

Stability conditions and the autoequivalence group on K3 surfaces

Kotaro Kawatani(Osaka Univ.)

§ 1. Stability conditions

Let X be a projective manifold over $\mathbb{C}$.

μ -stability: $E \in \text{Coh}(X)$, $\omega \in \text{Amp}(X)$

Generalize E is μ -stable $\stackrel{\text{def}}{\iff} \forall F \subset E, \frac{\deg_{\omega} F}{\text{rank} F} < \frac{\deg_{\omega} E}{\text{rank} E}$

Bridgeland's stability: $E^{\bullet} \in D^b(\text{Coh}(X))$, $\sigma \in \text{Stab}(D^b(X))$

$\sigma = (\mathcal{A}, Z)$, t-structure of $\mathcal{A} : D^b(X) (= \text{fullsub abelian category})$,
 $Z : K(D^b(X)) \rightarrow \mathbb{C}$, group hom.

$E^{\bullet}$ is σ -stable " $\stackrel{\text{def}}{\iff}$ " $\forall F^{\bullet} \subset E^{\bullet} \arg Z(F^{\bullet}) < \arg Z(E^{\bullet})$

§ 2. Known Results

Trivial	non-emptiness of $\text{Amp}(X)$
Difficult	non-emptiness of $\text{Stab}(X) (= \text{Stab}(D^b(X)))$.

- cf. • When X is a K3 or an abelian surface $\text{Stab}(X) \neq \emptyset$
 • When X is a 3-dim Calabi-Yau ???

In the following, we assume that X is a K3 or an abelian surface.

$$\text{Stab}(X) \supset \text{Stab}^{\dagger}(X) \supset U(X)$$

$\text{Stab}^{\dagger}(X) \stackrel{??}{=} \{\sigma \in \text{Stab}(X) | \forall \mathcal{O}_x \text{ is } \sigma\text{-stable}\}$

- Conn. & Open
- Explicit locus

X : K3 surface	$\text{Stab}^{\dagger}(X) \supsetneq U(X)$
X : Abelian surface	$\text{Stab}^{\dagger}(X) = U(X)$

§ 3. Motivation

- $\text{Stab}(X)$ is defined on the category $D^b(X)$.
- For $U(X)$ what's happen when we change X preserving $D(X)$?

cf. For some X , $\exists Y$ s.t. $Y \not\cong X$ but $\Phi : D^b(Y) \xrightarrow{\sim} D^b(X)$.

Question.

Does there exist an equivalence $\Phi : D^b(Y) \rightarrow D^b(X)$, so that
 $\Phi_* U(Y) = U(X)$??

Partial Answer (X : Abelian surface.)

$$\forall \Phi : D^b(Y) \rightarrow D^b(X), \Phi_*(U(Y)) = U(X).$$

§ 4. Main Results

Theorem 1. (= Answer)

Let X and Y be projective K3 surfaces, and $\Phi : D^b(Y) \xrightarrow{\sim} D^b(X)$ an equivalence. If $\Phi_*(U(Y)) = U(X)$ then

$$\Phi(-) = M \otimes f_*(-)[n].$$

where $M \in \text{Pic}(X)$, $f : Y \xrightarrow{\sim} X$, $[n]$: n -times shift.

We define the subgroup $\text{Aut } D^b(X)^{U(X)}$ of $\text{Aut}(D(X))$ by

$$\text{Aut}(D^b(X))^{U(X)} := \{\Phi \in \text{Aut}(D^b(X)) | \Phi_*(U(X)) = U(X)\}.$$

Corollary 2.

$$\text{Aut}(D^b(X))^{U(X)} = (\text{Aut}(X) \ltimes \text{Pic}(X)) \times \mathbb{Z}[1]$$