

Vibration-based Bayesian model updating of an actual steel truss bridge subjected to incremental damage

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Abstract: As the finite element (FE) model has become increasingly important in engineering, the model updating method has also received much attention as a means of improving the structural FE model accuracy. However, the objectives of many earlier studies have been laboratory experiments and numerical simulations. Actual structures have not been adequately investigated. This report describes work on a vibration-based Bayesian model updating for an actual steel truss bridge. The transitional Markov chain Monte Carlo (TMCMC) sampling method was used to estimate the posterior distribution. A field experiment assessing this target bridge was conducted under five damage scenarios. Then a fast Bayesian FFT method was used to identify the modal properties. Based on sensitivity analysis, three model classes were proposed for the model updating process of the target bridge. In all cases, the updated modal properties were found to fit well with the experimental data, whereas the updated model parameters cannot. The case with more prior information identified the structural damage. Results demonstrate that making full use of prior information can improve the model updating accuracy. Feasibility of damage detection was observed for an actual steel truss bridge based on the FE model updating method.

Keywords: Bayesian model updating, Damage detection, Field vibration test, Markov chain Monte Carlo, Steel truss bridge

1 Introduction

Effective maintenance of aged bridge structures has become a crucially important challenge for the civil engineering community. Structural health monitoring (SHM) has been investigated widely for damage detection and prediction of structural responses of bridges as an effective means of inspecting aged bridges. The FE model is also used widely in engineering for structural analysis, response prediction, and design support because FE analysis is less costly than field experiments. Nevertheless, discrepancies between FE model simulations and measurements of actual structures can often be found. Such discrepancies result from uncertainties of materials, geometric properties, and other information. In fact, FE model updating methods are also used widely to reduce discrepancies between measured and analytical structural responses of bridges for SHM [1, 2]. Model updating methods are aimed at minimizing these differences by calibrating the uncertain parameters of structural FE models based on

measured data. An updated model can provide more accurate information for structural response prediction and for safety evaluation.

Structural damage can be treated as changes in material and geometric properties that reduce structural performance. In other words, damage detection can be regarded as identification of structural changes [3, 4]. If the updated model can fit actual structural conditions, then changes in the model parameters can indicate the severity and location of structural damage.

Bayesian model updating is a popular model updating approach that considers uncertainties of model parameters attributable to measurement noise or modeling error [5, 6]. Uncertain parameters are assumed to follow a certain distribution based on experimentally obtained data. In other words, the Bayesian model updating process is fundamentally an optimization problem for which the criterion is the posterior distribution based on the level of the difference between the simulation and experiment. Au and Beck [7] and Beck and Au [8] investigated reliability analysis based on Bayesian model updating. Vanik et al. [9] discussed damage detection based on model updating. Beck and Yuen [10] and Muto and Beck [11] discussed model classes in the model updating method. Goller and Schueller [12] and Goller et al. [13] investigated uncertainties in Bayesian model updating and weighting factors of each mode in the objective function. Yuen et al. [14] and Lam et al. [15, 16] extended the applicability and efficiency of the Bayesian-based method. Also, Goller et al. [17] applied model updating to a complex aerospace structure. Yang and Lam [18] and Yang et al [19] reported the application of model updating on the laboratory experiments.

For Bayesian model updating, sampling procedures are invariably necessary because obtaining analytical solutions to describe a probability density function (PDF) of uncertain parameters is difficult. Among many sampling algorithms, the Markov chain Monte Carlo (MCMC) sampling algorithm [20, 21] has received the most attention, especially for the Metropolis–Hastings (MH) algorithm [22, 23]. When numerous uncertain parameters are considered, the high dimensions pose important difficulties for sampling efficiency. Advanced sampling methods of many types have been proposed. Beck and Au [8] proposed the Adaptive MH algorithm (AMH), which uses a sequence of intermediate distributions to converge on a target distribution. Ching and Chen [24] proposed the transitional Markov chain Monte-Carlo (TMCMC) method based on AMH but avoiding the difficulty of calculation of the kernel density estimation, especially in high-dimensional space. The different sampling methods will only affect the efficiency, not the accuracy. Considering both the efficiency and the ease of implementation, TMCMC has important benefits. For this reason, TMCMC has been used widely for recent studies [11, 12, 25].

Modal identification is the basis of the vibration-based model updating. Because vibration tests can be conducted conveniently [26], model updating methods using the modal properties identified from vibration data such as natural frequencies and mode shapes, called the vibration-based model updating methods, have received much attention. Accuracy of the identified structural modal properties is necessary for the model updating process. Many modal identification methods exist, for example, stochastic subspace identification (SSI) [27, 28] and fast Bayesian FFT [29–31]. SSI is a popular time-domain method, whereas fast Bayesian FFT is a recently developed frequency domain method. Fast Bayesian FFT, which has been applied widely, is also based on the Bayes' theorem to identify the posterior distribution of modal properties [15, 16, 32, 33, 34].

Most existing studies have verified the validity of model updating methods using numerical simulations [e.g., 24, 25] or laboratory experiments [e.g., 13, 16, 18, 19]. However, the numerical simulations or laboratory experiments often refer to well-controlled measurement noise and a simple FE model with

fewer modeling errors and fewer uncertain parameters, thereby showing a marked difference from actual structural conditions. Few studies have verified methods using field experiments for actual bridge structures. Even fewer studies have investigated the feasibility of applying a damage detection-aimed model update to actual bridge structures. The experimentally obtained data are often collected from a few sensors despite the greater degrees of freedom (DOF) of actual bridges. The wholly insufficient data cannot support the identification of many parameters. Moreover, sampling in an extremely high-dimensional solution space is a computationally expensive task. Therefore, it is impractical to conduct member-level or element-level model updating of an actual structure. An available approach to reduce the number of parameters is the grouping of multiple members into a block and assuming that values of uncertain parameters of the members in the same block are equal. A bridge structure with a complex model might have many available model classes with different grouping schemes. With these model classes, the updated results might also vary. To acknowledge these challenges, one must investigate the performance of the vibration-based FE model update method for actual bridge structures.

This study assesses the feasibility of damage detection of an actual steel truss bridge considering different damage scenarios using Bayesian FE model updating. Vibration and damage experiments were conducted on a steel truss bridge. A fast Bayesian FFT method is adopted as an operational modal analysis method. The TMCMC method is taken as the sampling method to generate a sample sequence from the posterior distribution. Based on sensitivity analysis, three model classes with different structural member grouping schemes were examined in FE model updates for bridge damage detection.

2 Bayesian Model Updating and Sampling

2.1 Bayesian model updating method

Bayesian model updating is an optimization problem based on the posterior distribution using available experimentally obtained data. In Bayesian model updating [5, 6], the formulation of the posterior PDF of the uncertain parameter vector θ^r ($\theta^r \in \mathbf{R}^{N_{\theta^r}}$, where N_{θ^r} is the number of uncertain parameters in the model class M^r) under the given system response D is expressed as

$$P(\theta^r | D, M^r) = \frac{P(D|\theta^r, M^r)P(\theta^r | M^r)}{P(D|M^r)}, \quad (1)$$

where M^r ($r = 1, 2, \dots, N_M$, where N_M represents the number of available model classes) is the assumed probabilistic model class for the structure, $P(D|M^r)$ denotes evidence of model class M^r , and $P(D|M^r) = \int P(D|\theta^r, M^r)P(\theta^r | M^r)d\theta^r$ is a normalizing constant. Also, $P(\theta^r | M^r)$ is the prior PDF. For convenience, it is often assumed as a uniform distribution. Finally, $P(D|\theta^r, M^r)$ is the likelihood function, which denotes the conditional probability of D given θ^r .

For this study, the posterior PDF has a similar formulation to that used for earlier studies [8, 9, 12, 13, 15]. For convenience, assuming that the prediction error of modal properties follows the normal distribution with the same variances, the likelihood function becomes

$$P(D|\theta^r, M^r) = c_l^r e^{-J(\theta^r)/2\sigma^2}, \quad (2)$$

where $J(\theta^r) = \sum_{i=1}^{N_m} \left((1 - \langle \varphi_i^r(\theta^r), \hat{\varphi}_i \rangle^2) + (1 - f_i^r(\theta^r)/\hat{f}_i)^2 \right)$ is the objective function indicating the prediction errors of the modal properties. Also, $f_i^r(\theta^r)$ and $\varphi_i^r(\theta^r)$ respectively represent the i th predicted natural frequency and normalized eigenvectors obtained from the FE model under the given uncertain parameter vector θ^r while $\hat{f}_i$ and $\hat{\varphi}_i$ respectively denote the natural frequency and mode shape identified from the experiments. Also, N_m denotes the total number of modes considered in model updating; c_i^r is the normalizing constant.

Substituting Eq. (2) into Eq. (1) provides the posterior PDF expressed as

$$P(\theta^r|D, M^r) = c^r e^{-J(\theta^r)/2\sigma^2}, \quad (3)$$

where $c^r = c_i^r \frac{P(\theta^r|M^r)}{P(D|M^r)}$ denotes a normalizing constant.

Model classes, which include all candidate models of the target structure, play an important role in the effectiveness and accuracy of the model updating process. In existing research [10, 11], the most probable model was selected from several model classes based on the posterior distribution obtained from the Bayesian model update method.

The complexity of the posterior PDF shown in Eq. (3) makes it difficult to describe the probability distribution and to calculate the probability directly. Therefore, the Monte Carlo simulation (MCS) is used to estimate the posterior distribution by the generated samples.

2.2 Transitional Markov chain Monte Carlo (TMCMC)

The FE model of a real bridge is often complex and inclusive of numerous uncertain parameters. For greater numbers of uncertain parameters, higher dimensionality of the solution space is necessary: more samples must be accumulated to produce a better approximation. Therefore, the computational cost of sampling might be staggeringly high. An efficient sampler can improve the efficiency of the model updating process by reducing the required samples. The TMCMC [24] sampling method was used in the sampling process as an improved MCMC sampling method. The key idea of the TMCMC is to improve the sampling efficiency using a series of asymptotic intermediate distributions to approach the target distribution and avoiding direct sampling from the complex target distribution. Compared to the AMH [8], the TMCMC shares the same key idea but does not need to calculate the kernel density estimation in a high-dimensional space. For the concerns of sampling efficiency and ease of implementation, the TMCMC is taken as the sampling method in this study.

In this section, for convenience, the model class M^r is shorthand for M . The intermediate distributions $P_j(\theta|D, M)$ can be expressed as

$$P_j(\theta|D, M) = c_j e^{-J(\theta)/2\sigma_j^2} \quad j = 1, 2 \dots m, \quad (4)$$

where c_j is the normalizing constant, and where $c_m = c$; $\sigma_1 > \dots > \sigma_m = \sigma$. Especially, one must assume that $P_0(\theta|D, M) = c_0$ follows a uniform distribution. With the value of j increasing, the

$P_j(\theta|D, M)$ would become closer to the target distribution $P(\theta|D)$, and when $j = m$, $P_m(D|\theta, M) = P(\theta|D)$.

Importance sampling is used for the transition of adjacent intermediate distributions. The formulation of the weighting $w_j(\theta)$ is given as presented below.

$$w_j(\theta) = \frac{P_{j+1}(\theta|D, M)}{P_j(\theta|D, M)} = \frac{c_{j+1}}{c_j} \exp\left(-\frac{J(\theta)}{2} \left(\frac{1}{\sigma_{j+1}^2} - \frac{1}{\sigma_j^2}\right)\right), \quad (5)$$

Because some unknown parameters exist, it is impractical to use $w_j(\theta)$ directly in the resampling process. Therefore, the normalized weighting $w_{j,k}^n$ for samples sequence $\{\theta_{j,k}, k = 1, 2, \dots, N_j\}$ is given as

$$w_{j,k}^n = \frac{w_j(\theta_{j,k})}{\sum_{l=1}^{N_j} w_l(\theta_{j,l})} = \frac{\exp\left(-\frac{J(\theta_{j,k})}{2} \left(\frac{1}{\sigma_{j+1}^2} - \frac{1}{\sigma_j^2}\right)\right)}{\sum_{l=1}^{N_j} \exp\left(-\frac{J(\theta_{j,l})}{2} \left(\frac{1}{\sigma_{j+1}^2} - \frac{1}{\sigma_j^2}\right)\right)}, \quad (6)$$

where $\{\theta_{j,k}, k = 1, 2, \dots, N_j\}$ represents the sample sequence. It follows the probability distribution of $P_j(\theta|D, M)$. Also, N_j denotes the sample length.

Given a target coefficient of variation (COV) of the normalized weight $w_{j,k}^n$ and given that σ_j is already obtained in the j -th step, the only unknown σ_{j+1} in Eq. (6) can be solved for the next step $j+1$. In this study, the target COV($w_{j,k}^n$) is empirically taken as 90% or smaller, for avoiding too large difference between adjacent intermediate distributions, and σ_{j+1} is solved numerically.

Then, one must resample $\{\theta_{j,k}, k = 1, 2, \dots, N_j\}$ with $w_{j,k}^n$ to obtain $\{\theta_{j+1,l}, l = 1, 2, \dots, N_{j+1}\}$, which is shown below as

$$\theta_{j+1,l} = \theta_{j,k} \quad \text{with the probability} \quad w_{j,k}^n, \quad (7)$$

With this resampling process, samples with heavy weighting would be repeated easily. The ratio of distinct ones would decrease. Eventually, the sequence might even be a clone of only one sample. Therefore, MCMC is imported to resolve this problem. For example, sample $\theta_{j,k}$ repeats n_d times in the sample sequence $\{\theta_{j,k}, k = 1, 2, \dots, N_j\}$. Then, the MH algorithm is used to generate a n_d sample length Markov chain started from $\theta_{j,k}$ to replace the original samples.

3 Field Vibration and Damage Experiments for a Steel Truss Bridge

3.1 Target bridge

The target bridge, the simply supported steel truss bridge depicted in Figure 1, consists of concrete slabs, main girders, and truss members. The length and width of the main span are, respectively, 59.2 m and 3.6 m. This bridge, built in 1959, was removed in 2012. Ambient and vehicle-induced vibration experiments were arranged before removal. To obtain results with less disturbance, the bridge was closed to public use before the experiment.

3.2 Field experiment

Eight uniaxial accelerometers were installed on the bridge deck, as shown in Figure 2(a), to measure the vertical bridge acceleration responses. Because this bridge would eventually be removed, artificial damage was allowed. Five scenarios (INT, DMG1, DMG2, RCV, and DMG3) were considered consecutively for this bridge, as shown in Figure 2(b) and as described briefly in Table 1. Damage in the DMG1, DMG2, and RCV states was imposed mid-span, whereas the damage in the DMG3 state was imposed in the 5/8 span. The form of damage appears as severance of the tension members. A full cut and half cut represent different degrees of damage. A 21 kN vehicle was used to excite the bridge with an average speed of about 20 km/h under each damage scenario. More details about the field experiment were described in earlier reports of the literature [35–37].

3.3 Modal property identification

Accurate identification of modal properties is necessary for model updating. The data from vehicle-induced vibration test described above is used for modal identification. For the discussion presented in this section, INT state is examined as an example illustrating the modal identification process.

The power spectral density (PSD) and singular value spectrum (SVS) of acceleration data measured by all eight sensors under the INT state are presented in Figure 3. Below 15 Hz, five peaks are readily apparent and are taken as the candidate modes. The frequencies at five peaks were approximately 3 Hz, 5 Hz, 7 Hz, 9.5 Hz, and 10.5 Hz. To reduce random errors, the same procedure for experiments under each state has run at least ten times. A fast Bayesian FFT method [30, 32] was used to identify the modal parameters. Their histograms with the normal distribution fit are shown in Figure 4 with those from damage cases (DMG1, DMG2, RCV, and DMG3). The most probable value (MPV) and coefficient of variation (COV) of the frequencies are presented in Table 2. The corresponding mode shapes of the INT state are presented in Figure 5(a). From top to bottom, they are the first bending mode, the first torsional mode, the second bending mode, the third bending mode, and the fourth bending mode. Those mode shapes for the DMG1, DMG2, RCV, and DMG3 states are also portrayed in Figure 5. Table 3 presents the modal assurance criterion (MAC) values of mode shapes between damage states (DMG1, DMG2, RCV, and DMG3) and the INT state. The MPVs of frequencies and most stable mode shapes in each state were treated as actual values to be used in the model updating process.

The frequencies and mode shapes of the same mode under INT, DMG1, and RCV state are similar. Under the DMG2 state, the frequencies of the first two modes changed to a readily apparent degree. According to the distribution of frequencies in Figure 4, the MAC value in Table 3, and mode shapes in Figure 5, it is noteworthy that mode shapes and frequencies under the DMG3 state differed from others. A possible reason is that the full cut of the 5/8 span broke the bridge symmetry.

4 FE Model and Model Classes

Model classes represent all candidate models of the target bridge. Differences among them arise in the definition and description of the structure, such as element types, connections, material parameters, and other prior information. Because of different empirical information, measurement noise, limited data, and model errors for each model class, the available prior information for each model class might be different. These model classes are based on different prior information. Therefore, the results of model classes based on diverse information might also be different. This study investigates differences in the design of uncertain parameters on the model class.

The ABAQUS finite element analysis software suite was used for analyses. In the FE model, concrete slabs were modeled by shell elements, main girders by beam elements, and truss members by truss elements, as shown in Figure 6. It is always an issue to trade off computational efficiency against accuracy in finite element modelling and updating. A too simple model may fail to reflect the properties of the real bridge, causing accuracy problems, while a too complicated model may take a huge amount of computation time and resources, causing efficiency problems. In this study, the following simplifications were introduced to reduce the computational burden without losing much mechanical generality: using truss elements rather than beam elements to model top chords, vertical and diagonal truss members; and using shell elements rather than solid elements to model the concrete slabs. The model included more than one hundred members and three thousand elements.

4.1 Uncertain parameters

The sources of error in the modeling process derive mainly from the simplified representation of the actual bridge and unavailability of material property information [8, 12, 15]. Uncertainties from boundary conditions and material properties of the FE model were considered in the FE model update.

Although no perfect roller and pin exist in actual bridges, they are used widely in the FE model. The uncertainties in boundary conditions can be parameterized as the constants of springs of three types added to the supports: horizontal, vertical, and torsional springs. When the spring constant is equal to zero, movement in this direction is free; when the spring constant approaches infinity, the movement is fixed. In this case, by modifying the spring constraint values, constraints of many types are available, including the perfect pin and roller. Field experiments were conducted briefly in a few days. The cut in the vertical member introduced no loss in mass. Therefore, it is reasonable to assume that the density of the structural members was unchanged. The candidates of material parameters mainly include the elastic moduli of steel and concrete.

Because artificial cuts to tension members were applied to the bridge as damage, it is reasonable to simulate the damage as a decrease of the elastic modulus of the member. A full cut means that the elastic modulus becomes zero. Changes in the elastic modulus distribution can be treated as structural damage that occurred at the same location. In this case, changes can also be reflected by uncertain parameters from the model updating process.

Model parameters of five types, consisting of two elastic moduli and three spring constants, were considered for model updating. Table 4 shows the design values of truss-bridge model parameters. Table 5 presents the frequencies and mode shapes from eigenvalue analysis using FEA considering the design values in Table 4. MAC values are compared with those from experiments conducted under INT and DMG2 states. The greatest discrepancy of natural frequencies between FEA and model property

identification from measured accelerations was observed in the first bending mode: approximately 8% under INT state and 10% under DMG2 states. For the largest discrepancy of mode shapes, approximately 6% difference of MAC values was observed in the third and fourth bending modes.

4.2 Initial model class

As described earlier, element-level or member-level model updating is impractical because of the limited experimental data and the unaffordable calculation cost. As an initial trial, we grouped multiple members into a block and assumed that values of uncertain parameters of the members in the same block are equal. Therefore, a block model class shown in Figure 7, called the initial class, was firstly used for model updating test. The main part of this model was divided into eight blocks based on their structural types and locations. In all, nineteen model parameters were considered, including elastic moduli of eight steel model parameters (SS400_DP1, SS400_DM, SS400_DP2, SS400_UP1, SS400_UM, SS400_UP2, SS400_T, and SS400_B), ten spring constants (spring-H_DP2, spring-H_UP2, spring-V_DP1, spring-V_DP2, spring-V_UP1, spring-V_UP2, spring-R_DP1, spring-R_DP2, spring-R_UP1, spring-R_UP2), and one concrete elastic modulus (RC). For convenience, two sides of this bridge were designated respectively as the upstream side and downstream side. All artificial damage had occurred on the downstream side. Each block included only one type of parameter. Therefore, the names of parameters also represent the corresponding blocks. The parameters starting with 'SS400' represent the elastic modulus of steel members. 'RC' denotes the elastic modulus of the reinforced concrete. Also 'spring-H', 'spring-V', and 'spring-R' respectively stand for the horizontal, vertical, and torsional springs. The 'D' and 'U' respectively signify the downstream and upstream sides; the 'P1', 'M', and 'P2' respectively denote the P1 end, mid-span, and P2 end. Characters 'B' and 'T' respectively represent members at the bottom and top of the bridge.

4.3 Sensitivity analysis

Comprehensive sensitivity analysis was conducted to investigate changes in dynamic response with each parameter in the initial model class. A model for which parameters were set as design values was taken as the reference state to illustrate changes in the mode shapes.

The ranges of model parameters about the elastic modulus were set as $\pm 25\%$ of the design value for SS400_T, SS400_B, and RC. They were set as $\pm 10\%$ for the remaining parameters because they were more sensitive. Figure 8 presents results of sensitivity analyses for SS400_DP1, SS400_UP1, SS400_DM, and SS400_B. The horizontal axis represents the elastic modulus in GPa. As the value of elastic modulus increases, the frequencies of all modes also increased. When the value of the elastic modulus is equal to 200 GPa, which is the reference state, MACs reached 1. It is noteworthy that although SS400_DP1 and SS400_UP1 were located symmetrically in the model, the trend of their MAC values was slightly different, especially for the first torsional mode, as shown as a blue line and orange line in the 2nd row of Figure 8(b). One reason is the asymmetry of the sensor layout. Therefore, it might not be proper to apply the same design of model parameters on both upstream and downstream sides in FE model updating. As shown in Figure 8(b), the change in MAC values of SS400_B was much slighter. Although omitted from discussion in this report, a similar trend was observed for SS400_T and RC. In other words, these three parameters (RC, SS400_T, and SS400_B) are not as important as the others for damage detection.

Figure 9 shows summaries of results obtained for spring-H_DP2, spring-H_UP2, spring-V_DP1, and spring-R_DP1. The horizontal axis shows the spring constant. The unit for the linear spring (spring-H

and spring-V) is $\text{kN} \cdot \text{m}^{-1}$. That for the torsional spring (spring-R) is $\text{kN} \cdot \text{m}^{-1} \cdot \text{rad}^{-1}$. Spring constants were set from zero to over $10^{12} \text{ kN} \cdot \text{m}^{-1}$ for the linear spring and as greater than $10^{12} \text{ kN} \cdot \text{m}^{-1} \cdot \text{rad}^{-1}$ for the torsional spring for sensitivity analysis. Indeed, when the spring constant values reach $10^{11} \text{ kN} \cdot \text{m}^{-1}$ and $10^{11} \text{ kN} \cdot \text{m}^{-1} \cdot \text{rad}^{-1}$ for the torsional spring, the response would be unchanged.

For springs of these three types, when the spring constant is between 10^3 and 10^6 , drastic changes in both frequencies and mode shapes are apparent; when the value of spring constants is less than 10^3 or greater than 10^6 , the changes are slight. With the increase of spring constants, the frequencies also show an increasing trend. Vertical springs such as spring-V_DP1 are apparently more sensitive than springs of the other two types. The first bending mode and first torsional mode are almost unaffected by torsional springs such as spring-R_DP1. Higher modes such as second and fourth bending modes were more sensitive to changes in the torsional spring constant. Comparison of the changes in frequencies and MACs, as shown in Figures 8 and 9, reveals that the spring constants are apparently more sensitive than the elastic modulus in the model updating process. However, considering that no sensors close to the P1 and P2 supports were installed for the experiment, it remains uncertain whether these spring constants are correctly identifiable based on the limited data.

Based on the results of sensitivity analysis, three model classes were investigated, as shown in Figures 7 and 10: Case 1, Case 2, and Case 3. The difference between these three model classes is their different assumptions about model parameters. Case 1, the initial model class, had no additional assumptions. Case 2 assumed that the vertical spring constants are equal to infinity and that the torsional spring constants are zero, in concerning the observation that the modal parameters are over-sensitive to these spring constants. The model of Case 3 further assumes that elastic moduli of SS400_UM, SS400_UP1, and SS400_UP2 are equal, in concerning the observation in the inconsistent MAC change due to the asymmetric sensor layout. It is designated as SS400_U, as shown in Figure 10(b). A parameter list of these three cases is presented in Table 6.

5 Modal parameter-based model updating

5.1 Initial model class: Case 1 model

All 19 parameters and all 10 springs were considered in model updating of the Case 1 model shown in Figure 7. The samples generated at the last sampling step could form a posterior distribution of model parameters. From the posterior distribution, the global maximum could be identified and the model parameter values that give this global maximum were termed most probable value (MPV). Table 7 presents the updated natural frequencies and MAC values of the Case 1 model corresponding to the MPVs of model parameters. In comparison with the FE model considering the design values presented in Table 5, the Case 1 model clearly reduced the discrepancy of the first bending mode. The mode shapes especially well fit the experimentally obtained data, whereas the discrepancies of frequencies might be readily apparent, especially for the first torsional mode. In general, the updated modal properties using the Case 1 model led to better prediction of the modal properties than the FE model considering design values in both the INT and DMG2 states.

Figures 11 and 12 portray marginal distributions of three updated spring constants (spring-H_DP2, spring-R_DP2, and spring-V_DP1) and six elastic moduli of the steel model parameters (SS400_DP1, SS400_DM, SS400_DP2, SS400_UP1, SS400_UM, and SS400_UP2). Note that in this study the marginal distribution is simply used to visualize the high-dimensional posterior distribution in a lower

dimension, and its peak is close to but not necessarily coinciding with the global maximum of the posterior distribution. Most spring constants were decreasing from the INT state to DMG2 state. A full cut was applied to the tension member at the mid-span, as shown in Figure 2. However, most model parameters of the steel showed an increasing tendency, except SS400_UP1, as might be apparent in Figure 12. The value of SS400_DM was expected to decrease, although it increased during model updating. Observations from FE model updating using the Case 1 model showed that the updated model parameters such as elastic moduli and spring constants only slightly link to damage detection although performance of the updated modal properties both for the intact and damage states was improved considerably.

5.2 Without vertical and torsional springs in initial model class: Case 2 model

Considering the modal parameters are too sensitive to the torsional and vertical springs might affect the updating process, the Case 2 model removed the vertical and rotation spring constants from the updating parameters and set vertical constants as infinite and torsional spring constants as zero. Table 8 presents the updated natural frequencies and MAC values of the Case 1 model corresponding to the MPVs of model parameters. The total discrepancy of modal properties of Case 2 model between those from experiment and the FE update was greater than that of the Case 1 model. However, for the INT scenario, the accuracy of the updated frequency of the first torsional mode of Case 2 model was improved markedly.

Figures 13 and 14 show marginal distributions of the updated parameters of the Case 2 model. Table 9 presents the MPVs of each parameter corresponding to the modal properties presented in Table 8. It is noteworthy that the spring constant of spring-H_UP2 was decreased drastically when comparing the INT state to the DMG2 state, as shown in Figure 13(b). The change in the spring constant of the undamaged upstream side was much greater than that of the damaged downstream side. For the model parameters, when comparing the INT state to DMG2 state, a decrease of elastic modulus of SS400_DM is visible; also, those for SS400_DP1 and SS400_DP2 were almost unchanged, which approximates an actual structural situation. However, the elastic modulus of SS400_UM was decreased, whereas those for SS400_UP1 and SS400_UP2 were increased.

5.3 Without vertical and torsional springs and with only a single model parameter on the upstream side in initial model class: Case 3 model

Considering the asymmetry of the sensor layout, the Case 3 model incorporates the assumption of a single model parameter for the upstream side of the bridge and the horizontal spring at P2, as shown in Figure 10(b). The updated natural frequencies and MAC values of the Case 1 model corresponding to the MPVs of model parameters in Case 3 model are presented in Tables 10 and 11. Figures 15 and 16 present histograms of the updated model parameters.

Two peaks in the marginal distributions of the updated model parameter of the DMG2 state are observed as shown in Figures 15 and 16: each peak is denoted as DMG2(1) and DMG2(2). The first and third bending modes of DMG2(1) are closer to experimentally obtained data, whereas the first torsional mode, second bending mode, and fourth bending modes of DMG2(2) are closer to experimental data, as shown in Table 10.

Because of artificial damage at mid-span on the downstream side, we expected that the SS400_DM stiffness decreases, whereas stiffness of other model parameters is nearly unchanged. The DMG2(2)

histogram is more like the expectation. From Table 11, the MPV of SS400_DM of DMG2(2) decreased about 16.2 GPa, whereas tiny changes in SS400_U, SS400_DP1, and SS400_DP2 are apparent. That fact demonstrates that the possible damage was located in the block of SS400_DM. The distribution of spring-H_UP2 was almost entirely unchanged, although a clear decrease of spring-H_DP2 was observed. Consequently, it is probable that Case 3 performed much better than the other two cases: Case 1 and Case 2 models.

5.4 Model updating under DMG1 and RCV states using the Case 3 model

In earlier discussions, Case 3 model was shown to be closely comparable with the engineering prediction under INT and DMG2 states. Therefore, this section presents an investigation of model updating under DGM1 and RCV states using the Case 3 model. For the DMG3 state, except for the first bending mode and first torsional mode, the other three modes were not observed using FE analysis. One probable reason is the difference in damage location from other states. The designed blocks cannot reflect the damage of DMG3 states well. Therefore, discussions presented in this section specifically address results of DMG1 and RCV states.

The MPVs of updated modal parameters are presented in Table 12, which shows that the values of the updated MACs and frequencies closely approximate those obtained from experiments. For the first bending frequency, the difference of updated frequencies was approximately 8% both for DMG1 and RCV states, similarly to results found for DMG2(2). Differences of updated frequencies for other modes were less than 6.7%.

In Table 13, updated parameters of DMG1 and RCV states are presented. Those of INT and DMG2(2) of Table 11 are shown again for convenience. In both DMG1 and RCV states, only a single peak was observed in the histogram, even though it is omitted here for brevity. From INT to DMG1, because the artificial damage is a half cut, the elastic modulus of each block is nearly the same. When the damage became a full cut, as in the DMG2 state, the value of SS400_DM decreased considerably. When the cut member was recovered as shown in RCV state, an increase of SS400_DM was observed. Except for SS400_DM, the changes of model parameters were slight.

The spring constants were also changed clearly in each state. The strength of the spring constant of spring-H_DP1 on the damaged side was much greater than that of spring-H_UP1 on the undamaged side.

6 Discussions

The Case 1 model considers 19 model parameters (10 spring constants and nine elastic moduli) during model updating. It is treated as an initial model. The Case 2 model appropriately reduced the number of spring parameters from the Case 1 model considering the lack of sensor close to the supports, i.e. the model without vertical and torsional springs in initial model class. The Case 3 model reduced the number of parameters further, i.e. without vertical and torsional springs and with only a single model parameter on the upstream side in initial model class.

The modal properties of the updated models all closely approximate those identified from experimental data. In terms of modal properties, model updating achieves the original purpose of obtaining a better model. However, the updated parameters of these three cases showed a different tendency. In the Case 1 model, although the updated modal properties fit better, all parameter distributions poorly approximated

the actual data. Then, in the Case 2 model, although the result was much better than that found for the Case 1 model, some unreasonable changes in parameters were found on the downstream side. The Case 3 model distribution of the updated parameters shows a similar trend to that obtained by experimentation.

The physical meaning should be considered in the model updating process for real structures. Most existing studies [e.g., 10, 11, 22] have considered the model class selection based only on probability. Therefore, the importance of physical meaning was not identified explicitly. In this study, the Case 1 model showed the best fitness with modal properties and the worst fitness with the engineering prediction, whereas the Case 3 model showed the worst fitness with modal properties and the best fitness with the engineering prediction. These results indicate that the fitness with modal properties and with the engineering prediction is not always equal. Moreover, because of a lack of data, this phenomenon would readily occur in actual structures. Therefore, it is extremely important to consider the physical meanings in model class selection, especially for actual structures.

Investigations demonstrated that the model class, which includes as much information as possible, can improve the model updating accuracy, especially for real structures for which data are lacking. As stated in [5,6], making full use of information such as geometric characteristics, and sensor deployment (sensor number, location, and type), is important for model updating processes. The difference between the three model classes considered in this study derives from their different prior information. The Case 3 model, which used the most information, produced the best fit for prediction by experimentation and therefore was regarded as the best selection among these three model classes.

Structural damage also affects model updating. The Case 3 model was suitable for the INT, DMG1, DMG2, and RCV states. However, the Case 3 model was found to be unsuitable for DMG3 state because of the different damage locations. On the other hand, correctly detecting structural damage is also significant for improving the accuracy of FEA. One approach to improving the damage simulation performance of the model is to obtain more available data for more detailed model updating with smaller blocks.

7 Conclusions

This report describes an investigation of a Bayesian model updating method for an actual steel truss bridge. Vibration and damage experiments were conducted on the target bridge. Then the modal properties such as natural frequencies and mode shapes were identified from the measured data and were taken into the objective function for the model updating process. Because of the limited data, the model was grouped to reduce the number of uncertain parameters. According to the sensitivity analysis, three model classes were proposed for model updating. Based on the updated modal properties and model parameters, the feasibility and limitation of the vibration-based model updating method on the real bridge were discussed.

This study demonstrates the significance of model class on the physical meaning of updated model parameters and proposes some recommendations for designing a suitable model class for the actual bridges. With different model classes used, although the updated modal parameters can always be close to those identified from experimental data, the physical meaning of updated model parameters might be different from experimental prediction. The physical meanings are the basic for the accuracy of FE analysis. Therefore, it should be considered when choosing model classes. Making full using of system

information for the design of model class can improve the performance of physical meanings. Not only the data measured by sensors, prior information, including geometric characteristics, sensor deployment, and damage information, can provide a powerful supplement.

The feasibility of damage detection on real structures using model updating method was also demonstrated. Structural damage is one type of uncertain information in the system. Therefore, it is also possible to identify the damage information by model updating. With a suitable model class, the damaged block can be identified correctly. With an unsuitable model class, although the occurrence of structural damage also can be indicated, the damaged location is often wrong.

Although these concluding remarks apply to bridge structures and damage scenarios similar to those in this experimental study, the methods presented herein demonstrate potential real-world implementation on bridges. However, further investigations of various structures (other types of bridges or buildings), damage scenarios, and damage severity will be necessary to support the general applicability of such methods.

Acknowledgments

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Table 1: Damage scenarios

Scenario	Description
INT	No damage
DMG1	Half cut in a vertical member at the mid-span
DMG2	Full cut in a vertical member at the mid-span
RCV	Recovery of the cut member at the mid-span
DMG3	Full cut in a vertical member at the 5/8 span

Table 2: Identified frequencies for the respective damage scenarios used for the field experiment

Scenario	1st bending mode		1st torsional mode		2nd bending mode		3rd bending mode		4th bending mode	
	Freq. (Hz)	COV (%)	Freq. (Hz)	COV (%)	Freq. (Hz)	COV (%)	Freq. (Hz)	COV (%)	Freq. (Hz)	COV (%)
INT	2.98	0.0932	5.21	0.3181	6.87	0.1188	9.61	0.1087	10.57	0.2472
DMG1	2.98	0.0397	5.22	0.5803	6.89	0.0508	9.69	0.0571	10.61	0.2229
DMG2	2.88	0.0259	4.99	0.2232	6.87	0.1308	9.67	0.0195	10.59	0.1623
RCV	2.97	0.0378	5.19	0.2603	6.84	0.0571	9.57	0.0603	10.46	0.1015
DMG3	2.92	0.0555	5.10	0.1870	6.46	0.1800	8.66	0.2152	10.07	0.1686

Freq., mean frequency for each mode; COV, coefficient of variation of the identified frequency

Table 3: MAC values of the mode shapes in DMG1, DMG2, RCV, and DMG3 states compared with the INT state

Scenario	1st bending mode	1st torsional mode	2nd bending mode	3rd bending mode	4th bending mode
DMG1	1.0000	0.9994	0.9899	0.9987	0.9966
DMG2	0.9987	0.9928	0.9915	0.9985	0.9990
RCV	0.9995	0.9983	0.9978	0.9974	0.9988
DMG3	0.9945	0.9709	0.8040	0.3109	0.6134

Table 4: Design values of parameters

Parameter	Young's modulus of steel	Young's modulus of RC	horizontal spring constant	vertical spring constant	rotation spring constant
Value	200 GPa	21 GPa	$10^9 \text{ kN} \cdot \text{m}^{-1}$	$\infty \text{ kN} \cdot \text{m}^{-1}$	$0 \text{ kN} \cdot \text{m}^{-1} \cdot \text{rad}^{-1}$

Table 5: Dynamic properties of the FE model with design values and with results of experimentation for INT and DMG2 states

		1st bending mode	1st torsional mode	2nd bending mode	3rd bending mode	4th bending mode
INT	Freq. (Hz) (experiment)	2.98	5.21	6.87	9.61	10.57
	Freq. (Hz) (FE update)	3.20	4.96	6.78	9.97	10.71
	Difference of Freq. (%)	7.49	-4.77	-1.25	3.68	1.37
	MAC	0.9948	0.9906	0.9703	0.9389	0.9464
DMG2	Freq. (Hz) (experiment)	2.88	4.99	6.87	9.67	10.59
	Freq. (Hz) (FE update)	3.20	4.96	6.78	9.97	10.71
	Difference of Freq. (%)	10.88	-0.57	-1.35	3.13	1.19
	MAC	0.9885	0.9715	0.9752	0.9431	0.9414

Table 6: Parameter list of three cases

Parameter	Case 1	Case 2	Case 3
SS400_DP1/DM/DP2	✓	✓	✓
SS400_UP1	✓	✓	Equal to SS400_U
SS400_UM	✓	✓	
SS400_UP2	✓	✓	
SS400_T, SS400_B, and RC	✓	✓	✓
spring-H constants	✓	✓	✓
spring-V constants	✓	NA	NA
spring-R constants	✓	NA	NA

Table 7: MPV of the updated frequencies and MAC of the Case 1 model

		1st bending mode	1st torsional mode	2nd bending mode	3rd bending mode	4th bending mode
INT	Freq. (Hz) (experiment)	2.98	5.21	6.87	9.61	10.57
	Freq. (Hz) (FE update)	3.04	4.69	6.70	9.59	10.54
	Difference of Freq. (%)	2.22	-9.95	-2.43	-0.23	-0.25
	MAC	0.9967	0.9968	0.9947	0.9982	0.9978
DMG2	Freq. (Hz) (experiment)	2.88	4.99	6.87	9.67	10.59
	Freq. (Hz) (FE update)	2.99	4.84	6.99	9.79	10.85
	Difference of Freq. (%)	3.52	- 2.99	1.63	1.25	2.51
	MAC	0.9986	0.9848	0.9912	0.9907	0.9909

Table 8: MPV of the updated frequencies and MAC of the Case 2 model

		1 st bending mode	1 st torsional mode	2 nd bending mode	3 rd bending mode	4 th bending mode
INT	Freq. (Hz) (experiment)	2.98	5.21	6.87	9.61	10.57
	Freq. (Hz) (FE update)	3.09	5.06	6.68	10.12	10.63
	Difference of Freq. (%)	3.95	-2.96	-2.67	5.23	0.60
	MAC	0.9997	0.9939	0.9808	0.9961	0.9668
DMG2	Freq. (Hz) (experiment)	2.88	4.99	6.87	9.67	10.59
	Freq. (Hz) (FE update)	3.09	4.79	6.56	9.83	10.28
	Difference of Freq. (%)	6.97	-4.01	-4.60	1.67	-2.89
	MAC	0.9992	0.9852	0.9921	0.9907	0.9886

Table 9: MPV of updated model parameters of the Case 2 model

	INT	DMG2
SS400_DP1(GPa)	169.7	164.6
SS400_DM(GPa)	168.6	153.6
SS400_DP2(GPa)	161.7	156.3
SS400_UP1(GPa)	173.0	216.2
SS400_UM(GPa)	192.8	161.1
SS400_UP2(GPa)	168.4	197.9
SS400_T(GPa)	131.1	160.0
SS400_B(GPa)	225.6	219.5
RC(GPa)	29.2	12.5
spring-H_DP2(kN·m ⁻¹)	8.1×10^8	5.5×10^8
spring-H_UP2(kN·m ⁻¹)	8.0×10^8	4.7×10^5

Table 10: MPV of the updated frequencies and MAC of the Case 3 model

		1st bending mode	1st torsional mode	2nd bending mode	3rd bending mode	4th bending mode
INT	Freq. (Hz) (experiment)	2.98	5.21	6.87	9.61	10.57
	Freq. (Hz) (FE update)	3.18	4.92	6.78	10.10	10.60
	Difference of Freq. (%)	7.01	-5.59	-1.32	5.05	0.29
	MAC	0.9991	0.9974	0.9819	0.9904	0.9766
DMG2(1)	Freq. (Hz) (experiment)	2.88	4.99	6.87	9.67	10.59
	Freq. (Hz) (FE update)	3.04	4.69	6.47	9.69	10.14
	Difference of Freq. (%)	5.37	-6.10	-5.89	0.20	-4.21
	MAC	0.9991	0.9824	0.9961	0.9909	0.9726
DMG2(2)	Freq. (Hz) (experiment)	2.88	4.99	6.87	9.67	10.59
	Freq. (Hz) (FE update)	3.14	4.82	6.64	9.90	10.29
	Difference of Freq. (%)	8.80	-3.38	-3.48	2.44	-2.76
	MAC	0.9992	0.9818	0.9957	0.9901	0.9732

Table 11: MPV of the updated parameters of the Case 3 model

Model parameter	INT	DMG2(1)	DMG2(2)
SS400_DP1(GPa)	172.6	165.2	179.7
SS400_DM(GPa)	168.8	143.3	152.6
SS400_DP2(GPa)	154.9	147.3	156.1
SS400_U(GPa)	205.0	185.0	199.8
SS400_T(GPa)	225.6	231.1	225.6
SS400_B(GPa)	247.3	224.6	203.5
RC(GPa)	14.8	13.3	15.0
spring-H_DP2(kN·m ⁻¹)	1.9×10^8	2.0×10^7	6.6×10^7
spring- H_UP2(kN·m ⁻¹)	2.8×10^5	2.1×10^5	1.7×10^5

Table 12: MPV of the updated frequencies and MAC under the Case 3 model in DMG1 and RCV states

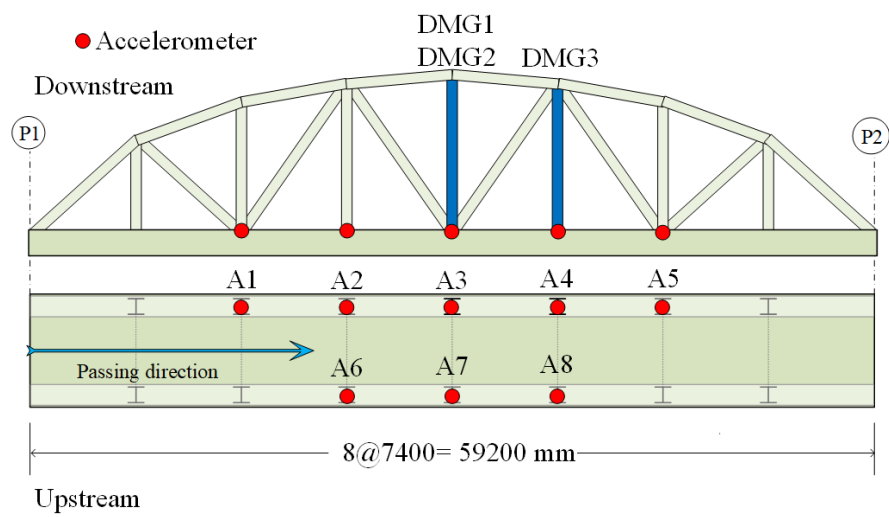
		1st bending mode	1st torsional mode	2nd bending mode	3rd bending mode	4th bending mode
DMG1	Freq. (Hz) (experiment)	2.98	5.21	6.87	9.61	10.57
	Freq. (Hz) (FE update)	3.20	4.93	6.80	10.11	10.60
	Difference of Freq. (%)	7.49	-5.46	-1.03	5.18	0.30
	MAC	0.9971	0.9958	0.9953	0.9945	0.9814
RCV	Freq. (Hz) (experiment)	2.97	5.19	6.84	9.57	10.46
	Freq. (Hz) (FE update)	3.22	4.97	6.84	10.22	10.71
	Difference of Freq. (%)	8.26	-4.16	0.05	6.74	2.36
	MAC	0.9986	0.9924	0.9876	0.9913	0.9824

Table 13: MPV of the updated parameters under Case 3 model in DMG1 and RCV states

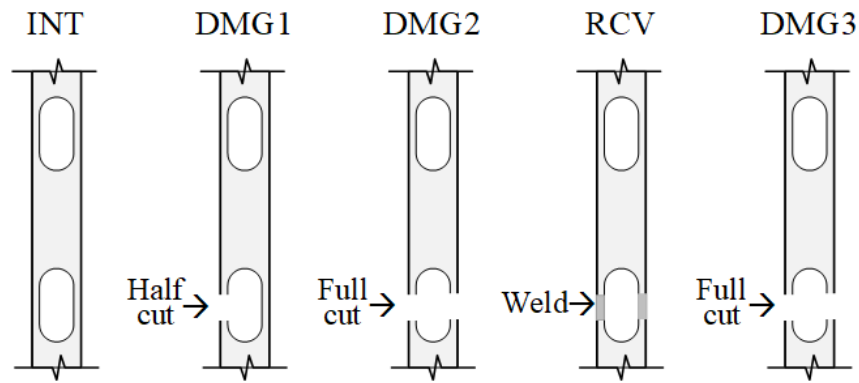
Model parameter	INT	DMG1	DMG2(2)	RCV
SS400_DP1(GPa)	172.6	172.7	179.7	179.8
SS400_DM(GPa)	168.8	169.5	152.6	178.0
SS400_DP2(GPa)	154.9	150.1	156.1	157.5
SS400_U (GPa)	205.0	211.6	199.8	203.1
SS400_T(GPa)	225.6	215.0	225.6	208.7
SS400_B(GPa)	247.3	243.4	203.5	243.4
RC(GPa)	14.8	15.6	15.0	17.5
spring-H_DP2(kN·m ⁻¹)	1.9×10^8	3.2×10^7	6.6×10^7	4.4×10^7
spring- H_UP2(kN·m ⁻¹)	2.8×10^5	1.8×10^5	1.7×10^5	2.9×10^5



Figure 1: Target bridge.



(a) Sensor deployment



(b) Sketch of damage scenarios

Figure 2: Sensor layout and damage scenario [35].

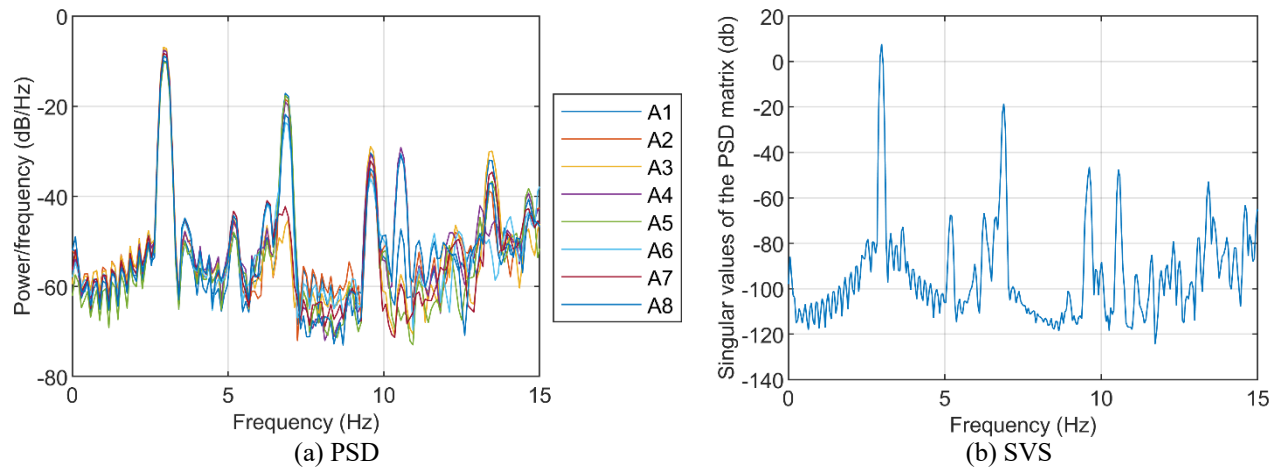


Figure 3: Power spectral density (PSD) and first singular value spectrum (SVS) from the acceleration data.

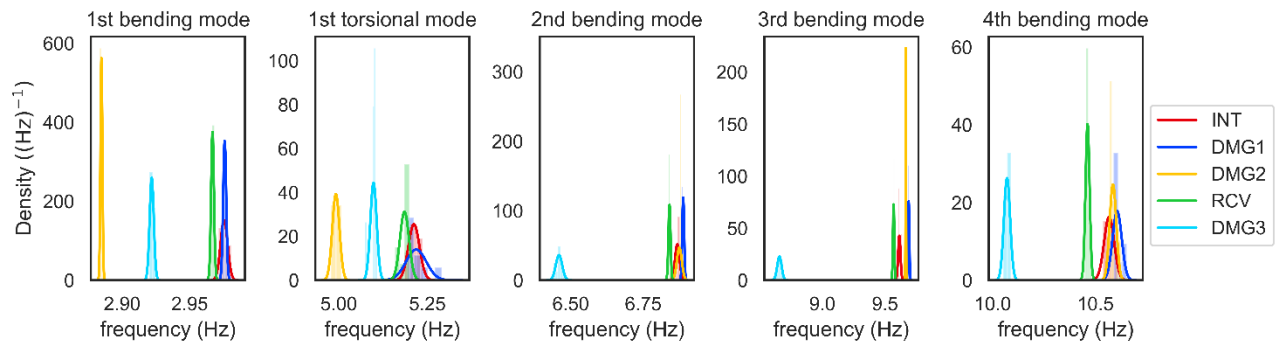
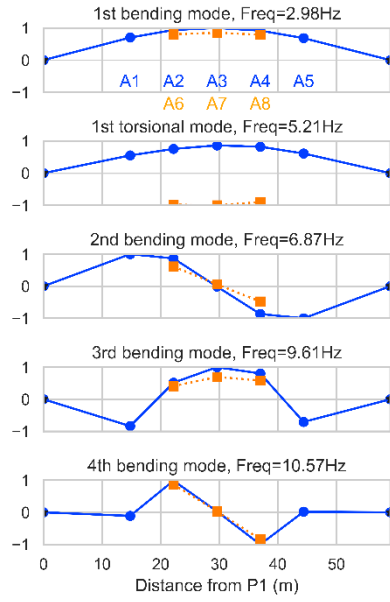
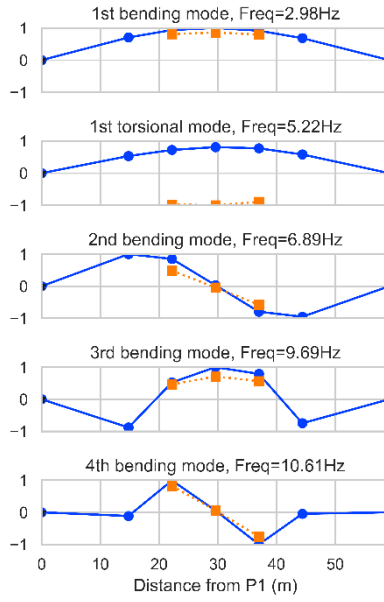


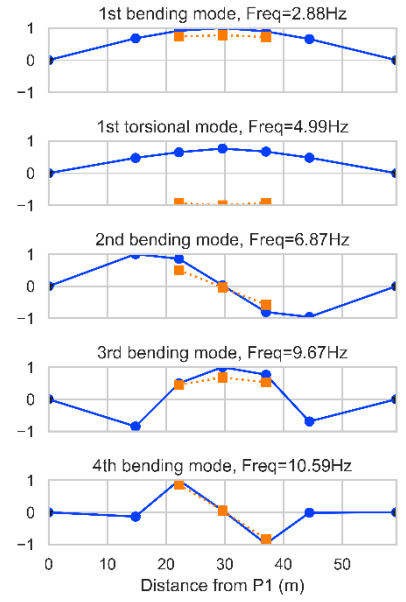
Figure 4: Histograms with the distribution fit of the identified frequencies.



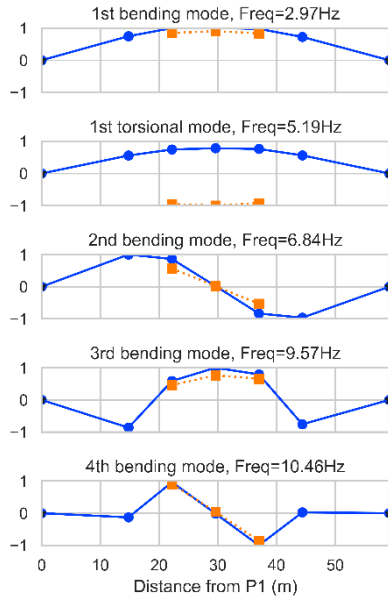
(a) INT state



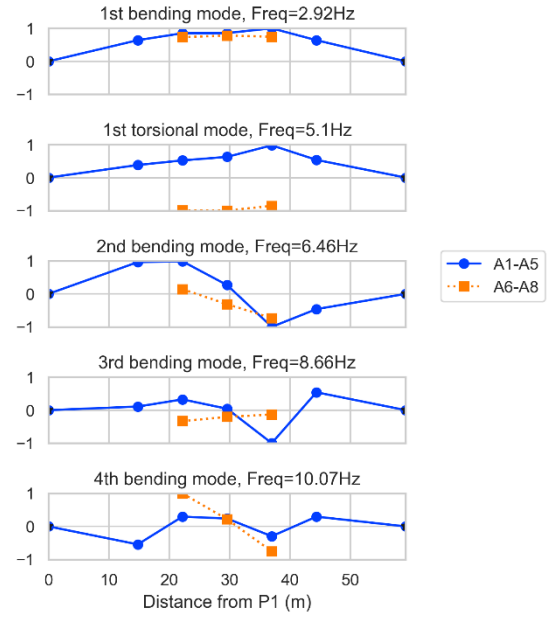
(b) DMG1 state



(c) DMG2 state



(d) RCV state



(e) DMG3 state

Figure 5: Identified mode shapes.

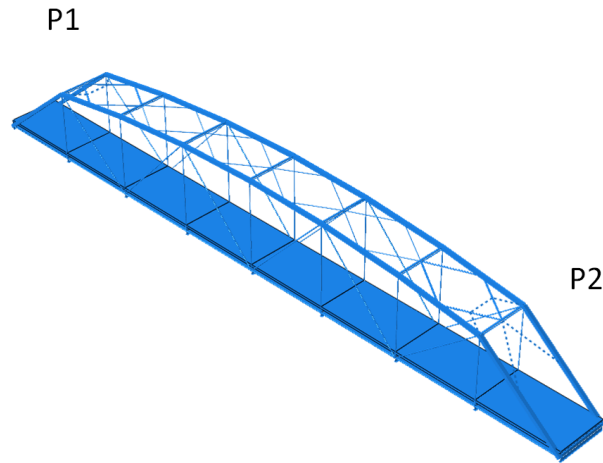


Figure 6: FE model.

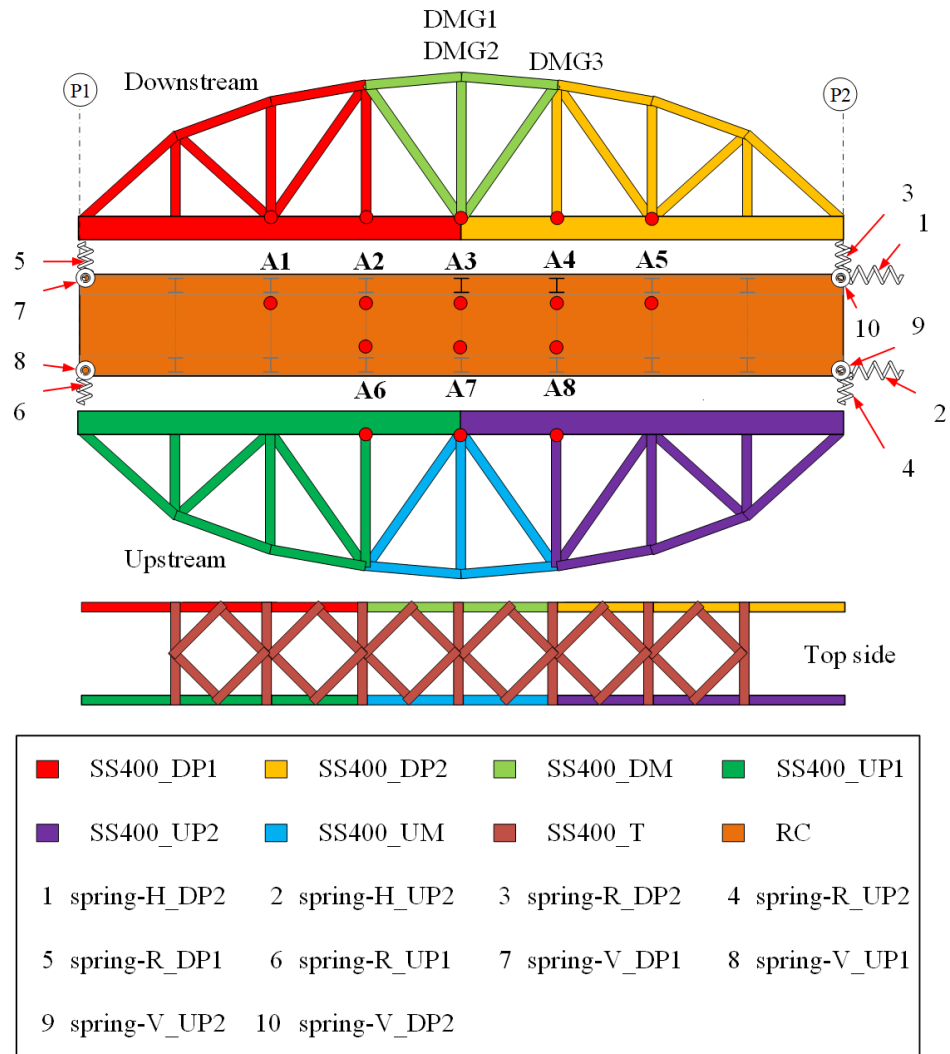


Figure 7: Initial model class – Case 1 model.

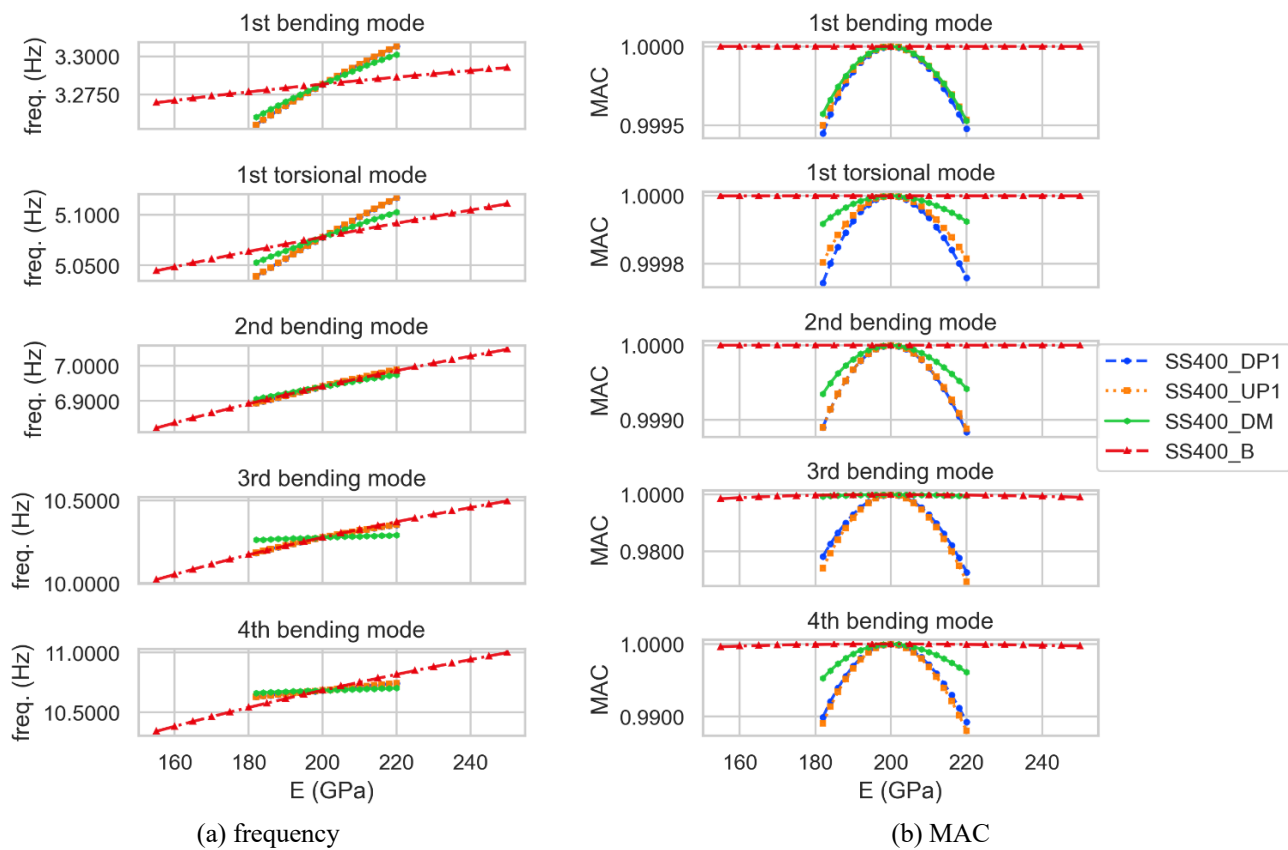


Figure 8: Sensitivity analysis of elastic modulus.

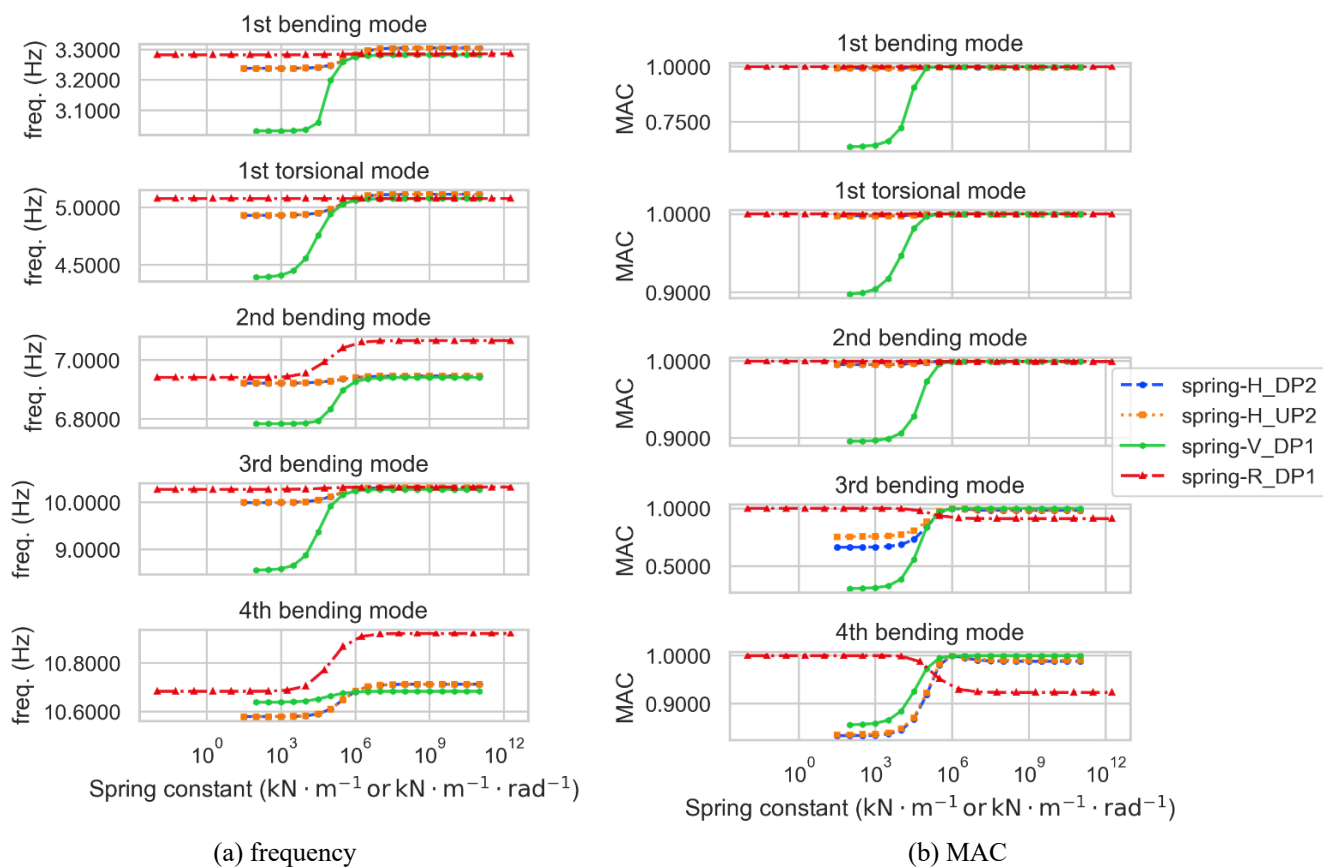
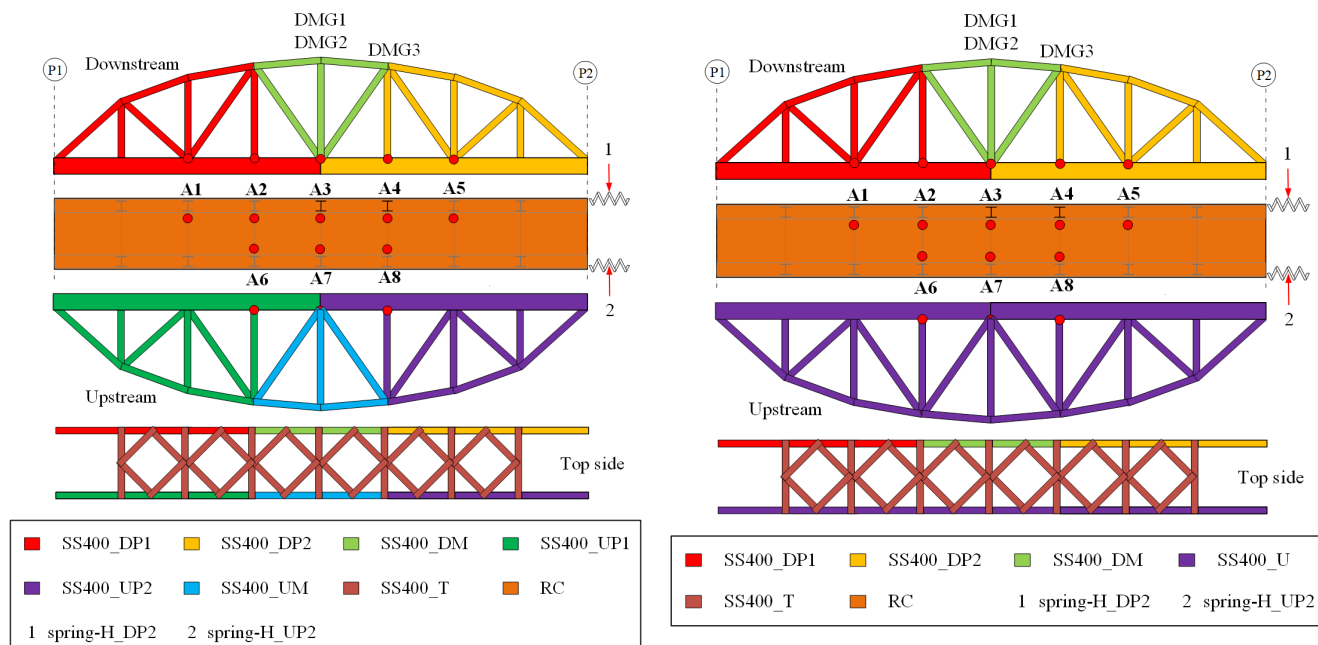


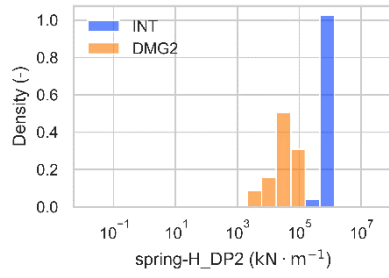
Figure 9: Sensitive analysis of springs.



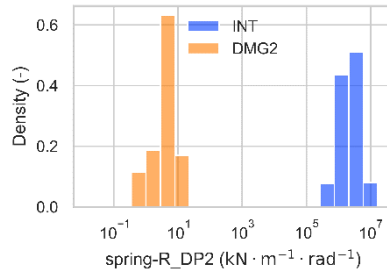
(a) Case 2 Model

(b) Case 3 model

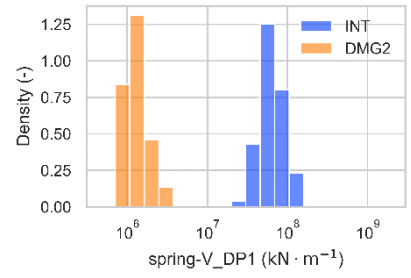
Figure 10: Model classes – Case 2 and Case 3 models.



(a) spring-H_DP2

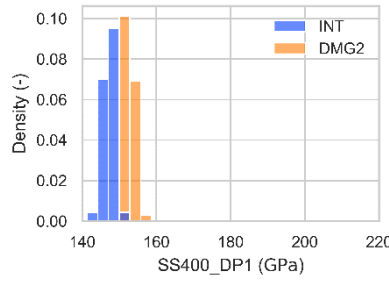


(b) spring-R_DP2

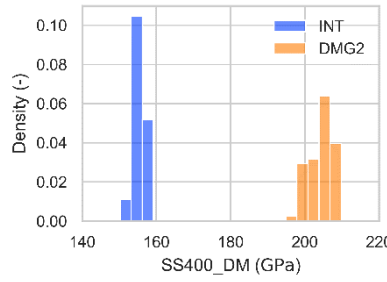


(c) spring-V_DP1

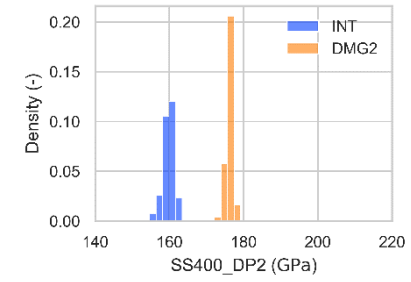
Figure 11: Marginal distributions of spring constants of the Case 1 model.



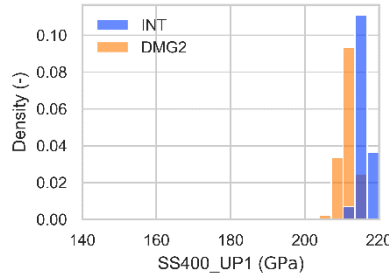
(a) SS400_DP1



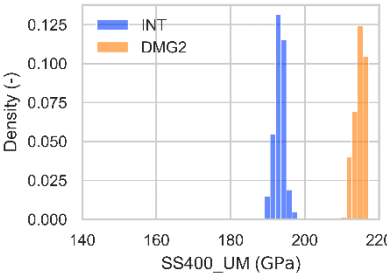
(b) SS400_DM



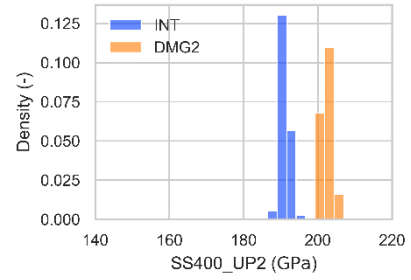
(c) SS400_DP2



(d) SS400_UP1

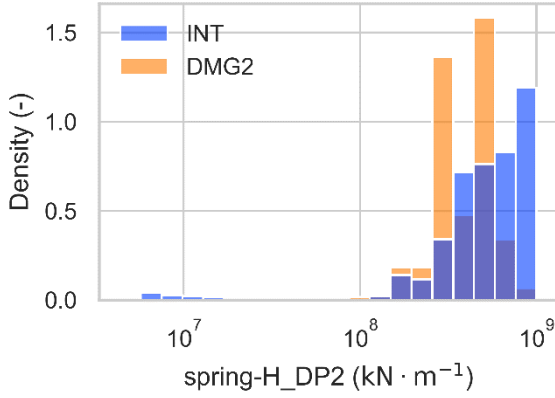


(e) SS400_UM

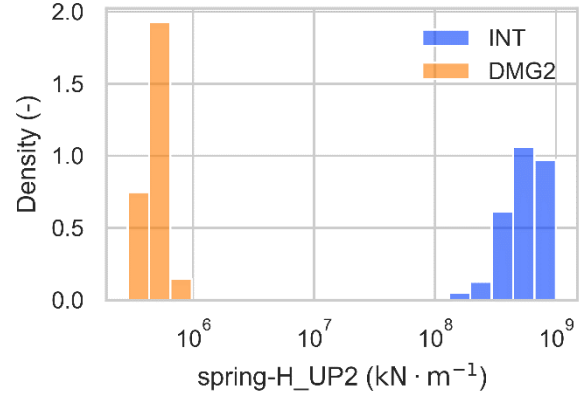


(f) SS400_UP2

Figure 12 Marginal distributions of elastic modulus of the Case 1 model.

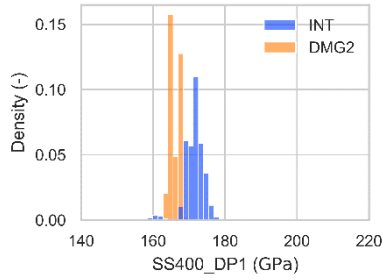


(a) spring-H_DP2

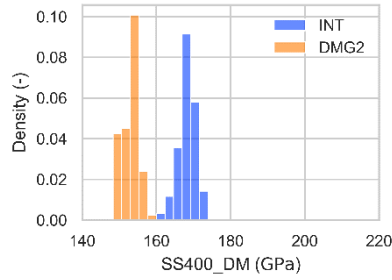


(b) spring-H_UP2

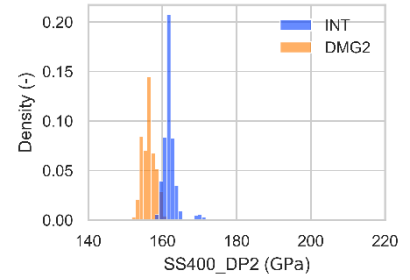
Figure 13: Marginal distribution of horizontal spring constants of the Case 2 model.



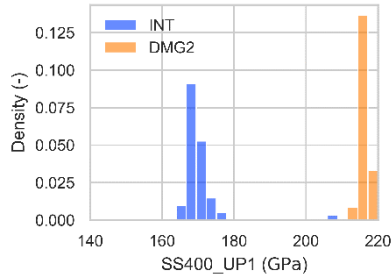
(a) SS400_DP1



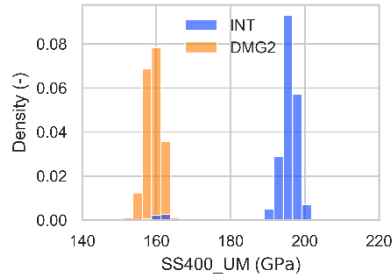
(b) SS400_DM



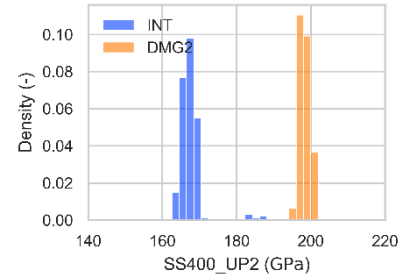
(c) SS400_DP2



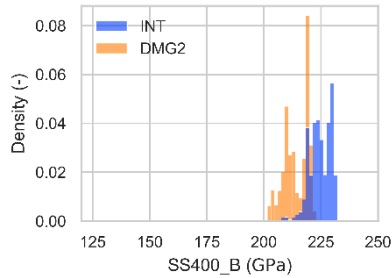
(d) SS400_UP1



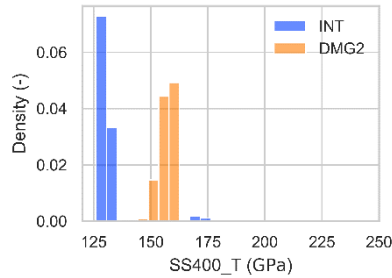
(e) SS400_UM



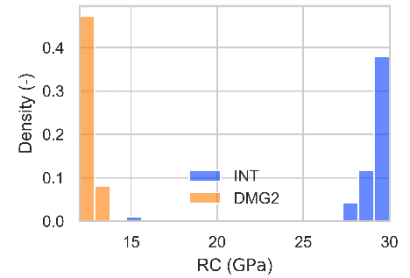
(f) SS400_UP2



(g) SS400_B

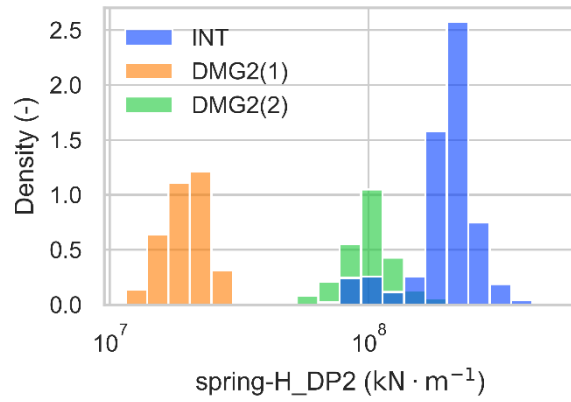


(h) SS400_T

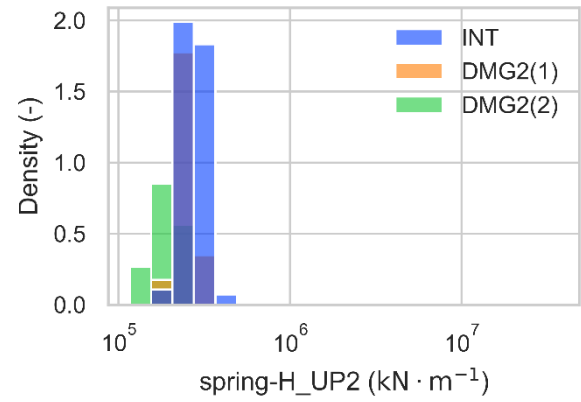


(i) RC

Figure 14: Marginal distribution of elastic modulus of the Case 2 model.

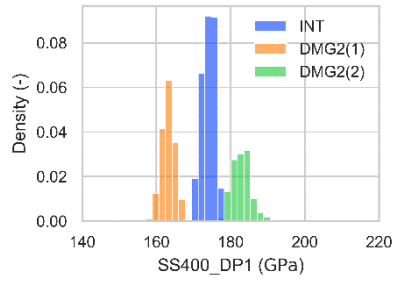


(a) spring-H_DP2

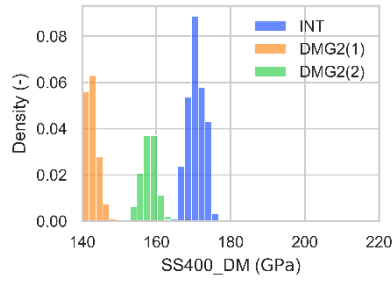


(b) spring-H_UP2

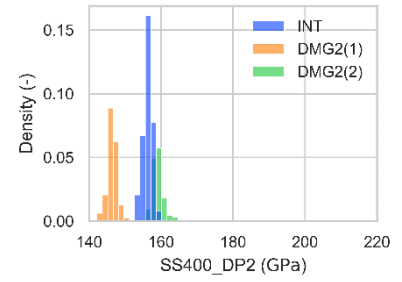
Figure 15 Marginal distribution of horizontal spring constants in the Case 3 model.



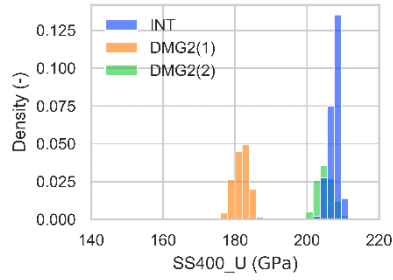
(a) SS400_DP1



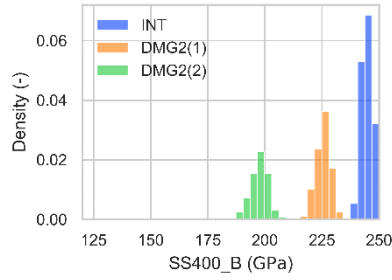
(b) SS400_DM



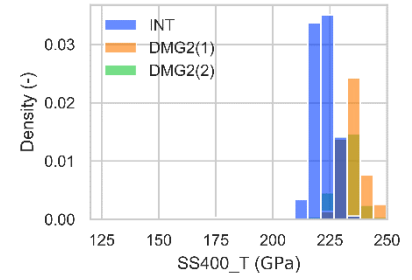
(c) SS400_DP2



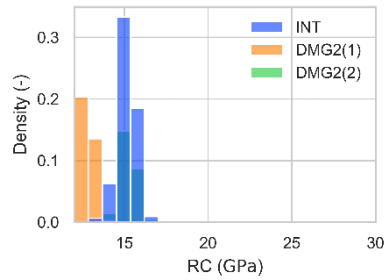
(d) SS400_U



(e) SS400_B



(f) SS400_T



(g) RC

Figure 16 Marginal distribution of elastic modulus in the Case 3 model