

**Risk-based scour assessment and
decision-making of a railway bridge pier
by use of ambient vibration**

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2025

Abstract

Scour is the localized erosion that undermines bridge foundations during flooding. Recently, scour has been one of the most critical threats to bridges crossing rivers around the world because of intense rainfall derived from climate change. Such a situation highlights the necessity of a quantitative assessment of the pier to enable quick but accurate decision-making during bridge operation. One novel technology to assess ground conditions around the pier during swollen river water periods is to identify its natural frequency using microtremors measured by accelerometers installed on top of the pier. Deploying a remote sensing system that includes sensors makes it possible for bridge authorities to make decisions regarding bridge operation in real time in their office by providing identified natural frequencies of the pier and the ratio of the observed frequency to the intact frequency. However, one of the main issues to deal with the ambient vibration of the bridge pier is that the effect of the signal noise and identification errors are relatively larger, which leads to variations in the identification of modal parameters due to the weak signal responses of the ambient vibration. Therefore, bridge authorities have to make decisions under uncertainty when using ambient vibration. Statistical approaches to deal with uncertainty are necessary, but few studies have addressed this issue in scour assessment.

This thesis, thus, intends to propose indices for scour assessment based on statistical approaches. Four data-driven stochastic indices and one scour depth prediction method using the concept of Digital Twins are proposed and applied to two case studies: an RC specimen experiment for scour simulation and a long-term vibration monitoring of a railway bridge pier, to demonstrate the validity of the proposed methods. Each index is proven to clarify abnormalities by the specimen experiment, and the validity and characteristics of the indices are summarized by applying them to the actual long-term observations.

Because the outputs of these indices potentially have statistical meanings, risk-based decision-making is also proposed as a comprehensive framework for scour assessment using ambient vibration. However, risk-based approaches are sometimes insufficient because decision makers do not always select actions with minimum costs when high safety is required like bridge operations. Therefore, this study considers an additional option of sending engineers to the site into the proposed framework and discusses its feasibility and challenges.

Acknowledgements

This dissertation presents the research that the author has carried out since the entrance to the PhD course in the Department of Civil and Earth Resources Engineering at Kyoto University. It is great honor to have an opportunity to receive profound lectures and advice from my supervisor, Professor Chul-Woo Kim. I would like to express my best gratitude and appreciation to his patient guidance for broad topics including the perspectives of the research area and how to apply our research idea to the real world. I also appreciate his acceptance of entrance and valuable help nevertheless the author was in the parallel career of the civil engineer in the venture company and research work in the University.

I would like to express my deep appreciation to Professor Kunitomo Sugiura and Professor Takashi Yamamoto for their exertion as vice supervisor in my PhD course.

I would like to express my appreciation to Emeritus Professor Tadanobu Sato in Kyoto University, Emeritus Professor Mitsuo Kawatani in Kobe University, and Associate Professor Kai-Chun Chang in Kyoto University for their helpful advice and insightful discussions in the meeting in our laboratory.

I would like to express my appreciation to Professor Sarhosis Vasilis in University of Leeds for his helpful advice of our collaborating research and many supports for short staying in U.K. in the chaos of a pandemic.

I would like to express my appreciation to Mr. Shinji Kitagawa in Fuji Electric Co., Ltd. and Mr. Yoshisada Hamada in West Japan Railway Company for their cooperation on the field experiment for scour monitoring and advice about the related topics. I would also like to express my appreciation to Mr. Zhihao Wang and Mr. Kei Akutsu, who are alumni of our laboratory, for their cooperation as a member of risk-based Structural Health Monitoring team.

I would like to express my appreciation to Dr. Yoshinao Goi in Kyoto University, Dr. Soichiro Hasegawa in Kawasaki Heavy Industries, Ltd., Dr. Gen Hayashi in Osaka Metropolitan University, Dr. Shinya Kimura in Eight-Japan Engineering Consultants Inc., and other members of our laboratory for their great advices on the research and supports for my research life.

I also would like to express my gratitude to my company members, Dr. Kazuo Takase, Dr. Hirotaka Oda, and Mr. Katsutoshi Nakano for giving me continuous supports to my research works.

Finally, I would like to show my appreciation to my family for continuous and kind supports and encouragements.

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Chapter 1. Introduction

1.1. Background

1.1.1. Railway bridges

Most of Japan's areas are mountainous, and the development of a network requires many bridges, tunnels, and other structures. The railway network in Japan has been rapidly developed since 1872 when the first railway was opened. The number of railway bridges in Japan is approximately 114,000, and the ratio of river crossing bridges is 31% [1]. Due to the geographical conditions of the land, river bridges are indispensable structures for improving railway networks. Therefore, many bridges were constructed at that time and are still in use. These old bridges are crucial to economic development and still play a vital role in supporting society. However, under immature technology and economic conditions, many old bridges were built in the simplest type with relatively short spans and shallow foundations. Although many civil engineering railways currently in service were developed and improved during the rapid economic growth after World War II, they are deteriorating with age. In particular, increasing the live loads of passing vehicles may cause a lack of redundancy in the load-bearing capacity. Furthermore, the amount of heavy and intensive rainfall over a short span (called “ゲリラ豪雨” in Japanese) has been increasing as shown in Figure 1.1. It is forecasted that extreme weather events will occur more frequently as global warming progresses [2]. These events may cause severe damage to bridges, and the repair cost may increase. With a sophisticated logistics network already in place, long-term suspended traffic would result in significant economic losses and user disruptions, despite less severe damage.

Thus, while railway bridges play a critical role in economic activities, they are at high risk of damage due to the fragility of the structure itself and of collapse due to localized heavy rainfall caused by climate change. In particular, river-crossing railway bridges may cause localized erosion, called “scour”, which undermines bridge foundations during flooding. Scour is considered as one of the most critical threats for bridge crossing rivers, not only in Japan where mountainous terrain and fast-flowing rivers exists, but also regions where scour risks have not been considered due to few rainfall events. The presents social issues highlight the need for the development of novel and reliable approaches for detecting scour effects to ensure the

continued safe operation of the aging bridge asset network.

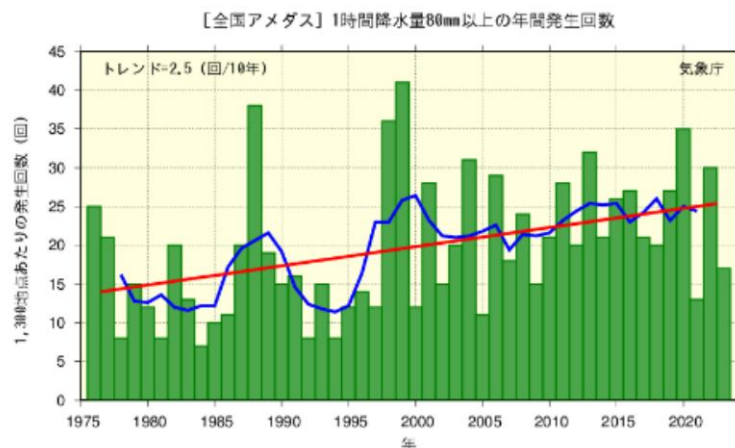


Figure 1.1 Time-series of the amount of annual intensive rainfalls (1976 to 2023) [2].

1.1.2. Bridge scour

This section briefly explains the phenomenon of bridge scour. Scour is defined as the excavation and removal of material from a stream bed or bank by the erosive action of flowing water. Scour has three main forms: General scour, contraction scour, and local scour. Among these three forms, local scour occurs around individual bridge piers and abutments. It is caused by horseshoe vortices forming at the base of the pier [1, 3, 4]. The flow obstruction created a stagnation line on the front of the pier. According to fundamental fluid dynamics, the stagnation pressure is proportional to the square of the local velocity and is lower near the bed. Therefore, a downward hydraulic gradient is developed in front of the pier, and the water flow goes down directed towards the riverbed, as shown in Figure 1.2. This vortex system removes riverbed material from around the pier's base, resulting in a local scour hole. Briaud et al. [5] proposed a method for calculating the maximum scour depth from water velocity, obstacle geometry, and soil type. The horseshoe vortex decreased with increasing scour strength. In summary, scouring is the excavation and removal of material from stream beds and banks by the erosive action of flowing water.

Many railway bridges in Japan are damaged by scouring. Murakami [6] summarized past cases of bridge scouring. In recent years, on October 22, 2017, an accident occurred in a train

derailment along the Nankai main line (from Tarui station to Ozaki station). Five passengers were injured in the accident. It is presumed that this accident was caused by scouring around the foot due to the increased river water, which led to subsidence and inclination of the bridge [7]. Many similar catastrophic damage occurred on railway bridges resulting from rising water levels, such as the Hitaka mainline inflicted by Typhoon No.10 in 2003, Etsumihoku line inflicted by heavy rain in July 2004 and Takayama mainline infected by Typhoon No.23 in October 2004. According to the Ministry of Land, Infrastructure, Transport and Tourism in Japan (hereafter MLIT) reported that totally 47 railway bridges of 6 Japan Railway companies were severely damaged due to intensive rainfall in the past 20 years [8].

For reference, road bridges across rivers in Japan have also been facing the risk of scouring. MLIT has published results on bridge damage due to intensive rainfall [9, 10]. Thus, bridge damage caused by extreme rainfall is frequent and poses serious challenges to stable operation.

In the United Kingdom, more than 100 bridges collapsed because of scouring during flooding, causing 15 fatalities and unquantified economic losses to bridge owners and the public. There are still several noticeable incidents such as the tragic railway bridge collapse in Lancashire in June 2002, which highlighted the need for the development of novel and reliable approaches for detecting scour effects to ensure the continued safe operation of the aging bridge asset network.

The analysis regarding scoured bridges by MLIT [8] should be noted here for the detailed understanding of the important characteristics of the scouring. As shown in Figure 1.3, most bridges damaged by intense rainfall were built before 1945. Among them, approximately 80 % of the bridges that suffered scour damage were of the shallow foundation type, as shown in Figure 1.4. These analyses show that railway bridges at high risk of scouring are older and of the shallow foundation type. Changes in the location of flowing water are also major causes of damage, and scour risk must be evaluated by comprehensively considering the foundation types and surrounding environments. Some studies [10-12] have summarized the key factors for extracting bridges with high scouring risks. In the USA, nearly 21,000 bridges are reported to be susceptible to have their foundations undermined during extreme storm events [13]. A large number of bridges face the threat of scouring.

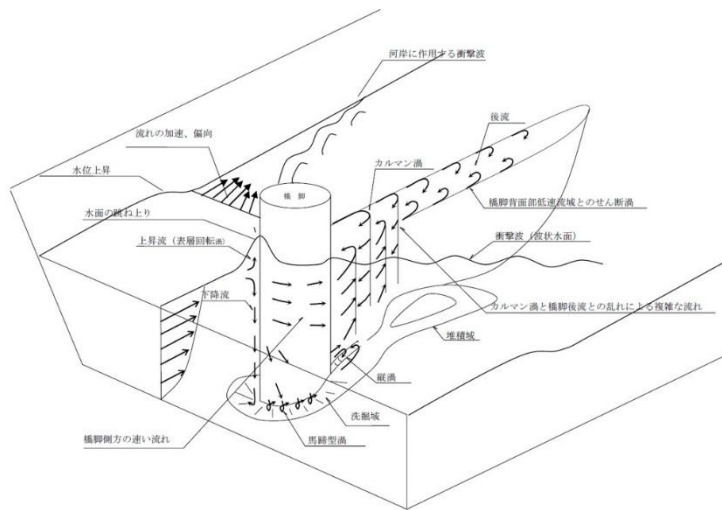


Figure 1.2 Image of local scour around the pier [3].

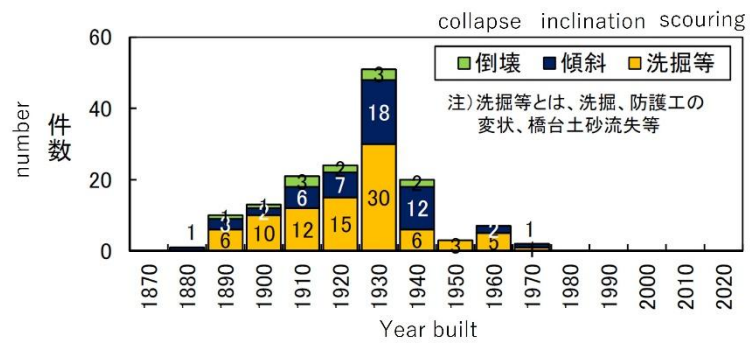


Figure 1.3 Number of damaged bridges by construction year [8].

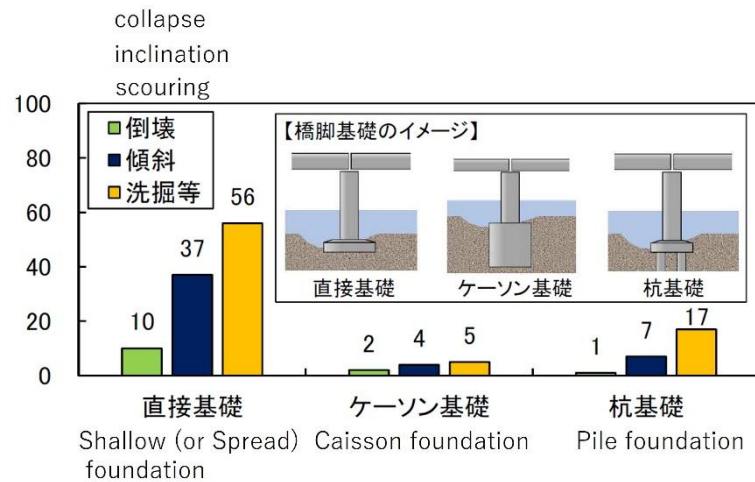


Figure 1.4 Number of damaged bridges by foundation type [8].

1.1.3. Scour assessment of railway bridges

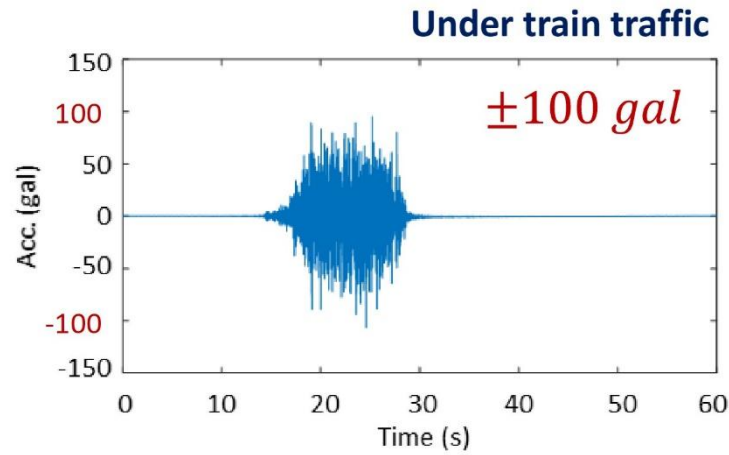
Various methods are available for assessing the stability of bridge pier including bridge scour. Generally, the methods for scour depth estimation of bridges are classified into direct inspection, nondestructive methods, and vibration-based methods. Direct inspection usually involves diving underwater to observe the conditions of the ground around the pier or abutments during the dry season. However, it is labor-consuming and impossible to implement during the water-rise period. To improve the accuracy of scour depth identification, nondestructive methods that install several pieces of equipment into the piers or abutments to estimate the scour depth, such as radar, Fiber Bragg Grating sensors [14], and sound-wave devices, have been developed. Although each device has advantages, all instruments need to be fixed on the pier or buried into the streambed. Therefore, they can only be used periodically for routine inspections.

Japanese railway companies have conducted a vibration-based method called impact test [1, 15-18]. The impact test first prepares the accelerometers and installs them on the pier. Next, around 30 kg weight is hit to the pier against the transverse direction to excite the vibration of the pier. The vibration is monitored by accelerometers. Then, considering the results of the Fast Fourier Transform and Phase differences of the acceleration responses, the rocking frequency at the transverse direction of the pier is identified. Finally, by comparing the previously identified frequency with the observed frequency, changes in the pier stability can be obtained. Since impact test uses the stiffness of the basement of the pier as an index, the stability of the pier can

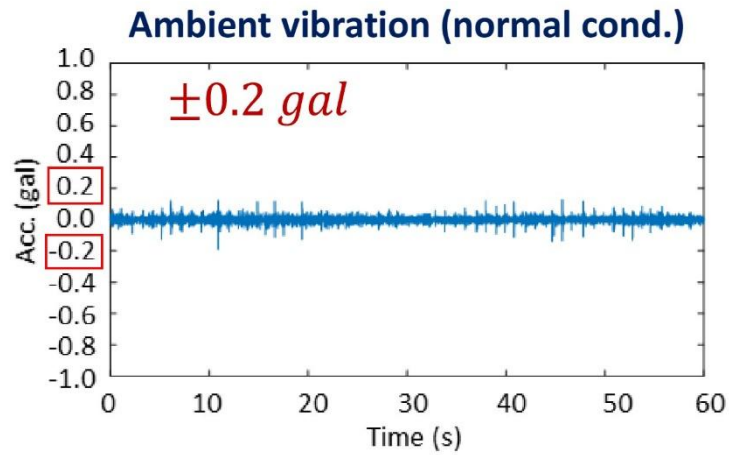
be quantitatively evaluated. However, it is a time- and labor-consuming method. It is inapplicable during the water-rise period for safety reasons. Therefore, when railway authorities make a decision whether operation would resume or not after the operation suspends due to water rising, it sometimes takes time to judge the operation resumption because they have no information on the state of the bridges. In the case of economically important bridges across rivers, economic impacts and user disruptions due to long-term operation suspension are significant. The impact test still has the improvement from the aspect of stable operation.

A real-time monitoring scheme using accelerometers installed on top of piers has been considered as an alternative method for scouring assessment during flooded rivers [19-22]. Using a system identification method with a remote sensing system, the natural frequency of the monitoring can be identified in a timely manner, regardless of the water level of the river. Real-time vibration monitoring can assist bridge owners and authorities to make decisions regarding traffic operation conditions more effectively, saving money and improving the resilience of transport networks.

When we consider the vibration of a railway bridge pier using an accelerometer installed on top of the pier, train-induced vibration and ambient vibration can be measured, as shown in Figure 1.5 as an example. Train-induced vibration cannot be measured during the operation suspension period due to rising water. Therefore, ambient vibration is used to identify the modal parameters of the monitored pier during flood period. However, the Signal to Noise Ratio (hereafter SN Ratio) of ambient vibration on top of the bridge pier at normal condition is relatively small because the exciting force against the pier is insufficient. This means that the effect of signal noise and identification error is relatively larger, and it is known that variations in the identification of modal parameters usually occur [20-22]. Bridge authorities must make decisions under uncertainty when modal parameters identified from the ambient vibration of bridge piers are used.



(a) Traffic-induced vibration at normal condition.



(b) Ambient vibration at normal condition.

Figure 1.5 Measurable vibrations using an accelerometer installed on top of the bridge pier.

1.1.4. Scour risk assessment of railway bridges using ambient vibration

One valid approach to address the difficulty associated with using ambient vibration is uncertainty quantification. Although there are several output-only operational modal analysis methods (hereafter OMA methods) like Stochastic Subspace Identification Method (hereafter SSI) [23, 24], Fast Bayesian Fast Fourier Transform (hereafter Fast Bayesian FFT) method [25-30] is a well-known and useful method because the identified modal parameters can be expressed as a posterior distribution in which the mean and variance values are regarded as Most Probable Values (hereafter MPVs) and the quantitative uncertainty information.

Reducing uncertainty is an important factor in reducing wrong decision-making. To reduce the uncertainty, it is effective to fuse the modal parameters of the monitored structure into different information. For example, the modal parameters of the monitored structure are affected by not only the signal noise but also temperature, wind, and other environmental factors [31-35]. Fusing these factors allows us to reduce the uncertainty of the identified modal parameters. However, if we consider a bridge vibration monitoring system that includes the time-series of such environmental information, the system would become huge and might require high running costs.

Thus, two approaches are considered. The first approach is to adapt the variances of the identified results in the modal parameters through a monitoring campaign as the uncertainty as well as the posterior distribution using the Fast Bayesian FFT method. The observation includes two uncertainties; one is the posterior distribution given at each time step, the other is the scattered MPVs obtained after a certain period. It is rational and convenient to discuss both uncertainties according to statistical theory. Although it is of great significance to statistically treat the natural frequencies of monitored piers identified by ambient vibration and to provide them as a probabilistic index, these studies are limited.

The second approach considers a decision-making of sending engineers to the site. The most fundamental and vital information regarding the situation of the site can be known by sending engineers. When the identified modal parameter by ambient vibration includes uncertainty, some of them are difficult to judge whether the output can be said to be “clearly healthy” or “clearly abnormal”. In the maintenance of infrastructure that requires a high level of safety like bridges, safety side decisions should be adapted like operation suspension when such information receives. However, the frequency of operation suspension will increase the number of intensive rainfall events in the future, leading to economic losses and user disruptions. If such ambiguous information is received continuously, an effective approach would be to send engineers to the site to check the situation of the bridge. Sending engineers can provide important information that assists in decision-making on the operation in most cases after arriving at the site. In other words, when comparing operations suspension decisions and sending engineers, it can be regarded as comparing the losses caused by economic losses by operation suspension with the risks reduced by the availability of accurate information through sending inspectors. Quantifying the risk due to uncertainty in the observed values and the effect of sending engineers can be used to make decisions regarding engineer sending to the site. There

are many factors we should consider like the location of the monitored bridge, its importance level, the cost of labor, etc. The development of such a framework would be very significant to improve the accessibility of the ambient vibration of the pier for scour detection and the accuracy of decision-making. However, few such frameworks have been systematically developed.

1.2. Objectives of this study

To judge the scour occurrence and the operation resumption of the monitored bridge based on the changes in the identified natural frequencies by ambient vibration, it is necessary to use a stochastic approach to deal with the uncertainty of the identified frequencies and build a comprehensive framework, including the action of sending engineers to the site. Therefore, this thesis has three major objectives.

The first objective is to propose a data-driven stochastic scour warning index that deals with the uncertainty of identified natural frequencies. First, two scour warning indices utilizing Fast Bayesian FFT method are proposed. Next, following the idea of the probability of failure in reliability engineering, Probability of Scour Occurrence (hereafter PSO) that calculates the probabilities that scour would occur using identified natural frequencies in real time, is proposed. Furthermore, because it is claimed that the conventional statistical hypothesis testing in Structural Health Monitoring that can obtain outputs sequentially potentially tends to show the false negative, the Bayes factor, which is one of the ideas to avoid wrong judges based on Bayesian stochastics, is applied to the observation for anomaly detection.

The second objective is to propose a statistical way of predicting scour depth as practical information on structural performance by identifying natural frequencies using Digital Twins. To output information on structural performance like scour depth, is more intuitively understandable than to show changes in the identified natural frequencies of the monitored pier. To realize accurate decision making, not only data-driven approaches but also providing information on structural performance using Digital Twins can play an important role in assisting bridge authorities in making decisions. Thus, this thesis proposes a method for predicting scour depth from identified natural frequencies during a flood period by using the relationship between scour depth and natural frequency obtained by Finite Element Model (hereafter FEM) of the target pier updated by observation at normal period.

The third objective is to build a comprehensive risk-based framework for decision-making on bridge operations related to scouring during the flood period. The selection of actions can be determined by comparing the estimated value of the risk given outputs, the reducing value of the risk by sending engineers, and the cost of sending engineers. This thesis summarizes comprehensive ideas of bridge scour monitoring and describes the necessary procedures for sequential decision-making during flood periods.

To demonstrate the validity of the proposed methods, this thesis conducted an RC specimen experiment to simulate scouring by excavating the ground around the specimen, and a long-term railway bridge pier vibration monitoring.

1.3. Outline

This thesis is composed of 6 chapters. Chapter 1 (this chapter) introduces the background and objectives of this thesis.

In Chapter 2, two case studies on bridge scouring are presented. The first case study involved an experiment with a Reinforced Concrete (hereafter RC) pier specimen to simulate the scour. We conducted an experiment to simulate bridge scouring by excavating the ground around the RC pier specimen. Ambient vibration was measured on top of the specimen at each excavation step to identify its natural frequency. Chapter 2 also introduces the FEM for numerical analysis. The second case study involved long-term vibration monitoring of a railway bridge pier. We performed long-term microtremor measurements on a single railway bridge pier. This chapter provides bridge information, flood information during monitoring campaigns, and FEM.

Chapter 3 discusses data-driven approaches. Two Scour Warning Indices that use the statistically modeled natural frequencies of the bridge pier by using ambient vibration are proposed and applied to two case studies. Then, a method for calculating the Probability of Scour Occurrence using the probability distribution of natural frequencies at normal conditions is proposed and applied. The objective of proposing Probability of Scour Occurrence is to facilitate an intuitive understanding of whether scour is occurring. Since we have seen the probability of rainfall tomorrow on the weather forecast, or the probability of failure in reliability engineering, showing the probability of scour occurrence makes it easier for engineers to decide whether they should act or take a wait-and-see. Furthermore, Bayes Factor, which is useful for comparing the two models, is also introduced and applied. Bayes Factor has been widely used as one of the solutions to the problems of conventional statistical hypothesis testing. Therefore, this chapter compares the Bayes factor with conventional testing and applies it to two case studies.

Chapter 4 proposes a method for predicting the scour depth around the river-crossed bridge pier where the monitoring system is deployed for real-time assessment during a flood period. In addition to data-driven scouring detection, real-time structural performance information, such as scour depth, is also crucial for faster and more accurate decision making when suspending or resuming traffic under uncertainty. As a fundamental case study, the RC specimen scour experiment is used to verify the validity of the proposed method. The proposed method is, then, applied to the second case study of real bridge monitoring measurements.

Chapter 5 discusses a comprehensive risk-based framework for decision-making on bridge

operations related to scouring during flood period.

In Chapter 6, the summary of this thesis is presented with future perspectives for the scour assessment of bridge piers using ambient vibration.

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Chapter 2. Experiment and Long-term Monitoring for the Assessment of Bridge Scour

2.1. Case study 1: RC pier specimen experiment for scour simulation

This study conducted an experiment to simulate bridge scouring by excavating the ground around the Reinforced Concrete (hereafter RC) pier specimen to express the effectiveness of using microtremors measured by accelerometers installed on the target pier. This chapter describes the experimental conditions and numerical model information.

2.1.1. Overview of the RC specimen and the ground

The RC specimen simulating a bridge pier has 2000mm wide, 1000mm deep (here after, “side depth” for avoiding complication), and 2200mm high. The weight is approximately 80kN. The geometry simulates a pier and footing. An overview of the specimen is shown in Figure 2.1. The support soil at the bottom of the footing was compacted after lime was stirred into the Narita Sand. The Portable Falling Weight Deflectometer test (hereafter PFWD test) [1] was conducted to obtain the ground stiffness, and the averaged converted coefficient of subgrade reaction K_{30} is $55.2MN/m^3$. After the test, the specimen was placed and backfilled to a specified height to simulate the embedded depth. The initial embedded depth was 1200 mm. The backfill material was Narita Sand with a compaction density of 90%. A photograph of the intact step is shown in Figure 2.2.

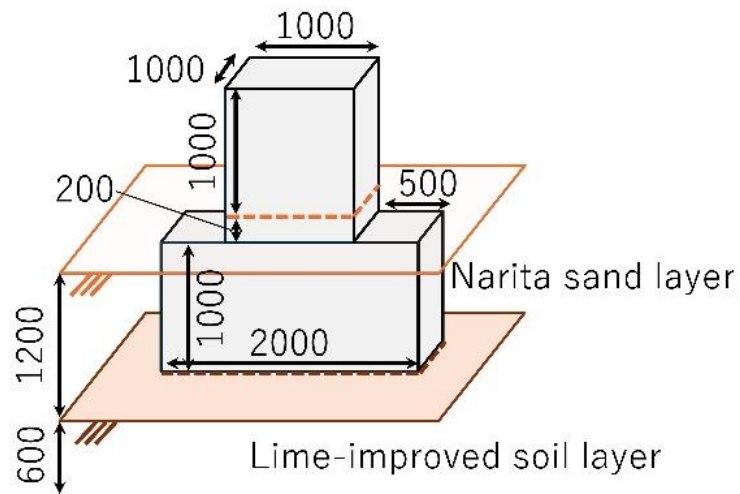


Figure 2.1 Size of the RC pier specimen (unit mm).



Figure 2.2 Photo at intact step with locations in accelerometers.

2.1.2. Ground excavation steps

To simulate the scour of the bridge pier, 11 excavation steps, including intact step, were considered. The excavation steps are summarized in Figure 2.3. Step 1 shows the intact step, and each step is followed by further excavation. Up to Step 4, scouring at embedded area is simulated. The excavation geometry was uniformly excavated around the specimen. Steps 7 and 8 simulate progressing scour at the bottom of the footing. Steps 9, 10, and 11 are backfilled to the locations of Step 5, 3, and 1, respectively.

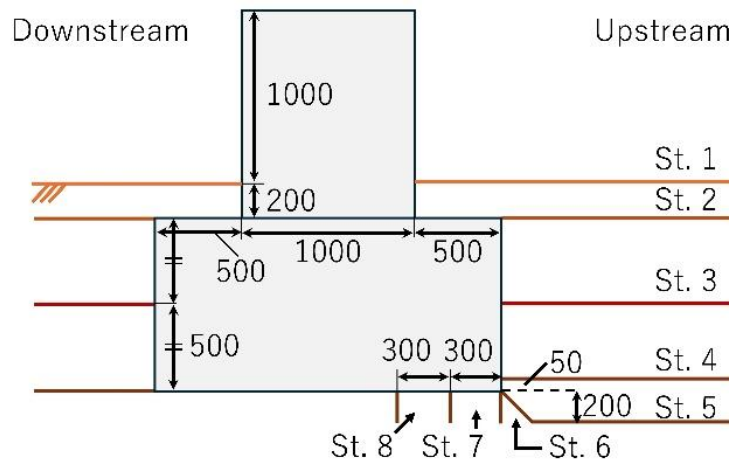


Figure 2.3 Excavation steps (unit mm).

2.1.3. Impact and sweep tests

To obtain the true natural frequencies of the specimen at each excavation step, impact and sweep tests were conducted. The impact test was performed by hitting the top of the downstream side of the specimen with a hammer. The vibration response was measured by accelerometers and the rocking frequency in the transverse direction of the pier was identified based on the analysis method described in the Annex. 7 in the Maintenance Standard for Railway Structures and Commentary [2]. Generally, a 30kg weight is used to excite the vibration of the bridge pier in site, but Human-induced impact tests using a wooden hammer were performed in this study.

The Sweep test is also performed at each excavation step. A small shaker is installed and fixed

on the central position at the top of the specimen and sweep vibration excitation was applied in the transverse direction. The rocking frequency in the transverse direction was identified by obtaining the maximum resonance point.

The identified rocking frequency of the specimen by impact and sweep tests at each excavation step is shown in Table 2.1. Both the impact and sweep test results in Step 1 which is the intact condition of the scour were 21.6 Hz. Furthermore, the frequencies decreased as the excavation steps proceeded. The lowest frequency by impact test was observed at Step 8, and the value was 9.3Hz. The identified frequencies at Steps 9, 10, and 11 of the backfilling processes were recovered, which illustrates an increase in the ground restraints. However, the identified frequency at Step 11 which backfilled up to the intact condition did not reach at 21.6 Hz of the frequency observed at Step 1.

Although the results of the impact and sweep tests showed an absolute difference of 0.9 Hz in Step 11, the differences were kept below 0.4 Hz in the other step. Therefore, it can be concluded that the rocking frequency identification results of both test methods are generally in agreement. This study, then, uses the results of an impact test that is adopted in field as true values.

Table 2.1 Results in identified rocking frequency by impact and sweep tests at each excavation step (unit Hz).

Step	1	2	3	4	5	6	7	8	9	10	11
Impact test	21.6	19.8	13.2	9.8	9.9	9.6	9.4	9.3	9.6	12.6	21.3
Sweep test	21.6	19.4	13.5	9.7	9.6	9.2	9.2	9.1	9.5	12.9	20.4
Absolute difference	0	0.4	0.3	0.1	0.3	0.4	0.2	0.2	0.1	0.3	0.9

2.1.4. Natural frequency identification by use of microtremors

Firstly, sensor information is introduced in this section. Two tri-axial accelerometers [3] were installed on top of the RC specimen, one on the upstream side and the other on the downstream side. The locations of each sensor are also noted in Figure 2.2. The sensor is a MEMS-type acceleration sensor with a capacity of about 14.71 m/s^2 , resolution of $2.00 \times 10^{-4} \text{ m/s}^2$ in horizontal directions (X and Y directions), and resolution of $7.00 \times 10^{-4} \text{ m/s}^2$ in the gravity direction (Z directions). The sampling frequency was 200 Hz, and 24 hours measurements are

carried out. This study divides acceleration responses of transverse direction of two sensors into one minute as one data sample, and identifies rocking frequencies for each sample. This study identifies modal parameters by use of Fast Bayesian Fast Fourier Transform (hereafter Fast Bayesian FFT) method which enables us to provide precise results with information on identification uncertainty. The details of the Fast Bayesian FFT method can be summarized in **Appendix A1**.

The result in identified rocking frequencies by use of microtremors is shown in Figure 2.4. The vertical axis represents frequency, and the horizontal axis represents excavation steps. Each black plots show the identification results, and the information on identification uncertainty is represented by the error bars in each plot. The blue lines are the same as results in impact test as shown in Figure 2.4. The reason why Steps 2,4,6, and 10 have many plots is that the acceleration responses were captured during nighttime. Identification results other than Steps 1, 2, and 11 have relatively small variations with respect to the identification results of Steps 1, 2 and 11. This may be caused by the vibration responses due to the removal of the specimen-side ground restraints.

In Steps 1, 2, and 11, identification results by microtremors tend to be lower than those by impact tests. On the contrary, results from Step 4 to 10 by microtremors tend to be higher than those by impact tests. However, through the experiment, the identification results by microtremors generally followed the impact test results. Thus, it is concluded that microtremors are useful for scour monitoring.

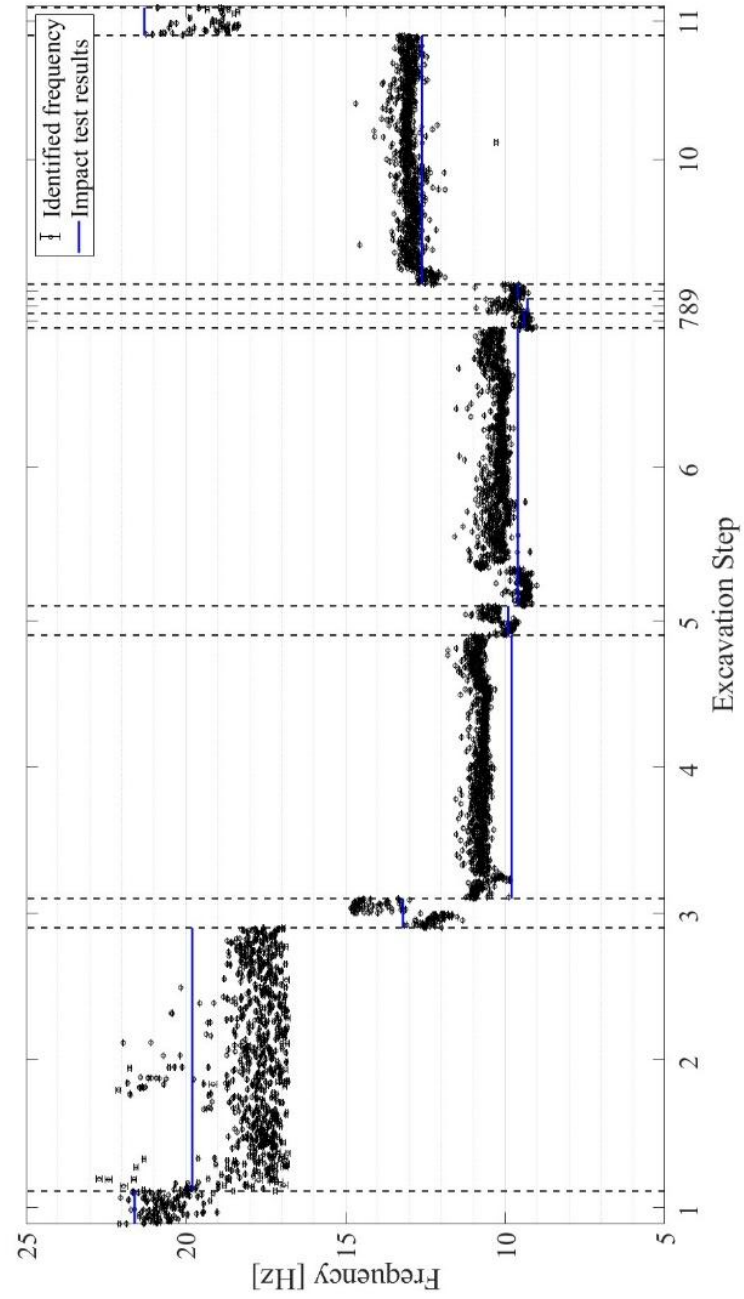


Figure 2.4 Results in identified rocking frequencies with uncertainties by use of microtremors.

2.1.5. Finite Element Model

This study also proposes a method for predicting scour depth with the aid of numerical analysis. Therefore, this chapter outlines the Finite Element Model (hereafter FEM) used in the numerical analysis of the RC pier specimen. Since the purpose of this study is to evaluate the scouring of the monitored pier, expressing the soil-structure interaction using FEM is important. Therefore, the RC pier specimen is modeled with three-dimensional beam elements and is not modeled with the reinforcing bars inside the concrete. Instead, the mass density of reinforced concrete is used. The material properties of the concrete are summarized in

Table 2.2. In addition, to express the soil-structure interaction, soil springs are introduced in the model. Based on the Design Standard for Railway Structures and Commentary [4], vertical, shear, rotational, and horizontal soil springs are placed in the positions shown in Figure 2.5. The shear, vertical, and rotational soil springs are placed at the center of gravity at the bottom of the footing, and the horizontal soil spring is placed at the center of the side of the footing. The values of the soil spring constants as design values are used as shown in Table 2.3 according to the PFWD test results and following the procedures in reference [1, 4]. The soil spring constants can be calculated as follows:

- Vertical soil spring constant (k_v)

Vertical soil spring constant can be expressed as:

$$k_v = \rho A_v^{\frac{5}{8}} \quad (2-1)$$

where ρ is the modified coefficient determined by soil properties and the soil penetration test. A_v denotes area at the bottom of the footing. Although the top area of the footing also resists vertical movement, its effect is negligible because the embedded depth is usually shallow. Therefore, k_v can be regarded as a function of the area of the bottom of the footing A_v . The PFWD test was already conducted in this study. Therefore, it is calculated based on the result.

- Shear soil spring constant (k_s)

Shear soil spring constant can be expressed as:

$$k_s = \lambda k_v \quad (2-2)$$

where λ is convention factor that has obtained around 1/3 by theoretical and experimental investigation in the past. This study uses $\lambda = 1/3$.

- Rotational soil spring constant (k_r)

Rotational soil spring constant can be expressed as:

$$k_r = k_v I_v = \frac{k_v B L^3}{12} \quad (2-3)$$

where I_v is a moment of inertia of the bottom of the footing, B and L denote side depth and width length of the footing, respectively.

- Horizontal soil spring constant (k_h)

Horizontal soil spring constant can be expressed as:

$$k_h = \rho A_h^{\frac{5}{8}} \quad (2-4)$$

where ρ is the modified coefficient that is determined by soil properties and the soil penetration test. A_h denotes the area at the front side of the footing.

Based on the above set-up conditions, an eigenvalue analysis was performed, resulting in a rocking frequency in the transverse direction of 15.17 Hz. Since the rocking frequency observed in the impact test is 21.6 Hz as shown in Table 2.1, and there is large difference between them, updating the values of soil spring constants are necessary.

Table 2.2 Material properties of the concrete.

Property name	value	unit
Young's modulus	3.0×10^4	N/mm^2
Poisson ratio	0.2	
Mass density	2.5×10^{-6}	kg/mm^3

Table 2.3 Design values of each soil spring constant.

Soil spring	Spring constant	unit
Vertical (k_v)	1.80×10^5	N/mm
Shear (k_s)	6.00×10^4	N/mm
Rotational (k_r)	6.75×10^{10}	$N \cdot mm$
Horizontal (k_h)	1.60×10^5	N/mm

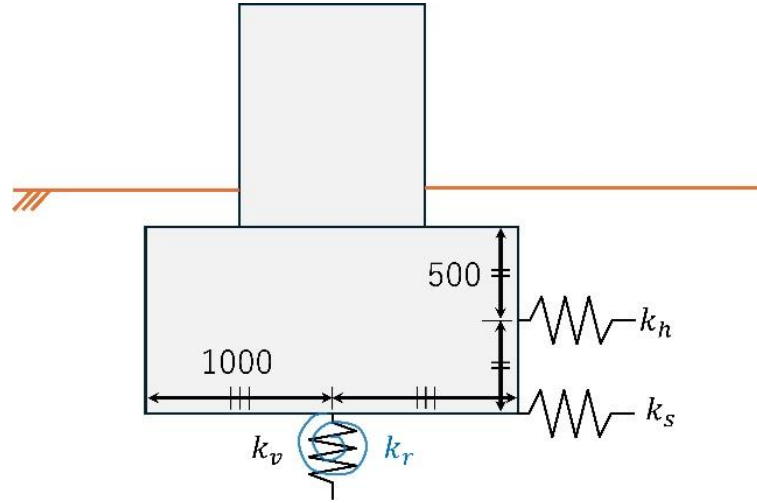


Figure 2.5 Locations of the soil springs in FE Model (unit: mm).

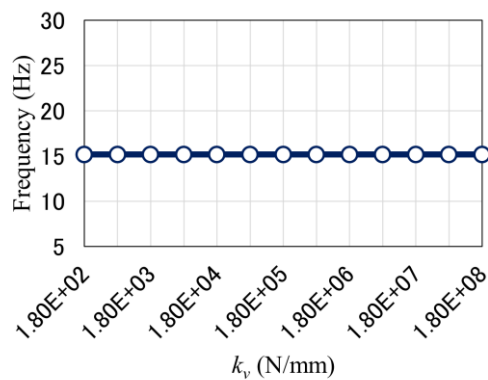
2.1.6. Sensitivity analyses and updating soil spring constants

Sensitivity analysis was conducted to investigate changes in rocking frequency in the transverse direction with each soil spring in Step 1. A model for which parameters were set as design values was taken as the reference state to illustrate changes in rocking frequency. The results of the sensitivity analysis are shown in Figure 2.6. The horizontal axis shows the spring constant, and the vertical axis represents the eigenvalue. The vertical and shear springs seldom affect change in eigenvalues as the spring constants change. On the other hand, the rotational and horizontal soil springs affect the eigenvalue as the spring constants change. The rotational soil spring constant is the most sensitive spring to the rocking frequency. The horizontal soil spring was the second most sensitive spring, and it did not reach at 21.6 Hz that is results in impact test shown in Table 2.1. In the Design Standard for Railway Structures and Commentary [4], the upper limit of the horizontal coefficient of subgrade reaction is defined based on the standard of degree of compaction because the soil resistance at the side of the footing is dominated by the physical properties of the backfill materials. Therefore, this study considers the horizontal soil spring constant converted from the PFWD test as the true value and does not update it.

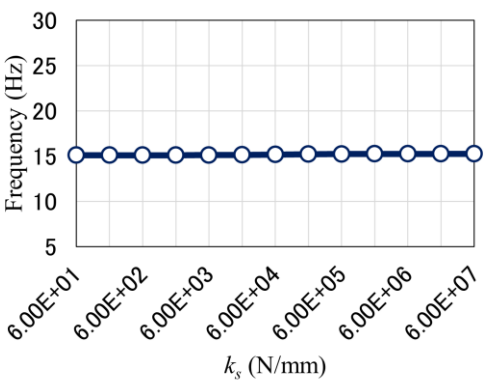
Consequently, this study updated only the rotational soil spring constant. The rotational soil spring constants and eigenvalues before and after the model updating are listed in Table 2.4.

Table 2.4 Rotational soil spring constants before and after model updating and results in eigenvalue analyses.

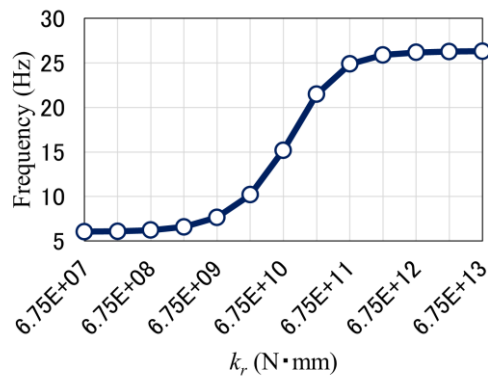
	Before model updating	After model updating
Rotational soil spring constant ($N \cdot mm$)	6.75×10^{10}	2.19×10^{11}
Rocking frequency (Hz)	15.2	21.6



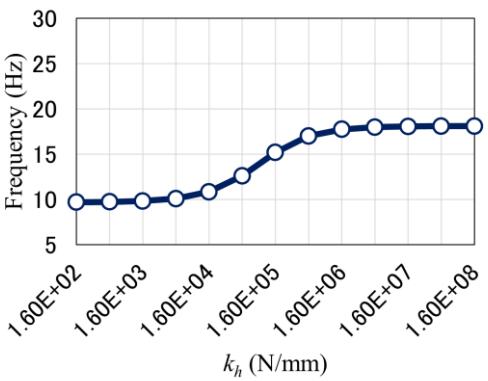
(a) Vertical soil spring



(b) Shear soil spring



(c) Rotational soil spring



(d) Horizontal soil spring

Figure 2.6 Results in sensitivity analyses of each soil spring against rocking frequency of the transverse direction.

2.1.7. Discussion of modeling the scour around the pier with shallow foundation

Since scour has a significant impact on the stability of piers, many studies reported on scour related research experimentally and analytically [5-8]. However, as Tubaldi et al. [9] points out, the study on scour for piers with shallow foundation, where the risk of scour is more significant, is surprisingly limited. This means that the influence of footings on the modal characteristics of piers when scour is in progress is not fully discussed. Generally, when the mass and volume of the footing are sufficiently large in relation to the pier, the resistance to rotation and sliding is so huge that the change in vibration characteristics is small when scour progresses within the embedded depth. Scozzese [7] et al. conducted an analytical study of an actual case of scour damage to a masonry arch bridge with shallow foundations and showed that the modal parameters may change rapidly after the scour reaches the bottom of the footing. In other words, they showed that the change in modal parameters is small in the progression within embedded depth. Tubaldi et al. [9] carried out a field experiment on EuroProteas, a structural prototype with shallow foundation that was subjected to increasing levels of scours and showed the modeling strategies for bridge scour with shallow foundations.

In previous studies, however, a clear way of setting soil springs numerically in scour situations of bridge pier with shallow foundations, specifically for design objectives, could not be found. This study, thus, aims to discuss a way to express the scour by decreasing the values of four soil spring constants of numerical model following the Design Standard [4].

For each of the four soil spring constants, the area of resistance is assumed as follows, using the area resisting each of the four soil spring constants. The schematic diagrams illustrating the decreasing trends of each soil spring constant in the RC pier specimen are shown in Figure 2.7.

- Vertical soil spring constant (k_v)
 k_v is a function of the area of the bottom of the footing A_v as shown in Equation (2-1). In other words, k_v does not change until the scour reaches at the bottom of the footing. For the RC pier specimen, k_v could not be changed until Step 6 but is rapidly decreased after Step 6 due to the loss of the bottom area of the footing.
- Shear soil spring constant (k_s)
 k_s does not change until the scour reaches at the bottom of the footing but is rapidly decreased after Step 6 in case of RC pier specimen like k_v .
- Rotational soil spring constant (k_r)
 k_r represents the rotational soil spring constant about the axis at the center of the bottom

area of the footing in the transverse direction. Therefore, the resisting area primarily consists of half of the bottom footing, the front area of the footing, a part of top area of the footing, and the front area of the pier, as shown in Figure 2.8. Since the resisting effect of top area of the footing is small due to shallow layer of embedded soil, top area of the footing can be ignored. Since the side depths of the footing and the pier of the RC pier specimen in this study are the same, the value of k_r decreases linearly and becomes zero when the scour reaches the center of the bottom footing.

- Horizontal soil spring constant (k_h)

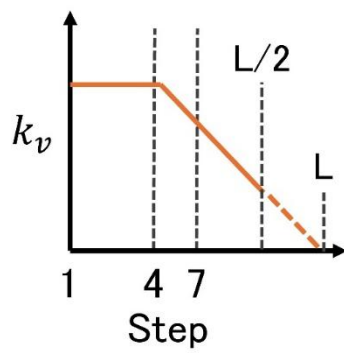
k_h is a function of the area of the front side of the footing A_h as shown in Equation (2-4). Since lateral movement is resisted by backfill soil above the bottom of the footing, k_h decreases linearly from Step 1 and it reaches 0 at Step 5 when the backfill soil is fully eroded.

Based on the above discussion, the decrease ratio in soil spring constants from Step 1 to Step 8 is shown in Table 2.5. Steps 9, 10, and 11 are removed due to backfilling steps. And Step 6 is regarded as the same process as Steps 5 and deleted. This is because it is difficult to distinguish Step 5 and 6 with only four soil springs.

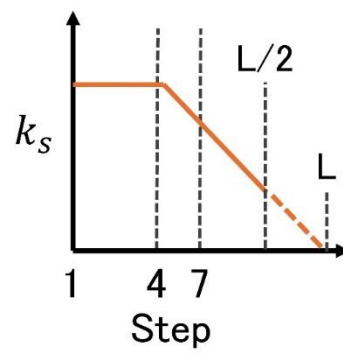
Consequently, the assumption can be summarized that the rate of decrease in the soil spring constant is proportional to the rate of decrease in the area resisting load in the direction carried by the soil spring.

Table 2.5 Decreasing ratio of soil spring constants at each excavation step.

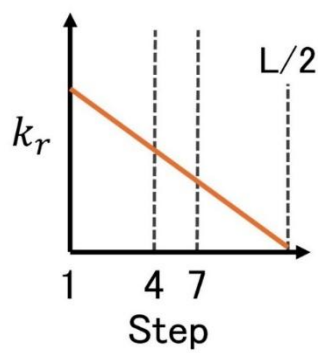
Step	1	2	3	4	5	6	7	8	9	10	11
k_v	1.00	1.00	1.00	1.00	1.00	/	0.85	0.70	/	/	/
k_s	1.00	1.00	1.00	1.00	1.00	/	0.85	0.70	/	/	/
k_r	1.00	0.91	0.68	0.48	0.45	/	0.32	0.18	/	/	/
k_h	1.00	0.83	0.42	0.04	0.00	/	0.00	0.00	/	/	/



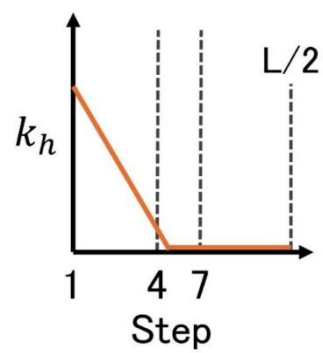
a) Vertical soil spring



b) Shear soil spring



c) Rotational soil spring



d) Horizontal soil spring

Figure 2.7 Schematic graphs of decreasing trends of each soil spring constant.

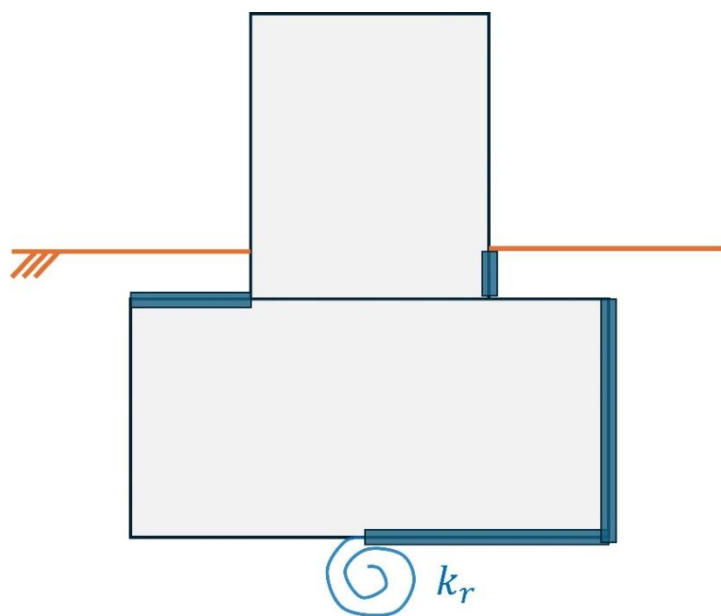


Figure 2.8 The resistance area of the rotational soil spring constant, k_r .

2.1.8. Eigenvalue analysis in scour situations

Frequencies from eigenvalue analyses at each excavation step are summarized in Figure 2.9 with those from the impact test. Although there is a maximum 3.00 Hz difference between results in eigenvalue analysis and the impact test at Step 3, the difference ratios at other steps were below 20%. It can be said that the eigenvalue analysis well predicts the experimental results. Therefore, the FE model expresses the experiment conditions well, and the assumptions discussed in 2.1.7 that the rate of decrease in the soil spring constants is proportional to the rate of decrease in the area resisting load in the direction of the soil spring.

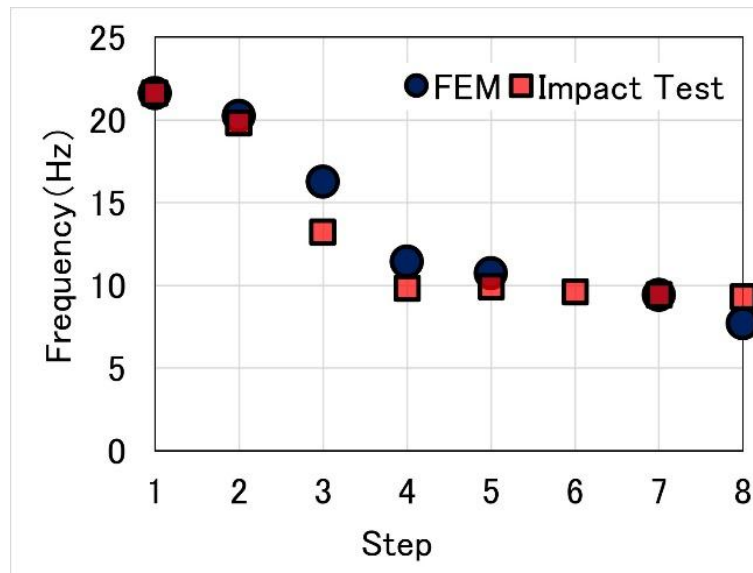


Figure 2.9 Frequencies from eigenvalue analyses and the impact tests.

2.2. Case study 2: A long-term vibration monitoring of a railway bridge pier

This study conducted a long-term vibration monitoring of a railway bridge pier from September 2017 to November 2022. This section introduces the monitoring conditions and numerical model information.

2.2.1. Bridge information

The monitoring bridge is a steel plate girder railway bridge with a span length of 22.5 m and an overall width of 9 m, designed for single railway track. An overview of the target pier is shown in Figure 2.10. The material of the target pier, whose height and width are 9 m and 3 m, respectively, is concrete. The target pier has a shallow foundation, and it is laid on very dense sand and gravel ground. The depth of the river water in Figure 2.10 is approximately 0.5 m from the bottom of the ground, which is the standard water level at the normal period. The Railway company has installed a water level measurement system and categorizes three train operation states based on the water level considering the scouring risks and the damages derived from debris; 2.5 m (operation suspended), 3.0 m (slow passing), and 3.5 m (caution). It should be noted that the water level represents the distance between the water surface and the lower flange of the bridge girder: i.e. as the water surface goes up, the water level decreases.

According to the impact test conducted in 2006, the natural frequency of the transverse direction was identified as 9.1 Hz, which is called target frequency from now on. In addition, the local scour did not progress through the period of monitoring campaign according to the measured inspection during the dry season.



Figure 2.10 Monitoring pier of the railway bridge.

2.2.2. Remote monitoring system

The tri-axial accelerometers were installed on top of the bridge pier for long-term monitoring to assess the scour of the pier. Sensors are MEMS-type accelerometers with a capacity of about 14.71 m/s^2 , an optical resolution of $2 \times 10^{-4} \text{ m/s}^2$ in the bridge axis and perpendicular direction, and an optical resolution of $7 \times 10^{-4} \text{ m/s}^2$ in the vertical direction. The sampling frequency was 200 Hz. This study uses the measurement period from 12th June in 2021 to 6th September in 2021 including a swollen river water period. The sensing system transmits observations via the Internet. The schematic figure of remote sensing system in this bridge is shown in Figure 2.11.

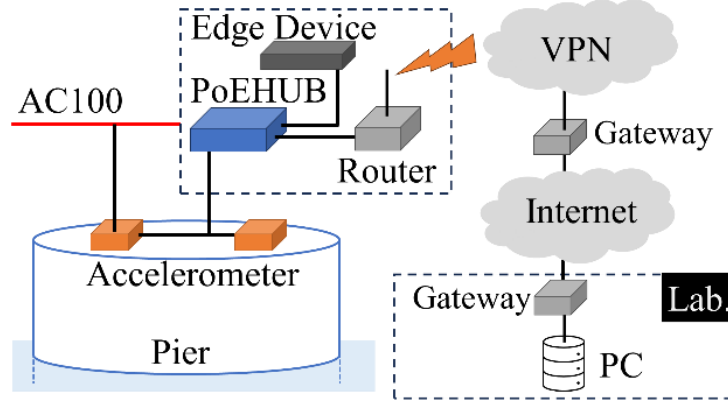


Figure 2.11 Remote monitoring and data transmission system.

2.2.3. Natural frequency from the impact test before the long-term monitoring

For the purposes of clarifying the modal parameters of the target bridge pier and girders on its pier before long-term monitoring campaigns, impact test [2] was carried out. Impact-induced vibration of the pier and the girders on the pier were measured to identify the rocking frequency of the pier. The locations of the accelerometers are shown in Figure 2.12. In addition to the sensors for long-term monitoring (No. 6 and 7 in Figure 2.12), sensors were installed at the main girders on the pier and the center on top of the pier to identify natural frequencies of the piers and adjacent girders during the impact-induced vibration.

Stochastic Subspace Identification (hereafter SSI) [11, 12] that is one of the time-series system identification methods is used to extract the modal parameters of the target structures. Furthermore, natural frequencies and mode shapes of the pier were extracted from Stabilization Diagram (hereafter SD) [13], which is one of the extracting methods for stable modes of the structure. The example of SD of impact-induced vibration is shown in Figure 2.13 in which the vertical axis represents the model order and the frequencies corresponding to the poles of the system identified at each model order are plotted as black dots. Since the system poles derived from the monitored structure can be obtained stably regardless of model orders, the black dots are plotted consecutively in the vertical direction, visually. Let f_p , f_{p+1} and f_ϵ be the frequencies corresponding to the poles of the system obtained at model orders p , $p + 1$, and the allowable bias of the natural frequencies, respectively, then the poles satisfying $|f_{p+1} - f_p| < f_\epsilon$ are extracted as stable modes. The bias of the modal parameters such as natural frequency, damping ratio, and mode shapes are set $f_\epsilon = 0.25 \text{ (Hz)}$, $\zeta_\epsilon =$

0.1(%), $MAC_l = 0.95$, respectively considering data length, sampling frequency and previous researches [14]. Only those frequencies that satisfy the three tolerances are circled in red. Blue vertical broken lines indicate the average value of consecutive red circles, and can be regarded as the dominant mode of the structure. Figure 2.13 shows two dominant modes, at 4.9 Hz and 9.1 Hz. The mode shapes corresponding to the dominant modes are shown in Figure 2.14. It is hard to see the motion of transverse direction of the target pier in spite of the weight hitting against transverse direction of the pier in Figure 2.14(a), while horizontal motion of the connected girders can be seen. Consequently, the mode in 4.9 Hz is relevant to the mode of the connected girders. Next, the mode shape in Figure 2.14(b) shows the motion of the transverse direction of the pier. According to the impact test results in 2006, the mode in 9.1 Hz is, therefore, the rocking mode of the pier in the transverse direction, and this frequency is called target frequency hereafter.

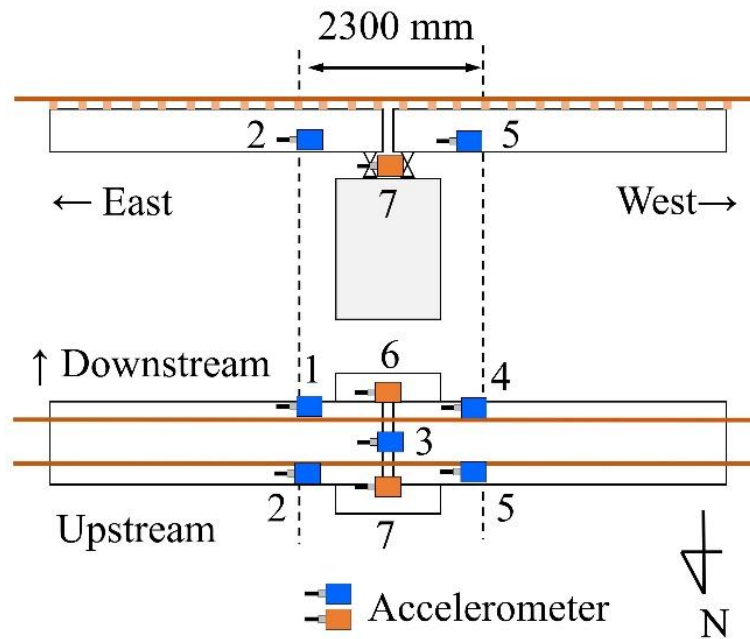


Figure 2.12 Locations of accelerometers for impact tests before long-term monitoring.
(No. 6 and 7 were used for long-term monitoring.)

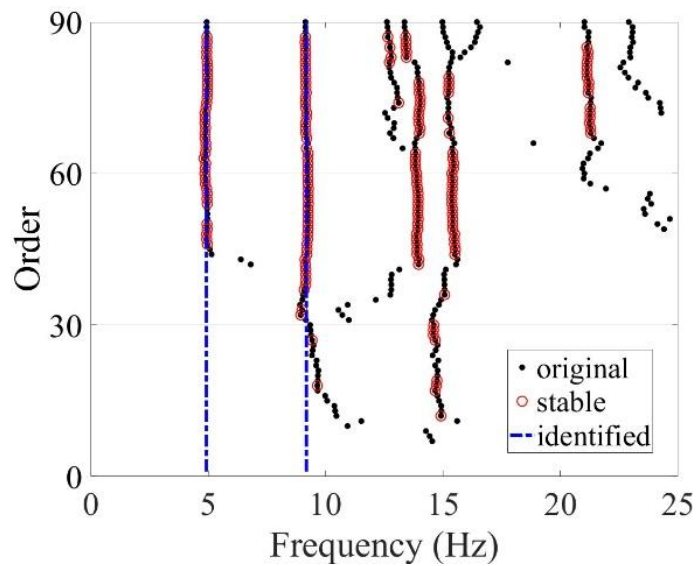
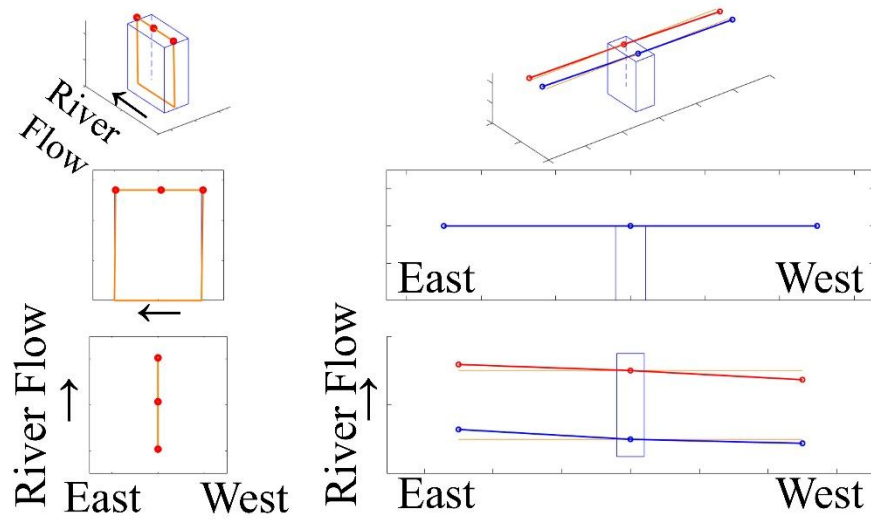
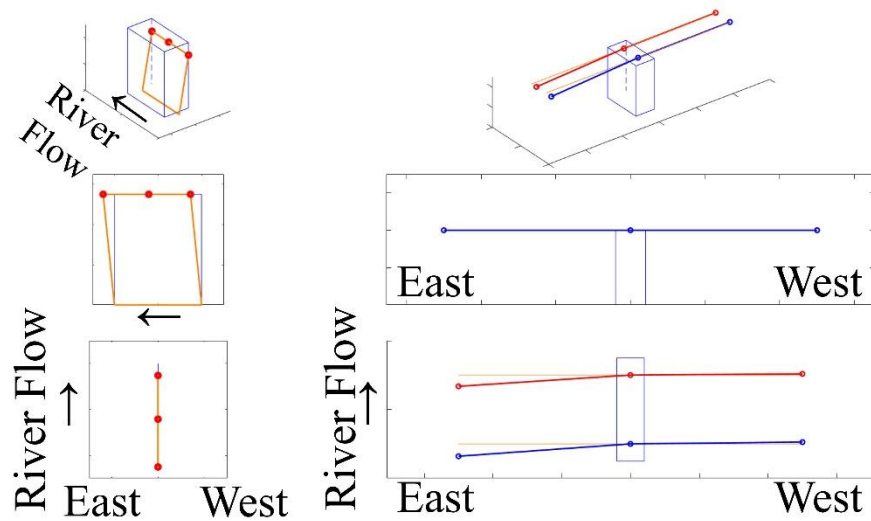


Figure 2.13 Stabilization Diagram from impact-induced vibration.



(a) Mode shape in 4.9 Hz.



(b) Mode shape in 9.1 Hz.

Figure 2.14 Mode shapes of the dominant frequencies from impact-induced vibration.

2.2.4. Natural frequency identification using microtremors from long-term monitoring

The target pier had been monitored 24 hours until November 2022, and the modal parameters were identified utilizing every one-minute data. Fast Bayesian FFT method, which enables us to provide high precise results with information on identification uncertainty, was applied to microtremors from the long-term monitoring. The details in Fast Bayesian FFT method are described in **Appendix A1**. Fast Bayesian FFT method can set the frequency band to be examined. The lower cutoff frequency was determined at 6 Hz because the dominant girder-derived mode is identified at around 4.9 Hz and the worst category of the soundness index κ ([2], see chapter 3.1) is less than 6.37 Hz. Further, the upper cutoff frequency was determined at 12 Hz considering the differences between the target frequency and the lower cutoff frequency.

- During normal river water period

Data measured from 12th June to 30th June in 2021 (i.e., 19 continuous days), were examined as the observation at a normal period where the floods events when the water level under the girder is within 6 m were not observed. The time-series of mean target frequencies with standard deviation identified using the Fast Bayesian FFT method at a normal period is shown in Figure 2.15. It is noted that the measurement data affected by train-induced vibration was excluded. Accelerations in the transverse direction of the two sensors on top of the pier are investigated. Each data sample comprises one minute observation. The vertical and horizontal axes denote mean frequency and day, respectively. Each black dot represents the identified Most Probable Value (hereafter, MPV), and its standard deviation is represented as error bar. Although there is some local variation in the identification results most plots were converged at around the target frequency, 9.1 Hz, as we discussed in the previous section 2.2.3.

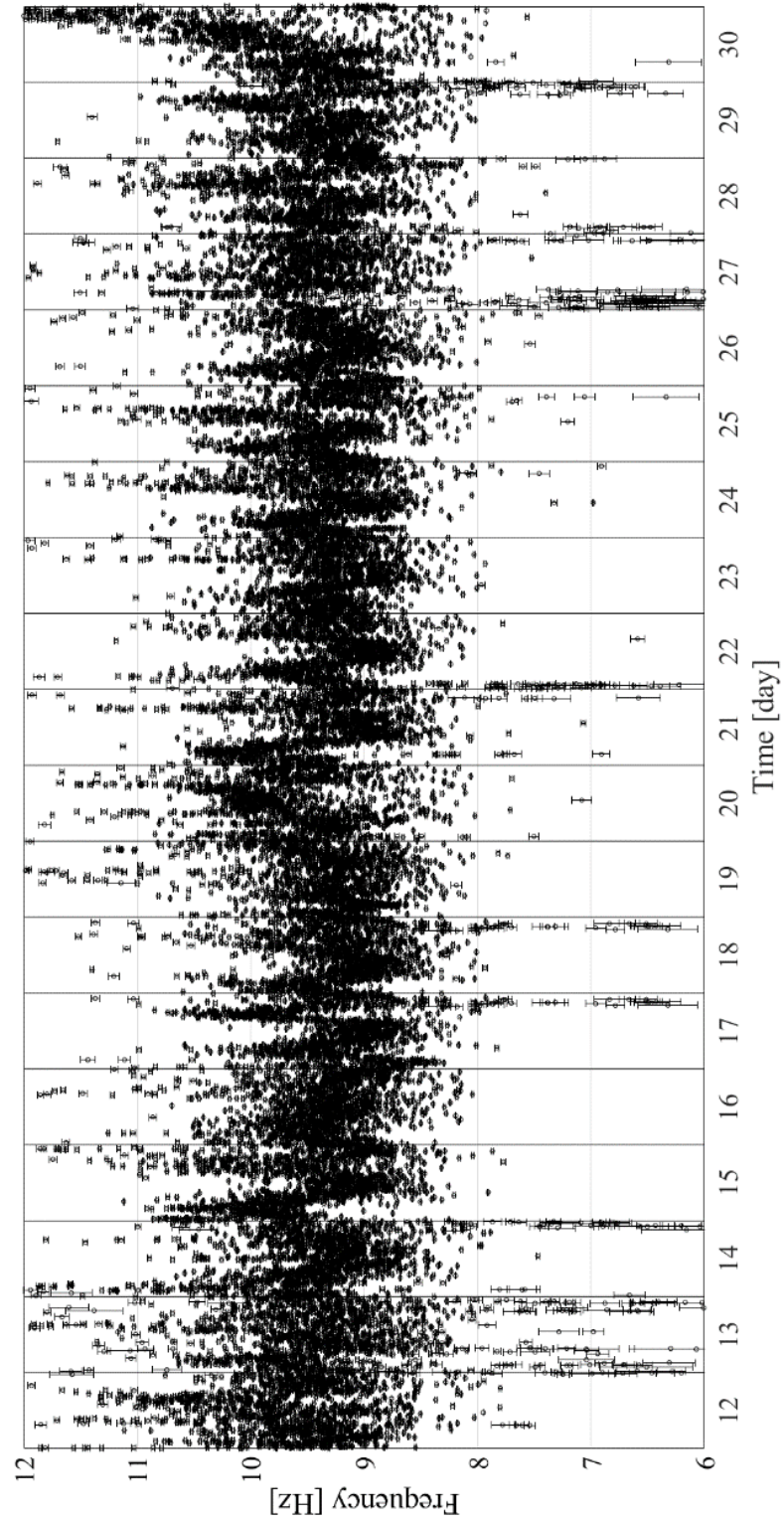


Figure 2.15 Time-series of identified MPV of the rocking frequencies and standard deviation

at a normal period.

- During swollen river water period

A water rising event, called Swollen River Water period (hereafter, SRW period), occurred in 3rd September 2021. Figure 2.16 shows time-series of MPVs and standard deviations of identified frequencies from microtremors from September 1st to September 5th including SRW period. The green line and right vertical axis are information on water level. Identified frequencies show large variations. This is mainly caused by the low SN ratio due to weak external force. However, during SRW period in 3rd September, identified frequencies converged compared to the normal water period. This is because the SN ratio becomes larger during SRW period as the pier vibration is activated by higher hydraulic pressure caused by water flow, and acceleration responses on top of the pier are relatively larger. It is noted that the similar trends can be observed at other SRW periods during 5-year long-term monitoring. Figure 2.17 shows those identified frequencies during SRW period on September 3rd for reference.

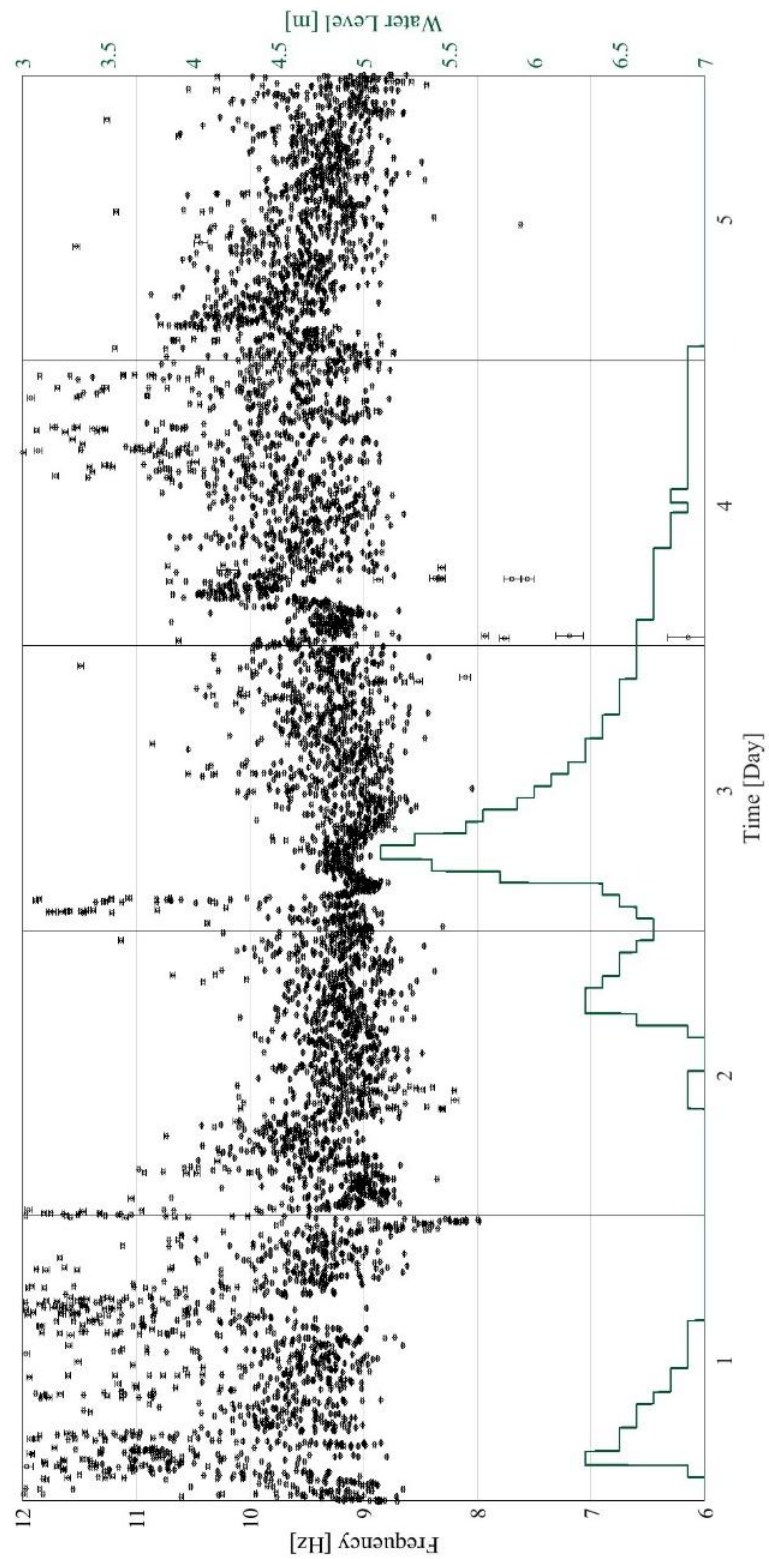


Figure 2.16 Time-series of identified rocking frequencies from microtremors from 1st to 5th September 2021 including a Swollen River Water period.

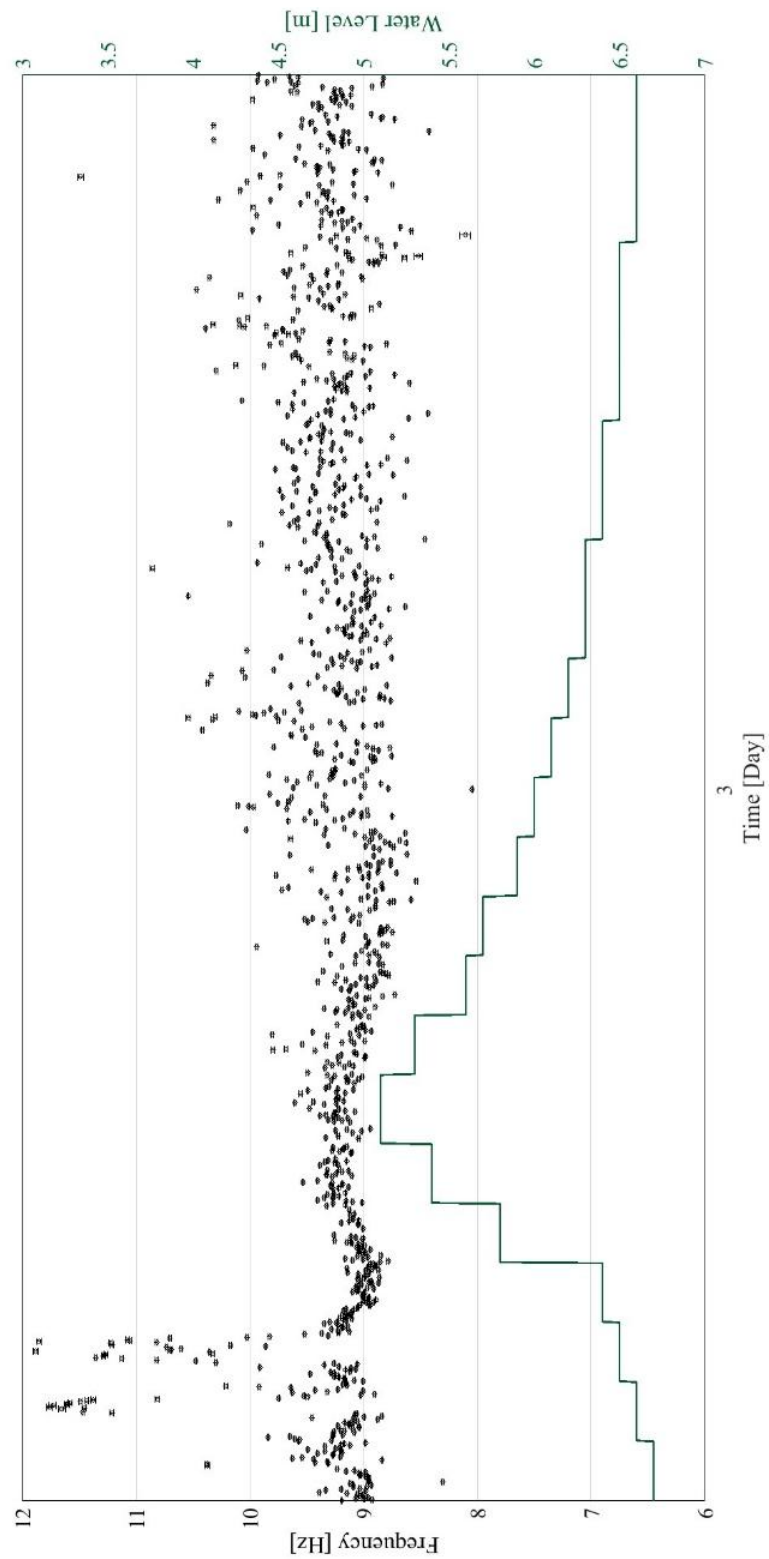


Figure 2.17 Time-series of identified rocking frequencies from microtremors in September 3rd 2021 during Swollen River Water Period.

2.2.5. Finite Element Model

This section outlines Finite Element (hereafter FE) model of the target concrete bridge pier to build scour depth prediction. Since the purpose of this study is to evaluate the scouring of the monitored pier, expressing the soil-structure interaction using FE model is crucial. This study uses OpenSees which is an object-oriented software framework created during the National Science Foundation-sponsored era of the Pacific Earthquake Engineering Research Center [15]. OpenSees allows users to create finite element applications for simulating the response of structural and geotechnical systems subjected to earthquakes. The bond-slip effect between reinforcement and concrete is hard to simulate indeed in other Finite Element software like ABAQUS, etc. which leads to discrepancies between simulation results and reality. While in OpenSees, the fiber beam model can simulate this phenomenon very well. Thus, the numerical models investigated in this study were modeled using OpenSees platform. The advantage of OpenSees is to simulate non-linear behavior, and while it does not offer much advantage in expressing scouring of the pier, it is worth using for the scalability of the numerical analysis.

The target concrete pier is modeled with three-dimensional beam elements. To model the monolithic piers, dispBeamColumn fiber-based element is used, as shown in Figure 2.18. Uniaxial material Concrete01 was chosen to simulate the confined and unconfined concrete parts and Steel01 was chosen to simulate the steel rebar. The material properties of concrete and steel rebar are shown in Table 2.6 and Table 2.7, respectively.

The superstructure whose weight is totally around $3.12 \times 10^4 kg$ is simplified as a point mass and attached to the pier top. The foundation is modeled as a rigid body, and for simplicity the shape is regarded as cuboid whose size is $L: 7.8 m \times W: 4.5 m \times H: 2.0 m$.

In addition, to express the soil-structure interaction, soil springs are introduced in the model. Based on Design Standard for Railway Structures and Commentary [4], vertical, shear, rotational, and horizontal soil springs are placed in the positions shown in Figure 2.20. The shear, vertical and rotational soil springs are placed at the center of gravity at the bottom of the footing, and the horizontal soil spring is placed at the center of the side of the footing. The values of soil spring constants as design values are used as shown in Table 2.8 according to the soil investigation conducted in 2016 and Design Standard [4]. The calculation of obtaining soil spring constants can be summarized as follows:

Table 2.6 Material properties of the concrete.

Element	Compressive strength f_{pc} (MPa)	Strain at maximum strength ε_{c0}	Crushing strength ε_U (MPa)	Strain at crushing strength f_{pcu}
Confined concrete	-30	-0.002	-10	-0.0035
Unconfined concrete	-27.5	-0.002	-10	-0.0035

Table 2.7 Material properties of the steel rebar.

Element	Yield strength f_y (MPa)	Modulus of elasticity E_0 (MPa)	Strain hardening ratio b
Steel rebar	400	205000	0.001

Table 2.8 Design values of each soil spring constant of the concrete bridge pier.

Soil spring	Spring constant	unit
Vertical (K_v)	8.528×10^6	N/mm
Shear (K_s)	2.842×10^6	N/mm
Rotational (K_r)	4.324×10^{13}	$N \cdot mm$
Horizontal (K_h)	7.225×10^4	N/mm

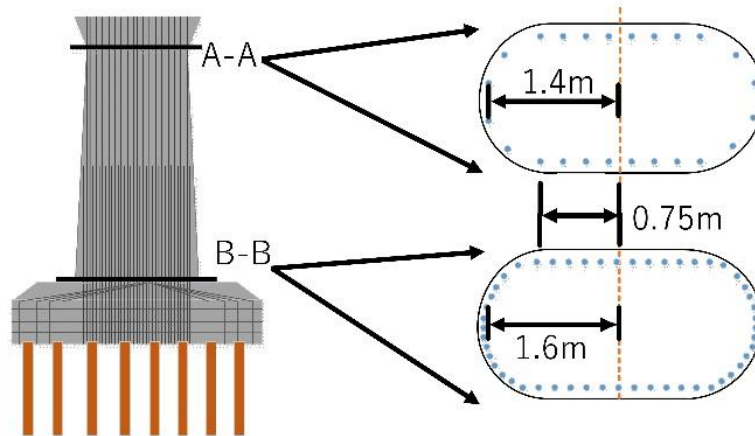


Figure 2.18 A schematic figure of the target pier.

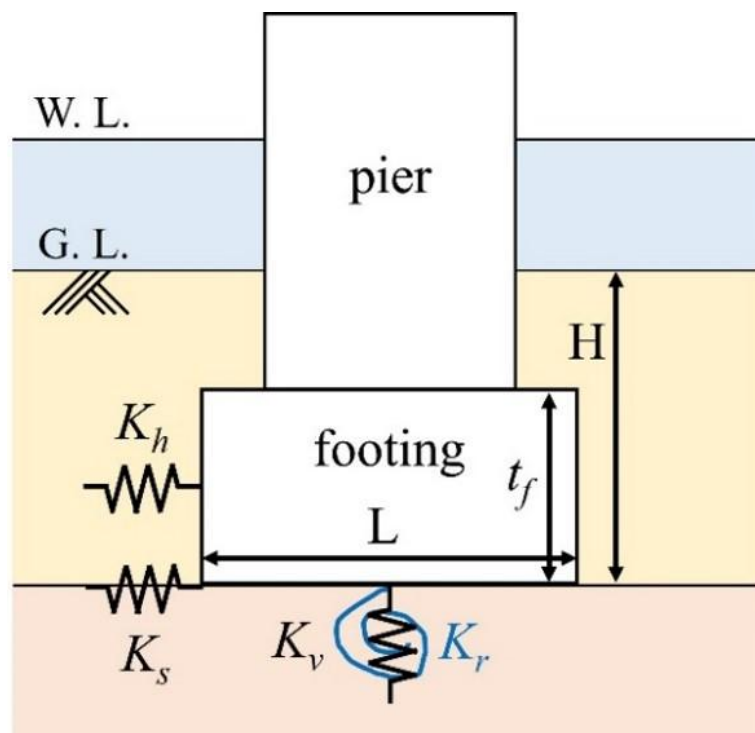


Figure 2.19 Locations in soil springs of the concrete pier. (W.L.: Water Level; G.L.: Ground Level)

2.2.6. Sensitivity analysis and updating soil spring constant

Sensitivity analysis was conducted to investigate changes in rocking frequency at transverse direction with each soil springs. A FE model considering parameters in Table 2.8 was taken as the reference state to illustrate changes in rocking frequency. The results of the sensitivity analysis are shown in Figure 2.20. The horizontal axis denote spring constant, and the vertical axis shows frequency. Black dashed lines correspond to reference spring constant in Table 2.8. It can be seen that the horizontal and the vertical soil spring constants seldom affect rocking frequency. For shear soil spring constant (k_s), changing ratio around dashed line looks high. However, it is not logical that the shear force of the ground is drastically change when scour progress above the bottom of the footing because shear force is mainly affected by the area of the bottom surface of the footing. Therefore, this study regards shear force as constant until scour reaches the bottom of the footing. On the other hand, it is observed that the frequencies of the rotational soil spring change as the spring constants change around the dashed line.

Consequently, this study updated only rotational soil spring constant. The rotational soil spring constants and eigenvalues before and after the model updating are shown in Table 2.9.

Table 2.9 Rotational soil spring constants before and after model updating and results in eigenvalue analyses of the concrete bridge pier.

	Before model updating	After model updating
Rotational soil spring constant ($N \cdot mm$)	4.324×10^{14}	5.600×10^{13}
Eigenvalue (Hz)	9.57	9.20

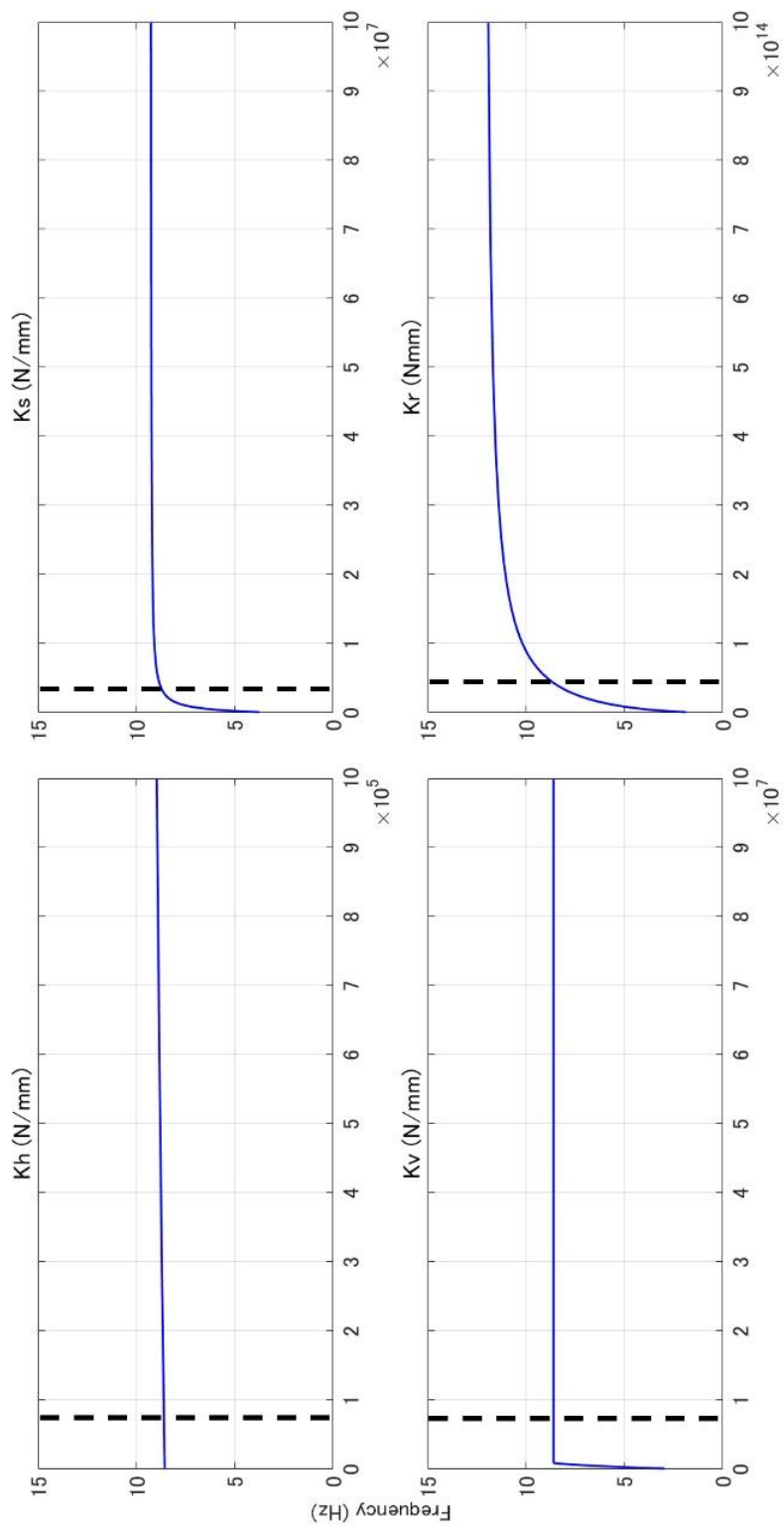


Figure 2.20 Results in sensitivity analyses of the concrete bridge pier.

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Chapter 3. Data-driven Stochastic Scour Assessment

The identified modal parameters obtained from weak acceleration responses have indispensable issues of identification uncertainty and the effect of environmental and operational variations (hereafter EOVs) [1-5]. When we measure the acceleration responses of the concrete bridge pier, we must address them properly because it results in a weak response. This chapter proposes several warning indices focusing on scour detection using acceleration responses. They are all considered by introducing a probabilistic approach to address weak responses.

Section 3.1 introduces the vibration-based scour soundness index κ [6], which has been widely used when the impact hammer test [7-10] is conducted in railway bridge pier. Section 3.2 proposes two Scour Warning Indices (hereafter SWI) and applies them to the two case studies depicted in Chapter 2. SWI uses probability density distributions at the normal condition, and it is revealed that they follow Levy-Flight distribution [11]. This section discusses Levy-Flight distribution and organizes the appropriate treatment methods. Section 3.3 proposes a new indicator called Probability of Scour Occurrence (hereafter PSO). The objective of proposing PSO is to facilitate an intuitive understanding of whether the scour is occurring. Since we have seen the probability of rainfall tomorrow on the weather forecast, or the probability of failure in reliability engineering field, showing the probability of scour occurrence makes it easier for engineers to decide whether they should act or take a wait-and-see. Section 3.4 introduces Bayes Factor (hereafter BF) [12, 13] for utilizing anomaly detection of bridge scour using acceleration observations. BF has been widely used as one of the solutions to the problems of conventional statistical hypotheses testing [14]. This study discusses the ways in which we apply BF to the real-time vibration monitoring data of the bridge pier as case studies.

3.1. Vibration-based scour soundness index: κ

According to the Maintenance Standard for Railway Structures and Commentary [6], a soundness index, which is called κ , is defined as shown in Table 3.1 from theoretical computation and statistical information on impact tests in the past. Here, κ is the ratio of the observed frequency (f_t) to the healthy frequency (f_h), and is expressed as follows.

$$\kappa = f_t / f_h \quad (3-1)$$

Considering the vibration of the pier as one-degree-of-freedom system, natural frequency at healthy condition can be obtained by.

$$f_h = C \sqrt{\frac{k}{m}} \quad (3-2)$$

Here, m , k , and C are mass, stiffness, and constant, respectively. When the stiffness decreases a half from k , the observed frequency f_t can be obtained by.

$$f_t = C \sqrt{\frac{1}{2} \times \frac{k}{m}} = \frac{1}{\sqrt{2}} f_h = 0.707 f_h \quad (3-3)$$

This is one of the reasons why the ratio of abnormal condition is set to 0.70.

Table 3.1 Scour soundness categories [6].

Soundness index κ	Category	Assessment
$1.00 < \kappa$	S	Healthy
$0.85 < \kappa \leq 1.00$	B	Low possibility of abnormal condition
$0.70 < \kappa \leq 0.85$	A2	Need to check progress of deterioration
$0.70 \leq \kappa$	A1	Abnormal condition

3.1.1. Application to case study 1

● Identified rocking frequencies by impact tests

Results in impact tests of the experiment with the RC specimen to simulate the scour are again shown in Table 3.2. This study considers the rocking frequency at healthy condition as 21.6 Hz, which corresponds to the impact test result at Step 1. κ and categories of each Step are also shown in Table 3.2. The borders of each category following Table 3.1 are calculated as below:

$$\begin{aligned} f_B &= 21.6 \text{ [Hz]} \\ f_{A2} &= 0.85 \times 21.6 = 18.4 \text{ [Hz]} \\ f_{A1} &= 0.70 \times 21.6 = 15.1 \text{ [Hz]} \end{aligned} \quad (3-4)$$

The rocking frequency at Step 3 when the ground is excavated by the middle of the side footing was drastically decreased and κ showed 0.61 and categorized as the worst zone, A1. This means that the backfilled ground overlying the RC specimen is dominant in the overall ground constraint of the specimen. Also, the lack of superstructure loading and constraints, unlike actual railway piers, might be a main factor in the large change in rocking frequencies.

Table 3.2 Results in identified rocking frequencies and soundness ranks by impact tests.

Step	1	2	3	4	5	6	7	8	9	10	11
frequency	21.6	19.8	13.2	9.8	9.9	9.6	9.4	9.3	9.6	12.6	21.3
κ	1	0.92	0.61	0.45	0.46	0.44	0.44	0.43	0.44	0.58	0.99
rank		B	A1	A1	A1	A1	A1	A1	A1	A1	B

● Identified rocking frequencies by microtremors

The identified soundness index from microtremors is summarized in Figure 3.1. Black circles, orange circles, red triangles, and purple crosses are categorized as S, B, A2, and A1, respectively. The horizontal lines show the borders of each category shown in Equation (3-1). In Step1, most identified frequencies were categorized as B despite the intact Step because the identified frequencies from microtremors were relatively smaller than identified frequencies by the impact test. A similar trend can be seen in Steps 2 and 11. In Step 2, the impact test resulted in category B, however, most identified frequencies from microtremors were categorized as A2. On the contrary, through Step 3 to Step 10 when the excavation progresses, both the results in the

impact test and microtremors show the same categories A1.

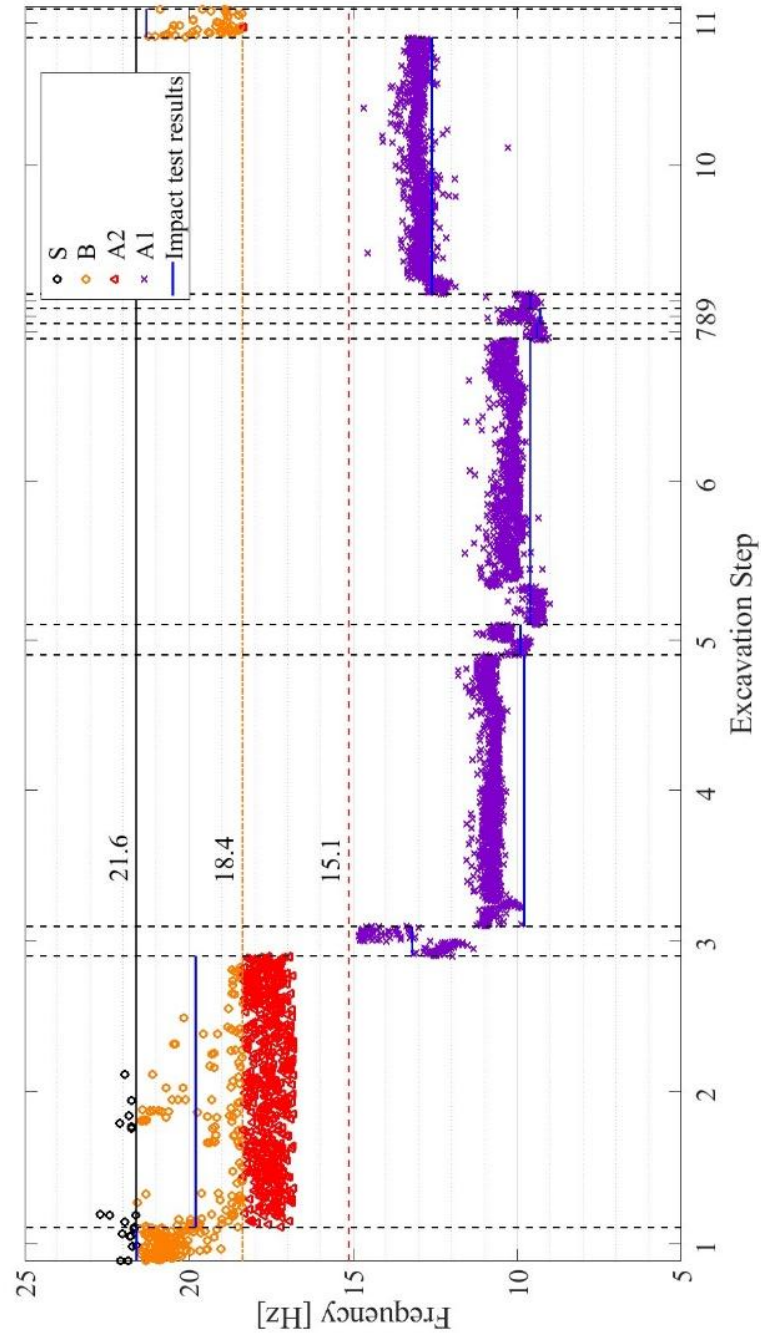


Figure 3.1 Identified soundness indices from microtremors.

3.1.2. Application to case study 2

Since the impact test at the target pier of the real bridge in 2006 resulted in 9.15 Hz, the borders of each category of soundness indices shown in **Table 3.1** were calculated as below.

$$\begin{aligned} f_B &= 9.15 \text{ [Hz]} \\ f_{A2} &= 0.85 \times 9.15 = 7.78 \text{ [Hz]} \\ f_{A1} &= 0.70 \times 9.15 = 6.41 \text{ [Hz]} \end{aligned} \quad (3-5)$$

κ is calculated using the identified frequencies of the bridge pier from September 1st to 5th 2021, including the SRW period and is shown in Figure 3.2. Black circles, orange circles, red triangles, and purple crosses are categorized as S, B, A2, and A1, respectively. The horizontal lines show the borders of each category shown in **Equation (3-5)**. Most plots are categorized as S or B. However, a few plots in 4th and 5th show A2 or A1 representing outliers. Since there was no scour progression during the monitoring campaign based on regular investigation, these results might be misleading regarding scour occurrence. This is one of the reasons why statistical approaches are necessary.

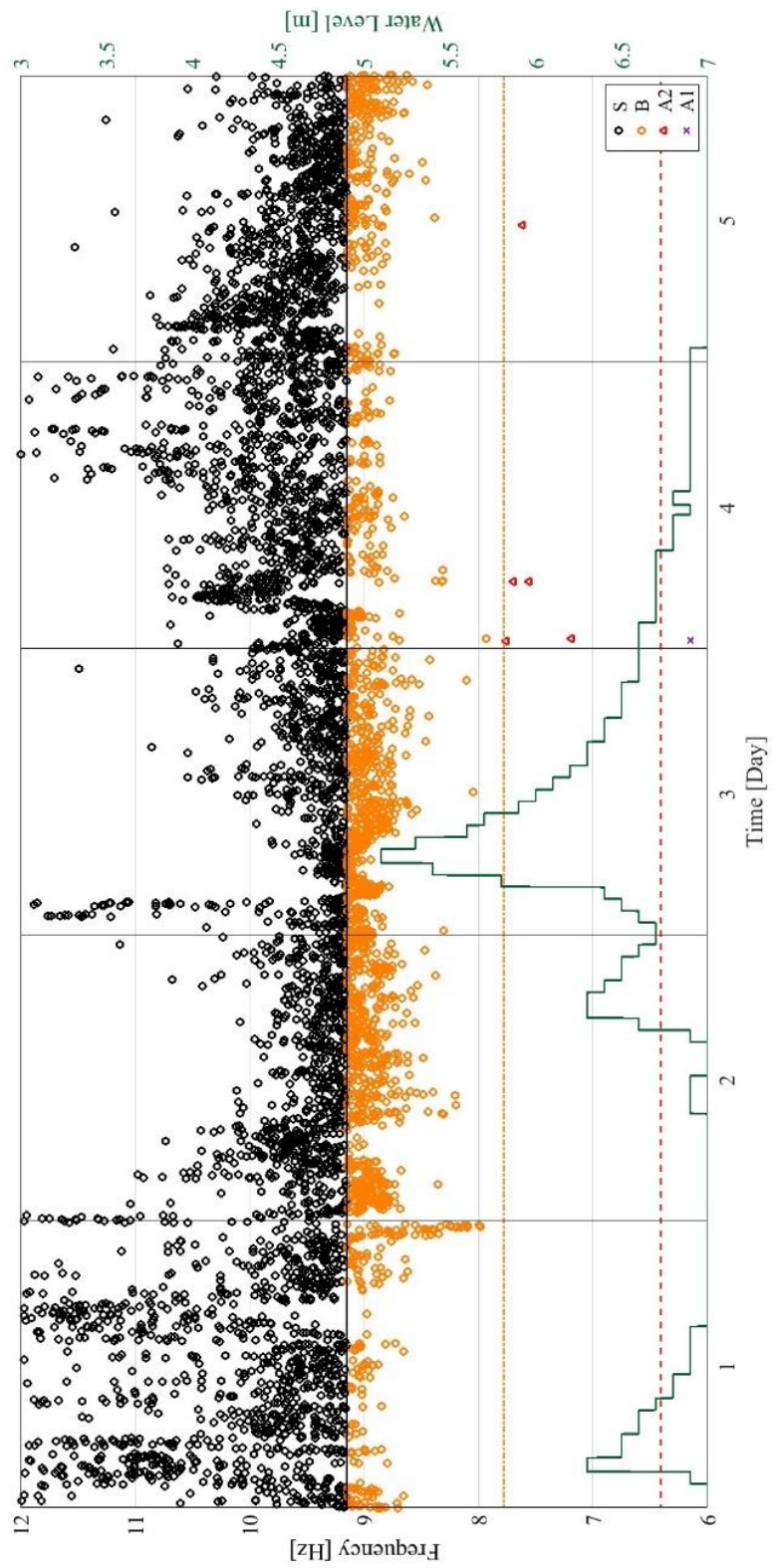


Figure 3.2 Identified soundness indices from microtremors of the real railway bridge pier.

3.2. Scour warning indices

There are variances in the identified rocking frequencies at normal condition as discussed in the previous chapters 2.1.4 and 2.2.4. Therefore, it is rational and convenient to assess the soundness of the pier by statistical approach. This study, thus, proposes statistical methods for evaluating the soundness for scouring of the target pier by use of probability distribution of the identified target frequencies at normal condition and the identified frequencies obtained in realtime during SRW period. Two Scour Warning Indices, SWI1 and SWI2, are explained in the next section and are applied to the two case studies.

3.2.1. Methodology

- SWI1: Probabilistic Scour Warning Index focusing on cumulative density function

When we accumulate identified frequencies of the target pier by use of microtremors, the results should be scattered. It means that identified frequencies have uncertainty. Therefore, accumulated frequencies can be modeled as probability density function (PDF). The Cumulative density function (CDF) $G(x)$ of probability density function $g(x)$ is written as

$$G(x) = \int_{-\infty}^x g(x) dx \quad (3-6)$$

Since $G(x)$ at the normal period can be determined using parameter group to determine shape, location, etc. and one observation information, $G(x)$ can be represented as a set of parameters. When we define n parameters as $\theta_i, i = 1, \dots, n$, $G(x)$ can be written as

$$\Psi(x) = G(x) = \{\theta_1, \dots, \theta_n, x\} \quad (3-7)$$

For simplicity, Ψ is introduced. Using **Equation (3-7)**, the borders of each soundness index can be written as follows.

$$\begin{aligned} SWI_B &= \Psi(f_B) \\ SWI_{A2} &= \Psi(f_{A2}) \\ SWI_{A1} &= \Psi(f_{A1}) \end{aligned} \quad (3-8)$$

SWI_B , SWI_{A2} , and SWI_{A1} can be interpreted as the probability that the observed information is within the boundary of the category at a normal period. For example, the probability that an observation less than or equal to f_{A1} is observed under normal conditions where the identified target frequency has uncertainty can be interpreted as SWI_{A1} . If the local scour around the target pier occurred, and the identified frequencies were decreased, $\Psi(x)$ would output a very low value continuously. It can be interpreted that events that occur with extremely low

probability under normal conditions continue to be observed now. Focusing on scouring, identified frequencies more than impact test results should not be considered, but rather the accuracy of fitting lower frequencies is much more important from the risk assessment point of view. Therefore, the $\Psi(x)$ itself estimated from the normal period, functions as a probabilistic indicator and can be used as a scour warning indicator having a probabilistic approach. This scour warning indicator is hereafter referred to as SWI1.

SWI1 makes it easier to interpret because of the value derived from the cumulative density function. Furthermore, the management level can be conveniently set to a probabilistic 1%, 5% for example. It is convenient because the bridge authorities can determine the management levels corresponding to the amount of traffic volumes and the importance of the monitoring bridge.

- SWI2: Probabilistic Scour Warning Index considering identification uncertainty

When we use a Fast Bayesian FFT method [15-19] to identify modal parameters, their outputs are expressed in multiple Normal Distributions as posterior probability distributions. Therefore, the quantitative uncertainty information on the identified modal parameters can also be obtained as standard deviations. This section, thus, proposes the second indicator using the marginal probability density function combining the probability distribution derived from the normal period with the identified Normal Distribution during SRW period in realtime.

Since this study only focuses on the target frequency, the posterior distribution of the identified modal parameters can be expressed as a univariate Normal Distribution. The marginal probability density function using the probability distribution in **Equation (3-6)** and the univariate Normal Distribution during SRW period can be written as

$$\Psi'(x) = \int_{-\infty}^{\infty} h(v) \int_{-\infty}^x g(u) du dv \quad (3-9)$$

$$h(v) \sim \text{Normal}(x, \sigma_t^2)$$

Here, $h(v)$ is the probability distribution of the identified frequency during SRW period in realtime. For distinguishing SWI1, Ψ' is introduced. Ψ' is possible to use the scour warning indicator considering the uncertainty of identification by calculating the marginal probability density function of the timely observed frequency. This scour warning indicator is hereafter referred to as SWI2.

3.2.2. Discussion of statistical modeling

The purpose of this discussion is to clarify the best performance of the statistical modeling of the identified target frequencies of the real bridge pier at a normal period. Since the accuracy of statistical modeling is critical for assessing the risk of the structure, it is necessary to investigate the preferable statistical modeling results. Using the accumulated MPVs of the real pier identified from microtremors at the normal period as shown in Figure 3.3, this Section investigates the preferable statistical modeling of them to cope with such variances derived from aleatory uncertainty. The histogram of the identified target MPVs at normal period is shown in Figure 3.4 with the log-scaled figure for visualization. As a fundamental statistical modeling, Normal Distribution is discussed at first, then utilizing Levy-Flight distribution (hereafter LF Distribution) is discussed because the heavy-tailed feature can be observed in Figure 3.4., According to Figure 3.3, the coefficient of variance tends to be higher if the MPV is farther away from the target frequency, 9.15 Hz. Therefore, this section also discusses the statistical modeling results for different treatments of the coefficient of variation.

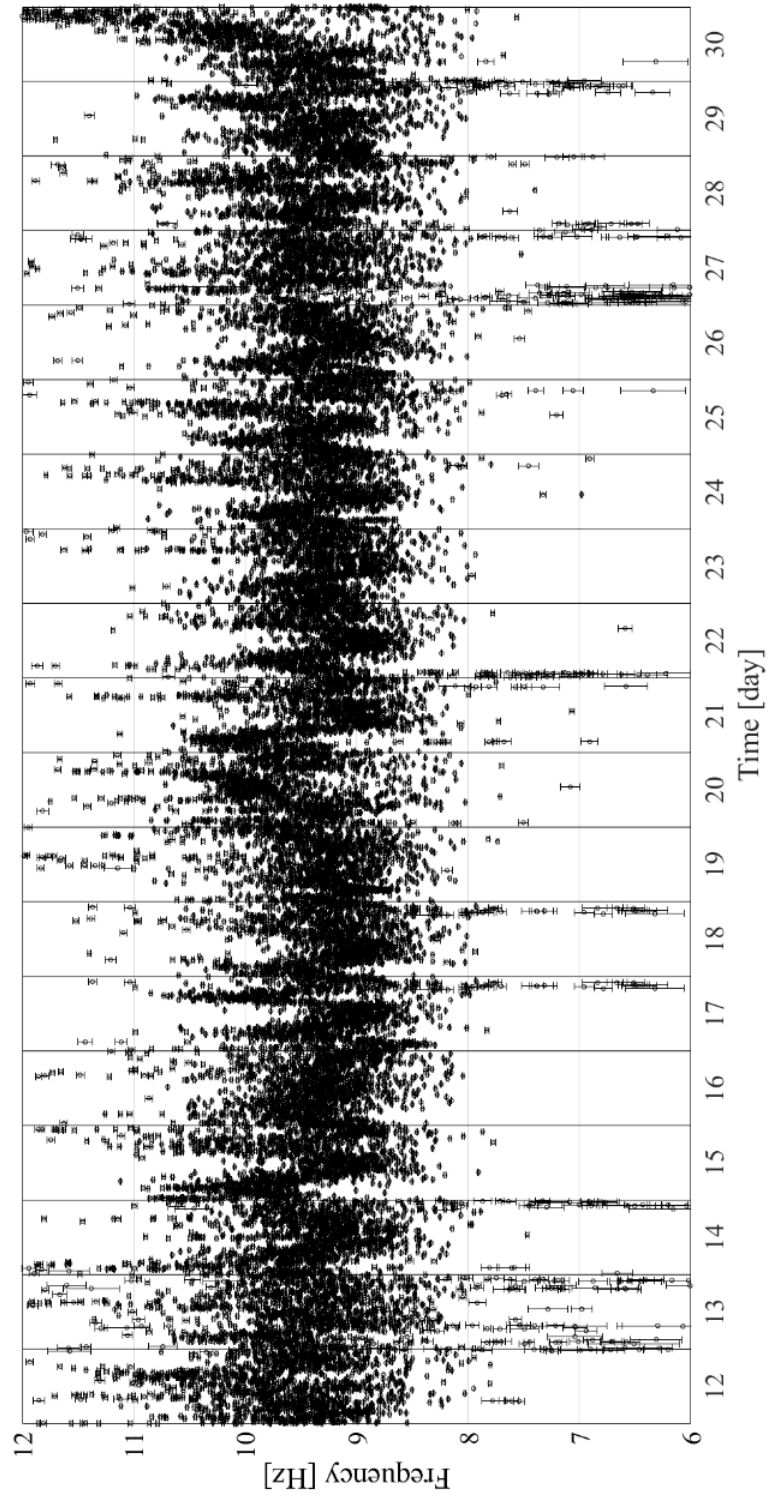


Figure 3.3 Time-series of identified MPVs of the rocking frequencies with standard deviation from microtremors during normal period (same as **Figure 2. 15**).

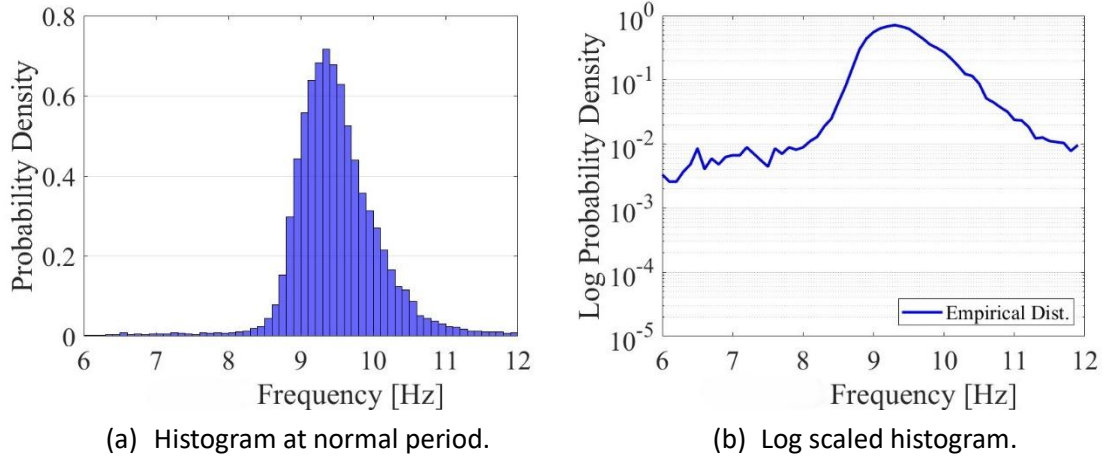


Figure 3.4 Accumulated identified target MPVs at normal period by use of microtremors

● Normal Distribution

The parameters of the Normal Distribution estimated by the Maximum Likelihood Estimation method (hereafter, MLE method) [20], and Normal Distribution fitting on the histogram of the identified MPVs is shown in Figure 3.5, in which the blue dashed line indicates the Normal Distribution fitting. The estimated parameters are shown below.

$$g(x) \sim N(\mu_N = 9.48, \sigma_N^2 = 0.61) \quad (3-10)$$

Where $g(x)$ stands for the probability density function. It is important to represent the distribution of the tail of the low-frequency part accurately from the viewpoint of risk assessment because the target frequency of the pier decreases when the scour occurs. Considering log-scaled plots in Figure 3.5(b), not only from around 8 Hz to 9 Hz, but also less than around 7.5 Hz show large difference between the empirical distribution and the Normal Distribution. This may not allow for sufficient accuracy in risk evaluation when modeling the probability distribution as the Normal Distribution.

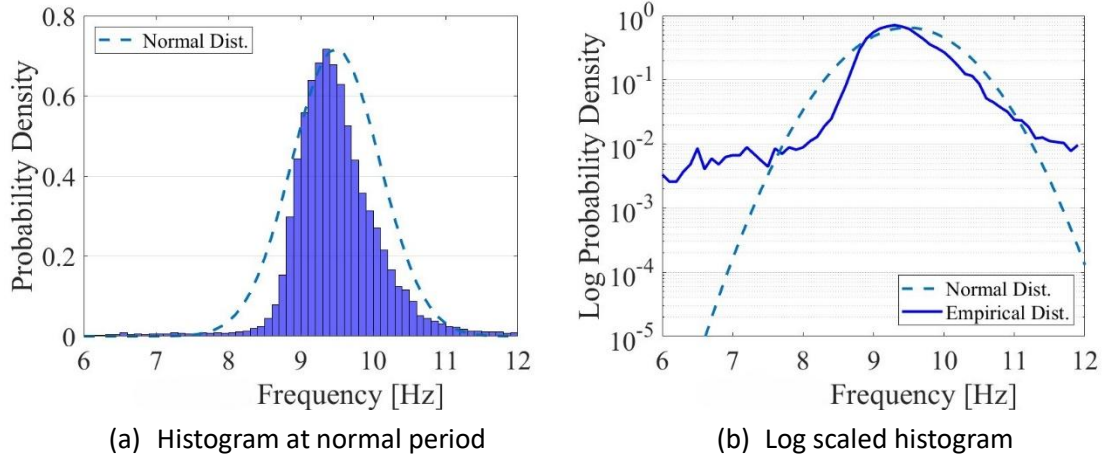


Figure 3.5 Accumulated identified rocking MPVs and modeling by Normal Distribution at normal period.

● Levy-Flight Distribution

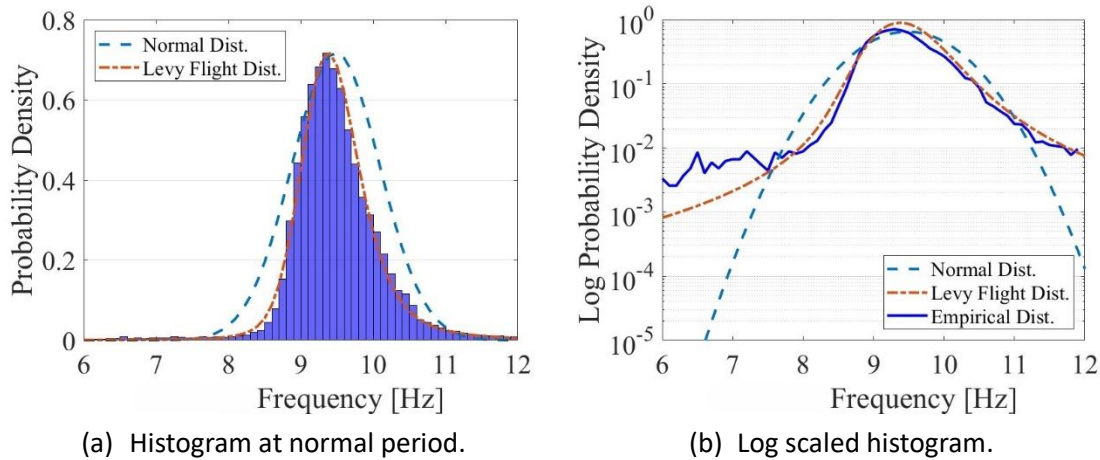
The parameters of the LF Distribution estimated by the MLE method and the results depicting the probability distribution of the LF Distribution on the histogram of the identified MPVs are shown in Figure 3.6 as orange chain lines. The estimated parameters of the LF Distribution are shown as follows.

$$g(x) \sim LF(\alpha_N = 1.58, \beta_N = 0.58, \gamma_N = 0.31, \delta_N = 9.40) \quad (3-11)$$

The parameters used in the LF distribution are summarized in Table 3.3. Focusing on the orange chain line in **Figure 3.6**, although gaps are still observed below 7.5 Hz between the empirical distribution and the LF Distribution, the LF Distribution at the low-frequency band fits well, indicating much higher accuracy than modeling with the Normal Distribution. It is expected that the accuracy of the risk evaluation would be improved if we used the LF Distribution for modeling the identified target MPVs by use of microtremors rather than the Normal Distribution. This study, thus, adopts the LF Distribution.

Table 3.3 Parameters of Levy-Flight Distribution

Parameter	Explanation	Range
α	Characteristic exponent	$0 < \alpha \leq 2$
β	Skewness parameter	$-1 \leq \beta \leq 1$
γ	Scale parameter	$0 < \gamma < \infty$
δ	Location parameter	$-\infty < \delta < \infty$

**Figure 3.6** Accumulated identified rocking MPVs of the identified frequency and modeling by the Levy-Flight Distribution during a normal period.

- Discussion of reducing uncertainty

In Figure 3.3, when the identified MPVs of the rocking frequency are less than 8 Hz, many plots have wide ranges of error bars. Several MPVs greater than 10 Hz have wide ranges of error bars. It can be seen in the trend that the farther the identified MPVs from 9.15 Hz are, the wider the ranges of error bars are. This means that the identified MPVs with large uncertainty are increasing when they are far from 9.15 Hz.

When the bearing capacity of the ground around the target pier decreases, the natural frequency also decreases. Therefore, a small variation of the identified natural frequency in the normal period should be preferred to accurately determine the amount of natural frequency reduction. This section, thus, considers a method for reducing the variability of identified natural frequencies that are far from 9.15 Hz. It is convenient to focus on the coefficients of variance and the SN ratio, which are the quantitative information on identification uncertainty. The

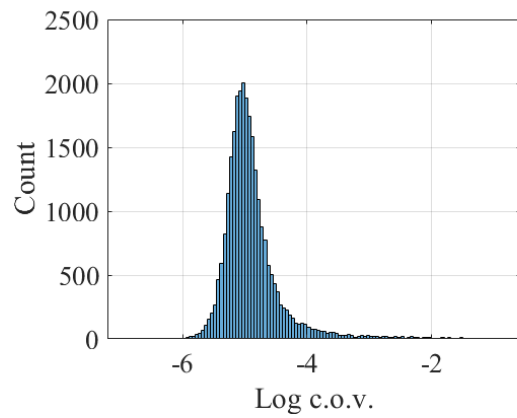
improvement of the accuracy of the fitting of the LF Distribution by eliminating MPVs with large uncertainty is discussed.

Log-scaled histograms of the coefficients of variance and the SN ratio of the identified target frequencies at the normal period are shown in Figure 3.7. When the coefficient of variance is large and the SN ratio is small, the uncertainty is large. Therefore, this study proposes to eliminate the identification cases exceeding one of the thresholds. The thresholds were set to $mean \pm 2\sigma$. The eliminated cases of the coefficients of variance and SN ratio were, *coefficient of variance* > 0.02 , and *SN ratio* < 7.40 , respectively. The eliminated result is shown in Figure 3.8. All MPVs below 7.8 Hz were filtered. Several plots greater than 11 Hz were also filtered. Figure 3.8 shows the reduction of the variances of identified MPVs during the normal period. The histogram of the filtered MPVs and probability density function of the LF Distribution estimated by the MLE method are shown in Figure 3.9. The parameters of LF Distribution are written as follows.

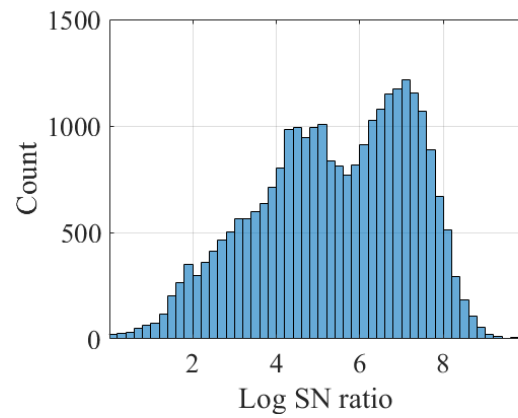
$$g(x) \sim LF(\alpha_N = 1.73, \beta_N = 1.00, \gamma_N = 0.30, \delta_N = 9.41) \quad (3-12)$$

Although the accuracy of fitting appears good, the LF Distribution showed large asymmetry. The skewness of distribution, β_N , reaches the upper limit, 1.00. In other words, the distribution of the identified MPVs of the rocking frequency may not be properly modeled using the LF Distribution. This modeling by eliminating the MPVs at the tails of lower and upper frequency bands may rather reduce the accuracy of statistical modeling.

Consequently, although there is a trend that the farther from 9.15 Hz the identified MPVs are, the larger the uncertainties are, parameter estimation of the LF Distribution including large uncertainty outputs should be better to satisfy the accuracy of modeling.



(a) Log-scaled histogram of coefficient of variance.



(b) Log-scaled histogram of SN ratio.

Figure 3.7 Histograms of quantitative information on identification uncertainty of identified rocking frequency during normal period.

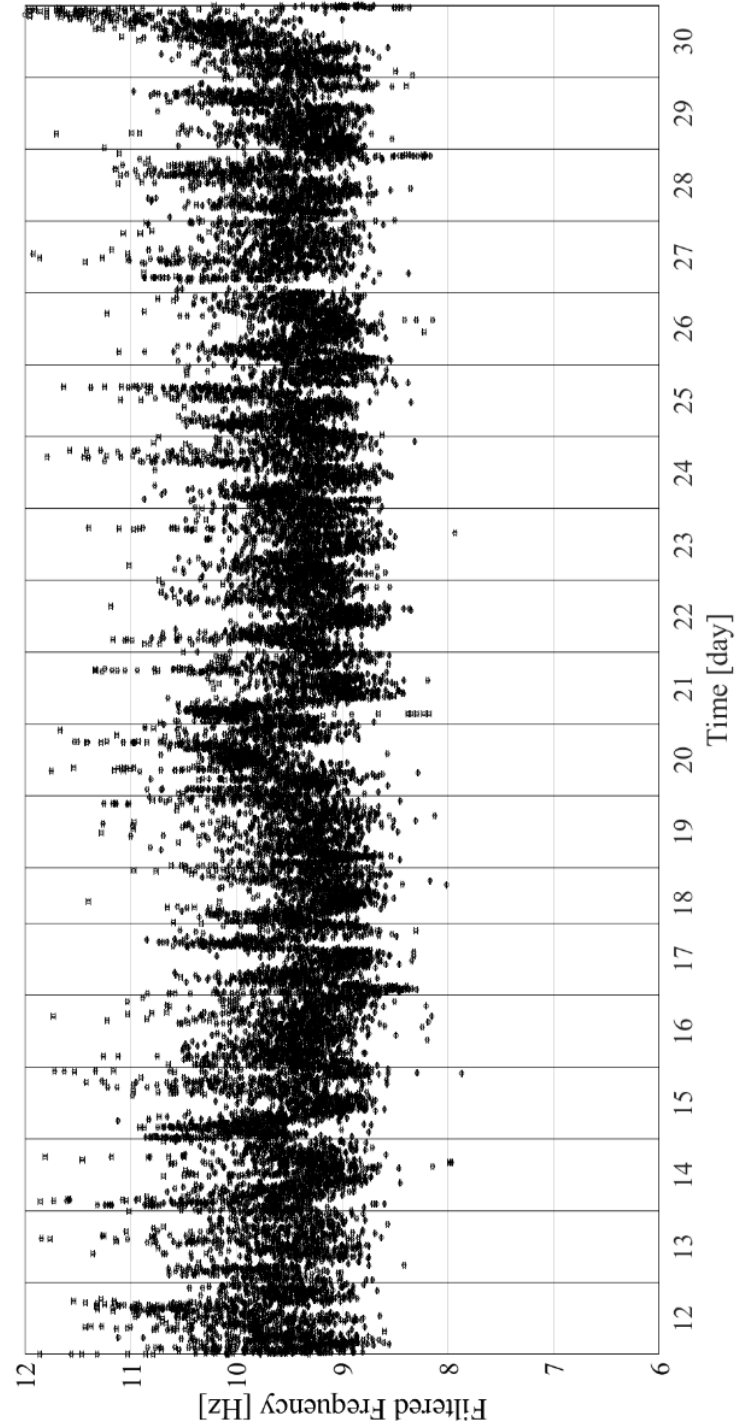


Figure 3.8 Time-series of identified MPVs with standard deviation of the rocking frequency during normal period from microtremors eliminating large identification uncertainty outputs.

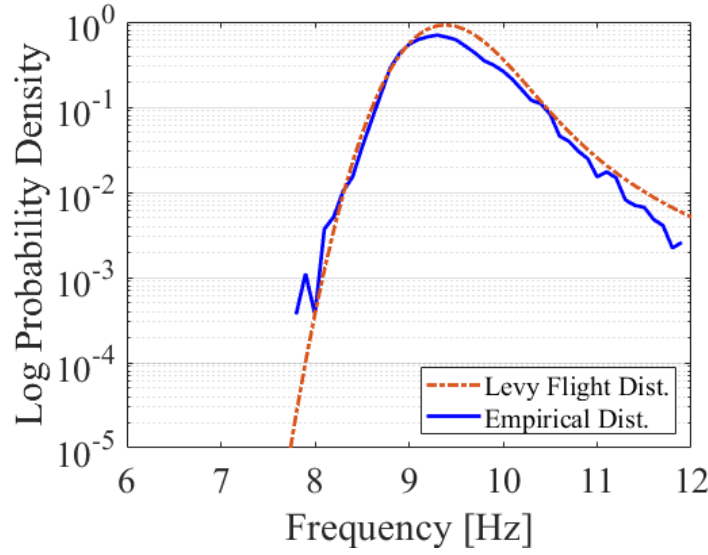


Figure 3.9 Log-scaled histogram of accumulated identified MPVs for rocking frequencies and probability density function of Levy-Flight Distribution

3.2.3. Application of Scour Warning Index to case study 1

In this section, SWI1 and SWI2 are applied to the RC pier specimen experiment. The histogram of the identified MPVs for rocking frequency from microtremors at Step 1 is shown in Figure 3.10. Since the identified MPVs of the rocking frequency during normal period of the real bridge pier follow the LF Distribution, as discussed in Section 3.2.2, the PDF following the LF Distribution is depicted as an orange chain line in Figure 3.10. The target MPVs of the rocking frequency were identified between 19 and 22 Hz. The parameters estimated by the MLE method are described as follows.

$$g(x) \sim LF(\alpha_N = 1.85, \beta_N = -1.00, \gamma_N = 0.39, \delta_N = 20.81) \quad (3-13)$$

The borders of each scour soundness category shown in **Table 3.1** are calculated from **Equation (3-8)** utilizing Equation (3-13). Finally, the borders are described as follows.

$$\begin{aligned} SWI_B &= \Psi(f_B) = \Psi(21.6) = 0.94 \\ SWI_{A2} &= \Psi(f_{A2}) = \Psi(18.36) = 6.29 \times 10^{-3} \\ SWI_{A1} &= \Psi(f_{A1}) = \Psi(15.12) = 1.06 \times 10^{-3} \end{aligned} \quad (3-14)$$

Estimated SWI1 and SWI2 are plotted in Figure 3.11, where the black circle and blue cross plots are SWI1 and SWI2, respectively. The horizontal lines correspond to borders in **Equation (3-14)**. In Step 2, most observations of SWI1 and SWI2 were continuously below SWI_{A2} , which indicates

that an event equivalent to category rank A2 occurred with high probability. Considering the impact test results in **Table 3.2** shows rank B. SWI1 and SWI2 provide safety-side decisions.

For visualization, Figure 3.12 shows the enlarged figure below 10^{-3} of SWI in Figure 3.11. All outputs of SWI1 and SWI2 from Steps 3 to 9 were below the purple line corresponding to the A1 rank, when the scour reached the middle of the side footing. These results match well with the impact test results shown in **Table 3.2**. Furthermore, both SWI1 and SWI2 values decrease following scour progression. It is concluded that although there is a safety side decision on scour warning at Step 2, SWI1 and SWI2 at all other Steps indicate the same decisions on scour warning as impact test results and SWI1 and SWI2 values follow scour progression.

Next, the differences between SWI1 and SWI2 are discussed. SWI1 and SWI2 in Step 2 show similar scour warning index, and it is difficult to find clear differences between them. On the other hand, from Steps 3 to 9, SWI2 is smaller than SWI1. SWI2 provides a stricter judge than SWI1. Identification uncertainty may affect the results because SWI2 considers identification uncertainty.

Consequently, SWI1 and SWI2 follow scour progressions corresponding to the scour soundness categories. In particular, SWI2 is more practical because it considers uncertainty and provides the safety-side decision.

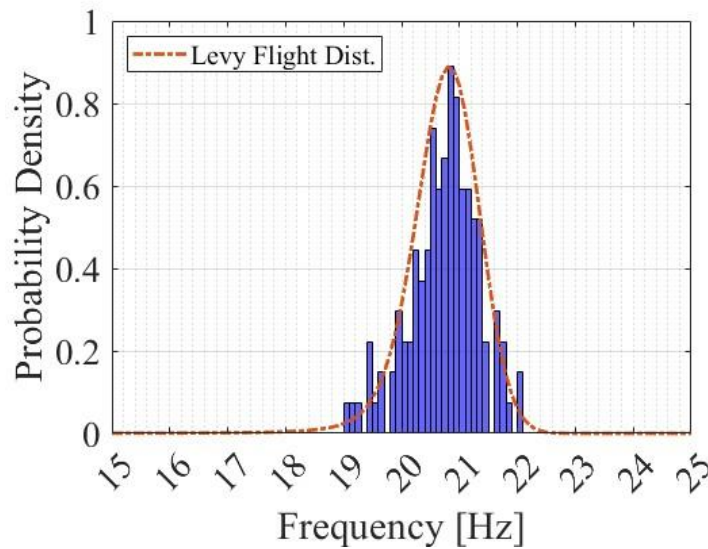


Figure 3.10 Identified MPVs for rocking frequency at Step 1 with Levy-Flight Distribution fitting.

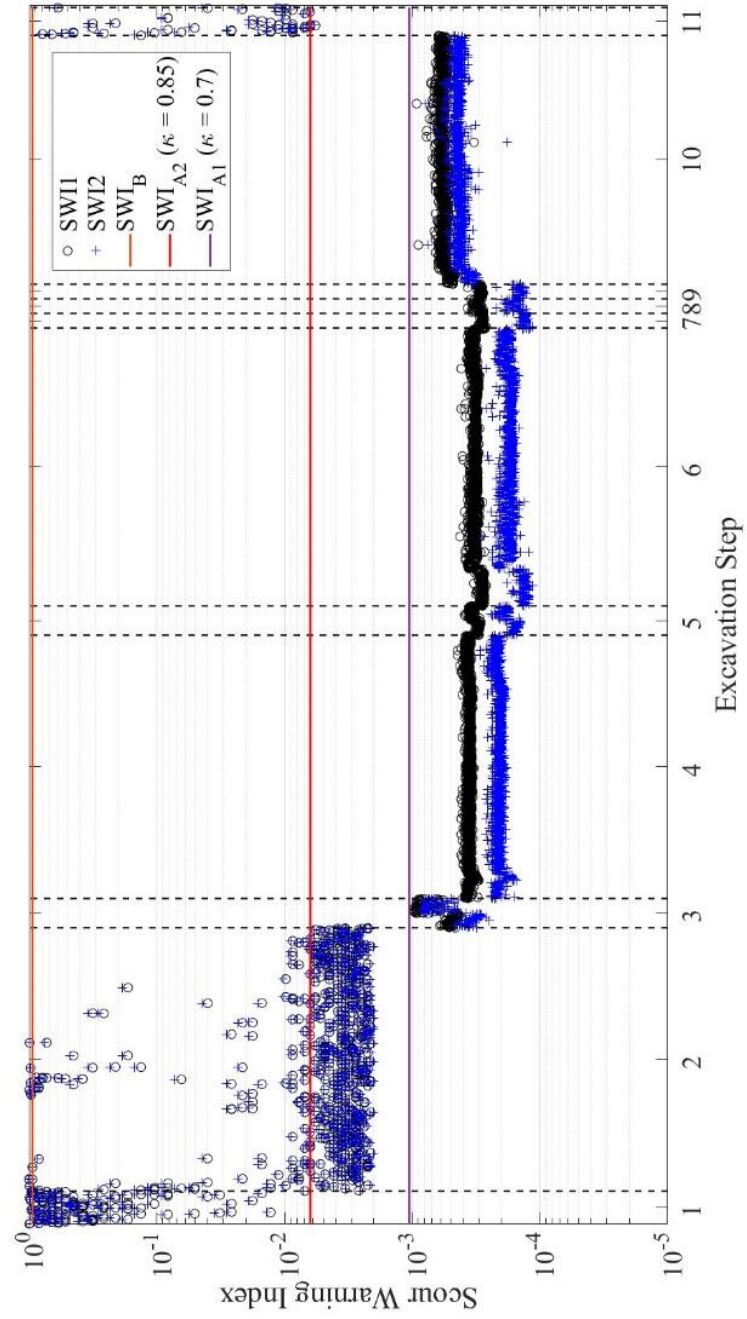


Figure 3.11 Results in applying SWI1 and SWI2 from microtremors.

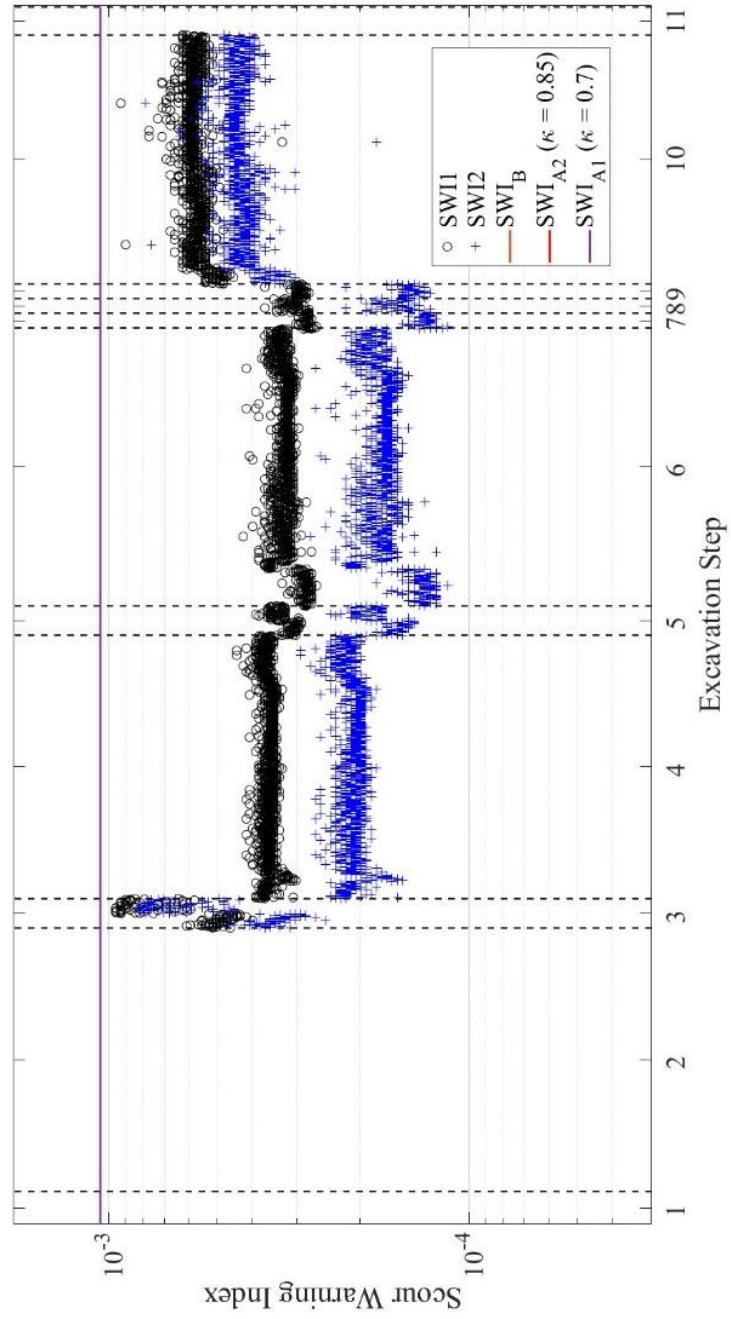


Figure 3.12 The enlarged figure of **Figure 3.11**.

3.2.4. Application of Scour Warning Index to case study 2

As discussed in Section 3.2.2, the LF Distribution is modeled for the distribution of identified MPVs during normal period and the parameters for the LF Distribution are shown in **Equation (3-11)**. The borders of each scour soundness category shown in **Table 3.1** are calculated in **Equation (3-8)**. Finally, the values of the borders are defined as follows.

$$\begin{aligned} SWI_B &= \Psi(f_B) = \Psi(9.15) = 0.25 \\ SWI_{A2} &= \Psi(f_{A2}) = \Psi(7.78) = 6.00 \times 10^{-3} \\ SWI_{A1} &= \Psi(f_{A1}) = \Psi(6.41) = 2.10 \times 10^{-3} \end{aligned} \quad (3-15)$$

● Application to SRW period

SWI1 and SWI2 were applied to the observation during the SRW period in 3rd September 2021. The result is shown in Figure 3.13. The black circles and plus markers correspond to SWI1 and SWI2, respectively. The horizontal lines denote the borders of each category in the scour assessment. During the SRW period when the water level (distance between water surface and lower flange of the bridge) was 6m, SWI1 and SWI2 were categorized as S rank and B rank. When the water level decreased gradually during the SRW period, the ratio of B rank plots increased approximately 48 %. After the SRW period, SWI1 and SWI2 plotted at S rank or B rank, and the ratio of S rank was 52 %. Focusing on values of the indices, they plotted from around 0.1 to 1. It means that an event of scour occurrence of about 0.1 to 1 % during the normal period is observed now. These outputs provide us an information that the probability of scour occurring during the SRW period is low.

Next, the differences between SWI1 and SWI2 are discussed. There were no significant differences in outputs between SWI1 and SWI2. This might be because the SN ratio became relatively high because of the water-induced vibration and the identification error would have been reduced during the water-rise period.

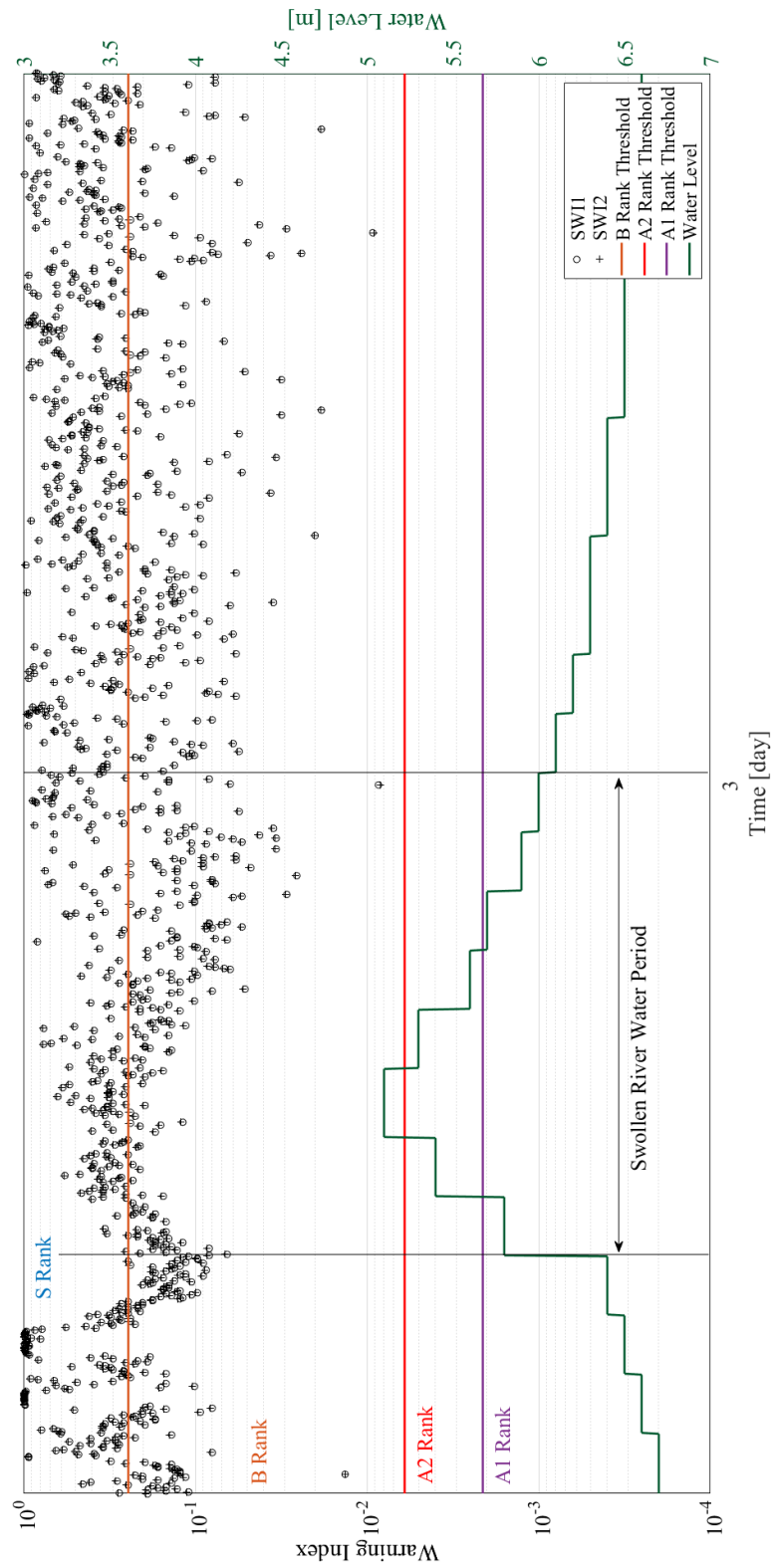


Figure 3.13 Results in applying SWI1 and SWI2 from microtremors of the concrete pier.

- Application to artificial acceleration to simulate scour occurrence

Scour progression had not been observed by regular inspection during the long-term monitoring campaign. To verify how the proposed SWIs behave when the scour occurs, this study creates artificial acceleration responses and applies them to the SWIs.

Because the natural frequency of a bridge pier decreases as the scour progresses, this study expresses the decreased natural frequency via frequency shift processing. First, the original observed acceleration is transformed into the frequency domain by Fast Fourier Transform (hereafter FFT). Second, frequency shifts are processed. According to the border of each category as shown in **Table 3.1**, -2 Hz is adapted as a shifted value. Finally, the inverse FFT is used to reproduce the time-domain acceleration. Then, fast Bayes FFT is used to identify the MPVs and standard deviations of the target frequency, which are then applied to the SWIs.

This study assumes that the scour rapidly progresses. Therefore, the natural frequency of the target pier is drastically decreasing when the water surface level is highest in 3rd September 2021. Artificial acceleration responses were used 20 minutes after the water level reached its highest point of 5.1 m. The time-series of the identified target frequency are shown in Figure 3.14. It is clear that changes in -2 Hz occur at the peaks of water level. In addition, there is a trend that large coefficients of variance are seen where the identified MPVs of the rocking frequency are low during the scour period.

The results of applying SWI1 and SWI2 to the artificial acceleration are shown in Figure 3.15. SWI1 and SWI2 were categorized between S rank and B rank during the normal period. However, both indices were categorized as A2 rank when the water surface level peaked, and thereafter, they were plotted between A2 and A1 ranks. Focusing on values of indices, they plotted around 5.00×10^{-3} . This means that a rare event where the probability of occurrence at the normal period is approximately 0.5 % is continuously observed now. It reveals that both indices reflect scour progression, and they alert the anomalous situation. Focusing on differences between SWI1 and SWI2, the values of SWI2 are smaller than SWI1. There is a tendency that SWI2 shows safety-side decisions rather than SWI1. These differences can be attributed to the fact that SWI2 takes into account the coefficients of variance of the identified target frequency. Observations that the coefficients of variance are large as shown by the error bars in Figure 3.14 have large uncertainties, which are reflected in the variance of $h(v)$ in **Equation (3-9)**. Therefore, SWI2 shows stricter decisions (i.e. safety-side decisions).

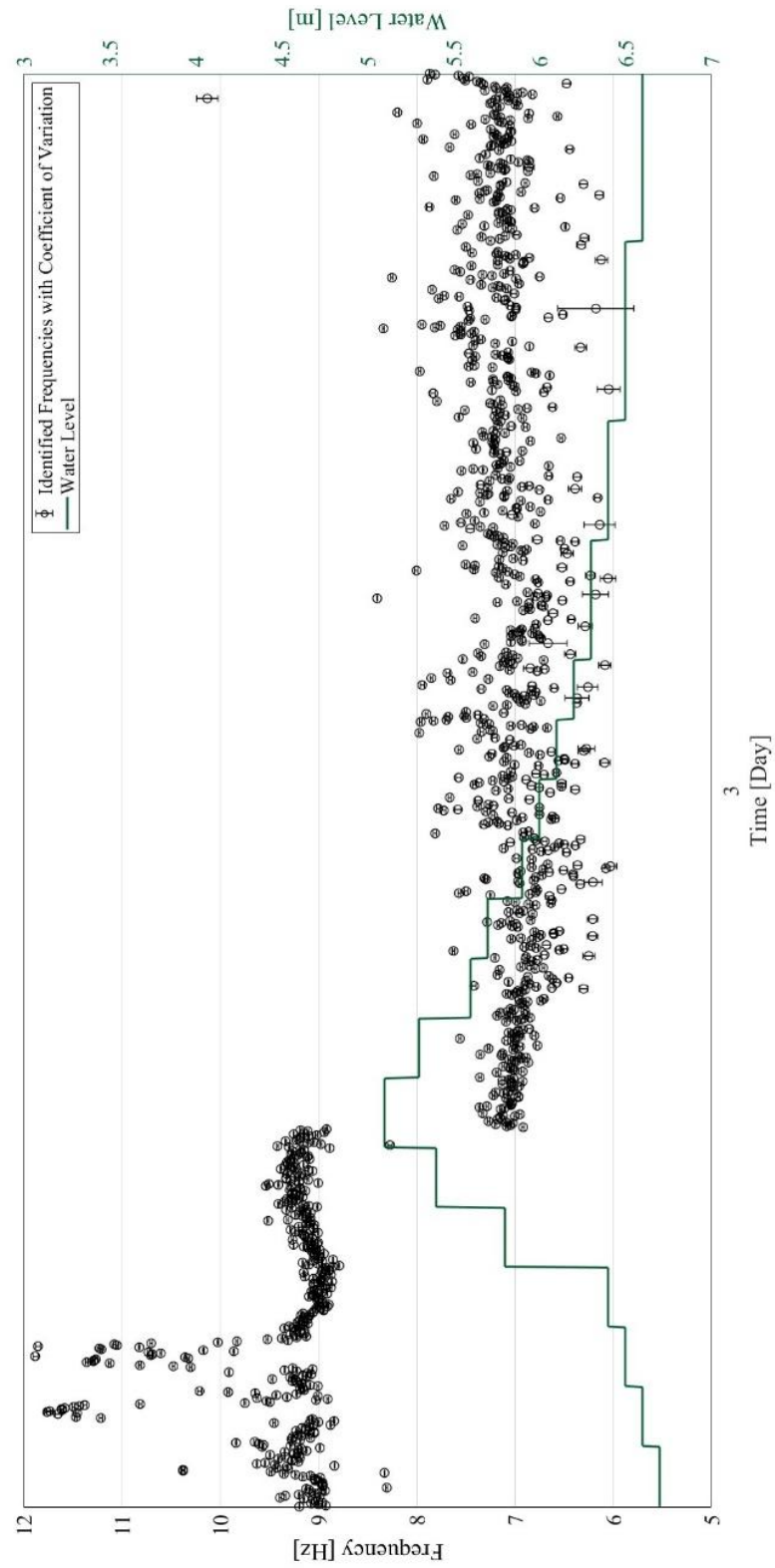


Figure 3.14 Time-series of identified MPVs of the rocking frequency with coefficients of variance during SRW period using artificial acceleration.

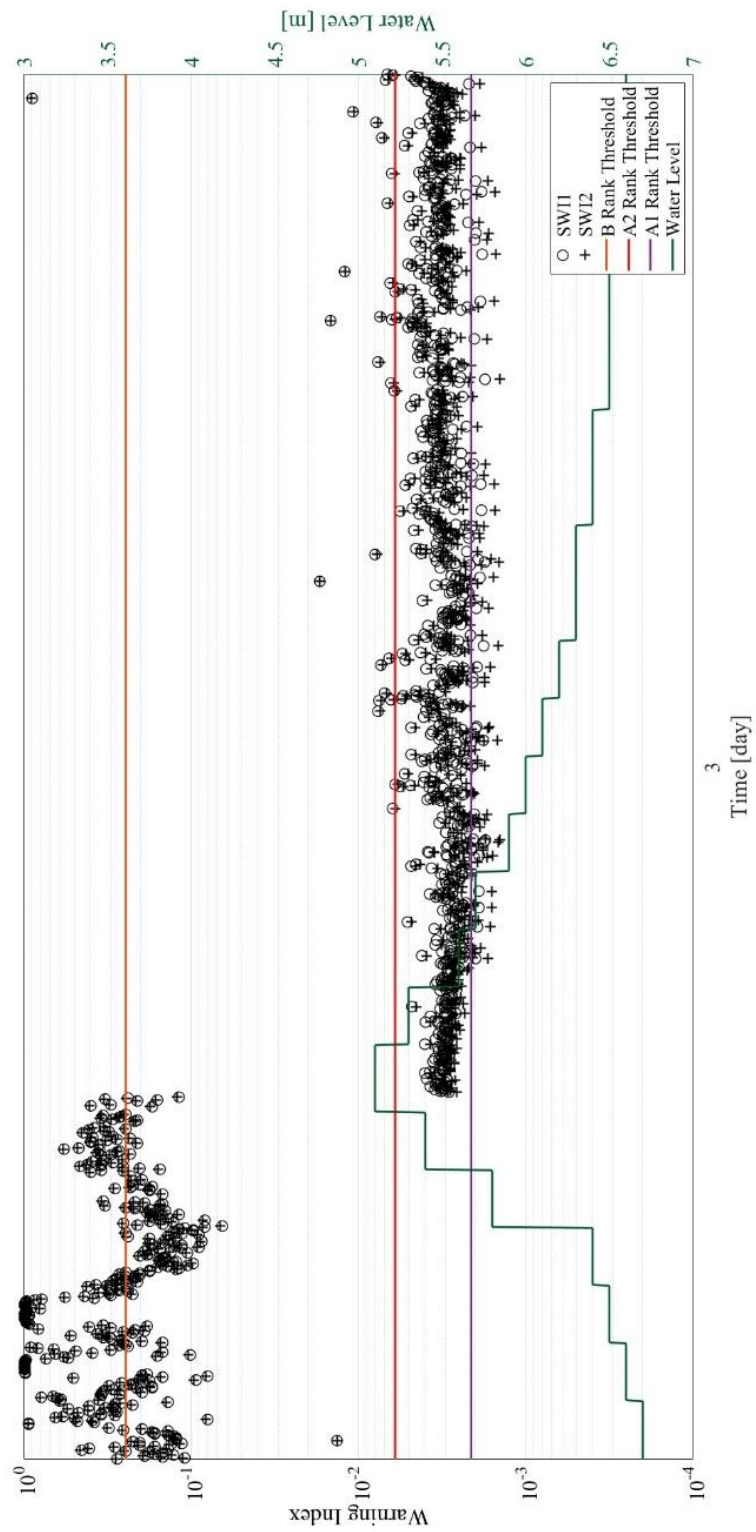


Figure 3.15 SWI1 and SWI2 are applied to identified MPVs of the rocking frequency with coefficients of variance using artificial acceleration.

3.3. Probability of scour occurrence (PSO)

In Section 3.2, the Scour Warning Index (SWI) was proposed to infer as the way of thinking that “there is a high possibility that scour would occur because the identified value of the target frequency, which can be obtained with extremely low probability during the normal period, has been continuously observed at the SRW period”. This section discusses a different concept of statistical interpretation from the SWIs. To provide more intuitively understandable information, this section proposes a statistical index for scour assessment called Probability of Scour Occurrence (hereafter PSO). A way to apply more familiar probabilistic ideas to scour phenomena, such as the probability of rain, the LF Distribution discussed in Section 3.2.2 is used to calculate PSO. The proposed methodology is explained in the next section and applied to two case studies.

3.3.1. Methodology

An event corresponding to the A1 rank as shown in Table 3.1 is recognized as severe local scour, and other events doing to other ranks as the normal condition. In other words, when identified frequency $x = f_t$ is observed, $f_t \leq f_{A1}$ is defined as occurring an A1 rank level event, and $f_t > f_{A1}$ is defined as no scouring. In this case, it can be considered as a qualitative data. We can assume the following ideas based on f_{A1} and the result in the impact test, 9.15 Hz.

- The probability of scour occurrence is extremely low and asymptotically approaches zero when the target frequency is identified around 9.15 Hz during the SRW period.
- The probability of scour occurrence is high when the target frequency is identified around f_{A1} .

Then, the Logistic function that the probability of scour occurrence asymptotically approaches 0 or 1 when the identified target frequency is decreased or increased, respectively, can be considered. By expressing the probability of scour occurrence as q , the Logistic function can be written as follows [20]:

$$q(\Gamma) = \frac{1}{1 + e^{-\Gamma}} \quad (3-16)$$

Where Γ is the linear predictor, which can be expressed as explanatory variables and partial regression coefficients. Because the explanatory variable is only the target frequency, x , Γ can be written as follows:

$$\Gamma = \beta_0 + \beta_1 x \quad (3-17)$$

This study uses Logit model that defines the probability distribution of the error term as the Logistic Distribution, and the partial regression coefficients, β_0 , and β_1 can be estimated using the MLE method.

In actual bridge monitoring, it is difficult to obtain the identified frequencies when the scour occurs due to the limitation of sensor installation works. Therefore, estimating the Logistic function, the following assumption is considered.

- When the natural frequency is decreased by scouring, shape and scale parameters of the LF Distribution shown in **Table 3.3** remain, but only the location parameter, δ changes.

Then the probability distribution of decreasing natural frequency can be expressed as

$$g(x) \sim LF(\alpha_N, \beta_N, \gamma_N, \delta_S) \quad (3-18)$$

where δ_S is a location parameter when the natural frequency is decreasing. Scale parameter δ of the LF Distribution can be estimated using the mean value of the LF Distribution as [11]

$$\delta = \mu_x + \beta \gamma \tan\left(\frac{\pi \alpha}{2}\right) \quad (3-19)$$

Substituting **Equation (3-19)** into **Equation (3-18)**, probability distribution can be calculated. Following **Equation (3-7)**, the cumulative density function can be written as follows.

$$\Psi_{A1}(x) = G(x) = \{\alpha_N, \beta_N, \gamma_N, \mu_x + \beta \gamma \tan\left(\frac{\pi \alpha}{2}\right), x\} \quad (3-20)$$

Now, the probability distribution when the natural frequency is decreasing can be estimated using decreasing target identified frequency and shape and scale parameters during the normal period. $\Psi_{A1}(x)$ corresponds to the probability that scour does not occur. Therefore, when the PSO given $x = f_x$ is written as $P(\theta_S|f_x)$, $P(\theta_S|f_x)$ is calculated as follows.

$$P(\theta_S|f_x) = 1 - \Psi_{A1}(x) \quad (3-21)$$

Because $P(\theta_S|f_x)$ is discrete information, Logistic function is fitted.

3.3.2. Application of Probability of Scour Occurrence to case study 1

The parameters of the LF Distribution in Step 1 of the RC specimen experiment, referring **Equation (3-13)**, are used to estimate the Logistic function at the A1 rank. The mean value of the LF Distribution in Step 1 in **Equation (3-13)** is $\mu_x = 20.7$ Hz. Following the soundness index categories in **Table 3.1**, the decreased natural frequency at the A1 rank is $21.6 \times 0.7 = 15.1$ Hz. Then, the cumulative density function of the LF Distribution at the A1 rank is estimated as shown in Equation (3-22)

$$\Psi_{A1}(x) = \begin{cases} \alpha_N = 1.85, \beta_N = -1.00, \gamma_N = 0.39, \\ \mu_x + \beta\gamma \tan\left(\frac{\pi\alpha}{2}\right) = 14.88, x \end{cases} \quad (3-22)$$

Cumulative density functions are shown in Figure 3.16 for visualization. The blue and orange lines are cumulative density functions of the LF Distribution in Step 1 (denoted as “LF Dist. at St. 1”) and at estimated A1 rank (denoted as “Est. Dist. at A1”). For example, when the identified frequency is 21.6 Hz, $\Psi_{A1}(x = 21.6) = 1$, while the identified frequency is 15.0 Hz, $\Psi_{A1}(x = 15.0) = 0.61$. Values of Ψ_{A1} correspond to probability that scour does not occur. Therefore, PSO, $P(\theta_S|x = 21.6)$ and $P(\theta_S|x = 15.0)$ can be calculated as

$$\begin{aligned} P(\theta_S|21.6) &= 1 - \Psi_{A1}(21.6) = 0 \\ P(\theta_S|15.0) &= 1 - \Psi_{A1}(15.0) = 0.39 \end{aligned} \quad (3-23)$$

These are plotted as orange circles in Figure 3.17. Using the MLE method, the PSO fitted the Logistic function can be estimated using Equation (3-24) and drawn as the blue line in Figure 3.17.

$$P(\theta_S|x) = q(\Gamma) = \frac{1}{1 + e^{-(40.71 - 2.70x)}} \quad (3-25)$$

Finally, PSO is applied to the experimental data and the result is shown in Figure 3.18. It is very clear that PSO in Steps 1, 2 and 11 are mostly close to zero, while other Steps show near 1. This means the probability that the A1 rank event occurs is almost zero at Steps 1, 2 and 11, and is extremely high at Steps 3 to 10. These results completely correspond to impact test results summarized in **Table 3.2**. It demonstrated that the proposed PSO successfully represents the soundness category A1 and corresponding results as the scour progression. Figure **3.19** shows enlarged plots of Steps 1, 2, and 11 shown in Figure 3.18.

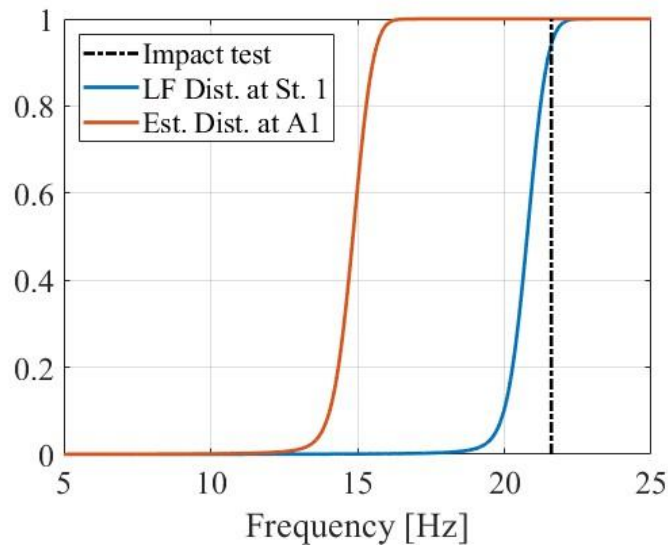


Figure 3.16 Cumulative density functions of the LF Distribution and estimated Distribution at A1 rank damaging.

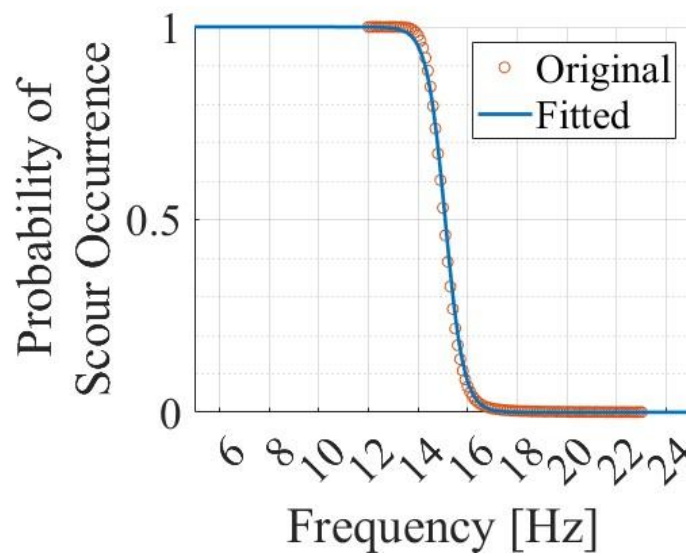


Figure 3.17 The logistic curve estimated from identified MPVs of rocking frequency at normal period.

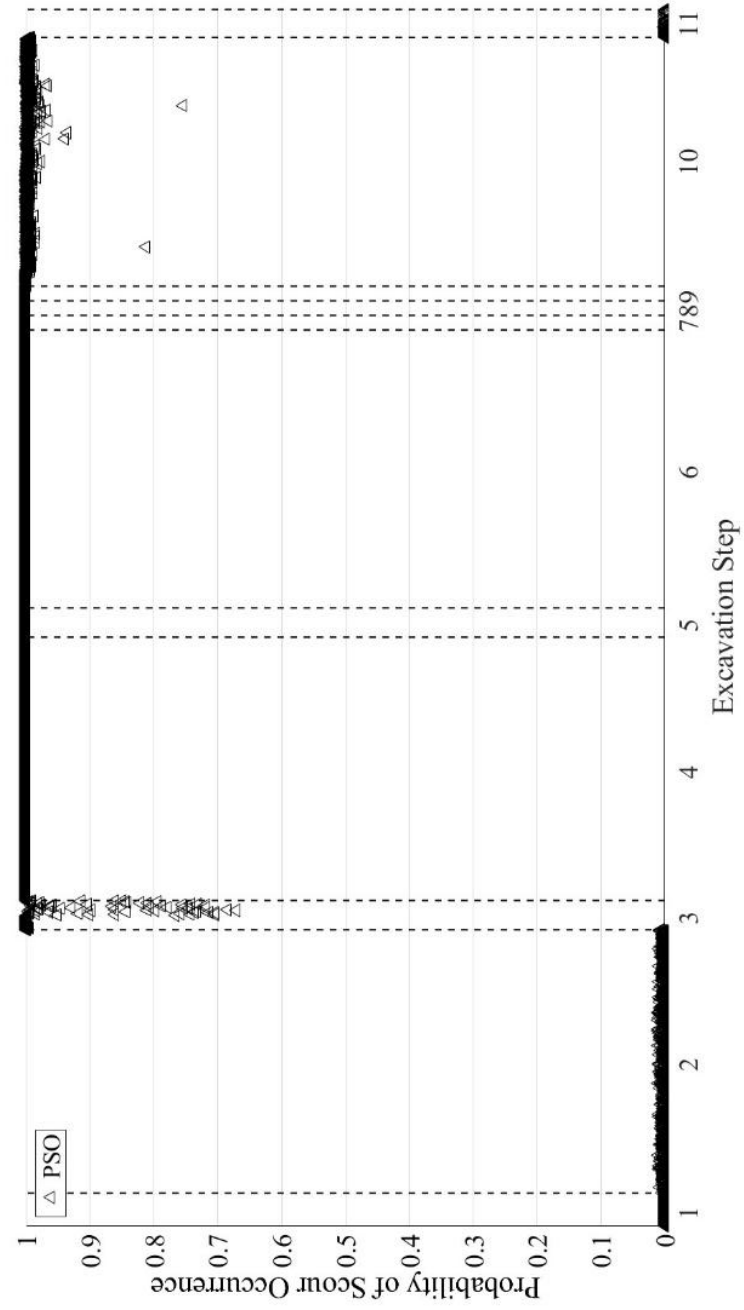


Figure 3.18 The result in applying PSO to the RC specimen experiment.

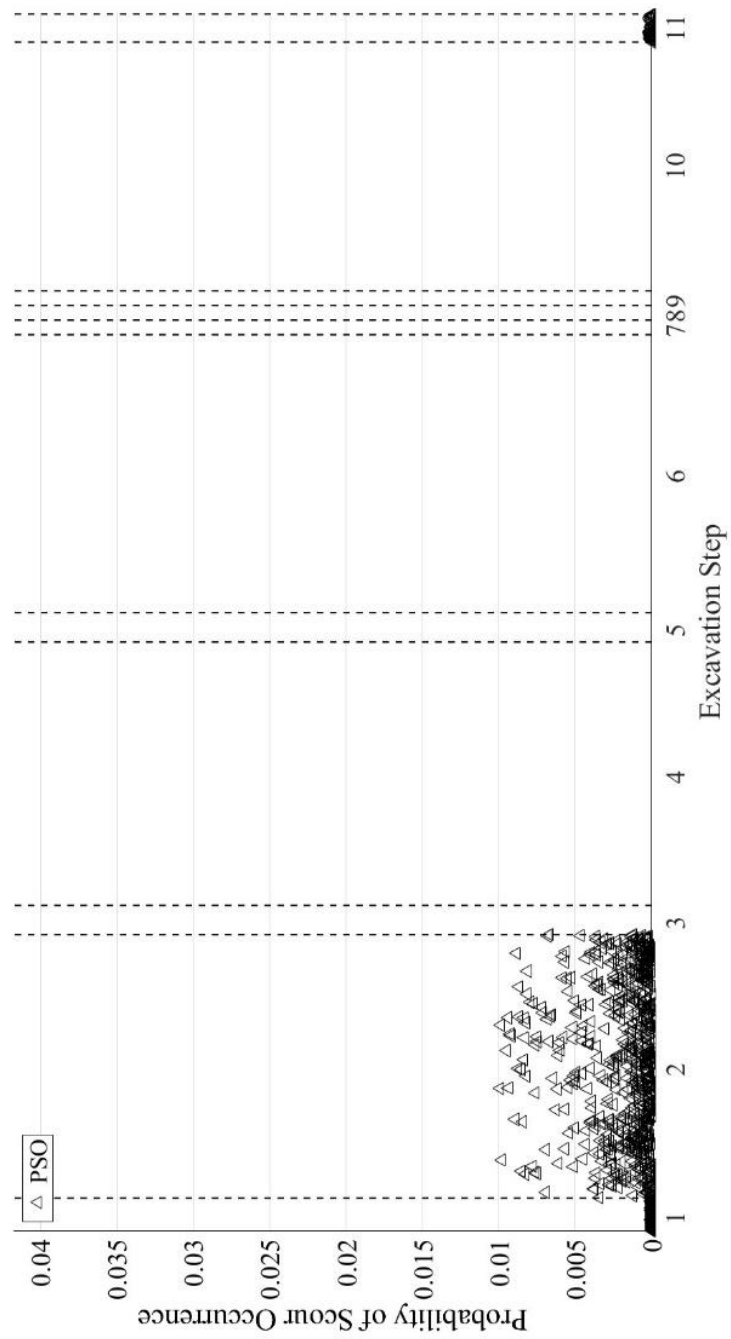


Figure 3.19 The enlarged result in **Figure 3.18** at around 0.

3.3.3. Application of Probability of Scour Occurrence to case study 2

Parameters of the LF Distribution during the normal period, referring **Equation (3-11)**, are used to estimate the Logistic function at A1 rank. The mean value of the LF Distribution during the normal period in **Equation (3-11)** is $\mu_x = 9.54$ Hz. Following the soundness index categories in **Table 3.1**, the decreased natural frequency at A1 rank is $9.15 \times 0.7 = 6.41$ Hz. Then, the cumulative density function of the LF Distribution at A1 rank is estimated as follows:

$$\Psi_{A1}(x) = \left\{ \begin{array}{l} \alpha_N = 1.58, \beta_N = 0.58, \gamma_N = 0.31, \\ \mu_x + \beta\gamma \tan\left(\frac{\pi\alpha}{2}\right) = 9.55, x \end{array} \right\} \quad (3-26)$$

By the MLE method, PSO fitted the Logistic function can be estimated as Equation (3-27) and drawn as blue line in Figure 3.20.

$$P(\theta_S|x) = q(\Gamma) = \frac{1}{1 + e^{-(21.00 - 3.12x)}} \quad (3-28)$$

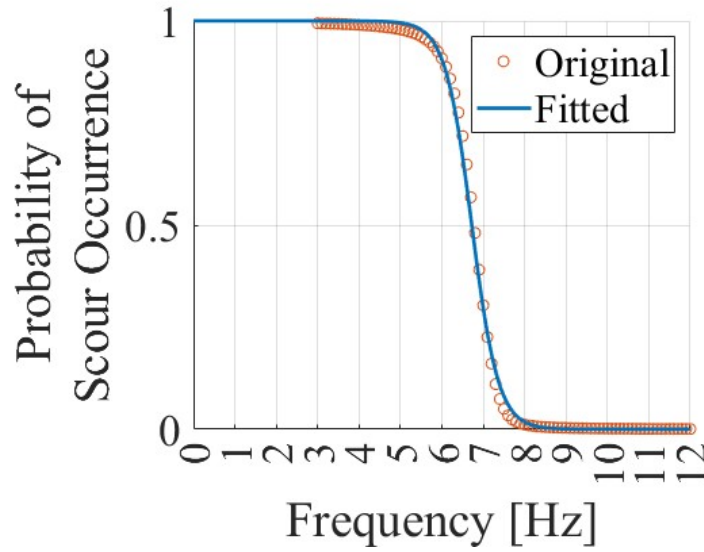


Figure 3.20 The logistic curve estimated from identified MPVs of rocking frequency during normal period.

- Application to SRW period

PSO were applied to the observation during the SRW period in 3rd September 2021. The result is shown in Figure 3.21. It is very clear that PSO at Steps 1, 2 and 11 are mostly close to zero, while other Steps show near 1. Values of PSO during the SRW period are converged around 10^{-3} . This means the probability of scour occurrence is around 10^{-3} . When the water surface level decreased after the peak water surface level, the PSO were gradually decreased around 5.0×10^{-3} , while values were scattered. As discussed in SWI1 and SWI2 in Section 3.2.4, water-induced vibration makes the SN ratio of observation response higher and identification error may be decreasing. Therefore, PSOs converge during highest water surface level.

Consequently, the probability of scour occurrence through the flood event is low.

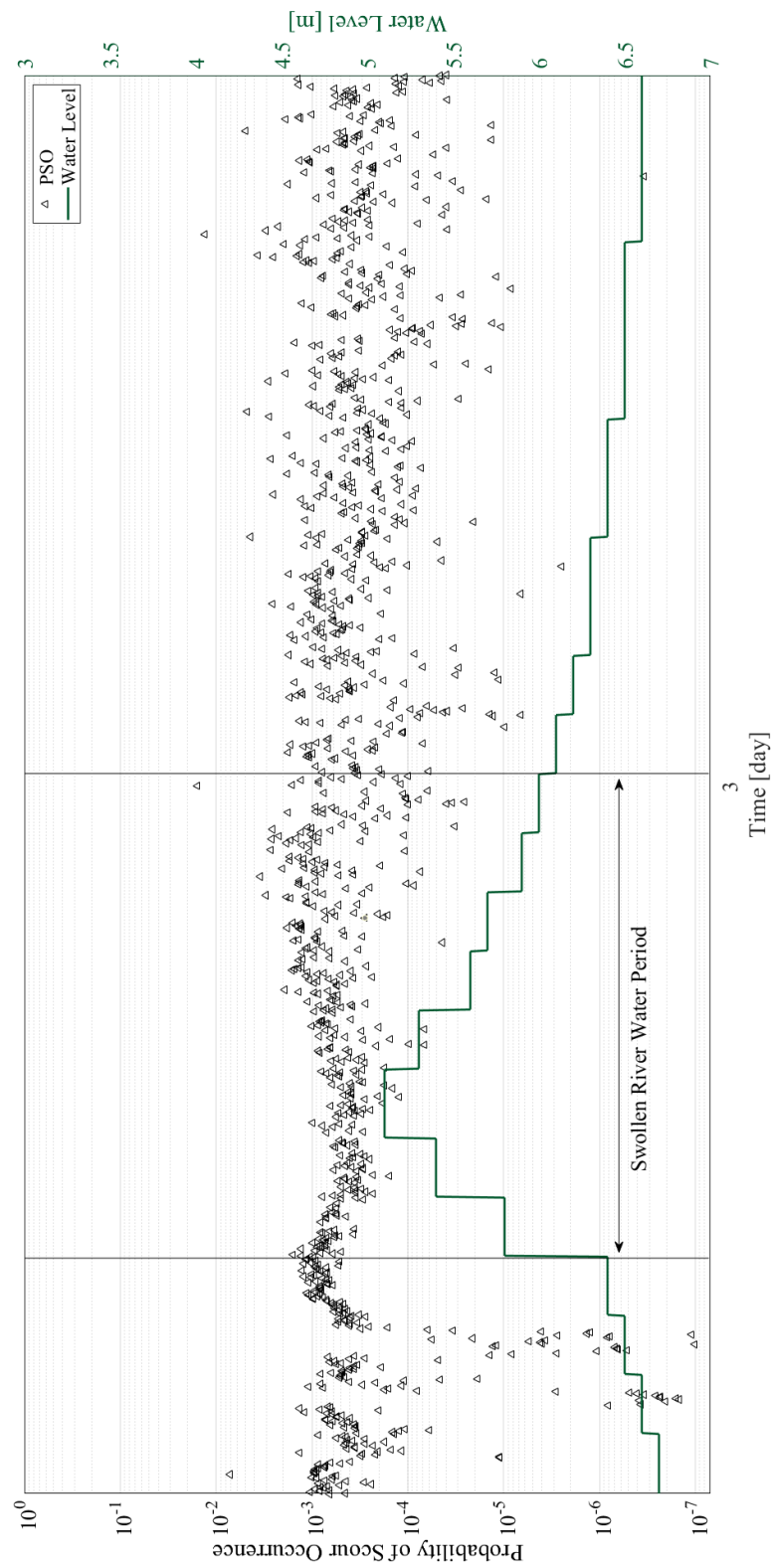


Figure 3.21 Results in applying PSO of the concrete bridge pier during SRW period.

- Application to artificial acceleration to simulate scour occurrence

Scour progression had not been observed by regular inspection during long-term monitoring campaign. To verify how the proposed PSO behaves when the scour occurs, this study creates artificial acceleration responses and applies them to the PSO. The way of building artificial acceleration is described in Section 3.2.4.. The result in applied PSO to the artificial acceleration in 3rd September 2021 is shown in Figure 3.22. Values of PSO after when water level reached highest level were around 0.1 to 1.0. It shows that the probability of scour occurrence is very high. It demonstrated that PSO reflects decreased frequency and alerts anomalous.

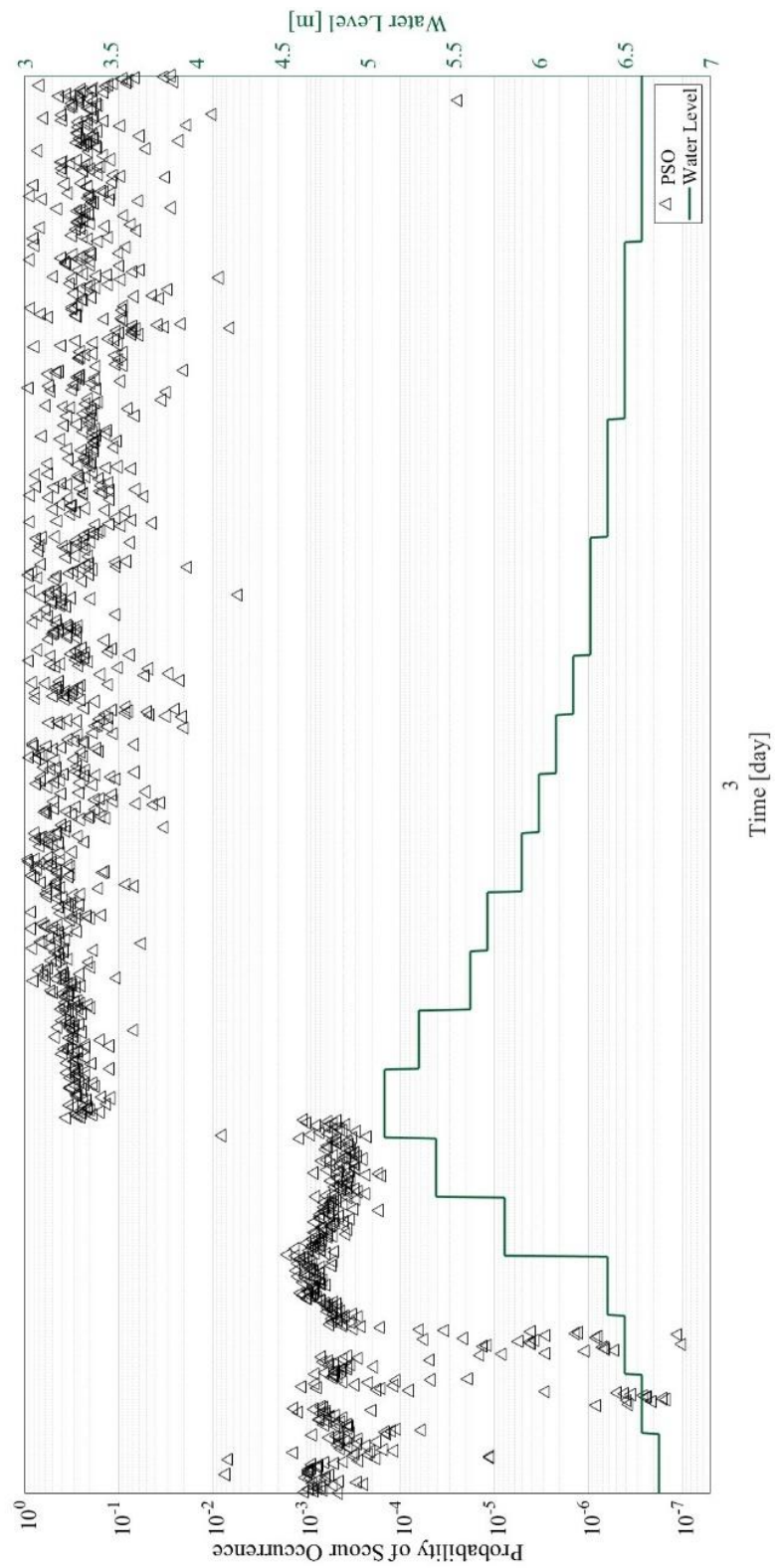


Figure 3.22 PSO of the concrete bridge pier to the artificial acceleration generated to represent SRW period.

3.4. Bayes Factor for anomaly detection

3.4.1. Bayes Factor

Using statistical hypothesis testing is one of the general anomaly detection approaches. However, in some cases, the conventional frequentist tests cannot express the significance of observations accurately [12, 21-23]. Frequentist tests tend to reject null hypotheses almost systematically in numerous samples. This has been a real problem in sociology, where data sets frequently have thousands of cases. In the case of long-term bridge health monitoring, the amount of data volume would become large because the observation is continuously provided by sensors installed on site. It must deal with the same issues that sociology has been facing. One alternative idea to realize rational anomaly detections is to use Bayes Factors (hereafter BF) which focuses on evaluating evidence in favor of a null hypothesis. There are several applications using BF in bridge health monitoring field [24-27]. Therefore, this chapter briefly introduces the definition of BF and the calculation method for the scour monitoring of bridge piers. Secondly, to present the rationality of BF, it is compared with conventional frequentist tests. Furthermore, since this study claims that identified frequencies of the real bridge pier are followed by the LF Distribution, BF using the LF Distribution is discussed by comparing BF using the Normal Distribution. Two case studies are conducted to present the performance of BF.

Definition

Kass et al. [12] provides the points of BF as follows.

- BF offers a way of evaluating evidence in favor of a null hypothesis.
- BF provides a way of incorporating external information into the evaluation of evidence about a hypothesis.
- BF is very general and does not require alternative models to be nested.

Now, $\mathbf{D}$ is data, and it is assumed to have arisen under one of the two hypotheses, H_0 , and H_1 according to a probability density $P(\mathbf{D}|H_0)$ or $P(\mathbf{D}|H_1)$. Given prior distributions $P(H_0)$ and $P(H_1)$, the data produces posterior distributions, $P(H_0|\mathbf{D})$ and $P(H_1|\mathbf{D})$. Because any prior opinion gets transformed to a posterior opinion through consideration of the data, the transformation itself represents the evidence provided by the data. From Bayes's theorem, the posterior distribution can be obtained by

$$P(H_k|\mathbf{D}) = \frac{P(\mathbf{D}|H_k)P(H_k)}{\sum_{i=0}^1 P(\mathbf{D}|H_i)P(H_i)}, k = 0,1 \quad (3-29)$$

The ratio of posterior distributions is written as

$$\frac{P(H_0|\mathbf{D})}{P(H_1|\mathbf{D})} = \frac{P(\mathbf{D}|H_0)}{P(\mathbf{D}|H_1)} \times \frac{P(H_0)}{P(H_1)} \quad (3-30)$$

Therefore, the transformation is simply multiplication by the ratio of likelihood functions as shown in Equation (3-31)

$$BF = \frac{P(\mathbf{D}|H_0)}{P(\mathbf{D}|H_1)} \quad (3-32)$$

This is called Bayes Factor. The likelihood functions $P(\mathbf{D}|H_k)$ are obtained by integrating over the parameter space, and are written as

$$P(\mathbf{D}|H_k) = \int P(\mathbf{D}|\theta_k, H_k)\pi(\theta_k|H_k)d\theta_k \quad (3-33)$$

where θ_k is the parameter under H_k , $\pi(\theta_k|H_k)$ is its prior distribution, and $P(\mathbf{D}|\theta_k, H_k)$ is the probability density of $\mathbf{D}$ given the value of θ_k , i.e. likelihood function of θ . The quantity $P(\mathbf{D}|H_k)$ is the marginal probability of the data, because it is obtained by integrating the joint density of $(\mathbf{D}, \theta_k)$ given $\mathbf{D}$ over θ_k . It is also the predictive probability of the data; that is, the probability of the data that actually were observed, calculated before any data became available. It is also sometimes called a marginal likelihood, or an integrated likelihood. Note that, one of the hypotheses is considered the null and thus is denoted by H_0 . In this case BF is traditionally calculated H_0 as denominator.

For simplicity, **Equation (3-33)** is rewritten as

$$I = \int P(\mathbf{D}|\theta, H)\pi(\theta|H)d\theta \quad (3-34)$$

Most cases are necessary to compute it by numerical methods. A useful approximation to the marginal density of the data as given by **Equation (3-34)** is obtained by assuming that the posterior density, which is proportional to $P(\mathbf{D}|\theta, H)\pi(\theta|H)$ is highly peaked about its maximum $\tilde{\theta}$, which is the posterior mode. The method using this assumption is called Laplace's method of approximation [28, 29].

On the other hand, the simplest Monte Carlo integration estimate of **Equation (3-34)** is

$$\hat{P}(\mathbf{D}) = \frac{1}{m} \sum_{i=1}^m P(\mathbf{D}|\theta^{(i)}) \quad (3-35)$$

where $\{\theta^{(i)}: i = 1, \dots, m\}$ is a sample from the prior distribution; this is the average of the likelihoods of the sampled parameter values [30]. Importance sampling is also discussed to realize the precision of integration process [31].

The BF is a summary of the evidence provided by the data in favor of one scientific theory, represented by a statistical model, as opposed to another. Jeffreys [21] suggested interpreting BF in half units on the log10 scale. Kass et al. [12] illustrated twice the natural logarithm of the BF for usefulness. The interpretation of Kass is summarized as Table 3.4.

Regarding to anomaly detection, the output would have enough evidence to illustrate “clearly anomalous” if the $2 \log_e BF$ is greater than 10, according to Table 3.4.

Table 3.4 Interpretation of Bayse Factor by Kass et al. [12].

$2 \log_e BF$	Evidence against H_0
0 to 2	Not worth more than a bare mention
2 to 6	Positive
6 to 10	Strong
>10	Very Strong

Comparing a conventional frequentist test with Bayes Factor

By using identified frequencies of the RC specimen experiment, the conventional frequentist test is compared with BF. For this comparison, the identified frequencies are assumed to follow Normal Distribution and assumed that the variance is known. The identified frequencies in Step 1 of the experiment are regarded as data at healthy condition. Besides, 300 data for validation are created by decreasing values of identified data to 30% in increments of 0.1%. For each data, P-value and BF are calculated by the following procedures.

- P-value of the conventional Frequentist test

For the conventional frequentist test, Z-test is applied as the mean of the population is unknown but the variance is known. Then, each sample follows Normal Distribution: $Normal(\mu, \sigma^2 =$

0.59^2), independently. The population at a healthy condition follows the Normal Distribution: $Normal(\mu_0 = 20.74, \sigma^2 = 0.59^2)$.

Here, H_0 and H_1 are defined as null and alternative hypotheses, respectively. Both tests are set as below.

Null hypothesis $H_0 : \mu = \mu_0$

Alternative hypothesis $H_1 : \mu < \mu_0$

As a frequency value should not increase from a healthy condition, the one-sided test is applied. where $\{x^{(i)} : i = 1, \dots, N\}$ is a sample from the data $\mathbf{D}$, standardization of data is conducted as below.

$$z = \frac{\frac{\sum x^{(i)}}{N} - \mu_0}{\frac{\sigma}{\sqrt{N}}} \quad (3-36)$$

Z-value follows the standard Normal Distribution: $Normal(0, 1^2)$ and P-value is estimated as Equation (3-37)

$$P_value = \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = \frac{\frac{\sum x^{(i)}}{N} - \mu_0}{\frac{\sigma}{\sqrt{N}}} \quad (3-38)$$

Using P-value and significance level α , results in Z-test are interpreted as

- P-value $< \alpha$: reject H_0 , which means data represent at “Not healthy” condition
- P-value $> \alpha$: fail to reject H_0 , which means data do not represent at “Not healthy” condition

The above calculation is applied to each data and the result is shown in **Figure 3.23**. The significance level α is set to be 0.01.

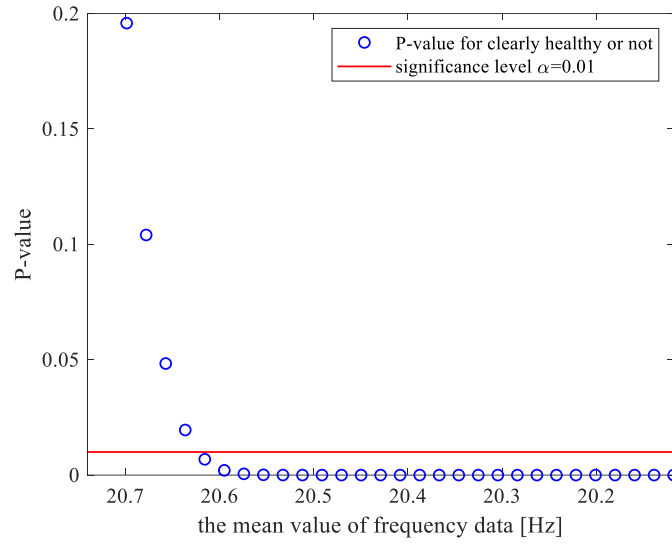


Figure 3.23 Results in conventional hypothesis testing of the RC specimen experiment.

● BF

For the Bayesian hypothesis test, Bayesian inference of estimating only an unknown μ of the Normal Distribution is adopted to calculate BF since a sample follows the Normal Distribution and its variance is known. Each two hypothesis represents a healthy and not healthy condition respectively. Here, the distribution of “not healthy condition” is set that the lower limit of 1% is around 14.5. This is because the identified frequency would be realistically no less than 50% from the healthy state, 21.6 Hz, no matter how the bridge scour progresses in real bridges. Therefore, the value of standard deviation is set to be large. The prior distributions are set as

$$H_0(\text{Healthy}): P(H_0) = \text{Normal}(\mu \mid 20.74, 0.59^2)$$

$$H_{1c}(\text{Not Healthy}): P(H_{1c}) = \text{Normal}(\mu \mid 20.74, 2.68^2)$$

BF for judging whether healthy or not is written as

$$BF = \frac{P(\mathbf{D} \mid H_{1c})}{P(\mathbf{D} \mid H_0)} \quad (3-39)$$

By using interpretation of BF by Kass et al. [12], above BF is interpreted as

- $2 \log_e BF < 2$: Evidence against H_{1c} is not worth more than a bare mention
- $2 \log_e BF > 2$: Evidence against H_{1c} is positive

The above calculation is applied to each data and the result is shown in **Figure 3.24**.

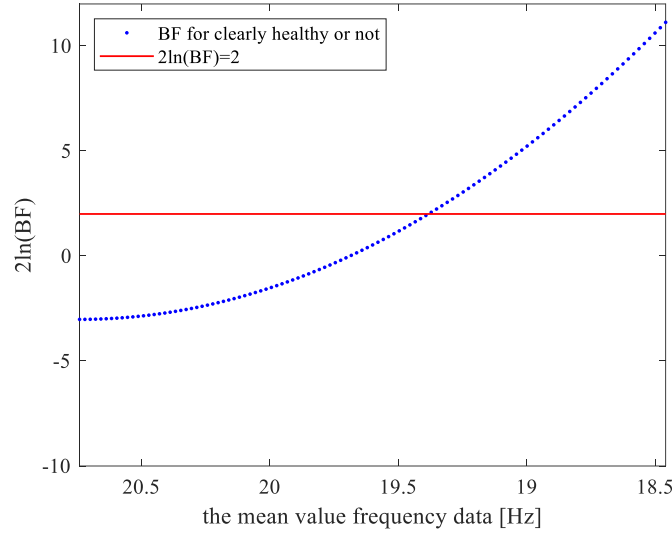


Figure 3.24 Results in BF of the RC specimen experiment.

- Discussion of both results and rationality of BF

As shown in **Figure 3.23**, when the mean value of frequency data decreases by 0.1 Hz from healthy condition, the null hypothesis H_0 is rejected. Frequentist tests tend to judge the structure as at not healthy condition almost systematically. On the other hand, by setting the prior distribution H_{1c} which represents not healthy condition adequately in BF, judgments become flexible and rational.

Additionally, for frequentist test, failures of rejecting null hypothesis H_0 do not mean adopting alternative hypothesis H_1 . In other words, even if P-value is greater than the significance level α , the structure is not judged as at healthy condition. This asymmetry of the test is another defect of frequentist test. In contrast, BF is used to determine which prior distribution the data provides stronger evidence for. If $2 \log_e BF$ is smaller than the threshold, it indicates that the evidence from the data supports the prior distribution H_0 , which represents a healthy condition.

For the above reasons, BF is more rational for a statistical judgement method in bridge health monitoring field compared to a conventional Frequentist test.

Normal Distribution vs Levy-Flight Distribution

This section discusses differences of results in BF when we adopt the Normal Distribution and the LF Distribution. Parameters of Normal Distribution and the LF Distribution at each state are summarized in Table 3.5 and Table 3.6. “Not healthy state” of the Normal Distribution and the LF Distribution are set to the standard deviation and the parameter, γ , so that the lower limit of 1% would include within a realistic value, 14.5 Hz. “A1 rank damaged state” of the Normal Distribution and the LF Distribution are set to the mean and the parameter, δ , so that the value satisfies the soundness categories shown in **Table 3.1**. All distributions are shown in Figure 3.25.

Results in BF are shown in Figure 3.26. Following Kass’s interpretation as shown in Table 3.4, $2 \log_e BF = 2$ and 10 are drawn as blue and red lines. The Normal Distribution result in Figure 3.26(a) shows increasing trends when the identified frequency is decreased. On the other hand, the result for the LF Distribution in Figure 3.26(b) shows different trends. In “not healthy” result, the yellow dashed line does not reach the red line, which indicates very strong evidence that the output is not healthy, when the identified frequency is low. However, it exceeds the blue line, which indicates positive evidence that output is not healthy. Therefore, when we apply BF of the LF Distribution for detecting “healthy” or “not healthy”, $2 \log_e BF = 2$ should be used.

In “A1 rank damaged” result, the orange chain line exceeded the red line between 12.5 and 16 Hz. This result is useful for detecting the damaged state. While not significantly affecting the engineers’ judgement, it should be noted that the result below 12.5 Hz is below the red line.

These results provide important information for establishing the applicable range of the LF Distribution with the identified frequency. Conversely, if an incorrect applicable range of the LF Distribution with identified frequency, the BF derived from the LF Distribution may indicate a incorrect anomaly detection.

Table 3.5 Parameters of the Normal Distribution at healthy, damaged, and not healthy states.

	Mean	Standard deviation
H_0 : Healthy	20.74	0.59
H_1 : A1 rank damaged	14.52	0.59
H_{1c} : Not healthy	20.74	2.68*

* Setting the lower limit of 1% to be 14.5.

Table 3.6 Parameters of the Levy-Flight Distribution at healthy, damaged, and not healthy states.

	α	β	γ	δ
H_0 : Healthy	1.85	-1.00	0.39	20.81
H_1 : A1 rank damaged	1.85	-1.00	0.39	14.60
H_{1c} : Not healthy	1.85	-1.00	1.24*	20.81

* Setting the lower limit of 1% to be 14.5.

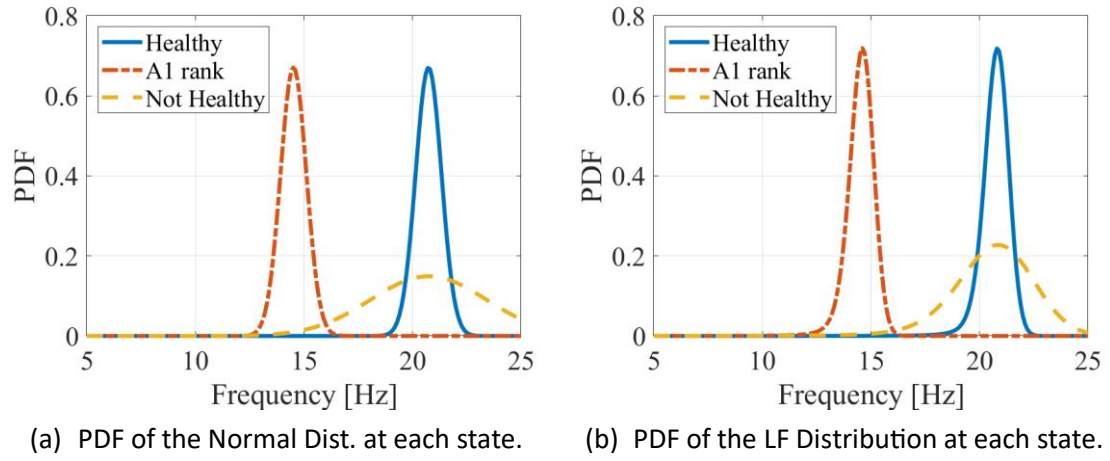


Figure 3.25 Distributions of each state.

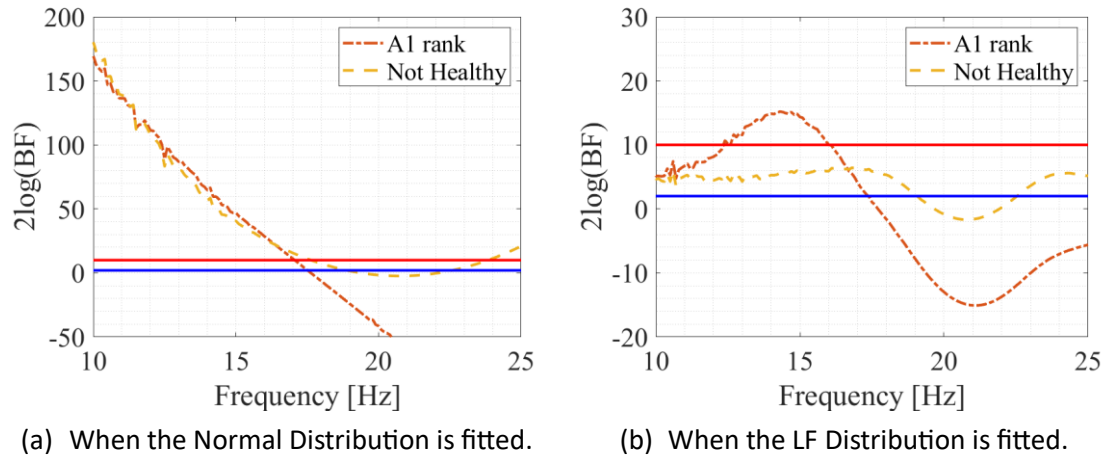


Figure 3.26 Changing trends of Bayes Factor when the identified frequency is observed.

Blue and red lines are $2\log_e \text{BF} = 2, 10$ following Kass interpretation [12].

3.4.2. Application of Bayes Factor to case study 1

In this section, BF is applied to the RC specimen experiment. Firstly, the result in applying BF of detecting “A1 rank damaged state” using the Normal Distribution is shown in Figure 3.27. Each blue square plot represents $2 \log_e(BF)$. The blue and red lines represent the borders following Kass’s interpretation as shown in Table 3.4. In Step 2, the plots are unstable and difficult to judge “clearly anomalous” because few plots exceed the red line, indicating that the scour reaches the A1 rank category. After Step 3, all plots exceed the red line, and it can be said that the output shows the evidence of reaching A1 rank category is very strong.

The result in applying BF of detecting “Not healthy state” using the Normal Distribution is shown in Figure 3.28. Each green inverted triangle plot represents $2 \log_e(BF)$. In Step 1, all plots are below the blue line. This means there is a lack of evidence that the output is anomalous. However, in Step 2, most plots exceed the blue line, indicating that the evidence that the output is anomalous is positive. To show that the output is not in a healthy state, the blue line is preferred to the red line. Therefore, BF using Normal Distribution shows that the output has evidence that the condition is not healthy in Step 2, and has very strong evidence that the scour reaches at A1 rank category after Step 3. This matches perfectly the result in impact test shown in Table 3.2.

Next, the result in applying BF of detecting “Not healthy state” using the LF Distribution is shown in Figure 3.29. In Step 2, although some plots exceed the red line, many plots do not reach the red line. This means that the evidence that the scour reaches A1 rank category is not sufficient in Step 2. In Steps 3 and 10, half plots exceed the red line, and it can be said that the output shows the evidence of reaching A1 rank category is very strong. However, after Step 4, results in BF were within the blue and red lines even though the scour progressed. This is because the impact test result in Step 4 is 9.8 Hz as shown in Table 3.2, and BF using the LF Distribution shows $2 \log_e(BF) < 10$ as shown in Figure 3.26(b). In fact, the identified frequencies and values of BF follows Figure 3.26(b).

The result in applying BF of detecting “Not healthy state” using the LF Distribution is shown in Figure 3.30. As well as Figure 3.29, the result follows Figure 3.26(b). In Step 2, most plots exceed the blue line, indicating that the evidence that the output is anomalous is positive. Therefore, BF using the LF Distribution shows that the output has evidence that the condition is not healthy in Step 2.

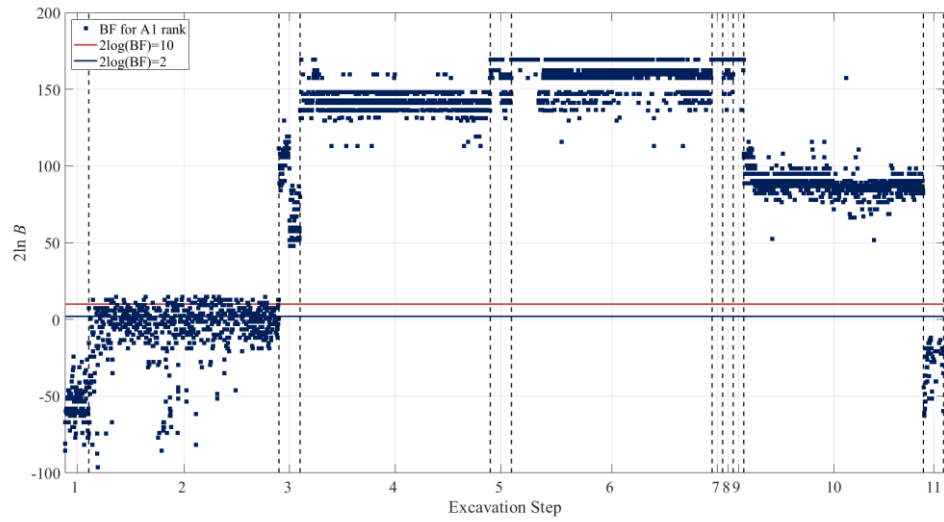


Figure 3.27 Results in applying Bayes Factor assuming Normal Distribution for A1 rank level detection.

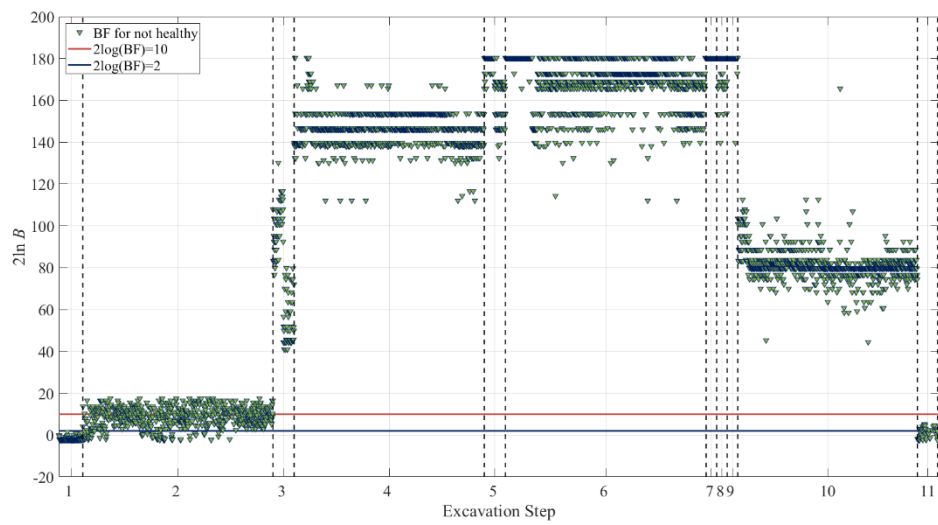


Figure 3.28 Results in applying Bayes Factor assuming Normal Distribution for not healthy detection.

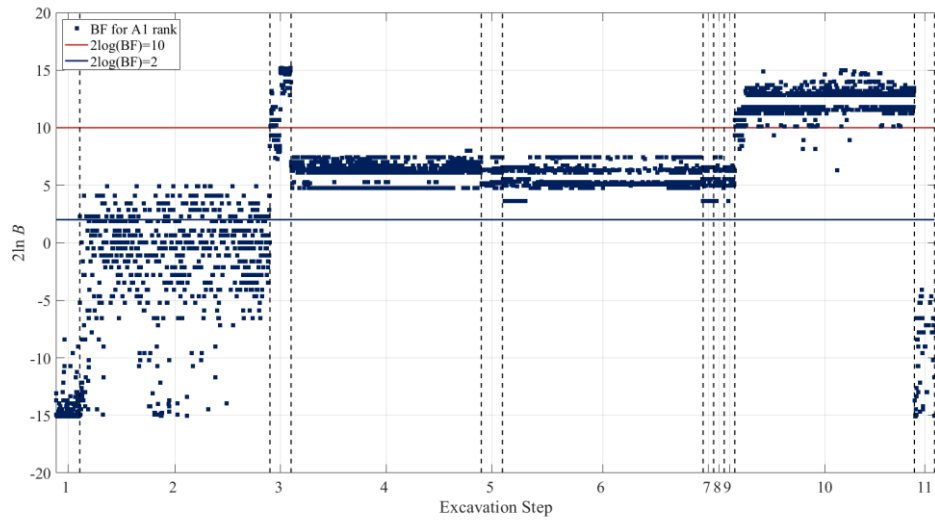


Figure 3.29 Results in applying Bayes Factor assuming Levy-Flight Distribution for A1 rank level detection.

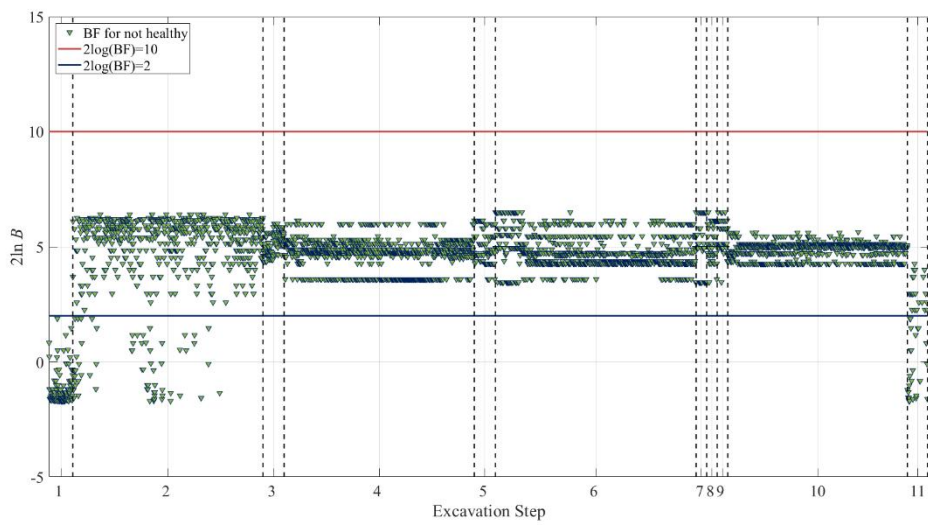


Figure 3.30 Results in applying Bayes Factor assuming Levy-Flight Distribution for not health detection.

3.4.3. Application of Bayes Factor to case study 2

In this section, BF is applied to the long-term monitoring of the bridge pier during the SRW period. Since the identified MPVs of the rocking frequency of the monitored pier during the normal period follow the LF Distribution as discussed in Section 3.2.2, this section considers only BF of the LF Distribution.

The LF Distribution parameters at each state are summarized in **Table 3.7**. The parameters in the healthy state are the same as **Equation (3-11)**. “Not healthy state” of the LF Distribution is set to the parameter, γ , so that the lower limit of 1% would include within a realistic value, 4.00 Hz. “A1 rank damaged state” of the LF Distribution are set to the mean and the parameter, δ , so that the value satisfies the soundness categories shown in **Table 3.1**. All distributions are shown in **Figure 3.31**. Results in BF using the LF Distribution are shown in **Figure 3.32**. Following Kass’s interpretation shown in **Table 3.4**, $2 \log_e BF = 2$ and 10 are drawn as blue and red lines, respectively. In “not healthy” result, the yellow line does not reach the red line, where the red line indicates very strong evidence that the output is not healthy, when the identified frequency is low. However, it exceeds the blue line, which indicates positive evidence that output is not healthy. Therefore, when we apply BF of the LF Distribution for detecting “healthy” or “not healthy”, $2 \log_e BF = 2$ should be used. It should be also mentioned that BF of “A1 rank damaged state” does not reach the red line that indicates very strong evidence.

The result of applying BF when using the LF Distribution to the identified MPVs of the rocking frequency of the concrete bridge pier during the SRW period is shown in **Figure 3.33**. According to the blue square plots in **Figure 3.33**, there were no plot exceeding the blue line. Therefore, the outputs do not provide sufficient evidence that the scour reaches the A1 rank category in this flood event. However, when we focus on the green inversed triangle plots, indicating “not healthy state”, some plots exceed the blue line, especially before the water level peak. In **Figure 3.2**, the some identified MPVs before the water level peak were observed greater than 10 Hz, and they correspond to BFs which exceed the blue line. For scour assessment, we do not care when the identified frequency becomes high. Therefore, these plots can be eliminated.

The results of applying SWI1 and SWI2 to the artificial acceleration shown in **Figure 3.14** are shown in **Figure 3.34**. Both plots exceed the blue line indicating there is evidence of “not healthy” just when scour occurs. However, the blue square plots do not exceed the red line at all even though there were identified MPVs of the rocking frequency that reach the A1 rank category.

These results indicate that there are challenges when setting BF thresholds.

Table 3.7 Parameters of the Levy-Flight Distribution at healthy, damaged, and not healthy states of the concrete pier.

	α	β	γ	δ
H_0 : Healthy	1.58	0.58	0.31	9.40
H_1 : A1 rank damaged	1.58	0.58	0.31	6.58
H_{1c} : Not healthy	1.58	0.58	1.35*	9.40

* Setting the lower limit of 1% to be 6.0.

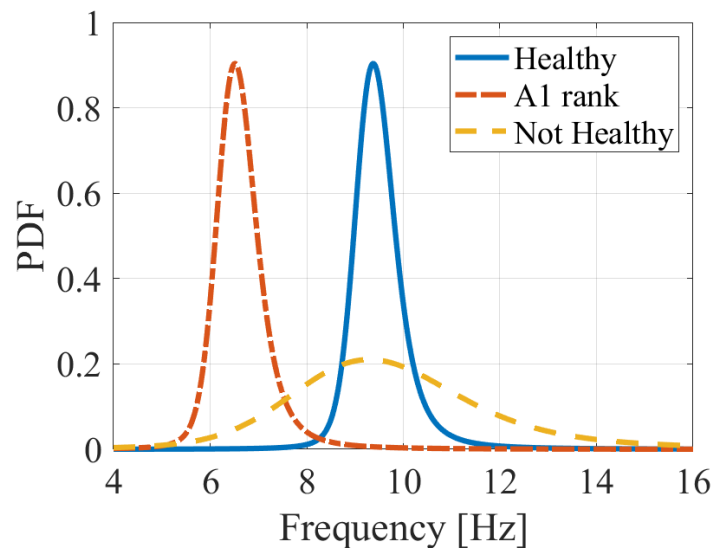


Figure 3.31 PDF of the Levy-Flight Distribution at each state of the concrete pier.

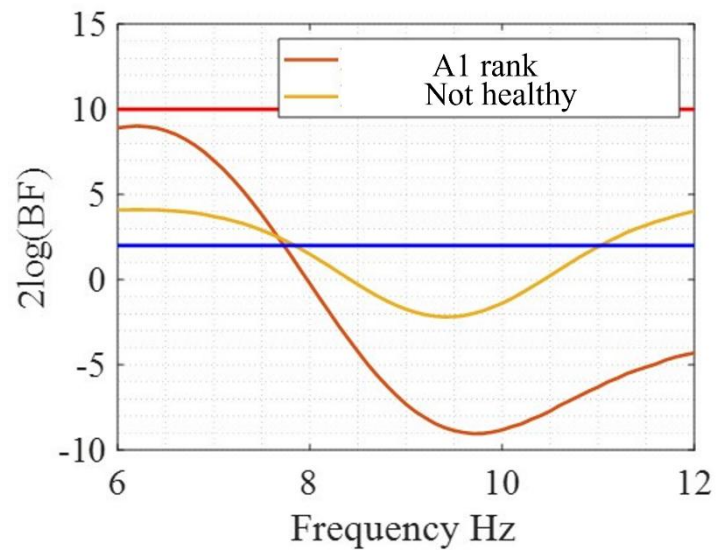


Figure 3.32 Changing trends of Bayes Factor using the Levy-Flight Distribution when the identified frequency of the concrete pier is observed.

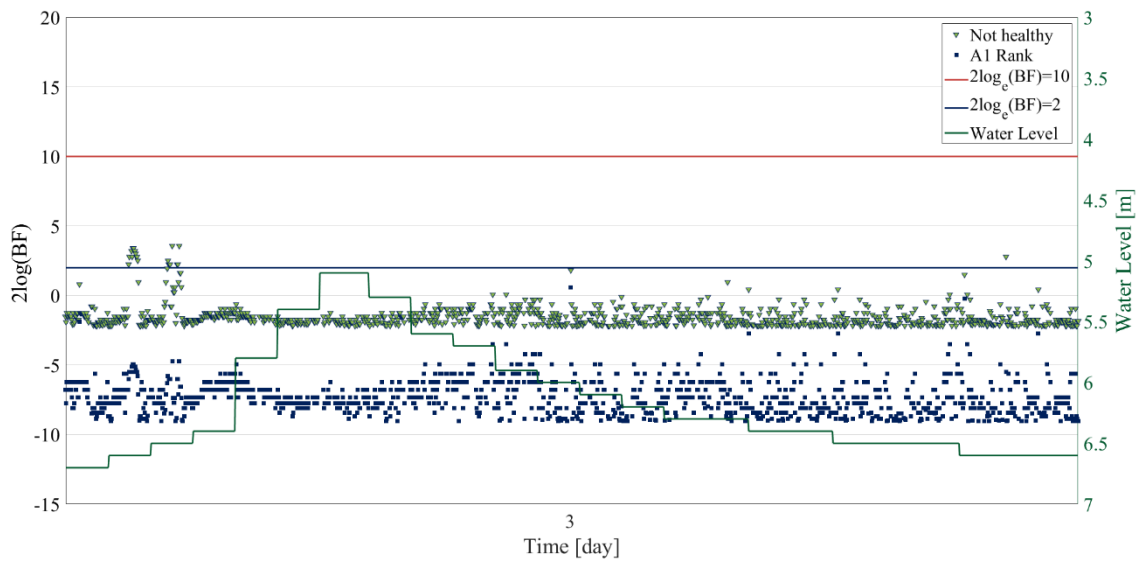


Figure 3.33 Results in applying Bayes Factor of the Levy-Flight Distribution to the identified MPVs during the Swollen River Water period of the concrete pier.

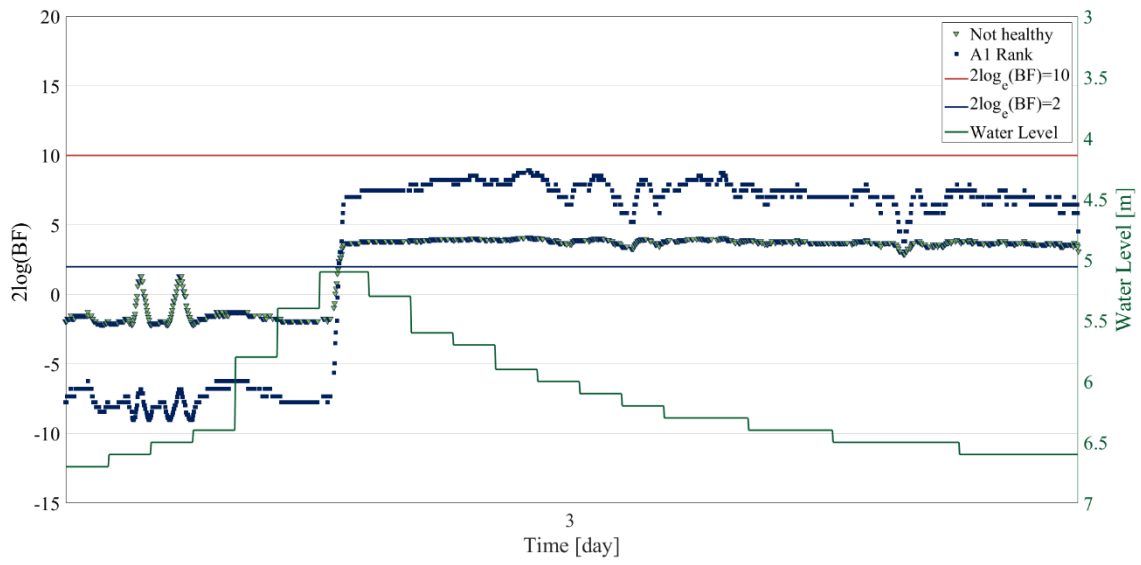


Figure 3.34 Results in applying Bayes Factor of the Levy-Flight Distribution to artificial acceleration during the Swollen River Water period of the concrete pier.

3.4.4. Discussion of thresholds of Bayes Factor

In the previous sections, Kass interpretation was used as the BF thresholds. However, when the observation follows the LF Distribution, it was found that the observations and the BF thresholds based on Kass's interpretation do not match well, especially for the A1 rank determination. Therefore, this section considers the thresholds modification. Based on the definition of BF equation shown in **Equation (3-32)**, BF corresponds to the distance between two probability distributions, H_0 , and H_1 . The overlapped area of them is useful to illustrate their distance. Usually, Bayes Theorem assumes the Normal Distribution. Therefore, the overlapped area of BF using probability distribution following the Normal Distribution should be equal to that of BF using probability distribution following the LF Distribution. Based on the threshold by Kass interpretation [12], the overlapped area of BF between the healthy state and A1 rank damaged state using the Normal Distribution is considered.

The values of BF using the Normal Distribution given observation D is shown in **Figure 3.35(a)**, and the corresponding overlapped area is shown in **Figure 3.35(b)**. In **Figure 3.35(b)**, the blue and red lines illustrate the overlapped areas at $2 \log_e(BF) = 2$ and 10 , respectively. The overlapped area is exponentially decreasing when the Normal Distribution is considered, which is the reason why BF shows the linearity in **Figure 3.35(a)**. Then, the results in BF using the LF Distribution which is equal the overlapped area to BF using the Normal Distribution is summarized in **Table 3.8**. For example, the BF using the LF Distribution corresponding to $2 \log_e(BF) = 10$ using Normal Distribution is $2 \log_e(BF) = 9.52$.

This study also considers the LF Distribution when $\beta = 0$ because one of the principal reasons why $2 \log_e(BF)$ does not reach the red line in **Figure 3.32** is heavy tail asymmetry. The heavy asymmetry characteristic of the LF Distribution makes the overlapped area relatively large rather than the Normal Distribution with symmetry. Therefore, the LF Distribution satisfying symmetry by setting the skewness parameter, β , as $\beta = 0$ is considered. The results of BF using the LF Distribution with $\beta = 0$ which is equal the overlapped area to BF using the Normal Distribution is summarized in **Table 3.9**. The overlapped area of $2 \log_e(BF)$ using the LF Distribution with $\beta = 0$ is 9.39, and it is smaller than in **Table 3.8**. Finally, values of BF using the LF Distribution with modified thresholds in **Table 3.8** and **Table 3.9** are shown in **Figure 3.36**.

In **Figure 3.36(a)**, BF for detecting the A1 rank category shown as the orange chain line does not reach the red line yet. What is more, BF for detecting the "not healthy condition" exceeds the

blue line at around 7 Hz, which is too late to make decisions about scour assessment. On the contrary, **Figure 3.36(b)**, BF for detecting the A1 rank category exceeds the red line at around 7 Hz, which is around middle of the borders of the A1 and A2 category shown in **Equation (3-5)**. BF for detecting the “not healthy condition” exceeds the blue line at around 8.5 Hz. These results show that BF using the LF Distribution with $\beta = 0$ and the modified thresholds based on the overlapped area between two probability densities, H_0 and H_1 , are well-designed so as to correspond to the scour assessment categories as shown in **Table 3.1**.

Figure 3.37 shows the results in applying the BF of the LF Distribution with modified thresholds shown in Table 3.9 to identified MPVs of the rocking frequency from ambient vibration during SRW period of the concrete pier. Comparing with Figure 3.33 with the original thresholds, all BFs for clearly healthy are below the blue line. It means that the observation always shows clearly healthy state thorough this SRW event.

Figure 3.38 shows the result in applying the BF of the LF Distribution with modified thresholds shown in Table 3.9 to artificial acceleration during SRW period of the concrete pier. Comparing with Figure 3.34 with the original thresholds, the cross point when BF for clearly healthy exceeds the blue line is almost same as the point in Figure 3.38. On the other hand, the point when BF for clearly abnormal exceeds the red line in Figure 3.38 although Figure 3.34 does not exceed the red line.

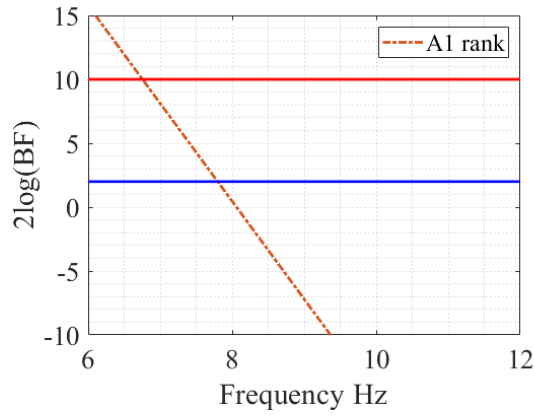
Consequently, BF using LF Distribution should be used with modified thresholds considering the overlapped area for scour assessment decision procedures.

Table 3.8 Bayes Factor thresholds for the Levy-Flight Distribution.

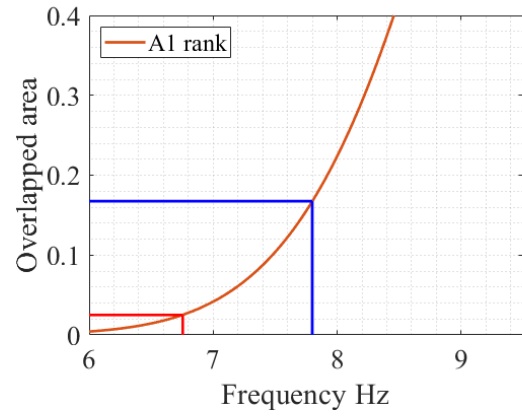
Normal distribution		Overlapped area	Levy Flight distribution			
$2 \log_e (BF)$	H_1		$2 \log_e (BF)$	H_1	Mode of H_1	Mean of H_1
10	$N(6.75, 0.61)$	0.025	9.52	$L.F. (1.58, 0.58, 0.31, 6.10)$	6.07	6.24
6	$N(7.28, 0.61)$	0.0708	6.57	$L.F. (1.58, 0.58, 0.31, 7.34)$	7.33	7.50
2	$N(7.80, 0.61)$	0.1675	4.02	$L.F. (1.58, 0.58, 0.31, 8.04)$	8.01	8.18

Table 3.9 Bayes Factor thresholds for the Levy-Flight Distribution when $\beta = 0$.

Normal distribution		Overlapped area	Levy Flight distribution			
$2 \log_e (BF)$	H_1		$2 \log_e (BF)$	H_1	Mode of H_1	Mean of H_1
10	$N(6.75, 0.61)$	0.025	9.39	$L.F. (1.58, 0, 0.31, 6.59)$	6.59	6.59
6	$N(7.28, 0.61)$	0.0708	6.44	$L.F. (1.58, 0, 0.31, 7.62)$	7.62	7.62
2	$N(7.80, 0.61)$	0.1675	3.87	$L.F. (1.58, 0, 0.31, 8.18)$	8.18	8.18

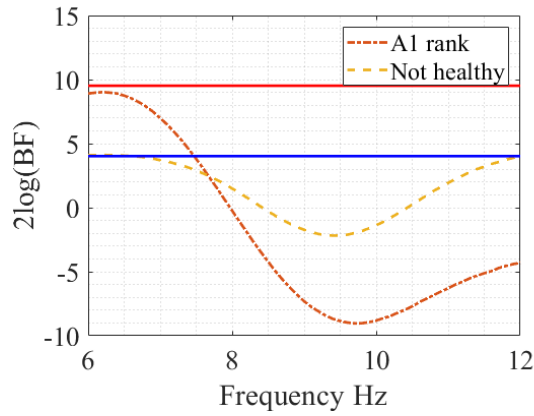


(a) BF using the Normal Distribution.

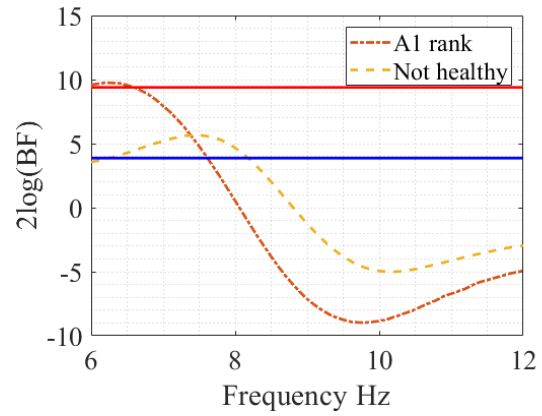


(b) Overlapped area between healthy state and damaged state.

Figure 3.35 The changing trend of Bayes Factor and corresponded overlapped area.



(a) BF with modified thresholds



(b) BF with modified thresholds when $\beta = 0$

Figure 3.36 Changing trends of Bayes Factor using the Levy-Flight Distribution with modified thresholds when identified frequency of the concrete pier is observed.

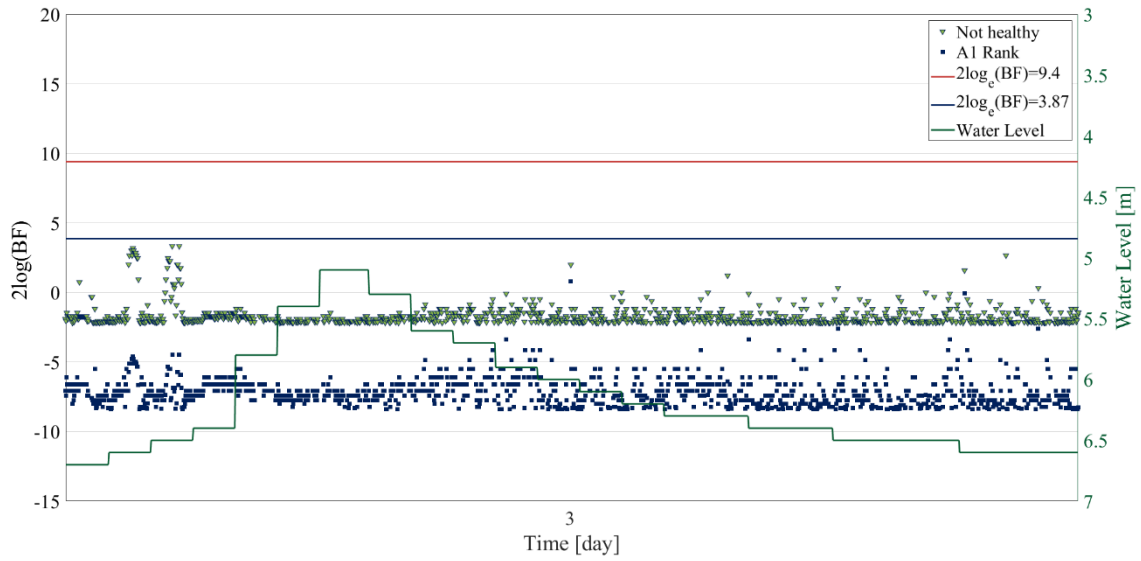


Figure 3.37 Results in applying Bayes Factor of the Levy-Flight Distribution with modified thresholds shown in Table 3.9 to the identified MPVs during the Swollen River Water period of the concrete pier.

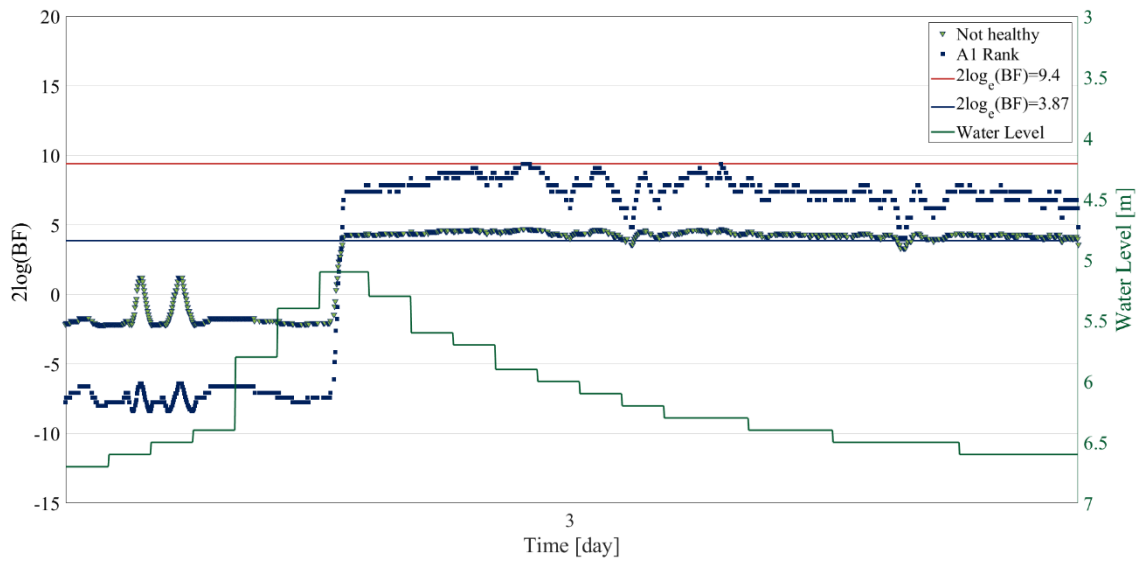


Figure 3.38 Results in applying Bayes Factor of the Levy-Flight Distribution with modified thresholds shown in Table 3.9 to the artificial acceleration during the Swollen River Water period of the concrete pier.

3.5. Discussions

This Chapter discusses the identified frequency of the real concrete bridge pier from ambient vibration following the LF Distribution, at first. Then four data-driven stochastic indices, SWI1, SWI2, PSO, and BF using LF Distribution obtained at normal period are proposed and applied to the two case studies; the RC specimen experiment for scour progress and long-term vibration monitoring of a railway bridge pier crossing river. This section summarizes the pros and cons of each proposed stochastic index for wide usage of vibration monitoring of a bridge monitoring. κ is also added for comparison as shown in Table 3.10 to Table 3.11.

Table 3.12 Scopes of proposed data-driven indices.

Index	Impact test	Ambient vibration	System Identification
κ	positive	-	Any method is possible
SWI1	negative	Pros	Any method is possible
SWI2	negative	Pros	Only Fast Bayesian FFT available
PSO	negative	Pros	Any method is possible
$2 \log_e(BF)$	negative	Pros	Any method is possible

Table 3.13 Thresholds setting.

Index	Threshold settings	dependency
κ	deterministic	Uniquely
SWI1	Probabilistic. Can be freely set at 1%, 5%, etc.	Uniquely
SWI2	Probabilistic. Can be freely set at 1%, 5%, etc.	Uniquely
PSO	Probabilistic. Can be freely set at 1%, 5%, etc.	Uniquely
$2 \log_e(BF)$	Probabilistic. Can be freely set at 1%, 5%, etc.	depending on alternative hypothesis

Table 3.14 Expandability to risk.

Index	Expandability to risk
κ	negative
SWI1	Modification is needed
SWI2	Modification is needed
PSO	positive
$2 \log_e(BF)$	positive

Table 3.15 Usability for stepwise judgement.

Index	Stepwise judgement
κ	Positive (already exists)
SWI1	Positive (Can be freely set statistically at 1%, 5%, etc.)
SWI2	Positive (Can be freely set statistically at 1%, 5%, etc.)
PSO	positive
$2 \log_e(BF)$	positive

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Chapter 4. Scour Depth Prediction with Ambient Vibration and Numerical Analysis

In chapter 3, data-driven stochastic scour anomaly detection methods have been proposed. In addition to data-driven scouring detection, real-time information on structural performance of the bridge pier, such as scour depth, is also crucial for faster and more accurate decision making when suspending or resuming traffic under uncertainty. It is, therefore, expected that the identified frequencies will be used to update the numerical models for estimating the structural performance of the pier. This can be called “Digital Twins for scour assessment”. However, there is insufficient research to propose a real-time scour assessment method that uses the modal parameters of the pier to provide information on the performance and safety of the structure.

This chapter proposes a method for predicting the scour depth around a river-crossed bridge pier where a monitoring system is deployed for real-time assessment during the SRW period. A FE model is being built to express a soil-structure interaction and clarify the relationship between the soil spring constants and the target frequency, which is the rocking mode in the transverse direction of the pier. Since the identified frequencies of a bridge pier using microtremors have a significant uncertainty due to weak signal responses, this chapter introduces a Bayesian model updating method [1-3] for expressing the uncertainty of its properties. Using the updated model, the relationship between eigenvalues and scour progression is determined by eigenvalue analysis of the FE model. Finally, the multivariate probability density function (PDF) to estimate the probability of scour depth is approximated using the polynomial curve fitting.

As a fundamental case study, the RC pier specimen experiment for simulating the scour described in Section 2.1 was used to demonstrate the validity of the proposed method. A railway bridge concrete pier with 5-year long-term microtremor monitoring was conducted for practical application in site. Both FE models are already described in Sections 2.1 and 2.2.

It should be noted that this study only focused on the shallow foundation type as many older railway bridges have shallow foundations.

4.1. Methodology

4.1.1. Bayesian model updating

The stability of a bridge pier due to scouring can be treated as changes in soil conditions that reduce structural performance. If the updated model fits the actual structural conditions, then changes in the model parameters can indicate the severity of scouring.

Bayesian model updating is a popular model updating approach that considers uncertainties in model parameters attributable to measurement noise or modeling errors [1, 2]. Uncertain parameters are assumed to follow a certain distribution based on experimentally obtained data. In other words, the process of Bayesian model updating is fundamentally an optimization problem for which the criterion is the posterior distribution based on the level of the difference between the simulation and experiment. For Bayesian model updating, sampling procedures are invariably necessary because it is difficult to obtain analytical solutions to describe a PDF of uncertain parameters. Among the many sampling algorithms, the Markov Chain Monte Carlo (hereafter MCMC) sampling algorithm [4, 5] has received the most attention, especially the Metropolis-Hasting (hereafter MH) algorithm [6, 7]. When considering bridge scouring, only a change in the rotational soil spring constant is generally critical because the rocking mode of the target pier is most sensitive to scouring [8]. Since only the rotational soil spring constant is considered as an uncertain parameter, not much attention needs to be paid to the sampling frequency. Therefore, the MH algorithm was used to update the model.

In Bayesian model updating, the formulation of the posterior PDF of the uncertain parameter θ under given system response D is expressed as

$$P(\theta|D, M) = \frac{P(D|\theta, M)P(\theta|M)}{P(D|M)} \quad (4-1)$$

where M is the assumed model for the structure, $P(D|M)$ denotes evidence of model M , and $P(D|M)$ is a normalized constant. Also, $P(\theta|M)$ is the prior PDF. For convenience, it is usually assumed as a uniform distribution. Finally, $P(D|\theta, M)$ is the likelihood function, which denotes the conditional probability of D given θ . Since $P(\theta|M)/P(D|M)$ is normalizing constant, and the posterior PDF, $P(\theta|D, M)$, is proportional to the likelihood function $P(D|\theta, M)$, Equation (4-1) is rewritten as

$$P(\theta|D, M) \propto P(D|\theta, M) \quad (4-2)$$

As discussed before, the complexity of posterior PDF makes it difficult to calculate the probability

and describe the probability distribution directly. Consequently, Monte Carlo Simulation is used to approximate and estimate the posterior distribution by the generated samples. This study briefly explains MH algorithm that is commonly used as MCMC solutions in the Bayesian model updating sampling problems. Presuming that the PDF of the target distribution is $P(\theta)$, θ_k is the k th sample, and θ_{k+1}^* is the candidate sample generated from $q(\theta_{k+1}^*|\theta_k)$, which is the PDF of proposed distribution with given θ_k . The θ_{k+1}^* is then accepted with probability α_k given by the following Equation (4-3).

$$\alpha_k = \alpha(\theta_k, \theta_{k+1}^*) = \min \left\{ 1, \frac{p(\theta_{k+1}^*)q(\theta_k|\theta_{k+1}^*)}{p(\theta_k)q(\theta_{k+1}^*|\theta_k)} \right\} \quad (4-4)$$

Therein, α_k is a constant to ensure that the stationary distribution of the Markov chain is the target distribution. If the candidate sample θ_{k+1}^* is accepted, then it would be set as the next sample of Markov chain, $\theta_k = \theta_{k+1}^*$; otherwise, the present sample is to be the next one: $\theta_{k+1} = \theta_k$. Then, this process can be repeated to generate the Markov chain samples $\theta_0, \theta_1, \dots, \theta_n$. The symmetric random-walk Metropolis algorithm is an important special case of the MH method in which the Markov chain will jump with a symmetric proposed distribution, as $q(\theta_{k+1}|\theta_k^*) = q(\theta_k|\theta_{k+1}^*)$. It is taken to be the sampling algorithm in this study. Equation (4-4), therefore, can be simplified as follow

$$\alpha_k = \alpha(\theta_k, \theta_{k+1}^*) = \min \left\{ 1, \frac{p(\theta_{k+1}^*)}{p(\theta_k)} \right\} \quad (4-5)$$

The basic ideas of Bayesian approaches are summarized in references [9, 10].

4.1.2. Scour depth prediction by Finite Element model

For the purpose of scour depth prediction, this section considers the relationship between scour progress and soil resistance. Usually, the types of footing of old railway bridges are shallow foundation which is high risk for rotation and collapsing due to scour. Since a large proportion of scour damage occupies shallow foundation type, this study considers such types of foundations. The analysis of the stability of bridge piers is considered 4 categories: sinking, sway motion, roll motion, and destruction. Based on the findings of a survey of past disaster cases in Japan, this study only considers roll motion. According to the Japanese guideline for railway bridge design document [11], the modeling of soil resistance for a bridge pier with a shallow foundation can be expressed as four soil springs as shown in Figure 4.1. Here, k_v , k_s , k_h , and k_r represent the vertical, shear, horizontal and rotational soil springs, respectively. The calculation procedures

are summarized in Section 2.1.5. In order to facilitate the discussion, the progress of scouring is distinguished from two stages as shown in Figure 4.2. Stages 1 and 2 are defined when the scour depth is still above the bottom of the footing and it erodes under the bottom of the footing, respectively.

As discussed in Section 2.1.7. and 2.1.8., since the rate of decrease in the soil spring constants is proportional to the rate of decrease in the area resisting load in the direction carried by the soil spring, changes in each soil spring constant are summarized in Figure 4.3. Using the eigenvalue analysis of FE model, the target frequency corresponding to the decreased soil spring constants and scour depth can be numerically obtained. Finally, the diagram that denotes the scour depth and frequency of the pier can be estimated as shown in Figure 4.4. In stage 1, as shown in Figure 4.4, the decreased trend of natural frequency occurs due to decreasing k_h , whereas a rapid change in natural frequency emerges at Stage 2 due to decreasing k_v , k_s , and k_r .

When we observed the identified MPVs of the rocking frequency, $x = f_x$, the PDF of scour depth given f_x can be calculated. The schematic figure of the PDF is shown in Figure 4.4b). Assuming $f_{x1} > f_{x2}$, and f_{x1} and f_{x2} are depicted as green dotted and orange dashed lines respectively, the probability that scour depth reaches Stage 2, $Pr(Stage2|f_{x1})$ or $Pr(Stage2|f_{x2})$ can be calculated by the enclosed areas of Stage 2 and PDFs. It should be noted that $Pr(Stage2|f_{x1})$ is smaller than $Pr(Stage2|f_{x2})$. From the results of eigenvalue analyses repeatedly, the probability of the scour depth given identified MPVs of the rocking frequency can be predicted. The whole process of this method is shown in Figure 4.5 as a flowchart.

It should be noted that the identified frequency in real situation varies due to EOVs and signal noises, which is the reason why sampling method is introduced to assess the variables of soil springs.

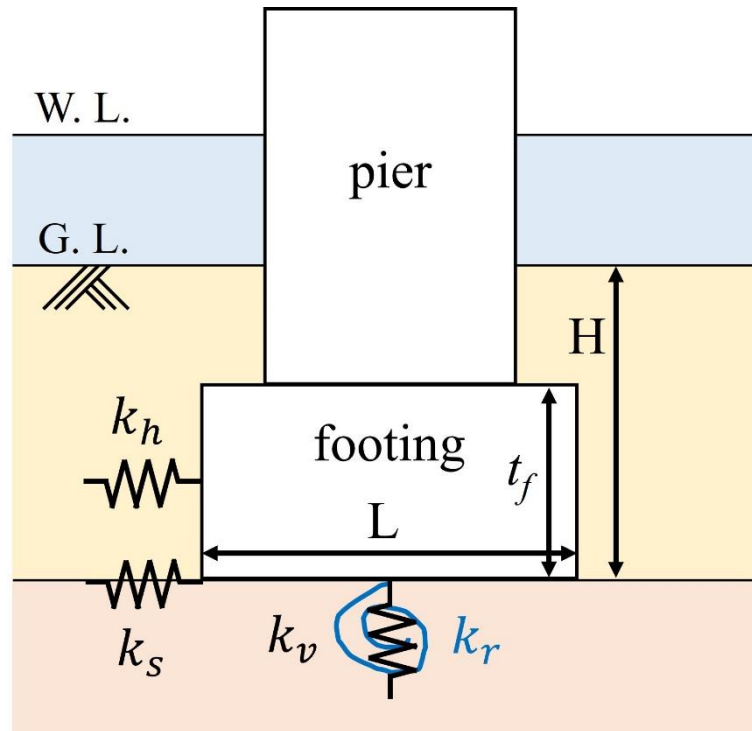


Figure 4.1 Four soil springs to illustrate the soil resistance based on Japanese Design document.
(W.L.: Water Level, G.L.: Ground Level)

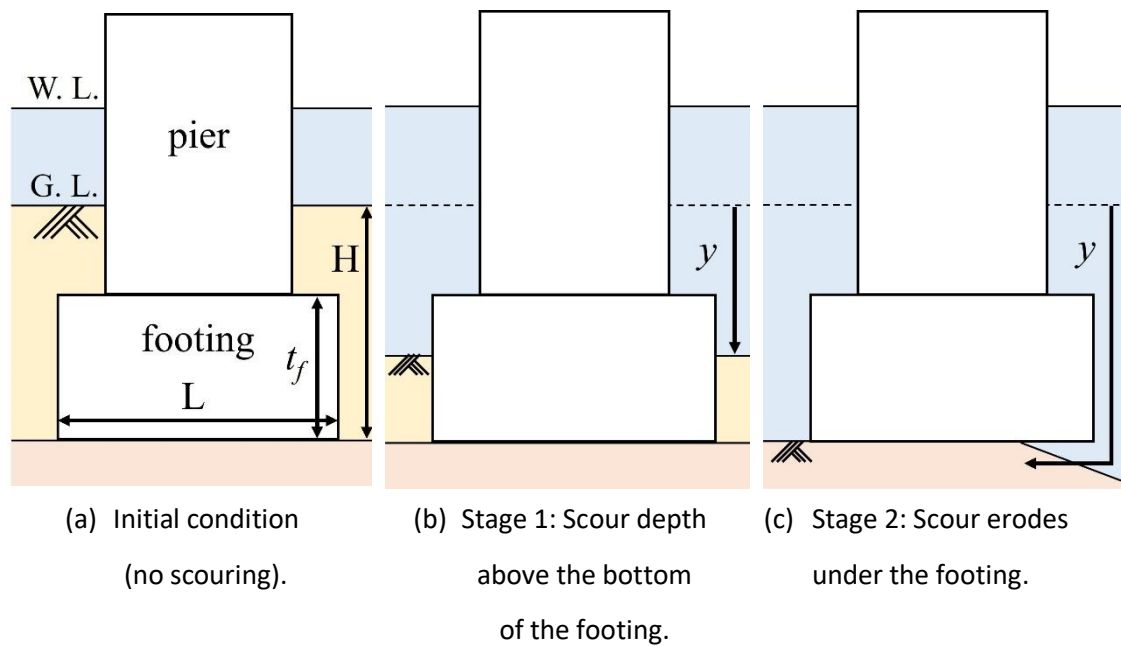


Figure 4.2 Two excavation steps of the scouring around the pier.

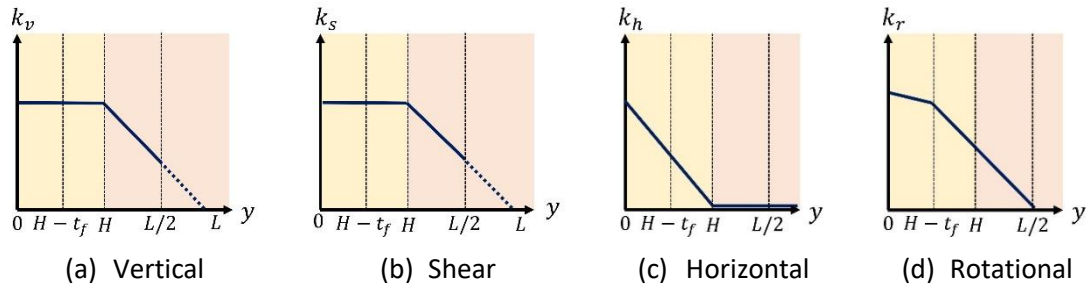
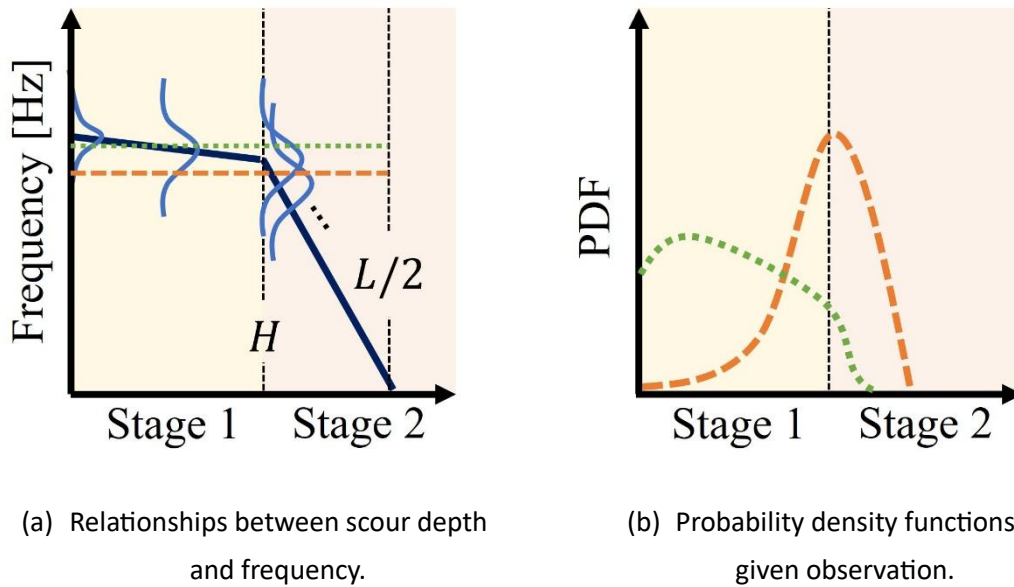


Figure 4.3 Schematic diagrams for the relationship between the soil spring constant and scour depth.



(a) Relationships between scour depth and frequency.
(b) Probability density functions given observation.

Figure 4.4 Schematic diagrams of scour depth progress with natural frequency of the pier.

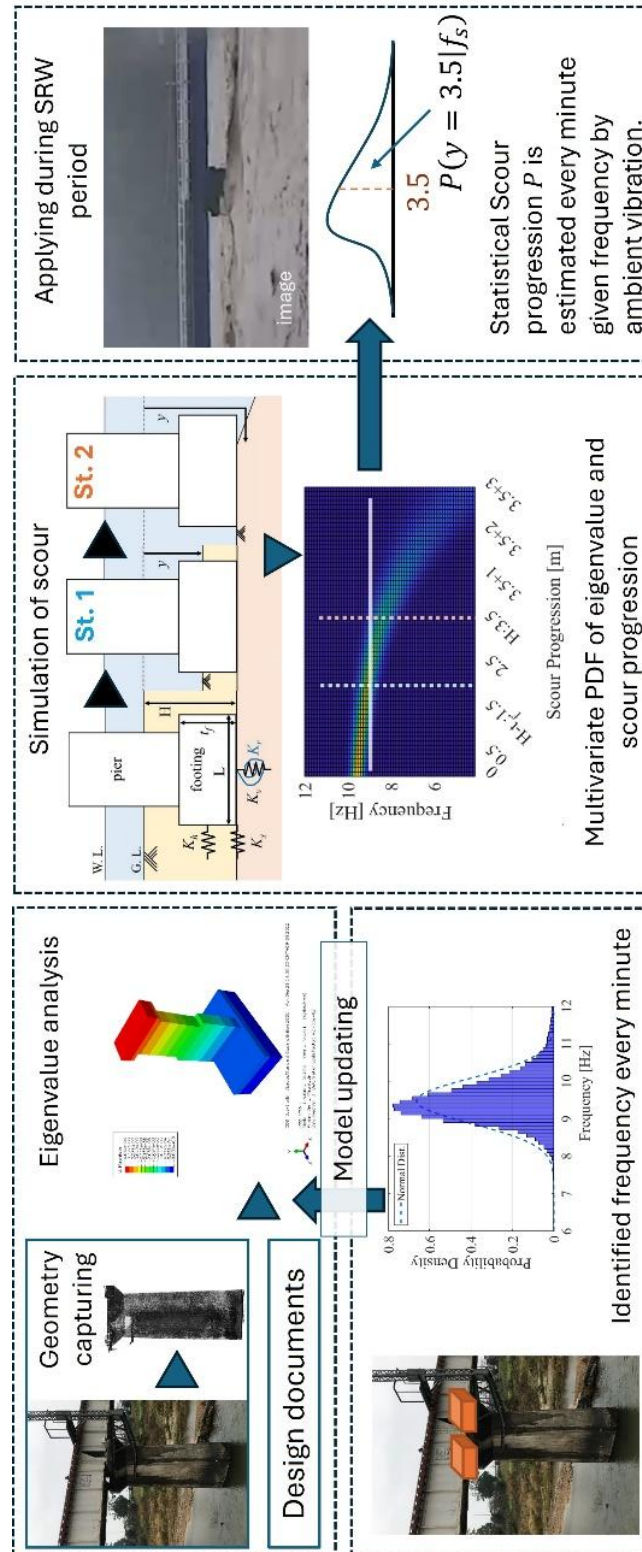


Figure 4.5 The image of the whole process of predicting the scour depth around the pier statistically.

4.2. Application of the scour depth prediction method to case study 1

4.2.1. Bayesian model updating in case study 1

This section investigates the validity of the proposed scour depth prediction method to the experiment on RC pier specimen. The updated FE model is summarized in Section 2.1.5. The four soil spring constants in Step 1 are listed in Table 4.1. The eigenvalue analysis of the target frequency was, then, 21.6 Hz. The identified MPVs of the rocking frequency in Step 1 is shown in Section 2.1.4. When they are modeled as Normal Distribution using MLE method, the parameters were estimated as follows, and the fitted figure is shown in Figure 4.6. Blue dashed line in Figure 4.6 represents Normal Distribution curve.

$$MPVs \sim N(\mu = 20.74, \sigma = 0.59) \quad (4-6)$$

Since the identified MPVs of the rocking frequency of the RC specimen scattered due to uncertainty, the distribution of the identified MPVs of the rocking frequency is expressed using Bayesian model updating. Based on the sensitivity analysis shown in Section 2.1.6., only one parameter, rotational soil spring constant (k_r), was updated as Normal Distribution. The amount of the randomly created k_r between $\pm 50\%$ from $2.19 \times 10^{11} N \cdot mm$ was 3000. The proposed distribution of MH algorithm shown in Equation (4-4) is set as standard Normal Distribution. Then, the result in updating k_r is shown in Figure 4.7. The red line in Figure 4.7 is modeled as Normal Distribution. The parameters are expressed as follows.

$$k_r \sim N(\mu = 1.86 \times 10^{11}, \sigma = 1.75 \times 10^{10}) (N \cdot mm) \quad (4-7)$$

Then, the eigenvalue is modeled using MLE method as follows.

$$Eigenvalues \sim N(\mu = 20.79, \sigma = 0.49) (Hz) \quad (4-8)$$

The eigenvalue in Equation (4-8) mostly matches that in MPVs of Step 1. Therefore, Bayesian updated model successfully expresses the uncertainty in MPVs. Finally, the updated soil spring constants in Step 1 are summarized in Table 4.2.

Table 4.1 Updated soil spring constants in Step 1.

Soil spring	Spring constant	unit
Vertical (k_v)	1.80×10^5	N/mm
Shear (k_s)	6.00×10^4	N/mm
Rotational (k_r)	2.19×10^{11}	$N \cdot mm$
Horizontal (k_h)	1.60×10^5	N/mm

Table 4.2 Soil spring constants in Step 1 using Bayesian model updating.

Soil spring	Spring constant	unit
Vertical (k_v)	1.80×10^5	N/mm
Shear (k_s)	6.00×10^4	N/mm
Rotational (k_r)	$N(\mu = 1.86 \times 10^{11}, \sigma = 1.75 \times 10^{10})$	$N \cdot mm$
Horizontal (k_h)	1.60×10^5	N/mm

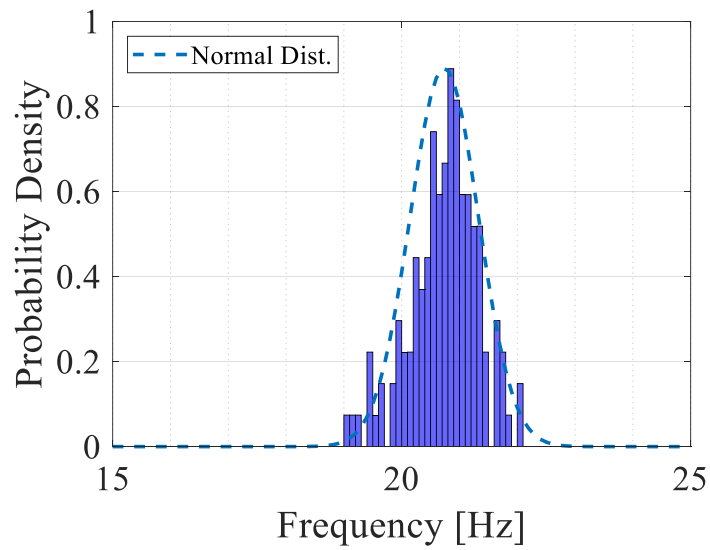


Figure 4.6 MPVs of the rocking frequency in Step 1 and Normal Distribution fitting.

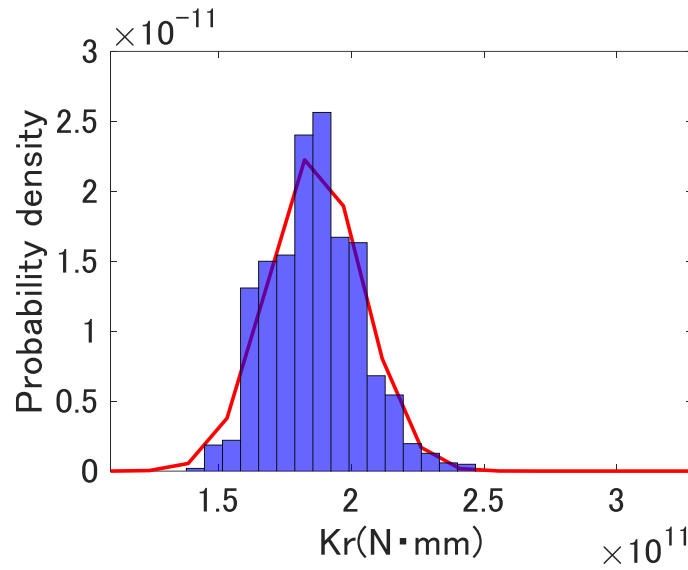


Figure 4.7 Bayesian updated rotational soil spring constant and modeling by Normal Distribution.

4.2.2. Scour depth prediction in case study 1

Following the scour progression around the target pier, the soil spring constants would decrease which leads to decrease the natural frequency of the pier as discussed in Figures 2.4 and 2.8. In Section 2.1.8., it figured out that the rate of decrease in the soil spring constants is proportional to the rate of decrease in the area resisting load in the direction carried by the soil spring. Therefore, this study uses the decreasing values of each soil spring constant shown in Figure 2.7 to predict scour progression. Since the probability distribution of the rotational soil spring constant is modeled with Normal Distribution, this study assumes that the mean value decreases following scour progression, but the variance does not change. The scour progression is set at 100 mm intervals, and at each scour depth, 50 random numbers of k_r are generated to follow the assumed distribution. Then, eigenvalue analyses are repeatedly conducted to obtain eigenvalues under samples of k_r .

The eigenvalue analysis results are shown in Figure 4.8. The horizontal axis represents the scouring depth. The point where scour depth is H:1200 indicates that the scour has reached the bottom of the footing. The values more than 1200 mm of scour depth means that scour excavates under the footing. Figure 4.8 clearly shows the decreasing trend following scour progression. When the scour depth is less than 1200 mm (scour does not reach the bottom of

the footing), the trend is mostly linear, with few changes in variance. While the scour excavates the bottom of the footing, the trend is still linear, but the variance gradually increases. The trend also appears bilinear.

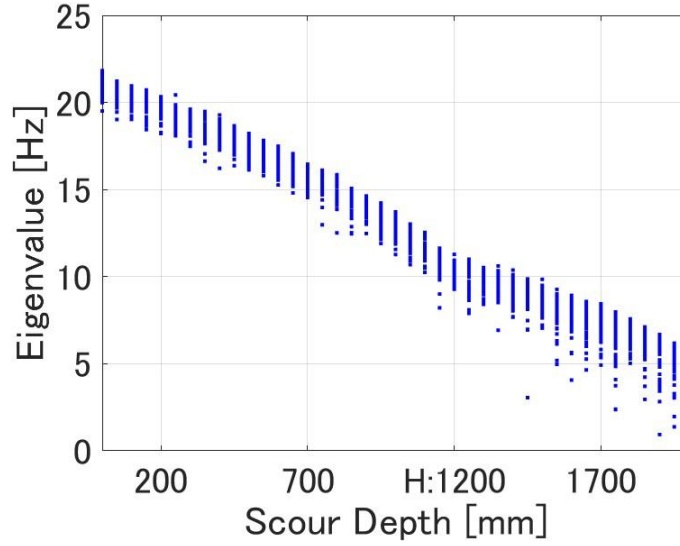


Figure 4.8 The relationship between the scour depth and eigenvalues.

4.2.3. Polynomial curve fitting and probability of scour depth

To illustrate the discrete outputs as deterioration functions, the curve fitting method is considered. Bilinear and expanding variance trends should be expressed. The joint probability distribution model of scour progression y and identified frequency x is calculated as follows.

$$f(y, x) = \frac{1}{\sqrt{2\pi}\beta(y)} e^{-\frac{1}{2}\left(\frac{x-\alpha(y)}{\beta(y)}\right)^2} \quad (4-9)$$

Therein, hyperparameters α and β are obtained from a procedure of fitting a polynomial curve. The optimal order of the polynomials for the hyperparameters is chosen taking into account the convergence of the root mean square error. The polynomials are shown as follows.

$$\alpha(y) = 3.40 \times 10^{-15}y^5 - 4.45 \times 10^{-12}y^4 - 3.25 \times 10^{-9}y^3 + 3.84 \times 10^{-6}y^2 + 9.22 \times 10^{-3}y + 10.65 \quad (4-10a)$$

$$\beta(y) = -4.02 \times 10^{-13}y^4 + 3.40 \times 10^{-10}y^3 + 4.67 \times 10^{-7}y^2 - 5.09 \times 10^{-4}y + 0.61 \quad (4-9b)$$

Figure 4.9 shows fitted polynomial curve.

When the identified frequency, x_t , is obtained, the probability that scour has reached y_t , which is written as $P(y_t|x_t)$, can be calculated as the cumulative density function of $f(y, x)$ as shown in Figure 4.4, and written as Equation (4-11).

$$P(y_t|x_t) = 1 - \int_0^{y_t} f(y, x_t) dy \quad (4-12)$$

Following the two stages described in Figure 4.2, Stage 2 is defined as the time when the scour depth has reached the bottom of the footing. Therefore, the probability that scour reaches Stage 2 when identified frequency is x_t , can be calculated as follows.

$$P(\text{Stage 2}|x_t) = 1 - \int_0^{y_t} f(y, x_t) dy \quad (4-13)$$

Consequently, it is possible to predict the statistical scour depth.

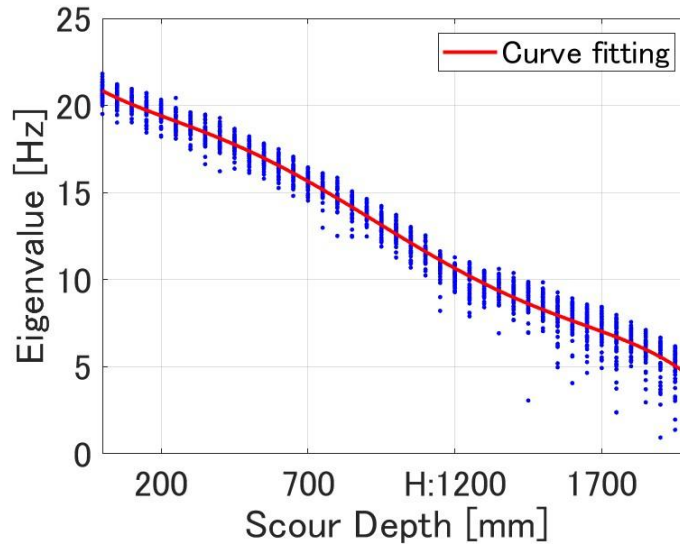


Figure 4.9 The relationship between the scour depth and eigenvalues with a polynomial curve fitting.

4.2.4. Results and conclusions

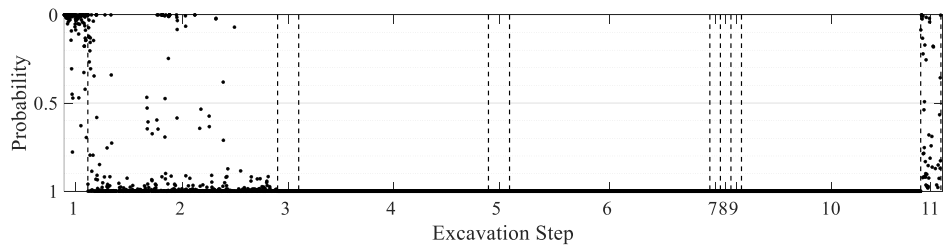
The results of predicting the probability of scour depth using the proposed method are shown in Figure 4.10. Each black plot represents the predicted probability of scour depth given identified MPVs of the rocking frequency. Figure 4.10(a) shows the probability that the scour reaches the top of the footing of the RC pier specimen. In Step 1, most plots are around 0, whereas most plots are near 1 after Step 3. This result indicates that after Step 3, the scour reaches the top of the footing. These results correspond to the excavation Steps. Figure 4.10(b) shows the probability that the scour reaches the middle of the footing of the RC specimen. In Steps 1 and 2, most plots are around 0, which means that the scour does not reach the middle of the footing. Since Step 2 excavates the top of the footing, the prediction corresponds to the excavation Step.

Figure 4.10(c) shows the probability that the scour reaches the bottom of the footing of the RC specimen corresponding to Stage 2 as shown in Figure 4.2. In Steps 1, 2, and 3, the probabilities are close to 0. However, in Step 4, most plots do not converge to 0, but rather are scattered around 0.5. The results indicate that the probability that the scour reaches the bottom of the footing is around 0.5. Since the ground around the specimen was excavated up to 50 mm above the bottom of the footing in Step 4, the predicted probability values were valid.

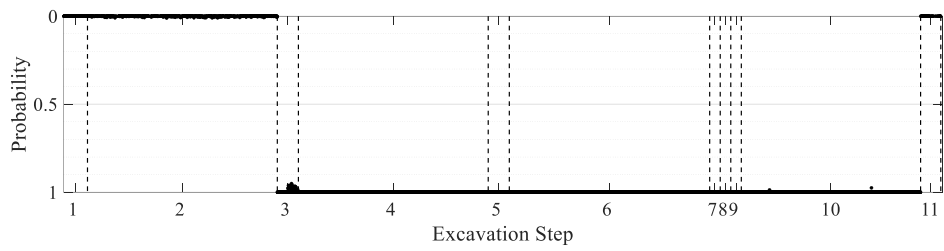
Additionally, Figure 4.10(d) shows the probability that the scour progresses the 300 mm excavation from the edge of the bottom footing corresponding to Step 7 in the experiment. The probabilities were around 0.1, not 0. Changes in the probabilities between Steps 6 and 7 are observed. Therefore, the degree of scour progression under the footing can be also calculated.

In practical situations, the risk of inclining the monitored pier is already severe when scour erodes under the footing. Therefore, from an engineering point of view, the use of Figure 4.10(c), which can determine whether the scour reaches the bottom of the footing, is sufficient rather than obtaining an exact probability and positions where the scour erodes under the footing.

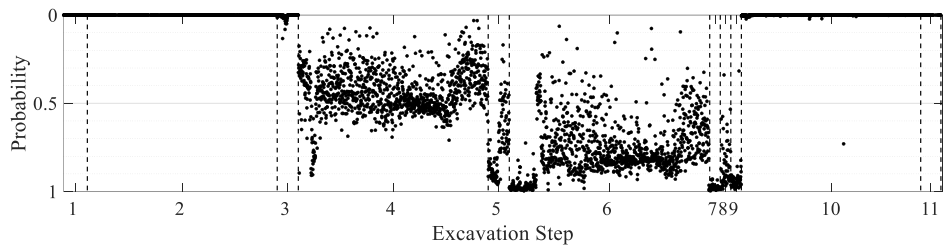
Consequently, the proposed scour depth prediction method can accurately predict the probability of scour depth, and it is shown that it is possible to predict the scour depth from the identified MPVs using microtremors.



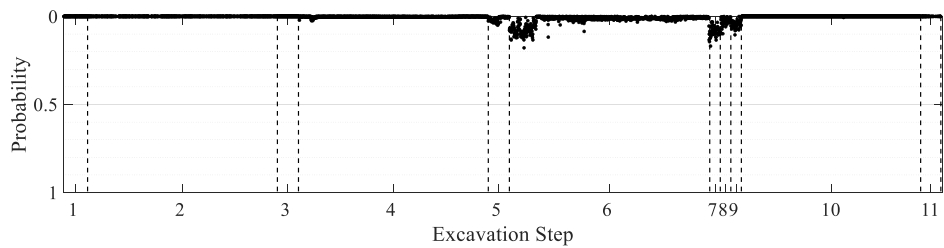
(a) Probability that the scour reaches the top of the footing.



(b) Probability that the scour reaches the middle of the footing.



(c) Probability that the scour reaches the bottom of the footing.



(d) Probability that the scour progresses the 300mm excavation from the edge of the footing.

Figure 4.10 Predicted probabilities that the scour reaches specific elevations.

4.3. Application of the scour depth prediction method to case study 2

4.3.1. Bayesian model updating in case study 2

This section applies the proposed scour depth prediction method to the real railway bridge pier where long-term vibration monitoring had been conducted for 5 years. The updated FE model is summarized in Section 2.2.5. The four soil spring constants during a normal condition are listed in Table 4.3. The eigenvalue analysis of the target frequency was 9.1 Hz. The identified MPVs of the rocking frequency during normal condition is shown in 2.2.4. Although the identified MPVs of the rocking frequency during normal condition follows the LF Distribution as discussed in 3.2.2., this section considers Normal Distribution for simplicity. When identified MPVs of the rocking frequency are modeled with Normal Distribution by means of MLE method, the parameters are estimated as Equation (4-14), and the fitted figure is shown in Figure 4.11, where Normal Distribution is depicted as a blue dashed line.

$$MPVs \sim N(\mu = 9.47, \sigma = 0.56) \text{ (Hz)} \quad (4-15)$$

Based on the sensitivity analysis shown in Section 2.2.6., only one parameter, rotational soil spring constant (k_r), is updated by means of Bayesian model updating. The amount of the randomly created k_r between $\pm 50\%$ from $4.32 \times 10^{14} \text{ N} \cdot \text{mm}$ was 3000. The proposed distribution of MH algorithm shown in Equation (4-4) is set as standard Normal Distribution. Then, the updating k_r is shown in Figure 4.12, where the bars denote updated k_r and the red line shows fitted Normal Distribution curve. The parameters are expressed as follows.

$$k_r \sim N(\mu = 6.72 \times 10^{13}, \sigma = 7.21 \times 10^{12}) \text{ (N} \cdot \text{mm)} \quad (4-16)$$

Then, the MLE provides the Normal Distribution of the eigenvalues as follows.

$$\text{Eigenvalues} \sim N(\mu = 9.50, \sigma = 0.24) \quad (4-17)$$

Although the standard deviation from the updated model is still smaller than that of identified frequencies in the field, the eigenvalues from the updated model can provide to assess the uncertainty. Finally, the updated soil spring constants at normal condition is summarized in Table 4.4.

Table 4.3 Updated soil spring constants at normal condition.

Soil spring	Spring constant	unit
Vertical (K_v)	8.528×10^6	N/mm
Shear (K_s)	2.842×10^6	N/mm
Rotational (K_r)	4.324×10^{13}	$N \cdot mm$
Horizontal (K_h)	7.225×10^4	N/mm

Table 4.4 Soil spring constants at normal condition using Bayesian model updating.

Soil spring	Spring constant	unit
Vertical (K_v)	8.528×10^6	N/mm
Shear (K_s)	2.842×10^6	N/mm
Rotational (K_r)	$N(\mu = 6.72 \times 10^{13}, \sigma = 7.21 \times 10^{12})$	$N \cdot mm$
Horizontal (K_h)	7.225×10^4	N/mm

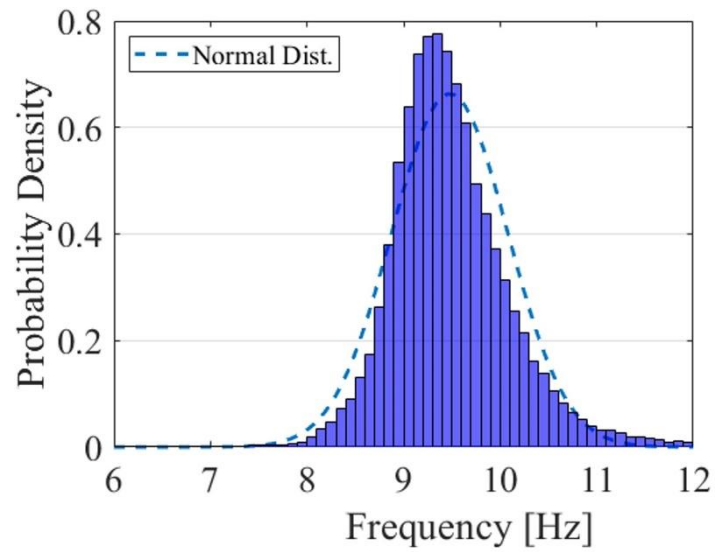


Figure 4.11 MPVs of the rocking frequency during normal condition and Normal Distribution fitting.

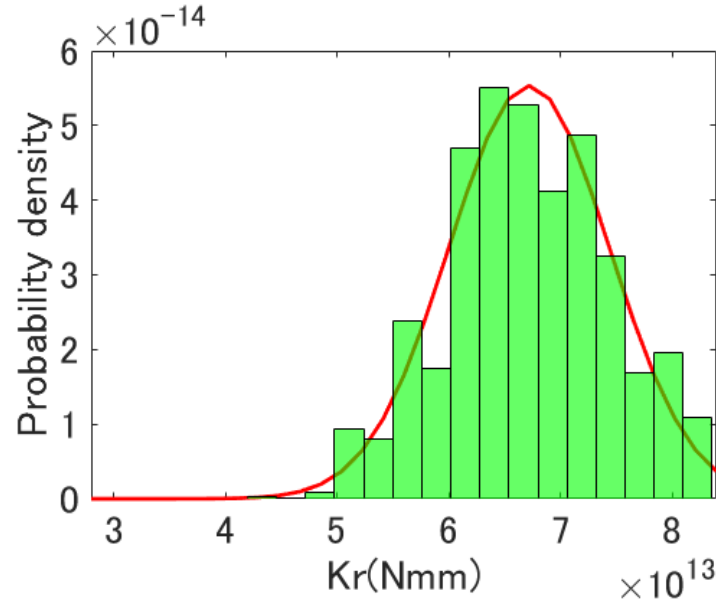


Figure 4.12 Updated rotational soil spring constant by means of Bayesian model updating and Normal Distribution fitting.

4.3.2. Scour depth prediction in case study 2

Following the scour progression around the target pier, soil spring constants would decrease which leads to decrease the natural frequency. In Section 2.1.8., it figured out that the rate of decrease in the soil spring constants is proportional to the rate of decrease in the load resisting area in the direction carried by the soil spring. Therefore, this study uses the decreasing values of each soil spring constant shown in Figure 4.13 to predict scour progression. Focusing on the rotational soil spring constant, the decreasing ratio changes after the scour depth reaches $H - t_f$ caused by the differences of the side depth of the footing and the pier. Since the probability distribution of the rotational soil spring constant is modeled with Normal Distribution, this study assumes that the mean value decreases following scour progression, but the variance does not change. The scour progression is set at 0.1 m intervals, and at each scour depth 40 random numbers of k_r to follow the assumed distribution are generated. Then eigenvalue analyses are repeatedly conducted to obtain eigenvalues of the target frequency.

The frequencies from eigenvalue analyses under sampled k_r are shown in Figure 4.14. The horizontal axis is scouring depth. The points where scour depths are $H - t_f: 1.5$ and $H: 3.5$ mean that scour reaches the top and bottom of the footing, respectively. The values greater than

$H: 3.5$ of scour depth means that scour excavates under the footing. Figure 4.14 clearly shows the decreasing trend following scour progression.

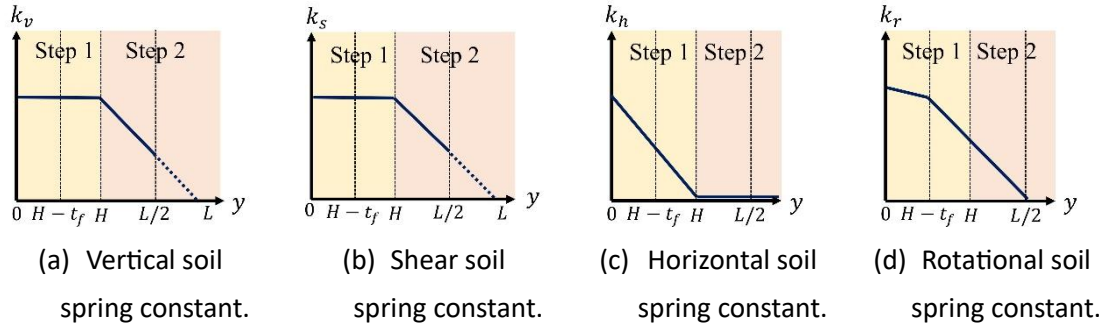


Figure 4.13 Schematic diagrams for the relationship between each soil spring constant and scour depth of the concrete pier.

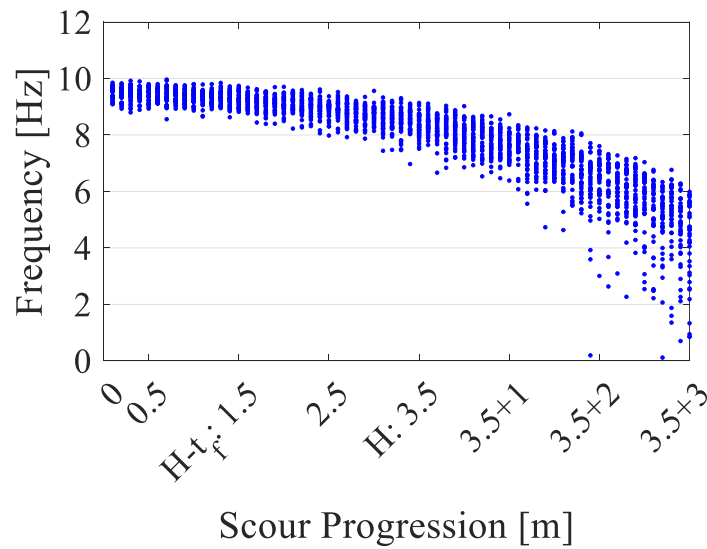


Figure 4.14 The relationship between the scour depth and rocking frequency.

4.3.3. Polynomial curve fitting and probability of scour depth

To illustrate the discrete outputs as deterioration function, curve fitting method is considered. Two trends of decreasing mean value and expanding variances should be expressed. The optimal order of the polynomials for the hyperparameters of Equation (4-18) is chosen considering the convergence of the root mean square error. The polynomial curves are shown in Equation (4-19).

$$\alpha(y) = 0.017y^3 - 0.161y^2 + 0.687y + 8.335 \quad (4-20a)$$

$$\beta(y) = -5.00 \times 10^{-4}y^4 - 5.00 \times 10^{-3}y^3 + 0.048y^2 - 0.137y + 0.389 \quad (4-15b)$$

The result in deterioration function is shown in Figure 4.15. The colormap shows the values of PDF.

When we obtain identified frequency, x_t , the probability that scour reaches y_t , which is written as $P(y_t|x_t)$, can be calculated as the cumulative density function of $f(y, x)$ as shown in Figure 4.4, and written as follows.

$$P(y_t|x_t) = 1 - \int_0^{y_t} f(y, x_t) dy \quad (4-21)$$

Following the two stages described in Figure 4.2, Stage 2 is defined as the time when the scour depth is the bottom of the footing. Therefore, the probability that scour reaches Stage 2 when identified frequency is x_t , can be written as

$$P(\text{Stage 2}|x_t) = 1 - \int_0^{y_t} f(y, x_t) dy \quad (4-22)$$

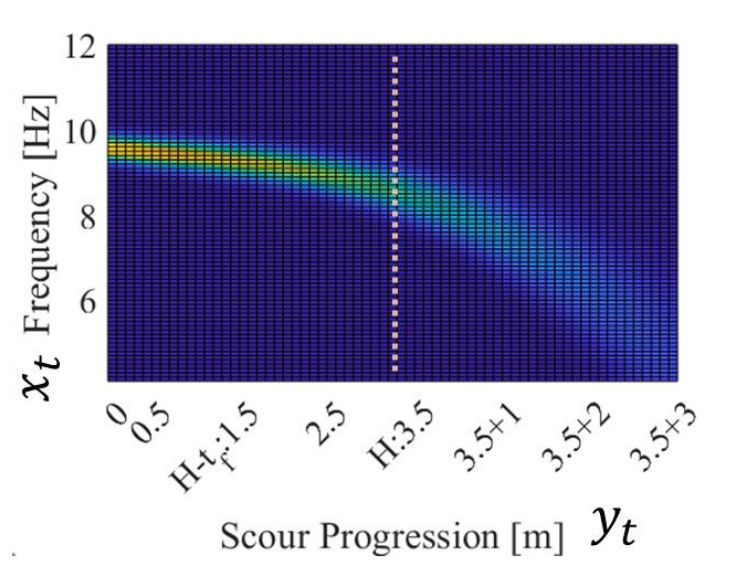


Figure 4.15 The deterioration function of the rocking frequency with scour progression.

4.3.4. Results and conclusions

The results in predicting the probability of scour depth at the SRW period shown in 2.2.4. are shown in Figure 4.16. Each black plot represents the predicted probability of scour depth given identified MPVs of the rocking frequency at every minute. The blue line represents the time-series of water surface level. The probabilities became a bit higher when the water surface level went high and just after the water surface level was at peak. However, the probability recovered near zero when the water surface level decreased at nighttime on the 3rd September. Although there are several outliers through the observation, there are few reasons that we can claim that scour reaches the bottom of the footing through this flood event. Because in-sight investigation after the event, no significant damage was found, it can be said that the result expresses the situation of scour level sufficiently.

In Section 3.2.4., artificial acceleration responses are created to investigate the performance of the proposed stochastic scour warning indices. This section also uses them to investigate the performance of the predicted scour depth using the proposed method. The results of applying the method to the artificial acceleration is shown in Figure 4.17. The probabilities that the scour reaches the bottom of the footing became almost 1 after the water surface level was at peak.

Consequently, it is demonstrated that the proposed scour depth prediction method is possible to apply the real bridge pier and can express the condition of bridge scour sufficiently.

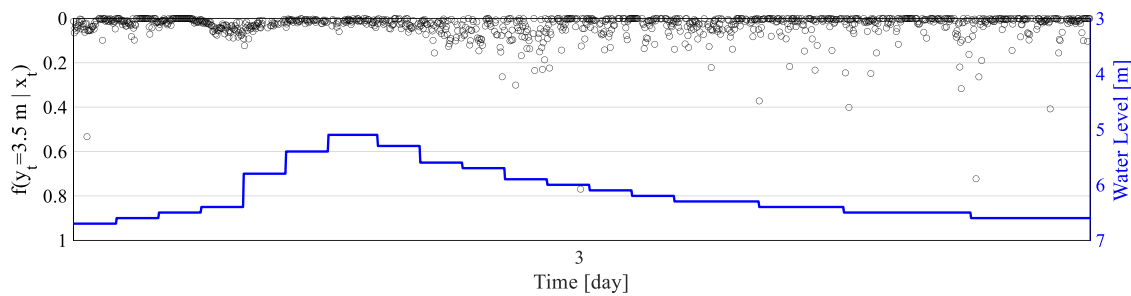


Figure 4.16 The result in time-series of probabilities that scour reaches the bottom of the footing.

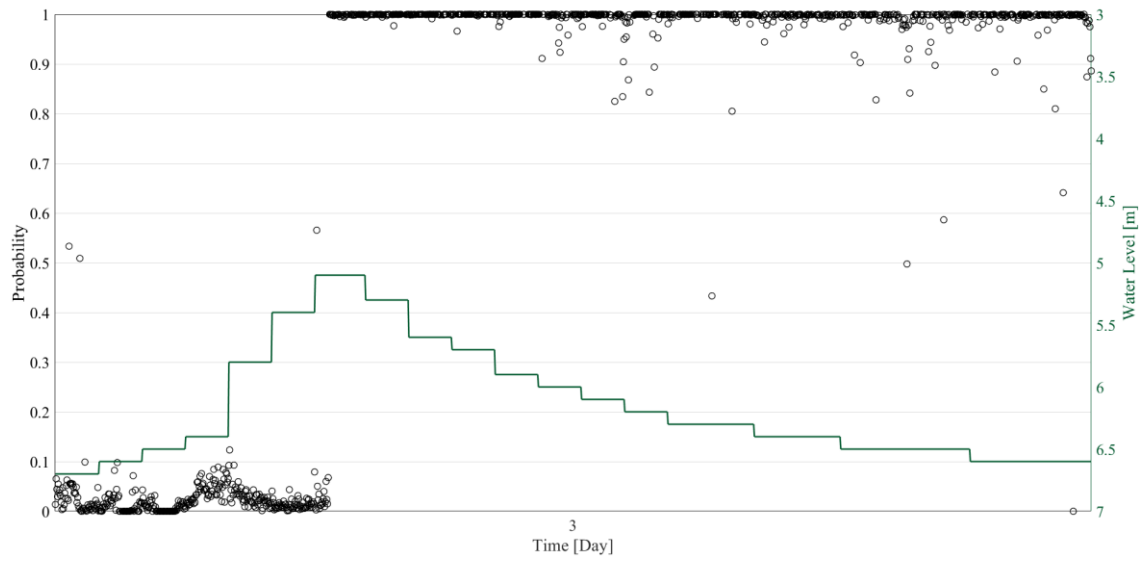


Figure 4.17 The result in time-series of probabilities that scour reaches the bottom of the footing using artificial acceleration.

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Chapter 5. Risk-based scour assessment

Owing to Structural Health Monitoring (hereafter SHM) which can obtain an information on physical characteristics of the monitored structure in realtime such as vibration monitoring of a railway bridge pier as discussed in previous chapters, recognizing the state of the monitored structures and making decisions on the operation of the structure are now possible without sending inspectors or engineers on site.

Some structures with long-term monitoring systems require quick decision making. For example, some severely damaged structures need to be monitored for their progress until retrofit. Some routes with large volumes of traffic must resume operation as soon as possible after intensive weather events. These situations require structure monitoring. This chapter focuses on situations where prompt decisions regarding the operation of the structure are necessary using realtime monitoring.

Although there are many demands for realtime monitoring of structures, limited examples of monitoring systems exist in practice. One of the main reasons is the necessity of dealing with uncertainty. Since observations of SHM have uncertainty, some SHM outputs have insufficient statistical evidence to determine whether they are healthy or abnormal. **Figure 5.1** illustrates the region where it is statistically difficult to determine “clearly healthy” nor “clearly abnormal” when the observations from SHM have uncertainty. In the case of long-term SHM of the structures, uncertainties derived from EOVs also have significant effects on decision making on the operation. Structure authorities see additional information and often make decisions on the safety-side. In the field of decision theory, this orientation is referred to as a Risk-averse orientation [1-6]. However, such actions lead to increased maintenance costs. It is essential to reduce the uncertainty where improvement and advancement of the accuracy of decision making are needed.

There are several ways to reduce the uncertainty of the SHM, and one of the practical ideas is that the structure authorities send engineers to the site. When they receive SHM outputs which cannot determine whether “clearly healthy” nor “clearly abnormal”, it would be reasonable to send engineers if the amount of reducing risks by sending engineers and receiving the correct information on the state of the structure was larger than the cost of sending engineers. **Figure 5.2** shows the flow of deciding engineer’s sending when SHM outputs are obtained. If it is

difficult to judge whether the SHM outputs show clearly anomalous, the flow would proceed the next node. The next node considers whether the authority would send engineers or not by comparing the costs of sending engineers with the risks that can be reduced by the action of sending them. It is necessary to define both thresholds, and the ways of defining them will be discussed using Bayes Factor in this chapter.

This chapter proposes such a framework with a process of SHM and engineer sending. When making decisions under uncertainty, Expected Utility Theory (hereafter EUT) [1-6] is generally used. There are several decision-making topics concerning the infrastructure maintenance using EUTs or risks. Straub[7], Kamariotis et al.[8], Luque et al.[9], etc., built a scheme of using the Value of Information calculated based on EUT for inspection strategies. However, no study has focused on the availability of engineers sending for real time decisions. This chapter firstly summarizes the definition of “Risk” in terms of reliability engineering, and secondly introduces EUT. Then the threshold between “clearly healthy” and “clearly abnormal” is discussed using Bayes Factors. Furthermore, this study explains the framework for judging whether engineers should be sent to the site or not when the SHM outputs cannot determine “clearly healthy” nor “clearly anomalous”. The proposed framework can be applied to the long-term vibration monitoring of a real railway bridge pier as introduced in Section 2.2.

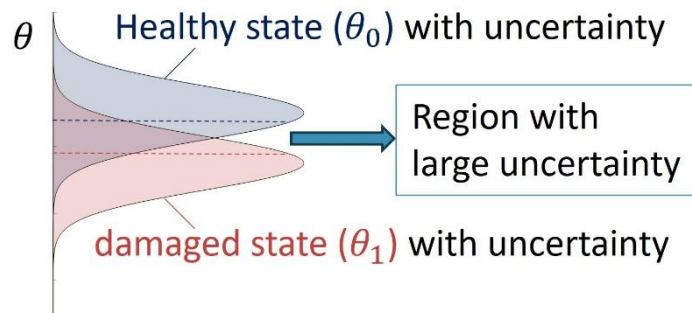


Figure 5.1 Schematic figure illustrating a region with large uncertainty.

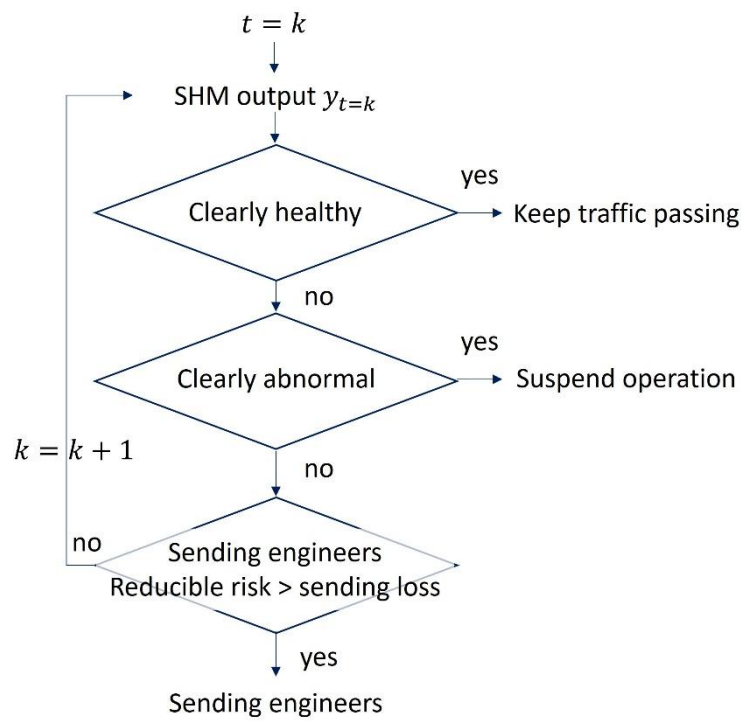


Figure 5.2 Flows of decision making of sending engineers when obtaining SHM outputs.

5.1. Risk in reliability engineering

To begin with the detailed discussions, the definition of risk should be described. The word, “Risk”, has been widely used in various fields, and “risk engineering” or risk analysis has been developed. One field in which the concept of risk has been considered is the mathematical theory of insurance. The most important issue when performing insurance campaigns is to maintain the ability to pay for claims. The main reason for the analysis is to calculate the probability of this ability losing, and to set the proper value of insurance premiums so that it does not exceed the allowable range of the probability. In this case, risk is defined as the probability that the ability to pay for claims will be lost. This situation is called “default”.

In reliability engineering, the main concern is to predict the probability of failure or destruction of an industrial product or system. The probability of the occurrence of default can be directly translated into the probability of failure of the products in reliability engineering. For reference, Alipour et al. [10] summarized the framework for a reliability-based study for scouring. However, conventionally, reliability engineering does not regard the probability of failure as “risk”. In reliability engineering, “risk” is usually used as the expected losses when the failure or destruction occurs. The Japanese Industrial Standards (JIS) provides definitions related to reliability engineering in JIS Z 8115 [11]. This study follows the definition of “risk” as the expected losses when the failure or destruction occurs. Risk can be calculated using Equation (5-1).

$$risk = Probability \times Cost \quad (5-2)$$

Therefore, the difference in the meaning of each term, “Reliability-based” and “Risk-based” should be careful by treated in the civil engineering field. For example, two identical bridges with the same structure and deterioration environment but different location have the same probability of failure. However, the costs due to operation suspension must differ because the locations of each bridge where different societies and economies have is different. Therefore, “Reliability-based” output may be different from “Risk-based” output. There are several researches related to risk-based methods for bridge scour management, especially Pregnolato et al. [12] surveyed published methods and summarized characteristics of their methods.

In situations where bridge authorities cannot decide whether the observation is “clearly healthy” nor “clearly abnormal”, decision making using risk or Expected Utility is considered.

5.2. Expected Utility Theory

5.2.1. Definition

Expected Utility Theory (EUT) is generally used for decision making under uncertainty in the economic field. Assuming D is the identified MPVs of the rocking frequency of the bridge pier, there are two actions of bridge authorities; keep traffic passing, or suspend operation. A group of actions is defined as $\mathbb{A} = \{a_p, a_s\}$. A group of the pier state, healthy and damaged, is defined as $\mathbb{H} = \{H_0, H_1\}$. Then the action tree is illustrated in Figure 5.3. The bridge authorities have two actions given observation D . Since this thesis has proposed data-driven based PSO in Section 3.3 and the prediction of scour depth using Finite Element Analysis in Chapter 4, the probability of damaged, $P(H_1|D)$ can be calculated. Then $P(H_0|D)$ can be calculated as below

$$P(H_0|D) = 1 - P(H_1|D) \quad (5-3)$$

The cost of each end node shown in Figure 5.3 is summarized in Table 5.1. When the bridge authority takes an action of keep operation, a_p , under healthy state, H_0 , no direct cost arises. However, when taking an action of keeping operation under damaged state, costs of fatalities, repairing works, and economic losses may arise.

Then, the utility of each node can be calculated as below

$$U(a_i, H_j) = u(C(a_i, H_j)); i = p, s; j = 0, 1 \quad (5-4)$$

where u denotes utility function of cost C to illustrate the preference of decision makers, and a convex shape function, u , is used when decision makers attribute to risk averse. Utility functions are usually drawn by the results in interview survey for decision makers. However, since bridge authorities who have experienced in facing the severe scour and fatalities are rare, building utility function for scour assessment from a survey like interview is not easy. Therefore, this thesis uses the utility function as below

$$u(C) = C \quad (5-5)$$

The expected utility of each end node is, thus, the same as the definition of the risk shown in Equation (5-2). Therefore, in this chapter, the meaning of the Expected Utility Values (hereafter EUV) and risk are the same.

Then, the EUV of each action can be calculated as Equation (5-6)

$$EUV[a_i] = \sum_{j=0}^1 P(H_j|D)u(a_i, H_j) \quad (5-7)$$

Since this chapter does not consider the benefits but the loss of the bridge operation, the optimum action is the action with minimum EUV as below

$$opt.action = minimum\{EUV[a_i]\}, i = p, s \quad (5-8)$$

Table 5.1 List of Costs relevant to actions of bridge authorities and bridge states for scour

Actions	States	Costs
a_p	H_0	0
a_p	H_1	Fatalities arise, structure recovery, long-term economic loss
a_s	H_0	Short-term economic loss
a_s	H_1	Structure recovery, long-term economic loss

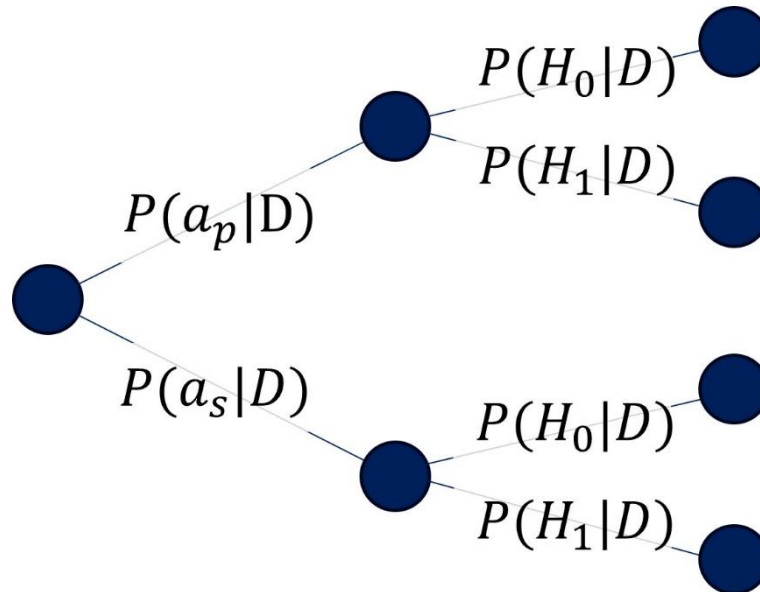


Figure 5.3 Decision tree.

5.2.2. Necessity to devise Expected Utility Theory for scour assessment

Figure 5.4 illustrates the schematic figure of EUV given observation D in case of vibration monitoring for bridge scour. The horizontal and vertical axes present the identified frequency, and EUVs, respectively. The blue and red lines are $EUV[a_p]$, and $EUV[a_s]$. When the identified frequency is high, $EUV[a_p]$ is smaller than $EUV[a_s]$. Therefore, bridge authorities keep operation. When the frequency is decreasing, the red and blue lines intersect in the middle, and $EUV[a_p]$ becomes larger than $EUV[a_s]$. Therefore, the proper decision could stop the operation. The EUV can achieve quantitative decision-making. However, EUV-based decision making is not always practical in situations where high safety is required like bridge operations. When a greater difference between $EUV[a_p]$ and $EUV[a_s]$ given D subject to $EUV[a_p] < EUV[a_s]$ exists, the bridge authorities make decision of keeping traffic passing with high confident. In contrast, when a difference between $EUV[a_p]$ and $EUV[a_s]$ given D subject to $EUV[a_p] < EUV[a_s]$ is short, the authorities may make decision of keeping traffic passing with less confidence because the probability of damage is also increasing. Under such situation, some authorities may select decision of suspending traffic even though $EUV[a_p] < EUV[a_s]$ from risk averse point of view. This content indicates that the level of confidence in decision making of keeping traffic passing or suspending traffic may correspond to the difference between $EUV[a_p]$ and $EUV[a_s]$ given D subject to $EUV[a_p] < EUV[a_s]$. The level of confidence is like a “certainty factor for selecting decision of keeping traffic passing”. In situations where high safety is required, the safety side decision should be selected when the certainty factor is weak regardless of EUVs. Mostly these situations happen when the observation has large uncertainty as shown in Figure 5.1.

The challenge is to develop the border where bridge authorities judge that the EUVs of two lines are close. It is firstly necessary to quantify the certainty factor based on the assumption that the decision maker prefers to select the action with the smaller EUV. Also, it is not easy to judge from only two actions, keeping operation and stopping operation when the observation and calculated EUVs have huge uncertainties. This study, thus, considers an additional option of the action of sending engineers on site, and the details are discussed in Section 5.5.

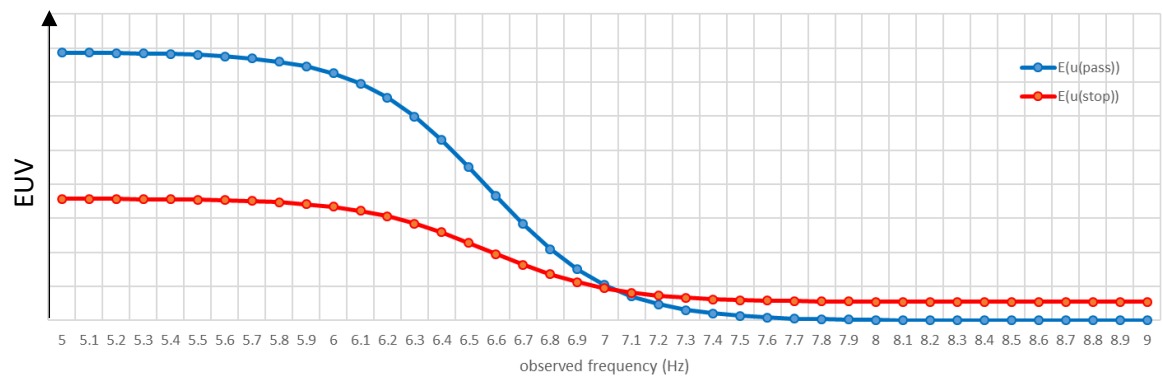


Figure 5.4 Schematic figure for Expected Utility Values given observation outputs.

5.3. Decision making for clearly anomalous

In vibration-based scour assessment, abnormal conditions, called A1 rank [13], are categorized when 70 % of the identified frequency of the pier is observed. However, the identified frequencies of the bridge pier during a normal period have uncertainty. The issue is how to determine whether the observation is clearly anomalous. In other words, the issue is how to define the red dashed line in **Figure 5.1**, which distinguishes “clearly anomalous” from the observation. When the observation shows clearly anomalous, only action of suspending operation must be selected regardless of the risk assessment.

In Section 3.4, Bayes Factor (hereafter BF) which evaluates the evidence in favor of a null hypothesis, is already considered for anomaly detection to address the issues of conventional hypothesis testing. Kass et al. [14] provided the interpretation of BF shown in **Table 3.4.**, i.e. when $2 \log_e BF > 10$, the evidence against the null hypothesis is very strong. Therefore, $2 \log_e BF > 10$ is the basic idea to determine the thresholds for clearly anomalous as shown below.

$$2 \log_e (BF) = 2 \log_e \frac{P(D|H_1)}{P(D|H_0)} > 10 \quad (5-9)$$

Section 3.4 discusses the practical way of usage of BF since the identified frequency of the bridge pier does not follow the Normal Distribution. Here is a summary of the discussions.

- Identified frequencies of a real concrete bridge pier follows Levy-Flight Distribution (hereafter LF Distribution).
- The thresholds based on Kass’s interpretation [14] do not match the categories of scour soundness categories [13] due to the asymmetry of the LF Distribution.
- The modified thresholds considering the overlapped area between the distribution at healthy and damaged states are introduced.
- The parameter, β , which illustrates the skewness of the LF Distribution, changes to zero to satisfy the symmetry is used for scour assessment.

5.4. Decision making for clearly healthy

Section 3.4 also discusses the usage of BF in situations where the bridge authorities cannot judge whether the observation is clearly healthy or not. It is very difficult to predict the changes in the distribution of the observation according to the gradual deterioration in civil infrastructures because there is lack of monitoring data which has experienced the severe damage through the monitoring period. One way to cope with this difficulty is to use a weak informative distribution [15] as an alternative hypothesis of BF. Several studies have proposed practical methods for detecting anomalies in a real bridge monitoring using BF with weak informative approach [16-19]. The concept of this approach is to clarify that the test observations do not follow the reference observations. Therefore, it is useful to judge whether the observations follow the reference observations or not. In scour monitoring, the observation during the SRW period follows the reference observations obtained at normal period.

Since the lower range of natural frequencies of the pier under a severe scour can be expected by scour soundness categories or numerical simulations, the skewness of the distribution of alternative hypotheses can be predefined. This study sets the lower limit of 1% as around half of the mean identified frequency during the normal period. The distribution is defined as H_{1c} .

Kass et al. [14] provides the interpretation of BF shown in **Table 3.4**, when $2 \log_e BF > 2$, the evidence against the null hypothesis is positive. Therefore, $2 \log_e BF \leq 2$ is the basic idea to determine the thresholds for clearly healthy as shown below.

$$2 \log_e (BF) = 2 \log_e \frac{P(D|H_{1c})}{P(D|H_0)} \leq 2 \quad (5-10)$$

As the discussion in Section 3.4, the modified thresholds for the LF Distribution and the skewness parameter $\beta = 0$ are used.

5.5. Decision making for sending engineers

When the observation does not satisfy the Equations (5-9) and (5-10), the observation is not “clearly healthy” nor “clearly abnormal”. In such situations, it is difficult to make decisions regarding bridge operation based only on the SHM outputs. Therefore, this section considers the additional action of sending engineers to the site. A group of actions is then; $\mathbb{A} = \{a_p, a_s, a_{se}, a_{wose}\}$. a_{se} is the action that bridge authorities decide to send engineers to the site. a_{wose} is the action that bridge authorities do not send engineers but keep investigating the SHM outputs continuously. The time-series of the action tree is shown in Figure 5.5. When the observation $y_{t=k}$ is obtained, firstly whether the observation is clearly abnormal or not by Equation (5-9). If $y_{t=k}$ did not satisfy Equation (5-9), secondly whether the $y_{t=k}$ is clearly healthy or not would be considered. If $y_{t=k}$ did not satisfy Equation (5-10), they must consider the possibility of sending engineers to the site.

In such a situation, bridge authorities would compare the merits of sending engineers with the labor required to send engineers to the site. Table 5.2 summarizes the cost and effectiveness of action of sending engineers or not. In the situation of sending engineers, in deed cost is required. However, the risks of keeping traffic of the bridge passing despite scouring, and stopping operations despite the healthy state, could be removed. On the other hand, without sending engineers, which saves engineer sending costs, bridge authorities have to take risks of keeping operations despite scouring, and suspending operations regardless of the bridge state. Therefore, the decision to send engineers can be made by comparing the risks associated with each action.

It is, thus, necessary to formulate the risk of each end node shown in Figure 5.3. Risk can be calculated by the result in selected action under the bridge state given observations D . Therefore, risk at each end node shown in Figure 5.3 can be written as

$$r(a_i, H_j | D) = P(H_j | D) u(C(a_i, H_j)); i = p, s; j = 0, 1 \quad (5-11)$$

The risk of each action is written as

$$r(a_i | D) = \sum_{j=0}^1 r(a_i, H_j | D); i = p \quad (5-12)$$

Here, $r(a_p | D)$ must be smaller than $r(a_s | D)$. If $r(a_p | D)$ is much smaller than $r(a_s | D)$, they could select a_p with much larger confidence. On the other hand, if $r(a_p | D)$ is only slightly smaller than $r(a_s | D)$, they would select a_p with little confidence. This means that the

distance between $r(a_p|D)$ and $r(a_s|D)$ shows the degree of confidence. In other words, it can be defined as “certainty factor for decision making”.

Based on the definition of certainty factor, the probability of selecting action subject to $r(a_p|D) < r(a_s|D)$ can be calculated as below

$$p(a_p|D) = \frac{r(a_s|D) - r(a_p|D)}{r(a_s|D)} \quad (5-13)$$

$$p(a_s|D) = 1 - p(a_p|D)$$

This study assumes that the correct state can be known when bridge authorities send engineers to the site subject to $r(a_p|D) < r(a_s|D)$. If engineers were not sent to the site, there would be potentially one risk of keeping traffic passing despite scouring in site. The intensive cost including compensation for casualties might arise. Therefore, the risk of the decision of not sending engineers is calculated as below

$$r(a_{wose}|D) = p(a_p|D) \times r(a_p, H_1|D) \quad (5-14)$$

On the other hand, when bridge authorities decide to send engineers to the site, there are potentially two risks. One is the case of bridge in a state of damage. However, engineers can make decision on the damaged condition just after investigating at the site. Since bridge authorities can avoid the casualties, the risk of this situation is the same as $r(a_s, H_1|D) + C_{se}$. C_{se} is the cost of sending engineers. Another risk is that the state of the bridge is healthy. This risk is written as $r(a_p, H_0|D) + C_{se}$. Therefore, the risk of sending engineers is calculated as follows.

$$r(a_{se}|D) = p(a_p|D) \times [(r(a_s, H_1|D) + C_{se}) + (r(a_p, H_0|D) + C_{se})] \quad (5-15)$$

By comparing Equation (5-14) with Equation (5-15), bridge authorities decide to send engineers to the site when $r(a_{se}|D)$ is smaller than $r(a_{wose}|D)$. This is named “Risk Comparison Viewpoint” (hereafter RCV).

Another approach to send engineers to the site is to compare the effects obtained by sending engineers with the reducible risks of sending engineers. This is named “risk reducible Viewpoint” (hereafter RRV). The simple explanation between RCV and RRV is shown in Figure 5.6. While RCV compares the absolute value of risks, RRV compares the cost of sending engineers shown as the green bar with the difference between the risk of sending and not sending engineers shown as the orange bar in Figure 5.6.

The two viewpoints do not have a distinct usage concept. However, when the uncertainty of SHM

outputs is larger or the estimated cost of fatalities and repairing works is more significant, RRV is preferred to RCV because of the easiness of intuitive understanding.

Table 5.2 Pros and cons comparison between sending engineers or not.

	Sending engineers, a_{se}	Without sending engineers, a_{wose}
Pros	Enabling $r(a_p, H_1)$ and $r(a_s, H_0)$ to remove	Ignoring cost of sending engineers
Cons	Cost of sending engineers	Having $r(a_p, H_1)$, $r(a_s, H_0)$, and $r(a_s, H_1)$

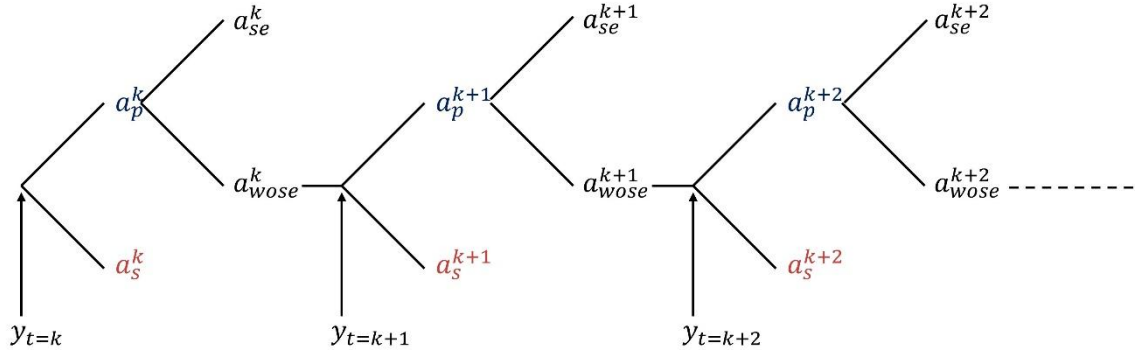
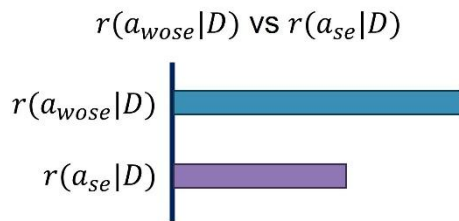


Figure 5.5 Time-series of the action tree of the monitored bridge operation.

Risk Comparison Viewpoint



Risk Reducible Viewpoint

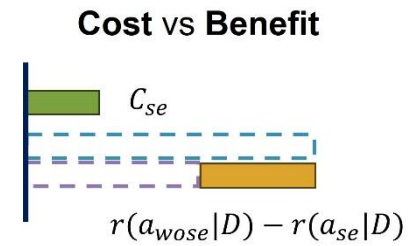


Figure 5.6 Two approaches to decide to send engineers to the site.

5.6. Application to case study 2

5.6.1. Settings

This section applies the proposed framework for decision making of bridge operation using SHM outputs, including the option of sending engineers. According to the discussion in Section 3.4.4., the modified thresholds of “clearly healthy” and “clearly abnormal” considering the LF Distribution with symmetricity are set to $2 \log_e(BF) < 3.87$, and $2 \log_e(BF) > 9.39$ as shown in Table 3.9. In the observation during the SRW period, all BFs throughout the event show clearly healthy as shown in Figure 3.37. Therefore, this section uses artificial accelerometers to simulate scour occurrence as shown in Figure 3.38.

To calculate the risk shown in Equations (5-14) and (5-15), losses and costs are surveyed through Web pages and related documents relevant to insurance, economic losses due to earthquakes, statistics of logistics around the bridge, etc. These surveyed values of cost setting are shown in Table 5.3. In general, costs can distinguish direct costs from indirect costs. The costs of fatalities arise, structure recovering and sending engineers are categorized as direct costs. Other costs relevant to the effect of railway bridge scours are ignored. The cost of each end node shown in Figure 5.3 can be obtained by the combination of costs shown in Table 5.3. Table 5.4 shows the calculated costs at each end node in Figure 5.3. Using the cost of each end node, risks are calculated and decision making for sending engineers are considered.

Table 5.3 The value of each cost setting in this study.

Cost	Notation (unit: thousand yen)
Fatalities arise	$C_{fa} = 492760$
Structure recovering	$C_{re} = 32000$
Economical loss due to suspended (short or long)	$C_{els}, C_{ell} = 3000, 96000$
Sending engineers	$C_{se} = 108$

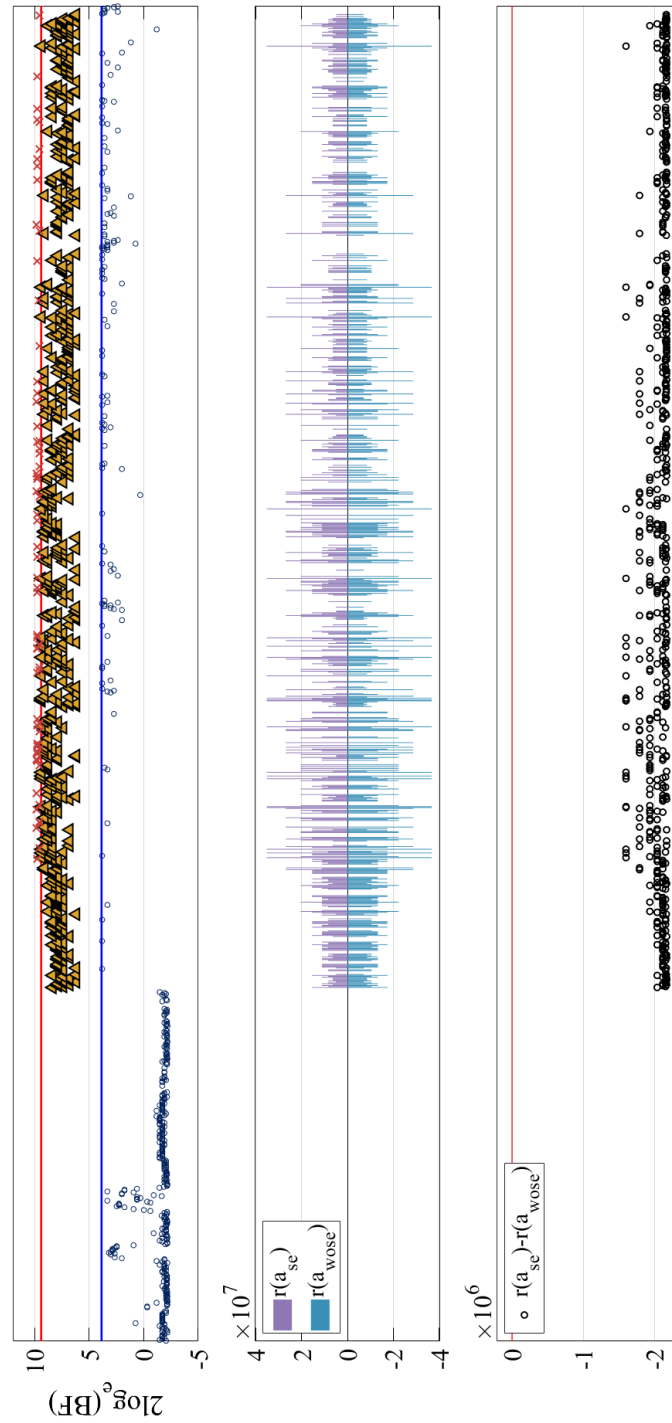
Table 5.4 Costs at each end node shown in Figure 5.3.

$C[a_p H_0] = 0$
$C[a_s H_0] = C_{els} = 3,000$
$C[a_p H_1] = C_{fa} + C_{rep} + C_{ell} = 620,760$
$C[a_s H_1] = C_{fa} + C_{rep} = 128,000$

5.6.2. Risk Comparison Viewpoint

This section utilizes the RCV assessment for sending engineers to the site during the SRW period. The results in applying the calculation of $r(a_{se}|D)$ and $r(a_{wose}|D)$ with BFs of A1 rank to the artificial acceleration of real bridge vibration monitoring data are shown in Figure 5.7. The top graph in Figure 5.7 illustrates the BFs for A1 rank judgement, and the yellow triangle plots indicate that it is necessary to consider sending engineers or not because the SHM outputs do not support “clearly healthy” nor “clearly abnormal”. Once scour occurs, BFs suddenly show yellow triangle plots. The intermediate graph illustrates the values of $r(a_{se}|D)$ as purple bars and $r(a_{wose}|D)$ as light blue bars corresponding to the triangle plots at the top graph. The black plots at the bottom graph in Figure 5.7 illustrate the difference between $r(a_{se}|D)$ and $r(a_{wose}|D)$ in the intermediate graph of Figure 5.7. The negative values show that the risk of sending engineers is smaller than that of not sending engineers. All plots show negatives. Therefore, RCV shows that the engineer should be sent to the site in this SRW event.

Figure 5.8 shows the behavior to be selected, on top of the identified frequencies with uncertainties. Horizontal lines of yellow, red, and purple indicate the border of each scour soundness category as shown in Table 3.1. The blue circle and red cross plots denote clearly healthy and clearly abnormal based on the SHM outputs, respectively. The yellow triangle plots show sending engineers to the site based on the RCV. Decisions mostly match the borders of the scour soundness categories. After scour occurrence, the decisions were varied because this study shows decision making using only identified frequencies measured every minute. Therefore, it may be useful to present smoothed information that also considers the previous observations for practical situations.



3

Figure 5.7 Decision making for sending engineers of Risk Comparison Viewpoint. Top, intermediate and bottom figures show, Bayes Factors of A1 rank judgement, the value of risks of sending engineers or not, and differences between their risks, respectively.

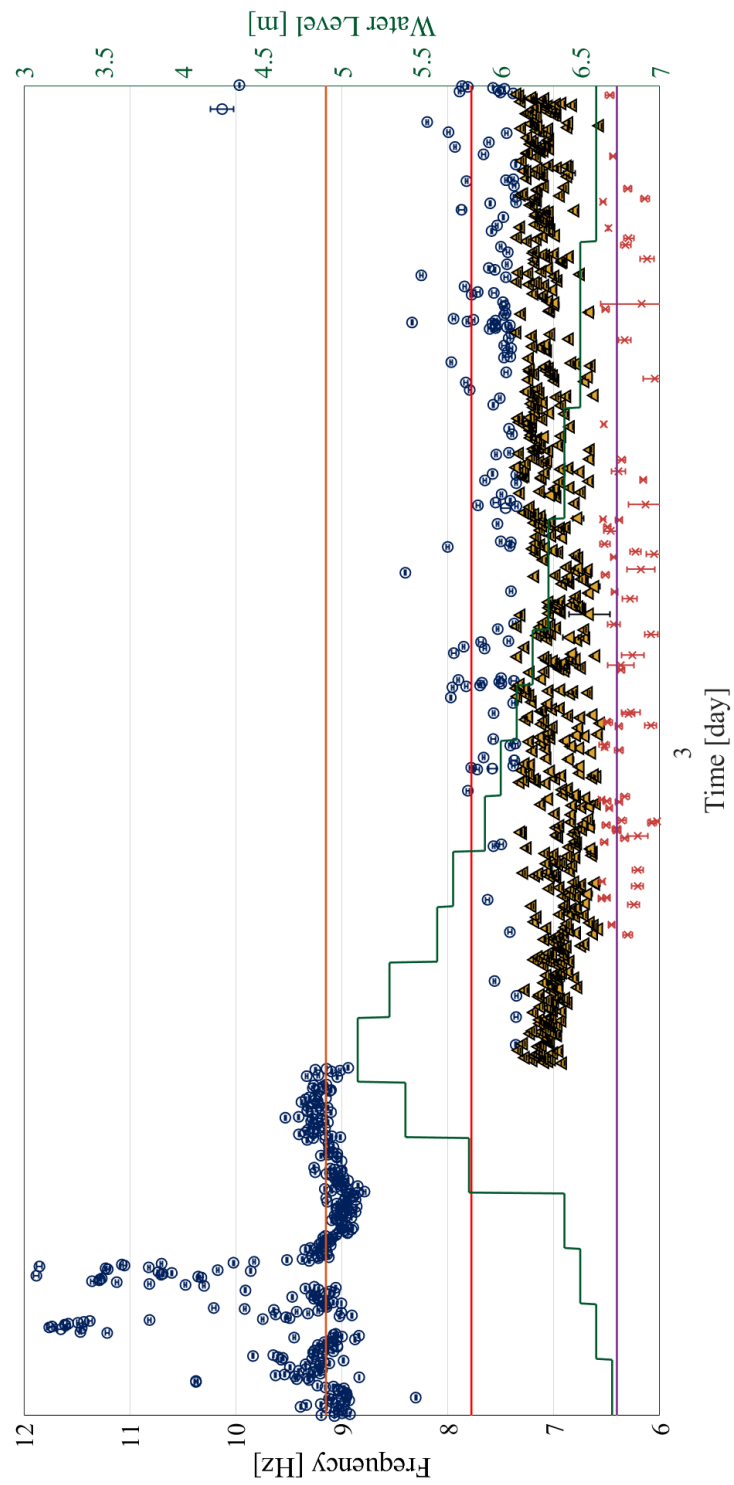


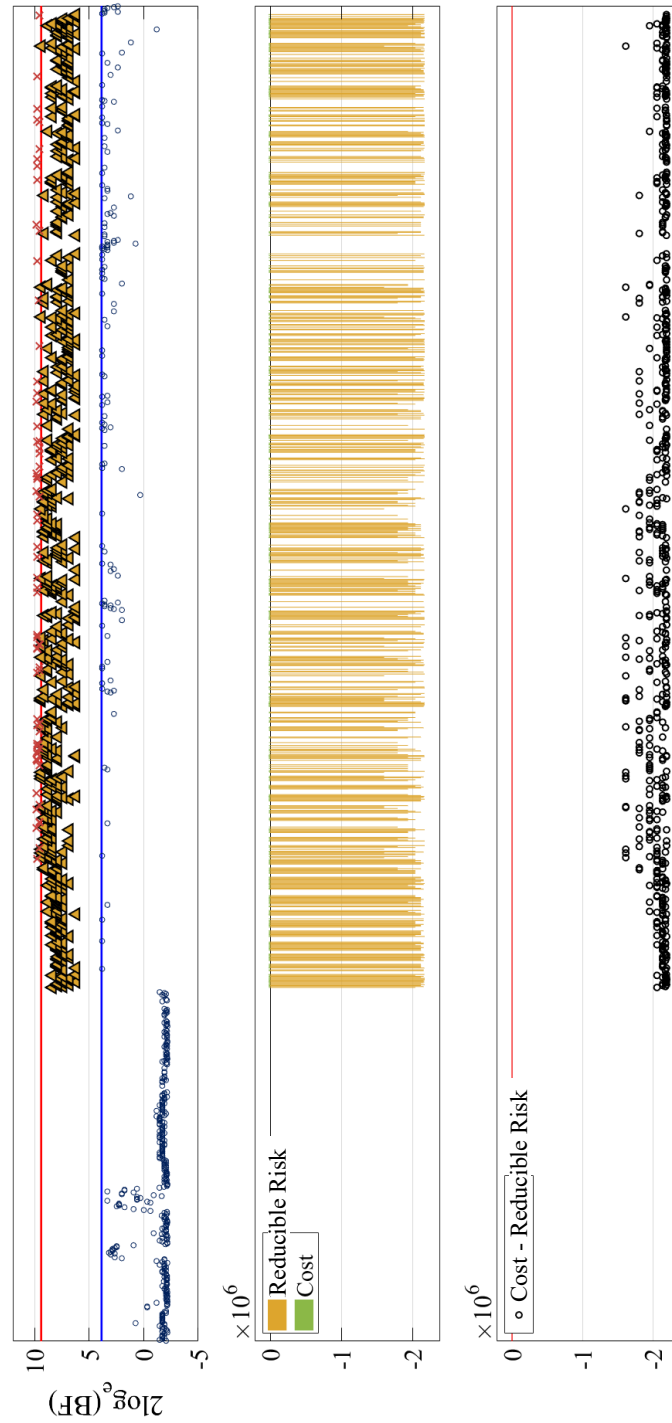
Figure 5.8 Time-series of identified MPVs of the rocking frequency with uncertainty and decisions of each plot when artificial accelerometer observations are considered.

5.6.3. Risk Reducible Viewpoint

This section utilizes the RRV assessment for sending engineers to the site during the SRW period. The results of applying the calculation of costs and reducible risks with BFs of A1 rank to the artificial acceleration of real bridge vibration monitoring data are shown in Figure 5.9. The top graph in Figure 5.9 illustrates the BFs for A1 rank judgement, and the yellow triangle plots indicate that it is necessary to consider sending engineers or not because the SHM outputs do not support “clearly healthy” nor “clearly abnormal”. Once scour occurs, BFs suddenly show yellow triangle plots. The intermediate graph illustrates the values of costs as green bars and reducible risks as orange bars corresponding to the triangle plots at the upper figure. Furthermore, the black plots at the bottom graph in Figure 5.9 illustrate the difference between costs and reducible risks in the intermediate graph of Figure 5.9. The negative values show that the reducible risks by sending engineers are higher than the costs of sending engineers. All plots show negatives. Therefore, RRV showed that engineers should be sent to the site in this SRW event.

Figure 5.10 shows the behavior to be selected, on top of the identified frequencies with uncertainties. The horizontal lines of yellow, red, and purple indicate the border of each scour soundness category as shown in Table 3.1. The blue circle and red cross plots indicate clearly healthy and clearly abnormal based on the SHM outputs, respectively. The yellow triangle plots show sending engineers to the site based on the RRV. Decisions mostly match the borders of the scour soundness categories. After scour occurrence, the decisions are varied because this study shows decision making using only identified frequencies measured every minute. Therefore, it may be useful to present smoothed information that also considers the previous observations for practical situations.

Figure 5.10 is completely the same result as Figure 5.8. This means that there is no significant difference between the outcomes of RCV and RRV. In this situation and assumptions, all outcomes when the SHM outputs do not support “clearly healthy” nor “clearly abnormal” show “ a_{se} ; sending engineers to the site”, not “ a_{wose} ; continue to obtain additional outputs”



3

Figure 5.9 Decision making for sending engineers of Risk Reducible Viewpoint. Top, intermediate and bottom figures show Bayes Factors of A1 rank judgement, the cost and reducible risk, and differences between them, respectively.

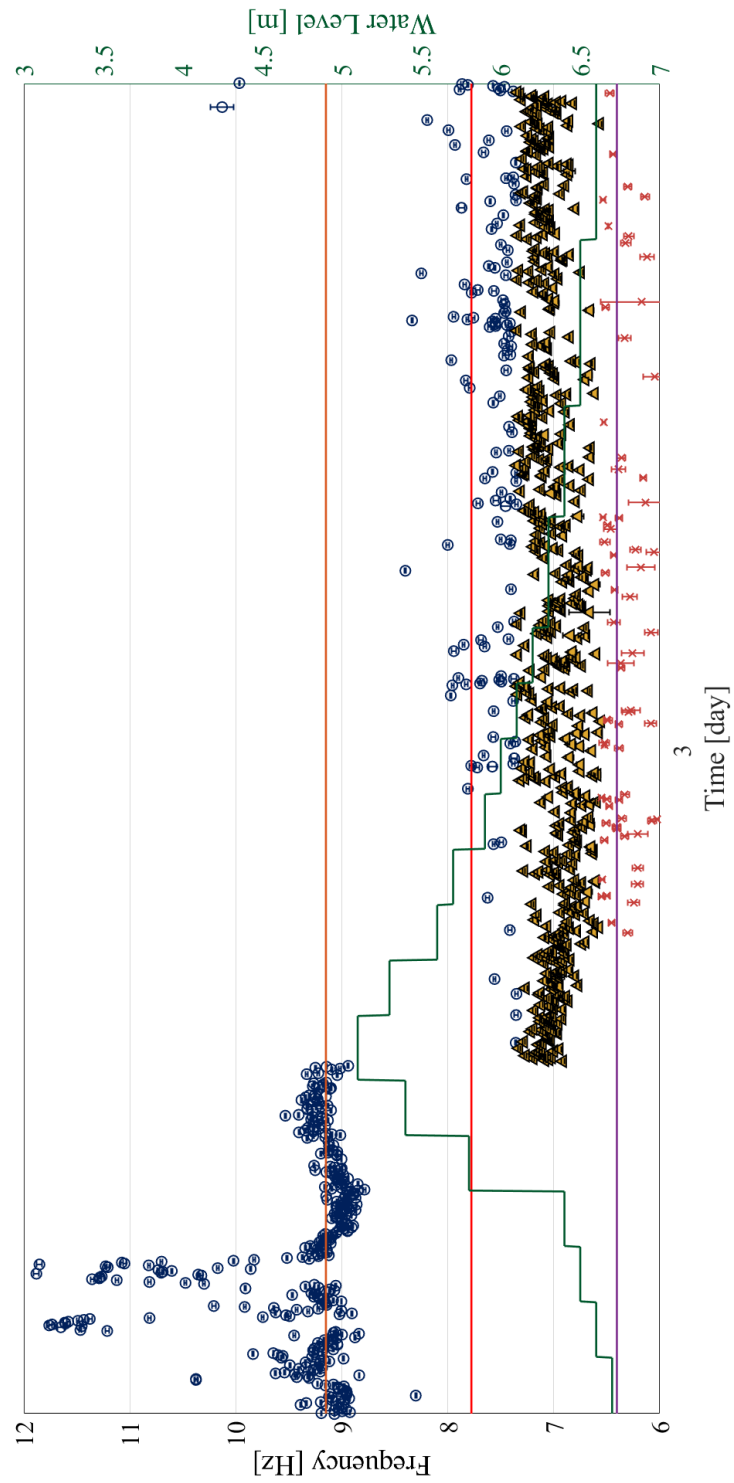


Figure 5.10 Time-series of identified MPVs of the rocking frequency with uncertainty and decisions of each plot when artificial accelerometer observations are considered.

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Chapter 6. Conclusions and future work

6.1. Conclusions

This study proposed novel scour assessment methods using the ambient vibration obtained from the accelerometers installed on top of the bridge pier to realize realtime decision-making for bridge operation. To demonstrate the validity of the proposed methods and investigate the pros and cons of each method, two case studies; an RC specimen experiment for simulating scouring by excavating the ground around the specimen, and a long-term vibration monitoring of a railway bridge pier, were conducted. The details of each study are summarized in Chapter 2. This chapter provides a summary of the entire study.

Chapter 1 provides the background and objectives of this study. It is emphasized that the intensive rainfall these days is a crucial factor in the maintenance of bridges across rivers. Vibration-based scour assessment methods using ambient vibration are introduced to realize a fast and accurate decision making of bridges' operation. The importance of considering uncertainty in utilizing ambient vibration is stated to assist the decision making.

Chapter 2 shows the information on the two case studies. The first study is an RC specimen experiment for scour simulation. The second study is a long-term vibration monitoring of a railway concrete bridge pier installed with remote sensing systems. Finite Element models of the piers are also introduced.

Chapter 3 proposes four data-driven stochastic scour assessment indices and verifies the validity of each index using two case studies. Firstly, it shows that the identified rocking frequency of the bridge pier at normal period follows the Levy-Flight Distribution, rather than Normal Distribution. Then, Scour Warning Index 1 (SWI1) and Scour Warning Index 2 (SWI2) to use the probability distribution of the Levy-Flight Distribution during normal period and a Fast Bayesian Fast Fourier Transform method as a system identification method are proposed. Next, Probability of Scour Occurrence (PSO) that is intuitively understandable is proposed to use a logistic curve. Since the outputs of these indices have statistical meanings, the characteristics and the interpretations are discussed through two case studies. In anomaly detection, the conventional statistical hypothesis is generally used, however it is sometimes not practical to apply to the case when we have plenty of observations like long-term monitoring deploying sensors in site. As one of the alternatives, Bayes Factor is applied to two case studies. Since the identified rocking frequency of the bridge

pier follows the Levy-Flight Distribution, this Chapter considered the thresholds of Bayes Factors to satisfy the scour soundness categories, and the Bayes Factors shows anomalies when the scour progressed in the experiment.

Chapter 4 proposes a stochastic method for predicting the scour depth using ambient vibrations. Providing scour depth that correlates with information on the structural performance is preferable rather than indirect outputs like changes in the natural frequency of the pier. Using a 3-dimensional beam-based finite element model of the bridge pier, the soil spring constants are modeled as the probability distribution by Bayesian model updating. Then, the relationship between the scour depth and the natural frequency of the pier is obtained. The proposed method is applied to the RC specimen experiment, and it figures out that the decreasing ratio of the soil spring constants follows the decreasing ratio of the resistance area of the pier. This outcome is important for assessing the bridge soundness with shallow foundations because the identified natural frequency would drastically decrease when the scour reaches the bottom of the footing.

Chapter 5 proposes a comprehensive risk-based framework for bridge operation including an option of sending engineers. Since the outputs of the observed information have uncertainty, which sometimes makes it difficult for bridge authorities to judge whether the bridge is safe or damaged, sending engineers is one of the practical solutions. This chapter discusses a way of identifying the border of “clearly healthy or not”, “clearly abnormal or not”, and “engineer should be sent to the site or not”. Bayes Factor is adopted to determine their borders. Specifically, two risk-based approaches, Risk Comparison Viewpoint (RCV) and Risk Reducible Viewpoint (RRV) to decide to send engineers are introduced and applied to the long-term monitoring outputs. The results in RCV and RRV are mostly the same, which indicates that there is little significant difference between the two approaches.

6.2. Future work

This study includes four data-driven stochastic indices. In the case of scour assessment during Swollen River Water Periods, quick and accurate decision making is required. Therefore, coping with outliers is always important when we utilize data-driven approach. The combination of smoothing methods should be considered as one of the future works.

This study also proposed a scour depth prediction method utilizing the Digital Twins approach. As mentioned, realtime assessment is always necessary during the Swollen River Water period. Therefore, this study used the 3-Dimensional finite element model to conduct eigenvalue analyses to reduce the computation time. However, to provide more accurate conditions of the ground around the bridge pier with a fast computation, applying a Physics-Informed Neural Network to the scour assessment of the bridge pier will be the next challenges. Also, since this study introduces the stochastic scour depth prediction methods, the resistance of the structure with uncertainty can be estimated. Then, the reliability index would enable bridge authorities to provide if the external force of water flow with uncertainty is quantified.

There will be many trials when we build the comprehensive framework for risk-based decision making for the bridge operation of scour. For simplicity, this study only focuses on three actions: keeping traffic passing, sending engineers, and suspending operation. However, in the real bridge operation, there are other options such as keeping operation with speed limitation, etc. Sometimes engineers may not be able to arrive at the site due to road suspension due to intensive rainfall. The framework needs to be updated as appropriate to the situation.

Modeling costs also have a significant effect on the decision making on sending engineers, especially unit labor costs. Unit labor costs and the amount of traffic vary by region. These will affect the management thresholds. The balance between unit labor costs, amount of traffic, and management thresholds are the big challenges to realize the comprehensive framework.

APPENDIX

A1. Fast Bayesian Fast Fourier Transform

The Fast Bayesian Fast Fourier Transform (hereafter Fast Bayesian FFT) algorithm is developed for output-only system identification in many fields. In civil structures, it is mainly used for operational modal analysis by associating the Bayesian Inference with the frequency domain feature of the output signals (generally dynamic response like acceleration).

Bayesian Inference

Given θ as a set of model parameters to be learned, D as the observed data, and M as a potential model of the system, Bayesian Inference properly describes the nature of the learning procedure of human beings with a form like

$$p(\theta|D, M) = p(D|M)^{-1}p(D|\theta, M)p(\theta|M) \quad (\text{A-16})$$

where the RHS of the equation depicts the learning procedure with the prior PDF $p(\theta|M)$ denoting the prior information about θ (perception from preceding experience, etc.), the likelihood $p(D|\theta, M)$ denoting the information about θ contained in the observed data D (evidence supported by data D , used for updating), the marginal likelihood $p(D|M)$ denoting the supporting information in the observed data D about the model class M (also called the model evidence about M , and generally a constant for given D and M). The LHS $p(\theta|D, M)$ is the posterior PDF of θ which indicates the updated knowledge about θ with the observations D and the prior PDF.

Formulation of Fast Bayesian FFT

In the formation of the Bayesian FFT approach, a key point is to connect the modal parameters θ to the observed acceleration data $D = \{\hat{x}_j \in R^n : j = 1, \dots, N\}$ with n channels and N samples in each channel, namely the formulation of likelihood $p(D|\theta, M)$, which can be realized by the theory of mode superposition and FFT. With all contributing modes considered, the data can be denoted as

$$\hat{x}_j = x_j + \varepsilon_j = \sum_i \Phi_i \eta_{ij} + \varepsilon_j \quad (\text{A-17})$$

where $\Phi_i \in R^n$ is the i^{th} partial mode shape related with the measured degree of freedoms; η_{ij} is the j^{th} sample of the i^{th} modal response; $\varepsilon_j \in R^n$ is the prediction error.

Then, transforming the data into the frequency domain by FFT and selecting a resonance frequency band containing m modes of interest in practice leads to

$$\hat{\mathbf{F}}_k = \sum_{i=1}^m \boldsymbol{\Phi}_i h_{ik} p_{ik} + \mathbf{e}_k = \boldsymbol{\Phi} \text{diag}(\mathbf{h}_k) \mathbf{p}_k + \mathbf{e}_k \quad (\text{A-18})$$

where $\hat{\mathbf{F}}_k \in \mathbb{C}^n$ is a vector of the scaled FFTs of data at frequency f_k ; $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1, \dots, \boldsymbol{\Phi}_m] \in \mathbb{R}^{n \times m}$ is the partial mode shape matrix; $\mathbf{p}_k = \{p_{ik}\}_{i=1}^m \in \mathbb{C}^m$ is a vector of the scaled FFTs of modal forces at frequency f_k ; $\mathbf{e}_k \in \mathbb{C}^n$ is the scaled FFTs of prediction error; $\text{diag}(\cdot)$ is the diagonal operation, and $\mathbf{h}_k \in \mathbb{C}^m$ is a vector of the modal transfer functions with the i^{th} element denoted as

$$h_{ik} = [(\beta_{ik}^2 - 1) + i(2\zeta_i \beta_{ik})]^{-1} \quad (\text{A-19})$$

where $\beta_{ik} = \frac{f_i}{f_k}$ is the frequency ratio with the i^{th} modal frequency f_i , and ζ_i denotes the corresponding damping ratio.

According to the research about spectral analysis, it is recognized the FFTs $\hat{\mathbf{F}}_k$ are asymptotically independent at different frequencies, following complex Gaussian distribution with zero mean and a covariance matrix given by

$$\mathbf{C}_k = \frac{1}{2} \begin{bmatrix} \boldsymbol{\Phi} \text{Re} \mathbf{H}_k \boldsymbol{\Phi}^T & -\boldsymbol{\Phi} \text{Im} \mathbf{H}_k \boldsymbol{\Phi}^T \\ \boldsymbol{\Phi} \text{Im} \mathbf{H}_k \boldsymbol{\Phi}^T & \boldsymbol{\Phi} \text{Re} \mathbf{H}_k \boldsymbol{\Phi}^T \end{bmatrix} + \frac{S_e}{2} \mathbf{I}_{2n} \quad (\text{A-20})$$

where S_e is the power spectral density (PSD) of the prediction error which is assumed constant within the selected frequency band and different channels, and $\mathbf{I}_{2n}$ is an identity matrix; $\mathbf{H}_k \in \mathbb{C}^{m \times m}$ is the theoretical spectral density matrix of the modal acceleration responses and is given by

$$\mathbf{H}_k = \text{diag}(\mathbf{h}_k) \mathbf{S} \text{diag}(\mathbf{h}_k^*) \quad (\text{A-21})$$

with $\mathbf{S} = \mathbf{p}_k \mathbf{p}_k^* \in \mathbb{C}^{m \times m}$ denoting the PSD matrix of modal forces which is assumed constant within the selected frequency band, and here the asterisk * is the notation of conjugate transpose.

Then, given a non-informative prior distribution (generally a uniform distribution) of $\boldsymbol{\theta}$ without loss of generality, the posterior distribution can be represented as

$$\begin{aligned} p(\boldsymbol{\theta}|D, M) &\propto p(D|\boldsymbol{\theta}, M) = \prod_k (2\pi)^{-n/2} [\det \mathbf{C}_k(\boldsymbol{\theta})]^{-1/2} \exp[-\mathbf{Z}_k^* \mathbf{C}_k(\boldsymbol{\theta})^{-1} \mathbf{Z}_k] \\ &= \exp[-L(\boldsymbol{\theta})] \end{aligned} \quad (\text{A-22})$$

where $L(\boldsymbol{\theta})$ is the objective function for the most probable value (hereafter MPV) $\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$, and called the ‘negative log-likelihood function’ (hereafter NLLF) given by

$$L(\boldsymbol{\theta}) = \frac{1}{2} \sum_k \ln \det \mathbf{C}_k(\boldsymbol{\theta}) + \frac{1}{2} \sum_k \mathbf{Z}_k^* \mathbf{C}_k(\boldsymbol{\theta})^{-1} \mathbf{Z}_k + \frac{n}{2} \sum_k \ln 2\pi \quad (\text{A-23})$$

with $\mathbf{Z}_k = [\text{Re} \hat{\mathbf{F}}_k; \text{Im} \hat{\mathbf{F}}_k] \in R^{2n}$, and the modal parameters to be identified can be assembled as a set $\boldsymbol{\theta} = \{\{f_i\}, \{\zeta_i\}, \boldsymbol{\Phi}, \mathbf{S}, S_e\}_{i=1}^m$ including modal frequencies, damping ratios, partial mode shape matrix, PSD matrix of modal forces, PSD of prediction error, correspondingly.

Identification uncertainty

As a major merit of Bayesian FFT, the identification uncertainty of OMA can be quantified by the posterior covariance matrix of $\boldsymbol{\theta}$. It is proved that a second-order Taylor approximation of the NLLF at the MPV $\hat{\boldsymbol{\theta}}$ engenders a Gaussian approximation of the posterior PDF as

$$p(\boldsymbol{\theta}|D, M) \approx (2\pi)^{-n_{\theta}/2} (\det \hat{\mathbf{C}}_{\theta})^{-1/2} \exp \left[-\frac{1}{2} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^* \hat{\mathbf{C}}_{\theta}^{-1} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \right] \quad (\text{A-24})$$

where $\hat{\mathbf{C}}_{\theta}$ denotes the posterior covariance matrix of $\boldsymbol{\theta}$, which is the inverse of the Hessian matrix of NLLF at the MPV $\hat{\boldsymbol{\theta}}$; and n_{θ} is the number of parameters in $\boldsymbol{\theta}$.

Algorithm summary

Overall, the estimation of modal properties by a fast Bayesian FFT algorithm turns out to be an optimization issue that minimizes the objective function NLLF by the gradient descent method. Therefore, initial value and frequency searching bands are required before running the optimization algorithm, which is proposed to use a singular value spectrum (SVS) to guide the initial specification. As a summary of the algorithm, it can be operated in the following procedure.

- (1) Specify initial probable values for modal properties and potential searching band (for FFTs the resonance band for each mode) by checking the PSD or SVS of multi-channel acceleration response $D = \{\hat{\mathbf{x}}_j \in R^n : j = 1, \dots, N\}$;
- (2) Iteratively optimize the NLLF $L(\boldsymbol{\theta})$ until convergence, to get the MPV of modal parameters $\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$;
- (3) Calculate the inverse of the Hessian matrix of NLLF $L(\boldsymbol{\theta})$ at the MPV $\hat{\boldsymbol{\theta}}$, as the posterior covariance matrix of modal parameters which offers a quantification of the identification uncertainty.

A2. Calculation of soil spring constants in FEM

This section presents how to calculate the soil spring constants of FEM of Soma bridge as written in TTT. Here, the soil-structure interaction of the pier is modeled using four springs: vertical, shear, rotational, and horizontal.

- Vertical soil spring constant (K_v)

The coefficient of vertical reaction force of the ground at the bottom of the footing $k_v (kN/m^2)$, if the ground is sandy, is expressed as

$$k_v = 6.9 \rho_{gk} E_d B_v^{-\frac{1}{2}} \quad (A-25)$$

Here, ρ_{gk} is modified coefficient of the ground regarding with the coefficient of reaction force of the ground and is equal to 1.0 when eternal reaction is not considered. E_d is designed value of deformation modulus of the ground and can be obtained from the results in geological survey. Since the ground around the bottom of the footing is sandy and the averaged N value at the bottom of the footing was 60, E_d is finally calculated 85,714 (kN/m^2). B_v is a conversion width of the bottom of the footing and calculated by $B_v = \sqrt{A_v}$. Here, A_v is the square of the footing. The size of the footing in Soma bridge is $L: 7.8 \text{ m} \times W: 4.5 \text{ m} \times H: 2.0 \text{ m}$. therefore, $B_v = 5.93 \text{ (m)}$.

Finally, the vertical spring constant of the bottom of the footing is expressed as:

$$K_v = k_v A_v = 8.528 \times 10^6 (N/mm) \quad (A-26)$$

- Shear soil spring constant (K_s)

The coefficient of shear reaction force of the ground at the bottom of the footing $k_s (kN/m^2)$, is expressed as

$$k_s = \lambda k_v \quad (A-27)$$

λ is conversion coefficient and is equal to 1/3. Therefore, the soil spring constant of the bottom of the footing is expressed as:

$$K_s = k_s A_v = 2.842 \times 10^6 (N/mm) \quad (A-28)$$

- Rotational soil spring constant (K_r)

The coefficient of rotational reaction force of the ground at the bottom of the footing $k_r (kN \cdot m^2)$, is expressed as:

$$K_r = k_r I_v \quad (A-29)$$

I_v is moment of inertia at the bottom of the footing and calculated $I_v = WL^3/12$. Finally,

$$K_r = 4.324 \times 10^7$$

- Horizontal soil spring constant (K_h)

The coefficient of horizontal reaction force of the ground at the bottom of the footing $k_h (kN \cdot m^2)$, is expressed as

$$k_h = 5.1 \rho_{gk} E_d B_h^{-\frac{3}{4}} \quad (A-30)$$

Here, B_h is a conversion width of the front of the footing and calculated by $B_h = \sqrt{A_h}$. Here, A_h is the square in front of the footing. This study does not consider the soil-pier interaction but does the soil-footing interaction. Therefore, A_h is equal to $9 m^2$.

However, the Design Standard specifies an upper limit of k_h because the soil resistance at the front and sides of the footing is dominated by the physical properties of the backfill materials and it corresponds to the $K_{30} = 70 (MN/m^3)$ which is a compaction control standard. The upper limit of the horizontal coefficient can be expressed as:

$$k_h = 18,300 \rho_{gk} B_h^{-\frac{3}{4}} \quad (A-31)$$

This study adapts the equation of the upper limit. Finally, the vertical spring constant of the bottom of the footing is expressed as:

$$K_h = k_h A_v = 7.225 \times 10^4 (N/mm) \quad (A-32)$$

