

Geometric methods in Asymptotic Analysis

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CONTACT GROUPOIDS AND PREQUANTIZATION

Pierre DAZORD

Université Claude Bernard - Lyon I

Introduction. The geometric prequantization of a general Poisson manifold (Γ_0, Λ_0) , as defined by Weinstein [16] and Xu, is an extension of Souriau's process [14] for symplectic manifolds. First, if possible, is associated to (Γ_0, Λ_0) a symplectic groupoid $(\hat{\Gamma}, \sigma)$ [2], which is a kind of nice desingularisation, and secondly the symplectic manifold is eventually prequantized [14], that is to say, one looks to the existence of a $\mathbb{S}^1$ -principal bundle $\pi : \Gamma \rightarrow \hat{\Gamma}$, with a connection λ , the curvature of which is $\pi^*\sigma$. If $\hat{\Gamma}$ is Hausdorff (which is a very special case), there exists a prequantization iff $[\sigma] \in H^2(\hat{\Gamma}, \mathbb{Z})$.

Actually, if (Γ_0, Λ_0) is prequantizable, (Γ, λ) has a nice structure : it is a contact groupoid, a notion which enlarges previous one due to Libermann [11] and Kerbrat and Souici [9].

This lecture is a survey of relations between contact groupoids and prequantization [4].

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1. CONTACT GROUPOIDS [4]

Definition 1.1 - A contact groupoid $(\Gamma, \mathcal{H})$ is a Lie groupoid endowed with a compatible contact structure.

• Γ is a Lie groupoid : Γ is a C^∞ (generally not Hausdorff) manifold, Γ_0 is a Hausdorff submanifold of Γ (the manifold of units), α and β two C^∞ surjective submersions of Γ on Γ_0 (α is the source and β the target) ; moreover on Γ is defined a C^∞ "inverse" map $i : x \rightarrow i(x) = x^{-1}$, and on $\Gamma_0 = \{(x, y) \mid \alpha(x) = \beta(y)\}$ a C^∞ product $m : (x, y) \rightarrow m(x, y) = x.y$ such that

- (i) $\forall x \in \Gamma \quad x.\alpha(x) = \beta(x).x = x$
- (ii) If $(x, y) \in \Gamma_2$ and $(y, z) \in \Gamma_2$, $(x.y, z)$ and $(x, y.z)$ are in Γ_2 and $(x.y).z = x.(y.z)$
- (iii) $\forall x \in \Gamma, \quad x^{-1}.x = \alpha(x) \quad x.x^{-1} = \beta(x).$

First examples of Lie groupoids

- (i) If Γ_0 is reduced to a point, Γ is a Lie group.
- (ii) If Γ_0 is any Hausdorff C^∞ manifold, $\Gamma_0 \times \Gamma_0$ is canonically endowed with a groupoid structure (the coarse groupoid of Γ_0) with the laws :

$$\begin{aligned} ((x, y)(y, z)) &= (x, z) \\ (x, y)^{-1} &= (y, x) \end{aligned}$$

and Γ_0 is identified to the diagonal of $\Gamma_0 \times \Gamma_0$.

- (iii) Any vector bundle $\pi : E \rightarrow \Gamma_0$ is a vector groupoid with addition in the fibers. In this case $\alpha = \beta = \pi$.

- (iv) If $\Gamma \xrightarrow[\beta]{\alpha} \Gamma_0$ is any Lie groupoid, the tangent groupoid of Γ , $T\Gamma$ is the Lie groupoid

$$T\Gamma \xrightarrow[T\beta]{T\alpha} T\Gamma_0 \text{ with the inverse law } X \rightarrow \Theta X = Ti(X) \text{ and the product law :}$$

$$Tm : (T\Gamma_2) \rightarrow T\Gamma \quad (X, Y) \rightarrow X \oplus Y = Tm(X, Y).$$

• Compatibility condition of Γ and $\mathcal{H}$

$(\Gamma, \mathcal{H})$ is a contact groupoid iff

- (i) $X \in \mathcal{H} \Rightarrow \Theta X \in \mathcal{H}$
- (ii) $(X, Y) \in (\mathcal{H} \times \mathcal{H}) \cap T\Gamma_2 \Rightarrow X \oplus Y \in \mathcal{H}$

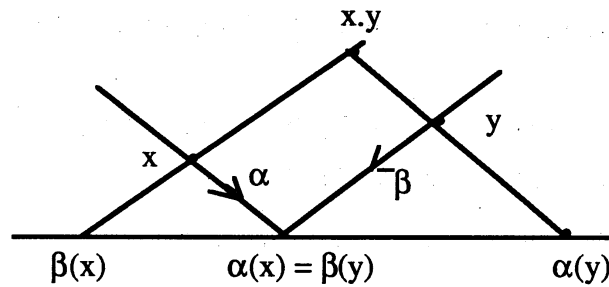
The first result is the following.

Theorem 1.1 [4] *If the dimension of Γ , $\dim \Gamma$, is strictly greater than the dimension of Γ_0, n ,*

- (i) $\dim \Gamma = 2n+1$
- (ii) Γ_0 is a Legendrian submanifold of $(\Gamma, \mathcal{H})$
- (iii) $\mathcal{H}_\alpha = \mathcal{H} \cap \text{Ker } T\alpha$ (resp. $\mathcal{H}_\beta = \mathcal{H} \cap \text{Ker } T\beta$) is right (resp. left) invariant.

The notion of left (or right) invariance is explicited in the next paragraph.

2. LIE ALGEBROID OF A CONTACT GROUPOID.



If $x \in \Gamma$ the left action by x , L_x is defined only from $\beta^{-1}(\alpha(x))$ into $\beta^{-1}(\beta(x))$:

$$L_x(y) = x.y$$

This leads to the notion of left invariance for objects in $\text{Ker } T\beta$, which makes clear theorem 1.1. In particular if $X \in \mathfrak{X}(\Gamma)$ Lie algebras of vector fields of Γ , X is left invariant iff $T\beta X \equiv 0$ and

$$L_x X_y = X_{xy},$$

which is equivalent to

$$X \in \mathfrak{X}^l(\Gamma) \text{ iff } 0 \oplus X = X$$

that is to say $X_{xy} = 0_x \oplus X_y$.

For left invariant vector fields we have the three nice properties

- (i) $\mathfrak{X}^l(\Gamma)$ is a sub Lie algebra of $\mathfrak{X}(\Gamma)$
- (ii) Even if Γ is not Hausdorff, it can be proved that for each x , $\beta^{-1}(x)$ is a Hausdorff submanifold so if $X \in \mathfrak{X}^l(\Gamma)$ it has a (local) flow.
- (iii) To $X \in \mathfrak{X}^l(\Gamma)$ is associated a unique vector field on Γ_0 , X_0 such that $T\alpha \circ X = X_0 \circ \alpha$ and X is complete iff X_0 is.

This leads to the notion of Lie algebroid due to Pradines [13] and which plays for Lie groupoid the same role than Lie algebras for Lie groups.

Definition 2.1. A Lie algebroid $E \rightarrow \Gamma_0$ is a C^∞ vector bundle over a C^∞ Hausdorff manifold together with a Lie algebra structure on the space $\mathfrak{A}(E)$ of sections of E and a vector bundle morphism, the anchor map ρ , from E to $T\Gamma_0$ such that, for each $s \in \mathfrak{A}(E)$ and $u \in C^\infty(\Gamma_0, \mathbb{R})$

$$(i) \quad \rho_0\{s_1, s_2\} = [\rho_0 s_1, \rho_0 s_2]$$

(ρ is a Lie algebra morphism from $\mathfrak{A}(E)$ to $\mathfrak{X}(\Gamma_0)$)

$$(ii) \quad \{s_1, u.s_2\} = u.\{s_1, s_2\} + (\mathfrak{L}_{\rho_0 s_1} u).s_2 \text{ where } \mathfrak{L} \text{ is the Lie derivative.}$$

Note that this implies that $\mathfrak{A}(E)$ is a class of Kirillov's algebras [10].

Going back to Γ , the Lie algebroid $\underline{\Gamma}$ of Γ can be described :

(i) As vector bundle $\underline{\Gamma}$ is the normal bundle of Γ_0 in Γ : $\underline{\Gamma} = T\Gamma|_{\Gamma_0}/T\Gamma_0$. So $\underline{\Gamma}$ is canonically isomorphic to $\text{Ker } T\beta|_{\Gamma_0} \rightarrow \Gamma_0$.

(ii) The anchor map is defined by the diagram :

$$\begin{array}{ccc} \underline{\Gamma} & \xrightarrow{\sim} & \text{Ker } T\beta|_{\Gamma_0} \\ \searrow \rho & & T\alpha \swarrow \\ & T\Gamma_0 & \end{array}$$

(iii) The Lie algebra structure of $\mathfrak{A}(\underline{\Gamma})$ comes from the canonical isomorphisms

$$\mathfrak{X}^l(\Gamma) \simeq \mathfrak{X}^l(\Gamma)|_{\Gamma_0} \simeq \mathfrak{A}(\underline{\Gamma}).$$

In the sequel we shall restrict ourselves to the special case of "oriented" contact groupoids, which is enough to our purpose. In this case $\mathfrak{H}$ is the Kernel of a contact form λ , which, more over, can be chosen such that :

$$\lambda(X_x \oplus Y_y) = f(x)\lambda(Y_y) + g(y)\lambda(X_x)$$

where f and g are morphisms of Γ into $\mathbb{R}_*$, multiplication group of non zero real numbers. Such a groupoid (Γ, λ) is called a *pfallian groupoid*, subordinate to $(\Gamma, \mathfrak{H})$.

Theorem 2.1. [4] If $(\Gamma, \mathfrak{H})$ is an oriented contact groupoid and $(\underline{\Gamma}, \lambda)$ a subordinate pfallian groupoid,

(i) There exists on Γ_0 a canonically defined Jacobi structure $(\Gamma_0, E_0, \Lambda_0)$ [8].

(ii) Γ is the Lie algebroid canonically defined by $(\Gamma_0, E_0, \Lambda_0)$: as vector bundle, $\Gamma = \mathbb{J}^1(\Gamma_0, \mathbb{R}) \simeq \mathbb{R} \times T^*\Gamma_0$; the anchor map ρ is defined by $\rho(u, \xi) = uE_0 + \Lambda_0^\# \xi$ and the bracket is the Kerbrat-Souici's bracket [9] :

$$\{(u, \xi), (v, \eta)\} = (\{u, v\} - \iota_{\Lambda_0}(\xi \cdot du) \wedge (\eta \cdot dv), u\mathfrak{L}_{E_0} \eta - v\mathfrak{L}_{E_0} \eta - \iota_{E_0}(\xi \wedge \eta) + [\xi, \eta]_p)$$

where $\{u, v\}$ is the Jacobi bracket on $C^\infty(\Gamma_0, \mathbb{R})$,

$$\{u, v\} = u\mathfrak{L}_{E_0} v - v\mathfrak{L}_{E_0} u + \iota_{\Lambda_0}(du \wedge dv)$$

$$\text{and where } [\xi, \eta]_p = \iota_{\Lambda_0^\# \xi} d\eta - \iota_{\Lambda_0^\# \eta} d\xi + d\iota_{\Lambda_0}(\xi \wedge \eta).$$

We have noted $\Lambda_0^\#$ the morphism $T^*\Gamma_0 \rightarrow T\Gamma_0$ given by $\iota(\Lambda_0^\# \xi)\eta \equiv \iota_{\Lambda_0}(\xi \wedge \eta)$.

Remark. If $(\Gamma_0, E_0, \Lambda_0)$ is a Jacobi manifold, generally speaking there exists only a "local" Lie algebroid in Van Est's sense [4]. We say that $(\Gamma_0, E_0, \Lambda_0)$ is *contact-integrable* if it is the units of a contact groupoid.

3. Lie groups of infinite dimension.

Before describing the relations between contact groupoids and prequantization, it is necessary to recall a notion introduced by Ehresmann [6] and which enlarges the usual notion of graph of a diffeomorphism. If $\Gamma \rightrightarrows \Gamma_0$ is a Lie groupoid, a *bisection* of Γ is a submanifold S such that $\alpha|_S$ and $\beta|_S$ are diffeomorphisms on Γ_0 . So a bisection of Γ is, equivalently, a section S^β of $\beta : \Gamma \rightarrow \Gamma_0$ such that $\alpha_0 S^\beta$ is a diffeomorphism ϕ_S^r of Γ_0 . If $\Gamma_0 \times \Gamma_0$ is the coarse groupoid, S is the graph of ϕ^{-1} . The set $\hat{G}_\Gamma$ of bisections is a group and $S \rightarrow \phi_S^r$ is an (anti) representation of $\hat{G}_\Gamma$ into $\text{Diffeo } \Gamma_0$, group of diffeomorphisms of Γ_0 . $\hat{G}_\Gamma$ has another (anti) representation in $\text{Diffeo } \Gamma$, given by $S \rightarrow \phi_S^r$

$$\phi_S^r(x) = x.S = x.S^\beta(\alpha(x)).$$

We note by G_Γ the subgroup of $\hat{G}_\Gamma$ of bisections S such that ϕ_S^r has compact support, and we endowed G_Γ with a diffeological structure in Souriau's sense [15] that is we say that a map f from an open subset U of some $\mathbb{R}^p$ in G_Γ is " C^∞ " if

- (i) $U \times \Gamma_0 \rightarrow \Gamma$ $(u, x_0) \rightarrow \phi_{f(u)}^r(x_0) = f(u)(x_0)$ is C^∞ .
- (ii) For each $u_0 \in U$ there exists a compact K in Γ_0 and a neighbourhood V of u_0 in U such that for each $u \in V$, the support of $\phi_{f(u)}^r$ is contained in K .

With this "diffeology", G_Γ is called the group of compactly controlled bisections.

If $E \rightarrow \Gamma_0$ is a Lie algebroid of anchor map ρ , we defined (mutatis mutandis) the Lie algebra $\mathfrak{A}_c(E)$ of compactly controlled sections s such that $\rho_0 s$ is a compactly supported vector field on Γ_0 .

If G is a subgroup of G_Γ we can define the tangent space to G in Γ_0 as the set of time derivative for $t=0$ of " C^∞ " maps $f: I \rightarrow G$, where I is a neighbourhood of 0 in $\mathbb{R}$, such that $f(0) = \Gamma_0$. It turns that $\underline{G}_\Gamma = T_{\Gamma_0} G_\Gamma$ is $\mathfrak{A}_c(\Gamma)$ and that for any subgroup $G \subset G_\Gamma$, $\underline{G} = T_{\Gamma_0} G$ is a vector subspace of $\mathfrak{A}_c(\Gamma)$. We say that a vector subspace V of $\mathfrak{A}_c(\Gamma)$ is full if $T_0 V = V$ where $T_0 V$ is the tangent space in 0. Moreover, for each $X \in \mathfrak{A}_c(\Gamma)$ we define $\exp X = \varphi_1^{X^\sharp}(\Gamma_0)$ where $\varphi_t^{X^\sharp}$ is the flow of the left invariant vector field $X^\sharp$ associated to X . $t \rightarrow \exp tX$ is a C^∞ map from $\mathbb{R}$ into G_Γ .

Definition 3.1. A subgroup G of G_Γ is an (infinite dimension) Lie group iff

- (i) $\underline{G} = T_{\Gamma_0} G$ is full. Then $\underline{G}$ is a subLie algebra of $\mathfrak{A}_c(\Gamma)$.
- (ii) For each $X \in \underline{G}$ $t \rightarrow \exp tX$ is a C^∞ map from $\mathbb{R}$ into G .

Remark. If Γ_0 is reduced to a point G_Γ is a Lie group (of finite dimension) and Yamabe's theorem [17] implies that (i) and (ii) are always satisfied.

Let $(\Gamma, \mathcal{H})$ be a contact groupoid and $G^\mathcal{H}$ the set of *Legendrian* bisections.

Theorem 3.1. [4] $G^\mathcal{H}$ is a Lie group.

In order to describe $\underline{G}^\mathcal{H}$, we assume that $(\Gamma, \mathcal{H})$ is oriented and we consider a subordinate pfaffian groupoid (Γ, λ) and the Jacobi structure $(\Gamma_0, E_0, \Lambda_0)$ on Γ_0 .

Then $j^1: C^\infty(\Gamma_0, \mathbb{R}) \rightarrow \mathfrak{A}_c(\Gamma) = \mathfrak{A}_c(j^1(\Gamma_0, \mathbb{R}))$ is an injective Lie algebra morphism and we note $C_c^\infty(\Gamma_0, \mathbb{R})$ the subLie algebra of $C^\infty(\Gamma_0, \mathbb{R})$ of u such that $j^1 u \in \mathfrak{A}_c(\Gamma)$ with the diffeology induced that is to say $f: U \rightarrow C_c^\infty(\Gamma_0, \mathbb{R})$ is " C^∞ " iff $(u, x_0) \rightarrow f_u(x_0)$ is C^∞ and $j^1 f_u$ compactly controlled.

Corollary. If $\mathcal{H}$ is oriented,

$$\underline{G}^\mathcal{H} = C_c^\infty(\Gamma_0, \mathbb{R}).$$

Remark. For an extension of these results if $\mathcal{H}$ is not oriented, cf. [4]. This extension solves the problem of integration of all Kirillov Lie algebras of rank one (cf. [8]).

4. PREQUANTIZATION

We restrict ourselves now to a special case of Jacobi manifolds, the Poisson ones in which $E_0 \equiv 0$. Then if (Γ_0, Λ_0) is contact integrable, the contact groupoid is always orientable and we can choose the contact form λ such that

$$\lambda(X_x \oplus Y_y) = \lambda(X_x) + \lambda(Y_y).$$

This peculiar situation has been studied by Libermann [11].

In this case the Reeb vector field E of (Γ, λ) is right and left invariant and so, E is complete. Then we can define a Lie groupoid morphism ψ in a trivial sense from the vector groupoid $\mathbb{R} \times \Gamma_0 \rightarrow \Gamma_0$ into Γ by mean of the E -flow, ϕ_t^E :

$$\begin{aligned} \psi : \mathbb{R} \times \Gamma_0 &\rightarrow \Gamma \\ \psi(t, x_0) &= \phi_t^E(x_0). \end{aligned}$$

ψ is an immersion and $\psi^* \lambda = dt$.

So if ψ is an embedding, λ restricted to $\mathcal{J}^E(\Gamma_0) = \psi(\mathbb{R} \times \Gamma_0)$ is a regular foliation, which in turns implies that all the maps $t \rightarrow \phi_t^E(x)$ have same period T independant of x (by convention $T = 0$ if $t \rightarrow \phi_t^E(x)$ is injective).

Then $\tilde{\Gamma} = \Gamma / \mathcal{J}^E(\Gamma_0)$ is a groupoid with a nice structure : it is a symplectic groupoid with the symplectic form σ induced by $d\lambda$. This means that, on $\tilde{\Gamma}$,

$$\sigma(X_x^1 \oplus Y_y^1, X_x^2 \oplus Y_y^2) = \sigma(X_x^1, X_x^2) + \sigma(Y_y^1, Y_y^2)$$

and $\Gamma \rightarrow \tilde{\Gamma}$ is a prequantization in Weinstein's sense.

We shall say that a Poisson manifold (Γ_0, Λ_0) is symplectic-integrable if it is the units of a symplectic groupoid $(\tilde{\Gamma}, \sigma)$. The Lie algebroid of $\tilde{\Gamma}$, $\tilde{\Gamma}$ is the so called Lie algebroid of the Poisson manifold (Γ_0, Λ_0) which, as fiber bundle, is $T^*\Gamma_0 \rightarrow \Gamma_0$, and for which the anchor map is $\Lambda_0^\#$ and the bracket $\{\xi, \eta\} = [\xi, \eta]_p$ (cf. supra). Then $\mathfrak{A}_c(\tilde{\Gamma}) = \Omega_c^1(\Gamma_0)$ the Lie algebra of 1-form ξ such that $\Lambda_0^\# \xi$ has a compact support. We note $Z\Omega_c^1(\Gamma_0)$ and $B\Omega_c^1(\Gamma_0)$ respectively the

subLie algebras of closed and exact one-forms, and $H_c^1(\Gamma_o, \mathbb{R})$ the quotient $Z\Omega_c^1(\Gamma_o, \mathbb{R})/B\Omega_c^1(\Gamma_o, \mathbb{R})$. Then the exact sequence of diffeological Lie algebras

$$0 \rightarrow B\Omega_c^1(\Gamma_o) \rightarrow Z\Omega_c^1(\Gamma_o) \rightarrow H_c^1(\Gamma_o, \mathbb{R}) \rightarrow 0$$

can be integrate in the exact sequence of Lie groups (in the sense of definition 3.1)

$$0 \rightarrow G^{\text{ex}} \rightarrow G_o^\sigma \xrightarrow{C} H_c^1(\Gamma_o, \mathbb{R})/D \rightarrow 0$$

where G_o^σ is the connected component of the Lie group of Lagrangian bisection and where G^{ex} is a sub Lie group the group of "hamiltonian isotopies" which is defined in the following way :

$S \in G^{\text{ex}}$ iff it exists a C^∞ map $t \rightarrow S_t$ from a neighbourhood I of 0 in $\mathbb{R}$ to G^σ such that

- (i) $S_o = \Gamma_o \quad S_1 = S$
- (ii) If $X_t \in X^\lambda(\Gamma)$ is the time dependant vector field defined by

$$\frac{d}{dt} \phi_{S_t}^r = X_t \circ \phi_{S_t}^r$$

then $\iota_{X_t} \sigma = -\alpha^* dH_t$ where $t \rightarrow H_t$ is a C^∞ map from I to $C_c^\infty(\Gamma_o, \mathbb{R})$.

C is defined by mean of an extended notion of Calabi's invariant and D is a discrete subgroup (in diffeological sense) of $H_c^1(\Gamma_o, \mathbb{R})$.

Remark. The above result is an extension of the one given in [3]. In fact there is no necessity of the assumptions of [3] to have it. The results extend Banyaga's one [1].

Assume now that (Γ_o, Λ_o) is prequantizable so it is contact and symplectic-integrable. Then we can integrate also the exact sequence

$$0 \longrightarrow \mathbb{R} \longrightarrow C_c^\infty(\Gamma_o, \mathbb{R}) \xrightarrow{d} B\Omega_c^1(\Gamma_o) \longrightarrow 0$$

in the exact sequence of Lie groups

$$0 \longrightarrow \mathbb{R}/T\mathbb{Z} \longrightarrow G_o^{\mathcal{H}} \longrightarrow G^{\text{ex}} \longrightarrow 0$$

where T (eventually 0) is the common period of the Reeb vector field of (Γ, λ) .

In fact, as usual, if $T = 0$ we can always by using a suitable $\mathbb{Z}$ -quotient of (Γ, Λ) assume that $T = 1$. So in any case, the prequantized case leads to the exact sequence

$$0 \longrightarrow \mathbb{S}^1 \longrightarrow G_0^{\mathcal{H}} \longrightarrow G^{\text{ex}} \longrightarrow 0$$

$$(\mathbb{S}^1 \simeq \mathbb{R}/T\mathbb{Z} \quad T > 0)$$

and prequantization appears as a $\mathbb{S}^1$ central extension of the hamiltonian isotopies.

5. CONCLUDING REMARKS

5.1. There exists Poisson manifolds which are contact integrable but not symplectic-integrable. For example if we look at $\mathfrak{G}^*$ dual of a semi-simple compact Lie algebra, and at $S = S(\mathfrak{G}^*)$ the unit sphere (for the Killing form), $S(\mathfrak{G}^*)$ is a Poisson manifold which is symplectic integrable iff $\mathfrak{G} = \mathfrak{so}(3)$ but is - trivially - always contact integrable, with $\Gamma = (T^*G|_S, \lambda)$ where G is a Lie group of Lie algebra $\mathfrak{G}$ and λ is the Liouville form.

Another example is given by $\Gamma_0 = \mathbb{S}^2 \times \mathbb{R}$ with $(S^1 \times \{t\}, \sigma_0 f(t))$ as symplectic leaves, where σ_0 is the standard form on $\mathbb{S}^2$ and f is a C^∞ function which is every where non zero. Then $\Gamma = \mathbb{S}^3 \times \mathbb{S}^3 / \mathbb{S}^1 \times T^*\mathbb{R}$ with $\lambda = [p_1^* \lambda_0 - p_2^* \lambda_0] + tdu$ when the bracket denotes the image in $\mathbb{S}^3 \times \mathbb{S}^3 / \mathbb{S}^1$ (i.e. quotiented by the diagonal action) of $p_1^* \lambda_0 - p_2^* \lambda_0$ where λ_0 is the contact form on $\mathbb{S}^3$ and where Γ has the product groupoid structure of the $\mathbb{S}^1$ -quotient structure of the coarse groupoid $\mathbb{S}^3 \times \mathbb{S}^3$, by the vector groupoid $T^*\mathbb{R}$. $(\Gamma_0, \sigma_0 f(t))$ is symplectic integrable iff $f(t)$ is constant either a submersion, in which case Γ_0 is isomorphic to an open subPoisson manifold of $\mathfrak{so}(3)^*$.

If we look at $\Gamma_0 = \mathbb{S}^2 \times \mathbb{S}^1$ with $\sigma_0 f(t)$ as above, $(\Gamma_0, \sigma_0 f(t))$ is symplectic and contact integrable iff f is constant.

5.2. If (Γ_0, Λ_0) is a symplectic manifold (Γ_0, σ_0) , it is prequantizable in Weinstein's sense iff it exists $T > 0$ and a covering $\tilde{\Gamma}_0$ of Γ_0 such that $(\tilde{\Gamma}_0, T^{-1} \tilde{\sigma}_0)$ is prequantizable in Souriau's sense.

5.3. On any regular Poisson manifold there exists a star-product [12] but even on a symplectic manifold, the problem of the L^2 -representation (in asymptotic sense) of the *-product cannot be solve if a cohomological condition is not satisfied [5] [7].

From these three remarks, it seems that a nice category of Poisson manifold to study problems of quantization is the category of Poisson manifolds, a covering of which is contact integrable.

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Pierre DAZORD
Université Claude Bernard - Lyon I
Institut Girard Desargues - UPRES A 5028
43, boulevard du 11 Novembre 1918
69622 VILLEURBANNE CEDEX