

Reaction-Diffusion Model of Somite Segmentation

Dept. of Physics, Kyoto Univ. Hiroki Nagahara ¹, Yoshiko Takenaka ²

脊椎動物の体節形成においては、時計のように周期的に現れる遺伝子の発現が、波のように中胚葉を伝播し、脊椎の周期的な構造を形成することが知られている [1]。本発表では、それらの興味深い遺伝子発現の挙動を単純な反応拡散方程式によって再現することを試み、さらに細胞間相互作用の実効的な距離など物理的な側面について議論する予定である。

There are a wide variety of spatio-temporal patterns in nature that appear in a self-organizing manner, such as the morphogenesis during the development and the skin patterns of mature animals. Since Turing proposed an epoch-making idea that morphogens self-organize into patterns based on the reaction-diffusion system in 1952, pattern formation based on reaction-diffusion systems has been extensively studied (see e.g. [2]).

Here, we focus on the dynamic process of morphogens in the somite genesis of vertebrate animals, where the expression of a gene sweeps across the presomitic mesoderm (PSM) from the posterior end toward the anterior end like a wave propagation (see. e.g. [1]). The gene-expression is periodically generated at the posterior end of the PSM, where a so-called "molecular oscillator" is working [1]. As the wave comes close to the anterior end, it gradually decreases its speed, shrinks its thickness and finally gets stabilized.

In order to understand these curious behaviors of the gene-expression (oscillation and wave propagation), we present a simple reaction-diffusion model, including two hypothetical substances which act as a oscillator and another material which controls or interacts with the oscillatory process, and demonstrate how the periodic pattern can be generated with the combination of the oscillation and the propagation. Based on this simple model, we will also discuss some physical aspects which would be free from the details in molecular reactions, such as the effective length of the interaction among the neighboring cells.

As a hypothetical model which describes the characteristics in somite genesis above, we introduce a set of reaction-diffusion equations:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{1}{\tau_1} \{f(u, v) - au\} + D_u \nabla^2 u, & \frac{\partial v}{\partial t} &= \frac{1}{\tau_2} \{g(u, v) - ev\} + D_v \nabla^2 v \\ f(u, v) &= H(u - \alpha) - v, & g(u, v) &= (u - b) + \gamma v \end{aligned} \quad (1)$$

¹E-mail: nagahara@chem.scphys.kyoto-u.ac.jp

²E-mail: takenaka.yoshiko@scphys.kyoto-u.ac.jp

where u and v are the concentration of activator and inhibitor respectively. The terms $\tau_1^{-1}\{f(u, v) - au\}$ and $\tau_2^{-1}\{g(u, v) - ev\}$ are the reaction terms which are based on a so-called "activator-inhibitor system" (see e.g. [2]). τ_1 and τ_2 are the time constants which measure the characteristic time scales of the local reaction kinetics of u and v , respectively. The function $H(u - \alpha)$ is the Heaviside step function with the threshold α . D_u and D_v are the reaction coefficients of u and v , respectively.

Figure 1 shows the results of the numerical simulations based on Eqs. 1, which is composed of the spatio-temporal profiles together with schematic drawings on the manner of the wave propagations in one-dimensional reaction-diffusion fields. In Fig. 1(a), the kinetic parameter γ is given so as to have a spatial gradient. In Fig. 1(a), a wave triggered at the left boundary propagates in the right direction, reducing its speed and thickness, and finally stands on a certain point. The standing wave does not vanish when it stops. In Fig. 1(b), in order to take the growth of the embryo into consideration, the kinetic parameter γ is taken to depend not only on space but also on time. In Fig. 1(b), a series of waves are periodically triggered at a oscillatory region at the left end, which moves into the left direction at a constant speed. It is clear that the the wave train forms a stationary, periodic structure in Fig. 1(b)

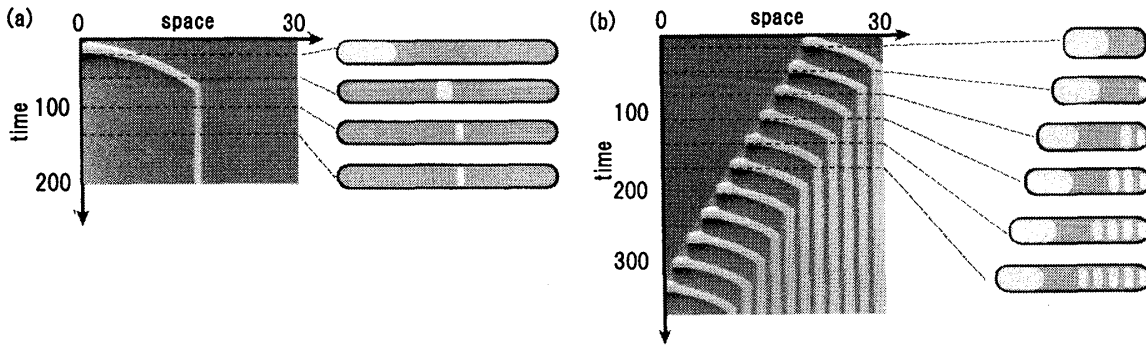


Figure 1: Spatio-temporal profiles together with schematic drawings on the manner of wave propagation in the numerical simulations. (a)The kinetic parameter γ is given so as to have a spatial gradient, and a wave triggered at the left boundary propagates in the left direction, decreasing its speed and thickness, and finally stands on a certain point. (b)In order to take the growth of the embryo into consideration, the kinetic parameter γ is taken to depend not only on space but also on time. A wave train makes a static, periodic structure.

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References

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