

**STUDIES ON
ECONOMIC EFFECTS OF TIME-VARYING PRICING
IN ENERGY SUPPLY SYSTEMS**

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SUMMARY

Recently, in urbanized areas of Japan, the energy systems such as electricity and town-gas supply systems are confronted with the remarkable growth of their peak loads. In the electric power systems, their salient peak load in summer afternoon is brought about by the demand for air conditioning. In contrast, the town-gas systems have their peak load in winter evening which is brought about by the demand for space heating and water heating. In future, much more increase in the load fluctuations due to growth of the peak loads is expected in these systems because of the sophistication of the human life and the industrial production.

Increase in fluctuation of the load has brought about low rate utilization of the capacity, and it makes the supply system inefficient and unstable as well. Lately, in order to keep the supply systems efficient, necessity of 'the load management', i.e., control of the load itself by the energy supplier, has been stressed. One of the principal ways of load management is an indirect control of the load by use of price incentives. That is, the load will be controlled through the responses of the consumers to the energy price, by setting, e.g., the price higher in the peak period (season or time-of-day). Principal pricing schemes for this method are the time-varying pricings such as the seasonal pricing (SP) and the time-of-use pricing (TOUP). Also, as a more sophisticated scheme, the load adaptive pricing (LAP) which adjusts the price according to the change of the load in an on-line manner is suggested.

In the present dissertation, the author investigates the economic effects of load management of the energy systems by means of these time-varying pricing schemes. The issue is discussed with three sorts of models

focusing on different aspects of the problem.

This dissertation consists of six chapters. Chapter 1 is an introductory one. Chapter 2 is a review of the peak load problems in the electricity and the town-gas supply systems, and the load management by means of the time-varying pricings. In this chapter, the marginal cost pricing principle, which is the basic idea of the optimal time-varying price, is also explained.

In Chapter 3, the load adaptive pricing in the electric power systems is studied by means of a dynamic Stackelberg game model. The supply/demand of electricity is modeled as a game between one electricity supplier and several consumers. Based on the model, an optimal LAP strategy is derived mathematically, and it is shown that the obtained pricing strategy forms the marginal cost price adaptively. Then, simulation based on the model is carried out by using data of a real electric power system in order to evaluate the economic effect of LAP quantitatively.

In Chapter 4, recognizing the difference between the load patterns of electricity and town-gas, and further considering possibility of mutual substitution of these loads for air conditioning and water heating, the effects of the cooperative supply of these two sorts of energy by means on time-of-use pricing (TOUP) are studied. To investigate the issue, an energy demand/supply model of nonlinear programming type is developed. In this model, the capacities of the supply systems are endogenized to take long term effects of the load management into account. Through a case study taking Kinki District in 2000 as a study area, the economic effect of TOUP is analyzed.

Chapter 5 is concerned with competition between the electricity and the town-gas suppliers. It complements the study in Chapter 4 which assumes a complete cooperation of these two energy suppliers. The problem of the inter-energy competition is modeled as a noncooperative games between the two monopolistic companies which supply partially substitutable goods or services under regulatory constraints. The nature of the equilibrium point of the game is studied analytically, and some numerical examples are also presented. Using these results, effectiveness and reasonability of the regulation to the public utility companies are discussed.

In Chapter 6, the general conclusions and some open problems for further study are summarized.

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CHAPTER 1 INTRODUCTION

1.1 Motivation of the Research

Recently, in urbanized areas of Japan, the peak loads of energy systems such as electricity and town-gas supply systems are growing remarkably according to spread of apparatuses for air conditioning and space heating. The electric power systems have their salient peak load in summer afternoon which is brought about by the demand for air conditioning. In one of the principal electric power companies, its load factor, i.e., the ratio of the average load to the peak load, has fallen down from 65.7% in 1965 to 56.5% in 1980. In contrast, the town-gas systems have their peak load in winter evening which is brought about by the demand for space heating and water heating.

At present, the suppliers of these energy utilities are required to supply their customers with whatever amount of energy they need at whatever time they desire. Hence the supplier must hold enough capacity to cover its peak load. Increase in fluctuation of the load due to aforesaid growth of the peak load has brought about low rate utilization of the capacity, and it makes the supply system inefficient and unstable as well. This difficulty is, what is called, 'the peak load problem' in the energy supply systems.

So far, the energy suppliers have coped with the fluctuating load by selection of fuel types, adjustment of the plant operation and the capacity expansion. In future, however, much more increase in the load fluctuation is expected because of the sophistication of the human life and the industrial production. Accordingly, necessity of 'the load management', i.e.,

control of the load itself by the energy supplier, has been lately stressed in order to keep the supply systems efficient/1.1, 1.2/.

In the load management, there are two principal ways. The one is a direct control of the load. For example, let customers install air conditioners remotely controllable by the supplier. If the peak load becomes serious for the supply system, the supplier reduces the operating level of the customers' air conditioners. Thus, the load is kept in an adequate level for the supply system.

The other way is an indirect one which uses price incentives. Principal pricing schemes for this method are the time-varying pricings such as the seasonal pricing (SP) and the time-of-use pricing (TOUP). In these pricing schemes, the price is set higher in the peak period (season or time-of-day). Through the responses of the consumers to the energy price, the load will be controlled. And also, as a more sophisticated scheme, the load adaptive pricing (LAP) which adjusts the price according to the change of the load in an on-line manner is suggested/1.3, 1.4/.

In the present dissertation, the author is concerned with the load management of the energy systems by means of these time-varying pricing schemes. Especially, economic effects of these pricing schemes on the energy systems are studied. The issue is discussed with three sorts of models focusing on different aspects of the problem. As well as mathematical study of the models, simulation analysis by using the data of the real energy systems is carried out in order to estimate quantitatively the effect of the policies. As the result, the economic effects of the load management by the time-varying pricing schemes are made clear in detail.

1.2 Overview

This dissertation consists of six chapters including this introductory one. In the next chapter, Chapter 2, the peak load problems in the electricity and the town-gas supply systems and the load management by means of the time-varying pricings are stated in more detail. Then the marginal cost pricing principle, which is the basic idea of the optimal time-varying price, is explained briefly by use of the surplus theory in welfare economics.

In Chapter 3, the load adaptive pricing in the electric power systems is studied by means of a dynamic Stackelberg game model. The supply/demand of electricity is modeled as a game between one electricity supplier and several consumers. The features of the present model are reflected in consideration of stochastic change of the load due to weather condition etc., and operation of the energy storage systems by the consumers. Based on the model, an optimal LAP strategy is derived mathematically. It is shown that the obtained pricing strategy forms the marginal cost price adaptively. Then, simulation based on the model is carried out by using data of a real electric power system. Through a comparative study with time-of-use pricing and other conventional pricing schemes, the economic effect of LAP is made clear quantitatively.

In Chapter 4, recognizing the difference between the load patterns of electricity and town-gas, and further considering possibility of mutual substitution of these loads for air conditioning and water heating, the effect of the cooperative supply of these two sorts of energy by means on time-of-use pricing (TOUP) is studied. To investigate the issue, an energy demand/supply model of nonlinear programming type is developed based

on the surplus theory. In this model, the capacities of the supply systems are endogenized to take long term effects of the load management into account. Substitution of electricity and town-gas in the demand for air conditioning, space heating and water heating is dealt with by demand functions having inter-energy cross price elasticities. Through a case study taking Kinki District in 2000 as a study area, the economic effect of TOUP is analyzed.

Chapter 5 is concerned with competition between the electricity and the town-gas suppliers. It complements the study in Chapter 4 which assumes a complete cooperation of these two energy suppliers. The problem of the inter-energy competition is modeled as a noncooperative games between the two monopolistic companies which supply partially substitutable goods or services under regulatory constraints. The nature of the equilibrium point of the game is studied analytically. Some numerical examples are also presented. Using these results, effectiveness and reasonability of the regulation to the public utility companies are discussed.

In the final chapter, Chapter 6, the general conclusions and some open problems for further study are summarized.

CHAPTER 2 PEAK LOAD PROBLEM IN ENERGY SUPPLY SYSTEMS AND TIME-VARYING PRICING

2.1 Introduction

In this chapter, the peak load problem faced by the electricity and town-gas systems is described in more detail. First, the peak load problem in energy supply systems and the general characteristics of the systems suffering from such a problem are summarized. Then, the concept of load management, i.e., a countermeasure of the problem, is introduced. The time-varying pricing schemes such as the time-of-use pricing and the load adaptive pricing are principal ways of the load management considered here. Finally, the marginal cost pricing principle, which is a theoretical basis of the optimal (time-varying) price, is reviewed including some problems accompanied with the actual implementation of the principle.

2.2 The Peak Load Problem in Electricity and Town-Gas Supply Systems

The electricity and the town-gas loads fluctuate largely in urbanized areas of Japan. The typical daily load curves of the both energy systems in summer and winter are shown in Fig. 2.1. Looking at the load curves, we notice that the electricity load has a salient peak in summer afternoon, which is brought about by the demand for air conditioning. Recently, these load fluctuations are increasing according to the spread of apparatuses for air conditioning. Table 2.1 shows the recent trend of change of the load factor, i.e. the ratio of the average load to the peak load, in an electric utility company. The load factor has fallen down by about 9% from 1965 to 1980. On the other hand, the town-gas system has its peak load in winter evening, which is brought about by the demand for space

and water heating. It is expected that these load fluctuations will grow significantly in future owing to sophistication of the human life style and industrial production technologies.

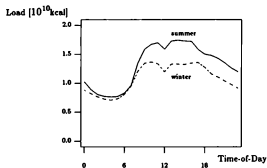
In order to supply electricity and town-gas, huge amount of equipments to produce, transfer and distribute the energy are needed. The suppliers of these energy utilities are required to supply their customers with whatever amount of energy they need at whatever time they desired. Consequently, the suppliers have to hold enough amount of capacities to meet their peak loads.

Because of the large fluctuations of the demands, the suppliers of the energy utilities are suffering from seriously low rate utilization of their capacities. Especially, in the electric power supply, owing to the difficulty of storing electricity, the supplier must hold enough amount of generating plants to meet only the keen peak load in spite of its very short duration. As a consequence, it raises the average supply cost of electric power. In the town-gas supply, the large seasonal fluctuation of the load is a serious problem though the time-of-day fluctuation of the load can be absorbed by gas-holders. These difficulties are called 'the peak load problem' in energy systems.

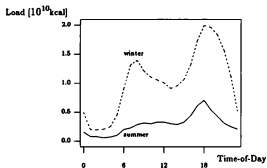
From the view point of supplier, the characteristics of the peak load problems are summarized as follows /2.1/:

- (1) The demand fluctuates largely.
- (2) Large surplus investment is needed to construct a supply system.
- (3) Storage of the products is technically or economically difficult.

These features are observed typically in public utility industries such as traffic and telecommunication services as well as energy utilities. In these



(a) Electricity Load



(b) Town-Gas Load

Fig. 2.1 Typical daily load curves of the electricity and the town-gas systems.

Table 2.1 Change of the Load Factor in an Electric Power System

Year	Load Factor(%)
1965	65.7
1970	61.0
1975	54.8
1980	56.8

Kansai Electric Power Co./2.2/

industries, the peak load problem is or can be an important problem to be resolved, too.

2.3 Load Management by Time-Varying Pricing

So far, the suppliers of electricity and town-gas have coped with the load fluctuations by 'supply-side management', i.e., selection of fuels, adjustment of operation and plant types, and capacity expansion. Optimization of the generating plant mix in the electric power system is a typical example of the supply-side management: In generation of electric power, several kinds of plants such as the nuclear power, the coal-fired steam power, the oil-fired steam power are available. The base load, a demand existing all day long, is supplied by generating plants with low operating cost such as the nuclear power or the coal-fired steam power. In the peak period, some peak load plants are operated additionally. For the peak load, plants with low capacity cost such as the gas-turbine or the small-size oil-fired steam power, are selected while such plants have high operating cost due to use of expensive fuels and low efficiency in energy conversion. The electric power system has conventionally coped with its load fluctuation. However, as the peak load problem is getting quite serious in

these days, such supply-side management is not enough to keep the supply system efficient, and the necessity of controlling the load itself, i.e., 'the load management' has been stressed especially in the electric power supply /1.1, 1.2/.

In load-management alternatives, there are two principal categories. The one is direct control of the load by the supplier. Utilization of remotely controllable air conditioner, water heater or electric pump is an example in this category which manages the load of individual consumer by the supplier. Reduction of the voltage is another alternative in this category which controls the demands of many consumers collectively.

The other category is indirect control by means of price incentives. Time-varying pricing is a typical method in this category. In this pricing scheme, the price is set higher in the peak period (season or time-of-day), and set lower in the off-peak period. Through the response of the consumers to the time-varying price, e.g., curtailment, temporal shift and inter-energy substitution, the load is expected to be controlled.

Seasonal pricing (SP), time-of-use pricing (TOUP) and load adaptive pricing (LAP) are the principal ways of the time-varying pricing schemes. In the following, the characteristics of these pricing schemes are summarized.

The Seasonal Pricing (SP) is a pricing scheme in which the price of the energy is set higher in the peak season and lower in the off-peak season. While this pricing scheme can be adopted without any additional implementation cost, ability of controlling the peak load is somewhat limited because the same price is charged all day long. At present, this pricing scheme is adopted to the commercial customers in the electric power sys-

tems in Japan. In the town-gas system, the discount for air conditioning use /2.3/ have the nature of this pricing scheme.

The Time-of-Use Pricing (TOUP) is a pricing scheme which is considered to be the most notable pricing style for the load management. It is also called the '(seasonal-) time-of-day pricing' (STDP), or the 'peak load pricing' (PLP). In TOUP, a day is divided into several periods according to the load levels. The price in each period is set reflecting the supply cost in the period. Hence this pricing scheme has an effect on leveling time-of-day load fluctuation as well as seasonal one. It should be noted that, in this pricing scheme, the time-of-use load of each customer must be measured, and therefore some additional implementation cost is needed.

TOUP is adopted effectively in the electric power system in France /2.4/. In electric power systems in the United States, this pricing scheme has been introduced after some experiments /2.5/. In electric power systems in Japan, discount for adjustable demand applied to large-size customers has the nature of this pricing scheme /2.6/. TOUP itself has also been introduced as an optional contract to such customers since 1988.

The Load Adaptive Pricing (LAP) is a pricing scheme in which price is adjusted according to change of the load in realtime and in on-line manner. Then, it is also called the 'spot pricing'. This pricing scheme is suggested by Schweppe /1.4/ for electric power systems recognizing the progress in telecommunication and computation technologies, while the same concept has already been suggested by Vickrey in more general context of the pricing in public utilities /1.3/.

The peak loads of the electricity and town-gas are largely affected by weather condition because they are brought about by the demand for heat, i.e., air conditioning, space heating etc. /2.7, 2.8, 2.9/. LAP works well even if the load fluctuates irregularly while TOUP cannot cope with such a situation. Further, LAP can manage the load when the supplying condition is changed irregularly by some accident in the system. On the other hand, the implementation cost of LAP may be much bigger than that of TOUP because the price must be announced in realtime, and the demand must be measured also in realtime.

2.4 Marginal Cost Pricing Principle

The load management by means of price incentives makes the supply system efficient and reduces the supply cost. At the same time, it has influences on the consumers due to the controlled load and the change of payment. Hence the price must be chosen appropriately with consideration of the effects on both the supply and demand sides. In this section, the marginal cost pricing principle, which gives an optimal pricing in the aforesaid point of view, is reviewed using the framework of the surplus analysis /2.5, 2.10/.

First, we consider the demand and supply of energy in a single period and assume that the energy demand changes according to the change of the price. Let $p(q)$ be the inverse demand function, i.e., the relation between the price, p , and quantity of the demand, q . As illustrated in Fig. 2.2, $p(q)$ is usually a decreasing function. In the figure, let $mc(q)$ be the marginal supply cost of the energy, i.e., the marginal change of the supply cost according to the marginal change of the demand. The plants operated additionally in the peak period are usually less efficient than the base-

load plants. Hence the marginal cost mc increases with the quantity q as shown in Fig. 2.2.

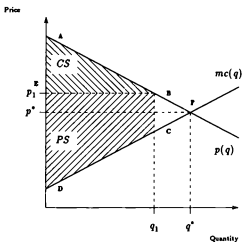


Fig. 2.2 The inverse demand function $p(q)$ and the marginal supply cost function $mc(q)$.

We can interpret the inverse demand function $p(q)$ as follows: 'At the quantity q , the consumer considers or takes the marginal increase in his demand, i.e., additional increase in the demand by one unit, is worth paying $p(q)$.' Hence the maximum amount of the money that the consumer will pay for the quantity q_1 is

$$\int_0^{q_1} p(q) dq. \quad (2.1)$$

We use this quantity as a measure of the utility obtained by the energy consumption. As the actual payment of the consumer is $p_1 q_1$, the net benefit obtained by the consumer becomes

$$CS(q_1) = \int_0^{q_1} p(q) dq - p_1 q_1. \quad (2.2)$$

We call it the consumer's surplus CS . In Fig. 2.2, CS is equal to the area of the triangle ABE .

As a matter of course, it is not realistic to use the inverse demand function in extreme region of the demand. The consumer's surplus expressed by Eq. (2.2) itself, therefore, seems to be nonsense because the integration starts from the null point of the demand. Yet the difference between the consumer's surpluses at the two different realistic demands is still meaningful, for the integrations in unrealistic region of the demand are canceled. It is noteworthy that in the case study, the results must be evaluated not by the consumer's surplus itself but by the difference between those at adequate demands. In the succeeding part of the section, the values expressed by Eqs. (2.1) and (2.2) are used keeping the above note in our minds.

The benefit of the supplier, on the other hand, is expressed as follows, and it is called the producer's surplus PS :

$$PS(q_1) = p_1 q_1 - \int_0^{q_1} mc(q) dq. \quad (2.3)$$

In Fig. 2.2, the producer's surplus is represented by the area of quadrangle $BCDE$.

Suppose that the social welfare SW is expressed by the sum of PS and CS , and let the price p_{OPT} which maximizes SW be the optimal price. Correspondingly, let q_{OPT} be the optimal demand when the optimal price p_{OPT} is taken, i.e.:

$$\begin{aligned} q_{OPT} &= \arg \max_q SW(q) \equiv CS(q) + PS(q) \\ p_{OPT} &= p(q_{OPT}). \end{aligned} \quad (2.4)$$

Then, the necessary condition for q_{OPT} is given by

$$\left. \frac{dSW}{dq} \right|_{q_{OPT}} = 0. \quad (2.5)$$

By substituting Eqs. (2.2) and (2.3) into Eq. (2.5), we get

$$p_{OPT} = mc(q_{OPT}). \quad (2.6)$$

Equation (2.6) shows that the social welfare is maximized by setting the price always equal to the marginal supply cost. It is called 'the marginal cost pricing principle'. In Fig. 2.2, the optimal demand and the price are those associated with the point F .

Next, we consider the demand and supply in two periods, i.e., the peak and the off-peak periods. Let $p_p(q)$ and $p_o(q)$ be the inverse demand functions in the peak and off-peak periods, respectively, as shown in Fig. 2.3. As discussed above, the optimal demand and the price of each period are associated with the crossing point of the demand function in the period and the marginal cost function. In Fig. 2.3, (q_p^*, p_p^*) and (q_o^*, p_o^*) are the optimal points in the peak and off-peak periods, respectively. Under the constant pricing scheme, this price setting is impossible. In other words, to achieve those two optimal points, the time-varying pricing is needed. If the price remains the constant price at p_c , it yields welfare loss

corresponding to the area $ABC + DEF$.

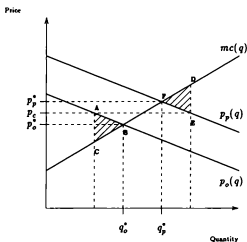


Fig. 2.3 Optimal prices in the peak and off-peak periods.

2.5 Problems in the Marginal Cost Price

As stated in the previous section, the social welfare is optimized by adopting the marginal cost price. Nevertheless, the marginal cost price has some difficulties in actual implementation. In the following, two of the problems in this pricing principle are pointed.

The first one is the problem how to estimate the marginal cost of the real energy system. Under a short-term situation, the marginal cost is mainly consist of the fuel cost and other operating cost of the plant sup-

plying marginally. Then, if the capacity is fully utilized at the peak period, the marginal cost at that time will be infinite. By contrast, from the long-term point of view, the amount of capacity is adjustable, and the marginal cost includes the cost for capacity expansion, reduction or replacement. The problem is which marginal cost to be used for the pricing. In the next chapter, Chapter 3, the short-term marginal cost is adopted because the model considers only the short-term situation. In Chapters 4 and 5, the optimal pricing is investigated from the long-term point of view including adjustment of the capacities. Kaya /2.11/ has discussed this problem by means of a dynamic optimization model.

The second problem to be pointed is imbalance between the total supply cost and the revenue of the supplier. The revenue by the marginal cost pricing does not always balance with the total supply cost. Appendix A indicates that it requires certain condition on the structure of supply cost. Hence some means are needed to achieve the balance of revenue and cost. The following ideas have been proposed to this problem /2.10/:

- (1) To let the price deviate from the optimal one. The social welfare obtained by this method is no longer the optimal but the second-best one.
- (2) To adopt nonlinear prices such as two-part tariff which consists of a fixed charge and an unit price.
- (3) To support the supplier by taxation and/or subsidiary (financial aid).

At the same time, this problem relates to the fair distribution of the welfare gain obtained by a new pricing scheme /2.12, 2.13/.

The study in the next chapter does not take the revenue-cost balance into account. The studies in Chapters 4 and 5, the balance of revenue and

cost are satisfied by the deviations of prices.

CHAPTER 3 A STUDY ON THE LOAD ADAPTIVE PRICING IN ELECTRIC POWER SYSTEMS BY MEANS OF A MULTIFOLLOWER STACKELBERG GAME MODEL

3.1 Introduction

In this chapter, economic effects of the load adaptive pricing (LAP) in electric power systems are studied. To investigate the issue, a multifollower Stackelberg game model is developed, which is an extended version of the game model formulated by Luh et al. /3.1/. An optimal LAP strategy for the extended model is obtained mathematically, and the qualitative nature of the optimal pricing strategy is discussed. Then the parameters of the model are estimated by using data of a real power system, and a variety of simulations are carried out. Through this case study, economic effects of LAP are evaluated quantitatively.

3.2 A Game Model of the Load Adaptive Pricing

Luh et al. have modeled the electric power market as a game between an electricity supplier and a consumer. The electricity supplier decides the price of electricity and tells it to the consumer. Knowing this price, the consumer decides his consumption level of the electric power. This situation can be modeled as a game of Stackelberg type, which assume an order of decision making among the players. Taking demand/supply in multiple periods and random fluctuation of the demand into account, they have formulated the model as a stochastic dynamic Stackelberg game model. Then, they have obtained an optimal LAP strategy for the model, and have pointed its advantage over TOUP from the game theoretic point of view /3.1/.

Since the formulation of their game model aims at the conceptual explanation of the difference between LAP and TOUP, it is too simple to carry out a case study and to make a numerical evaluation of LAP. For example, in their model, the number of the consumers is restricted to only one, and the temporal pattern of the demand variation is a simple repetition of a peak and an off-peak period. To reflect the real situation better, the model presented in this chapter is expanded as follows:

- 1) The time span treated in the model is divided into N periods of equal duration admitting arbitrary patterns of the demand variation. Let $N = \{1, \dots, N\}$ be a set of the periods. Each period is assumed short enough to suppose the demand is kept constant during the period. Hence we assume a constant price in each period.
- 2) The electric power is supplied by a player, Player 0. On the other hand, multiple sorts of the consumers are taken into account. The consumers are categorized into K classes such as industrial, commercial or household sectors according to their load characteristics. Each class is represented by one player in the game. Let $K = \{1, \dots, K\}$ be the set of the consumer classes, and $\hat{K} = \{0, \dots, K\}$ be the set of the whole players including the supplier. Figure 3.1 illustrates the players of the game.
- 3) In the model of Luh et al., interperiod demand substitution is taken into account by introducing several terms which express the effect of interperiod demand substitution into the consumer's surplus. This approach has some difficulty in the parameter estimation and the interpretation of the results. Instead of it, in the extended model, dynamic behaviors of the consumers are represented by utilization of

the energy storage systems on the consumer side.

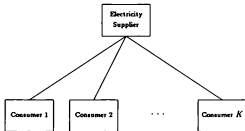


Fig. 3.1 Players of the game model.

a) Generating Cost and Utility Function

In the electric power system, some cost is needed to generate the power on one hand. On the other hand, the consumers obtain utilities by consuming the generated electricity. Considering the short-run situation in which the capacity of the supply system is not adjusted, and also considering that the peak load is supplied by less efficient plants than the base load, we assume the generating cost C is approximated by the following quadratic function:

$$C = \sum_{i=1}^N \left[\frac{1}{2} c_1 (q_i^d)^2 + c_2 q_i^d \right] \quad (3.1)$$

where

$$q_i^d = \sum_{j=1}^K q_{ij}^d, \quad (3.2)$$

c_1, c_2 are positive constants, and q_{ij}^d denotes the electricity demand of consumer j at period i .

Consumer j 's utility S_j obtained by consuming electricity is assumed to be the following quadratic function:

$$S_j = - \sum_{i=1}^N \frac{1}{2} w_{ij} [q_{ij}^* - (\bar{q}_{ij1}\xi_i + \bar{q}_{ij2})]^2, \quad j \in K \quad (3.3)$$

where q_{ij}^* is the net consumption of consumer j at period i , and w_{ij} , $\bar{q}_{ij1}$ and $\bar{q}_{ij2}$ are positive constants. In Eq. (3.3), $\bar{q}_{ij1}\xi_i + \bar{q}_{ij2}$ expresses the potential demand, which fluctuates owing to weather condition, e.g. The variable ξ_i represents this random factor, which obey the following equations:

$$\begin{aligned} \xi_i &= \alpha \xi_{i-1} + (1 - \alpha^2)^{\frac{1}{2}} \nu_i, \quad i \in N \\ \xi_0 &= \nu_0 \end{aligned} \quad (3.4)$$

where $\alpha \in (0,1)$ is a constant, and ν_i is a white Gaussian with null mean and unit variance. The utility S_j defined by Eq. (3.3) has negative value. It may seem peculiar, but it is just due to the reference point of the utility. As mentioned in the previous chapter, it is noteworthy that not the value of S_j itself but the difference between those at adequate demands must be evaluated.

b) Energy Storage

If the price of the energy varies temporally as in LAP or TOUP, it motivates the consumers to take some dynamic behaviors, e.g., to utilize heat storage equipment, or to shift the demand from one period to another. In this model, such dynamic behaviors of the con-

sumers are represented by the operation of energy storage systems on the consumer sides. The consumers are assumed to have some sorts of energy storage equipments and to operate them according to the varying price.

The net energy consumption q_{ij}^n and the level of the stored energy x_{ij} of consumer j at period i are assumed to obey the following equations:

$$q_{ij}^n = q_{ij}^d - q_{ij}^s, \quad i \in N, j \in K \quad (3.5)$$

$$x_{ij} = \beta x_{i-1,j} + q_{ij}^s, \quad i \in N, j \in K \quad (3.6)$$

where q_{ij}^s is input to the storage system (or if it is negative, it means output). $\beta \in (0,1)$ is a loss factor of the storage system. x_{0j} is an initial level of the stored energy. To set limits to q_{ij}^s and x_{ij} due to the capacity of the storage system, the following penalty function P_{Nj} is introduced:

$$P_{Nj} = \sum_{i=1}^N \left[-\frac{1}{2} p_{ij} (x_{ij} - \bar{x}_j)^2 - \frac{1}{2} c_{Lj} (q_{ij}^s)^2 \right], \quad j \in K \quad (3.7)$$

where p_{ij} , $\bar{x}_j$ and c_{Lj} are nonnegative constants.

c) Pricing Scheme

A two part tariff consisting of an unit energy price and a fixed charge are assumed. Let v_{ij} and h_{ij} be the unit energy price and the fixed charge to consumer j at period i , respectively. The revenue of the supplier from consumer j , R_j , is given by

$$R_j = \sum_{i=1}^N (v_{ij} q_{ij}^d + h_{ij}) + h_{N+1,j}, \quad j \in K. \quad (3.8)$$

In Eq. (3.8), the final term $h_{N+1,j}$ is needed to control the behaviors of the consumers at the final period. It must be noted that the 'fixed charge' means the charge which must be paid in each period regardless of the amount of demand in the period, and that dynamic adjustment of it is admitted in LAP as well as the unit price.

d) Payoff Function

The consumer j 's surplus CS_j , the producer's surplus PS and the social welfare SW are defined as follows:

$$CS_j = S_j + P_{Nj} - R_j, \quad j \in K \quad (3.9)$$

$$PS = \sum_{j=1}^K R_j - C \quad (3.10)$$

$$SW = PS + \sum_{j=K}^K CS_j = \sum_{j=1}^K (S_j + P_{Nj}) - C. \quad (3.11)$$

These definitions are same with those in the previous chapter except P_{Nj} which is introduced to set limit to the operation of the energy storage systems. Suppose consumer j acts to maximize the expectation of his surplus CS_j , and the supplier acts to maximize the expectation of SW , then the payoff function of consumer j , J_j and that of the supplier J_0 are given as follows:

$$J_j = E[CS_j], \quad j \in K \quad (3.12)$$

$$J_0 = E[SW] \quad (3.13)$$

where $E[\cdot]$ denote the expectation.

e) Information Structure

In TOUP, the prices of the all period must be decided in advance. From a game theoretic point of view, it means that the supplier has no available information as he decides the prices. It is represented by an information structure of the open-loop type:

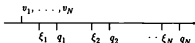
$$\begin{aligned} \text{(IS1)} \quad \eta_{10} &= \eta_{20} = \dots = \eta_{N+1,0} = \phi \\ \eta_{1j} &= (v_1, \dots, v_N, h_1, \dots, h_{N+1}, \xi_1), \quad j \in K \\ \eta_{ij} &= (\eta_{i-1}, q_{i-1}, \xi_i), \quad 2 \leq i \leq N, j \in K \end{aligned}$$

where η_{ij} , $j \in \bar{K}$ denotes the available information of player j for his decision at period i . v_i , h_i and q_i denote $(v_{ij}, \dots, v_{iK})^T$, $(h_{i1}, \dots, h_{iK})^T$ and $(q_{i1}^d, \dots, q_{iK}^d, q_{i1}^s, \dots, q_{iK}^s)^T$, respectively, where the superscript ' T ' means transposition of a vector.

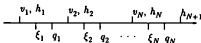
In LAP, in contrast, the supplier is permitted to decide the prices of each period just before it. In the game, it is represented by an information structure of the closed-loop type:

$$\begin{aligned} \text{(IS2)} \quad \eta_{10} &= \phi, \\ \eta_{1j} &= (v_1, h_1, \xi_1), \quad j \in K \\ \eta_{i0} &= (\eta_{i-1,j}, q_{i-1}), \quad 2 \leq i \leq N+1 \\ \eta_{ij} &= (\eta_{i,0}, v_i, h_i, \xi_i), \quad 2 \leq i \leq N, j \in K. \end{aligned}$$

It should be noted that the random factor ξ_i should not be included in the supplier's information η_{i0} because the prices at period i must be decided just before the period. The difference between the information structure (IS1) and (IS2) is illustrated in Fig. 3.2.



IS1 (Open Loop)



IS2 (Closed Loop)

Fig. 3.2 Difference between the information structures IS1 (for TOUP and CP, the open-loop type) and IS2 (for LAP, the closed-loop type) in the sequences of the decisions.

7) Multifollower Stackelberg Game

Let γ_0 and γ_j be the supplier's and consumer j 's strategies, respectively, i.e., mappings from their available informations to their decisions:

$$\gamma_0 = (\gamma_{01}, \dots, \gamma_{0,N+1})$$

$$\gamma_{01}: \eta_{i0} \mapsto v_i, h_i, \quad i \in N$$

$$\gamma_{0,N+1}: \eta_{N+1,0} \mapsto h_{N+1}$$

$$\gamma_j = (\gamma_{j1}, \dots, \gamma_{jN})$$

$$\gamma_{ji}: \eta_{ij} \mapsto q_{ij}^d, q_{ij}^s, \quad j \in K, i \in N.$$

Suppose the supplier declares his strategy, i.e., the pricing formulas, first, and then the consumers declare their strategies knowing the supplier's strategy before the operation of the game. Then the game becomes a Stackelberg game of multistage and multifollower type in which the supplier and the consumers play the rolls of a leader and followers, respectively. In this framework, the problem of optimal pricing is formulated as follows:

$$\max_{\gamma_0} J_0(\gamma_0, \gamma_1^*(\gamma_0), \dots, \gamma_K^*(\gamma_0)) \quad (3.14)$$

where $\gamma_j^*(\gamma_0)$ denotes the optimal response of consumer j to the pricing strategy γ_0 . That is:

$$\gamma_j^* = \arg \max_{\gamma_j} J_j(\gamma_0, \gamma_j, \gamma_k^*(\gamma_0), k \in K, k \neq j), \quad j \in K \quad (3.15)$$

where $J_j(\gamma_0, \dots, \gamma_K)$ denotes the payoff function of player j as a functional of the strategies. In the definition of the optimal response, Eq. (3.15), Nash equilibrium /3.2/ among the consumers is assumed.

The variables, constants and functions of the model are listed in Table 3.1.

3.3 Optimal Strategies

Methods to solve the multistage Stackelberg game with the closed loop information structure have developed by Ho et al. /3.3, 3.4/ and Basar et al. /3.2, 3.5/. In this section, an optimal LAP strategy of the model formulated in the previous section is obtained by the method used

Table 3.1 Variables, Constants and Functions of the Model

Name	Meaning
N	Number of the periods
K	Number of the consumers
q_{ij}^d	Electricity demand
q_{ij}^s	Input to the storage system
q_{ij}^*	Net consumption
v_{ij}	Unit price
h_{ij}	Fixed charge
C	Generating cost
S_j	Utility obtained by electricity consumption
P_{Nj}	Penalty function for energy storage
R_j	Revenue
CS_j	Consumer's surplus
PS_j	Producer's surplus
SW	Social welfare
J_j	Consumer's payoff
J_0	Producer's payoff
c_1, c_2	Parameters in C
$\bar{q}_{ij1}, \bar{q}_{ij2}, w_{ij}$	Parameters in S_j
α	Correlation of ξ_i
β	Loss factor of the stored energy
ξ_i	Random factor in the potential demand
ν_i	Random variable of white Gaussian type
$\bar{x}_j, c_{Lj}, p_{ij}$	Parameters in P_{Nj}
F_i, z_{i1}, z_{i2}	Coefficients of the optimal team strategy
a_{ij1}, a_{ij2}	
b_{ij1}, b_{ij2}	Parameters in the incentive strategy
b_{ij3}, b_{ij4}	
q_i^d	Aggregated demand
x_{ij}	Stored energy
η_{ij}	Available information
γ_j	Strategy

In the above, the indexes i and j represent the period and the player, respectively.

by Luh et al. /3.1/. Then it is shown that the obtained optimal pricing strategy forms the marginal cost price adaptively. Additionally, the methods to calculate the optimal time-of-use pricing (TOUP) strategy and the optimal constant pricing (CP) strategy are described, which are used for comparative study in the numerical simulation presented in Section 3.6.

a) A Two Step Method for the Closed Loop Stackelberg Games

The method used by Luh et al. to solve the closed-loop multistage Stackelberg game consists of the following two steps:

- 1) Suppose the problem where the followers (the consumers) act cooperatively with the leader (the supplier) to maximize the leader's payoff. We call it 'the team problem'. Obtain the solution of the team problem, which is called 'the optimal team strategies'. The optimal team strategies give the upper limit of the leader's payoff in the original problem.
- 2) An adequate function form as a strategy of the leader is assumed. It is called an incentive strategy. Adjust the parameters of the incentive strategy in such a way to make the followers' strategies maximizing their own payoffs coincide with the optimal team strategies obtained in step 1). Then the most desirable payoff of the leader is achieved, and therefore this incentive strategy is a solution to the Stackelberg game.

It should be noted that the incentive strategy may not be unique /3.3/, and that the function form of it must be selected empirically in step 2) of the above method.

b) Optimal Team Strategies

In the supplier's payoff J_0 , the prices are not appeared explicitly. Hence the team problem is a problem only to decide consumption strategies of the consumers. That is,

$$\max_{\gamma_j, j \in K} J_0. \quad (3.16)$$

Looking at the formulation in the previous section, this problem is a variation of the LQ regulator problem, and it can be solved by the backward dynamic programming.

Necessary conditions for the optimal team strategies are

$$\nabla_{q_i} E_{j/i}[SW] = 0, \quad i \in N \quad (3.17)$$

where $E_{j/i}[\cdot]$ denotes the conditional expectation when the information available for the consumers in period i is known. By solving Eq. (3.17) step by step from the final period to the first, the following optimal team strategies in a feedback form are obtained containing (x_{i-1}, ξ_i) as state variables:

$$q_i = F_i x_{i-1} + z_{i1} \xi_i + z_{i2}, \quad i \in N \quad (3.18)$$

where F_i are $2K \times K$ constant matrix, and z_{i1} and z_{i2} are $2K$ constant vectors. z_i denotes $(z_{i1}, \dots, z_{iK})^T$. The formulas which give F_i , z_{i1} and z_{i2} are presented in Appendix B.

c) Optimal LAP Strategy

Next, we obtain a pricing strategy which induces the consumers' behavior to the optimal team strategies, when the consumers act to maximize their own payoffs. Considering that the payoff functions of the con-

sumers are quadratic functions with respect to their decisions, and that the optimal team strategies have linear forms, the pricing strategy is assumed as follows:

$$v_{ij} = a_{ij1}^T x_{i-1} + b_{ij1}, \quad i \in N, j \in K \quad (3.19)$$

$$\begin{aligned} h_{ij} &= 0, \text{ for } i = 1, j \in K \\ &= b_{ij2} \xi_{i-1} q_{i-1,j}^d \\ &\quad + a_{ij2}^T x_{i-2} q_{i-1,j}^d + b_{ij3} \xi_{i-1} q_{i-1,j}^d + b_{ij4} q_{i-1,j}^d, \\ &\text{for } 2 \leq i \leq N+1, j \in K \end{aligned} \quad (3.20)$$

where $a_{ij\mu}$, $b_{ij\mu}$ are K -vector and scalar parameters, respectively.

If the supplier adopts the above strategy, the game becomes a multistage one among the consumers. It is known that in games of this type, the Nash equilibrium are not strategically unique [3.2]. In the following discussion, the concept of the feedback Nash equilibrium [3.2] which guarantees the uniqueness of the equilibrium is used for simplicity. The feedback Nash equilibrium is a Nash equilibrium which forms a Nash equilibrium in any stage of the game when the decisions in the proceeding stages are given. With this restriction, the optimal responses of the consumers are also obtained by the backward dynamic programming. The necessary conditions for the optimal responses are

$$\begin{aligned} \frac{\partial E_{ji}[S_j + P_{Nj} - R_j]}{\partial q_{ij}^d} &= 0 \\ \frac{\partial E_{ji}[S_j + P_{Nj} - R_j]}{\partial q_{ij}^d} &= 0, \quad i \in N, j \in K. \end{aligned} \quad (3.21)$$

The solution of Eq. (3.21) has the same function form with the optimal team strategies, Eq. (3.18). By selecting appropriately the parameters of Eqs. (3.19) and (3.20), the optimal LAP strategy is obtained. The optimal parameters of Eqs. (3.19) and (3.20), are shown in Appendix C.

As shown in Appendix D, in the obtained optimal LAP strategies, the unit price v_{ij} forms the marginal cost price omitting the influences of the random variable ξ_i . And the first term of the fixed charge $h_{i+1,j}$ compensates the influences of ξ_i after the fact. The remaining terms of the fixed charge $h_{i+1,j}$ compensate the influences of the prices in the succeeding periods on the consumers' decisions of the energy storage q'_{ij} .

d) Optimal TOUP Strategy and Optimal CP Strategy

In this subsection, methods to obtain the optimal TOUP and CP strategies are described. As to the optimal TOUP and CP strategies which have the open-loop information structure, the following proposition holds.

Proposition Let a deterministic model be a model which has a modification of $\xi_i \equiv 0$ in the original model. The optimal TOUP and SP strategies of the original model coincide with those of the deterministic model.

The proof of the proposition is shown in Appendix E.

The open loop-multistage Stackelberg game can be solved by a method which uses the discrete-time maximum principle [3.2]. However, the optimal TOUP strategy of the present (deterministic) model is obtained more easily by using the marginal cost pricing principle. The procedure to obtain it is as follows:

- 1) Obtain the optimal team strategies of the deterministic model.
- 2) Substitute the optimal team solution into cost function C , and calculate the marginal generating cost, $\partial C / \partial q_{ij}^d$, and make it the unit price v_{ij} .

In TOUP, arbitrary fixed charge is admitted because constant change of the payment has no influence on the consumers' decisions. For simplicity, the fixed charge is set null.

In CP, owing to the constraint that the price must be constant throughout the all periods, the above method is not applicable. Considering that the variables to be decided are few in CP, the optimal CP strategy is obtained by solving the optimization problem (3.14) numerically for the deterministic model in the simulation study presented in Section 3.6. The fixed charge is set null in CP similarly in TOUP.

3.4 Estimation of the Model Parameters

In order to evaluate the effects of LAP quantitatively, the model parameters are estimated taking an electric company in Japan as a study case. We call it 'A' company. The case is studied regarding the summer weekday, that is the time when the system has the highest peak load. The parameters are estimated under the following basic settings:

- 1) The demand/supply level of A company on summer weekday in 1981 is considered.
- 2) The length of a period is taken to one hour, and 24 periods (a day) are considered.
- 3) The short-run situation in which the capacity of the supply system cannot be adjusted is assumed.

- 4) The consumers are categorized into four sectors, i.e., large-size industrial, small-size industrial, commercial and household sectors. Each sector is represented by one player in the model.

a) Generating Cost Function

In the case where the capacity cannot be adjusted, the varying part of the generating cost is just the operating cost such as cost for fuel. The generating plant of A company is categorized into four types, i.e., the hydraulic power, the nuclear power, the LNG-fired power and the oil-fired power. The operating cost of each generating plant type is assumed to be same with the calculation by the Agency of Resource and Energy in 1982 /3.6/. Considering that the generating plants of these types are put into the system in the increasing order of their operating costs according to the load, the generating cost function is estimated as a piecewise linear function shown by a solid line in Fig. 3.3. By the least square method, this function is approximated by the following quadratic function:

$$C = 32.49q^2 + 207.1q - 1284.5 \quad (3.22)$$

where the units of C and q are [10^4 yen] and [GWh], respectively. It is also shown by a dashed line in Fig. 3.3. Thus the parameters of (3.1) are obtained, i.e., $c_1 = 64.98 (= 2 \times 32.49)$ [10^4 yen/GWh²] and $c_2 = 207.1$ [10^4 yen/GWh].

b) Utility Function

As stated in the previous chapter, the utility function $S(q)$ are related with inverse demand function $\pi(q)$ as follows:

$$\frac{dS(q)}{dq} = \pi(q) \quad (3.23)$$

Generating Cost [10^8 yen]

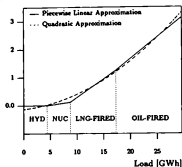


Fig. 3.3 Estimated cost function of the power generation.

The assumption of consumer j 's utility function as Eq. (3.3), is equivalent to assumption of the following inverse demand function π_{ij} :

$$\pi_{ij}(q_{ij}^n) = -w_{ij}(q_{ij}^n - \bar{q}_{ij1}\xi_i - \bar{q}_{ij2}). \quad (3.24)$$

Consequently, the inverse demand function (3.24) is needed to be identified. The parameters of Eq. (3.24) are estimated in the following manner:

- 1) Obtain the mean hourly load $\bar{q}_{ij}^n$ and unit price $\bar{\pi}_{ij}$ of each sector in the service area of A company on summer weekdays in 1981. Owing to availability of the data, the load pattern of each sector in August 1975 is used, and they are rescaled by multiplying a constant to make the total demand coincide with that in 1981. As the value of the unit

price, the average price 23.9[yen/kWh] of the company is used regardless of the sectors.

- 2) So far, the price elasticity of the time-of-use electricity demand have not been measured in Japan. Considering its value measured in the United States /3.7, 3.8, 3.9/ and also the measurement of the short- and the long-term price elasticity of the aggregated electricity demand in Japan /3.10/, the price elasticity η is set to -0.2 regardless of the time-of-day or the consumer sector.

Then the parameters w_{ij} and $\bar{q}_{ij2}$ are obtained by the following formulas:

$$\begin{aligned} w_{ij} &= -\frac{\bar{\pi}_{ij}}{\eta \bar{q}_{ij}^*} \\ \bar{q}_{ij2} &= \bar{q}_{ij}^* (1 - \eta), \quad i \in N, j \in K. \end{aligned} \quad (3.25)$$

- 3) The parameter $\bar{q}_{ij1}$, which indicates the fluctuating level of the demand, is estimated as follows:

First, the standard deviation of the daily maximum load in the summer weekdays is obtained. Second, the difference between the mean time-of-day loads in the summer and that in the spring/autumn is calculated, which is taken as 'the mean hourly demand for air conditioning'.

The fluctuating level of the load is assumed to be proportional to the mean hourly demand for air conditioning, and it is rescaled to coincide with the standard deviation of the maximum load obtained before at their maximum. Owing to availability of the data, this process is done only about the total demand of the studied case, and

then it is divided into the parameter $\overline{q}_{ij1}$ of each sector according to the air conditioning demand of the sector estimated by the monthly load data.

The parameter α is set to 0.976 based on the correlation of the daily maximum load.

The obtained parameters w_{ij} , $\overline{q}_{ij1}$ and $\overline{q}_{ij2}$ are shown in Fig. 3.4.

3.5 Simulation Analysis

Using the parameter values estimated in the previous section, effects of the LAP are studied quantitatively in comparison with those of other pricing schemes. The following four pricing schemes are considered:

- (1) Load Adaptive Pricing (LAP).
- (2) Time-of-Use Pricing (TOUP). The price is altered in every hour as well as LAP.
- (3) Constant Pricing Discriminating the Consumers' Sector (CP-D).
- (4) Constant Pricing Common to the All Consumers (CP-C).

a) Parameter Selection of the Energy Storage

The parameters β , c_{Lj} , p_{ij} and $\overline{x}_j$ concerning the energy storage are set as follows:

- 1) The parameter β is set equal to $0.9^{1/24}$. It implies that 10% of the stored energy is lost in one day.
- 2) Several values are assigned to the parameter $\overline{x}_j$ in order to investigate the influences of size of the storage.

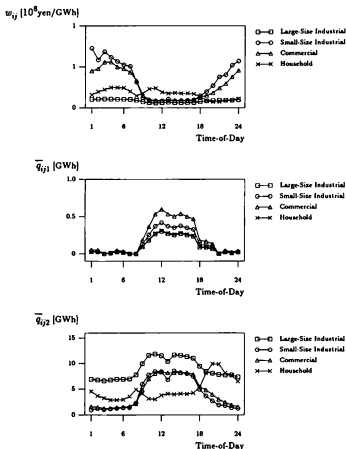


Fig. 3.4 Estimated parameter values of the utility functions.

- 3) In the case of $\bar{x}_j = 0$, large values are set to c_{Lj} and p_{ij} to inhibit the energy storage.
- 4) In the case of $\bar{x}_j > 0$, the parameters c_{Lj} and p_{ij} for $i \neq N$ are selected so that the minimum of the stored energy coincides with 20% of $\bar{x}_j$ in TOUP in the deterministic model. It implies that the assumed size of the storage system is about twice of $\bar{x}_j$. In the above procedure, a constraint $c_{Lj} = p_{ij}$ is assumed for simplicity. The coefficient at the final period, p_{Nj} is set large enough to make the stored energy x_{Nj} at the period close to $\bar{x}_j$. The initial value of the stored energy x_{0j} is set equal to $\bar{x}_j$.

There are some transitional effects on the decisions of energy consumption and storage near the starting and the final periods due to termination. To reduce these effects on the simulation results, the simulation is carried out with extension of the time span to 96 periods (four days). The payoffs are evaluated by the mean of 100 repetitive simulations using random numbers.

b) Effects of the Pricing Schemes and the Energy Storage

A simulation result is presented in Fig. 3.5. The figure shows the variation of the supplier's payoff (the social welfare), the consumers' payoffs, the generating cost and the load factor when the $\bar{x}_1$ is altered from 0 to 5GWh while the other $\bar{x}_j$ are kept null, i.e., the energy storage system is used only in the sector of large-size industry. Figure 3.6 illustrates the temporal variations of the total demand/supply, the unit price and the fixed charge for the large-size industrial sector and the stored energy level of the sector in LAP and CP-C for $\bar{x}_1=0$ and 5GWh. The total load of

the studied area is about 25GWh in its peak hour and about 12GWh in its bottom hour. Hence the investigated capacity of the energy storage (about $2 \times \bar{x}_1$, 10GWh at maximum) amounts to the demand in 0.4 hour at the peak period.

Looking at the Fig. 3.5, the social welfare is improved by about $0.3[10^6 \text{yen/day}]$ by LAP to compare with that by CP-C when $\bar{x}_1$ is null. It amounts to about 0.7% of the generating cost in CP-C. At the same time, the load factor is improved by about 2.5%. This improvement becomes more salient according to the increase in $\bar{x}_1$. When $\bar{x}_1$ is 5GWh, the improvement of the social welfare by LAP, or by TOUP as well, amounts to $0.76[10^6 \text{yen/day}]$. In contrast, by CP-C and CP-D which give no incentive of energy storage, the social welfare becomes slightly worse. It is due to the cost needed to keep the level of the stored energy in the model, and it is not substantial loss in CPs. In CPs, the social welfare should be considered to be unchanged.

The difference between the social welfares achieved by LAP and TOUP is also magnified according to the increase in $\bar{x}_1$ though it does not appear clearly in Fig. 3.5. It is brought about by the ability of LAP to induce the consumption and the energy storage responding to the irregular change of the potential demand. In this simulation, the difference between LAP and TOUP may be underestimated because of the following reasons:

- 1) In the model, the supplying cost is formulated as a quadratic function, and the evaluation of the increasing cost for the peak load is rather mild. If the lack of supplying capacity at the peak period occurs, it causes more serious loss of the social benefit. LAP will be more advantageous than TOUP to relieve the system from such seri-

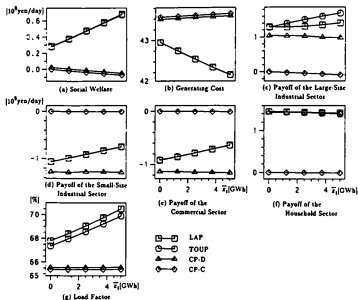


Fig. 3.5 Results of the simulation.

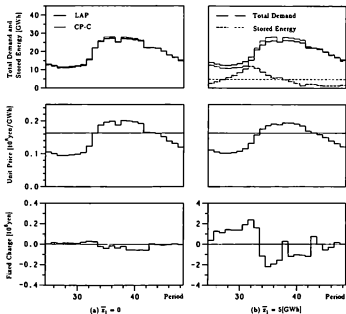


Fig. 3.6 Temporal variations of the total demand, the stored energy, and the unit price and the fixed charge to the large-size industrial sector under LAP and CP-C.

ous difficulty.

- 2) The price is altered every hour in TOUP as well as LAP in this simulation. In TOUP used actually in France or in the U.S., the price is altered only two or three times a day. In such a case, the welfare loss will be larger than that shown by this simulation.

Looking at the influences of the pricing schemes on each consumer's sector, adoption of LAP instead of CP-C, for example, is advantageous for the consumers in the sectors of the large-size industry and the household, and in contrast, it is disadvantageous for the consumers in the other sectors. It is due to the difference between the daily load patterns of the both groups.

Figure 3.5 shows that the payoff of the large-size industrial sector, which is able to cope with the varying price by managing its own storage system, is increasing according to $\bar{x}_1$. At the same time the payoffs of the commercial and the small-size industrial sectors are also increasing because these sector has their salient peaks at the peak periods of the total demand, and the high prices in the peak periods are relaxed according to the expansion of $\bar{x}_1$. In contrast, the payoff of the household sector, whose peak demand is in the off-peak periods of the total demand, is decreased.

These results indicate that altering the pricing scheme may not be beneficial for all the sectors even if it improves the social welfare. Under such situation, some means for benefit reallocation are needed to stimulate smooth introduction of a new pricing schemes.

The social welfare itself is not influenced by the sector having the storage system as far as the size of the energy storage is same. But it has influences on the consumers' payoffs. To examine this point, the simula-

tion in the case where all the sectors have energy storage systems is carried out. The results are compared with those of the above simulation in Fig. 3.7. The size of energy storage system $\bar{x}_j$ of each sector is set proportional to the amount of its demand for air conditioning. The payoff of the large-size industrial sector, whose storage system is made small, decreases. In contrast, the payoffs of the other sectors increase slightly. This tendency is more remarkable in TOUP than LAP.

c) Influences of the Price Elasticity of the Demand and the Growth of the Demand for Air Conditioning

Among the parameters of the model used for the simulation, the most indefinite one is the price elasticity of the demand η . In order to evaluate the influence of η , simulations in the cases of $\eta = -0.3$ (high elasticity case) and $\eta = -0.1$ (low elasticity case) are carried out in addition to that of $\eta = -0.2$ (standard case) presented before. The results are summarized in Table 3.2. As shown in the table, the social welfare varies remarkably depending on the price elasticity of the demand. The difference between the social welfares in the high and the low elasticity cases amounts to about 0.29[10⁸yen/day]. It shows that the more precise estimation of the price elasticity is needed to make the analysis more definite.

The amount of the demand for air conditioning may also have large influences on the estimated welfare gain by LAP. This demand is expected to grow remarkably in future. The influence of the growth of the air conditioning demand is also investigated. Simulations in the cases of 50% growth and of 100% growth of air conditioning demand are carried out. The results are shown in Table 3.3. The welfare gain by LAP will increase by 16 through 30% (according to $\bar{x}_1$) in the case of 100% growth of the air

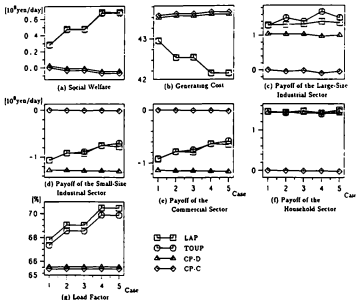


Fig. 3.7 Comparison between the concentrated and the distributed energy storages. Case 1: No energy storage, Case 2: Concentrated, $\bar{x}_1 = 2.5[\text{GWh}]$, Case 3: Distributed, $\sum_j \bar{x}_j = 2.5[\text{GWh}]$, Case 4: Concentrated, $\bar{x}_1 = 5[\text{GWh}]$, Case 5: Distributed, $\sum_j \bar{x}_j = 5[\text{GWh}]$.

Table 3.2 Influence of Price Elasticity of Demand

Case	Elasticity	$\Delta SW(\bar{x}_1 = 0)$	$(\bar{x}_1 = 2.5\text{GWh})$
Low	-0.1	0.143	0.372
Standard	-0.2	0.285	0.515
High	-0.3	0.429	0.659

ΔSW represents the difference between the social welfares (SW 's) by LAP and by CP-C. Its unit is [10^4 yen/day].

Table 3.3 Influence of Growth of Air Conditioning Demand

Case	Demand Growth	$\Delta SW(\bar{x}_1 = 0)$	$(\bar{x}_1 = 2.5\text{GWh})$
Standard	0%	0.285	0.515
Mid	50%	0.325	0.557
High	100%	0.372	0.607

The meaning and the unit of ΔSW are same with those in Table 3.2.

conditioning demand. In short, this factor also has large influences on the simulation results.

3.6 Concluding Remarks

In this chapter, effects of LAP in the electric power system is studied by means of a multifollower Stackelberg game model. The principal findings of the study are summarized as follows:

- 1) The optimal LAP strategy has a nature of marginal cost pricing which is adjusted adaptively in response to the random fluctuation of

the potential demand.

- 2) The social welfare is improved by about $0.3[10^8 \text{yen/day}]$ by adopting LAP or TOUP compared with CP-C, in the studied case if no energy storage is used. The improvement amounts to 0.7% of the generating cost.
- 3) The effect of LAP or TOUP becomes more remarkable if the consumers respond to the varying prices by their management of energy storage.
- 4) Generally, LAP is more advantageous than TOUP because of its adaptive nature. However, the effect of TOUP can get very close to that of LAP if TOUP is adopted with fine time division.
- 5) The influences of introduction of LAP and TOUP on the consumers are different by the demand characteristics of the consumers. To some consumers, even a disadvantage may be forced. Thus, to make the new pricing scheme acceptable to all the consumers, some means to reallocate the benefit are needed.

Finally, the following is to be pointed. To implement LAP or TOUP, some cost is needed for reporting the price and measuring the time-of-day demand. It is called 'the metering cost'. Especially the concept of LAP is based on utilization of a communication network to report the price in realtime. Hence LAP needs some additional cost for the communication network. From the viewpoint of cost benefit analysis, it is a necessary condition for the feasibility of LAP (or TOUP as well) that the welfare gain obtained by the new pricing scheme exceeds its metering cost. Since it is difficult to estimate the metering cost of LAP at present, this cost is not treated in the presented study.¹ In future it is expected that the me-

tering cost will be reduced well because of the innovation of the telecommunication and computation technologies.

The problem of benefit reallocation pointed above is closely related to the problem of cross-subsidization among the industries having the economy of scope, which has studied from the viewpoint of the cooperative game theory /2.12, 2.13, 3.11, 3.12/.

¹For the TOUP, the metering cost is evaluated in the study presented in the next chapter.

CHAPTER 4 A STUDY ON THE COOPERATIVE SUPPLY OF ELECTRICITY AND TOWN-GAS UNDER TIME-OF-USE PRICING SCHEME

4.1 Introduction

As mentioned in Chapter 2, the electric power and the town-gas systems in Japan have contrastive load patterns, and therefore mutual substitution of their loads can be an effective way to solve the peak load problems in the both energy systems. In this chapter, the effects of cooperative supply of electricity and town-gas by means of the time-of-use pricing (TOUP) are studied. Based on the framework of surplus analysis, an energy supply/demand model of nonlinear programming type is developed. Then, a case study using this model is carried out. The social welfare obtained by TOUP is estimated and compared with those by other pricing schemes such as the seasonal pricing (SP) and the constant pricing (CP) through numerical simulations. By those analyses and simulations, the welfare economical effect of TOUP adopted cooperatively in the electricity and the town-gas systems is made clear quantitatively.

4.2 Cooperation of Electricity and Town-Gas Supply

As mentioned in Chapter 2, the electric power systems in the urbanized areas of Japan have their peak loads in summer afternoon which are brought about by the demand for air conditioning. Contrary to this, the town-gas systems have their salient peak loads in winter evening which are brought about by the demand for space and water heating. Namely, these energy systems have their peak loads in different seasons or time-of-days, and their peak loads are both brought about by the demand for heat.

From the modern technological point of view, these demands are substitutable between the two energy utilities. Recently a remarkable progress of the technology in this field is seen. For example:

- (1) Space heating by using the Electric Heat Pump,
- (2) Air conditioning by using the Gas Engine Heat Pump or the Gas Absorption Heat Pump /4.1/, and
- (3) Cogeneration by using the Gas Engine or the Gas Turbine

are attracting our attentions. In future, the improvement of the performance of the heat pump and the development of the fuel cell will make the inter-energy substitution much easier.

Taking notice of the aforesaid possibility of inter-energy substitution between the electricity and the town-gas demands, it can be an effective policy to solve the peak load problems faced by the two supply systems by encouraging substitution of their demands. In the present chapter, a cooperative adoption of the time-of-use pricing (TOUP) as means to induce a satisfactory inter-energy substitution is studied.

First, an energy supply/demand model of nonlinear programming type is developed to investigate the issue. Then a simulation study is carried out by taking Kinki District at the year of 2000 as a study area, and the effectiveness of this policy is evaluated quantitatively.

4.3 The Energy Supply/Demand Model of Nonlinear Programming Type

(a) Structure of the Model

So far, energy models of linear programming type (LP model) have often been used for normative studies of the energy supply/demand structure /4.2, 4.3, 4.4/. An LP Model is a model which minimizes the total cost needed to meet the prespecified final energy demand by fuel selection and capacity adjustment of the supply system. Hence, a model of this type is not adequate to study the effect of load management because the final demand itself should be altered in this policy. Concerning the price, the marginal costs to supply the final demands can be measured as the simplex multipliers with the LP model/4.2/. But it is difficult in the LP model to treat the pricing scheme itself explicitly.

In this chapter, an energy model of nonlinear programming (NLP) type is presented. The formulation of the model is based on the surplus theory, and it can cope with the aforesaid difficulty faced by the LP model. In the NLP model, the final demand of the various sorts of energy and their prices are endogenized as well as the operations and the capacities of the energy supply systems. Considering the demand-side influences of the load management, the sum of the consumers' surplus and the producers' surplus is taken as an objective function to be optimized instead of the total supply cost, which is the objective function in the LP model. Thus the present NLP model is a sort of the extended version of the energy model of LP type /4.5/. The structure of the NLP model is illustrated in Fig. 4.1.

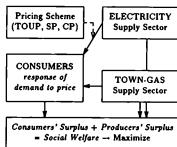


Fig. 4.1 Structure of the NLP model.

(b) Time Division

The time span treated in the present model is one year. Considering the seasonal variation of the electricity and the town-gas loads, one year is divided into three seasons, i.e., summer, winter and spring/autumn. Furthermore, each season is divided into five periods reflecting the daily load variations of the both energy loads. Namely, one year is divided into 15 periods. In Table 4.1, the time division is shown.

In the present model, we use a time division corresponding directly to season and time-of-day instead of a time division based on the load duration curves which has often been used in energy optimization models. It is because the inter-energy substitution considered in the model should occur only between the electricity and town-gas demands at the same time-of-day in the same season. With the time division based on the load dura-

Table 4.1 Time Division of the NLP Model

Summer	Period		Time-of-Day	Duration D_i [hour]
	Winter	Spring/Autumn		
1	6	11	0:00- 8:00	638.75
2	7	12	8:00-13:00	456.25
3	8	13	13:00-18:00	456.25
4	9	14	18:00-22:00	365.0
5	10	15	22:00-24:00	275.75

* The figures in the column 'Duration' represent the duration of the periods in summer and winter. Those in spring/autumn are twice as long as the values shown in the table.

tion curve, it is difficult to represent such simultaneous change of the both energy demands.

(c) Endogenized Variables

The time-of-use demands and prices of electricity and town-gas are endogenized as well as the operation and capacities of the supply systems of these two sorts of energy. In the electricity supply, four sorts of generating plants are considered, i.e., the nuclear power, the LNG-fired power, the oil-fired power and the hydraulic power. Their capacities and the time-of-use operations are endogenized except those of the hydraulic power plant, which are given exogenously. The town-gas is supposed to be supplied by one sort of plant. For simplicity, utilization of the gas-holder is not considered. The endogenized variables are listed in Table 4.2. All the endogenized variables are constrained to be nonnegative.

Table 4.2 Endogenized Variables

Variable	Meaning	Unit
q_B	Electricity Demand	$[10^{12} \text{ kcal}]$
q_G	Town-Gas Demand	$[10^{12} \text{ kcal}]$
$x_{NUC,t}$	Operation of Nuclear Power	$[10^{12} \text{ kcal}]$
$x_{LNG,t}$	Operation of LNG-Fired Power	$[10^{12} \text{ kcal}]$
$x_{OIL,t}$	Operation of Oil-Fired Power	$[10^{12} \text{ kcal}]$
k_{NUC}	Capacity of Nuclear Power	$[10^{12} \text{ kcal/year}]$
k_{LNG}	Capacity of LNG-Fired Power	$[10^{12} \text{ kcal/year}]$
k_{OIL}	Capacity of Oil-Fired Power	$[10^{12} \text{ kcal/year}]$
k_{GAS}	Capacity of Town-Gas Plant	$[10^{12} \text{ kcal/year}]$
p_B	Electricity Price	$[10^{10} \text{ yen}/10^{12} \text{ kcal}]$
p_G	Town-Gas Price	$[10^{10} \text{ yen}/10^{12} \text{ kcal}]$

(d) Constraints

The following constraints are considered in the energy supply/demand.

Inverse Demand Function The time-of-use energy demands and prices are related by the following inverse demand function:

$$p = -Aq + p_0 \quad (4.1)$$

where $p = (p_{E1}, \dots, p_{E15}, p_{G1}, \dots, p_{G15})^T$ is a time-of-use price vector, and $q = (q_{E1}, \dots, q_{E15}, q_{G1}, \dots, q_{G15})^T$ is a time-of-use demand vector. A is a 30×30 constant matrix, and p_0 is a 30-dimensional constant vector. The matrix A is assumed to be symmetric. It is required to define the consumers' surplus /4.6/.

Demand Supply Balance The sum of the outputs of the four electricity generating plants must exceed the electricity demand in each period:

$$q_{Ei} \leq x_{NUC,i} + x_{LNG,i} + x_{OIL,i} + x_{HYD,i}, \quad i = 1, \dots, 15 \quad (4.2)$$

where $x_{NUC,i}$, $x_{LNG,i}$ and $x_{OIL,i}$ denote the outputs of the nuclear, the LNG-fired and the oil-fired power plants at the period i , respectively, and $x_{HYD,i}$ is the output of the hydraulic power plant in the period i given exogenously.

Fair Preparation of Electricity Supply Capacity The sum of the capacities of the electricity generating plants must keep a fair preparation rate to the peak load:

$$q_{Ei}(1 + \epsilon) \leq (k_{NUC} + k_{LNG} + k_{OIL} + k_{HYD})D_i/8760, \quad (4.3)$$

$$i = 1, \dots, 15$$

where D_i is the duration of the period i (the number 8760 means the total hours in a year), ϵ is a fair rate of the preparatory capacity to the peak load. k_{NUC} , k_{LNG} and k_{OIL} are the capacities of the nuclear, the LNG-fired and the oil-fired power plants, respectively, and k_{HYD} is the capacity of the hydraulic power given exogenously.

Upper Bound of Plant Operation The output of each plant cannot exceed the limit decided by its capacity:

$$x_{j,i} \leq k_j D_i / 8760,$$

$$j = NUC, LNG \text{ and } OIL, \quad i = 1, \dots, 15 \quad (4.4)$$

$$q_{Gi} \leq k_{GAS} D_i / 8760, \quad i = 1, \dots, 15$$

where k_{GAS} is the capacity of the town-gas supply plant.

Upper and Lower Bounds of Plant Construction The capacities of the electricity and the town-gas supply plants must be larger than the existing

levels, and the capacity of the nuclear power plant is also constrained by a prespecified upper bound:

$$\begin{aligned} \underline{k}_j &\leq k_j, \\ j &= NUC, LNG, OIL \text{ and } GAS \\ k_{NUC} &\leq \bar{k}_{NUC} \end{aligned} \quad (4.5)$$

where $\underline{k}_j$ is the existing capacity, and $\bar{k}_{NUC}$ is the upper limit of the construction of the nuclear power plant.

Fair Annual Outputs of Electric Power Plants In each type of plants, the ratio of the annual output to the output with the full operation through the year must be below a prespecified fair value:

$$\sum_{i=1}^{15} x_{j,i} \leq k_j r_j, \quad j = NUC, LNG \text{ and } OIL \quad (4.6)$$

where r_j is a fair ratio of the annual operation of the plant j to the total energy produced by the full operation through the year.

Constant Operation of Nuclear Power Plant The output of the nuclear power plant cannot follow load fluctuation so quickly. Considering this, the nuclear power plant should be operated at a constant level in each season:

$$\begin{aligned} x_{NUC,i}/D_i &= x_{NUC,i+1}/D_{i+1}, \\ i &= 1, \dots, 4, 6, \dots, 9, 11, \dots, 14. \end{aligned} \quad (4.7)$$

Balance of the Revenue and the Cost The revenue and supply cost should be balanced in each of electricity and town-gas:

$$\sum_{i=1}^{15} P_{Ei} q_{Ei} = C_E \quad (4.8)$$

$$\sum_{i=1}^{15} p_{Gi} q_{Gi} = C_G \quad (4.9)$$

where C_E and C_G are, respectively, the supply costs of electricity and town-gas. These are defined by the following linear functions:

$$C_E = \sum_j \left(\sum_{i=1}^{15} c_{Oj} x_{ji} + c_{Cj} k_j \right) + c_{FE} \quad (4.10)$$

$$C_G = \sum_{i=1}^{15} c_{OGi} q_{Gi} + c_{CG} k_{GAS} + c_{FG} \quad (4.11)$$

The summation with respect to j in Eq. (4.10) means that for $j = NUC, LNG$ and OIL . The coefficients c_{Oj} and c_{Cj} are, respectively, the unit operating cost and the unit capacity cost of plant j . The coefficients c_{FE} , c_{OG} , c_{CG} and c_{FG} are the fixed cost of the electricity supply, the unit operating cost, the unit capacity cost and the fixed cost of the town-gas supply, respectively.

Pricing Scheme The pricing schemes such as the constant pricing (CP) or the seasonal pricing (SP) are represented by adding some constraints on the time-of-use pricing:

$$\begin{aligned} p_{Ei} &= p_{E,i+1} \\ p_{Gi} &= p_{G,i+1}, \\ i &= 1, \dots, 14 \text{ for CP;} \\ i &= 1, \dots, 4, 6, \dots, 9, 11, \dots, 14 \text{ for SP} \end{aligned} \quad (4.12)$$

(e) Objective Function

The sum of the consumers' surplus and the producers' surplus is taken as an objective function to be maximized, which is called 'the social welfare SW ':

$$\begin{aligned} SW &= \int_L p dq - C_E - C_G \\ &= -\frac{1}{2} q^T A q + p_0^T q - C_E - C_G. \end{aligned} \quad (4.13)$$

The assumption of symmetricity of the matrix A is required to make the line integral $\int_L p dq$ in Eq. (4.13) independent of the integration path /4.6/.

4.4 Estimation of the Model Parameters

The parameters of the model presented in the previous section are estimated by taking the Kinki District at the year of 2000 as a study area. The chosen area includes three large cities, i.e., Osaka, Kobe and Kyoto, and the peak load problem of the electric power system in summer is quite serious in the area. The service area of the electric power system and that of the town-gas system are overlapping favorably in the urbanized area of the district. Because of these characteristics, the chosen area is adequate for the simulation study with the present model. Data for parameter estimation are picked up mainly from the references /2.2/, /2.3/, /2.6/, /3.6/ and /4.7/.

(a) Estimation of the Inverse Demand Function

Since TOUP for energy utilities has not been adopted yet in Japan, it is impossible to estimate the inverse demand function statistically using the real data of energy demand/supply. In the present study, it is estimated under the following assumptions:

Assumption 1

The electricity demands in the peak (13:00-18:00), the middle (8:00-13:00 and 18:00-22:00) and the off-peak (22:00-8:00) periods will grow at 3.5%, 3.0% and 2.5% a year, respectively, until 2000 if the prices of electricity and town-gas are kept at the levels in 1982 /2.2, 2.3/. At the same time, the town-gas demands in all the periods will grow at 4.0% a year.

Assumption 2

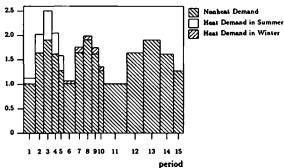
The energy demands in spring/autumn are assumed to be nonheat demands. The incremental demands in summer or winter periods from the corresponding period in spring/autumn are assumed to be heat demands. Though the incremental demands of town-gas in summer is a bit negative, they are assumed to be null. See Fig. 4.2.

Assumption 3

The nonheat demand function is assumed to be linear and to have no inter-period and inter-energy cross price elasticities. The own price elasticity of the nonheat demand is assumed to be η . Selection of the value of η is mentioned later.

Under the assumptions 1 through 3, the following nonheat demand functions are obtained:

Electricity Demand [10^{10} kcal/hour]



Town-Gas Demand [10^{10} kcal/hour]

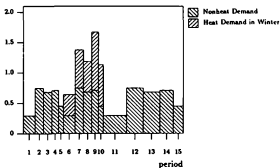


Fig. 4.2 Separation of the demands for electricity and town-gas into the heat and the nonheat demands.

$$q_{NEi} = \alpha_{NEi} p_{Ei} + \bar{q}_{NEi} \quad (4.14)$$

$$q_{NGi} = \alpha_{NGi} p_{Gi} + \bar{q}_{NGi}, \quad i = 1, \dots, 15$$

where q_{NEi} and q_{NGi} are the nonheat demands of electricity and town-gas in period i , respectively. α_{N*} and $\bar{q}_{N*}$ are the constants determined based upon the predicted demands in 2000, the energy prices in 1982, and the price elasticity of the demand η . The formulas giving these constants are described in Appendix F.

Assumption 4

The efficiencies (COPs, coefficients of performance) of the electric and the town-gas air conditioners are assumed to be 4.0 and 1.0, respectively /4.8/. If the price of town-gas is reduced and the operating cost of the air conditioner is equal to that of the electric air conditioner with the electricity price in 1982, the heat demand in summer will be shared at the ratio $s : 1 - s$ ($0 \leq s \leq 1$) between electricity and town-gas measured in the final calories when the total amount of the heat demand is kept unchanged. The parameter s stands for inter-energy substitutability of the heat demand. A small value of s means that the heat demand can be easily substituted between the two sorts of energy. Selection of the value of parameter s is mentioned later.

Assumption 5

The efficiencies of the electric and the town-gas space heaters are assumed to be 0.9 and 0.72, respectively /4.8/. If the price of electricity is reduced and the operating cost of the electric space heater is equal to that of the town-gas space heater with the town-gas price in 1982, the heat demand in winter will be shared at the ratio

1 - s : s ($0 \leq s \leq 1$) between electricity and town-gas measured in the final calories when the total amount of the heat demand is kept unchanged. For simplicity, the parameter s is assumed same with that appeared in Assumption 4.

Assumption 6

If the prices of electricity and town-gas are changed at the same ratio simultaneously, the heat demands in summer and in winter will be changed at the elasticity η near the prices in 1982. For simplicity, the parameter η is assumed same with that appeared in Assumption 3.

Under Assumptions 4 through 6, the following heat demand functions in linear form are obtained:

$$q_{HEi} = \alpha_{HEEi} p_{Ei} + \alpha_{HEGi} p_{Gi} + \bar{q}_{HEi} \quad (4.15)$$

$$q_{HG_i} = \alpha_{HGEi} p_{Ei} + \alpha_{HGGi} p_{Gi} + \bar{q}_{HG_i}, \quad i = 1, \dots, 15$$

where q_{HEi} and q_{HG_i} are the heat demand for electricity and town-gas in period i . α_{HEi} and $\bar{q}_{HEi}$ are the constants determined based on the predicted demands in 2000, the energy prices in 1982, and the parameters s and η . The formulas which give these constants are also described in Appendix F.

In the present study, the two sets of values are assumed for the parameters s and η . They are shown in Table 4.3. The total demand function are obtained by summing the nonheat demand function (4.14) and the heat demand function (4.15). In order to define the consumers' surplus, the demand functions are symmetrized as follows:

$$\begin{aligned}
\alpha'_{HEGi} &= \alpha'_{HGEi} = (\alpha_{HEGi} + \alpha_{HGEi})/2 \\
\bar{q}'_{HEi} &= \bar{q}_{HEi} - (\alpha_{HGEi} - \alpha'_{HEGi})\bar{p}_{Gi} \\
\bar{q}'_{HGi} &= \bar{q}_{HGi} - (\alpha_{HGEi} - \alpha'_{HEGi})\bar{p}_{Ei}, \quad i = 1, \dots, 10
\end{aligned} \tag{4.16}$$

where $\alpha'_{H.}$ and $\bar{q}'_{H.}$ are the coefficients in the demand function after symmetrization. $\bar{p}_{Ei}$ and $\bar{p}_{Gi}$ are the prices of electricity and town-gas in 1982. As the matter of course, the above symmetrization should not distort the demand function largely. This point is checked numerically for the cases used in the simulation. Inverting the estimated demand function, the inverse demand function (4.1) is obtained.

(b) Parameters of the Supply System

The unit operating costs and the unit capacity costs of the considered generating plants are determined base on the calculation by the Agency of Resource and Energy in 1982 /3.6/. The fixed cost of the electric power system is estimated from the financial statements of the company which supply the electricity in the region. The unit operating cost, the unit capacity cost and the fixed cost of the town-gas supply are estimated from the financial statements of the company which supply the town-gas in most part of the urbanized areas.

The lower bounds of the capacities are set as much as the existing capacities in 1982. The upper bound of the nuclear power capacity is determined based on the long-run energy supply/demand estimation in 1982. The capacity and the operation of the hydraulic power are as much as those in 1981. The fair preparation ratio of the electric power capacity, ϵ is set to 0.15. The fair operation ratios of the generating plants, r_{NUC} ,

r_{LNG} and r_{OIL} are set to 0.7, 0.8 and 0.8, respectively.

4.5. Results of the Simulation

Simulation is carried out considering the two cases shown in Table 4.3. The following four options are considered as the combination of the pricing schemes:

(Pricing in Electric Power, Pricing in Town-Gas) =
(TOUP, TOUP), (TOUP, SP), (SP, SP) and (CP, CP)

The nonlinear optimization problems are solved by the computer program based on Powell's method /4.9, 4.10/.

Table 4.3 Values of the parameters η and s

Case	η	s	Notes
1	-0.4	0.5	Case of a high price elasticity and a high inter-energy substitutability
2	-0.2	0.7	Case of a low price elasticity and a low inter-energy substitutability

(a) Demand/Supply of the Energy

The results of the simulation are presented in Table 4.4 and Table 4.5. The demand/supplies and the prices in case 1 are illustrated in Fig. 4.3. Table 4.4 and Table 4.5 show that the improvement of the social welfare amounts to 14.3 ~ 20.1 [10^{10} yen/year] when the pricing option is changed from (CP, CP) to (SP, SP), and that it is 18.2 ~ 25.2

[10^{10} yen/year] when the pricing option is changed from (CP, CP) to (TOUP, TOUP). It amounts to 5 ~ 6% of the total supply cost. At the same time, the load factors are improved by 13 ~ 16% in the electric power system, and by 4 ~ 5% in the town-gas system. It must be noted that the obtained load factors are calculated from the 15-period rectangular approximation of the load, and the influences of the keen peak load with short duration are not considered. Therefore the presented values of the load factor get higher than the published value /2.2/.

Concerning the capacities of the generating plants, it is shown that the expenditure for capacities is reduced by adopting SP or TOUP. The nuclear power plant is constructed to its upper bound in any pricing options. The construction of the other power plants needed in the option (CP, CP) is suppressed in the other options. On the capacity of the town-gas system, remarkable differences cannot be seen. It is because the existing capacity of the town-gas is considerably large.

(b) Structure of the Demand

On the structure of the energy demand, the inter-energy substitution of the heat demand in summer, i.e., the demand for air conditioning, is remarkable. By adopting TOUP or SP, the air conditioning by town-gas is induced. It contributes to suppression of constructing the electricity generating plant, and then to improvement of the load factor. In contrast, the heat demand in winter does not move remarkably from town-gas to electricity. It can be interpreted that, according to reduction of the summer electricity load, the electric power plants are fully operated even in winter, and then increase in winter load needs capacity expansion which is not economic any longer.

Table 4.4 Results of Simulation (Case 1)

Pricing Scheme	Electricity	TOUP	TOUP	TOUP	SP	CP
	Town-Gas	TOUP	SP	CP	SP	CP
Social Welfare ¹⁾	[10 ¹⁰ yen/year]	25.19	24.87	23.33	20.09	0.0
Capacity & Fixed Cost	Electricity	143.29	143.29	143.38	147.64	170.96
	Town-Gas	26.86	26.86	26.86	26.86	26.86
Operating Cost	Electricity	143.59	144.64	146.65	141.80	143.35
	Town-Gas	61.92	61.04	58.04	63.74	51.77
Newly Constructed Plant	[10 ¹³ kcal/year]					
	Nuclear ²⁾	5.68	5.68	5.68	5.68	5.68
	LNG-Fired	0.0	0.0	0.0	0.0	4.32
	Oil-Fired	0.0	0.0	0.02	1.09	1.64
	Town-Gas	0.0	0.0	0.0	0.0	0.0
Heat Demand	[10 ¹⁶ kcal/year]					
Summer	Electricity	0.623	0.607	0.712	0.467	0.932
	Town-Gas	0.398	0.443	0.119	0.732	-0.337 ³⁾
Winter	Electricity	0.282	0.381	0.370	0.366	0.447
	Town-Gas	1.207	1.058	1.048	1.065	0.915
Nonheat Demand	Electricity	14.011	14.008	14.007	13.966	13.701
	Town-Gas	4.927	4.938	4.955	4.927	4.883
Load Factor [%]	Electricity	78.2	78.9	81.3	73.9	62.2
	Town-Gas	49.0	48.3	45.9	50.4	44.8

- 1) The social welfares are represented by the relative deviation from that in the option (CP, CP).
- 2) The amount of newly constructed nuclear power plant meets its upper bound.
- 3) Heat demand can be negative because the nonlinear programming is solved with consideration of the inverse demand function of the total energy demand only.

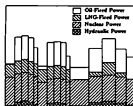
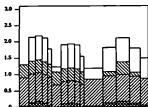
Table 4.5 Results of Simulation (Case 2)

Pricing Scheme	Electricity	TOUP	TOUP	TOUP	SP	CP
	Town-Gas	TOUP	SP	CP	SP	CP
Social Welfare	[10 ¹⁰ yen/year]	18.24	18.19	17.87	14.33	0.0
Capacity & Fixed Cost	Electricity	143.29	143.29	143.38	144.89	165.01
	Town-Gas	26.86	26.86	26.86	26.86	27.11
Operating Cost	Electricity	135.58	133.81	135.46	131.16	131.70
	Town-Gas	60.53	60.34	58.33	63.77	55.56
Newly Constructed Plant	[10 ¹³ kcal/year]					
	Nuclear	5.68	5.68	5.68	5.68	5.68
	LNG-Fired	0.0	0.0	0.0	0.0	3.27
	Oil-Fired	0.0	0.0	0.0	0.35	1.44
	Town-Gas	0.0	0.0	0.0	0.0	0.34
Heat Demand	[10 ¹⁰ kcal/year]					
Summer	Electricity	0.648	0.647	0.720	0.453	0.852
	Town-Gas	0.293	0.296	0.098	0.727	-0.182
Winter	Electricity	0.279	0.293	0.304	0.328	0.297
	Town-Gas	1.180	1.155	1.135	1.094	1.140
Nonheat Demand	Electricity	13.419	13.418	13.428	13.407	13.287
	Town-Gas	4.927	4.912	4.915	4.920	4.906
Load Factor [%]	Electricity	75.2	75.3	75.8	73.2	62.3
	Town-Gas	47.9	47.7	46.1	50.4	42.8

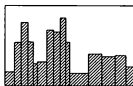
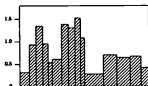
(c) The influences of the revenue-cost balance

To examine the influences of the revenue-cost balance constraints, simulations with and without the constraints are carried out. The results are compared in Table 4.6. If the revenue-cost balance is not imposed, the prices in the option (TOUP, TOUP) are equal to the marginal supply costs/4.11, 4.12/. By imposing the revenue-cost balance, the social welfare

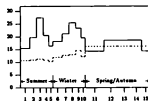
Electricity Demand [10^{16} kcal/hour]



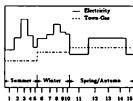
Town-Gas Demand [10^{16} kcal/hour]



Energy Price [yen/ 10^6 kcal]



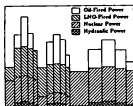
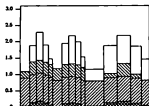
(a) period



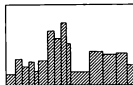
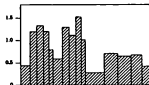
(b) period

Fig. 4.3 Demand/supplies and prices in case 1,
(a): (TOUP, TOUP), (b): (TOUP, SP).

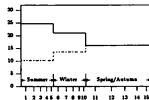
Electricity Demand [10^{14} kcal/hour]



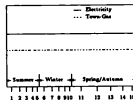
Town-Gas Demand [10^{14} kcal/hour]



Energy Price [yen/ 10^6 kcal]



(c)



(d)

Fig. 4.3 Demand/supplies and prices in case 1,
(c): (SP, SP), (d): (CP, CP).

is lost in the option (CP, CP) by $11 \sim 17$ [10^{10} yen/year]. The more flexible the pricing scheme be, the less such a welfare loss becomes. In the option (TOUP, TOUP), the welfare loss is reduced to $2.5 \sim 4$ [10^{10} yen/year].

Table 4.6 Influences of the Revenue-Cost Balance Constraints

Pricing of Electricity	TOUP	TOUP	TOUP	SP	CP
Pricing of Town-Gas	TOUP	SP	CP	SP	CP
Case 1	3.98	3.95	5.06	5.51	17.08
Case 2	2.48	2.35	2.77	3.52	10.96

The figures in the table show the incremental social welfare obtained by removing the revenue-cost balance constraints. The unit is [10^{10} yen/year].

(d) Consideration of the metering cost

To implement the time-of-use pricing, the metering cost to measure the time-of-day load is needed while the seasonal pricing does not need such an additional cost. For feasibility of TOUP, the welfare gain obtained by this pricing scheme must exceed the metering cost. The cost of TOUP meter (for electricity) is estimated at 1.4[\$/Month-Customer]/4.13/. The study area has about 8 million customers of electricity and 4.5 million customers of town-gas. If the TOUP meters are installed at all the customers, the total cost will be about 40 billion [yen/year]. On the other hand, The welfare gain estimated by the previous simulation is about $39 \sim 51$ billion [yen/year] (the difference between the SWs in (TOUP, TOUP) and (SP, SP)). This welfare gain is able to justify the aforesaid metering cost. If TOUP is adopted only to the large size custo-

mers, the metering cost will be reduced remarkably with slight loss of the effect of TOUP. Considering that the welfare gain obtained by the option (TOUP, SP) is not so different from that by (TOUP, TOUP), the former option may be more advantageous.

4.6 Concluding Remarks

In this chapter, economical effects of the cooperative supply of electricity and town-gas in the framework of time-of-use pricing (TOUP) scheme is studied. An energy supply/demand model of nonlinear programming type is developed, and the simulation study is carried out using the model by taking Kinki District in 2000 as a study area. The results of the study is summarized as follows:

- 1) By adopting the seasonal pricing (SP) instead of the constant pricing (CP), the improvement of the social welfare will be 3.8 ~ 5.1% of the total supply cost. If the time-of-use pricing (TOUP) is adopted, the incremental improvement of the social welfare is about 1% of the supply cost.
- 2) The welfare gain obtained by TOUP is enough to justify the metering cost needed for implementation of TOUP.
- 3) Considering the metering cost, the option of TOUP for electricity and SP for town-gas is the most advantageous one.

In the present study, as we pointed out in the previous chapter, the most indefinite factor is the response of the load to the price. To carry out the simulation, the demand functions are estimated with several assumptions. The simulation results reflect some drawbacks of the present approach. One is neglect of the inter-period relation of the demand. In

estimating the demand function, the inter-period cross price elasticity of the demand is not considered. Hence the shares of the electricity and town-gas in the heat demand change largely by time-of-day in one season. Such a behavior may not be realistic, and time-of-day energy demands for heat in a season should be complementary to some extent because each consumer would use the same apparatus for air conditioning and for space heating and, accordingly, he would use one energy source.

Another point is disregard of the advanced technologies in heat utilization. The demand function is estimated with the assumption that the electricity is used with a conventional electric heater for space heating. The utilization of electric heat-pump is not considered in the present study while it is expected to play an important role in future space heating.

Another defect in the present study is lack of consideration of the dynamical process of the load management and capacity expansion. By expanding the present model to the multistage one, the problem of the dynamical process can be treated. However, in such a model, the optimization will be difficult much more because of the increase in numbers of the variables and the constraints. For the large scale nonlinear optimization models, we might have to give up getting the optimal price, and to be satisfied with evaluation and comparison of the prespecified pricing options /4.14/.

CHAPTER 5 A STUDY ON THE COMPETITION BETWEEN ELECTRICITY AND TOWN-GAS SUPPLIERS UNDER TIME-OF-USE PRICING

5.1 Introduction

In the previous chapter, we have investigated a policy of supplying electricity and town-gas cooperatively with use of the time-of-use pricing. It has assumed a complete cooperation of the suppliers of the two sorts of energy. However, actually in Japan, they are supplied by different private companies having their own goals, e.g., profit and sale. Hence there exists a certain competition between these companies with strategic time-of-use pricing for the mutually substitutable demands.

Since the supply system needs huge amount of investments as mentioned in Chapter 2, these energy utility companies are allowed to supply the energy monopolistically in their service areas. At the same time, some regulations are imposed on their pricing policies in order to prevent the suppliers from getting monopolistic profits and to protect the public welfare. The most typical regulation is to require the profit of the supplier to keep a fair ratio to its investment.

Behaviors of a regulated monopolistic company have been studied by Averch and Johnson /5.1/. They have shown that the regulation of the profit causes over-capitalization because the company increase investment in order to raise the ceiling of its profit (A-V effect), and then it gives a negative effect on the supply/demand efficiency. Bailey /5.2/, and Bailey and White /5.3/ have studied time-of-use price made by a regulated monopolistic company. They have compared several regulation rules from

the viewpoint of welfare economics. They have pointed out a possibility of reversals in the peak and the off-peak prices in such situation (B-W effect).

In this chapter, the competition between an electricity and a town-gas supplier under TOUP is studied. The situation is formulated into a competition problem between two regulated companies which supply utilities partially substitutable. For this, a game model of static type is proposed. Then, the characteristics of the equilibrium prices are discussed analytically. Some numerical examples are also presented to illustrate the results.

5.2 Game Model of Inter-energy Competition

(a) Formulation of the model

Let us consider the supply/demand of electricity and town-gas in N periods. The electricity supplier decides the time-of-use price of electricity $p_E = (p_{E1}, \dots, p_{EN})^T$ in these N periods, and the town-gas supplier decides the time-of-use price of town-gas $p_G = (p_{G1}, \dots, p_{GN})^T$ as well. Responding to these prices, the time-of-use demand of electricity $q_E = (q_{E1}, \dots, q_{EN})^T$ and that of town-gas $q_G = (q_{G1}, \dots, q_{GN})^T$ in the N periods are determined according to demand functions D_E and D_G , respectively:

$$\begin{aligned} q_E &= D_E(p_E, p_G) \\ q_G &= D_G(p_E, p_G). \end{aligned} \tag{5.1}$$

As a matter of course, the vectors p_E , p_G , q_E and q_G are constrained to be nonnegative.

The goal of each supplier is assumed to be maximization of its total sale. Namely, the payoff function of the electricity supplier f_E and that of the town-gas supplier f_G are, respectively:

$$\begin{aligned} f_E &\equiv p_E^T q_E \\ f_G &\equiv p_G^T q_G. \end{aligned} \quad (5.2)$$

We consider a regulation that each supplier is required to keep a fair ratio of its profit to the total supply cost:

$$\begin{aligned} f_E - C_E(q_E) &\leq \epsilon C_E(q_E) \\ f_G - C_G(q_G) &\leq \epsilon C_G(q_G) \end{aligned} \quad (5.3)$$

where ϵ is an upper limit of the profit ratio to the total supply cost, and $C_E(q_E)$ and $C_G(q_G)$ are the total supply costs of the electricity and the town-gas, respectively.

The problem of the competition between the suppliers are formulated as the following game problem:

$$\begin{aligned} \max_{p_E} \quad & f_E \quad \text{for the Electricity Supplier} \\ \max_{p_G} \quad & f_G \quad \text{for the Town-Gas Supplier} \\ \text{sub. to} \quad & g(p_E, p_G) \leq 0 \end{aligned} \quad (5.4)$$

where a vector function $g(p_E, p_G) = (g_1, \dots, g_{4N+2})^T$ is defined as follows:

$$g_1 \equiv f_E - (1 + \epsilon)C_E \quad (5.5a)$$

$$g_2 \equiv f_G - (1 + \epsilon)C_G \quad (5.5b)$$

$$(g_3, \dots, g_{2+N})^T \equiv -p_E \quad (5.5c)$$

$$(g_{3+2N}, \dots, g_{2+2N})^T \equiv -p_G \quad (5.5d)$$

$$(g_{3+2N}, \dots, g_{2+2N})^T \equiv -q_E = -D_E(p_E, p_G) \quad (5.5e)$$

$$(g_{3+2N}, \dots, g_{2+2N})^T \equiv -q_G = -D_G(p_E, p_G). \quad (5.5f)$$

Equations (5.5a) and (5.5b) correspond to the regulatory constraints (5.3), and Eqs. (5.5c) through (5.5f) are for nonnegativity of the prices and the demands.

(b) Concept of the solution

Let us consider the problem (5.4) with an assumption that the both suppliers behave noncooperatively, and confine our discussion only to pure strategies. In noncooperative game problems, the most acceptable concept of the solution is the Nash equilibrium /5.5/. For the problem (5.4), the Nash equilibrium (p_E^N, p_G^N) is defined as follows:

$$\begin{aligned} f_E(p_E^N, p_G^N) &\geq f_E(p_E, p_G^N), \text{ for all } p_E \text{ such that } g(p_E, p_G^N) \leq 0 \\ f_G(p_E^N, p_G^N) &\geq f_G(p_E^N, p_G), \text{ for all } p_G \text{ such that } g(p_E^N, p_G) \leq 0. \end{aligned} \quad (5.6)$$

However, in a problem with constraints such as problem (5.4), it is known that there can exist infinite number of the Nash equilibria on the boundary of its feasible region /5.4/. In order to avoid this difficulty, we introduce a more strict concept of equilibrium, 'the normalized Nash equilibrium' which is proposed by Rosen /5.4/. The normalized Nash equilibrium (p_E^*, p_G^*) for the problem (5.4) is defined as follows /5.5/:

$$\begin{aligned}
& f_E(p_E^*, p_G^*) + f_G(p_E^*, p_G^*) = \\
& \max_{p_E, p_G} [f_E(p_E, p_G) + f_G(p_E, p_G)] \\
& \text{sub. to } g(p_E, p_G) \leq 0.
\end{aligned} \tag{5.7}$$

Rosen has shown that the normalized Nash equilibrium is unique under certain conditions /5.4/. He has also shown that the equilibrium is achieved by a process in which the both players adjust their decision variables according to the projected gradients of their payoff functions to the feasible region /5.4/. This process is quite similar to a famous process proposed by Cournot as a model of duopoly /5.6/.

5.3 Analytical Study

In order to discuss the natures of the equilibrium prices of the problem (5.4), let us make some assumptions as follows:

- 1) There is no inter-period cross price elasticity of the demand, while there are some nonzero inter-energy cross price elasticities of the simultaneous demands.
- 2) The equilibrium prices (and the associated demands) are positive.
- 3) The fair profit ratio to the cost, ϵ , is assumed to be 0 for simplicity. This assumption can be easily relaxed.
- 4) The functions f_E , f_G , C_E , C_G , D_E and D_G are continuously differentiable with respect to both the price and the demand.
- 5) There exists at least one equilibrium point, and at that point, Kuhn-Tucker constraint qualification /5.5, 5.7/ holds.

With the above assumptions, the necessary conditions for the normalized Nash equilibrium* for the problem (5.4) are:

$$\begin{aligned}
 & -\nabla_{p_E} f_E + \lambda_E \nabla_{p_E} g_1 + \lambda_G \nabla_{p_E} g_2 = 0 \\
 & -\nabla_{p_G} f_G + \lambda_E \nabla_{p_G} g_1 + \lambda_G \nabla_{p_G} g_2 = 0 \\
 & g_1 \leq 0, \quad g_2 \leq 0 \\
 & \lambda_E g_1 = \lambda_G g_2 = 0 \\
 & \lambda_E, \lambda_G \geq 0
 \end{aligned} \tag{5.8}$$

where λ_E and λ_G are Lagrange multipliers for the regulatory constraints (5.3).

First, let us consider the problem without the regulatory constraints. The necessary conditions (5.8) are simplified as follows:

$$\begin{aligned}
 MR_E &= 0 \\
 MR_G &= 0
 \end{aligned} \tag{5.9}$$

where $MR_E \equiv \nabla_{p_E} f_E$ and $MR_G \equiv \nabla_{p_G} f_G$, i.e., the marginal change of the revenues according to the marginal changes of the prices. Here we call these quantities 'marginal revenues' for simplicity, while the word, marginal revenue, indicates usually the marginal change of the revenue according to the marginal change of the supply.

Eq. (5.9) means that, at an equilibrium point, each supplier could not change his revenue by unilateral change of his offering price.

* This set of necessary conditions is derived from a theorem which applies the Kuhn-Tucker condition for a nonlinear programming problem to a game problem /5.5/.

Then, let us consider the regulatory constraints. From the first equation of Eqs. (5.8), the following is derived readily:

$$MR_{Ei} = \frac{\lambda_E}{\lambda_E - 1} MC_{Ei} \frac{\partial q_{Ei}}{\partial p_{Ei}} + \frac{\lambda_G}{\lambda_E - 1} (MC_{Ei} - p_{Gi}) \frac{\partial q_{Gi}}{\partial p_{Ei}} \quad (5.10)$$

where MR_{Ei} is the i -th element of MR_E . $MC_{Ei} \equiv \partial C_E / \partial q_{Ei}$ and $MC_{Gi} \equiv \partial C_G / \partial q_{Gi}$, i.e., the marginal supply costs of electricity and town-gas, respectively. Taking the symmetry of the model into consideration, a similar equation for the marginal revenue of the town-gas is obtained as well.

The first term of the RHS of Eq. (5.10) means that the marginal revenue of the supplier deviates from the equilibrium without regulatory constraints according to his marginal supply cost. At the same time, the second term of the RHS implies that the equilibrium marginal revenue is also influenced by the deviation of the price offered by the competitor from his marginal supply cost.

It depends on the sign of factor of each term in the RHS of Eq. (5.10) whether the term raises or reduces the equilibrium marginal revenue from that without regulatory constraints. Let us examine the signs of these terms. The partial derivatives of the demands with respect to the price, i.e., $\partial q_{Ei} / \partial p_{Ei}$ or $\partial q_{Gi} / \partial p_{Ei}$ are determined according to the characteristics of the market. Ordinarily, the partial derivative with respect to the own price, i.e., $\partial q_{Ei} / \partial p_{Ei}$ is negative. Further, if the market of the electricity and the town-gas are substitutable, the partial derivative $\partial q_{Gi} / \partial p_{Ei}$ will be positive. Hence, to decide the signs of the factors in the RHS of Eq. (5.10), we must know the ranges of the Lagrange multipliers λ_E and λ_G . The following proposition shows that the values of the Lagrange multi-

pliers are between null and unity under some conditions.

Proposition

Under the following conditions, the values of the Lagrange multipliers appeared in Eq. (5.10) are between null and unity, i.e.:

$$0 < \lambda_E, \lambda_G < 1. \quad (5.11)$$

Conditions

- 1) At the equilibrium point, the both regulatory constraints are active.
- 2) Considering a problem which maximize C_E with respect to p_E under constraints $g_1(p_E, p_G^*) \geq 0$ and $g_2(p_E, p_G^*) \leq 0$, and with a fixed price of town-gas p_G^* , the former constraint becomes active, and the Lagrange multiplier associated with the constraint is not degenerated. Similar conditions hold for the symmetric problem with respect to C_G and p_G .
- 3) With an adequate selection of the periods i and j , the following condition holds at the equilibrium:

$$\begin{aligned} (MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) \neq \\ (MR_{Ej} - \overline{MC}_{Ej})(\overline{MR}_{Gi} - \overline{MC}_{Gi}) \end{aligned} \quad (5.12)$$

where $\overline{MC}_{Ei}$ and $\overline{MC}_{Gi}$ are $\partial C_E / \partial p_{Ei}$ and $\partial C_G / \partial p_{Gi}$, respectively. $\overline{MR}_{Gi}$ stands for $\partial f_G / \partial p_{Ei}$.

Proof

Due to the Kuhn-Tucker condition (5.8), the following equation is derived:

$$\begin{aligned} \{ (MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - \\ (MR_{Ej} - \overline{MC}_{Ej})(\overline{MR}_{Gi} - \overline{MC}_{Gi}) \} \lambda_E = \end{aligned} \quad (5.13)$$

$$MR_{Ei}(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - MR_{Ej}(\overline{MR}_{Gi} - \overline{MC}_{Gi}).$$

Then let us consider two unilateral optimization problems to choose p_E with a fixed town-gas price p_G^* :

$$\max_{p_E} f_E(p_E, p_G^*) \quad (5.14)$$

$$\text{sub. to } g_1(p_E, p_G^*) = f_E - C_E|_{p_G=p_G^*} \leq 0,$$

$$g_2(p_E, p_G^*) = f_G - C_G|_{p_G=p_G^*} \leq 0$$

and

$$\max_{p_E} C_E(p_E, p_G^*) \quad (5.15)$$

$$\text{sub. to } g_1(p_E, p_G^*) = f_E - C_E|_{p_G=p_G^*} \geq 0,$$

$$g_2(p_E, p_G^*) = f_G - C_G|_{p_G=p_G^*} \leq 0.$$

The first constraints in the both problems are active due to the required Conditions 1) and 2). Then the optimal p_E of the problem (5.14) and (5.15) coincide with each other. The equilibrium price p_E^* of the original problem (5.10) is the optimal price of the problem (5.14) because a normalized Nash equilibrium is a solution to the unilateral optimization problem (5.14), and consequently it is also a solution to the problem (5.15). Following the Kuhn-Tucker condition for the problem (5.15), the following equation is derived:

$$\begin{aligned} & \{(MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - \\ & (MR_{Ej} - \overline{MC}_{Ej})(\overline{MR}_{Gi} - \overline{MC}_{Gi})\} \mu_E = \\ & - MR_{Ei}(\overline{MR}_{Gj} - \overline{MC}_{Gj}) + MR_{Ej}(\overline{MR}_{Gi} - \overline{MC}_{Gi}) \end{aligned} \quad (5.16)$$

where μ_E is a Lagrange multiplier associated with the first constraint of the problem (5.15).

According to Eqs. (5.13), (5.16) and Condition 3), the Lagrange multipliers λ_E and μ_E satisfy the relation:

$$\lambda_E + \mu_E = 1. \quad (5.17)$$

Considering that the Lagrange multipliers are positive under Condition 2), the range of λ_E ,

$$0 < \lambda_E < 1 \quad (5.18)$$

is obtained. Because of the symmetricity of the model, the range of λ_G is also derived similarly.

QED.

If the aforesaid proposition holds, the sign of each factor appearing in the RHS of Eq. (5.10) is determined. The factor of the first term,

$\frac{\lambda_E}{\lambda_E - 1} \frac{\partial q_{Ei}}{\partial p_{Ei}}$ is nonnegative because $\lambda_E > 0$, $\lambda_E - 1 < 0$ and $\partial q_{Ei} / \partial p_{Ei} \leq 0$. Consequently, it raises the equilibrium marginal revenue MR_{Ei} from that in the case without the regulatory constraints. If the marginal revenue MR_{Ei} is decreasing with respect to the price p_{Ei} (See footnote), and if the influence of the price offered by the competitor is small, raising of the marginal revenue means that the price is discounted according to the marginal supply cost at that period.

The partial derivative of MR_D with respect to p_D is:

$$\partial MR_D / \partial q_D = 2 \partial q_D / \partial p_D + p_D \partial^2 q_D / \partial p_D^2$$

If the second term, i.e., the second order partial derivative of q_D with respect to p_D is negligible, and if $\partial q_D / \partial p_D$ is negative, which holds ordinarily, the marginal revenue MR_D is decreasing with respect to p_D .

The factor of the second term $\frac{\lambda_G}{\lambda_E - 1} \frac{\partial q_{Gi}}{\partial p_{Ei}}$ is nonpositive because $\lambda_G > 0$, $\lambda_E - 1 < 0$ and $\frac{\partial q_{Gi}}{\partial p_{Ei}} \geq 0$. Table 5.1 shows that the influences of the price offered by the competitor and his marginal supply cost on the equilibrium marginal revenue and the corresponding equilibrium price.

Table 5.1 Influence of the Competitor's Price

$MC_G - p_G$	MR_D	p_D
+	↓	↑
-	↑	↓

It is assumed that the marginal revenue $MR_D = \partial p_{Ei}^T / \partial p_D$ is decreasing with respect to p_D .

Let us examine the natures of the equilibrium prices from the viewpoint of the supply/demand efficiency. The first term of the RHS of Eq. (5.10) has an effect of discounting the price according to the marginal supply cost. If the partial derivative of the demand with respect to the price does not vary remarkably by the time-of-use, it implies that the price in the peak period is reduced much more than those in the off-peak periods. It is because the marginal supply cost in the peak period is usually higher than those in the off-peak periods. Consequently it magnifies the difference between the demands in the peak and off-peak periods, and it makes the supply/demand more inefficient. This fact has been pointed out by Bailey and White /5.3/ while their model does not take the competition between the regulated companies into consideration.

The effect of the competition appears through the second term of the RHS of Eq. (5.10). Let us suppose the situation in which a price lower than the marginal price is offered at the peak period in one energy market by the associated supplier. As shown in Table 5.1, it has an effect of raising the price of the competitor, and consequently the more demand will shift to the considering market from the other energy market. Thus the peak load can be magnified much more. The implication of the second term of the RHS of Eq. (5.10) is that the substitutable structure of the energy markets does not make the demand/supply efficient by itself if the both sorts of energy are supplied by regulated monopolistic suppliers.

5.4 Numerical Examples

In this section, a case study is carried out on the basis of the game model. First, the study area, the time division and the forms of the demand and cost functions are mentioned. Then, an algorithm to obtain the normalized Nash equilibrium of the game problem is explained briefly. Finally, the simulation results are presented.

(a) Study Area and Time Division

The study area is the same with that used in the previous chapter, i.e., Kinki district in 2000. The time division is reduced from 15 periods to 5 because of the difficulty of numerical computation. See Table 5.2. In the followings, we call the electric power company in the region 'A-company', and the dominant town-gas company 'B-company'.

Table 5.2 The Time Division of the Model

Period	Duration (hour)	Season	Time-of-Day	Comments
1	456.25	Summer	12:00-17:00	Peak Load of Electricity
2	365	Summer	17:00-21:00	Middle Load of Electricity
3	365	Winter	17:00-21:00	Peak Load of Town-Gas
4	1186.25	Winter	12:00-17:00 & 21:00-24:00	Middle Load of Town-Gas
5	6387.5	the others		Base Loads

(b) Demand and Cost Functions

Similarly to the function forms used in the previous chapter, let us consider the electricity demand $q_E = (q_{E1}, \dots, q_{E5})^T$ consisting of the nonheat demand $q_{NE} = (q_{NE1}, \dots, q_{NE5})^T$ and the heat demand $q_{HE} = (q_{HE1}, \dots, q_{HE5})^T$ of a linear form. Likewise, the town-gas demand $q_G = (q_{G1}, \dots, q_{G5})^T$ consists of nonheat demand $q_{NG} = (q_{NG1}, \dots, q_{NG5})^T$ and the heat demand $q_{HG} = (q_{HG1}, \dots, q_{HG5})^T$:

$$q_{Ei} = q_{NEi} + q_{HEi} \quad (5.19)$$

$$q_{Gi} = q_{NGi} + q_{HGi} \quad (5.20)$$

$$q_{NEi} = \alpha_{NEEi} p_{Ei} + \bar{q}_{NEi} \quad (5.21)$$

$$q_{NGi} = \alpha_{NGGi} p_{Gi} + \bar{q}_{NGi} \quad (5.22)$$

$$q_{HEi} = \alpha_{HEEi} p_{Ei} + \alpha_{HEGi} p_{Gi} + \bar{q}_{HEi} \quad (5.23)$$

$$q_{HGi} = \alpha_{HGEi} p_{Ei} + \alpha_{HGGi} p_{Gi} + \bar{q}_{HGi}, \quad i=1, \dots, 5 \quad (5.24)$$

where α_s , $\bar{q}_s$ are constants. These parameters are estimated in a similar manner to that in the previous section except the symmetrization procedure, because the present model does not require the symmetry of the

demand function. The energy demand is measured in the unit [kcal/hour]. The constraints of the nonnegativity of the demands are considered on both the nonheat and the heat demand, separately:

$$q_{NEi} \geq 0, q_{NGi} \geq 0, q_{HEi} \geq 0, q_{HG_i} \geq 0, i=1, \dots, 5. \quad (5.25)$$

The present prices and forecast demands in future which are needed to estimate the demand functions are given in Table 5.3. As to the elasticity parameter η and the substitutability parameter s introduced in the previous chapter, three cases shown in Table 5.4 are considered.

Table 5.3 Present Prices and Forecast Time-of-Use Demands

	Period	Electricity	Town-Gas
Price [yen/kcal]		0.0252	0.0144
Nonheat Demand [10^{10} kcal/hour]	1	1.6387	0.60208
	2	1.4387	0.56919
	3	1.4387	0.56919
	4	1.5014	0.57951
	5	1.2911	0.54496
Heat Demand [10^{10} kcal/hour]	1	0.51663	0.16383
	2	0.39559	0.30169
	3	0.31380	1.1688
	4	0.15839	0.55492
	5	0.0	0.0

As the supply costs of electricity and town-gas, the following function forms are used:

$$C_E = k_{EO} \sum_{i=1}^5 T_i q_{Ei} + k_{EC} (\sum_{i=1}^5 q_{Ei}^{10})^{1/10} + k_{EF} \quad (5.26)$$

$$C_G = k_{GO} \sum_{i=1}^5 T_i q_{Gi} + k_{GC} (\sum_{i=1}^5 q_{Gi}^{10})^{1/10} + k_{GF} \quad (5.27)$$

Table 5.4 Parameters Used to Estimate the Demand Functions

Parameter	Meaning	Value	Comments
η	price elasticity of demand	-0.4	in all cases
s	inter-energy substitutability	0.7	case 1 (low case)
		0.6	case 2 (middle case)
		0.5	case 3 (high case)

where T_i is the duration of the period i , and k_s are constants. The first terms of the RHSs of Eqs. (5.26) and (5.27), being proportional to the total energy demands, stand for the operating costs. The second terms stand for the capacity costs which are decided mainly by the peak demand, and the final terms are the fixed costs.

The parameter k_{EO} is estimated based on the expenditure of A-company for fuel. The parameter k_{EC} is a weighted average of the estimations of the several sorts of power plants by the Agency of Energy and Resource /3.6/ while the weights are decided according to the existing plant mixture of A-company. The parameter k_{EP} is selected to make the LHS of Eq. (5.26), taken from the financial statement of the company, balance with the RHS at the demand and capacity in 1982.

The cost parameters k_{GO} and k_{GC} are same with those used in the previous chapter, and the parameter k_{GF} are chosen in a similar manner to k_{EP} . The values of these parameters are listed in Table 5.5.

Table 5.5 Parameters of the Supply Cost Functions

Parameter	Value	Unit
c_{OE}	8.82	[10^{10} yen/ 10^{12} kcal]
c_{OG}	9.48	[10^{10} yen/ 10^{12} kcal]
c_{CE}	5.1×8760	[10^{10} yen·hour/ 10^{12} kcal]
c_{CG}	2.1×8760	[10^{10} yen·hour/ 10^{12} kcal]
c_{FE}	78.81	[10^{16} yen]
c_{FG}	-9.89	[10^{16} yen]

(c) Algorithm for Numerical Calculation

To obtain a normalized Nash equilibrium numerically, we use an algorithm which applies penalty functions /5.7/ to the pseudogradient method /5.4/. As a penalty function method to obtain Nash equilibrium, Shimizu /5.5/ has proposed a method which uses interior penalty functions. In the present study, his method is used with some modifications, i.e. exterior penalty functions are used instead of the interior penalty functions. Superiority of the exterior penalty functions to the interior penalty functions is flexibility in choosing the initial values. Justification of using exterior penalty functions in game problems is given by Kawano /5.8/. Since the model is not convex, the normalized Nash equilibrium point may not be unique. In the following simulation, an equilibrium point obtained with an initial value equal to the present prices are regarded as the solution.

(d) Results of Simulation

Simulation is carried out for the three cases of the demand functions shown in Table 5.3. These demand functions have different inter-energy substitutability. Simulation results without and with the regulatory constraints are shown in Fig. 5.1 and Fig. 5.2, respectively.

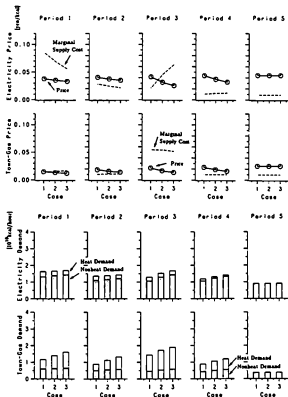


Fig. 5.1 The results of the simulation without regulatory constraints.

Case 1: Low inter-energy substitutability.

Case 2: Middle inter-energy substitutability.

Case 3: High inter-energy substitutability.

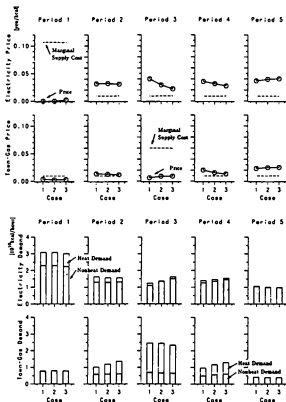


Fig. 5.2 The results of the simulation with regulatory constraints.

Case 1: Low inter-energy substitutability.

Case 2: Middle inter-energy substitutability.

Case 3: High inter-energy substitutability.

Comparing Fig. 5.2 with Fig. 5.1, the price of each energy is reduced remarkably at its peak period, i.e., Period 1 in electricity supply and Period 3 in town-gas supply, in the case with regulatory constraints. Then the peak demand is magnified much more than the case without regulation. Consequently, the B-W effect is observed even under the competitive situation.

The variations of the prices and demands according to the change of the substitutability of the demand are not clear. However, even in the case of the highest substitutability, the peak prices are still discounted remarkably, and therefore the peak demands are still large. The effect of competition summarized in Table 5.1 is not clearly observed. It may be because of the fact that the town-gas heat demand in Period 1 is null. That is to say, since one of the nonnegativity constraints (5.25) is active, the second assumption made in the beginning of the previous section does not hold in the simulation.

5.5 Concluding Remarks

This chapter is concerned with a competitive supply of electricity and town-gas under the time-of-use pricing. This situation of supply is modeled as a game problem between the regulated companies which supply the partially substitutable utilities. The analytical study on the model and the numerical simulations are presented. The main findings of the study are as follows:

- (1) If the both energy suppliers adopt the time-of-use pricing aiming at maximization of their sales, the regulation of the profit rate causes reduction of the peak price, and consequently makes the

demand/supply still more inefficient. Namely, the B-W effect is observed even in the competitive situation.

- (2) Under the regulatory constraints, the relation between the inter-energy substitutability and the demand/supply efficiency is not clear. There observed a case where the demand/supply remains still inefficient even under high inter-energy substitutability.

The implication of the above findings is that, when the time-of-use price is offered by a monopolistic energy utility company, the regulation of profit ratio is not sufficient to make the supply and demand efficient even if there exists a competition between the regulated companies. Hence, some other regulations on the time-of-use pricing, e.g., a regulation that the time-of-use price must be decided based on the time-of-use marginal supply cost, is needed to achieve the efficiency.

CHAPTER 6 GENERAL CONCLUSIONS

In this dissertation, the typical time-varying pricing schemes in the energy supply industries are studied as a way of load management from a viewpoint of welfare economics.

In Chapter 2, a brief review is made on the concepts of several sorts of the time-varying pricing schemes such as the time-of-use pricing (TOUP) and the load adaptive pricing (LAP), and the marginal cost pricing principle which gives a welfare economic basis to these pricing schemes.

In Chapter 3, the load adaptive pricing in electric power system is studied by means of a multifollower dynamic Stackelberg game model. An optimal pricing scheme is derived based on the model, and it is shown that the obtained optimal strategy forms the marginal cost price adaptively. Through a case study, effectiveness of LAP under the fluctuating load and its influences on the consumers having different load characteristics are evaluated quantitatively.

In Chapter 4, cooperative supply of electricity and town-gas under the time-of-use pricing is studied considering the difference of the load patterns in the two energy utilities, and possibility of the mutual load substitution which will make the supply more efficient. To investigate the issue, an energy supply/demand model of nonlinear programming type is developed based on the surplus theory. A case study is carried out, and the effectiveness of this policy is made clear quantitatively. The results of the simulation show that the inter-energy substitution between electricity and town-gas with TOUP or SP suppresses the construction of new electric power plants needed for the peak load. It is also shown that the es-

timated welfare gain can justify the implementation cost of TOUP.

In Chapter 5, a competition of an electricity and a town-gas supplier is studied by means of a noncooperative static game model. It complements the study in Chapter 4 which assumes a complete cooperation between the suppliers. Through an analytical study and numerical simulations, it is shown that the regulation of the profit may make the demand and supply inefficient even if there exists a competition between the suppliers. Further, it is also shown that the substitutability of the energy demand does not make the demand and supply efficient by itself if the both of the competing companies are regulated. The implication of the result is that the regulation of the profit ratio is not sufficient any more under TOUP, and some other regulations are required to achieve efficient demand and supply.

The studies presented in the dissertation show the effectiveness of load management by the time-varying pricing schemes. Especially, the cooperation of the different sorts of energy utilities, i.e., electricity and town-gas, by means of time-of-use pricing is expected to be an effective policy to relieve the peak load problem in the energy systems. On the load management of the energy systems and pricing strategies for it, some further studies are needed to make their effectiveness and defects in more definite ways. For example, the manageable load in the industrial, commercial and household sectors, supply-side benefit of the load management, response of the consumers to the time-varying price and technical feasibility of TOUP and LAP should be clarified more in detail.

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Appendix A Condition for Balance of the Revenue and the Cost under the Marginal Cost Pricing

Suppose the demand/supply in N periods. Let q_i , $i=1, \dots, N$ be the quantity of the demand/supply in the period i , and $C(q_1, \dots, q_N)$ be the total supply cost. C is assumed to be continuously differentiable. Then, the following proposition holds.

Proposition If and only if the cost function C is a homogeneous function of one degree in the positive region of the quantity, then the revenue by the marginal cost pricing is always equal to the total cost.

Proof) Due to Euler's theorem on homogeneous functions and continuously differentiability of C , the following equation is necessary and sufficient with the homogeneous function of one degree in the positive region of the demand/supply:

$$\text{for all positive } q_i, \quad \sum_{i=1}^N \frac{\partial C}{\partial q_i} q_i = C. \quad (\text{A.1})$$

Since the price p_i in the period i is set equal to the marginal cost $\partial C / \partial q_i$,

$$\sum_{i=1}^N p_i q_i = C. \quad (\text{A.2})$$

QED.

Implication of this proposition is that the balance of the revenue and the cost under the marginal cost pricing holds if the supply cost has neither economy nor diseconomy of scale. It must be noted that the condition for the balance permits the existence of 'joint cost' which is a cost needed commonly to supply two or more kinds of goods. In electric power or town-gas systems, the capacity cost is a joint cost for the time-of-use supply, i.e., the capacity and its cost is decided by the maximum load.

The revenue by the marginal cost pricing balances with the supply cost if it has neither economy nor diseconomy of scale regardless of the existence of joint cost.

Appendix B Formulas of the Optimal Team Strategies

The coefficients of the optimal team strategies (3.18), i.e., F_i , z_{i1} and z_{i2} are given by the followings:

$$\begin{aligned} F_i &= (B^T \hat{E}_i B + D_i)^{-1} (-B^T \hat{E}_i B_i) \\ z_{im} &= (B^T \hat{E}_i B + D_i)^{-1} (y_{im} + B^T e_{im}), \quad i \in N, \\ m &= 1 \text{ and } 2 \end{aligned} \quad (\text{B.1})$$

where

$$\begin{aligned} E_i &= \text{diag}(p_{i1}, \dots, p_{iK}) \\ D_i &= \begin{bmatrix} \text{diag } w_i + \hat{C} & -\text{diag } w_i \\ -\text{diag } w_i & \text{diag } w_i + \text{diag}(c_{L1}, \dots, c_{LK}) \end{bmatrix} \\ y_{i1} &= (\bar{q}_{i11} w_{i1}, \dots, \bar{q}_{iK1} w_{iK}, -\bar{q}_{i11} w_{i1}, \dots, -\bar{q}_{iK1} w_{iK})^T \\ y_{i2} &= (\bar{q}_{i12} w_{i1} - c_2, \dots, \bar{q}_{iK2} w_{iK} - c_2, -\bar{q}_{i12} w_{i1}, \dots, -\bar{q}_{iK2} w_{iK})^T \\ e_{i2} &= (\bar{x}_1 p_{i1}, \dots, \bar{x}_K p_{iK})^T, \quad i \in N. \end{aligned}$$

In the above, $B = [O|U]$, $B_i = \beta U$ where O and U are the $K \times K$ zero and identity matrices, respectively. w_i denotes $(w_{i1}, \dots, w_{iK})^T$. $\hat{C}$ is the $K \times K$ matrix whose all elements are c_1 . $\hat{E}_i$, $\hat{e}_{i1}$ and $\hat{e}_{i2}$ are determined recursively as follows:

$$\begin{aligned} \hat{E}_N &= E_N \\ \hat{E}_{i-1} &= E_{i-1} + (F_i^T D_i F_i + (B_i + B F_i)^T \hat{E}_i (B_i + B F_i)) \\ &\quad \text{for } 2 \leq i \leq N \\ \hat{e}_{N1} &= 0 \\ \hat{e}_{i-1,1} &= -[z_{i1}^T D_i F_i - y_{i1}^T F_i + (z_{i1}^T B^T \hat{E}_i - e_{i1}^T)(B_i + B F_i)]^T \alpha \end{aligned}$$

$$\dot{e}_{N2} = e_{N2} \quad (\text{B.2})$$

$$\begin{aligned} \dot{e}_{i-1,2} = & -[z_{i2}^T D_i F_i - y_{i2}^T F_i + (z_{i2}^T B^T \dot{E}_i - \dot{e}_{i2}^T)(B_i + B F_i)]^T \\ & + e_{i-1,2} \\ & \text{for } 2 \leq i \leq N-1. \end{aligned}$$

When the all the consumers adopt these optimal team strategies, the optimal value of the supplier's payoff, J_0^* , is given by the following:

$$J_0^* = \tilde{d}_{00} + \sum_{i=0}^N \tilde{d}_{3i} E[\nu_i] + \sum_{i=0}^N \tilde{d}_{4i} E[\nu_i^2] \quad (\text{B.3})$$

where $\tilde{d}_{00}$, $\tilde{d}_{3i}$ and $\tilde{d}_{4i}$ are determined recursively as follows:

$$\begin{aligned} \tilde{d}_{00} = & \left(-\frac{1}{2} z_{12}^T D_1 z_{12} + y_{12}^T z_{12} - \frac{1}{2} z_{12}^T B^T \dot{E}_1 B z_{12} + e_{12} B z_{12} + \tilde{d}_{01} \right) \\ \tilde{d}_{30} = & \tilde{d}_{10} \\ = & \left(-z_{12}^T D_1 z_{11} + y_{11}^T z_{12} + y_{12}^T z_{11} - z_{11}^T B^T \dot{E}_1 B z_{12} \right. \\ & \left. + \dot{e}_{11}^T B z_{12} + \dot{e}_{12} B z_{11} + \tilde{d}_{11} \right) \alpha \\ \tilde{d}_{20} = & \tilde{d}_{40} \\ = & \left(-\frac{1}{2} z_{11}^T D_1 z_{11} + y_{11}^T z_{11} - \frac{1}{2} z_{11}^T B^T \dot{E}_1 B z_{11} + \dot{e}_{11}^T B z_{11} + \tilde{d}_{21} \right) \alpha^2 \\ \tilde{d}_{0N} = & d_{0N}, \quad \tilde{d}_{1N} = d_{1N}, \quad \tilde{d}_{2N} = d_{2N} \end{aligned}$$

For $2 \leq i \leq N$,

$$\begin{aligned} \tilde{d}_{0,i-1} = & d_{0,i-1} + \\ & \left(-\frac{1}{2} z_{i2}^T D_i z_{i2} + y_{i2}^T z_{i2} - \frac{1}{2} z_{i2}^T B^T \dot{E}_i B z_{i2} + \dot{e}_{i2}^T B z_{i2} + \tilde{d}_{0i} \right) \\ \tilde{d}_{1,i-1} = & d_{1,i-1} + \left(-z_{i2}^T D_i z_{i1} + y_{i1}^T z_{i2} + y_{i2}^T z_{i1} \right. \end{aligned}$$

$$\begin{aligned}
& - z_{i1}^T B^T \bar{E}_i B z_{i2} + \bar{e}_{i1}^T B z_{i2} + \bar{e}_{i2}^T B z_{i2} + \bar{d}_{1i}) \alpha \\
\bar{d}_{2,i-1} = & d_{2,i-1} + (-\frac{1}{2} z_{i1}^T D_i z_{i1} + y_{i1}^T z_{i1} - \frac{1}{2} z_{i1}^T B^T \bar{E}_i B z_{i1} \\
& + \bar{e}_{i1}^T B z_{i1} + \bar{d}_{2i}) \alpha^2
\end{aligned}$$

Further, for $i \in N$

$$\begin{aligned}
\bar{d}_{3i} = & (-z_{i2}^T D_i z_{i1} + y_{i1}^T z_{i2} + y_{i2}^T z_{i1} - z_{i1}^T B^T \bar{E}_i B z_{i2} \\
& + \bar{e}_{i1}^T B z_{i2} + \bar{e}_{i2}^T B z_{i1} + \bar{d}_{1i}) (1-\alpha^2)^{1/2} \\
\bar{d}_{4i} = & (-\frac{1}{2} z_{i1}^T D_i z_{i1} + y_{i1}^T z_{i1} - \frac{1}{2} z_{i1}^T B^T \bar{E}_i B z_{i1} \\
& + \bar{e}_{i1}^T B z_{i1} + \bar{d}_{2i}) (1-\alpha^2)
\end{aligned} \tag{B.4}$$

In the above, d_{0i} , d_{1i} and d_{2i} are constants defined as follows:

$$\begin{aligned}
d_{0i} = & \sum_{j=1}^K (-\frac{1}{2} w_{ij} (\bar{q}_{ij2})^2 + \bar{x}_j p_{ij} \bar{x}_j) \\
d_{1i} = & -\sum_{j=1}^K w_{ij} \bar{q}_{ij2} \bar{q}_{ij1} \\
d_{2i} = & -\frac{1}{2} \sum_{j=1}^K w_{ij} (\bar{q}_{ij1})^2, \quad i \in N.
\end{aligned}$$

Appendix C Optimal Responses of the Consumers and Formulas of the Optimal LAP Strategy

When the pricing strategy given by Eqs. (3.19) and (3.20) is adopted by the supplier, the optimal responses of the consumers q_i^* are given by the followings:

$$q_i^* = \tilde{F}_i x_{i-1} + \tilde{z}_{i1} \xi_i + \tilde{z}_{i2}, \quad i \in N \quad (C.1)$$

where

$$\begin{aligned} \tilde{F}_i &= [D_i + \begin{bmatrix} -\tilde{C} & 0 \\ 0 & A_i \end{bmatrix}]^{-1} [a_i + \begin{bmatrix} 0 \\ \text{diag } p_i - A_i \end{bmatrix}] \\ \tilde{z}_{i1} &= [D_i + \begin{bmatrix} -\tilde{C} & 0 \\ 0 & A_i \end{bmatrix}]^{-1} [-y_{i1} + b_{i1} - \zeta_{i1}] \\ \tilde{z}_{i2} &= [D_i + \begin{bmatrix} -\tilde{C} & 0 \\ 0 & A_i \end{bmatrix}]^{-1} [-\bar{y}_{i2} + b_{i2} - \zeta_{i2}], \quad i \in N \end{aligned}$$

Furthermore, for $i \in N$

$$\begin{aligned} A_i &= [2A_{i1}^T u_1, \dots, 2A_{iK}^T u_K]^T \\ \zeta_{im} &= (\underbrace{0, \dots, 0}_K, u_1^T \zeta_{i1m}, \dots, u_K^T \zeta_{iKm})^T, \quad m = 1 \text{ or } 2 \\ a_i &= [a_{i11}, \dots, a_{iK1}, a_{i+1,1,2}, \dots, a_{i+1,K,2}]^T \\ b_{i1} &= (b_{i+1,1,2}, \dots, b_{i+1,K,2}, b_{i+1,1,3}, \dots, b_{i+1,K,3})^T \\ b_{i2} &= (b_{i11}, \dots, b_{iK1}, b_{i+1,1,4}, \dots, b_{i+1,K,4})^T \\ \bar{y}_{i2} &= (w_{i1} \bar{q}_{i12}, \dots, w_{iK} \bar{q}_{iK2}, \\ &\quad -w_{i1} \bar{q}_{i12} + p_{i1} \bar{x}_1, \dots, -w_{iK} \bar{q}_{iK2} + p_{iK} \bar{x}_K)^T. \end{aligned}$$

In the above, u_j denotes an unit vector having the unit value in the j -th element. The $K \times K$ matrix A_{ij} and the K -vectors ζ_{ij1} and ζ_{ij2} are defined recursively as follows:

$$\begin{aligned}
 A_{Nj} &= 0, \quad j \in K \\
 A_{i-1,j} &= -\frac{1}{2} \{ w_{ij} \text{prd}(\dot{f}_{ij}^d - \dot{f}_{ij}^e) + c_{Lj}^T \text{prd}(\dot{f}_{ij}^e) + p_{ij} \text{prd}(\beta u_j^T + \dot{f}_{ij}^e) \\
 &\quad + a_{ij1} \dot{f}_{ij}^d + (\dot{f}_{ij}^d)^T a_{ij1}^T + a_{i+1,j,2} \dot{f}_{ij}^e + (\dot{f}_{ij}^e)^T a_{i+1,j,2}^T \} \\
 &\quad + (B_i + B\hat{F}_i)^T A_{ij} (B_i + B\hat{F}_i) \quad (C.2) \\
 &\text{for } 2 \leq i \leq N, j \in K
 \end{aligned}$$

$$\zeta_{Njm} = 0, \quad j \in K, m=1 \text{ and } 2$$

$$\begin{aligned}
 \zeta_{i-1,j,m}^T &= [-w_{ij}(\dot{z}_{ijm}^d - \dot{z}_{ijm}^e - \bar{q}_{ijm})(\dot{f}_{ij}^d - \dot{f}_{ij}^e) \\
 &\quad - c_{Lj}^T \dot{z}_{ijm}^e \dot{f}_{ij}^e - p_{ij} \dot{z}_{ijm}^e (\beta u_j^T + \dot{f}_{ij}^e) \\
 &\quad - \dot{z}_{ijm}^e a_{ij1}^T - \begin{Bmatrix} b_{i+1,j,2} \\ b_{i,j,1} \end{Bmatrix} \dot{f}_{ij}^d - \dot{z}_{ijm}^e a_{i+1,j,2}^T - \begin{Bmatrix} b_{i+1,j,3} \\ b_{i+1,j,4} \end{Bmatrix} \dot{f}_{ij}^e \\
 &\quad + 2(\dot{B} \dot{z}_{im})^T A_{ij} (B_i + B\hat{F}_i) \\
 &\quad + \zeta_{ijm}^T (B_i + B\hat{F}_i)] \times \begin{cases} \alpha: & \text{upper is for } m=1, \\ 1: & \text{lower is for } m=0, \end{cases} \quad (C.3)
 \end{aligned}$$

$$\text{for } 2 \leq i \leq N, j \in K$$

where $\text{prd}(x)$ denotes the operator which produces a matrix $x^T x$ from a row vector x . $\dot{f}_{ij}^d$ and $\dot{f}_{ij}^e$ denote the j -th and the $(j+K)$ -th rows of the ma-

trix $\hat{P}_i$, respectively. $\hat{z}_{ijm}^d$ and $\hat{z}_{ijm}^s$ denote the j -th and the $(j+K)$ -th elements of the vector $\hat{z}_{ijm}$, respectively, for $m = 1$ and 2 .

The necessary conditions for the optimal responses, Eq. (3.21), is sufficient when the following matrices are negative definite:

$$\begin{bmatrix} -w_{ij} & w_{ij} \\ w_{ij} & -w_{ij} - c_{Lj} - p_{ij} + 2u_j^T A_{ij} u_j \end{bmatrix}, \quad i \in N, j \in K.$$

It is equivalent to the following inequalities:

$$2u_j^T A_{ij} u_j < c_{Lj} + p_{ij}, \quad i \in N, j \in K. \quad (C.4)$$

The optimal parameters of the pricing strategy which make the optimal responses of the consumers, Eq. (C.1), coincide with the optimal team strategies (3.18) are as follows:

$$\begin{aligned} a_{ij1} &= -w_{ij}(f_{ij}^d - f_{ij}^s)^T \\ b_{ij1} &= -w_{ij}(z_{ij2}^d - z_{ij2}^s - \bar{q}_{ij2}) \\ b_{i+1,j,2} &= -w_{ij}(z_{ij1}^d - z_{ij1}^s - \bar{q}_{ij1}) \\ a_{i+1,j,2} &= [w_{ij}(f_{ij}^d - f_{ij}^s) - c_{Lj} f_{ij}^s \\ &\quad - p_{ij}(\beta u_j^T + f_{ij}^s) + 2u_j^T A_{ij}(B_i + BF_i)]^T \\ b_{i+1,j,3} &= w_{ij}(z_{ij1}^d - z_{ij1}^s - \bar{q}_{ij1}) - c_{Lj} z_{ij1}^s \\ &\quad - p_{ij} z_{ij1}^s + 2u_j^T A_{ij} B z_{i1} + \zeta_{ij1}^T u_j \\ b_{i+1,j,4} &= w_{ij}(z_{ij2}^d - z_{ij2}^s - \bar{q}_{ij2}) - c_{Lj} z_{ij2}^s \\ &\quad - p_{ij}(z_{ij2}^s - \bar{x}_j) + 2u_j^T A_{ij} B z_{i2} + \zeta_{ij2}^T u_j, \\ &\quad i \in N, j \in K \end{aligned} \quad (C.5)$$

where A_{ij} , ζ_{ij1} and ζ_{ij2} are given by Eqs. (C.2) and (C.3) replacing $\hat{F}_i$, $\hat{z}_{i1}$ and $\hat{z}_{i2}$ by the coefficients of the optimal team strategies, i.e., F_i , z_{i1} and z_{i2} , respectively.

Appendix D A Meaning of the Optimal LAP Strategy

From the necessary condition for the optimal demand, Eq. (3.17), the following equation is derived:

$$\begin{aligned} & -(D_i + B^T \bar{E}_i B) q_i - B^T \bar{E}_i B_i x_{i-1} + y_{i2} \\ & + y_{i1} \xi_i + B^T \bar{e}_{i1} \xi_i + B^T \bar{e}_{i2} = 0, \quad i \in N. \end{aligned} \quad (D.1)$$

The j -th element of Eq. (D.1) is:

$$-w_{ij}(q_{ij}^d - q_{ij}^s) + \bar{q}_{ij2} w_{ij} + \bar{q}_{ij1} w_{ij} \xi_i - c_1 q_i^d - c_2 = 0, \quad i \in N, j \in K$$

where q_i^d is the total demand at period i . Replacing q_{ij}^d and q_{ij}^s by the optimal team strategies, the following equation is obtained:

$$\begin{aligned} & -w_{ij}(f_{ij}^d - f_{ij}^s) x_i - w_{ij}(z_{ij1}^d - z_{ij1}^s) \xi_i \\ & - w_{ij}(z_{ij2}^d - z_{ij2}^s) + \bar{q}_{ij2} w_{ij} + \bar{q}_{ij1} w_{ij} \xi_i = c_1 q_i^d + c_2, \quad i \in N, j \in K. \end{aligned}$$

Considering the optimal parameters of the pricing strategy, Eqs. (C.5), it becomes:

$$v_{ij} + b_{i+1,j,2} \xi_i = c_1 q_i^d + c_2. \quad (D.2)$$

The RHS of Eq. (D.2) is the marginal generating cost. The implication of Eq. (D.2) is that the unit price v_{ij} forms the marginal cost price along with the term containing $b_{i+1,j,2}$ in the formula of the fixed charge $h_{i+1,j}$.

Appendix E Proof of the Proposition

According to the notations of the LAP strategy, Eq. (3.19), let the unit price of TOUP (or CP) be

$$v_{ij} = b_{ij1}. \quad (\text{E.1})$$

As shown in Appendix C, the optimal responses of the consumers to the above pricing are given by

$$q_i^* = \hat{F}_i z_{i-1} + \hat{z}_{i1} \xi_i + \hat{z}_{i2}. \quad (\text{E.2})$$

Looking at the definition of $\hat{F}_i$, $\hat{z}_{i1}$ and $\hat{z}_{i2}$ in Appendix C, it is known that b_{ij1} appears only in $\hat{z}_{i2}$. Further, as shown in Appendix B, when the consumers take the strategies given by Eq. (E.2), the suppliers payoff J_0 becomes

$$J_0 = \tilde{d}_{00} + \sum_{i=1}^N \tilde{d}_{3i} E[\nu_i] + \sum_{i=1}^N \tilde{d}_{4i} E[\nu_i^2] \quad (\text{E.3})$$

where the coefficients $\tilde{d}_{00}$, $\tilde{d}_{3i}$ and $\tilde{d}_{4i}$ are defined by (B.4) substituting $\hat{F}_i$, $\hat{z}_{i1}$ and $\hat{z}_{i2}$ for F_i , z_{i1} and z_{i2} , respectively. The definition of $\tilde{d}_{4i}$ shows that it does not contain $\hat{z}_{ij2}$ (and accordingly b_{ij1}). Hence b_{ij1} optimizing J_0 is independent of $E[\nu_i^2]$. Considering the assumption $E[\nu_i] \equiv 0$, the optimal TOUP (or CP) strategy of the original model coincides with that of the deterministic model.

QED.

Appendix F Coefficients in the Demand Function

The coefficients in the nonheat and the heat demand functions are given by the following equations:

Nonheat Demand

$$\begin{aligned}
 \alpha_{NEi} &= \eta \bar{q}_{NEi} / \bar{p}_{Ei} \\
 \alpha_{NGi} &= \eta \bar{q}_{NGi} / \bar{p}_{Gi} \\
 \bar{q}_{NEi} &= (1 - \eta) \bar{q}_{NEi} \\
 \bar{q}_{NGi} &= (1 - \eta) \bar{q}_{NGi}, \quad i = 1, \dots, 15
 \end{aligned} \tag{F-1}$$

Heat Demand in Summer

$$\begin{aligned}
 \alpha_{HEEi} &= \frac{\eta \bar{q}_{HEi} - \alpha_{HEGi} \bar{p}_{Gi}}{\bar{p}_{Ei}} \\
 \alpha_{HEGi} &= \frac{(s - 1) \bar{q}_{HEi}}{\bar{p}_{Gi} - \bar{p}_{Gi}} \\
 \alpha_{HGEi} &= - \frac{\alpha_{HGGi} \bar{p}_{Gi}}{\bar{p}_{Ei}} \\
 \alpha_{HGGi} &= \frac{(1 - s) e_{ES} \bar{q}_{HEi}}{e_{GS} (\bar{p}_{Gi} - \bar{p}_{Gi})} \\
 \bar{q}_{HEi} &= (1 - \eta) \bar{q}_{HEi} \\
 \bar{q}_{NGi} &= 0, \quad i = 1, \dots, 5
 \end{aligned} \tag{F-2}$$

Heat Demand in Winter

$$\begin{aligned}
 \alpha_{HEEi} &= \frac{(1-s)\bar{q}_{Hi}/e_{EW} - \bar{q}_{HEi}}{\bar{p}_{Ei} - \bar{p}_{Gi}} \\
 \alpha_{HEGi} &= \frac{\eta\bar{q}_{HEi} - \alpha_{HEEi}\bar{p}_{Ei}}{\bar{p}_{Gi}} \\
 \alpha_{HGEi} &= \frac{s\bar{q}_{Hi}/e_{GW} - \bar{q}_{HG_i}}{\bar{p}_{Ei} - \bar{p}_{Gi}} \\
 \alpha_{HGG_i} &= \frac{\eta\bar{q}_{HG_i} - \alpha_{HGEi}\bar{p}_{Ei}}{\bar{p}_{Gi}} \\
 \bar{q}_{HEi} &= (1 - \eta)\bar{q}_{HEi} \\
 \bar{q}_{HG_i} &= (1 - \eta)\bar{q}_{HG_i}, \quad i = 6, \dots, 10
 \end{aligned} \tag{F-3}$$

where

$$\begin{aligned}
 \bar{p}_{Gi} &= \bar{p}_{Ei}e_{GS}/e_{ES}, \quad i = 1, \dots, 5 \\
 \bar{p}_{Ei} &= \bar{p}_{Gi}e_{EW}/e_{GW}, \quad i = 6, \dots, 10 \\
 \bar{q}_{Hi} &= \bar{q}_{HEi}e_{EW} + \bar{q}_{HG_i}e_{GW}, \quad i = 6, \dots, 10
 \end{aligned}$$

$\bar{p}_{Ei}$ and $\bar{p}_{Gi}$ are the prices of electricity and town-gas in 1982, respectively. $\bar{q}_{NEi}$ and $\bar{q}_{NG_i}$ are, respectively, the forecast nonheat demands for electricity and town-gas in 2000 when the prices of the two sorts of energy are kept as those in 1982. $\bar{q}_{HEi}$ and $\bar{q}_{HG_i}$ are, respectively, the forecast heat demand for electricity and town-gas in 2000 as well. e_{ES} and e_{GS} are, respectively, the efficiencies (COPs) of the electric and the town-gas air conditioners. e_{EW} and e_{GW} are, respectively, the efficiencies of the electric and the town-gas space heaters.