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“Stochastic Dominance Statistics for Risk Averters
and Risk Seekers: An Analysis of Stock Preferences
for USA and China”

Zhidong Bai, Hua Li, Michael McAleer and Wing-Keung Wong

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Zhidong Bai

*KLASMOE and School of Mathematics and Statistics
Northeast Normal University
Department of Statistics and Applied Probability
and Risk Management Institute
National University of Singapore*

Hua Li

*Department of Statistics and Applied Probability
National University of Singapore*

Michael McAleer

*Econometric Institute, Erasmus School of Economics, Erasmus
University Rotterdam
Tinbergen Institute, The Netherlands
Department of Quantitative Economics, Complutense University of
Madrid
Institute of Economic Research, Kyoto University*

Wing-Keung Wong

*Department of Economics
Hong Kong Baptist University*

June 6, 2012

Corresponding author: Wing-Keung Wong, Department of Economics, Hong Kong Baptist University, Kowloon Tong, Hong Kong. Tel: (852)-3411-7542, Fax: (852)-3411-5580, Email: awong@hkbu.edu.hk

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Stochastic Dominance Statistics for Risk Averters and Risk Seekers: An Analysis of Stock Preferences for USA and China

Abstract We derive the limiting process of the stochastic dominance statistics for risk averters as well as for risk seekers when the underlying processes might be dependent or independent. We take account of the dependency of the partitions and propose a bootstrap method to decide the critical point. In addition, we illustrate the applicability of the stochastic dominance statistics for both risk averters and risk seekers to analyze the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the financial crises, including the internet bubble and the recent sub-prime crisis. The findings could be used to draw inferences on the preferences of risk averters and risk seekers in investing in the Chinese and US stock markets. The results also enable us to examine whether there is any arbitrage opportunity in these markets and whether these markets are efficient and investors are rational.

Keywords: Stochastic dominance, risk aversion, risk seeking, test statistic, hypothesis testing.

JEL Classification: C12, G1, G12, G15

1 Introduction

The stochastic dominance (SD) tests have been developing to test for statistically significant differences between any two cumulative density functions, say F and G , of two assets, Y and Z , drawing inference for the preferences for risk averters. Davidson and Duclos (hereafter DD, 2000) develop the SD statistic, $T_j(x)$ ($j = 1; 2; 3$), to determine the levels of significance for the first-, second-, and third-order SD for two distributions F and G . The DD test compares distributions at a finite number of grid points, and under the null hypothesis of equality of F and G , Davidson and Duclos show that T_j is asymptotically distributed as the studentized maximum modulus (SMM) distribution. Barrett and Donald (2003) comment that too few grids will miss information about the distributions between any two consecutive grids whereas, according to Richmond (1982), one may argue that too many grids will violate the independence assumption required by the SMM distribution. Fong et al. (2005) and others suggest using 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison. We note that the limiting distributions of $T_j(x)$ are not independent even for two points of partition. More importantly, the limiting correlations would depend on the null distribution $F = G$. To solve this problem, in this paper we modify the DD statistic by taking care of the dependency of the partitions. We suggest investigating the limiting distribution of $T_j(x)$; $a < x < b$, regarded as a stochastic process. We derive the limiting process of the DD test statistic $T_j(x)$, $j = 1; 2; 3$, and hence, the DD test can be performed by $\sup_x |T_j(x)|$.

It is well known that investors may not be risk-averse as they could be risk-seeking (see Meyer (1977), Stoyan (1983), Wong (2007) and the references therein for properties of the behavior of risk seekers, and investors with convex utility functions). In this paper, we also develop SD statistics to test for statistically significant differences between any two density

functions of two assets, Y and Z , drawing inference for the preferences for risk seekers. In addition, in many real data analyses, the random variables Y and Z being examined may not be independent. To take care of both dependent and independent situations, in this paper we develop the modified DD tests to cover the dependent and independent situations and provide the limiting correlations on the null distribution $F = G$. We note that Bai et al. (2012) have extended the DD test to SD test for investors with S-shaped and reverse S-shaped utility functions, our paper extend their work in the sense that we (1) compliment their work and the work by Davidson and Duclos (2000) to develop SD tests for risk averters and risk seekers, and (2) take care of the dependent and independent situations. In this paper we also propose a bootstrap method to decide the critical point of the modified DD tests so that their critical values will be closer to the true critical values.

Thereafter, we illustrate the applicability of the SD statistics for both risk averters and risk seekers analyze the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the financial crises, including the internet bubble and the recent sub-prime crisis. The findings could be used to draw inferences on the preferences of risk averters and risk seekers in investing in the Chinese and US stock markets. In addition, the results enable us to examine whether there is any arbitrage opportunity in these markets and whether these markets are efficient and investors are rational. We also note that a hierarchical relationship exists in SD mathematically (Sriboonchitta et al., 2009) but, in our analysis, we observe that the SD hierarchical relationship may not exist statistically. We will discuss this issue in our illustration section and propose an inference to this new finding in the literature (see, for example, Wong and Chan (2008), Broll et al. (2010), Egozcue et al. (2011), and the references therein for further information).

The remainder of the paper is organized as follows: Section 2 introduces definitions and notation, and discusses some basic SD properties. We will develop the SD tests for risk averters and risk seekers in Section 3. In Section 4, we discuss the bootstrap method to decide the critical points of the tests. Section 5 illustrates the applicability of our proposed SD test statistics to study the preferences of risk averters and risk seekers for their investment in the Chinese and USA stock markets. Section 6 concludes our findings.

2 Definitions, Notations, and Basic Properties

Let $\overline{\mathbb{R}}$ be the set of extended real numbers and $\Omega = [a, b]$ be a subset of $\overline{\mathbb{R}}$. Let $\mathbb{B}$ be the Borel σ -field of Ω and μ be a *measure* on $(\Omega, \mathbb{B})$. We first define the functions F^A and F^D of the measure μ on the support Ω as $F^A(x) = F_1^A(x) = \mu(a, x]$ and $F^D(x) = F_1^D(x) = \mu[x, b)$ for all $x \in \Omega$. Function F is a *cumulative distribution function (CDF)* and μ is a *probability measure* if $\mu(\Omega) = 1$. From the basic probability theory, for any random variable X with an associated probability measure P , there exists a unique induced probability measure μ on $(\Omega, \mathbb{B})$ and a CDF F such that $\mu(B) = P(X^{-1}(B)) = P(X \in B)$ for any $B \in \mathbb{B}$. An integral written in the form of $\int_A f(t) d\mu(t)$ or $\int_A f(t) dF(t)$ is understood as a Lebesgue-Stieltjes integral for integrable function $f(t)$. If the integral has the same value for all A among $(c, d]$, $[c, d)$ or $[c, d]$, we use the notation $\int_c^d f(t) d\mu(t)$ instead. In addition, if μ is a Borel measure with $\mu(c, d] = d - c$, we write the integral as $\int_c^d f(t) dt$. Random variables, denoted by Y and Z , defined on Ω , are considered together with their corresponding CDFs F and G , respectively. The following notations will be used throughout this paper:

$$\begin{aligned} \mu_F = \mu_Y = E(Y) &= \int_a^b x dF(x), \quad \mu_G = \mu_Z = E(Z) = \int_a^b x dG(x); \text{ and} \\ H_j^A(x) &= \int_a^x H_{j-1}^A(y) dy, \quad H_j^D(x) = \int_a^x H_{j-1}^D(y) dy, \quad j = 2, 3; \end{aligned} \quad (2.1)$$

where $H = F$ or G .¹ In (2.1), $\mu_F = \mu_Y$ is the mean of Y , whereas $\mu_G = \mu_Z$ is the mean of Z .

We note that the definition of H_j^A in (2.1) can be used to develop the SD theory for risk averters (see, for example, Quirk and Saposnik, 1962; Fishburn, 1964) and thus we call this type of SD ascending stochastic dominance (ASD) and call H_j^A the j^{th} order ASD integral or the j^{th} order cumulative probability, since H_j^A is integrated from H_{j-1}^A in ascending order from the leftmost point of downside risk. On the other hand, H_j^D can be used to develop the SD theory for risk seekers (see, for example, Meyer, 1977; Stoyan, 1983; Li and Wong, 1999) and thus we call this type of SD descending stochastic dominance (DSD) and call H_j^D the j^{th} order DSD integral or the j^{th} order reversed cumulative probability, since H_j^D is integrated H_{j-1}^D in descending order from the rightmost point of upside profit. Typically, risk averters prefer assets that have a smaller probability of loss whereas risk seekers prefer assets that have a higher probability of gain, especially in upside profit. To make a choice between two assets F and G , risk averters will compare their corresponding j^{th} order ASD in integrals F_j^A and G_j^A and choose F if F_j^A is smaller, since it has a smaller probability of loss. On the other hand, risk seekers will compare their corresponding j^{th} order DSD integrals F_j^D and G_j^D and choose F if F_j^D is bigger, since it has a higher probability of gain. We first state the definitions of SD for risk averters and risk seekers as follows:

Definition 2.1 *Given two random variables Y and Z with F and G as their respective CDFs, Y dominates Z and F dominates G in the sense of FASD (SASD, TASD), denoted by $Y \succeq_1^A Z$ or $F \succeq_1^A G$ ($Y \succeq_2^A Z$ or $F \succeq_2^A G$, $Y \succeq_3^A Z$ or $F \succeq_3^A G$), if and only if $F_1^A(x) \leq G_1^A(x)$ ($F_2^A(x) \leq G_2^A(x)$, $F_3^A(x) \leq G_3^A(x)$) for each x in $[a, b]$, where FASD, SASD, and TASD stand for first-, second-, and third-order ASD, respectively. In addition, if there*

¹see, for example, Wong and Li (1999), Anderson (2004), and Wong and Chan (2008).

exists strict inequality for any x in $[a, b]$, we say that Y dominates Z and F dominates G in the sense of SFASD, SSASD, and STASD, denoted by $Y \succ_1^A Z$ or $F \succ_1^A G$, $Y \succ_2^A Z$ or $F \succ_2^A G$, and $Y \succ_3^A Z$ or $F \succ_3^A G$, respectively, where SFASD, SSASD, and STASD stand for strictly first-, second-, and third-order SD, respectively.

Definition 2.2 Given two random variables Y and Z with F and G as their respective CDFs, Y dominates Z and F dominates G in the sense of FDSD (SDSD, TDSD), denoted by $Y \succeq_1^D Z$ or $F \succeq_1^D G$ ($Y \succeq_2^D Z$ or $F \succeq_2^D G$, $Y \succeq_3^D Z$ or $F \succeq_3^D G$), if and only if $F_1^D(x) \geq G_1^D(x)$ ($F_2^D(x) \geq G_2^D(x)$, $F_3^D(x) \geq G_3^D(x)$) for each x in $[a, b]$, where FDSD, SDSD, and TDSD stand for first-, second-, and third-order DSD, respectively. In addition, if there exists strict inequality for any x in $[a, b]$, we say that Y dominates Z and F dominates G in the sense of SFDSD, SSDSD, and STDSD, denoted by $Y \succ_1^A Z$ or $F \succ_1^A G$, $Y \succ_2^A Z$ or $F \succ_2^A G$, and $Y \succ_3^A Z$ or $F \succ_3^A G$, respectively, where SFDSD, SSDSD, and STDSD stand for strictly first-, second-, and third-order SD, respectively.

The SD theory is related to utility maximization. To discuss the issue, we first state the definition of utility functions for risk averters and risk seekers as follows:

Definition 2.3 For $j = 1, 2, 3$, U_j^A , U_j^D , U_j^{SA} and U_j^{SD} are the sets of the utility functions² u such that

$$\begin{aligned} U_j^A(U_j^{SA}) &= \{u : (-1)^i u^{(i)} \leq (<) 0, i = 1, \dots, j\}; \\ U_j^D(U_j^{SD}) &= \{u : u^{(i)} \geq (>) 0, i = 1, \dots, j\}; \end{aligned}$$

where $u^{(i)}$ is the i^{th} derivative of u .

We note that investors in U_j^A are risk averse, while investors in U_j^D are risk seeking. Choosing between F and G in accordance with a consistent set of preferences will

²We note that the theory can be easily extended to satisfy utilities defined to be non-differentiable and/or non-expected utility functions (Wong and Ma, 2008; Sriboonchitta, et al., 2009).

satisfy the von Neumann-Morgenstern (1944) consistency properties. Accordingly, F is (strictly) preferred to G , or equivalently, Y is (strictly) preferred to Z if $\Delta Eu \equiv u(F) - u(G) \equiv u(Y) - u(Z) \geq 0 (> 0)$, where $u(F) \equiv u(Y) \equiv \int_a^b u(x)dF(x)$ and $u(G) \equiv u(Z) \equiv \int_a^b u(x)dG(x)$. The ASD and DSD approaches are regarded as two of the most useful tools for ranking uncertain investment prospects, since ranking assets have been proven (Li and Wong, 1999) to be equivalent to utility maximization for the preferences of risk-averse and risk-seeking investors as shown in the following proposition:

Proposition 2.1 *Let Y and Z be random variables with CDFs F and G , respectively. Suppose u is a utility function. For $j = 1, 2$ and 3 , $F \succeq_j^A (\succ_j^A)G$ if and only if $u(F) \geq (>)u(G)$ for any u in $U_j^A (U_j^{SA})$, and $F \succeq_j^D (\succ_j^D)G$ if and only if $u(F) \geq (>)u(G)$ for any u in $U_j^D (U_j^{SD})$.*

The existence of ASD implies that the expected utility of the risk-averse investor is always higher when holding the dominant asset than when holding the dominated asset, and consequently, the dominated asset would not be chosen. We note that a hierarchical relationship exists in ASD: FASD implies SASD, which, in turn, implies TASD. However, the converse is not true. Similarly, the hierarchical relationships also exist in DSD. Thus, it is a common practice to report only the lowest dominance order of ASD and DSD.

3 SD Tests for Risk Averters and Risk Seekers

The tests developed by Davidson and Duclos (DD, 2000), Barrett and Donald (BD, 2003) and Linton, et al. (LMW, 2005) are the most commonly used statistics to investigate the preference for risk averters. Since the test developed by DD is found³ to be one of the most powerful statistics to test the significance of stochastic dominance and yet one of the least conservative in size, in this paper we discuss only the DD test and extend the

³See, for example, Wei and Zhang (2003), Tse and Zhang (2004) and Lean, et al. (2008) for more information.

theory by modifying only the DD test for risk averters and risk seekers in the following subsections. One could easily use the idea in our paper to extend the BD and LMW tests or other SD tests to test for the preferences for risk seekers.

3.1 ASD Tests for Risk Averters

In light of the definition of ASD's, one may want to test the following hypothesis, for $j = 1, 2, 3$,

$$H_0^A : F_j^A \equiv G_j^A,$$

against three alternatives

$$\begin{aligned} H_1^A : & \quad F \not\equiv_j^A G, \text{ two sided test,} \\ H_{1l}^A : & \quad F \succ_j^A G, \text{ one sided test,} \\ H_{1r}^A : & \quad F \prec_j^A G, \text{ one sided test.} \end{aligned} \tag{3.1}$$

The three hypotheses are equivalent to

$$\begin{aligned} H_1^A : & \quad F_j^A(x) \neq G_j^A(x), \text{ for some } x, \\ H_{1l}^A : & \quad F_j^A(x) \leq G_j^A(x), \forall x \text{ and the inequality is strict for at least one } x, \\ H_{1r}^A : & \quad F_j^A(x) \geq G_j^A(x), \forall x \text{ and the inequality is strict for at least one } x. \end{aligned}$$

Now, assume the data $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) are observations drawn from the independent random variables Y and Z with distribution functions F and G , respectively. The integrals F_j^A and G_j^A for F and G are defined in (2.1) for $j = 1, 2$ and 3 . For a grid of pre-selected points $\{x_k, k = 1, \dots, K\}$, Davidson and Duclos propose to use the following j^{th} order ascending DD test statistic, $T_j^A(x)$ ($j = 1, 2$ and 3) to test for H_1^A , H_{1l}^A , and H_{1r}^A :

$$T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}}, \tag{3.2}$$

where

$$\begin{aligned}
\hat{V}_j^A(x) &= \hat{V}_{F_j}^A(x) + \hat{V}_{G_j}^A(x), \\
\hat{H}_j^A(x) &= \frac{1}{N_h(j-1)!} \sum_{i=1}^{N_h} (x - h_i)_+^{j-1}, \\
\hat{V}_{H_j}^A(x) &= \frac{1}{N_h} \left[\frac{1}{N_h((j-1)!)^2} \sum_{i=1}^{N_h} (x - h_i)_+^{2(j-1)} - \hat{H}_j^A(x)^2 \right], \quad H = F, G; \quad h = f, g.
\end{aligned} \tag{3.3}$$

It is not difficult to show that under the null hypothesis H_0^A , for each fixed $x \in (a, b)$, $T_j^A(x) \rightarrow N(0, 1)$, the standard normal distribution. However, the limiting joint distribution of $(T_j^A(x_1), T_j^A(x_2))$ does not have independent components for any $x_1 \neq x_2$. More importantly, the limiting correlation depends on the null in which the distribution functions $F = G$. Therefore, it is empirically difficult to test the hypotheses described in (3.1). One may incorporate the idea proposed by Bishop, et al. (1992) to test the null hypothesis for a pre-designated finite number of values $\{x_k, k = 1, \dots, K\}$ and then test the following weaker hypotheses:

$$\begin{aligned}
H_0^{AK} &: F_j^A(x_k) = G_j^A(x_k) \text{ for all } x_k; \\
H_1^{AK} &: F_j^A(x_k) \neq G_j^A(x_k) \text{ for some } x_k; \\
H_{1l}^{AK} &: F_j^A(x_k) \leq G_j^A(x_k) \text{ for all } x_k, \quad F_j^A(x_k) < G_j^A(x_k) \text{ for some } x_k; \\
H_{1r}^{AK} &: F_j^A(x_k) \geq G_j^A(x_k) \text{ for all } x_k, \quad F_j^A(x_k) > G_j^A(x_k) \text{ for some } x_k.
\end{aligned}$$

Under the null hypothesis H_0^{AK} , DD show that T_j^A is asymptotically distributed as the studentized maximum modulus (SMM) distribution (Richmond, 1982). To implement the DD test, the T_j^A is computed at each grid point and the null hypothesis, H_0^{AK} , is rejected if

$$\begin{aligned}
\max_{k \leq K} |T_j^A(x_k)| &> M_{\infty, \alpha/2}^K, \text{ for the alternative } H_1^{AK}; \\
\min_{k \leq K} T_j^A(x_k) &< -M_{\infty, \alpha}^K, \text{ for the alternative } H_{1l}^{AK}; \\
\max_{k \leq K} T_j^A(x_k) &> M_{\infty, \alpha}^K, \text{ for the alternative } H_{1r}^{AK};
\end{aligned}$$

where $M_{\infty, \alpha}^K$ is the $1 - \alpha$ percentile of M_{∞}^K tabulated by Stoline and Ury (1979):

Remark 3.1 *The DD test compares distributions at a finite number of grid points. Various studies examine the choice of grid points. For example, Tse and Zhang (2004) show that an appropriate choice of K for reasonably large samples ranges from 6 to 15. Too few grids will miss information about the distributions between any two consecutive grids (Barrett and Donald, 2003), and too many grids will be said to violate the independence assumption required by the SMM distribution (Richmond, 1982). To make the comparisons comprehensive without too much violating the independence assumption, Fong, et al. (2005), Gasbarro, et al. (2007), and Wong, et al. (2008) suggest to use the 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison and draw statistical inference based on the SMM distribution for $K = 10$ and infinite degrees of freedom.*

As we have pointed out earlier that the limiting distributions of $T_j^A(x)$ are not independent even for two points partition. More importantly, the limiting correlations would depend on the null in which distributions $F = G$. To solve these problems, we suggest to investigate the limiting distribution of $T_j^A(x)$, $a < x < b$, regarded as a stochastic process. To do so, we establish the following theorem.

Theorem 3.1 *Let $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) be random observations drawn from the independent random variables Y and Z , with continuous distribution functions F and G , respectively. Under the null hypothesis $F \equiv G$, the j^{th} order ASD test statistics, $T_j^A(x)$ ($j = 1, 2$ and 3), weakly tends to a limiting Gaussian process with mean 0, variance 1, and correlation function $r^A(x, y)$ in which for the case $j = 1$, we get*

$$r_1^A(x, y) = \frac{F(x \wedge y) - F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have

$$r^A(x, y) = \frac{\int_a^x \int_a^y (x-t)^{j-2} (y-s)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds}{\sqrt{V_j^A(x) V_j^A(y)}},$$

where $\hat{H}_j^A(x)$ and $\hat{V}_j^A(x)$ for $H = F$ and G are defined in (3.2),

$$V_1^A(x) = F(x)(1 - F(x)) ,$$

and

$$V_j^A(x) = \int_a^x \int_a^x (x-t)^{j-2} (x-s)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds .$$

Based on this theorem, to test the hypotheses in (3.1), we propose to reject the null hypothesis H_0^A if

$$\begin{aligned} \max_{a < x < b} |T_j^A(x)| &> M_{\alpha/2}^A, \text{ for the alternative } H_1^A; \\ \min_{a < x < b} T_j^A(x) &< -M_{\alpha}^A, \text{ for the alternative } H_{1l}^A; \\ \max_{a < x < b} T_j^A(x) &> M_{\alpha}^A, \text{ for the alternative } H_{1r}^A; \end{aligned}$$

in which we suggest to obtain the critical values M_{α}^A by a bootstrap approach that will be described later in the paper.

In many cases, the asset random variables Y and Z are not independent of each other. Assume that they have a joint distribution $Q(y, z)$ with marginal distributions F and G . Suppose that the sample is drawn in a way that $\{(f_i, g_i), i = 1, \dots, m, f_k, g_l, k = m+1, \dots, N_f; l = m+1, \dots, N_g\}$ are mutually independent. In such a case, the covariance of $\hat{F}_j^A(x)$ and $\hat{G}_j^A(x)$ can be estimated by

$$\hat{V}_{FG_j}^A(x) = \frac{1}{N_f N_g ((j-1)!)^2} \sum_{i=1}^m (x - f_i)_+^{j-1} (x - g_i)_+^{j-1} - \frac{m}{N_f N_g} \hat{F}_j^A(x) \hat{G}_j^A(x).$$

Hence, the variance of $\hat{F}_j^A(x) - \hat{G}_j^A(x)$ can be estimated by

$$\hat{V}_j^A(x) = \hat{V}_{F_j}^A(x) + \hat{V}_{G_j}^A(x) - 2\hat{V}_{FG_j}^A(x), \quad (3.4)$$

in which the integrals F_j^A and G_j^A are defined in (2.1) for $j = 1, 2$ and 3 . Thereafter, one may define the test statistic $T_j^A(x)$ by (3.2) with the new $\hat{V}_j^A(x)$ defined in (3.4). We summarize the result in the following theorem:

Theorem 3.2 *Let $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) be random observations drawn from the dependent random variables Y and Z , with continuous joint distribution function $Q(x, y)$ and marginal distribution functions F and G , respectively. Suppose the samples are drawn as described earlier; that is, we assume that $(f_i, g_i); i = 1, \dots, m$ are pairwise drawn and the rest are independent. Suppose that $m/(N_f + N_g) \rightarrow \lambda$. Then, under the null hypothesis $F \equiv G$, the j^{th} order ASD test statistic, $T_j^A(x)$ ($j = 1, 2$ and 3), weakly tends to a limiting Gaussian process with mean 0 , variance 1 , and correlation function $r_{j, \text{dep}}^A(x, y)$ in which for the case $j = 1$, we get*

$$r_{1, \text{dep}}^A(x, y) = \frac{F(x \wedge y) - \lambda Q(x, y) - (1 - 2\lambda)F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have

$$r_{j, \text{dep}}^A(x, y) = \frac{V_j^A(x, y)}{\sqrt{V_j^A(x)V_j^A(y)}},$$

where $\hat{H}_j^A(x)$ and $\hat{V}_j^A(x)$ for $H = F$ and G are defined in (3.3) and (3.4), respectively, and

$$\begin{aligned} V_j^A(x) &= \int_a^x \int_a^x (x-t)^{j-2}(x-s)^{j-2}(F(t \wedge s) - F(t)F(s))dtds \quad \text{and} \\ V_j^A(x, y) &= \int_a^x \int_a^y (x-t)^{j-2}(y-s)^{j-2}[(F(t \wedge s) - \lambda(Q(t, s) + Q(s, t)) - (1 - 2\lambda)F(t)F(s))]dtds. \end{aligned}$$

3.2 DSD Tests for Risk Seekers

In this paper, we modify the DD test for risk averters to be DD test statistic for risk seekers and we call it the descending DD test statistic. Let $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) be observations drawn from the independent random variables Y

and Z with distribution functions F and G , respectively. The integrals F_j^D and G_j^D for F and G are defined in (2.1) for $j = 1, 2$ and 3 . For a grid of pre-selected points $\{x_k, k = 1, \dots, K\}$, the j^{th} order descending DD test statistic, $T_j^D(x)$ ($j = 1, 2$ and 3), is:

$$T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}}, \quad (3.5)$$

where

$$\begin{aligned} \hat{V}_j^D(x) &= \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x), \\ \hat{H}_j^D(x) &= \frac{1}{N_h(j-1)!} \sum_{i=1}^{N_h} (h_i - x)_+^{j-1}, \\ \hat{V}_{H_j}^D(x) &= \frac{1}{N_h} \left[\frac{1}{N_h((j-1)!)^2} \sum_{i=1}^{N_h} (h_i - x)_+^{2(j-1)} - \hat{H}_j^D(x)^2 \right], \quad H = F, G; \quad h = f, g; \end{aligned}$$

in which the integrals F_j^D and G_j^D are defined in (2.1) for $j = 1, 2$ and 3 . For $k = 1, \dots, K$, the following hypotheses are tested for risk seekers:

$$\begin{aligned} H_0^D &: F_j^D(x_k) = G_j^D(x_k) \text{ for all } x_k; \\ H_1^D &: F_j^D(x_k) \neq G_j^D(x_k) \text{ for some } x_k; \\ H_{1l}^D &: F_j^D(x_k) \geq G_j^D(x_k) \text{ for all } x_k, \quad F_j^D(x_k) > G_j^D(x_k) \text{ for some } x_k; \\ H_{2r}^D &: F_j^D(x_k) \leq G_j^D(x_k) \text{ for all } x_k, \quad F_j^D(x_k) < G_j^D(x_k) \text{ for some } x_k. \end{aligned}$$

Under the null hypothesis H_0^D , one may believe that T_j^D is asymptotically distributed as the studentized maximum modulus (SMM) distribution (Richmond, 1982). Similar to the argument we correct the ASD test in last subsection, this is not correct and we correct the theory for the DSD test as follows:

To implement the DD test, T_j^D , for risk seekers for $j = 1, 2, 3$, one should test the following hypothesis at each grid point being computed:

$$H_0^D : F_j^D \equiv G_j^D,$$

against three alternatives

$$\begin{aligned}
H_1^D : \quad & F \not\equiv_j^D G, \text{ two sided test,} \\
H_{1l}^D : \quad & F \succ_j^D G, \text{ one sided test,} \\
H_{1r}^D : \quad & F \prec_j^D G, \text{ one sided test.}
\end{aligned} \tag{3.6}$$

The three hypotheses are equivalent to

$$\begin{aligned}
H_1^D : \quad & F_j^D(x) \neq G_j^D(x), \text{ for some } x, \\
H_{1l}^D : \quad & F_j^D(x) \geq G_j^D(x), \forall x \text{ and the inequality is strict for at least one } x, \\
H_{1r}^D : \quad & F_j^D(x) \leq G_j^D(x), \forall x \text{ and the inequality is strict for at least one } x.
\end{aligned}$$

Using a similar technique as the development of Theorem 3.1, we obtain the following theorem for DSD test:

Theorem 3.3 *Let $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) be random observations drawn from the independent random variables Y and Z , with continuous distribution functions F and G , respectively. Under the null hypothesis $F \equiv G$, the j^{th} order DSD test statistic, $T_j^D(x)$ ($j = 1, 2$ and 3), weakly tends to a limiting Gaussian process with mean 0, variance 1, and correlation function $r^A(x, y)$ in which for the case $j = 1$, we get*

$$r_1^D(x, y) = \frac{F(x \wedge y) - F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have

$$r^D(x, y) = \frac{\int_x^b \int_y^b (t - x)^{j-2} (s - y)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds}{\sqrt{V_j^D(x) V_j^D(y)}},$$

where $\hat{H}_j^D(x)$ and $\hat{V}_j^D(x)$ are defined in (3.5) for $H = F$ and G ,

$$V_1^D(x) = F(x)(1 - F(x)) ,$$

and

$$V_j^D(x) = \int_x^b \int_x^b (t-x)^{j-2} (s-x)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds .$$

The proof of this theorem is given in Appendix. Based on this theorem, to test the hypotheses (3.6), we propose to reject the null hypothesis H_0^D if

$$\begin{aligned} \max_{a < x < b} |T_j^D(x)| &> M_{\alpha/2}^D, \text{ for the alternative } H_1^D; \\ \max_{a < x < b} T_j^D(x) &> M_{\alpha}^D, \text{ for the alternative } H_{1l}^D; \\ \min_{a < x < b} T_j^D(x) &< -M_{\alpha}^D, \text{ for the alternative } H_{1r}^D; \end{aligned}$$

where critical values M_{α}^D is suggested to be found by a bootstrap approach which will be described later in our paper.

Similar to the case of the ASD tests, the asset random variables Y and Z may not be independent of each other. Now, we assume that the samples have the same structure as described in last subsection. In such a case, the covariance of $\hat{F}_j^D(x)$ and $\hat{G}_j^D(x)$ can be estimated by

$$\hat{V}_{FG_j}^D(x) = \frac{1}{N_f N_g ((j-1)!)^2} \sum_{i=1}^m (f_i - x)_+^{j-1} (g_i - x)_+^{j-1} - \frac{m}{N_f N_g} \hat{F}_j^D(x) \hat{G}_j^D(x).$$

Hence, the variance of $\hat{F}_j^D(x) - \hat{G}_j^D(x)$ can be estimated by

$$\hat{V}_j^D(x) = \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x) - 2\hat{V}_{FG_j}^D(x). \quad (3.7)$$

Then, one may define the test statistic $T_j^D(x)$ by (3.5) with the newly defined $\hat{V}_j^D(x)$. The proof of Theorem 3.3 is given in the Appendix.

Theorem 3.4 *Let $\{f_i\}$ ($i = 1, 2, \dots, N_f$) and $\{g_i\}$ ($i = 1, 2, \dots, N_g$) be random observations drawn from the dependent random variables Y and Z , with continuous joint distribution function $Q(x, y)$ and marginal distribution functions F and G , respectively. Suppose the samples are drawn as described earlier; that is, we assume that $(f_i, g_i); i =$*

$1, \dots, m$ are pairwise drawn and the rest are independent. Suppose that $m/(N_f + N_g) \rightarrow \lambda$. Then, under the null hypothesis $F \equiv G$, the j^{th} order DSD test statistic, $T_j^D(x)$ ($j = 1, 2$ and 3), weakly tends to a limiting Gaussian process with mean 0, variance 1, and correlation function $r_{j, \text{dep}}^D(x, y)$ in which for the case $j = 1$, we get

$$r_{1, \text{dep}}^D(x, y) = \frac{F(x \wedge y) - \lambda Q(x, y) - (1 - 2\lambda)F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have

$$r_{j, \text{dep}}^D(x, y) = \frac{V_j^D(x, y)}{\sqrt{V_j^D(x)V_j^D(y)}},$$

where $\hat{H}_j^D(x)$ and $\hat{V}_j^D(x)$ for $H = F$ and G are defined in (3.3) and (3.7), respectively, and obtain

$$\begin{aligned} V_j^D(x) &= \int_x^b \int_x^b (t - x)^{j-2} (s - x)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds \\ V_j^D(x, y) &= \int_x^b \int_y^b (t - x)^{j-2} (s - y)^{j-2} [(F(t \wedge s) - \lambda(Q(t, s) + Q(s, t)) - (1 - 2\lambda)F(t)F(s))] dt ds. \end{aligned}$$

4 Determination of Critical Values

In this section, we discuss only the situation in which the variables are independent. The situation in which the variables are dependent could be obtained similarly. Suppose the variables being examined are independent and the sample series $\{f_i, i = 1, 2, \dots, N_f\}$ and $\{g_i, i = 1, 2, \dots, N_g\}$ are iid. We draw two resamples $\{f_i^*, i = 1, 2, \dots, N_f\}$ and $\{g_i^*, i = 1, 2, \dots, N_g\}$ from the pooled sample $\{f_{(i)}, g_{(j)}, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$, the DD test statistics ($\hat{T}_j^A$ and $\hat{T}_j^D$) ($j = 1, 2, 3$) defined in (3.2) and (3.5) can then be bootstrapped. Using this method, one can approximate the null distribution of the test statistics. The details to obtain the **critical values** of statistic $\hat{T}_j^A$ and $\hat{T}_j^D$, $j = 1, 2, 3$, are as follows:

Step 1 Draw a sample set $\{f_i^*, i = 1, 2, \dots, N_f\}$ from $\{f_i, g_j, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$ with replacement and draw another sample set $\{g_i^*, i = 1, 2, \dots, N_g\}$ in the same way.

Step 2 Compute: $A_j = \hat{T}_j^{*A}(x) = [F_j^A(x) - \hat{G}_j^A(x)]/\sqrt{\hat{V}_j^A(x)}$, and $D_j = \hat{T}_j^{*D}(x) = [F_j^D(x) - \hat{G}_j^D(x)]/\sqrt{\hat{V}_j^D(x)}$, where $\hat{H}_j(x)$ and $\hat{V}_j(x)$ for $H = F$ and G , respectively, are defined in (3.2). Repeat this process N times.

Step 3 Find $A_j(\alpha)$ such that $\sharp\{|A_{jk}| \geq A_j(\alpha), k \leq N\} = [N_\alpha]$; that is, the α percentile of the distribution of $\hat{T}_j^{*A}$.

Step 4 Find $D_j(\alpha)$ such that $\sharp\{|D_{jk}| \geq D_j(\alpha), k \leq N\} = [N_\alpha]$; that is, the α percentile of the distribution of $\hat{T}_j^{*D}$.

We make the following remarks in obtaining the critical values:

Remark 4.1 *The samples $\{f_i^*, g_j^*, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$ are iid random samples with distribution function $F_{N_f+N_g}^* = \frac{N_f}{N_f+N_g}F_{N_f} + \frac{N_g}{N_f+N_g}G_{N_g}$. So, under the null hypothesis $F^* = F = G$, the sample sets $\{f_i^*, g_j^*, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$ and $\{f_i, g_j, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$ are almost the same as the samples which come from the same distribution only if the sum of N_f and N_g is large enough. Under this situation, the critical value obtained by using the above procedure will be almost the same as the real critical point value. But under the relative hypothesis, because the sample sets $\{f_i, i = 1, 2, \dots, N_f\}$ and $\{g_j, j = 1, 2, \dots, N_g\}$ come from different distributions F and G , respectively, the absolute value of $\hat{T}_j$ with the sample set $\{f_i^*, g_j^*, i = 1, 2, \dots, N_f, j = 1, 2, \dots, N_g\}$ could be different from the real critical point value.*

Remark 4.2 *Start with $N=10000$, $N_f = 200$, $N_g = 200$, and $\alpha = 0.05$. Given that F and G are from some specific distributions, we compare the probability of the error of the first kind and the power for some varieties for pairs of F and G . For example, suppose*

$F = N(0, 1)$, $G = N(0, 1)$, $N(0.1, 1)$, $N(0.2, 1)$, $N(0.3, 1)$, $N(0.4, 1)$, and $N(0.5, 1)$, respectively, we obtain the simulation results in Table 1. From the table, one could easily conclude that when F and G are further apart, the difference between the errors of the first kind and the power become greater.

Table 1: the error of the first kind and the power of SD tests

ASD test						
j=1	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.057	0.139	0.405	0.700	0.934	0.993
j=2	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.060	0.122	0.430	0.733	0.956	0.997
j=3	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.036	0.074	0.085	0.174	0.208	0.423
DSD test						
j=1	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.0499	0.1227	0.3566	0.6906	0.9156	0.9921
j=2	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.0542	0.1434	0.4191	0.7726	0.9577	0.9965
j=3	N(0,1)	N(0.1,1)	N(0.2,1)	N(0.3,1)	N(0.4,1)	N(0.5,1)
N(0,1)	0.0511	0.1362	0.4097	0.7594	0.9495	0.9962

Note: ASD test is defined in (3.2) while DSD test is defined in (3.5).

5 Empirical Results

In this section, we will illustrate the applicability of the SD statistics for both risk averters and risk seekers developed in this paper to analyze the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the financial crises, including the internet bubble and the recent sub-prime crisis. We are also interested in studying their preferences in these markets in the bull runs as well as in the bear markets. To do so, we apply the modified DD test statistics developed above to examine the daily returns of both SSE and S&P 500 for the full sample period (02/01/1996 to 19/05/2011) and the 5 sub-periods: 01/1996 to 03/2000, 03/2000 to 10/2002, 10/2002 to 10/2007, 10/2007 to 11/2008, and 11/2008 to 05/2011. SSE denotes the Shanghai Stock Exchange Composite Index, while S&P 500 denotes the Standard & Poor 500 index, a free-float capitalization-weighted index of the prices of 500 large-cap common stocks actively traded in the USA.

The Chinese stock markets, initiated in early 1990's, have been expanding tremendously in the past decade. China has experienced dramatic economic growth in the past decade. Its average annual growth rate is about 9%, much higher than that of the world economy. As one important component of the Chinese economy, Chinese stock markets, have also expanded rapidly. There are many studies that have analyzed the Chinese and USA stock markets. For example, Chakravarty et al. (1998) report the bivariate return correlations among the Chinese stock indices, as well as the U.S. market indices, and suggest that the Chinese market is still isolated.

We now ready to discuss our analysis for the performance of the daily returns of Chinese (SSE) and USA (S&P) stock markets for risk averters as well as for risk seekers in the full sample period as well as all the sub-periods. To analyze the preferences for risk averters, we adopt the modified DD statistic T_j^A (we call it ASD test) for risk averters for $j = 1, 2$, and 3 where T_j^A is defined in (3.2) for $j = 1, 2$, and 3 with F and G be the

CDFs of the returns of *S&P* and *SSE*, respectively. We display the results in Table 2. In addition, we plot the ASD tests together with the CDFs, F and G , of the daily returns of *S&P* and *SSE* in Figure 2. On the other hand, to analyze the preferences for risk seekers, we employ the modified DD statistic T_j^D (we call it DSD test) for risk seekers for $j = 1, 2$, and 3 where T_j^D is defined in (3.5) for $j = 1; 2$, and 3 with $F = S\&P$ and $G = SSE$. We exhibit the results in Table 3. In addition, we plot the DSD tests together with the DSD integrals F_1^D and G_1^D defined in (2.1) in Figure 3. At last, we summarize all SD relationship between SSE and S&P 500 for the full sample period as well as all the 5 sub-periods in Table 4.

5.1 First Order Stochastic Dominance Analysis

We first our analysis for the performance of the daily returns of the Chinese and USA stock markets for risk averters in the full sample period. To minimize the Type II error and to avoid SD (Leshno and Levy, 2002), we use a 5 per cent cut-off point ⁴ to the proportion of the values of the test statistic in (3.2) for SSE and S&P to draw statistical inferences. Using the 5 per cent cut-off point, if S&P dominates SSE, one should find at least 5 per cent of T_j^A to be significantly negative and no portion of T_j^A to be significantly positive. The reverse holds if SSE dominates S&P. From Table 2, we find that 34.92 per cent (27.92 per cent) of T_1^A is significantly positive (negative) for the entire period. Thus, the results lead us reject the hypothesis that S&P stochastically dominates SSE or vice versa at the first order in the sense of ASD.

Together with the plot of the modified ASD test for risk averters exhibited in Figure 2, the results from Table 2 show that T_1^A is significantly negative in the downside risk and significantly positive in the upside profit, inferring that investors with increasing utility functions will prefer to invest in S&P when facing with the downside risk, whereas they

⁴We note that Leshno and Levy (2002) use an example of 1 per cent to state the problem of almost SD. We choose a more conservative 5 per cent cut-off point to avoid the problem of almost SD. The conclusion drawn in our paper holds if one uses any less conservative cut-off point, say 1 per cent.

will prefer to invest in SSE in anticipation of the upside profit. This could also infer that investors prefer to invest in USA stock market in the bear markets and prefer to invest in Chinese stock market in the bull runs. Table 3 shows that 27.92 per cent (34.92 per cent) of T_1^D is significantly positive (negative), leading us reject the hypothesis that S&P stochastically dominates SSE or vice versa in the sense of FDSD. Together with the plot of the modified DSD test for risk seekers displayed in Figure 3, the results reveal that T_1^D is significantly positive at the downside risk and significantly negative at the upside profit. This result draws the same inference as the result of T_1^A that S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits. These results do not reject the market efficiency hypothesis. In order to explore whether the market is efficient and investors are rational, we need to examine the higher-order SD.

5.2 Higher-Order Stochastic Dominance

We now turn to examine whether there is any higher-order stochastic dominance relationship between the Chinese and USA stock markets. From Table 2, we observe that 89.57 per cent of the second-order modified ASD statistic T_2^A is significantly negative and none of it is significantly positive at the 5 per cent bootstrap simulated critical level. Similarly, from Table 2, we find that 98.98 per cent of the third-order modified ASD statistic T_3^A is significantly negative and none of it is significantly positive at the 5 per cent bootstrap simulated critical level. Hence, we conclude that there is a dominance of S&P over SSE in terms of both SASD and TASD at the 5 per cent significant level, inferring that second- and third-order risk averters prefer to invest in the USA stock market rather than the Chinese stock market. In addition, we also apply the testing procedure by using $\max_x |T_j^A(x)|$. The inference drawn from this approach leads to the same conclusion.

From the results discussed in the above, one may follow the suggestion from Falk and Levy (1989) and others and conclude that the markets are not efficient and investors are

not rational. In order to check whether the market is efficient and investors are rational, in this paper we suggest studying the preference of risk seekers.

5.3 Preference of Risk Seekers

In order to study the preferences of risk seekers between the Chinese and USA stock markets, we adopt the DSD theory and employ the DSD statistic, T_j^D , for risk seekers as stated in (3.5) for $j = 2$ and 3 to conduct the analysis. From Table 3, we find that 96.42 per cent of T_2^D is significantly negative and no portion of T_2^D is significantly positive at the 5 per cent significant level for the whole sample period. This implies that SSE stochastically dominates S&P in the sense of SDSD, and thus we conclude that second-order risk seekers prefer SSE to S&P for the whole sample period. Similarly, from Table 3, we find that 98.96 per cent of T_3^D is significantly negative and no portion of T_3^D is significantly positive at the 5 per cent level for the whole sample period. This implies that SSE stochastically dominates S&P in the sense of TDSD, and thus third-order risk seekers prefer SSE to S&P for the whole sample period.

Different from the conclusion drawn in the ASD test in which one could conclude that risk averters prefer to invest in the USA stock market rather than the Chinese stock market, our DSD analysis reveals the reverse preference for risk seekers that they are attracted to the Chinese stock market to maximize their expected utilities. Is there any inference on market efficiency and rationality from our findings? We will discuss the issue in next section.

5.4 Robustness Analysis in the Sub-Periods

We turn to investigate the preferences for risk averters and risk seekers in the Chinese and USA stock markets in each of the sub-periods. We first discuss their relationship in the sense of the first-order stochastic dominance.

Table 2 indicates that 36.9 per cent, 38.8 per cent, 30.3 per cent, 0 per cent, and 15.9

per cent of T_1^A are significantly positive in the first, second, third, fourth, and fifth sub-periods, respectively, and all are occurred in the positive domain. On the other hand, 31.6 per cent, 4.0 per cent, 23.8 per cent, 25.3 per cent, and 0 per cent of T_1^A are significantly negative for the first, second, third, fourth, and fifth sub-periods, respectively, and all are occurred in the negative domain. Since the analysis of the FSDS is the same as that of the FASD, we skip the discussion of the FSDS analysis. From our finding, we notice that only the first and the third sub-periods (both are in the bull runs) are similar to the ASD results for the entire period that the results lead us to reject the hypothesis that S&P stochastically dominates SSE or vice versa in the sense of FSD. Thus, for the first and third sub-periods, the conclusion drawn from our FSD analysis is the same as that drawn for the entire period that there is no FSD between S&P and SSE but S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits.

For the second sub-period from March 9, 2000 to October 7, 2002, the period after the burst of internet bubble, investors may expect that the stock market in the USA does not perform well and the Chinese stock market will be doing better than the USA stock market. Our FSD analysis supports this argument. From Table 2, we find that 38.8 per cent of T_1^A are significantly positive in the positive domain whereas 4.0 per cent of T_1^A are significantly negative in the negative domain. One may infer that, same as those obtained in the first and third sub-periods, there is no FSD between S&P and SSE but S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits. However, if one uses a 5 per cent cut-off point to exclude the possibility of almost SD (Leshno and Levy, 2002), one could claim that SSE stochastically dominates S&P in the sense of FSD.

From FSD analysis in the period after the burst of the sub-prime crisis from October 16, 2007 to November 4, 2008, one may recommend investors to invest in the Chinese stock market but not to invest in the USA stock market when the USA is in a financial

crisis. However, this may not be true. We turn to conduct our FSD analysis to these market during the period after the burst of the sub-prime crisis support this argument. From Table 2, we notice that no T_1^A is significantly positive whereas it possesses 25.3 per cent to be significantly negative, inferring that, surprisingly, the S&P stochastically dominates SSE in the sense of FSD and thus, investors should invest in the USA stock market rather than the Chinese stock market during the sub-prime crisis.

Furthermore, one may believe to invest in the USA stock market is better than the Chinese stock market since the investment is better even during the sub-prime crisis. Our SD analysis finds that this may not be true either. We make such inference from our SD analysis in the period during recovery after the burst of sub-prime crisis from November 4, 2008 to May 19, 2011. Table 2 exhibits that 15.9 per cent of T_1^A are significantly positive whereas none of it is significantly negative. One could conclude that SSE stochastically dominates the S&P in the sense of FSD and thus, investors should invest in the Chinese stock market instead of investing in the USA stock market in the bull run during the recovery of the sub-prime crisis.

However, the situation is not that simple because though we find that SSE stochastically dominates the S&P in the sense of FASD, our findings indicate that SSE does not stochastically dominate the S&P in the sense of both SASD and TASD. One may believe that this result contradicts the hierarchical property for ASD that FASD implies SASD which, in turn, implies TASD and thus it is recommended that only the lowest dominance order of ASD is reported. However, our findings show that FASD does not imply SASD or TASD! We note that there is no contradiction. The hierarchical property does, in fact, hold for both ASD and DSD mathematically but this does not imply that the hierarchical property must hold statistically. Our finding in the fifth subperiod shows that it is possible that FASD does not imply SASD or TASD and thus in this paper we suggest that academics and practitioners report FSD, SSD, and TSD, in case, say, X stochastically

dominates Y FASD, but not SASD and TASD. We suggest to call it X stochastically dominates Y *marginally* in the sense of FASD.

6 Inference from our Findings

Without identifying any risk index or any specific model, the SD rules can be used to determine whether there is any opportunity for arbitrage and whether the markets are efficient and investors are rational. We begin by examining arbitrage opportunities and then turn to the questions of market efficiency and investor rationality.

6.1 Inference on Arbitrage Opportunity

Jarrow (1986) and Falk and Levy (1989) claim that if FSD (FASD or FDSD) exists, under certain conditions, arbitrage opportunities also exist, and investors will increase their wealth and expected utilities if they shift from holding the dominated asset to the dominant one. However, Wong, et al. (2008) have shown that if FSD exists statistically, arbitrage opportunities may not exist, but investors can increase their expected wealth as well as their expected utilities if they shift from holding the dominated asset to the dominant one.

To check whether there is any arbitrage opportunity, one can apply the FSD analysis to the assets for comparison. In our analysis discussed in Section 5.1, although we find that S&P FSD dominates SSE in the downside returns while SSE FSD dominates S&P in the upside returns, S&P does not FSD dominate SSE over the entire distribution and vice versa. This implies that there is no arbitrage opportunity in the USA and Chinese stock markets over the entire period studied in this paper.

However, in Section 5.5, we find that (1) in the second sub-period from March 9, 2000 to October 7, 2002, the period after the burst of internet bubble, if one adopts a 5 per cent cut-off point to exclude the possibility of almost SD, one could claim that SSE

stochastically dominates S&P in the sense of FSD; (2) in the period after the burst of the sub-prime crisis from October 16, 2007 to November 4, 2008, the S&P stochastically dominates SSE in the sense of FSD; and (3) in the period during recovery from the sub-prime crisis from November 4, 2008 to May 19, 2011, one could conclude that SSE stochastically dominates the S&P marginally in the sense of FSD.

Could these findings infer that there are arbitrage opportunities in the USA and Chinese stock markets and these markets are not efficient? We will say, yes, there could exist some arbitrage opportunities in the USA and Chinese stock markets in some short periods and we will discuss the issue of market efficiency in the next subsection.

6.2 Inference on Market Efficiency and Rationality

In Section 5.5, in some short periods SSE stochastically dominates S&P in the sense of FSD, whereas in some other short periods, the first order dominance relationship reverses. Indeed, these results do infer that there are some arbitrage opportunities in some short periods of time. However, since there is no FSD for a long period in these markets, we will say that our FSD findings do not reject that markets are efficient and investors are rational.

However, in order to further explore whether the market is efficient and investors are rational, we need to examine the higher order of SD as studied in Section 5.2. If no SASD is found in the market containing X and Y, this suggests that risk-averse investors are indifferent between X and Y, so they will not switch X to Y, or vice-versa, to increase their expected utility. In this situation, we claim that the market is rational and efficient. Similarly, if no TASD is found in the market containing X and Y, this implies that risk-averse investors with DARA are indifferent between X and Y. In this situation, we claim that the market is both rational and efficient.

Nonetheless, Falk and Levy (1989) claim that, given two assets, X and Y, if by switch-

ing from X to Y (or by selling X short and holding Y long), an investor can increase expected utility, the market is inefficient. SSD does not imply any arbitrage opportunities, but it does imply the preference of one asset over another by risk-averse investors. For example, as we found in Section 5.2, S&P stochastically dominates SSE in the sense of both SASD and TASD. Thus, one may not make an expected profit by switching from SSE to S&P, but switching would allow risk-averse investors to increase their expected utility. In this situation, should we claim that the USA and Chinese stock markets are inefficient and investors are irrational?

This claim could be made if one believes that the markets only contain risk-averse investors. However, it is well known that the market could have other types of investors (see, for example, Friedman and Savage, 1948, Markowitz, 1952, Thaler and Johnson, 1990, Broll et al., 2010, and Egozcue et al., 2011 for more discussion). Under the assumption that the markets could contain more than one type of investors, such as risk averters as well as risk seekers, one could find that one asset dominates another asset by ASD but is dominated by that asset by DSD. These are exactly the findings we obtained in this paper: S&P stochastically dominates SSE in the sense of both SASD and TASD, while SSE stochastically dominate S&P strictly in the sense of SDSD and TDSO.

Thus, risk averters could prefer to invest in the USA stock market rather than the Chinese stock market, while risk seekers prefer to invest in the Chinese stock market rather than the USA market. Then, in equilibrium, the number of trades that risk averters, who go long in the USA market and/or short sell the Chinese stock market, would match the number of trades that risk seekers, who go long in the Chinese stock market and/or short sell the USA market. In this situation, there is no pressure to push up or down the indices in the USA and Chinese stock markets when both risk averters and risk seekers can attain what they seek. Under these conditions, we argue that the market remains efficient and investors are rational.

At last, we note that, conceptually, market rationality within the SD framework is not different from the conventional concepts captured by some of the rational asset pricing models, such as the CAPM. The only difference is that the latter approach defines an abnormal return as an excess return adjusted to some specific risk measure, while SD market rationality tests employ the whole distribution of returns. Given the imprecise knowledge of the best model, the SD approach with fewer restrictions on both investors' preferences and return distributions seems to help us understand the markets better.

7 Concluding Remarks

There are two basic approaches to the problem of portfolio selection under uncertainty. One approach is based on the concept of utility theory (see, for example, Markowitz (1952a), Post and Levy (2005), and the references therein for more information). Several SD test statistics⁵ have been developed by using this approach. This approach offers a mathematically rigorous treatment for portfolio selection. The other is the mean-risk (MR) analysis.⁶ In this approach, the portfolio choice is made with respect to two measures: the expected portfolio mean return and portfolio risk. A portfolio is preferred if it has higher expected return and smaller risk. There are convenient computational recipes and geometric interpretations of the trade-off between the two measures. A disadvantage of the latter approach is that it is derived by assuming the von Neumann-Morgenstern quadratic utility function and that returns are normally distributed (Feldstein, 1969). Thus, it cannot capture the richness of the former. In addition, the SD test statistics are superior to the MR test statistics because the conclusions drawn by these SD test statistics between the assets being examined could be used by investors to compare their expected utility on these assets since they do not require investors to possess a quadratic

⁵see, e.g. Schechtman, et al. (2008) and the references therein for more information.

⁶See, for example, Leung and Wong (2008) and Bai, et al. (2009, 2012) and the references therein for more discussion.

utility function nor any form of the distribution for the assets being analyzed.

We note that Post and Versijp (2007) have developed a new SD test for multiple comparisons. Recently, Egozcue and Wong (2010) present a general theory and a unifying framework for determining the second-order SD efficient set. Further research could include extending the DD tests to determine the SD efficient set statistically. In addition, Lam, et al. (2010, 2012) develop a pseudo-Bayesian model to explain some anomalies such as the short-run underreaction and long-run overreaction. Further research could also apply the DD tests to explore the short-run underreaction and long-run overreaction phenomena. In addition, the theory of SD has been widely used to explain well-known economic and financial phenomena or financial anomalies and used to test for market efficiency (Lean, et al., 2010; Chan, et al., 2012). It could also be used to assist investors for decision making.

Appendix

Proof of Theorem 3.3:

Without loss of generality, we assume $N_f = N_g = N$ in the proof. According to empirical process theorem⁷, for any continuous distribution function $F(x)$, we have

$$\sqrt{N}(F_N(x) - F(x)) \longrightarrow B(F(x)),$$

where F_N is the empirical distribution of N iid observations drawn from F and $B(\cdot)$ is the standardized Brownian bridge on the interval $[0, 1]$.

Recall that

$$\hat{F}_j^D(x) = \frac{1}{N(j-1)!} \sum_{i=1}^N (f_i - x)_+^{j-1} = \frac{1}{(j-1)!} \int_x^b (t-x)^{j-1} dF_N(t) \text{ if } a \leq x \leq b.$$

Applying the empirical process theorem, for $j \geq 1$, we have

$$\sqrt{N}(\hat{F}_j^D(x) - F_j^D(x)) \rightarrow \frac{1}{(j-1)!} \int_x^b (t-x)^{j-1} dB(F(t)) \text{ if } a \leq x \leq b.$$

Because $F_j^D(x) = G_j^D(x)$ under null hypothesis, and $\hat{F}_j^D(x)$ and $\hat{G}_j^D(x)$ are independent, we get

$$\sqrt{N}(\hat{F}_j^D(x) - \hat{G}_j^D(x)) \rightarrow \frac{\sqrt{2}}{(j-1)!} \int_x^b (t-x)^{j-1} dB(F(t)) \text{ if } a \leq x \leq b.$$

By the law of large numbers, with probability 1, one could easily find that

$$\begin{cases} N\hat{V}_{FG_j}^D \rightarrow 0 \\ N\hat{V}_{F_j}^D \rightarrow \frac{1}{((j-1)!)^2} \int_a^x (t-x)^{2j-2} dF(t) - F_j^D(x)^2, \\ N\hat{V}_{G_j}^D \rightarrow \frac{1}{((j-1)!)^2} \int_a^x (t-x)^{2j-2} dG(t) - G_j^D(x)^2. \end{cases}$$

Thereafter, under the null hypothesis, we get

$$\begin{aligned} N(\hat{V}_{F_j}^D + \hat{V}_{G_j}^D) &\rightarrow \frac{2}{((j-1)!)^2} \int_x^b (t-x)^{2j-2} dF(t) - F_j^D(x)^2 \\ &\stackrel{j \geq 1}{=} \frac{2}{((j-2)!)^2} \int_x^b \int_x^b (t-x)^{j-2} (s-x)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds. \end{aligned}$$

⁷Readers may refer to Donsker (1952) and Wolfowitz (1954) for more information about empirical process theorem.

Using integration by parts, when $j > 1$, we have

$$\frac{\sqrt{2}}{(j-1)!} \int_x^b (t-x)^{j-1} dB(F(t)) = -\frac{\sqrt{2}}{(j-2)!} \int_x^b (t-x)^{j-2} B(F(t)) dt.$$

Hence, when $j > 1$, we obtain

$$T_j^D(x) \rightarrow -\frac{\int_x^b (t-x)^{j-2} B(F(t)) dt}{\sqrt{\int_x^b \int_x^b (t-x)^{j-2} (s-x)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds}} \quad \text{if } a \leq x \leq b.$$

From this expression, we know that the limiting process of T_j is Gaussian with mean 0, variance 1, and covariance function

$$r_j^D(x, y) = \frac{\int_x^b \int_a^y (t-x)^{j-2} (s-y)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds}{\sqrt{\mathbb{V}_j^D(x) \mathbb{V}_j^D(y)}} \quad \text{if } a \leq x, y \leq b,$$

where $\mathbb{V}_j^D(x) = \int_a^x \int_a^x (t-x)^{j-2} (s-x)^{j-2} (F(t \wedge s) - F(t)F(s)) dt ds$. For the case $j = 1$, we have $T_1^D(x, y) = \frac{B(F(x))}{\sqrt{F(x) - F^2(x)}}$. The limiting process is also Gaussian with mean 0, variance 1, and covariance function $r_1^D(x, y) = \frac{F(x \wedge y) - F(x)F(y)}{\sqrt{F(x)F(y)(1-F(x))(1-F(y))}}$ if $a < x, y < b$.

□

Figure 1: Time series plot of SSE and S&P500 from 02/01/1996 to 19/05/2011.

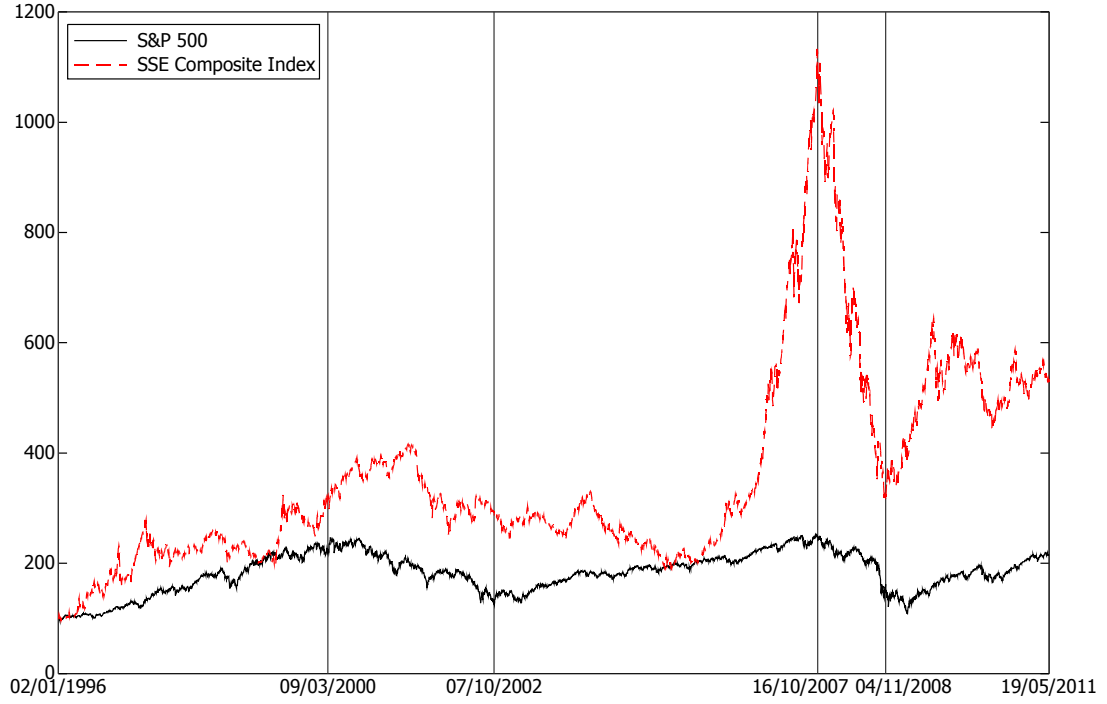
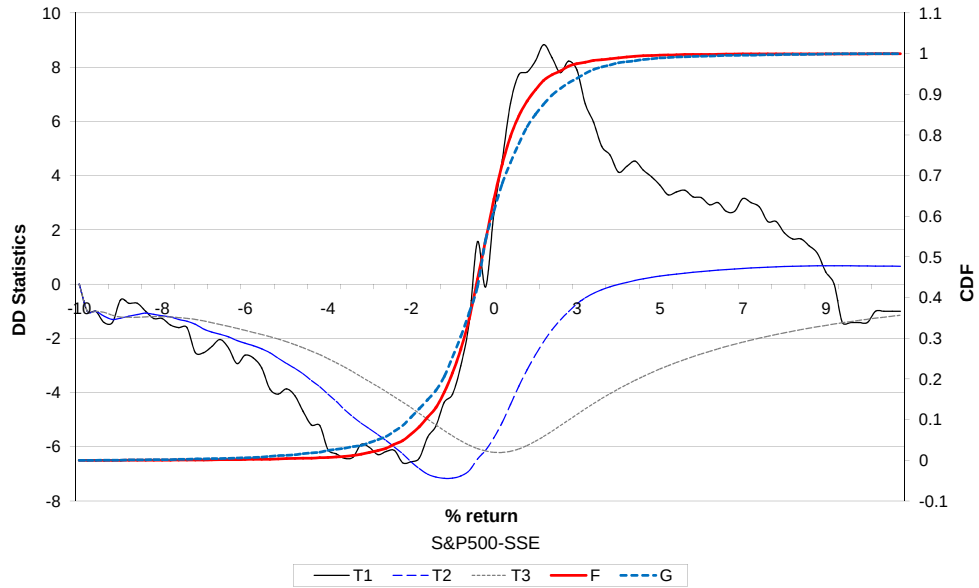


Figure 2: The modified ASD test statistics and the distribution functions F and G from 02/01/1996 to 19/05/2011



Note: T_j is the test statistic $T_j^A(x)$ defined in (3.2) for $j = 1, 2$, and 3 with $F = F_1^A = S\&P$ and $G = G_1^A = SSE$. F_1^A and G_1^A are defined in (2.1).

Table 2: Results of the modified ASD test statistic for the Risk Averters

02/01/1996-19/05/2011 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	34.92	27.92	0	89.57	0	98.98
Positive Domain(%)	34.92	0	0	0	0	0
Negative Domain(%)	0	27.92	0	89.57	0	98.98
$\max(T_j^A)$	9.10	7.36	0.68	7.16	1.00	6.20
02/01/1996-09/03/2000 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	36.91	31.59	0	91.29	0	99.32
Positive Domain(%)	36.91	0	0	0	0	0
Negative Domain(%)	0	31.59	0	91.29	0	99.32
$\max(T_j^A)$	8.35	5.94	0.54	6.35	1.00	5.46
09/03/2000-07/10/2002 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	38.82	4.02	63.98	0	20.21	0
Positive Domain(%)	38.82	0	63.98	0	20.21	0
Negative Domain(%)	0	4.02	0	0	0	0
$\max(T_j^A)$	4.83	3.59	3.52	1.00	2.14	1.00
07/10/2002-16/10/2007 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	30.34	23.81	0	87.25	0	98.98
Positive Domain(%)	30.34	0	0	0	0	0
Negative Domain(%)	0	23.81	0	87.25	0	98.98
$\max(T_j^A)$	8.86	7.48	1.04	6.84	0.74	5.42
16/10/2007-04/11/2008 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	0	25.28	0	78.86	0	61.13
Positive Domain(%)	0	0	0	0	0	0
Negative Domain(%)	0	25.28	0	78.86	0	61.13
$\max(T_j^A)$	2.79	4.39	1.41	3.49	1.39	2.74
04/11/2008-19/05/2011 $F = S\&P500, G = SSE$	FASD		SASD		TASD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	15.94	0	0	0	0	0
Positive Domain(%)	15.94	0	0	0	0	0
Negative Domain(%)	0	0	0	0	0	0
$\max(T_j^A)$	3.73	2.41	1.00	1.14	1.00	0.56

Note: This table summarizes the modified ASD test results for risk averters. The table reports the percentage of modified ASD statistics that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method discussed in Section 4. Tj is the test statistic $T_j^A(x)$ defined in (3.2) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. FASD, SASD, and TASD stand for first-, second-, and third-order ASD, respectively.

Table 3: Results of the modified DSD test statistic for the Risk Seekers

02/01/1996-19/05/2011 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	27.92	34.92	0	96.42	0	98.96
Positive Domain(%)	27.92	0	0	0	0	0
Negative Domain(%)	0	34.92	0	96.42	0	98.96
$\max(T_j^A)$	7.36	9.10	1.35	8.72	1.24	6.92
02/01/1996-09/03/2000 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	31.59	36.91	0	96.95	0	99.46
Positive Domain(%)	31.59	0	0	0	0	0
Negative Domain(%)	0	36.91	0	96.95	0	99.46
$\max(T_j^A)$	5.94	8.35	0.54	8.06	1.00	6.96
09/03/2000-07/10/2002 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	4.02	43.38	0	0	0	0
Positive Domain(%)	4.02	0	0	0	0	0
Negative Domain(%)	0	43.38	0	0	0	0
$\max(T_j^A)$	3.59	4.83	2.23	1.63	0.73	1.51
07/10/2002-16/10/2007 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	23.81	30.34	0	96.30	0	98.36
Positive Domain(%)	23.81	0	0	0	0	0
Negative Domain(%)	0	30.34	0	96.30	0	98.36
$\max(T_j^A)$	7.48	8.86	1.00	8.64	1.00	7.32
16/10/2007-04/11/2008 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	25.28	0	0	0	0	0
Positive Domain(%)	25.28	0	0	0	0	0
Negative Domain(%)	0	0	0	0	0	0
$\max(T_j^A)$	4.39	2.79	1.53	1.75	1.31	1.09
04/11/2008-19/05/2011 $F = S\&P500, G = SSE$	FDSD		SDSD		TDSD	
	$T1 > 0$	$T1 < 0$	$T2 > 0$	$T2 < 0$	$T3 > 0$	$T3 < 0$
Total(%)	0	15.94	0	0	0	0
Positive Domain(%)	0	0	0	0	0	0
Negative Domain(%)	0	15.94	0	0	0	0
$\max(T_j^A)$	2.41	3.73	0.81	2.01	0.74	1.00

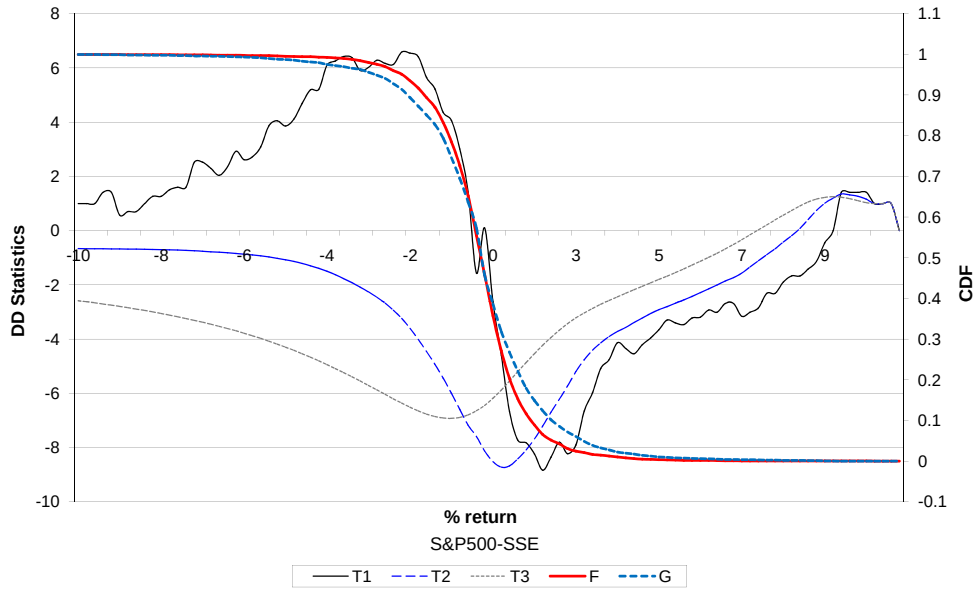
Note: This table summarizes the modified DSD test results for risk seekers. The table reports the percentage of modified DSD statistics that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method discussed in Section 4. Tj is the test statistic $T_j^D(x)$ defined in (3.5) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. FDSD, SDSD, and TDSD stand for first-, second-, and third-order DSD, respectively.

Table 4: Results of ASD and DSD Tests for the Risk Averters and Risk Seekers

01/1996-05/2011	G=S&P500	SSE
F=S&P500 SSE	SDSD	SASD
01/1996-03/2000	G=S&P500	SSE
F=S&P500 SSE	SDSD	SASD
03/2000-10/2002	G=S&P500	SSE
F=S&P500 SSE		FASD/FDSD#
10/2002-10/2007	G=S&P500	SSE
F=S&P500 SSE	SDSD	SASD
10/2007-11/2008	G=S&P500	SSE
F=S&P500 SSE		FASD/FDSD*
11/2008-05/2011	G=S&P500	SSE
F=S&P500 SSE	FASD*/FDSD*	

Note: Here all the “SD” results are “F” (in the first column) SD “G” (in the first row). * means SD marginally and # means SD when taking the concept of almost SD, Readers may refer to the text for more details.

Figure 3: The modified DSD test statistics and the DSD integrals F_1^D and G_1^D from 02/01/1996 to 19/05/2011



Note: T_j is the test statistic $T_j^D(x)$ defined in (3.5) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. F_1^D and G_1^D are defined in (2.1).

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