

4 次方程式における 4 重根の精度

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$$x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \quad (1)$$

解は、 $x_1 = x_2 = x_3 = x_4 = \alpha \cdot 10^\beta$ である。

解と係数の関係より、次式が得られる。

$$\begin{cases} a_3 = -4\alpha \cdot 10^\beta + [10^{\beta-m}] \\ a_2 = 6\alpha^2 \cdot 10^{2\beta} + [10^{2\beta-m}] \\ a_1 = -4\alpha^3 \cdot 10^{3\beta} + [10^{3\beta-m}] \\ a_0 = \alpha^4 \cdot 10^{4\beta} + [10^{4\beta-m}] \end{cases} \quad \begin{array}{l} [\quad] \text{ 内の桁以下は} \\ \text{計算に用いられ} \\ \text{ない桁} \\ m : \text{ 計算桁数} \end{array}$$

$x = y + \bar{x}$ を用いて、4 次方程式 (1) を 3 次の係数が零である 4 次方程式 (2) に変換する。

$$y^4 + k'y^2 + l'y + m' = 0 \quad (2)$$

4 次方程式 (2) の係数は、次のように、誤差のみになる。

$$\begin{cases} \bar{x} = -\frac{1}{4} a_3 = \alpha \cdot 10^\beta + [10^{\beta-m}] \\ k' = -\frac{3}{8} a_3^2 + a_2 = -6\alpha^2 \cdot 10^{2\beta} + 6\alpha^2 \cdot 10^{2\beta} + [10^{2\beta-m}] \\ \quad = \varepsilon_k \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \end{cases}$$

$$\left\{ \begin{aligned}
 \ell' &= \frac{1}{8} a_3^3 - \frac{1}{2} a_3 a_2 + a_1 \\
 &= -8 \alpha^3 \cdot 10^{3\beta} + 12 \alpha^3 \cdot 10^{3\beta} - 4 \alpha^3 \cdot 10^{3\beta} + [10^{3\beta-m}] \\
 &= \varepsilon_{\ell'} \cdot 10^{3\beta-m} + [10^{3\beta-2m}] \\
 m' &= -\frac{3}{4} a_3^4 + \frac{1}{4} a_3^2 a_2 - \frac{1}{4} a_3 a_1 + a_0 \\
 &= -3 \alpha^4 \cdot 10^{4\beta} + 6 \alpha^4 \cdot 10^{4\beta} - 4 \alpha^4 \cdot 10^{4\beta} + \alpha^4 \cdot 10^{4\beta} + [10^{4\beta-m}] \\
 &= \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}]
 \end{aligned} \right.$$

4次方程式 (2) が4次方程式 $(y^2 + u)^2 - (vy + w)^2 = 0$ に変形できるように、 u, v, w を求める。

$$\left\{ \begin{aligned}
 v^2 &= 2u - k' = 2u - \varepsilon_{k'} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \\
 v^2 w^2 &= \ell'^2 / 4 = \frac{1}{4} \varepsilon_{\ell'}^2 \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \\
 w^2 &= u^2 - m' = u^2 - \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}]
 \end{aligned} \right. \quad (3)$$

$$v^2 w^2 = (2u - k')(u^2 - m') = \ell'^2 / 4$$

$$\begin{aligned}
 & (2u - \varepsilon_{k'} \cdot 10^{2\beta-m} + [10^{2\beta-2m}]) (u^2 - \varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}]) \\
 &= \frac{1}{4} \varepsilon_{\ell'}^2 \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 & u^3 + \left\{ -\frac{\varepsilon_{k'}}{2} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \right\} u^2 + \left\{ -\varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \right\} u \\
 & + \left\{ \left(\frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{2} - \frac{\varepsilon_{\ell'}^2}{8} \right) \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \right\} = 0 \quad (5)
 \end{aligned}$$

座標変換 (6) により、(5) を2次の係数が零である3次方程式 (7) に変換する。

$$u = z + \left\{ \frac{\varepsilon_{k'}}{6} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \right\} \quad (6)$$

$$z^3 + 3pz + q = 0 \quad (7)$$

$$\begin{cases} p = -\frac{\varepsilon_m}{3} \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \\ q = \left(\frac{\varepsilon_k \cdot \varepsilon_{m'}}{3} - \frac{\varepsilon_{\ell'}^2}{8} \right) \cdot 10^{6\beta-2m} + [10^{6\beta-3m}] \end{cases}$$

$$q^2 + 4p^3 = -\frac{4}{27} \varepsilon_m^2 \cdot 10^{12\beta-3m} + [10^{12\beta-4m}]$$

$$\sqrt{q^2 + 4p^3} = 2 \sqrt{\frac{\varepsilon_m^3}{27}} i \cdot 10^{6\beta-3m/2} + [10^{6\beta-5m/2}]$$

$$\begin{aligned} (\sqrt{q^2 + 4p^3} - q)/2 &= \sqrt{\frac{\varepsilon_m^3}{27}} i \cdot 10^{6\beta-3m/2} + \left(\frac{\varepsilon_{\ell'}^2}{16} - \frac{\varepsilon_k \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{6\beta-2m} \\ &\quad + [10^{6\beta-5m/2}] \end{aligned}$$

$$\begin{aligned} (-\sqrt{q^2 + 4p^3} - q)/2 &= -\sqrt{\frac{\varepsilon_m^3}{27}} i \cdot 10^{6\beta-3m/2} \\ &\quad + \left(\frac{\varepsilon_{\ell'}^2}{16} - \frac{\varepsilon_k \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{6\beta-2m} + [10^{6\beta-5m/2}] \end{aligned}$$

$$\begin{aligned} S &= \sqrt[3]{(\sqrt{q^2 + 4p^3} - q)/2} \\ &= \frac{(\sqrt{3} + i)}{2} \cdot \sqrt{\frac{\varepsilon_m}{3}} \cdot 10^{2\beta-m/2} \left\{ 1 - \frac{1}{3} \sqrt{\frac{27}{\varepsilon_m^3}} \left(\frac{\varepsilon_{\ell'}^2}{16} - \frac{\varepsilon_k \cdot \varepsilon_{m'}}{6} \right) i \cdot 10^{-m/2} + [10^{-m}] \right\} \\ &= \frac{(\sqrt{3} + i)}{2} \cdot \sqrt{\frac{\varepsilon_m}{3}} \cdot 10^{2\beta-m/2} + \frac{(1 - \sqrt{3}i)}{2 \varepsilon_m} \left(\frac{\varepsilon_{\ell'}^2}{16} - \frac{\varepsilon_k \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} \\ &\quad + [10^{2\beta-3m/2}] \end{aligned}$$

$$\begin{aligned} t &= \sqrt[3]{(-\sqrt{q^2 + 4p^3} - q)/2} \\ &= \frac{(\sqrt{3} - i)}{2} \cdot \sqrt{\frac{\varepsilon_m}{3}} \cdot 10^{2\beta-m/2} \left\{ 1 + \frac{1}{3} \sqrt{\frac{27}{\varepsilon_m^3}} \left(\frac{\varepsilon_{\ell'}^2}{16} - \frac{\varepsilon_k \cdot \varepsilon_{m'}}{6} \right) i \cdot 10^{-m/2} + [10^{-m}] \right\} \end{aligned}$$

$$= \frac{(\sqrt{3}-i)}{2} \cdot \sqrt{\frac{\varepsilon_{m'}}{3}} \cdot 10^{2\beta-m/2} + \frac{(1+\sqrt{3}i)}{2\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} \\ + [10^{2\beta-3m/2}]$$

$$\omega \cdot S = \frac{(-\sqrt{3}+i)}{2} \cdot \sqrt{\frac{\varepsilon_{m'}}{3}} \cdot 10^{2\beta-m/2} + \frac{(1+\sqrt{3}i)}{2\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} \\ + [10^{2\beta-3m/2}] \quad (\omega^3 = 1.0)$$

$$\omega \cdot t = i \cdot \sqrt{\frac{\varepsilon_{m'}}{3}} \cdot 10^{2\beta-m/2} - \frac{1}{\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$\omega^2 \cdot S = -i \cdot \sqrt{\frac{\varepsilon_{m'}}{3}} \cdot 10^{2\beta-m/2} - \frac{1}{\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$\omega^2 \cdot t = -\frac{(\sqrt{3}+i)}{2} \cdot \sqrt{\frac{\varepsilon_{m'}}{3}} \cdot 10^{2\beta-m/2} + \frac{(1-\sqrt{3}i)}{2\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} \\ + [10^{2\beta-3m/2}]$$

しに於て、(7) の解 Z_1, Z_2, Z_3 は (8) である。

$$\left\{ \begin{aligned} Z_1 &= S + t \\ &= \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{1}{\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\ Z_2 &= \omega S + \omega^2 t \\ &= -\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{1}{\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{16} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{6} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\ Z_3 &= \frac{-\eta}{Z_1 \cdot Z_2} = -\frac{1}{\varepsilon_{m'}} \left(\frac{\varepsilon_{l'}^2}{8} - \frac{\varepsilon_{k'} \cdot \varepsilon_{m'}}{3} \right) \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \end{aligned} \right. \quad (8)$$

$$\{ Z_1 \cdot Z_2 = (s + t)(\omega s + \omega^2 t) = -\varepsilon_{m'} \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \}$$

Z_3 を $\omega^2 s + \omega t$ より計算すると桁落ち誤差が入るので、 Z_3 は解と係数の関係より求める。(8) を (6) に代入して、3次方程式 (5) の解 u_1, u_2, u_3 (9) を求める。

$$\begin{cases} u_1 = \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{\varepsilon_{\ell'}^2}{16 \varepsilon_{m'}} \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\ u_2 = -\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{\varepsilon_{\ell'}^2}{16 \varepsilon_{m'}} \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\ u_3 = \left(\frac{\varepsilon_{\kappa}}{2} - \frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} \right) \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \end{cases} \quad (9)$$

u_1 (9) を用いて (2) を因数分解した2つの2次方程式 (12), (13) の係数を求める。

$$2u_1 - k = 2\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$v_1 = \sqrt{2u_1 - k}$$

$$\begin{aligned} &= \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} + \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa} \right) \cdot 10^{\beta-3m/4} \\ &\quad + [10^{\beta-5m/4}] \end{aligned} \quad (10)$$

$$u_1^2 = \varepsilon_{m'} \cdot 10^{4\beta-m} + \frac{\varepsilon_{\ell'}^2}{8 \sqrt{\varepsilon_{m'}}} \cdot 10^{4\beta-3m/2} + [10^{4\beta-2m}]$$

w_1 を $\sqrt{u_1^2 - m'}$ より計算すると桁落ち誤差が入るので、(11) で求める。

$$w_1 = -\frac{\ell'}{2\nu_1} = -\frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} 10^{2\beta-3m/4} \\ - \frac{\varepsilon_{\ell'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-5m/4} + [10^{2\beta-7m/4}] \quad (11)$$

$$y^2 + \nu_1 y + (u_1 + w_1) = 0 \quad (12)$$

$$y^2 - \nu_1 y + (u_1 - w_1) = 0 \quad (13)$$

2次方程式 (12) より (2) の解 y_1, y_2 を求める。

$$\nu_1 = \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} + \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}]$$

$$u_1 + w_1 = \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} - \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \cdot 10^{2\beta-3m/4} + \frac{\varepsilon_{\ell'}^2}{16\varepsilon_{m'}} \cdot 10^{2\beta-m} \\ + \frac{\varepsilon_{\ell'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}]$$

$$\nu_1^2 = 2\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$-4(u_1 + w_1) = -4\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{\sqrt{2} \cdot \varepsilon_{\ell'}}{\sqrt[4]{\varepsilon_{m'}}} \cdot 10^{2\beta-3m/4} - \frac{\varepsilon_{\ell'}^2}{4\varepsilon_{m'}} \cdot 10^{2\beta-m} \\ - \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}]$$

$$\nu_1^2 - 4(u_1 + w_1) = -2\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} \left\{ 1 - \frac{\varepsilon_{\ell'}}{\sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \cdot 10^{-m/4} \right. \\ + \frac{1}{2\sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} + \varepsilon_{k'} \right) \cdot 10^{-m/2} \\ \left. + \frac{\varepsilon_{\ell'}}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \varepsilon_{m'}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{-3m/4} + [10^{-m}] \right\}$$

$$\sqrt{v_1^2 - 4(u_1 + w_1)} = \sqrt{2} \cdot \sqrt[4]{\varepsilon_m} \cdot i \cdot 10^{\beta-m/4} - \frac{\varepsilon_{l'}}{2 \cdot \sqrt[4]{\varepsilon_m}} \cdot i \cdot 10^{\beta-m/2} \\ - \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot i \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}]$$

$$\left\{ \begin{aligned} y_1 &= \frac{1}{2} \left\{ -v_1 + \sqrt{v_1^2 - 4(u_1 + w_1)} \right\} = -\frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_m} \cdot 10^{\beta-m/4} \\ &\quad - \frac{\varepsilon_{l'}}{4 \cdot \sqrt[4]{\varepsilon_m}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_2 &= \frac{1}{2} \left\{ -v_1 - \sqrt{v_1^2 - 4(u_1 + w_1)} \right\} = -\frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_m} \cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_{l'}}{4 \cdot \sqrt[4]{\varepsilon_m}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned} \right. \quad (14)$$

2次方程式 (13) より (2) の解 y_3, y_4 を求める。

$$u_1 - w_1 = \sqrt{\varepsilon_m} \cdot 10^{2\beta-m/2} + \frac{\varepsilon_{l'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \cdot 10^{2\beta-3m/4} + \frac{\varepsilon_{l'}^2}{16 \varepsilon_m} \cdot 10^{2\beta-m} \\ - \frac{\varepsilon_{l'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m} \cdot \sqrt{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}] \\ -4(u_1 - w_1) = -4\sqrt{\varepsilon_m} \cdot 10^{2\beta-m/2} - \frac{\sqrt{2} \cdot \varepsilon_{l'}}{\sqrt[4]{\varepsilon_m}} \cdot 10^{2\beta-3m/4} - \frac{\varepsilon_{l'}^2}{4 \varepsilon_m} \cdot 10^{2\beta-m} \\ + \frac{\varepsilon_{l'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m} \cdot \sqrt{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}] \\ v_1^2 - 4(u_1 - w_1) = -2\sqrt{\varepsilon_m} \cdot 10^{2\beta-m/2} \left\{ 1 + \frac{\varepsilon_{l'}}{\sqrt{2} \cdot \sqrt[4]{\varepsilon_m} \cdot \sqrt{\varepsilon_m}} \cdot 10^{-m/4} \right. \\ \left. + \frac{1}{2\sqrt{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} + \varepsilon_{k'} \right) \cdot 10^{-m/2} \right. \\ \left. - \frac{\varepsilon_{l'}}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m} \cdot \varepsilon_m} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_m} - \varepsilon_{k'} \right) \cdot 10^{-3m/4} + [10^{-m}] \right\}$$

$$\begin{aligned}
\sqrt{v_1^2 - 4(u_1 - w_1)} &= \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot i \cdot 10^{\beta-m/4} + \frac{\varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&\quad - \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
\left\{ \begin{aligned}
y_3 &= \frac{1}{2} \left\{ v_1 + \sqrt{v_1^2 - 4(u_1 - w_1)} \right\} = \frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} \\
&\quad + \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
y_4 &= \frac{1}{2} \left\{ v_1 - \sqrt{v_1^2 - 4(u_1 - w_1)} \right\} = \frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} \\
&\quad - \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}]
\end{aligned} \right. \quad (15)
\end{aligned}$$

u_2 (9) を用いて (2) を因数分解して 2 つの 2 次方程式 (18), (19) の係数を求める。

$$2u_2 - k' = -2\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$v_2 = \sqrt{2u_2 - k'}$$

$$\begin{aligned}
&= \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot i \cdot 10^{\beta-m/4} - \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{\beta-3m/4} \\
&\quad + [10^{\beta-5m/4}] \quad (16)
\end{aligned}$$

$$u_2^2 = \varepsilon_{m'} \cdot 10^{4\beta-m} - \frac{\varepsilon_{l'}^2}{8 \sqrt{\varepsilon_{m'}}} \cdot 10^{4\beta-3m/2} + [10^{4\beta-2m}]$$

w_2 を $\sqrt{u_2^2 - m'}$ より計算すると桁落ち誤差が入るので. (17)

で求める。

$$w_2 = -\frac{\ell'}{2v_2} = \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \cdot i \cdot 10^{2\beta-3m/4} \\ + \frac{\varepsilon_{\ell'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{2\beta-5m/4} + [10^{2\beta-7m/4}] \quad (17)$$

$$y^2 + v_2 y + (u_2 + w_2) = 0 \quad (18)$$

$$y^2 - v_2 y + (u_2 - w_2) = 0 \quad (19)$$

2次方程式(18)より(2)の解 y_1, y_2 を求める。

$$v_2 = \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot i \cdot 10^{\beta-m/4} - \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{2\beta-3m/4} \\ + [10^{2\beta-5m/4}]$$

$$u_2 + w_2 = -\sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \cdot i \cdot 10^{2\beta-3m/4} + \frac{\varepsilon_{\ell'}^2}{16\varepsilon_{m'}} \cdot 10^{2\beta-m} \\ + \frac{\varepsilon_{\ell'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{2\beta-5m/4} + [10^{2\beta-7m/4}] \\ v_2^2 = -2 \cdot \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$-4(u_2 + w_2) = 4 \cdot \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} - \frac{\sqrt{2} \cdot \varepsilon_{\ell'}}{\sqrt[4]{\varepsilon_{m'}}} \cdot i \cdot 10^{2\beta-3m/4} - \frac{\varepsilon_{\ell'}^2}{4\varepsilon_{m'}} \cdot 10^{2\beta-m} \\ - \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{2\beta-5m/4} + [10^{2\beta-7m/4}]$$

$$v_2^2 - 4(u_2 + w_2) = 2 \cdot \sqrt{\varepsilon_{m'}} \cdot 10^{2\beta-m/2} \left\{ 1 - \frac{\varepsilon_{\ell'}}{\sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{-m/4} \right. \\ \left. - \frac{1}{2 \cdot \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8\varepsilon_{m'}} + \varepsilon_{k'} \right) \cdot 10^{-m/2} \right.$$

$$\begin{aligned}
& - \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot \mathcal{E}_{m'}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot i \cdot 10^{-3m/4} + [10^{-m}] \} \\
\sqrt{V_2^2 - 4(u_2 + w_2)} &= \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} - \frac{\mathcal{E}_{\ell'}}{2 \cdot \sqrt{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&+ \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
\left\{ \begin{aligned} y_4 &= \frac{1}{2} \left\{ -V_2 + \sqrt{V_2^2 - 4(u_2 + w_2)} \right\} = \frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} \\ &- \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_2 &= \frac{1}{2} \left\{ -V_2 - \sqrt{V_2^2 - 4(u_2 + w_2)} \right\} = -\frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} \\ &+ \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned} \right. \quad (20)
\end{aligned}$$

2次方程式 (19) より (2) の解 y_3, y_1 を求める。

$$\begin{aligned}
u_2 - w_2 &= -\sqrt{\mathcal{E}_{m'}} \cdot 10^{2\beta-m/2} - \frac{\mathcal{E}_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{2\beta-3m/4} + \frac{\mathcal{E}_{\ell'}^2}{16 \mathcal{E}_{m'}} \cdot 10^{2\beta-m} \\
&- \frac{\mathcal{E}_{\ell'}}{8 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot \sqrt{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot i \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}] \\
-4(u_2 - w_2) &= 4 \sqrt{\mathcal{E}_{m'}} \cdot 10^{2\beta-m/2} + \frac{\sqrt{2} \cdot \mathcal{E}_{\ell'}}{\sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{2\beta-3m/4} - \frac{\mathcal{E}_{\ell'}^2}{4 \mathcal{E}_{m'}} \cdot 10^{2\beta-m} \\
&+ \frac{\mathcal{E}_{\ell'}}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot \sqrt{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot i \cdot 10^{2\beta-5m/4} + [10^{2\beta-3m/2}] \\
V_2^2 - 4(u_2 - w_2) &= 2 \cdot \sqrt{\mathcal{E}_{m'}} \cdot 10^{2\beta-m/2} \left\{ 1 + \frac{\mathcal{E}_{\ell'}}{\sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot \sqrt{\mathcal{E}_{m'}}} \cdot i \cdot 10^{-m/4} \right. \\
&- \frac{1}{2 \cdot \sqrt{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} + \mathcal{E}_{k'} \right) \cdot 10^{-m/2}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot \mathcal{E}_{m'}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot i \cdot 10^{-3m/4} + [10^{-m}] \} \\
\sqrt{V_2^2 - 4(u_2 - w_2)} &= \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} + \frac{\mathcal{E}_{\ell'}}{2 \cdot \sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
& + \frac{1}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
\left\{ \begin{aligned} y_3 &= \frac{1}{2} \left\{ V_2 + \sqrt{V_2^2 - 4(u_2 - w_2)} \right\} = \frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} \\ & + \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_1 &= \frac{1}{2} \left\{ V_2 - \sqrt{V_2^2 - 4(u_2 - w_2)} \right\} = -\frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\mathcal{E}_{m'}} \cdot 10^{\beta-m/4} \\ & - \frac{\mathcal{E}_{\ell'}}{4 \cdot \sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\mathcal{E}_{m'}}} \left(\frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} - \mathcal{E}_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned} \right. \quad (21)
\end{aligned}$$

u_3 (9) を用いて (2) を因数分解した 2 つの 2 次方程式 (24)

, (25) の係数を求める。

$$2u_3 - k' = -\frac{\mathcal{E}_{\ell'}^2}{4 \mathcal{E}_{m'}} \cdot 10^{2\beta-m} + [10^{2\beta-2m}]$$

$$v_3 = \sqrt{2u_3 - k'} = \frac{\mathcal{E}_{\ell'}}{2 \cdot \sqrt[4]{\mathcal{E}_{m'}}} \cdot i \cdot 10^{\beta-m/2} + [10^{\beta-3m/2}] \quad (22)$$

$$u_3^2 = \left(\frac{\mathcal{E}_{k'}}{2} - \frac{\mathcal{E}_{\ell'}^2}{8 \mathcal{E}_{m'}} \right) \cdot 10^{4\beta-2m} + [10^{4\beta-3m}]$$

$$w_3 = \sqrt{u_3^2 - m'} = \sqrt{\mathcal{E}_{m'}} \cdot i \cdot 10^{2\beta-m/2} + [10^{2\beta-3m/2}] \quad (23)$$

$$y^2 + v_3 y + (u_3 + w_3) = 0 \quad (24)$$

$$y^2 - v_3 y + (u_3 - w_3) = 0 \quad (25)$$

2 次方程式 (24) より (2) の解 y_1, y_4 を求める.

$$\begin{aligned}
 v_3 &= \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + [10^{\beta-3m/2}] \\
 u_3 + w_3 &= \sqrt{\varepsilon_{m'}} \cdot i \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{\kappa'}}{2} - \frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\
 v_3^2 &= -\frac{\varepsilon_{\ell'}^2}{4 \varepsilon_{m'}} \cdot 10^{2\beta-m} + [10^{2\beta-2m}] \\
 -4(u_3 + w_3) &= -4 \sqrt{\varepsilon_{m'}} \cdot i \cdot 10^{2\beta-m/2} - 4 \left(\frac{\varepsilon_{\kappa'}}{2} - \frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\
 v_3^2 - 4(u_3 + w_3) &= -4 \sqrt{\varepsilon_{m'}} \cdot i \cdot 10^{2\beta-m/2} \left\{ 1 + \frac{1}{2 \sqrt{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa'} \right) \cdot i \cdot 10^{-m/2} \right. \\
 &\quad \left. + [10^{-m}] \right\} \\
 \sqrt{v_3^2 - 4(u_3 + w_3)} &= -\sqrt{2} \cdot (1-i) \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} \\
 &\quad - \frac{(1+i)}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
 \left\{ \begin{aligned} y_1 &= \frac{1}{2} \left\{ -v_3 + \sqrt{v_3^2 - 4(u_3 + w_3)} \right\} = -\frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} \\ &\quad - \frac{\varepsilon_{\ell'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_4 &= \frac{1}{2} \left\{ -v_3 - \sqrt{v_3^2 - 4(u_3 + w_3)} \right\} = \frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} \\ &\quad - \frac{\varepsilon_{\ell'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{\ell'}^2}{8 \varepsilon_{m'}} - \varepsilon_{\kappa'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned} \right. \quad (26)
 \end{aligned}$$

2 次方程式 (25) より (2) の解 y_3, y_2 を求める.

$$\begin{aligned}
u_3 - w_3 &= -\sqrt{\varepsilon_m} \cdot i \cdot 10^{2\beta-m/2} + \left(\frac{\varepsilon_{k'}}{2} - \frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\
-4(u_3 - w_3) &= 4\sqrt{\varepsilon_m} \cdot i \cdot 10^{2\beta-m/2} - 4 \left(\frac{\varepsilon_{k'}}{2} - \frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}] \\
v_3^2 - 4(u_3 - w_3) &= 4\sqrt{\varepsilon_m} \cdot i \cdot 10^{2\beta-m/2} \left\{ 1 - \frac{1}{2\sqrt{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot i \cdot 10^{-m/2} \right. \\
&\quad \left. + [10^{-m}] \right\}
\end{aligned}$$

$$\begin{aligned}
\sqrt{v_3^2 - 4(u_3 - w_3)} &= \sqrt{2} \cdot (1+i) \cdot \sqrt[4]{\varepsilon_m} \cdot 10^{\beta-m/4} \\
&\quad + \frac{(1-i)}{2 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\
\left\{ \begin{aligned} y_3 &= \frac{1}{2} \left\{ v_3 + \sqrt{v_3^2 - 4(u_3 - w_3)} \right\} = \frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_m} \cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_l}{4 \cdot \sqrt{\varepsilon_m}} \cdot i \cdot 10^{\beta-m/2} + \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \\ y_2 &= \frac{1}{2} \left\{ v_3 - \sqrt{v_3^2 - 4(u_3 - w_3)} \right\} = -\frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_m} \cdot 10^{\beta-m/4} \\ &\quad + \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_m}} \cdot i \cdot 10^{\beta-m/2} - \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{8\varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-5m/4}] \end{aligned} \right.
\end{aligned} \tag{27}$$

3次方程式(5)の解 u_1, u_2, u_3 (9)のいずれを用いても、当然のことながら、4次方程式(2)の解 y_1, y_2, y_3, y_4 は等しい値が得られる。したがって、4次方程式(1)の解は(28)である。

$$\begin{aligned}
x_1 = y_1 + \bar{x} &= \alpha \cdot 10^\beta - \frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} - \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&\quad - \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_2 = y_2 + \bar{x} &= \alpha \cdot 10^\beta - \frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} + \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&\quad - \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_3 = y_3 + \bar{x} &= \alpha \cdot 10^\beta + \frac{(1+i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} + \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&\quad + \frac{(1-i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-m}] \\
x_4 = y_4 + \bar{x} &= \alpha \cdot 10^\beta + \frac{(1-i)}{\sqrt{2}} \cdot \sqrt[4]{\varepsilon_{m'}} \cdot 10^{\beta-m/4} - \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} \\
&\quad + \frac{(1+i)}{4 \cdot \sqrt{2} \cdot \sqrt[4]{\varepsilon_{m'}}} \left(\frac{\varepsilon_{l'}^2}{8 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{\beta-3m/4} + [10^{\beta-m}]
\end{aligned} \tag{28}$$

解と係数の関係を用いて、解より係数を求める。

$$y_1 + y_2 + y_3 + y_4 = \bar{\varepsilon} \cdot 10^{\beta-5m/4} + [10^{\beta-9m/4}] \quad \{=0\}$$

$$y_1 \cdot y_4 = \sqrt{\varepsilon_{m'}} \cdot i \cdot 10^{2\beta-m/2} - \frac{1}{2} \left(\frac{\varepsilon_{l'}^2}{4 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$y_2 \cdot y_3 = -\sqrt{\varepsilon_{m'}} \cdot i \cdot 10^{2\beta-m/2} - \frac{1}{2} \left(\frac{\varepsilon_{l'}^2}{4 \varepsilon_{m'}} - \varepsilon_{k'} \right) \cdot 10^{2\beta-m} + [10^{2\beta-3m/2}]$$

$$y_1 + y_4 = -\frac{\varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \delta_1 \cdot 10^{\beta-5m/4} + [10^{\beta-3m/2}]$$

$$y_2 + y_3 = \frac{\varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + \delta_2 \cdot 10^{\beta-5m/4} + [10^{\beta-3m/2}]$$

$$\begin{aligned}
& \underline{y_1 \cdot y_4 + y_2 \cdot y_3 + y_1 \cdot y_2 + y_1 \cdot y_3 + y_4 \cdot y_2 + y_4 \cdot y_3} \\
&= y_1 \cdot y_4 + y_2 \cdot y_3 + (y_1 + y_4)(y_2 + y_3) \\
&= \varepsilon_k \cdot 10^{2\beta-m} + \delta_3 \cdot 10^{2\beta-3m/2} + \frac{\varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_m}} (\delta_1 - \delta_2) \cdot 10^{2\beta-7m/4} + [10^{2\beta-2m}] \{ = k \} \\
& y_1 \cdot y_4 \cdot (y_2 + y_3) = -\frac{\varepsilon_{l'}}{2} \cdot 10^{3\beta-m} - \frac{\varepsilon_l}{4 \cdot \sqrt{\varepsilon_m'}} \left(\frac{\varepsilon_{l'}^2}{4 \varepsilon_m} - \varepsilon_k \right) \cdot i \cdot 10^{3\beta-3m/2} \\
& \quad + \delta_2 \cdot \sqrt{\varepsilon_m} \cdot i \cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}] \\
& y_2 \cdot y_3 \cdot (y_1 + y_4) = -\frac{\varepsilon_{l'}}{2} \cdot 10^{3\beta-m} + \frac{\varepsilon_{l'}}{4 \cdot \sqrt{\varepsilon_m}} \left(\frac{\varepsilon_{l'}^2}{4 \varepsilon_m} - \varepsilon_k \right) \cdot i \cdot 10^{3\beta-3m/2} \\
& \quad - \delta_1 \cdot \sqrt{\varepsilon_m} \cdot i \cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}]
\end{aligned}$$

$$\begin{aligned}
& \underline{y_1 \cdot y_2 \cdot y_3 + y_1 \cdot y_2 \cdot y_4 + y_1 \cdot y_3 \cdot y_4 + y_2 \cdot y_3 \cdot y_4} \\
&= y_1 \cdot y_4 \cdot (y_2 + y_3) + y_2 \cdot y_3 \cdot (y_1 + y_4) \\
&= -\varepsilon_{l'} \cdot 10^{3\beta-m} + (\delta_2 - \delta_1) \cdot \sqrt{\varepsilon_m} \cdot i \cdot 10^{3\beta-7m/4} + [10^{3\beta-2m}] \{ = -l' \}
\end{aligned}$$

$$\underline{y_1 \cdot y_2 \cdot y_3 \cdot y_4} = \varepsilon_m \cdot 10^{4\beta-m} + [10^{4\beta-2m}] \{ = m' \}$$

$$x_1 + x_2 + x_3 + x_4 = 4\alpha \cdot 10^\beta + [10^{\beta-m}] \{ = -a_3 \}$$

$$x_1 \cdot x_4 = \alpha^2 \cdot 10^{2\beta} + \left(\sqrt{\varepsilon_m} - \frac{\alpha \cdot \varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_m'}} \right) \cdot i \cdot 10^{2\beta-m/2} + [10^{2\beta-m}]$$

$$x_2 \cdot x_3 = \alpha^2 \cdot 10^{2\beta} - \left(\sqrt{\varepsilon_m} - \frac{\alpha \cdot \varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_m'}} \right) \cdot i \cdot 10^{2\beta-m/2} + [10^{2\beta-m}]$$

$$x_1 + x_4 = 2\alpha \cdot 10^\beta - \frac{\varepsilon_{l'}}{2 \cdot \sqrt{\varepsilon_m'}} \cdot i \cdot 10^{\beta-m/2} + [10^{\beta-m}]$$

$$\chi_2 + \chi_3 = 2\alpha \cdot 10^\beta + \frac{\varepsilon_{\ell'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \cdot i \cdot 10^{\beta-m/2} + [10^{\beta-m}]$$

$$\begin{aligned} & \frac{\chi_1 \cdot \chi_4 + \chi_2 \cdot \chi_3 + \chi_1 \cdot \chi_2 + \chi_1 \cdot \chi_3 + \chi_4 \cdot \chi_2 + \chi_4 \cdot \chi_3}{=} \\ &= \chi_1 \cdot \chi_4 + \chi_2 \cdot \chi_3 + (\chi_1 + \chi_4)(\chi_2 + \chi_3) \\ &= 6\alpha^2 \cdot 10^{2\beta} + [10^{2\beta-m}] \{ = a_2 \} \end{aligned}$$

$$\begin{aligned} & \chi_1 \cdot \chi_4 \cdot (\chi_2 + \chi_3) \\ &= 2\alpha^3 \cdot 10^{3\beta} + \left\{ 2\alpha \cdot \sqrt{\varepsilon_{m'}} - \frac{\alpha^2 \cdot \varepsilon_{\ell'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \right\} \cdot i \cdot 10^{3\beta-m/2} + [10^{3\beta-m}] \end{aligned}$$

$$\begin{aligned} & \chi_2 \cdot \chi_3 \cdot (\chi_1 + \chi_4) \\ &= 2\alpha^3 \cdot 10^{3\beta} - \left\{ 2\alpha \cdot \sqrt{\varepsilon_{m'}} - \frac{\alpha^2 \cdot \varepsilon_{\ell'}}{2 \cdot \sqrt{\varepsilon_{m'}}} \right\} \cdot i \cdot 10^{3\beta-m/2} + [10^{3\beta-m}] \end{aligned}$$

$$\begin{aligned} & \frac{\chi_1 \cdot \chi_2 \cdot \chi_3 + \chi_1 \cdot \chi_2 \cdot \chi_4 + \chi_1 \cdot \chi_3 \cdot \chi_4 + \chi_2 \cdot \chi_3 \cdot \chi_4}{=} \\ &= \chi_1 \cdot \chi_4 \cdot (\chi_2 + \chi_3) + \chi_2 \cdot \chi_3 \cdot (\chi_1 + \chi_4) \\ &= 4\alpha^3 \cdot 10^{3\beta} + [10^{3\beta-m}] \{ = -a_1 \} \end{aligned}$$

$$\chi_1 \cdot \chi_2 \cdot \chi_3 \cdot \chi_4 = \alpha^4 \cdot 10^{4\beta} + [10^{4\beta-m}] \{ = a_0 \}$$

$\bar{\varepsilon}, \delta_1, \delta_2, \delta_3$: 誤差のみの数値の数値部

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