

SPECTRAL STUDY OF VOLCANIC MICRO-TREMORS (1)

PROPAGATION OF THE MICRO-TREMORS OF THE 1ST KIND OBSERVED AT VOLCANO ASO

By

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Abstract

Sassa classified the volcanic micro-tremors of Volcano Aso into four kinds. The tremor of the 1st kind is expected to be originated near or at the 1st crater, but its properties of propagation are not completely determined. By analyzing the records obtained through tripartite system into Fourier components and taking the correlation between every pair of them, the properties of propagation are examined. It is resulted that the volcanic micro-tremors of the 1st kind originate near the boundary between the 1st and 2nd crater and travels with the phase velocity of about 1,200 m/sec at 1~2 cps.

1. Introduction

Many studies of the volcanic micro-tremors with the seismometric methods have been published by various investigators. Since the phase of the volcanic micro-tremors cannot always be identified clearly as in the case of the earthquake waves, it is not feasible to make use of the initial arrivals on investigating the propagation properties of the micro-tremors. Furthermore it occurs numerously in a short period. Then we need the special methods for its data processes. Analyzing only the particular phases may contribute to large error because of small sampling. It is desired that we use the statistical methods such as picking up the arbitrary portion of the micro-tremors and obtaining the informations contained in it.

Under the same idea, Aki *et al.* dealt with microseisms (Aki, 1957) and near-earthquakes (Aki *et al.*, 1958; 1959) by resolving wave trains observed at several points into Fourier components and computing the correlation between waves of the same components. In the sense stated above, we are interesting to apply this method for studying in the volcanic micro-tremors.

2. Observation

Last of August, 1961, we observed the volcanic micro-tremors with a

tripartite net near the Hondo observational station of Kyoto University that is about 1 km westward from the Naka-dake crater, Volcano Aso. Each leg of the tripartite net was limited to about 100 m due to topography. The observing site and the distribution of the seismographs are shown in Figure 1. The seismographs we used have the horizontal pendulum of moving coil type. Its natural period is 4 sec and its sensitivity is 2.9 volt/kine. The output of the seismograph was amplified in voltage and recorded simultaneously in three channels magnetic data recorder. The characteristic of the amplifier is shown in Figure 2.

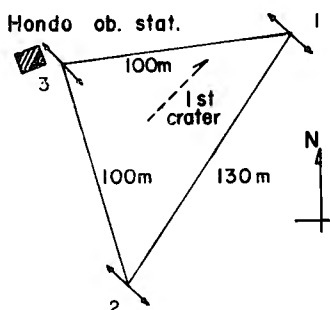


Fig. 1. Map of tripartite site; small arrows represent the direction of vibration of seismographs.

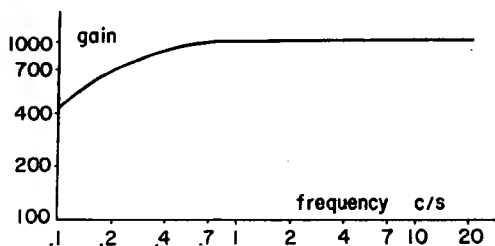


Fig. 2. Response curve of amplifier.

We observed for about 10 days and in the night because of the artificial ground noise in the daytime. Besides we were restricted to a few days since we selected the fine and calm days for observation. The magnetic data recorder can record continuously for 30 minutes. So we recorded freely under good conditions, that is, during the periods that the micro-tremors occur frequently. We set the another seismograph recording on the smoked paper for monitoring. An example of records was reproduced on the electro-magnetic

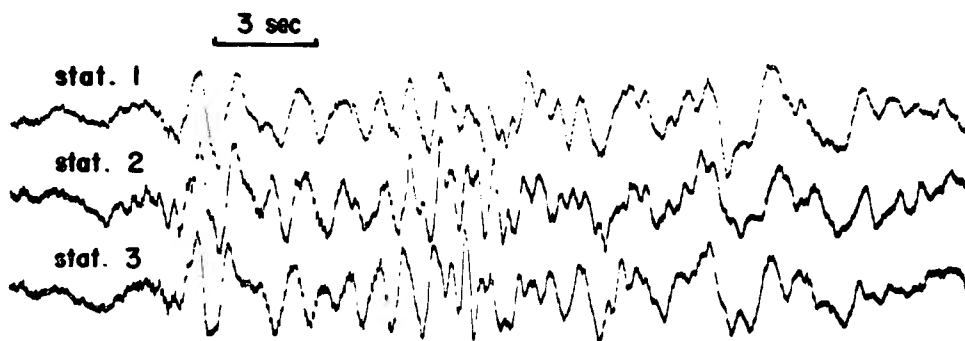


Fig. 3. An example of the records reproduced from data recorder.

oscillograph and is shown in Figure 3.

3. Method of Analysis

Analysis is divided in following four processes.

(1) Wave trains stored in the magnetic data recorder are resolved into Fourier components.

(2) For a pair of seismographs of different sites, the cross-correlation coefficient between the corresponding Fourier components of same frequency is calculated and the correlation coefficient versus frequency curve (ρ - f curve) is obtained.

(3) We gain the apparent velocity with which waves propagate from a observing point to another.

(4) Then the phase velocity and the direction of propagation of the micro-tremors are determined from three pairs of the apparent velocities by means of the tripartite net observation.

Let us consider two observing points A , B of a tripartite system and the waves traveling from A to B . We resolve the waves into Fourier components and remark the component wave with frequency f . On the point A , if it is crest at $t=0$, its phase is $2\pi ft$ at an arbitrary time $t=t$. At the same time, the phase on the point B is $2\pi ft + \varphi$, where φ is the phase difference between the point A and the point B .

The cross-correlation coefficient ρ_{AB} of the component waves with frequency f and the phase difference φ between A and B are

$$\rho_{AB} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \cos(2\pi ft) \cdot \cos(2\pi ft + \varphi) dt = \cos \varphi \quad (1)$$

$$\varphi = 2\pi f \frac{r}{V}$$

where r is the distance between A and B , V the velocity of the wave propagating from A to B . If V is independent of frequency, r/V is constant c and (1) becomes

$$\rho_{AB} = \cos\left(\frac{r}{V} 2\pi f\right) = \cos(2\pi cf) = \rho_{AB}(f) \quad (2)$$

In general, since the direction of leg AB is not coincident with the directions of traveling wave, the velocity V obtained is the apparent velocity V_{AB} along the leg AB .

On observing with a tripartite system, (2) becomes

$$\rho_{ij} = \cos\left(2\pi f \cdot \frac{r_{ij}}{V_{ij}}\right) \quad (i, j = 1, 2, 3 \quad i \neq j)$$

where the subscripts i, j are referred to the three points of the tripartite net. In the case of plane wave, if three apparent velocities V_{12} , V_{23} , and V_{31} are drawn along each leg from an origin in vector, the heads of three vectors are on a line. Therefore the line perpendicular to this is the direction of wave propagation and the distance from the origin to this line represents the velocity of plane wave.

If the waves observed are composed of two or more than two kinds of wave trains, which have different propagation characters each other, we can make use of this method. For the complicated wave train as above mentioned, equation (2) is transformed into the following

$$\rho = \frac{\sum_n P_n(f)}{P(f)} \cos\left(2\pi f \frac{r}{V_n}\right)$$

where $P(f)$ and $P_n(f)$ represent the power of the composed wave train and the constituent wave train referred to the suffix n . If the frequency ranges of their $P_n(f)$ are not severely overlapping each other, it is possible to analyze such a composed wave and determine simultaneously the propagation characters of the constituent waves. In practice, we could treat only a wave train containing a few constituent waves.

We made a filter for carrying out the first process of analysis to resolve wave train into Fourier components. The filter is a tuned amplifier with negative feedback by RC twin T network and is covered with 15 central frequency components from 0.5 to 10 cps. This filter is two channels whose characteristics are the same and shown in Figure 4.

The component waves obtained through the filter from the magnetic data recorder are fed in two channels endless tape recorder again and calculated with the correlator of analog type. Block diagram in Figure 5 shows the processes of analysis. Photographs in Figure 6 show the analysers.

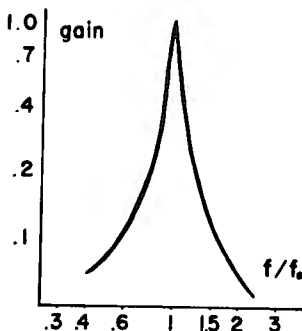


Fig. 4. Characteristic of filter.

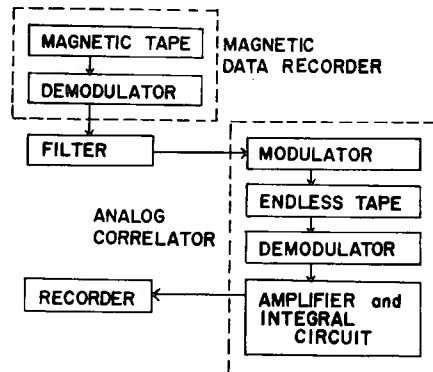


Fig. 5. Block diagram of analysis system.

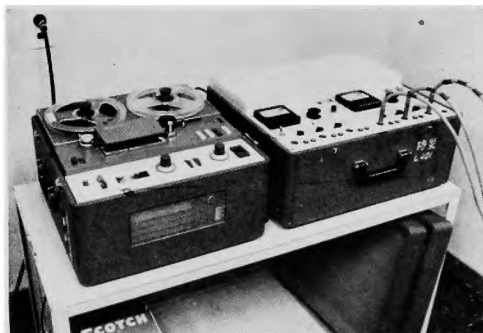


Fig. 6(a). Magnetic data recorder.

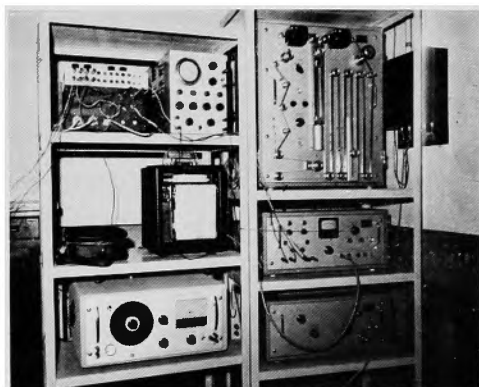


Fig. 6(b). Correlator on right and filter, recorder, and oscillator on left, up to down.

We must normalize the cross-correlation coefficient calculated with analyser because the amplitudes of the component waves affect the values of the cross-correlation coefficient. Let the functions stored in 2 channels A, B be denoted by $a(t)$ and $b(t)$ respectively. Then the auto-correlation coefficients ρ'_A, ρ'_B and the cross-correlation coefficient ρ'_{AB} are expressed as follows :

$$\rho'_A = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \{a(t)\}^2 dt, \quad \rho'_B = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \{b(t)\}^2 dt$$

$$\rho'_{AB} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a(t) \cdot b(t) dt$$

Then the normalized cross-correlation coefficient is calculated by following formula :

$$\rho_{AB} = \frac{\rho'_{AB}}{\sqrt{\rho'_A \cdot \rho'_B}}$$

4. Analysis

As stated in previous section, we need to plot the cross-correlation coefficient ρ_{ij} for the frequency f of the filter in order to determine the apparent velocity V_{ij} . The values obtained for eight examples of the wave trains as shown in Figure 3 were plotted and shown in Figure 7. According to Sassa (1935), the frequency of the volcanic micro-tremor of the 1st kind is 0.7~1.0 sec. So we need to remark the low frequency parts of $\rho-f$ curves. On these eight examples, all ρ_{12} and ρ_{31} cross the abscissa at nearly same point. But regard as ρ_{23} , since the wave travels nearly perpendicularly to this leg, that is, apparent velocity V_{23} is large, there is the case that the $\rho-f$ curve does not cross the abscissa, and then the determination of V_{23} is less accurate than

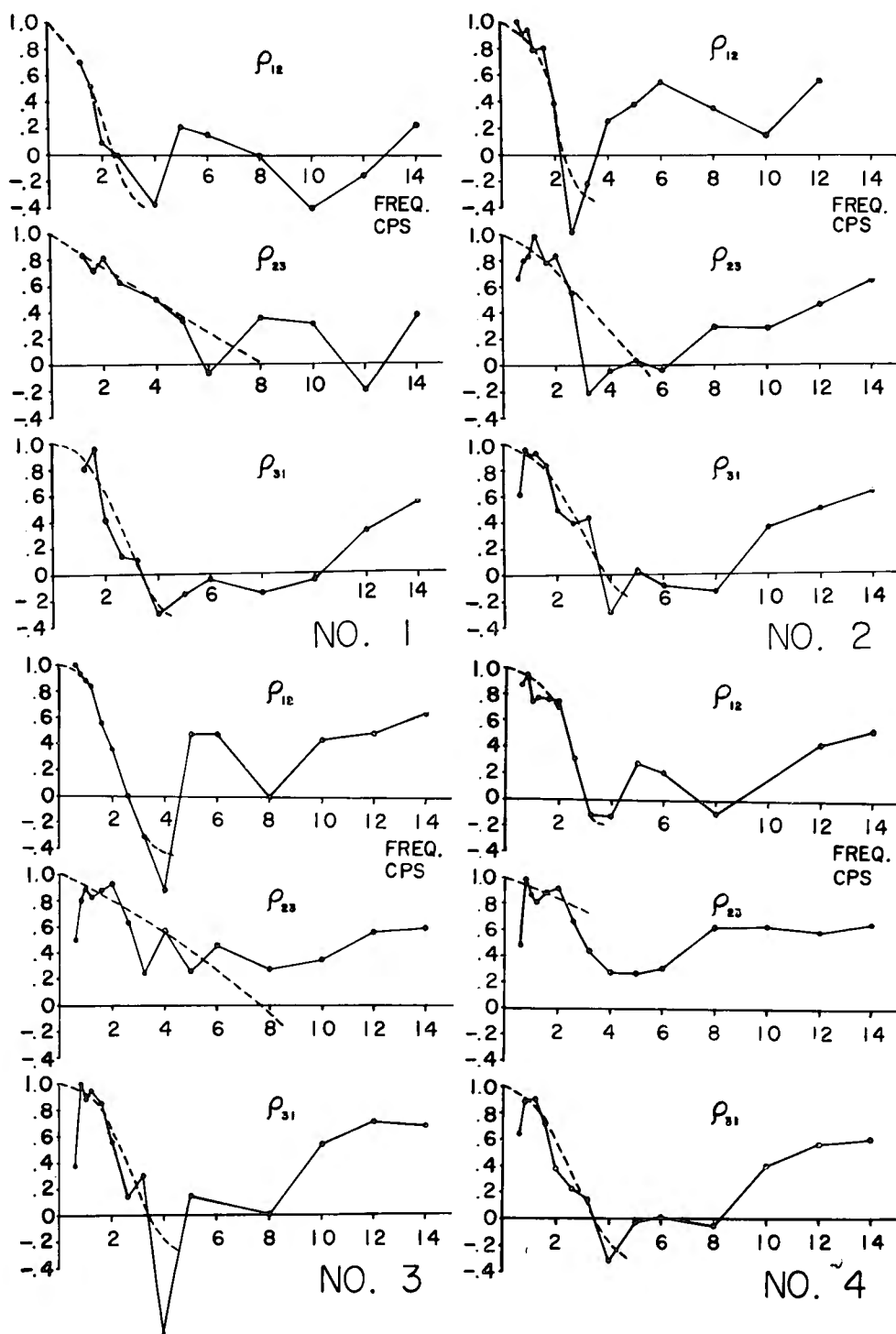


Fig. 7. Correlation curves.

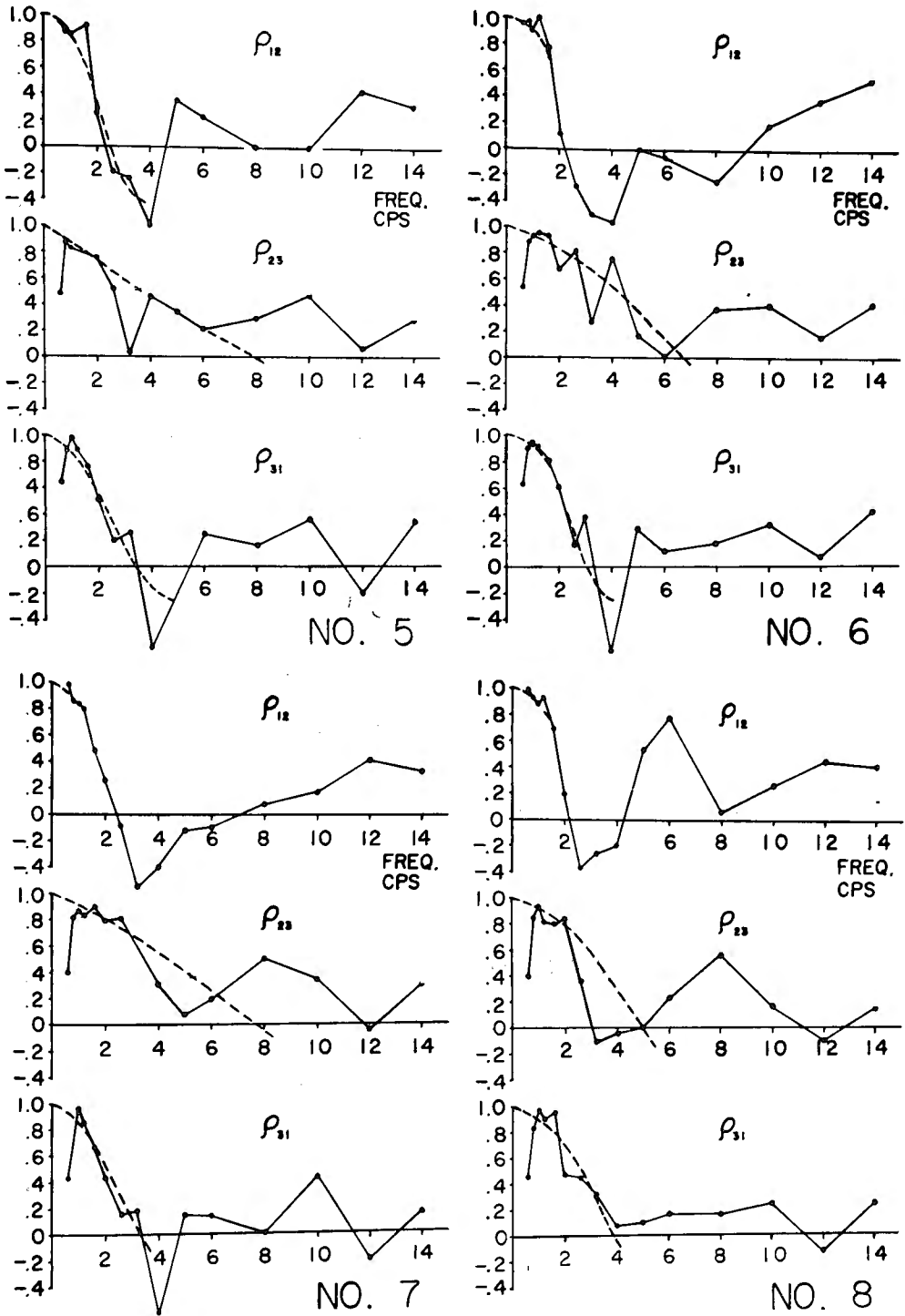


Fig. 7. (Continued).

that of V_{12} or V_{31} . As stated below, because it seems that the high frequency parts are mixed with the high frequency volcanic micro-tremors such as the 3rd and or 4th kind, we used mainly ρ_{12} and ρ_{31} which cross the abscissa at small frequency and determined the velocity and the direction of propagation.

From $f=f_0$ at $\rho_{ij}=0$, we calculated the apparent velocity V_{ij} by

$$V_{ij}=4f_0r_{ij}$$

and determined the velocity and the direction of propagation by drawing V_{ij} along the leg ij from the origin as shown in Figure 8. The results obtained are shown together in Figure 9. The average values of the velocity and the direction of traveling from origin are 1165 ± 49 m/sec and $S50.5^\circ W\pm 6.0^\circ$ respectively. The accuracy of determination of wave velocity is higher than by usual phase method, that is, equivalent to identification of each phase within the error $\approx 10^{-3}$ sec. We may conclude that the volcanic micro-tremors ap-

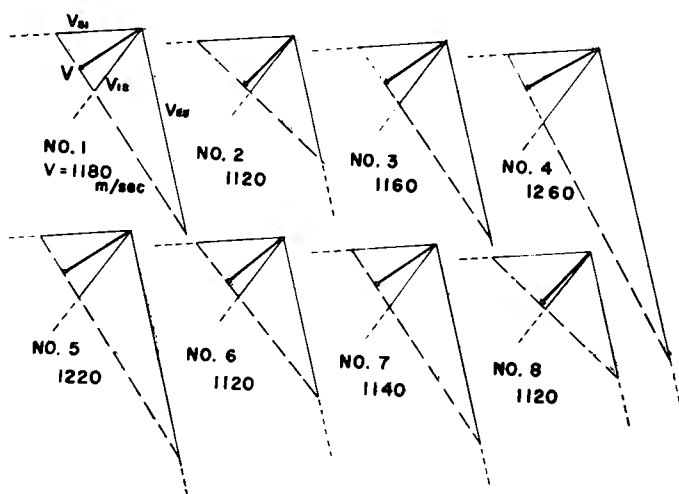


Fig. 8. Graphical determination of phase velocity vector.

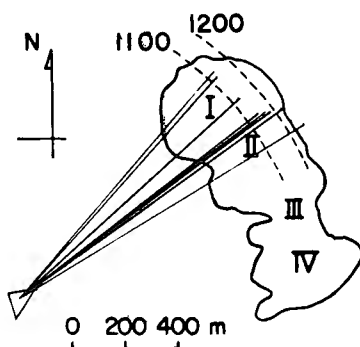


Fig. 9. Phase velocity vectors of the volcanic micro-tremors of the 1st kind.

proach from near the 1st and the 2nd crater of the Naka-dake, the Volcano Aso. The velocities are also almost equal to about 1,200 m/sec. This value is compatible with 1,000 m/sec of Sassa (1935) and 1,200 and 1,350 m/sec of Kamo (1962), in consideration of disagreement of observing points.

If the micro-tremor of this kind is a certain of surface wave of Love type (Sassa, 1935), the dispersive property must be considered. The velocity obtained above corresponds to the phase velocity of 1~2 cps. On estimating the correlation between two points we compute the cross-correlation function $\rho_{ij}(\tau)$ for time lag τ between two records and then can find the time lag τ_{max} , where $\rho_{ij}(\tau_{max})$ is the maximum value of $\rho_{ij}(\tau)$. We may consider that this τ_{max} implies the travel time between two points. These τ_{max} thus obtained for some frequency range were not constant as shown in Figure 10. This suggests that wave is dispersive. But the legs of tripartite net was too short and then the τ_{max} was very small. We could not obtain the values with sufficient accuracy.

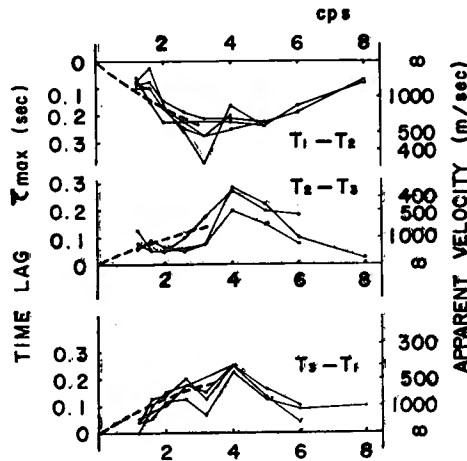


Fig. 10. Some examples of τ_{max} curves.

Seeing Figure 7, irregularities appear from about 4 cps. It suggests that different kinds of wave trains exist and that these are the volcanic micro-tremors of the 3rd and the 4th kind. We also cannot conclude about such facts like dispersive characters. But we may resolve these kind waves from the volcanic micro-tremors of the 1st kind if the legs of tripartite system are longer and the filtering is divided into finer and denser system.

5. Conclusion

By spectral analysis, we concluded that the volcanic micro-tremors of the 1st kind were dispersive, approached from near the 1st and the 2nd crater of

Naka-dake and had the phase velocity of about 1,200 m/sec at 1~2 cps. So the volcanic micro-tremors of the 1st kind, as Sassa (1935) pointed out, occur near the 1st crater and are the surface waves of Love type. Though the high frequency waves, corresponding to the volcanic micro-tremors of the 3rd and the 4th kind, might exist, we could not investigate these waves with this analysis at detail. But if the legs of tripartite net were enlarged more than 300 m and the intervals of central frequency of the filter became smaller, we could know the properties of these waves. On this study we need to discern the spectral types of tremors and intend to practise in near future. But it is shown that the method used here is effective to study in volcanic micro-tremors.

Acknowledgment

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