

多重線形 Littlewood-Paley 作用素と多重線形 Fourier Multiplier

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表題の多重線形 Littlewood-Paley 作用素は次の形のもの

$$T(f_1, f_2, \dots, f_m)(x) = \int_0^\infty ((\varphi_1)_t * f_1)(x) ((\varphi_2)_t * f_2)(x) \cdots ((\varphi_m)_t * f_m)(x) b(t) \frac{dt}{t},$$

但し, $\varphi_j(x) \in L^1(\mathbb{R}^n)$ で適当な条件を満たし, 少なくとも一つの j に対し $\int_{\mathbb{R}^n} \varphi_j(x) dx = 0$ であり, $b(t) \in L^\infty(0, \infty)$. 又, ここでも以下でも, $\mathbb{R}^n$ 上の函数 $f(x)$ と $t > 0$ に対して, $f_t(x) = t^{-n} f(x/t)$ とする. $\varphi * f$ は φ と f の合成積を表す. Poisson 核 $P(x) = c_n(1 + |x|^2)^{-\frac{n+1}{2}}$ を用いて $\psi(x) = \frac{\partial P_t(x)}{\partial t} \Big|_{t=1}$ として,

$$\left(\int_0^\infty |(\psi_t * f)(x)|^2 \frac{dt}{t} \right)^{1/2}$$

が, $\mathbb{R}^n$ での Littlewood-Paley の g 函数である.

また, 多重線形 Fourier Multiplier は次の形のもの

$$\begin{aligned} M_\sigma(f_1, f_2, \dots, f_m)(x) \\ = \frac{1}{(2\pi)^{nm}} \int_{(\mathbb{R}^n)^m} e^{ix \cdot (\xi_1 + \xi_2 + \cdots + \xi_m)} \sigma(\xi_1, \xi_2, \dots, \xi_m) \hat{f}_1(\xi_1) \hat{f}_2(\xi_2) \cdots \hat{f}_m(\xi_m) d\xi_1 d\xi_2 \cdots d\xi_m. \end{aligned}$$

ここで, 表象 $\sigma(\xi_1, \xi_2, \dots, \xi_m)$ は, 例えば, 次を満たす.

$$|\partial^\alpha \sigma(\xi_1, \xi_2, \dots, \xi_m)| \leq C_\alpha (|\xi_1| + |\xi_2| + \cdots + |\xi_m|)^{-|\alpha|}, \quad |\alpha| \leq nm + 1$$

on $(\mathbb{R}^n)^m \setminus \{(0, 0, \dots, 0)\}$.

もう一つ, 関連したもので, 次の多重線形 Calderón-Zygmund 特異積分がある.

$$\begin{aligned} T_K(f_1, f_2, \dots, f_m)(x) \\ = \text{p.v.} \int_{\mathbb{R}^{nm}} K(x - y_1, x - y_2, \dots, x - y_m) f_1(y_1) f_2(y_2) \cdots f_m(y_m) dy_1 dy_2 \cdots dy_m. \end{aligned}$$

ここで, $K(x)$ は, 例えば, 次の条件を満たす.

- (i) $\int_{S^{nm-1}} K(y_1, y_2, \dots, y_m) dy_1 dy_2 \cdots dy_m = 0,$
- (ii) $|K(y_1, y_2, \dots, y_m)| \leq C/(|y_1| + |y_2| + \cdots + |y_m|)^{nm},$
- (iii) $|\nabla K(y_1, y_2, \dots, y_m)| \leq C/(|y_1| + |y_2| + \cdots + |y_m|)^{nm+1}.$

これら3つは, Coifman-Meyer [3, 4, 5] の研究以来, 特にいろいろな人が関心を寄せている. Coifman-Meyer [3, 4, 5] では, 適当な条件の下で $1 < p_j < \infty$ ($j = 1, 2, \dots, m$) と $p_0 \geq 1 : 1/p_0 = 1/p_1 + \dots + 1/p_m$ に対して上記の作用素が $L^{p_1} \times L^{p_2} \times \dots \times L^{p_m} \rightarrow L^{p_0}$ 有界になるということである. 最近の Grafakos-Torres [7] によれば, 積分核, あるいは表象の適当な滑らかさの下に, 例えば最初に挙げた条件下で, 多重線形 Calderón-Zygmund 特異積分と多重線形 Fourier multiplier の場合には, $p_0 \geq 1$ の制限は取れる, つまり, $p_0 > 1/m$ としてよいということである. (表象に対する条件に現れる滑らかさの指数 $nm+1$ については, 例えば, Yabuta [16] 参照) また, 多重線形 Littlewood-Paley 作用素については, Sato-Yabuta [13] で, $b(t) \equiv 1$ の場合に, 同様のことを示している.

ここでは, Grafakos-Torres の枠外になる必ずしも滑らかでない表象の多重線形 Fourier multiplier を扱ってみる. 具体的には, $n = 1, m = 2$ の場合を扱う. 滑らかでない表象の2重線形 Fourier multiplier として, よく知られているものに Calderón の交換子 $C(a, f)$ と2重線形 Hilbert 変換 $H_s(a, f)$ がある.

$$C(a, f)(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{\int_y^x a(u) du}{(x-y)^2} f(y) dy = .$$

$$H_s(a, f)(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{a(x-s(x-y))}{x-y} f(y) dy.$$

これらを2重線形 Fourier multiplier として表現したときの表象 σ_C, σ_{H_s} は次のようになる

$$\sigma_C(\xi, \alpha)/(-\pi i) = \left\{ 1 - \left(1 - \left| \frac{\xi}{\alpha} \right| \right)^+ \right\} \text{sgn } \xi + \left(1 - \left| \frac{\xi}{\alpha} \right| \right)^+ \text{sgn } \alpha$$

$$\sigma_{H_s}(\xi, \alpha)/(-\pi i) = \text{sgn}(\xi + s\alpha)$$

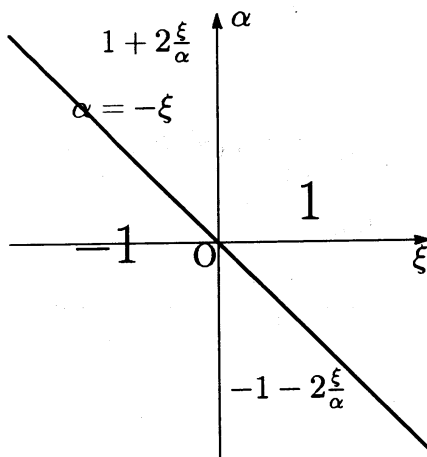


Fig 1. $\sigma_C(\xi, \alpha)/(-\pi i)$

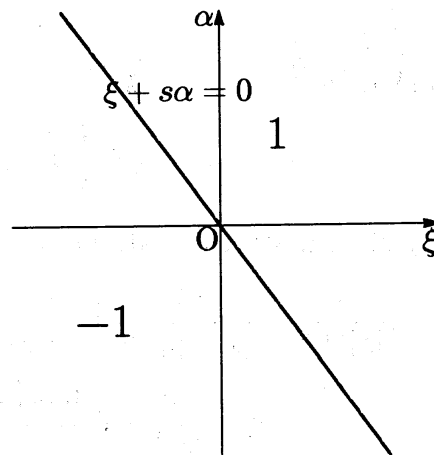


Fig 2. $\sigma_{H_s}(\xi, \alpha)/(-\pi i)$

Calderón の交換子 $C(a, f)$ については C. P. Calderón [2] により, $1 < p_1, p_2 < \infty$, $p_0 > 1/2 : 1/p_0 = 1/p_1 + 1/p_2$ に対して $L^{p_1} \times L^{p_2} \rightarrow L^{p_0}$ 有界性が成り立つことが示されている. また, 2重線形 Hilbert 変換 $H_s(a, f)$ については, ごく最近 Lacey-Thiele [10, 11] により, 上のことが $p_0 > 2/3$ の時, 成り立つことが示されている (ただし, $s \neq 1$). $2/3$ が最良かどうかは, まだ未解決である.

表象の特徴としては、 $\sigma_C(\xi, \alpha)$ は 0 次斉次で単位円周上で連続、区分的に C^2 であり、 $H_s(a, f)$ の方は 0 次斉次で単位円周上で区分的に C^2 だが、不連続点があることである。

以下で、Calderón の交換子と同じような表象の 2 重線形 Fourier multiplier については、同じ結果が成り立つことを検証してみる。目標は次の定理である (Yabuta [15] では $r \geq 1$ であった)。以下は、英文で記すこととする。

Theorem 1. *Let $\sigma(\xi, \alpha)$ be a continuous and homogeneous function of degree zero in $\mathbb{R}^2 \setminus \{(0, 0)\}$, such that $\omega(\theta) = \sigma(\cos \theta, \sin \theta)$ is differentiable except at most countably many points and $\omega'(\theta)$ is of bounded variation on $[0, 2\pi]$. Let T be the bilinear Fourier multiplier defined by*

$$T(f, g)(x) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ix(\xi+\alpha)} \sigma(\xi, \alpha) \hat{f}(\xi) \hat{g}(\alpha) d\xi d\alpha.$$

Then, for $1 < p, q < \infty$, and $r : \frac{1}{r} = \frac{1}{p} + \frac{1}{q}$, there exists $C > 0$ such that

$$\|T(f, g)\|_r \leq C \|f\|_p \|g\|_q.$$

To prove this we prepare some lemmas.

Let $(M_\gamma f)(\xi) = |\xi|^{i\gamma} \hat{f}(\xi)$. Then, the distributional kernel $K_\gamma(x)$ of M_γ is given by

$$K_\gamma(x) = c_\gamma |x|^{-1-i\gamma}, \quad c_\gamma = \frac{2^{i\gamma} \Gamma(\frac{1}{2} + \frac{i\gamma}{2})}{\pi^{\frac{1}{2}} \Gamma(-\frac{i\gamma}{2})}, \quad |c_\gamma| \sim \sqrt{\frac{|\gamma|}{2\pi}} \text{ as } |\gamma| \rightarrow \infty.$$

Lemma 1. *Let $v(\gamma)$ be a nonnegative measurable function on $\mathbb{R}$, and $A \subset \mathbb{R}$ be a measurable set. Then,*

$$\begin{aligned} \int_{|x| \geq 2|y|} \left(\int_A \left| \frac{1}{|x|^{1+i\gamma}} - \frac{1}{|x-y|^{1+i\gamma}} \right|^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} dx \\ \leq C \left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}} \left(1 + \log_2 \left(\int_A \gamma^2 v(\gamma) d\gamma / \int_A v(\gamma) d\gamma \right) \right). \end{aligned}$$

Proof. By elementary calculations, we have for $|x| > 2|y|$

$$\begin{aligned} \left| \frac{1}{|x|^{1+i\gamma}} - \frac{1}{|x-y|^{1+i\gamma}} \right| &\leq \left| \frac{1}{|x|} - \frac{1}{|x-y|} \right| + \frac{1}{|x-y|} \left| \frac{1}{|x|^{i\gamma}} - \frac{1}{|x-y|^{i\gamma}} \right| \\ &\leq C \frac{|y|}{|x|^2} + C \frac{\min(1, \frac{|\gamma y|}{|x|})}{|x|}. \end{aligned}$$

Hence

$$\begin{aligned} I &= \int_{|x| \geq 2|y|} \left(\int_A \left| \frac{1}{|x|^{1+i\gamma}} - \frac{1}{|x-y|^{1+i\gamma}} \right|^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} dx \\ &\leq \int_{|x| \geq 2|y|} \left(\int_A \left(C \frac{|y|}{|x|^2} \right)^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} dx + \int_{|x| \geq 2|y|} \left(\int_A \left(C \frac{\min(1, \frac{|\gamma y|}{|x|})}{|x|} \right)^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} dx \\ &=: I_1 + I_2. \end{aligned}$$

As for I_1 ,

$$I_1 \leq C \left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}} |y| \int_{2|y|}^{\infty} \frac{1}{r^2} dr \leq C \left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}}.$$

As for I_2 ,

$$\begin{aligned} I_2 &= C \sum_{l=1}^{\infty} \int_{2^l|y| \leq |x| \leq 2^{l+1}|y|} \left(\int_A \left(\frac{\min(1, \frac{|y|}{|x|})}{|x|} \right)^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} dx \\ &\leq C \sum_{l=1}^{\infty} \left(\int_{2^l|y| \leq |x| \leq 2^{l+1}|y|} \int_A \left(\frac{\min(1, \frac{|y|}{|x|})}{|x|} \right)^2 v(\gamma) d\gamma dx \right)^{\frac{1}{2}} \left(\int_{2^l|y| \leq |x| \leq 2^{l+1}|y|} dx \right)^{\frac{1}{2}} \\ &\leq C \sum_{l=1}^{\infty} (2^{l+1}|y|)^{1/2} \left(\int_A \left[\int_{2^l|y| \leq |x| \leq 2^{l+1}|y|} \left(\frac{\min(1, \frac{|y|}{|x|})}{|x|} \right)^2 dx \right] v(\gamma) d\gamma \right)^{\frac{1}{2}} \\ &\leq C \sum_{l=1}^{\infty} (2^{l+1}|y|)^{1/2} \left(\int_A \left[\left(\frac{\min(1, \frac{|y|}{2^l|y|})}{2^l|y|} \right)^2 2^l|y| \right] v(\gamma) d\gamma \right)^{\frac{1}{2}} \\ &\leq C \sum_{l=1}^{\infty} \left(\int_A \min(1, \gamma^2 2^{-2l}) v(\gamma) d\gamma \right)^{1/2} \\ &\leq C \sum_{l=1}^{\infty} \min \left(\left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}}, \left(\int_A \gamma^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} 2^{-l} \right) \\ &\leq C \sum_{1 \leq l \leq \log_2(\int_A \gamma^2 v(\gamma) d\gamma / \int_A v(\gamma) d\gamma) / 2} \left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}} \\ &\quad + \sum_{\log_2(\int_A \gamma^2 v(\gamma) d\gamma / \int_A v(\gamma) d\gamma) / 2 < l} \left(\int_A \gamma^2 v(\gamma) d\gamma \right)^{\frac{1}{2}} 2^{-l} \\ &\leq C \left(\int_A v(\gamma) d\gamma \right)^{\frac{1}{2}} \left(1 + \log_2 \left(\int_A \gamma^2 v(\gamma) d\gamma / \int_A v(\gamma) d\gamma \right) \right). \end{aligned}$$

□

Taking $v(\gamma) = (1 + \sqrt{|\gamma|})^2 / (1 + \gamma^2)$ in Lemma 1, we have

Lemma 2. Let $b(\gamma) \in L^\infty(\mathbb{R})$, $A_0 = \{|\gamma| < 1\}$ and $A_j = \{2^j \leq |\gamma| < 2^{j+1}\}$ ($j = 1, 2, \dots$). Let $K_\gamma(x)$ be the distributional kernel of M_γ , where $(M_\gamma f)(\xi) = |\xi|^{i\gamma} \hat{f}(\xi)$. Then, there exists $C > 0$ such that

$$\int_{|x| \geq 2|y|} \left(\int_{A_j} |K_\gamma(x) - K_\gamma(x-y)|^2 \frac{|b(\gamma)|}{1 + \gamma^2} d\gamma \right)^{\frac{1}{2}} dx \leq C j^{3/2}, \quad j = 0, 1, 2, \dots$$

Lemma 3. Let $(M_\gamma f)(\xi) = |\xi|^{i\gamma} \hat{f}(\xi)$, $b(\gamma) \in L^\infty(\mathbb{R})$, and

$$T(f, g)(x) = \int_{-\infty}^{\infty} M_\gamma f(x) M_{-\gamma} g(x) b(\gamma) \frac{d\gamma}{1 + \gamma^2}.$$

Then, for $1 < p, q < \infty$, and $r : \frac{1}{r} = \frac{1}{p} + \frac{1}{q}$, there exists $C > 0$ such that

$$\|T(f, g)\|_r \leq C \|f\|_p \|g\|_q.$$

Proof. In the case $1 \leq r < \infty$, one can easily show the above by using Minkowski's inequality. We treat the case $1/2 < r < 1$. We treat first the case $1 < p, q \leq 2$. Let $A_0 = \{|\gamma| < 1\}$ and $A_j = \{2^j \leq |\gamma| < 2^{j+1}\}$ ($j = 1, 2, \dots$). Put $u(\gamma) = b(\gamma)/(1 + |\gamma|^2)$. Then,

$$\begin{aligned} \int_{\mathbb{R}} |T(f, g)(x)|^r dx &= \int_{\mathbb{R}} \left| \sum_{j=0}^{\infty} \int_{A_j} M_{\gamma} f(x) M_{-\gamma} g(x) u(\gamma) d\gamma \right|^r dx \\ &\leq \sum_{j=0}^{\infty} \int_{\mathbb{R}} \left| \int_{A_j} M_{\gamma} f(x) M_{-\gamma} g(x) u(\gamma) d\gamma \right|^r dx \\ &\leq \sum_{j=0}^{\infty} \int_{\mathbb{R}} \left[\left(\int_{A_j} |M_{\gamma} f(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right]^r dx \\ &\leq \sum_{j=0}^{\infty} \left(\int_{\mathbb{R}} \left(\int_{A_j} |M_{\gamma} f(x)|^2 |u(\gamma)| d\gamma \right)^{\frac{p}{2}} dx \right)^{\frac{r}{p}} \left(\int_{\mathbb{R}} \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)| d\gamma \right)^{\frac{q}{2}} dx \right)^{\frac{r}{q}} \\ &\quad (\because 1 = \frac{r}{p} + \frac{r}{q}). \end{aligned}$$

Now,

$$\left\| \left(\int_{A_j} |M_{\gamma} f(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right\|_2 = c_n \left(\int_{A_j} |u(\gamma)| d\gamma \right)^{1/2} \|f\|_2 = C 2^{-j/2} \|f\|_2.$$

So, since $\frac{1}{p} = (1 - (2 - \frac{2}{p})) + \frac{2-\frac{2}{p}}{2}$, by Lemma 2 and a result of Hörmander (M_{γ} is an $L^2(A_j, |u(\gamma)| d\gamma)$ -valued singular integral),

$$\left\| \left(\int_{A_j} |M_{\gamma} f(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right\|_p \leq C (j^{3/2} + 2^{-2j/2})^{\frac{2}{p}-1} (2^{-j/2})^{2-\frac{2}{p}} \leq C j^{\frac{3}{p}-\frac{3}{2}} 2^{-j(1-\frac{1}{p})} \|f\|_p.$$

Similarly we have

$$\left\| \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right\|_q \leq C j^{\frac{3}{q}-\frac{3}{2}} 2^{-j(1-\frac{1}{q})} \|g\|_q.$$

Hence, using $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$ and $r > 1/2$ we have

$$\begin{aligned} \int_{\mathbb{R}} |T(f, g)(x)|^r dx &\leq C \sum_{j=0}^{\infty} (j^{\frac{3}{p}-\frac{3}{2}} 2^{-j(1-\frac{1}{p})} j^{\frac{3}{q}-\frac{3}{2}} 2^{-j(1-\frac{1}{q})})^r \|f\|_p^r \|g\|_q^r \\ &\leq C \sum_{j=0}^{\infty} j^{3-3r} 2^{-j(2r-1)} \|f\|_p^r \|g\|_q^r \leq C \|f\|_p^r \|g\|_q^r. \end{aligned}$$

Next, we treat the case $1 < p \leq 2$, $2 \leq q$ or $2 \leq p$, $1 < q \leq 2$. We may assume $1 < p \leq 2$, $2 \leq q$. For $1 < p \leq 2$, we can use

$$\left\| \left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right\|_p \leq C j^{\frac{3}{p} - \frac{3}{2}} 2^{-j(1 - \frac{1}{p})} \|f\|_p.$$

For $q \geq 2$, we have by duality

$$\left\| \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)| d\gamma \right)^{1/2} \right\|_q \leq C j^{3(1 - \frac{1}{q}) - \frac{3}{2}} 2^{-\frac{j}{q}} \|g\|_q \leq C j^{\frac{3}{2} - \frac{3}{q}} 2^{-\frac{j}{q}} \|g\|_q.$$

Since $1 - \frac{1}{p} > 0$, we have

$$\begin{aligned} \int_{\mathbb{R}} |T(f, g)(x)|^r dx &\leq C \sum_{j=0}^{\infty} (j^{\frac{3}{p} - \frac{3}{2}} 2^{-j(1 - \frac{1}{p})} j^{\frac{3}{2} - \frac{3}{q}} 2^{-\frac{j}{q}})^r \|f\|_p^r \|g\|_q^r \\ &\leq C \sum_{j=0}^{\infty} j^{3r(\frac{1}{p} - \frac{1}{q})} 2^{-jr(1 - \frac{1}{p} + \frac{1}{q})} \|f\|_p^r \|g\|_q^r \leq C \|f\|_p^r \|g\|_q^r. \end{aligned}$$

□

Proof of Theorem 1. Let $\sigma_1(\xi, \alpha)$ be a C^∞ homogeneous function of degree zero in $\mathbb{R}^2 \setminus \{(0, 0)\}$ such that $\sigma_1(\pm 1, 0) = \sigma(\pm 1, 0)$ and $\sigma_1(0, \pm 1) = \sigma(0, \pm 1)$. Let T_1 be the bilinear Fourier multiplier defined by

$$T_1(f, g)(x) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ix(\xi + \alpha)} \sigma_1(\xi, \alpha) \hat{f}(\xi) \hat{g}(\alpha) d\xi d\alpha.$$

Then, by a theorem of Grafakos and Torres, the conclusion of Theorem 1 holds for this bilinear operator T_1 . Hence, to prove Theorem 1, we may assume $\sigma(\pm 1, 0) = \sigma(0, \pm 1) = 0$. Thus, as in the proof of Theorem 4.1 in Yabuta [15, pp. 552-553], there exist four bounded function $b_j(\gamma)$ ($j = 1, 2, 3, 4$) such that

$$\begin{aligned} T(f, g)(x) &= \int_{-\infty}^{\infty} M_\gamma f(x) M_{-\gamma} g(x) b_1(\gamma) \frac{d\gamma}{1 + \gamma^2} + \int_{-\infty}^{\infty} M_\gamma H f(x) M_{-\gamma} g(x) b_2(\gamma) \frac{d\gamma}{1 + \gamma^2} \\ &\quad + \int_{-\infty}^{\infty} M_\gamma f(x) M_{-\gamma} H g(x) b_3(\gamma) \frac{d\gamma}{1 + \gamma^2} + \int_{-\infty}^{\infty} M_\gamma H f(x) M_{-\gamma} H g(x) b_4(\gamma) \frac{d\gamma}{1 + \gamma^2}, \end{aligned}$$

where H is the Hilbert transform, defined by $(Hf)(\xi) = \frac{\xi}{|\xi|} \hat{f}(\xi)$. Using the L^p -boundedness of the Hilbert transform and Lemma 3, we get the desired conclusion. □

Remark 1. It was my misunderstanding that I could prove the assertion in Remark 1 in Yabuta [15, p. 553]. It is still an open problem whether Theorem 1 holds for $0 < p, q < \infty$ (L^p replaced by H^p), even in the case $\sigma(\pm 1, 0) = \sigma(0, \pm 1)$.

参考文献

- [1] A. Benedek, A. P. Calderón, and R. Panzone, Convolution operators on Banach space valued functions, *Proc. Nat. Acad. Sci. U. S. A.*, 48 (1962), 356–365.
- [2] C. P. Calderón, On commutators of singular integrals, *Studia Math.*, 53 (1975), 139–174.
- [3] R. R. Coifman and Y. Meyer, On commutators of singular integrals and bilinear singular integrals, *Trans. Amer. Math. Soc.*, 212 (1975), 315–331.
- [4] R. R. Coifman and Y. Meyer, Commutateurs d'intégrales singulières et opérateurs multilinéaires, *Ann. Inst. Fourier*, 28 (1978), 177–202.
- [5] R. R. Coifman and Y. Meyer, Au-delà des opérateurs pseudo-différentiels, *Astérisque*, 57 (1978), 1–185.
- [6] L. Grafakos and N. J. Kalton, The Marcinkiewicz multiplier condition for bilinear operators, preprint. (<http://www.math.missouri.edu>)
- [7] L. Grafakos and R. H. Torres, Multilinear Calderón-Zygmund theory, preprint. (<http://www.math.missouri.edu>)
- [8] L. Grafakos and R. H. Torres, Maximal operators and weighted norm inequalities for multilinear singular integrals, preprint. (<http://www.math.missouri.edu>)
- [9] C. Kenig and E. M. Stein, Multilinear estimates and fractional integrals, *Math. Res. Let.*, 6 (1999), 1–15.
- [10] M. T. Lacey and C. M. Thiele, L^p bounds for the bilinear Hilbert transform, $2 < p < \infty$, *Ann. Math.*, 146 (1997), 693–724.
- [11] M. T. Lacey and C. M. Thiele, On Calderón's conjecture, *Ann. Math.*, 149 (1999), 475–496.
- [12] S. Sato, Remarks on square functions in the Littlewood-Paley theory, *Bull. Austral. Math. Soc.*, 58 (1998), 199–211.
- [13] S. Sato and K. Yabuta, Multilinearized Littlewood-Paley operators, preprint.
- [14] K. Yabuta, A multilinearization of Littlewood-Paley's g -function and Carleson measures, *Tôhoku. Math. J.*, 34 (1982), 251–275.
- [15] K. Yabuta, Bilinear Fourier multipliers, *Tôhoku Math. J.*, 35 (1983), 541–555.
- [16] K. Yabuta, Calderón-Zygmund operators and pseudo-differential operators, *Comm. Partial Differential Equations*, 10 (1985), 1005–1022.
- [17] K. Yabuta, m -linearized Littlewood-Paley operators, *Proceedings of the conference "Singular Integrals and Related Topics, III"*, pp. 1–7.