

A REMARK ON ASSOCIATED PRIMES IN THE COHOMOLOGY ALGEBRA OF A FINITE GROUP

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Introduction

Let G be a finite group and $H^*(G, k)$ be the cohomology algebra of G over an algebraically closed field k of characteristic $p > 0$. It is well known that $H^*(G, k)$ is a commutative noetherian graded k -algebra. In my talk we discussed properties of the associated primes in $H^*(G, k)$ and gave a slight generalization of a result of Green [7].

Theorem 0.1. *If $\mathfrak{p}$ is an associated prime in $H^*(G, k)$ with $\dim H^*(G, k)/\mathfrak{p} = s$, then there exists an elementary abelian p -subgroup E of rank s of G such that the depth of $H^*(C_G(E), k)$ is s and $\mathfrak{p} = \text{Res}_{G,E}^{-1}(\sqrt{0})$.*

Conversely, let E be an elementary abelian p -subgroup E of rank s of G and assume that the depth of $H^(C_G(E), k)$ is s . Then $\text{Res}_{G,E}^{-1}(\sqrt{0})$ is an associated prime in $H^*(G, k)$.*

The first assertion was known to be true using the notion of the Steenrod Algebra [4]. We shall give a different proof using arguments by Carlson [3]. And the second assertion is known to be true if $C_G(E) = G$ by Green [7].

1. The Rank-Restriction Condition

Let r be the p -rank of G . Let $\mathcal{A}_s = \mathcal{A}_s(G)$ be the set of elementary abelian p -subgroups of G ($1 \leq s \leq r$) and set $\mathcal{H}_s = \mathcal{H}_s(G) = \{ C_G(E) ; E \in \mathcal{A}_s \}$. And for $1 \leq s \leq r$, let $\mathcal{K}_s = \mathcal{K}_s(G)$ be the family of subgroups K of G such that the Sylow p -subgroups of $C_G(K)$ are not conjugate to a subgroup of any of the groups in $\mathcal{H}_s$. (If such a subgroup K does not exist, then we set $\mathcal{K}_s = \{ 1 \}$.) Notice that any elementary abelian p -subgroup of $K \in \mathcal{K}_s$ have rank at most $s - 1$. For a nonempty family $\mathcal{P}$ of subgroups of G , set $\text{Im Tr}_{\mathcal{P}, G} = \sum_{P \in \mathcal{P}} \text{Im Tr}_{P, G}$ and $\text{Ker Res}_{G, \mathcal{P}} = \cap_{P \in \mathcal{P}} \text{Ker Res}_{G, P}$. Then by a theorem of Benson (Theorem 1.1 [1]) $\sqrt{\text{Im Tr}_{\mathcal{H}_s, G}} = \sqrt{\text{Ker Res}_{G, \mathcal{K}_s}}$.

Let $\zeta_1, \dots, \zeta_r$ be a homogeneous system of parameters in $H^*(G, k)$ satisfying the rank-restriction condition. Then for $1 \leq s \leq r$, $\text{Res}_{G, E}(\zeta_s) = 0$ for any $E \in \mathcal{A}_{s-1}$, we see that $\zeta_s \in \sqrt{\text{Ker Res}_{G, \mathcal{K}_s}} = \sqrt{\text{Im Tr}_{\mathcal{H}_s, G}}$. Thus replacing ζ_s 's by a suitable p -power, we may assume that $\zeta_s \in \text{Im Tr}_{\mathcal{H}_s, G}$ for each $1 \leq s \leq r$. Thus we always have a homogeneous system of parameters $\zeta_1, \dots, \zeta_r$ in $H^*(G, k)$ satisfying the following conditions. See Lemma 8.5 [2] for the condition 3 and Lemma 3.4 and Theorem 9.6 [2] for the condition 4.

Condition 1.1.

- (1) $\zeta_1, \dots, \zeta_r$ satisfy the rank-restriction condition.

- (2) $\zeta_s \in \text{Im Tr}_{\mathcal{H}_s, G}$
- (3) For each s and $E \in \mathcal{A}_s$, the restrictions of $\zeta_1, \dots, \zeta_s$ to $H^*(E, k)$ form a homogeneous system of parameters (and $\zeta_{s+1}, \dots, \zeta_r$ restrict to 0). In particular, their restrictions to $H^*(C_G(E), k)$ form a regular sequence for $H^*(C_G(E), k)$.
- (4) If $H^*(G, k)$ has depth s , then $\zeta_1, \dots, \zeta_s$ form a regular sequence.
- (5) $\dim H^*(G, k)/(\zeta_{s+1}, \dots, \zeta_r)$ is s .

Lemma 1.2 (Carlson [3]). *Let $0 \neq \eta \in H^*(G, k)$ be a homogeneous element.*

Assume $\dim H^(G, k)/\text{Ann}_{H^*(G, k)}(\eta) = s$. Then the following statements hold.*

- (1) *If $t > 0$, then for any $H \in \mathcal{H}_t$, $\text{Res}_{G, H}(\eta) = 0$.*
- (2) *For some $H \in \mathcal{H}_s$, $\text{Res}_{G, H}(\eta) \neq 0$.*

Proof. Set $\mathfrak{a} = \text{Ann}_{H^*(G, k)}(\eta)$.

1. Suppose that $\text{Res}_{G, H}(\eta) \neq 0$ for some $H \in \mathcal{H}_t$ with $t > 0$ and set $\mathfrak{b} = \text{Ann}_{H^*(G, k)}(\text{Res}_{G, H}(\eta))$. Then as $\text{Res}_{G, H}^{-1}(\mathfrak{b}) \subset \mathfrak{a}$,

$$s = \dim H^*(G, k)/\mathfrak{a} \geq \dim H^*(G, k)/\text{Res}_{G, H}^{-1}(\mathfrak{b}) = \dim H^*(H, k)/\mathfrak{b} \geq \text{depth } H^*(H, k)$$

H has a central elementary abelian p -subgroup of rank t and $\text{depth } H^*(H, k) \geq t$ by a theorem of Duflot [5]. Thus we have a contradiction.

2. Suppose that $\text{Res}_{G, H}(\eta) = 0$ for all $H \in \mathcal{H}_s$. Then by the statement 1, $\text{Res}_{G, H}(\eta) = 0$ for all $H \in \mathcal{H}_t$ with $t \geq s$. Thus $\text{Ann}_{H^*(G, k)}(\eta)$ contains $\zeta_s, \dots, \zeta_r$ and

$$s = \dim H^*(G, k)/\text{Ann}_{H^*(G, k)}(\eta) \leq \dim H^*(G, k)/(\zeta_s, \dots, \zeta_r) \leq s - 1$$

This is a contradiction. □

2. Results of D.J. Green

In this section we shall prove some results on cohomology algebra of a finite group having a normal elementary abelian p -subgroup. A discussion here depends heavily on investigations by D.J.Green [7].

2.1. In this section let C be a finite group with a central elementary abelian p -subgroup E of order p^s ($s > 0$) and assume that a p -rank of C is larger than s . Set $\overline{C} = C/E$ and for a subgroup $E \subset H \subset C$, set $\overline{H} = H/E \subset \overline{C}$. Set $\mathcal{A} = \mathcal{A}_1(E)$ be the set of elementary abelian p -subgroups F of C containing E with $[F : E] = p$ and $\mathcal{H} = \{ C_C(F); F \in \mathcal{A} \}$. And let $\mathcal{K}$ be the family of subgroups $K \subset C$ such that the Sylow p -subgroups of $C_C(K)$ are not conjugate to a subgroup of any of the groups in $\mathcal{H}$ (If $\mathcal{H}$ has a subgroup H with p' -index in C , then we set $\mathcal{K} = \{ 1 \}$ as before). Notice that if $K \in \mathcal{K}$, then any elementary abelian p -subgroup of K is contained in E .

Let $F_1, \dots, F_k$ be a complete set of representatives of the C -conjugacy classes in $\mathcal{A}$. Then $\overline{F_i} \subset \overline{C}$ ($1 \leq i \leq k$) are not $\overline{C}$ -conjugate. let $\sigma'_i \in H^2(\overline{F_i}, GF(p)) \subset H^2(\overline{F_i}, k)$ be a nonzero fixed element and set $\sigma_i = (\sigma'_i)^{p-1}$. Then using the Evens' norm map, there can be constructed a homogeneous element $\lambda \in H^*(\overline{C}, k)$ such that $\text{Res}_{\overline{C}, \overline{F_i}}(\lambda)$ is a p -power of σ_i for each i (see a discussion in Section 7.1 [7]). Now set $\kappa = \text{Inf}_{\overline{C}, C}(\lambda) \in H^*(C, k)$. κ is a primitive element. We know that $\text{Res}_{C, K}(\kappa) = 0$ for any $K \in \mathcal{K}$ and therefore, replacing κ by some power, we may assume that $\kappa \in \text{Im Tr}_{\mathcal{H}, C}$ (see a proof of Lemma 2.6 [7]). Among the result of D.J.Green [7], the following fact will be used in this note.

Proposition 2.1. *The homogeneous element $\kappa \in H^*(C, k)$ given above satisfies the following properties.*

- (1) $\kappa \in \text{Im Tr}_{\mathcal{H}, C}$.
- (2) $\text{Res}_{C, H}(\kappa) \in H^*(H, k)$ is a regular element for every $H \in \mathcal{H}$.
- (3) $\text{Ann}_{H^*(C, k)}(\kappa) = \text{Ker Res}_{C, \mathcal{H}}$
- (4) Suppose that $\xi_1, \dots, \xi_t$ is a sequence of homogeneous elements of $H^*(C, k)$ whose restrictions form a regular sequence in $H^*(E, k)$. Then
 - (a) $\xi_1, \dots, \xi_t$ is a regular sequence for $H^*(C, k)/H^*(C, k) \cdot \kappa$. In particular, as a $k[\xi_1, \dots, \xi_t]$ -module, $H^*(C, k)/H^*(C, k) \cdot \kappa$ is free.
 - (b) $\text{Ker Res}_{C, \mathcal{H}}$ is a free $k[\xi_1, \dots, \xi_t]$ -module
- (5) If the depth of $H^*(C, k)$ is s , then κ is a zero divisor in $H^*(C, k)$. In particular, $\text{Ker Res}_{C, \mathcal{H}} \neq 0$.

Proof. All the statements in the proposition except the assertion (b) in the statement 4 are included in [7]. We shall prove the statement 4, (b). Set $R = k[\xi_1, \dots, \xi_t]$. Then $H^*(C, k)$ is a free R -module by a result of Duflot [5]. We have the following exact sequence of R -modules

$$0 \rightarrow \text{Ann}_{H^*(C, k)}(\kappa) \rightarrow H^*(C, k) \xrightarrow{\cdot \kappa} H^*(C, k) \rightarrow H^*(C, k)/H^*(C, k) \cdot \kappa \rightarrow 0$$

and $\text{Ann}_{H^*(C, k)}(\kappa) = \text{Ker Res}_{C, \mathcal{H}}$. Thus by the assertion (a), 4, it follows that $\text{Ker Res}_{C, \mathcal{H}}$ is a projective R -module. So the result follows because a projective R -module is a free R -module (see, for example, Theorem 2.5 [8]). $\square$

2.2. In this section let N be a finite group with normal elementary abelian subgroup E of rank s and set $C = C_N(E)$. We use notations in the previous section for C . We shall prove the following proposition.

Proposition 2.2. *Assume that the depth of $H^*(C, k)$ is s . Then*

- (1) As a kN/C -module $\text{Ker Res}_{C, \mathcal{H}}$ contains a regular module kN/C .
- (2) $\text{Ker Res}_{C, \mathcal{H}}$ contains a nonzero N -invariant element.

Set $E = \langle a_1, \dots, a_s \rangle$, $\alpha_i \in H^1(E, k) = \text{Hom}(E, k)$ be the element dual to a_i and $\beta_i = \beta(\alpha_i)$, where β is the Bockstein map. We have a polynomial subalgebra $k[\beta_1, \dots, \beta_s]$ in $H^*(E, k)$. Using Evens' norm map, we obtain homogeneous elements $\xi_1, \dots, \xi_s \in H^*(C, k)$ such that $\text{Res}_{C, E}(\xi_i) = \beta_i^{p^a}$ for some p -power p^a . The subalgebra $k[\beta_1, \dots, \beta_s] \subset H^*(E, k)$ is N -stable, but the subalgebra $k[\xi_1, \dots, \xi_s] \subset H^*(C, k)$ may not be N -stable. For $g \in N$, let $\beta_i^g = \sum_{j=1}^s \lambda_{ij} \beta_j$, $\lambda_{ij} \in GF(p)$ and consider the element $\xi = \xi_i^g - (\sum_{j=1}^s \lambda_{ij} \xi_j)$. Then $\text{Res}_{C, E}(\xi) = 0$ and therefore $\text{Res}_{C, K}(\xi)$ is nilpotent for any $K \in \mathcal{K}$ because elementary abelian p -subgroups of such K are contained in E . Hence by a theorem of Benson (Theorem 1.1 [1]) some p -power of ξ is contained in $\text{Im Tr}_{\mathcal{H}, C}$. Replacing the ξ_i 's by suitable p -powers, we have proved the following.

Lemma 2.3. *There exist homogeneous elements ξ_i ($1 \leq i \leq s$) and a p -power p^b satisfying the following.*

- (1) $\text{Res}_{C, E}(\xi) = \beta_i^{p^b}$ for each i ($1 \leq i \leq s$).

- (2) $k[\xi_1, \dots, \xi_s] + \text{Im Tr}_{\mathcal{H}, C}$ is N -stable and $k[\xi_1, \dots, \xi_s] + \text{Im Tr}_{\mathcal{H}, C} / \text{Im Tr}_{\mathcal{H}, C} \cong k[\beta_1^{p^b}, \dots, \beta_s^{p^b}]$ as N -modules.

Proof of Proposition 2.2

Now we shall prove the proposition. First we prove the statement 1. Set $R_0 = k[\beta_1^{p^b}, \dots, \beta_s^{p^b}]$ and $R = k[\xi_1, \dots, \xi_s]$. Then it is well known that the sequence $\xi_1, \dots, \xi_s$ is a regular sequence for $H^*(C, k)$ and $H^*(C, k)$ is a free R -module (see [5]).

By Proposition 2.1, $\text{Ker Res}_{C, \mathcal{H}} \neq 0$. Let n be the least integer such that $V = \text{Ker Res}_{C, \mathcal{H}} \cap H^n(C, k) \neq 0$. Then V is N -stable and any k -basis of V can be extended to a set of R -free generators of $\text{Ker Res}_{C, \mathcal{H}}$. Then by Lemma 1.2, Proposition 2.1 and the fact that $\text{Im Tr}_{\mathcal{H}, C}$ annihilates $\text{Ker Res}_{C, \mathcal{H}}$, it follows that $V \cdot R$ is N -stable and $V \cdot R \cong V \otimes_k R_0$ as N -modules. It is known that R_0 contains kN/C as a kN/C -module (see [9]). Therefore the statement (1) follows.

The statement 2 is an easy consequence of the statement 1.

3. Proof of Theorem 0.1

First we shall show that the first statement of the theorem hold. Let $\mathfrak{p}$ be an associated prime in $H^*(G, k)$ with $\dim H^*(G, k)/\mathfrak{p} = s$ and take a homogeneous element $0 \neq \eta \in H^*(G, k)$ such that $\text{Ann}_{H^*(G, k)}(\eta) = \mathfrak{a}$. Then by Lemma 1.2, there exists $E \in \mathcal{A}_s$ such that $\text{Res}_{G, C_G(E)}(\eta) \neq 0$. Set $C = C_G(E)$, $\eta_0 = \text{Res}_{G, C}(\eta)$ and we shall use the notations in Section 2. Again by Lemma 1.2, $\eta_0 \in \text{Ker Res}_{C, \mathcal{H}}$.

Let $\alpha \in H^*(C, k)$ such that $\text{Res}_{C, E}(\alpha) = 0$. Then $\alpha \in \sqrt{\text{Ker Res}_{C, \mathcal{K}}}$ by an argument in Section 1 and therefore $\alpha \in \sqrt{\text{Im Tr}_{\mathcal{H}, C}}$. Thus $\alpha \in \sqrt{\text{Ann}_{H^*(C, k)}(\eta_0)}$.

By a result of Benson in [1] there exists a homogeneous element $\tau \in H^*(C, k)$ such that $\text{Res}_{N, C} \text{Tr}_{C, N}(\tau)$ is a regular element in $H^*(C, k)$ and restricts to zero on every subgroup A of C with $A \not\supseteq E$, where $N = N_G(E)$. Set $\sigma = \text{Tr}_{C, G}(\tau)$ and consider the element $\alpha = \eta\sigma$. We shall show the following equality hold.

$$\text{Res}_{G, C}(\alpha) = \eta_0 \text{Res}_{N, C} \text{Tr}_{C, N}(\tau)$$

As $\alpha = \eta \text{Tr}_{C, G}(\tau) = \text{Tr}_{C, G}(\text{Res}_{G, C}(\eta)\tau) = \text{Tr}_{C, G}(\eta_0\tau)$, the Mackey double coset formula shows that

$$\text{Res}_{G, C}(\alpha) = \sum_{g \in C \backslash G / C} \text{Tr}_{C^g \cap C, C} \text{Res}_{C^g, C^g \cap C}((\eta_0\tau)^g) = \sum_{g \in C \backslash G / C} \text{Tr}_{C^g \cap C, C} \text{Res}_{C^g, C^g \cap C}(\eta_0\tau^g)$$

Suppose that $C^{g^{-1}} \not\supseteq E$. Then as $\text{Res}_{C, C \cap C^{g^{-1}}}(\tau) = 0$, it follows that $\text{Res}_{C, C \cap C^{g^{-1}}}(\eta_0\tau) = 0$ and therefore $\text{Res}_{C^g, C^g \cap C}(\eta\tau^g) = 0$. If $E \subset C^{g^{-1}}$ and $E \neq E^{g^{-1}}$, then $F = EE^{g^{-1}}$ is elementary abelian, $F \supsetneq E$ and $C \cap C^{g^{-1}} = C_G(F)$. So $\text{Res}_{C, C \cap C^{g^{-1}}}(\eta_0) = 0$ and therefore $\text{Res}_{C^g, C^g \cap C}(\eta\tau^g) = 0$. Thus we have the desired equality. $\alpha \neq 0$ and therefore $\mathfrak{p} = \text{Ann}_{H^*(G, k)}(\eta) = \text{Ann}_{H^*(G, k)}(\alpha)$. In these notations, we shall show that for $\rho \in H^*(G, k)$, $\rho\alpha = 0$ if and only if $\text{Res}_{G, C}(\rho)\eta_0 = 0$.

Assume that $\text{Res}_{G, C}(\rho)\eta_0 = 0$, then $\text{Res}_{G, C}(\rho)\eta_0\tau = 0$ and $\rho\alpha = \text{Tr}_{C, G}(\text{Res}_{G, C}(\rho)\eta_0\tau) = 0$. Thus we have $\mathfrak{p} = \text{Res}_{G, C}^{-1}(\text{Ann}_{H^*(C, k)}(\eta_0))$.

As $\text{Res}_{C,E}^{-1}(\sqrt{0}) \subset \sqrt{\text{Ann}_{H^*(C,k)}(\eta_0)}$, we have

$$\text{Res}_{G,E}^{-1}(\sqrt{0}) \subset \text{Res}_{G,C}^{-1}(\text{Ann}_{H^*(C,k)}(\eta_0)) = \mathfrak{p}$$

As $\text{Res}_{G,E}^{-1}(\sqrt{0})$ is a prime ideal and $\dim H^*(G, k)/\text{Res}_{G,E}^{-1}(\sqrt{0}) = s$, we can conclude that $\text{Res}_{G,E}^{-1}(\sqrt{0}) = \mathfrak{p}$ and the first statement in the theorem follows.

We next shall prove the second statement in the theorem. Let $E \in \mathcal{A}_s$ and assume that the depth of $H^*(C_G(E), k)$ is s . Set $C = C_G(E)$, $N = N_G(E)$ and we shall use the notations Section 2. As before let τ be a homogeneous element in $H^*(C, k)$ such that $\text{Res}_{N,C} \text{Tr}_{C,N}(\tau)$ is a regular element in $H^*(C, k)$ and restricts to zero on every subgroup A of C with $A \not\supset E$,

By Proposition 2.2, there exists a non zero homogeneous element $\eta_0 \in \text{Ker Res}_{C,H}$ which is N -invariant. Notice that $\text{Res}_{C,E}^{-1}(\sqrt{0}) \subset \sqrt{\text{Ann}_{H^*(C,k)}(\eta_0)}$. Consider the element $\gamma = \eta_0 \tau$. Then an entirely same argument as above, we have

$$\text{Res}_{G,C} \text{Tr}_{C,G}(\gamma) = \text{Res}_{N,C} \text{Tr}_{C,N}(\gamma) = \eta_0 \text{Res}_{N,C} \text{Tr}_{C,N}(\tau)$$

Set $\alpha = \text{Tr}_{C,G}(\gamma) \in H^*(G, k)$. We shall show that for $\rho \in H^*(G, k)$, $\rho\alpha = 0$ if and only if $\text{Res}_{G,C}(\rho)\eta_0 = 0$.

Assume that $\rho\alpha = 0$. Then $\text{Tr}_{C,G}(\text{Res}_{G,C}(\rho)\gamma) = 0$. Hence $\text{Res}_{G,C} \text{Tr}_{C,G}(\text{Res}_{G,C}(\rho)\gamma) = 0$. By the similar argument in the above we have that

$$\text{Res}_{G,C} \text{Tr}_{C,G}(\text{Res}_{G,C}(\rho)\gamma) = \text{Res}_{N,C} \text{Tr}_{C,N}(\text{Res}_{G,C}(\rho)\gamma) = (\text{Res}_{G,C}(\rho)\eta_0) \text{Res}_{N,C} \text{Tr}_{C,N}(\tau)$$

As $\text{Res}_{N,C} \text{Tr}_{C,N}(\tau)$ is a regular element in $H^*(C, k)$, it follows that $\text{Res}_{G,C}(\rho)\eta_0 = 0$.

Conversely, if $\text{Res}_{G,C}(\rho)\eta_0 = 0$, then $\text{Res}_{G,C}(\rho)\gamma = 0$ and $\rho\alpha = \text{Tr}_{C,G}(\text{Res}_{G,C}(\rho)\gamma) = 0$.

A standard argument in commutative noetherian rings says that there exists $\delta \in H^*(G, k)$ such that $\eta = \alpha\delta \neq 0$ and $\mathfrak{p} = \text{Ann}_{H^*(G,k)}(\eta)$ is a prime ideal. Then $\eta_1 = \text{Res}_{G,C}(\delta)\eta_0 \neq 0$ and by an entirely same argument as before, we have

$$\text{Ann}_{H^*(G,k)}(\eta) = \text{Res}_{G,C}^{-1}(\text{Ann}_{H^*(C,k)}(\eta_1))$$

As $\text{Res}_{C,E}^{-1}(\sqrt{0}) \subset \sqrt{\text{Ann}_{H^*(C,k)}(\eta_0)} \subset \sqrt{\text{Ann}_{H^*(C,k)}(\eta_1)}$, $\text{Res}_{G,E}^{-1}(\sqrt{0}) \subset \sqrt{\text{Ann}_{H^*(G,k)}(\eta)} = \mathfrak{p}$ and $\dim H^*(G, k)/\mathfrak{p} = \dim H^*(H, k)/\text{Ann}_{H^*(C,k)}(\eta_1) = s$. Thus $\text{Res}_{G,E}^{-1}(\sqrt{0}) = \mathfrak{p}$ and the second statement in the theorem follows.

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