

**A STUDY ON SYSTEMS APPROACH TO SOME INDUSTRIAL
MANAGEMENT PROBLEMS**

JUNJI NOMURA

OCTOBER, 1987



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ACKNOWLEDGEMENTS

The author owes a lasting debt of his gratitude to Dr. Yoshikazu Nishikawa, Professor of Kyoto University, for his constant guidance and encouragement in promoting this work. Especially, the author has been given precise and helpful advice by Professor Nishikawa in completing the present dissertation. Acknowledgment must also be made to Dr. Yoshikazu Sawaragi, Professor Emeritus of Kyoto University, and Mr. Kaoru Kobayashi, Corporate Technology Counselor of Matsushita Electric Works, Ltd., for their stimulating and useful discussions of all kinds forwarded to the author. The study of Chapter 4 was motivated by and discussed based on the stuff of the System Analysis Group of Matsushita Electric Works, Ltd. The author is also willing to express many thanks to them. The author is likewise grateful to the members of Professor Nishikawa's Laboratory for their excellent cooperation.

GENERAL SUMMARY

The text consists of four chapters. Analytical methods and theory of systems approach are described in the first three chapters. The chapter that follows is concerned with the application to some industrial management problems. Complementary remarks are provided in three appendixes to the text.

Chapter 1 is concerned with the concepts of systems approach. Systems approach is an approach based upon systems concepts and it is a key step to solve the issues with complexity. The process of systems approach and the role of systems approach to solve the problems in a firm are described.

Chapter 2 describes structural modeling methods for modeling analysis. Structural modeling methods aim to facilitate communicability of people with different backgrounds and then to make a common recognition of the fundamental structure of the subject issue. ISM (Interpretive Structural Modeling) method and DEMATEL (Decision Making Trial and Evaluation Laboratory) method are typical structural modeling method [8, 16, 51, 52]. But ISM considers only existence or not, corresponding to one or zero, of the pairwise relation. DEMATEL, on the other hand, does take care of a strength of the relation, but it does not generate a clear hierarchical structure. This chapter further extends the concept and the procedure of ISM to cope with an issue in which the strength of relation among system elements is quantitatively defined or estimated. The proposed method is called IWSM (Interpretive Weighted Structural Modeling) [33]. The mathematical procedure of IWSM and the comparison of IWSM, DEMATEL and ISM through a simple example are described.

Chapter 3 deals with decision analysis with multiobjectives. There are basically two approaches to the solution of problems with multiple objectives. One is to find the preferred solution directly, the other is to generate the non-inferior set and then find the preferred solution from among these. The first type of direct approach is the utility function approach [10, 17, 18, 19, 20, 43]. The typical one of the second type is the surrogate worth tradeoff method [12, 13]. The procedure of SWT method and MAUF method are mentioned in this chapter and a modified decision procedure is proposed.

Chapter 4 is concerned with the application to some industrial management problems. To real five problems in a private sector, systems approach technique is applied [31, 33, 34, 35, 36]. In Section 4.2, multiobjective optimization procedure is applied to real blending problem of an industrial material with six objectives. The idea of surrogate worth tradeoff method and the method of multiattribute utility function are applied to the problem. The solution of SWT method is compared with that of MAUF method. In Section 4.3 and 4.4, real applications of IWSM are described. The first application treats a scheduling of a data transmission test and the second one is concerned with an allocation of budget to various section in a Research and Development Laboratory. In Section 4.5, multiobjective optimization procedure is applied to a decision problem in an inventory management. A multiattribute utility function is constructed for totally evaluating the degree of satisfaction of objectives, and it is used, together with a nonlinear programming technique, for optimizing the decision variables. In Section 4.6, multiobjective assessment procedure is applied to the assessment of investment plans in a production firm. The total assessment formula is derived considering both the economical and the technical terms by use of a multiattribute utility function.

These real problems have successfully been dealt with systems approach technique. Several comments are given in the methodologies and our experiences stressing the practical feasibility and effectiveness view points.

As has been mentioned earlier, three appendixes are annexed to the text. Appendix I describes the multiplier method for nonlinear programming. Appendix II shows the Winters exponential smoothing method. Finally, Appendix III explains demand prediction by GMDH and IWSM algorithm.

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CHAPTER 1

A SYSTEMS APPROACH

1.1 Introduction

There are many theories, methods or axiomatics concerned with the identification simulation, optimization as well as control of a system. With the emergence of computer development, it has become easy to apply these methodologies to the problems of the real world, and the usefulness and availability of such has already been reported in several papers. These theories and methodologies for the analysis and solution of problems are almost assured of a place in any engineering system. On the other hand it is true that a variety of critical problems of considerable complexity have emerged. These complex problems include such problems as environmental pollution, energy capacities and food shortage all of which require analysis and solutions at either national or international level. The conventional approach to problem solving will not apply to these issues in a living, behavioral or social system, an interdisciplinary approach is necessarily demanded. The concepts and methodologies for dealing rationally with complex issues fall under what is called either a systems approach, systematic approach or systemic approach. A systems approach is a subclassification of the systems theory. There are many systems theories [3] [4] [5], and many systems approaches are defined respectively. They differ from one another, but no matter what they may be, each is defined. In general it may be said that any systems approach depends upon the concepts of each system. In this paper we shall use as our departure point based on the fact that systems approaches are already defined. The process of systematic approach is discussed in Section 1.2. Although this approach does not directly relate to the systems theory, it is useful in analyzing and solving complex problems. Numerous studies have been done on the matter of the use of a systematic approach in the public sector. Problems endemic to a production firm are not those found in the public sector, but it is nevertheless important to be able to apply a systematic approach even to the problems of the private sector.

1.2 The Process of the Systems Approach

The process of a systems approach (or systematic approach) is depicted in Figure 1.1.

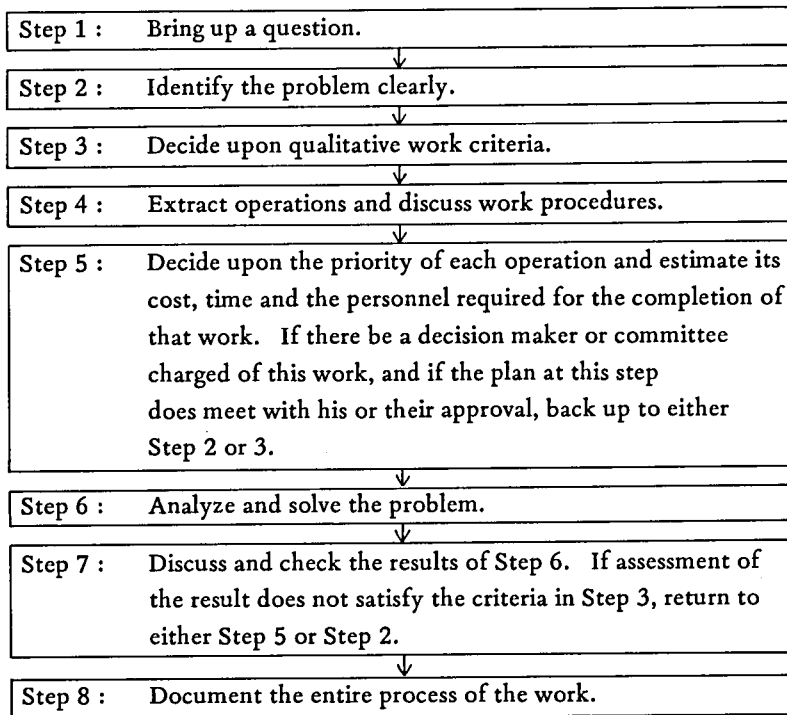


Figure 1.1. The Scheme of the Systems Approach.

Brain storming, the KJ method - a kind of group dynamic decision making process - , structural modeling methods, multi-dimensional scaling methods and so on are usually applied from Step 1 to Step 5. Mathematical programming, multi-objective optimization techniques, scenario writing or the cross-impact method and so on are applied in Steps 6 and 7. Among all these methods and techniques the most often applied are structural modeling methods and multiobjective optimization techniques. They are discussed in Chapters 2 and 3.

1.3 Applying the Systems Approach to the Problems of the Private Sector

The basic model of the systems approach most often applied to social systems is a hierarchical open system. This system is shown in Figure 1.2 [28].

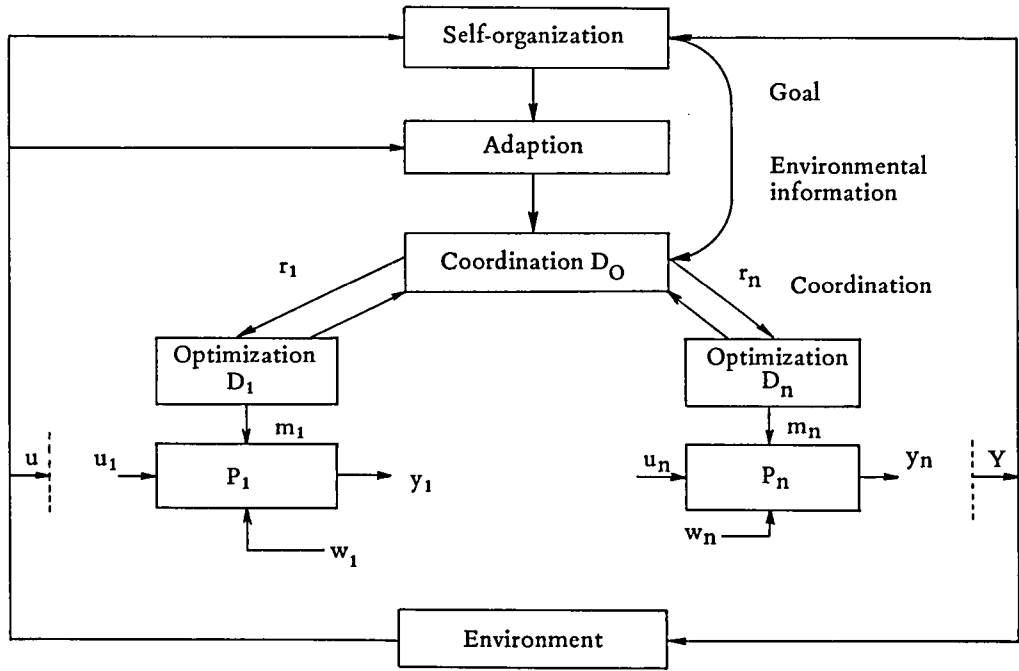


Figure 1.2. Hierarchical open system [28].

This hierarchical open system is applicable to private sector systems. If the system in Figure 1.2 is presumed to be a private sector system, the following symbol systems will be applied. The self-organization subsystem that regulates the structure of the total system and supervises the performance of the system may be presumed to be the president and chairman of the board. The coordinator D_O who coordinates interaction among subprocesses, $P_1, P_2, \dots, P_n$, is presumed to be a vice-president or managing director. D_i is the decision maker of a subprocess P_i , and D_i is presumed to be the head of the operational division or the head of a research and development laboratory or head of a finance department or the like. P_i is the subprocess which is presumed to be the work of a division, department or laboratory. m_i is input decision maker D_i can manipulate assuming m_i is a budget. y_i is output. u_i is input decision maker D_i cannot manipulate and is presumed to be sales amount. w_i is the interaction among subprocesses and w_i represents instances of conflict or cooperation among divisions. An adaption for communicating information concerning the environment to coordinator D_O is

presumed to be an information division.

The theory of hierarchical open systems provides many ramifications for solving the problems of the private sector, but the fact remains that there are many problems that cannot be solved simply by the application of the theory of a hierarchical open system. The problems of the private sector may often be characterized something which possess a hierarchy or that of a multi-layered nature. There are many decision makers at every level and they apply many criteria to any problem. Using these criteria they will often decide things intuitively. The optimization procedures in each subsystem, $P_1, P_2, \dots, P_n$, are not often formulated and neither are the criteria that decision makers D_i apply very clearly defined. When attempts are made to apply the systems theory to the problems of the private sector, the primary difficulty that must be recognized at the outset is that coordination and optimization procedures and so on are not formally defined. Even if the criteria are clearly defined, there is nevertheless the problem of Arrow's paradox [1].

If total optimization of system in firm is effective, the systems approach should be carried out step by step and its usefulness and availability should be reported according to the amounts of money that it saves to the coordinator D_O or decision maker in the self-organization subsystem. The role of a systems approach is depicted in Figure 1.3. First, the coordinator D_O analyzes the present situation according to his principles, sets up a workable strategy and lays down the best scenario. Second, the coordinator discusses the scenario intensively with his staff to see if the prospective product will be unique and optimal. Third, he applies the idea to every stage of the corporate activities such as planning, designing, manufacturing, selling, and so forth. The systems approach serves as a "catalyst" in this scheme. The commitment of the coordinator in optimizing the system and the enthusiasm of the staffs react to each other catalytically in the systems approach. Without the commitment of the coordinator and the enthusiasm of the staffs, no reaction occurs and systems approach is useless. The most important thing for system analysts and systems engineers to keep in mind in the systems approach is that it is only a "catalyst". Without that "catalyst", however or without the carrying out of much delineation of the systems approach to

problems, total optimization of the system is impossible.

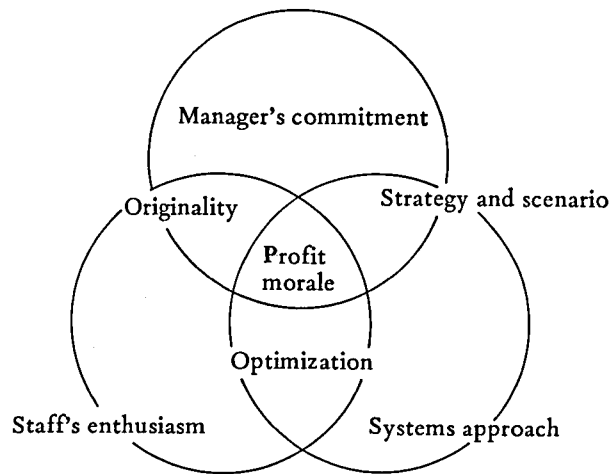


Figure 1.3. An active role of systems approach in a firm.

CHAPTER 2

STRUCTURAL MODELING METHODS

2.1 Introduction

Model analysis is an important step in the systems approach. Technical models are usually constructed by using the Newtonian theory of Physics, Euclidean Geometry or the like and the procedure in the structural model depends upon a specialist. Such process of technical modeling is called dynamic modeling and it is useful when the problem to be solved within a technical field, but quite useless if the problem is compounded by other issues among which are included energy reserves, environmental pollution, risk assessment and so on. No complex issue can be solved by just one expert. Modeling analysis of complex problems must be performed by specialists in various fields together with the persons directly involved. Important points to be considered when performing model analysis include the followings:

- (1) Modeling analysis can be performed by the persons directly concerned, despite the fact they may possess no knowledge of mathematics or computers.
- (2) Opinions and ideas of the participants may be accepted regardless of their positions, status or interests.
- (3) Participants may discuss freely with each other and the results of those discussions may be reported either quantitatively or qualitatively.

Converting a completely intuitive process of model analysis into the more systematic approach after the points mentioned above is called structural modeling. This process is illustrated in Figure 2.1 [27].

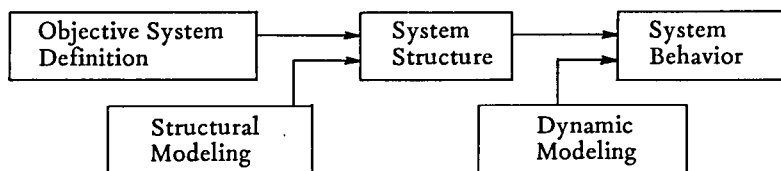


Figure 2.1 The Process of Systematic Modeling.

Recently structural modeling techniques have been emerging as useful tools for complex issues. Among the several methods developed there are those which generally aim at facilitating communications among peoples of different backgrounds and then reaching at a common recognition point with fundamental structures for the problem at hand [2]. Of those, two of the more typical are Interpretive Structural Modeling (ISM) method and Decision Making Trial and Evaluation Laboratory (DEMATEL) method.

In ISM and DEMATEL, the relationship among systems elements is considered. Work is commenced with the existence (or nonexistence) of corresponding pairs of judgments in the relationship and in the case of ISM, results in the generation of a hierarchical digraph which represents a total structure as it exists among the given set of elements. ISM, however, only considers the existence or non-existence of pairwise relationships by representing them as either one or zero. DEMATEL, on the other hand, makes careful note of the strength of any relationship. It does not, however, generate any clear hierarchical structure.

ISM, DEMATEL and improved ISM procedures have been described in this chapter and we should now like to assert that the concept and procedures of ISM may be extended to cope with any issue in which the strength of the relationships between and among systems elements to include quantitative definitions or estimation. Such method can be termed Interpretive Weighted Structural Modeling (IWSM). In Section 2.4 a mathematical procedure for IWSM is set forth with particular attention paid to the relationships possessing characteristics capable of addition or multiplication. Section 2.5 provides comments on similarity and simultaneously notes dissimilarities among IWSM, DEMATEL and ISM. Here a comparison made by means of a simple example will assist in understanding the features of IWSM.

2.2 Interpretive Structural Modeling Method [16] [50] [52]

Interpretive Structural Modeling (ISM) is a technique to assist individuals and small groups enhance the ability to communicate among themselves while they work on complex issues. The mental image of complex issues held individually or

collectively are represented in a hierarchical digraph. The digraph depicts the pattern of relationships among the various elements in the exploratory system [14].

The processes involved in ISM are shown below in Figure 2.2.

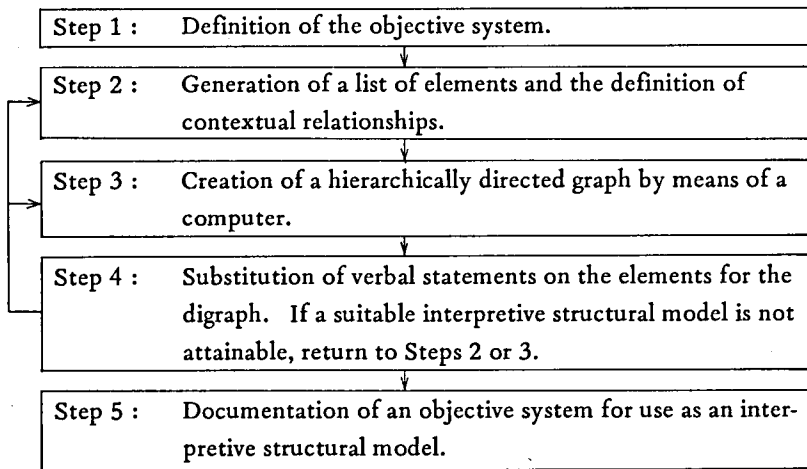


Figure 2.2. The Process of ISM Application.

A list of elements may be generated using either the brainstorming technique or the nominal group technique [7]. The contextual relation is a verb phrase expressing possible transitive relationships among elements. Typical contextual relationships are noted by such phrases as “ precedes”, “ supports ”, “ aggravates ” and so on.

2.2.1 Procedures for creating a digraph

The procedure for creating a hierarchical directed graph is depicted in Figure 2.3 and the mathematical procedures involved in it are described in the following:

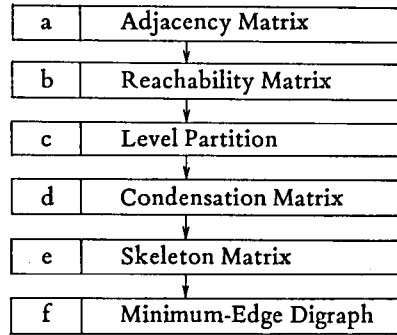


Figure 2.3. Procedures to be followed in the creation of a digraph.

(a) Adjacency matrix

If S be a set of elements from an objective system and there is a contextual relationship R^* among some members of that set, contextual relationship R^* may be represented as being present in the contextual relationship and absent from the contextual relationship $\bar{R}$. Consider any two elements of a system, s_i and s_j ($i, j = 1, 2, \dots, n$). It is possible to say either that s_i and s_j are related in a certain way or that they are not. That may be expressed as either

$$s_i R s_j \quad (2.1)$$

or

$$s_i \bar{R} s_j \quad (2.2)$$

In conducting an exercise, a person or group is subjected to a series of inquiries which take this form: "Is system element s_i related to system element s_j ?" or similar statements more appropriate to any given context. The replies of each respondent are arranged in a matrix, A , whose rows and columns correspond to the system element. If the system element s_i is related to the system element s_j , the numeral 1 is inserted into place of the i, j entries, a_{ij} of the matrix. If s_i is not related to s_j , the numeral 0 is inserted in a_{ij} . This $n \times n$ matrix $A = (a_{ij})$ ($i, j = 1, 2, \dots, n; i \neq j$) is called the adjacency matrix.

(b) Reachability matrix

For any square binary matrix A , a matrix M exists such that

$$(A + I)^{l-1} \neq (A + I)^l = (A + I)^{l+1} = M, \quad (2.3)$$

where ℓ is a positive integer less than n and I is the $n \times n$ identity matrix. The operations of Equation (2.3) are Boolean. The matrix M is the reachability matrix of A , and is also called the transitive closure of A . A hierarchical directed graph is structured from the reachability matrix in the ways next described.

(c) Level partition

Every element s_i in a transitive matrix has the reachability set $R(s_i)$ and an antecedent set $A(s_i)$. The reachability set $R(s_i)$ consists of all S elements lying on paths that may have originated from s_i and an antecedent set $A(s_i)$ consists of all elements of S lying on paths that include s_i . The levels may be found iteratively beginning with L_1 , by means of the equation:

$$\begin{aligned} L_j = \{ & s_i \in S - L_0 - L_1 - \dots - L_{j-1} \mid R_{j-1}(s_i) \\ & = R_{j-1}(s_i) \cap A_{j-1}(s_i) \} \end{aligned} \quad (2.4)$$

where $L_0 = 0$, the null set and $R_{j-1}(s_i)$ and $A_{j-1}(s_i)$ are the subset of $S - L_0 - L_1 - \dots - L_{j-1}$ respectively that are reachable from s_i , antecedent to s_i .

(d) Condensation matrix

By replacing elements in each cycle with a proxy element and deleting isolated elements in the matrix M , the condensation matrix, $C = (c_{pq})$, ($p, q = 1, 2, \dots, m$; $m \leq n$) is defined.

(e) Skeleton matrix

A matrix known as a skeleton matrix, $H = (h_{pq})$, ($p, q = 1, 2, \dots, m$), has the following property:

$$H^{\ell-2} \neq H^{\ell-1} = H^\ell = C \quad (2.5)$$

where the operations of multiplication are Boolean.

(f) Minimum-edge digraph

The digraph formed from the skeleton matrix is a minimum-edge digraph that preserves reachability. Any other digraph will have additional edges that interconnect with nonadjacent vertexes.

2.2.2 Transitive inference

To fill an adjacency matrix with pairwise comparisons requires $(n^2 - n)$ entries. If the number of elements increase, this could take a very long time. J. N. Warfield proposed two methods to deal with this -- the Scanning method and Iterative Bordering. Both reduce the inquiries for filling matrices by using transitive inference [53] [56]. With these methods, a reachability matrix is generated jointly by a man and a computer program. Figure 2.4 shows the procedure involved in the scanning method.

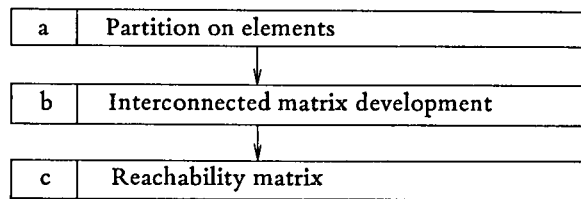


Figure 2.4. Procedure involved in the scanning method.

(a) Partition on elements [54]

Suppose an arbitrary element s_i is selected from an initial set S and the computer asks a man a series of inquiries. Set $S - s_i$ is divided at s_i into four subsets. They include (1) the lift set, (2) the feedback set, (3) the vacancy set and (4) the drop set. Figure 2.5 indicates how those four subsets are defined in terms of element s_i .

In the reachability matrix this partitioning may be carried as shown in Figure 2.6.

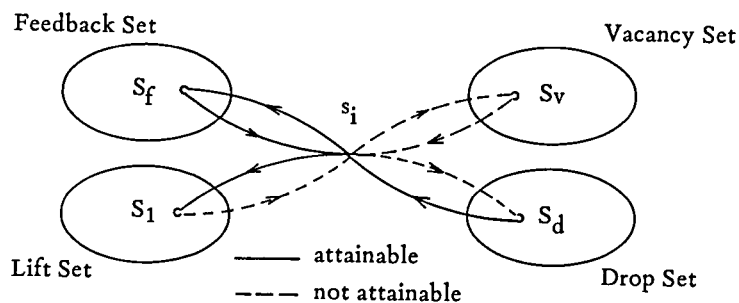


Figure 2.5. Interrelation among the elements of the four subsets.

L	M_{LL}	$\emptyset$	$\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}$	$\emptyset$	$\emptyset$
F	$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$	$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$	$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$	$\emptyset$	$\emptyset$
	$1 \dots 1$	$1 \dots 1$	1	$0 \dots 0$	$0 \dots 0$
V	M_{VL}	$\emptyset$	$\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}$	M_{VV}	$\emptyset$
D	$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$		$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$	M_{DV}	M_{DD}

$\begin{matrix} 1 \\ \vdots \\ 1 \end{matrix}$ transitively inferred 1's $\emptyset$ transitively inferred 0's

Figure 2.6. Four subsets in the reachability matrix and transitive inference.

In Figure 2.6, submatrices are filled with inferred data by means of transitivity except in the three diagonal submatrices M_{LL} , M_{VV} , M_{DD} and the two submatrices M_{VL} and M_{DV} which are known as interconnection matrices. The partitioning is repeated in the three diagonal submatrices until every submatrix is either empty or composed of a single element. The greater part of reachability matrix is completed as a result of this step except for the interconnection matrices.

(b) Interconnection matrix development [55]

Let A and B be subsets in a cascade connection. Suppose an interconnection matrix M_{BA} such that two reachability matrices M_{AA} and M_{BB} interconnect where all interconnections are oriented from subset B to subset A. The reachability matrix of the union of A and B may then be expressed as

$$M_{CC} = \begin{pmatrix} M_{AA} & M_{AB} = 0 \\ M_{BA} & M_{BB} \end{pmatrix} \quad (2.6)$$

where each entry in M_{AB} may be taken as zero. Since $M_{AA} = M_{AA}^2$, $M_{BB} = M_{BB}^2$ and $M_{CC} = M_{CC}^2$, we obtain

$$M_{BA} = M_{BA}M_{AA} + M_{BB}M_{BA} \quad (2.7)$$

Equation (2.7) is the matrix form of the characteristic logic equation. The characteristic logic equation expresses the necessary and sufficient condition in terms of the entries in the interconnection matrix M_{BA} . If the value of an entry in M_{BA} is specified, by filling the condition, values of the other entries are simultaneously determined. Different strategies, max-max-min strategy or maximum inference potential strategy for example, may be used to develop the data required for filling in the interconnection matrix.

2.2.3 Improved ISM method

By resorting to transitivity inference to fill in matrices with computer assistance, a user's burden may be diminished. The computer will perform many operations that are either distasteful to or difficult for the user. There are, however, for every advantage obtained attendant disadvantages, drawbacks for the advantage obtained.

(a) Intransitivity in human judgements

Suppose a user's mental model is illustrated as in Figure 2.7.



Figure 2.7. Example digraph of a user's mental model.

Suppose the interaction is indirect and little weak. When a paired comparison involving s_a and s_d is made a man will tend to think that no relationship exists. Partitioning the set at s_d , s_a will yield a vacant element set and s_c will become the drop set. Therefore in the interconnected matrix development stage, a computer will provide an inquiry in the form, "Is element s_c related to element s_a ? to which the user may simply provide a "Yes" ($s_c R s_a$) reply or "No" ($s_c \bar{R} s_a$). In this case the user's response will of course be "No." Because of this the developed structural model will be as shown in Figure 2.8.

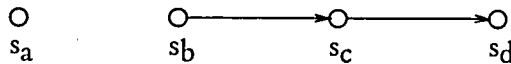


Figure 2.8. A developed digraph.

This developed digraph does not reflect the metal model shown in Figure 2.7.

(b) Improving the scanning method [30]

This problem may be lessened by resorting to a bidirectional form of inquiry and if the user responds with $s_v R s_d$ or $s_i R s_v$, the corresponding cell of M_{VD} or M_{LV} is filled with 1. 1's are disregarded, however, when characteristic logic equation are derived. The reachability matrix is finally defined as the transitive closure of the matrix thus developed. Y. Nishikawa and A. Udo proposed revised procedures which took into consideration intransitivity in the human judgment process and applied revised methods to several real problems which are illustrated with comments [32].

2.3 Decision Making Trial and Evaluation Laboratory Method [8]

Decision Making Trial and Evaluation Laboratory is a project name at the Battelle Research Center in Geneva and the DEMATEL method is an analytical tool for the study of individual perceptions of the problematical world. Procedures involved in the application of DEMATEL are depicted in Figure 2.9 and its mathematical procedure is described below.

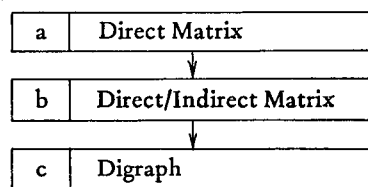


Figure 2.9. Procedure for applying DEMATEL.

(a) Direct matrix

In a DEMATEL exercise the respondent is given inquiries in the form “Is system element s_i related to system element s_j ? If yes, how great is the relationship?” or a similar statement more appropriate to any given context and asked to indicate the degree of direct relationship between s_i and s_j according to a scale run by some integer. The replies x_{ij}^* ($i, j = 1, 2, \dots, n; i \neq j$) of the respondent are arranged in the direct matrix, $X^* = (x_{ij}^*)$.

(b) Direct/indirect matrix

The normalized direct matrix, $X = (x_{ij})$ ($i, j = 1, 2, \dots, n; i \neq j$), is defined by

$$X \triangleq \frac{1}{\lambda} X^*, \quad (2.8)$$

where

$$\lambda = \text{Max}_i \left\{ \sum_{j=1}^n x_{ij}^* \right\}. \quad (2.9)$$

The direct/indirect matrix, $X_A = (x_{ij}^a)$ ($i, j = 1, 2, \dots, n; i \neq j$) is defined by

$$X_A = X + X^2 + X^3 + \dots = X(I - X)^{-1}, \quad (2.10)$$

assuming a monotone decrease in the indirect relation effect.

(c) Digraph

The digraph is produced from the direct/indirect matrix X_A . The coordinates of the i th element s_i are determined according to the following values:

$$c_i = c_i^* / \text{Max}_k \left\{ c_k^* \right\}, \quad c_k^* = \sum_{j=1}^n x_{jk}^a, \quad (2.11)$$

$$r_i = r_i^* / \text{Max}_k \left\{ r_k^* \right\}, \quad r_k^* = \sum_{j=i}^n x_{kj}^a. \quad (2.12)$$

An arrow is drawn from the i th element to the j th element if i, j -entry of X^* is nonzero.

2.4 Interpretive Weighted Structural Modeling Method [33]

Interpretive Weighted Structural Modeling (IWSM) is a technique for coping with an issue where the strength among system elements is either quantitatively defined or estimated. A measure of expressing the strength of the relationship will be defined in some appropriate way. Its value is usually chosen from among a set of positive numbers whether they be integers or not.

2.4.1 IWSM application procedure

The procedure for applying IWSM is shown in Figure 2.10.

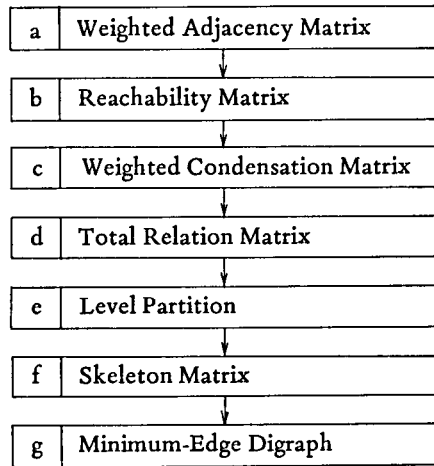


Figure 2.10. IWSM application procedure.

The IWSM mathematical procedure will be described below by means of a simple example.

(a) Weighted adjacency matrix

In the conduct of an IWSM exercise persons or groups are subjected to a series of inquiries which take the form “Is system element s_i related to system element s_j ? If so, how great is the relationship?” Replies of each respondent are arranged in a matrix, A^* , in which the rows and columns correspond to the system elements. If the system element s_i is related to the system element s_j , the replied value of the weight is inserted into the i, j -entry, a_{ij}^* , of the matrix. If s_i is not related to s_j , the value of the weight is thought of as zero and inserted into a_{ij}^* . This $n \times n$ matrix $A^* = (a_{ij}^*)$ ($i, j = 1, 2, \dots, n; i \neq j$) is called a weighted adjacency matrix. An example of a weighted adjacency matrix is shown in Figure 2.11.

	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈
S ₁								
S ₂	1							1
S ₃	2	2						2
S ₄	3		2			2		
S ₅	2							
S ₆	3							
S ₇								
S ₈		1						

Figure 2.11. Example Matrix.

(b) Reachability Matrix

The adjacency matrix $A = (a_{ij})$ is generated by substituting unit for the nonzero entries of A^* . There corresponds an unique reachability matrix M to the matrix A . These two matrices are related in Equation (2.3).

The reachability matrix of the example is obtained as follows:

$$A + I = \begin{array}{c|cccccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 \\ \hline S_1 & 1 & & & & & & & \\ S_2 & 1 & 1 & & & & & & 1 \\ S_3 & 1 & 1 & 1 & & & & & 1 \\ S_4 & 1 & & 1 & 1 & & 1 & & \\ S_5 & 1 & & & & 1 & & & \\ S_6 & 1 & & & & & 1 & & \\ S_7 & & & & & & & 1 & \\ S_8 & & 1 & & & & & & 1 \end{array} \cdot (A+I)^2 = \begin{array}{c|cccccccc} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 \\ \hline S_1 & 1 & & & & & & & \\ S_2 & 1 & 1 & & & & & & 1 \\ S_3 & 1 & 1 & 1 & & & & & 1 \\ S_4 & 1 & 1 & 1 & 1 & & 1 & & 1 \\ S_5 & 1 & & & & 1 & & & \\ S_6 & 1 & & & & & 1 & & \\ S_7 & & & & & & & 1 & \\ S_8 & 1 & 1 & & & & & & 1 \end{array}$$

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
s_1	1							
s_2	1	1						1
s_3	1	1	1					1
$(A+I)^3 = s_4$	1	1	1	1		1		1
s_5	1				1			
s_6	1					1		
s_7							1	
s_8	1	1						1

$(A+I)^2 = (A+I)^3 = M$

(c) Weighted condensation matrix

The condensation matrix $C = (c_{pq})$ is defined by replacing the elements in each cycle with a proxy element and deleting isolated elements in Matrix M . Furthermore by replacing each of the off-diagonal unity entries of C with corresponding values of the weights appearing in A^* and putting zero into all the diagonal entries, a matrix known as a weighted condensation matrix, $C^* = (c_{pq}^*)$ ($p, q = 1, 2, \dots, m$), is obtained. Figure 2.12 shows a weighted condensation matrix.

	s_1	s_2	s_3	s_4	s_5	s_6
s_1						
s_2	1					
s_3	2	2				
s_4	3	1	2			2
s_5	2					
s_6	3					

s_2, s_8 : cycle
 s_p : proxy element, $s_p = s_2$
 s_7 : isolated element

Figure 2.12. Weighted condensation matrix.

(d) Total relation matrix

A total relation matrix, $T = (t_{pq})$ ($p, q = 1, 2, \dots, m$), is defined as

follows. Before the definition of T, the special operational symbols $\odot$ and $\oplus$, and the matrix ${}^{(K)}C^* = ({}^{(K)}C_{pq}^*)$ are defined as follows:

$${}^{(K)}C^* \triangleq {}^{(K-1)}C^* \odot C^*, \text{ and } {}^{(1)}C^* \triangleq C^* \quad (2.13)$$

where provided the relation among system elements is regarded as having an additive nature,

$${}^{(K)}C_{pq}^* \triangleq \text{Max}_r \left\{ {}^{(K-1)}C_{pr}^* \oplus C_{rq}^* \right\} \quad (2.14)$$

$${}^{(K-1)}C_{pr}^* \oplus C_{rq}^* \triangleq \begin{cases} {}^{(K-1)}C_{pr}^* + C_{rq}^* & \text{if } {}^{(K-1)}C_{pr}^* C_{rq}^* \neq 0, \\ 0 & {}^{(K-1)}C_{pr}^* C_{rq}^* = 0, \end{cases} \quad (2.15)$$

and if the relation is regarded as having a multiplicative nature,

$${}^{(K)}C_{pq}^* \triangleq \text{Max}_r \left\{ {}^{(K-1)}C_{pr}^* C_{rq}^* \right\}. \quad (2.16)$$

Then ${}^{(K)}C^*$ satisfies the equation

$${}^{(K)}C^* = 0 \quad \text{for } \ell \leq K \leq m-1, \quad (2.17)$$

Where ℓ represents the number of levels in the associated hierarchical digraph.

Depending upon the definitions obtained by Equations (2.13) through (2.16), t_{pq} of the total relation matrix is defined as follows:

$$t_{pq} \triangleq \text{Max} \left\{ C_{pq}^*, {}^{(2)}C_{pq}^*, \dots, {}^{(t-1)}C_{pq}^* \right\}, \quad (p, q = 1, 2, \dots, m). \quad (2.18)$$

The notion of matrix T is relevant to the notion of the distance between the system elements at varying levels. That is to say the distance between the bottom level element and the i th element is obtained by

$$d_1^C = \text{Max}_K \left\{ t_{Ki} \right\}, \quad (2.19)$$

and the distance between the top level element and the k th element is given by

$$d_1^r = \text{Max}_K \left\{ t_{iK} \right\}. \quad (2.20)$$

Where the relationship has an additive nature, the total relation matrix for the example is obtained as followed:

② $c^* = c^* \odot c^* =$

	s_1	s_2	s_3	s_4	s_5	s_6
s_1						
s_2						
s_3	3					
s_4	5	4				
s_5						
s_6						

② $c_{31}^* = c_{32}^* + c_{21}^* = 3$

② $c_{41}^* = \text{Max} \{ c_{42}^* + c_{21}^*, c_{43}^* + c_{31}^*, c_{46}^* + c_{61}^* \} = 5$

② $c_{42}^* = c_{43}^* + c_{32}^* = 4$

③ $c^* = \text{② } c^* \odot c^* =$

	s_1	s_2	s_3	s_4	s_5	s_6
s_1						
s_2						
s_3						
s_4	5					
s_5						
s_6						

③ $c_{41}^* = \text{② } c_{42}^* + c_{21}^* = 5$

T =

	s_1	s_2	s_3	s_4	s_5	s_6
s_1						
s_2	1					
s_3	3	2				
s_4	5	4	2			2
s_5	2					
s_6	3					

$d_i^c = (5, 4, 2, 0, 0, 2)$

$d_i^r = (0, 1, 3, 5, 2, 3)$

Figure 2.13. Total relation matrix.

(e) Level partition

Levels are derived iteratively, beginning with L_1 , by means of the equation:

$$L_j = \left\{ s_i \in S - L_0 - L_1 - \dots - L_{j-1} \mid D_{\min}(s_i) = \min_k \left\{ d_k^r \{ L_{j-1}(s_i) \} \right\} \right\} \quad (2.21)$$

where $L_0 = 0$, the null set and $d_k^r \{ L_{j-1}(s_i) \}$ and $D_{\min}(s_i)$ are the distances between the top level element and the elements in level L_{j-1} , minimum distance from $d_k^r \{ L_{j-1}(s_i) \}$ respectively.

The levels in the example are:

$$L_1 = \{s_1\}, L_2 = \{s_2, s_8\}, L_3 = \{s_5\}, L_4 = \{s_3, s_6\}, L_5 = \{s_4\}.$$

(f) Skeleton Matrix

A skeleton matrix $S = (s_{pq})$ is defined:

$$S \triangleq C - \textcircled{2}C - \textcircled{3}C - \dots - \textcircled{l}C, \quad (2.22)$$

where $\textcircled{k}C$ is the matrix obtained by substituting unity for all nonzero elements in $\textcircled{k}C^*$. The operation symbol “-” in Equation (2.22) means that,

$$\left. \begin{array}{ll} 1 - 0 = 1, & 1 - 1 = 0, \\ 0 - 1 = 0, & 0 - 0 = 0. \end{array} \right\} \quad (2.23)$$

The skeleton matrix in the example is obtained as follows:

$$\textcircled{2}C = \begin{array}{c|cccccc} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \hline s_1 & & & & & & \\ s_2 & & & & & & \\ s_3 & 1 & & & & & \\ s_4 & 1 & 1 & & & & \\ s_5 & & & & & & \\ s_6 & & & & & & \end{array} \quad \textcircled{3}C = \begin{array}{c|cccccc} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \hline s_1 & & & & & & \\ s_2 & & & & & & \\ s_3 & & & & & & \\ s_4 & 1 & & & & & \\ s_5 & & & & & & \\ s_6 & & & & & & \end{array}$$

$$S \triangleq C - \textcircled{2}C - \textcircled{3}C = \begin{array}{c|cccccc} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \hline s_1 & 1 & & & & & \\ s_2 & 1 & 1 & & & & \\ s_3 & & 1 & 1 & & & \\ s_4 & & & 1 & 1 & & 1 \\ s_5 & 1 & & & & 1 & \\ s_6 & 1 & & & & & 1 \end{array}$$

(g) Minimum-edge digraph

The digraph $D(S-I)$ formed from the skeleton matrix is a minimum-edge digraph.

In the digraph a coordinate of the i th system element is calculated by using Equations (2.19) and (2.20). An arrow is drawn from the i th element to the j th element when the i, j -entry of $S-I$ is nonzero. Figure 2.14 shows the minimum-edge digraph for the example.

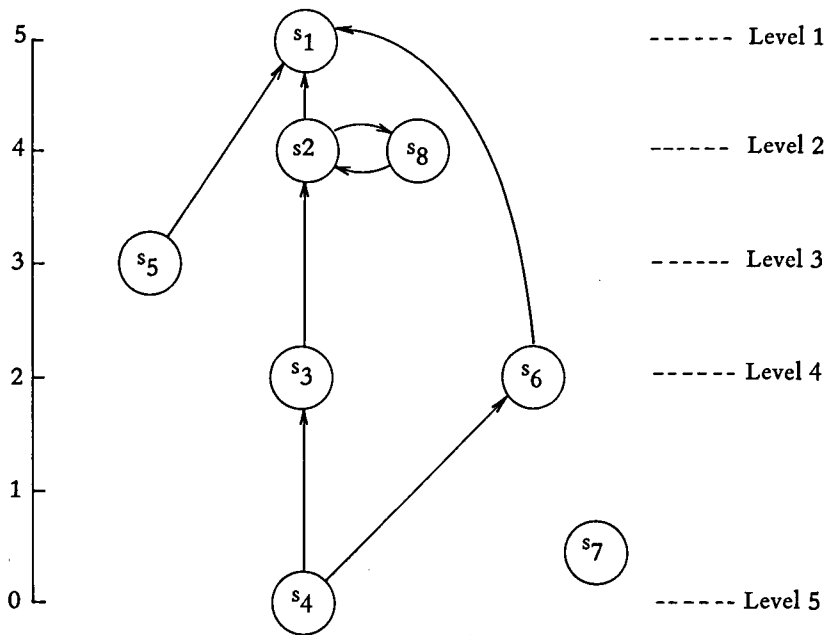


Figure 2.14. Minimum-edge digraph.

2.4.2 Transitive inference

Scanning method, Iterative Bordering method and their revised methods mentioned in Section 2.2.2 may be used in the IWSM process and the inquiries in the filling matrices may be reduced by making use of transitive inference. An additional procedure is required only the insertions of the replied value of the weight to the corresponding cell of the weighted adjacency matrix. The reachability and weighted adjacency matrices may be obtained simultaneously.

2.5 Numerical Examples

In this section two numerical examples are examined. One involves the application of the improved scanning method and the other compares ISM, DEMATEL and IWSM by means of the example.

2.5.1 Application of the scanning method

The Improved Scanning Method is applied to priority setting for fifteen construction material test items in such procedures as the bending, percussion and adiabatic tests. In this exercise respondents are given inquiries of the form “Is text s_i more significant than text s_j ?”. At the element partition step, the relationship of replies between s_1 and s_7 is $s_1 \bar{R} s_7$ because the interaction is indirect and rather weak towards the respondent. At the development of the interconnection matrix stage, respondent is given an inquiry that takes the form “Is text s_7 more significant than s_{15} ?” to which he would provide either a simple “Yes” ($s_{15} R s_7$) or “No” ($s_{15} \bar{R} s_7$) and make the replied relationship $s_{15} \bar{R} s_7$. The digraph developed by ISM is shown in Figure 2.15. Figure 2.16 shows the digraph developed by means of the improved method. This user must admit that he considers the digraph developed with the improved method to be much better. It may be noted that the digraph in Figure 2.16 appears more simple than the digraph in Figure 2.15.

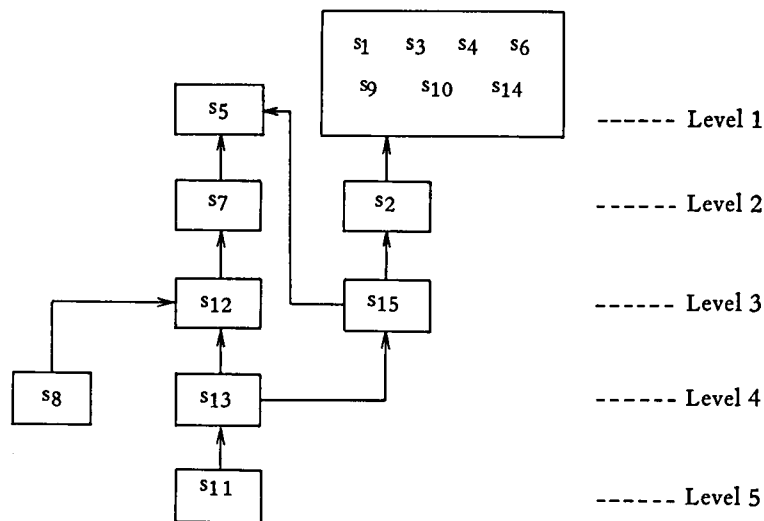


Figure 2.15. Digraph developed with ISM.

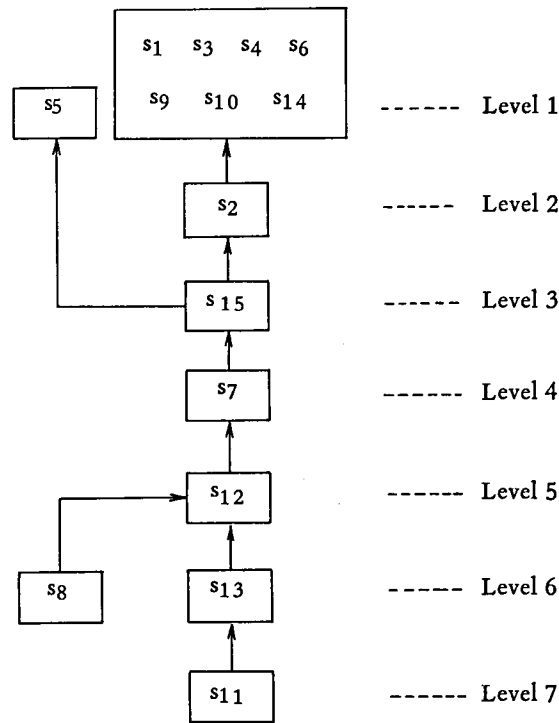


Figure 2.16. Digraph developed with improved ISM.

2.5.2 Comparison of ISM, DEMATEL and IWSM by means of examples

Inquiries were placed before freshmen entering a Research and Development Laboratory about how they felt about the usefulness of a training program.

Table 2.1 shows the topics presented in that program. When the i th topic and the j th topic were compared they were asked if they felt that the j th topic was more useful than the i th topic and they were asked to indicate that difference with an integer between 1 and 3. The relationships of the different levels of usefulness they indicated were regarded as having an additive nature.

There were eighteen freshmen so in this exercise we defined an average matrix as being $V = (v_{ij})$ ($i, j = 1, 2, \dots, 13$) for the respondents, i. e.,

$$V_{ij} = \frac{1}{18} \sum_{k=1}^{18} x_{ij}^k / 18, \quad (2.24)$$

where x_{ij}^k was the k th freshman's reply. V was taken as being a direct matrix in DEMATEL and the weighted adjacency matrix in IWSM. Substituting unity for

all the entries in the matrix where more than half the freshmen indicated a difference in usefulnesses, the adjacency matrix in ISM was obtained. The V entries in this exercise are shown in Figure 2.17. The adjacency matrix was the same matrix obtained by substituting unity for all the nonzero elements in V.

Table 2.2 shows the values c_j , r_i and c_j-r_i . Figure 2.18 shows the digraph obtained by DEMATEL and Figure 2.19 shows the digraph obtained with IWSM in which the additive rule was adopted. Figure 2.20 shows the digraph obtained with ISM.

Table 2.1. Topics in the training program

No.	Topics (elements)
1	Introduction to each section
2	Explanation R&D Laboratory conventions
3	Explanation of a patent
4	Presentations by senior researches
5	Explanation of digital computer use
6	Explanation of analog computer use
7	Exercise in numerical analysis of differential equations
8	Exercise in automatic drawing
9	Exercise with the Newton-Raphson method
10	Exercise in the least square method
11	Exercise in multiple regression analysis
12	Exercise in mathematical programming
13	Exercise in the finite element method

j i	1	2	3	4	5	6	7	8	9	10	11	12	13
1			1	2	3	2	2	3	2	2	1	2	2
2	2		2	2	3	2	2	2	2	1	1	2	2
3					2	1	1	2	2	2	1	1	
4					2			1					
5													
6					2			2	1	1		1	
7					1								
8													
9					1			2					
10					1			1					
11				1	2			2					
12				1	2			2					
13			1		2		1	2	1				

Figure 2.17. Matrix V.

Table 2.2. Values of c_i , r_i and $c_i - r_i$

Element	c_i	r_i	$c_i - r_i$
1	0.0804	0.8728	-0.7924
2	0.0	1.0	-1.0
3	0.1717	0.4562	-0.2845
4	0.2656	0.1033	0.1623
5	1.0	0.0	1.0
6	0.2156	0.2562	-0.0406
7	0.2631	0.0344	0.2287
8	0.9122	0.0	0.9122
9	0.3604	0.1038	0.2571
10	0.2726	0.0689	0.2038
11	0.1316	0.1767	-0.0450
12	0.2652	0.1767	0.0885
13	0.1679	0.2669	-0.0990

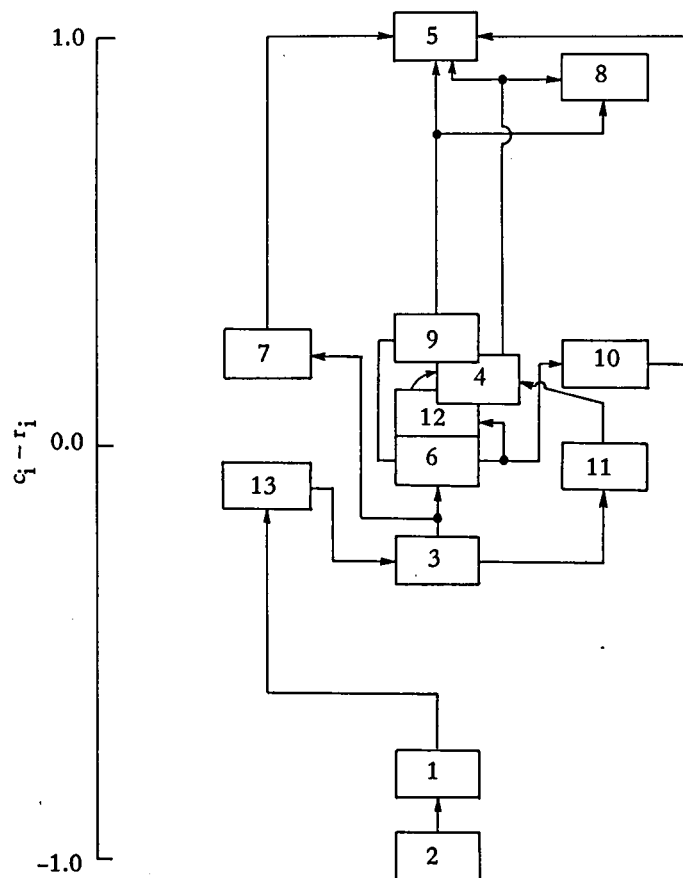


Figure 2.18. Relative usefulness of the topics in the training program as determined by DEMATEL.

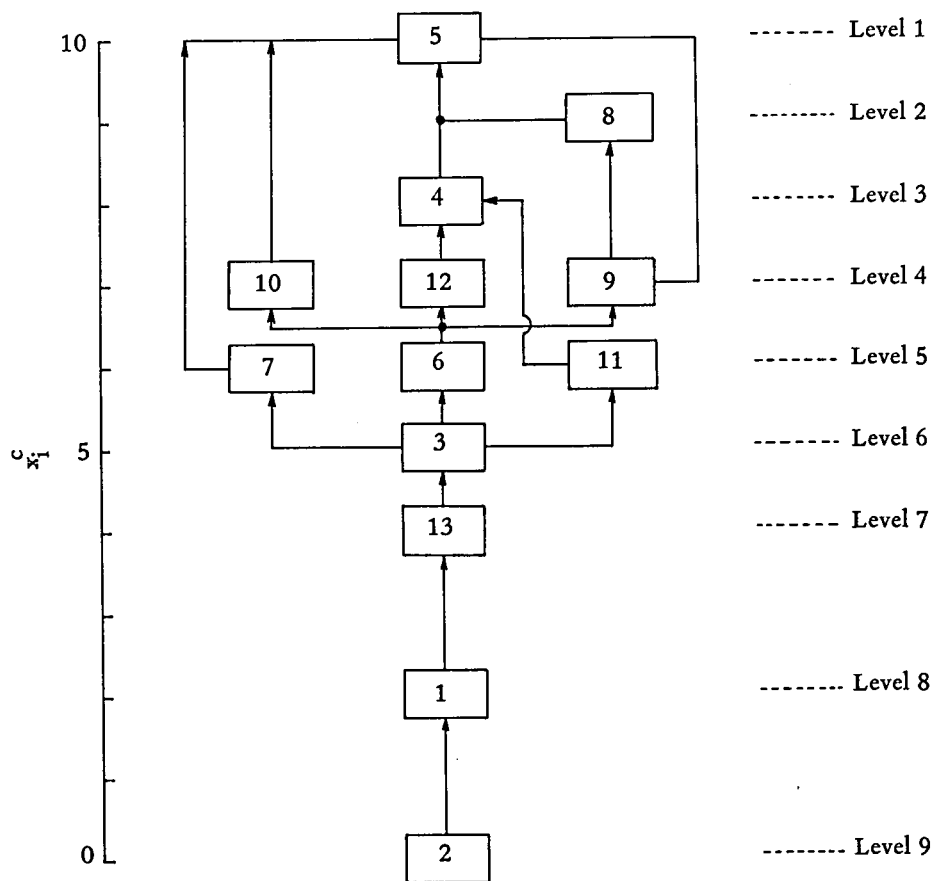


Figure 2.19. Relative usefulness of the topics in the training program as determined by IWSM.

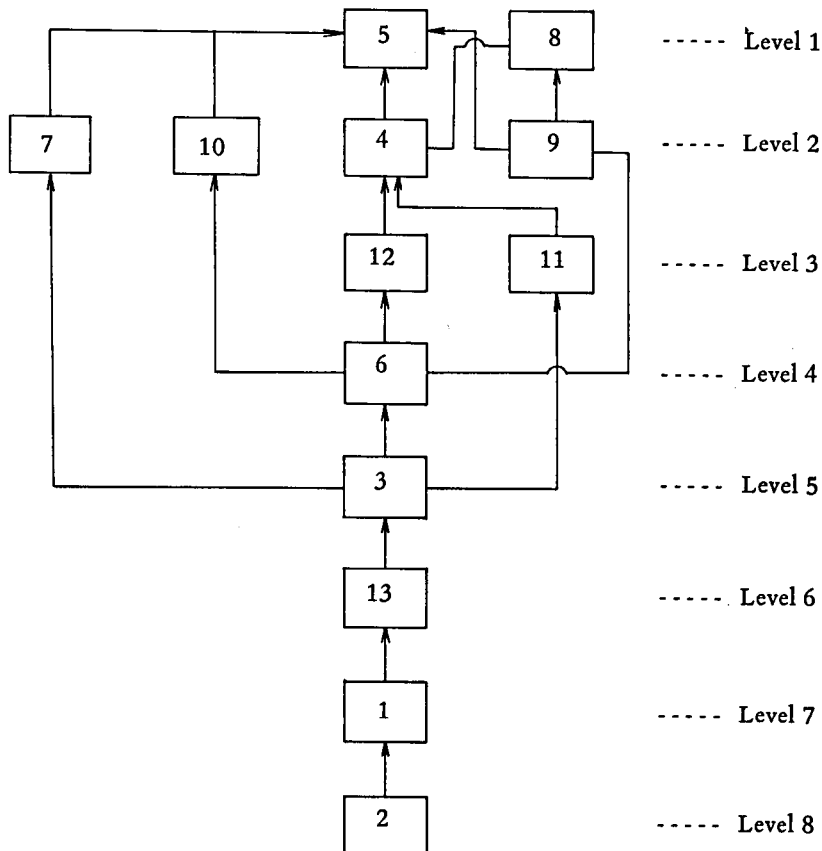


Figure 2.20. Relative usefulness of the topics in the training program as determined by ISM.

Comparing Figures 2.18 and 2.19 we see that the contextual relationship between the third and thirteenth elements reveals itself in quite different ways. The matrix V shows clearly that the third element is regarded as more useful than the thirteenth element. This fact is correctly reflected in the IWSM digraph but not in that of DEMATEL. Comparing Figures 2.19 and 2.20 we see that the contextual relationship of each elements reveals itself in a very similar same way. Only the position of each element appears in a different manner. The IWSM digraph has many more levels and information than an ISM digraph.

2.6 Concluding Remarks

In this chapter a proposal regarding a new method of structural modeling known as IWSM for coping with problems having relationships among system elements with either an additive or multiplicative nature has been made.

IWSM characteristics as well as those of DEMATEL and ISM have been analyzed by means of a simple example. Comparision of those methods has revealed that IWSM is able to generate a clearer hierarchical structure than the other method.

This method has been applied to real problems and the results have shown the feasibility and effectiveness of the method [see Chapter 4]. Thus far the method has been successfully applied to numerous other problem areas among which are priority analysis of research and development subjects, evaluation of development strategies for merchandising and determining parameters in utility analysis.

CHAPTER 3

DECISION ANALYSIS THROUGH MULTIOBJECTIVES

3.1 Introduction

When analyzing a complicated system, defining objectives or criteria can be uniquely difficult. Real problems generally include more than one criterion and often criteria are in conflict with one another. In dam construction, for instance, at least two such criteria exist - - safety and construction costs. If the plan attaches too much importance to safety, costs will become impossibly high. If on the other hand though emphasis is laid on cost, the probability that safety concerns will be given their due is substantially reduced. This sort of a relationship is known as a tradeoff relationship. If a tradeoff relationship exists between and among criteria, optimal solutions do not generally exist and one must be content with obtaining non-inferior solutions (Pareto optimum or efficient solutions). Determination of a non-inferior set, however, is not sufficient. Preferred solutions therefore are those defined as distinguishing the best of several non-inferior solutions. This is known as the problem of multiobjective optimization or making a decision that involves multiple objectives.

Finding solutions for problems with multiple objectives basically involves two approaches. One is to find a preferred solution directly and the other is to generate a non-inferior set and then find the preferred solution from among these. The first one or the direct approach is a utility function approach. In the early days R. L. Keeney offered a multiattribute utility function approach (MAUF) for such problems. A typical program for the second type of problem was the surrogate-worth tradeoff (SWT) method proposed by Professor Y. Y. Haimes. Numerous studies which include multiobjectives in public sector problem solution have been conducted. They included for example the multiobjective analysis used in water and related land resources planning [13], selection of nuclear power plant sites in the Pacific Northwest [18] and so on.

The procedures involved in the SWT and MAUF methods are discussed in the

chapter below and a modification of the decision procedure is proposed. Several comments are made upon the methodologies and our experience which center on viewpoints having to do with practical feasibility and effectiveness.

3.2 The Surrogate Worth Tradeoff Method [12]

Suppose objective function, f_1 , is the cost of the construction of a dam and objective function, f_2 , is the safety factors involved in that construction. The non-inferior region for that construction is indicated by Figure 3.1. A non-inferior solution is one in which no increase in worth can be obtained in any objective without causing a simultaneous decrease of the worth in at least one other of the objectives. The preferred solution in this case is one selected from among those offered in the non-inferior set. The preferred solution, therefore, is decided through two phases -- generating a non-inferior set and then finding the preferred solution.

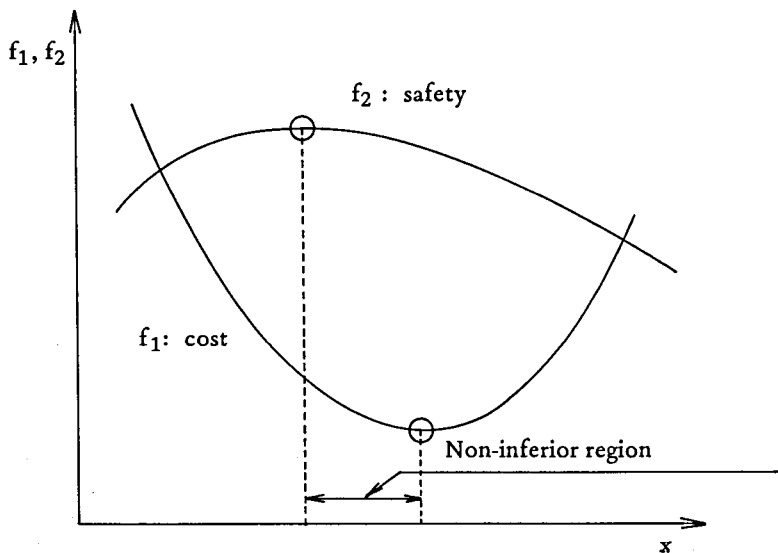


Figure 3.1. The Non-inferior Region.

There are a number of approaches possible when it comes to finding the preferred solution. They include such means as the lexicographic approach, the ϵ -constraint approach, the parametric approach, goal programming, surrogate worth tradeoff

and so on.

The procedure used to first generate a non-inferior set using the ϵ -constraint method then finding the preferred solution from among these with the surrogate-worth tradeoff (SWT) method operates as follows [13]:

Step 1: Construct a mathematical model and define the objectives. For notational convenience, define the general vector optimization as:

$$\text{Problem 1: } \min_x [f_1(x), f_2(x), \dots, f_n(x)] \quad (3.1)$$

subject to

$$g_k(x) \leq 0 \quad k = 1, 2, \dots, m \quad (3.2)$$

where x is an N -dimensional vector of decision variables.

$f_i(x)$, $i = 1, 2, \dots, n$, are n objective functions.

$g_k(x)$, $k = 1, 2, \dots, m$, are m constraint functions.

Step 2: Generate the non-inferior set. The ϵ -constraint approach is used for finding the non-inferior set. The algorithm of the ϵ -constraint approach goes as follows:

Step 2.1: Find the minimum value of f_j , $f_{j\min}$, by solving: $\min [f_j(x)]$ subject to (3.2).

This step will be repeated for all j . The minimum value of f_j is determined by ignoring all other objectives. If possible the maximum value f_j , $f_{j\max}$, should be found here.

Step 2.2: Set the initial values for $\epsilon_j = f_{j\min} + \Delta\epsilon_j$, $\Delta\epsilon_j > 0$, where $\Delta\epsilon_j$ is a deviation from the value of $f_{j\min}$.

Step 2.3: Solve $\min [f_i(x)]$ subject to $f_j(x) \leq \epsilon_j$, $i \neq j$; $j = 1, 2, \dots, n$ and (3.2)

Let x^* be the decision vector which solves this problem. The solution is $f_i^*(\epsilon) = f_i(x^*)$; each solution should also contain $\lambda_i(\epsilon)$, the vector of Lagrange multiplies for the constraints. If all of the ϵ_j constraints are binding, then $\epsilon_j = f_j$ so that the output at this step are $f_i^*(\epsilon)$ and $\lambda_i(\epsilon)$ where ϵ is a vector of ϵ_j variables. If any of the ϵ -constraints are not binding then ignore these values and continue this step until ϵ -constraints are binding.

Step 3: Obtain each tradeoff ratio λ_{ij} . The tradeoff rate function between the i^{th} and j^{th} functions denoted by T_{ij} are defined as follows:

$$T_{ij}(x) = \frac{df_i(x)}{df_j(x)} \quad (3.3)$$

where

$$df_i(x) = \sum_{k=1}^N \frac{\partial f_i(x)}{\partial x_k} dx_k \quad (3.4)$$

Equation (3.4) however, is impractical and an alternative approach must be sought. Reformulate the system therefore in problem 1 as follows:

Problem 2: $\min_x [f_i(x)]$ subject to

$$f_j(x) \leq \epsilon_j, \quad i \neq j; \quad j = 1, 2, \dots, n,$$

and

$$g_k(x) \leq 0, \quad k = 1, 2, \dots, m$$

where

$$\epsilon_j = f_{j\min} + \Delta\epsilon_j, \quad i \neq j; \quad j = 1, 2, \dots, n$$

$$\Delta\epsilon_j > 0, \quad i \neq j; \quad j = 1, 2, \dots, n$$

Form the generalized Lagrangian L , for problem 2:

$$L = f_i(x) + \sum_{k=1}^m \mu_k g_k(x) + \sum_{\substack{j=1 \\ i \neq j}}^n \lambda_{ij} (f_j(x) - \epsilon_j) \quad (3.5)$$

where $\mu_k, k = 1, 2, \dots, m$ and $\lambda_{ij}, i \neq j; j = 1, 2, \dots, n$, are generalized Lagrange multipliers. The subscript ij in λ is the Lagrange multiplier associated with the i^{th} objective function and j^{th} constraint. The Kuhn-Tucker conditions here are:

$$\lambda_{ij} (f_j(x) - \epsilon_j) = 0 \quad i \neq j; \quad j = 1, 2, \dots, n$$

$$\lambda_{ij} \geq 0 \quad i \neq j, \quad j = 1, 2, \dots, n$$

The following is derived from Equation (3.5):

$$\lambda_{ij}(\epsilon_j) = -\frac{\partial L}{\partial \epsilon_j}, \quad i \neq j; \quad j = 1, 2, \dots, n$$

If these constraints are active, $f_i(x) = L$ and $f_j(x) = \epsilon_j$.

Therefore:

$$\lambda_{ij} = -\frac{\partial L}{\partial \epsilon_j} = -\frac{\partial f_i(x)}{\partial \epsilon_j} = -\frac{\partial f_i(x)}{\partial f_j(x)} \quad (3.6)$$

Thus T_{ij} can be found by calculating $-\lambda_{ij}$ which is obtained from the overall system Lagrangian.

Step 4: Develop the surrogate-worth function W_{ij} as follows:

A surrogate-worth function W_{ij} , $i \neq j$; $j = 1, 2, \dots, n$, may be defined as any monotonic function of λ_{ij} estimating the desirability of the tradeoff λ_{ij} , or as a function of $n-1$ of the f_j estimating the desirability of the additional units of f_i . For example, the scale can range from -10 to $+10$, with a -10 indicating that λ_{ij} marginal units of objective i are much less desirable than an additional unit of j , a $+10$ meaning just the opposite and zero signifying an even trade (i. e., belongs to the indifference band). The preferred solution is where the surrogate-worth functions are simultaneously equal to zero.

Step 5: If enough information concerning surrogate-worth function W_{ij} has been generated then proceed to step 6; otherwise select new values for j and go back to Step 2.3.

Step 6: Finding the indifference band

One can either resolve problem 2 to find the precise tradeoff ratio or either use curve-fitting interpolation or regression techniques on known non-inferior values to approximate the tradeoff ratio.

Step 7: The preferred decision vector x^* is found by solving:

Problem 3: Min $[f_i(x)]$

Subject to

$$f_j(x) \leq f_j^*, \quad i \neq j, \quad j = 1, 2, \dots, n$$

$$g_k(x) \leq 0, \quad k = 1, 2, \dots, m$$

3.2.1 The surrogate-worth function [12]

The surrogate-worth function provides an interface between the decision maker and the mathematical model. The value of the surrogate-worth function W_{ij} is an assessment of the decision maker as to how much he prefers trading λ_{ij} marginal units of f_i for one marginal unit of f_j , given the values of all the objectives $f_1, f_2, \dots, f_n$ corresponding to λ_{ij} . A formal definition of W_{ij} is given below:

$$W_{ij} \left\{ \begin{array}{ll} > 0 & \text{when } \lambda_{ij} \text{ marginal units of } f_i(x) \text{ are preferred to one} \\ & \text{marginal unit of } f_j(x), \text{ given the level of achievement of all} \\ & \text{the objectives.} \\ = 0 & \text{when } \lambda_{ij} \text{ marginal units of } f_i(x) \text{ are equivalent to one} \\ & \text{marginal unit of } f_j(x), \text{ given the level of all the objectives.} \\ < 0 & \text{when } \lambda_{ij} \text{ marginal unit of } f_i(x) \text{ are not preferred over one} \\ & \text{marginal unit of } f_j(x), \text{ given the level of achievement of all} \\ & \text{the objectives.} \end{array} \right.$$

The scale may range from -10 to +10 on an ordinal scale.

3.2.2 The modified decision procedure [31]

The surrogate-worth function is very useful, but in practice it usually takes a great deal of time to identify the band of indifference. Besides that decision makers often feel that the procedure is quite groublesome. Taking that into consideration, we have modified the decision procedure with tradeoff curves and the IWSM method as follows [33]:

Step 1: Rank the objectives by their orders of importance after discussion with decision makers. We have made use of the IWSM method to obtain the rank of the objectives.

Step 2: Choose a limited number of objectives after their ranking. Let f_i , f_j and f_k be the most important, second most important and third most important objectives respectively.

Step 3: Draw tradeoff curves on the f_i , f_j -plane. The value of objective f_k is the parameter of the tradeoff curves and the values of other objectives are fixed.

Step 4: Develop the assessment function W_k for each tradeoff curve. The definition of W_k is given below:

$$W_k \left\{ \begin{array}{ll} > 0 & \text{The value of the objective } f_k \text{ is preferable, but there is no} \\ & \text{indifference band on the tradeoff curve.} \\ = 0 & \text{The value of the objective } f_k \text{ is preferable and there is an} \\ & \text{indifference band on the tradeoff curve.} \\ < 0 & \text{There is an indifference band on the tradeoff curve, but the} \\ & \text{value of the objective is not preferable.} \end{array} \right.$$

Step 5: Find the value of f_k for which the value of W_k is equal to zero and affix the value of f_k to it.

Step 6: Choose the fourth most important objective, f_1 . Replace f_k with f_1 and return to Step 3.

Step 7: Repeat Step 6 until the least important objective has been taken into account.

These steps as described above are not so troublesome as they may seem and with them the outline of the indifference band is easily discovered. After this procedure the indifference band may also be clearly outlined by use of the surrogate-worth function.

3.3 Utility Function [57]

There are several concepts of utility and much work has been done in decision theory on how individual and societal utility functions may be approximated. Utility models are classified according to certain of their formal properties. The main classes of utility models are deterministic models as well as probabilistic models. In probabilistic models utilities are assessed by determining probabilities of preference. Deterministic models assume no randomness whatever in utilities or preferences and they are special cases of probabilistic models in which only one and zero are permissible.

The other main classes of utility models are ordinal models and interval models. Ordinal models produce utility functions that make statements about the order of preferences only. Interval models produce utility functions that also make statements about the relative strength of preferences. Utility functions of interval models contain meaningful information about the modeled preferences.

Utility models are classified also according to decision situations. These situations include: static decision environments or dynamic decision environments, single decision maker or multiple decision makers, single attributed choice or multiattributed choice, risk free choice or risk attended choice, time invariant choice and time variable choice and so on. These classification schemes lead to many distinct decision situations and generate a huge number of utility models. The model categories are: probabilistic models [24], semi-orders [22], interval orders [10], lexicographic orders [11], weak orders [21], difference measurement [21], bisymmetric measurement [42], conjoint measurement [23], expected utility measurement [29] and so on. Expected utility theory has been applied to many real problems among these categories. Expected utility theory is both one of the deterministic models and one of the interval models and its decision situations are risk attendant. The crucial assumptions that go with expected utility theory concerning preferences among risky choices are: (1) the weak order assumption, (2) independence assumptions and (3) the certainty equivalence assumption. The first means that the decision maker can order risky alternatives transitively. The second means that preferences among risky alternatives should be independent of events in which these alternatives have common outcomes. The final assumption means that the decision maker is able to find a riskless entity that is just as valuable to him as the risky entity. Although there are a number of expected utility axiomatizations, basically different approaches to measuring the utility of risky choice entities include (1) von Neumann and Morgenstern's expected utility theory [29], (2) Savage's subjective expected utility theory [44], (3) Davidson, Suppes and Sigel's finite utility theory [6] and (4) Luce and Krantz's conditional expected utility theory [25].

Based upon these approaches R. L. Keeney proposed the multiattribute utility function [17]. This model is an expected utility decomposition model which decomposes the utility function into utility functions of single attributes using independence assumptions about preferences and then aggregates them according to some rules which depend upon the types of independence assumptions. The multiobjective optimization procedure by multiattribute utility function and its construction procedure are described below.

3.3.1 Optimization procedure with the multiattribute utility function

Let the set X be the product of m attributes (criterial) $X_i: X \triangleq X_1 \times X_2 \times \dots \times X_m$. If x and y are elements in X and x is not preferred to y , a preference relation is denoted as $x \leq y$. Assuming that $\leq$ on X is connected (i. e., $x \leq y$ or $y \leq x$) and transitive (i. e., if $x \leq y$ and $y \leq z$, then $x \leq z$) and that there is a countable subset of X that is $\leq$ -dense in X , real-valued functions $u(x)$ and $u(y)$ can be assigned in such a way that for all x and y in X ,

$$x \leq y \text{ if and only if } u(x) \leq u(y).$$

The function $u(x)$ is a utility function which transforms the preference structure of a person into a corresponding numerical utility structure [10].

Let X_i - denote $X_1 \times X_2 \times \dots \times X_{i-1} \times X_{i+1} \times \dots \times X_m$, and X_{ij} - denote $X_1 \times X_2 \times \dots \times X_{i-1} \times X_{i+1} \times \dots \times X_{j-1} \times X_{j+1} \times \dots \times X_m$. For $m \geq 3$, if for some X_i , $X_i \times X_j$ is preferentially independent of X_{ij} - for all $i \neq j$, and X_i is utility independent of X_i - for all i , then we have either

$$U(X) = \sum_{i=1}^m k_i u_i(x_i), \quad (3.7)$$

or

$$U(X) = \left(\prod_{i=1}^m (1 + k_i u_i(x_i)) - 1 \right) / k. \quad (3.8)$$

where U and all the u_i are scaled for zero to one k_i are scaling constants such that $0 < k_i < 1$, and $k > -1$ is a nonzero constant. The utility function of Equation (3.7) is called the additive utility function while that of Equation (3.8) is the multiplicative utility function.

This multiattribute utility function is defined using the preferential independence and utility independence conditions [19]. Two attributes X_i, X_j are preferentially independent of the other attributes if the preference order for (X_i, X_j) combinations do not depend upon the fixed levels of the other attributes. Attribute X_i is defined as utility independent of the other attributes if the preference order for lotteries on X_i does not depend upon fixed levels of the other attributes. This implies that the conditional utility functions over X_i are the same regardless of the levels of the other attributes. In the case that the utility independence is not satisfied, the multiattribute utility function is defined using the convex

dependence [49].

If the multiattribute utility function is capable of construction, the preferred solution is defined as the solution which maximizes the multiattribute utility function. Then the multiobjective optimization problem can be solved as the one-objective optimization problem.

3.3.2 Construction of the multiattribute utility function (1) [20]

The procedure for constructing the multiattribute utility function is as follows:

Step 1: Assess the single-attribute utility functions. Let f_i^0 and f_i^* denote the least and the most desirable values of attribute f_i respectively. For each individual utility function $u_i(f_i)$ we shape the normalization condition as

$$u_i(f_i^0) = 0.0, \quad (3.9)$$

$$u_i(f_i^*) = 1.0. \quad (3.10)$$

The value of u_i for f_i between f_i^0 and f_i^* have been decided by questionnaires to the decision maker using the standard 50–50 lottery technique like that shown in Figure 3.2. In Figure 3.2 the certainty equivalent of the lottery is a riskless entity such that the decision maker is indifferent between the lottery which is the risky entity and the amount for certain. The functional form of the utility function implies the decision maker's basic attitudes toward risk. If the utility function is concave, the decision maker is risk averse. If the utility function is convex, the decision maker is risk seeking or risk prone. If the utility function is linear, the decision maker is risk neutral [43].

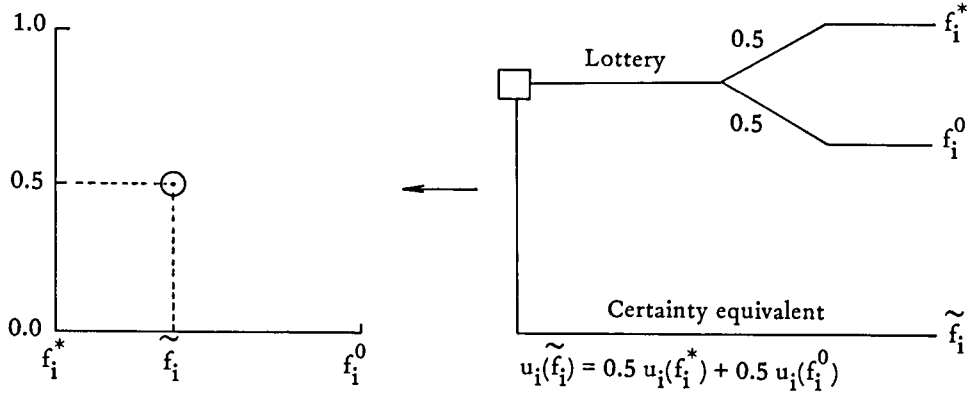


Figure 3.2. 50-50 lottery.

Step 2: Evaluate the scaling constants. The scaling constants k_i are evaluated by use of the lottery shown in Figure 3.3. In Figure 3.3, f_{i-} implies the set of all attributes other than f_i . If, at a certain value p^* of the probability p , the lottery and the certainty equivalent become indifferent to each other and we have the following equalities:

$$p^* U(f_1^*, f_2^*, \dots, f_m^*) + (1 - p^*) U(f_1^0, f_2^0, \dots, f_m^0) = U(f_i^*, f_{i-}^0)$$

and

$$U(f_1^0, f_2^0, \dots, f_m^0) = 0.0, \quad U(f_1^*, f_2^*, \dots, f_m^*) = 1.0,$$

$$U(f_i^*, f_{i-}^0) = ((1 + k k_i) - 1) / k = k_i.$$

Hence

$$k_i = p^*. \quad (3.11)$$

The value of parameter k can be calculated by the following equation which is derived from Equation (3.8) with $f_i = f_i^*$ for all i .

$$1 + k = (1 + k k_1) (1 + k k_2) \dots (1 + k k_m) \quad (k > -1) \quad (3.12)$$

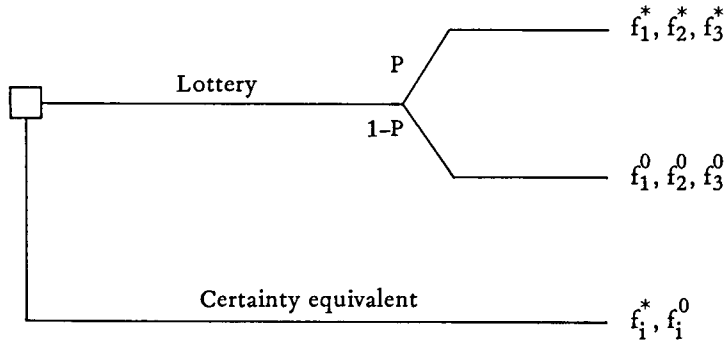


Figure 3.3. Evaluating the scaling constants.

3.3.3 Construction of the multiattribute utility function (2) [31]

If the certainty equivalent in Figure 3.3 is fixed as f_i, f_i^0 , to one another, then we have the following equality

$$p = k_i u_i(f_i). \quad (3.13)$$

Equation (3.13) means that the relation between the individual utility function $u_i(f_i)$ and the probability p is linear. In real problems, however, when the relationship is checked by questionnaires to decision makers using the lottery, many decision makers violate it. We think that this fact is caused by a lack of experience in lotteries such as those shown in Figure 3.3. The use therefore of indifference curves combined with a lottery is investigated.

At the outset of this procedure the most important attribute, f_j , is decided and the utility function, $u_j(f_j)$, and the scaling constant, k_j , are assessed by use of the lotteries. The significance of attributes can be clarified with the IWSM procedure as described earlier. Then the procedure for drawing an indifference curve is as follows:

Step 1: Define the point P_s on the f_i, f_j -plane whose coordinates are f_i^* and f_j^0 . A decision maker is given an inquiry about the location of the P_i which is indifferent to point P_s .

Step 2: Ask the decision maker about the location of the point P_{i+1} which is indifferent to point P_i .

Step 3: Repeat Step 2 until the point whose f_i coordinate is equal to f_i is found.

Step 4: Define the point P_e whose coordinates are f_i^0 and f_j^e .
The scaling constant k_i and the individual utility function $u_i(f_i)$ are assessed using the indifferent curve without resort to the lotteries.

(A) Evaluating the scaling constants

By completion of the above inquiries point P_s is discovered to be indifferent to point P_e , and the following equalities emerge:

$$\begin{aligned} (1 + k k_1 u_1(f_1^0)) \dots (1 + k k_i u_i(f_i^*)) \dots (1 + k k_j u_j(f_j^0)) \dots \\ (1 + k k_m u_m(f_m^0)) = (1 + k k_1 u_1(f_1^0)) \dots (1 + k k_i u_i(f_i^0)) \dots \\ (1 + k k_j u_j(f_j^e)) \dots (1 + k k_m u_m(f_m^0)) \end{aligned}$$

and

$$u_i(f_i^*) = 1.0, u_i(f_i^0) = 0.0, i = 1, 2, \dots, m.$$

Hence

$$k_i = k_j u_j(f_j^e). \quad (3.14)$$

(B) Assessing the individual utility functions

Point P_s is indifferent to every point on the indifference curve and there are the following equalities:

$$\begin{aligned} (1 + k k_1 u_1(f_1^0)) \dots (1 + k k_i u_i(f_i^*)) \dots (1 + k k_j u_j(f_j^0)) \dots \\ (1 + k k_m u_m(f_m^0)) = (1 + k k_1 u_1(f_1^0)) \dots (1 + k k_i u_i(f_i^0)) \dots \\ (1 + k k_j u_j(f_j^e)) \dots (1 + k k_m u_m(f_m^0)) \end{aligned}$$

and

$$u_i(f_i^*) = 1.0, u_i(f_i^0) = 0.0, i = 1, 2, \dots, m.$$

Hence

$$u_i(f_i) = ((1 + k k_j) / (1 + k k_j u_j(f_j^e)) - 1) / (k k_j). \quad (3.15)$$

3.4 Concluding Remarks

There are many approaches to the solution of problems possessing multiple objectives. Among the several multiobjective optimization methods the SWT and MAUF methods have here been described and a modified procedure outlined for them. Some applications for these methods as they relate to real problems are discussed in Chapter 4.

CHAPTER 4

APPLICATION TO SOME INDUSTRIAL MANAGEMENT PROBLEMS

4.1 Introduction

Industrial management problems are featured, in most cases, by their multi-level and hierarchical structures. At each level, there are many decision makers with different criteria. Ways of their decision and coordination are usually intuitive rather than logical, but those are well based on their experiences. Meanwhile, for real problems in a private sector, conventional operations research techniques do not work so effectively. The main causes may be summarized as follows:

- (1) A system analyst or a model builder conducts the procedure of structural modeling in his own way, and then a decision maker does not understand or agree the model quite enough.
- (2) The model is usually formulated in a complicated mathematical way, and then it is difficult to understand for a decision maker.
- (3) Though a decision maker has usually various criteria, but the model is often formulated with only one objective. Further, the objective is set by a system analyst to make solution of the problem easy by using a conventional mathematical programming technique, and then it is often different from the criteria of the decision maker.

Considering these causes, the most important point is to pay a severe attention to get a good structural model. For that, the procedure of structural modeling should be performed by an intimate cooperation of the decision maker and the analyst, even if the decision maker does not have enough knowledge of mathematics and/or a computer. Through this procedure of structural modeling, the decision maker acquires confidence of the model and has intention for optimization depending on it. In all the applications to real problems in this chapter, modelings have been performed by the persons concerned, including decision makers. Thus the procedures shown will be applicable to many other problems, though some modification will be needed depending upon features of the problems.

In Section 4.2, a multiobjective optimization procedure is applied to a blending problem of an industrial material formulated with six objectives. The idea of surrogate-worth tradeoff (SWT) method and the method of multiattribute utility function are applied to the problem and the solution of SWT method is compared with that of MAUF method. A modified SWT procedure is used where the solution is sought for by observing the tradeoff curves. In Section 4.3 and 4.4, real applications of interpretive weighted structural modeling (IWSM) method are described. The first application treats a scheduling of a data transmission test and the second one is concerned with an allocation of budget to various sections in a Research and Development Laboratory. The feasibility and effectiveness of IWSM method have been verified through these applications.

In Section 4.5, multiobjective optimization procedure is applied to a decision problem in an inventory management. Decision variables to the problem are production quantities and ordering points in the next period (month). A multiattribute utility function is constructed for evaluating totally the degree of satisfaction of objectives, and it is used, together with a nonlinear programming technique, for optimizing the decision variables. In Section 4.6, multiobjective assessment procedure is applied to the assessment of investment plans in a production firm. The total assessment formula is derived considering both the economical and the technical terms by use of a multiattribute utility function. A real problem of assessing the investments has successfully been dealt with by this procedure.

4.2 Application To Material Blending Problem [31]

In the recent industrial world, many kinds of industrial materials are used and they are mostly made from some kinds of raw materials. By ratios of the adopted raw materials, production cost and various characteristics (mechanical characteristic, chemical characteristic, electrical characteristic, etc.) of a resulting material are changed. Hence, the production cost and the various characteristics are the criteria for a blending problem of industrial materials, and they are generally conflicting with each other. For example, if the production cost is reduced, the values indexing the characteristics will be deteriorated. If the value of

mechanical characteristic is ameliorated at the same production cost, the value of electrical characteristic will be sacrificed. Although, the blending problem in a production firm is different from the problem of public sector, application of optimization methods used in the public sector seems interesting and fruitful as well.

This section first formulates the blending problem of an industrial material as an optimization problem with six objectives. The characteristic values of the material are represented by nonlinear functions of the blending ratios. Second, the idea of the surrogate-worth tradeoff (SWT) method is applied to finding preferred solutions of four decision makers. A modified decision procedure which is mentioned in Section 3.2 is applied where the solution is sought for by observing tradeoff curves. Third, the method of multiattribute utility function (MAUF) is applied to the same problem and the solution is compared with that of SWT method. Several comments are given on the methodologies and the present experiences stressing the viewpoints of practical feasibility and effectiveness.

4.2.1 Formulation of the Blending Problem

The industrial material considered here is a plastic material and it is often used in household appliances. It is made from six raw materials. Those raw materials are polyvinyl chloride resin and fillers (calcium carbonate, stabilizer, etc.). The characteristic values of the material are represented by nonlinear functions of the raw material blending-ratios x_i ($i = 1, 2, \dots, 6$) as follows:

(a) Cost: f_1 (yen)

$$\begin{aligned} f_1 = & (0.231 \cdot 10^1 x_1 + 0.583x_2 + 0.813 \cdot 10^1 x_3 + 0.671x_4 + 0.240x_5 \\ & + 0.10x_6) \cdot 10^3 / (0.696x_1 + 0.620x_2 + 0.299x_3 + 0.102 \cdot 10^1 x_4 \\ & + 0.60x_5 + 0.561x_6), \end{aligned} \quad (4.1)$$

(b) Melting point: f_2 ($^{\circ}\text{C}$)

$$\begin{aligned} f_2 = & 0.140x_2^2 + 0.203x_3^2 + 0.114x_1 x_3 - 0.105x_1 x_4 + 0.768 \cdot 10^{-1}x_1 x_6 \\ & - 0.148 \cdot 10^1 x_1 - 0.112 \cdot 1x^2 x_2 - 0.167 \cdot 10^2 x_3 + 0.307 \cdot 10^1 x_4 \\ & - 0.444 \cdot 10^1 x_6 + 0.540 \cdot 10^3, \end{aligned} \quad (4.2)$$

(c) Coefficient of thermal expansion: f_3 ($10^{-5}/^{\circ}\text{C}$)

$$f_3 = 0.153 \cdot 10^1 x_2 + 0.106 \cdot 10^1 x_3 - 0.227 x_4 + 0.351 x_6 + 0.206 \cdot 10^2 \quad (4.3)$$

(d) Acid-resisting characteristic: f_4 (%)

$$\begin{aligned} f_4 = & 0.117 \cdot 10^1 x_6 - 0.160 x_1 x_4 - 0.881 x_2 x_3 - 0.830 \cdot 10^{-1} x_2 x_4 \\ & - 0.990 x_2 x_4 - 0.218 x_4 x_5 + 0.336 x_1 + 0.177 \cdot 10^2 x_2 + 0.115 \cdot 10^2 x_3 \\ & + 0.986 \cdot 10^1 x_4 + 0.505 \cdot 10^2 x_5 - 0.174 \cdot 10^2 x_6 - 0.189 \cdot 10^3, \end{aligned} \quad (4.4)$$

(e) Alkaline-resisting characteristic: f_5 (%)

$$\begin{aligned} f_5 = & 0.230 \cdot 10^{-1} x_1^2 - 0.735 \cdot 10^{-1} x_4^2 + 0.445 \cdot 10^{-1} x_1 x_3 - 0.915 \cdot 10^{-1} x_2 x_5 \\ & - 0.124 x_2 x_4 + 0.255 \cdot 10^{-1} x_3 x_4 - 0.10 x_4 x_5 - 0.164 \cdot 10^{-1} x_1 \\ & + 0.336 \cdot 10^1 x_2 - 0.146 \cdot 10^1 x_3 + 0.543 \cdot 10^1 x_4 + 0.259 \cdot 10^4 x_5 \\ & + 0.114 \cdot 10^4 x_6 - 0.728 \cdot 10^2, \end{aligned} \quad (4.5)$$

(f) Aqua-resisting characteristic: f_6 (%)

$$\begin{aligned} f_6 = & 0.148 x_6^2 + 0.258 \cdot 10^{-1} x_1 x_2 - 0.765 \cdot 10^{-1} x_5 x_6 + 0.189 \cdot 10^1 x_1 \\ & - 0.70 \cdot 10^{-1} x_1 x_5 - 0.830 x_2 - 0.75 x_3 + 0.714 x_5 + 0.314 \cdot 10^1 x_6 \\ & + 0.839 \cdot 10^2. \end{aligned} \quad (4.6)$$

The functions f_1 through f_6 are obtained by using the multiple regression analysis. These are considered to be the objective functions of the problem.

The constraints on x_i are given by:

(g) Blending constraint

$$\sum_{i=1}^6 x_i = 100. \quad (4.7)$$

$$0 \leq x_i \leq 100, \quad i = 1, 2, \dots, 6. \quad (4.8)$$

Hence, our blending problem is formulated into the following optimization problem with six objectives.

$$\begin{aligned} & \text{Minimize } [f_1(x), f_2(x), f_3(x), f_4(x), f_5(x), f_6(x)], \\ & \text{subject to} \end{aligned} \quad (4.9)$$

$$\sum_{i=1}^6 x_i = 100. \quad (4.10)$$

$$0 \leq x_i \leq 100, \quad i = 1, 2, \dots, 6. \quad (4.11)$$

4.2.2 Ranking the Objectives by IWSM Method

There are four decision makers who are specialists of the present material. We have had a discussion about an optimization procedure of this problem. The objectives of this blending problem are ranked in the degree of importance by use of the IWSM method. Each of the decision makers is given a series of inquiries of the form. "Is f_i more important than f_j ? If yes, how much is the degree? The numeral 2 means that the degree is 'clearly more important' and the numeral means that the degree is 'some what more important'." The replied degrees of each decision maker are arranged in a matrix such as shown in Figure 4.1. Figure 4.2 shows the digraph obtained by the IWSM method. By discussion about the each digraph, relative rank of the objectives is decided as shown in Figure 4.3. Then the optimization procedure is separated into two steps. The first step is an optimization procedure with three objectives and the second step is an optimization procedure with all six objectives.

$f_i \backslash f_j$	f_1	f_2	f_3	f_4	f_5	f_6
f_1						
f_2						
f_3	1	1				
f_4	1	2	2		2	
f_5	2	2	1			
f_6	2	2	2	2	1	

Figure 4.1. An example of individual matrix obtained from replies.

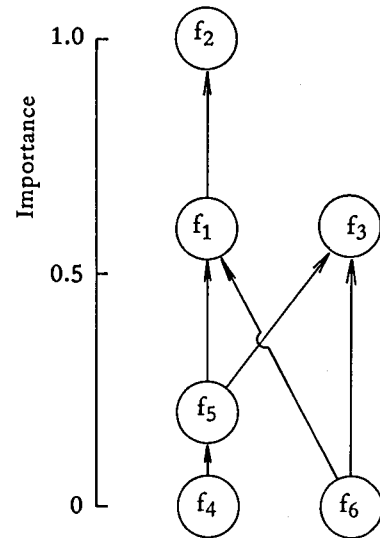


Figure 4.2. Relative significance of the objectives (by a decision maker).

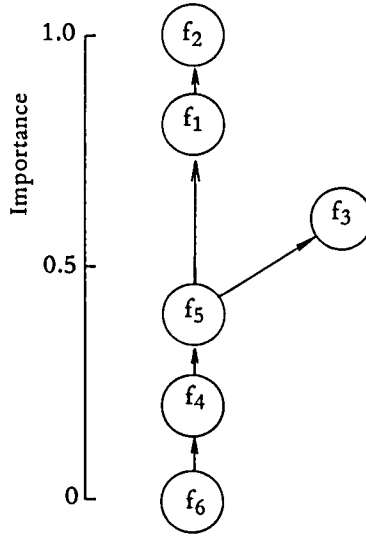


Figure 4.3. Rank of the objectives.

4.2.3 Optimization Problem with Three Objectives

According to the result of ranking objectives, first our blending problem is formulated into the following optimization problem with three objectives:

$$\text{Minimize } [f_1(x), f_2(x), f_3(x)] \quad (4.12)$$

subject to

$$0 \leq f_j(x) \leq 5. \quad j = 4, 5, 6 \text{ and } (4.10), (4.11). \quad (4.13)$$

(A) SWT procedure

The surrogate-worth tradeoff method which is mentioned in Section 3.2 is applied to finding preferred solution of the problem. The ϵ -constraint approach is used for finding noninferior points. Table 4.1 shows noninferior points and the responses of a decision maker. The first two columns of Table 4.1 show the selected values of ϵ_2 and ϵ_3 (or equivalently of f_2 and f_3). The third column indicates the corresponding noninferior value of the objective f_1 , while the fourth and the fifth columns tradeoff ratios. The values of the surrogate-worth functions given by the decision maker are tabulated in the last two columns. Figure 4.4 shows the preference bands of the four decision makers. In the figure, there is no common band where all the four decision makers can find some parts of

their own preference band. Hence, by repeating discussions with them we have decided the common band to be:

$$f_1 = 1500, \quad f_2 = 270, \quad f_3 = 55.$$

Table 4.1. Noninferior points and the responses of a decision maker

f_2	f_3	f_1	λ_{12}	λ_{13}	W_{12}	W_{13}
310	45	1590	20	96	+10	+5
300	45	1720	21	97	+6	+5
300	50	1410	13	60	+5	0
290	50	1550	14	62	+3	0
280	50	1680	16	70	0	-1
270	50	1860	19	85	-2	-3
260	50	2060	20	108	-5	-5
280	55	1400	9	42	+1	0
270	55	1500	11	44	0	0
260	55	1650	12	48	0	-1
250	55	1800	14	66	0	-3
240	55	1960	19	83	-3	-5

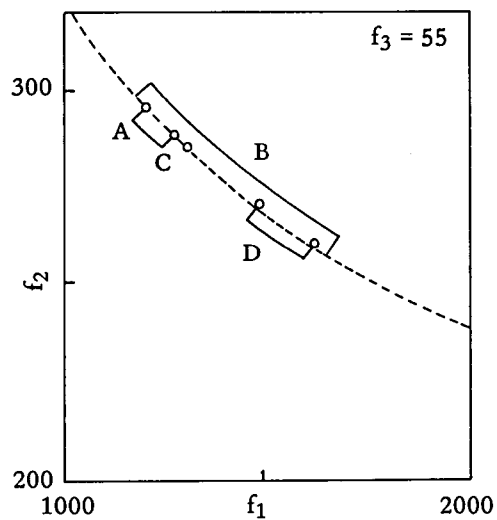


Figure 4.4. Preference bands of the decision makers (1).

After that, the preferred decision vector is found by solving the following optimization problem:

$$\begin{aligned} & \text{Minimize } f_1(x), \\ & \text{subject to} \\ & 0 \leq f_2(x) \leq 270, 0 \leq f_3(x) \leq 55, \\ & 0 \leq f_j(x) \leq 5, j = 4, 5, 6 \text{ and (4.10), (4.11).} \end{aligned}$$

The resulting values of the decision vector and the objective functions are:

$$\begin{aligned} x_1 &= 20.0, & x_2 &= 18.8, & x_3 &= 3.2, \\ x_4 &= 19.8, & x_5 &= 19.5, & x_6 &= 18.7, \\ f_1 &= 1500, & f_2 &= 270, & f_3 &= 55, \\ f_4 &= 5, & f_5 &= 5, & f_6 &= 5. \end{aligned} \tag{4.14}$$

Although this procedure for finding preferred solution is very useful, it takes a lot of time. Also the decision makers feel it very troublesome. Therefore a modified procedure mentioned below is applied to the same problem.

(B) Modified procedure

By considering the result of objective ranking, tradeoff curves are on the f_1, f_2 -plane as shown in Figure 4.5. The value of the objective f_3 is taken to be the running parameter and the values of other objectives are fixed as $f_j(x) = 5, j = 4, 5, 6$. By observing the tradeoff curves, the preferred solution is sought with help of the assessment function W_k to tradeoff curves. Figure 4.6 shows the tradeoff curves and the responses of a decision maker. In this example, the value of W_3 is equal to zero when f_3 is equal to 55. Hence the preference band is decided on the tradeoff curve with $f_3 = 55$. The other decision makers have also decided their preference bands on the tradeoff curve also with $f_3 = 55$. The preference bands of the four decision makers are shown in Figure 4.7. By comparing Figure 4.7 with Figure 4.4, the preference bands in Figure 4.7 are wider than those in Figure 4.4. This means that the modified procedure can not locate a preferred point so sharply. But this procedure has some desirable features as follows:

- (1) It is less time consuming and the decision makers do not feel so troublesome.
- (2) The decision by viewing the tradeoff curves is much easier than the decision

by reading the values of objectives and tradeoff ratios.

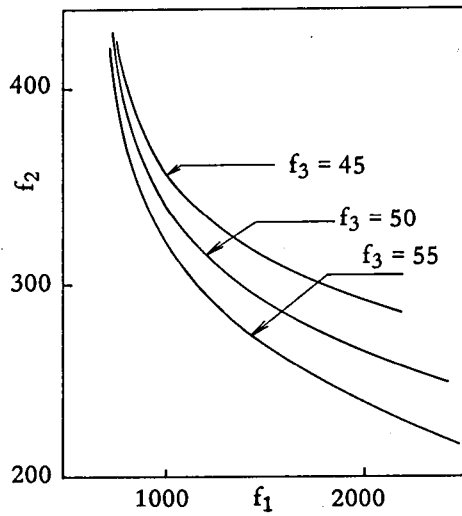


Figure 4.5. Tradeoff curves.

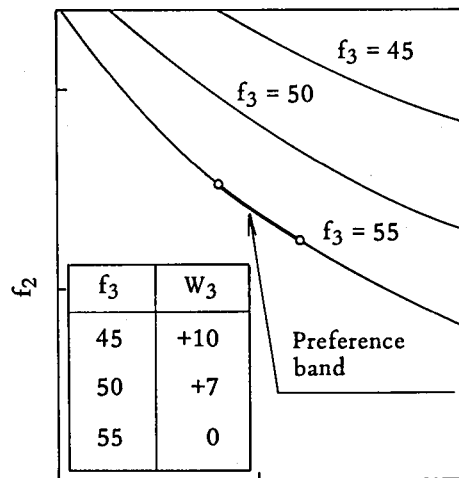


Figure 4.6. Tradeoff curves and response of a decision maker.

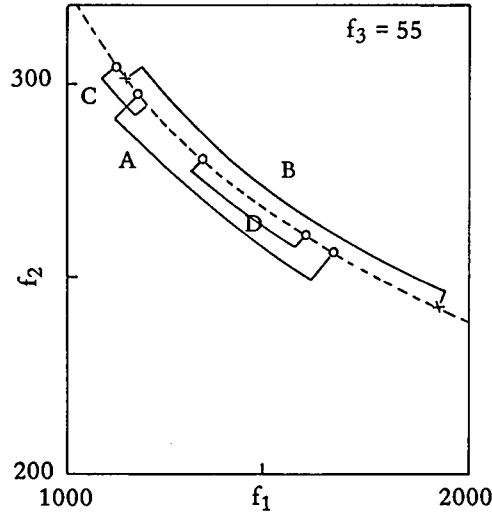


Figure 4.7. Preference bands of the decision makers (2).

4.2.4 Six-Objective Optimization Problem

As a result of investigation of the three-objective optimization problem, the modified procedure is adopted here and the value of objective f_3 is fixed as 55. According to the rank of objectives, the objective f_5 is taken as the running parameter of the tradeoff curves. Observing the tradeoff curves, the solution is sought for an each decision maker has decided the preference band on the tradeoff curves by use of the modified procedure. After all of the decision maker have decided the preference bands, we have had a discussion about each preference band. As a result of this discussion, the value of objective f_5 is fixed as 2.5. In the same procedure the preference band on the tradeoff curves are decided and the value of objectives are fixed as $f_4 = 2.5$ and $f_6 = 2.5$. Table 4.2 shows the responses of the four decision makers.

Through these steps, an outline of the preference band for each decision maker has been found and the final information about each preference band have been gotten by use of the surrogate-worth function. Figure 4.8 shows the each preference band. In Figure 4.8, there is no common band, and we have had a discussion again about each preference band. As a result of the discussion, the common band is decided as follows:

$$f_1 = 1500, \quad f_2 = 270, \quad f_3 = 55,$$

$$f_4 = 2.5, \quad f_5 = 2.5, \quad f_6 = 2.5.$$

Table 4.2. Responses of the decision makers.

		w_k			
f_k		A	B	C	D
f_5	2.5	0	0	2	0
	10	-5	-2	-8	-5
	15	-10	-10	-10	-7
f_4	2.5	0	0	-2	0
	5	-2	0	-10	-4
	5	-5	-10	-10	-10
f_6	2.5	0	0	-2	0
	5	-1	0	-5	-5

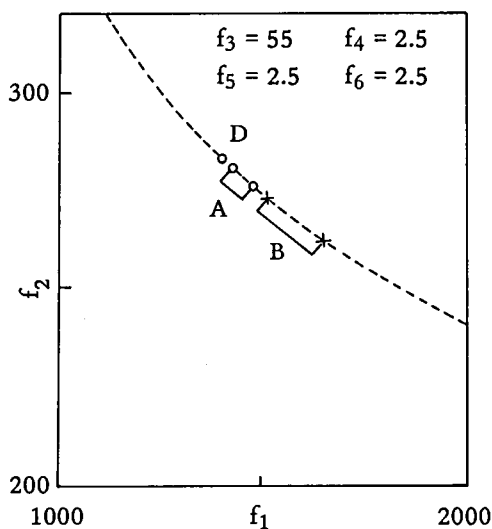


Figure 4.8. Preference bands of the decision makers (3).

The preference band of a decision maker, Mr. C, can not be found in Figure 4.8, but he has agreed to accept the common band after another discussion. The preferred decision vector has been found by solving the following problem.

$$\begin{aligned}
& \text{Minimize } f_1(x), \\
& \text{subject to} \\
& 0 \leq f_2(x) \leq 270, 0 \leq f_3 \leq 55. \\
& 0 \leq f_j(x) \leq 2.5, j = 4, 5, 6 \text{ and (4.10), (4.11).}
\end{aligned}$$

Thus, the following decision vector and the values of the objective functions are decided:

$$\begin{aligned}
x_1 &= 21.5, & x_2 &= 18.8, & x_3 &= 3.1, \\
x_4 &= 17.7, & x_5 &= 21.8, & x_6 &= 17.8, \\
f_1 &= 1530, & f_2 &= 270, & f_3 &= 55, \\
f_4 &= 2.5, & f_5 &= 2.5, & f_6 &= 2.5.
\end{aligned} \tag{4.15}$$

4.2.5 Comparison of the Solution of MAUF Method with That of SWT Method

Our blending problem is formulated into the optimization problem with three objectives or with six objectives, and the SWT method has been already applied to this problem. In this section, the three-attribute utility function is constructed and the indifference curves of the utility function are compared graphically with the preference band on the tradeoff curve obtained by SWT method.

(A) Construction of multiattribute utility function using the lotteries

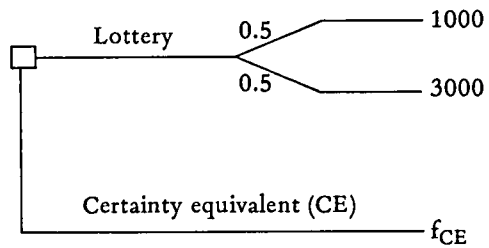
Let f_i^0 and f_i^* denote the least and the most desirable values of attribute f_i , respectively. Table 4.3 shows the ranges of the three attributes decided by discussions among persons in charge. The individual utility function $u_i(f_i)$ have been decided by questionnaires to the decision maker using the 50-50 lottery. Figure 4.9 shows the 50-50 lottery to the individual utility function $u_1(f_1)$. Figure 4.10 through 4.12 show the individual utility functions $u_i(f_i)$ ($i = 1, 2, 3$).

Table 4.3. Allowable ranges of f_i

f_i	f^*	f^0
f_1 (yen)	1000	3000
f_2 ($^{\circ}\text{C}$)	150	300
f_3 ($10^{-5}/^{\circ}\text{C}$)	50	56

f_{CE}	CE	Lottery
1000	○	
1200		
1400		
1600		
1800		
2000		
2200		
2400		
2600		
2800		
3000		○

○ : preferable



Consider a choice between a 50-50 lottery yielding either $f_1 = 1000$ or $f_1 = 3000$ and an option giving you the value of f_{CE} for sure. Which would you prefer? If you think that the lottery is indifferent to the value of f_{CE} , write the value of f_{CE} .

Figure 4.9. The 50-50 lottery to $u_1(f_1)$.

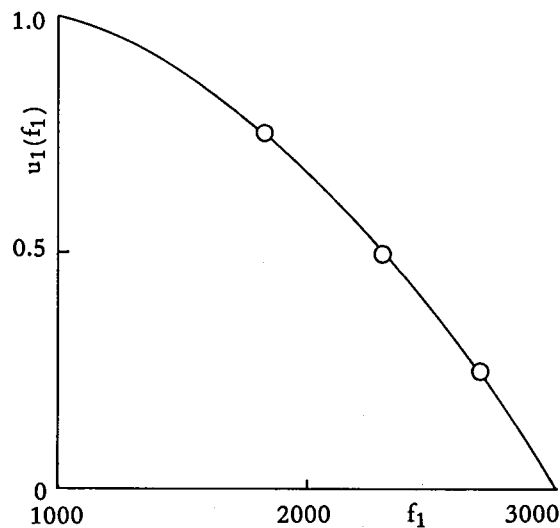


Figure 4.10. Utility function for f_1 .

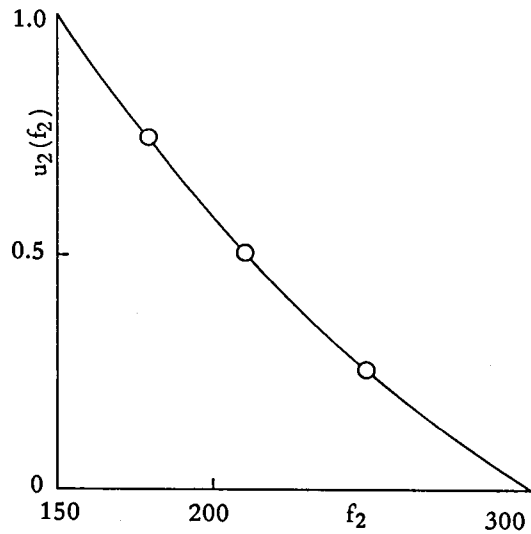


Figure 4.11. Utility function for f_2 .

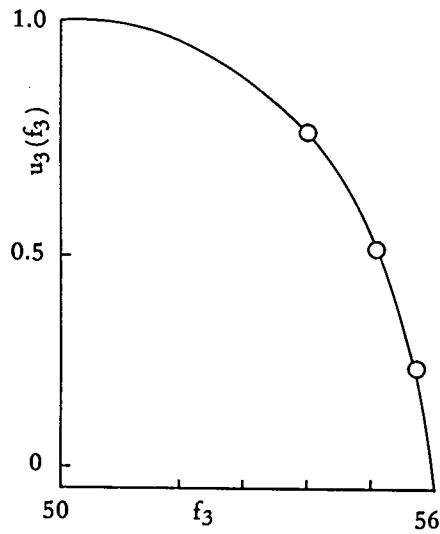


Figure 4.12. Utility function for f_3 .

The scaling constants k_i are evaluated by using the lottery such as shown in Figure 3.3. Table 4.4 shows the values of k_i and k , evaluated from the responses of the decision makers.

Table 4.4. Values of k_i and k

k_i, k	A	B	C	D
k_1	0.35	0.50	0.10	0.25
k_2	0.35	0.60	0.80	0.90
k_3	0.25	0.20	0.10	$0.1 \cdot 10^{-4}$
k	0.165	-0.621	0	-0.667

As a result of the SWT procedure, the preference bands of decision makers are obtained along the tradeoff curves with $f_3 = 55$ on the f_1, f_2 -plane. The indifference curves of the utility function are depicted on the f_1, f_2 -plane. Figure 4.13 shows the preference band together with the indifference curves obtained from the utility function of Mr. A. In Figure 4.13, we see that the tangency point of the tradeoff curve and the indifference curve is close to the preference band.

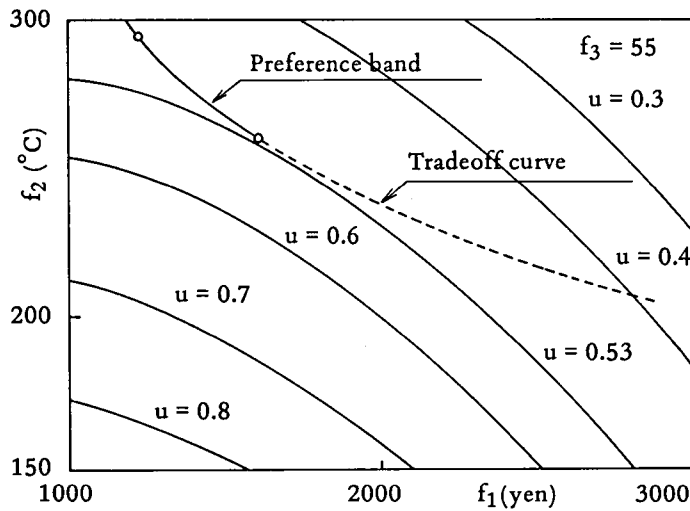


Figure 4.13. Indifference curves of the utility function and preference band on the tradeoff curve (by Mr. A).

By comparing the results of four decision makers, we have found that the tangency points of Mr. A and Mr. B are very close to their preference bands, but the tangency points of Mr. C and Mr. D are far from their preference bands. The

gaps are caused by some factors. We have had some discussions about the gaps and have concluded that the main factor is the lack of experience to the lottery.

Hence, we have verified this conclusion as follows.

First of all the lottery such as shown in Figure 4.14 have been prepared. In Figure 4.14, f'_2 satisfies the following equation:

$$u_2(f'_2) = 0.5. \quad (4.16)$$

If the replied probability is symbolized as $p_2^{0.5}$, $p_2^{0.5}$ satisfies the following equation:

$$p_2^{0.5} = p_2^*/2. \quad (4.17)$$

Table 4.5 shows the replied probabilities $p_2^{0.5}$ and p_2^* . In Table 4.5, the replied probabilities of Mr. A and Mr. B satisfy Equation (4.17), but those of Mr. C and Mr. D do not satisfy it. The reason of this inconsistency is thought as the error in assessing the individual utility functions using the 50-50 lottery or the error in evaluating the scaling constants using the lottery such as shown in Figure 3.3.

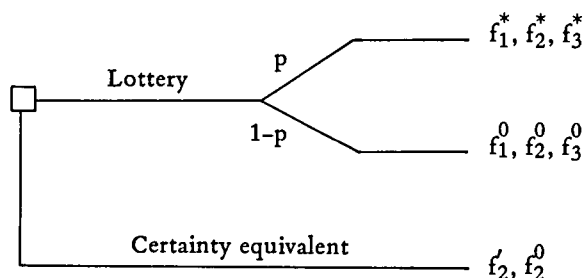


Figure 4.14. Checking the scaling constants.

Table 4.5. Probability corresponding to the utility

P	A	B	C	D
p_2^*	0.35	0.60	0.80	0.90
$p_2^{0.5}$	0.25	0.30	0.70	0.15

(B) Construction of multiattribute utility function using the indifference curves

The multiattribute utility function is constructed by using the indifference

curves which are drawn by the decision makers, Mr. C and Mr. D. The procedure to draw an indifference curve is shown in Section 3.3.3. Figure 4.15 shows an indifference curve drawn by Mr. D. The scaling constant is evaluated by Equation (3.14) and the individual utility functions are assessed by Equation (3.15). Figure 4.16 shows the preference band on the tradeoff curve and the indifference curves of the utility function by Mr. D. In Figure 4.16, the tangency point of the tradeoff curve and the indifference curve is close to the preference band. As for Mr. C, the tangency point is also close to the preference band.

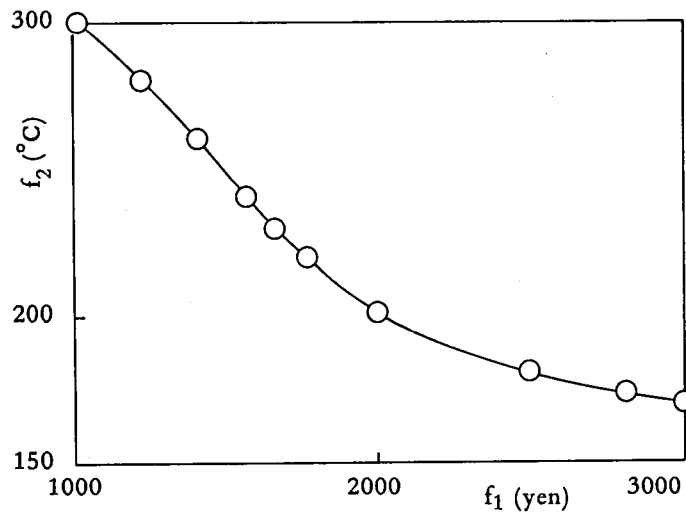


Figure 4.15. Indifference curve given by Mr. D.

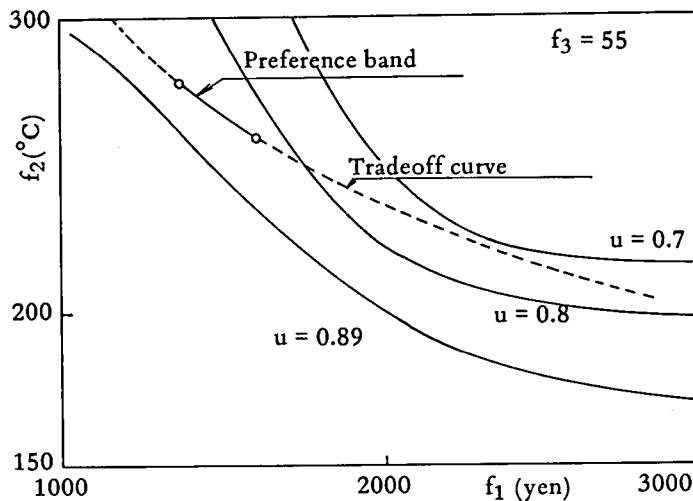


Figure 4.16. Indifference curves and the preference by Mr. D.

4.2.6 Concluding Remarks

The surrogate-worth tradeoff method and the multiattribute utility function method are applied to the real blending problem with six objectives. The preference bands of four decision makers are compared with the indifference curves due to their utility functions. As a result of real application, we have found that these methods work satisfactorily.

The SWT method is useful to find the preference band, but in practice, it takes a lot of time to identify the indifference band. The proposed modified procedure using IWSM method works more effectively.

The MAUF method is useful to find the preferred solution. Using the MAUF, the multiobjective optimization problem can be solved as the one-objective optimization problem. But in practice, the numerical construction of MAUF is often not so easy. In the present study, two decision makers among four can not construct the MAUF so nicely. To these decision makers, we have applied the procedure of constructing the MAUF by use of their indifference curves. This procedure seems effective for helping the decision makers when they are puzzled by the lotteries.

4.3 Application to a Scheduling Problem [33]

IWSM has been applied to a lot of real complex problems. Among them, in this section, a scheduling problem is discussed. For a scheduling problem, the program evaluation and review technique (PERT) is usually applied. By using the PERT, the critical path, the start times, the finish times and the floats can be easily calculated, but an arrow diagram of a problem has to be constructed previously by a user. If the number of elements is not so large, it is not troublesome to construct an arrow diagram. But, if the number of elements is large, it is very difficult to construct an arrow diagram.

For scheduling of a data transmission test, a flow diagram and its critical path are drawn up by IWSM. The start times, the finish times and the floats of each activity are calculated by use of the total relation matrix which is defined in Section 2.4.

4.3.1 Flow Diagram and Critical Path

A person in charge of the data transmission test has given the list of activities and estimated the duration (in days) of each activity. Inquiries about the contextual relations among activities are put to him. His replies are arranged in a matrix as shown in Figure 4.17. If he thinks that the i th activity must be finished to start the j th activity, a check mark “√” is inserted to the i, j -entry of the matrix. By replacing all the check marks on the i th row by the duration of the i th activity, the weighted adjacency matrix of Figure 4.18 is obtained. Then the total relation matrix of Figure 4.19 is constructed by means of Equations (2.13) ~ (2.15). The additive rule is used in the present problem.

$i \backslash j$	1	2	3		16	17	18	Duration (days)
1		√	√					5
2			√					5
3								15
⋮								
⋮								
⋮								
⋮								
16								10
17								15
18								0

Figure 4.17. Replies to the inquiries on time scheduling.

i \ j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		5	5	5														
2			5						5									
3				15														
4																	20	
5				2		2												
6				5														
7				15													15	
8																		
9													2	2				
10												2						
11									2							2		
12													5		5			
13								30										
14								20										
15								40										
16										10								
17																		15
18																		

Figure 4.18. Weighted adjacency matrix.

i \ j	18	17	4	8	3	6	7	13	14	15	5	9	12	2	10	1	16	11
18																		
17	15																	
4	35	20																
8	30	15																
3	50	35	15															
6	40	25	5															
7	50	35	15															
13	60	45		30														
14	50	35		20														
15	70	55		40														
5	42	27	7			2												
9	62	47		32				2	2									
12	75	60		45				5		5								
2	67	52	20	37	5			7	7			5						
10	77	62		47				7		7			2					
1	72	57	25	42	10			12	12			10		5				
16	87	72		57				17		17			12		10			
11	89	74		59				19		19			14		12		2	

Figure 4.19. Total relation matrix.

The flow diagram is drawn in the following way. The coordinates d_i^c and d_i^r of the i th activity are calculated by Equations (2.19) and (2.20), respectively. Table 4.6 shows the values of d_i^c and d_i^r . If the initial time is taken to be zero, d_i^c represents the coordinate of the i th activity on the real time axis; while, if the final time is taken to be zero, d_i^r represents the coordinate of the i th activity on the reverse time axis. Figure 4.20 shows the flow diagram obtained with use of d_i^c . The arrows in the diagram are drawn by referring to the skelton matrix.

The critical path is found as follows. The duration of the critical path, L , is given by

$$L = \text{Max} \{d_i^c\}. \quad (4.18)$$

The necessary and sufficient condition for the activities contained in the critical path is

$$L - d_i^c - d_i^r = 0. \quad (4.19)$$

In the present problem, L is eighty-nine days and the activities in the critical path are 11, 16, 10, 12, 15, 8, 17 and 18. The critical path is shown in Figure 4.20 by thick arrows.

Table 4.6. d_i^c and d_i^r

$i \backslash$	1	2	3	4	5	6	7	8	9
d_i^c	0	5	10	25	0	2	0	59	10
d_i^r	72	67	50	35	42	20	50	30	62

$i \backslash$	10	11	12	13	14	15	16	17	18
d_i^c	12	0	14	19	12	19	2	74	89
d_i^r	77	89	75	60	50	70	87	15	0

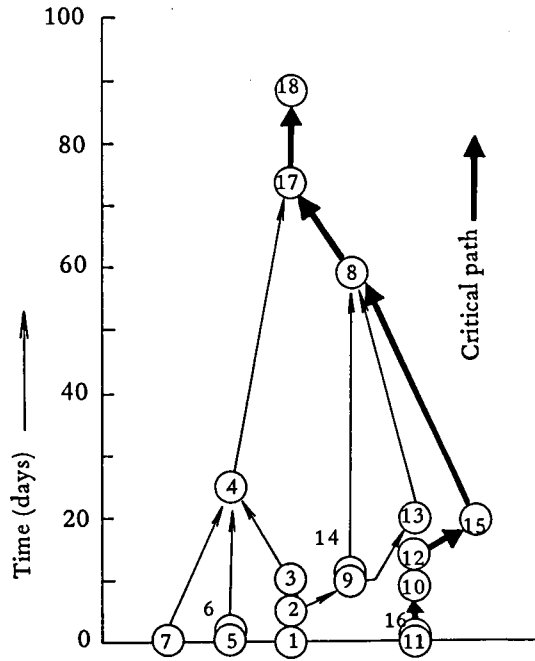


Figure 4.20. Flow diagram for the data transmission test.

4.3.2 Start Times, Finish Times and Floats

The start times, the finish times and the floats of the i th activity are defined as follows:

- 1) Earliest start time: $ES = d_i^c$, (4.20)

- 2) Latest start time: $LS = L - d_i^r$, (4.21)

- 3) Earliest finish time: $EF = d_i^c + n_i$, (4.22)

- 4) Latest finish time: $LF = L - d_i^r + n_i$, (4.23)

- 5) Total float: $TF = L - d_i^r - d_i^c$, (4.24)

- 6) Free float: $FF = ES_j - d_i^c$, (4.25)

- 7) Independent float: $IF = L - d_i^r - ES_j$. (4.26)

Where n_i is the duration of the i th activity, and ES_j is the earliest start time of the j th activity which must start right after the i th activity. Table 4.7 lists the start times, the finish times and the floats of the activities.

Table 4.7. Start times, finish times and floats.

Activity Item	1	2	3	4	5	6	7	8	9
ES	0	5	10	25	0	2	0	59	10
LS	17	22	39	54	47	39	39	59	27
EF	5	10	25	45	2	17	15	74	12
LF	22	27	54	74	49	54	54	74	29
TF	17	17	29	29	47	37	39	0	17
FF	0	0	0	29	0	8	10	0	0
IF	17	17	29	0	47	29	29	0	17

Activity Item	10	11	12	13	14	15	16	17	18
ES	12	0	14	19	12	19	2	74	89
LS	12	0	14	29	39	19	2	74	89
EF	14	2	19	49	32	59	12	89	89
LF	14	2	19	59	59	59	12	89	89
TF	0	0	0	10	27	0	0	0	0
FF	0	0	0	10	27	0	0	0	0
IF	0	0	0	0	0	0	0	0	0

4.3.3 Concluding Remarks

The scheduling by IWSM has some features as compared with the PERT:

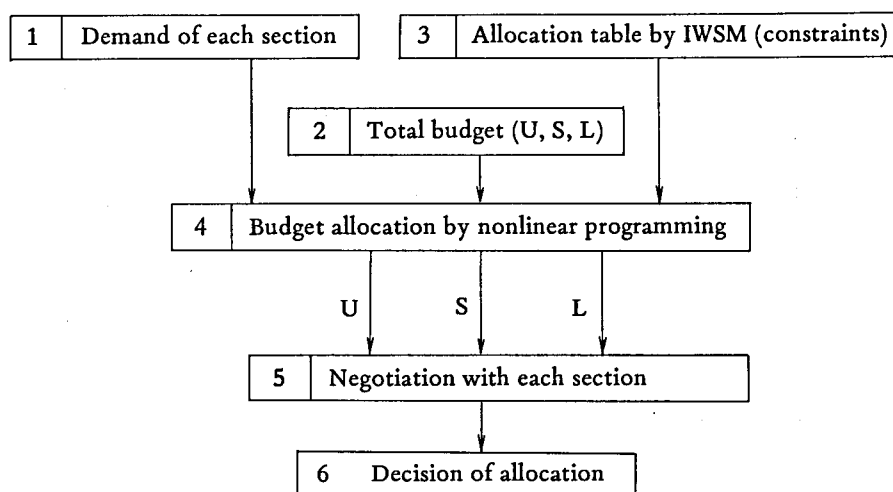
- (1) Owing to the inquiry about the contextual relations among activities, construction of the flow diagram can be started smoothly.
- (2) The whole procedure is proceeded quite systematically. Moreover, an interactive revision of the flow diagram can easily be made, e. g., in case of cycling.
- (3) The activity-on-node diagram does not need a dummy activity.

Applying IWSM method to a scheduling problem, the persons concerned can get easily each flow diagram, critical path, start times, finish times and floats of

the problem and they can discuss each other about each diagram. If there are some parties among the persons concerned, diagrams of each party can be gotten and they can discuss about the diagrams. As a result of this, a scheduling problem will be solved by common consent.

4.4 Application To A Budget--Allocation Problem [33]

An allocation of budget is always a matter of serious concern of participants. This section presents an application of IWSM to a budget-allocation procedure in a Research and Development Laboratory of a firm. The flow diagram of Figure 4.21 illustrates the whole procedure of budget-allocation. The problem is how to allocate the total budget to each of the twenty-two sections in the Laboratory.



U: Upper limit, S: Standard, L: Lower limit

Figure 4.21. Procedure of budget-allocation by IWSM.

In the first step, each section chief lays a demand for his budget. The second step is to determine the total budget T of the Laboratory between the maximum T_U and the minimum T_L . The third step, a key step of the procedure, is dealt with by IWSM and it will be detailed in the following.

4.4.1 Relative Allocation Ratio

At the outset of the third step, the section chiefs are given a series of inquiries of the form “Should the budget of the j th section S_j be more than or equal to that of the i th section S_i ? If yes, how much is the ratio?” The replied ratio a_{ij} ($i, j = 1, 2, \dots, 22; i \neq j$) of each chief are arranged in a matrix, called the *individual relative allocation-ratio matrix* such as shown in Figure 4.22. If the reply is “No” to the first question, a_{ij} is zero. Hence, $a_{ij} \geq 1$ or $a_{ij} = 0$.

From the twenty-two individual relative allocation-ratio matrices is derived the *aggregated relative allocation-ratio matrix* whose i, j -entry is the median a_{ij}^* of the i, j -entries of the individual matrices. Figure 4.23 shows the aggregated matrix thus obtained, and this is taken for the weighted adjacency matrix in IWSM. Then the total relation matrix is constructed by use of Equations (2.13), (2.14) and (2.16). The multiplicative rule is used in the present problem because the relation is *ratio*. Correspondingly, d_i^C and d_i^R of Equations (2.19) and (2.20) are also to be considered as *ratio*.

	S_1	S_2	S_3		S_j		S_N
S_1		a_{12}	a_{13}		a_{1j}		a_{1N}
S_2	a_{21}		a_{23}		a_{2j}		a_{2N}
S_3	a_{31}	a_{32}			a_{3j}		a_{3N}
$\vdots$							
S_i	a_{i1}	a_{i2}	a_{i3}		a_{ij}		a_{iN}
$\vdots$							
S_N	a_{N1}	a_{N2}	a_{N3}		a_{Nj}		

Figure 4.22. Individual relative allocation-ratio matrix.

i \ j	S ₁	S ₂₁	S ₃	S ₄	S ₅		S ₁₀	S ₂₀	S ₂₁	S ₂₂
S ₁		1	20	3	4		6	1	1	4
S ₂	0		19	3	3		6	1	1	4
S ₃	0	0		0	0		0	0	0	0
S ₄	0	0	9		1		2	0	0	1
S ₅	0	0	7	0			1	0	0	0
⋮										
S ₁₉	0	0	4	0	0			0	0	0
S ₂₀	0	0	10	1	2		4		0	3
S ₂₁	0	0	10	1	2		4	0		2
S ₂₂	0	0	6	0	0		1	0	0	

Figure 4.23. Aggregated allocation-ratio matrix obtained from the replies.

Now we are going to define a rate of allocation to each section. As a preliminary, a new ratio R_i ($i = 1, 2, \dots, 22$) are defined by

$$R_i \triangleq \begin{cases} 1, & \text{if the } i\text{th section is at the problem level,} \\ d_i^c, & \text{if otherwise.} \end{cases} \quad (4.27)$$

The ratio R_i , the *ratio to the bottom*, implies the ratio of the i th sectional budget to the least sectional budget. In terms of R_i , a rate of allocation to the i th section is defined by

$$X_i \triangleq R_i / \sum_{i=1}^{22} R_i, \quad i = 1, 2, \dots, 22. \quad (4.28)$$

The rate X_i may seem to give the final solution. However, in practice, we have found the following modification is more appropriate. That is, the sum of the i th column entries of the aggregated relative allocation-ratio matrix,

$$P_i = \sum_{j=1}^{22} a_{ji}^*, \quad (4.29)$$

and its normalized value

$$Y_i \triangleq P_i / \sum_{k=1}^{22} P_k, \quad (4.30)$$

are calculated. The value of Y_i also gives an evaluation of the i th sectional budget

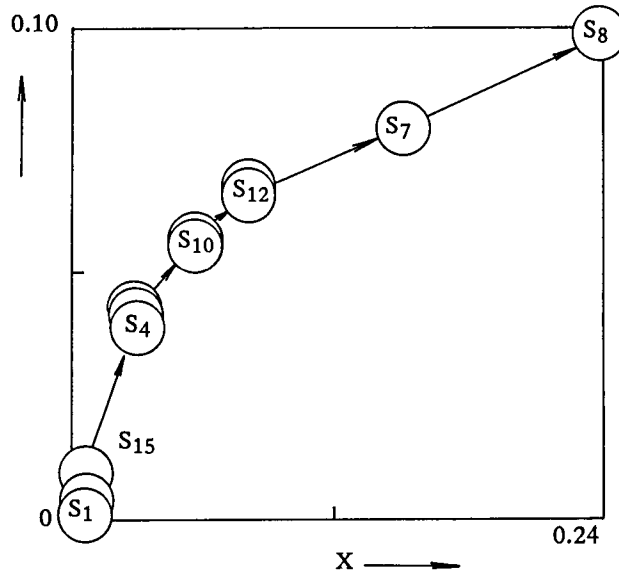
relative to the others. Then, after some discussions and trials, we have decided to define the following rate for practical use:

$$W_i \triangleq Q_i / \sum_{k=1}^{22} Q_k, \quad i = 1, 2, \dots, 22, \quad (4.31)$$

where

$$Q_i \triangleq (2X_i^2 + Y_i^2)^{1/2}. \quad (4.32)$$

Figure 4.24 shows the configuration of the sectional budgets on the X, Y-plane and Table 4.8 is the list of W_i . The cycles listed on Figure 4.24 means the cycles (i. e., groups of equivalents) appeared by using only X_i .



Cycle 1: S1, S2, S15, S18, S20, S21

Cycle 2: S4, S5, S6, S8, S9, S11, S14, S16, S17, S22

Cycle 3: S10, S19

Cycle 4: S12, S13

Figure 4.24. Configuration of the sectional budgets.

Table 4.8. Values of W_i (%)

Section	W_i	Section	W_i	Section	W_i	Section	W_i
S ₁	0.5	S ₇	11.2	S ₁₃	7.0	S ₁₉	5.7
S ₂	0.6	S ₈	3.9	S ₁₄	4.4	S ₂₀	1.7
S ₃	17.0	S ₉	3.8	S ₁₅	1.7	S ₂₁	1.6
S ₄	3.5	S ₁₀	5.7	S ₁₆	4.4	S ₂₂	4.0
S ₅	3.8	S ₁₁	4.2	S ₁₇	4.2		
S ₆	3.6	S ₁₂	7.0	S ₁₈	0.5		

4.4.2 Upper Limit and Lower Limit of the Allocation Amount

For the budget allocation to each section, two additional rules are laid down. The first is that, for the i th section, a demand less than some lower limit, say L_i , will be approved unconditionally. The second is that any budget can not exceed some upper limit, say U_i .

The amounts of U_i and L_i for each section are determined as follows. First, the upper limit of the total budget, T_U , and its lower limit. T_L , are set up by discussions among the persons in charge. Second, the maximum value of U_i for any section i . U_{MAX} , and the maximum value of L_i , L_{MAX} , are set up again by discussions. Then U_i and L_i for each section are determined by

$$U_i = (U_{MAX} - a) W_i / W_{MAX} + a, \quad (4.33)$$

$$L_i = (L_{MAX} - b) W_i / W_{MAX} + b, \quad i = 1, 2, \dots, 22, \quad (4.34)$$

where

$$a = (T_U W_{MAX} - U_{MAX}) / (22W_{MAX} - 1), \quad (4.35)$$

$$b = (T_L W_{MAX} - L_{MAX}) / (22W_{MAX} - 1), \quad (4.36)$$

$$W_{MAX} = \max_i W_i \quad (4.37)$$

Figure 4.25 illustrates the ways of determination described above. Table 4.9 is the list of U_i and L_i thus determined.

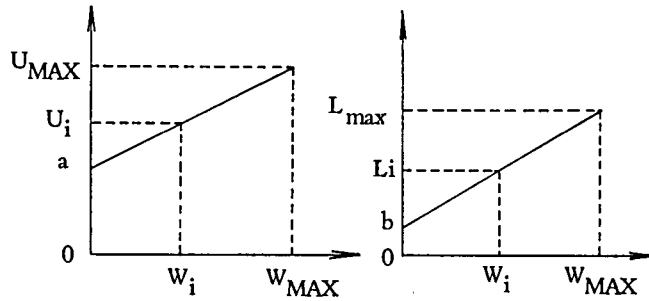


Figure 4.25. Determination of the upper and lower limit amounts of the allocation.

Table 4.9. Values of U_i , L_i for each section

Section	U_i	L_i
S_1	70	10
S_2	80	10
S_3	300	100
S_4	120	20
S_5	120	30
⋮	⋮	⋮
S_{20}	90	20
S_{21}	90	20
S_{22}	120	30

4.4.3 Final allocation by a Nonlinear Programming

Here the final procedure of the budget allocation is described. At the outset, all the sections are divided into two groups. The first group consists of the sections whose demands are less than or equal to their lower limit amounts L_i , and the second group consists of the others, i. e., the sections whose demands are more than their lower limit amounts L_i .

To the sections in the first group, the demanded budgets are allocated

unconditionally. Let T_s be the sum of those unconditionally allocated amounts. Then, the problem is how to allocate the remained total budget $T_R = T - T_s$, to the sections in the second group. The problem will be formulated into a type of mathematical programming. Let M be the number of those sections and x_j ($j = 1, 2, \dots, M$) be an amount to be allocated to the j th section in the second group. With use of these notations, the constraints to the allocation are represented as follows:

$$L_j \leq x_j \leq U_j, \quad (4.38)$$

$$x_j \leq d_j, \quad j = 1, 2, \dots, M, \quad (4.39)$$

$$\sum_{j=1}^M x_j \leq T_R, \quad (4.40)$$

where d_j is the demand of the j th section.

After a discussion among the section chiefs, the objective function to be minimized is taken to be

$$f = \sum_{j=1}^M \lambda_j (x_j - d_j)^2. \quad (4.41)$$

where

$$\lambda_j = \left(\sum_{j=1}^M x_j / x_j \right)^{3/2}. \quad (4.42)$$

The objective represents a *regret* for cut of the budgets. Hence the aim is to minimize the regret or discontent of the participants. Now the budget allocation problem is formulated into the following nonlinear programming problem:

$$\text{Minimize } [f], \quad (4.43)$$

Subject to (4.38) ~ (4.40).

Table 4.10 shows the budgets thus determined together with the demands.

Table 4.10. Values of d_i , x_i for each section.

Section	x_i	d_i
S	10	10
S	8	8
S	280	310
S	22	25
S	30	34
⋮	⋮	⋮
S_{20}	44	50
S_{21}	3	3
S_{22}	0	0

4.4.4 Concluding Remarks

The results obtained in this application are summarized as follows:

- (1) Relative budget-allocation ratio of each section can be decided by the section chiefs not by a decision maker, using IWSM procedure, and the ratios of the budget allocation are decided with their consent.
- (2) The budget-allocation in a Research and Development Laboratory of a firm can be decided smoothly using budget allocation by nonlinear programming.

The procedure of budget allocation seems a little complicated because of some subprocedures added to IWSM. But, by experience, we are sure that the present procedure is practically feasible and well convincing to the party concerned.

4.5 Application to Inventory Management Problem [34] [36]

For a production firm, it is one of the most important but difficult problems to decide production quantities and ordering points in the next period, because they must be decided quickly and precisely with consideration of uncertainties in future demand, quantities in stock, capacity of production, effect of sales effort,

and so on. There are many approaches to solve an inventory management problem. They are fixed-period control system, fixed-order-quantity system [26], multi-stage system or multi-echelon system [38], material requirements planning system [37], and so on. So far, this problem has been formulated and solved as a problem of conventional mathematical programming type which has only one objective, i. e., linear programming or nonlinear programming. But in reality, the inventory management problem includes more than one criterion, and those criteria are usually conflicting with each other. Then formulating and solving the problem as multiobjective optimization is more suitable for decision making in production planning and inventory management.

The first part of this section discusses a prediction of future demand using IWSM method and GMDH (Group Method of Data Handling) [15]. In the second part, a multiobjective optimization procedure is applied to a replenishment decision problem in a production-inventory management. In the third part, a multiobjective optimization procedure is applied to a decision of ordering points. The result of decision by using the present procedure is compared with that of by a human decision in the past. The comparison proves the effectiveness of our multiobjective optimization.

4.5.1 Prediction of Future Demand

The prediction of demand in the next period is an important problem for the inventory-management, because a prediction error will cause a shortage of goods or excessive stock. It is also a difficult problem, because it should be decided with consideration of uncertainties of future trend of the market, sales effort, quantities in stock and so on.

There are many methods for prediction of the future demand. Among them, the Winters exponential smoother or the simple exponential smoother has been used in the firm. Figure 4.26 shows the distributions of errors in demand prediction observed in the past data. The results of prediction for each item by using these methods are not so bad, but for the main items or for a group of some items, the accuracy is not satisfactory. Hence, we have some discussions on the procedure and methods for prediction. As a result of discussions, a combined

method of IWSM and GMDH is adopted.

At the outset of this procedure, forty-four factors for the prediction are come out by brainstorming among the persons in charge. Eliminating redundant factors, twenty-two factors are selected as shown in Table 4.11. Next, to make clear contextual relations among the factors, we have inquired to each person in charge about three kinds of relations, i. e., relative importance, influential relation and measurability among factors. For example, we have inquired to each person, “Is the factor a_i more important than the factor a_j ?”. Then he has replied in the following four degrees, i. e., “much more important”, “more important”, “equally important” and “less important”. After repeating such pairwise questionnaires among these twenty-two factors, by using IWSM, we have constructed the hierarchical digraph representing the relative importance among factors as shown in Figure 4.2.7.

By similar procedures, the hierarchical digraphs representing the influential relation and the measurability are also constructed, as shown in Figure 4.28 and Figure 4.29, respectively. By observing Figure 4.27 and Figure 4.29, we know that there are some factors important but difficult to be measured (or quantified). For those factors, appropriate substitutes are selected by using Figure 4.28.

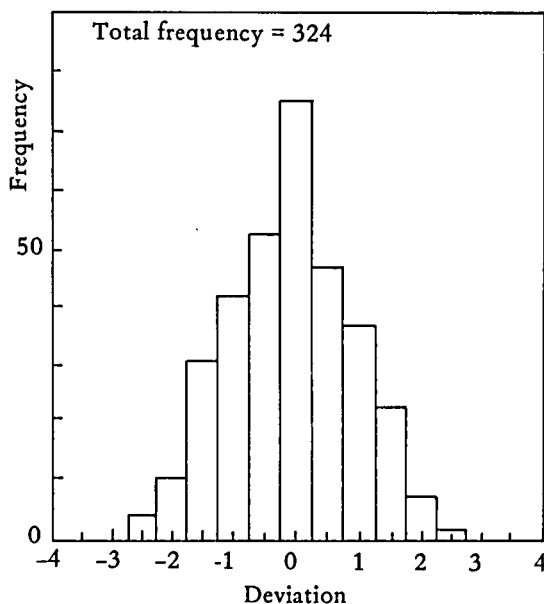


Figure 4.26. Distribution of errors in demand prediction.

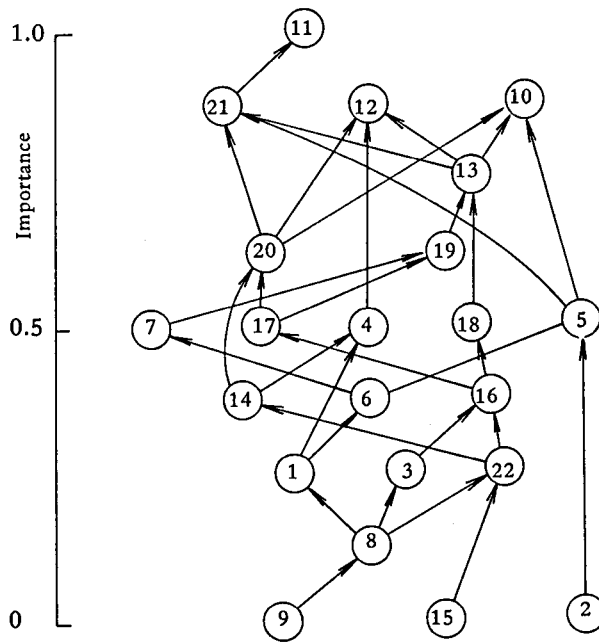


Figure 4.27. Relative importance among factors.

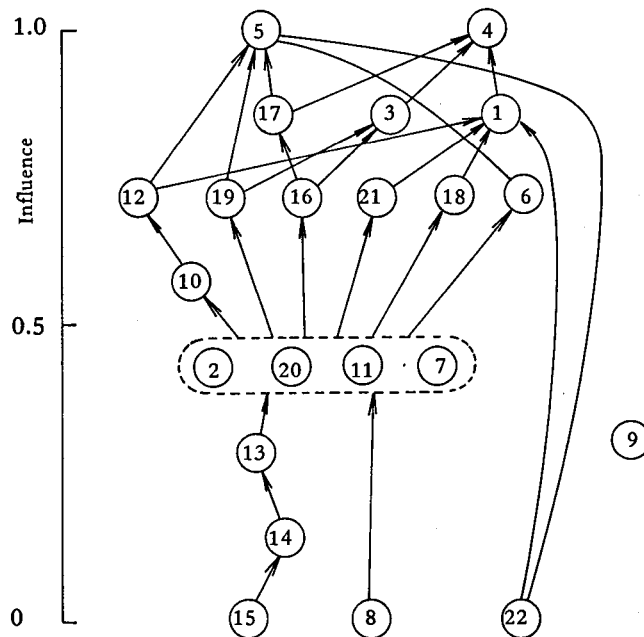


Figure 4.28. Influential relation among factors.

Table 4.11. Prediction factors.

No.	Factors
1	Sales quantity in the past.
2	Policy of the head.
3	Annual sales plan of the operational division.
4	Sales plan in the next period.
5	Achievement ratio of the operational division's sales plan.
6	Achievement ratio of the total sales plan.
7	Achievement ratio of the group sales plan.
8	Group sales policy.
9	Sales quantity of the operational division.
10	Assumed demand.
11	Price of merchandise.
12	Price policy.
13	Price of other firm's merchandise.
14	Price of the material.
15	Price of the raw material.
16	Plan of new merchandises.
17	Sales plan of new merchandises.
18	Sales effort to new merchandises.
19	Sales effort to principal merchandises.
20	Sales campaign.
21	Shortage of goods.
22	Ratio of returned goods.

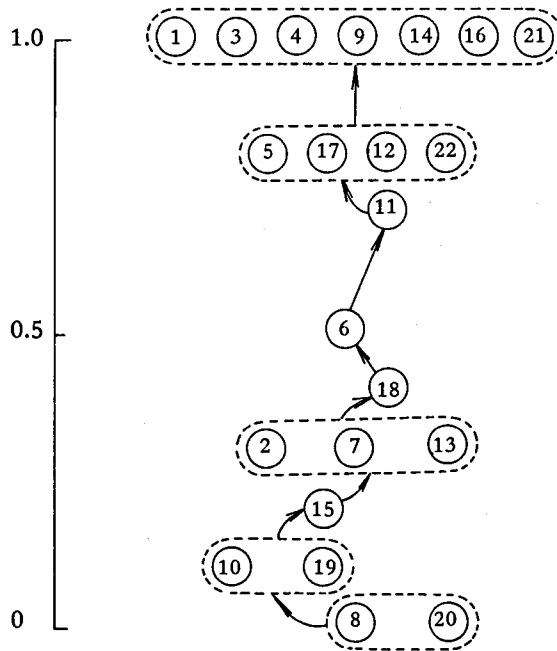


Figure 4.29. Measurability of factors.

Due to the procedure explained above, a set of factors are finally selected for the prediction of future demand of the main item or a group of some factors. By way of example, Figure 4.30 shows a comparison of the prediction by GMDH algorithm [44] including the factors 1, 4, 5, 11, 12, 14 and 21 with that by a person on duty for about ten years. This is a typical example which illustrates the effectiveness of the present procedure.

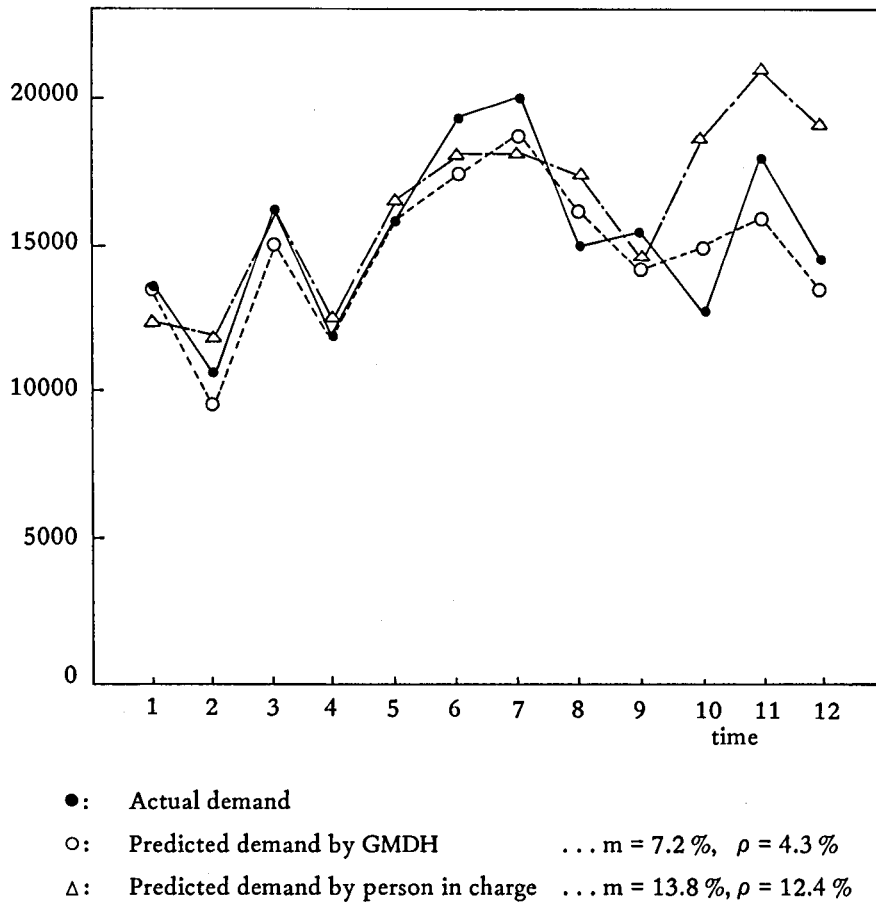


Figure 4.30. Comparison of the predictions by person and GMDH algorithm.

4.5.2 Multiobjective Optimization of Fixed-Period Control System

This section discusses a multiobjective optimization procedure as applied to a fixed-period control system. Decision variables to the problem are production quantities to be produced in the next period (month). Criteria of optimization are selected by discussions among persons concerned to be shortage of goods, excessive stock and a period of stock. Their expected values are quantified under the assumption that future demands obey normal distributions. Among several methodologies applied to this problem, the most successful one, i. e., a combined use of multiattribute utility function with nonlinear programming technique is

presented. A real problem of production-inventory of walling materials for architecture has successfully been dealt with by the present procedure.

(A) Mathematical model and optimization procedure

The inventory-management problem investigated in this section is defined as finding orders for stock replenishment that maximize a set of given criteria of effectiveness subject to certain constraints. The flows of orders and goods among the factory, the operational division, the business office etc. are shown in Figure 4.31. The following symbols are used to describe the production-inventory system.

- I_i^t : total stock of the i th item at the end of this month (t),
- D_i^t : stock of the i th item in the operational division at the end of this month (t),
- P_i^t : ordered quantity of the i th item to be produced in this month (t),
- S_i^t : predicted demand of the i th item in this month (t).

Figure 4.32 shows various factors affecting inventory management. An ordering point, the time point to make decision for order, is set at the beginning of this month (t) to make the order for production in the next month ($t + 1$).

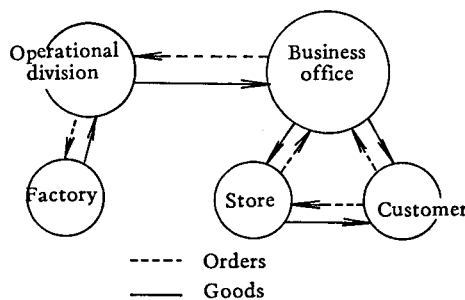


Figure 4.31. Flows of orders and goods.

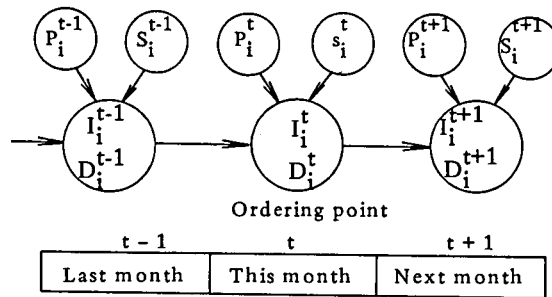


Figure 4.32. Ordering point and factors affecting the decision for order.

(A. 1) Formulation of the criteria

Selecting criteria for optimization is the first essential issue in our whole procedure. The criteria to our problem are selected by discussions with inventory managers, accountants and the head of the operational division. Those criteria finally selected are (1) shortage of goods, (2) excessive stock, and (3) a period of stock, for each item of goods. Their definitions are as follows.

- (1) Shortage of goods: For each item, the sum of demands from the business offices that can not be filled from the stock in the operational division.
- (2) Excessive stock: For each item, when the total stock exceeds the sum of demands in the past three months, the excessive quantity.
- (3) Period of stock: For each group of goods (the goods are divided into some groups according to their characters), an average period kept in warehouses.

Usually, the effectiveness of an inventory-management system is measured by various sorts of cost, i. e., inventory-carrying cost, ordering cost, etc. But, in reality, these costs vary greatly depending upon the characters of goods, sales activity, financial environment, and many other factors. Hence we have defined above criteria which the participants have in their minds as simple and operational decision rules for evaluations.

However, because of uncertainties in the future demand, the shortage of goods and excessive stock can not be formulated definitely. So far, the future demand has been predicted by using the Winters exponential smoother or the simple exponential smoother or the procedure which is mentioned in Section 4.5.1.

From Figure 4.26, it can be seen that the distribution of the demand-prediction errors is well fitted by a normal distribution $N(0, \sigma^2)$, where σ represents the standard deviation from the mean, and it is checked by chi-square goodness of the fit test.

(1) Shortage of goods

Under the assumption that the demand prediction error obeys the normal distribution, the quantity ordered for production in the next month ($t + 1$) is here determined by

$$P_i^{t+1} = S_i^t + S_i^{t+1} - P_i^t - D_i^t - \sqrt{2} n_i \sigma_i, \quad (4.44)$$

where σ_i represents the standard deviation of prediction errors, and n_i is a parameter to be decided later for optimization. The subscript i stands for the i th item of goods, and the superscript $t-1$, t and $t+1$ represent the last, the present and the next month, respectively. Accordingly, the total stock of the i th item at the end of the next month

$$I_i^{t+1} = I_i^{t-1} - D_i^{t-1} + S_i^{t+1} - S_i^a + \sqrt{2} n_i \sigma_i, \quad (4.45)$$

where S_i^a is the actual demand for the i th item in the next month. When I_i^{t+1} is null, the actual demand is to be

$$S_i^0 = I_i^{t-1} - D_i^{t-1} + S_i^{t+1} + \sqrt{2} n_i \sigma_i. \quad (4.46)$$

Then, the shortage of the i th item of goods is defined as follows:

$$SG_i \triangleq \begin{cases} S_i^a - S_i^0, & \text{if } S_i^a > S_i^0, \\ 0 & \text{if } S_i^a \leq S_i^0. \end{cases} \quad (4.47)$$

Since the probability density function of the above event is given by

$$P_i = \frac{1}{\sqrt{2\pi} \sigma_i} e^{-\frac{(S_i^a - S_i^0)^2}{2\sigma_i^2}}, \quad (4.48)$$

the expected value of the shortage of the i th item, ESG_i , is given by

$$ESG_i = \int_{S_i^0}^{\infty} (S_i^a - S_i^0) \frac{1}{\sqrt{2\pi} \sigma_i} e^{-\frac{(S_i^a - S_i^0)^2}{2\sigma_i^2}} dS_i^a. \quad (4.49)$$

(2) Excessive stock

Let MS_i be the sum of demands for the i th item in the past three months. Owing to Equation (4.45), when I_i^{i+1} , the total stock at the end of the next month, is equal to MS_i , the value of S_i^a is to be

$$S_i^a = S_i^e \triangle I_i^{t-1} - D_i^{t-1} + S_i^{t+1} + \sqrt{2} n_i \sigma_i - MS_i. \quad (4.50)$$

Then, the excessive stock of the i th item is defined as follows:

$$ES_i \triangle \begin{cases} 0, & \text{if } S_i^a > S_i^e, \\ S_i^e - S_i^a, & \leq S_i^e. \end{cases} \quad (4.51)$$

Since the probability of the event is again given by Equation (4.48), the expected value of the excessive stock, EES_i , is given by

$$EES_i = \int_{S_i^*}^{\infty} (S_i^e - S_i^a) \frac{1}{\sqrt{2\pi} \sigma_i} e^{-\frac{(S_i^* - S_i^{i+1})^2}{2\sigma_i^2}} \quad (4.52)$$

(A. 2) Formulating to the multiobjective optimization problem

We define the three objective functions as follows:

(1) Shortage in percent

$$f_2(n_i) = \frac{\sum_{i=1}^M C_i ESG_i}{\sum_{i=1}^M C_i S_i^{t-1}} \quad (\%) \quad (4.53)$$

(2) Excessive stock in percent

$$f_2(n_i) = \frac{\sum_{i=1}^M C_i EES_i}{\sum_{i=1}^M C_i S_i^{t-1}} \quad (\%) \quad (4.54)$$

(3) Period of stock

$$f_3(n_i) = 30 \frac{\sum_{i=1}^M C_i (I_i^{t-1} + \sqrt{2} n_i \sigma_i)}{\sum_{i=1}^M C_i S_i^{t+1}} \text{ (days)}. \quad (4.55)$$

As the total production can not exceed the production capacity T , and P_i^{t+1} in Equation (4.44) and I_i^{t+1} in Equation (4.45) must be nonnegative, the constraints to the problem are

$$\sum_{i=1}^M (S_i^t + S_i^{t+1} - P_i^t - D_i^{t-1} + \sqrt{2} n_i \sigma_i) \leq T, \quad (4.56)$$

$$n_i \geq (P_i^t + D_i^{t-1} - S_i^t - S_i^{t+1}) / (\sqrt{2} \sigma_i), \quad (4.57)$$

$$n_i \geq (D_i^{t-1} - I_i^{t-1}) / (\sqrt{2} \sigma_i), \quad i = 1, 2, \dots, M. \quad (4.58)$$

Hence, our inventory-management problem is formulated into the following optimization problem with three objectives:

$$\begin{aligned} & \text{Minimize } [f_1(n_i), f_2(n_i), f_3(n_i)], \\ & \text{subject to (4.56) } \sim \text{ (4.58)}. \end{aligned} \quad (4.59)$$

After solving this problem, the quantity orderd for production in the next month is calculated by Equation (4.44) for each item.

By use of the multiattribute utility function which is mentioned in Section 3.3, the multiobjective problem defined above is transformed into the following problem of nonlinear programming type:

$$\begin{aligned} & \text{Maximize } [U(f_1(n_i), f_2(n_i), f_3(n_i))], \\ & \text{subject to (4.56) } \sim \text{ (4.58)}. \end{aligned} \quad (4.60)$$

(B) Example

A real problem considered here is from the inventory management of walling materials for architecture. There are eighty-nine items of goods.

(B. 1) Approximate expressions of the shortage of goods and the excessive stock Equation (4.49) and (4.52) are transformed into, respectively,

$$ESG_i = \frac{\sigma_i}{\sqrt{2\pi}} (e^{-\frac{1}{2}a^2} - a \int_a^\infty e^{-\frac{1}{2}v^2} dv), \quad (4.61)$$

$$EES_i = \frac{\sigma_i}{\sqrt{2\pi}} (e^{-\frac{1}{2}b^2} - b \int_b^\infty e^{-\frac{1}{2}v^2} dv + \sqrt{2\pi} b), \quad (4.62)$$

where

$$a = (I_i^{t-1} - D_i^{t-1} + \sqrt{2} n_i \sigma_i) / \sigma_i, \quad (4.63)$$

$$b = (I_i^{t-1} - D_i^{t-1} + \sqrt{2} n_i \sigma_i - MS_i) / \sigma_i. \quad (4.64)$$

For convenience of the numerical calculation, we approximate the integration term in the following way:

$$\frac{1}{\sqrt{2\pi}} \int_K^{\infty} e^{-\frac{1}{2}v^2} dv = \begin{cases} \frac{1}{2} e^{g(K)}, & \text{if } K \geq 0, \\ 1 - \frac{1}{2} e^{g(-K)}, & \text{if } K < 0, \end{cases} \quad (4.65)$$

where

$$g(K) = -0.0058 K^4 + 0.0165 K^3 - 0.4013 K^2 - 0.7602 K. \quad (4.66)$$

Substituting Equations (4.65) and (4.66) into Equations (4.61) and (4.62) yields

$$f_1(n_i) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{M}{\sum_{i=1}^M C_i \sigma_i} (e^{-\frac{1}{2}a^2} - \frac{\sqrt{2\pi}}{2} a e^{g(a)}) / \frac{M}{\sum_{i=1}^M C_i S_i^{t+1}}, & \text{if } a \geq 0, \\ \frac{1}{\sqrt{2\pi}} \frac{M}{\sum_{i=1}^M C_i \sigma_i} (e^{-\frac{1}{2}a^2} + \frac{\sqrt{2\pi}}{2} a e^{g(-a)} - \sqrt{2\pi} a) / \frac{M}{\sum_{i=1}^M C_i S_i^{t+1}}, & \text{if } a < 0. \end{cases} \quad (4.67)$$

$$f_2(n_i) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{M}{\sum_{i=1}^M C_i \sigma_i} (e^{-\frac{1}{2}b^2} - \frac{\sqrt{2\pi}}{2} b e^{g(b)} + \sqrt{2\pi} b) / \frac{M}{\sum_{i=1}^M C_i S_i^{t+1}}, & \text{if } b \geq 0, \\ \frac{1}{\sqrt{2\pi}} \frac{M}{\sum_{i=1}^M C_i \sigma_i} (e^{-\frac{1}{2}b^2} + \frac{\sqrt{2\pi}}{2} b e^{g(-b)}) / \frac{M}{\sum_{i=1}^M C_i S_i^{t+1}}, & \text{if } b < 0. \end{cases} \quad (4.68)$$

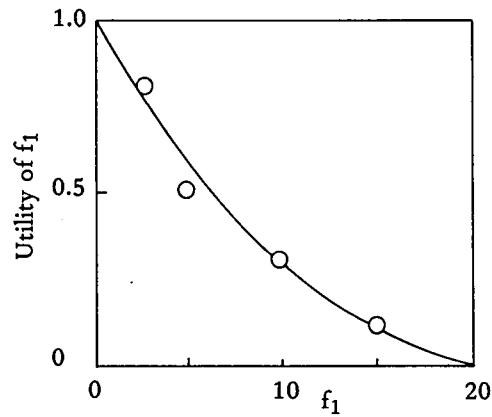
(B. 2) Construction of multiattribute utility function

Let f_{α}^0 and f_{α}^* denote the least and the most desirable values of attribute f_{α} , respectively. Table 4.12 shows the ranges of attributes decided by discussions among the persons in charge. For each individual utility function u_{α} , we put the normalization condition as $u(f_{\alpha}^*) = 1$ and $u(f_{\alpha}^0) = 0$.

The values of u_{α} for f_{α} between zero and one have been decided by questionnaires to the decision makers. Figures 4.33 through 4.35 show the individual utility functions obtained from the responses of the head of the operational division.

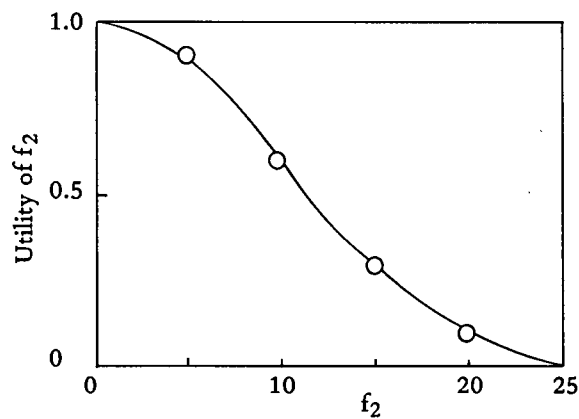
Table 4.12. Allowable ranges of f

f_{α}	f_{α}^*	f_{α}^0
f_1	0 %	20 %
f_2	0 %	25 %
f_3	20 days	50 days



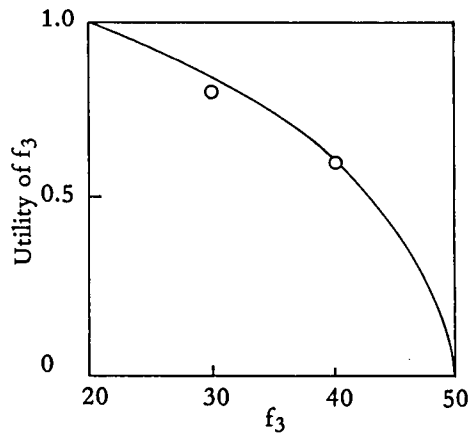
$$u_1(f_1) = 1.0 - 0.1150 \cdot f_1 + 0.5010 \cdot 10^{-2} \cdot f_1^2 - 0.8971 \cdot 10^{-4} \cdot f_1^3$$

Figure 4.33. Utility function for the percent shortage of goods.



$$u_2(f_2) = 1.0 - 0.5476 \cdot 10^2 \cdot f_2 - 0.4714 \cdot 10^{-2} \cdot f_2^2 + 0.1333 \cdot 10^{-3} \cdot f_2^3$$

Figure 4.34. Utility function for the percent excessive stock.



$$u_3(f_3) = 2.0 - 0.9833 \cdot 10^{-1} \cdot f_3 + 0.325 \cdot 10^{-2} \cdot f_3^2 - 0.4166 \cdot 10^{-4} \cdot f_3^3$$

Figure 4.35. Utility function for the period of stock.

Table 4.13 shows the values of k_α and k evaluated from the responses of the head of the operational division. Figure 4.36 shows some of the indifferent curves between f_1 and f_3 .

Table 4.13. Values of k_α and k

k_1	0.45	k_3	0.20
k_2	0.25	k	0.38

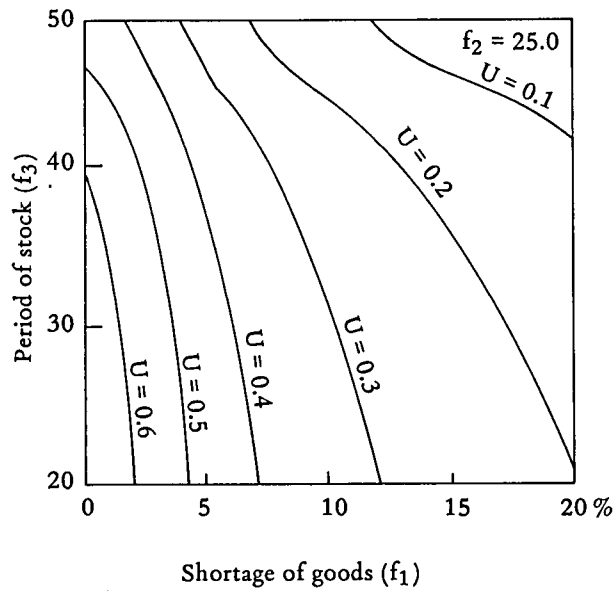


Figure 4.36. Indifference curves between f_1 and f_3 .

(B. 3) Result of simulation

As mentioned earlier, there are eighty-nine items of goods in total, and they are classified into four groups according to their characters. By way of example, we have solved the problem for a group including twenty-one items. Table 4. 14 compares the result of decision made by an inventory manager in the past with the result of our multiobjective optimization under the same condition. We can see the effectiveness of the present optimization method, as both the shortage of goods and the excessive stock are reduced while the period of stock is almost same.

Simulating the system for the past one year in like way, we have found that the the shortage of goods and the excessive stock are almost halved with keeping the same period of stock.

Table 4.14. Comparison of the decisions by person and the present method

i	σ_i	MS_i	D_i^{t-1}	I_i^{t-1}	I_i^t	P_i^t	S_i^{t+1}	P_i^{t+1}		SG_i		ES_i	
								(I)	(II)	(I)	(II)	(I)	(II)
1	65	777	203	345	200	250	260	150	220	0	0	0	0
2	109	972	293	443	250	350	320	200	311	0	0	0	0
3	16	222	14	75	50	80	70	50	75	0	0	0	0
4	13	132	69	113	30	50	40	0	0	0	0	0	0
5	27	432	121	194	120	180	140	100	56	0	0	0	0
6	50	519	239	364	140	200	190	100	60	0	0	38	0
7	51	633	57	185	180	350	221	100	170	0	0	0	0
8	113	1644	292	501	470	700	540	400	445	0	0	0	0
9	94	1065	325	479	320	600	390	300	130	0	0	0	0
10	97	1239	350	574	400	545	460	350	292	0	0	0	0
11	66	1203	170	336	350	480	390	250	310	0	0	0	0
12	30	636	229	351	200	250	210	150	38	0	0	0	0
13	49	588	156	260	140	250	180	150	44	0	0	0	0
14	78	903	82	217	250	300	281	300	435	0	0	0	0
15	85	774	31	177	220	300	230	150	377	0	0	0	0
16	11	147	7	75	25	40	41	20	54	0	0	0	0
17	18	171	44	86	38	30	46	50	65	13	0	0	0
18	19	180	7	81	40	100	65	50	56	0	0	0	0
19	28	219	26	85	61	60	75	60	142	6	0	0	0
20	29	336	43	151	60	100	60	0	77	45	0	0	0
21	47	441	175	312	100	100	120	50	102	3	0	0	0

(I): by person on duty, (II): by the present method

$f_3 = 39$ days by (I) and 40 days by (II)

4.5.3 Multiobjective Optimization of Fixed-Order-Quantity System

The section deals with a multiobjective optimization procedure as applied to a fixed-order-quantity system. Decision variables to the problem are ordering points in the next period (month). Objectives of optimization are examined by persons concerned, and those finally selected are shortage of goods during a month, expected production period in factories and deviations from standard stocks through a month. A multiattribute utility function is constructed for evaluating totally the degree of satisfaction of these objectives, and it is used, together with a nonlinear programming technique, for optimizing the decision variables. A real problem of inventory management of ceiling materials for architecture has successfully been dealt with by the present procedure.

(A) Mathematical model and optimization procedure

The inventory management problem investigated in this section is defined as follows: Find ordering points which maximize a set of given criteria of effectiveness subject to certain constraints.

The flows of orders and goods among the factory, the operational division, the business office etc. are shown in Figure 4.31. The time point to make decision is set at the beginning of the present month to decide the ordering points in the next month for each item of goods.

(A. 1) Mathematical formulation of the criteria

Selecting criteria for optimization is the first essential issue in our whole procedure. The criteria to our problem have been selected by the following procedure. First, we have obtained the seven criteria by discussions with some inventory managers. The criteria are as follows:

- f_a : Shortage of goods during a month,
- f_b : Maximum stock in a month,
- f_c : Average stock in a month,
- f_d : Holding cost of stock in a month,
- f_e : Expected production period in the factories,
- f_f : Differential quantity between the maximum stock and the minimum stock in a month.

f_g : Deviation from a standard stock through a month.

Next, in order to make easy the decision of inventory managers, we have tried to eliminate unimportant or duplicating criteria among these seven ones by use of the IWSM procedure. In the first part of this procedure, we have inquired to each inventory manager about the contextual relations of the relative importance among criteria. Namely, we have inquired to each person, for example. "Is the criterion f_a more important than the criterion f_b ?", then he has replied in the following three degrees, i. e., "much important", "important" and "equally important or not important". After repeating such pairwise questionnaires among these seven criteria, by using IWSM, we have constructed the hierarchical digraph representing the relative importance among criteria as shown in Figure 4.37. From Figure 4.37, the criteria f_a , f_b , f_f and f_g have been adopted as the important criteria to this problem. However, since the criteria f_b and f_f are discontinuous functions, these two criteria have not been adopted from the viewpoint of numerical analysis. Then, by discussions with inventory managers, the following three criteria have been finally selected:

- (1) Shortage of goods during a month,
- (2) Expected production period in the factories,
- (3) Deviation from the standard stock through a month.

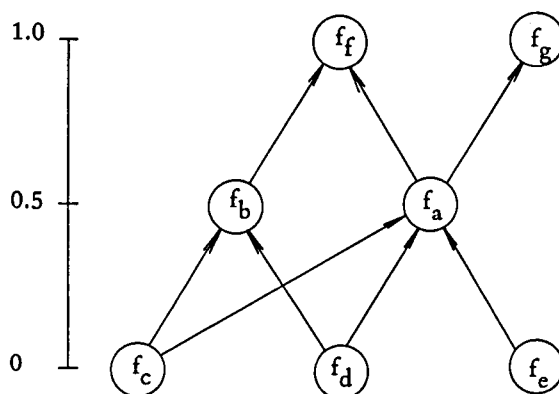


Figure 4.37. Relative importance among criteria.

Their definition are such as follows:

(1) Shortage of goods during a month

For each item of goods, the sum of orders from the business offices that can not be sent out from the operational division, because the stock in the operational division runs short due to the delay of ordering points.

(2) Expected production period in the factories

For each group of goods (the goods are divided into some groups according to their characters), the average period from the beginning of a month untill a day of delivery from the factories to the operational division.

(3) Deviation from the standard stock through a month

For each item of goods, differential quantity between an actual stock and the standard stock in the operational division.

However, because of uncertainties in the future demand, the shortage of goods during a month can not be formulated definitely. So far, the future demand has been predicted by using the Winters exponential smoother or the simple exponential smoother or the procedure which is mentioned in Section 4.5.1. Then, under the assumption that the distribution of the demand prediction errors is fitted by a normal distribution $N(O, \sigma^2)$, where σ represents the standard deviation from the mean, we formulate the shortage of goods during a month. Furthermore, in the following formulations, arrival time points from the factories to the operational division are considered as decision variables.

Definitions of symbols:

The following symbols are used to describe the ordering point decision problem.

- t : Present month,
- $t + 1$: Next month,
- i : i th item of goods,
- r : Time (normalized to zero to one between the beginning and the end of a month),
- P_i^t : Production quantity of the i th item in the present month.
- $S_i^t(r)$: Predicted demand of the i th item at time r in the present month,
- $H_i^t(r)$: Actual demand of the i th item at time r in the present month,

- σ_i : Standard deviation of the demand prediction errors of the i th item,
 $D_i^t(r)$: Stock of the i th item at time r in the operational division,
 C_i : Unit price of the i th item,
 B_i : Standard stock of the i th item in the operational division,
 M : Number of goods,
 T_i : Arrival time point of the i th item from factories to the operational division (normalized to zero to one between the beginning and the end of a month).

(a) Shortage of goods during a month

The predicted cumulative demand and the actual cumulative demand of the i th item from the beginning of the present month untill r in the next month is given by

$$S_i^{\text{sum}}(r) = S_i^t(1) + S_i^{t+1}(r), \quad (4.69)$$

$$H_i^{\text{sum}}(r) = H_i^t(1) + H_i^{t+1}(r). \quad (4.70)$$

In the above, it is assumed that $H_i^{\text{sum}}(r)$ obeys the normal distribution $N(S_i^{\text{sum}}(r), [\sigma_i S_i^{\text{sum}}(r)]^2)$. The standard deviation of prediction error, σ_i , is a fixed quantity obtained by the past data.

When $D^{t+1}(T_i)$ is null, the actual cumulative demand is to be

$$H_i^0 = D_i^t(0) + P_i^t. \quad (4.71)$$

Then, the shortage of the i th item of goods during the next month is defined as follows:

$$SG_i(T_i) = \begin{cases} H_i^{\text{sum}}(T_i) - H_i^0, & \text{if } H_i^{\text{sum}}(T_i) > H_i^0, \\ 0, & \text{if } H_i^{\text{sum}}(T_i) \leq H_i^0. \end{cases} \quad (4.72)$$

Since the probability density function of the event is given by

$$P_i(T_i) = \frac{1}{\sqrt{2\pi} \sigma_i S_i^R(T_i)} \exp \left\{ -\frac{[H_i^{\text{sum}}(T_i) - S_i^{\text{sum}}(T_i)]^2}{2\sigma_i^2 [S_i^R(T_i)]^2} \right\}, \quad (4.73)$$

where

$$S_i^R(T_i) = \sqrt{[S_i^t(1)]^2 + [S_i^{t+1}(T_i)]^2}, \quad (4.74)$$

the expected value of the shortage of the i th item during the next month, $ESG_i(T_i)$, is given by

$$ESG_i(T_i) = \int_{H_i^0}^{\infty} [H_i^{\text{sum}}(T_i) - H_i^0] P_i(T_i) d[H_i^{\text{sum}}(T_i)]. \quad (4.75)$$

(b) Deviation from the standard stock through a month

The deviation from the standard stock of the i th item at r is defined by

$$\text{where } \begin{cases} d_i^{t+1}(r) = |D_i^{t+1}(r) - B_i|, \\ D_i^{t-1}(r) = \begin{cases} D_i^t(0) + P_i^t - S_i^{\text{sum}}(r), & \text{if } r \leq T_i \\ D_i^t(0) + P_i^t + P_i^{t+1} - S_i^{\text{sum}}(r), & \text{if } r > T_i \end{cases} \end{cases} \quad (4.76)$$

$$(4.77)$$

Consequently, the deviation from the standard stock of the i th item, $EES_i(T_i)$, is given by

$$\begin{aligned} EES_i(T_i) &= \int_0^1 d_i^{t+1}(r) dr \\ &= \int_0^{T_i} |D_i^t(0) + P_i^t - S_i^{\text{sum}}(r) - B_i| dr \\ &\quad + \int_{T_i}^1 |D_i^t(0) + P_i^t + P_i^{t+1} - S_i^{\text{sum}}(r) - B_i| dr. \end{aligned} \quad (4.78)$$

(A. 2) Formulation of the multiobjective optimization problem

We define the three objective functions as follows:

(1) Shortage ratio of goods during the next month

$$f_1(T_i) = \frac{\sum_{i=1}^M C_i ESG_i(T_i)}{\sum_{i=1}^M C_i S_i^{t+1}(1)}, \quad (4.79)$$

(2) Expected production period in the factories

$$f_2(T_i) = \frac{\sum_{i=1}^M P_i^{t+1} T_i}{\sum_{i=1}^M P_i^{t+1}}, \quad (4.80)$$

(3) Deviation ratio from the standard stock through the next month

$$j_3 (T_i) = \frac{\sum_{i=1}^M EES_i (T_i)}{\sum_{i=1}^M B_i} . \quad (4.81)$$

The constraints to the problem are

$$0 \leq T_i \leq 1, \quad i = 1, 2, \dots, M. \quad (4.82)$$

Hence, our inventory management problem is formulated into the following optimization problem with three objectives:

$$\begin{aligned} &\text{Minimize } [f_1 (T_i), -f_2 (T_i), f_3 (T_i)] , \\ &\text{subject to (4.82).} \end{aligned} \quad (4.83)$$

By use of the multiattribute utility function which is mentioned in Section 3.3, the multiobjective problem defined above is transformed into the following problem of nonlinear programming type:

$$\begin{aligned} &\text{Maximize } [U (j_1 (T_i), f_2 (T_i), f_3 (T_i))] , \\ &\text{subject to (4.82).} \end{aligned} \quad (4.84)$$

(B) Application to a real problem

A real problem considered here is from the inventory management of ceiling materials for architecture.

There are one hundred and sixty-five items of goods.

(B.1) Approximate expressions of the shortage of goods during the next month

Equation (4.75) is transformed into

$$ESG_i (T_i) = \sigma_i S_i^R (T_i) \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_i^2}, -\frac{a_i}{\sqrt{2\pi}} \int_{a_i}^{\infty} e^{-\frac{1}{2}v_i^2} dv_i \right), \quad (4.85)$$

where

$$\sigma_i = [H_i^0 - S_i^{\text{sum}} (T_i)] / [\sigma_i S_i^R (T_i)] . \quad (4.86)$$

For convenience of the numerical calculation, we approximate the integration term in the following way:

$$\frac{1}{\sqrt{2\pi}} \int_{a_i}^{\infty} e^{-\frac{1}{2}v_i^2} dv_i = \begin{cases} \frac{1}{2} e^{g(a_i)}, & \text{if } a_i \geq 0, \\ 1 - \frac{1}{2} e^{g(-a_i)}, & \text{if } a_i < 0, \end{cases} \quad (4.87)$$

where

$$g(a_i) = -0.0058a_i^4 + 0.0165a_i^3 - 0.4013a_i^2 - 0.760a_i. \quad (4.88)$$

Substituting Equations (4.87) and (4.88) into Equation (4.85) yields

$$f_1(T_i) = \begin{cases} \left[\sum_{i=1}^M C_i \sigma_i S_i^R(T_i) \frac{1}{-\sqrt{2\pi}} e^{-\frac{1}{2}a_i^2} - \frac{1}{2}a_i e^{g(a_i)} \right] \\ / \sum_{i=1}^M C_i S_i^{t+1}(1), \quad \text{if } a_i \geq 0, \\ \left[\sum_{i=1}^M C_i \sigma_i S_i^R(T_i) \frac{1}{-\sqrt{2\pi}} e^{-\frac{1}{2}a_i^2} - \frac{1}{2}a_i e^{g(-a_i)} \right] \\ / \sum_{i=1}^M C_i S_i^{t+1}(1), \quad \text{if } a_i < 0. \end{cases} \quad (4.89)$$

(B. 2) Construction of multiattribute utility function

Let f_α^0 and f_α^* denote the least and the most desirable values of attribute f_α , respectively. Table 4.15 shows the ranges of attributes decided by the inventory manager. Before the construction of the multiattribute utility function in this problem, we verified the independence assumptions concerning preferences by questionnaires to the inventory manager. After some discussion with him, the following results were obtained.

Table 4.15. Allowable ranges of f

f_α	f^*	f_α^0
f_1	0 %	30 %
f_2	30 days	0 day
f_3	0	100 %

Figure 4.38 shows the result of verification of the preferential independence assumption. In Figure 4.38 (a), when f_3 was held fixed throughout at f_3^* , the manager considered that the point A was in different to the point C, i. e.,

$$(f_1^0, f_2^*, f_3^*) \sim (27.5\%, f_2^0, f_3^*),$$

and in Figure 4.38 (b), when f_3 was held fixed at f_3^0 , he considered that the point A' was indifferent to the point C' , i. e..

$$(f_1^0, f_2^*, f_3^0) \sim (27.5\%, f_2^0, f_3^0).$$

Also, in Figure 4.38 (c) and Figure 4.38 (d), he considered that the point D was indifferent to the point F and the point D' was indifferent to the point F' , i. e..

$$(f_1^0, f_2^*, f_3^*) \sim (22.5\%, f_2^*, f_3^0),$$

$$(f_1^0, f_2^0, f_3^*) \sim (22.5\%, f_2^0, f_3^0).$$

Therefore the set of attributes (f_1, f_2) is preferentially independent of f_3 and the set of attributes (f_1, f_3) is preferentially independent of f_2 .

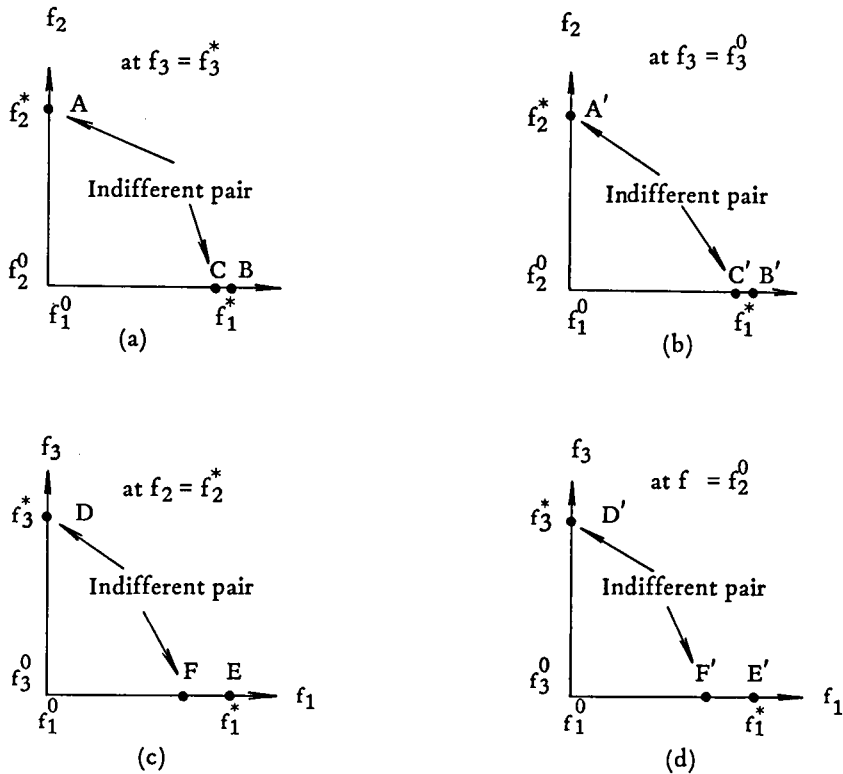


Figure 4.38. Verification of the preferential independence assumption.

The utility independence assumption was verified by using the lotteries as shown in Figure 4.39. In Figure 4.39 (a), when f_{1-} was held fixed through at f_{1-}^* , the inventory manager considered that his certainty equivalent for a 50-50 gamble yielding values f_1^* and $f_1' = 12.5\%$. And as shown in Figure 4.39 (b), when f_{1-} was held fixed at some other value, say f_{1-}^0 , his certainty equivalent was $f_1'' = 12.5\%$. Then f_1 .

As the results of these questionnaires, we can assume that attributes f_α are mutually utility independent, hence the multiattribute utility function of the form of Equation (3.7) or (3.8) can be adopted.

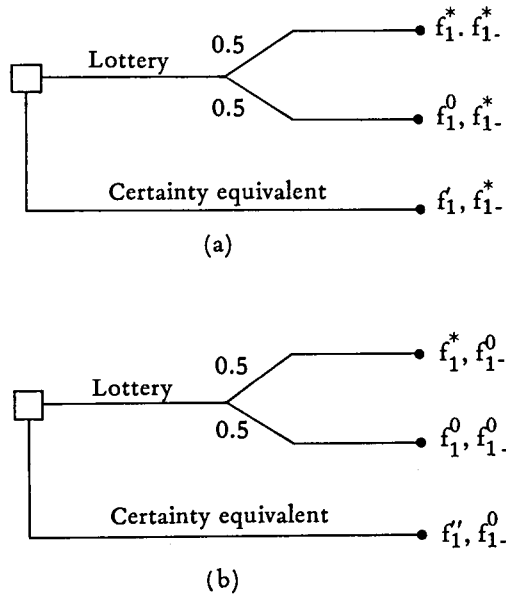


Figure 4.39. Verification of the utility independence assumption.

Next, to verify the additive independence assumption, we gave the questionnaires to the inventory manager as shown in Figure 4.40. In Figure 4.40, when we asked him whether Lottery A and B were equally preferable (i. e., indifferent) or not, he answered that Lottery B was preferable for the case of $\alpha = 2$. Hence the additive independence assumption is not held.

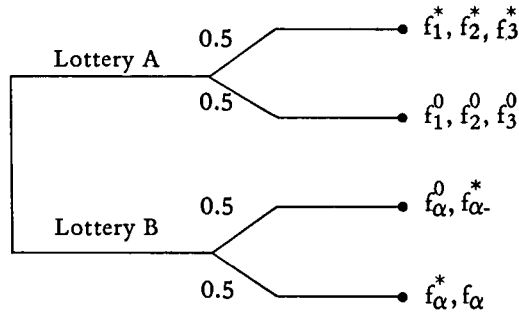


Figure 4.40. Verification of the additive independence assumption.

As the results of these questionnaires to the inventory manager, the multiattribute utility function has been constructed in the multiplicative form, i. e.,

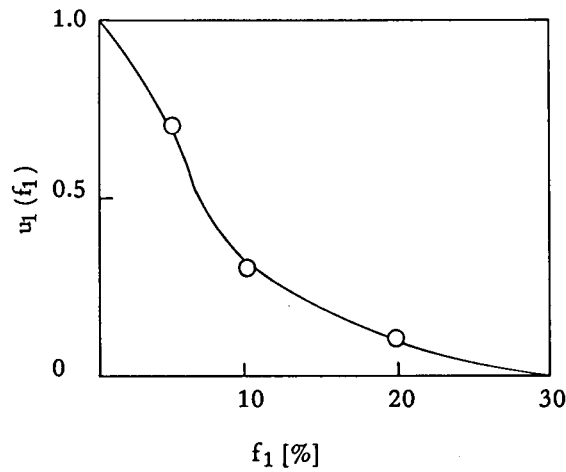
$$U(f) = \left\{ \prod_{a=1}^3 [(1 + k k_a u_a(f_a))] - 1 \right\} / k. \quad (4.90)$$

For each individual utility function $u_\alpha(f_\alpha)$, we put the normalization condition as

$$u_a(f_a^0) = 0 \quad (4.91)$$

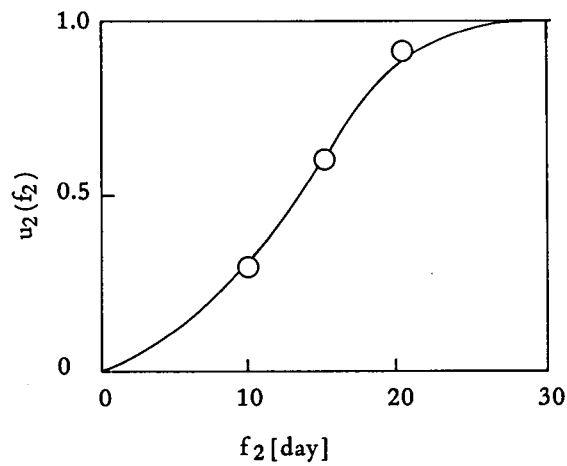
$$u_a(f_a^*) = 1. \quad (4.92)$$

The values of $u_\alpha(f_\alpha)$ for f_α between f_α^0 and f_α^* have been decided by questionnaires to the inventory manager. Figures 4.41 through 4.43 show the individual utility functions obtained from his responses. Table 4.16 shows the values of k_α and k , evaluated from the responses of the inventory manager.



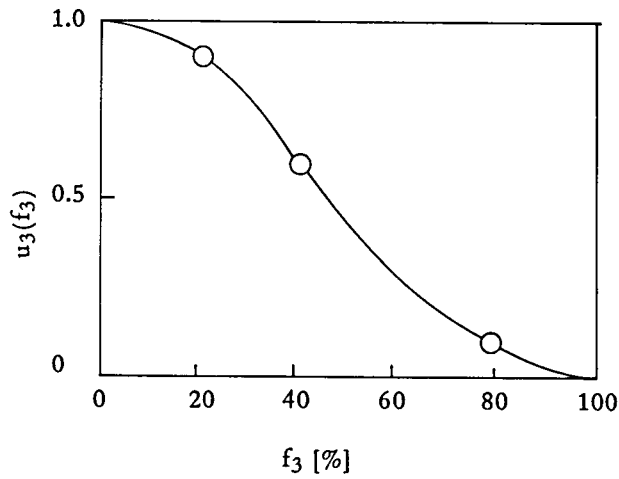
$$u_1(f_1) = 1.0 - 0.903 \cdot 10^{-1} \cdot f_1 + 0.272 \cdot 10^{-2} \cdot f_1^2 - 0.281 \cdot 10^{-4} \cdot f_1^3$$

Figure 4.41. Utility function for the shortage ratio of goods.



$$u_2(f_2) = -0.124 \cdot 10^{-1} f_2 + 0.552 \cdot 10^{-2} \cdot f_2^2 - 0.133 \cdot 10^{-3} \cdot f_2^3$$

Figure 4.42. Utility function for the expected production period in the factories.



$$u_3(f_3) = 1.0 - 0.153 \cdot 10^{-2} \cdot f_3 - 0.281 \cdot 10^{-3} \cdot f_3^2 + 0.196 \cdot 10^{-5} \cdot f_3^3$$

Figure 4.43. Utility function for the deviation ratio from

Table 4.16. Values of k_α and k

k_1	0.7	k_2	0.3
k_3	0.4	k	-0.73

(B. 3) Result of simulation

As mentioned earlier, there are one hundred and sixty-five items of goods in total, and they are classified into five groups according to their characters. By way of example, we have solved the problem for a group including twenty-seven items. Table 4.17 compares the result of decision made by an inventory manager in the past with the result of our multiobjective optimization under the same condition. We can see the effectiveness of the present optimization method, as both the shortage of goods during a month and the deviations from the standard stocks are reduced. Simulating the same problem for the past five months in a like way, we have found that the shortage of goods is almost equal to zero and the deviations from the standard stocks are slightly reduced.

Table 4.17. Comparison of the decisions by person and by the present method

i	$D_i^r(1)$	P_i^r	$S_i^r(1)$	P_i^{r-1}	$S_i^{r+1}(1)$	*) T_i'		SG (T)	
						(I)	(II)	(I)	(II)
1	88	200	180	300	250	25	6	5	0
2	61	200	220	250	280	20	1	0	0
3	50	60	50	70	70	15	17	0	0
4	139	100	85	50	100	13	30	0	0
5	103	50	105	100	110	24	3	12	0
6	180	300	210	150	295	9	16	0	0
7	281	100	105	0	100			0	0
8	235	200	135	50	150	5	30	0	0
9	222	300	210	250	280	8	24	0	0
10	346	100	95	0	150			0	0
11	90	150	105	100	110	23	20	0	0
12	256	200	170	150	210	5	22	0	0
13	134	100	105	100	100	13	27	0	0
14	12	100	55	100	80	27	8	34	0
15	84	50	65	80	80	27	12	0	0
16	111	50	75	150	120	11	7	44	33
17	150	50	85	100	120	14	11	0	0
18	218	200	190	250	250	16	11	0	0
19	58	20	30	60	60	22	10	0	0
20	323	400	530	600	600	8	4	0	0
21	226	150	135	100	150	17	30	0	0
22	190	200	170	200	200	23	23	0	0
23	22	200	135	350	200	28	8	46	0
24	163	100	85	100	120	28	27	40	28
25	327	500	310	400	450	29	21	0	0
26	8	60	40	60	60	27	8	0	0
27	235	100	105	50	130	10	28	0	0

(I) : by person on duty, (II) : by the present method

$f_3 = 68\%$ by (I) and 63% by (II)

*) T_i'

4.5.4 Concluding Remarks

We have formulated the replenishment decision problem and the ordering point decision problem in the inventory management as multiobjective optimization problems, and have solved them by using the multiattribute utility theory. As a result of real applications, we have found that the proposed procedure works so well. The main findings are as follows.

- (1) The IWSM procedure is very useful for screening the criteria.
- (2) As the shortage of goods, the excessive stock in future etc. are predicted quantitatively, the person in charge can get ideas, at an early stage, how to cope with that situation.
- (3) The decision of the inventory manager can be reflected more easily and quickly than before by use of the utility theory.

This procedure is now adopted to real inventory management problem, and the production quantities and the ordering points of about three hundred and fifty items are decided satisfactorily.

4.6 Application to Investment Assessment Problem [35]

For a production firm, it is one of the important yet difficult problems to assess and decide investment plans, because they are necessary and indispensable for a stabilization and magnification of the production firm or for an increase of profit. Once the investment plans are executed, they give a significant effect upon the production quantities, the quality of goods, the unit costs of materials, the cost of dynamic force, etc. After beginning the performance, a change of the plan will be impossible practically and will ensue a lot of losses.

So far, there have been developed many methods for assessment of investments. Among them, the rate of return method, MAPI (Machinery and Allied Products Institute) method [45] and the payout period method assess the investments from the economical point of view. Further, there are many reports about simulation models which relate the number of laborers, the rates of operation, the production quantities, the sales quantities, etc. All of the above methods, however, assess the investments only from the economical viewpoint and the simulation models are effective for assessing the investments under uncertainties of

economical situation. Thus, it is not easy to make a decision on the real problem, without technical considerations, by using these assessment methods or simulation models.

In this section, we have defined the criterion from the technical viewpoint in addition to the economical viewpoint, and then have derived a total assessment function by use of multiattribute utility functions. As an application, a real problem of annual investments has successfully been dealt with by the present procedure.

The first part of this section defines three criteria (objectives) of the assessment, i. e., the rate of return, the payout period and the technical level of new equipment. Generally, the rate of return and the payout period can readily be defined and formulated quantitatively. But, the technical level of equipment is difficult to quantify. Hence, it is graded into ten classes where each class is defined by the technical level in comparison with the equipments of other production firms.

In the second part, the multiattribute utility function associated with the three objectives is constructed by using the utility theory. Based on answers to questionnaires put to the officer in charge, individual utility functions are assessed and parameters in the multiattribute utility function are also evaluated.

In the third part, a real problem is investigated. The problem is to assess the four hundred and forty-one kinds of investments. The consequences of the decision with use of the present procedure are compared with those of a heuristic of an intuitional decision in the past. The comparison proves the effectiveness of the present procedure.

4.6.1 Criteria of the Investments

Selecting criteria for the assessment is the first important step in our whole procedure. The criteria to our problem are selected by discussions with several administrators, the accountant general and the vice-president of the firm. Fundamental criteria finally selected are (1) economical and technical assessment, (2) top management policy and industrial policy assessment, and (3) assessment of future circumstances. Figure 4.44 illustrates these three factors affecting the total

assessment. For making a decision definitely, quantification of these criteria is needed. Considerations for the quantification are detailed in what follows.

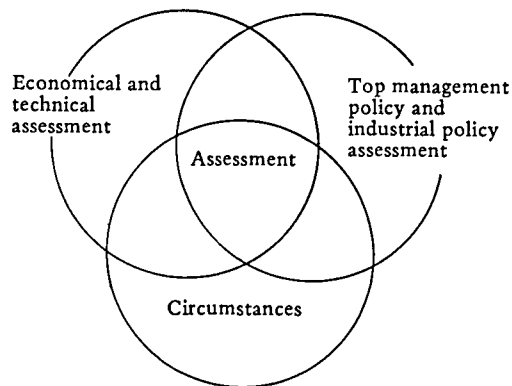


Figure 4.44. Factors in the assessment of investments.

(1) Economical criteria

There have been developed many methods which assess the investment from the economical point of view. The outlines of typical methods are as follows.

a) Minimum-cost rule

By comparing the annual cost of the present equipments with that of the new equipment, the replacement investment is judged. MAPI is a revised method of the minimum-cost rule. This method only estimates the cost saving and it is indifferent to the goal rate of production firm. Hence, it is difficult to decide whether investment is appropriate or not.

b) Rat of return

By examining ratio of the net profit to the amount of invested money, the investment is judged. For a production firm, this method is better than the other methods from the viewpoint of profit, but there are a lot of uncertainties about economical data in the future.

c) Payout period

By examining the payout period, the investment is judged. This method is

useful to judge the safety of investment and floating capital, but it is not useful to judge whether the investment is profitable or not.

d) Capital turnover

By examining the capital turnover, the investment is judged. This method is useful to compare the investment plans if they are for the development of new products and for the increase of productive power, but it is meaningless if they are for rationalization of the production.

There are many methods for economical estimation of the investment. However, as mentioned above, they are similar more or less [42]. For a real problem of assessing the investment, we have to define and formulate the economical criteria more effectively. We have had some discussions about the economical criteria with the persons in charge (accountants, administrators). In order to get a persuasive conclusion out of the discussions, we have made use of the method of IWSM. At the outset of this step, we selected six criteria which had been used rather independently in different divisions. They are 1) minimum-cost rule, 2) rate of return, 3) payout period, 4) capital turnover, 5) MAPI, and 6) revised MAPI. The person in charge are given a series of inquiries of the form, "Is the j th criterion more useful than the i th criterion? If yes, how much is the degree?" The replied degree a_{ij} ($i, j = 1, 2, \dots, 6; i \neq j$) of each person are arranged in a matrix such as shown in Figure 4.45.

$i \backslash j$	1	2	3	4	5	6
1			2			
2	1		3		1	1
3						
4	2		3		2	2
5			2			
6			2			

Figure 4.45. Individual matrix obtained from the replies.

In the matrix, the numeral 3 means that the j th criterion is much more useful than the i th criterion, the numeral 2 means that the j th criterion is more useful than the i th criterion, and the numeral 1 means that the j th criterion is a little more useful than the i th criterion. From this matrix, the digraph is obtained by IWSM and is shown in Figure 4.46. After representing the digraph of each person, we have had a discussion about the digraphs and concept of the economical criteria. Finally, we have defined the economical criteria as follows:

Rate of return f_1 [%],

$$f_1 = (P/I) \times 100, \quad (4.93)$$

where I : investment amount [yen].

P : annual average of profit [yen].

Payout period f_2 [year],

$$f_2 = I/(P + D), \quad (4.94)$$

where D : annual depreciation [yen].

Many formulas to the rate of return and the payout period have been proposed so far, but the simplest formula in each method is adopted here. Since there are a lot of uncertainties in the future forecast of economical situations, it may be meaningless to use complicated formulas.

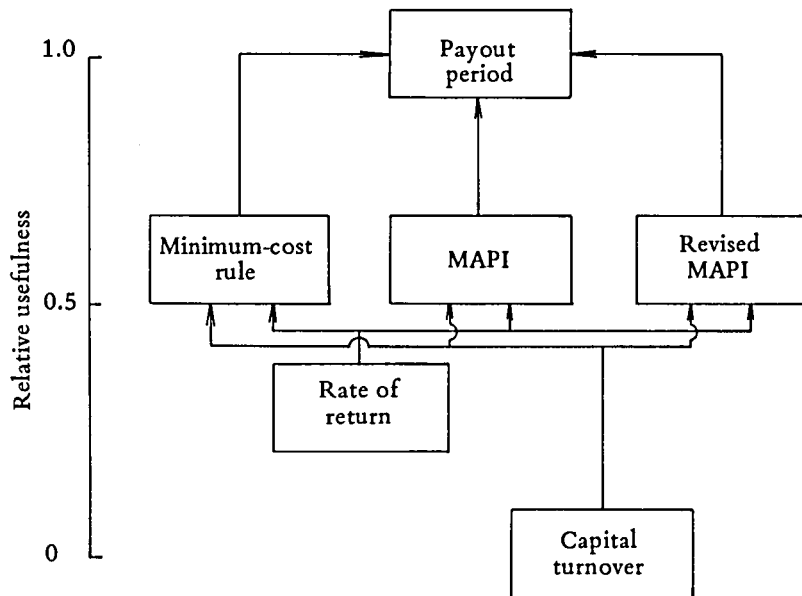


Figure 4.46. Relative usefulness of the economical criteria (by an accountant).

(2) Technical criterion

New equipments have direct influence upon the production technique, the labor requirement, the unit costs of materials, the cost for dynamic force, etc. They have also indirect influence upon the information of production technique and the activity of human resources (researchers, engineers), etc. Hence it is important to assess the investment not only from the economical viewpoint but also from the technical viewpoint. As a first step of technical assessment, an investment plan is graded into ten classes by comparing the new equipment with similar ones in other production firms.

The classes are as follows.

- A1 class: The new equipment is unprecedented in the world and it is possible to apply for an international patent.
- A2 class: The new equipment is unprecedented inside the country and it can be boasted in the world.
- B1 class: There is no similar equipment to the new equipment in the country and it is possible to apply for a domestic patent.
- B2 class: There are quite few similar equipments to the new equipment in the country and it can be boasted in the country.
- C1 class: There are similar equipments to the new equipment in the country but it has many better functions than the similar equipments of other production firms.
- C2 class: There are many similar equipments to the new equipment in the country and it has almost equivalent functions to the similar equipments of other production firms.
- D1 class: The new equipment has a few worse functions than the similar ones of other production firms.
- D2 class: The new equipment has worse functions than the similar ones of other production firms.
- E1 class: The new equipment gets behind in its functions compared with the similar ones of other production firms.
- E2 class: The new equipment gets behind a lot in its functions compared with the similar ones of other production firms.

4.6.2 Construction of Assessment Formula

The assessment is the preference representation of a decision maker, and its probability means that the decision maker has the preference structure to the objectives. If the preference structure of the decision maker is represented explicitly, the quantitative analysis will be made possible to the assessment procedure. R. L. Keeney proposed the multiattribute utility functions in 1974. In his description, the preference of the decision maker is represented explicitly if the utility independence and the preferential independence are confirmed. By use of the multiattribute utility function which is mentioned in Section 3.3, for the economical and technical criteria, the quantitative analysis can be done for assessing the investment.

Let f_i^0 and f_i^* denote the least and the most desirable values of attribute f_i , respectively. Table 4.18 shows the ranges of the three attributes decided by discussions among the person in charge, where f_3 denotes the technical criterion.

Table 4.18. Allowable ranges of f

f_i	f_i^*	f_i^0
f_1	200 %	0 %
f_2	0.5 year	8 years
f_3	A1 class	E2 class

(A) Verifying mutual utility independence and the additive structure [19], [20]

Because it is very important to verify independence conditions and the additive structure, we have a short dialogue with the decision maker for verification. First, we ask him about the certainty equivalent for the lottery which is shown in Figure 4.47. Figure 4.47 is the 50–50 lottery to the attribute f_1 where attributes f_2 and f_3 are fixed as $f_2 = 0.5$, $f_3 = A1$. His answer is thirty-eight percent. Further, we put some other questions to him by changing the values of f_2 and f_3 . As a result of those questions, we know that the value of certainty equivalent is always thirty-eight percent and it is indifferent to the values of f_2 and f_3 . In the

same manner, we ask him about the certainty equivalents for the 50–50 lottery to the attribute f_2 and to f_3 . Due to those investigations, we are able to verify the condition of mutual utility independence.

To verify the additive structure, we ask the decision maker whether the lotteries to the attributes f_1 and f_3 are indifferent or not. Figure 4.48 shows the lotteries. His answer is that the lottery L1 is preferred to the lottery L2. This result violates the condition for additive independence, and then the multiattribute utility function is not of an additive form.

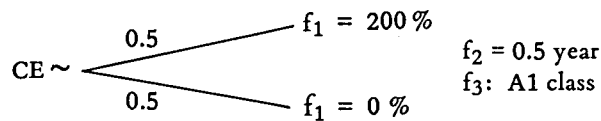


Figure 4.47. 50–50 lottery to the attribute f_1 .

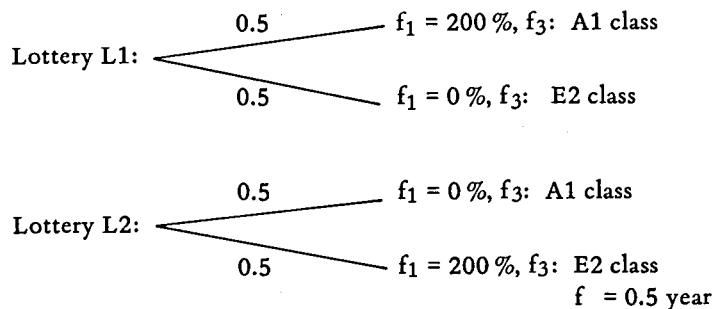


Figure 4.48. Lotteries to the attributes f_1 and f_3 .

(B) Construction of multiattribute utility function

For each individual utility function $u_i(f_i)$, we put the normalization condition as

$$\begin{aligned} u_i(f_i^0) &= 0.0, \\ u_i(f_i^*) &= 1.0. \end{aligned}$$

The values of u_i for f_i between f_i^0 and f_i^* have been decided by questionnaires to the decision maker, i. e., the vice-president of the firm.

Figures 4.49 through 4.51 show the individual utility functions obtained from his responses. Table 4.19 shows the values of k_i and k , evaluated from the responses of the vice-president.

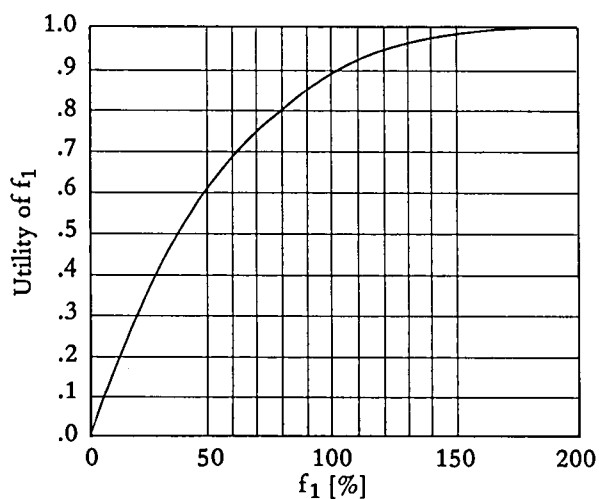


Figure 4.49. Utility function for the rate of return.

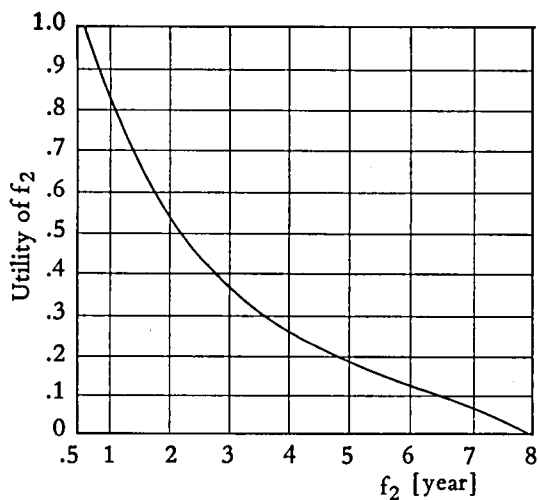


Figure 4.50. Utility function for the payout period.

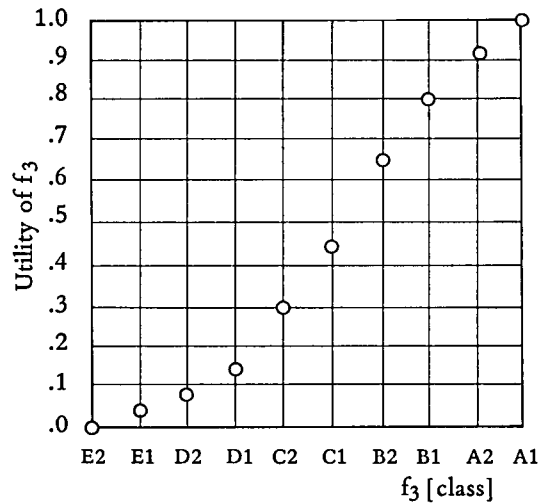


Figure 4.51. Utility function for the technical level of equipment.

Table 4.19. Values of k_i and k .

k_1	0.30	k_2	0.30
k_3	0.35	k	0.16

4.6.3 Example

A real problem considered here is from the assessment of investment plans of the production firm in 1980. There are four hundred and forty-one investment plans.

(A) Transformation of the assessment formula

Before applying the assessment formula to the real problem, we have had discussions among the persons in charge about the problems. Their opinions to be noted are such as follows:

- (1) Since the persons in charge (accountants, administrators) are not familiar to the complicated mathematical presentation, it is important to define the assessment formula as simple as possible.
- (2) If the assessment formula is not simple, the top-managers will not be able to

understand its meaning, then it will not be applicable to the real problems.

As a result of the discussions, we have decided to simplify the assessment formula. We substitute the values k_i and k into Equation (3.8) giving

$$\begin{aligned} U = & 0.30 u_1(f_1) + 0.30 u_2(f_2) + 0.35 u_3(f_3) + 0.015 u_1(f_1)u_2(f_2) \\ & + 0.017 u_2(f_2)u_3(f_3) + 0.017 u_1(f_1)u_3(f_3) \\ & + 0.001 u_1(f_1)u_2(f_2)u_3(f_3). \end{aligned} \quad (4.95)$$

The Equation (4.95) consists of the linear terms and the nonlinear terms, and the coefficients of the nonlinear terms are relatively small. Because of this, we have defined a simplified assessment formula as follows:

$$U = 0.30 u_1(f_1) + 0.30 u_2(f_2) + 0.40 u_3(f_3). \quad (4.96)$$

The coefficients of $u_i(f_i)$ are evaluated again by the questionnaires put to the decision maker.

This procedure may seem meaningless from the result of non-additive independence, but this is quite useful for the practical application. The above formula is used for setting a kind of standard for the assessment.

(B) Result of assessment

According to the purpose of investments, the investment plans are classified into into ten categories. They are for the new products, for the increase of production power, for the rationalization, for the research and development, for the welfare, etc. Among these categories, the investment plans for the research and development, for the welfare of the employee and for the safety are paid no regard at present. The total percentage of these is twenty-seven.

The investment plans from fourteen operational divisions are handled by using a computer. The input and output system is shown in Figure 4.52. Figure 4.53 shows the configuration of the operational divisions on the x, y-plane where x-axis indicates the utility of technical attribute $u_3(f_3)$ and y-axis indicates the utility of economical attribute $[u_1(f_1) + u_2(f_2)] / 2$. The investment plans are classified into three ranks according to the value of U , i. e.,

- A rank . . . $0.7 \leq U$,
- B rank . . . $0.5 \leq U < 0.7$, and

C rank ... $U < 0.5$.

Figure 4.54 shows the percentage of the plans put in each rank.

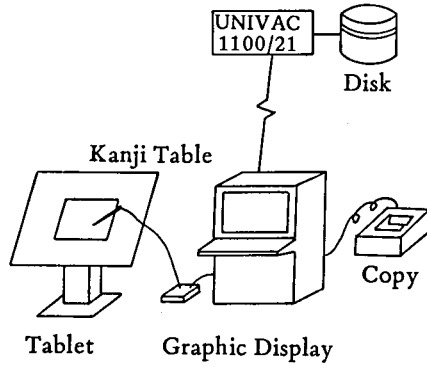


Figure 4.52. I-O system.

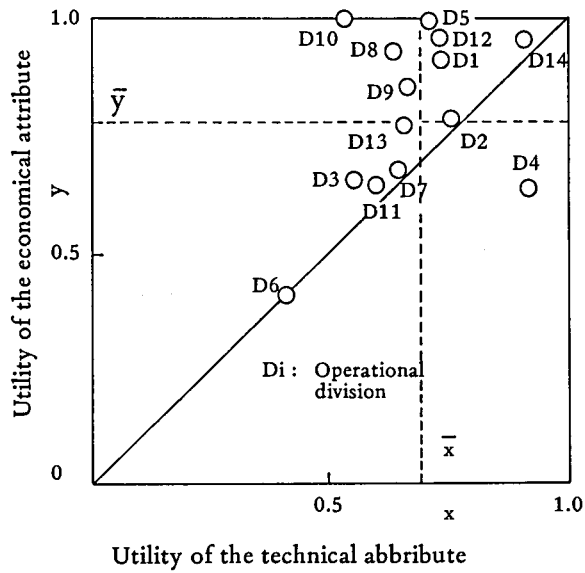


Figure 4.53. Configuration of the investment plans by divisions.

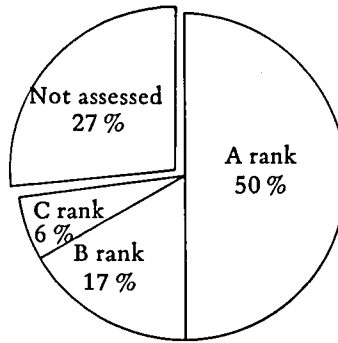


Figure 4.54. Ratio of the A, B and C ranks.

4.6.4 Concluding Remarks

We have derived a formula for the assessment of investment plans by use of the multiattribute utility function. As a result of real application, we have found that the proposed assessment formula works so well. The main findings are such as follows.

- (1) The decision of the top-managers can be made more readily and quickly than before because of the definite assessment criteria.
- (2) Since the assessment standard is defined clearly, everyone can give easily his ideas or advices on the investments.
- (3) As all the investment plans are represented on the x, y-plane (x: the utility of technical attribute, y: the utility of economical attribute), it is easy to see the relative utilities of those plans. Moreover, the principal intention of each operational division can easily be understood.

4.7 Conclusion and Future Recommendation

Some industrial management problems in a private sector have been successfully dealt with by using structural modeling procedure and multiobjective optimization procedure. As a result of real applications, we have found that these procedures work so well. The main findings are as follows.

- (1) Modeling of a real problem can successfully be performed by cooperation of all the persons concerned. Opinions and ideas of participants can be accepted by

a systems method without regard to their position, status or interests. IWSM method has demonstrated an excellent ability for structural modeling.

(2) Through the procedure of structural modeling, the decision maker has acquired confidence of the model and then a good motivation for optimization, because he has made out ideas for himself.

(3) The decision of the top-manager can be reflected more easily and quickly than before by using multiobjective optimization procedure, because once his criteria are made explicit, the decision procedure after that can be made automated to much extent.

Then we have been investigating some other problems such as man-power planning and reliable system design. Executing much iteration of systems approach to real problems, we can say with confidence that a total optimization of management in a firm will become finally possible.

CHAPTER 5

GENERAL CONCLUSION

This text has presented some studies on systems approach to industrial management problems.

In Chapter 1, the text has described the process of a systematic approach. The problems of the private sector are characteristic of something which possesses a hierarchy or that of a multi-layered nature. The systems approach here discussed serves as a “catalyst” in terms of this scheme to bring about the production profits.

Chapter 2 deals with the structural modeling method for model analysis. Development of a new method of the structural modeling is aimed at coping with the problems which have mutual relationships among system elements, each having an additive or multiplicative nature. This new method is termed as Interpretive Weighted Structural Modeling (IWSM). Here are analyzed the IWSM characteristics as well as those of Interpretive Structural Modeling (ISM) method and Decision Making Trial and Evaluation Laboratory (DEMATEL) method by referring to a simple example to be cited. Comparison among those methods has proved that IWSM is able to generate a clearer hierarchical structure than the others.

Chapter 3 is concerned with the procedures involved in the surrogate-worth tradeoff (SWT) method and a multiattribute utility function (MAUF) approach. The surrogate-worth function is found very useful, but, in practice, it usually takes a lot of time to identify the band of indifference. Besides that, its procedures are likely to have the decision makers often feel quite troublesome in taking them. With that put into our consideration, the decision making procedure with some modifications on tradeoff curves and IWSM method has been developed.

The procedure for constructing the multiattribute utility function is derived on the condition that the relationship between the individual utility function and the probability is linear. In dealing with real problems, however, when the relationship is checked with use of the questionnaires handed to decision makers

using the lottery, many of them violate it. Why this is caused is by a lack of our experiences in lottery. Therefore, using the indifference curve in combination with a lottery is investigated.

Chapter 4 is concerned with the real application to some industrial management problems. Multiobjective optimization procedure is applied to a blending problem of an industrial material formulated with six objectives. The idea of SWT method and the method of MAUF are applied to the problem, and the solution of SWT method is compared with that of MAUF method. The SWT method is useful to find the preference band, but, in practice, it takes a lot of time to identify the indifference band. The proposed modified procedure using the IWSM method works more effectively and the MAUF method is useful to find the preferred solution. With use of the MAUF, it is possible to solve the multiobjective optimization problem as the one objective optimization problem. And yet, in practice, the numerical construction of MAUF is often not so easy. In the present study, we have found that two decision makers among the four can not construct the MAUF so nicely. To these decision makers, the procedure of constructing the MAUF with the aid of their indifference curves has therefore been applied. This procedure seems effective for helping the decision makers when they are puzzled with the lotteries.

IWSM method is applied to real two problems. Its first application treats a scheduling of a data transmission test and the second one is concerned with an allocation of budget to various sections in a Research and Development Laboratory. The scheduling with use of the IWSM has some features as compared with the program evaluation and review technique (PERT). The persons concerned can get easily either of flow diagram, critical path, start times, finish times and floats of their own problems, and they can discuss among each other about their respective diagrams obtained. If there are some parties among the persons concerned, each party can bring out their diagrams and they can proceed with discussion about those diagrams. As a result of this, a scheduling problem will be solved with common consent reached among all of the persons concerned.

The budget allocation in a Research and Development Laboratory of a firm can be decided using the IWSM procedure and nonlinear programming. The deciding procedure of budget allocation seems a little complicated because of

some subprocedures having had to be added to the IWSM. However, by experience, the present procedure is practically feasible and well convincing to the parties concerned.

Multiobjective optimization procedure is applied to a decision problem in an inventory management. Criteria of optimization of fixed-period control system are selected through discussions among the persons concerned who are facing the problem inclusive of shortage of goods, excessive stock and a period of stock. Among several methodologies applied to this problem, the most successful one, i. e., a combined use of the MAUF method and nonlinear programming technique is presented. The same procedure is applied to a fixed-order-quantity system. A real problem of inventory management of the building materials (interior) for architecture has successfully been dealt with by the present procedure. Along with simulation of the system for the past one year, it is found that the shortage of goods and the excessive stock are each almost halved compared with the same period of stock earlier. The system was already put in operation in June 1984, and has since then been well accepted by the persons concerned. It enables them to save their job time drastically too; the time required for completing the inventory plan has been reduced from four days to a half day.

Multiobjective assessment procedure is applied to the assessment of investment plans in a production firm. The total assessment formula is derived considering both the economical and the technical terms by use of the MAUF method. As a result of the real application, we have found that the proposed assessment formula works so well. The top manager can make his decision more readily and quickly than ever before, owing to the definite assessment criteria thus established. Some problems in a private sector have been successfully dealt with by use of a systems approach technique. The industrial management problems are quite interesting from the viewpoint of a total optimization. There may be case that the optimum experimental achievements obtained at the product development stage could either be judged as totally useless or neglected at all at the production stages. Hence, it is worthwhile to examine the problems, possibly occurring in any of the production stages, in a more strict mathematical manner or by use of a heuristic approach.

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APPENDIX I

MULTIPLIER METHOD FOR NONLINEAR PROGRAMMING

In Chapter 4, some problems are solved by use of a nonlinear programming technique. The mathematical programming problem is to determine a vector $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ that solves the problem

$$\text{Minimize } f(x) \quad (\text{I. 1})$$

subject to

$$g_i(x) \geq 0, \quad i = 1, 2, \dots, m. \quad (\text{I. 2})$$

If any of the functions is not linear, the problem is called a nonlinear programming problem.

Multiplier method is a transformation approach which transforms a given constrained minimization problem into a sequence of unconstrained minimization problem. This transformation is accomplished by defining an appropriate auxiliary function to define a new objective function whose minima are unconstrained in some domain. By gradually removing the effect of the constraints in the auxiliary function by controlled changes in the value of parameters, a sequence unconstrained problem is generated that have solutions converging to a solution of the original constrained problem [9].

The multiplier function is defined as follows [46].

$$L(x, t) = f(x) + \sum_{i=1}^m \begin{cases} tg_i^2 - \sigma_i g_i(x) & , \quad g_i(x) \leq 0 \\ -\frac{\sigma_i^2 g_i(x)}{tg_i(x) + \sigma_i} & , \quad g_i(x) > 0 \end{cases} \quad (\text{I. 3})$$

where t is a penalty parameter and σ_i ($i = 1, 2, \dots, m$) is Lagrange multipliers
Figure I. 1 shows a flow chart of multiplier method.

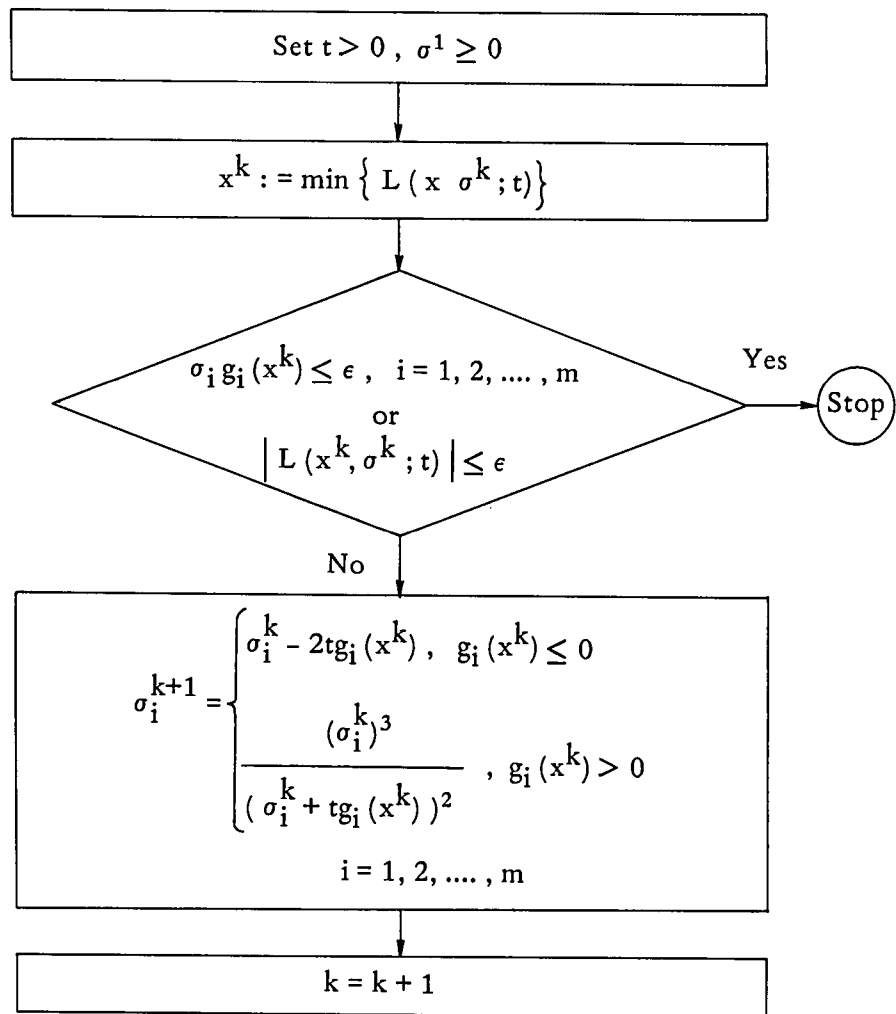


Figure I. 1. Flow chart of multiplier method.

APPENDIX II

WINTERS EXPONENTIAL SMOOTHING METHOD

In Chapter 4, future demand is predicted by use of Winters exponential smoothing method. Using this method, demand in ℓ months hence from this month, $\hat{X}_{t+\ell}$, is predicted as follows.

$$\hat{X}_{t+\ell} = (S_t + \ell \cdot R_t) \cdot F_{t+\ell-L} \quad (\text{II. 1})$$

$$S_t = A \cdot X_t / F_{t-L} + (1 - A) S_{t-1} + R_{t-1} \quad (\text{II. 2})$$

$$R_t = C \cdot (S_t - S_{t-1}) + (1 - C) \cdot R_{t-1} \quad (\text{II. 3})$$

$$F_t = B \cdot X_t / S_t + (1 - B) \cdot F_{t-L} \quad (\text{II. 4})$$

where

X_t = actual demand at the end of this month (t)

S_t = basic predicted demand at the end of this month (t)

R_t = trends factor at the end of this month (t)

$\hat{X}_{t+\ell}$ = Predicated demand in ℓ months hence from this month (t+ ℓ)

L = seasonal cycle ($L = 12$)

F_t = seasonal effects factor at the end of this month (t)

A, B, C = smoothing parameter

Initial values of S_t, R_t, F_t are assessed using time series past data. Dividing past data into two parts and supposing that the first part has H periods data which are some times as much as seasonal cycle, initial values of them can be assessed as follows.

(a) Initial value of S_t

The mean value of actual demand in the i th period ($i = 1, 2, \dots, H/L$), V_i , is defined as

$$V_i = \frac{L}{\sum_{j=1}^L} V'_{ij} / L, \quad (\text{II. 5})$$

where

V'_{ij} = the actual demand of j th month in the i th period.

The initial value of S_t is defined as

$$S_1 = V_1. \quad (\text{II. 6})$$

(b) Initial value of R_t

Considering the mean value of trends, the initial value of R_t is defined as

$$R_1 = (V_{H/L} - V_1) (H - L). \quad (\text{II. 7})$$

(c) Initial value of F_t

The basic value of seasonal effects factor of j th month in the i th period is defined as

$$F_{ij} = (X_L \cdot (i-1) + j) / (V_i - ((L+1)/2 - j) R_1). \quad (\text{II. 8})$$

Then, the initial values of F_{j-L} ($j = 1, 2, \dots, L$) is defined as follows.

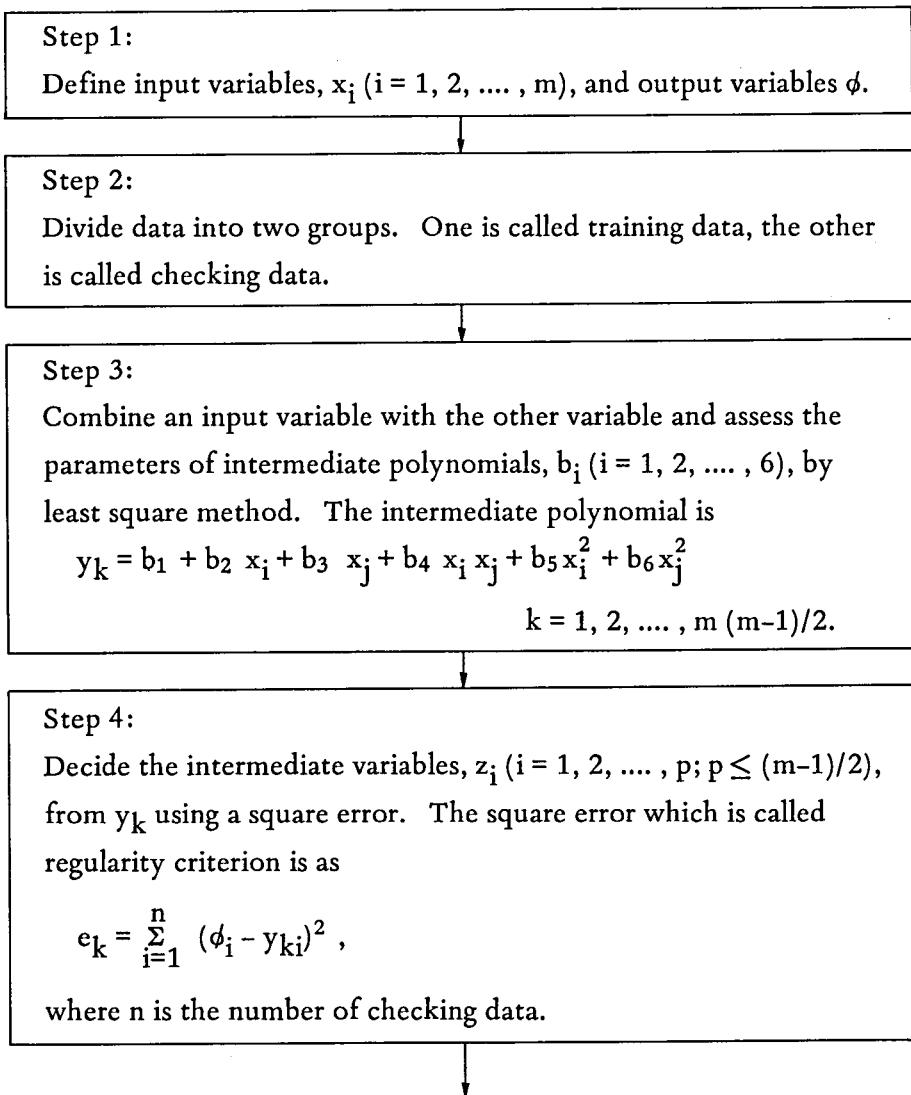
$$F_{j-L} = F'_{j-L} L / \sum_{j=1}^L F'_{j-L} \quad (\text{II. 9})$$

$$F'_{j-L} = \sum_{i=1}^{H/L} F_{ij} / (H/L) \quad (\text{II. 10})$$

APPENDIX III

DEMAND PREDICTION BY GMDH AND IWSM ALGORITHM

In Chapter 4, future demand is predicted by use of GMDH (Group Method of Data Handling) and IWSM (Interpretive Weighted Structural Modeling) [15] [33] [48]. A GMDH algorithm is shown in Figure III. 1.



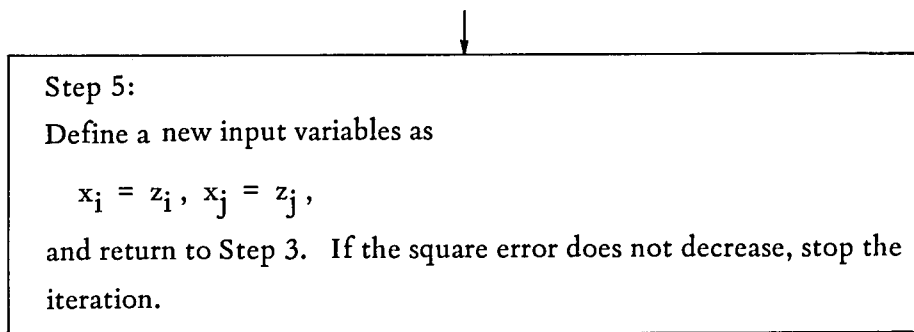


Figure III. 1. Procedure of GMDH.

The most important step to predict future demand using GMDH is the selection of input variables, because if future demand is predicted using many factors which are indifferent to the prediction, the value of prediction error will increase. Sample correlation analysis is often used to select input variables, but it is not so useful because the structure of models for demand prediction is usually nonlinear. IWSM is used to select effective input variables. The effectiveness of this procedure is shown in Chapter 4.