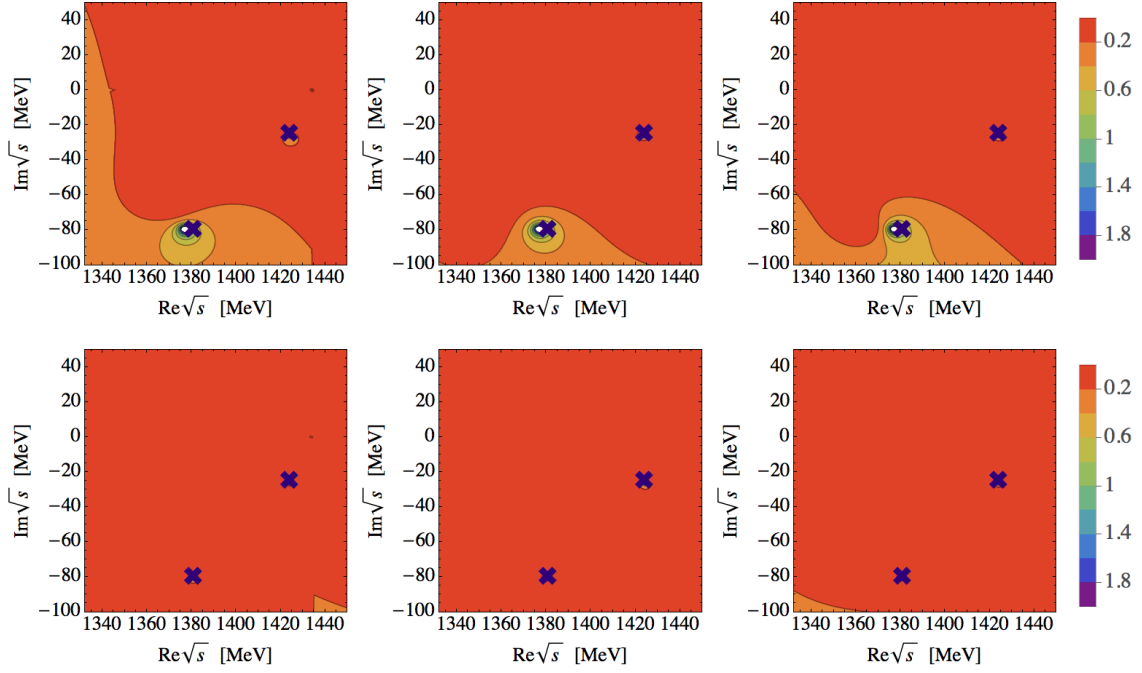


FIGURE 2.21: ΔF_{ij} in the complex energy plane with the Gaussian potential (IHW). Upper and lower figures respectively represent the results with the first and second order polynomials. From the left, each figure represents $\Delta F_{\pi\Sigma,\pi\Sigma}$, $\Delta F_{\pi\Sigma,\bar{K}N}$, and $\Delta F_{\bar{K}N,\bar{K}N}$.

G in Eq. (2.47) is also small. Thus, the linear dependence of the potential would not be strongly contaminated, and the first or second order potential is enough to reproduce the original amplitudes.

The most favorable benefit to use the first order potential is that we may be able to reduce the uncertainty in few-body calculations. As explained in Sec. 2.3.1, in the single-channel case with the tenth order polynomial, we suffer from the uncertainty from the energy dependence of a potential especially for the decay width. It is an intrinsic problem in the variational method with an energy-dependent potential. This problem may be solved only in the first order case by renormalizing the energy dependent potential to obtain an energy-independent one. In Ref. [196], the validity to employ the renormalized potential in a few-body system is indicated in the three α calculation with an originally energy-dependent α - α potential. In a similar manner, we may be able to calculate the $\bar{K}$ nuclei with less ambiguity.

Finally, in Table 2.11, we show the coefficients of the best potentials, which are used in Eq. (2.51). In this table, the results of the first and second order polynomials in the energy with the Gaussian potential are summarized.

2.4.4 $I = 1$ channel

For the application to few-body calculations, the potential in $I = 1$ channel as well as the above discussed $I = 0$ channel is necessary. In $I = 1$, there is the additional $\pi\Lambda$ channel below the $\bar{K}N$ and $\pi\Sigma$ thresholds. Hence, here, we construct the $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ coupled-channel potential for the $I = 1$ channel in just the same way as for the $I = 0$ channel.

Following Sec. 2.4.1, we first determine range parameters in Eq. (2.49) without the adjustment term ΔV_{ij} to minimize the sum of the deviations between F^{Ch} and

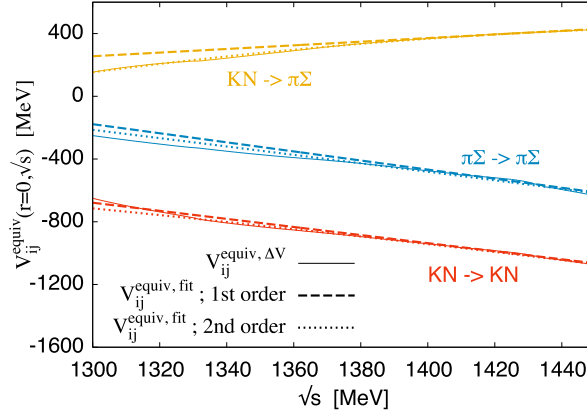


FIGURE 2.22: $V_{ij}^{\text{equiv,par}}$ is compared with $V_{ij}^{\text{equiv},\Delta V}$ (thin lines) with the Gaussian potential in the IHW model. The dashed (dotted) lines represent $V_{ij}^{\text{equiv,par}}$ with first (second) order polynomial and 1403-1440 MeV (1362-1511 MeV) parametrized range. The blue, yellow, and red lines respectively represents the $\pi\Sigma$ -diagonal, off-diagonal, and $\bar{K}N$ -diagonal potential.

polynomial type	channel	K_0 [MeV]	K_1 [MeV ⁰]	K_2 [MeV ⁻¹]
1st order	$\pi\Sigma, \pi\Sigma$	-5.6749×10^2	-2.8956×10^0	—
	$\pi\Sigma, \bar{K}N$	4.1101×10^2	1.1565×10^0	—
	$\bar{K}N, \bar{K}N$	-1.0253×10^3	-2.5759×10^0	—
2nd order	$\pi\Sigma, \pi\Sigma$	-5.7557×10^2	-2.7396×10^0	-3.9289×10^{-4}
	$\pi\Sigma, \bar{K}N$	4.1072×10^2	1.0500×10^0	-6.4195×10^{-3}
	$\bar{K}N, \bar{K}N$	-1.0297×10^3	-2.5907×10^0	-1.8604×10^{-3}

TABLE 2.10: Coefficients $K_{\alpha,ij}$ in Eq. (2.51) of the strength of the best potentials in $I = 0$. The results of the first and second order polynomials in the energy with the Gaussian potential are summarized. In both cases, we use the following range parameters, $b_{\pi\Sigma} = 0.80$ fm and $b_{\bar{K}N} = 0.43$ fm.

F_{ij}^{equiv} at thresholds in the $\pi\Lambda$, $\pi\Sigma$, and $\bar{K}N$ channels,

$$\Delta F_{\text{threshold}}^{I=1} \equiv \sum_{i=\pi\Lambda, \pi\Sigma, \bar{K}N} \left| F_{i,i}^{\text{Ch}}(\sqrt{s} = m_i + M_i) - F_{i,i}^{\text{equiv}}(\sqrt{s} = m_i + M_i) \right|. \quad (2.103)$$

The resulting ranges are

$$b_{\pi\Lambda} = 0.43 \text{ fm}, \quad b_{\pi\Sigma} = 0.51 \text{ fm}, \quad b_{\bar{K}N} = 0.35 \text{ fm}. \quad (2.104)$$

$b_{\pi\Sigma}$ in $I = 1$ is much smaller than in $I = 0$ where $b_{\pi\Sigma} = 0.80$ fm is roughly twice as large as $b_{\bar{K}N} = 0.43$ fm. This may be caused by the absence of the $\pi^0\Sigma^0$ component, which leads to the Pauli blocking, in the $I = 1$ channel. The results of the scattering amplitude $F_{ij}^{\text{equiv, eff}}$ from the potential without ΔV_{ij} are summarized in Fig. 2.23 together with the original amplitude F_{ij}^{Ch} of the IHW model [23, 24]²⁹. As in $I = 0$ case, F_{ij}^{Ch} is qualitatively reproduced at this stage.

²⁹We note that the non-analytic behavior around 1360 MeV in Fig. 2.23 is related to the treatment of the artificial cut of the u -channel Born term with the on-shell approximation [161, 197].

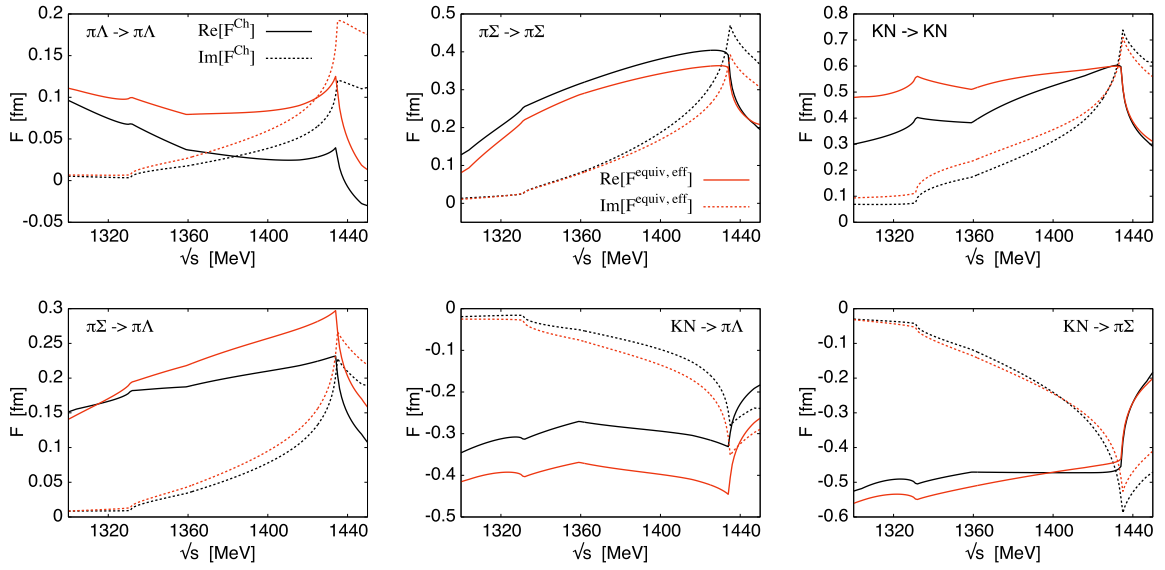


FIGURE 2.23: Scattering amplitudes $F^{\text{equiv, eff}}$ from the potential without the adjustment ΔV_{ij} (Eq. (2.49)) in the IHW model in the $I = 1$ channel. Black lines (red lines) represent the ones from chiral unitary approach (the equivalent potential). Solid lines and dashed lines are real and imaginary parts, respectively. Upper figures denote the diagonal amplitudes, $F_{\pi\Lambda, \pi\Lambda}$, $F_{\pi\Sigma, \pi\Sigma}$, and $F_{\bar{K}N, \bar{K}N}$, respectively. Lower figures denote the off-diagonal amplitudes, $F_{\pi\Lambda, \pi\Sigma}$, $F_{\pi\Lambda, \bar{K}N}$, and $F_{\pi\Sigma, \bar{K}N}$, respectively.

Next, following Secs. 2.4.2 and 2.4.3, we make an adjustment with ΔV_{ij} (see Eq. (2.50)) and parametrize the strength of the equivalent potential by some polynomial in the energy. The optimum ΔV_{ij} is determined to minimize the deviation between the original amplitude F^{Ch} and the one from the equivalent potential $F^{\text{equiv, } \Delta V}$ in Eq. (2.97) in each energy. As the type of the parametrizing polynomial and the parametrized range, we use the optimized ones, 1403-1440 MeV for the first order polynomial and 1362-1511 MeV for the second order one (see Table 2.9), to reproduce F^{Ch} in the complex energy plane in the $I = 0$ channel (Sec. 2.4.3). The reason why we do not reoptimize them is that the scattering amplitude cannot be analytically continued to the complex energy plane naturally due to the prescription to avoid the unphysical cut from the u -channel Born term in the $I = 1$ channel [161, 197]. However, the reuse of the set in $I = 0$ would be sufficient for the application to the few-body calculations considering the weaker interaction and the larger uncertainty in $I = 1$ than in $I = 0$. We summarize the results of the amplitude from the parametrized equivalent potential $F^{\text{equiv, par}}$ in Fig. 2.24. Thanks to the adjustment, the original amplitude F^{Ch} is quantitatively well reproduced considering the theoretical uncertainty of F^{Ch} in $I = 1$ which is larger than in $I = 0$ [24]. Finally, we summarize the coefficients of the parametrized potentials $V^{\text{equiv, par}}$ in $I = 1$ (see Eq. (2.51)) in Table , and the strength of $V^{\text{equiv, par}}$ at $r = 0$ in Fig. 2.25.

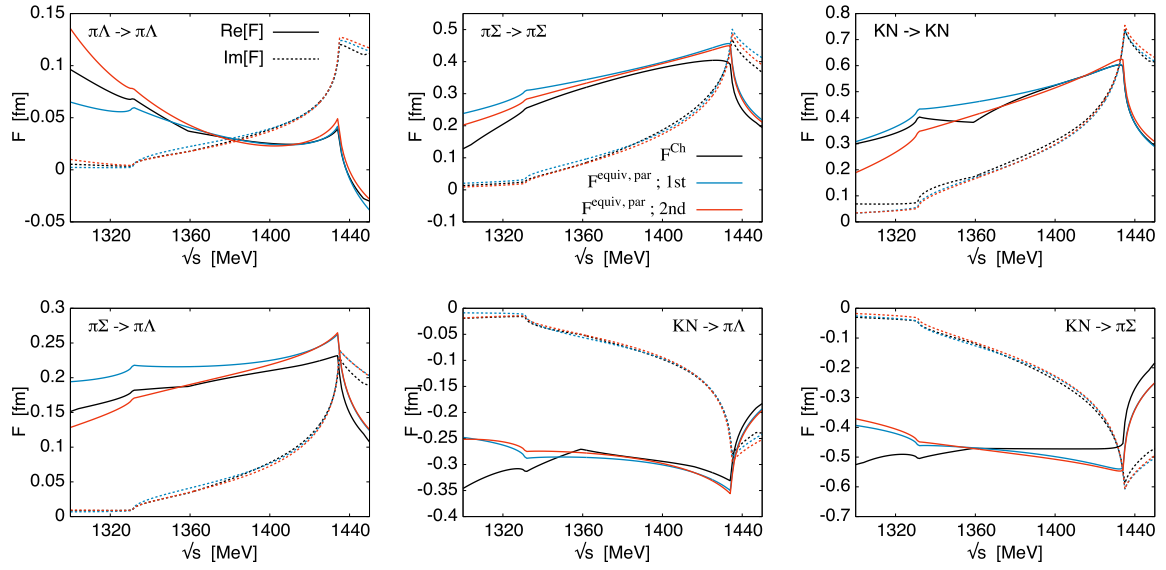


FIGURE 2.24: Scattering amplitudes $F^{\text{equiv, par}}$ from the equivalent potential in Eq. (2.51) after parametrizing the adjusted potential in Eq. (2.50) in the IHW model in the $I = 1$ channel. Black, blue, and red lines respectively represent the amplitudes from chiral unitary approach, the first order equivalent potential (parametrized range is 1403-1440 MeV), and the second order one (1362-1511 MeV). Solid lines and dashed lines are real and imaginary parts, respectively. Upper figures denote the diagonal amplitudes, $F_{\pi\Lambda, \pi\Lambda}$, $F_{\pi\Sigma, \pi\Sigma}$, and $F_{\bar{K}N, \bar{K}N}$, respectively. Lower figures denote the off-diagonal amplitudes, $F_{\pi\Lambda, \pi\Sigma}$, $F_{\pi\Lambda, \bar{K}N}$, and $F_{\pi\Sigma, \bar{K}N}$, respectively.

polynomial type	channel	K_0 [MeV]	K_1 [MeV ⁰]	K_2 [MeV ⁻¹]
1st order	$\pi\Lambda, \pi\Lambda$	4.7304×10^2	3.5828×10^0	—
	$\pi\Sigma, \pi\Sigma$	-4.8658×10^2	-1.7718×10^0	—
	$\bar{K}N, \bar{K}N$	-5.6846×10^3	-2.6948×10^0	—
	$\pi\Lambda, \pi\Sigma$	3.2464×10^2	4.1096×10^{-1}	—
	$\pi\Lambda, \bar{K}N$	6.0458×10^2	6.3015×10^{-1}	—
	$\pi\Sigma, \bar{K}N$	6.3748×10^2	-2.6152×10^{-1}	—
2nd order	$\pi\Lambda, \pi\Lambda$	4.3503×10^2	1.6328×10^0	-2.9915×10^{-2}
	$\pi\Sigma, \pi\Sigma$	-4.6518×10^2	-1.7384×10^0	1.5941×10^{-4}
	$\bar{K}N, \bar{K}N$	-5.8292×10^2	-3.7810×10^0	9.1253×10^{-3}
	$\pi\Lambda, \pi\Sigma$	-3.2709×10^2	-2.8261×10^{-2}	9.0911×10^{-3}
	$\pi\Lambda, \bar{K}N$	5.9513×10^2	2.8877×10^{-1}	2.1955×10^{-3}
	$\pi\Sigma, \bar{K}N$	6.3549×10^2	-1.3703×10^0	-2.8676×10^{-3}

TABLE 2.11: Coefficients $K_{\alpha, ij}$ in Eq. (2.51) of the strength of the best potentials in $I = 1$. The results of the first and second order polynomials are summarized. In both cases, we use the following range parameters, $b_{\pi\Lambda} = 0.43$ fm, $b_{\pi\Sigma} = 0.51$ fm and $b_{\bar{K}N} = 0.35$ fm.

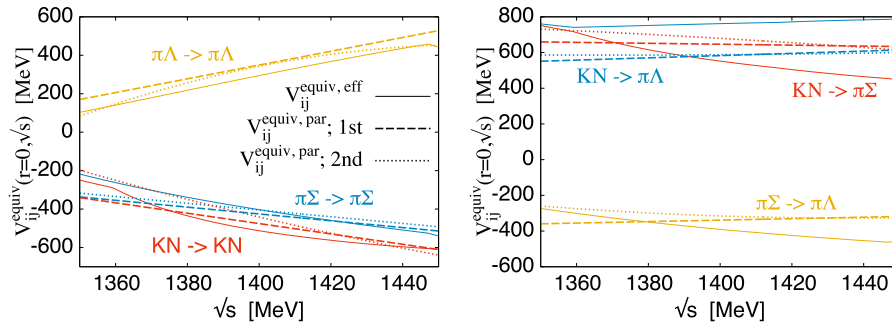


FIGURE 2.25: $V_{ij}^{\text{equiv}, \text{par}}$ is compared with $V_{ij}^{\text{equiv}, \text{eff}}$ (thin lines) with the Gaussian potential in the IHW model in the $I = 1$ channel. The dashed (dotted) lines represent $V_{ij}^{\text{equiv}, \text{par}}$ with first (second) order polynomial and 1403-1440 MeV (1362-1511 MeV) parametrized range. The yellow, blue, and red lines respectively represent the $\pi\Lambda$ -diagonal, $\pi\Sigma$ -diagonal, and $\bar{K}N$ -diagonal potentials.

Chapter 3

Analysis of $\Lambda(1405)$ wave function

In Chapter 2, we constructed reliable $\bar{K}N$ and $\bar{K}N-\pi\Sigma$ local potentials. There, we found that the strength of the potentials depends on the energy in the two-body system. The main origin of the energy dependence is the leading-order term, WT term, in chiral unitary approach as in Eq. (2.19), which is a linear form in the energy (See Sec. 2.1.1). Furthermore, additional energy dependence comes from the effective elimination of some coupling channels (Feshbach projection) conducted for convenience. In Sec. 3.1.1, we will establish a general formulation to treat the system with an energy dependent potential. Subsequently, to interpret the additional term of the normalization in the new formulation, we will reconsider the origin of the energy dependence and find the direct connection of the formulation to the compositeness and elementariness in Sec. 3.1.2. Applying the formulation to the $\Lambda(1405)$ system, we will analyze the spatial structure and the compositeness of the $\Lambda(1405)$ with the single-channel potential in Sec. 3.2.1. and with the coupled-channel potential in Sec. 3.2.2. The results of this work are summarized in Ref. [195]¹.

3.1 Formulation

3.1.1 Normalization of wave function for E -dep. complex potential

As seen from Eqs. (2.19) and (2.50), we find that our local potential has energy dependence. For a system with an energy dependent and real potential, it has been known that the normalization scheme and orthogonality condition should be modified to satisfy essential requirements in the quantum mechanics [199–205]. Here, based on the work for a real potential [203], we derive a modified scheme for an energy-dependent potential in the non-Hermitian system² like the $\Lambda(1405)$. It is an extended version of a single-channel system in Refs. [119, 198] to a general coupled-channel system.

The important point is to search for a probability density $P_{E'E}$ and a current $j_{E'E}$ for eigenstates with the energies E and E' that satisfy the continuity equation,

$$\frac{\partial}{\partial t} P_{E'E}(\mathbf{r}, t) = -\nabla \cdot \mathbf{j}_{E'E}(\mathbf{r}, t). \quad (3.1)$$

¹The results of the normalization scheme in the energy-dependent and non-Hermitian system with a single-channel potential is summarized in Refs. [119, 198].

²The non-Hermitian system means not only the system with a complex potential but also the one with the outgoing boundary condition with a complex eigenenergy. Actually, in the $\bar{K}N-\pi\Sigma$ system, the local potential is real but the complex eigenenergies are considered.

This equation should be respected because it leads to the norm conservation and the orthogonality relation. Actually, if $j_{E'E}$ satisfies

$$j_{E'E}(\mathbf{r}, t) \rightarrow 0, \quad (r \rightarrow \infty), \quad (3.2)$$

integrating Eq. (3.1) with respect to $\mathbf{r}$ and taking $E' = E$, we obtain the norm conservation,

$$\begin{aligned} \frac{\partial}{\partial t} N_E(t) &= 0, \\ N_E(t) &\equiv \int d^3r P_{EE}(\mathbf{r}, t). \end{aligned} \quad (3.3)$$

Furthermore, if the probability density is in the standard form with a time-dependent (time-independent) wave function Ψ_E (ψ_E),

$$\begin{aligned} P_{E'E}(\mathbf{r}, t) &= \Psi_{E'}^*(\mathbf{r}, t) \Psi_E(\mathbf{r}, t) \\ &= e^{-i(E-E')t} \psi_{E'}^*(\mathbf{r}) \psi_E(\mathbf{r}), \end{aligned} \quad (3.4)$$

the integration of Eq. (3.1) results in the orthogonality condition as follows:

$$\begin{aligned} \frac{\partial}{\partial t} \int d^3r P_{E'E}(\mathbf{r}, t) &= -i(E - E') e^{-i(E-E')t} \int d^3r \psi_{E'}^*(\mathbf{r}) \psi_E(\mathbf{r}) \\ &= 0, \end{aligned} \quad (3.5)$$

hence, if $E' \neq E$ then,

$$\int d^3r \psi_{E'}^*(\mathbf{r}) \psi_E(\mathbf{r}) = 0. \quad (3.6)$$

Because the norm conservation and the orthogonality relation are fundamental requirements for the quantum mechanics, we try to retain it in the E -dep. complex potential systems, where the standard probability density and current violate the continuity equation as shown later.

To this end, we introduce the adjoint wave function (or the Gamow vector) $\Psi^\dagger$ [206–208], which is used in the non-Hermitian system. Consider an usual wave function $\Psi = (\Psi_1, \Psi_2, \dots)^T$ satisfying the coupled-channel Schrödinger equation with the E -dependent complex potential,

$$\begin{aligned} i \frac{\partial}{\partial t} \Psi_i(\mathbf{r}, t) &= \sum_j H_{ij} \Psi_j(\mathbf{r}, t), \\ H_{ij} &= -\frac{\nabla^2}{2\mu_i} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}(\mathbf{r}, i \frac{\partial}{\partial t}), \end{aligned} \quad (3.7)$$

where μ_i and ΔM_i represent the reduced mass of channel i and the mass difference of channel i from a reference threshold, and an eigenfunction of H , $\Psi_{E,i}(\mathbf{r}, t) = e^{-iEt} \psi_{E,i}(\mathbf{r})$, satisfying

$$\sum_j H_{ij} \psi_{E,j}(\mathbf{r}) = \sum_j \left[-\frac{\nabla^2}{2\mu_i} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}(\mathbf{r}, E) \right] \psi_{E,j}(\mathbf{r}) = E \psi_{E,i}(\mathbf{r}). \quad (3.8)$$

Then, the adjoint functions $\Psi_E^\dagger$ and $\psi_E^\dagger$ are defined so that a redefined current,

$$\mathbf{j}_E^G(\mathbf{r}, t) \equiv - \sum_i \frac{i}{2\mu_i} \left[\Psi_{E,i}^{\dagger*}(\mathbf{r}, t) \nabla \Psi_{E,i}(\mathbf{r}, t) - \{ \nabla \Psi_{E,i}^{\dagger*}(\mathbf{r}, t) \} \Psi_{E,i}(\mathbf{r}, t) \right], \quad (3.9)$$

vanishes at $r \rightarrow \infty$ (see Ref. [206])³. This requirement is met for

$$\begin{aligned} \Psi_{E,i}^\dagger(\mathbf{r}, t) &= e^{-iE^*t} \psi_{E,i}(\mathbf{r}), \\ \psi_{E,i}^\dagger(\mathbf{r}) &= \psi_{E,i}^*, \end{aligned} \quad (3.10)$$

which satisfy the Schrödinger equations,

$$\begin{aligned} i \frac{\partial}{\partial t} \Psi_{E,i}^\dagger(\mathbf{r}, t) &= \sum_j H_{ij}^* \Psi_{E,j}^\dagger(\mathbf{r}, t), \\ \sum_j H_{ij}^* \psi_{E,j}^\dagger(\mathbf{r}) &= \sum_j \left[-\frac{\nabla^2}{2\mu_i} \delta_{ij} + V_{ij}^*(\mathbf{r}, E) \right] \psi_{E,j}^\dagger(\mathbf{r}) = E^* \psi_{E,i}^\dagger(\mathbf{r}). \end{aligned} \quad (3.11)$$

The second equation is just the complex conjugate of Eq. (3.8). Here, we note $\Psi_{E,i}^\dagger \neq \Psi_{E,i}^*$.

Using this Gamow vector, we reconstruct a naive candidate of the probability density as

$$P_{E'E}^{G, \text{naive}}(\mathbf{r}, t) = \sum_i \left[\Psi_{E',i}^\dagger(\mathbf{r}, t) \right]^* \Psi_{E,i}(\mathbf{r}, t). \quad (3.12)$$

With the Schrödinger equations (3.7) and (3.11), the time-derivative of $P_{E'E}^G$ becomes

$$\begin{aligned} \frac{\partial}{\partial t} P_{E'E}^{G, \text{naive}} &= \sum_i \left(\frac{\partial \Psi_{E',i}^{\dagger*}}{\partial t} \Psi_{E,i} + \Psi_{E',i}^{\dagger*} \frac{\partial \Psi_{E,i}}{\partial t} \right) \\ &= \sum_{i,j} \left[-i \left\{ -\frac{\nabla^2}{2\mu_i} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}^*(E') \right\} \Psi_{E',j}^\dagger \right]^* \Psi_{E,i} \\ &\quad + \sum_{i,j} \Psi_{E',i}^{\dagger*} \left[-i \left\{ -\frac{\nabla^2}{2\mu_i} \delta_{ij} + \Delta M_i \delta_{ij} + V_{ij}(E) \right\} \Psi_{E,j} \right] \\ &= -\nabla \cdot \mathbf{j}_{E'E}^G + i \sum_{i,j} \left[V_{ij}(E') \Psi_{E',j}^{\dagger*} \Psi_{E,i} - V_{ij}(E) \Psi_{E',i}^{\dagger*} \Psi_{E,j} \right] \\ &= -\nabla \cdot \mathbf{j}_{E'E}^G + i \sum_{i,j} \Psi_{E',i}^{\dagger*} [V_{ij}(E') - V_{ij}(E)] \Psi_{E,j}, \end{aligned} \quad (3.13)$$

where

$$\mathbf{j}_{E'E}^G(\mathbf{r}, t) = - \sum_i \frac{i}{2\mu_i} \left[\Psi_{E',i}^{\dagger*}(\mathbf{r}, t) \nabla \Psi_{E,i}(\mathbf{r}, t) - \{ \nabla \Psi_{E',i}^{\dagger*}(\mathbf{r}, t) \} \Psi_{E,i}(\mathbf{r}, t) \right]. \quad (3.14)$$

Because of the E -dependence of the potential, there appears the second term in the right-hand-side of Eq. (3.13), which breaks the naive continuity equation, $\frac{\partial}{\partial t} P =$

³In Ref. [206], from the definition of the Hermite conjugate of H , the vanishment of the current directly leads to $H^\dagger = H^*$, and $\psi_E^\dagger = \psi_E^*$ can be defined as the eigenstate for $H^\dagger$, resulting in the norm conservation and the orthogonal relation. On the other hand, in our case, because of the E -dependence of the potential, the relation between $\mathbf{j}^G(\mathbf{r} \rightarrow 0, t) = 0$ and $H^\dagger = H^*$ does not hold. Thus, the Gamow state should be introduced as the eigenstate of H^* .

$-\nabla \cdot \mathbf{j}$. In a similar way to Ref. [203], we can rewrite this term in the form of a time derivative by the relation,

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\{ \Psi_{E',i}^{*} \left[\frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \Psi_{E,j} \right\} \\
&= \frac{\partial \Psi_{E',i}^{*}}{\partial t} \left[\frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \Psi_{E,j} + \Psi_{E',i}^{*} \left[\frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \frac{\partial \Psi_{E,j}}{\partial t} \\
&= (-iE'^{*} \Psi_{E',i}^{*})^{*} \left[\frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \Psi_{E,j} + \Psi_{E',i}^{*} \left[\frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] (-iE \Psi_{E,j}) \\
&= i \Psi_{E',i}^{*} [V_{ij}(E') - V_{ij}(E)] \Psi_{E,j}, \tag{3.15}
\end{aligned}$$

where we use the Schrödinger equations (3.7), (3.8), and (3.11). Hence, the additional term can be regarded as a part of the probability density, and the following modified continuity equation holds,

$$\frac{\partial}{\partial t} P_{E'E}^G \equiv \frac{\partial}{\partial t} \left(\sum_{i,j} \Psi_{E',i}^{*} \left[\delta_{ij} - \frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \Psi_{E,j} \right) = -\nabla \cdot \mathbf{j}_{E'E}^G. \tag{3.16}$$

This means that the norm N_E^G and the orthogonality relation should also be modified as⁴

$$\begin{aligned}
N_E^G &= \int d^3r P_{EE}^G = \int d^3r \left(\sum_{i,j} \Psi_{E,i}^{*} \left[\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(E) \right] \Psi_{E,j} \right) \\
&= \int d^3r \left(\sum_{i,j} \psi_{E,i} \left[\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(E) \right] \psi_{E,j} \right), \tag{3.17}
\end{aligned}$$

$$\begin{aligned}
& \int d^3r \left(\sum_{i,j} \Psi_{E',i}^{*} \left[\delta_{ij} - \frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \Psi_{E,j} \right) \\
&= e^{-i(E-E')t} \int d^3r \left(\sum_{i,j} \psi_{E',i} \left[\delta_{ij} - \frac{V_{ij}(E') - V_{ij}(E)}{E' - E} \right] \psi_{E,j} \right) = 0 \quad (E' \neq E), \tag{3.18}
\end{aligned}$$

where the first equation for the norm is obtained by taking a limit, $E' \rightarrow E$. The norm conservation and the realization of the orthogonality relation can be checked as in Eqs. (3.3) and (3.6). In the following, we use this modified normalization.

If we started from $\Psi_{E'}^* \Psi_E$, not from $\Psi_{E'}^{*} \Psi_E$ in Eq. (3.12), as in the E -dependent real potential case [203], two differences would appear in the continuity equation (3.16). The first is the additional term to the probability density (second term in the left-hand side),

$$-\Psi_{E',i}^{*} \frac{V_{ij}^{*}(E') - V_{ij}(E)}{E'^{*} - E} \Psi_{E,j}, \tag{3.19}$$

⁴We note that generally, $\mathbf{j}_{E'E}^G$ does not vanish at $r \rightarrow \infty$ unless $E' = E$ in the non-Hermitian system. However, when resonant wave functions are regularized by, for instance, the complex scaling method, $\mathbf{j}_{E'E}^G$ vanishes at the boundary and the orthogonality relation holds for the regularized resonances. It would be related to the extended completeness relation in the complex scaling method, which consists of the regularized resonant states as well as the bound and continuum states [209].

and the second is the current $j_{E'E}^G$ changed to the standard one in the right-hand side,

$$j_{E'E}(\mathbf{r}, t) = - \sum_i \frac{i}{2\mu_i} [\Psi_{E',i}^*(\mathbf{r}, t) \nabla \Psi_{E,i}(\mathbf{r}, t) - \{\nabla \Psi_{E',i}(\mathbf{r}, t)\}^* \Psi_{E,i}(\mathbf{r}, t)]. \quad (3.20)$$

When we consider $E \in \mathbb{R}$ as in a physical scattering state and take the limit, $E' \rightarrow E$, the term (3.19) diverges for the complex potential, $\text{Im}[V_{ij}] \neq 0$, and the norm cannot be well defined. Furthermore, when we treat a resonance state, whose wave function diverges at $r \rightarrow \infty$, the standard current (3.20) also diverges. Thus, neither the norm conservation nor the orthogonality relation are guaranteed. On the other hand, in our new form with Eq. (3.16), these divergences do not appear, and both the essential requirements are guaranteed⁵.

3.1.2 Relation between new normalization and elementariness

In this section, we aim to extract the meaning of the additional term to the norm in Eq. (3.17),

$$\int d^3r \sum_{i,j} \psi_{E,i} \left(-\frac{\partial V_{ij}}{\partial E}(E) \right) \psi_{E,j}, \quad (3.21)$$

in association with the compositeness and the elementariness discussed in Ref. [11]. The compositeness and elementariness respectively represent the norm of some degrees of freedom in a model space and outside the model space. A formulation to calculate the compositeness of a resonance from scattering amplitudes was derived via wave functions, and applied with some model in Ref. [194]. In Ref. [210], the compositeness was related to observables model-independently in the weak binding limit, based on an early study of a stable bound state, the deuteron [10]. In our work, we reconsider the elementariness, paying attention to the energy dependence of interactions, with minimum assumption in the non-relativistic framework.

Here, we consider a coupled-channel system with an ordinary energy-independent Hamiltonian. Only assumption is that the energy dependence of a practical potential comes from the elimination of some channels from the initial "full" system. In our case for meson-baryon interactions, the eliminated effects may be related to the bare Λ^* or interactions in the quark and gluon level. Thus, we first prepare the "full" channel state $|\psi\rangle$ and the corresponding Gamow state $|\psi^\dagger\rangle$ ⁶,

$$|\psi\rangle = \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \\ \vdots \end{pmatrix}, \quad |\psi^\dagger\rangle = \begin{pmatrix} |\psi_1^\dagger\rangle \\ |\psi_2^\dagger\rangle \\ \vdots \end{pmatrix}, \quad (3.22)$$

conceptually including all possible states. Furthermore, we do not constrain our discussion to the two-body systems. That is, $|\psi_i\rangle$ can represent from one-body to many-body systems of any degrees of freedom. For such "full" channel state, we consider the Schrödinger equation with an energy-independent interactions $\hat{V}$ ⁷,

$$\hat{H}|\psi\rangle = (\hat{H}_0 + \hat{V})|\psi\rangle = E|\psi\rangle, \quad (3.23)$$

$$\langle\psi^\dagger|\hat{H} = \langle\psi^\dagger|(\hat{H}_0 + \hat{V}) = \langle\psi^\dagger|E, \quad (3.24)$$

⁵With the Gamow vector in Eq.(3.10), $j_{E'E}^G = 0$ holds even for a resonance state.

⁶Because we are interested in the non-Hermitian systems, as in Sec. 3.1.1, we introduce the Gamow vector $|\psi^\dagger\rangle$.

with a free part $\hat{H}_0$, which is diagonal for each channel.

What we actually handle is a system where some channels are eliminated from the full state. Here, we define $\hat{P}$ as a projection operator to the subspace whose degrees of freedom is explicitly treated in a model. The projection to eliminated channels are denoted by $\hat{Q}$. This kind of projection is well known as the Feshbach projection [138, 139]. These operators meet the usual relations for general projection operators,

$$\hat{P} + \hat{Q} = 1, \quad (3.25)$$

$$\hat{P}\hat{Q} = 0, \quad (3.26)$$

$$\hat{P}^2 = \hat{P}, \quad \hat{Q}^2 = \hat{Q}. \quad (3.27)$$

Here, we define the quantities X_i and Z as the norms of the channel i in the model space projected by P (P space) and of all states in eliminated space (Q space), respectively,

$$X_i = \langle \psi_i^\dagger | \psi_i \rangle \quad (i \in P), \quad (3.28)$$

$$Z = \langle \psi^\dagger | \hat{Q}\hat{Q} | \psi \rangle. \quad (3.29)$$

Inserting Eq. (3.25) to the normalization of the full $|\psi\rangle$, $\langle \psi^\dagger | \psi \rangle = 1$, we find the sum rule for X_i and Z ,

$$\begin{aligned} \langle \psi^\dagger | \hat{P} + \hat{Q} | \psi \rangle &= \langle \psi^\dagger | \hat{P}\hat{P} | \psi \rangle + \langle \psi^\dagger | \hat{Q}\hat{Q} | \psi \rangle \\ &= \sum_{i \in P} X_i + Z = 1. \end{aligned} \quad (3.30)$$

In our practical application to the $\Lambda(1405)$ where the P space consists only of two-body states, X_i and Z respectively corresponds to the compositeness and the elementariness [11]. Operating $\hat{P}$ or $\hat{Q}$ from the left (right) to Eqs. (3.23) (Eq. (3.24)) and inserting $1 = \hat{P} + \hat{Q}$, we obtain

$$(\hat{P}\hat{H}\hat{P} + \hat{P}\hat{H}\hat{Q})|\psi\rangle = E\hat{P}|\psi\rangle, \quad (3.31)$$

$$(\hat{Q}\hat{H}\hat{P} + \hat{Q}\hat{H}\hat{Q})|\psi\rangle = E\hat{Q}|\psi\rangle, \quad (3.32)$$

and

$$\langle \psi^\dagger | (\hat{P}\hat{H}\hat{P} + \hat{Q}\hat{H}\hat{P}) = \langle \psi^\dagger | \hat{P}E, \quad (3.33)$$

$$\langle \psi^\dagger | (\hat{P}\hat{H}\hat{Q} + \hat{Q}\hat{H}\hat{Q}) = \langle \psi^\dagger | \hat{Q}E. \quad (3.34)$$

⁷In Ref. [206], the Gamow vector turns out to satisfy the Schrödinger equation, $\hat{H}^\dagger |\psi^\dagger\rangle = E^* |\psi^\dagger\rangle$, for energy-independent potentials. This is equivalent to Eq. (3.24). We note that the Hermitian conjugate of $\hat{H}$ can be well defined with the proper boundary condition for both ψ and $\psi^\dagger$ (See Ref. [206] for more detail).

From Eqs. (3.32) and (3.34), the eliminated channels can be expressed by the remaining channels as

$$\begin{aligned}\hat{Q}|\psi\rangle &= \frac{1}{E - \hat{Q}\hat{H}\hat{Q}}(\hat{Q}\hat{H}\hat{P})\hat{P}|\psi\rangle \\ &= \frac{1}{E - \hat{Q}\hat{H}\hat{Q}}(\hat{Q}\hat{V}\hat{P})\hat{P}|\psi\rangle,\end{aligned}\quad (3.35)$$

$$\begin{aligned}\langle\psi^\dagger|\hat{Q} &= \langle\psi^\dagger|\hat{P}(\hat{P}\hat{H}\hat{Q})\frac{1}{E - \hat{Q}\hat{H}\hat{Q}} \\ &= \langle\psi^\dagger|\hat{P}(\hat{P}\hat{V}\hat{Q})\frac{1}{E - \hat{Q}\hat{H}\hat{Q}},\end{aligned}\quad (3.36)$$

where we use Eq. (3.27) and the diagonality of $\hat{H}_0$. Inserting Eq. (3.35) to Eq. (3.31), the Schrödinger equation only for the remaining channels $\hat{P}|\psi\rangle$ is obtained as,

$$(\hat{P}\hat{H}^0\hat{P})\hat{P}|\psi\rangle + \left[\hat{P}\hat{V}\hat{P} + (\hat{P}\hat{V}\hat{Q})\frac{1}{E - \hat{Q}\hat{H}\hat{Q}}(\hat{Q}\hat{V}\hat{P}) \right] \hat{P}|\psi\rangle = E\hat{P}|\psi\rangle. \quad (3.37)$$

Hence, the effective potential $\hat{V}^{\text{eff}}$ for $\hat{P}|\psi\rangle$ is

$$\hat{V}^{\text{eff}}(E) = \hat{P}\hat{V}\hat{P} + (\hat{P}\hat{V}\hat{Q})\frac{1}{E - \hat{Q}\hat{H}\hat{Q}}(\hat{Q}\hat{V}\hat{P}). \quad (3.38)$$

$\hat{V}^{\text{eff}}$ operates only to $\hat{P}|\psi\rangle$ and exactly reproduce $|\psi\rangle$ with the full Schrödinger equation regarding the corresponding channels in the P space.

From Eqs. (3.32) and (3.34), we find that the total norm of the eliminated channels Z can be calculated only with the explicit channels (P space) as

$$\begin{aligned}Z &= \langle\psi^\dagger|\hat{Q}\hat{Q}|\psi\rangle \\ &= \langle\psi^\dagger|\hat{P}(\hat{P}\hat{V}\hat{Q})\frac{1}{E - \hat{Q}\hat{H}\hat{Q}}\frac{1}{E - \hat{Q}\hat{H}\hat{Q}}(\hat{Q}\hat{V}\hat{P})\hat{P}|\psi\rangle \\ &= \langle\psi^\dagger|\hat{P}\left(-\frac{\partial\hat{V}^{\text{eff}}}{\partial E}(E)\right)\hat{P}|\psi\rangle.\end{aligned}\quad (3.39)$$

This is nothing but the additional term in Eq. (3.17). Considering that $\hat{V}^{\text{eff}}$ operates only to the P space, we find that the norm of the missing channels in Q space can be calculated only from the explicitly treated channels in the P space.

So far, we have discussed the meaning of $\frac{\partial V^{\text{eff}}}{\partial E}$ in general. In the following, with the application to the $\Lambda(1405)$ system in mind, we make some assumptions. First, we restrict the P space only to the two-body system such as the $\bar{K}N$ and $\pi\Sigma$, and push contribution from other channels like the $\eta\Lambda$, $K\Xi$, and one-body systems into the Q space. In this situation, X_i and Z correspond to the so called compositeness and elementariness [11]. Here, we denote the completeness relation as

$$\begin{aligned}1 &= \hat{P} + \hat{Q} \\ &= \sum_{i \in P} \int d^3r_i |r_i\rangle\langle r_i| + \hat{Q},\end{aligned}\quad (3.40)$$

Inserting the completeness relation to Eq. (3.39), we obtain the elementariness as

$$\begin{aligned} Z &= \sum_{i,j \in P} \int \int d^3 r_i d^3 r_j \langle \psi^\dagger | \hat{P} | \mathbf{r}_i \rangle \langle \mathbf{r}_i | \left(-\frac{\partial \hat{V}^{\text{eff}}}{\partial E}(E) \right) | \mathbf{r}_j \rangle \langle \mathbf{r}_j | \hat{P} | \psi \rangle \\ &= \sum_{i,j \in P} \int \int d^3 r_i d^3 r_j \psi_i(\mathbf{r}_i) \left(-\frac{\partial V_{ij}^{\text{eff}}}{\partial E}(\mathbf{r}_i, \mathbf{r}_j; E) \right) \psi_j(\mathbf{r}_j), \end{aligned} \quad (3.41)$$

where the indices i and j run only in the P space. As in Chapter 2, assuming a local effective interaction, $V^{\text{eff}} \propto \delta^{(3)}(\mathbf{r}_i - \mathbf{r}_j)$, for convenience, this general relation can be reduced to

$$Z = \sum_{i,j \in P} \int d^3 \mathbf{r} \psi_i(\mathbf{r}) \left(-\frac{\partial V_{ij}^{\text{eff}}}{\partial E}(\mathbf{r}; E) \right) \psi_j(\mathbf{r}). \quad (3.42)$$

This is exactly same as Eq. (3.21). The compositeness X_i for channel i in the P space can be simply written as

$$\begin{aligned} X_i &= \langle \psi_i^\dagger | \psi_i \rangle \quad (i \in P) \\ &= \int d^3 r \psi_i^2(\mathbf{r}). \end{aligned} \quad (3.43)$$

Here, we should note that ψ_i in X_i and Z obey the new normalization condition as in Eq. (3.17). Actually, it can be seen from the normalization of the full $|\psi\rangle$ or the sum rule in Eq. (3.30),

$$\sum_{i \in P} X_i + Z = \int d^3 r \left(\sum_{i,j} \psi_i(\mathbf{r}) \left[\delta_{ij} - \frac{\partial V_{ij}^{\text{eff}}}{\partial E}(\mathbf{r}; E) \right] \psi_j(\mathbf{r}) \right) = 1. \quad (3.44)$$

In this way, in terms of the channel elimination, we have verified the validity of the new normalization in Sec. 3.1.1 for systems with energy-dependent potentials and the interpretation of the energy-derivative term as the elementariness. The same interpretation of X_i and Z has been numerically confirmed in Ref. [211] recently.

In the similar manner, we consider the method to calculate the expectation value of an operator $\hat{O}$. It can be written with the full state $|\psi\rangle$ as

$$\begin{aligned} \langle \hat{O} \rangle &= \langle \psi^\dagger | \hat{O} | \psi \rangle \\ &= \langle \psi^\dagger | (\hat{P} + \hat{Q}) \hat{O} (\hat{P} + \hat{Q}) | \psi \rangle \\ &= \langle \psi^\dagger | \hat{P} \hat{O} \hat{P} | \psi \rangle + \langle \psi^\dagger | \hat{Q} \hat{O} \hat{Q} | \psi \rangle + \langle \psi^\dagger | \hat{P} \hat{O} \hat{Q} | \psi \rangle + \langle \psi^\dagger | \hat{Q} \hat{O} \hat{P} | \psi \rangle. \end{aligned} \quad (3.45)$$

If $\hat{O}$ is diagonal with respect to the channel such as $\hat{r}_i^2$, the last two terms vanish. The first term can be easily calculated as in X_i case. On the other hand, the second term, which seems the component of the Q space, causes some problem. Actually, when we substitute Eqs. (3.35) and (3.36), the second term turns out to be

$$\langle \psi^\dagger | \hat{Q} \hat{O} \hat{Q} | \psi \rangle = \langle \psi^\dagger | \hat{P} (\hat{P} \hat{V} \hat{Q}) \frac{1}{E - \hat{Q} \hat{H} \hat{Q}} \hat{O} \frac{1}{E - \hat{Q} \hat{H} \hat{Q}} (\hat{Q} \hat{V} \hat{P}) \hat{P} | \psi \rangle. \quad (3.46)$$

Naively we expect that this term can be also expressed with the energy derivative of the effective interaction $\langle \psi^\dagger | \hat{P} \hat{O} \left(-\frac{\partial \hat{V}^{\text{eff}}}{\partial E} \right) \hat{P} | \psi \rangle$. However, unless $\hat{O}$ commutes with all other operators, such derivative form cannot be obtained, which means that some

information in Q space aside from $\hat{V}^{\text{eff}}$ is necessary to extract the eliminated-channel component of the expectation value. For example, in the case of $\hat{O} = \sum_i \hat{r}_i^2$, the operator does not commute with $\hat{H}$ at least. Hence, we can conclude that in general the whole expectation value as well as the Q -space component $\langle \hat{Q} \hat{O} \hat{Q} \rangle$ cannot be calculated only with the P space wave function and V^{eff} , in contrast to the norm, or Z . Only information which we can extract as the expectation value is the i th component $\langle \hat{O} \rangle_i$ in the P space. For example, the mean-squared distance can be calculated as

$$\langle \hat{r}^2 \rangle_i = \int d^3r r^2 \psi_i(\mathbf{r})^2 \quad (i \in P). \quad (3.47)$$

3.2 Analysis of $\Lambda(1405)$ wave function

3.2.1 $\bar{K}N$ single-channel system

Applying the new formulation in Secs. 3.1.1 and 3.1.2 to the $\bar{K}N$ single-channel potential in Sec. 2.2, we can analyze the two-body wave function, which is directly related to the $\Lambda(1405)$ in the $I = 0$ channel. Here, applying our realistic $\bar{K}N$ potential ($I = 0$), we estimate the $\bar{K}N$ distance and analyze the $\Lambda(1405)$ spatial structure.

As in the analysis of a bound state, we calculate the wave function at the resonance pole positions. The asymptotic behavior of the s -wave part of the radial wave function, $u(r)$, solved in the Schrödinger equation (2.44), is

$$u(r) \propto e^{-ikr} - (1 + 2ikF_{\bar{K}N})e^{+ikr} \quad (r \rightarrow \infty), \quad (3.48)$$

where $k = \pm\sqrt{2\mu E}$ is the eigenmomentum. For a complex energy E , the choice of the sign of k is related to the Riemann sheet: $k = +\sqrt{2\mu E}$, that is, $\text{Im}k > 0$, represents the first sheet, and $k = -\sqrt{2\mu E}$, that is, $\text{Im}k < 0$, represents the second sheet. At a resonance energy, where $F_{\bar{K}N} \rightarrow \infty$, the asymptotic behavior is determined only from the outgoing wave, e^{ikr} , and vanishes (diverges) at large r for the first (second) Riemann sheet. In our $\bar{K}N$ single-channel case, the wave function vanishes at $r \rightarrow \infty$ because the $\Lambda(1405)$ lies below the $\bar{K}N$ threshold in the first Riemann sheet for the $\bar{K}N$ channel, which is known as the quasibound state and the finite decay width appears due to the complex potential⁸. However, if a chosen energy is deviated from the resonance even a little, the convergence cannot be achieved. This is because the other divergent term e^{-ikr} in Eq. (3.48) begins to affect u as r goes to infinity when we numerically solve the differential equation. Thus, we have to precisely choose the resonance energy for the wave function analysis. Here, the two resonance energies from Kyoto $\bar{K}N$ potential ($I = 0$) are chosen as

$$\begin{aligned} \text{higher pole: } & 1423.97 - 26.28i \text{ MeV,} \\ \text{lower pole: } & 1380.7043281 - 80.5647905i \text{ MeV,} \end{aligned} \quad (3.49)$$

to achieve sufficient convergence at $r \sim 10$ fm.

As discussed in Sec. 3.1.1⁹, for a resonant state with a complex energy, we encounter a problem about the normalization or the calculation of an expectation value.

Following Sec. 3.1.1, we normalize the $\bar{K}N$ wave function in s wave, $\psi(r) = u(r)/(\sqrt{4\pi r})$, as

$$\int d^3r \left[1 - \frac{\partial V_{\bar{K}N}^{\text{equiv}}(r, E)}{\partial E} \right] \psi_G^2(r) = 1, \quad (3.50)$$

where the index G denotes the Gamow vector [206, 207] introduced to treat the non-Hermitian system. The additional term, $\partial V_{\bar{K}N}^{\text{equiv}}/\partial E$, is related to the elementariness, or the effect of missing channels like the $\pi\Sigma$ channel which are just effectively treated

⁸In contrast, in the coupled-channel case in Sec. 2.4, because the $\Lambda(1405)$ lies in the second Riemann sheet for the $\pi\Sigma$ channel, the $\pi\Sigma$ component of the wave function diverges.

⁹Discussion for a single-channel case is summarized in the appendix in Ref. [119] and in Refs. [198, 212].

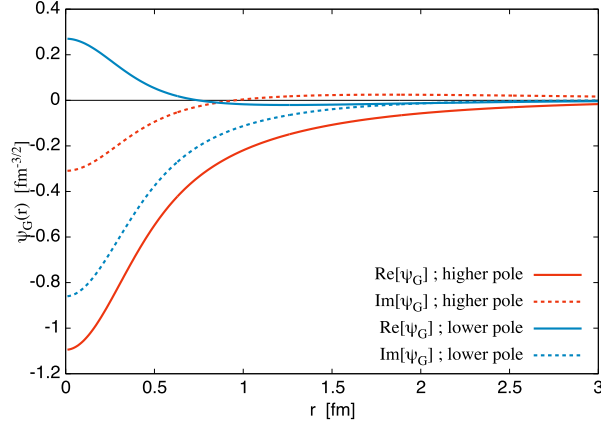


FIGURE 3.1: $\bar{K}N$ wave function ψ_G in $I = 0$ at the two $\Lambda(1405)$ resonances (Eq. (3.49)). The information at the higher (lower) pole is represented by the red (blue) lines. The real (imaginary) parts are shown by the solid (dashed) lines.

in the $\bar{K}N$ single-channel system. Thus, the norm of the $\bar{K}N$ component (compositeness $X_{\bar{K}N}$) and the elementary component (elementariness Z) are (See Sec. 3.1.2)

$$X_{\bar{K}N} = \int d^3r \psi_G^2, \quad (3.51)$$

$$Z = \int d^3r \left(-\frac{\partial V_{\bar{K}N}^{\text{equiv}}}{\partial E} \psi_G^2 \right). \quad (3.52)$$

While the elementary component of an expectation value of a general local operator $\hat{O}$ cannot be calculated, the $\bar{K}N$ component can be calculated as (See Sec. 3.1.2)¹⁰

$$\langle \hat{O} \rangle_G = \int d^3r \mathcal{O}(r) \psi_G^2(r). \quad (3.53)$$

In Fig. 3.1, we show ψ_G at the $\Lambda(1405)$ resonance energies in Eq. (3.49) with the normalization in Eq. (3.50). Furthermore, to clarify the spatial structure in each component, we define the density distribution of the $\bar{K}N$ component $\rho_{\bar{K}N}$ as

$$\rho_{\bar{K}N}(r) = 4\pi r^2 \psi_G^2(r). \quad (3.54)$$

We show the density distribution in Fig. 3.2. It turns out that the magnitudes of both the real and imaginary parts of $\rho_{\bar{K}N}$ at the higher pole are larger than the ones at the lower pole, which would indicate the larger contribution of the $\bar{K}N$ component in the higher pole. Furthermore, $\rho_{\bar{K}N}$ at the higher pole broadly extends over 1.5 fm, where it almost vanishes at the lower pole. In terms of the compositeness ($X_{\bar{K}N}$), we find $X_{\bar{K}N} = 1.01 - 0.07i$ for the higher pole and $X_{\bar{K}N} = -0.33 - 0.03i$ for the lower pole. Though the complex value is generally difficult to interpret, here we can see the clear tendency that the higher pole is dominated by the $\bar{K}N$ channel ($X_{\bar{K}N} \sim 1$) and the lower pole is by other missing channels ($X_{\bar{K}N} \sim 0$). We will discuss the compositeness for more detail in Sec. (3.2.2). With the wave functions and Eq. (3.53),

¹⁰We note that in Refs. [119, 198, 212] we calculated the expectation value with $\langle \hat{O} \rangle_G = \int d^3r \mathcal{O}(r) \left(1 - \frac{\partial V_{\bar{K}N}^{\text{equiv}}(r, E)}{\partial E} \right) \psi_G^2(r)$, following Ref. [203].

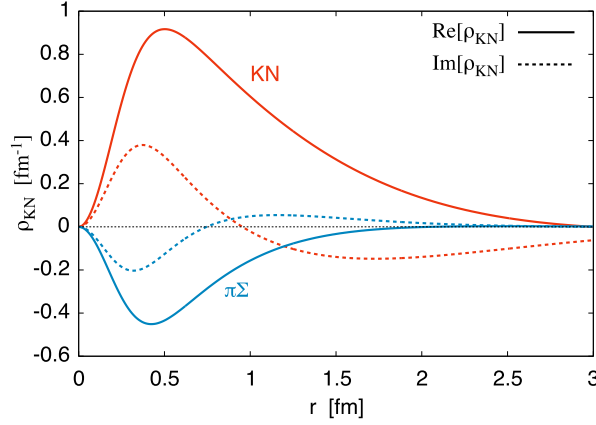


FIGURE 3.2: Density distributions of the $\bar{K}N$ component $\rho_{\bar{K}N}$ in $I = 0$. The results at the higher and lower poles of the $\Lambda(1405)$ (Eq. (3.49)) are respectively represented by the red and blue lines. The real (imaginary) parts are shown by the solid (dashed) lines.

	$\sqrt{\langle r^2 \rangle}_G$ at higher pole [fm]	$\sqrt{\langle r^2 \rangle}_G$ at lower pole [fm]
Kyoto $\bar{K}N$ potential	$1.04 - 0.61i$	$0.13 + 0.41i$
Ref. [12]	$1.22 - 0.63i$	—
Ref. [215] ¹¹	$1.22 - 0.47i$	$0.87 - 0.13i$
zero range limit (Eq. (3.56))	$0.84 - 0.61i$	$0.14 + 0.29i$
Kyoto $\bar{K}N$ potential (standard $\sqrt{\langle r^2 \rangle}$)	1.44	0.84

TABLE 3.1: Root-mean-squared distance at two $\Lambda(1405)$ poles. For comparison, we attach the previous results [12, 215].

we obtain the mean-square-distances $\langle r^2 \rangle_G$ (root-mean-squared distances $\sqrt{\langle r^2 \rangle}_G$) as $0.71 - 1.27i \text{ fm}^2$ ($1.04 - 0.61i \text{ fm}$) and $-0.15 + 0.11i \text{ fm}^2$ ($0.13 + 0.41i \text{ fm}$) for the higher and lower poles, respectively. In previous works, the distances are calculated from the $\Lambda(1405)$ form factor in chiral unitary approach [12, 213, 214] and by the complex scaling method with the coupled-channel potential model [215]. We compare our results with the previous ones in Table 3.1. It turns out that the tendency for the higher pole, that the real parts of $\sqrt{\langle r^2 \rangle}_G$ are dominant and near 1 fm, is similar. Though the results for the lower pole have not converged, we do not have to care too much considering that the position of the lower pole has not been well converged¹². Taking into account the previous works do not include the SIDDHARTA constraints, it can be said that for higher pole our quantitative results confirm the previous estimates.

From the definition of the expectation value in Eq. (3.53), we obtain a complex value as a physical quantity, which is generally difficult to interpret¹³. To extract some information from the complex $\sqrt{\langle r^2 \rangle}_G$, we first compare it with the spatial size estimated by assuming that ψ_G extends much larger than the potential range. When

¹¹We pick up the results with $f_\pi = 90 \text{ MeV}$.

¹²In Ref. [215], the normalization without $\partial V / \partial E$ term is employed.

¹³If a resonance has a tiny decay width and is approximately regarded as a bound state, the imaginary part of the expectation value would be also tiny and can be interpreted by its real part or the absolute value.

the potential range is negligibly smaller than the ψ_G extent, the wave function can be approximated by the asymptotic form,

$$\psi_G(r) \sim A \frac{e^{ikr}}{r}, \quad (3.55)$$

with the normalization constant A . We call this approximation the zero-range limit. Here we set the energy at the resonance, which leads to the pure outgoing wave. Using Eq. (3.47) for the expectation value and Eq. (3.44) for the normalization condition with an energy-dependent potential, the mean-squared distance can be approximately calculated by the compositeness X and the eigenenergy E as

$$\begin{aligned} \langle r^2 \rangle_G &= \frac{\int d^3r r^2 \psi_G(r)^2}{\int d^3r \left(1 - \frac{\partial V}{\partial E}\right) \psi_G(r)^2} \\ &= X \cdot \frac{\int d^3r r^2 \psi_G(r)^2}{\int d^3r \psi_G(r)^2} \\ &\sim X \cdot \frac{-ik}{2\pi} 4\pi \int_0^\infty dr r^2 e^{2ikr} \\ &= \frac{X}{2(-ik)^2} = \frac{X}{-4\mu E}. \end{aligned} \quad (3.56)$$

Taking the square root, the results for the two $\Lambda(1405)$ poles in Eq. (3.49) are $\sqrt{\langle r^2 \rangle_G} = 0.84 - 0.61i$ MeV for the higher one, and $\sqrt{\langle r^2 \rangle_G} = 0.14 + 0.29i$ for the lower one. Both values are close to the complex $\sqrt{\langle r^2 \rangle_G}$, which may mean that the spatial extent of the $\bar{K}N$ system is relatively larger than the potential range (~ 0.4 fm).

As another prescription, we extract a real value as the distance aiming at more straightforward interpretation [119, 198, 212]. Here, we respect that the asymptotic wave function e^{ikr} dumps with the factor $e^{-\text{Im}[k]r}$, that is, the factor $1/\text{Im}[k]$ represents the spatial size. In the zero range limit, when we calculate the expectation value without the Gamow vector at a resonance energy,

$$\langle r^2 \rangle = \frac{\int d^3r r^2 |\psi(r)|^2}{\int d^3r \left(1 - \frac{\partial V}{\partial E}\right) |\psi(r)|^2} \quad (3.57)$$

$$\begin{aligned} &\sim \frac{\int d^3r r^2 |\psi(r)|^2}{\int d^3r |\psi(r)|^2} \\ &\sim \frac{\text{Im}[k]}{2\pi} 4\pi \int_0^\infty dr r^2 e^{-2\text{Im}[k]r} = \frac{1}{2(\text{Im}[k])^2}, \end{aligned} \quad (3.58)$$

it corresponds to the spatial extent estimated by the dumping of the wave function. Expecting that $\langle r^2 \rangle$ reflects the spatial extent even for the finite range potential, we calculate the root-mean-squared distance with the exact wave function as in Eq. (3.57). The most favorable feature is that it gives an interpretable real value. The results are

$$\begin{aligned} \text{higher pole: } \sqrt{\langle r^2 \rangle} &= 1.44 \text{ fm}, \\ \text{lower pole: } \sqrt{\langle r^2 \rangle} &= 0.84 \text{ fm}. \end{aligned} \quad (3.59)$$

Taking into consideration the charge radii of the proton (~ 0.85 fm) and K^- (~ 0.55 fm) [216], it turns out that the $\bar{K}N$ distance is larger than an usual hadron size. We

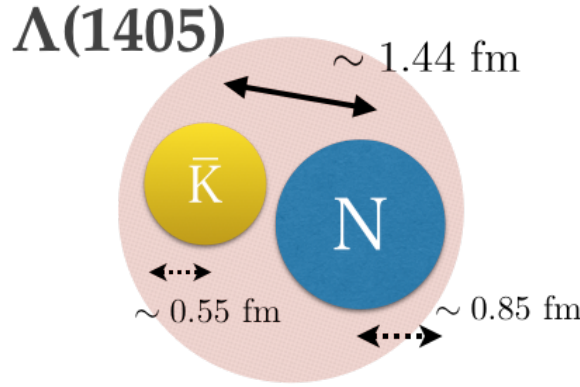


FIGURE 3.3: Picture of the $\Lambda(1405)$ considering our result $\sqrt{\langle r^2 \rangle} = 1.44$ fm and the rms radii of the proton (~ 0.85 fm) and the K^- (~ 0.55 fm).

b [fm]	$\sqrt{\langle r^2 \rangle}_G$ [fm]	$\sqrt{\langle r^2 \rangle}$ [fm]
0.2	$0.96 - 0.58i$	1.35
0.38	$1.06 - 0.57i$	1.44
0.4	$1.07 - 0.57i$	1.48
0.6	$1.18 - 0.57i$	1.57
0.8	$1.29 - 0.57i$	1.67

TABLE 3.2: Dependence of the $\bar{K}N$ distances at the higher pole, $\sqrt{\langle r^2 \rangle}_G$ with Eq. (3.53) and $\sqrt{\langle r^2 \rangle}$ with the standard definition, on the potential range $b_{\bar{K}N}$.

visualize the simple picture of the $\Lambda(1405)$ in Fig. 3.3, which supports the $\Lambda(1405)$ molecular picture consisting of $\bar{K}N$.

Before closing this section, we investigate the dependence of the distance against the range parameter of the potential, $b_{\bar{K}N}$. We vary $b_{\bar{K}N}$ by readjusting ΔV to reproduce the original amplitude $F_{\bar{K}N}^{\text{Ch}}$. The dependence of $\sqrt{\langle r^2 \rangle}_G$ and $\sqrt{\langle r^2 \rangle}$ is summarized in Table 3.2. The $b_{\bar{K}N}$ dependence is not so strong, and the qualitative nature that $\sqrt{\langle r^2 \rangle}$ is larger than an usual hadron size is preserved for all $b_{\bar{K}N}$. Thus, the molecular picture of the $\Lambda(1405)$ is valid regardless of the potential range.

3.2.2 $\bar{K}N$ - $\pi\Sigma$ coupled-channel system

Using the realistic $\bar{K}N$ - $\pi\Sigma$ potentials in $I = 0$, we analyze the spatial structure and the compositeness of the $\Lambda(1405)$ with the new normalization in Sec. 3.1. The wave function of each channel in s -wave,

$$\psi_i(r) = \frac{1}{\sqrt{4\pi}} \frac{u_i(r)}{r}, \quad (3.60)$$

is calculated at the higher or lower pole energy of the $\Lambda(1405)$.

We first consider how to normalize resonance states. The asymptotic behavior of the wave function is

$$\psi_{i,\alpha}(r) \xrightarrow{r \rightarrow \infty} \frac{1}{\sqrt{4\pi}} \frac{1}{r} \frac{1}{A} \left[e^{-ik_\alpha r} \delta_{i\alpha} - \left(\frac{\mu_i k_\alpha}{\mu_\alpha k_i} \right)^{1/2} S_{i\alpha} e^{+ik_i r} \right], \quad (3.61)$$

where the index α stands for the incident channel, and A is the normalization factor for the radial wave function u . $S_{i\alpha}$ denotes the S matrix in s wave, where α and i represent the channels in the initial and final states respectively. In the following, we only consider the energies at the resonances. The Riemann sheet of the complex energy plane determines the sign of the eigenmomentum, $k_i = \pm \sqrt{2\mu_i E}$. At such energies, $S_{i\alpha}$ diverges and the first term in Eq. (3.61) becomes negligible,

$$\psi_{i,\alpha}(r) \propto e^{+ik_i r} = e^{+i\text{Re}[k_i]r} e^{-\text{Im}[k_i]r}. \quad (3.62)$$

($r \rightarrow \infty$, at a resonance energy)

Therefore, after the normalization, $\psi_{i,\alpha}$ does not depend on the choice of the incident channel α . The more important point is that the qualitative behavior of ψ is determined from the Riemann sheet of the complex energy plane in channel i where the resonance lies. From Eq. (3.62), it turns out that the wave function converges to zero for the first sheet ($\text{Im}[k_i] > 0$), but goes to infinity for the second sheet ($\text{Im}[k_i] < 0$), in the $r \rightarrow \infty$ limit. In Fig. 3.4, we show the $\Lambda(1405)$ pole positions of each channel in the complex energy and momentum planes. From the right figure, we find that both the resonance poles lie on the first sheet for the $\bar{K}N$ channel, but on the second sheet for the $\pi\Sigma$ channel, which means

$$\begin{aligned} \psi_{\bar{K}N,\alpha} &\xrightarrow{r \rightarrow \infty} 0, \\ \psi_{\pi\Sigma,\alpha} &\xrightarrow{r \rightarrow \infty} \infty, \end{aligned} \quad (3.63)$$

irrespective of the incident channel α . In the $\bar{K}N$ single-channel case as in Sec. 3.2.1, because the wave function converges to zero at $r \rightarrow \infty$, we can easily normalize it. On the other hand, in this coupled-channel case, because of the divergence of $\psi_{\pi\Sigma}$, we cannot simply normalize the wave function.

A well known procedure to treat such a divergent wave function for a resonance is the complex scaling method [169, 170], which is often applied to many-body systems [209, 217] as in atomic and molecular physics [218, 219]. In this complex scaling method, a relative coordinate r is transformed as

$$r \rightarrow r e^{i\theta}. \quad (3.64)$$

¹⁴The pole positions are the ones in chiral unitary approach. From the equivalent potentials, the positions are slightly deviated, but the qualitative statement is not changed.

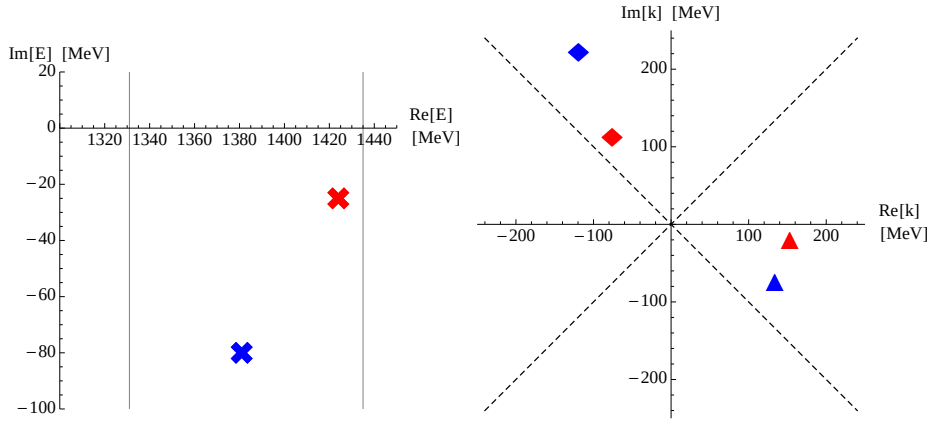


FIGURE 3.4: Resonance positions in the E (left panel) and k (right panel) planes¹⁴. Red and blue symbols represent the higher and lower pole, respectively. In the right panel, the rhombus (triangle) symbol represents the resonance momentum in the $\bar{K}N$ ($\pi\Sigma$) channel, $k_i = \pm\sqrt{2\mu_i\{\sqrt{s} - (M_i + m_i)\}}$, where the sign in k_i determine the Riemann sheet.

Accordingly, the wave function (3.61) is also transformed as

$$\psi_{i,\alpha}^\theta(r) \equiv e^{i\frac{3\theta}{2}} \psi_{i,\alpha}(re^{i\theta})$$

$$\xrightarrow{r \rightarrow \infty} e^{i\frac{3\theta}{2}} \frac{1}{\sqrt{4\pi}} \frac{1}{re^{i\theta}} \frac{1}{A} \left[e^{-ik_\alpha re^{i\theta}} \delta_{i\alpha} - \left(\frac{\mu_i k_\alpha}{\mu_\alpha k_i} \right)^{1/2} S_{i\alpha} e^{+ik_i re^{i\theta}} \right]. \quad (3.65)$$

At a resonance energy, $k_i = k_R \equiv |k_R|e^{-i\theta_R}$ ¹⁵,

$$\begin{aligned} \psi_i^\theta(r) &\propto e^{+ik_R re^{i\theta}} \\ &= e^{+i|k_R|r e^{i(\theta-\theta_R)}} = e^{i|k_R|r \cos(\theta-\theta_R)} e^{-|k_R|r \sin(\theta-\theta_R)}. \end{aligned} \quad (3.66)$$

Therefore, if $\theta > \theta_R$, the wave function can be regularized. Here, we have to note a requirement for θ to keep a potential analytic. A Gaussian potential used in this work is changed under the complex scaling as

$$V(re^{i\theta}) \propto e^{-(re^{i\theta})^2/b^2} = e^{-ir^2/b^2 \sin 2\theta} e^{-r^2/b^2 \cos 2\theta}. \quad (3.67)$$

To prevent the divergence of the potential, we should restrict the scaling angle to $\theta < \pi/4$ for a Gaussian potential. Hence, we should choose an angle among $\theta_R < \theta < \pi/4$. With this method, though the wave function is changed, the resonance energy, the divergent point of $S_{i,\alpha} \rightarrow \infty$, is not changed as seen from Eq. (3.66). Furthermore, with the Gamow vector [206–208],

$$\psi_i^G(r) = \psi_i^*(r), \quad (3.68)$$

¹⁵As shown in the so-called ABC theorem [169, 170], the resonance energy does not depend on the scaling angle θ .

which is used to treat a non-Hermitian system like a resonant state (see Sec. 3.1.1), an expectation value of an operator $\hat{\mathcal{O}}$ is independent of θ [220],

$$\begin{aligned}\langle \hat{\mathcal{O}} \rangle &\equiv \frac{\sum_i \int d^3r \mathcal{O}(\mathbf{r}) \{\psi_i^G(\mathbf{r})\}^* \psi_i(\mathbf{r})}{\sum_i \int d^3r \{\psi_i^G(\mathbf{r})\}^* \psi_i(\mathbf{r})} = \frac{\sum_i \int d^3r \mathcal{O}(\mathbf{r}) \psi_i^2(\mathbf{r})}{\sum_i \int d^3r \psi_i^2(\mathbf{r})} \\ &= \frac{\sum_i \int d^3r \mathcal{O}(re^{i\theta}) \{\psi_i^\theta(\mathbf{r})\}^2}{\sum_i \int d^3r \{\psi_i^\theta(\mathbf{r})\}^2} = \langle \hat{\mathcal{O}}^\theta \rangle^\theta.\end{aligned}\quad (3.69)$$

for a system with an E -independent potential.

On the other hand, for an E -dependent potential as in our case, we should modify the normalization scheme and the expectation value. In Secs. 3.1.1 and 3.1.2, we showed that the i -channel component of an expectation value can be calculated as in Eq. (3.47) with the new normalization in Eq. (3.17). Applying the scheme to the complex scaling method, we can calculate the expectation value of a resonant state as

$$\langle \hat{\mathcal{O}}^\theta \rangle_i^\theta = \frac{\int d^3r \mathcal{O}(re^{i\theta}) \psi_{E,i}(re^{i\theta}) \psi_{E,i}(re^{i\theta})}{\sum_{i,j} \int d^3r \psi_{E,i}(re^{i\theta}) \left[\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(re^{i\theta}, E) \right] \psi_{E,j}(re^{i\theta})}. \quad (3.70)$$

As discussed in Sec. 3.1.2, the component of missing channels, which are not explicitly treated in a model, would not be able to be generally calculated. In contrast, regarding the norm, we can calculate the one of the missing channel called the elementariness Z as well as the ones of the explicitly treated channels called the compositeness X_i as

$$X_i = \frac{\int d^3r \psi_{E,i}^2(re^{i\theta})}{\sum_{i,j} \int d^3r \psi_{E,i}(re^{i\theta}) \left[\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(re^{i\theta}, E) \right] \psi_{E,j}(re^{i\theta})}, \quad (3.71)$$

$$Z = \frac{\sum_{i,j} \int d^3r \psi_{E,i}(re^{i\theta}) \left(-\frac{\partial V_{ij}^{\text{eff}}}{\partial E}(re^{i\theta}, E) \right) \psi_{E,j}(re^{i\theta})}{\sum_{i,j} \int d^3r \psi_{E,i}(re^{i\theta}) \left[\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(re^{i\theta}, E) \right] \psi_{E,j}(re^{i\theta})}. \quad (3.72)$$

For the two poles of the $\Lambda(1405)$, we calculate the above quantities with the realistic coupled-channel potentials (Gaussian type) in Sec. 2.4. The results of X_i and Z are summarized in Table 3.3 and the results of $\sqrt{\langle r^2 \rangle}_i$ are in Table 3.4, respectively. For comparison, we attach the results from the $\bar{K}N$ single-channel potential in Sec. 2.2. It turns out that all quantities are quite similar against the different potentials, qualitatively and even quantitatively. In particular, it would be worthwhile to compare the single and coupled-channel potentials. As seen in Tables 3.3 and 3.4, $X_{\bar{K}N}$ and $\sqrt{\langle r^2 \rangle}_{\bar{K}N}$ in the single-channel case are quite close to the ones in the coupled-channel case. If the single-channel potential is obtained from the coupled-channel one by the exact Feshbach projection, these quantities should be equal from the discussion in Sec. 3.1.2. Considering that both the potentials are constructed to reproduce the same amplitudes based on the same interaction in chiral unitary approach, the single channel potential would be approximately regarded as the one obtained by effectively eliminating the $\pi\Sigma$ channel from the coupled-channel potential with the Feshbach projection. The tiny deviation would be caused by the loss of

¹⁶In Ref. [194], the $\eta\Lambda$ and $K\Xi$ channels are explicitly treated, and the meaning of Z is different from ours. What we show in this table is $Z + X_{\eta\Lambda} + X_{K\Xi}$ in the previous work.

Pole	Model	Polynomial type	$X_{\pi\Sigma}$	$X_{\bar{K}N}$	Z
higher	$\bar{K}N$ - $\pi\Sigma$	first order	$-0.04 - 0.23i$	$0.95 - 0.14i$	$0.08 + 0.37i$
		second order	$-0.02 - 0.25i$	$1.01 - 0.13i$	$0.01 + 0.37i$
	$\bar{K}N$ single	tenth order	—	$1.01 - 0.07i$	—
	Ref. [194]	—	$-0.19 - 0.22i$	$1.14 + 0.01i$	$0.05 + 0.21i$
lower	$\bar{K}N$ - $\pi\Sigma$	first order	$0.30 + 0.86i$	$-0.21 + 0.03i$	$0.91 - 0.89i$
		second order	$0.19 + 0.96i$	$-0.30 + 0.08i$	$1.11 - 1.04i$
	$\bar{K}N$ single	tenth order	—	$-0.33 - 0.03i$	—
	Ref. [194]	—	$0.66 + 0.52i$	$-0.39 - 0.07i$	$0.73 - 0.45i$

TABLE 3.3: Compositeness X_i and elementariness Z for each pole of the $\Lambda(1405)$ from the $\bar{K}N$ - $\pi\Sigma$ potential in the Gaussian type in Sec. 2.4. Both polynomials in the energy (the first and second order ones) are shown. For comparison, we attach the results from the $\bar{K}N$ single-channel potential in Sec. 2.2 and the ones in the previous work [194] where X_i and Z ¹⁶ are directly calculated from F^{Ch} in the IHW model.

Pole	Model	Polynomial type	$\sqrt{\langle r^2 \rangle_{\pi\Sigma}}$ [fm]	$\sqrt{\langle r^2 \rangle_{\bar{K}N}}$ [fm]	$\sqrt{\langle r^2 \rangle_{\pi\Sigma + \bar{K}N}}$ [fm]
higher	$\bar{K}N$ - $\pi\Sigma$	first order	$0.45 + 0.17i$	$1.03 - 0.61i$	$1.07 - 0.52i$
		second order	$0.45 + 0.20i$	$1.05 - 0.62i$	$1.07 - 0.52i$
	$\bar{K}N$ single	tenth order	—	$1.04 - 0.61i$	—
lower	$\bar{K}N$ - $\pi\Sigma$	first order	$0.39 - 0.56i$	$0.11 + 0.36i$	$0.29 - 0.60i$
		second order	$0.42 - 0.57i$	$0.17 + 0.42i$	$0.28 - 0.61i$
	$\bar{K}N$ single	tenth order	—	$0.13 + 0.41i$	—

TABLE 3.4: Each channel component of the root-mean-squared distance $\sqrt{\langle r^2 \rangle_i}$ calculated with Eq. (3.70). The Gaussian type $\bar{K}N$ - $\pi\Sigma$ potentials in Sec. 2.4 with the first and second order polynomials are employed. For comparison, the results of the $\bar{K}N$ single-channel potential in Sec. 2.2 are shown.

the rigorousness of the Feshbach projection due to the local approximation of the intrinsically non-local projected potential, while the non-local effect is effectively taken into account in the strength of the single-channel potential.

On the other hand, regarding the missing channels like the $\pi\Sigma$ channel, we pointed out in Sec. 3.1.2 that the expectation value cannot be calculated with the naively expected way as $\sum_{i,j} \int d^3r \mathcal{O}(r) \psi_{E,i}(r) [-\frac{\partial V_{ij}}{\partial E}(r, E)] \psi_{E,j}(r)$, which is deduced from the norm calculation in Eq. (3.42). It also means that the total expectation value including the contribution of all channels cannot be obtained in the similar way, $\sum_{i,j} \int d^3r \mathcal{O}(r) \psi_{E,i}(r) [\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(r, E)] \psi_{E,j}(r)$, which is suggested in Ref. [203]. To confirm it, we calculate the "total" root-mean-squared distance of the $\Lambda(1405)$ in the naive way, which is named as $\sqrt{\langle r^2 \rangle_{\text{naive}}} = \sum_{i,j} \int d^3r \mathcal{O}(r) \psi_{E,i}(r) [\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(r, E)] \psi_{E,j}(r)$, and summarize the results in Table 3.5. Because the higher pole is dominated by the $\bar{K}N$ channel as shown in Table 3.3, $\sqrt{\langle r^2 \rangle_{\text{naive}}}$ in single and coupled channels naturally become similar. Contrarily, the lower pole is dominated by other channels, and it would be a good benchmark. The result indicates that $\sqrt{\langle r^2 \rangle_{\text{naive}}}$ is quite sensitive to the elimination of the channels. This convinces us the discussion in Sec. 3.1.2 that the naive expectation value is invalid.

Pole	Model	Polynomial type	$\sqrt{\langle r^2 \rangle}_{\text{naive}}$ [fm]
higher	$\bar{K}N-\pi\Sigma$	first order	$1.13 - 0.64i$
		second order	$1.15 - 0.65i$
	$\bar{K}N$ single	tenth order	$1.04 - 0.61i$
lower	$\bar{K}N-\pi\Sigma$	first order	$0.87 - 0.26i$
		second order	$0.87 - 0.24i$
	$\bar{K}N$ single	tenth order	$0.22 + 0.21i$

TABLE 3.5: Naive "total" root-mean-squared distance $\sqrt{\langle r^2 \rangle}_{\text{naive}}$. The naive mean-squared distance is calculated as $\sum_{i,j} \int d^3r r^2 \psi_{E,i}(r) [\delta_{ij} - \frac{\partial V_{ij}}{\partial E}(r, E)] \psi_{E,j}(r)$. The results of the Gaussian type $\bar{K}N-\pi\Sigma$ potentials in Sec. 2.4 with the first and second order polynomials and of the $\bar{K}N$ single-channel potential in Sec. 2.2 are shown.

Generally, the physical interpretation of the quantities in the non-Hermitian system is difficult, because they are complex values due to the unstable nature. However, in some situation, they can be somehow interpreted [194, 221]. For example, $X_i(Z)$ would represent dominant contribution of channel i (missing channels) for $X_i(Z) \sim 1 + 0i$, and negligible contribution for $X_i(Z) \sim 0 + 0i$. Furthermore, when the decay width of a resonance is tiny, it would reveal a similar properties to a bound state, which leads to interpretable real values for X_i and Z satisfying $0 \leq X_i, Z \leq 1$. In fact, for the higher pole with a relatively smaller decay width, the imaginary parts of the above quantities are relatively small, and $X_{\bar{K}N} \sim 1 + 0i$ and $X_{\pi\Sigma}, Z \sim 0 + 0i$. Thus, we can conclude that the higher pole is dominated by the $\bar{K}N$ channel. On the other hand, in the lower pole, the imaginary parts are larger, which obstructs the physical interpretation. However, $X_{\bar{K}N} \sim 0 + 0i$ would represent the smaller contribution of the $\bar{K}N$ channel to the lower pole. Concerning the root-mean-squared distance, we can interpret the $\bar{K}N$ distance in the higher pole is relatively larger around 1 fm from the dominant real part.

In general, the compositeness and the elementariness are considered to be model dependent [211], while they are determined model-independently only in the weak binding limit [210]. In our approach, the ambiguity would come from the local approximation of the potential and the choice of the Gaussian spatial distribution. As the previous work, in Ref. [194], X_i and Z in the IHW model are calculated with a different scheme where X_i and Z are derived from the two-body loop function and the coupling constant of each channel to the poles of the $\Lambda(1405)$. In the previous approach, the ambiguity would be caused by the assumption of the separable interaction, $\langle q'_j | \hat{V} | q_k \rangle = v_{jk} f_j(q'^2) f_k(q^2)$, and the choice of the regularization scheme for the loop function, the dimensional regularization. Hence, it is worth comparing our results with the previous ones. We show the X_i and Z in Ref. [194] in Table 3.3. We find that the results of the two approaches surprisingly coincide with each other. Regarding the higher pole, the consistency may be caused by the smaller model dependence in the weaker binding energy. We can say that our results confirm the $\bar{K}N$ dominance of the higher pole indicated in the previous work with the different scheme. Concerning the lower pole, while there appears relatively large difference related to the stronger model dependence with the larger binding energy, the less contribution of the $\bar{K}N$ channel to the lower pole is confirmed in both the models.

Chapter 4

Summary

In this thesis, we have constructed quantitatively reliable $\bar{K}N$ single-channel and $\bar{K}N$ - $\pi\Sigma$ coupled-channel local potentials, and analyzed the $\Lambda(1405)$ and $\bar{K}$ nuclei. The reliability in the $\Lambda(1405)$ energy region below the $\bar{K}N$ threshold, which cannot be directly accessed by experiments, is guaranteed by chiral unitary approach whose uncertainty is significantly reduced with the precise SIDDHARTA data. These local potentials enable the quantitative discussion of the $\Lambda(1405)$ and $\bar{K}$ nuclei systems which are the subjects of current and future experiments.

In Chapter 2, we have established the way to construct the general coupled-channel local potential used in the Schrödinger equation, after the brief review of chiral unitary approach. With this method, we have constructed the $\bar{K}N$ single-channel and $\bar{K}N$ - $\pi\Sigma$ coupled-channel potentials. The main strategy is to reproduce the results in chiral unitary approach by the equivalent local potential. The problem lies in connecting originally different frameworks, chiral unitary approach and quantum mechanics. We utilize the Born approximation and the matching condition at each threshold, based on the Fourier transform of the interaction kernel in chiral unitary approach, that is, the tree-level meson-baryon amplitude. Here, in the similar sense that the loop function in chiral unitary approach is regularized, we smear a delta-type potential to the Gaussian one. We first applied this prescription to the $\bar{K}N$ single channel system. Here, the original scattering amplitudes from chiral unitary approach was roughly reproduced on the real energy axis. Next, we constructed the $\bar{K}N$ - $\pi\Sigma$ coupled-channel potential, considering that the $\pi\Sigma$ channel is expected to largely contribute to the $\Lambda(1405)$. The results almost quantitatively reproduced the original amplitudes on the real energy axis, which would indicate the importance of the $\pi\Sigma$ channel. The validity of our prescription would be supported by the success of the coupled-channel potential in reproducing the original amplitude. Moreover, when we examine the same prescription with the coupled-channel Yukawa-type potential assuming implicit one-boson exchange picture, it turns out that the exchanged boson mass is around 500 MeV which is consistent with the mass of the σ meson (two-pion exchange).

However, when we analytically continue the amplitudes to the complex energy plane, we find that the reproducibility becomes worse, and for instance, the pole positions are relatively deviated from the original ones even in the coupled-channel system. Such deviation would be caused by the difference of the framework of the quantum mechanics and chiral unitary approach. Considering that the pole information is essential for a resonant state, we adjusted the strength of the potentials to reproduce the original amplitudes even in the complex energy plane. The strategy is based on the uniqueness of the analytic continuation, that is, the amplitude in the complex energy plane would be reproduced if that on the real energy axis were perfectly reproduced on the whole real energy axis. Hence, we elaborate the strength of the potential in the energy range as wide as possible. After the adjustment of the

strength, we parametrize the potential strength by a polynomial in the energy. We should note that the boundaries of the real energy range for the adjustment, named "parametrized range", is limited from the choice of the polynomial. Besides, because we effectively eliminated the much higher channels than the $\Lambda(1405)$ energy region like the $\eta\Lambda$, the non-analytic threshold effect of such channels also limits the boundaries. With the above strategy, we have constructed the realistic potentials, and found that both the single and coupled-channel potentials can reproduce the original amplitude even in the complex energy plane including the positions of the two $\Lambda(1405)$ poles. Though the single-channel potential is sufficiently realistic, it needed the unnaturally high order polynomial (tenth order polynomial) with the large parametrized range to broaden the precise region, where the original amplitude is well reproduced within 20 % deviation, to near the lower resonance pole as well as the higher pole. This problem is caused by the elimination of the $\pi\Sigma$ channel, and has been resolved by the coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential. By explicitly treating the $\pi\Sigma$ channel, the energy dependence of the potential can be parametrized by the second order polynomial for the precise region to perfectly cover the $\Lambda(1405)$ energy region.

In Sec. 2.3, we have applied the single-channel potential to some systems. First application is the study of few-body $\bar{K}$ nuclei. For quantitative discussion, the accurate few-body calculation method, the stochastic variational method with correlated Gaussian basis, is employed. To determine the complex eigenenergy in the variational method, we established the strategy: the basis functions are determined only with the real part of the Hamiltonian to minimize the real eigenenergy, and the full complex Hamiltonian is diagonalized with the basis functions, which results in the complex eigenenergy corresponding to the $\Lambda(1405)$. Here, a problem arises from the energy dependence of the $\bar{K}N$ potential, because the two-body energy in many body systems cannot be uniquely determined. In our work, we adopted two prescriptions developed in previous works to connect the two-body energy with calculable quantities, and imposed the self-consistency. As a result, we obtained several quasibound states in the three to seven-body systems. For instance, in the $\bar{K}NN$ system, relatively small binding energy (20-30 MeV) was obtained, which is consistent with the recent experimental data, while the decay width is sensitive to the choice of the two-body energy. In the seven-body system, there is the possibility that the level ordering of the original ground state ($J^P = 1^+$) and excited state ($J^P = 0^+$) in the six-nucleon system is inverted for the strong $\bar{K}N$ attraction, depending on the strength of the $\bar{K}N$ potential. This means that the information of the quantum number of the ground state would lead to the constraint to the $\bar{K}N$ interaction. We note that the explicit treatment of the lower channels is necessary for further discussion, considering the large binding energy of the seven-body system around 70-80 MeV.

As another application of the $\bar{K}N$ potential, we have calculated the K^-p correlation function in heavy-ion collisions. The correlation function reflects both $I = 0$ and $I = 1$ $\bar{K}N$ interactions due to the channel-coupling of K^-p and $\bar{K}^0n$. An interesting point is the different isospin combination from the ordinary K^-p scattering, which is caused by the characteristic outgoing boundary condition with the channel coupling. In our theoretical prediction, the difference appears as a characteristic dip and bump structure which is not seen in the ordinary scattering data. We find that this qualitative behavior is insensitive to the source size. To directly compare with future experiments, we also calculated the K^-p correlation with the Coulomb interaction in K^-p channel.

In Chapter 3, we have built up the new normalization scheme for the energy-dependent potential, as in the $\bar{K}N$ and $\bar{K}N-\pi\Sigma$ systems, with the complex eigenenergy. Furthermore, we have revealed the connection of the new normalization scheme with the compositeness and elementariness. What we respect to consider in the normalization scheme are the norm conservation and the orthogonality relation, which are the essential requirements in quantum mechanics. These requirements can be guaranteed by the continuity equation. However, in the system with an energy-dependent potential, it is known that the naive continuity equation does not hold. Extending the previous work for the energy-dependent but Hermitian potential by introducing the Gamow vector, we developed the modified continuity equation for the system with the energy-dependent and non-Hermitian potential with a complex eigenenergy. This leads to the new normalization scheme with the additional term of the energy derivative of the potential. To clarify the meaning of this term, we have reconsidered the new scheme, based on the assumption that the energy dependence comes from the elimination of channels which are not explicitly treated in the model space. Introducing the Feshbach projection, we have found that the new term with the energy derivative of the potential is just the norm of the missing channels, namely, the elementariness. In the same way, it turns out that the expectation value of an operator by an explicitly treated channel can be calculated, but that of a missing channel cannot be directly obtained by the effective interaction in the reduced model space.

In Sec. 3.2, applying the above formulation and the interpretation, we have calculated the compositeness and the root-mean-squared distance of the $\Lambda(1405)$. It is worth comparing our results with the previous work in another approach, because the compositeness is model dependent in general. We have confirmed that both the results well coincide with each other. Though the quantities are complex values which are difficult to straightforwardly interpret, we have found the tendency that the higher pole is dominated by the $\bar{K}N$ channel and the $\bar{K}N$ distance would be around 1 fm. On the other hand, we find that the $\bar{K}N$ contribution to the lower pole is small, which is consistent with the $\pi\Sigma$ origin of this state.

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