

Blow-up at space infinity for nonlinear heat equations

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1 Introduction and main theorems

In this paper we gather the papers [5], [6] and [12] for our talk at Kyoto University. In particular we make the proofs of theorems in [5] easier by using the methods in [12] and other.

We consider solutions of the initial value problem for the equation

$$\begin{cases} u_t = \Delta u + f(u), & x \in \mathbf{R}^n, t > 0, \\ u(x, 0) = u_0(x), & x \in \mathbf{R}^n. \end{cases} \quad (1)$$

The nonlinear term $f \in C^1(\bar{\mathbf{R}}_+)$ satisfies that

$$\int_C^\infty \frac{d\xi}{f(\xi)} < \infty \text{ with some } C \geq 0, \quad (2)$$

and

$$\left\{ \begin{array}{l} \text{there exists a function } \Phi \in C^2(\mathbf{R}_+) \text{ such that} \\ \Phi(v) > 0, \Phi'(v) > 0 \text{ and } \Phi''(v) \geq 0 \text{ for } v > 0, \\ \int_1^\infty \frac{d\xi}{\Phi(\xi)} < \infty, \\ \text{and } f'(v)\Phi(v) - f(v)\Phi'(v) \geq c\Phi(v)\Phi'(v) \text{ for } v > b \\ \text{with some } b \geq 0 \text{ and } c \geq 0. \end{array} \right. \quad (3)$$

Remark. The conditions (2) and (3) were used in [12]. They are weaker than the conditions used in [5] and [6]:

$$f(\delta b) \leq \delta^p f(b)$$

for all $b \geq b_0$ and for all $\delta \in (\delta_0, 1)$ with some $b_0 > 0$, some $\delta_0 \in (0, 1)$ and some $p > 1$.

The initial data u_0 is assumed to be a measurable function in $\mathbf{R}^n$ satisfying

$$0 \leq u_0(x) \leq M \text{ a.e. in } \mathbf{R}^n \quad (4)$$

for some positive M . We are interested in initial data such that $u_0 \rightarrow M$ as $|x| \rightarrow \infty$ for x in some sector of $\mathbf{R}^n$. We assume that there exists a sequence $\{x\}_{m=1}^\infty \subset \mathbf{R}^n$ such that

$$\lim_{m \rightarrow \infty} u_0(x + x_m) = M \text{ a.e. in } \mathbf{R}^n. \quad (5)$$

Remark. The condition (5) was given in [12]. This condition is equivalent to the condition in [5] with [6]:

$$\operatorname{ess\,inf}_{x \in \tilde{B}_m} (u_0(x) - M_m(x - x_m)) \geq 0 \quad \text{for } m = 1, 2, \dots,$$

where $\tilde{B}_m = B_{r_m}(x_m)$ with a sequence $\{r_m\}_{m=1}^\infty$, a sequence of functions $\{M_m(x)\}_{m=1}^\infty$ satisfying

$$\lim_{m \rightarrow \infty} r_m = \infty, \quad M_m(x) \leq M_{m+1}(x) \quad \text{for } m \geq 1$$

$$\lim_{m \rightarrow \infty} \inf_{s \in [1, r_m]} \frac{1}{|B_s|} \int_{B_s(0)} M_m(x) dx = M,$$

and some sequence of vectors $\{x_m\}_{m=1}^\infty$. Here $B_r(x)$ denotes the opened ball of radius r centered at x .

Problem (1) has a unique bounded solution at least locally in time. However, the solution may blow up in finite time. For a given initial value u_0 and nonlinear term f let $T^* = T^*(u_0, f)$ be the maximal existence time of the solution. If $T^* = \infty$, the solution exists globally in time. If $T^* < \infty$, we say that the solution blows up in finite time. It is well known that

$$\limsup_{t \rightarrow T^*} \|u(\cdot, t)\|_\infty = \infty, \quad (6)$$

where $\|\cdot\|_\infty$ denotes the L^∞ -norm in space variables.

In this paper we are interested in behavior of a blowing up solution near space infinity as well as location of blow-up directions defined below. A point $x_{BU} \in \mathbf{R}^n$ is called a *blow-up point* if there exists a sequence $\{(x_m, t_m)\}_{m=1}^\infty$ such that

$$t_m \uparrow T^*, \quad x_m \rightarrow x_{BU} \quad \text{and} \quad u(x_m, t_m) \rightarrow \infty \quad \text{as } m \rightarrow \infty.$$

If there exists a sequence $\{(x_m, t_m)\}_{m=1}^{\infty}$ such that

$$t_m \uparrow T^*, \quad |x_m| \rightarrow \infty \quad \text{and} \quad u(x_m, t_m) \rightarrow \infty \quad \text{as} \quad m \rightarrow \infty,$$

then we say that the solution blows up to at space infinity.

A direction $\psi \in S^{n-1}$ is called a *blow-up direction* if there exists a sequence $\{(x_m, t_m)\}_{m=1}^{\infty}$ with $x_m \in \mathbb{R}^n$ and $t_m \in (0, T^*)$ such that $u(x_m, t_m) \rightarrow \infty$ as $m \rightarrow \infty$ and

$$\frac{x_m}{|x_m|} \rightarrow \psi \quad \text{as} \quad m \rightarrow \infty. \quad (7)$$

We consider the solution $v(t)$ of an ordinary differential equation

$$\begin{cases} v_t = f(v), & t > 0, \\ v(0) = M. \end{cases} \quad (8)$$

Let $T_v = T^*(M, f)$ be the maximal existence time of solutions of (8), i. e.,

$$T_v = \int_M^{\infty} \frac{ds}{f(s)}.$$

We are now in position to state our main results.

Theorem 1. Assume that $f \in C^1(\mathbb{R}_+)$ is nondecreasing function and locally Lipschitz in $\bar{\mathbb{R}}_+$. Let u_0 be a continuous function satisfying (4) and (5). Then there exists a subsequence of $\{x_m\}_{m=1}^{\infty}$, independent of t such that

$$\lim_{m \rightarrow \infty} u(x + x_m, t) = v(t) \quad \text{in } \mathbb{R}^n. \quad (9)$$

The convergence is uniform in every compact subset of $\mathbb{R}^n \times [0, T_v)$. Moreover, the solution blows up at T_v .

For this theorem we should introduce the results of Gladkov [7]. In his paper there is the result [7, Theorem 1] relative to our first theorem. He considered the initial-boundary value problem:

$$\begin{cases} u_t = u_{xx} + f(x, t, u), & x > 0, 0 < t < T_0, \\ u(x, 0) = u_0(x), & x > 0, \\ u(0, t) = \mu(t) & 0 < t < T_0, \end{cases}$$

and the ordinary differential equation

$$\begin{cases} v_t = \tilde{f}(t, u), & 0 < t < T_0, \\ v(0) = M, \end{cases}$$

where $T_0 \in (0, \infty]$, $0 \leq f(x, t, u) \leq \tilde{f}(t, u)$, $\lim_{x \rightarrow \infty} f(x, t, u) = \tilde{f}(t, u)$, $0 \leq u_0 \leq M$ and $\lim_{x \rightarrow \infty} u_0(x) = M$. For the equations he had $u(x, t) \rightarrow v(t)$ as $x \rightarrow \infty$ uniformly for $[0, T]$ with $T < T_0$. For the proof of this result, he used the fundamental solution of the heat equation.

In [5] the expression (9) was the weak sense:

$$\lim_{n \rightarrow \infty} u(x_n, t) = v(t). \quad (10)$$

After [5], (9) was used in [12]. However, for proving Theorems 2 and 3, we can select even the expression (10).

Our second main result is on the location of blow-up points.

Theorem 2. *Assume the same hypotheses of Theorem 1 and that f satisfies (2) and (3). Let $u_0 \not\equiv M$ a.e. in $\mathbf{R}^n$. Then the solution of (1) has no blow-up points with ∞ in $\mathbf{R}^n$. (It blows up only at space infinity.)*

There is a huge literature on location of blow-up points since the work of Weissler [15] and Friedman-McLeod [1]. (We do not intend to list references exhaustively in this paper.) However, most results consider either bounded domains or solutions decaying at space infinity; such a solution does not blow up at space infinity [2].

As far as the authors know, before the result of [4] the only paper discussing blow-up at space infinity is the work of Lacey [8]. He considered the Dirichlet problem in a half line. He studied various nonlinear terms and proved that a solution blows up only at space infinity. His method is based on construction of suitable subsolutions and supersolutions. However, the construction heavily depends on the Dirichlet condition at $x = 0$ and does not apply to the Cauchy problem even for the case $n = 1$.

As previously described, the Giga-Umeda [4] proved the statement of Theorems 1 and 2 assuming that $\lim_{|x| \rightarrow \infty} u_0(x) = M$ for positive solutions of $u_t = \Delta u + u^p$. Later, Simojō [13] had the same results as in [4] by relaxing the assumptions of initial data $u_0 \geq 0$ which is similar to that in the present paper. His approach is a construction of a suitable supersolution which implies that $a \in \mathbf{R}^n$ is not a blow-up point. Although he restricted himself for $f(s) = s^p$, his idea works for f under slightly strong assumption on u_0 . Here we give a different approach.

By Simojō's results [13] it is natural to consider a problem of "blow-up direction" defined in (7). We next study this "blow-up direction" for the value ∞ .

Theorem 3. *Assume the same hypotheses of Theorem 1. Let a direction $\psi \in S^{n-1}$. If and only if there exists sequences $\{y_m\}_{m=1}^\infty$ and satisfying*

$\lim_{m \rightarrow \infty} y_m/|y_m| = \psi$ such that

$$\lim_{m \rightarrow \infty} u_0(x + y_m) = M \text{ a.e. in } \mathbf{R}^n, \quad (11)$$

then ψ is a blow-up direction.

After [5] there are some results in this field. Shimojō had the result of the upperbound and the lowerbound:

$$v(t - \eta(x, t)) \leq u(x, t) \leq v(t - c\eta(x, t))$$

with some function η and $c \in (0, 1)$. Moreover, he proved the complete blow-up of the solution. Seki-Suzuki-Umeda [12] and Seki [11] improved the results of [5] for the quasilinear parabolic equation:

$$u_t = \Delta \varphi(u) + f(u).$$

In particular they had more results for more general case. In [3] some of the proofs of theorems in [5] were corrected.

This paper is organized as follows. In section 2 we prove Theorem 1 by using the fundamental solution of the heat equation. The proof of Theorem 2 is given in section 3 by using the argument used in [12]. In section 4 we show Theorem 3 using Theorem 1 and Lemma 3.2.

2 Behavior at space infinity

In this section we prove Theorem 1. We give proof of Theorem 1 which is inspired in private communication with Y. Seki and M. Shimojō.

Proof of Theorem 1. Put $w = v - u$. Then, we have for $t \in (0, T_0]$ with $T_0 \in (0, T(M))$,

$$w_t = \Delta w + f(v(t)) - f(u(\cdot, t)) \leq \Delta w + C(v - u),$$

where

$$C = \sup_{t \in [0, T_0]} \left\| \int_0^1 f'(\theta v(t) + (1 - \theta)u(\cdot, t)) d\theta \right\|_{\infty}.$$

Then, by comparison we obtain

$$w(x, t) \leq e^{CT_0} e^{\Delta t} (M - u_0(x)) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbf{R}^n} e^{-|x-y|^2/4t} (M - u_0(y)) dy.$$

From (5) we have

$$\lim_{m \rightarrow \infty} u(x + x_m, t) = v(t) \quad \text{in } \mathbf{R}^n. \quad (12)$$

It remains to prove that u blows up at $t = T_v$. For this purpose it suffices to prove that $\lim_{m \rightarrow \infty} u(x_m, t_m) = \infty$ for some sequence $t_m \rightarrow T_v$. We argue by contradiction. Suppose that $\lim_{m \rightarrow \infty} u(x_m, t_m) \leq C$ for some $C \in [M, \infty)$. Then we could take $t_0 \in (0, T_v)$ satisfying $v(t_0) \geq C$ and $v_t(t) > 0$ for $t \geq t_0$. By (12) we have

$$\lim_{m \rightarrow \infty} u\left(x_m, \frac{t_0 + T_v}{2}\right) = v\left(\frac{t_0 + T_v}{2}\right) > C,$$

which yields a contradiction. We thus proved that $\lim_{m \rightarrow \infty} u(x_m, t_m) = \infty$, so that $u(x, t)$ blows up at T_v . $\square$

3 No blow-up point in $\mathbf{R}^n$

In this section we prove Theorem 2. We use three lemmas for proving the theorem..

Lemma 3.1. *Assume the same hypothesis of Theorem 1. Let u and v be solutions of (1) and (8) with u_0 , M and f satisfying (2), (3) and (4). Then there exist $\delta = \delta(a, t_0, u_0, f) \in (0, 1)$ such that for $(x, t) \in B_1(a) \times [t_0, T_v)$,*

$$u(x, t) \leq \delta v(t)$$

with $t_0 \in [0, T_v)$.

Proof. By (2) there exist $M_f = M_f(f) > M$ and $\delta_f = \delta_f(f) \in (0, 1)$ satisfying for $r \geq M_f$ and $\delta \in (\delta_f, 1)$,

$$f(\delta r) \leq \delta f(r). \quad (13)$$

Let $T_0 = T_0(u_0, f) \in (0, T_v)$ such that $v(T_0) = M_f$. Since $u_0 \leq M$ and $u_0 \not\equiv M$ a.e. in $\mathbf{R}^n$, we have $u(x, T_0) < v(T_0)$. Note that $u(x, t) < v(t)$ for $t \in (0, T_0]$. Let w be the solution of

$$\begin{cases} w_t = \Delta w, & x \in \mathbf{R}^n, t \in (T_0, T^*), \\ w(x, T_0) = \max\{u(x, T_0)/v(T_0), \delta_f\}, & x \in \mathbf{R}^n. \end{cases}$$

Put $\bar{u} = v w$. Then we have

$$\begin{cases} \bar{u}_t = \Delta \bar{u} + w f(v), & x \in \mathbf{R}^n, t \in (T_0, T^*), \\ \bar{u}(x, T_0) = \max\{u(x, T_0), \delta_f v(T_0)\}, & x \in \mathbf{R}^n. \end{cases}$$

Since $w(x, t) \in [\delta_f, 1)$ and $v(t) \geq M_f$, we have

$$wf(v) \geq f(wv) = f(\bar{u})$$

by (13). This $\bar{u}$ is supersolution of (1).

Since for any $x \in \mathbf{R}^n$, $\sup_{t \in [T_0, T^*)} w(x, t) < 1$, we can take $\delta = \delta(a, T_0, u_0, f) \in (0, 1)$ satisfying $w(x, t) \leq \delta$ for $(x, t) \in B_1(a) \times [T_0, T_v)$. Thus, we obtain

$$u(x, t) \leq \bar{u}(x, t) = w(x, t)v(t) \leq \delta v(t)$$

and Lemma 3.1 is proved. $\square$

For any $a \in \mathbf{R}^n$, we consider the solution $\phi = \phi_a$ of the equation:

$$\begin{cases} \phi_t = \Delta\phi + f(\phi), & x \in B_1, t \in (t_1, T_v), \\ \phi(x, 0) = \phi_0(x), & x \in B_1, \\ \phi(x, t) = v(t), & x \in \partial B_1, t \in (t_1, T_v), \end{cases} \quad (14)$$

where $\phi_0(x) = v(t_1)(1 - \varepsilon \cos \frac{\pi|x|}{2})$ with $\varepsilon = \varepsilon(u_0, f, a) > 0$ sufficiently small satisfying

$$\phi_0(x) \geq u(x + a, t_1) \quad (15)$$

and B_1 denotes the open ball of radius 1 and centered at 0. It is easily seen that

$$\Delta\phi_0(x) + f(\phi_0(x)) \geq 0.$$

By the maximum principle [10] we have

$$\phi(x, t) \geq u(x + a, t) \quad \text{and} \quad \phi_t \geq 0 \quad \text{for } x \in \bar{B}_1, t \in [t_1, T_v). \quad (16)$$

If w has no blow-up point in $\mathbf{R}^n$, the u has no blow-up point in $\mathbf{R}^n$, neither. We should show that w has no blow-up point.

Lemma 3.2. *Assume the same hypotheses of Lemma 3.1. Let $\Omega \in B_1$ be a domain. If $\partial_t \phi(x, t) \geq 0$ in $\Omega \times (t_1, T_v)$ and there exist $\nu \in S^{n-1}$ and $\delta > 0$, such that*

$$\nu \cdot \nabla \phi(x, t) \leq -\delta |\nabla \phi(x, t)| < 0 \quad \text{in } \Omega \times (t_1, T_v),$$

then ϕ does not uniformly blow-up in Ω :

$$\inf_{x \in \Omega} \phi(x, t) \leq L < \infty \quad \text{for } t \in (t_1, T_v).$$

Proof of Lemma 3.2. This lemma is proved in [9] (See [9, Lemma 4.1]). $\square$

Proof of Theorem 2. Put $r \in (0, 1)$. Define

$$\mu(x, t) = \phi(2r - x_1, x', t) - \phi(x_1, x', t),$$

where $x = (x_1, x')$ with $x' = (x_2, x_3, \dots, x_n) \in \mathbf{R}^{n-1}$. Then, we obtain

$$\begin{cases} \mu_t \geq \Delta\mu + C(x, t)\mu, & x \in D_r, t \in (t_1, T_v), \\ \mu(x, 0) = \phi_0(2r - x_1, x') - \phi_0(x_1, x') \geq 0, & x \in D_r, \\ \mu(x, t) \geq 0, & x \in \partial D_r, t \in (t_1, T_v), \end{cases}$$

where

$$C(x, t) = \int_0^1 \{ \theta \phi(2r - x_1, x', t) + (1 - \theta) \phi(x_1, x', t) \} d\theta$$

$$D_r = \{x : x_1 < r\} \cap \{x : (x - 2r)^2 < 1\}.$$

Thus, by the maximum principle [10] we have

$$\mu \geq 0 \quad \text{in} \quad D \times [t_1, T_v)$$

and

$$\phi(2r - x_1, x', t) \geq \phi(x_1, x', t) \quad \text{in} \quad D \times [t_1, T_v).$$

Since $r \in (0, 1)$ is arbitrary, we obtain that $\phi_{x_1} \geq 0$ for $x \in \{x | x_1 > 0\}$ and

$$-e_1 \cdot \nabla \phi \leq -\phi_{x_1} \leq -\frac{\delta x_1}{|x|} |\nabla \phi|, \quad \text{in} \quad D \cup \{x | x_1 \geq 0\}$$

with some $\delta > 0$, where $e_1 = {}^t(1, 0, 0, \dots, 0)$. Since $\phi_t \geq 0$ and $\inf_{x \in B_1} \phi(x, t) = \phi(0, t)$, by Lemma 3.2 we have

$$\lim_{t \rightarrow T_v} \phi(0, t) \leq L \text{ with some } L < \infty.$$

Thus

$$\lim_{t \rightarrow T_v} u(a, t) \leq L \text{ with same } L.$$

Since $a \in \mathbf{R}^n$ is arbitrary, u does not blow up at $t = T_v$ in $\mathbf{R}^n$. $\square$

4 On blow-up direction

We shall prove Theorem 3 which gives a condition for blow-up direction.

Proof of Theorem 3. We first prove that if u_0 satisfies (11), then ψ is a blow-up direction. By assumption we obtain that $u_0(x)$ satisfies (5) with some sequences $\{x_m\}_{m=1}^{\infty}$ satisfying $\lim_{m \rightarrow \infty} x_m/|x_m| = \psi$. Then, from the proof of Theorem 1 it follows that

$$\lim_{m \rightarrow \infty} u(x_m, t_m) = \infty$$

with the sequence $\{t_m\}_{m=1}^{\infty}$ satisfying $\lim_{m \rightarrow \infty} t_m = T_v$. Since $\lim_{m \rightarrow \infty} x_m/|x_m| = \psi$ by the assumption we obtain that ψ is a blow-up direction.

We next show that if ψ is a blow-up direction, then there exist $\{x_m\}_{m=0}^{\infty} \subset \mathbf{R}^n$ such that $x_m/|x_m| \rightarrow \psi$, $t_m \rightarrow T_v$ and $u(x_m, t_m) \rightarrow \infty$ as $m \rightarrow \infty$. In contrary it says that if for any sequences $\{x_m\}_{m=1}^{\infty} \subset \mathbf{R}^n$ satisfying $\lim_{m \rightarrow \infty} x_m/|x_m| = \psi$, u_0 does not satisfy (11), then ψ is not a blow-up direction.

Since $\lim_{m \rightarrow \infty} u_0(x + x_m) = M$ a.e. in $\mathbf{R}^n$, we have

$$\lim_{m \rightarrow \infty} \sup_{x \in B_3(x_m)} \frac{1}{(4\pi t)^{n/2}} \int_{\mathbf{R}^n} e^{-(x-y)^2/4t} u_0(y) dy < M \quad (17)$$

for $t > 0$. Since the solution of (1) satisfies the integral equation

$$u(x, t) = e^{\Delta t} u_0(x) + \int_0^t e^{\Delta(t-s)} f(u(x, s)) ds,$$

we have

$$u(x, t) \leq e^{\Delta t} u_0(x) + \int_0^t f(v(s)) ds = v(t) - M + e^{\Delta t} u_0(x)$$

for $(x, t) \in \mathbf{R}^n \times [0, T^*)$.

Let M_f , δ_f and T_0 be the same as proof of Lemma 3.1. We consider the solution w of

$$\begin{cases} w_t = \Delta w, & x \in \mathbf{R}^n, t \in (T_0, T_v), \\ w(x, T_0) = \max\{\{v(T_0) - M + e^{\Delta T_0} u_0(x)\}/v(T_0), \delta_f\}, & x \in \mathbf{R}^n. \end{cases}$$

We now introduce $\tilde{u} = vw$. From the proof of Lemma 3.1, it follows that $\tilde{u} \geq u$ for $(x, t) \in \mathbf{R}^n \times [T_0, T^*)$. Then we have

$$u(x, t) \leq v(t) e^{\Delta(t-T_0)} \max\{\{v(T_0) - M + e^{\Delta T_0} u_0(x)\}/v(T_0), \delta_f\}$$

for $(x, t) \in \mathbf{R}^n \times [T_0, T_v)$.

Put $U_m = \sup_{x \in B_2(x_m)} e^{T_0} u(x)$. From (17), there exists $M_0 \in (0, M)$ such that

$$\lim_{m \rightarrow \infty} U_m \leq M_0 (< M).$$

There exists a sequence $\{V_k\}_{k=1}^\infty$ such that $V_k = (M_0 + M)/2$, $\lim_{k \rightarrow \infty} V_k = M_0$, $V_{k+1} \leq V_k$ and $V_k \geq U_{m_k}$ with a sequence $\{m_k\}_{k=1}^\infty$ satisfying $u_{k+1} > u_k$ for $k \in \mathbf{N}$. Thus, since $(x - y)^2 \leq 2x^2 + 2y^2$, we obtain

$$\begin{aligned} & \sup_{x \in B_1(\tilde{x}_k)} w(x, t) \leq W_k(t) \\ & = e^{\Delta(t-T_0)} \max \left\{ \frac{v(T_0) - (M - V_k)e^{-|x|^2/2t} \int_{|y|<2} e^{-|y|^2/2t} u_0(y) dy}{(4\pi T_0)^{-n/2} v(T_0)}, \delta_f \right\} < 1 \end{aligned}$$

for $t \in [T_0, T_v)$, where $\tilde{x}_k = x_{m_k}$. By comparison we have $W_{k+1}(t) \leq W_k(t)$ for $t \in [T_0, T_v)$ and $k \in \mathbf{N}$. From Lemma 3.2 and comparison it follows that there exist the sequence $\{\eta_k\}_{k=1}^\infty$ satisfying $0 < \eta_{k+1} \leq \eta_k < \infty$ such that

$$\lim_{t \rightarrow T_v} u(x_{m_k}, t) \leq \eta_k.$$

Since the sequence $\{x_m\}_{m=1}^\infty$ is arbitrary, we obtain that ψ is not blow-up direction. $\square$

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