

Invariant Hyperfunctions on a Rank One Semisimple Symmetric Space

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1. Introduction

Let G be a connected semisimple Lie group, σ an involutive automorphism of G and H an open subgroup of the group of fixed points of σ . Then G/H is called a semisimple symmetric space. Let $\mathfrak{g}$ be the Lie algebra of G and $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$ the decomposition into $+1$ and -1 eigenspaces with respect to σ . Choose a maximal abelian subspace $\mathfrak{a} \subset \mathfrak{q}$ consisting of semisimple elements. Then all such $\mathfrak{a}$ have the same dimension, which we call the rank of G/H . In what follows, we assume that the rank of G/H is one. It is clear that $\mathfrak{q}$ can be identified with the tangent space at the origin of G/H . Moreover, H acts on $\mathfrak{q}$ by means of the restriction of the adjoint representation to $\mathfrak{q}$.

If E is a real vector space, we put $E_{\mathbb{C}}$ the complexification of E and $S(E_{\mathbb{C}})$ the symmetric algebra of $E_{\mathbb{C}}$. For any $p \in S(\mathfrak{q}_{\mathbb{C}})$, we define, as usual, the differential operator $\partial(p)$ on $\mathfrak{q}$. Let $B(\ , \)$ be the Killing form on $\mathfrak{g}$, and write $\omega(X) = B(X, X)$ for all $X \in \mathfrak{q}$, the *Casimir polynomial* of $\mathfrak{q}$, then the polynomial ω is an H -invariant nondegenerate

quadratic form on $\mathfrak{q}$. Identifying $\mathfrak{q}$ with its dual by means of the Killing form, $\square = \partial(\omega)$ can be viewed as an H -invariant constant coefficients second order differential operator on $\mathfrak{q}$. We call $\square$ the *pseudo-Laplacian* of $\mathfrak{q}$. We set

$$(\tilde{H})_1 = \{T \in GL(\mathfrak{q}); \omega(Tx) = \omega(x) \text{ for all } x \in \mathfrak{q}\}.$$

Let $(\tilde{H})_0$ be the identity component of $(\tilde{H})_1$, and $\tilde{\mathfrak{h}}$ be the Lie algebra of $(\tilde{H})_0$, then $\text{ad}_{\mathfrak{q}}(\tilde{\mathfrak{h}}) \subset \tilde{\mathfrak{h}}$.

The dimensions of H -invariant eigendistributions of $\square$ on $\mathfrak{q}$ is given by careful considerations to the radial part of pseudo-Laplacian $\square$ by van Dijk [2]. On the other hand by a result of Cerezo [1] we can also give the dimensions of $\tilde{H}$ -invariant eigenhyperfunction of $\square$ on $\mathfrak{q}$. By comparison of these results, Kowata introduce the conjecture that every H -invariant eigenhyperfunction of $\square$ on $\mathfrak{q}$ is $(\tilde{H})_0$ -invariant, and he has shown that this conjecture holds if the eigenvalue of $\square$ is not equal to zero.

In this paper we give the answer to this conjecture except for the case of one of the exceptional type $F_{4(4)}/Spin(4, 5)$. In §2 we rewrite the problem in terms of the systems of differential equations ($\mathcal{D}$ -modules) and in §3 we compare these two $\mathcal{D}$ -modules related to H and $(\tilde{H})_0$ in the case of the classical type. In §4 we make more precise investigation by using the H -orbits decomposition of $\mathfrak{q}$. We must remark that the conjecture of Kowata does not hold in one case (Proposition 18).

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2. Preliminaries

Let $\tilde{H}$ be the subgroup of $GL(\mathfrak{q})$ generated by $(\tilde{H})_0$ and $\text{Ad}_{\mathfrak{q}}(H)$. Then H is contained in $\tilde{H}$ and $(\tilde{H})_0$ is the identity component of $\tilde{H}$. Let $\mathcal{B}$ be the sheaf of hyperfunctions on $\mathfrak{q}$.

We define $\mathcal{B}^H(\mathfrak{q})$, $\mathcal{B}_{\lambda}^H(\mathfrak{q})$, $\mathcal{B}^{\tilde{H}}(\mathfrak{q})$ and $\mathcal{B}_{\lambda}^{\tilde{H}}(\mathfrak{q})$ for $\lambda \in \mathbb{C}$ by

$$\mathcal{B}^H(\mathfrak{q}) = \{f \in \mathcal{B}(\mathfrak{q}); f \text{ is } H\text{-invariant}\}$$

$$\mathcal{B}_{\lambda}^H(\mathfrak{q}) = \{f \in \mathcal{B}^H(\mathfrak{q}); (\square - \lambda)f = 0\}$$

$$\mathcal{B}^{\tilde{H}}(\mathfrak{q}) = \{f \in \mathcal{B}(\mathfrak{q}); f \text{ is } \tilde{H}\text{-invariant}\}$$

$$\mathcal{B}_{\lambda}^{\tilde{H}}(\mathfrak{q}) = \mathcal{B}^{\tilde{H}}(\mathfrak{q}) \cap \mathcal{B}_{\lambda}^H(\mathfrak{q}).$$

A hyperfunction $u \in \mathcal{B}^H(\mathfrak{q})$ is said to be *H-invariant* and a hyperfunction $u \in \mathcal{B}_{\lambda}^H(\mathfrak{q})$ is called an *invariant spherical hyperfunction*. In general, we have $\mathcal{B}_{\lambda}^{\tilde{H}}(\mathfrak{q}) \subset \mathcal{B}_{\lambda}^H(\mathfrak{q})$ and $\mathcal{B}^{\tilde{H}}(\mathfrak{q}) \subset \mathcal{B}^H(\mathfrak{q})$.

Let $\mathcal{D}$ be the sheaf of the ring of holomorphic coefficient differential operators on $\mathfrak{q}_{\mathbb{C}}$. For any $X \in \mathfrak{gl}(\mathfrak{q})$, define a vector field D_X on $\mathfrak{q}$ by

$$(D_X f)(y) = \left. \frac{d}{dt} f(e^{-tX} y) \right|_{t=0} \quad (y \in \mathfrak{q}, f \in C^{\infty}(\mathfrak{q})).$$

Then D_X is a first order differential operator with polynomial coefficients on $\mathfrak{q}_{\mathbb{C}}$. We define the systems of differential equations *defining H-invariants* and *$\tilde{H}$ -invariants*

$$\mathcal{M} = \mathcal{D}/\mathcal{I} = \mathcal{D} / \sum_{X \in \mathfrak{h}} \mathcal{D} D_X$$

$$\mathcal{N} = \mathcal{D}/\mathcal{J} = \mathcal{D} / \sum_{X \in \tilde{\mathfrak{h}}} \mathcal{D} D_X,$$

and also define

$$\mathcal{M}_\lambda = \mathcal{D}/\mathcal{I}_\lambda = \mathcal{D}/(\mathcal{I}_\lambda + \mathcal{D}(\square - \lambda))$$

$$\mathcal{N}_\lambda = \mathcal{D}/\mathcal{J}_\lambda = \mathcal{D}/(\mathcal{J}_\lambda + \mathcal{D}(\square - \lambda)).$$

In general, we have surjective homomorphisms $\mathcal{M} \rightarrow \mathcal{N}$ and $\mathcal{M}_\lambda \rightarrow \mathcal{N}_\lambda$.

In these notations, our problems are written as follows.

PROBLEM I. When is $\mathcal{B}_\lambda^H(\mathfrak{q}) = \mathcal{B}_\lambda^{\tilde{H}}(\mathfrak{q})$?

PROBLEM II. When is $\mathcal{B}^H(\mathfrak{q}) = \mathcal{B}^{\tilde{H}}(\mathfrak{q})$?

PROBLEM III. When is $\mathcal{M}_\lambda \xrightarrow{\sim} \mathcal{N}_\lambda$?

PROBLEM IV. When is $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$?

REMARK 1.

- (1) If (IV) holds, then (II) and (III) hold. If (II) or (III) hold, then (I) hold.
- (2) Problem III and IV depend only on the complexification $(\mathfrak{g}_\mathbb{C}, \mathfrak{h}_\mathbb{C})$ of $(\mathfrak{g}, \mathfrak{h})$. ■

Our approach to these problems is case by case calculation. The following table shows the non-Riemannian semisimple symmetric spaces G/H of rank one ([4, Chapter X], [2, § 6.3]).

TABLE 2.

No.	G/H
1°	$SO_0(p, q+1)/SO_0(p, q)$
2°	$SO(p, q+1)/S(O(p, q) \times O(1))$
3°	$SU(p, q+1)/S(U(p, q) \times U(1))$
4°	$Sp(p, q+1)/Sp(p, q) \times Sp(1)$
5°	$F_{4(-20)}/Spin(1, 8)$
6°	$SL(m+1, \mathbf{R})/S(GL_+(m, \mathbf{R}) \times GL_+(1, \mathbf{R}))$
7°	$SL(m+1, \mathbf{R})/S(GL(m, \mathbf{R}) \times GL(1, \mathbf{R}))$
8°	$Sp(m+1, \mathbf{R})/Sp(m, \mathbf{R}) \times Sp(1, \mathbf{R})$
9°	$F_{4(4)}/Spin(4, 5)$

Here $p \geq 1, q \geq 1, m \geq 2$.

We sometimes write $p+q=m$ in cases 1°, 2°, 3°, 4°.

In cases 1° and 2°, then we have $\text{ad}_q(\mathfrak{h}) = \tilde{\mathfrak{h}}$, hence Problem IV and therefore all problems are true. On the other hands, we have $\text{ad}_q(\mathfrak{h}) \neq \tilde{\mathfrak{h}}$ in the rest cases, hance the whole problems have meanings.

3. Problem III and IV

3.1. The Spaces 6° : $G/H = SL(m+1, \mathbf{R})/S(GL_+(m, \mathbf{R}) \times GL_+(1, \mathbf{R}))$. ■

Let $(x, y) = (x_1, \dots, x_m, y_1, \dots, y_m)$ be a coordinate system in $\mathfrak{q} \cong \mathbf{R}^{2m}$. We denote the action of $T \in GL(\mathfrak{q})$ on $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathfrak{q}$ by left multiplication. The Casimir polynomial is $\omega = x_1 y_1 + \dots + x_m y_m$. We realize $\mathfrak{h}$ and $\tilde{\mathfrak{h}}$ as subalgebras of $\mathfrak{gl}(\mathfrak{q})$.

$$\left\{ \begin{array}{l} \mathfrak{h} = \mathfrak{gl}(m, \mathbf{R}) \cong \left\{ \begin{pmatrix} A & 0 \\ 0 & -{}^t A \end{pmatrix} : A \in M(m, \mathbf{R}) \right\} . \\ \tilde{\mathfrak{h}} = \mathfrak{so}(m, m) \\ = \left\{ \begin{pmatrix} A & B \\ C & -{}^t A \end{pmatrix} : A, B, C \in M(m, \mathbf{R}), {}^t B = -B, {}^t C = -C \right\} . \end{array} \right.$$

First we deal with Problem IV.

PROPOSITION 3. If $m \geq 3$, then $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.

PROOF: We write $\mathcal{I}$ and $\mathcal{J}$ explicitly on $\mathfrak{q}_{\mathbb{C}}$ with the above coordinates

$$\mathcal{I} = \sum_{i,j=1}^m \mathcal{D}(x_i \partial_{x_j} - y_j \partial_{y_i}).$$

$$\mathcal{J} = \mathcal{I} + \sum_{i,j=1}^m \mathcal{D}(x_i \partial_{y_j} - x_j \partial_{y_i}) + \sum_{i,j=1}^m \mathcal{D}(y_i \partial_{x_j} - y_j \partial_{x_i}).$$

We will show $x_i \partial_{y_j} - x_j \partial_{y_i} \in \mathcal{I}$ for $1 \leq i < j \leq m$. Since $m \geq 3$, there is an integer k ($1 \leq k \leq m$) such that i, j, k are different to one another. Since the right hand side of the formula

$$\begin{aligned} x_i \partial_{y_j} - x_j \partial_{y_i} &= (x_j \partial_{y_k} - x_k \partial_{y_j})(x_i \partial_{x_k} - y_k \partial_{y_i}) \\ &\quad + (x_k \partial_{y_i} - x_i \partial_{y_k})(x_j \partial_{x_k} - y_k \partial_{y_j}) \\ &\quad + (x_i \partial_{y_j} - x_j \partial_{y_i})(x_k \partial_{x_k} - y_k \partial_{y_k}) \end{aligned}$$

is contained in $\mathcal{I}$, then we have $x_i \partial_{y_j} - x_j \partial_{y_i} \in \mathcal{I}$. Hence Proposition 3 is proved. ■

Next we deal with the case $m = 2$. We define two left $\mathcal{D}$ -Modules

$$\mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbb{C}}} = \mathcal{D}/\mathcal{D}(x_1, x_2, \partial_{y_1}, \partial_{y_2})$$

$$\mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbb{C}}} = \mathcal{D}/\mathcal{D}(y_1, y_2, \partial_{x_1}, \partial_{x_2}),$$

and two morphisms as left $\mathcal{D}$ -Modules

$$(3.1) \quad \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbb{C}}} \longrightarrow \mathcal{M}, \quad P \longmapsto P(y_1 \partial_{x_2} - y_2 \partial_{x_1})$$

$$(3.2) \quad \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbb{C}}} \longrightarrow \mathcal{M}, \quad P \longmapsto P(x_1 \partial_{y_2} - x_2 \partial_{y_1}).$$

By (3.1) and (3.2), we get the complex of left $\mathcal{D}$ -modules.

$$(3.3) \quad 0 \longrightarrow \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{c}}} \oplus \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{c}}} \longrightarrow \mathcal{M} \longrightarrow \mathcal{N} \longrightarrow 0.$$

PROPOSITION 4. *The complex (3.3) is exact.*

PROOF: It is sufficient to prove the injectivity of the first morphism, which follows from Proposition 6 below. ■

Next we consider Problem III.

PROPOSITION 5. *If $m = 2$ and $\lambda \neq 0$, then $\mathcal{M}_\lambda \xrightarrow{\sim} \mathcal{N}_\lambda$.*

PROOF: Since the pseudo-Laplacian $\square$ is written as $\square = \sum_{i=1}^m \partial_{x_i} \partial_{y_i}$ by the explicit coordinates, then $\square - \lambda = \partial_{x_1} \partial_{y_1} + \partial_{x_2} \partial_{y_2} - \lambda$. We find that $(y_1 \partial_{x_2} - y_2 \partial_{x_1}) \in \mathcal{I}_\lambda$ by the formula

$$\begin{aligned} \lambda(y_1 \partial_{x_2} - y_2 \partial_{x_1}) &= -(y_1 \partial_{x_2} - y_2 \partial_{x_1})(\square - \lambda) \\ &\quad + \partial_{x_1}^2 (x_1 \partial_{x_2} - y_2 \partial_{y_1}) - \partial_{x_2}^2 (x_2 \partial_{x_1} - y_1 \partial_{y_2}) \\ &\quad - \partial_{x_1} \partial_{x_2} (x_1 \partial_{x_1} - y_1 \partial_{y_1}) + \partial_{x_1} \partial_{x_2} (x_2 \partial_{x_2} - y_2 \partial_{y_2}). \end{aligned}$$

Hence Proposition 5 is proved. ■

The rest is in case $\lambda = 0$.

PROPOSITION 6. *The following complex (3.4) induced by the complex (3.3) is exact.*

$$(3.4) \quad 0 \longrightarrow \mathcal{B}_{\{x=0\}|\mathfrak{q}_{\mathbf{c}}} \oplus \mathcal{B}_{\{y=0\}|\mathfrak{q}_{\mathbf{c}}} \longrightarrow \mathcal{M}_0 \longrightarrow \mathcal{N}_0 \longrightarrow 0.$$

PROOF: It is sufficient to prove the injectivity of the first morphism. First we consider the complex (3.4) outside the origin. We may assume

$y_2 \neq 0$ without loss of generality. Then remark that $\mathcal{B}_{\{y=0\}}|_{\mathfrak{q}_{\mathbf{c}}} = 0$ on the open set $\{y_2 \neq 0\}$. Moreover the following coordinate transformation makes sense on the open set $\{y_2 \neq 0\}$:

$$(3.5) \quad \left\{ \begin{array}{l} \tilde{x}_1 = x_1 \\ \tilde{x}_2 = x_1 y_1 + x_2 y_2 \\ \tilde{y}_1 = y_1 \\ \tilde{y}_2 = y_2 \end{array} \right\} \quad \left\{ \begin{array}{l} x_1 = \tilde{x}_1 \\ x_2 = \frac{\tilde{x}_2 - \tilde{x}_1 \tilde{y}_1}{\tilde{y}_2} \\ y_1 = \tilde{y}_1 \\ y_2 = \tilde{y}_2 \end{array} \right\}.$$

We rewrite everything in the new coordinates. Then

$$\begin{aligned} \mathcal{B}_{\{x=0\}}|_{\mathfrak{q}_{\mathbf{c}}} &= \mathcal{D}/\mathcal{D}(\tilde{x}_1, \tilde{x}_2, \partial_{\tilde{y}_1}, \tilde{y}_2 \partial_{\tilde{y}_2} - 1), \\ \mathcal{M}_0 &= \mathcal{D}/\mathcal{D}(\partial_{\tilde{y}_1}, \partial_{\tilde{y}_2}, \tilde{x}_1 \partial_{\tilde{x}_1}, \tilde{x}_2 \partial_{\tilde{x}_1}, \partial_{\tilde{x}_2}^2 \tilde{x}_2), \end{aligned}$$

and the morphism (3.1) is the right multiplication by $\tilde{y}_2 \partial_{\tilde{x}_1}$, which is the composition of the following two injective morphisms

$$\mathcal{B}_{\{x=0\}}|_{\mathfrak{q}_{\mathbf{c}}} \xrightarrow[\sim]{\tilde{y}_2} \mathcal{D}/\mathcal{D}(\tilde{x}_1, \tilde{x}_2, \partial_{\tilde{y}_1}, \partial_{\tilde{y}_2}) \xrightarrow{\cdot \partial_{\tilde{x}_1}} \mathcal{M}_0.$$

This implies the complex (3.4) is exact outside the origin, i.e. the kernel of $\mathcal{B}_{\{x=0\}}|_{\mathfrak{q}_{\mathbf{c}}} \oplus \mathcal{B}_{\{y=0\}}|_{\mathfrak{q}_{\mathbf{c}}} \longrightarrow \mathcal{M}_0$ is supported in the origin. Moreover it follows from the involutivity of the characteristic varieties that the kernel must be zero. Thus we finish the proof of Proposition 6 and Proposition 4. ■

3.2. The Spaces $3^\circ : SU(p, q+1)/S(U(p, q) \times U(1))$

By Remark 1, we can get the following Proposition 7 for spaces 3° as a corollary of the previous results for spaces 6° . Let $m = p + q$, $\mathfrak{q} = \mathbb{C}^m$. We take coordinates $z = (z_1, \dots, z_m)$ of $\mathfrak{q}$ and those of $\mathfrak{q}_{\mathbb{C}}$ as $(z, w) \in \mathbb{C}^{2m}$ with $z, w \in \mathbb{C}^m$, $\mathfrak{q} = \{(z, w) \in \mathfrak{q}_{\mathbb{C}}; z = \bar{w}\}$.

PROPOSITION 7.

(1) If $m \geq 3$, then $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.

If $m = 2$, then we have the following exact sequence,

$$0 \longrightarrow \mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbb{C}}} \oplus \mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbb{C}}} \longrightarrow \mathcal{M} \longrightarrow \mathcal{N} \longrightarrow 0.$$

(2) If $m \geq 3$ or $\lambda \neq 0$, then $\mathcal{M}_\lambda \xrightarrow{\sim} \mathcal{N}_\lambda$.

If $m = 2$ and $\lambda = 0$, then we have the exact sequence,

$$0 \longrightarrow \mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbb{C}}} \oplus \mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbb{C}}} \longrightarrow \mathcal{M}_0 \longrightarrow \mathcal{N}_0 \longrightarrow 0.$$

(3) We have an isomorphism $\underline{Hom}_{\mathcal{D}}(\mathcal{N}, \mathcal{B}) \xrightarrow{\sim} \underline{Hom}_{\mathcal{D}}(\mathcal{M}, \mathcal{B})$.

In particular, $\mathcal{B}^{\tilde{H}}(\mathfrak{q}) = \mathcal{B}^H(\mathfrak{q})$ and $\mathcal{B}_\lambda^{\tilde{H}}(\mathfrak{q}) = \mathcal{B}_\lambda^H(\mathfrak{q})$.

PROOF: (1) and (2) follow from Proposition 3, 4, 5 and 6.

(3) is led by $\underline{Hom}_{\mathcal{D}}(\mathcal{B}_{\{z=0\}|\mathfrak{q}_{\mathbb{C}}}, \mathcal{B}) = \underline{Hom}_{\mathcal{D}}(\mathcal{B}_{\{w=0\}|\mathfrak{q}_{\mathbb{C}}}, \mathcal{B}) = 0$. ■

3.3. The Spaces $8^\circ : Sp(m+1, \mathbb{R})/Sp(m, \mathbb{R}) \times Sp(1, \mathbb{R})$

We set $Sp(m, \mathbb{R}) = \left\{ g \in SL(2m, \mathbb{R}); g \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} {}^t g = \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} \right\}$. We take coordinates

$$\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \in M(2m, 2; \mathbb{R}) \cong \mathfrak{q}, \quad x_1 = \begin{pmatrix} x_{11} \\ \vdots \\ x_{m1} \end{pmatrix}, x_2, y_1, y_2 \in \mathbb{R}^m.$$

In this coordinates, the Casimir polynomial ω is written as

$$\omega = {}^t \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \begin{pmatrix} 0 & 1_m \\ -1_m & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = {}^t x_1 \cdot y_2 - {}^t y_2 \cdot x_2 = \sum_{k=1}^m (x_{k1} y_{k2} - y_{k1} x_{k2}).$$

The group $H = Sp(m, \mathbb{R}) \times Sp(1, \mathbb{R})$ acts on $\mathfrak{q}$ by

$$(3.6) \quad (A, B) \cdot C = ACB^{-1} \quad (A \in Sp(m, \mathbb{R}), B \in Sp(1, \mathbb{R}), C \in \mathfrak{q}).$$

THEOREM 8. For $m \geq 1$, we have $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.

PROOF: First we consider the problem outside the origin. This step contains rather tedious calculation. We may assume $x_{11} \neq 0$ and the following coordinate transformation makes sense on the locus $\{x_{11} \neq 0\}$:

$$\begin{cases} \tilde{x}_{j1} = x_{j1}, & \tilde{y}_{j1} = y_{j1} & (1 \leq j \leq m) \\ \tilde{x}_{12} = x_{12}, & \tilde{y}_{12} = \omega(x, y) \\ \tilde{x}_{k2} = x_{k2} - \frac{x_{12}x_{k1}}{x_{11}}, & \tilde{y}_{k2} = y_{k2} - \frac{x_{12}y_{k1}}{x_{11}} & (2 \leq k \leq m). \end{cases}$$

We recall the generators of $\mathcal{I}$ by old coordinates (cf. (3.6)).

$$\left\{ \begin{array}{ll} (a): & x_{j1}\partial_{x_{i1}} - y_{i1}\partial_{y_{j1}} + x_{j2}\partial_{x_{i2}} - y_{i2}\partial_{y_{j2}} \quad (1 \leq i, j \leq m) \\ (b): & y_{j1}\partial_{x_{i1}} + y_{i1}\partial_{x_{j1}} + y_{j2}\partial_{x_{i2}} + y_{i2}\partial_{x_{j2}} \quad (1 \leq i, j \leq m) \\ (c): & x_{j1}\partial_{y_{i1}} + x_{i1}\partial_{y_{j1}} + x_{j2}\partial_{y_{i2}} + x_{i2}\partial_{y_{j2}} \quad (1 \leq i, j \leq m) \\ (d): & \sum_{i=1}^m (x_{i1}\partial_{x_{i1}} + y_{i1}\partial_{y_{i1}} - x_{i2}\partial_{x_{i2}} - y_{i2}\partial_{y_{i2}}) \\ (e): & \sum_{i=1}^m (x_{i1}\partial_{x_{i2}} + y_{i1}\partial_{y_{i2}}) \\ (f): & \sum_{i=1}^m (x_{i2}\partial_{x_{i1}} + y_{i2}\partial_{y_{i1}}). \end{array} \right.$$

In new coordinates, we exchange generators of $\mathcal{I}$:

$$(3.7) \quad \left\{ \begin{array}{ll} (e) = x_{11}\partial_{\bar{x}_{12}}, & \partial_{\bar{x}_{12}} \in \mathcal{I}. \\ ((c), i=j=1) = 2x_{11}\partial_{\bar{y}_{11}}, & \partial_{\bar{y}_{11}} \in \mathcal{I}. \\ ((a), i=j=1) = x_{11}\partial_{\bar{x}_{11}} + x_{12}\partial_{\bar{x}_{12}} - y_{11}\partial_{\bar{y}_{11}}, & \partial_{\bar{x}_{11}} \in \mathcal{I}. \\ ((c), i \geq 2, j=1) = x_{11}\partial_{\bar{y}_{i1}} + x_{i1}\partial_{\bar{y}_{11}}, & \partial_{\bar{y}_{i1}} \in \mathcal{I}. \\ ((a), i \geq 2, j=1) = x_{11}\partial_{\bar{x}_{i1}} - y_{i1}\partial_{\bar{y}_{11}}, & \partial_{\bar{x}_{i1}} \in \mathcal{I}. \end{array} \right.$$

$$(3.8) \quad (d) = - \sum_{k=2}^m (\tilde{x}_{k2}\partial_{\bar{x}_{k2}} + \tilde{y}_{k2}\partial_{\bar{y}_{k2}}) \\ + \sum_{k=2}^m (x_{k1}\partial_{\bar{x}_{k1}} + y_{k1}\partial_{\bar{y}_{k1}}) + x_{11}\partial_{\bar{x}_{11}} + y_{11}\partial_{\bar{y}_{11}}, \\ \sum_{k=2}^m (\tilde{x}_{k2}\partial_{\bar{x}_{k2}} + \tilde{y}_{k2}\partial_{\bar{y}_{k2}}) \in \mathcal{I}.$$

$$(3.9) \quad \left\{ \begin{array}{l} ((a), i \geq 2, j \geq 2) = \tilde{x}_{j2} \partial_{\tilde{x}_{i2}} - \tilde{y}_{i2} \partial_{\tilde{y}_{j2}} + x_{j1} \partial_{\tilde{x}_{i1}} - y_{i1} \partial_{\tilde{y}_{j1}}, \\ \qquad \qquad \qquad \tilde{x}_{j2} \partial_{\tilde{x}_{i2}} - \tilde{y}_{i2} \partial_{\tilde{y}_{j2}} \in \mathcal{I}. \\ ((b), i \geq 2, j \geq 2) = \tilde{y}_{j2} \partial_{\tilde{x}_{i2}} + \tilde{y}_{i2} \partial_{\tilde{x}_{j2}} + y_{j1} \partial_{\tilde{x}_{i1}} + y_{i1} \partial_{\tilde{x}_{j1}}, \\ \qquad \qquad \qquad \tilde{y}_{j2} \partial_{\tilde{x}_{i2}} + \tilde{y}_{i2} \partial_{\tilde{x}_{j2}} \in \mathcal{I}. \\ ((c), i \geq 2, j \geq 2) = \tilde{x}_{j2} \partial_{\tilde{y}_{i2}} + \tilde{x}_{i2} \partial_{\tilde{y}_{j2}} + x_{j1} \partial_{\tilde{y}_{i1}} + x_{i1} \partial_{\tilde{y}_{j1}}, \\ \qquad \qquad \qquad \tilde{x}_{j2} \partial_{\tilde{y}_{i2}} + \tilde{x}_{i2} \partial_{\tilde{y}_{j2}} \in \mathcal{I}. \end{array} \right.$$

Since ω is $\tilde{H}$ -invariant, then

$$\mathcal{J} \subset \mathcal{D}(\partial_{\tilde{x}_{ij}}, \partial_{\tilde{y}_{ij}}; 1 \leq i \leq m, 1 \leq j \leq 2, \text{ except for } \partial_{\tilde{y}_{12}}).$$

If we can prove

$$(3.10) \quad \mathcal{I} \supset \mathcal{D}(\partial_{\tilde{x}_{ij}}, \partial_{\tilde{y}_{ij}}; 1 \leq i \leq m, 1 \leq j \leq 2, \text{ except for } \partial_{\tilde{y}_{12}}),$$

then $\mathcal{I} = \mathcal{J}$. We prepare two Lemmas.

LEMMA 9. Two groups $Sp(n, \mathbf{R}) \subset SL(2n, \mathbf{R})$ act on $\mathbf{R}^{2n}$ by left multiplication ($n \geq 1$). Then the system $\mathcal{M}_1$ of differential equations defining $Sp(n, \mathbf{R})$ -invariants equals the system $\mathcal{M}_2$ of differential equations defining $SL(2n, \mathbf{R})$ -invariants.

PROOF: We set $\mathcal{M}_1 = \mathcal{D}/\mathcal{I}_1$, $\mathcal{M}_2 = \mathcal{D}/\mathcal{I}_2$. We can express $\mathcal{I}_1$ and $\mathcal{I}_2$

in coordinates.

$$\mathcal{I}_1 = \mathcal{D} \begin{pmatrix} x_i \partial_{x_j} - y_j \partial_{y_i} & ; & 1 \leq i, j \leq m \\ x_i \partial_{y_j} + x_j \partial_{y_i} & ; & 1 \leq i, j \leq m \\ y_i \partial_{x_j} + y_j \partial_{x_i} & ; & 1 \leq i, j \leq m \end{pmatrix}$$

$$\mathcal{I}_2 = \mathcal{D} \begin{pmatrix} x_i \partial_{x_i} - y_j \partial_{y_j} & ; & 1 \leq i, j \leq m \\ x_i \partial_{y_j}, & y_j \partial_{x_i} & ; & 1 \leq i, j \leq m \\ x_i \partial_{x_j}, & y_i \partial_{y_j} & ; & i \neq j \end{pmatrix}.$$

One after another,

$$x_i^2 \partial_{x_i} = x_i(x_i \partial_{x_i} - y_i \partial_{y_i}) + y_i x_i \partial_{y_i} \in \mathcal{I}_1.$$

$$x_i^2 \partial_{y_j} = x_i(x_i \partial_{y_j} + x_j \partial_{y_i}) - x_j x_i \partial_{y_i} \in \mathcal{I}_1.$$

$$x_i \partial_{y_j} = \frac{1}{2}(\partial_{x_i} x_i^2 \partial_{y_j} - \partial_{y_j} x_i^2 \partial_{x_i}) \in \mathcal{I}_1, \quad y_i \partial_{x_j} \in \mathcal{I}_1.$$

Lie brackets of these elements in $\mathcal{I}_1$ span all generators of $\mathcal{I}_2$. ■

LEMMA 10. *Let us consider that the group $GL_+(n, \mathbb{R})$ acts on $\mathbb{R}^n$ by left multiplication. Then the system of differential equations $\mathcal{M}_1$ defining $GL_+(n, \mathbb{R})$ -invariants is de Rham system.*

PROOF: We set $\mathcal{M}_1 = \mathcal{D}/\mathcal{I}_1$, $\mathcal{I}_1 = \sum_{i,j=1}^n \mathcal{D}(x_i \partial_j)$.

$$\partial_j = \partial_i x_i \partial_j - \partial_j x_i \partial_i \in \mathcal{I}_1 \quad (\text{for } i \neq j).$$

Then $\mathcal{I}_1 = \sum_{j=1}^m \mathcal{D} \partial_j$. ■

Now we return to the proof of (3.10). Applying Lemma 9 to (3.9) and then applying Lemma 10 to (3.8), we conclude

$$(3.11) \quad \partial_{\bar{x}_{i2}}, \quad \partial_{\bar{y}_{i2}} \in \mathcal{I} \quad (2 \leq i \leq m).$$

(3.7) and (3.11) imply (3.10). Thus $\mathcal{M}|_{\mathfrak{q}-\{0\}} \xrightarrow{\sim} \mathcal{N}|_{\mathfrak{q}-\{0\}}$. This isomorphism and following Proposition 11 imply $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$. Hence Theorem 8 is proved. ■

PROPOSITION 11. Let V be an n -dimensional vector space over $\mathbb{R}$ and $H_1 \subset H_2 \subset SL(V)$ be two closed subgroups. Suppose H_1 and H_2 are reductive in $SL(V)$, by definition ${}^tH_1 = H_1$, ${}^tH_2 = H_2$, here ${}^tH_i = \{h^{-1} \in SL(V); h \in H_i\}$. Let $\mathcal{M}_i$ be the system of differential equations defining H_i -invariants ($i = 1, 2$). We assume

$$(3.12) \quad \mathcal{M}_1|_{V_{\mathbb{C}}-\{0\}} \xrightarrow{\sim} \mathcal{M}_2|_{V_{\mathbb{C}}-\{0\}},$$

then $\mathcal{M}_1 \xrightarrow{\sim} \mathcal{M}_2$.

PROOF: $\mathcal{M}_i = \mathcal{D}/\mathcal{I}_i$, $\mathcal{I}_i = \sum_{X \in \mathfrak{h}_i} \mathcal{D}D_X$, here $\mathfrak{h}_i$ is the Lie algebra of H_i ($i = 1, 2$).

Let A be a Weyl algebra over $V_{\mathbb{C}}$ and $F : A \rightarrow A$ be the algebra automorphism defined by $x_i \mapsto -\partial_i$, $\partial_i \mapsto x_i$ (cf. [5]). By the assumption (3.12) and Hilbert zero point theorem, there exists a positive integer N such that $(x_1, \dots, x_n)^N D_X \subset \mathcal{I}_1$ for all $X \in \mathfrak{h}_2$, i.e. for any multi-index α satisfying ($|\alpha| \geq N$), there are $P_j(x, \partial) \in A$,

$$x^\alpha D_X = \sum_{X_j \in \mathfrak{h}_1} P_j(x, \partial) D_{X_j}.$$

Let $P_j^\circ(x, \partial) \in A$ be the homogeneous part of $P_j(x, \partial)$ of degree $-|\alpha|$, here we assign the degree 1 and -1 to the element ∂_i and x_i respectively. Then

$$(3.13) \quad x^\alpha D_X = \sum_{X_j \in \mathfrak{h}_1} P_j^\circ(x, \partial) D_{X_j}.$$

The transform of (3.13) by F is

$$(-\partial)^\alpha D_{-X} = \sum_{X_j \in \mathfrak{h}_1} P_j^\circ(-\partial, x) D_{-X_j}.$$

Then the left ideal $\{P \in A; PD_X \in \mathcal{I}_1 \text{ for any } X \in \mathfrak{h}_2\}$ of A contains both $(x_1, \dots, x_n)^N$ and $(\partial_1, \dots, \partial_n)^N$, but such an ideal must be A itself. This means that $D_X \in \mathcal{I}_1$ for any $X \in \mathfrak{h}_2$, hence Proposition 11 is proved. ■

3.4. The Spaces $4^\circ : Sp(p, q+1)/Sp(p, q) \times Sp(1)$

As a corollary of Theorem 8 and Remark 1,

THEOREM 12. *For $p, q \geq 1$, we have $\mathcal{M} \xrightarrow{\sim} \mathcal{N}$.* ■

4. Problem I and II

In this section, we treat Problem I and II with the H -orbit decomposition of $\mathfrak{q}$. Define $\mathfrak{q}_{reg} = \{x \in \mathfrak{q}; \text{codim}_{\mathfrak{q}} H \cdot x = 1\}$ and $S = \mathfrak{q} - \mathfrak{q}_{reg}$, then $\mathfrak{q}_{reg}$ is H -invariant open subset of $\mathfrak{q}$. Define $\omega' = \omega|_{\mathfrak{q}_{reg}}$ and $\omega_1 = \omega|_{\mathfrak{q} - \{0\}}$.

4.1.

We quote the following Proposition from [2, § 6.3].

PROPOSITION 13. In cases $3^\circ \sim 9^\circ$ except for the case 6° and $m = 2$.

- (1) $\omega' : \mathfrak{q}_{reg} \longrightarrow \mathbf{R}$ gives H -orbit parametrization, by definition, $\text{grad } \omega' \neq 0$ everywhere and $\omega^{-1}(t)$ is H -orbit for any $t \in \mathbf{R}$.
- (2) $S \subset \omega^{-1}(0)$ and S decomposes finite H -orbits.
- (3) $\omega_1 : \mathfrak{q} - \{0\} \longrightarrow \mathbf{R}$ gives $\tilde{H}$ -orbit parametrization.
- (4) As a consequence,

$$(4.1) \quad \begin{cases} \mathcal{B}^H(\mathfrak{q}_{reg}) = \omega'^*(\mathcal{B}_{\mathbf{R}}(\mathbf{R})) \\ \mathcal{B}^{\tilde{H}}(\mathfrak{q} - \{0\}) = \omega_1^*(\mathcal{B}_{\mathbf{R}}(\mathbf{R})). \end{cases} \quad \blacksquare$$

We quote the following theorem due to Cerezo [1].

THEOREM 14. A connected Lie group $SO_0(p, q)$ acts on $\mathbf{R}^{p+q}$ naturally.

If $(p, q) \neq (1, 1)$, then both

$$\mathcal{B}^{SO_0(p, q)}(\mathbf{R}^{p+q}) \longrightarrow \mathcal{B}^{SO_0(p, q)}(\mathbf{R}^{p+q} - \{0\}) \text{ and}$$

$$\mathcal{B}^{SO(p, q)}(\mathbf{R}^{p+q}) \longrightarrow \mathcal{B}^{SO(p, q)}(\mathbf{R}^{p+q} - \{0\}) \text{ are surjective.} \quad \blacksquare$$

Now we prepare Lemma 15 for the proof of Proposition 16.

LEMMA 15. Set $\mathcal{F}[0] = \{u \in \mathcal{F}(\mathfrak{q}); \text{supp } u \subset \{0\}\}$. Then in case $1^\circ \sim 9^\circ$,

$$\mathcal{B}^{\tilde{H}}[0] = \mathcal{B}^H[0].$$

PROOF: After Fourier transformation, Lemma 15 is easily shown by the fact that the ring of H -invariant polynomial on $\mathfrak{q}$ is generated by Casimir polynomial ω . $\blacksquare$

PROPOSITION 16. *In the same case in Proposition 13, if the assumption (A) : $\mathcal{B}^H(\mathfrak{q} - \{0\})[S - \{0\}] = 0$ holds, then we have $\mathcal{B}^{\tilde{H}}(\mathfrak{q}) = \mathcal{B}^H(\mathfrak{q})$.*

PROOF: Take $f \in \mathcal{B}^H(\mathfrak{q})$. For the restriction $f|_{\mathfrak{q}_{reg}} \in \mathcal{B}^H(\mathfrak{q}_{reg})$, there is a $g \in \mathcal{B}(\mathbf{R})$ such that $f|_{\mathfrak{q}_{reg}} = g \circ \omega'$ by (4.1). For $g \circ \omega_1 \in \mathcal{B}^{\tilde{H}}(\mathfrak{q} - \{0\})$, there is an $\tilde{f} \in \mathcal{B}^{\tilde{H}}(\mathfrak{q})$ such that $\tilde{f}|_{\mathfrak{q} - \{0\}} = g \circ \omega'$ by Theorem 14. Then $f - \tilde{f} \in \mathcal{B}^H(\mathfrak{q})[S]$. For $(f - \tilde{f})|_{\mathfrak{q} - \{0\}} \in \mathcal{B}^H(\mathfrak{q} - \{0\})[S - \{0\}]$, it must be zero by the assumption (A). Then $f - \tilde{f} \in \mathcal{B}^H[0] = \mathcal{B}^{\tilde{H}}[0]$, therefore $f \in \mathcal{B}^{\tilde{H}}(\mathfrak{q})$. ■

We can check the assumption (A) in some cases.

PROPOSITION 17.

- (1) *The singular set $S = \{0\}$ in cases 1°, 2°, 3°, 4° and 5°. In particular, the assumption (A) is satisfied.*
- (2) *The assumption (A) is satisfied in cases 6°, 7°.*

PROOF:

- (1) This is the definition of isotropic symmetric pairs. [2]
- (2) If $m \geq 3$, then Theorem 3 implies this. Thus we may assume $m = 2$. Since the problem is outside the origin, we adopt the coordinates (3.5). In this coordinates, $S = \{\tilde{x}_1 = \tilde{x}_2 = 0, \tilde{y}_2 \neq 0\}$ and $\mathcal{M} = \mathcal{D}/\mathcal{I} = \mathcal{D}/\mathcal{D}(\partial_{\tilde{y}_1}, \partial_{\tilde{y}_2}, \tilde{x}_1 \partial_{\tilde{x}_1}, \tilde{x}_2 \partial_{\tilde{x}_1})$, then $\underline{Hom}_{\mathcal{D}}(\mathcal{M}, \Gamma_S \mathcal{B}) \subset \underline{Hom}_{\mathcal{D}}(\mathcal{D}/\mathcal{D}\tilde{x}_1 \partial_{\tilde{x}_1}, \Gamma_{\{\tilde{x}_1=0\}} \mathcal{B}) = 0$. ■

Summing up the results, we have $\mathcal{B}^{\tilde{H}}(\mathfrak{q}) = \mathcal{B}^H(\mathfrak{q})$ in cases 3°-8° except for the case 6° and $m = 2$.

4.2. The Space : $SL(3, \mathbf{R})/S(GL_+(2, \mathbf{R}) \times GL_+(1, \mathbf{R}))$

We consider case 6°, $m = 2$. We adopt the notations in § 3.1. We introduce an element $f \in \Gamma(\mathbf{R}^4 - \{0\}, \mathcal{B})$

$$f = \begin{cases} \delta(x_1 y_1 + x_2 y_2) \{Y(x_1)Y(y_2) + Y(x_2)Y(y_1)\} \\ \hspace{15em} \in \Gamma(\{x_1 \neq 0 \text{ or } y_2 \neq 0\}, \mathcal{B}) \\ \delta(x_1 y_1 + x_2 y_2) \{Y(x_2)Y(-y_1) + Y(-x_2)Y(y_1)\} \\ \hspace{15em} \in \Gamma(\{x_2 \neq 0 \text{ or } y_1 \neq 0\}, \mathcal{B}) \end{cases}$$

Then f is H -invariant, $\square f = 0$, and

$$(x_1 \partial_{y_2} - x_2 \partial_{y_1})f = \delta(y) = \delta(y_1, y_2)$$

$$(y_1 \partial_{x_2} - y_2 \partial_{x_1})f = \delta(x),$$

thus f is not $\tilde{H}$ -invariant. Let $\theta = x_1 \partial_{x_1} + x_2 \partial_{x_2} + y_1 \partial_{y_1} + y_2 \partial_{y_2}$, then $(\theta + 2)f = 0$. We take $\tilde{f} \in \mathcal{B}(\mathbf{R}^4)$ such that $\tilde{f}|_{\mathbf{R}^4 - \{0\}} = f$. Since the mapping $(\theta + 2) : \mathcal{B}[0] \rightarrow \mathcal{B}[0]$ is bijective, there is a unique $g \in \mathcal{B}[0]$ such that $(\theta + 2)g = (\theta + 2)\tilde{f}$. We define $f_0 = \tilde{f} - g \in \mathcal{B}(\mathfrak{q})$, then $(\theta + 2)f_0 = 0$. For any $X \in \mathfrak{h}$, $[D_X, \theta] = 0$ implies that $(\theta + 2)D_X f_0 = D_X(\theta + 2)f_0 = 0$, then $D_X f_0 = 0$, i.e. $f_0 \in \mathcal{B}^H(\mathfrak{q})$. Moreover $\square f_0 \in \mathcal{B}[0]$ is homogeneous of degree -4 , then $\square f_0 = c\delta(x, y)$ by some constant $c \in \mathbf{C}$. The relation $(\square f_0)(-x_1, x_2, y_1, -y_2) = -\square f_0(x, y)$ and $c\delta(-x_1, x_2, y_1, -y_2) = c\delta(x, y)$ imply $\square f_0 = 0$. Therefore $f_0 \in \mathcal{B}_0^H(\mathfrak{q})$, $f_0 \notin \mathcal{B}^{\tilde{H}}(\mathfrak{q})$ and $\text{supp}(f_0) = \overline{N_+} = \{(x, y) \in \mathbf{R}^4; \omega(x, y) = 0, x_1 y_2 - x_2 y_1 \geq 0\}$. Conversely an element $\mathcal{B}^H(\mathfrak{q} - \{0\})$ whose support is contained in $\overline{N_+} - \{0\}$ is a constant multiple of $f_0|_{\mathfrak{q} - \{0\}}$.

Unfortunately $\omega' : \mathfrak{q}_{reg} \rightarrow \mathbf{R}$ does not give H -orbit parametrization because $\omega'^{-1}(0)$ has two connected components $N_{\pm} = \{(x, y) \in$

$\mathbb{R}^4; \omega(x, y) = 0, \pm(x_1 y_2 - x_2 y_1) > 0\}$, Therefore Proposition 13 must be modified in this case. Let $\mathbf{R}'$ be the non-Hausdorff manifold obtained by taking copies of $\mathbf{R}$, say $\mathbf{R}_+$, $\mathbf{R}_-$, and sticking together the positive parts of $\mathbf{R}_+$ and $\mathbf{R}_-$ and the negative parts of those [1]. Let $\mathbf{R}' \xrightarrow{\rho} \mathbf{R}$ be the canonical map and define $\mathcal{B}_{\mathbf{R}'} = \rho^{-1}\mathcal{B}_{\mathbf{R}}$, the sheaf of hyperfunctions on $\mathbf{R}'$. We define a real analytic mapping $\omega'' : \mathfrak{q}_{reg} \rightarrow \mathbf{R}'$ by $\omega''^{-1}(t) = \omega'^{-1}(t)$ for any $t \in \mathbb{R}^\times$ and $\omega''^{-1}(0\pm) = (N_\pm)$.

$$\begin{array}{ccccc} \mathfrak{q}_{reg} & \subset & \mathfrak{q} - \{0\} & \subset & \mathfrak{q} \\ \omega'' \downarrow & \omega' \searrow & \downarrow \omega_1 & \swarrow \omega & \\ \mathbf{R}' & \xrightarrow{\rho} & \mathbf{R} & & \end{array}$$

Then $\omega'' : \mathfrak{q}_{reg} \rightarrow \mathbf{R}'$ gives H -orbit parametrization, and therefore

$$(4.2) \quad \mathcal{B}^H(\mathfrak{q}_{reg}) = \omega''^*(\mathcal{B}_{\mathbf{R}'}(\mathbf{R}')).$$

PROPOSITION 18. *Retain the above notation.*

$$(1) \quad \mathcal{B}^H(\mathfrak{q}) = \mathcal{B}^{\tilde{H}}(\mathfrak{q}) \oplus \mathbb{C}f_0.$$

$$(2) \quad \mathcal{B}_0^H(\mathfrak{q}) = \mathcal{B}_0^{\tilde{H}}(\mathfrak{q}) \oplus \mathbb{C}f_0.$$

PROOF: (1) Take $f \in \mathcal{B}^H(\mathfrak{q})$. For $f|_{\mathfrak{q}_{reg}} \in \mathcal{B}^H(\mathfrak{q}_{reg})$, there is a $g \in \mathcal{B}(\mathbf{R}')$ such that $f|_{\mathfrak{q}_{reg}} = g \circ \omega''$ by (4.2). For $g|_{\mathbf{R}_-} \in \mathcal{B}_{\mathbf{R}'}(\mathbf{R}_-) = \mathcal{B}_{\mathbf{R}}(\mathbf{R})$, $(g|_{\mathbf{R}_-}) \circ \omega_1 \in \mathcal{B}^{\tilde{H}}(\mathfrak{q} - \{0\})$, then there is an $\tilde{f} \in \mathcal{B}^{\tilde{H}}(\mathfrak{q})$ such that $\tilde{f}|_{\mathfrak{q} - \{0\}} = (g|_{\mathbf{R}_-}) \circ \omega_1$ by Theorem 14. Then $f - \tilde{f} \in \mathcal{B}^H(\mathfrak{q})[\overline{N_+}]$. For $(f - \tilde{f})|_{\mathfrak{q} - \{0\}} \in \mathcal{B}^H(\mathfrak{q} - \{0\})[\overline{N_+} - \{0\}]$, there is a constant $c \in \mathbb{C}$ such that $(f - \tilde{f})|_{\mathfrak{q} - \{0\}} = cf_0$. Then $(f - \tilde{f} - cf_0) \in \mathcal{B}^H[0] = \mathcal{B}^{\tilde{H}}[0]$. Therefore $f \in \mathcal{B}^{\tilde{H}}(\mathfrak{q}) \oplus \mathbb{C}f_0$.
(2) It is obvious from (1) because $f_0 \in \mathcal{B}_0^H$. ■

Because f_0 is a distribution and tempered, the table in [2, § 6.3] must be corrected : case 7, $m = 3$ and $\lambda = 0$, in his notation, then $\dim D'_{\lambda, H}(q) = 3$, and $\dim S'_{\lambda, H}(q) = 3$.

以前 [8] で記したことのうち、この § 4.2 に相当する部分は誤っています。正しくは、このようになります。おわびして訂正します。

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