

Doctoral Thesis

Numerical simulation of the tsunami-induced electromagnetic
field using a time-domain finite element method:
application to the 2011 Tohoku Earthquake tsunami

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Abstract

Conductive seawater moving in the geomagnetic main field drives an electromotive force and induces secondary electromagnetic (EM) fields. This effect is well known as “oceanic dynamo effect” and has been investigated for many years, especially for low-frequency phenomena such as tides and steady oceanic flows. However, it was recently found that tsunamis are also significant sources of the oceanic dynamo effect. Toh et al. (2011) reported tsunami-induced EM field data observed at the northwest Pacific seafloor EM station (NWP) at the time of the 2006/2007 Krill tsunamigenic earthquakes. Ever since, many oceanic dynamo events associated with tsunamis, we hereafter call “tsunami dynamo effect”, have been reported (e.g., Manoj et al. 2011; Suetsugu et al., 2012; Ichihara et al., 2013). To explain the tsunami dynamo effects, most of the preceding studies adopted analytical approaches in the frequency domain (e.g., Tyler, 2005). However, frequency-domain analyses are not enough because tsunamis are inherently transient phenomena. In addition, it is difficult to include real bathymetry and complicated conductivity structures beneath the seafloor in analytical solutions of the frequency domain.

In this study, we developed a two-dimensional (2-D) tsunami dynamo simulation code using the finite element method (FEM) in the time domain in order to reproduce the tsunami-induced magnetic field variation observed at NWP at the time of the 2011 off the Pacific Coast of Tohoku Earthquake. We adopted triangular elements for our FEM calculations so that we can express complicated bathymetry and include arbitrary conductivity structures beneath the seafloor. Our simulation is composed of two steps. In the first step, we conducted hydrodynamic calculations to obtain oceanic flows due to tsunamis. In the second step, we calculated EM fields induced by the tsunamis using the obtained

oceanic flows as a source field. Our simulation successfully reproduced the horizontal and vertical component of the magnetic field as large as 3 nT induced by the 2011 Tohoku earthquake tsunami. It was also revealed that an initial rise in the horizontal magnetic component was present about 5 minutes prior to its arrival. We, therefore, classified the tsunami-induced magnetic signals into three types for the 2011 Tohoku tsunami. The first type is the initial rise in the horizontal component parallel to the tsunami propagation direction as large as 1 nT, detected approximately 5 minutes before the tsunami arrival. Second one is the negative main peak of the upward component as large as 3 nT in phase with the tsunami peak itself. Third one is the negative main peak of the horizontal component as large as 3 nT reached about 5 minutes after the tsunami peak. The initial rise in the horizontal component, may enable us to detect tsunami arrival prior to the actual tsunami arrival, because it was as large as 1 nT even at the seafloor site as far as 1500 km from the epicenter. Our additional simulation using solitary waves, however, revealed that the initial rise in the horizontal magnetic component becomes smaller, as the conductivity beneath the seafloor increases. The initial rise was found to become undetectable for the sub-seafloor conductivity of 0.1 S/m. This indicates that it is still possible to apply the initial rise to future tsunami early warnings in regions with resistive substrata beneath the seafloor. On the other hand, it was also found that, as the conductivity beneath the seafloor increases, the main peak in the vertical magnetic component becomes smaller, while that of the horizontal magnetic component larger. This aspect of tsunami dynamo effects provides us a new method of using the tsunami-induced EM field for inferring the conductivity structure beneath the seafloor.

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1. Introduction

When tsunamis propagate in the geomagnetic main field, EM fields are induced. Although the so-called “oceanic dynamo effect” itself had been already predicted in the early 19th century (Faraday, 1832), no EM observation data associated with tsunamis were reported until Toh et al. (2011), which reported EM field variations on the northwest Pacific seafloor, induced by the 2006/2007 Kuril earthquake tsunamis. The dynamo effect due to tsunamis, called hereafter “Tsunami Dynamo Effect (TDE)”, has a possibility to be applied to tsunami early warnings. Toh et al. (2011) pointed out that seafloor vector EM observations enable us to know the tsunami propagation direction by a single site observation alone.

Conductive seawater moving in the geomagnetic main field couples with the geomagnetic components perpendicular to the seawater motion to induce secondary EM fields in the ocean. This effect is well known as “Oceanic Dynamo Effect (ODE)”, which includes TDE. ODE has been studied for many decades and its origin dates back to the 19th century. Faraday (1832) first recognized ODE and attempted to detect the effect. Unfortunately, he could not detect any potential difference across the Thames River probably due to unsophisticated instruments at his age, when he conducted observations using two copper plates suspended from the Waterloo Bridge. It was not until late 1940s that ODE began to attract interests of scientists again. Many analytical studies (e.g., Stommel, 1948; Longuet-Higgins, 1949) led to seafloor potential difference observations using submarine cables in the 1950s (e.g., Wertheim, 1954). On the other hand, thanks to the development of the new seafloor magnetometer (Filloux and Cox, 1967), Larsen (1968) demonstrated that EM field variations observed on the seafloor are influenced by ODE associated with the lunar tide. Since then, many important

analytical studies on ODE were made. As for ODE due to surface gravity waves, Weaver (1965) derived expressions of EM fields induced by short waves and swells, while Larsen (1971) presented analytical solutions for long and intermediate waves. Sanford (1971) derived a most comprehensive theory on EM fields and low-frequency oceanic velocity fields, including effects of bathymetry and conductivity structures beneath the seafloor. Chave (1983) presented a solution of the EM field associated with ODE, using the poloidal/toroidal decomposition (Price, 1950) and the Green function expression, which was later extended to include vertically varying seawater conductivity (Chave and Luther, 1990). These useful analytical solutions were mainly used to explain ‘low-frequency’ seafloor potential difference variation data obtained by submarine cable observations (e.g., Larsen and Sanford, 1985; Sanford, 1986; Luther et al., 1987) or by point sensors on the seafloor (e.g., Luther et al., 1991; Fujii and Chave, 1999), although some analytical solutions including self-induction terms (e.g., Larsen, 1971; Chave, 1983) are applicable to high-frequency phenomena such as TDE. Since most of the previous works on ODE focused on low-frequency phenomena such as tides and steady oceanic flows, possibilities of detecting EM fields induced by tsunamis were not argued in the 20th century. This was partly because it was difficult to conduct long-term continuous EM observations in the ocean.

However, in the 21st century, thanks to development of sophisticated seafloor EM instruments and the expansion of the high-resolution magnetic surveys by low earth orbit (LEO) satellites (e.g., CHAMP; <http://op.gfz-potsdam.de/champ/>), TDE gradually became an attractive scientific subject. For instance, after Tyler (2003) recognized that the ocean lunar semidiurnal (M2; 12.42h period) tide affected magnetic field variations observed at the

CHAMP satellite altitude (430km), Tyler (2005) derived a useful simple formula for tsunami-induced EM variations including the self-induction and investigated feasibility of detecting TDE signals by satellite-borne magnetometers. On the other hand, new seafloor EM instruments, such as Ocean Bottom Electro Magnetometer (OBEM; e.g., Shiobara et al., 2009) and SeaFloor Electro-Magnetic Station (SFEMS; Toh et al., 2004, 2006) enabled us to conduct long-term seafloor EM vector observations in the 2000s, which led to the first TDE report by Toh et al. (2011). In recent years, many reports of tsunami-induced EM variations observed on the seafloor followed Toh et al. (2011), mostly because the two devastating tsunamis were generated by the Chilean earthquake on 27 February 2010 (M8.8) and the Off the Pacific Coast of Tohoku earthquake on 11 March 2011 (M9.0) (e.g., Manoj et al., 2011; Utada et al., 2011; Suetsugu et al., 2012; Ichihara et al., 2013).

It is also noticeable, for the development of ODE studies, that the global/three-dimensional (3-D) numerical simulation techniques, which demand very high computation power, have been promoting the ODE studies. With the aid of the thin-shell approximation (Price, 1949), a number of global simulations for the EM fields induced by the steady oceanic circulation flows were carried out, using the thin sheet composed of laterally variable conductivity and the insulating mantle underneath (e.g., Stephenson and Bryan, 1992; Tyler et al., 1997, 1999; Vivier et al., 2004). Tyler (2003) also conducted global-scale simulations for the EM field induced by the M2 tide oceanic flow using the thin shell with the insulating mantle. After Kuvshinov et al. (2002) applied the modified iterative dissipative method (MIDM), originally formulated by Singer (1995), to global induction problems, it was enabled to appreciate the effect of conducting mantle and 3-D conductive anomalies beneath the ocean in 3-D global ODE simulations. By means of the applied MIDM method, many 3-D global simulations

were performed in order to estimate the EM field variations induced by the steady ocean circulation flows (Manoj et al., 2006) as well as the tide oceanic flows (Kuvshinov and Olsen, 2005; Kuvshinov et al., 2006), in the presence of the 1-D conducting mantle based on the 3-layer model of Schumucker (1985). Maus and Olsen (2004) compared the predicted magnetic fields with observatory and satellite measurements in detail (see Kuvshinov (2008) for more detailed review about the 3-D ODE simulations). Eventually, the MIDM global simulation technique led to the first numerical simulation associated with TDE, performed by Manoj et al. (2010), which investigated feasibility of detecting the electric voltage induced by the 2004 Sumatra earthquake tsunami by submarine cable observations. Zhang et al. (2013) tried 3-D numerical simulations of the EM field induced by the 2011 Tohoku earthquake tsunami, applying a local 3-D MIDM simulation code, originally coded by Koyama et al. (2002).

Many analytical studies associated with TDE have been conducted so far, and some seafloor EM observations succeeded in detecting tsunami EM signals in recent years. Furthermore, numerical simulations of ODE associated with tsunamis as well as the tide and ocean circulation have increased recently. Using these analytical studies, observations, and numerical simulations, we are now able to investigate TDE in detail. However, it is noticeable that all of the numerical simulations associated with TDE are in the frequency domain because most of them are extended from studies on the low-frequency periodic phenomena. In addition, many useful analytical solutions of EM field associated with TDE (e.g., Tyler, 2005) are also derived in the frequency domain. Since tsunamis are inherently transient waves, especially for the first arrival of tsunamis, it is difficult to predict the tsunami-induced EM field by employing analyses in the frequency domain.

In this study, we developed a two-dimensional (2-D) time-domain tsunami-dynamo simulation code using the finite element method (FEM), which is capable of including actual bathymetry and arbitrary conductivity structures beneath the seafloor. We adopted FEM using triangular elements because unstructured triangular meshes can express actual bathymetry, which may be virtually impossible by rectangular ones (Minami and Toh, 2012). With our new time-domain code, we successfully reproduced seafloor magnetic variations, as large as 3 nT both in the horizontal and downward component, observed at the northwest Pacific seafloor induced by the passage of the 2011 Tohoku earthquake tsunami (Minami and Toh, 2013). It was also found that an initial rise in the horizontal magnetic component as large as 1 nT, which appeared observed about 5 minutes prior to the tsunami arrival, were also induced by the tsunami passage. Although it is necessary to conduct 3-D simulations to appreciate effects of 3-D tsunami sources and real bathymetry, our 2-D numerical simulations revealed fundamental physics of the TDE in the northwest Pacific driven by the 2011 Tohoku earthquake tsunami. In this thesis, we will discuss how the tsunami EM signals are generated by the tsunami passage.

Tsunami-induced EM fields depend on many physical quantities, e.g., the oceanic velocity field, the bathymetry, the ambient geomagnetic main field, the conductivity of the seawater, and the conductivity structures beneath the seafloor, and so on. Among these quantities, the oceanic velocity field and the conductivity structure beneath the seafloor are most difficult to determine precisely. If we know accurate oceanic velocity field, say, by hydrodynamic numerical simulations, observation of tsunami-induced EM fields enables us to obtain information on the conductivity structures, by comparing observed EM fields with those calculated. On the other hand, if we succeed in reproducing past tsunami-induced EM

variations employing some probable conductivity structure beneath the seafloor, we can obtain information of tsunami velocity field from observed EM variations. Our new time-domain tsunami dynamo simulation code may contribute to accurately predict tsunami-induced EM fields accurately for given conductivity structures and actual bathymetry.

In the following, we first describe analytical solutions of TDE in the frequency domain. Secondly, we present the tsunami-induced magnetic variation data observed by our research group in the northwest Pacific. Thirdly, we explain the method of our 2-D time-domain TDE simulation and present the simulation results. Finally, we compare the simulation results with the observed tsunami magnetic signals and discuss the application of TDE to future tsunami early warning.

2. Theory of the tsunami dynamo effect

Actual tsunamis are always 3-D due to complicated bathymetry and coastlines in addition to the 3-D distribution of initial sea surface topography. However, it is useful to understand 2-D phenomena of the tsunami-induced EM fields before working on 3-D problems. Here we consider two modes of 2-D tsunami dynamo effects, transverse electric mode (TE mode) and transverse magnetic mode (TM mode).

In 2-D configurations, it is well known that the induction problems decouple into TE and TM mode. In our 2-D tsunami dynamo studies, we further assume that the tsunami propagates in the direction perpendicular to the structure strike. When we assume the strike is along the x -direction, say $\partial/\partial x = 0$, the velocity of sea water is expressed as $\mathbf{v} = (0, v_y, v_z)$. Then, the three of quasi-static Maxwell's equations and the Ohm's law for a moving conductor are given by;

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2-1)$$

$$\nabla \times \mathbf{H} = \mathbf{i}, \quad (2-2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2-3)$$

$$\mathbf{i} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (2-4)$$

and decouple into TE-mode equations,

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}, \quad (2-5)$$

$$\frac{\partial E_x}{\partial y} = \frac{\partial B_z}{\partial t}, \quad (2-6)$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu\sigma(E_x + v_y B_z - v_z B_y), \quad (2-7)$$

$$\frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0, \quad (2-8)$$

and TM-mode equations,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}, \quad (2-9)$$

$$\frac{\partial B_x}{\partial z} = \mu\sigma(E_y + v_z B_x), \quad (2-10)$$

$$\frac{\partial B_x}{\partial y} = -\mu\sigma(E_z - v_y B_x), \quad (2-11)$$

Where $\mathbf{E}$, $\mathbf{B}$, $\mathbf{i}$, and $\mathbf{v}$ are the electric and magnetic field, the electric current density, and the oceanic velocity, respectively. In TE mode, horizontal and vertical component of oceanic flows couple with the vertical and horizontal component of the geomagnetic main field respectively and generate horizontal electromotive forces (emfs) perpendicular to the tsunami propagation direction as shown in Fig. 2-1. On the other hand, in TM mode, the horizontal and vertical component of the oceanic flows produce emfs within a vertical cross section along the tsunami propagation direction in the presence of the horizontal geomagnetic main field, as shown in Fig. 2-2. Actually, emfs driven by ODE is significant only in TE mode because those in TM mode are negligible (Larsen, 1971). This is probably the reason why most scientists have focused only on analytical studies in TE mode (e.g., Sanford, 1971; Tyler, 2005; Ichihara et al., 2013).

In this section, we first derive analytical solutions of the EM field induced in TE mode in Section 2.1. Secondly, we demonstrate emfs in TM mode negligible by referring Larsen's (1971) theory in Section 2.2.

2.1 TE mode theory

Sanford (1971) derived an analytical solution, including effects of bathymetry but neglecting the self-induction term. On the other hand, Larsen (1971) assumed a flat

seafloor and derived analytical expression of tsunami-induced magnetic fields with the self-induction term, assuming source velocity field is generated by intermediate/long waves. Tyler (2005) simplified Larsen's (1971) expression to obtain useful relationship between the tsunami height and the tsunami-induced magnetic field. Recently, Ichihara et al. (2013) derived the EM field on the ocean bottom with a homogeneously conducting half-space beneath the ocean. Here we derive analytical expressions for tsunami-induced EM field in TE mode in which a homogeneous conductivity is present beneath the flat seafloor. We first derive the EM field induced by tsunamis using the long wave approximation in Section 2.1.1. In Section 2.1.2, we further derive the EM field induced by linear dispersive waves, which have vertically varying horizontal velocities, in order to discuss effects of dispersion on tsunami-induced EM fields.

2.1.1 EM fields induced by long waves

Let us start with the induction equation in terms of the vertical component of the magnetic field,

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial y}(v_y B_z - v_z B_y) + K \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) B_z, \quad (2-1-1)$$

which is derived using Eqs. (2-6) – (2-8). Here $K = (\mu\sigma)^{-1}$ is the magnetic diffusivity.

We decompose the magnetic field $\mathbf{B}$ as $\mathbf{F} + \mathbf{b}$, where $\mathbf{F}$ and $\mathbf{b}$ are the geomagnetic main field and the tsunami-induced magnetic field, respectively. $\mathbf{F}$ obeys $\nabla^2 \mathbf{F} = 0$ and $\nabla \times \mathbf{F} = 0$. If we further assume $|\mathbf{b}| \ll |\mathbf{F}|$ and tsunamis can be approximated by linear long surface waves such that $|v_y| \gg |v_z|$, Eq. (2-1-1) is reduced to

$$\frac{\partial b_z}{\partial t} = -F_z \frac{\partial v_y}{\partial y} + K \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) b_z. \quad (2-1-2)$$

Now we assume a flat seafloor ($z = -h$) and place a half-space insulator and a homogeneous conductor (σ_1) above and beneath the ocean layer (σ_s) for the air and crust/mantle layer, respectively. If we express $v_y, b_z \propto e^{i(ky - \omega t)}$, where k and ω are the wave number and the angular frequency, Eq. (2-1-2) in each layer is rewritten as

$$\left(\frac{\partial^2}{\partial z^2} - k^2\right)b_z = 0 \quad (0 \leq z), \quad (2-1-3)$$

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2\right)b_z = \frac{ikv_y F_z}{K_s} \quad (-h \leq z \leq 0), \quad (2-1-4)$$

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_1^2\right)b_z = 0 \quad (z \leq -h), \quad (2-1-5)$$

where $\alpha_s = \sqrt{k^2 - i\omega\mu\sigma_s}$, $K_s = (\mu\sigma_s)^{-1}$ and $\alpha_1 = \sqrt{k^2 - i\omega\mu\sigma_1}$. Adopting the long wave approximation, the relationship between the horizontal flow and the surface elevation is expressed as

$$v_y = c \frac{\eta}{h}, \quad (2-1-6)$$

where $c = \omega/k = \sqrt{gh}$ is the phase velocity of the tsunami under the long wave approximation. Using Eq. (2-1-6) and the boundary condition that b_z and b_y are continuous at the sea surface and seafloor, Eq. (2-1-3) – (2-1-5) are solved to obtain the expression for the b_z in the ocean layer,

$$b_z = \left\{ \frac{-\sinh(\alpha_s z) + C_1 \cosh(\alpha_s z) + C_2 \cosh(\alpha_s(z+h)) + \sinh(\alpha_s(z+h))}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} - 1 \right\} \times \frac{i\omega\eta F_z}{\alpha_s^2 K_s h} \quad (-h \leq z \leq 0), \quad (2-1-7)$$

where $C_1 = \alpha_s/k$ and $C_2 = \alpha_s/\alpha_1$. Using the relation $b_y = (i/k)\partial b_z/\partial z$, derived from Eq. (2-8), b_y in the ocean is expressed as

$$b_y = \frac{-\cosh(\alpha_s z) + C_1 \sinh(\alpha_s z) + C_2 \sinh(\alpha_s(z+h)) + \cosh(\alpha_s(z+h))}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)}$$

$$\times \left(-\frac{c\eta F_z}{\alpha_s K_s h} \right) \quad (-h \leq z \leq 0), \quad (2-1-8)$$

The electric field parallel to the strike, $e_x + (\mathbf{v} \times \mathbf{B})_x$, is obtained using the relation $e_x = -cb_z$, derived from Eq. (2-6), as

$$e_x + (\mathbf{v} \times \mathbf{B})_x = -cb_z + \frac{c\eta F_z}{h}, \quad (2-1-9)$$

where b_z is given by Eq. (2-1-7). Figure 2-3 shows the predicted b_z , b_y , and $e_x + (\mathbf{v} \times \mathbf{B})_x$, assuming $h = 6000$ m, $F_z = 30000$ nT, and $\eta = 1.0$ m. It is shown in Fig. 2-3 that the amplitude of b_z on the seafloor decreases, while that of b_y increases, when the conductivity beneath the ocean increases. This is because highly conductive medium beneath the seafloor allows much electric currents to flow beneath the ocean, which prevents the vertical magnetic field from penetrating into the seafloor while reinforcing the tangential magnetic field.

Substituting $z = -h$ to Eqs. (2-1-7) and (2-1-8), we can obtain useful relationships between the seafloor magnetic components and the sea level change,

$$\begin{aligned} \frac{b_z}{F_z} &= \frac{i\omega}{\alpha_1 \alpha_s K_s} \frac{1 - \cosh(\alpha_s h) - C_1 \sinh(\alpha_s h)}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} \times \frac{\eta}{h} \\ &= Z(T, h, \sigma_s, \sigma_1) \times \frac{\eta}{h} \end{aligned} \quad (2-1-10)$$

$$\begin{aligned} \frac{b_y}{F_z} &= -\frac{c}{\alpha_s K_s} \frac{1 - \cosh(\alpha_s h) - C_1 \sinh(\alpha_s h)}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} \times \frac{\eta}{h} \\ &= i \left(\frac{\alpha_1}{k} \right) \times Z(T, h, \sigma_s, \sigma_1) \times \frac{\eta}{h}. \end{aligned} \quad (2-1-11)$$

Equations (2-1-10) and (2-1-11) are identical to Eqs. (C.5) and (C.6) in Appendix C of Ichihara et al. (2013), respectively. If we put the conductivity beneath the ocean to nil, Eq. (2-1-10) is reduced to Tyler's (2005) simple relationship on the seafloor,

$$\frac{b_z}{F_z} = \frac{c}{c + ic_d} \times \frac{\eta}{h}, \quad (2-1-12)$$

where $c_d = 2K/h$. Here $kh/2 \ll 1$ and $h/\delta \ll 1$ are assumed, where δ is the skin depth

of the sea water.

2.1.2 EM field induced by linear dispersive waves

Procedure of deriving the EM field induced by linear dispersive waves is similar to that of the previous section up to Eq. (2-1-5). For linear dispersive waves, we assume irrotational and incompressible tsunami flows and solve the Laplace equation of the stream potential,

$$\Delta\Phi = 0, \quad (2-1-13)$$

with the following linear boundary conditions,

$$\frac{\partial\Phi}{\partial t} + g\eta = 0 \quad (z = 0), \quad (2-1-14)$$

$$\frac{\partial\eta}{\partial t} - \frac{\partial\Phi}{\partial z} = 0 \quad (z = 0), \quad (2-1-15)$$

$$\frac{\partial\Phi}{\partial z} = 0 \quad (z = -h). \quad (2-1-16)$$

Provided that $\Phi \propto e^{i(ky-\omega t)}$, and the initial phase of η is zero, the solution is written as

$$\Phi = -iac \frac{\cosh(k(z+h))}{\sinh(kh)} \quad (2-1-17)$$

$$\eta = a \quad (2-1-18)$$

$$v_y = a\omega \frac{\cosh(k(z+h))}{\sinh(kh)} \quad (2-1-19)$$

$$v_z = -ia\omega \frac{\sinh(k(z+h))}{\sinh(kh)}, \quad (2-1-20)$$

where $e^{i(ky-\omega t)}$ is omitted for simplicity, c and a are the phase velocity and the amplitude of the sea level change, respectively. Substituting Eq. (2-1-19) into Eq. (2-1-4), the governing equation in the ocean layer is rewritten as

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_z = A \cosh(k(z+h)) \quad (-h \leq z \leq 0) \quad (2-1-21)$$

$$\left(A = \frac{F_z}{K_s} \frac{ia\omega k}{\sinh(kh)} \right) .$$

We can express solutions in each layer using unknown complex constants, A_0 , B_0 , A_1 , and B_1 , as

$$b_z = A_0 e^{-kz} \quad (0 < z) \quad (2-1-22)$$

$$b_z = A_1 e^{\alpha_s z} + B_1 e^{-\alpha_s(z+h)} - \frac{A \cosh(k(z+h))}{\alpha_s^2 - k^2} \quad (-h < z < 0) \quad (2-1-23)$$

$$b_z = B_0 e^{\alpha_1(z+h)} \quad (z < -h). \quad (2-1-24)$$

On the other hand, using the relation, $b_y = (i/k)(\partial b_z / \partial z)$, the horizontal magnetic field in each layer is derived as

$$b_y = -iA_0 e^{-kz} \quad (0 < z) \quad (2-1-25)$$

$$b_y = \frac{i\alpha_s}{k} (A_1 e^{\alpha_s z} - B_1 e^{-\alpha_s(z+h)}) - \frac{iA \sinh(k(z+h))}{\alpha_s^2 - k^2} \quad (-h < z < 0) \quad (2-1-26)$$

$$b_y = \frac{i\alpha_1}{k} B_0 e^{\alpha_1(z+h)} \quad (z < -h). \quad (2-1-27)$$

Adopting continuity of b_z and b_y at the ocean surface and bottom as boundary conditions, the resulting simultaneous equation for A_0 , B_0 , A_1 , and B_1 is expressed as

$$A_0 = A_1 + B_1 e^{-\alpha_s h} - \frac{A}{\alpha_s^2 - k^2} \cosh(kh) \quad (b_z \text{ at } z = 0) \quad (2-1-28)$$

$$B_0 = A_1 e^{-\alpha_s h} + B_1 - \frac{A}{\alpha_s^2 - k^2} \quad (b_z \text{ at } z = -h) \quad (2-1-29)$$

$$-A_0 = \frac{\alpha_s}{k} (A_1 - B_1 e^{-\alpha_s h}) - \frac{A}{\alpha_s^2 - k^2} \sinh(kh) \quad (b_y \text{ at } z = 0) \quad (2-1-30)$$

$$\frac{\alpha_1}{k} B_0 = \frac{\alpha_s}{k} (A_1 e^{-\alpha_s h} - B_1) \quad (b_y \text{ at } z = -h). \quad (2-1-31)$$

A_0 , B_0 , A_1 , and B_1 can be obtained by solving Eqs. (2-1-28) - (2-1-31). It follows that the vertical and horizontal component of the magnetic field induced by linear dispersive waves

are expressed as

$$b_z = \left\{ \frac{-\sinh(\alpha_s z) + C_1 \cosh(\alpha_s z) + e^{kh} \{C_2 \cosh(\alpha_s(z+h)) + \sinh(\alpha_s(z+h))\}}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} - \cosh(k(z+h)) \right\} \\ \times \frac{ia\omega k F_z}{(\alpha_s^2 - k^2) K_s \sinh(kh)} \quad (-h \leq z \leq 0), \quad (2-1-32)$$

$$b_y = \left\{ \frac{-\cosh(\alpha_s z) + C_1 \sinh(\alpha_s z) + e^{kh} \{C_2 \sinh(\alpha_s(z+h)) + \cosh(\alpha_s(z+h))\}}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} - \frac{k}{\alpha_s} \sinh(k(z+h)) \right\} \\ \times \frac{-\alpha_s a \omega F_z}{(\alpha_s^2 - k^2) K_s \sinh(kh)} \quad (-h \leq z \leq 0). \quad (2-1-33)$$

Expression for the electric field is derived in the similar manner to the previous section,

$$e_x + (\mathbf{v} \times \mathbf{B})_x = -c b_z + \omega a F_z \frac{\cosh(k(z+h))}{\sinh(kh)}, \quad (2-1-34)$$

where Eq. (2-1-19) is used for the horizontal oceanic velocity, and b_z is given by Eq. (2-1-32). The EM field expressed by Eqs. (2-1-32) – (2-1-34) asymptotes to the long wave solution in Eqs. (2-1-7) - (2-1-9), where $kh \ll 1$.

2.1.3 Effects of the horizontal component of the geomagnetic main field

In the preceding two sections, we assumed $|v_y| \gg |v_z|$ in the induction equation in terms of b_z to derive Eq. (2-1-2), considering only coupling of the horizontal oceanic velocity, v_y , with the vertical component of the geomagnetic main field, F_z . However, the coupling of v_z with the horizontal field, F_y , also has an influence on the tsunami-induced EM field. Larsen (1971) and Wang and Liu (2013) have already explicitly mentioned contributions of F_y . However, Larsen's (1971) expression assumed actual complicated layered conductivity model beneath the seafloor, while Wang and Liu (2013) failed to appreciate the combination of F_y and F_z . In this section, we evaluate the effect of coupling of v_z and F_y , assuming a simple half-space conductor beneath the seafloor.

Let us first consider meanings of the EM field solutions obtained in the previous two sections. In Section 2.1.1, we derived the EM field induced by long waves, as in Eqs. (2-1-7) - (2-1-9). If we differentiate Eq. (2-1-8) twice w.r.t. z , we obtain the following relation,

$$\frac{\partial^2 b_y}{\partial z^2} = \alpha_s^2 b_y \quad (-h < z < 0). \quad (2-1-35)$$

This indicates that the solutions for the EM field induced by a long wave tsunami obey consistently the induction equations in terms of b_z and b_y ,

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_z = \frac{1}{K_s} \frac{\partial}{\partial y} (v_y F_z) \quad (-h \leq z \leq 0), \quad (2-1-36)$$

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_y = 0 \quad (-h \leq z \leq 0), \quad (2-1-37)$$

where v_y is a long-wave oceanic flow given by Eq. (2-1-6). As for the EM field induced by a linear dispersive wave, Eq. (2-1-33) gives,

$$\begin{aligned} \frac{\partial^2 b_y}{\partial z^2} &= \alpha_s^2 b_y - \frac{\eta \omega k F_z \sinh(k(z+h))}{K_s \sinh(kh)} \\ &= \alpha_s^2 b_y - \frac{1}{K_s} \frac{\partial}{\partial z} (v_y F_z) \quad (-h < z < 0), \end{aligned} \quad (2-1-38)$$

where v_y is a linear dispersive horizontal flow, given by Eq. (2-1-19). The solutions derived for a linear dispersive wave were found to obey Eq. (2-1-36) and

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_y = - \frac{1}{K_s} \frac{\partial}{\partial z} (v_y F_z) \quad (-h \leq z \leq 0), \quad (2-1-39)$$

instead of Eq. (2-1-37). It is evident from comparison of Eq. (2-1-39) with Eq. (2-1-37) that the source term due to the vertical change of v_y on the r.h.s. of Eq. (2-1-39) was neglected in Eq. (2-1-37) in the case of the long wave approximation. Note that the absence of the source term on the r.h.s. of Eq. (2-1-37) does not lead to the absence of b_y , which is determined by both the induction equations and the divergence less nature of the magnetic field. Wand and Liu's (2013) formulation is misleading, because it discusses only b_z due to $v_y F_z$ and b_y due to

$v_z F_y$, neglecting b_y caused by $v_y F_z$. Actually, b_y induced by the $v_y F_z$ coupling is comparable to the induced b_z , as shown in Fig. 2-3.

Now we consider the effect of the horizontal component of the geomagnetic main field.

The induction equation in terms of b_z with the source term originated from F_y ,

$$\frac{\partial b_z}{\partial t} = \frac{\partial}{\partial y} (v_z F_y) + K \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) b_z, \quad (2-1-40)$$

Adopting the dependence, $e^{i(ky - \omega t)}$, and the oceanic velocity due to a linear dispersive wave in Eq. (2-1-20), Eq. (2-1-40) is reduced to

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_z = B \sinh(k(z+h)) \quad (-h \leq z \leq 0) \quad (2-1-41)$$

$$\left(B = \frac{a\omega k F_y}{K_s \sinh(kh)} \right).$$

The magnetic field associated with F_y can be expressed in each layer using arbitrary complex constants, A'_0 , B'_0 , A'_1 , and B'_1 , as

$$b_z = A'_0 e^{-kz} \quad (0 < z), \quad (2-1-42)$$

$$b_z = A'_1 e^{\alpha_s z} + B'_1 e^{-\alpha_s(z+h)} - \frac{B \sinh(k(z+h))}{\alpha_s^2 - k^2} \quad (-h < z < 0), \quad (2-1-43)$$

$$b_z = B'_0 e^{\alpha_1(z+h)} \quad (z < -h). \quad (2-1-44)$$

Corresponding b_y is expressed as,

$$b_y = -iA'_0 e^{-kz} \quad (0 < z), \quad (2-1-45)$$

$$b_y = \frac{i\alpha_s}{k} (A'_1 e^{\alpha_s z} - B'_1 e^{-\alpha_s(z+h)}) - \frac{iB \cosh(k(z+h))}{\alpha_s^2 - k^2} \quad (-h < z < 0), \quad (2-1-46)$$

$$b_y = \frac{i\alpha_1}{k} B'_0 e^{\alpha_1(z+h)} \quad (z < -h). \quad (2-1-47)$$

For the boundary condition, the continuity of b_z and b_y are adopted as follows;

$$A'_0 = A'_1 + B'_1 e^{-\alpha_s h} - \frac{B}{\alpha_s^2 - k^2} \sinh(kh) \quad (b_z \text{ at } z = 0), \quad (2-1-48)$$

$$B'_0 = A'_1 e^{-\alpha_s h} + B'_1 \quad (b_z \text{ at } z = -h), \quad (2-1-49)$$

$$-A'_0 = \frac{\alpha_s}{k} (A'_1 - B'_1 e^{-\alpha_s h}) - \frac{B}{\alpha_s^2 - k^2} \cosh(kh) \quad (b_y \text{ at } z = 0), \quad (2-1-50)$$

$$\frac{\alpha_1}{k} B'_0 = \frac{\alpha_s}{k} (A'_1 e^{-\alpha_s h} - B'_1) - \frac{B}{\alpha_s^2 - k^2} \quad (b_y \text{ at } z = -h). \quad (2-1-51)$$

We can solve Eqs. (2-1-48) – (2-1-51) w.r.t. A'_0 , B'_0 , A'_1 , and B'_1 . Eventually, the EM fields

induced by the coupling of v_z with F_y are expressed as follows:

$$b_z = \left\{ \frac{(k/\alpha_1)(\sinh(\alpha_s z) - C_1 \cosh(\alpha_s z)) + e^{kh}\{C_2 \cosh(\alpha_s(z+h)) + \sinh(\alpha_s(z+h))\}}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} - \sinh(k(z+h)) \right\} \\ \times \frac{a\omega k F_y}{(\alpha_s^2 - k^2) K_s \sinh(kh)} \quad (-h \leq z \leq 0), \quad (2-1-52)$$

$$b_y = \left\{ \frac{(k/\alpha_1)(\cosh(\alpha_s z) - C_1 \sinh(\alpha_s z)) + e^{kh}\{C_2 \sinh(\alpha_s(z+h)) + \cosh(\alpha_s(z+h))\}}{(1 + C_1 C_2) \sinh(\alpha_s h) + (C_1 + C_2) \cosh(\alpha_s h)} - \frac{k}{\alpha_s} \cosh(k(z+h)) \right\} \\ \times \frac{i\alpha_s a\omega F_y}{(\alpha_s^2 - k^2) K_s \sinh(kh)} \quad (-h \leq z \leq 0), \quad (2-1-53)$$

$$e_x + (\mathbf{v} \times \mathbf{B})_x = -cb_z + ia\omega F_y \frac{\sinh(k(z+h))}{\sinh(kh)} \quad (-h \leq z \leq 0). \quad (2-1-54)$$

where, $C_1 = \alpha_s/k$ and $C_2 = \alpha_s/\alpha_1$ are used again. By differentiating b_y twice w.r.t. z , one can also easily find that b_z and b_y in Eqs. (2-1-52) and (2-1-53) obey

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_z = -\frac{1}{K_s} \frac{\partial}{\partial y} (v_z F_y) \quad (-h \leq z \leq 0), \quad (2-1-55)$$

$$\left(\frac{\partial^2}{\partial z^2} - \alpha_s^2 \right) b_y = -\frac{1}{K_s} \frac{\partial}{\partial y} (v_y F_y) \quad (-h \leq z \leq 0). \quad (2-1-56)$$

Here v_y and v_z are given by Eqs (2-1-19) and (2-1-20) respectively.

Now we have derived both expressions for the EM fields associated with F_z and F_y . The observable EM fields are given by the sum of the EM fields originated from the two sources.

Figure 2-4 shows the comparison of the two sources, assuming a flat seafloor and a variable

half-space conductivity beneath the ocean. It can be seen in Fig. 2-4 that amplitudes of b_y and b_z induced by $v_z F_y$ are much less than those by $v_y F_z$ almost irrespective to the ocean depth and the conductivity beneath the ocean, when we assign comparable values in the horizontal and vertical components of the geomagnetic main field. As the period increases, the difference in amplitude gets larger between the two sources, especially for tsunami periods longer than 10 min.

Based on the comparison above, we adopted only the term, $v_y F_z$, as a source field for the induction equation in our numerical simulations (the simulation method will be described in detail in Section 5). Here we simply estimate the orders of b_z induced by $v_y F_z$ and $v_z F_y$ at the time of the 2011 Tohoku tsunami. At the geographical coordinate of NWP, (41.1026°N, 159.9518°E, -5616m), the three components of the geomagnetic main field in 2011 were (X,Y,Z) = (26712.5, -1470.7, 35808.1) nT, according to the International Geomagnetic Reference Field - 11 (IGRF11; refer Geophys. J. Int., 183, 1216-1230, 2010).

The propagation direction of the 2011 Tohoku tsunami is roughly estimated as N77°E, which is the angle between the straight line from the epicenter to NWP and a meridian. The horizontal component of the geomagnetic main field parallel to the tsunami propagation direction is, therefore,

$$F_y = 26712.5 \cos(77^\circ) - 1470.7 \sin(77^\circ) \approx 5035 \text{ nT}, \quad (2-1-57)$$

while the vertical component, F_z , is 35808 nT, directly obtained from IGRF11. If we assign these two values to F_y and F_z in Eq. (2-1-32) and (2-1-53), respectively, the amplitudes of b_z on the seafloor are obtained as,

$$|b_{z_{Fz}}| = 5.95 \text{ nT}, \quad (2-1-58)$$

$$|b_{z_{Fy}}| = 5.13 \times 10^{-2} \text{ nT}, \quad (2-1-59)$$

where we assumed that the tsunami period, the ocean depth, the sea level amplitude, and the conductivity beneath the seafloor are 20 min, 5616 m, 1.0 m, 0.0 S/m, respectively. The amplitude of the b_z induced by the $v_z F_y$ coupling is less than 1 percent of that induced by the $v_y F_z$ coupling. Hence, we can neglect the $v_z F_y$ coupling in our simulation.

2.2 TM mode theory

Here we consider tsunami dynamo effects in TM mode. Figure 2-2 shows a tsunami propagating in the y -direction and oceanic flows in the y - z -direction coupling with the x -component of the geomagnetic main field. It is expected that emfs driven in the y, z -plane induce the y - z -component of the electric field and the x -component of the magnetic field. However, we cannot actually detect tsunami-induced EM fields in TM mode, if the oceanic flows are incompressible (Larsen, 1971). We will demonstrate the fact in this section.

Let us start with the induction equation in terms of the magnetic field in the ocean:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + K \nabla^2 \mathbf{B}. \quad (2-2-1)$$

Here $\mathbf{B}$, $\mathbf{v}$, and K are the magnetic field, the oceanic flow, and the magnetic diffusivity, respectively. If we assume oceanic flows are incompressible, $\nabla \cdot \mathbf{v} = 0$, and recall $\nabla \cdot \mathbf{B} = 0$, the source term of Eq. (2-2-1) reduces to

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = (\mathbf{F} \cdot \nabla) \mathbf{v} - (\mathbf{v} \cdot \nabla) \mathbf{b}. \quad (2-2-2)$$

Here we used a vector identity, $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B}$ and decomposed the magnetic field, $\mathbf{B}$, into the geomagnetic main field and the tsunami-induced magnetic field, $\mathbf{F} + \mathbf{b}$, assuming $\nabla^2 \mathbf{F} = 0$, $\nabla \times \mathbf{F} = 0$, $\partial \mathbf{F} / \partial t = 0$, and $|\mathbf{F}| \gg |\mathbf{b}|$. Using Eq. (2-2-2), the x -component of Eq. (2-2-1) is reduced to

$$\frac{\partial b_x}{\partial t} = -(\mathbf{v} \cdot \nabla) b_x + K \nabla^2 b_x, \quad (2-2-3)$$

since $v_x = 0$. If we assume tsunamis to be linear long waves, orders of each term in Eq. (2-2-3) are estimated as follows:

$$\frac{\partial}{\partial t} \sim \frac{1}{T} \sim 10^{-2}, \quad (2-2-4)$$

$$|\mathbf{v} \cdot \nabla| \sim c \frac{A}{h^2} \sim 10^{-5}, \quad (2-2-5)$$

$$K \nabla^2 = \frac{K}{h^2} \sim 10^{-2}, \quad (2-2-6)$$

where T , c , A , and h are the period, the phase velocity, and the height of the tsunami, and the ocean depth, respectively. Here we set T , c , A , h , and K to 10^3 s, 200 m/s, 1 m, 10^2 m, and 10^{-6} m²s⁻¹, respectively. Eqs. (2-2-4) - (2-2-6) indicate that the first terms in r.h.s. of Eq. (2-2-1) and (2-2-3) are negligible compared with the second term. This means that tsunami-induced EM fields are very small in TM mode.

The source term in TM mode becomes insignificant because $(\mathbf{F} \cdot \nabla)\mathbf{v}$, which is the contribution of $\mathbf{v} \times \mathbf{F}$, cannot remain in Eq. (2-2-3), while, in TE mode, the term is very significant because

$$|(\mathbf{F} \cdot \nabla)\mathbf{v}| / |(\mathbf{v} \cdot \nabla)\mathbf{b}| \gg 1. \quad (2-2-7)$$

Larsen (1971) simply demonstrated why emfs driven by $\mathbf{v} \times \mathbf{F}$ vanish in TM mode, by evaluating a line integral of $\mathbf{v} \times \mathbf{F}$ about an arbitrary loop in y,z -plane using Stokes' theorem:

$$\oint (\mathbf{v} \times \mathbf{F}) \cdot d\mathbf{s} = \iint [\nabla \times (\mathbf{v} \times \mathbf{F})] \cdot \hat{\mathbf{x}} dydz = \iint (\mathbf{F} \cdot \nabla) v_x dydz = 0. \quad (2-2-8)$$

Equation (2-2-8) indicates the net emf driven by $\mathbf{v} \times \mathbf{F}$ about any y,z -loop vanishes. In other words, short circuits of emfs don't allow the electric potential difference to exist in incompressible oceanic flows. Consequently, the term, $\mathbf{v} \times \mathbf{F}$, cannot contribute to motional inductions in TM mode.

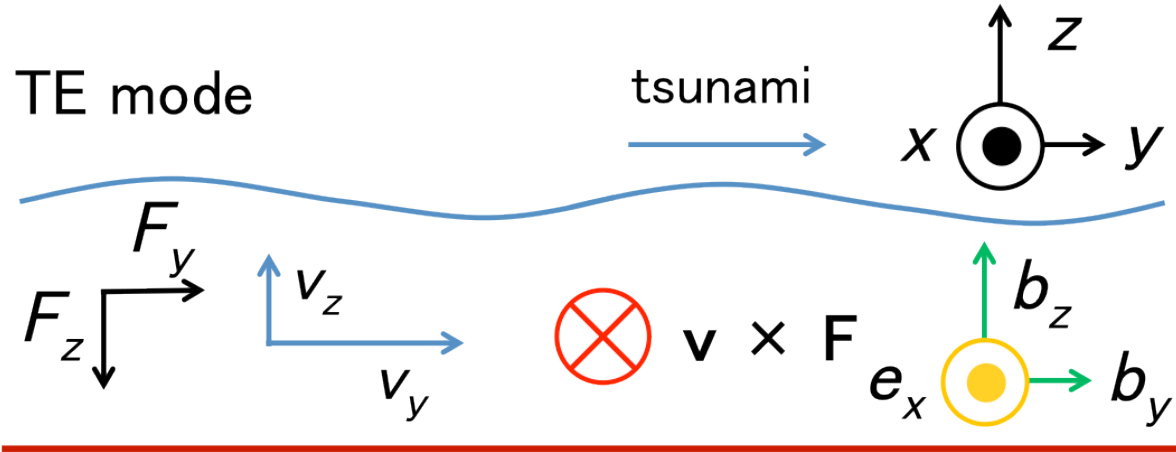


Fig. 2-1 Configuration of motional induction in TE mode. Tsunami propagates in the positive y -direction as plane waves, whose particle motion lies in the y -direction independent of x -coordinate. Throughout this thesis, oceanic flows in the z -direction, which couples with the horizontal component of the geomagnetic field, F_y , are omitted because $|v_y| \gg |v_z|$. Hence, emfs in the y,z -plane are driven by the term, $v_y F_z$, inducing electric fields along the x -axis (e_x) and the magnetic field in the y,z -plane (b_y, b_z).

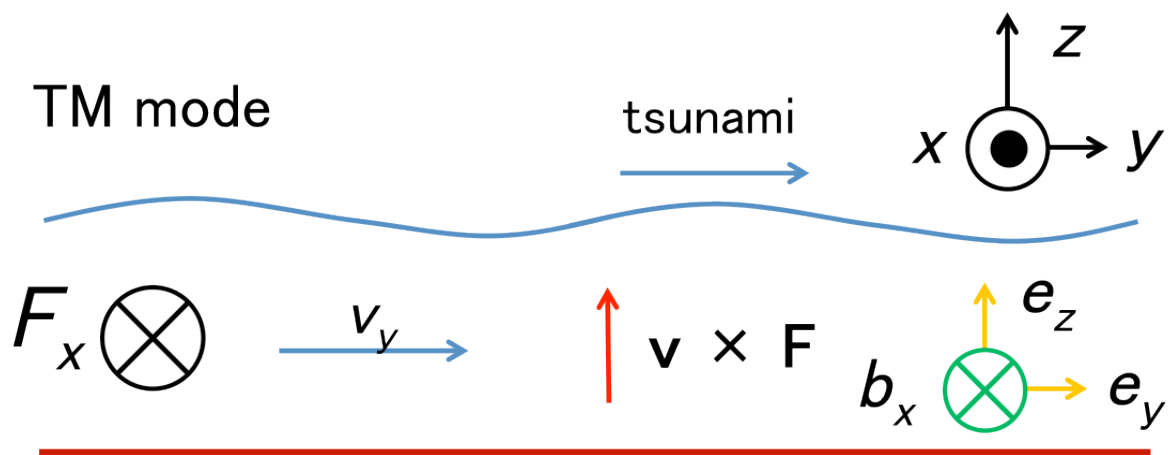


Fig. 2-2 Configuration of motional induction in TM mode. Tsunami propagates in the positive y -direction as plane waves, which contains oceanic flows in the y - and z -directions independent of x -coordinate. In the presence of x -component of the geomagnetic main field, emfs in the y,z -plane are driven, inducing electric fields in y,z -plane (e_y , e_z) and the x -component of the magnetic field (b_x).

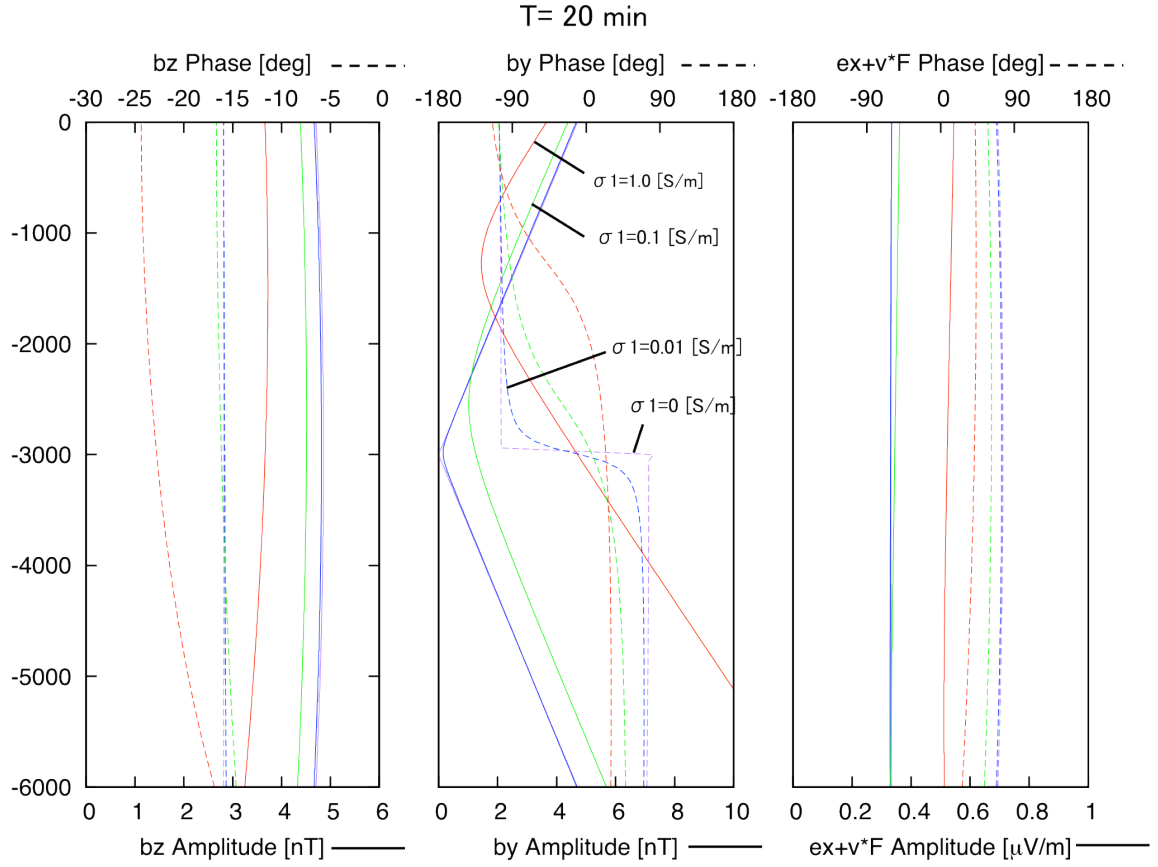


Fig. 2-3 Tsunami-induced EM fields in TE mode for different half-space conductivities beneath the seafloor. From left to right, each panel shows the vertical component of the induced magnetic field, b_z , the horizontal component, b_y , and the sum of the induced electric field in the strike direction, e_x , and the source field, $\mathbf{v} \times \mathbf{F}$. Red, green, blue, and purple lines indicate the sub-seafloor conductivity of 1.00, 0.10, 0.01, and 0.00 S/m, respectively. Solid lines and dashed lines indicate the amplitude and phase difference to the sea surface elevation, respectively. In the estimation, $h = 6000$ m, $F_z = 30000$ nT, $T=20$ min, and $\eta = 1.0$ m were adopted.

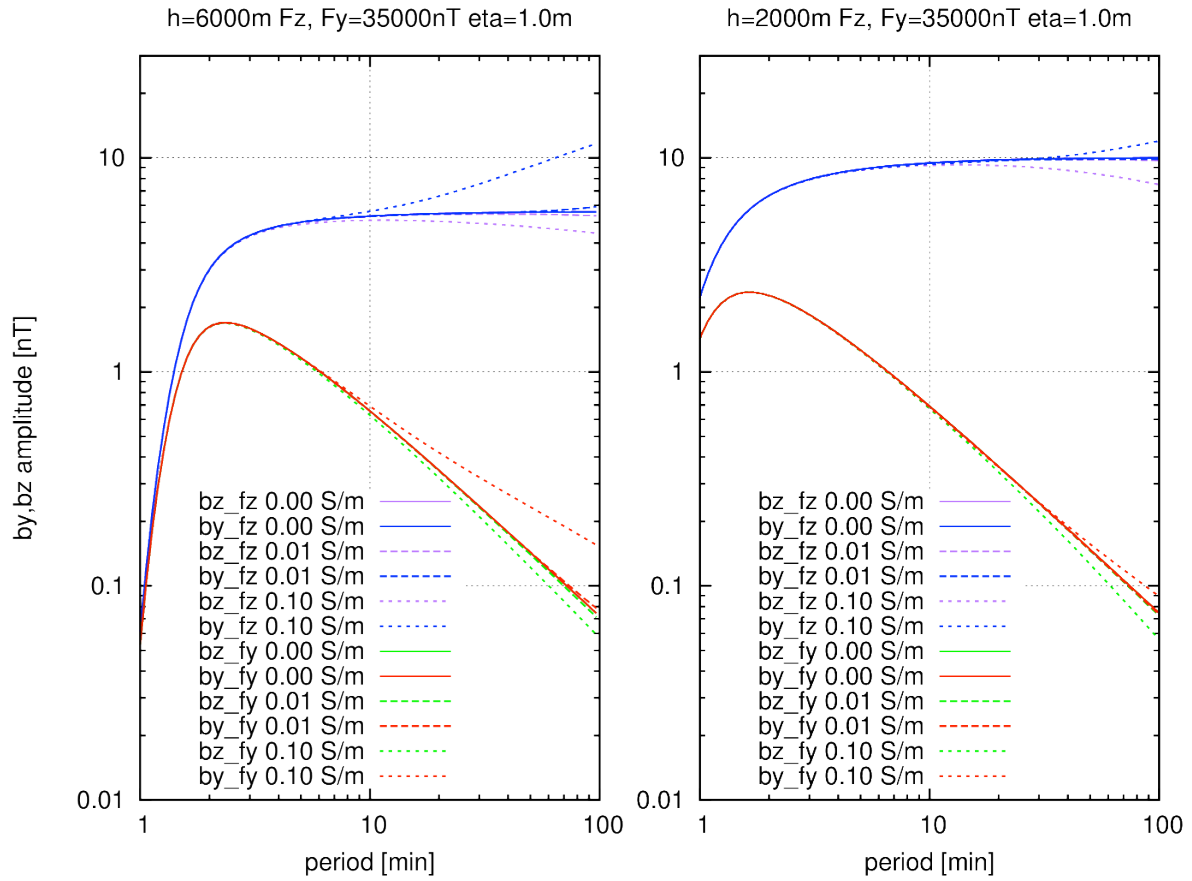


Fig. 2-4 Amplitudes of b_z and b_y on the seafloor induced by the coupling of $v_y F_z$ and $v_z F_y$. Blue and purple lines indicate the magnetic field induced by $v_y F_z$, while red and green lines corresponds to those induced by $v_z F_y$. Eqs. (2-1-32) and (2-1-33) were used for calculations associated with F_z . On the other hand, Eqs. (2-1-52) and (2-1-53) were used for F_y calculations. Left and right panels are the results for the ocean depths of 6000m and 2000m, respectively.

3. Data

Our SeaFloor ElectroMagnetic Station (SFEMS) (Toh et al., 2004, 2006), installed at site NWP in the northwest Pacific, successfully recorded the tsunami-induced magnetic field at the time of the 2011 Tohoku earthquake occurred on March 11, 2011. Although SFEMS has a horizontal electrometer as well as a three-component Fluxgate and a scalar Overhauser magnetometer (Toh et al., 2006), it could not detect tsunami-induced electric fields due to large electric noises. Figure 3-1 shows the location of NWP and the epicenter of the 2011 Tohoku earthquake. NWP is about 1500km from the epicenter and the ocean depth there is 5616m. Figure 3-2 shows the three-hour plot of raw data of the downward component of the magnetic field observed by our SFEMS, the column seawater height, corresponding to the sea level change, observed at DART 21401 and 21419. It is seen that the profiles of two sea level changes and the vertical component of the magnetic field on the seafloor have very similar forms, as predicted by the Tyler's relationship expressed in Eq. (2-1-12). In the following section, we introduce our EM observation instrument, SFEMS, in Section 3.1, and demonstrate that the magnetic variations obtained at NWP were induced by the 2011 Tohoku earthquake tsunami in Section 3.2.

3.1 Instruments

Our research group deployed a SFEMS on the seafloor at NWP, which can observe the vector magnetic field and the horizontal vector electric field. Figure 3-3 shows the SFEMS retrieved by the research vessel "Kairei", which is operated by Japan Agency for Marine-earth Science Technology (JAMSTEC), on July 27, 2011, during the research cruise named "KR11-07". Figure 3-4 shows the plan and side views of the SFEMS. The SFEMS

is composed of six 17' glass spheres containing various instruments. For EM observations, Fluxgate and Overhauser magnetometers and an electrometer with two mutually orthogonal dipoles are installed. On the other hand, a fiber optical gyro gyro and tilt-metes are installed to measure the instruments' orientation and tilt changes.

SFEMSs have been in continuous operation since August 1, 2001 (Toh et al., 2004). Its original purpose was to capture the secular variation of the geomagnetic main field in the northwest Pacific and investigate the mantle conductivity. After Toh et al. (2011) first reported the tsunami-induced EM field variations observed by the SFEMS observation system, long-term seafloor EM observations acquired a new role to detect tsunami passages. As we anticipated before the cruise KR11-07, the SFEMS deployed at NWP in October 2009 succeeded in detecting the magnetic variations induced by the devastating 2011 Tohoku earthquake tsunami. Now we can investigate tsunami dynamo phenomena by SFEMS data, and possibly extract some properties regarding tsunamis or the sub-seafloor conductivity structures from the observed magnetic signals.

3.2 Magnetic signals of the 2011 Tohoku tsunami

Here we show that the magnetic field variation observed at NWP was actually caused by the 2011 Tohoku earthquake tsunami. Figure 3-5 shows the 3-hour plots of the vector magnetic field and horizontal tilts at NWP together with the simultaneous vector magnetic fields observed at three land geomagnetic observatories operated by Japan Meteorological Agency (JMA). In the top panel of Fig. 3-5, colored lines with dots show time series of the vector magnetic components observed at NWP for 3 hours after the earthquake. About 100 minutes after the earthquake, the downward component of the magnetic field, b_z , started to

vary and has a positive peak as large as approximately 3 nT. On the other hand, the horizontal magnetic component parallel to the tsunami propagation direction, b_y , has a negative peak as large as approximately 3 nT a few minutes after the positive peak of b_z , while the horizontal component perpendicular to the tsunami, b_x , doesn't vary considerably. In addition, the occurrence time of these magnetic variations, approximately 100 minutes after the earthquake, coincide well with the estimated time of tsunami arrival (ETA) predicted by National Oceanic and Atmospheric Administration (NOAA) (http://www.ngdc.noaa.gov/hazard/honshu_11mar2011.shtml). These variations can, therefore, be magnetic signals of the tsunami passage. The middle panel of Fig. 3-5 indicates that motions of the seafloor instrument are due mainly to seismic waves, which started a few minutes after the earthquake, and it is hard to explain the large magnetic signature at ETA by instrumental motions because the tilt variations caused before ETA. In the bottom panel, there are time series of three vector magnetic field variations observed at Memambetsu (MMB), Kakioka (KAK), and Kanoya (KNY). The vector geomagnetic fields show similar profiles among the three observatories, which implies that external sources caused large-scale magnetic variations over the Japanese Islands after the earthquake. Comparing the magnetic variations at NWP with those at the three land observatories before ETA, it is evident that the land geomagnetic variations as large as 5 nT with a period of about 40 minutes attenuate to less than 1 nT at NWP with a depth of 5616 m. Consequently, we can claim that there were no external geomagnetic variations more than 1 nT at NWP for 3 hours after the earthquake, because the amplitudes of the land geomagnetic variations decreased towards ETA, and the magnetic variations at NWP was larger than 3 nT.

The magnetic field variations, therefore, can be regarded as tsunami signals because (1) variations of horizontal magnetic components appeared only in the predicted tsunami propagation direction, (2) the onset time of the magnetic variations almost coincided with the 2011 Tohoku earthquake tsunami arrival time, (3) any significant instrumental motions are not recognized during the magnetic variations, and (4) strong magnetic variations due to external sources were not observed at three on-land magnetic observatories around the estimated time of tsunami arrival. Toh et al. (2011) pointed out large sensitivity of the horizontal magnetic field to propagation directions of tsunamis, which enables us to estimate vector tsunami properties by single site observation alone. In the vicinity of NWP, Ocean Bottom Pressure gauges (OBPs) operated by NOAA at DART21401 and 21419, shown in Fig. 3-1, observed sea level changes due to the tsunami passage. It was found that profiles of the sea level changes at the two DART stations are very similar to the downward magnetic component observed at NWP (Fig. 3-2). In Section 5, we try to explain both the sea level changes at the DART stations and the magnetic variations at NWP by 2-D tsunami dynamo simulations, assuming that tsunamis can be approximated by plane waves.

For accurate comparison between observed data and simulation results, it is necessary to subtract effects of external sources from the magnetic field variations observed at NWP. We defined a transfer function (T_x) between the horizontal magnetic components at MMB and NWP as follows:

$$T_x(f) = X_{NWP}(f)/X_{MMB}(f). \quad (3-1)$$

Here, X_{NWP} and X_{MMB} indicate Fourier components of the northward component of the magnetic field at NWP and MMB, respectively, both of which are assumed free from TDE. The transfer function of the eastward component (T_y) can be defined in the same manner.

Correction of the external source effects was conducted as follows. First, we calculated T_x and T_y using the time series at MMB and NWP before the tsunami arrived at NWP. Second, horizontal magnetic fields due to the external sources at NWP during the tsunami passage were estimated from the transfer functions, T_x and T_y . Finally, we subtracted the external field from the raw horizontal magnetic components at NWP. Figure 3-6 shows the results of the subtraction of external source effects from the magnetic field data at NWP. In Section 5, we will compare simulation results with the magnetic field variations obtained here, which are free from external source effects.

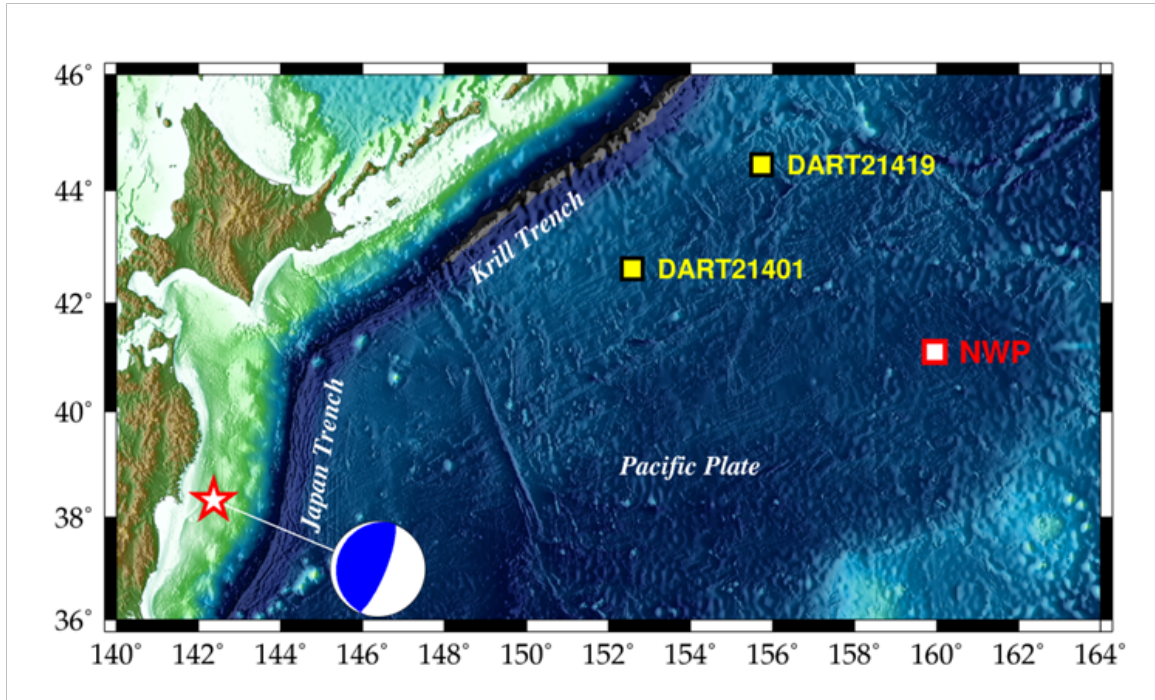


Fig. 3-1 The epicenter of the 2011 Tohoku earthquake occurred on 11 March 2011 (the open red star), its W-phase MT solution by USGS, and the seafloor EM observatory installed in the northwest Pacific (the red rectangle). The epicentral distance to NWP (41.1026°N, 159.9518°E) is approximately 1500 km. Two DART stations operated by NOAA (yellow rectangles) are also depicted. Sea level changes at the DART stations were fitted by our 2-D hydro-dynamic simulations in order to calculate the magnetic signals observed at NWP.

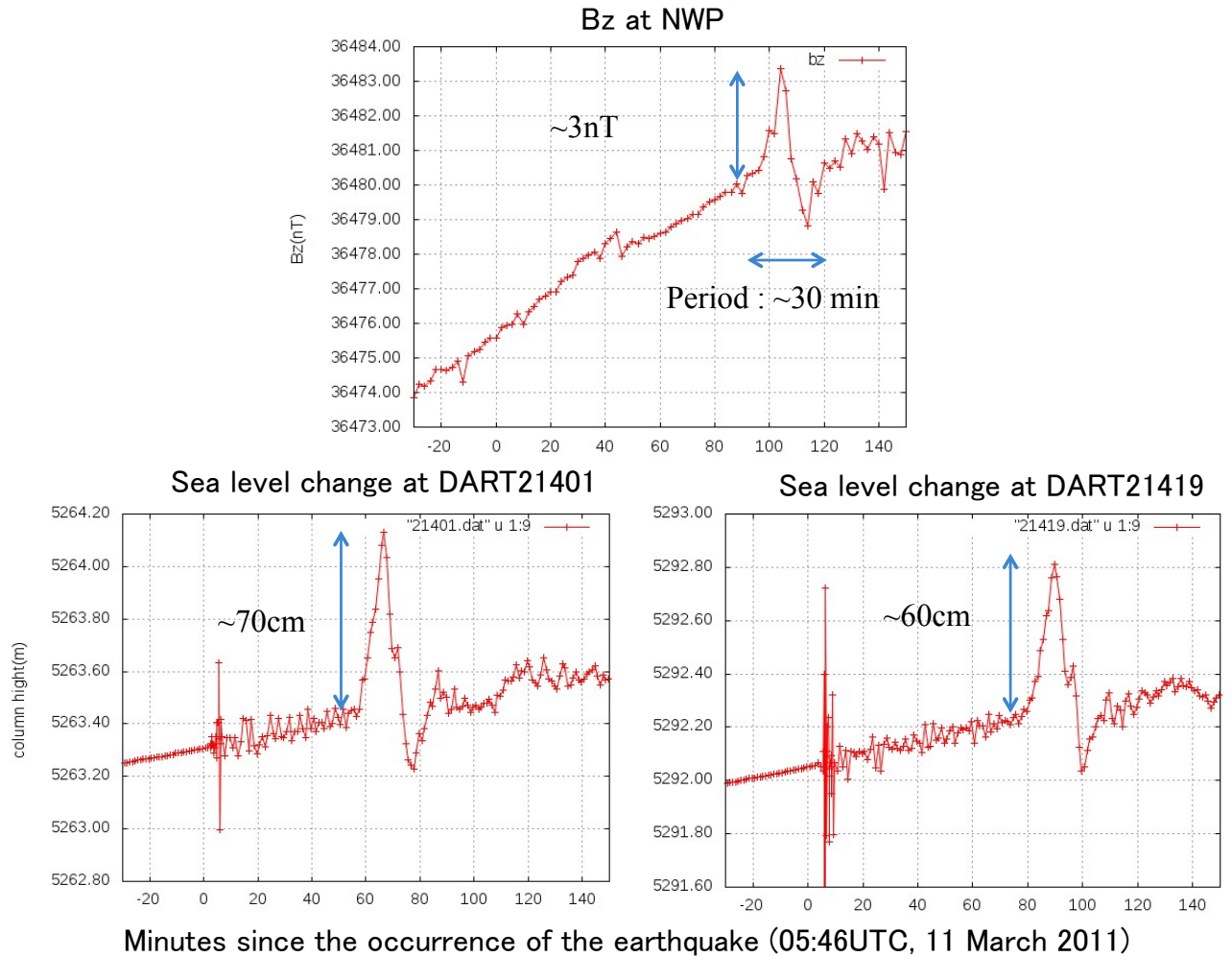


Fig. 3-2 Raw time series data of the downward component of the magnetic field observed at NWP (top), sea level changes observed at DART 21401 (bottom left), and DART21419 (bottom right). In these three panels, the horizontal axis indicate minutes since the occurrence of the 2011 Tohoku earthquake, namely 05:46 UTC on March 11, 2011.

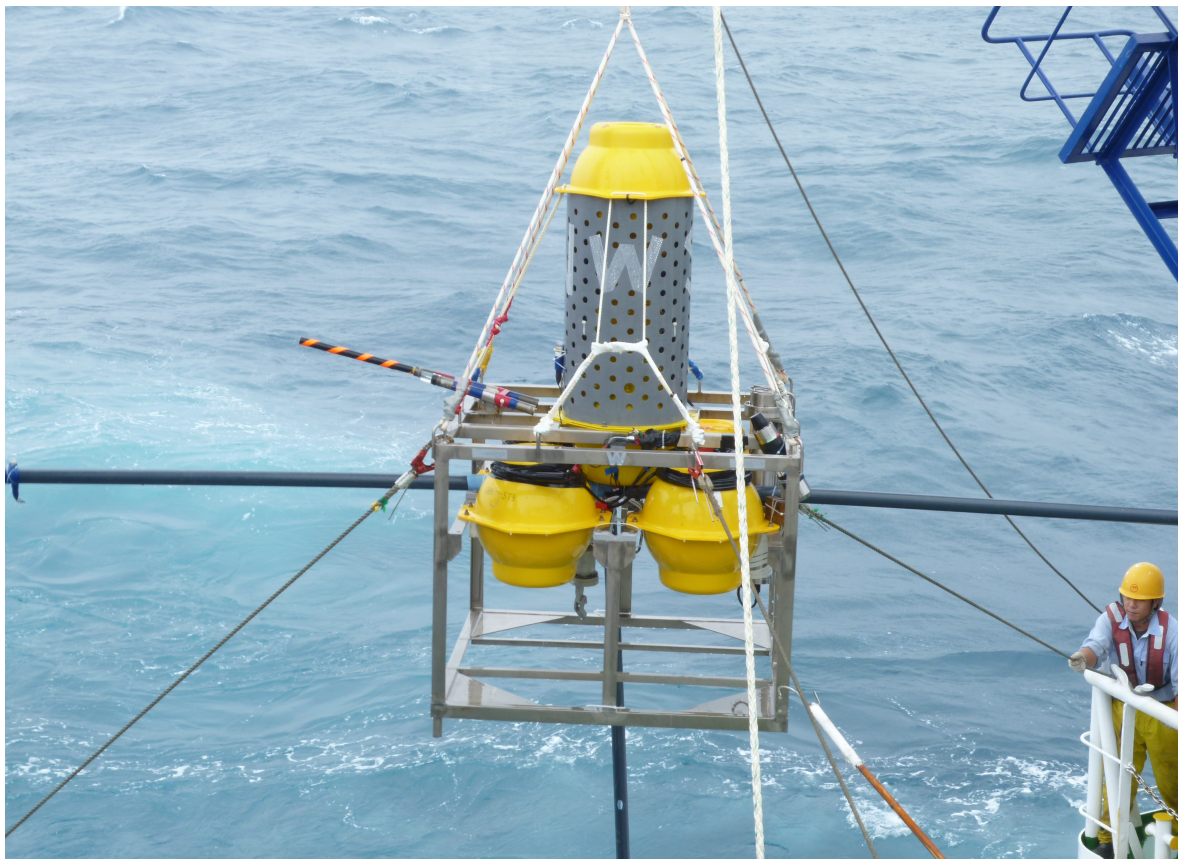


Fig. 3-3 SFEMS retrieved at NWP on July 27, 2011, which successfully recorded magnetic tsunami signals on the seafloor at the time of the 2011 Tohoku earthquake tsunami. Unfortunately, it failed to observe horizontal electric field variations because of large electric noises.

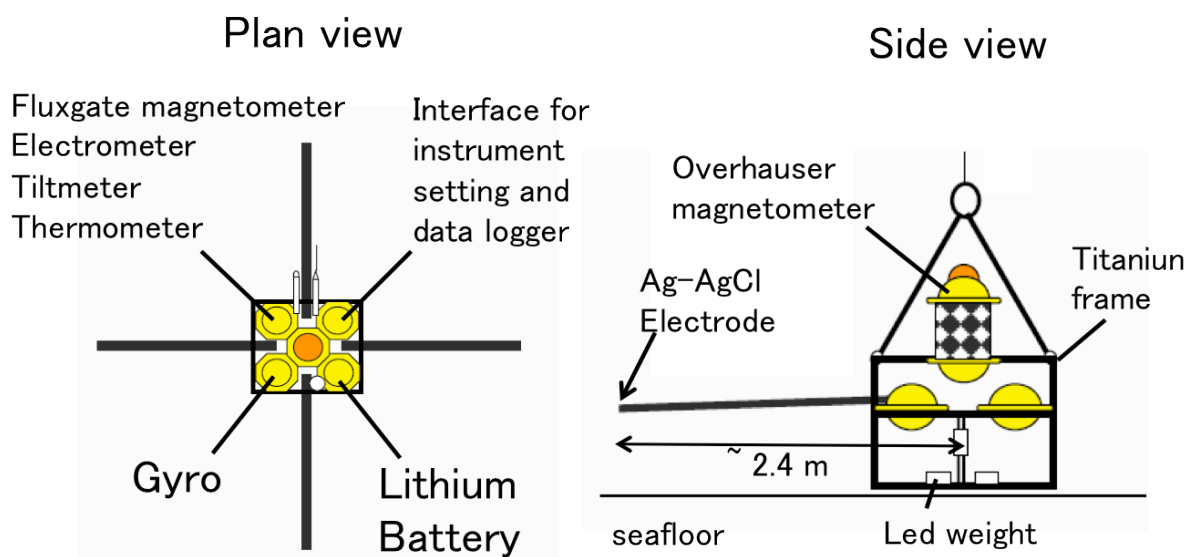


Fig. 3-4 Schematic figures of SFEMS installed at NWP. Yellow spheres indicate 17' pressure-tight glass housings, every one of which is filled with instruments.

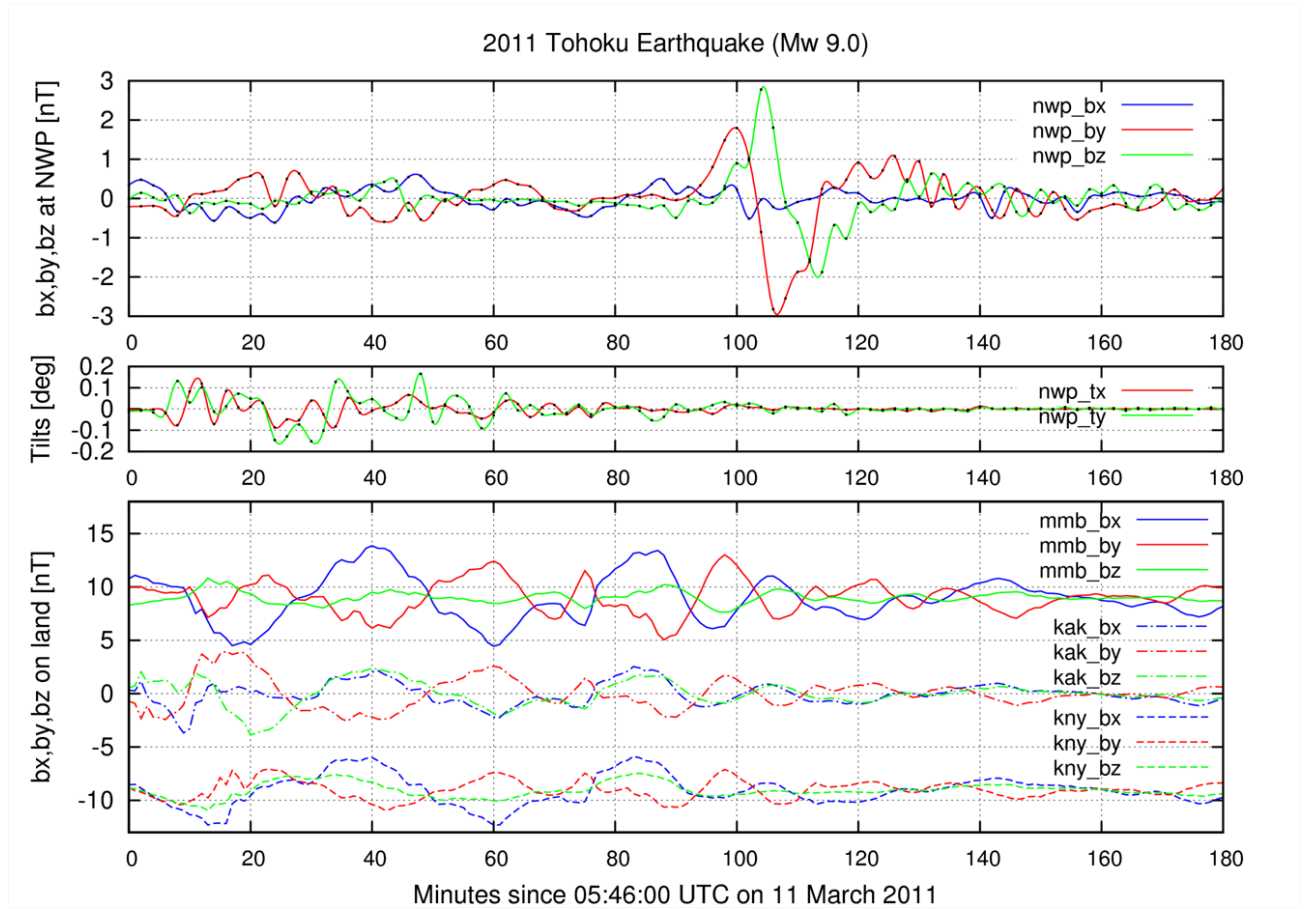


Fig. 3-5 The 3-hour plots of the observed time series at the time of the 2011 Tohoku earthquake. (top to bottom) The vector geomagnetic field observed at NWP, the horizontal tilts at NWP, and the three vector geomagnetic fields observed at land geomagnetic observatories: Memambetsu (MMB; 43.910°N, 144.189°E), Kakioka (KAK; 36.232°N, 140.186°E), and Kanoya (KNY; 31.424°N, 130.88°E). In each panel, the x-, y-, and z-components indicate the northward, eastward, and downward components, respectively. All the time series in the three panels are high-pass filtered with a 1-hour cut off period.

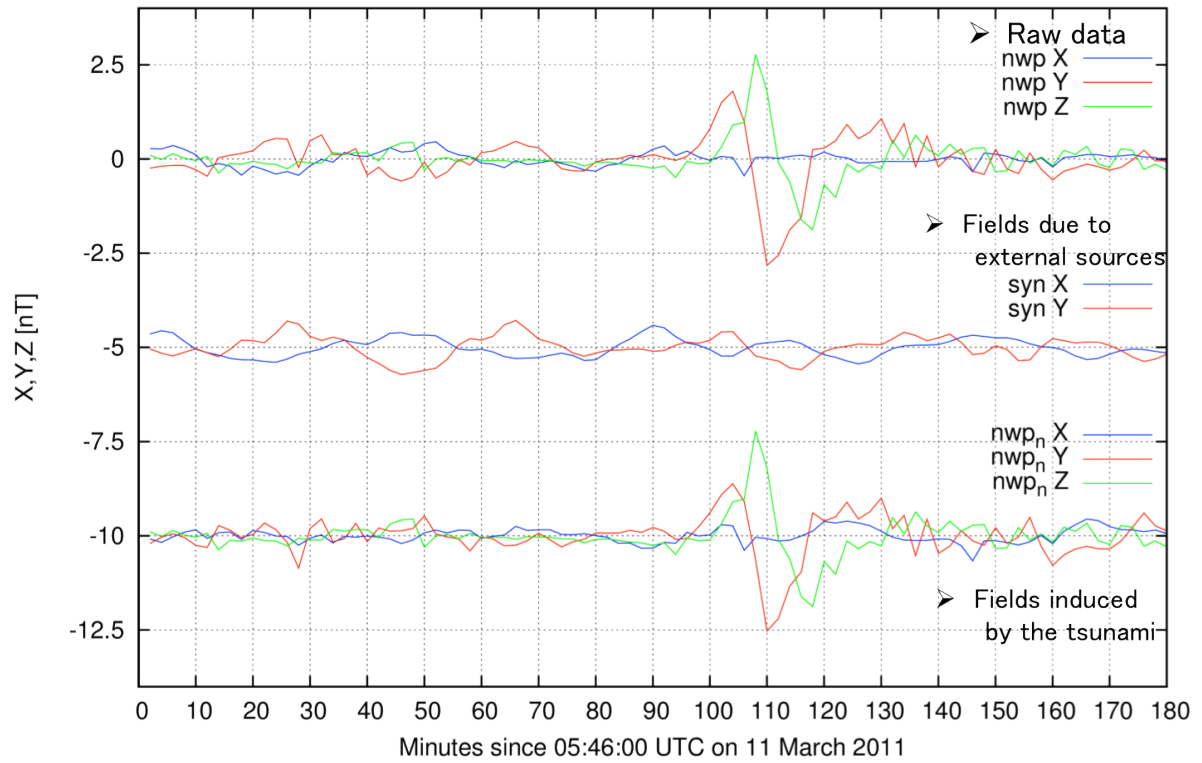


Fig. 3-6 Correction of external source effects from the observed time series at NWP. The three lines in the top panel indicate the raw vector magnetic field data observed at NWP. The two lines in the middle are the expected horizontal magnetic fields due to external sources, which were estimated from the horizontal magnetic field variations at MMB and the horizontal transfer functions between MMB and NWP. The three at the bottom indicate the time series without external source effects.

4. Effects of 2-D solitary waves

In order to understand basic physics of TDE, we investigate EM fields induced by a 2-D solitary plain wave. Understanding how a single solitary wave induces EM fields in the ocean will help us to achieve insight into TDE phenomena before working on realistic simulations of TDE phenomena. Furthermore, in many cases, the first arrival of tsunamis is most devastating compared with following waves. For mitigation of tsunami disasters, it is important to investigate effects of a solitary wave as the first arrival of tsunamis. In this section, we first explain our simulation method associated with solitary waves in Section 4.1, explain the discretization for 2-D FEM simulations in Section 4.2, and consider three particular tsunami dynamo cases in Section 4.3.

4.1 Formulation

For simplicity, we adopt a solitary wave with Gaussian waveform and applied the long-wave approximation. Assuming that the solitary wave propagates in the positive y -direction, the waveform and the corresponding seawater flow distribution can be expressed as follows:

$$\eta = A \exp\left(-\frac{(y - ct)^2}{2\chi^2}\right), \quad (4-1-1)$$

$$u_y = c \frac{A}{h} \exp\left(-\frac{(y - ct)^2}{2\chi^2}\right). \quad (4-1-2)$$

Here, $c = \sqrt{gh}$ is the phase velocity, where h , A , and χ are the ocean depth, the height of the solitary wave, and the horizontal scale length of the tsunami dynamic field. If we further assume a constant ocean depth, induced EM fields can be considered as functions of $y - ct$, which gives a relation between derivatives of b_z , viz., $\partial b_z / \partial t = -c \partial b_z / \partial y$. It follows that

this relation, Eq. (2-1-2) is reduced to

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \mu\sigma c \frac{\partial}{\partial y} \right) b_z = \mu\sigma F_z \frac{\partial v_y}{\partial y}. \quad (4-1-3)$$

Here we assumed $|v_y| \gg |v_z|$. Equation (4-1-3) can easily be solved numerically by FEM because of its independence of time. The boundary condition for b_z is the nil condition (Dirichlet type) on the boundaries far from the center grid.

When we recall b_z and e_x are functions of $y - ct$, the relationship between them in TE mode, $\partial e_x / \partial y = \partial b_z / \partial t$, is reduced to

$$e_x = -cb_z. \quad (4-1-4)$$

Therefore, the total electric field,

$$e_x + [\mathbf{v} \times \mathbf{F}]_x = e_x + v_y F_z, \quad (4-1-5)$$

can be calculated easily using Eqs. (4-1-2) and (4-1-4).

In FEM calculations of solitary waves, we adopt triangular elements as shown in Fig. 4-1, and use a 2-D FEM solver originally coded by Utada (1987). For EM induction problems, rectangular elements are often adopted in 2-D FEM calculations (e.g., Rodi, 1976; Uchida, 1993; Ogawa and Uchida, 1996). However, rectangular grids cost more expensive to express actual bathymetry than triangular grids. Although we deal only with a flat seafloor in this section, actual bathymetries should be included to reproduce the observed magnetic tsunami signals. In the next sub-section, we will describe discretization of Eq. (4-1-3) using triangular elements.

4.2 Galerkin discretization for 2-D FEM simulations

For the 2-D FEM simulations associated with solitary waves, we started with deriving the Galerkin equation of Eq. (4-1-3). Let us consider an arbitrary differential equation in terms of

a function, u , defined in the y,z -plane,

$$R(u; y, z) = 0. \quad (4-2-1)$$

If an approximated value, w , is substituted into Eq. (4-2-1), the solution does not exactly satisfy Eq. (4-2-1), namely:

$$R(w; y, z) \neq 0. \quad (4-2-2)$$

If we assume the weighted integral of $R(w; y, z)$ over whole space is equal to zero, the following equation is obtained;

$$\iint_S \delta w \cdot R(w; y, z) dydz = 0. \quad (4-2-3)$$

Here δw is a weight function. When we divide the whole space into finite number of elements, Eq. (4-2-3) is rewritten as the sum of the finite number (n) of integrals over each element,

$$\sum_{e=1}^n \iint_{S_e} \delta w_e \cdot R(w; y, z) dydz = 0, \quad (4-2-4)$$

where e denotes the e -th element. Eq. (4-2-4) indicates that the total of the integrated residual multiplied by a weight function becomes nil. By use of Eq. (4-1-3) for the differential equation, $R = 0$, Eq. (4-2-4) is written as follows:

$$\sum_{e=1}^n \iint_{S_e} \left\{ \delta w_e \cdot \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \mu \sigma c \frac{\partial}{\partial y} \right) b_z^e - \delta w_e \mu \sigma F_z \frac{\partial v_y^e}{\partial y} \right\} dydz = 0. \quad (4-2-5)$$

By integration by parts using Green's theorem, the Galerkin equation of Eq. (4-1-3) is obtained as

$$\sum_{e=1}^n \left[\iint_{S_e} \left\{ \left(\frac{\partial \delta w_e}{\partial y} \frac{\partial b_z^e}{\partial y} + \frac{\partial \delta w_e}{\partial z} \frac{\partial b_z^e}{\partial z} - \delta w_e \mu \sigma c \frac{\partial b_z^e}{\partial y} \right) + \delta w_e \mu \sigma F_z \frac{\partial v_y^e}{\partial y} \right\} dydz - \oint \delta w_e \frac{\partial b_z^e}{\partial n} dl \right] = 0, \quad (4-2-6)$$

where $\oint dl$ and $\partial/\partial n$ denote a line integral along the e -th element's boundary and a spatial

derivative outward normal to the element's boundaries, respectively. Eq. (4-2-6) is a so-called weak form, where all the partial derivatives are the first order. If we set numerical grid boundaries far enough from the center grid, the last term in Eq. (4-2-6) can be eliminated, and Eq. (4-2-6) becomes:

$$\sum_{e=1}^n \iint_{S_e} \left\{ \left(\frac{\partial \delta w_e}{\partial y} \frac{\partial b_z^e}{\partial y} + \frac{\partial \delta w_e}{\partial z} \frac{\partial b_z^e}{\partial z} - \delta w_e \mu \sigma c \frac{\partial b_z^e}{\partial y} \right) + \delta w_e \mu \sigma F_z \frac{\partial v_y^e}{\partial y} \right\} dy dz = 0. \quad (4-2-7)$$

In our 2-D FEM simulation, we adopted triangular elements and linear interpolation functions to express field variations in each element. The b_z and v_y in the e -th element can be expressed as follows;

$$b_z^e(y, z) = \mathbf{N}_e^T \mathbf{b}_z^e, \quad v_y^e(y, z) = \mathbf{N}_e^T \mathbf{v}_y^e, \quad (4-2-8)$$

$$\mathbf{N}_e = \begin{pmatrix} N_{e1} \\ N_{e2} \\ N_{e3} \end{pmatrix}, \quad \mathbf{b}_z^e = \begin{pmatrix} b_{z1}^e \\ b_{z2}^e \\ b_{z3}^e \end{pmatrix}, \quad \mathbf{v}_y^e = \begin{pmatrix} v_{y1}^e \\ v_{y2}^e \\ v_{y3}^e \end{pmatrix}, \quad (4-2-9)$$

where, N_{ei} , b_{zj}^e and v_{yj}^e are the linear interpolation function, and values for b_z and v_y given at each node in the e -th element, respectively. $\mathbf{N}_e^T$ is the transpose of $\mathbf{N}_e$. Figure 4-1 shows the configuration of the e -th element. As for the linear interpolation function, N_{ei} , we used;

$$\mathbf{N}_e = \frac{1}{2\Delta_e} (\mathbf{a}_e + \mathbf{b}_e y + \mathbf{c}_e z), \quad (4-2-10)$$

$$\mathbf{a}_e = (y_2 z_3 - y_3 z_2, y_3 z_1 - y_1 z_3, y_1 z_2 - y_2 z_1)^T, \quad (4-2-11)$$

$$\mathbf{b}_e = (z_2 - z_3, z_3 - z_1, z_1 - z_2)^T, \quad (4-2-12)$$

$$\mathbf{c}_e = (y_3 - y_2, y_1 - y_3, y_2 - y_1)^T, \quad (4-2-13)$$

$$\Delta_e = \frac{1}{2} \{ (y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1) \}, \quad (4-2-14)$$

where Δ_e is the area of the e -th element and $y_{1,2,3}$ and $z_{1,2,3}$ are the y and z coordinates of the nodes, respectively. In the Galerkin method, the interpolation function given by Eq. (4-2-10) works as the weight function, δw_e . Substituting Eqs. (4-2-8) and (4-2-9) into b_z^e , v_y^e , and

δw_e in Eq. (4-2-7), we can obtain the discretized form of Eq. (4-1-3),

$$\sum_{e=1}^n \iint_{S_e} \left[\left\{ \left(\frac{\partial \mathbf{N}_e}{\partial y} \frac{\partial \mathbf{N}_e^T}{\partial y} + \frac{\partial \mathbf{N}_e}{\partial z} \frac{\partial \mathbf{N}_e^T}{\partial z} \right) - P \mathbf{N}_e \frac{\partial \mathbf{N}_e^T}{\partial y} \right\} \mathbf{b}_z^e + Q \mathbf{N}_e \frac{\partial \mathbf{N}_e^T}{\partial y} \mathbf{v}_y^e \right] dydz = 0, \quad (4-2-15)$$

where $P = \mu\sigma c$ and $Q = \mu\sigma F_z$. Equation (4-2-15) includes dyadic products. By use of Eqs.

(4-2-10) - (4-2-13), Eq. (4-2-15) is further reduced to :

$$\sum_{e=1}^n \iint_{S_e} \left[\left\{ \frac{1}{2\Delta_e} (\mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T) - P \mathbf{N}_e \mathbf{b}_e^T \right\} \mathbf{b}_z^e + Q \mathbf{N}_e \mathbf{b}_e^T \mathbf{v}_y^e \right] dydz = 0. \quad (4-2-16)$$

Here we introduce a useful formula regarding area integrals over a triangular element,

$$\iint_S N_1^l N_2^m N_3^n dS = \frac{l! m! n!}{(l + m + n + 2)!} \cdot 2\Delta, \quad (4-2-17)$$

where l, m , and n are integers. With the aid of Eq. (4-2-17), we can calculate area integrals in

Eq. (4-2-16) as follows;

$$\iint_{S_e} 1 dydz = \frac{0! 0! 0!}{(0 + 0 + 0 + 2)!} \cdot 2\Delta_e = \Delta_e, \quad (4-2-18)$$

$$\iint_{S_e} \mathbf{N}_e dydz = \left(\frac{1}{3!} \cdot 2\Delta_e, \frac{1}{3!} \cdot 2\Delta_e, \frac{1}{3!} \cdot 2\Delta_e \right)^T = \frac{\Delta_e}{3} \mathbf{1}, \quad (4-2-19)$$

where $\mathbf{1} = (1, 1, 1)^T$. Using Eqs. (4-2-18) and (4-2-19), Eq. (4-2-16) becomes,

$$\sum_{e=1}^n \{ \mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T - P' \mathbf{1} \mathbf{b}_e^T \} \mathbf{b}_z^e = \sum_{e=1}^n Q' \mathbf{1} \mathbf{b}_e^T \mathbf{v}_y^e, \quad (4-2-20)$$

where $P' = 2\mu\sigma c \Delta_e / 3$ and $Q' = -2\mu\sigma F_z / 3$. Now we can assemble the final form of the simultaneous linear equations in terms of b_z as;

$$\mathbf{A} \mathbf{b}_z = \mathbf{B}, \quad (4-2-21)$$

where $\mathbf{A}$ and $\mathbf{B}$ are an $n \times n$ matrix assembled from $\{ \mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T - P' \mathbf{1} \mathbf{b}_e^T \}$ and a vector of size n composed of the sum of the r.h.s., which includes boundary conditions when necessary.

$\mathbf{b}_z$ corresponds to a variable vector to be obtained by solving Eq. (4-2-21). Note that the coefficient matrix, $\mathbf{A}$, is not symmetric because of the form of $\{ \mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T - P' \mathbf{1} \mathbf{b}_e^T \}$. To solve Eq. (4-2-21), we modified the Utada's (1987) 2-D FEM solver based on manipulation

of symmetric matrices. Figure 4-2 illustrates how we modified the solver. Our band matrix is twice as wide as that of Utada's solver in order to store unsymmetric part of the coefficient matrix. We have found that our new simulation code works well with various ocean depth, tsunami heights, and wavelengths. In the next sub-section, we will discuss three tsunami-dynamo phenomena using solitary waves in the simulation.

4.3 Three typical cases classified by the ocean depth

In order to appreciate effects of ocean depth, h , on TDE phenomena, we conducted simulations for three ocean depths of 1km, 2.5km, and 5km. A numerical grid shown in Fig. 4-3 was used. The horizontal grid spacing is fixed to 1.0km. On the other hand, the vertical spacing gets sparse exponentially upward/downward towards boundaries. Figure 4-4 shows the results, where colors and black lines indicate the x -component of the electric field and the upward component of the magnetic field on the seafloor, respectively. In these calculations, we assumed $\chi = 20$ km for the solitary wave, namely, a wave with a horizontal extent of approximately 100 km, and half-space insulators above the sea surface and beneath the seafloor. For the calculations with $h = 1.0, 2.5$, and 5.0 km, we put the tsunami height, η , to 2.236, 1.414, and 1.000m, respectively, so that $v_y h = \text{constant}$. This led to almost the same profiles of the vertical component of the magnetic field induced by $\mathbf{v} \times \mathbf{F}$ on the seafloor among the three calculations. This is equivalent to the case in which the volume integral of the source term in the induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + K \nabla^2 \mathbf{B}, \quad (4-3-1)$$

is constant. Note in Eq. (4-3-1) that the sign of the diffusion term, $K \nabla^2 \mathbf{B}$, is opposite to that of the source term, $\nabla \times (\mathbf{v} \times \mathbf{B})$, because the magnitude of the self-induction term, $\partial \mathbf{B} / \partial t$, is

always smaller than that of the source term. Provided that the source term is fixed, we can consider that TDE phenomena are strongly dependent on the balance between the self-induction term and the diffusion term. Furthermore, the order of the diffusion term under the influence of the ocean depth is given by,

$$K\nabla^2\mathbf{B}\sim K\left(\frac{1}{h^2}+\frac{1}{\lambda^2}\right)|\mathbf{B}|\sim\frac{K}{h^2}|\mathbf{B}|, \quad (4-3-2)$$

because tsunami wavelengths are usually much longer than ocean depths. Equation (4-3-2) implies that as the ocean depth increases, the influence of the diffusion term on TDE phenomena attenuates, while the self-induction term, $\partial\mathbf{B}/\partial t$, becomes larger. The three cases in Fig. 4-4 indicate typical three phenomena of TDE for different ocean depths. From left to right, we call the results as “*Diffusion dominant case*”, “*Intermediate case*”, and “*Self-induction dominant case*”, respectively, in terms of contribution of the diffusion term. Figure 4-5 shows different plots of the same calculations in terms of b_z and e_x . From top to bottom, the color lines indicate the tsunami-induced magnetic field (left) or the electric field (right) on the seafloor for the ocean depth of 1.0, 2.5, and 5.0 km, respectively. Figure 4-5 explicitly shows that the both magnetic and electric fields on the seafloor significantly change as the ocean depth increases. Hence, we discuss the three cases one by one as follows:

-*Diffusion dominant case*- The phenomena shown in the left column of Fig. 4-4 can be classified as “*Diffusion dominant case*”. When the ocean depth is small enough, the diffusion term becomes comparable with the source term, which leads to

$$0 \approx \nabla \times (\mathbf{v} \times \mathbf{B}) + K\nabla^2\mathbf{B}. \quad (4-3-3)$$

In this case, the self-induction term is negligible. As a result, the observable magnetic field is almost the same as that induced by the inducing electric current, $\sigma(\mathbf{v} \times \mathbf{F})$. This can be seen

by comparing the upper two profiles of the magnetic field on the seafloor, b_{z_total} and $b_{z_v \times F}$, in the left column of Fig. 4-4. Figure 4-6 shows how EM fields are induced in the “*diffusion dominant case*”. The peaks of the downward and upward magnetic field induced by $v \times F$ appear almost 90 degrees ahead of and behind the peak of the sea level change, respectively. The induced electric field, e_x , is also present on both sides of the solitary wave as implied by Eq. (4-1-4), although the amplitudes of e_x are tiny. We further found in the left panel of Fig. 4-5 that the negative peak of b_{z_total} for $h = 1.0$ km prior to the tsunami peak is slightly reinforced compared with that of $b_{z_v \times F}$, while the positive peak behind the tsunami attenuates. This is because the induced electric fields prior to and behind the tsunami are in the mutually opposite direction, both of which induce the downward component of the magnetic field at the center of the solitary wave, as shown in the bottom left panel of Fig. 4-4. This is a very particular case, because the self-induction basically attenuates the amplitudes of the vertical component of the tsunami-induced magnetic field, as shown in the following other cases.

-Intermediate case- As the ocean depth becomes deeper compared with the “*Diffusion dominant case*”, the self-induction term, $\partial \mathbf{B} / \partial t$, increases and induces the electric field, e_x , due to the Faraday’s Law of induction. We can regard the case in which both the diffusion and self-induction terms are important as “*Intermediate case*”. The center column of Fig. 4-4 shows the results corresponding to the “*Intermediate case*”, while Fig. 4-7 shows a schematic figure explaining this case. The electric field is induced in the direction opposite to the $v \times F$ field ahead of the solitary wave, which causes a phase delay of the electric current compared with the “*diffusion dominant case*”. We can find in lines for $h = 2.5\text{km}$ of Fig. 4-5 that both

of the phases of the downward peak of the vertical magnetic component, b_{z_total} , and the total electric field, $e_x + v \times F$, are delayed compared with the results for $h = 1.0$ km. Both the amplitudes of b_{z_total} and $e_x + v \times F$ were also attenuated compared with those of $b_{z_v \times F}$ and $v \times F$, respectively. These are because the increased self-induction term, which was comparable with the diffusion term due to the intermediate ocean depth of 2.5 km, induced the electric field opposite to $v \times F$ ahead of the tsunami peak.

-Self-induction dominant case- The right most panels of Fig. 4-4 show the results for the deepest ocean depth of 5km. When the ocean depth is large enough, the diffusion term becomes much smaller than the self-induction term. We can regard this phenomenon as “*Self-induction dominant case*”. In this case, the self-induction term is comparable with the source term, namely,

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times (\mathbf{v} \times \mathbf{B}). \quad (4-3-4)$$

Figure 4-8 shows the mechanism of how EM fields are induced in this case. The induced electric field, e_x , almost counter-balances $v \times F$, which makes the observable magnetic field much smaller than that induced by $v \times F$. The magnetic field change coincides with the sea level change, because the phase delay of the induced magnetic field is approximately 90 degrees compared with the “*Diffusion dominant case*”.

Our simulation using solitary waves revealed that tsunami dynamo phenomena are strongly controlled by the ocean depth, and they can be classified into three types. We will discuss these three cases in more detail later in Section 6.3.

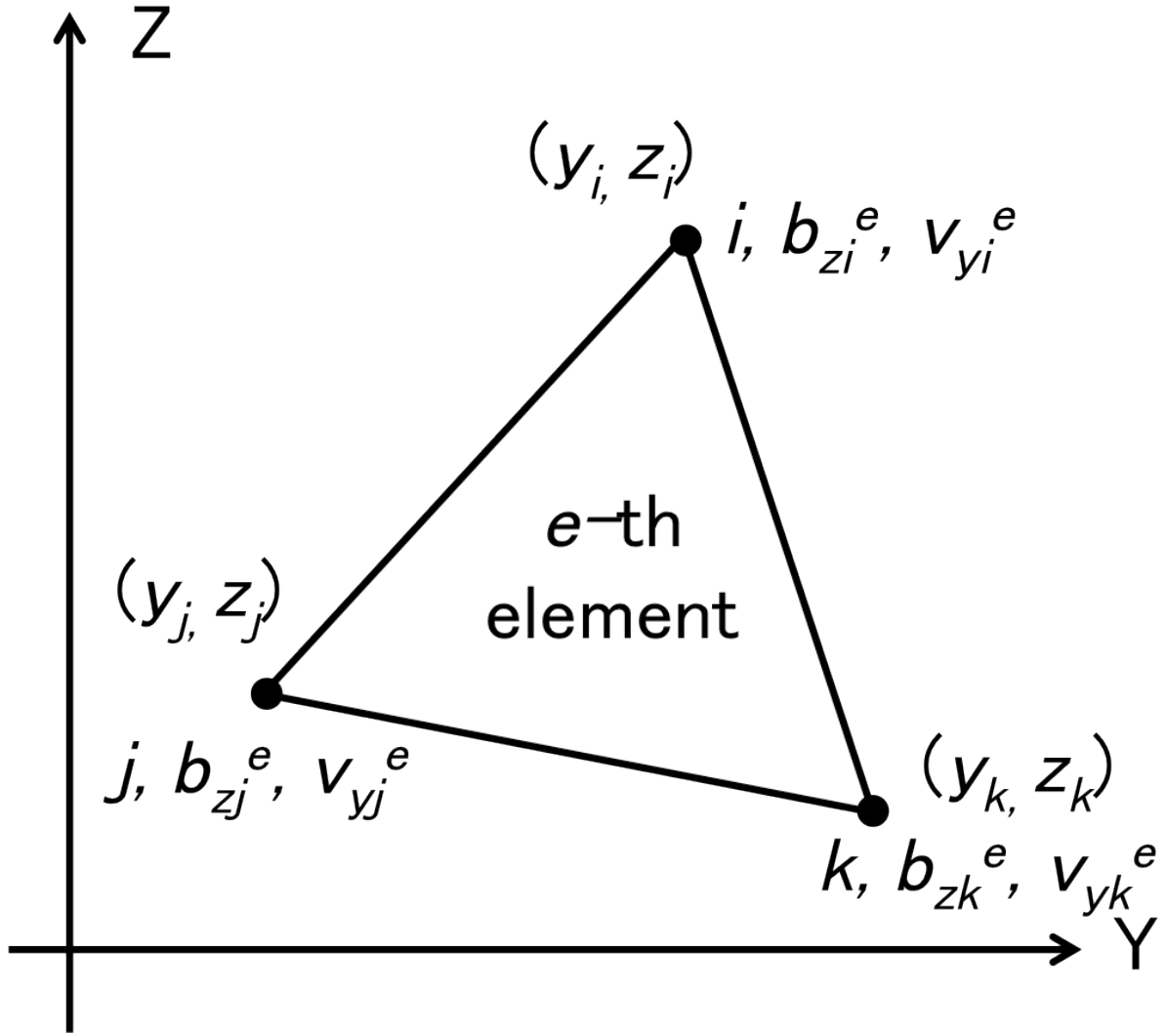


Fig. 4-1 Configuration of the e -th element for 2-D FEM simulations. i, j , and k denote the node numbers. (y, z) 's are the coordinates of nodes, at which the vertical component of the magnetic field, b_z , and the horizontal oceanic velocity, v_y , are given. Within the e -th element, b_z and v_y are calculated by linear interpolation functions expressed by Eq. (4-2-8) using nodal values.

Modification of the band matrix for unsymmetric matrix

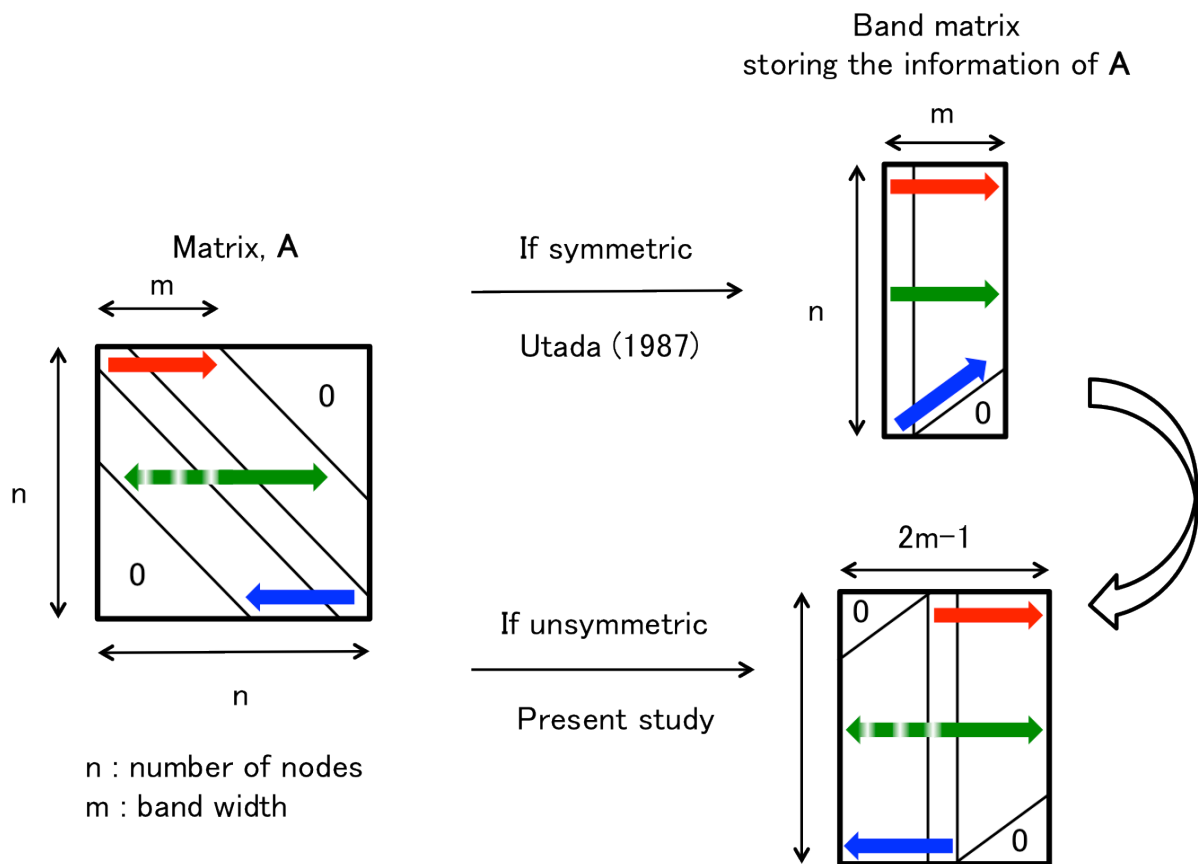


Fig. 4-2 A schematic figure of modification of the Utada's (1987) FEM solver. For storing the unsymmetric part of the matrix, **A**, our band matrix is almost twice as wide as Utada's (1987).

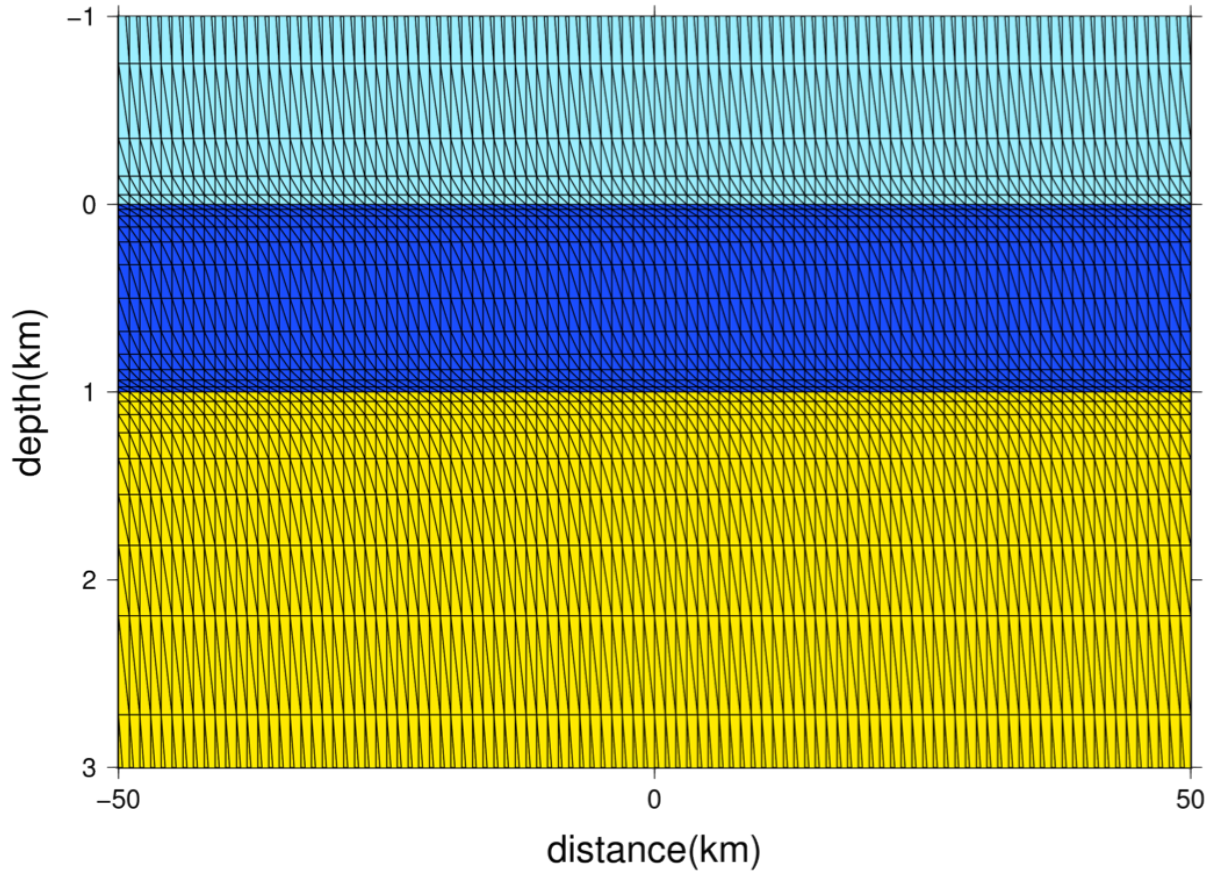


Fig. 4-3 Numerical mesh for FEM calculations for solitary waves. The light-blue, blue, and yellow regions correspond to the air, ocean, and crust/mantle beneath the ocean, respectively. The grid spacing in the horizontal direction is constant (1.0km). The vertical spacing are finest near the sea surface and seafloor, while it gets larger exponentially upward and downward.

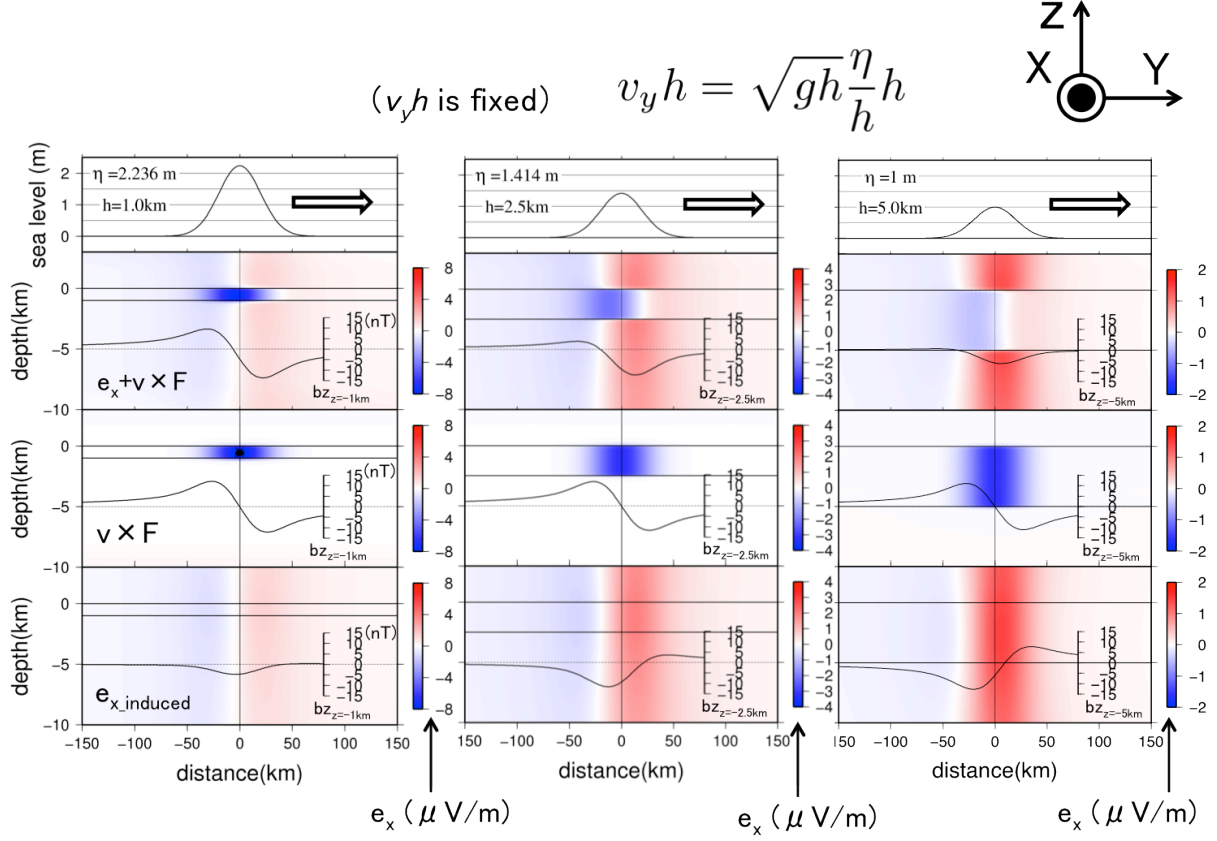


Fig. 4-4 Three cases of EM fields induced by solitary waves. From top to bottom, the panels show the sea level change, total electric field, $e_x + v \times F$, the inducing electric field, $v \times F$, and the induced electric field, e_x . Except for the top row, the vertical components of the magnetic field on the seafloor are plotted in each panel. From left to right, (h, η) was set to be (1.0km, 2.236m), (2.5km, 1.414m), and (5.0km, 1.0m), respectively, where $v_y h \propto \eta \sqrt{h} = \text{const.}$ so that the vertical components of the magnetic field induced by $v \times F$ have almost the same profiles on the seafloor, as shown in the third row from the top.

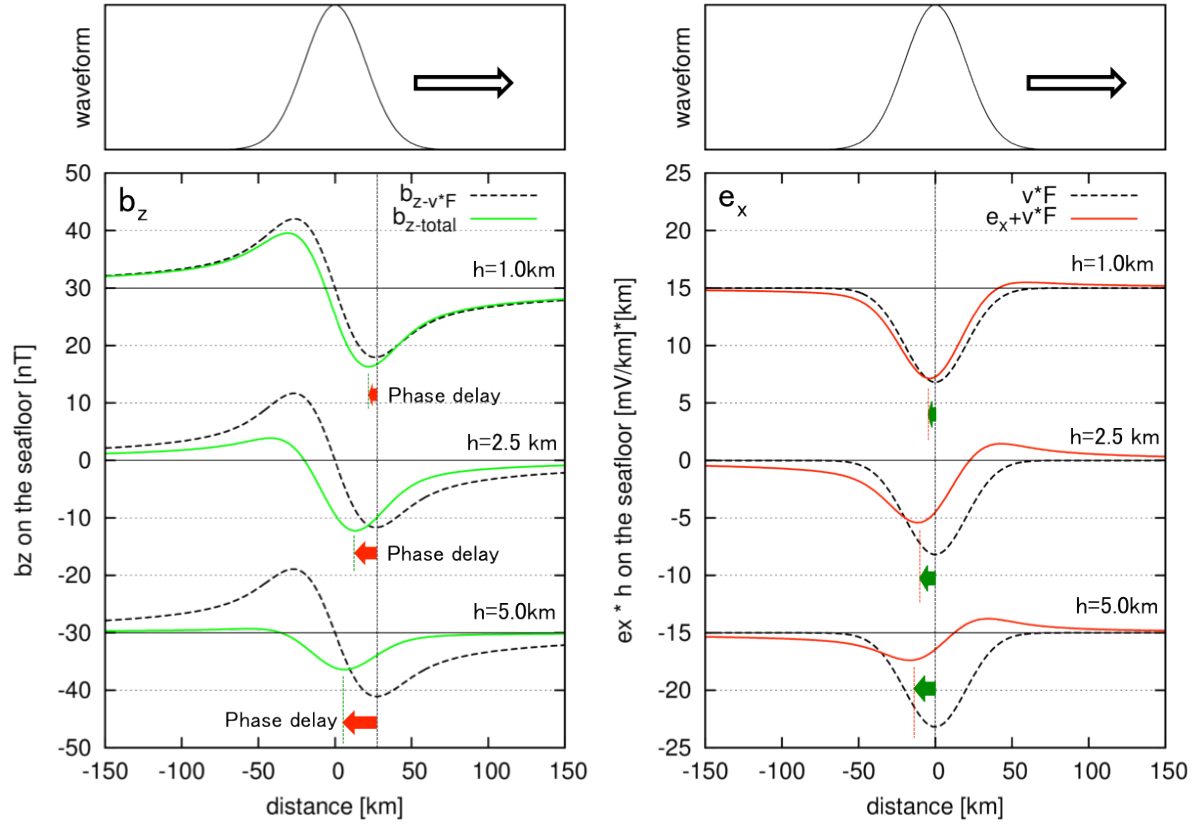


Fig. 4-5 The vertical component of the tsunami-induced magnetic field on the seafloor (left) and the horizontal component of the electric field multiplied by each ocean depth (right) in color solid lines. Dashed lines in the left and right column correspond to the magnetic field induced only by the $v \times F$ field and the $v \times F$ field itself on the seafloor, respectively. In both the left and right column, color arrows indicate phase delays of the vertical magnetic component (red) and the horizontal electric field (green), respectively.

-Diffusion dominant case-

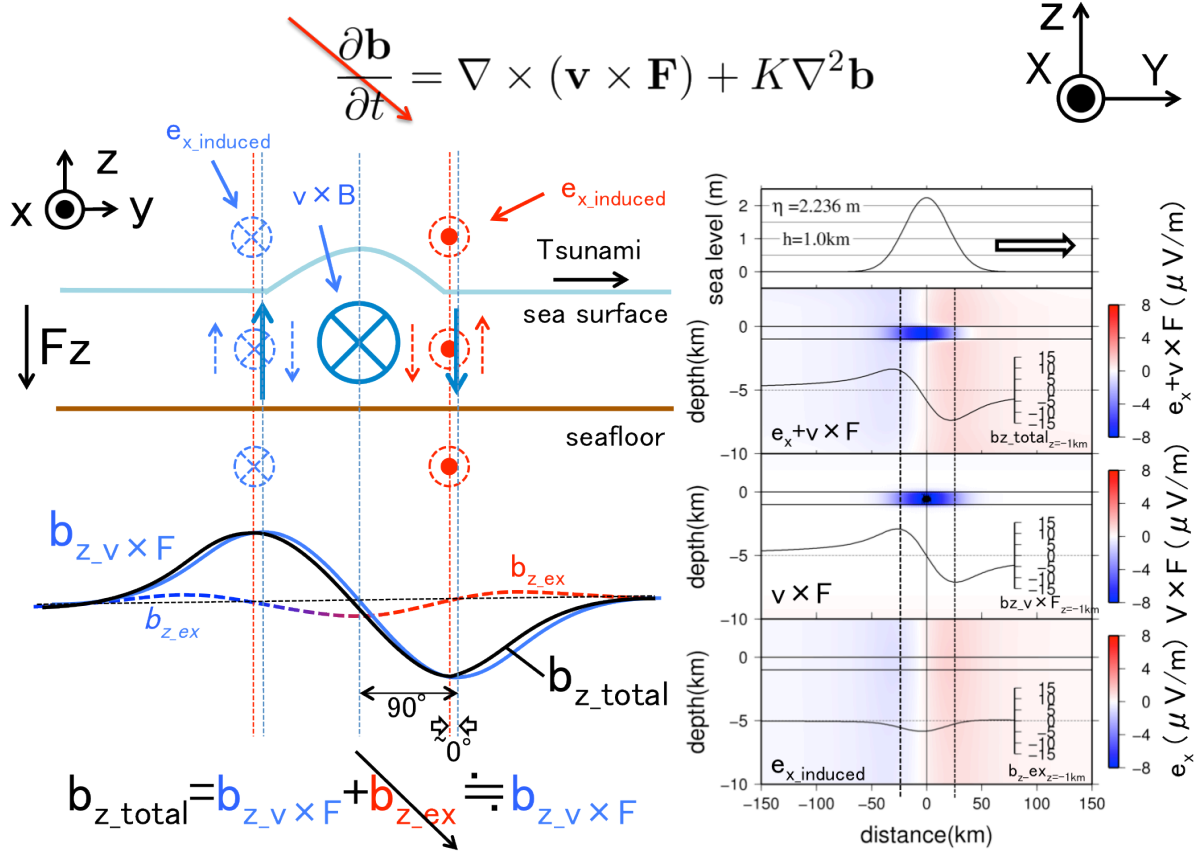


Fig. 4-6 A schematic figure of how EM fields are induced by a solitary tsunami when the ocean is very shallow. The right panels are the same as the left column panels of Fig. 4-4. In this case, the total magnetic field induced by the tsunami is influenced little by the self-induction so that the resulting total magnetic field is almost the same as those induced by $v \times F$. As a result, the magnetic field on the seafloor is probably detectable approximately T/4 prior to the first arrival of tsunamis.

-Intermediate case-

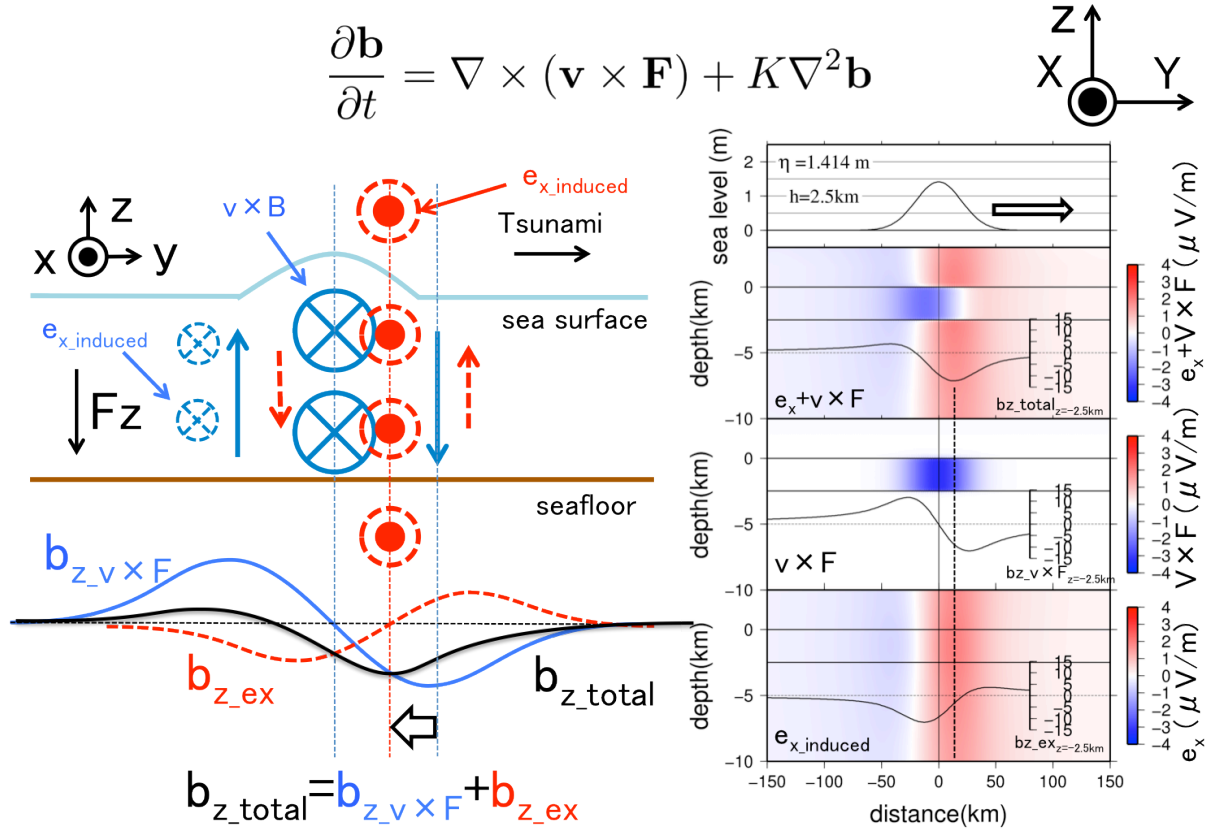


Fig. 4-7 A schematic figure of how EM fields are induced by a solitary wave when the ocean has an intermediate depth. Right panels are the same as the middle column panels of Fig. 4-4. In this case, since the self-induction and the diffusion terms are comparable, the vertical component of the induced magnetic field, B_{z_total} , is delayed in phase and reduced in amplitude compared with those of the magnetic field induced by $\mathbf{v} \times \mathbf{F}$.

-Self-induction dominant case-

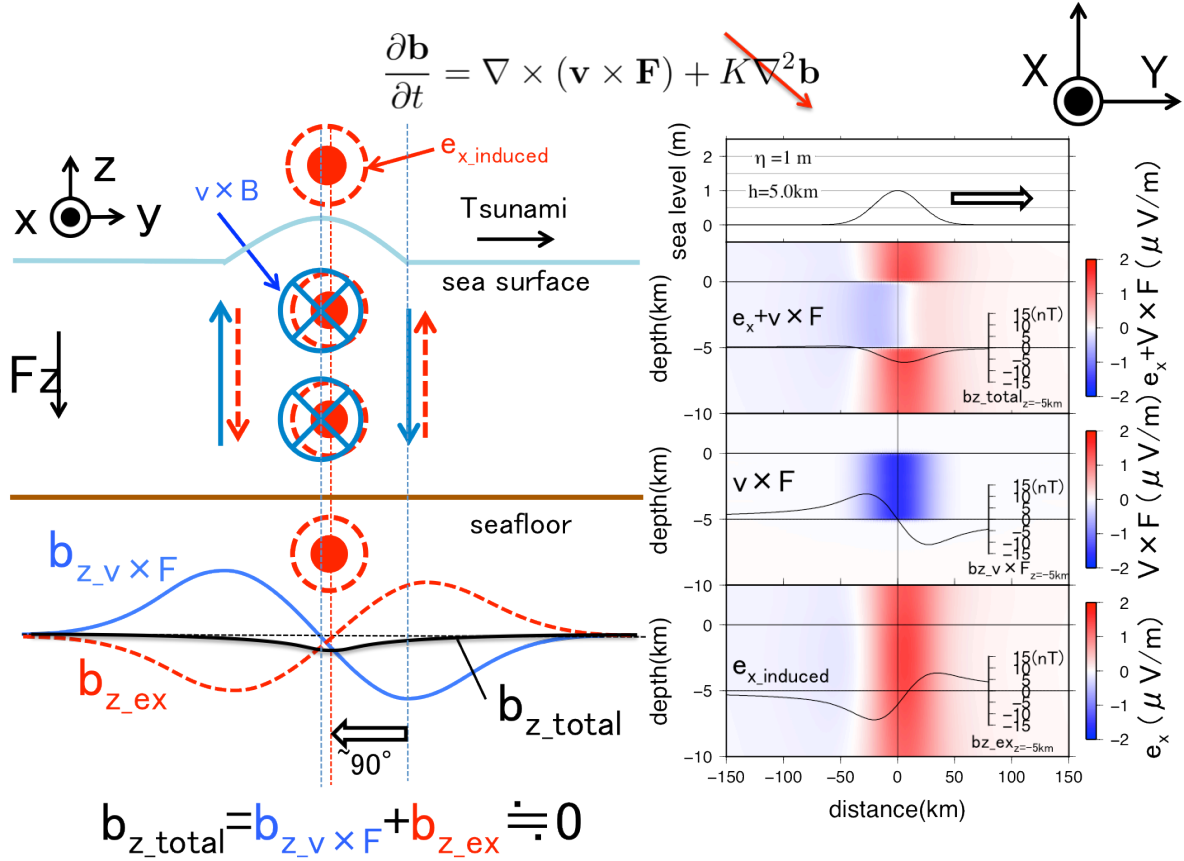


Fig. 4-8 A schematic figure of how EM fields are induced by a solitary wave when the ocean is deep. Right panels are the same as the right column panels of Fig. 4-4. In this case, the self-induction term is comparable with the source field. The magnetic field induced by $\mathbf{v} \times \mathbf{F}$ is significantly attenuated by those generated by the induced electric field. As a result, the total magnetic field on the seafloor, B_{z_total} , is much smaller compared with the magnetic field induced by $\mathbf{v} \times \mathbf{F}$ and almost in phase with the sea level change.

5. 2-D tsunami dynamo simulations in the time domain

We developed a 2-D FEM tsunami dynamo simulation code in order to explain the sea level changes at DART21401 and 21419 and the tsunami-induced magnetic variations at NWP simultaneously. We adopted simulations in the time domain, because transient waves such as solitary waves, cost a lot to reproduce in the frequency domain. We further adopted FEM using triangular elements because unstructured triangular meshes can express actual bathymetry. Minami and Toh (2012) reported that FEM using rectangular meshes costs too much to achieve the same accuracy as triangular ones. Our simulation method consists of two steps. In the first step, oceanic flows associated with tsunami propagation were calculated. In the second step, the induction equation in terms of the magnetic field was solved numerically for given oceanic flows. In order to obtain consistent results between the two steps, the same numerical mesh was used for both finite element calculations. Figure 5-1 shows an unstructured triangular mesh adopted in our simulation, in which actual bathymetry was well included. In the following, we first derive formulations for our tsunami dynamo simulations in Section 5.1. Second, we discretize the obtained formulations by using the Galerkin method, in the same manner as Section 5.2. Finally, tsunami dynamo simulations for the 2011 Tohoku earthquake tsunami are conducted in Section 5.3.

5.1 Formulation

In the first step of our simulation, we conducted hydrodynamic tsunami simulations for the source field of the tsunami dynamo EM simulation. In the field of hydrodynamics, various tsunami simulation techniques have been presented. We can classify most of them into simple two types. One type consists of linear horizontal 2-D simulations, and the other is

composed of non-linear 3-D simulations. The former is mainly dedicated to tsunami waveform inversions to investigate tsunami source processes by explaining many observed sea level change data. For the tsunami inversions, the computation time per one cycle is considerably important, which led to many simple 2-D linear long wave (LLW) tsunami inversions (e.g., Fujii and Satake, 2007; Satake et al., 2013). On the other hand, non-linear 3-D tsunami simulations concentrate on enhancing accuracy of their forward simulation results. Although they take much more time than the horizontal 2-D simulations, they can well reproduce dispersive or non-linear tsunami behaviors even near shore regions, allowing the oceanic velocity field to vary with the vertical coordinate (e.g., Saito and Furumura, 2009a and 2009b; Oishi et al., 2013).

The purpose of our tsunami dynamo simulation is to reproduce the magnetic tsunami signals observed at NWP. For this purpose, we need calculation accuracy near the seafloor, while non-linearity is not necessary because NWP is far enough from the coastal regions. We, therefore, adopted 2-D tsunami simulations in a vertical plane, assuming that the 2011 Tohoku tsunami can be approximated by plane waves at NWP. The Laplace equation in terms of the velocity potential was solved in a 2-D vertical plane, taking irrotational oceanic flows. This simulation method allowed us to remain the vertical coordinate for accuracy on the seafloor. Since NWP is in an open ocean, we eliminated non-linearity by taking linear boundary conditions at the sea surface. For a boundary condition on the seafloor, we assumed negligible bottom friction. This assumption was proved suitable by 3-D Navier-Stokes tsunami simulations of Oishi et al. (2013). The governing equation to be solved and the related boundary conditions are summarized as follows:

$$\Delta\Phi = 0, \quad (5-1-1)$$

$$\frac{\partial\Phi}{\partial t} + g\eta = 0 \quad (z = 0), \quad (5-1-2)$$

$$\frac{\partial\eta}{\partial t} - \frac{\partial\Phi}{\partial z} = 0 \quad (z = 0), \quad (5-1-3)$$

$$\frac{\partial\Phi}{\partial n} = 0 \quad (z = h(y)). \quad (5-1-4)$$

Here, Φ , g , η , and $h(y)$ denote the velocity potential, the gravity acceleration, the sea surface elevation, and the bathymetry, respectively. Operators, $\partial/\partial t$, $\partial/\partial z$, and $\partial/\partial n$ denote differentiations w.r.t. time, the vertical coordinate, and the direction normal to the seafloor, respectively. Note that η and z are upward positive throughout this paper. The leap-frog method (e.g., Dao and Tkalich, 2007; Sayama et al., 1987) was adopted to solve Eqs. (5-1-2) and (5-1-3), while Eq. (5-1-1) was solved by FEM in order to calculate Φ with the given boundary values. Figure 5-2 shows the flow of the leap-frog method in our hydrodynamic simulation. The method consists of three sub-steps. In the first sub-step, Eq. (5-1-2) was used to obtain $\Phi_{z=0}$ in the next time step from known $\Phi_{z=0}$ and η in the last time steps. In the second sub-step, Eq. (5-1-1) was solved by 2-D FEM calculation using obtained $\Phi_{z=0}$ as a boundary condition at the sea surface ($z=0$). By use of the obtained 2-D Φ distribution, we calculated $\partial\Phi/\partial z_{z=0}$ in the same time step as $\Phi_{z=0}$. In the last sub-step, Eq. (5-1-3) was used to get η in the new time step from the obtained $\partial\Phi/\partial z_{z=0}$ and η in the last time step. By iterating these three sub-steps, we can obtain time series of Φ at every node in the ocean. Finally, we can obtain the horizontal and vertical velocity of the oceanic flow, v_y and v_z , at every node and in every time step, by differentiating the obtained Φ w.r.t. y and z . In our tsunami dynamo simulation, obtained v_y was used as a source field in the second step.

In the second step, we calculated the tsunami-induced EM fields using the tsunami flows

of the first step as follows: We began with Eq. (9) in Tyler (2005), the induction equation in terms of the tsunami-induced vertical magnetic component,

$$\frac{\partial b_z}{\partial t} = -\nabla_H \cdot (F_z \mathbf{v}_H) + K \nabla^2 b_z. \quad (5-1-5)$$

$K = (\mu\sigma)^{-1}$ is the magnetic diffusivity, where μ and σ are the magnetic permeability and the conductivity respectively. b_z , $\mathbf{v}_H$, and F_z denote the tsunami-induced vertical magnetic component, the horizontal oceanic velocity, and the vertical component of the geomagnetic main field, respectively. In 2-D configuration with tsunamis propagating only in the y -direction, we can put the x -component of ∇_H and $\mathbf{v}_H$ to nil. Under this circumstance, Eq. (5-1-5) is reduced to

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \mu\sigma \frac{\partial}{\partial t} \right) b_z = \mu\sigma \frac{\partial}{\partial y} (F_z v_y). \quad (5-1-6)$$

Equation (5-1-6) is applicable wherever the conductivity is homogeneous, including the regions above and beneath the ocean layer. When Eq. (5-1-6) is used in the air, $\sigma = 0$ and $v_y = 0$ are substituted to make the Laplace equation for b_z . For time evolution, the Crank-Nicolson method was adopted. It follows that Eq. (5-1-6) is discretized to give

$$\begin{aligned} \frac{1}{2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) (b_z^n + b_z^{n+1}) - \mu\sigma \frac{1}{\Delta t} (b_z^{n+1} - b_z^n) \\ = \frac{1}{2} \mu\sigma \frac{\partial}{\partial y} \{ F_z (v_y^n + v_y^{n+1}) \}, \end{aligned} \quad (5-1-7)$$

where b_z^n , v_y^n , and Δt denote b_z and u_y at the n -th time step, and the time interval between two time steps, respectively. In our Galerkin FEM solver, Eq. (5-1-7) was integrated over each element, which will be described in detail in the next sub-section. All the boundary conditions under which Eq. (5-1-7) was solved are $b_z = 0$ (Dirichlet-type) at enough distances from the center grid. The other components of tsunami-induced EM fields, b_y and e_x , can be calculated using the following relations:

$$\frac{\partial b_y}{\partial y} = -\frac{\partial b_z}{\partial z} \quad (5-1-8)$$

$$\frac{\partial e_x}{\partial y} = \frac{\partial b_z}{\partial t} \quad (5-1-9)$$

Note here that our simulations corresponded to so-called TE mode calculations in magneto-telluric (MT) studies. As for TM mode, electromotive forces (emf) seem to be generated mainly by coupling of the horizontal oceanic flow, v_y , with the horizontal component of the geomagnetic main field perpendicular to the tsunami propagation direction, F_x . However, incompressible oceanic flows are not able to generate motional induction in TM mode as described in Section 2.2. As a result, in 2-D configuration, tsunami-induced EM variations at NWP can be obtained by calculations in TE mode alone.

5.2 Galerkin discretization for 2-D FEM simulations in the time domain

We first briefly explain the discretization of Eq. (5-1-1) for our hydrodynamic simulation in the first step. In the second sub-step shown in Fig. 5-2, we solved Eq. (5-1-1) by using a 2-D FEM calculation and obtain $\partial\Phi/\partial z_{z=0}$ from the obtained 2-D Φ distribution. Using the same manner as Section 4.2, we can obtain the discretized form of Eq. (5-1-1) as

$$\sum_{e=1}^{n_s} \{\mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T\} \boldsymbol{\Phi}^e = 0, \quad (5-2-1)$$

where $\boldsymbol{\Phi}^e = \{\Phi_1^e, \Phi_2^e, \Phi_2^e\}^T$, and n_s is the number of nodes within the ocean. Eq. (5-2-1) corresponds to Eq. (4-2-21) in Section 4.2, although it is much simpler than Eq. (4-2-20). Assembling a coefficient matrix from $\{\mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T\}$, Eq. (5-2-1) is reduced to the form of simultaneous linear equations,

$$\mathbf{A}'\boldsymbol{\Phi} = 0, \quad (5-2-2)$$

where $\mathbf{A}'$ and $\boldsymbol{\Phi}$ are a coefficient matrix composed of the sum of $\{\mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T\}$ and a variable vector of size n_s . By solving Eq. (5-2-2), we obtained the oceanic flow potential, Φ , at every node in the ocean. In order to calculate η in the next time step, we need $\partial\Phi/\partial z$ at $z=0$ to be substituted in Eq. (5-1-3). We adopted Li et al.'s (2008) extrapolation method, in order to calculate $\partial\Phi/\partial z$ at the sea surface. Figure 5-3 illustrates the method in detail. With the aid of Li et al.'s (2008) method, $\partial\Phi/\partial z$ at the sea surface is expressed as,

$$\frac{\partial\Phi}{\partial z}_{z=0} = \frac{(\Phi_0 - \Phi_1)(h_1 + H_2)}{h_1 h_2} - \frac{h_1(\Phi_0 - \Phi_2)}{h_2(h_1 + h_2)}, \quad (5-2-3)$$

where the notations are shown in Fig. 5-3. In this method, two $\partial\Phi/\partial z$ at $z = -h_1/2$ and $z = -(h_1 + h_2)/2$ are used to obtain $\partial\Phi/\partial z$ at $z = 0$. This method has an advantage of enhancing accuracy by using values on three nodes in a row. Hence, we obtained $\partial\Phi/\partial z_{z=0}$ at $z = 0$ from given $\Phi_{z=0}$ in the second sub-step of our hydrodynamic simulation.

Let us move on deriving the Galerkin discretization form of Eq. (5-1-7) for the second step. We cannot directly discretize Eq. (5-1-7) by using the Galerkin method, because the source term on the r.h.s. of Eq. (5-1-7) requires v_y as the nodal values of the e -th element. Our hydrodynamic simulation in the first step provides not v_y but Φ at every node. In order to obtain v_y at every node, we have to differentiate Φ w.r.t. the horizontal coordinate, y . However, it is difficult to calculate $\partial\Phi/\partial y$ at nodes, since we adopted an unstructured triangular grid without fixed horizontal straight lines, as shown in Fig. 5-1. In Section 4.1, v_y at any node was directly calculated by using Eq. (4-1-2), because we adopted linear long waves. For the pursuit of discretizing r.h.s. of Eq. (5-1-7), we transformed Eq. (5-1-7) to

$$\begin{aligned}
& \frac{1}{2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) (b_z^n + b_z^{n+1}) - \mu\sigma \frac{1}{\Delta t} (b_z^{n+1} - b_z^n) \\
& = \frac{1}{2} \mu\sigma \frac{\partial}{\partial z} \left\{ F_z \left(\frac{\partial \Phi^n}{\partial z} + \frac{\partial \Phi^{n+1}}{\partial z} \right) \right\},
\end{aligned} \tag{5-2-4}$$

where the Laplace equation in terms of Φ (Eq. (5-1-1)) was used. The source term in Eq. (5-2-4) is easy to discretize, because $\partial\Phi/\partial z$ at nodes can be calculated using vertically aligned three nodes as shown in Fig. 5-4. The Li et al.'s (2008) extrapolation method was used again, for nodal values at the sea surface and seafloor. As for the other nodes within the ocean, we first calculated two $\partial\Phi/\partial z$ on upside and downside of a node in concern, and obtained the value at the node by interpolating from the two values. Then we obtained the discretized form of Eq. (5-2-4) by using the same manner as Section 4.2,

$$\begin{aligned}
& \sum_{e=1}^n \iint_{S_e} \left\{ \left(\frac{\partial \mathbf{N}_e}{\partial y} \frac{\partial \mathbf{N}_e^T}{\partial y} + \frac{\partial \mathbf{N}_e}{\partial z} \frac{\partial \mathbf{N}_e^T}{\partial z} \right) (\mathbf{b}_z^{en} + \mathbf{b}_z^{en+1}) + P'' \mathbf{N}_e \mathbf{N}_e^T (\mathbf{b}_z^{en} - \mathbf{b}_z^{en+1}) \right. \\
& \quad \left. + Q'' \mathbf{N}_e \frac{\partial \mathbf{N}_e^T}{\partial z} (\mathbf{v}_z^{en} + \mathbf{v}_z^{en+1}) \right\} dydz = 0,
\end{aligned} \tag{5-2-5}$$

where $P'' = -2\mu\sigma/\Delta t$ and $Q'' = -\mu\sigma F_z$. $\partial\Phi/\partial z$ at three nodes of the e -th element are expressed as $\mathbf{v}_z^e$. By use of the relation, Eq. (4-2-10), Eq. (5-2-5) is reduced to

$$\begin{aligned}
& \sum_{e=1}^n \iint_{S_e} \left\{ (\mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T) (\mathbf{b}_z^{en} + \mathbf{b}_z^{en+1}) + 4\Delta e^2 P'' \mathbf{N}_e \mathbf{N}_e^T (\mathbf{b}_z^{en} - \mathbf{b}_z^{en+1}) \right. \\
& \quad \left. + 2\Delta e Q'' \mathbf{N}_e \mathbf{c}_e^T (\mathbf{v}_z^{en} + \mathbf{v}_z^{en+1}) \right\} dydz = 0.
\end{aligned} \tag{5-2-6}$$

The new integral present in Eq. (5-2-6) can be calculated by using the theorem for an area integral of triangular elements (Eq. (4-2-17)) as follows;

$$\iint_{S_e} \mathbf{N}_e \mathbf{N}_e^T dydz = \frac{\Delta_e}{12} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \equiv \frac{\Delta_e}{12} \mathbf{L}_e. \tag{5-2-7}$$

Eqs. (4-2-18), (4-2-19), and (5-2-7) enabled us to calculate all the area integrals in Eq. (5-2-5) and we obtained the form corresponding to Eq. (4-2-20) as;

$$\sum_{e=1}^n (\mathbf{K}_e + P''' \mathbf{L}_e) \mathbf{b}_z^{e^{n+1}} = \sum_{e=1}^n \{(-\mathbf{K}_e + P''' \mathbf{L}_e) \mathbf{b}_z^{e^n} + Q''' \mathbf{1} \mathbf{c}_e^T (\mathbf{v}_z^{e^n} + \mathbf{v}_z^{e^{n+1}})\}, \quad (5-2-8)$$

where $\mathbf{K}_e = \mathbf{b}_e \mathbf{b}_e^T + \mathbf{c}_e \mathbf{c}_e^T$, $P''' = 2\Delta_e^2 \mu u \sigma / 3\Delta t$, and $Q''' = 2\Delta_e \mu \sigma F_z / 3$. From Eq. (5-2-8),

we finally assembled simultaneous linear equations expressed as

$$\mathbf{A}'' \mathbf{b}_z^{n+1} = \mathbf{B}'(\mathbf{b}_z^n, \mathbf{v}_z^n, \mathbf{v}_z^{n+1}). \quad (5-2-9)$$

Here $\mathbf{B}'$ includes contributions of the magnetic field in the last time step and the source oceanic flows in the last and next time step. In Eq. (5-2-9), $\mathbf{A}''$ is symmetric because of the forms of $\mathbf{K}_e$ and $\mathbf{L}_e$ in Eq. (5-2-8). Hence, we obtained b_z distribution in the $n+1$ -th time step, by solving Eq. (5-2-9) with given $\mathbf{b}_z^n$, $\mathbf{v}_z^n$, and $\mathbf{v}_z^{n+1}$. In the next sub-section, we will describe our tsunami dynamo simulations for the 2011 Tohoku earthquake tsunami.

5.3 2-D tsunami dynamo simulation of the 2011 Tohoku tsunami

Here we describe our tsunami dynamo simulations in the time domain to reproduce the tsunami dynamo phenomenon at NWP driven by the 2011 Tohoku tsunami. Although many ODE simulations associated with the tides and steady flows have been made, there are few instances of those associated with tsunamis. To our knowledge, Manoj et al. (2010) is the only instance of conducting tsunami dynamo simulations. They investigated the possibility of detecting tsunami propagation by submarine electric cable measurements. However, their simulation is not able to reproduce small-scale phenomena and dispersive characteristics of tsunamis because of its coarse spatial resolution of $1^\circ \times 1^\circ$. This was caused by their application of a global MIDM frequency-domain code to tsunami dynamo phenomena. It is difficult to achieve high spatial resolution using global simulation code. Furthermore, simulations in the frequency domain cost a lot to reproduce transient tsunami properties. We can recognize Manoj et al.'s (2010) simulation as an example implying some limitations of

applying existing frequency-domain codes to reproduce tsunami dynamo phenomena.

Our time-domain simulation code is probably the first time-domain simulation code dedicated only to investigating TDE. By use of our new time-domain tsunami dynamo simulation code, we succeeded in reproducing the observed tsunami-induced magnetic fields. In the following, we describe the simulation set-up for the 2011 Tohoku earthquake tsunami in Section 5.3.1. Then we report the simulation results in Section 5.3.2.

5.3.1 Set-up for the 2011 Tohoku Tsunami

In order to explain the magnetic signals observed at NWP, we first reproduced the sea level changes at DART21401 and 21419. There are many tsunami source models for the 2011 Tohoku Tsunami inferred by tsunami waveform inversions (e.g., Fujii et al., 2011; Maeda et al., 2011; Satake et al., 2013). We assigned 4 sub-faults with a length of 180km in total and width of 50 km based on Maeda et al.'s (2011) sub-fault setting. In our simulations, the initial seawater elevation was assumed to be equal to the coseismic vertical displacement of the seafloor which was computed by the Okada's (1985) analytical formulas assuming a homogeneous half-space with $V_p = 5.8 \text{ km s}^{-1}$, $V_s = 3.2 \text{ km s}^{-1}$, and $\rho = 2.6 \times 10^3 \text{ kg m}^{-3}$ (Dziewonski and Anderson, 1981), while the initial velocity field was assumed to be nil everywhere.

For the 2-D numerical grid, 200 km wide bathymetry data were averaged and re-sampled at every 5km along a straight line drawn from the epicenter to NWP. Figure 5-1 shows the numerical mesh adopted to reproduce sea level changes and tsunami-induced EM signals in the northwest Pacific. In both hydrodynamic and EM calculations, the same mesh shown in Fig. 5-1 was used. It is noticeable that the actual bathymetry is expressed well by the

triangular mesh. Furthermore, the fine mesh around the seafloor and in the ocean enabled us to calculate accurate particle motions of the ocean flows and tsunami-induced electromagnetic (EM) fields at the same time.

5.3.2 Results

Figure 5-5 shows comparison results between observed and calculated time series of sea level changes and tsunami-induced magnetic fields. The simulation results were calculated for a homogeneous seawater conductivity of 4 S/m and a homogeneous resistivity of 100 ohm.m beneath the ocean. In both the hydrodynamic and tsunami-dynamo EM simulations, the same time step of $\Delta t = 5$ s was adopted. We adopted an initial sea level distribution calculated for slip lengths of each sub-fault of 1.1, 0.4, 5.3, and 6.6 m from the coast to trench side. The sea level changes at the two DART sites were well reproduced by our 2-D hydrodynamic simulations, especially for the tsunami arrival times and shapes of the first waves. As for the EM components, the observed magnetic field was also well reproduced. The presence of a negative peak in the vertical magnetic component as large as 3 nT, and an initial rise and the subsequent larger negative peak in the horizontal component as large as 1 nT and 3 nT, respectively, were confirmed. It is noticeable in Fig. 5-5 that the waveform of the predicted b_z on the seafloor is very similar to an upside-down waveform of the calculated sea level change. This feature will be discussed in detail in the next section.

Figure 5-6 shows a side view of predicted EM field distribution at the time of the 2011 Tohoku tsunami. The figure features a snapshot of an instant at approximately 104 minutes after the earthquake occurrence. It is evident that an initial rise in b_y , which can be observed only on the seafloor, had already passed over NWP prior to the first arrival of the tsunami.

We can see symmetry on the upper and lower side in the color maps of b_z and e_x , while the color of b_y is inverse on the upper and lower side. The symmetry was caused by the resistive (100 Ohm.m) substratum beneath the seafloor, while the inversed color of b_y was due to the electric current flowing in the ocean layer. It is also noticeable in Fig. 5-6 that the total electric field, $e_x + [v \times B]_x$, is weak only in the ocean layer. This indicates that the magnitude of the sum of e_x and $v \times B$ is smaller than that of e_x alone, because $v \times B$ is present only within the ocean layer. However, note that electric currents within the ocean layer dominantly induce the tsunami-induced magnetic field, because the conductivity of sea water (4 S/m) is much higher than that beneath the seafloor (100 Ohm.m). Then we can consider that the initial rise in b_y on the seafloor was due to the positive weak electric currents in front of the tsunami peak (indicated in Fig. 5-6).

Figure 5-7 illustrates how the initial rise in seafloor b_y is generated. The initial rise is caused by the induced electric field, e_x , which slightly precedes the inducing $v \times B$ field and points the direction opposite to $v \times B$. Although the induced e_x is weaker than the inducing $v \times B$, difference in phase between them makes e_x significant in front of $v \times B$, which causes an initial rise in seafloor b_y prior to b_z variations.

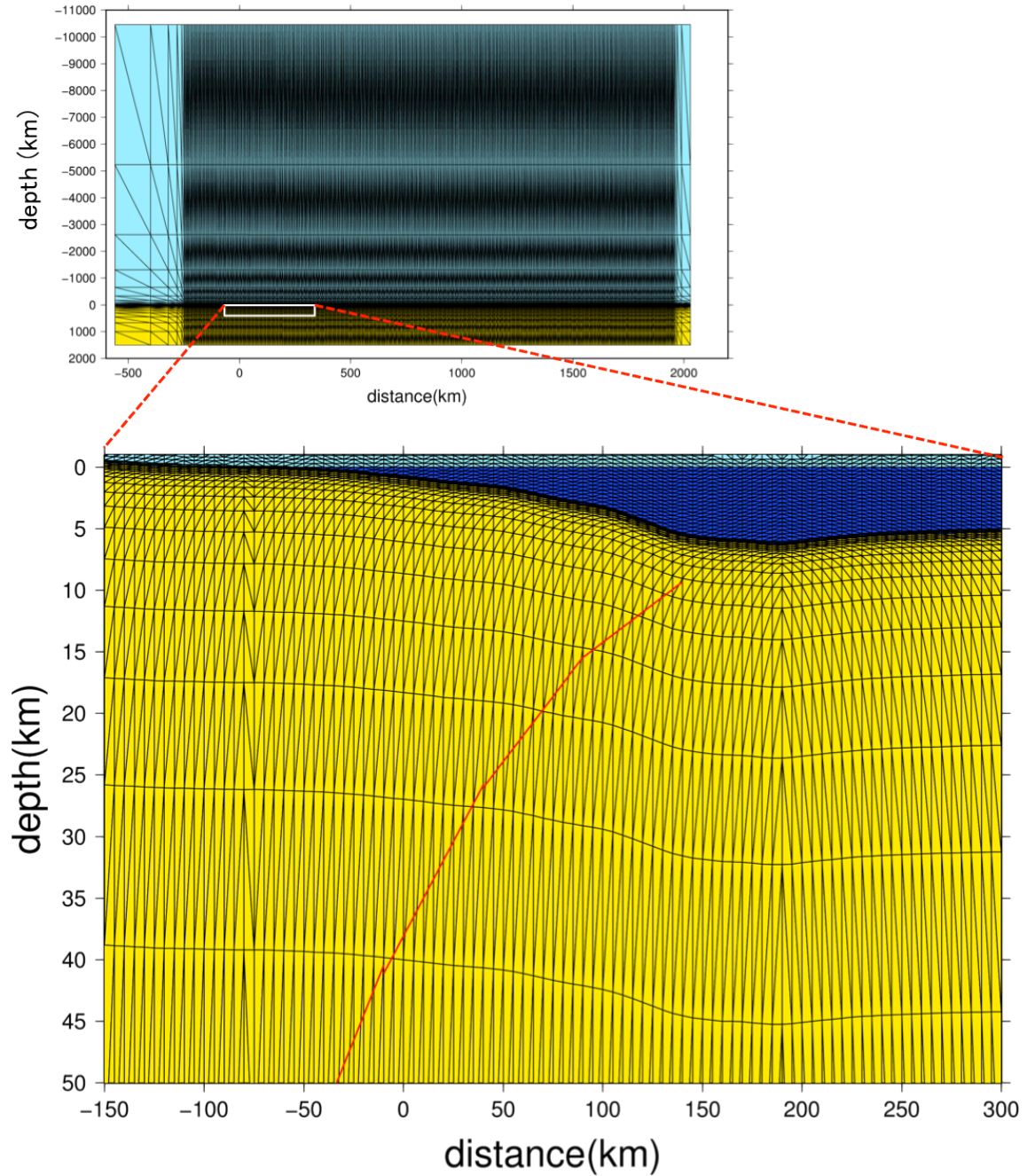


Fig. 5-1 The numerical mesh for 2-D FEM calculations using triangular elements. The light blue, blue and yellow regions indicate the air, the ocean, and the solid Earth beneath the seafloor, respectively. The top and bottom panels indicate the whole mesh used in the simulations and its zoom-in plot in the vicinity of the coastline, respectively. The red line in the bottom panel denotes the sub-faults based on Maeda et al. (2011) for calculations of the initial sea surface elevations.

Leap-frog method

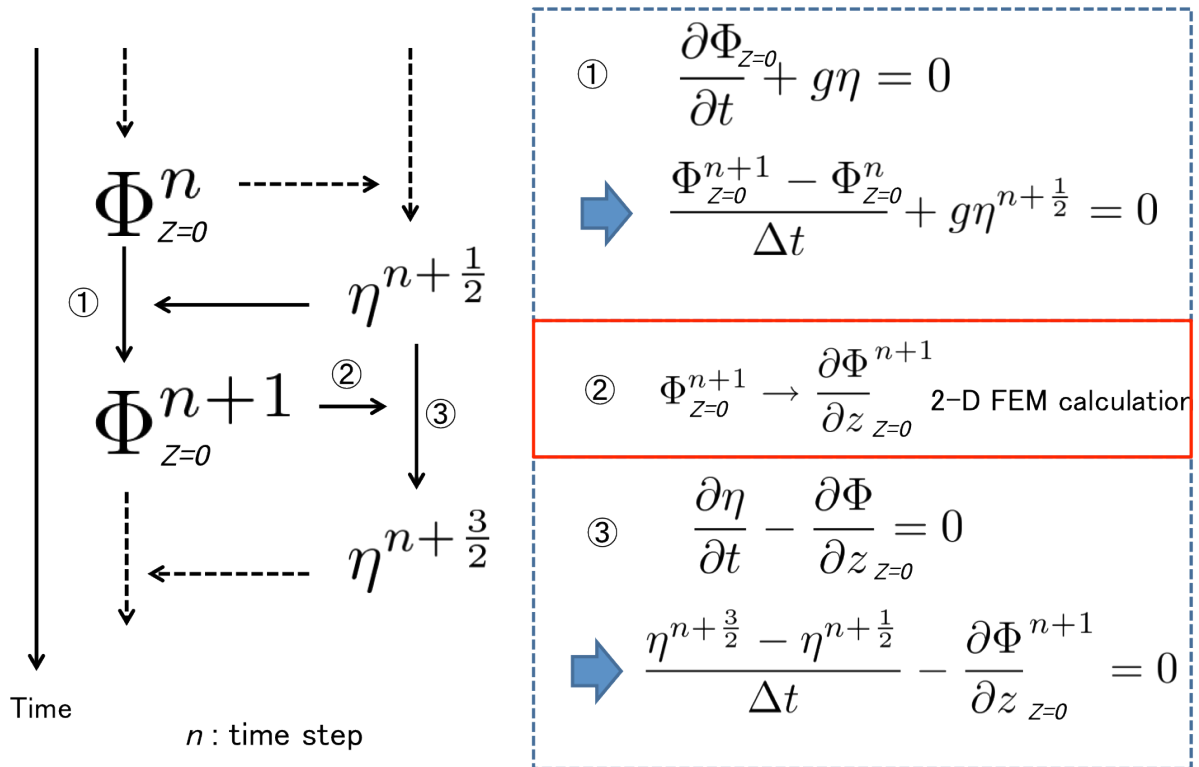


Fig. 5-2 The flow of the leap-frog method using 2-D FEM simulations. At the second step, $\Phi_{z=0}^{n+1}$ is used as a boundary condition at $z=0$ in order to calculate Φ distribution within the ocean. Li et al's (2008) method is adopted to calculate the partial derivative of Φ w.r.t. the vertical coordinate, z .

Li et al.'s (2008) extrapolation

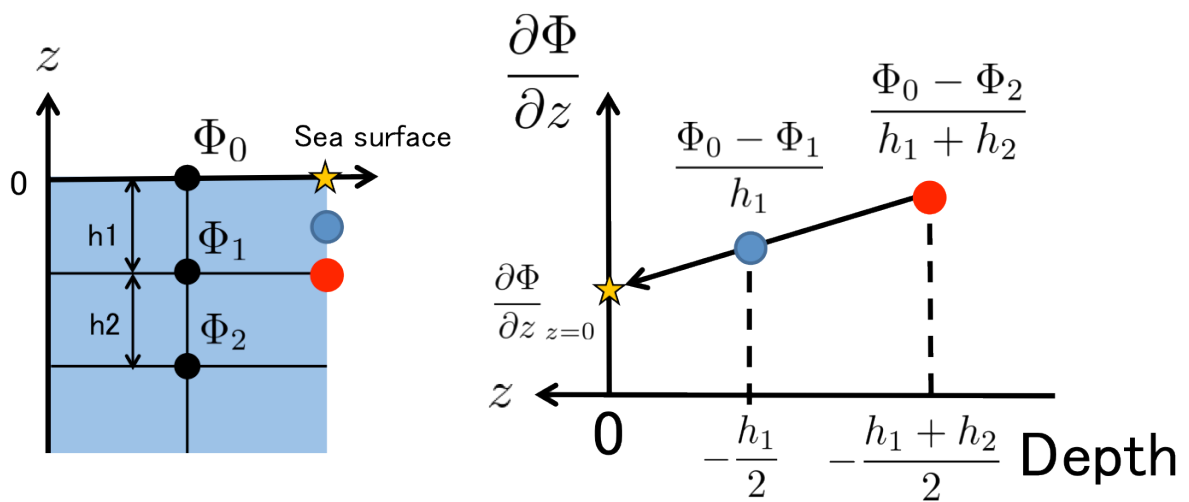


Fig. 5-3 A schematic figure of Li et al.'s (2008) extrapolation method. $\partial\Phi/\partial z$ at $z = -h_1/2$ and $-(h_1 + h_2)/2$ are first calculated using Φ at three nodes in a row. Then $\partial\Phi/\partial z$ at $z = 0$ is obtained by extrapolating upward using the calculated two values. In Li et al. (2008), this method is originally used to calculate the differentiation of the magnetic field with regard to z in TM mode.

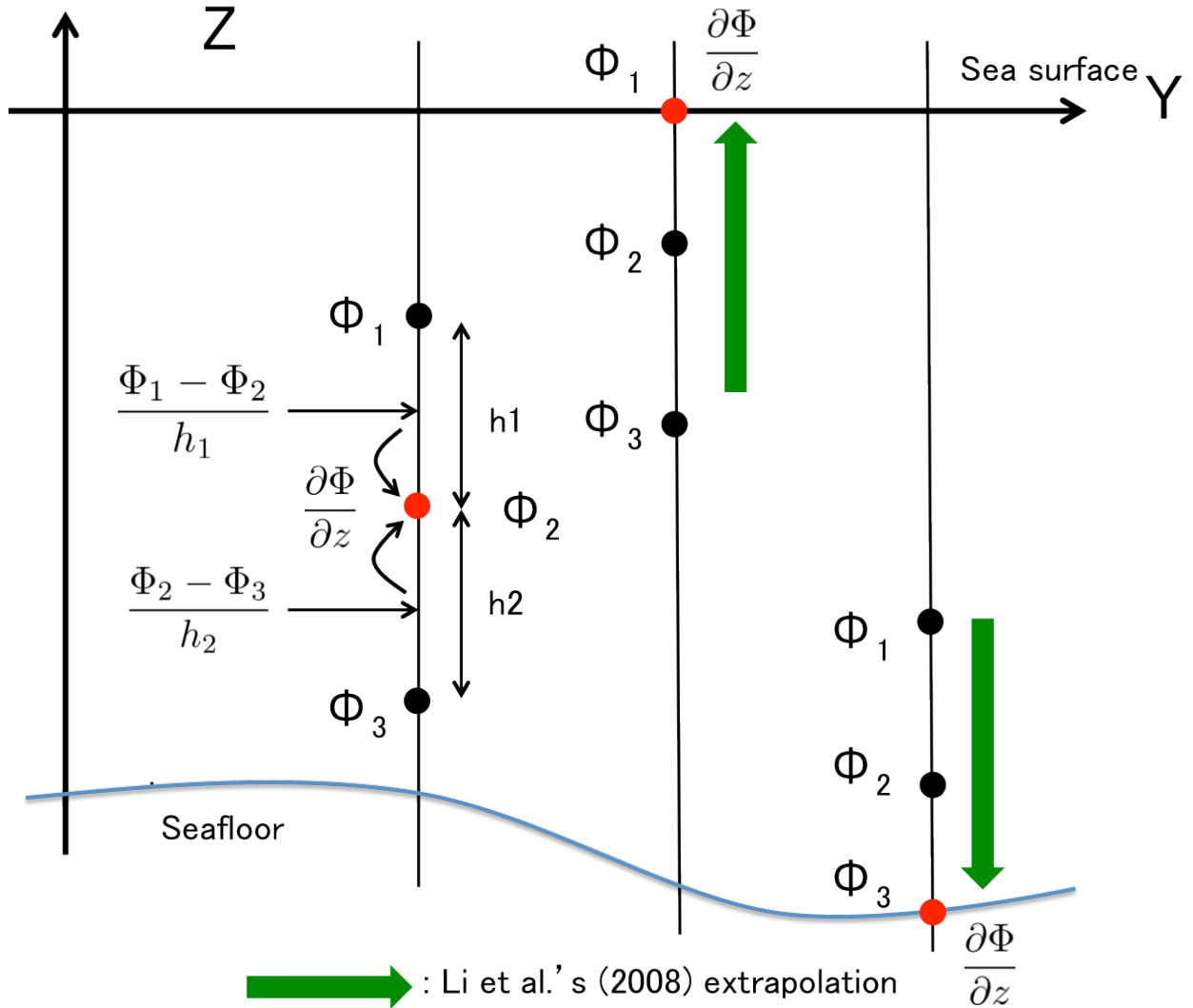


Fig. 5-4 A schematic figure of how to calculate $\partial\Phi/\partial z$ at nodes within the ocean. When $\partial\Phi/\partial z$ is necessary at the sea surface or seafloor (see the upward and downward arrows), we can calculate them by using Li et al.'s (2008) method as shown in Fig. 5-3. For the other nodes within the ocean, we first calculate the two derivatives, $(\Phi_1 - \Phi_2)/h_1$ and $(\Phi_2 - \Phi_3)/h_2$, and obtain $\partial\Phi/\partial z$ at the node of interest by interpolating from the two values. In any case, three nodes in a column are used to calculate $\partial\Phi/\partial z$ at desired nodes.

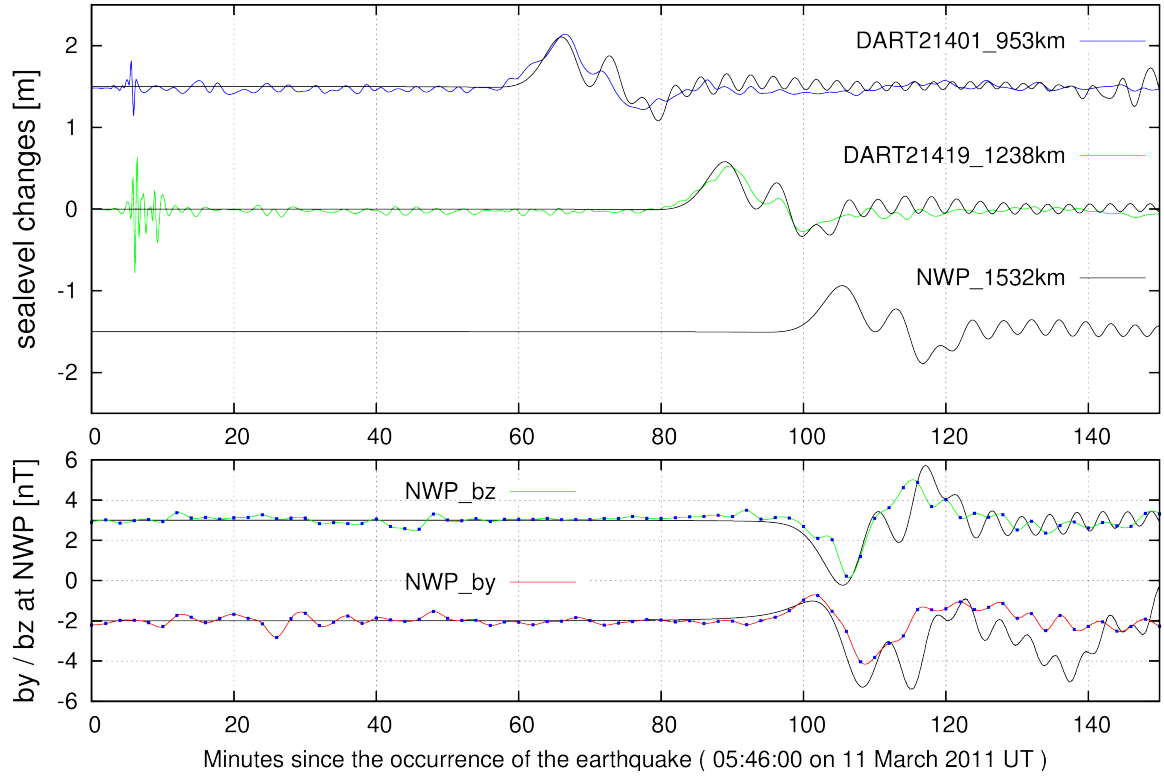


Fig. 5-5 Comparison of the simulation results with the observed sea level changes and magnetic variations. The top panel shows sea level changes at DART21401, 21419 and NWP, while the bottom panel shows the vertical and horizontal component of the magnetic field at NWP. In the bottom panel, b_y corresponds to the horizontal component parallel to the tsunami propagation direction, while b_z is the upward component. Here, effects of external sources have already been subtracted from b_y (see Section 3.2). In both panels, colored and black lines denote observed and simulated time series, respectively. In the top panel, distances written next to the station names are the epicentral distances of each station.

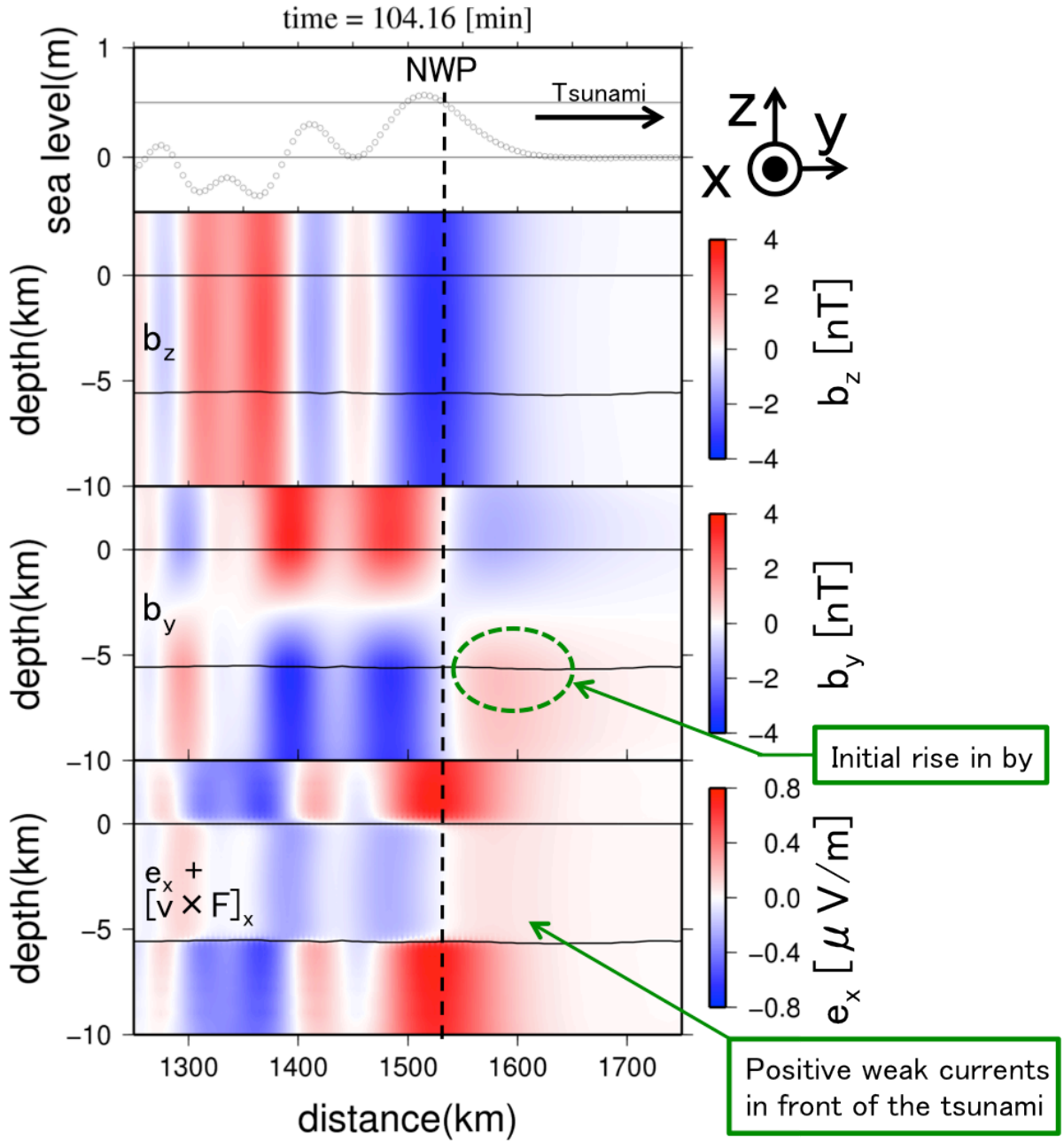


Fig. 5-6 Side view of the calculated tsunami-induced EM fields in TE mode when the 3.11 tsunami passed over the NWP site. The red and blue colors denote positive and negative values in each component, respectively. In the lowest panel, the sum of the inducing $v \times B$ field and the induced electric field, e_x , is shown. The vertical dashed line indicates the location of NWP. Green dotted line indicates the initial rise in b_y .

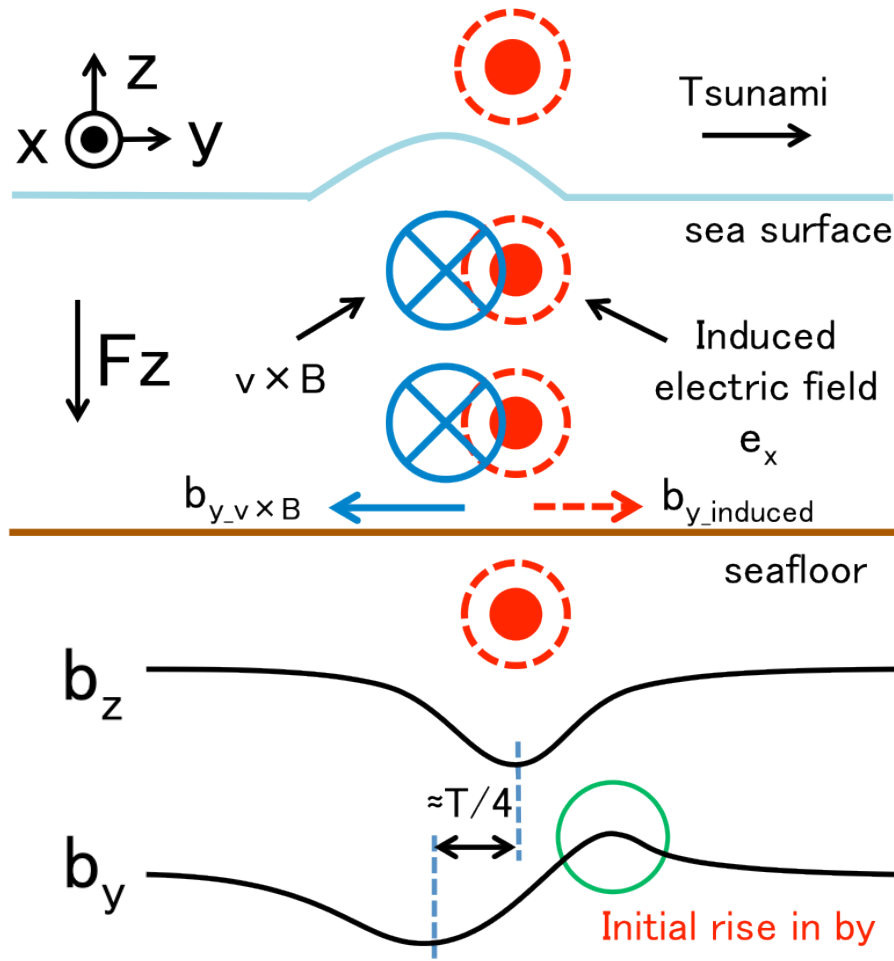


Fig. 5-7 Generation mechanism of an initial rise in seafloor b_y prior to the peak of b_z . The emf itself (the blue circles with crosses inside) is driven by coupling of horizontal tsunami flows with the vertical component of the geomagnetic main field. Tsunami-induced b_z and e_x slightly precede the sea level change in deep oceans due to self-induction. Because e_x is opposite to $v \times B$, the slight phase lead of e_x to $v \times B$ causes the initial rise in seafloor b_y preceding the peak of the sea level change. Difference in phase between the peak of b_z and the secondary peak of b_y becomes approximately $T/4$ due to the electric current consisting of e_x and $v \times B$.

6. Discussion

We derived theories associated with TDE in Section 2, and investigated typical three TDE phenomena in Section 4. In the previous section, we reproduced the tsunami-induced magnetic signals observed at NWP by using time-domain tsunami dynamo simulations in Section 5. Here we first interpret EM fields induced by the 2011 Tohoku earthquake tsunami, by comparing the simulation results with the TDE theories and additional simulations using solitary waves, in Section 6.1. Second, we explain an estimation of errors due to the Colioris force in our 2-D simulations, in Section 6.2. Finally, we analytically investigate where is suitable for detecting large EM tsunami signals, using the analytical solutions and additional simulations of solitary waves, in Section 6.3.

6.1 EM fields induced by the 2011 Tohoku tsunami

In Section 5, we succeeded in well reproducing the tsunami-induced magnetic signals observed at NWP at the time of 2011 Tohoku tsunami. For further investigation of the effect of the first arrival of the 2011 Tohoku tsunami, we conducted additional simulations associated solitary waves using the method described in Section 4. Figure 6-1 shows the vertical and horizontal magnetic components (b_z and b_y) and a sum of the inducing $\mathbf{v} \times \mathbf{B}$ field and the induced horizontal electric field (e_x) when the solitary wave passes. We found that the color distribution in Fig. 6-1 was very similar to colors near the first wave in the Fig. 5-6, which implies that we can interpret the effect of the first arrival of the 2011 Tohoku tsunami by using the solitary wave simulation results. In Fig. 6-1, the profile of b_z on the seafloor is very similar to an upside-down wave-form of the solitary wave, although b_z is not symmetric due to the balance of the self-induction and diffusion. This similarity also appears in Tyler's

(2005) frequency-domain solution. Namely, the relation between the sea level change, η , and the vertical magnetic field, b_z , can be expressed as;

$$b_z \approx \frac{\eta}{h} F_z e^{k(z+h)}, \quad (z < -h) \quad (6-1-1)$$

in deep oceans. Here, z is upward positive and $z = -h$ corresponds to the flat seafloor. This equation indicates that b_z and η vary almost in-phase as if the vertical magnetic field were frozen into the tsunami flow. As for the horizontal component of the induced magnetic field (b_y) and the total electric field ($e_x + v \times B$), it looks difficult to interpret them by the analytical solution in the frequency domain, because Tyler's (2005) solution requires the same amplitude for b_z and b_y .

Furthermore, it is noticeable in Figs. 5-6 and 6-1 that b_y initially starts to rise prior to the arrival of the tsunami wave and the b_z variation. As shown in Fig. 6-1, the initial rise in seafloor b_y is caused by the induced electric field, e_x , which slightly precedes the inducing $v \times B$ field and points the direction opposite to $v \times B$. In the bottom panels of Fig. 6-1, the preceding phase in e_x is clearly seen above and beneath the ocean layer thanks to the absence of the inducing $v \times B$ field. Although the induced e_x is weaker than the inducing $v \times B$, difference in phase between them makes a positive x -component current in front of $v \times B$, which causes an initial rise in seafloor b_y prior to the b_z variations.

The results shown in Fig. 5-6 and 6-1 are explained as the “*Self-induction dominant case*”, which was described in Section 4.3. When the ocean depth is shallow enough, the downward component of the tsunami-induced magnetic field precedes the sea level change in phase by almost 90 degrees, because of the negligible self-induction. However, in the case of NWP, the deep ocean allows the self-induction to exert a dominant influence on the magnetic field, which causes a frozen-flux regime, where the downward component of the magnetic

field, b_z , almost synchronize in phase with the sea level change. We can understand why the induced electric field, e_x , in the opposite direction to $\mathbf{v} \times \mathbf{F}$ slightly precedes the $\mathbf{v} \times \mathbf{F}$ itself in phase, by considering contributions of the self-induction. The induced electric field (e_x) always precedes $\mathbf{v} \times \mathbf{F}$, in order to generate a phase delay of the total electric currents in the ocean, because e_x is generated by the Faraday's law of induction. We can confirm in Fig. 4-5 that the self-induction increases a phase lag of the total electric current within the ocean, as the self-induction becomes large due to the increasing ocean depth. In the “*Self-dominant case*”, the slightly preceding e_x indicates the dominant influence of the self-induction. It is important to understand how the phase and amplitude of e_x varies with the ocean depth and the conductivity beneath the seafloor, because they are directly related to the initial rise in b_y . Further studies on this initial rise may enable us to detect tsunami arrivals by seafloor magnetic observations before actual arrivals of tsunami peaks.

Figure 6-2 shows the dependence of the TDE due to solitary waves on the sub-seafloor conductivity. As for the main peak of b_z and b_y , the negative peak of b_z gets smaller, while the positive b_y peak becomes larger, as the sub-seafloor conducting increases. This is attributed to the effect of the increasing electric current beneath the seafloor. It is evident in the bottom panel of Fig. 6-2 that the electric current beneath the seafloor, $\sigma_1(e_x + [\mathbf{v} \times \mathbf{F}]_x)$, prevents the downward magnetic field from penetrating into the seafloor, and reinforces the horizontal magnetic component on the seafloor, as σ_1 increases. We further found in Fig. 6-2 that when we assigned 0, 0.01, 0.1, and 1S/m to the conductivity beneath the seafloor, the peak of the initial rise in b_y became approximately 3, 2, <1, and 0 nT, respectively. The initial rise, therefore, vanished with the conductivity of 1S/m beneath the seafloor, because the induced current beneath the seafloor generates a horizontal magnetic field opposite to the initial rise.

Our numerical results implies that the initial rise may enable us to detect the tsunami passage prior to the tsunami arrival itself only with resistive sub-seafloor situations.

Consequently, we can claim that the tsunami-induced magnetic field variation, especially the ratio of b_z and b_y , significantly depends on the conductivity structure beneath the seafloor. When we extract tsunami properties from EM observation data, we should be careful about how accurate the given conductivity structures are. On the other hand, it may be possible to infer the actual conductivity structure using the amplitude ratio of b_z to b_y because the peak amplitude of b_z and b_y have opposite sense to the conductivity beneath the seafloor.

6.2 Effects of the Coriolis force on the 2011 Tohoku tsunami

In our 2-D simulations, we could not include effects of the Coriolis force because the Coriolis force points to the strike direction. Here, we estimate the error due to the Coriolis force in the tsunami dynamics. The governing momentum equation in terms of the particle motion including the Coriolis force is expressed as,

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla_H) \mathbf{v} = -g \nabla_H \eta - 2\mathbf{\Omega} \times \mathbf{v}, \quad (6-2-1)$$

where $\mathbf{v}$, η , and $\mathbf{\Omega}$ are the depth averaged horizontal velocity, the sea surface elevation, and the angular velocity of the Earth's rotation, respectively. ∂_t and ∇_H are the derivatives in terms of time and the horizontal coordinate, respectively. Since we neglected the 2nd term on the r.h.s. of Eq. (6-2-1) in our simulations, here we compare the magnitudes of those terms quantitatively. Amplitudes of the first and second term on the r.h.s of Eq. (6-2-1) are estimated as follows:

$$|g \nabla_H \eta| = gA/\lambda = 5 \times 10^{-5}, \quad (6-2-3)$$

$$|2\boldsymbol{\Omega} \times \mathbf{v}| = 2\Omega \sin\theta \cdot v = 4 \times 10^{-6}. \quad (6-2-4)$$

Here g , A , λ , θ , and v are the gravity acceleration, the wave height, the wavelength, the latitude, and the particle velocity, respectively. We adopted $g = 9.8 \text{ m/s}^2$, $A = 1 \text{ m}$, $\lambda = 200 \text{ km}$, $\theta = 45^\circ$, and $v = 4 \text{ cm/s}$, assuming the 2011 off the Tohoku earthquake tsunami passing through the northwest Pacific. Equation (6-2-3) and (6-2-4) show that the effects of the Coriolis force is less than 10 % of that of the pressure gradient. This order estimation is consistent with other preceding numerical studies (e.g., Dao and Tkalich, 2007). Furthermore, it is reported that the Coriolis force significantly influences the height of the tsunami rather than the arrival time (Shuto, 1991). We, therefore, can say that there are at most 10 % errors in the tsunami height due to the Coriolis force in our simulations.

Although our simulation results successfully reproduced the observed sea level changes and the tsunami-induced magnetic field simultaneously, it is difficult to constrain the source model of the 2011 Tohoku tsunami. Tsunami phenomena and their source models are always three-dimensional (3-D) in practice, because realistic fault geometry, bathymetry and the electrical conductivity structure in the Earth are always 3-D. It was found difficult in our 2-D simulations to reproduce later tsunami phases both in the hydrodynamic and in EM fields, which implies that we should include contribution of 3-D bathymetry and coastlines.

6.3 Where is suitable for detecting tsunami EM signals?

Although we succeeded in detecting tsunami magnetic signals as large as 3 nT at NWP at the time of 2011 Tohoku tsunami, it is also important to predict how large signals we could obtain in other places. We recognized the tsunami dynamo phenomenon at NWP as the “*Self-induction dominant case*” in Section 6.1. However, we found in Section 4 that

tsunami-induced magnetic signals are totally different among the three typical cases, “*Diffusion dominant case*”, “*Intermediate case*”, and “*Self-induction dominant case*”. Here we discuss where is suitable place for detecting large EM tsunami signals in terms of the ocean depth.

The three cases are determined by the balance between the diffusion and self-induction term in the induction equation in terms of the magnetic field. For a simple measure of contributions of the diffusion and self-induction, we refer to the Tyler’s (2005) relation between the sea level change and the vertical component of the magnetic field,

$$\frac{b_z}{F_z} = \frac{c}{c + ic_d} \frac{\eta}{h} \quad (\text{at the seafloor or sea surface}). \quad (6-3-1)$$

Here $c = \sqrt{gh}$ is the phase velocity of the long-wave tsunami and $c_d = 2/\mu\sigma h$ is the lateral diffusion velocity defined by Tyler (2005). Equation (6-3-1) is independent of the tsunami period, because Tyler (2005) assumed the wavelength of the tsunami and the skin depth in the ocean are much larger than the ocean depth, namely $kh \ll 1$ and $h/\delta \ll 1$. From Eq. (6-3-1), we define the phase delay of the magnetic field relative to that in “*Diffusion dominant case*” as:

$$\theta = \arctan(c/c_d) \text{ [deg]}. \quad (6-3-2)$$

The phase delay, θ , directly represents what extent the self-induction delays the magnetic fields induced by the source term, $v \times F$. In the “*Diffusion dominant case*”, $\theta \approx 0^\circ$ because $c_d \gg c$ due to shallow oceans, which clearly means absence of the self-induction. On the other hand, in the “*Self-induction dominant case*”, $\theta \approx 90^\circ$ because $c_d \ll c$ due to large ocean depths, which indicates absence of the diffusion term. Figure 6-3 shows how θ varies with the ocean depth. In Fig. 6-3, phase delays calculated from the solution for linear dispersive waves, Eq. (2-1-32), are also plotted, in addition to those calculated by Eq. (6-3-2).

The results calculated using linear-long wave (Eq. (2-1-10)) was omitted in Fig 6-3, because they were almost the same as the results for linear dispersive waves. It was found that the ocean depth has much more influence on the phase delays than the tsunami period and the conductivity beneath the seafloor. Only for ocean depths ranging from 1 km to 5km, mutual induction due to conductive substrata is capable of changing the phase of the tsunami-induced magnetic field by a few degrees. From Fig. 6-3, we can roughly classify the three particular cases in terms of the ocean depth as shown in Table 6-1, by defining the *Diffusion* and *Self-induction dominant cases* as phenomena whose θ ranges $0^\circ < \theta < 20^\circ$ and $70^\circ < \theta < 90^\circ$, respectively. When the ocean depth is smaller than 1km, we can regard the phenomena as “*Diffusion dominant case*”, because the phase delay, θ , is less than 20 degrees. In the same manner, phenomena with the ocean depth longer than 5 km, in which θ is larger than 70 deg, corresponds to the “*Self-induction dominant case*”. The case with remaining ocean depths ranging from 1km to 5km is regarded as the “*Intermediate case*”. In this context, we find in Fig. 6-3 that the mutual induction is important only in the “*Intermediate case*”.

Now, let us consider cases with the tsunami height fixed. Figure 6-4 shows the tsunami-induced EM field calculated with a steady tsunami height of 1m, where the other parameters including ocean depths are the same as Fig. 4-4. As the ocean depth increases, the volume integral of the inducing electric current in the ocean, $\int \int \sigma(\mathbf{v} \times \mathbf{F}) dy dz$, increases, because it is proportional to the ocean depth. As a result, amplitudes of EM fields in Fig. 6-4 are different from those in Fig. 4-4, while the phase delays are the same between the two figures. It is noticeable in Fig. 6-4 that the amplitude of the observable magnetic field on the seafloor, $|b_{z_total}|$, is largest in the case with the intermediate ocean depth of 2.5km, although

the vertical magnetic field induced by $\mathbf{v} \times \mathbf{F}$ increases as the ocean becomes deeper. This is because, in the case with the largest ocean depth of 5km, the effect of the self-induction overcomes the increasing source current in the ocean. Figure 6-5 shows how $|b_{z_total}|$ varies with the ocean depth, when the tsunami height is fixed to 1m. All profiles in Fig. 6-5 were calculated using the same analytical solutions and parameter settings as Fig. 6-3. We can understand the results shown in Fig. 6-5 in terms of the self-induction. In the “*Diffusion dominant case*”, up to the ocean depth of 1km, $|b_{z_total}|$ increases proportional to the ocean depth, because the almost negligible self-induction allows the increasing electric currents in the ocean to reinforce $|b_{z_total}|$. However, in the “*Intermediate case*”, in which h exceeds 1km, the self-induction has an influence on $|b_{z_total}|$ due to the intermediate ocean depths. Then, around $h = 2\text{km}$, the enhanced self-induction due to increasing ocean depth balances with that of the source current within the ocean, where $|b_{z_total}|$ has a peak. When h exceeds 2km, the self-induction has more influence than the increasing oceanic source current, continuously attenuating $|b_{z_total}|$ as the ocean depth increases. Since actual tsunami heights increase as the ocean depth decreases, it may be possible that tsunami magnetic signals are largest where the ocean depth is less than 2 km.

It is also noticeable in Fig. 6-5 that discrepancies among the 7 colored lines are largest around $h = 2\text{km}$, which implies that tsunami magnetic signals are most sensitive to the tsunami period and the conductivity beneath the seafloor around $h = 2\text{km}$.

Based on the consideration above, we can conclude that tsunami magnetic signals in the “*Intermediate case*”, whose ocean depth ranges from 1km to 5km, is seemingly most suitable to extract information on tsunamis and conductivity structures beneath the seafloor. This is partly because amplitudes of the magnetic field in this case are larger than the other cases

when the tsunami-height is fixed. In the “*intermediate case*”, furthermore, the mutual induction and the tsunami period can cause large discrepancies in phases and amplitudes of b_z on the seafloor, viz., approximately 5 degrees in phase and almost 1nT in amplitude, as shown in Figs. 6-3 and 6-5, respectively. Although tsunami EM signals in shallow oceans are subject to more noise originated from external sources than deep oceans, large observable amplitudes of tsunami-induced magnetic fields may possibly allow us to identify effects of conductivities beneath the seafloor and the tsunami period. Although we succeeded in acquiring the magnetic tsunami signals as large as 3 nT at the time of the 2011 Tohoku earthquake tsunami, we could have detected larger signals, if we had deployed our SFEMS in shallower ocean. This is because the ocean depth at NWP is as deep as 5616m, where the self-induction significantly attenuates magnetic tsunami signals.

We have found that if the tsunami height is constant, the tsunami magnetic signal becomes largest around $h = 2\text{km}$ (Fig. 6-5). However, we have to note that actual tsunami heights are not spatially constant and its spatial distribution and time evolutions differ depending on the tsunami events. The actual sea level distribution due to tsunamis can be obtained only by conducting 3-D tsunami simulations, because 3-D complicated source regions and 3-D bathymetry strongly affect actual tsunami propagations. In order to predict amplitudes of tsunami-induced EM fields accurately, it is necessary to conduct 3-D hydrodynamic simulations, assuming some typical tsunami source regions and processes.

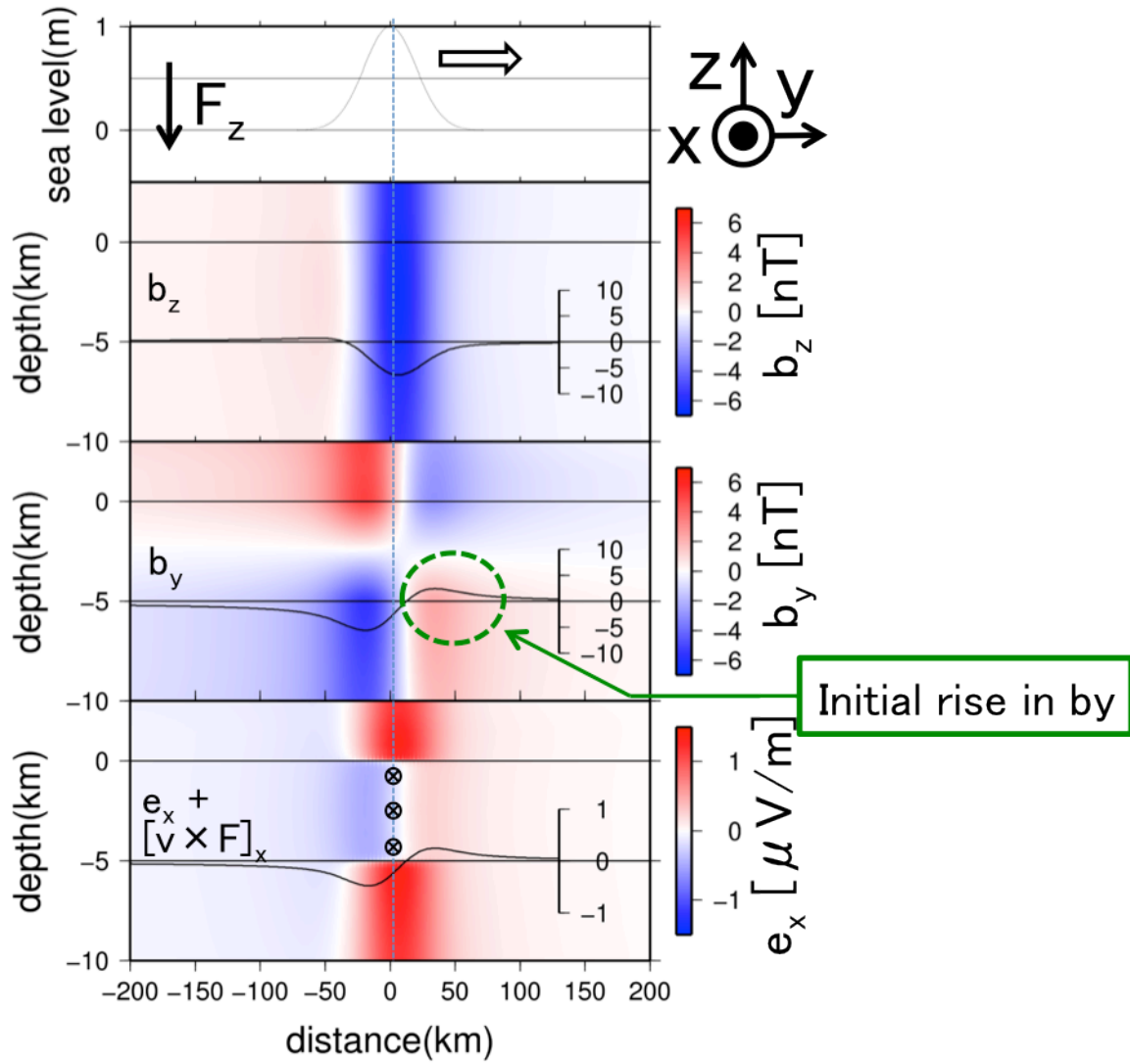


Fig. 6-1 Magnetic and electric fields induced by a rightward propagating solitary tsunami, which has a Gaussian waveform. In the lower three panels, the sea surface and seafloor are depicted by horizontal black lines at depths of 0 km and -5 km, respectively. The red and blue colors denote positive and negative values of each component, respectively. Black solid lines in each panel indicate EM component on the seafloor. In the lowest panel, the sum of the inducing $\mathbf{v} \times \mathbf{B}$ field and the induced electric field, e_x , is shown. The vertical blue dashed line corresponds to the center of the symmetric waveform, where the center of the inducing $\mathbf{v} \times \mathbf{B}$ field resides as drawn in the lowest panel.

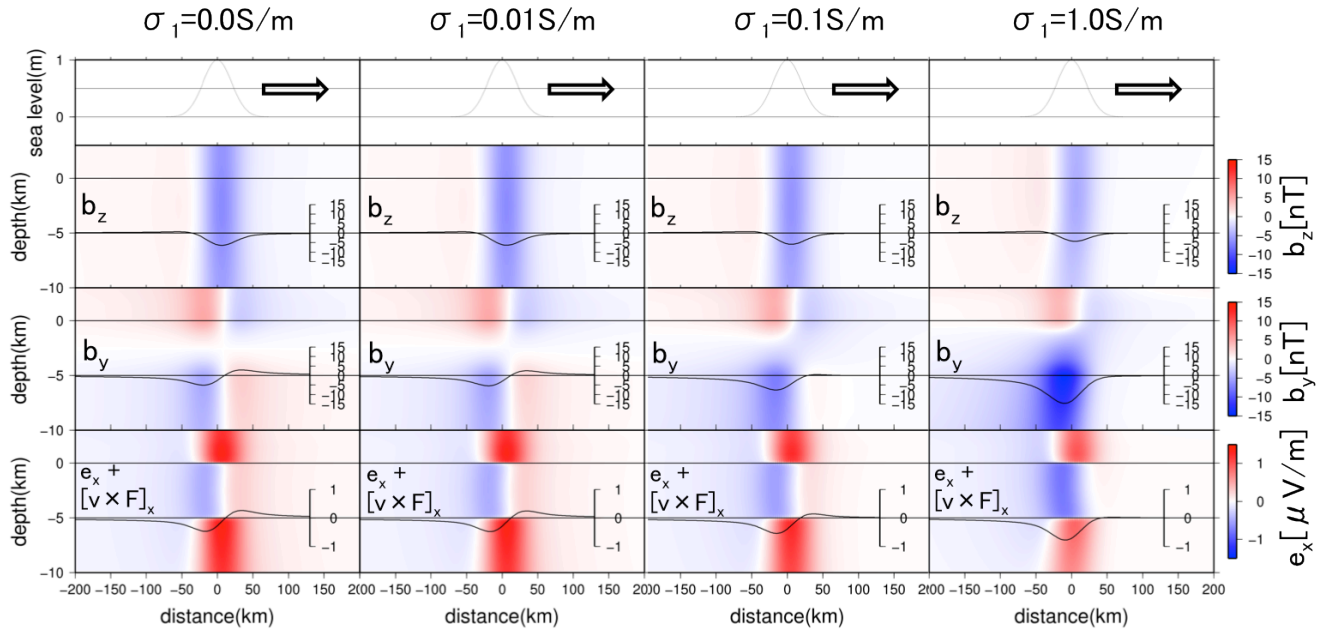


Fig. 6-2 EM variations induced by a solitary wave with varying conductivity beneath the seafloor. Each panel is depicted in the same manner as Fig. 5-6. From left to right, EM variations were calculated assigning 0, 0.01, 0.1, and 1 S/m to the half-space conductivity beneath the seafloor. It is noticeable that initial rise in b_y vanished if the conductivity became 0.1 S/m, while the magnitude of b_y itself variation got larger.

Diffusion dominant case

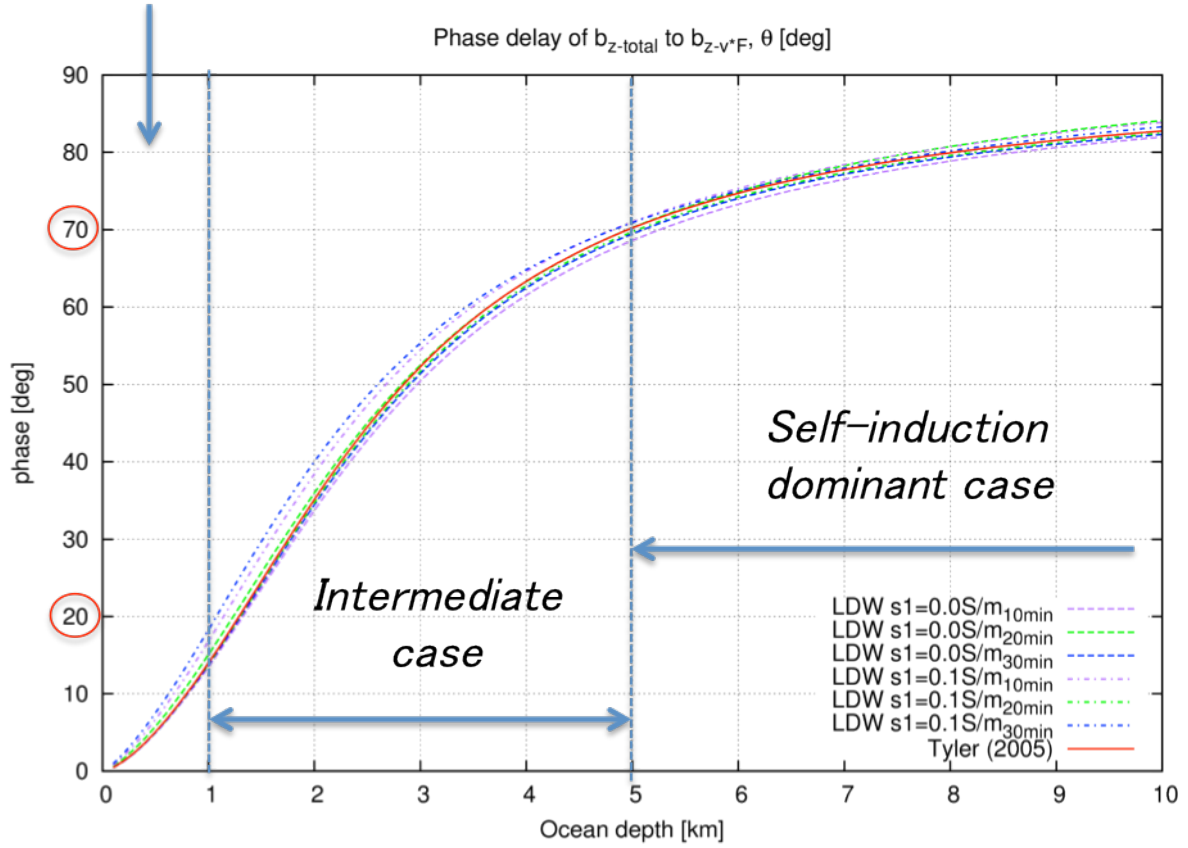


Fig. 6-3 The phase delay of the total magnetic field on the seafloor, b_{z_total} , against $b_{z_v \times F}$. Red line is calculated by Tyler's (2005) linear-long-wave solution (Eq. (4-3-5)), which is independent of the tsunami period. Tyler (2005) assumed the ocean layer is sandwiched between insulators above and underneath. The other lines are calculated by Eq. (2-1-32), adopting linear dispersive waves (LDW). As for dispersive waves, the phase delays are calculated for two sub-seafloor conductivities of 0.0 and 0.1 S/m, and three periods of 10, 20, and 30 min. When the ocean is shallow, phase discrepancy between $b_{z_v \times F}$ and b_{z_total} is small, because the diffusion term is dominant (*Diffusion dominant case*). As the ocean gets deeper, phase delay of b_{z_total} increases due to self-induction, which asymptotes to 90 degrees in deep oceans (*Self-induction dominant case*).

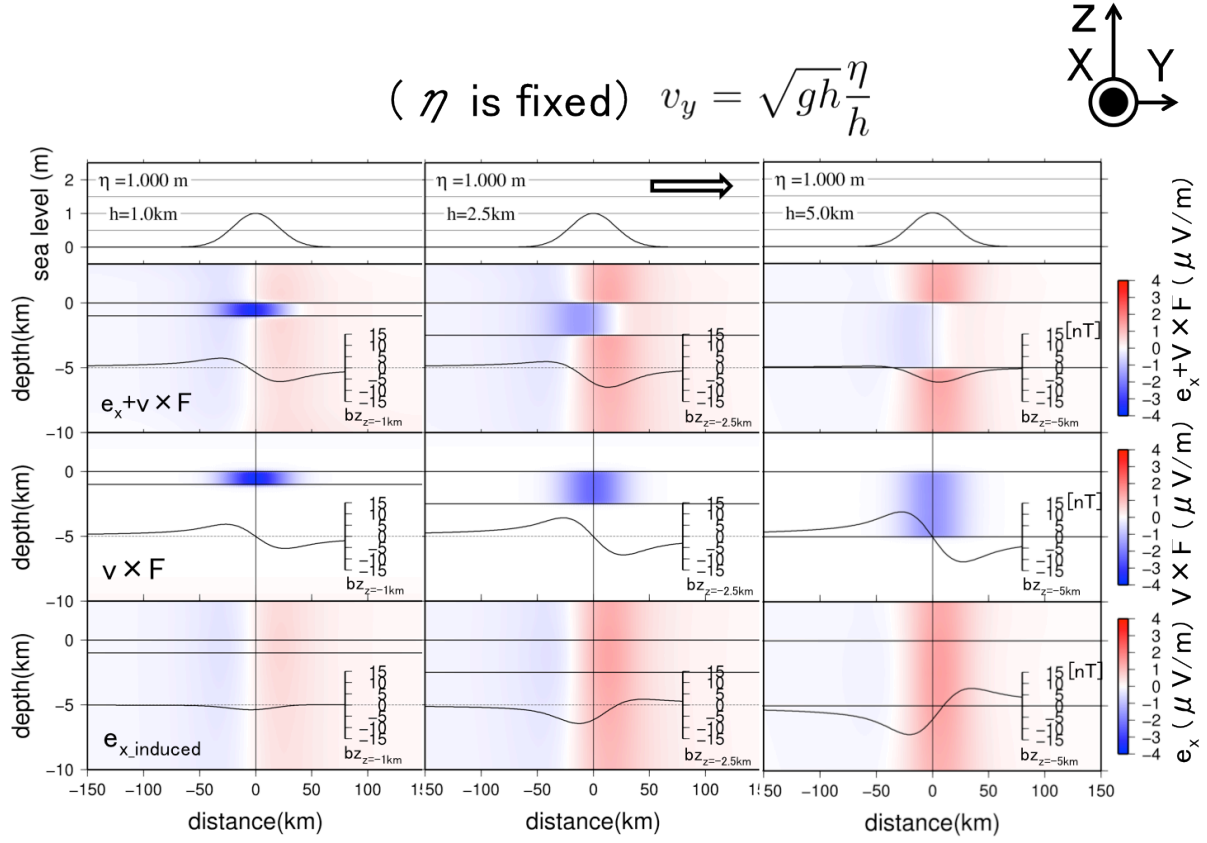


Fig. 6-4 Tsunami-induced EM fields with a constant tsunami height. Three panels from top to bottom are depicted in the same manner as Fig. 4-4 for the uniform tsunami height of 1 m. The ocean depths, however different from left to right. Note that the amplitude of $b_{z_v \times F}$ on the seafloor increases, as the ocean becomes deeper. This is because the volume of moving seawater within the ocean layer increases.

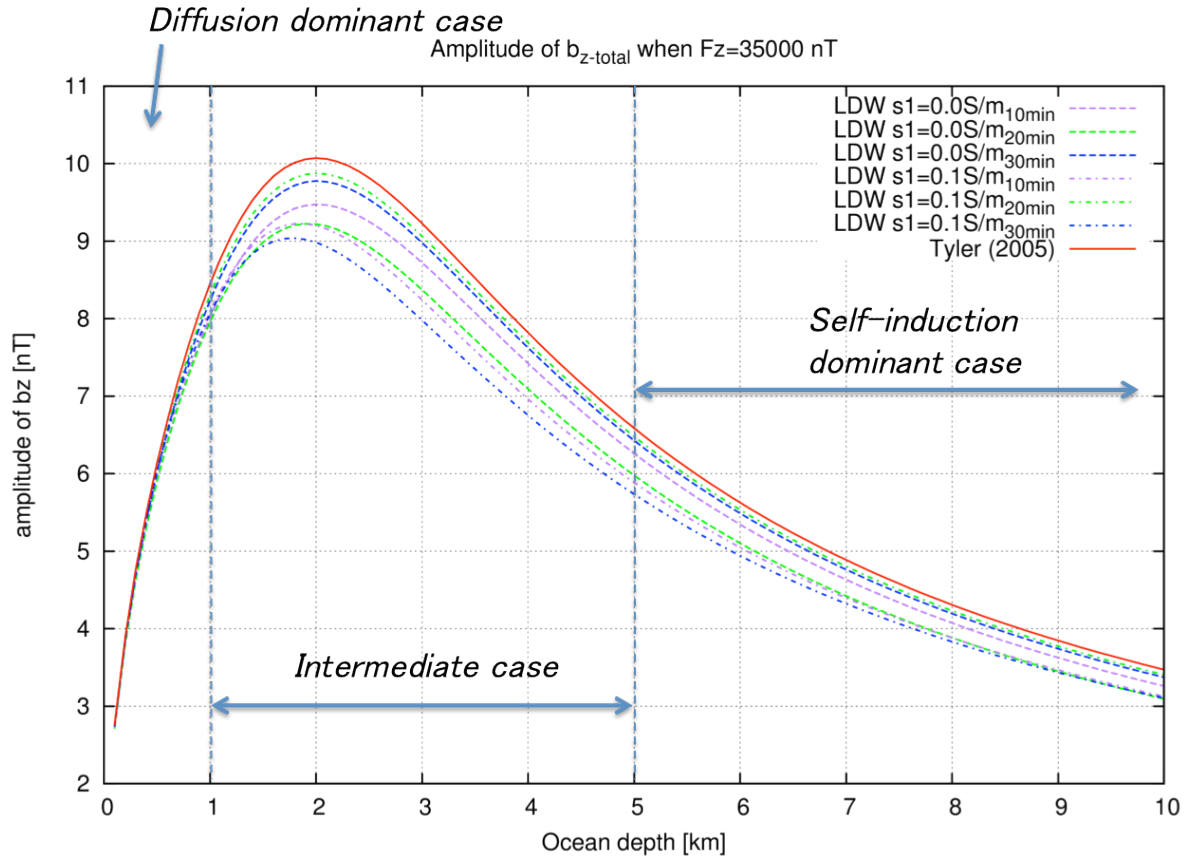


Fig. 6-5 The amplitudes of the vertical magnetic component on the seafloor for the different ocean depths. Red line is Tyler's (2005) linear-long-wave solution (Eq. (4-3-5)), which is independent of the tsunami period. The other lines are calculated by Eq. (2-1-32), adopting LDW. As for dispersive waves, the amplitudes are calculated for the same pairs of periods and sub-seafloor conductivities as Fig. 6-3.

Three particular cases of tsunami dynamo effects

	Diffusion dominant case	Intermediate case	Self-induction dominant case
Ocean depth, h	$< \sim 1$ km	~ 1 km $< h < \sim 5$ km	~ 5 km $< h$
Diffusion speed, c_d	$\gg c$	$\sim O(c)$	$\ll c$
Phase delay of b_{z_total} to $b_{z_v \times F}$, θ	$0 \sim 20$ [deg]	$20 \sim 70$ [deg]	$70 \sim 90$ [deg]
$ b_{z_total} / b_{z_v \times F} $	≈ 1	< 1	< 1
Diffusion, $ K\Delta\mathbf{b} $	$\approx \nabla \times (\mathbf{v} \times \mathbf{F}) $	Comparable with $ \partial\mathbf{b}/\partial t $	$\ll \partial\mathbf{b}/\partial t $
Self-induction, $ \partial\mathbf{b}/\partial t $	$\ll K\Delta\mathbf{b} $	Comparable with $ K\Delta\mathbf{b} $	$\approx \nabla \times (\mathbf{v} \times \mathbf{F}) $
$R_m = \nabla \times (\mathbf{v} \times \mathbf{F}) $ $/ K\Delta\mathbf{b} $	≈ 1	> 1	$\approx \infty$

Table 6-1 A summary chart of the three particular tsunami-dynamo phenomena. c and R_m are the phase velocity of tsunamis and the magnetic Reynolds number. The ocean depth classifies the three phenomena. As the ocean depth increases, the self-induction becomes more effective.

7. Conclusions

At the time of the gigantic tsunamis associated with the 2011 Tohoku earthquake, SFEMS observed vector magnetic signals as large as 3 nT on the northwest Pacific seafloor approximately 1500 km away from the epicenter. The horizontal magnetic variations appeared only in the anticipated tsunami propagation direction, which implies that even the single site EM observation on the seafloor enables us to extract vector properties of tsunamis.

We developed a 2-D FEM time-domain tsunami-dynamo simulation code in order to reproduce the sea level changes at DART21401 and 21419 and magnetic tsunami signals at NWP simultaneously. By adoption of FEM, our simulations can include realistic bathymetry and arbitrary conductivity structures beneath the seafloor.

Our 2-D tsunami-dynamo simulation well reproduced the profiles of the sea level changes observed at the two DART sites and the vector tsunami-induced magnetic field observed at NWP. It was found that the significant peaks both in the vertical and horizontal components as large as 3 nT, are very likely to be induced by the 2011 Tohoku tsunami. In addition, our simulation also revealed that an initial rise in the horizontal magnetic field as large as 1 nT was also induced by the 2011 Tohoku tsunami about 5 minutes prior to their passage. The generation of the initial rise in b_y observed at NWP can be understood by considering a simple solitary wave case. It, therefore, is possible that tsunami arrivals can be detected before its actual arrival, using the initial rise/retreat in the horizontal magnetic component corresponding to the rise/retreat waves, respectively. More detailed analysis of the initial rise may lead to significant improvement of the existing global tsunami early warning systems.

Our additional tsunami dynamo simulation associated with the EM field induced by a solitary tsunami revealed that the initial rise in b_y significantly depends on the conductivity

beneath the seafloor. The amplitude of the initial rise decreases as the sub-seafloor conductivity increases, eventually vanishing with the conductivity of 0.1 S/m. For applications of TDE to future tsunami early warnings, the first step we have to make is to know the conductivity structure beneath the seafloor precisely. On the other hand, it was also revealed by our simulations that the seafloor b_z and b_y vary in totally different ways w.r.t. the variable sub-seafloor conductivity, which implies there is a possibility of utilizing the motionally-induced magnetic field for inferring the conductivity structures beneath the ocean, even though it is necessary to know, in turn, accurate oceanic tsunami flows in the latter case.

We have classified the tsunami dynamo phenomena into three types: “*Diffusion dominant case*”, “*Intermediate case*”, and “*Self-induction dominant case*”. These three types were classified in terms of the ocean depth, because it has a dominant influence on the balance between the diffusion and self-induction. As the ocean depth increases, the self-induction increases. The tsunami dynamo phenomenon at NWP driven by the 2011 Tohoku tsunami was regarded as the “*Self-induction dominant case*”. In this case, the self-induction delays the phase of the magnetic field due to $\mathbf{v} \times \mathbf{F}$ by almost 90 degrees, so that the peak of the downward magnetic component coincides with the tsunami peak. We further analytically found that “*Intermediate case*”, whose ocean depth ranges from 1km to 5km, is most suitable to extract information of tsunami periods and conductivity beneath the seafloor from tsunami-induced EM variations. This is partly because the amplitude of the vertical magnetic component becomes largest at the ocean depth of approximately 2 km when the tsunami height is fixed, and partly because the tsunami period and the conductivity beneath the seafloor significantly varies the phase and amplitudes of the magnetic field rather than the other cases. Since the actual tsunami height increases as the ocean depth becomes shallower,

it may be possible that the largest tsunami magnetic signals are detected at the depth of less than 2 km. Although we have to conduct hydrodynamic tsunami simulations in order to predict the actual sea level distribution accurately, our classification of TDE in terms of provides preliminary insight into actual tsunami dynamo phenomena.

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List of Publications

1. Minami, T. and H. Toh (2013), Two-dimensional simulations of the tsunami dynamo effect using the finite element method, *Geophys. Res. Lett.*, 40, 4560-4564, doi:10.1002/grl.50823.
2. Minami, T. and H. Toh (2012), An improved forward modeling method for two-dimensional electromagnetic induction problems with bathymetry, *Earth and Planets Space*, 64, e9–e12, doi:10.5047/eps.2012.04.012.
3. Toh, H. and T. Minami (2012), Chapter 9 : Electromagnetic View of the Seismogenic Zones Beneath Island Arcs, in *Earthquake Research and Analysis – Seismology, Seismotectonic and Earthquake Geology*, edited by S. D’Amico, pp.183-198, InTech, Croatia.

This thesis is based on papers 1 and 2.