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The b-function of a prehomogeneous vector space $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1)$.

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概均質ベクトル空間 $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1)$ の b-関数は，尾関育三によるデータをもとにして，矢野環・関口次郎により，軌道の局所的構造が決定され，さらに b 関数の局所的双対性を用いて最終的に global な b 関数が決定された．

以下には，以前に準備したこの最終段階の部分の原稿をおさめた．今では b 関数の双対性はさらに一般化されており，より見通しのよい議論も可能であるが，それはいずれ整理して発表する．又，以下の原稿は b 関数の双対性を前提としている．その部分も別に発表する予定である．

末尾に holonomy diagram をつけた．これも1979年の尾関育三の論文（Proc. of Japan Acad. 37-40, +1 1979）のものを用いた．交わりは書き込まれ，いくつか補充されている。（この図は私の手書きである）

Microlocal Structure of the Regular Prehomogeneous Vector Space Associated with SL(5) x GL(4). II.

By Tamaki Yano and Ikuzo Ozeki

The microlocal structure of the triplet $(SL(5) \times GL(4), \Lambda_2 \otimes \Lambda_1, V(10) \otimes V(4))$ is investigated in [O 1] and [O 2]. The triplet is a reduced irreducible prehomogeneous vector space (abbreviated as PV in the sequel) , having 63 orbits. The b-function $b(s)$ of this PV is of degree 40. The second author made a conjecture that

$$\begin{aligned} b(s) = & (s+1)^8\{(s+\tfrac{5}{6})(s+\tfrac{7}{6})(s+\tfrac{3}{4})(s+\tfrac{5}{4})\}^4\{(s+\tfrac{7}{10})(s+\tfrac{9}{10})(s+\tfrac{11}{10})(s+\tfrac{13}{10})\}^2 \\ & \{(s+\tfrac{2}{3})(s+\tfrac{5}{3})\}^4 \\ = & \ \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^4 \ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5})^2 \ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6})^4 \end{aligned}$$

In this article, we will show that the b-functions of the orbits $S_{6,8}$, $S_{7,7}$ and $S_{12,10}$ are given by

$$\begin{aligned} b_{6,8}(s) &= (s+1) \ \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2 \ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5}) \ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6}) , \\ b_{7,7}(s) &= \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2 \ \Pi_{k=1}^{4}(s+\tfrac{1}{2}+\tfrac{k}{5}) \ \Pi_{k=1}^{5}(s+\tfrac{1}{2}+\tfrac{k}{6})^2 , \\ b_{12,10}(s) &= (s+1)^2 \ \Pi_{k=1}^{3}(s+\tfrac{1}{2}+\tfrac{k}{4})^2 \ \Pi_{k=1}^{2}(s+\tfrac{1}{2}+\tfrac{k}{3})^2 , \end{aligned}$$

and prove the Ozeki's conjecture.

Both $b_{6,8}(s)$, $b_{7,7}(s)$ and $b_{12,10}(s)$ coinside with micro-local b functions on corresponding holonomic varieties of those orbits.

In § 1, we prepare machinary from the microlocal calculus (see [SKKO]) and the locally prehomogeneous spaces (see [Y 3]).

Applying the latter, we get a duality

$$b_\Lambda(-s-2) = (-1)^{\deg b_\Lambda} b_\Lambda(s),$$

for each good holonomic variety Λ of our PV.

In § 2, we calculate $b_{6,8}(s)$ using the machinary in § 1.

In § 3, we calculate $b_{7,7}(s)$ and explain the backgroud of the conjecture.

In § 4, we collect the data of transverse localization, which will be needed in §§ 2-3.

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§ 1. Preliminaries

Let (G,ρ,V) be an irreducible regular PV, and let f be its fundamental relative invariants. Let S be an orbit of (G,ρ,V). We denote by Λ_S (or $T^*_S V$) the closure of the conormal bundle of S, and call it the holonomic variety.

We refer the reader [SKKO] for detailed discussion of micro-local calculus. We quote the following Theorem.

Theorem 1.1 ([Theorem 7.5 in SKKO]). Let Λ_0 and Λ_1 be good holonomic varieties whose intersection is of codimension one with the intersection exponent $(m:n)$. Assume that $\mathcal{M} = \mathcal{E}f^{\alpha}$ is a simple holonomic system with support $\Lambda_0 \cup \Lambda_1$. Assume that $m_0 > m_1$ where $\operatorname{ord}_{\Lambda_i} f^{\alpha} = -m_i\alpha - \mu_i/2$. Then we have

$$b_{\Lambda_0}(s)/b_{\Lambda_1}(s) = \Pi_{k=0}^{n} \left[\frac{1}{n+1}(\operatorname{ord}_{\Lambda_1} f^{\alpha} - \operatorname{ord}_{\Lambda_0} f^{\alpha})\Big|_{\alpha=s} + \frac{m+2k}{2(m+n)} \right]^{\frac{m_0-m_1}{n+1}}$$

where $[a]^b = a(a+1)\ldots(a+b-1)$.

Let $(\mathcal{G},U)$ be a locally prehomogeneous space that is weighted homogeneous and logarithmically free (abbrev. WH LGF LPH). We refer the reader [Y 3] for the micro-local calculus of LPH.

Set $\mathcal{N}(s) = \mathcal{D}[s] / \mathcal{J}(s)$, $\mathcal{J}(s) = \{P(s) \in \mathcal{D}[s] \ ; \ Pf^s = 0\}$,

$\mathcal{N}_{\alpha} = \mathcal{D} / \mathcal{J}(\alpha)$, $\mathcal{J}(\alpha) = \{Q \in \mathcal{D} \ ; \ Q = P(\alpha),\ P(s) \in \mathcal{J}(s)\}$,

$\mathcal{M}(s) = \mathcal{N}(s) / \mathcal{N}(s+1)$.

We define

$b_{(S)}$ = the minimal polynomial of s in $\mathcal{H}om_{\mathcal{D}}(\mathcal{M}(s),\mathcal{B}_{S|U})_p$ $p\in S$,

where $\mathcal{B}_{S|U}$ is the sheaf of delta functions supported on S.

Let S be an orbit at $0 \in U$. Set

$\mathscr{E}(S) = \{ S' \ ; \ S'$ is a local orbit with $S \subset \overline{S'} \ \}$,

$$b_{<\Lambda_S>} = \underset{\substack{S' \in \mathscr{E}(S) \\ S':\text{good} \ , \ S' \neq S}}{\text{l.c.m.}} (b_{\Lambda_{S'}}) \ ,$$

$$b_{<S>} = \underset{\substack{S' \in \mathscr{E}(S) \\ S' \neq S}}{\text{l.c.m.}} (b_{S'}) \ ,$$

$$b^{g}_{(S)} = \prod_{k=1}^{\text{codim } S} \ \underset{\substack{S' \in \mathscr{E}(S), \text{codim } S'=k \\ S':\text{good} \ , \ S' \neq S}}{\text{l.c.m.}} (b_{(S')}) \ ,$$

$$b^{ng}_{(S)} = \prod_{k=1}^{\text{codim } S} \ \underset{\substack{S' \in \mathscr{E}(S), \text{codim } S'=k \\ S':\text{not good} \ , \ S' \neq S}}{\text{l.c.m.}} (b_{(S')}) \ ,$$

The following theorem is proved in [Y 3].

Theorem 1.2. Let $(\mathscr{G}, U)$ be a WH LGF LPH with finite orbits. Then, the following properties hold.

(1) Let S be a good orbit. Then $b_{\Lambda_S} \mid b_S$.

(2) For all good holonomic variety Λ , we have

$$(1.1) \quad b_{\Lambda}(-s-2) = (-1)^{\deg b_{\Lambda}} b_{\Lambda}(s).$$

(3) Let S be an orbit. Then,

(1.2) $b_S \mid \text{l.c.m.}\ (b_{<\Lambda_S>} , b_{(S)} b^{ng}_{(S)})$.

If S is good, we have also

(1.3) $b_S \mid \text{l.c.m.}\ (b_{\Lambda_S} , b_{<\Lambda_S>} , b^{ng}_{(S)})$.

If $\mathscr{C}(S)$ consists of good orbits, we have

(1.4) $b_S = \text{l.c.m.}\ (b_{\Lambda_S}, b_{<\Lambda_S>})$.

(4) $b_{<S>} \mid b_S$, $b_{(S)} \mid b_S$.

Theorem 1.3. [Prop. Y3] Let S be a good orbit of LGF LPH. Then, if $(s + 1) \mid b_{(S)}$, we have

m_S - codim S $\in 2\mathbb{Z}$.

Theorem 1.4. Let S be an orbit of PV $(SL(5)xGL(4), \Lambda_2 \otimes \Lambda_1, V)$.

(1) [Thm 3.4 in O 2] If Λ_S is G-prehomogeneous, then S is good.

(2)[O 1, Prop in Y 3] $\text{ord}_{\Lambda_S} f^{\alpha} = - m_S (\alpha + \frac{1}{2})$, $m_S \geq 0$.

Our main concern is the orbits appearing in the following Figure 1. Each orbit $S_{i\ j}^{\ k}$ (if k=0, k is omitted) corresponds to a vertex encircling $i\ {}^{k}_{j}$. See [O 1] for details. Orbits $S_{5,21}$ and $S_{6,14,2}$ are not good. All the others are good orbits.

§ 2. B-functions .

In this section, we will determine micro-local b-functions and b-functions of orbits appearing in the following holonomy diagram (Figure 1, extracted from [O 1]) except $S^1_{5,21}$ and $S^2_{6,14}$ which are not good.

Theorem 2.1. Let S be an good orbit appearing in Figure 1. The micro-local b-function is given by the following Table 1. Here, if $b_\Lambda = \Pi_{i=1}^{m}(s+\alpha_i)$, we exhibit $\{\alpha_i\}_i$. The underlined factor $\underline{\alpha}$ denotes the fact that $b_{(S)} = \prod_{\alpha:\text{underlined}} (s+\alpha)$.

m	orbit	α_i
0	0 40	ϕ
1	1 30	$\underline{1}$
2	2 24	$\underline{1}$ 1
3	2 21	1 $\underline{5/6}$ $\underline{7/6}$
6	3 15	1 1 5/6 7/6 $\underline{3/4}$ $\underline{5/4}$
4	3 18	1 1 5/6 7/6
4	4 20	$\underline{1}$ 1 5/6 7/6
5	5 16	$\underline{1}$ 1 1 5/6 7/6
8	4 14	$\underline{1}$ 1 5/6 7/6 $\underline{3/4}$ $\underline{5/4}$ 3/4 5/4
9	5 12	$\underline{1}$ 1 1 5/6 7/6 3/4 5/4 3/4 5/4
10	4 11	1 1 5/6 7/6 3/4 5/4 $\underline{7/10}$ $\underline{9/10}$ $\underline{11/10}$ $\underline{13/10}$
10	6 14	1 1 5/6 7/6 5/6 7/6 3/4 5/4 3/4 5/4
15	5 9	$\underline{1}$ 1 1 5/6 7/6 3/4 5/4 3/4 5/4 7/10 9/10 11/10 13/10 $\underline{2/3}$ $\underline{4/3}$
16	6 8	$\underline{1}$ 1 1 1 5/6 7/6 3/4 5/4 3/4 5/4 7/10 9/10 11/10 13/10

							2/3	4/3						
11	8	11		1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
12	12	10	$\underline{1}$	1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
20	7	7	1	1	1	1	5/6	7/6	5/6	7/6	3/4	5/4	3/4	5/4
							7/10	9/10	11/10	13/10	2/3	4/3	2/3	4/3

Table 1.

The proof is given simultaneously in the proof of the following:

Theorem 2.2. Let S be a good orbit appearing in Fig.1.

(1) If $S \neq S_{6,14}$, we have

$b_S = b_{\Lambda_S}$, where b_{Λ_S} is given by Table 1.

(2) $b_{6,14} = (s+1)\, b_{\Lambda_{6,14}}$.

We prove theorems for $S \neq S_{7,7}$ in this section. The proof for $S_{7,7}$ will be given in § 3.

Proof. We proceed in 6 steps.

Step 1. $S = S_{0,40}, S_{1,30}, S_{2,24}, S_{2,21}, S_{3,18}, S_{4,20}, S_{5,16}$.

b_Λ is known by Thm 1.1.

Owing to Thm 1.2 (1.4), we have $b_\Lambda = b_S$.

Step 2. $S = S_{3,15}, S_{4,11}$.

The transverse localization along S is the discriminant $d_4(1,0,x_2,x_3,x_4)$ (resp. $d_5(1,0,x_2,x_3,x_4,x_5)$). We determine $b_{(S)}$ by the method explained in [YS].

$b_{(3,15)} = (s+\frac{3}{4})(s+\frac{5}{4})$

$b_{(4,11)} = \Pi_{k=0}^{3}(s+\frac{7+2k}{10})$.

By Thm 1.2 (1.4) and (4), we get b_S. Since b_S satisfies deg b_S =

m_S, we have $b_S = b_\Lambda$.

Step 3. $S = S_{4,14}, S_{5,12}$.

First, set $S = S_{4,14}$. Using the transverse localization in § 4, we know that

$b_{(4,14)} = (s+1)(s+\frac{3}{4})(s+\frac{5}{4})$.

Now set $b' = (s+1)^2(s+\frac{5}{6})(s+\frac{7}{6})(s+\frac{3}{4})^2(s+\frac{5}{4})^2$.

Owing to Thm 1.2 (1) and (1.2), we get

$$b_\Lambda \mid b_{4,14} \mid (s+1)b' .$$

Therefore, by Thm 1.2 (2) and the fact that deg b_Λ=8, we get $b_\Lambda = b'$. By Thm 1.2 (1.4), we have $b_{4,14} = b_\Lambda$.

Next, set $S = S_{5,12}$. Since $b_{\Lambda_{5,12}} / b_{\Lambda_{4,14}} = (s+1)$, we get $b_{\Lambda_{5,12}}$. By Thm 1.2 (1.4), $b_{5,12} = b_{\Lambda_{5,12}}$.

Step 4. $S = S_{5,9}$.

We can show directly that the possible α such that $(s+\alpha) | b_{(S)}$ and $\alpha \le 1$ is, counting multiplicity, $\alpha = \frac{2}{3}$, 1.

Set $b' = (s+1)^2(s+5/6)(s+7/6)(s+3/4)^2(s+5/4)^2(s+7/10)(s+9/10)$
$(s+11/10)(s+13/10)(s+2/3)(s+4/3)$.

Then, using Thm 1.2 (1.2) and (4), we see that $b' \mid b_S \mid (s+1)b'$. Since deg $b' = 14$, deg $b_\Lambda = 15$ and $b_\Lambda \mid b_S$, we have $b_\Lambda = b_S = (s+1)b'$.

Thus we get b_Λ and by Thm 1.2 (1.4) $b_S = b_\Lambda$.

Step 5. $S = S_{6,8}$.

By Thm 1.1, $b_{\Lambda_{6,8}} = (s+1)b_{\Lambda_{5,9}}$. By Thm 1.2 (1.4), we get $b_S = b_\Lambda$.

Step 6. $S = S_{6,14}, S_{8,11}, S_{12,10}$.

Since $b_{\Lambda_{8,11}}/b_{\Lambda_{5,12}} = (s+\frac{5}{6})(s+\frac{7}{6})$, $b_{\Lambda_{8,11}}/b_{\Lambda_{6,14}} = (s+1)$, and $b_{\Lambda_{12,10}} / b_{\Lambda_{8,11}} = (s+1)$, we get $b_{\Lambda_{8,11}}$, $b_{\Lambda_{6,14}}$ and $b_{\Lambda_{12,10}}$.

Since the transverse localization along $S_{12,10}$ is isomorphic to a regular irreducible PV $(SL(3)xGL(2), 2\Lambda_1 \otimes \Lambda_1, V)$ (cf.[O 2]), we have $b_{12,10} = b_{\Lambda_{12,10}}$.

We do not know whether $\Lambda_{5,21} \subset W$ or not. But, we can calculate that $b_{(5,21)} | (s+5/6)(s+7/6)$.

Applying Thm 1.2 (1.3) and (4) on $S_{6,14}$, we get $b_{S_{6,14}} = (s+1)b_{\Lambda_{6,14}}$. Especially, we also proved that $\Lambda_{5,21} \subset W$ and $b_{(5,21)} = (s+5/6)(s+7/6)$.

Then, by Thm 1.2 (1) and (1.3) proves $b_{8,11} = b_{\Lambda_{8,11}}$.

Hence the result.

Q.E.D.

Remark 2.3. From the orbits $S_{1,30}$, $S_{2,21}$, $S_{3,15}$, $S_{4,11}$, $S_{5,9}$, we get factors $s + \frac{1}{2} + \frac{1}{k}$, $k = 2,3,4,5,6$ respectively. The first four orbits give discriminants $d_k(1,0,x_2,...,x_k)$ whereas the last is not.

Remark 2.4. The b-function of $(SL(3)xGL(2), 2\Lambda_1 \otimes \Lambda_1, V)$ is anounced in [KM]. However, they used the 3-Lagrangeans formula, whose proof has not yet published. The above proof does not appeal to the 3-Lagrangeans formula, and hence gives another proof of the b-function of that PV.

§ 3. B-functions of $S_{7,7}$

The orbit $S_{7,7}$ intersects with $S_{6,8}$, $S_{5,9}$, $S_{4,11}$ and $S_{5,12}$ at one place on $\Lambda_{7,7}$ and with $S_{6,14,2}$ at another place.

The former intersection is so complicated and we cannot apply standard method of micro-local calculus. As for the latter, $S_{6,14,2}$ is not a simple holonomic variety.

Assume that $\Lambda_{6,14,2} \subset W$. Using the structure of the isotropy subalgebra at a point of $S_{6,14,2}$, we can see that

(3.1) $b_{(6,14,2)} \mid b'(s)$
where $b'(s)=(s+1)(s+2/3)(s+4/3)(s+5/6)(s+7/6)$.

We can also see, using the data in 4.5 and applying Thm 1.3,

(3.2) $b_{(7,7)} = 1$.

Consider $\mathcal{M}' = b_{5,9}(s)\mathcal{M}(s)$. Then, using the result in §2, we see that codim $\mathrm{Supp}(\mathcal{M}') \geq 6$ near a generic point of $S_{7,7}$. Since $b_{(6,8)}=(s+1)$, codim $\mathrm{Supp}(\, b'(s)\mathcal{M}') \geq 7$. Then, by (3.2), $b_{7,7} \mid b'b_{5,9}$. Since $b_{\Lambda_{7,7}} \mid b_{7,7}$ and deg $b_{\Lambda_{7,7}}=20$, we get $b_{\Lambda_{7,7}}=b_{7,7}=b'b_{5,9}$. Note that we also proved $\Lambda_{6,14,2} \subset W$.

Hence, we proved Thm 2.1 and Thm 2.2 for $S_{7,7}$.

Corollary 3.1. (1) $\Lambda_{5,21}$, $\Lambda_{6,14,2} \subset W$.

(2) Suppose $b_{5,21}(-s-2)= \pm b_{5,21}(s)$ and $b_{6,14,2}(-s-2)=\pm b_{6,14,2}(s)$.
Then,

$$b_{5,21} = (s+1)^2(s+3/4)(s+5/4)\{(s+5/6)(s+7/6)\}^2 ,$$

$$b_{6,14,2} =(s+1)^{3+a}(s+2/3)(s+4/3)\{(s+3/4)(s+5/4)(s+5/6)(s+7/6)\}^2,$$

where $a=0$ or 1.

Proof. (1) has already proved. Using Thm 1.2 (1.3),

$$b_{5,21} = (s+1)^2(s+3/4)(s+5/4)\{(s+5/6)(s+7/6)\}^{1+b} ,$$

where $b = 0$ or 1. Suppose $b=0$. Then, if we set

$$b' = (s+1)^3\{(s+3/4)(s+5/4)\}^2(s+5/6)(s+7/6) ,$$

we see that codim $\mathrm{Supp}(b'\mathcal{M}(s)) \geq 6$ at a generic point of $S_{6,14}$. Since $b_{(6,14)}=1$, we have $b_{6,14}|b'$, which is a contradiction because $\deg b'=9 < 11=\deg b_{6,14}$. The proof for $b_{6,14,2}$ is similar.

Q.E.D.

Quite generally, when (G,ρ,V) is a reduced irreducible regular PV other than ours, for any good Lagrangean Λ and its dual $\Lambda^{\vee}$, we have

$$(3.3)\qquad b_{\Lambda^{\vee}}\left(-s-\frac{\dim V}{\deg f}-1\right) = \pm\ b(s)/b_{\Lambda}(s)\ .$$

This global duality is proved in [Y 3] using Kashiwara's lemma. Then, combining with Thm 1.2 (2), we have

$$(3.4)\qquad b(s) = \{b_{\Lambda_{7,7}}(s)\}^2\ .$$

Therefore, the following Ozeki's conjecture is proved.

$$(3.5)\qquad b(s) = (s+1)^8\{(s+3/4)(s+5/4)\}^4\ \{(s+5/6)(s+7/6)\}^4$$
$$\{\Pi_{k=0}^{3}(s+\frac{7+2k}{10})\}^2\{(s+2/3)(s+4/3)\}^4\ .$$

There is another approach to prove (3.5). We quote the recent results of Professors K.Kawanaka and A.Gyoja.

Proposition 3.2. (S.Kawanaka)

(1) $(s+1)^8$ divides $b(s)$ but $(s+1)^9$ does not.

(2) $(s+\frac{1}{2})$ or $(s+\frac{3}{2})$ dose not divide $b(s)$.

Proposition 3.3. (A.Gyoja)

Set $b^{\exp}(t) = \Pi_{i=1}^{40}(t-\exp(2\pi\sqrt{-1}\ \alpha_i))$, for $b(s)=\Pi_{i=1}^{40}(s+\alpha_i)$.

(1) $b^{\exp}(t)$ is a product of cyclotomic polynomials.

(2) If we denote by f_n the n-th cyclotomic polynomial,

$$b^{\exp}(t) \mid f_1^8 f_2^8 f_3^4 f_4^4 f_5^2 f_6^4 f_7 f_8^2 f_9 f_{10}^2 f_{12}^2 f_{14} f_{15} f_{18} f_{20} f_{24} f_{30}.$$

Suppose that we get an estimate of the following form.

$$(3.6)\quad b(s) \mid (s+1)^{a_1}\{(s+3/4)(s+5/4)\}^{a_4}\{(s+5/6)(s+7/6)\}^{a_6}$$
$$\{\Pi_{k=0}^{3}(s+\tfrac{7+2k}{10})\}^{a_{10}}\{(s+2/3)(s+4/3)\}^{a_3},$$

where $a_1,\ldots,a_{10}$ are non-negative integers.

Then, Prop.3.3 (1)(2) and (3.6) prove (3.5) , because (3.6) leads

$$(3.7)\quad b^{\exp}(t) \mid f_1^{a_1} f_3^{a_3} f_4^{a_4} f_6^{a_6} f_{10}^{a_{10}},$$

and counting degree's , we must have $a_1=8$, $a_3=4$, $a_4=4$, $a_6=4$, $a_{10}=2$.

Using a result in [Y 3] and Prop.3.3 (1) we can show that the factor of $b^{\exp}(t)$ derived from good orbits is at most

$$f_1^{13} f_3^2 f_4^2 f_6^2 f_{10}.$$

Therefore, we must study non good orbits.

The related topics will be discussed in the forthcoming article.

Note: We get an estimate (3.6) with $a_1=20$, $a_3=4$, $a_4=5$, $a_6=6$, $a_{10}=2$.

§ 4. Transverse localizations

In this section, we list up transverse localizations needed in § 2. For that of $S_{4,11}$ the reader should refer [Y 1] or [O 2].

In the following paragraphs, v_0 denotes a point of the orbit, $p(x)$ a transverse direction and $L(x)$ denotes the coefficients of a basis of $\mathscr{H}$ of the transverse localization $(\mathscr{H},H)$ along S at v_0

$\sigma(f^{\alpha})$: the principal symbol of f^{α} on $T^*_{\{v_0\}}H$,

$v_{12,10}$: a point of $S_{12,10}$, $v_{12,10}=236 - 137 + 128 + 459$.

4.1. $S_{6,8}$

$v_0 = 256 - 346 + 157 - 148 + 238 - 129$.

basis of $T^*_{v_0}V$: 247, 359, 459, 259 + 349 - 458,

457 - 249 , 239 + 149 - $\frac{1}{2}$(348+258-456+3<357>).

$$p(x) = -\frac{2}{3}x_2\langle 239\rangle+x_3\langle 249\rangle-\frac{4}{3}x_4\langle 259\rangle+\frac{4}{3}x_5\langle 359\rangle+8x_6\langle 459\rangle-x_0\langle 247\rangle$$

$$L(x)=\begin{pmatrix} 2x_2 & 0 & 3x_3 & 4x_4 & 5x_5 & 6x_6 \\ 3x_3 & x_3 & x_0(4x_4-\frac{4}{3}x_2^2) & \frac{40}{3}x_0x_5-\frac{2}{3}x_2x_3 & 16x_6-\frac{16}{9}x_2x_4 & -\frac{8}{9}x_0x_2x_5+\frac{1}{9}x_3x_4 \\ 4x_4 & 0 & 5x_0x_5-x_2x_3 & 36x_6-\frac{4}{3}x_2x_4 & -2x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 \\ 5x_5 & -x_5 & 6x_6-\frac{2}{3}x_2x_4 & -2x_2x_5 & 0 & -\frac{1}{9}x_4x_5 \\ 6x_6 & 0 & -\frac{1}{3}x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 & -\frac{1}{9}x_4x_5 & -\frac{2}{27}(-5x_0x_5^2+12x_4x_6+4x_2^2x_6+\frac{1}{3}x_2x_4^2-x_2x_3x_5) \\ 0 & 2x_0 & 0 & 0 & -x_3 & 0 \end{pmatrix}$$

$$^t(X_{0\cdot}, X_{\cdot 0}, X_{11}, X_{20}, X_{3-1}, X_{40}) = {}^tL(x)\,{}^t(\partial_2,\partial_3,\partial_4,\partial_5,\partial_6,\partial_0),$$

	$X_{0\cdot}$	$X_{\cdot 0}$	X_{11}	X_{20}	X_{3-1}	X_{40}
$X(\log f)$	30	2	0	$-\frac{4}{3}x_2$	0	$2x_4+\frac{8}{27}x_2^2$
$\mathrm{div}^{(x)}X$	20	2	0	$-\frac{4}{3}x_2$	0	$\frac{16}{9}x_4+\frac{8}{27}x_2^2$
$\mathrm{Tr}^B_{\mathfrak{g}}\ \mathrm{ad}\ X$	10	0	0	0	0	$\frac{2}{9}x_4$

$$
\begin{aligned}
&[X_{0\cdot},X_{ij}] = iX_{ij}\ ,\ [X_{\cdot 0},X_{ij}] = jX_{ij}\ ,\\
&[X_{11},X_{20}] = -4x_0X_{3-1} - \tfrac{2}{3}x_2X_{11} - x_3X_{\cdot 0},\\
&[X_{11},X_{3-1}] = -3X_{40} + \tfrac{1}{3}x_2X_{20} - \tfrac{1}{2}x_3X_{11} + \tfrac{1}{3}x_4X_{0\cdot} + 4(x_4 - \tfrac{1}{3}x_2^2)X_{\cdot 0},\\
&[X_{11},X_{40}] = \tfrac{1}{6}x_3X_{20} - \tfrac{1}{3}x_4X_{11} + \tfrac{1}{3}x_5x_0X_{0\cdot},\\
&[X_{20},X_{3-1}] = -\tfrac{2}{3}x_2X_{3-1} + \tfrac{2}{3}x_5X_{0\cdot} + \tfrac{40}{3}x_5X_{\cdot 0},\\
&[X_{20},X_{40}] = \tfrac{2}{9}x_4X_{20} - \tfrac{8}{9}x_5X_{11} + \tfrac{8}{3}x_6X_{0\cdot},\\
&[X_{3-1},X_{40}] = -\tfrac{1}{9}x_4X_{3-1} + \tfrac{1}{9}x_5X_{20} - \tfrac{8}{9}x_2x_5X_{\cdot 0}\ .
\end{aligned}
$$

Set $X_3' = x_0X_{3-1} + \frac{1}{2}x_3X_{\cdot 0}$. Then we have

$$
\begin{aligned}
&[X_{0\cdot},X_3'] = 3X_3'\ ,\\
&[X_{11},X_{20}] = -4x_0X_3' - \tfrac{2}{3}x_2X_{11}\ ,\\
&[X_{11},X_3'] = x_0\{-3X_{40} + \tfrac{1}{3}x_2X_{20}\} - \tfrac{1}{2}x_3X_{11} + \tfrac{1}{3}x_0x_4X_{0\cdot}\ ,\\
&[X_{20},X_3'] = -\tfrac{2}{3}x_2X_3' + \tfrac{2}{3}x_0x_5X_{0\cdot}\ ,\\
&[X_3',X_{40}] = -\tfrac{1}{9}x_4X_3' + \tfrac{1}{9}x_0x_5X_{20}\ .
\end{aligned}
$$

$$\sigma(f^{\alpha}) = \xi_0^{-\alpha-1}\ \xi_2^{-15\alpha-10}\ \sqrt{d\xi}\ /\ \sqrt{dx}\ .$$

$D_2^5\delta$ is an eigenfunction belonging to an eigenvalue -1.

$b_{(6,8)} = s + 1$.

4.2. $S_{5,9}$

$v_0 = 256 - 346 + 157 - 247 - 148 + 238 - 129$.

basis of $T^*_{v_0}V$: 359, 459, 259 + 349 - 458,

$159 + 249 - 457 - 2<358>,\ 239 + 149 - \frac{1}{2}(348+258-456+3<357>)$.

$p(x) = -\frac{2}{3}x_2<239>+x_3<249>-\frac{4}{3}x_4<259>+\frac{4}{3}x_5<359>+8x_6<459>$

The localization is given by setting $x_0=1$ in 4.1.

$$L(x)=\begin{pmatrix} 2x_2 & 3x_3 & 4x_4 & 5x_5 & 6x_6 \\ 3x_3 & 4x_4-\frac{4}{3}x_2^2 & \frac{40}{3}x_5-\frac{2}{3}x_2x_3 & 16x_6+\frac{1}{2}x_3^2-\frac{16}{9}x_2x_4 & -\frac{8}{9}x_2x_5+\frac{1}{9}x_3x_4 \\ 4x_4 & 5x_5-x_2x_3 & 36x_6-\frac{4}{3}x_2x_4 & -2x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 \\ 5x_5 & 6x_6-\frac{2}{3}x_2x & -2x_2x_5 & -\frac{1}{2}x_3x_5 & -\frac{1}{9}x_4x_5 \\ 6x_6 & -\frac{1}{3}x_2x_5 & \frac{4}{9}x_4^2-\frac{4}{3}x_3x_5+\frac{8}{3}x_2x_6 & -\frac{1}{9}x_4x_5 & \frac{2}{27}(-5x_5^2+12x_4x_6+4x_2^2x_6 \\ & & & & +\frac{1}{3}x_2x_4^2-x_2x_3x_5) \end{pmatrix}$$

$${}^t(X_0,\ X_1,\ X_2,\ X_3,\ X_4) = {}^tL(x)\,{}^t(\partial_2,\ \partial_3,\ \partial_4,\ \partial_5,\ \partial_6),$$

	X_0	X_1	X_2	X_3	X_4
$X(\log f)$	30	0	$-\frac{4}{3}x_2$	x_3	$2x_4+\frac{8}{27}x_2^2$
$\mathrm{div}^{(x)}X$	20	0	$-\frac{4}{3}x_2$	$\frac{1}{2}x_3$	$\frac{16}{9}x_4+\frac{8}{27}x_2^2$
$\mathrm{Tr}^B_{\mathfrak{g}}$ ad X	10	0	0	$\frac{1}{2}x_3$	$\frac{2}{9}x_4$

where $f = \det L(x)$.

$$[X_0,X_k] = kX_k \quad , k=0,1,\ldots,4,$$

$$[X_1,X_2] = -4X_3 - \frac{2}{3}x_2X_1,$$

$$[X_1,X_3] = -3X_4 + \frac{1}{3}x_2X_2 - \frac{1}{2}x_3X_1 + \frac{1}{3}x_4X_0,$$

$$[X_1,X_4] = \frac{1}{6}x_3X_2 - \frac{1}{3}x_4X_1 + \frac{1}{3}x_5X_0,$$

$$[X_2,X_3] = -\frac{2}{3}x_2X_3 + \frac{2}{3}x_5X_0,$$

$$[X_2,X_4] = \frac{2}{9}x_4X_2 - \frac{8}{9}x_5X_1 + \frac{8}{3}x_6X_0,$$

$$[X_3,X_4] = -\frac{1}{9}x_4X_3 + \frac{1}{9}x_5X_2 .$$

$$\sigma(f^{\alpha}) = \xi_2^{-15\alpha-10}\ \sqrt{d\xi}\ /\ \sqrt{dx} \quad .$$

$$b_{(3,15)} = (s+1)(s+\tfrac{2}{3})(s+\tfrac{4}{3}).$$

4.3. $S_{4,14}$

$v_0 = v_{12,10} + 146 + 257$.

basis of $T^*_{v_0}V$: 347 + 248, 348, 358, 356 + 158

$p(x) = x_0(\langle 347\rangle+\langle 248\rangle)+ x_2\langle 348\rangle+ x_4\langle 358\rangle+x_3(\langle 356\rangle+\langle 158\rangle)$.

$$L(x) = \begin{pmatrix} 0 & 3x_0 & 0 & x_2 \\ 2x_2 & 4x_2 & 6x_0^2x_3 & 4x_0x_4 \\ 4x_4 & 2x_4 & 4x_2x_3 & 6x_0x_3^2 \\ 3x_3 & 0 & x_4 & 0 \end{pmatrix} ,$$

$${}^t(X_{0\cdot}\ ,\ X_{\cdot 0}\ ,\ X_{12}\ ,\ X_{21}) = {}^tL(x)\,{}^t(D_0\ ,\ D_2\ ,\ D_4\ ,\ D_3\ ) ,$$

	$X_{0\cdot}$	$X_{\cdot 0}$	X_{12}	X_{21}
$X(\log f)$	12	12	0	0
$\mathrm{div}^{(x)}X$	9	9	0	0
$\mathrm{Tr}^B_{\mathfrak{g}}$ ad X	3	3	0	0

where $f = \det L(x)$.

$$[X_{0\cdot},X_{12}] = X_{12}\ ,\ [X_{0\cdot},X_{21}] = 2X_{21},$$
$$[X_{\cdot 0},X_{21}] = X_{21}\ ,\ [X_{\cdot 0},X_{12}] = 2X_{12},$$
$$[X_{12},X_{21}] = 2x_0x_3\ (\ X_{0\cdot} - X_{\cdot 0}).$$

The $f(x)$ is the discriminant $d_3(x_0^2\ ,\ x_2\ ,\ x_4\ ,\ x_3^2\ )$ of the binary cubic form $x_0^2\,u^3 + x_2u^2v + x_4uv^2 + x_3^2\,v^3$.

$$f(x) = 12(\ x_2^2x_4^2 -4x_0^2x_4^3 -4x_2^3x_3^2 +18x_0^2x_2x_3^2x_4 - 27x_0^4x_3^4\ ) .$$

$$\sigma(f^{\alpha}) = (\xi_0\xi_3)^{-4\alpha-3}\sqrt{d\xi}\,/\,\sqrt{dx}\ .$$

We can see directly that

$$b_{(4,14)} = (s+1)(s+\tfrac{3}{4})(s+\tfrac{5}{4}).$$

Here, eigenvectors δ, $D_0D_3\delta$, $(D_0^2D_3^2 - 24D_2D_4)\delta$ belong to eigenvalues $-3/4$, -1, $-5/4$, respectively.

4.4. $S_{3,15}$

Under the situation in $S_{4,14}$, set $x_0 = 1$. Then, we get a localization along $S_{3,15}$.

$$L(x) = \begin{pmatrix} 2x_2 & 3x_3 & 4x_4 \\ 3x_3 & \frac{1}{2}x_4 & 2x_2x_3 \\ 4x_4 & 2x_2x_3 & 6x_3^2 + 2x_2x_4 \end{pmatrix}.$$

$${}^t(X_0, X_1, X_2) = {}^tL(x)\ {}^t(D_2, D_3, D_4).$$

	X_0	X_1	X_2
$X(\log f)$	12	0	$4x_2$
$\mathrm{div}^{(x)}X$	9	0	$4x_2$
$\mathrm{Tr}^B_{\mathfrak{g}}$ ad X	3	0	0

where $f = \det L(x) = 2(-4x_4^3 + x_2^2x_4^2 + 8x_2x_3^2x_4 - 27x_3^4 - 4x_2^3x_3^2)$.

$$[X_0, X_k] = kX_k\ ,\quad [X_1, X_2] = x_3X_0\ .$$

$$\sigma(f^{\alpha}) = \xi_2^{-6\alpha-9/2}\sqrt{d\xi}\,/\,\sqrt{dx}\ .$$

$f(x) = d_3(1, x_2, x_4, x_3^2)$ is the discriminant of type D_3, hence isomorphic to that of type A_3 . This polynomial was treated in [YS I,§6], and we proved the following.

$$b_{(3,15)} = (s+\frac{3}{4})(s+\frac{5}{4}),$$

$$b_{3,15} = (s+1)(s+\frac{5}{6})(s+\frac{7}{6})b_{(3,15)} .$$

4.5. $S_{7,7}$

The transverse localization is determined by J.Sekiguchi and the second author.

$v_0 = 356 + 137 + 128 + 458 + 149 + 239$

Basis of $T^*_{v_0}V$: 246, 247, 257, 157-259, 256-127+457, 236-248+347-146, 2<456>-2<126>-147+237-249.

$p(x)=x_{20}\langle 157\rangle+x_{11}\langle 237\rangle+x_{22}\langle 247\rangle+x_{31}\langle 257\rangle+x_{01}\langle 347\rangle+x_{21}\langle 457\rangle+x_{12}\langle 246\rangle.$

$$L(x) = \left(\begin{matrix} 0 & 2x_{20} & 0 & x_{21}+x_{20}x_{01} & x_{31}-x_{20}x_{11} \\ x_{01} & 0 & x_{11}/2 & 0 & -3x_{12} \\ x_{11} & x_{11} & 3x_{21}+\frac{5}{2}x_{20}x_{01} & 5x_{12} & -2x_{22} \\ x_{21} & 2x_{21} & \frac{5}{2}x_{31}-3x_{20}x_{11} & 2x_{22} & 5x_{12}x_{20} \\ x_{31} & 3x_{31} & \frac{1}{2}x_{20}x_{21} & 3x_{12}x_{20} & 0 \\ 2x_{12} & x_{12} & x_{22} & -2x_{12}x_{01} & 2x_{12}x_{11} \\ 2x_{22} & 2x_{22} & -2x_{31}x_{01}-x_{12}x_{20} & -3x_{12}x_{11} & 3x_{12}x_{21}+x_{11}x_{22} \\ & & -2x_{11}x_{21} & -3x_{01}x_{22} & \end{matrix}\right.$$

$$\left.\begin{matrix} -x_{31}+x_{20}x_{11} & 2x_{20}x_{21} \\ -2x_{12}-\frac{1}{2}x_{11}x_{01} & -2x_{22}-\frac{1}{2}x_{11}^2-2x_{01}x_{21} \\ -2x_{22}-\frac{1}{2}x_{01}x_{21} & \frac{3}{2}x_{31}x_{01}+7x_{12}x_{20} \end{matrix}\right) .$$

$$\begin{matrix} 2x_{12}x_{20}+x_{11}x_{21}+\frac{1}{2}x_{31}x_{01} & 4x_{20}x_{22}+\frac{1}{2}x_{11}x_{31} \\ \frac{1}{2}x_{21}^2 - 2x_{22}x_{20} & \frac{1}{2}x_{31}x_{21}+5x_{12}x_{20}^2 \\ x_{12}x_{11} & 0 \\ x_{11}x_{22}+\frac{5}{2}x_{12}x_{21}+\frac{1}{2}x_{12}x_{20}x_{01} & \frac{5}{2}x_{12}x_{31}-\frac{11}{2}x_{12}x_{20}x_{11} \end{matrix}\Bigg)$$

$${}^t(X_{\cdot 0}\ ,\ X_{0\cdot}\ ,\ X_{10}\ ,\ X_{01}\ ,\ X_{11}^{(1)},\ X_{11}^{(2)},\ X_{21}) = {}^tL(x)\,{}^t(D_{20},\ldots,D_{22}).$$

	$X_{\cdot 0}$	$X_{0\cdot}$	X_{10}	X_{01}	$X_{11}^{(1)}$	$X_{11}^{(2)}$	X_{21}
$X(\log f)$	12	16	0	$-2x_{01}$	$2x_{11}$	$4x_{11}$	$-2x_{21}$
$\mathrm{div}^{(x)}X$	8	11	0	$-4x_{01}$	$2x_{11}$	$\frac{7}{2}x_{11}$	$\frac{1}{2}x_{21}$
$\mathrm{Tr}_{\mathfrak{g}}^{B}\ \mathrm{ad}\ X$	4	5	0	0 $2x_{01}$	0	$\frac{1}{2}x_{11}$	$-\frac{5}{2}x_{21}$

where $f = \det L(x)$. We omit the commutation relations.

$$\sigma(f^{\alpha}) = \xi_{20}^{-8\alpha-\frac{11}{2}}\ \xi_{01}^{-12\alpha-8}\ \sqrt{d\xi}\,/\,\sqrt{dx}\ .$$

Using $X_{\cdot 0}$, $X_{0\cdot}$, X_{10} and $\sigma(f^{\alpha})$, we get $b_{(7,7)}=1$.

Using $L(x)$, we see that rank $L(x) = 1$ if and only if $x=(c,0,\ldots,0)$ or $(0,c,0,\ldots,0)$, $c\neq 0$, which correspond to $S_{6,14,2}$ and $S_{6,8}$.

4.6. $S_{5,21}$

$V_0 = V_{12,10} + 346 + 148 + 247$

basis of $T^*_{V_0}V$: 156, 256+157, 356+158-257, 357+258, 358

$p(x) = x_0\langle 156\rangle - \frac{1}{2}x_1\langle 256\rangle - x_2(\langle 356\rangle+\langle 158\rangle) + x_3(\langle 357\rangle+\langle 258\rangle) +$
$4x_4\langle 358\rangle$

$$L(x) = \begin{pmatrix} 0 & 4x_0 & x_1 & 0 & 0 \\ x_1 & 3x_1 & 2x_2 & 4x_0 & \frac{1}{2}x_1x_2 - 3x_0\,x_3 \\ 2x_2 & 2x_2 & 3x_3 & 3x_1 & x_2^2 - 4x_0x_4 - 2x_1x_3 \\ 3x_3 & x_3 & 4x_4 & 2x_2 & \frac{1}{2}x_2x_3 - 3x_1x_4 \\ 4x_4 & 0 & 0 & x_3 & 0 \end{pmatrix},$$

$${}^t(X_0^{(1)},\ X_0^{(2)},\ X_{1-1},\ X_{-1\,1},\ X_{22}) = {}^tL(x)\,{}^t(D_0,\ldots,D_4).$$

	$X_0^{(1)}$	$X_0^{(2)}$	X_{1-1}	$X_{-1\,1}$	X_{22}
$X(\log f)$	12	12	0	0	$4x_2$
$\mathrm{div}^{(x)}\,X$	10	10	0	0	$3x_2$
$\mathrm{Tr}^B_{\mathfrak{g}}\ \mathrm{ad}\,X$	2	2	0	0	x_2

$$[X_0^{(1)}, X_{ij}] = iX_{ij}\ ,\quad [X_0^{(2)}, X_{ij}] = jX_{ij}\ ,$$
$$[X_{1-1}\ ,\ X_{-11}] = X_0^{(1)} - X_0^{(2)}\ ,$$
$$[X_{1-1}\ ,\ X_{22}\ ] = \tfrac{1}{4}x_3X_0^{(1)} + \tfrac{3}{4}x_3X_0^{(2)} - \tfrac{1}{2}x_2X_{1-1} - x_4X_{-11}\ ,$$
$$[X_{-11}\ ,\ X_{22}\ ] = \tfrac{3}{4}x_3X_0^{(1)} + \tfrac{1}{4}x_3X_0^{(2)} - x_0X_{1-1} - \tfrac{1}{2}x_2X_{-11}\ .$$

$$b_{(5,21)} \mid (s+5/6)(s+7/6)\ .$$

4.7. $S_{5,12}$

$v_0 = v_{12,10} + 146 + 256 + 157$

basis of $T^*_{v_0}V$: 346+148-357-258, 247, 347+248, 348, 358

$p(x) = x_{20}\langle 247\rangle + x_{01}(\langle 346\rangle+\langle 148\rangle) + x_{11}(\langle 347\rangle+\langle 248\rangle) + x_{02}\langle 348\rangle$
$-x_{-12}\langle 358\rangle$.

$$L(x) = \begin{pmatrix} 2x_{20} & 0 & 0 & 0 & 2x_{11} \\ 0 & x_{01} & 3x_{11} & x_{02}-x_{01}^2 & x_{-12} \\ x_{11} & x_{11} & 3x_{20}x_{01} & x_{20}x_{-12} & x_{02} \\ 0 & 2x_{02} & 8x_{01}x_{11}+2x_{20}x_{-12} & 3x_{11}x_{-12} & 3x_{01}x_{-12} \\ -x_{-12} & 2x_{-12} & 2x_{02} & x_{01}x_{-12} & 0 \end{pmatrix},$$

$${}^t(X_0^{(1)},\ X_0^{(2)},\ X_{10}\ ,\ X_{01}\ ,\ X_{-11}) = {}^tL(x)\,{}^t(D_{20},\ldots,D_{-12})\ .$$

$X_0^{(1)}$	$X_0^{(2)}$	X_{10}	X_{01}	X_{-11}

X(log f)	2	8	0	$-2x_{01}$	0
$\mathrm{div}^{(x)}X$	2	6	0	$-x_{01}$	0
$\mathrm{Tr}^{B}_{\mathscr{g}}$ ad X	0	2	0	$-x_{01}$	0

$$[X_0^{(1)}, X_{ij}] = iX_{ij}\ ,\ [X_0^{(2)}, X_{ij}] = jX_{ij}\ ,$$
$$[X_{10}, X_{01}] = x_{11}(X_0^{(1)} - X_0^{(2)}) + x_{01}X_{10} - x_{20}X_{-11}\ ,$$
$$[X_{10}, X_{-11}] = 3x_{01}X_0^{(1)} - x_{01}X_0^{(2)} - X_{01}\ ,$$
$$[X_{01}, X_{-11}] = x_{-12}X_0^{(1)}\ .$$

$$\sigma(f^{\alpha}) = \xi_{20}^{-\alpha-1}\ \xi_{01}^{-8\alpha-6}\ \sqrt{d\xi}\ /\ \sqrt{dx}\ .$$

$$b_{(5,12)} = 1\ .$$

4.7. $S_{6,14,0}$, $S_{6,14,2}$

$S_{6,14,0}$

linear part of the transverse localization

$$L'(x) = \begin{pmatrix} 2x_{222} & 2x_{222} & 2x_{222} & 0 & 0 & 0 \\ x_{131} & 3x_{131} & x_{131} & -2x_{222} & 0 & 0 \\ x_{113} & x_{113} & 3x_{113} & 0 & -2x_{222} & 0 \\ 2x_{200} & 0 & 0 & 0 & 0 & x_{222} \\ 0 & 4x_{040} & 0 & x_{131} & 0 & 0 \\ 0 & 0 & 4x_{004} & 0 & x_{113} & 0 \end{pmatrix},$$

$${}^{t}(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_{1-11}, X_{11-1}, X_{022}) = {}^{t}L'(x)\,{}^{t}(D_{222}, \dots, D_{004}).$$

	$X_0^{(1)}$	$X_0^{(2)}$	$X_0^{(3)}$
X(log f)	8	12	12
$\mathrm{div}^{(x)}X$	6	10	10
$\mathrm{Tr}^{B}_{\mathscr{g}}$ ad X	2	2	2

$$\sigma(f^{\alpha}) = \xi_{200}^{-4\alpha-3}\ (\xi_{040}\xi_{004})^{-3\alpha-\frac{5}{2}}\ \sqrt{d\xi}\ /\ \sqrt{dx}\ .$$

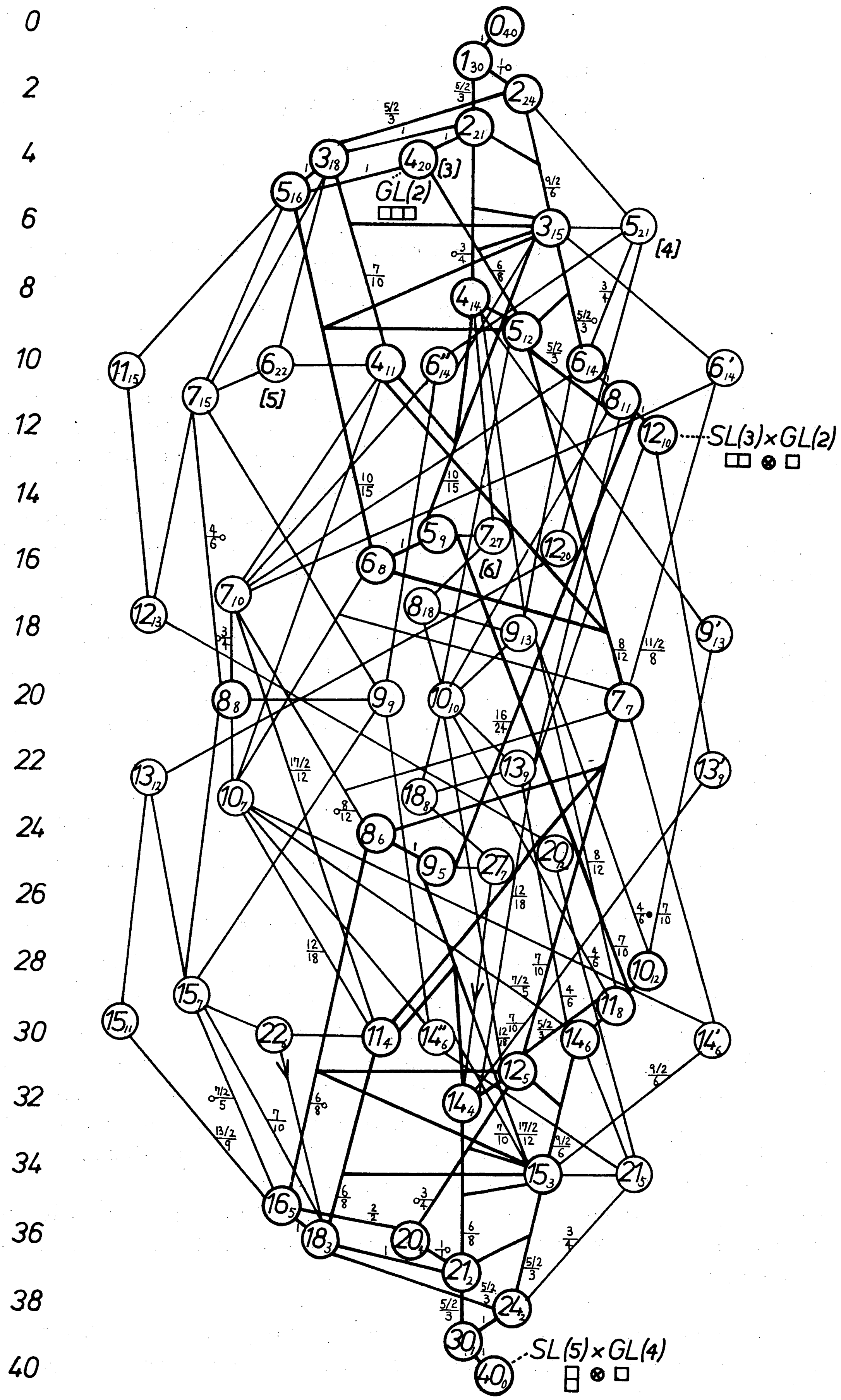
$S_{6,14,2}$

Linear part of the transverse localization

$$L'(x) = \begin{pmatrix} 2x_{20} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2x_{02} & 0 & 0 & 0 & 0 \\ 3x_{3-2} & -2x_{3-2} & x_{20} & 0 & 0 & 0 \\ -2x_{-23} & 3x_{-23} & 0 & x_{02} & 0 & 0 \\ 0 & x_{01} & 0 & -3x_{20} & 0 & x_{02} \\ x_{10} & 0 & 3x_{02} & 0 & x_{20} & 0 \end{pmatrix},$$

$${}^t(X_0^{(1)}, X_0^{(2)}, X_{-12}, X_{2-1}, X_{10}, X_{01}) = {}^tL'(x)\,{}^t(D_{20},\ldots,D_{10}) .$$

	$X_0^{(1)}$	$X_0^{(2)}$
$X(\log f)$	6	6
$\mathrm{div}^{(x)}X$	4	4
$\mathrm{Tr}_{\mathfrak{g}}^{B}\ \mathrm{ad}\ X$	2	2



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