

Tool Path Modification Approaches to Enhance Machining Geometric Accuracy in 3-Axis and 5-Axis Machining

Mohammad Sharif Uddin

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Tool Path Modification Approaches to Enhance Machining Geometric Accuracy in 3-Axis and 5-Axis Machining

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Mohammad Sharif Uddin
(*B. Sc. Eng., M. Eng.*)

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Kyoto University**

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Chapter 1

Introduction

Precision manufacture of components has become a necessity in the present day manufacturing sectors. The ever-increasing demands of humankind have forced researchers to come up with more improved innovations in technology; achieving higher levels of integration in microprocessors, and creating more versatile and precision multitasking systems being just a few of the major drivers in this area. All these, however, have one rudimentary requirement, namely, the need to use very high precision components. Hence, it can be very safely concluded that the success of each of these industries hinges on the ability to produce such components.

Recently, as the tremendous demands for mechanical parts with high geometric and dimensional accuracy increase, an exigency to produce those parts with such accuracy is greatly comprehended by today's manufacturing industries. To this end, CNC machine tools are the most important means of production for the manufacturing industries. CNC machine tools have been widely applied to a range of applications, for example, in the aerospace industries. With the recent advancement of the machine tools manufacturing technologies including high speed feed drives and high speed spindles, high speed end milling on the CNC machine tools has become constantly popular, and is being performed to manufacture the components with the required contour geometry and dimensional accuracy.

However, the geometric accuracy of the machined surface is greatly affected by the numerous errors sources ranging from errors existing in the machine tool system itself to the errors due to the cutting process. Figure 1-1 shows the general error sources that influence the machining geometric accuracy [Kakino et al., 1993]. Broadly, machining geometric errors are caused by two major error sources: motion errors of the machine tool system and errors due to

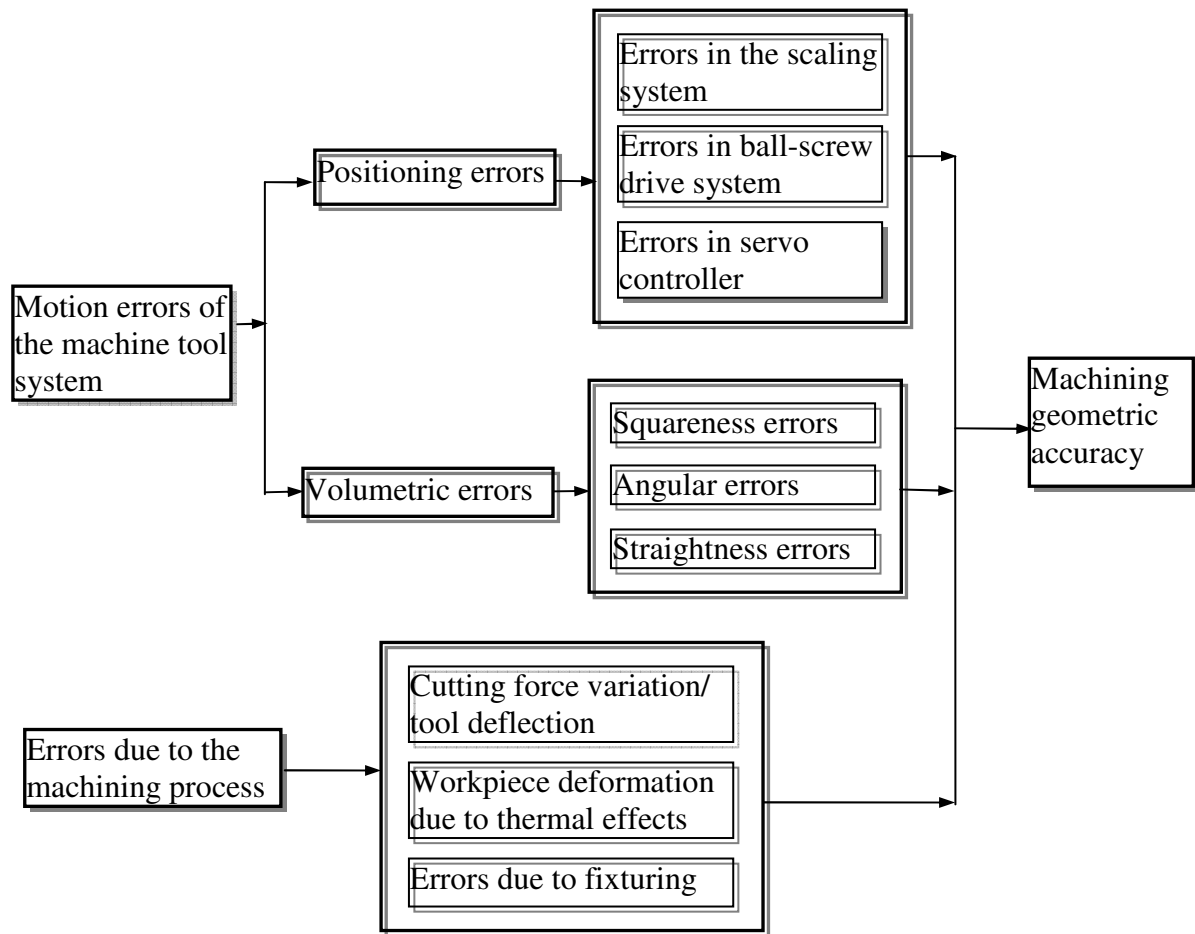


Figure 1-1 Relationship between major error sources that affect the machining geometric accuracy

the machining process. The key factors among error sources in the machine tool system that cause deviation of tool tip position relative to workpiece, and hence machining geometric errors are positioning errors and volumetric errors. Here, positioning errors are defined as the linear errors of the positioning mechanism, whose directions are in parallel with the direction of axis movement required for desired positioning. On the other hand, volumetric errors are here defined as error components whose directions are perpendicular to the direction of axis movement.

Major causes of positioning errors of the machine tools are: (1) errors in the scaling system (closed-loop type machine) which are caused by uniform expansion or contraction of the linear scale , (2) errors in the ball screw driving system (semi-closed loop type machine) which are caused by uniform expansion

or contraction of the ball-screw, pitch error, backlash, errors in coupling, and etc, and (3) errors in the servo-control system which include mainly lost motion, stick motion, stick-slip, mismatch of position loop gain, response lag, and etc. Errors in the servo-control system are mostly dependent on the feedrate and its direction.

Volumetric errors of the machine tools include straightness errors, squareness or orthogonality errors, and angular errors. Major causes of the straightness errors include imperfections and misalignments of guide-ways of the sliding stages. The squareness or orthogonality errors are mostly caused by the assembly errors of the alignment of each axis. The angular errors that include roll, pitch, and yaw, are caused by non-parallelity of guide-ways and profile errors of guide-ways. Among volumetric errors, squareness errors are defined as the first order type errors while straightness errors are defined as the second or higher order type errors. Compared to 3-axis machine tools having three orthogonal linear axes, in particular, the assembly error of the alignment of each axis is more significant in the multi-axis machine tools, especially 5-axis machine tools, which have two rotary axes in addition to orthogonal linear axes. These errors usually do not change with time meaning that the machine repeats the same error again and again for some duration. It has been reported that 75% of initial errors of a new machine tool arise as a result of manufacture and assembly [Cecil and Sutherland, 1998].

In order to improve the machine's motion accuracy and hence the machining geometric accuracy, the compensation of linear positioning errors is relatively easy, since the measurement of linear positioning errors is easier, and its compensation can be implemented in a servo-controller that drives the erroneous axis. Indeed, it is widely done in practical applications, by proper re-tuning servo parameters, or applying backlash and pitch compensations in the servo-controller of the machine tools.

On the other hand, compensating volumetric errors due to mechanical structures of the machine tool is more difficult. But, with the accurate measurement of volumetric errors and proper mathematical modeling of the machine's kinematics, it is possible to significantly reduce these errors such that

the appropriate tool tip position can be reached as the machine commands. In other words, once volumetric errors existing in the machine tools are measured and identified reliably by any measuring method, the effect of these errors on the geometric accuracy can be predicted and evaluated by developing a simulator of machining errors. Then, an error in the tool location with respect to the workpiece location can be reduced by applying the compensation to command trajectories for each axis in a way such that the effect of volumetric errors can be cancelled. Therefore, in this study, we will focus on the enhancement of machining geometric accuracy by identifying, evaluating and compensating these volumetric errors on a multi-axis machining center, for example, a 5-axis machining center. It is to be noted that, in this research work, we consider only volumetric errors of first order type because this type of errors can be physically defined and identifiable easily, and hence compensation for these errors can be done reliably for the improvement of machining geometric accuracy on 5-axis machining centers.

Besides, error sources in the machine tool system as described earlier, errors due to the machining process have a direct effect on the geometric accuracy of the machined surface (see also Figure 1-1). They include mainly cutting force variation to cause tool deflection, workpiece deformation due to thermal effects, fixturing errors, and etc. However, the most dominant factors among them are the cutting force variation that causes the variation in the tool deflection. Depending on the type of cutting processes, cutting tool path planning, cutting conditions, the combination of the workpiece and the tool materials, and the workpiece setup, the degree of cutting force variation and tool deflection during actual cutting varies. While all other machining parameters are kept constant, one significant parameter that causes the cutting force variation and tool deflection is the cutting tool path patterns to be used for machining the workpiece. In two-dimensional (2D) end milling process on a 3-axis machining center, contour parallel offset tool path patterns, generated by commercial CAM software, are widely adopted because they are both geometrically appealing and computationally tractable. Even efficient algorithms have recently been

developed for generating such contour parallel offsets, which have resulted in robust tool path generation routines dealing with machining of the workpiece with arbitrarily shape contour.

However, when the mechanics of the cutting process are taken into account, it is apparent that these conventional contour parallel offset tool paths often offer the cutting problem such as varying cutting engagement with workpiece. In actual cutting operation, this varying cutting engagement causes the variation in cutting force and tool deflection, which consequently leads to the geometric error on the machined workpiece surface. Further, from the process stability perspective, this variation of cutting force and tool deflection resulted from contour parallel paths can be a critical issue.

In order to address and overcome these cutting problems with contour parallel tool paths and hence to improve machining geometric accuracy, a number of research works has been reported. The most commonly used approach to this end is feedrate scheduling. However, the performance of feedrate scheduling scheme is limited by the lack of the sophisticated servo-controller of machine tools that possesses a rapid acceleration and deceleration control mechanism to response to the frequent and quick change of feedrate in actual machining operation. On such a situation, an approach to tool path modification becomes promising to tackle the varying cutting forces during machining with contour parallel offset paths. This approach basically aims to adjust the cutting force (i.e. chip load) by using a special tool path trajectory generated based on the physical geometry of the cutting process. Hence, the ultimate expectation with the tool path modification is to control the cutting force at a desired level and hence to enhance the geometric accuracy of the machined surface.

Motivated with the background and issues on errors in the machine tool system and the cutting process which cause machining geometric errors as discussed above, the current research work presents the tool path modification approaches for the improvement of the machining geometric accuracy in 3-axis and 5-axis machining.

With this aim, the first part of this dissertation proposes a new offset tool path generation by forward and backward tool path modification approaches in order to regulate the cutting engagement angle and hence the cutting force at a desirable constant level, which will consequently improve the machining geometric accuracy in 2D end milling on a 3-axis machining center.

The second part of the dissertation proposes a simulator of machining geometric errors in 5-axis machining by considering the effect of kinematic errors in a 5-axis machining center on the three-dimensional interference of the tool and the workpiece. Such simulator is crucial to quantitatively evaluate the significance of each kinematic error on the machine's overall machining geometric accuracy. An error compensation scheme by modifying the tool path trajectory of tool center position and orientation is presented for the improvement of the machining geometric accuracy in 5-axis machining.

The remainder of this dissertation is organized as follows: Chapters 2 to 4 constitute the first part of this dissertation. Chapter 2 presents a new constant engagement tool path generation by the algorithm for forward tool path modification. An illustrating example of the cutting force regulation by the presented approach is shown. Some critical issues with the proposed forward tool path modification approach are described in the chapter.

To tackle problems associated with the forward tool path modification approach, Chapter 3 presents an algorithm for backward tool path modification such that the cutting engagement is regulated at a desired constant level. The proposed algorithm modifies the semi-finishing path such that a constant cutting engagement angle is regulated on the machining along finishing path. Case studies with cutting experiments are shown to demonstrate the significance of the proposed algorithm in terms of suppression of cutting force variation and improved geometric accuracy of the machined surface.

An application of the proposed algorithm for backward tool path modification under a feedrate optimization approach is shown in Chapter 4. By applying both the feedrate optimization and the tool path modification, it has

been shown that both the geometric accuracy and the surface quality of the machined surface can be improved.

Chapters 5 to 6 constitute the second part of this dissertation. Chapter 5 presents modeling and identification of kinematic errors on 5-axis machining centers. A DBB measurement technique is applied to practically identify kinematic errors associated with linear and rotary axes on a 5-axis machining center of the tilting rotary table type.

A simulator of machining geometric errors by considering the effect of identified kinematic errors on the three-dimensional interference of the tool and the workpiece is introduced in Chapter 6. In an aim to improve the geometric accuracy of the machined surface, an error compensation scheme by modifying the trajectory of tool position and orientation such that the effect of kinematic errors is canceled, is presented in this chapter.

Finally, Chapter 7 closes the thesis with a summary of all the above chapters following some recommendations for future work.

Chapter 2

Constant Engagement Tool Path Generation by Forward Tool Path Modification

2.1 Introduction

With the recent advancement of high speed machining technology, two-dimensional (2D) contour end milling has gained an increasing demand in the manufacturing of die and mold products. This is partially due to the fact that a surprisingly larger number of mechanical parts are made of two-dimensional contour and even more complex objects are generally created from a billet by using two-dimensional roughing, semi-finishing and finishing processes. In two-dimensional contour end milling, conventional offset contour CNC tool paths generated by commercial CAM software are extensively used to machine these mechanical parts. However, depending on the geometry of the workpiece surface contour, these conventional offset contour tool paths often offer a varying cutting engagement with workpiece, which causes change in cutting load and tool deflection. This change in cutting load and tool deflection will consequently cause geometric error on the machined workpiece surface. This chapter will first discuss this issue with offset contour tool paths, and show that the variation in cutting engagement can be characterized by geometrically computing the engagement angle, a parameter defining the two-dimensional geometric interference between the tool and the workpiece.

To avoid the variation in cutting load during machining on offset contour tool paths, and consequently to improve the geometric accuracy of the machined surface, there have been many efforts available in the literature. Commonly applied approaches to regulate or reduce the variation in cutting load are feedrate scheduling and tool path modification.

For the application to a finishing tool path to improve geometric accuracy of the machined workpiece surface, tool path modification approaches have a practical advantage compared to feedrate scheduling approaches. From this view point, a notably unique tool path modification approach was proposed by Stori and Wright [Stori and Wright, 2000] that generates a contour parallel tool path with constant cutting engagement. In this chapter, this approach will be first briefly reviewed, and then present an extension of this approach such that it can be applied to tool paths of arbitrary geometry. This approach has, however, an inherent critical limitation in the application to a finishing path. Hence, the main objective of this chapter is to discuss the limitation of this tool path modification approach, which motivates the proposal of the new tool path modification approach which will be presented in Chapter 3.

The remainder of the chapter is organized as follows. Section 2.2 briefly discusses the conventional tool path patterns used in two-dimensional contour end milling. The mechanics of the contour milling such as variation of cutting engagement and its effect on the machining accuracy is addressed in Section 2.3. A thorough literature review focusing on regulation of the cutting engagement and cutting force by different techniques, are discussed in Section 2.4. A numerical procedure to compute the cutting engagement angle for an arbitrary contour geometry is illustrated in Section 2.5. A new constant engagement tool path generation by forward tool path modification is introduced and some critical issues with the proposed scheme are addressed in Section 2.6. Section 2.7 concludes the chapter with a brief summary.

2.2 NC Tool Paths for Two-Dimensional (2D) End Milling Processes

Two tool path patterns commonly used in two-dimensional end milling operations are direction parallel tool paths and contour parallel tool paths, also known as boundary parallel tool paths. The direction parallel tool paths are linear

tool paths that can further be classified as: one-way or zig tool path, zig-zag tool path and smooth zig-zag [Choy and Chan, 2003]. Figures 2-1, 2-2, and 2-3 show the three different types of direction parallel tool path patterns.

On the other hand, an example of non-linear tool paths is the contour parallel tool path. Figure 2-4 shows the contour parallel tool path pattern for machining a pocket. In contour parallel tool path patterns, a series of offset contours of the workpiece (e.g. a pocket) boundary are first generated. The tool travels along these offsets one by one, until the entire pocket is machined. As shown in Fig. 2-4, contour parallel tool path is derived from the boundary of the concerned machining region. It is a coherent tool in the sense that the cutter is kept in contact with the cutting material most of the time. So it incurs less idle times such as those spent in lifting, positioning and plunging the cutter. At the same time it can also maintain the consistent use of either up-cut or down-cut method throughout the cutting process. Contour-parallel tool path is therefore widely used as a cutting tool path especially for large-scale material removal in 2D end milling.

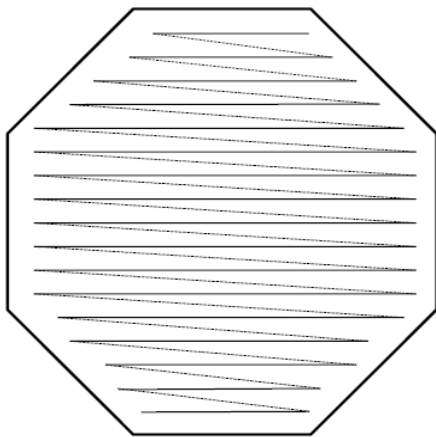


Figure 2-1 Zig tool path pattern

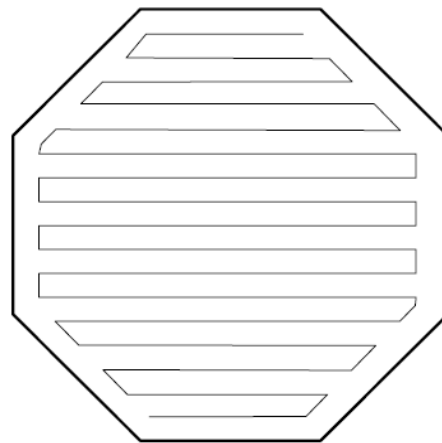


Figure 2-2 Zig-zag tool path pattern

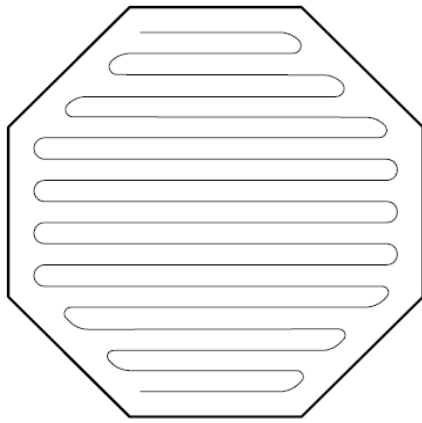


Figure 2-3 Smooth zig-zag tool path pattern

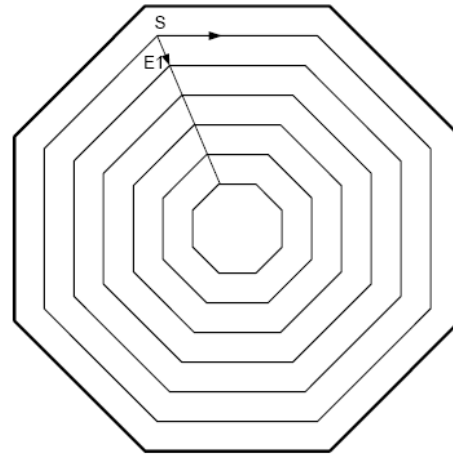


Figure 2-4 Contour parallel tool path pattern

The algorithms to generate the contour parallel offset tool paths can be divided into three different approaches: (1) ‘pair-wise intersection [e.g. Hansen and Arbab, 1992] (2) ‘Voronoi diagram’ [Persson, 1978], (3) ‘pixel-based’ (e.g. [Choi and Kim, 1997]). Among these three approaches, the contour parallel tool path generation based on ‘Voronoi diagram’ is known to be more efficient and robust since the steps in offsetting the tool path segments can be subdivided in an organized manner [Choy and Chan, 2003]. In this case, the individual offset segments are trimmed to their intersections using the Voronoi diagram of the original pocket boundary.

Almost all commercial CAM software available in the market uses one of these three approaches to generate contour parallel tool paths for any 2D contour geometry. Recently, many numerically robust and efficient algorithms have been developed to generate contour parallel offset paths for any given contour geometries and adopted in commercial CAM software. No matter which one of the above approaches is used for tool path generation, it is clearly evident that depending on the geometry of workpiece boundary, generated contour parallel tool paths will always contain many offset contours with convex and concave arc formed by the intersection of different types of geometric entities such as line and arc.

Contour parallel offset tool paths have been extensively adopted in 2D contour end milling. However, in real machining, these traditional contour parallel offset tool paths often cause many cutting problems such as variation in cutting engagement angle, etc. The following section will highlight the details of cutting problems with traditional contour parallel tool paths and their effects on the machining accuracy.

2.3 Cutting Problems Associated with Contour Parallel Tool Paths

As mentioned in earlier section, contour parallel offset paths are most widely adopted in 2D contour milling because it produces lesser idle tool path portions and can maintain a consistent use of down-cut (or up-cut) milling. However, these contour parallel offset paths inherently contain many convex and concave arcs, producing cutting problems in actual machining. Before addressing the cutting problems associated with contour parallel paths, it is important to introduce a measure of two-dimensional (2D) cutting process that defines cutting force or cutting load.

Figure 2-5 shows a schematic of two-dimensional end milling on a straight path. Kline et al. [Kline et al., 1982] reported that cutting force or cutter load is directly related with undeformed chip thickness, t_m as shown in Fig. 2-5, where undeformed chip thickness is geometrically defined to be the maximum thickness of material in the radial direction that a cutting edge of the tool encounters. Furthermore, the undeformed chip thickness is related with the cutting engagement angle or cutting engagement arc length [Choy and Chan, 2003, Kramer, 1992]. As can be also seen in Fig. 2-5, the cutting engagement angle, α_{en} or engagement arc length, L can be defined as the interaction between the cutting tool and the machined workpiece surface. In this study, we consider the cutting engagement angle, which can be computed from geometric interference of the tool and the workpiece in two-dimensional end milling, as a dominant process parameter in evaluating the cutting load acting on the tool.

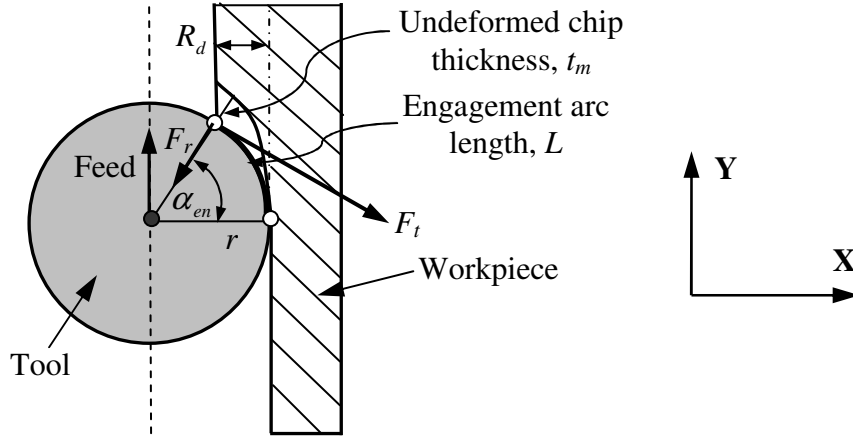


Figure 2-5 Schematic of two-dimensional end milling on a straight path (α_{en} : cutting engagement angle, r : tool radius, R_d : radial depth of cut, F_t : tangential cutting force, F_r : radial cutting force)

Contour parallel tool paths have been traditionally approached from a purely geometric perspective. When the basic mechanism of the cutting process is considered, they often inherently create many cutting problems in real machining operation. The cutting problems with conventional contour parallel tool paths now can be illustrated as follows: Figure 2-6 illustrates the change in cutting engagement angle with respect to different tool path geometry in two-dimensional end milling. Figure 2-6(b) shows a situation where the cutting tool moves along a linear path and makes the cutting engagement angle, α_{en}^0 . As shown in Fig. 2-6(a), when the cutting tool approaches from a linear path to a concave corner, the cutting engagement angle increases from α_{en}^0 to α_{en}^1 ($\alpha_{en}^1 > \alpha_{en}^0$). Under the assumption that the feedrate at the cutting point is constant, this means that the cutting force will increase when the tool machines a concave corner. On the other hand, when the cutting tool moves from a linear path into a convex corner, the cutting engagement angle decreases from α_{en}^0 to α_{en}^2 ($\alpha_{en}^2 < \alpha_{en}^0$) as shown in Fig. 2-6 (c). The decrease in cutting engagement angle will decrease the cutting force. In addition, referring to Fig. 2-6, for tool motions along linear and circular arc segments, the cutting engagement angle can be computed analytically as follows:

$$\text{Linear path segment: } \alpha_{en}^0 = \cos^{-1}(1 - s/r) \quad (2-1)$$

$$\text{Concave arc segment: } \alpha_{en}^1 = \cos^{-1}(1 - s/r - \Gamma) \quad (2-2)$$

$$\text{Convex arc segment: } \alpha_{en}^2 = \cos^{-1}(1 - s/r + \Gamma) \quad (2-3)$$

where, $\Gamma = s(r - 0.5s)/(Rr)$, and, s , r , R are the step-over distance, tool radius, and the curvature radius of the arc of tool path, respectively.

From the above description, it is clear that depending on the geometry of the workpiece or tool path, there is a variation in the cutting engagement angle, which will cause the variation in material removal rate, and consequently cutting force. This change in cutting force will cause the tool deflection, which will consequently affect the geometric accuracy of machined workpiece surface. Further, this may lead to the whole machining process an unsteady state. Tlustý et al. [Tlustý et al., 1990] have shown that the change in the radial depth of cut during corner cutting had an adverse effect on the machining stability such as causing high frequency chatters.

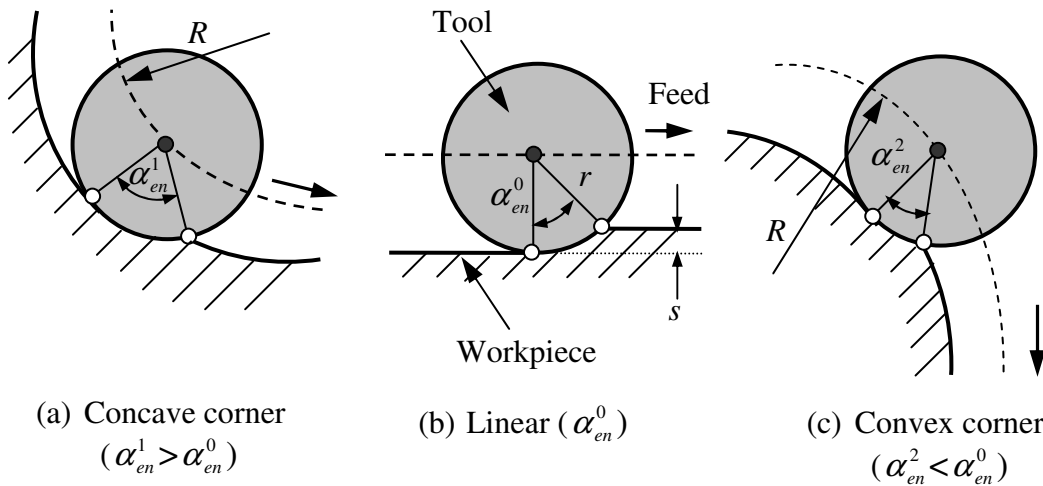


Figure 2-6 Variation of cutting engagement angle with respect to different tool path geometry in two-dimensional end milling (r : tool radius, α_{en} : engagement angle, s : step-over distance (i.e. radial depth of cut), R : curvature radius of circular arc of tool path)

To avoid these adverse consequences with conventional contour parallel tool paths, machining practitioners usually resort to using a lower feedrate and/or depth of cut which leads to a reduction of machining efficiency. Therefore, it is a great concern among researchers in the area of die and mold manufacturing how to regulate the variation in cutting load and hence, the tool deflection in actual machining without sacrificing the machining efficiency and productivity. It is to be expected that reduced variations of the cutting force only can improve the geometric accuracy and surface finish of machined workpiece [Farouki et al., 1998].

2.4 Literature Review: Regulation of Cutting Forces in 2D End Milling

While conventional contour parallel tool paths always create the variation in the cutting force, as described in the previous section, the next challenge to be taken by the researchers is how to reduce the change in cutting force, which will eventually lead to an improvement of geometric accuracy of machined surface. Two major approaches can be identified in the literature for tackling cutting problems with conventional contour parallel offset paths. Namely, they are: (1) feedrate scheduling or adaptive feedrate control, (2) the modification of tool path trajectory. This section will briefly review research works available in literature, focusing on these two methods to control or reduce variation in the cutting force in 2D contour end milling.

2.4.1 Regulation of Cutting Forces by Feedrate Scheduling

This approach focuses on regulating cutting forces by adjusting feedrates such that the cutting force is regulated at a desired level regardless of the tool path geometry. A majority of works found in the literature is on the subject of adaptive feedback control techniques. Various adaptive control techniques have

been tested on the cutting force regulation in end milling process. To be noted among them are the adaptive generalized predictive control (AGPC) [Altintas, 2000], the adaptive pole placement control (APPC) [Elbestawi, 1990], the robust adaptive control [Kooi, 1995], the extended model reference adaptive control (MRAC) [Rober and Shin, 1996]. Landers and Ulsoy [Landers and Ulsoy, 2000] presented a stability and performance analysis for an MRAC controller explicitly accounting for modeling errors. By identifying the cutting geometry such as radial depth of cut from NC routines, Tarng et al. [Tarng et al., 1993] attempted to maintain a constant cutting force level in pocket milling by adjusting feedrates adaptively. Other adaptive feedback control techniques applied to this problem include the neural network based control [Tarng et al., 1994] and the fuzzy logic controls [Kim et al., 1994]. However, there have been, by far, not many practical applications of the adaptive feedback control system in the manufacturing industries. The difficulty to implement an accurate, reliable, and economic way to monitor cutting forces in process and to ensure the reliability of the control has prevented feedback control schemes from being widely implemented in industrial applications [Rehorn et al., 2005].

A simpler but more practically feasible way is in-priori adjustment of feedrate on an NC program. In this case, feedrates are scheduled based on several regime of cutting engagement or continuously varied to regulate constant material removal rate (MRR) or cutting force. For example, Spence and Altintas [Spence and Altintas, 1994] scheduled feedrates in order to satisfy the cutting force, torque, and dimensional constraints. In order to regulate the MRR at a possible maximum level, Yamazaki et al. [Yamazaki et al., 1991] and Fussell et al. [Fussell et al., 2001] used a feedrate planning system to select feedrate for 3-axis sculpture milling by integrating a Z-buffer geometric model with a discrete mechanistic model of the cutting tool. By using a simplified cutting force model and 2D chip load analysis, Bae et al. [Bae et al., 2003] proposed an automatic but off-line feedrate adjustment method to regulate cutting load for pocket machining. Budak [Budak, 2000] used the nonlinear mechanistic model proposed by Spence and Altintas [Spence and Altintas, 1991], for the regulation of peak cutting force,

and showed that the model-based feed-forward control approach exhibited a similar control performance as an adaptive feedback control approach. Recently some latest commercial CAD/CAM software adopt a simple geometric process simulator to determine the feedrate such that an abrupt change in cutting forces can be avoided particularly at sharp corners.

An important pre-requisite for feedrate scheduling approaches should be that a process model to predict cutting forces is given, and accurate enough. And the performance of feedrate scheduling approaches is dependent on the sophisticated servo-controller of machine tools that possesses a rapid acceleration and deceleration control mechanism to response to the frequent and quick change of feedrate in actual machining operation. On commercial CNCs, it is often the case that the feedrate tracking performance is sacrificed in order to secure the contouring accuracy under the prescribed tolerance. Due to this sort of uncertainty in feedrate control performance, it is often difficult to apply a feedrate scheduling scheme on a finishing path, where unsmooth or abrupt transient change in feedrate often deteriorates the surface finish.

2.4.2 Regulation of Cutting Forces by Tool Path Modification

While feedrate control scheme has limitation of its application, the tool path modification approach has become promising to regulate cutting force and hence improve geometric accuracy of the machined surface particularly in finishing process. This approach aims to control the cutting forces by modifying the tool path geometry or using a special tool path trajectory. Iwabe et al. [Iwabe et al., 1989] proposed to add additional circular arcs to regulate the prescribed cutting engagement in convex corner cutting. Tsai and Takata [Tsai and Takata, 1991] enhanced Iwabe's idea to remove excess material in cornering but the improved tool path could deal with the complicated corner shapes. By using a pixel-based simulation technique, later Kim et al. [Kim et al., 2006] proposed optimized tool path to regulate constant MRR in corner cutting by inserting additional looping paths. The method reviewed in the above researches is an

effective ad-hoc way to avoid an abrupt and large increase in the cutting force at sharp convex corners, which often becomes a potential cause of the tool damage, by reducing the radial of depth of cut there. Since it is intended to apply only to sharp corners, it cannot be applied to continuously regulate cutting engagement angle on an arbitrary curve.

Recently, Stori and Wright [Stori and Wright, 2000] presented a notable approach to offset tool path modification for keeping constant cutting engagement on a tool path of a convex geometry. Their algorithm basically aims to modify the tool path by shifting the tool center location at a distance to the direction normal to the original path, such that a desirable cutting engagement angle is regulated. However, since the algorithm modifies the final tool path, it no longer removes required geometry, leaving excess material in corner cutting. Therefore, their approach would also be effectively realized mainly for a high speed and stable steady-state roughing operation, but not for a finishing process. Further, their approach is only applicable to the limited set of corner shapes like convex arc. Later Wang et al. [Wang et al., 2005] proposed a quantifiable metric-based approach to 2D tool-path optimization by considering instantaneous path curvature and cutter engagement.

Stori and Wright's approach [Stori and Wright, 2000] will be reviewed in Section 2.6.1 in more details. The modified algorithm, where a limitation of their algorithm that can be applied only to tool paths of convex geometry is addressed, will be proposed in Section 2.6.2. An inherent critical limitation of these approaches will be discussed in more details in Section 2.6.3, and Section 3 will present a new approach to address this issue.

2.5 Computation of Cutting Engagement Angle

The cutting engagement angle is an influential cutting parameter which dominates the cutting load in two-dimensional contour end milling. To evaluate the cutting load, geometric simulation is required to estimate the instantaneous

cutting engagement angle. It is to be noted that for any arbitrary tool paths, the cutting engagement angle cannot be computed analytically. This is because the cutting engagement at a particular point in time is critically dependent on the in-process geometry, which is a function of the *history* of the tool path. Therefore, to perform a simulation of the cutting engagement angle, a numerical procedure is presented to compute the cutting engagement angle for the given standard format machining NC programs.

2.5.1 Algorithm for Computation of the Engagement Angle

For a given tool center trajectory extracted from machining NC programs for two-dimensional contour milling process, the following outlines an algorithm for the computation of the cutting engagement angle. It is to be noted that the present algorithm can be seen as the simplification of geometric modeling algorithms for a general three-dimensional (3D) swept volume (e.g. [Wang et al., 1986]).

To start the computation, assume that (1) a trajectory of tool center location, $O(i) \in \mathfrak{R}^2 (i = 1, \dots, N_o)$ (CL data), and (2) an initial geometry of the previously machined workpiece surface on the same plane, $P(i) \in \mathfrak{R}^2 (i = 1, \dots, N_p)$ are given. Figure 2-7 shows a simple 2D end milling geometry illustrating the computation of the cutting engagement angle. When the tool center position is located at $O(i)$ at any i , the cutting engagement angle, $\alpha_{en}(i)$ can be computed as follows.

(a) *The intersection of the tool and newly machined surface (Point $Q(i)$)*

For the given tool center location of the tool path trajectory, $O(i) \in \mathfrak{R}^2 (i = 1, \dots, N_o)$, compute the intersection point of the tool circumference with the newly machined surface, $Q(i) \in \mathfrak{R}^2 (i = 1, \dots, N_o)$ as shown in Fig. 2-7, by parallel offsetting $O(i)$ to the workpiece's side by the tool radius, r . This operation can be written by:

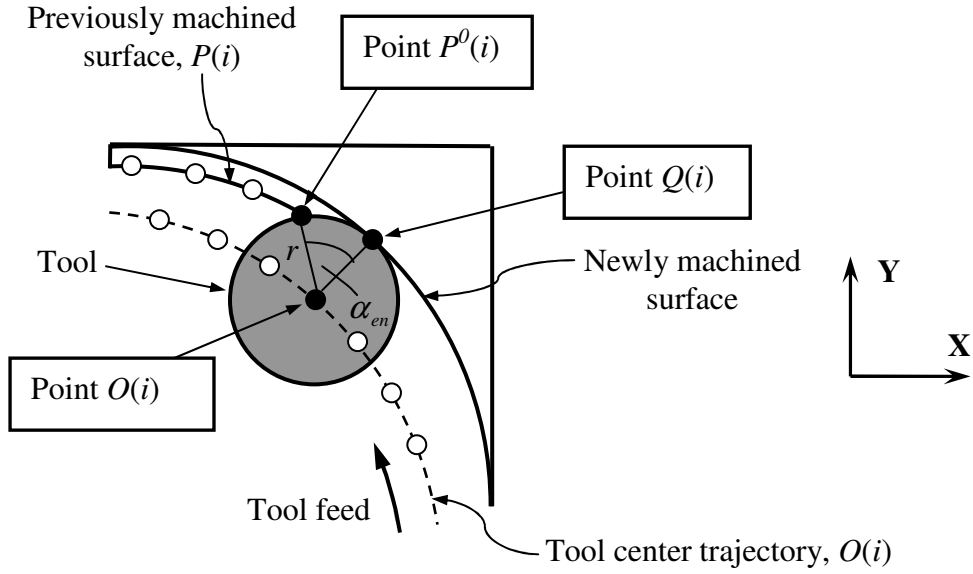


Figure 2-7 Computation of the cutting engagement angle, α_{en}

$$Q(i) = \text{offset}(O(i), +r), \text{ where } (i=1, \dots, N_o) \quad (2-4)$$

where, the function “offset($O(i)$, x)” represents the computation of the trajectory that is generated by parallel offsetting the trajectory, $O(i)$, to the outside by the distance x .

For the computation of parallel offsets, there have been numerous research efforts to build algorithms with higher robustness and smaller computational complexity [Dragomatz and Mann, 1997]. In this research work, we adopt the algorithm developed by Held to compute parallel offsets based on the Voronoi diagram [Held, 2001].

(b) *The intersection of the tool and previously machined surface (Point $P^0(i)$)*

The intersection of the tool and previously machined workpiece surface (Point $P^0(i)$ in Fig. 2-7) is first computed such that the distance between the tool center, $O(i)$ and a point on the previously machined surface trajectory, $P(i) \in \mathfrak{R}^2 (i=1, \dots, N_p)$ becomes equal to the radius of the cutting tool, r . If there exists no point among $P(i)$ within the distance r from the tool center, then it is assumed that there is no interference between the tool and the workpiece, i.e. it is

an air cut. Otherwise, several points in $P(i)$ are chosen from the nearest neighbor of the tool center, and their trajectory is curve-fit to a polynomial function. Then, the location of the point, $P^0(i) \in \mathbb{R}^2$ can be computed as the intersection of this curve and a circle representing the tool surface.

(c) The cutting engagement angle, α_{en}

Once the point on the newly machined surface, $Q(i)$ and the point on the previously machined surface, $P^0(i)$ are computed, then for the cutting geometry shown in Fig. 2-7, the cutting engagement angle, α_{en} at any i can be computed as follows:

$$\alpha_{en}(i) = 2 \sin^{-1} \left(\frac{\|P^0(i) - Q(i)\|}{2r} \right) \quad (2-5)$$

where, “ $\|\cdot\|$ ” represents the 2-norm of a vector.

(d) *Updating the previously machined surface*

The points in $P(i)$ that are within the distance r from the tool center, are eliminated from the array. Point, $P^0(i)$ is then added to the array, $P(i)$, which forms the “new” previously machined workpiece surface.

By repeating the steps from (a) to (d) until the end of NC program, a profile of the cutting engagement angle, $\alpha_{en}(i)$ for any 2D cutting geometry can be computed.

2.5.2 An Illustrative Example

In this section, an illustrative example of computation of the cutting engagement angle with the above algorithm for a simple 2D contour cutting will be described. Figure 2-8 shows the cutting geometry for a 2D contour path. The workpiece to be machined contains three linear segments (marked by **A**, **C**, and **E** in Fig. 2-8) of length 10 mm, a concave arc (marked by **B**) of curvature radius

15 mm, and a convex arc (marked by **D**) of curvature radius 15 mm. During the computation of the engagement angle with the above algorithm, the tool radius of 5 mm and a radial depth of cut of 0.5 mm are considered.

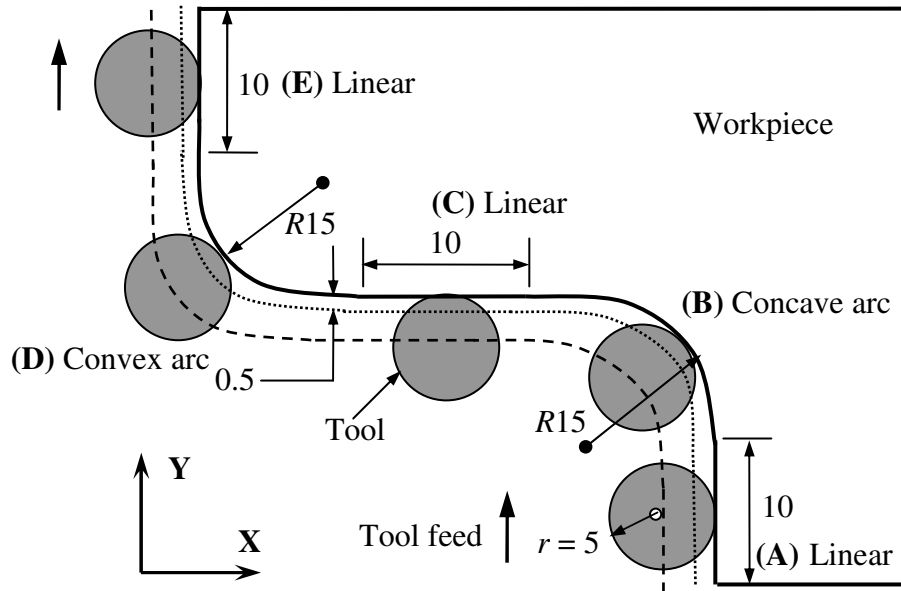


Figure 2-8 Cutting geometry for a 2D contour path

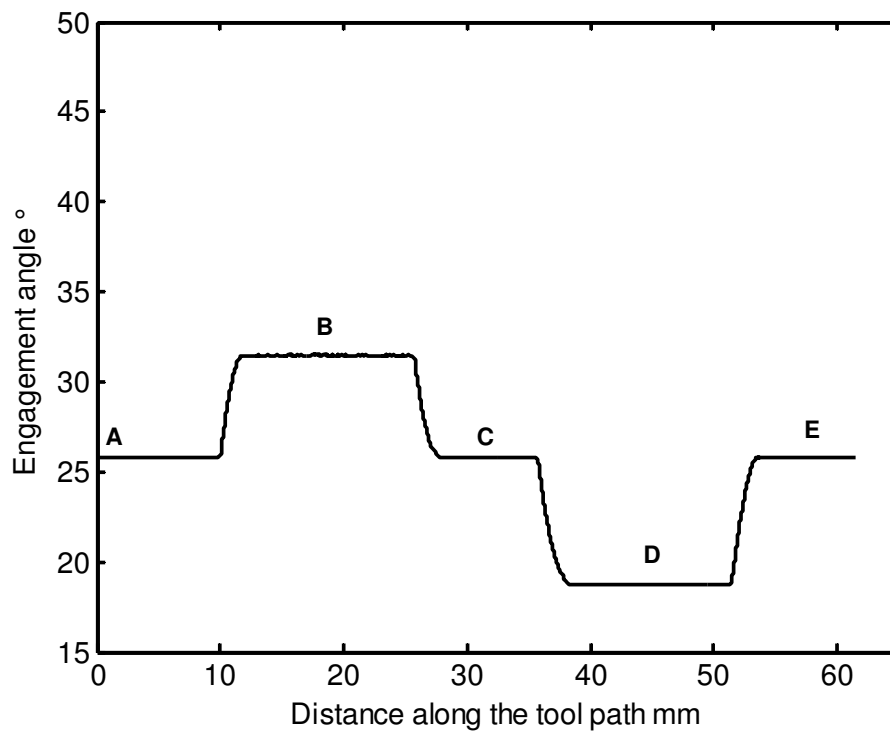


Figure 2-9 Engagement angle profile for a 2D contour path shown in Fig. 2-8

Figure 2-9 show a profile of the cutting engagement angle computed for the above cutting geometry. It is seen from Fig. 2-9 that the cutting engagement angle changes with the geometry of the workpiece. The engagement angle increases when the cutting tool approaches from a linear segment to a concave arc, and decreases when the cutting tool gets to a convex arc.

2.6 Forward Tool Path Modification for the Regulation of Cutting Engagement

Besides feedrate scheduling, the tool path modification approach has been an attractive and more reliable method to regulate the cutting engagement angle and consequently, cutting force at a desired level in 2D contour end milling. As is described earlier, in the case of tool path modification, the feedrate remains constant during machining along the entire path, and hence, the cutting engagement and cutting force will be regulated even without requiring advanced performance of a highly sophisticated servo controller of the machine tool. Stori and Wright [Stori and Wright, 2000] first proposed the forward tool path modification approach. Here, the term, “forward tool path modification” is realized in a sense that the proposed algorithm for tool path modification aims to start modifying innermost tool path in a forward direction until the final tool path (i.e. finishing path), which will give the final geometry of the workpiece to be machined. This section will first review their approach in Section 2.6.1, and then present the modification of the algorithm in Section 2.6.2 such that it can be applied to tool paths of arbitrary geometry.

2.6.1 Conventional Forward Tool Path Modification Algorithm

In an aim to maintain constant cutting engagement, and hence, steady-state cutting process for a convex geometry, Stori and Wright [Stori and Wright, 2000] proposed a notable approach to offset tool path modification. Since the

approach proposed by Stori and Wright [Stori and Wright, 2000] has been quite pioneer in generating modified tool paths for keeping constant engagement in two-dimensional contour milling and has motivated the works presented in this dissertation, the key idea of their approach to offset tool path modification must be required to be illustrated briefly.

Given an initial planer curve representing the previously machined surface denoted by a set of points, $P(i) \in \mathbb{R}^2 (i = 1, \dots, N_p)$, the main aim of the algorithm is to generate a offset newly machined surface and a series of tool center positions such that a constant engagement is maintained at all the times. Figure 2-10 shows a simplified illustration of the algorithm for tool path modification proposed by Stori and Wright. Assume that the following variables are known at a given point in time.

$P(i), P(i+1)$: the current and next points of intersection of the tool with the original workpiece surface, and

$Q(i) \in \mathbb{R}^2 (i = 1, \dots, N_q)$: the point of intersection of the tool with the newly machined workpiece surface.

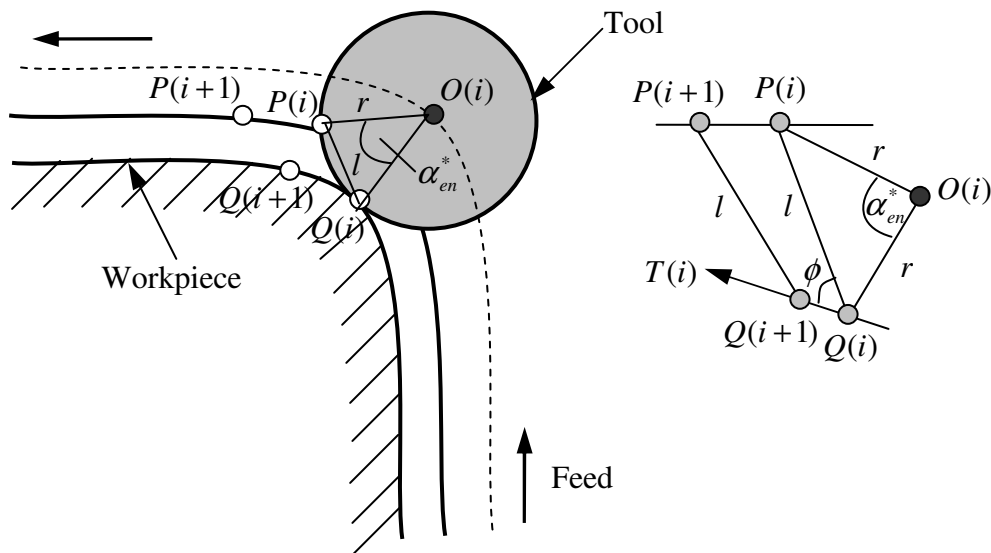


Figure 2-10 Simplified illustration of the algorithm for constant engagement tool path proposed by Stori and Wright [Stori and Wright, 2000]

Hence, given a tool radius, r and the desired cutting engagement angle, α_{en}^* to be regulated, the new offset surface point, $Q(i+1) \in \mathfrak{R}^2 (i=1, \dots, N_Q)$ and the modified tool center position, $O(i+1) \in \mathfrak{R}^2 (i=1, \dots, N_O)$ can be computed as follows:

(a) Compute the new offset surface point, $Q(i+1)$, by using the following equation:

$$Q(i+1) = Q(i) + \lambda T(i) \quad (2-6)$$

such that,

$$\|Q(i) + \lambda T(i) - P(i+1)\| = l \quad (2-7)$$

where l is the length of the base of the isosceles triangle formed with two sides of length, r and the desired cutting engagement angle, α_{en}^* , which can be calculated as:

$l = \sqrt{2r^2(1 - \cos \alpha_{en}^*)}$, and $T(i) \in \mathfrak{R}^2$ is a unit vector tangent to the new offset surface (see Fig. 2-10).

(b) Compute the unit tangent vector at the new offset surface, $T(i+1)$ by using the following equation:

$$T(i+1) = \frac{P(i+1) - Q(i+1)}{\|P(i+1) - Q(i+1)\|} \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \quad (2-8)$$

where, ϕ is an inclination angle of the unit tangent vector, $T(i)$, with the vector, $P(i+1) - Q(i+1)$ (see also Fig. 2-10).

(c) Compute the tool center position, $O(i+1)$, by the following equation:

$$O(i+1) = Q(i+1) + r \cdot T(i+1) \begin{bmatrix} \cos \frac{\pi}{2} & -\sin \frac{\pi}{2} \\ \sin \frac{\pi}{2} & \cos \frac{\pi}{2} \end{bmatrix} \quad (2-9)$$

(d) Set $i = i + 1$ and repeat the steps from (a) to (d) until the end of the whole machining path.

It must be noted that tool center position generated by the above algorithm is only applied when the given set of points defining the previously machined surface, $P(i)$, forms a convex geometry. The algorithm fails when it is not convex. This is a very strong limitation in practical applications, since tool paths in practical applications especially in die and mold manufacturing usually have a complicated geometry and almost never satisfy the convexity condition. Machining results of the constant engagement tool path generated by the algorithm can be found in [Stori and Wright, 2000].

2.6.2 Proposed Forward Tool Path Modification Algorithm

Motivated with the idea introduced by Stori and Wright [Stori and Wright, 2000] as described earlier, we propose a new approach to constant engagement tool path generation such that it can be applied to any kind of contour geometry containing convex and concave arcs.

Given the planner curve representing the geometry of the final contour to be machined, the objective of the proposed algorithm is to modify an original contour parallel tool path such that the cutting engagement angle is regulated at the given desired level throughout the path. Figure 2-11 illustrates the concept of the proposed algorithm for forward tool path modification. Assume that the tool radius, r , a sequence of points representing an initial contour parallel tool path (i.e. original finishing tool path hereafter), $O(i) \in \mathbb{R}^2 (i = 1, \dots, N_o)$, and a sequence of points representing the previously machined surface, $P(i) \in \mathbb{R}^2 (i = 1, \dots, N_p)$ are given. At each step, the tool center location, $O(i)$ is moved to the direction normal to the tool path by a distance, $x(i)$ such that the cutting engagement angle, $\alpha_{en}(i)$ is regulated at the given desired level, α_{en}^* . The detailed algorithm for the computation of the modified tool path can be described as follows:

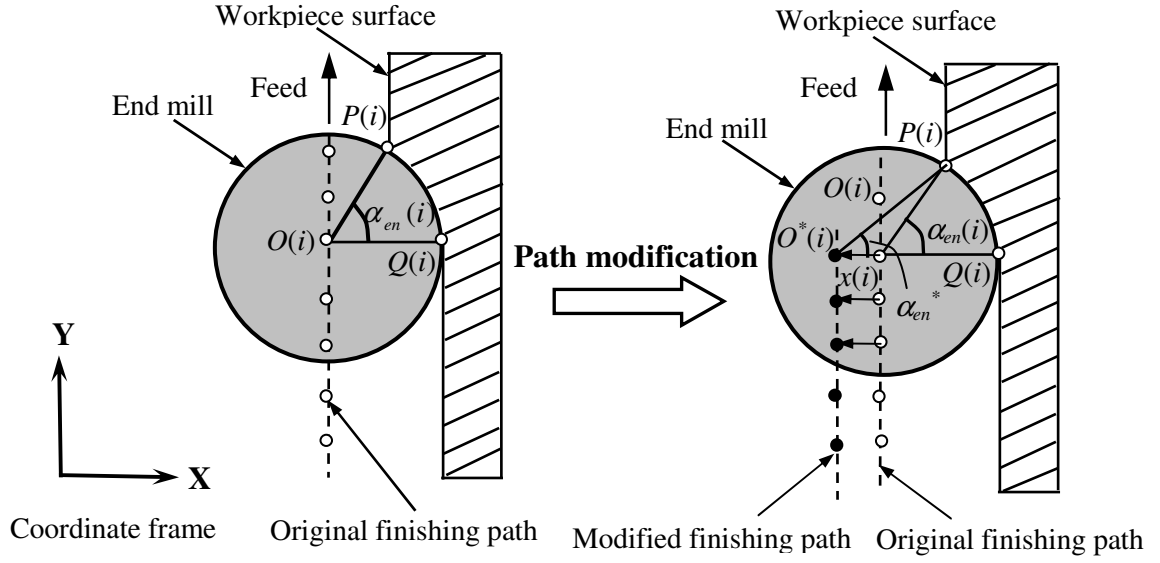


Figure 2-11 Concept of the proposed algorithm for forward tool path modification

(a) For a point in the tool center trajectory, $O(i)$, and in the previously machined surface, $P(i)$, compute the point in the offset surface or newly machined surface, $Q(i) \in \mathcal{R}^2(i, \dots, N_Q)$ in the same manner presented in Step (a) in Section 2.5.1.

(b) Move the original tool center location, $O(i)$ to $O^0(i)$ by an initial modification distance, $x(i-1)$. Compute the cutting engagement angle, $\alpha_{en}^0(i)$ for the tool center location, $O^0(i)$ by using the algorithm presented in Section 2.5.1.

(c) Compare the computed engagement angle, $\alpha_{en}^0(i)$ with the given desired engagement angle, α_{en}^* . Based on an error between $\alpha_{en}^0(i)$ and α_{en}^* , compute a new modification distance, $x(i)$ by performing one step of the Newton-Raphson method as follows:

$$e(i) = \alpha_{en}^* - \alpha_{en}^0(i) \quad (2-10)$$

$$x(i) = x(i-1) - \frac{e(i)}{\frac{de(i)}{dx(i)}} \quad (2-11)$$

(d) Set $i = i + 1$ and repeat the steps from (a) to (d) until $i = N$

Hence, by executing the above algorithm for forward tool path modification, a new modified finishing tool path, $O^*(i) \in \mathbb{R}^2 (i = 1, \dots, N)$ is generated such that the given desired cutting engagement angle, α_{en}^* is regulated throughout the path.

It is to be noted that, in this approach, the step of Newton-Raphson method is not repeated more than once to simply shorten the computation time. In Step (b), the cutting engagement angle, $\alpha_{en}(i)$, can be computed from the geometric interaction of the tool and the workpiece surface as described in Section 2.6. In Step (c), the engagement angle gradient, $\frac{de(i)}{dx(i)}$ is computed only numerically by using this function. Unlike the algorithm given by Stori and Wright [Stori and Wright, 2000], the present algorithm does not mathematically guarantee the convergence to optimal solutions. However, a significant advantage of the present algorithm is that it can be applied to an arbitrary 2D contour geometry, unlike the algorithm in Stori and Wright [Stori and Wright, 2000], which is restricted to only convex contours.

2.6.3 An Illustrative Example of Forward Tool Path Modification

An illustrative example of tool path modification by the proposed algorithm for forward tool path modification on a simple corner geometry is shown in Fig. 2-12. In the figure, three original contour parallel paths, namely Path-1, 2 and 3, are modified by the proposed algorithm such that a constant engagement is regulated. Path-3 in Fig. 2-12 is the final path which gives the final workpiece contour.

By using the modified tool path generated by the proposed algorithm and original contour parallel tool path, cutting experiments on a commercial 3-axis vertical machining center (VM4-II by OKK) are conducted. The machining conditions used in the experiments are shown in Table 2-1. Figure 2-13 shows

cutting force profile on the machining along the path-3 for the workpiece geometry shown in Fig. 2-12. It is seen from Fig. 2-13 that compared to contour parallel path, the variation in cutting force is significantly reduced when the modified tool path generated by the proposed algorithm is applied.

Table 2-1 Machining conditions used in experiments

Cutting tool	Sintered carbide (SSUP4100ZX) Radius, $r = 5\text{mm}$, 4 flutes
Workpiece	Carbon steel (S50C) Corner arc radius, $R = 15\text{mm}$
Feedrate, F (mm/min)	1000
Radial depth of cut, R_d (mm)	1
Axial depth of cut, A_d (mm)	10
Spindle speed, N (min^{-1})	2800
Cutting direction	Down cut
Coolant	Dry air

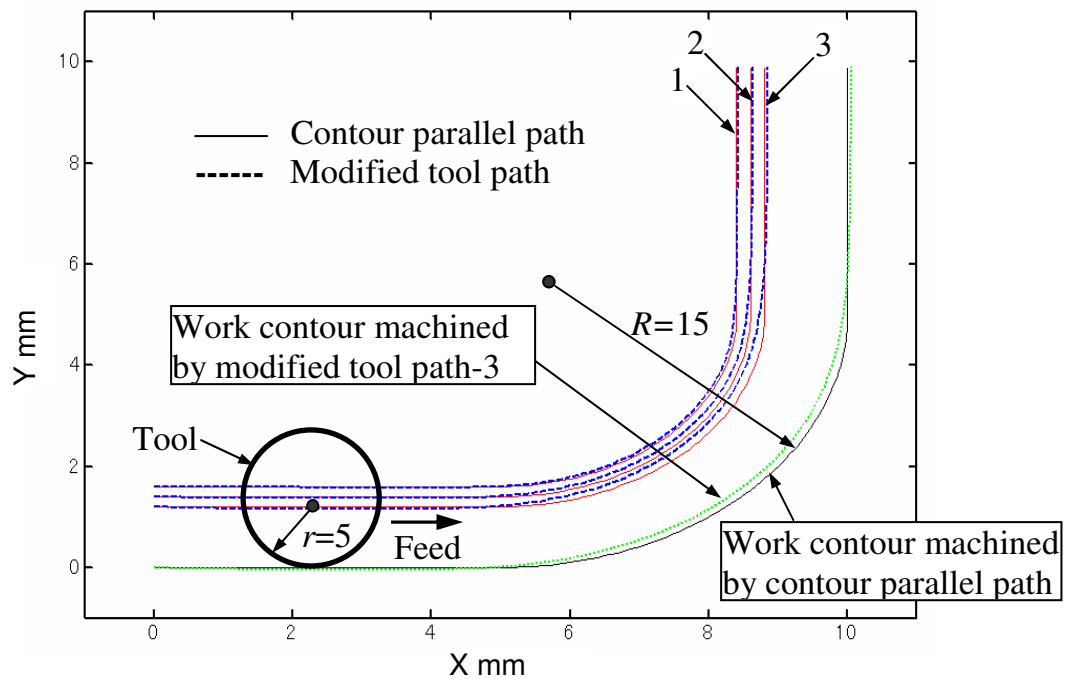


Figure 2-12 Modified tool path generation on a simple corner by the proposed forward tool path modification algorithm

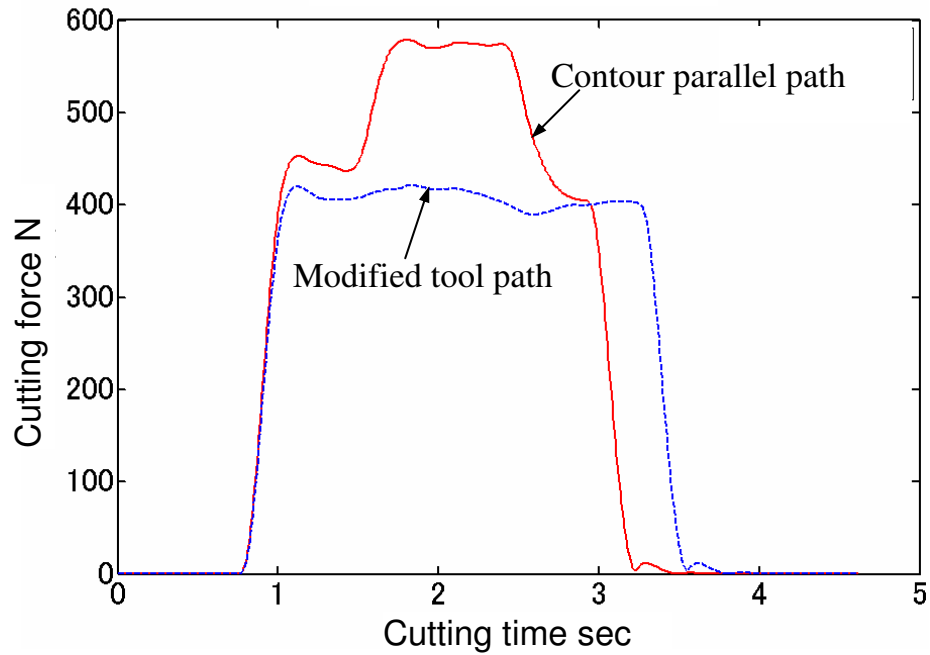


Figure 2-13 Cutting force profiles on the machining along path 3 in Fig. 2-12

2.6.4 Critical Issues in the Forward Tool Path Generation Approach

Consider an application of the proposed algorithm for forward tool path modification to a spiral-out tool path for pocket machining. As an example, conventional contour parallel tool paths are applied for pocket machining as can be seen in Fig. 2-14. By applying the proposed algorithm, original contour parallel tool paths are modified from the innermost path to the outermost path such that a given desired cutting engagement angle is regulated for each path. Figure 2-15 shows an example of modified constant engagement tool path generated by the proposed algorithm. As is clear from the figure, the outermost modified tool path is subject to the largest modification, and thus the required pocket contour cannot be made completely. Since the algorithm modifies the tool center position of a final or finishing path itself to keep constant engagement on it, the modified tool path no longer removes required geometry, leaving excess material in corner cutting. To achieve the required pocket geometry, additional machining will be required to remove excess corner materials.

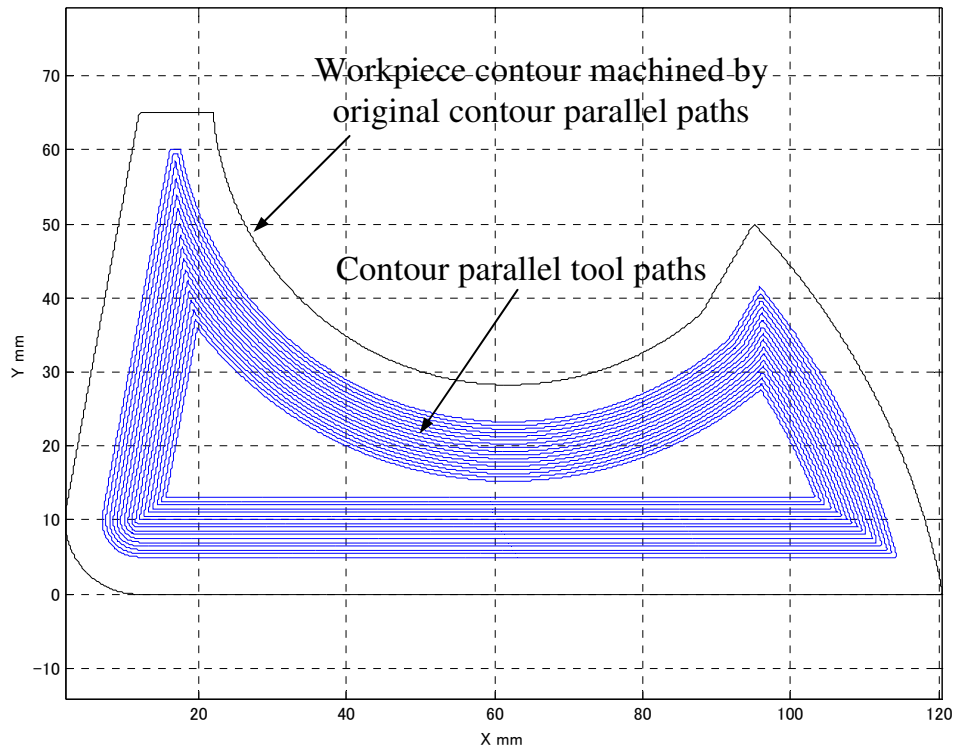


Figure 2-14 Original contour parallel tool paths in pocket machining

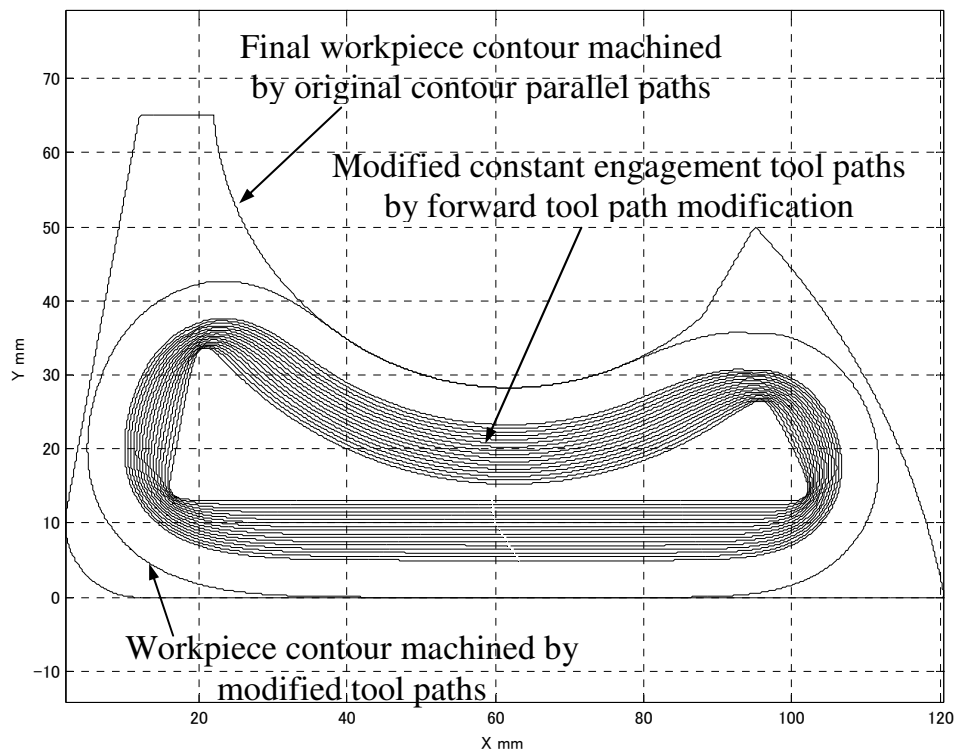


Figure 2-15 Modified constant engagement tool path generated by the proposed algorithm for forward tool path modification

From the modified tool paths shown in Fig. 2-15, hence, it can be said that although the proposed approach to tool path modification may be justified in cases where efficient material removal (i.e. roughing processes) is of the primary objective, it is not possible to be applied in finishing processes. Stori and Wright [Stori and Wright, 2000] presented an application of the present approach to roughing processes by using spiral-in approaches.

2.7 Conclusion

In this chapter, conventional contour parallel tool path used in 2D contour end milling and cutting problems associated it, such as variation in cutting engagement and cutting force are described and addressed. Research works to tackle cutting problems associated with contour parallel paths are reviewed. While the performance of feedrate scheduling to regulate cutting engagement can be limited by the lack of advanced performance of servo-controller of machine tools, the tool path modification approach becomes more promising. Hence, in order to overcome the cutting problems, an algorithm to generate constant engagement tool path by forward tool path modification is proposed. However, the critical issues with the proposed forward tool path modification approach are that, for spiral-out paths geometry, the outermost tool path is subject to the largest modification, hence leaving excessive material at the corners. Therefore, while the approach can be justified in roughing, it may not be applied to the finishing process.

Chapter 3

Constant Engagement Tool Path Generation by Backward Tool Path Modification

3.1 Introduction

In Chapter 2, we presented the forward tool path modification approach to generate offset tool paths subject to a constant engagement angle, and it was experimentally verified that the variation of cutting forces can be significantly suppressed by applying the constant engagement tool path. However, since the present approach modifies the finishing tool path itself, the modified tool path no longer generates the required workpiece contour. Therefore, it is clear that the present forward tool path modification approach cannot be applied to a finishing path in spiral-out pocketing.

To overcome this issue, in this chapter, we propose an algorithm to generate a new offset tool path trajectory by the backward tool path modification, which will regulate cutting engagement at a constant level on the finishing path. Unlike the algorithm to generate offset path by the forward tool path modification as described in Chapter 2, the inherent idea of the proposed algorithm is to modify the previous tool path trajectory (i.e. semi-finishing path) with an aim that a desired cutting engagement angle is regulated on the machining with the final path while the geometry of the final path itself is preserved. While the forward tool path modification approach has focused on mostly stable roughing operation, the proposed algorithm mainly deals with efficient finishing operation in order to produce an improved geometric accuracy and surface quality of the final machined contour.

The remainder of the chapter is organized as follows. Section 3.2 describes the proposed algorithm to generate a new offset tool path by the backward tool path modification. An illustrating example of the tool path modification is given.

In order to evaluate and compare the performance of the proposed algorithm with the conventional approaches, a feedrate scheduling approach based on the cutting force prediction model is illustrated in Section 3.3. In Section 3.4, several case studies with cutting tests on the workpieces made of different contour geometries and workpiece materials are carried out to experimentally verify the effectiveness of the proposed approach. Lastly Section 3.5 concludes the chapter with a brief summary.

3.2 Proposed Approach

3.2.1 Algorithm of Backward Tool Path Modification for Constant Cutting Engagement

Given an initial planar curve representing the desired geometry of the final contour to be machined, and an original contour-parallel (CP) tool path to achieve the desired contour (referred to as the finishing path hereafter) extracted from NC code, the main aim of the algorithm is to compute the previous tool path trajectory (the path prior to the finishing path) such that the engagement angle can be regulated at a desired level on the machining along the final tool path trajectory (i.e. finishing path). It should be emphasized that this study focuses on the machining by a straight end mill, and thus only the two-dimensional interference between a tool and workpiece is considered.

Here, the term “backward tool path modification” is comprehended in the sense that, for a given final tool path and the desired engagement angle to be regulated along the final path, the proposed algorithm aims to modify the previous tool path in backward direction.

Assume that a trajectory of the tool center location in the finishing path, $O_k(i) \in \mathbb{R}^2 (i = 1, \dots, N_k)$, is given by offsetting the final workpiece contour to be machined. As illustrated in Fig. 3-1, the engagement angle, $\alpha_{en}(i) \in \mathbb{R}^2 (i = 1, \dots, N_k)$, is defined by the tool center location, $O_k(i)$, the

intersection point of the tool circumference with the newly generated offset surface, $Q_k(i) \in \mathfrak{R}^2 (i=1,...,N_k)$, and the intersection point of the tool circumference with the semi-finish surface, $P_k(i) \in \mathfrak{R}^2 (i=1,...,N_k)$. The semi-finish surface, $P(i)$, is generated by the path prior to the finishing path, $O_{k-1}(i) \in \mathfrak{R}^2 (i=1,...,N_{k-1})$ (referred to as the semi-finish path hereafter). The intension of the proposed algorithm is to modify the location of the intersection point between the tool circumference and the semi-finish surface, $P_k(i)$, to the new location $P_k^*(i)$, in a way such that the cutting engagement angle is modified from $\alpha_{en}(i)$ to its desired value, $\alpha_{en}^*(i)$. Notice that the modification of the semi-finish surface, $P_k(i)$, can be done by the modification of the semi-finish path, $O_{k-1}(i)$. The new semi-finish path, $O_{k-1}^*(i)$, can be given simply by offsetting $P_k^*(i)$. The detailed algorithm of the computation of new modified offset tool path trajectory, $O_{k-1}^*(i)$, can be summarized into the following steps.

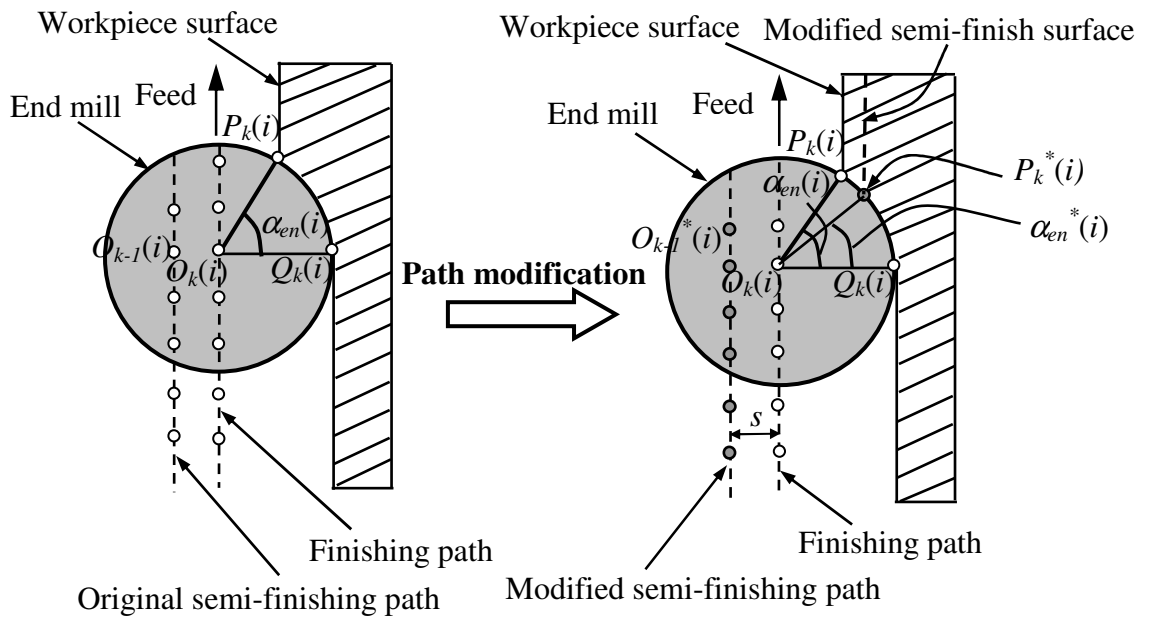


Figure 3-1 Concept of the algorithm for tool path modification to regulate cutting engagement angle

Step1: For the given tool center location of the finishing path, $O_k(i) \in \mathfrak{R}^2 (i = 1, \dots, N_k)$, compute the intersection point of the tool circumference with the newly generated offset surface, $Q_k(i) \in \mathfrak{R}^2 (i = 1, \dots, N_k)$, by offsetting $O_k(i)$ to the workpiece's side by the tool radius, r . This operation can be written by:

$$Q_k(i) = \text{offset} (O_k(i), +r), \text{ where } (i = 1, \dots, N_k) \quad (3-1)$$

where, as has been defined in Section 2.5.1, the function “offset($O(i)$, x)” represents the computation of the trajectory that is generated by parallel offsetting the trajectory $O(i)$ by the distance x .

Step2: Compute the “desired” intersection point of the tool circumference with the semi-finish surface, $P_k^*(i) \in \mathfrak{R}^2 (i = 1, \dots, N_k)$, such that the engagement angle, $\alpha_{en}(i)$, can be maintained at the desired cutting engagement angle, $\alpha_{en}^*(i)$. In other words, find $P_k^*(i)$ such that:

$$\angle P_k^*(i) \cdot O_k(i) \cdot Q_k(i) = \alpha_{en}^*(i), \text{ and } \|P_k^*(i) - O_k(i)\| = r, (i = 1, \dots, N_k)$$

where, the symbol “ $\angle$ ” represents the angle formed by the points.

Notice that $P_k^*(i) \in \mathfrak{R}^2 (i = 1, \dots, N_k)$ defines the trajectory of modified semi-finish surface (see Fig. 3-1).

Step3: Set $i = i + 1$ and repeat the steps (1) and (2) till $i = N_k$.

Step4: Then, by offsetting the modified semi-finish surface trajectory, $P_k^*(i)$, by the tool radius r , compute the modified tool center trajectory of the semi-finish path, $O_{k-1}^*(i) \in \mathfrak{R}^2 (i = 1, \dots, N_{k-1})$.

$$O_{k-1}^*(i) = \text{offset} (P_k^*(i), -r), \text{ where } (i = 1, \dots, N_k) \quad (3-2)$$

To summarize, a simplified flowchart of the proposed algorithm for backward tool path modification is illustrated in Fig. 3-2. The parallel offset of the location $O(i)$ by the distance x , denoted by offset ($O(i)$, x), can be computed by shifting $O(i)$ to the angle bisector direction between the vectors $O(i)-O(i-1)$ and $O(i+1)-O(i)$, as shown in Fig.3-3. Therefore, notice that in the computation of the modified semi-finish path, $O_{k-1}^*(i)$, in Eq. (3-2), its offsetting direction may

not be the same as that in the computation of $Q_k(i)$ in Eq. (3-1). In this research work, for the computation of parallel offsets, we adopted the algorithm based on the Voronoi diagram as stated in Section 2.5.1.

The desired engagement angle, $\alpha_{en}^*(i)$, along the final tool path trajectory must be given by considering proper machining conditions for the given tool and the workpiece such that an expected cutting force is maintained all the times. More details will be given in case studies presented in Section 3.4.

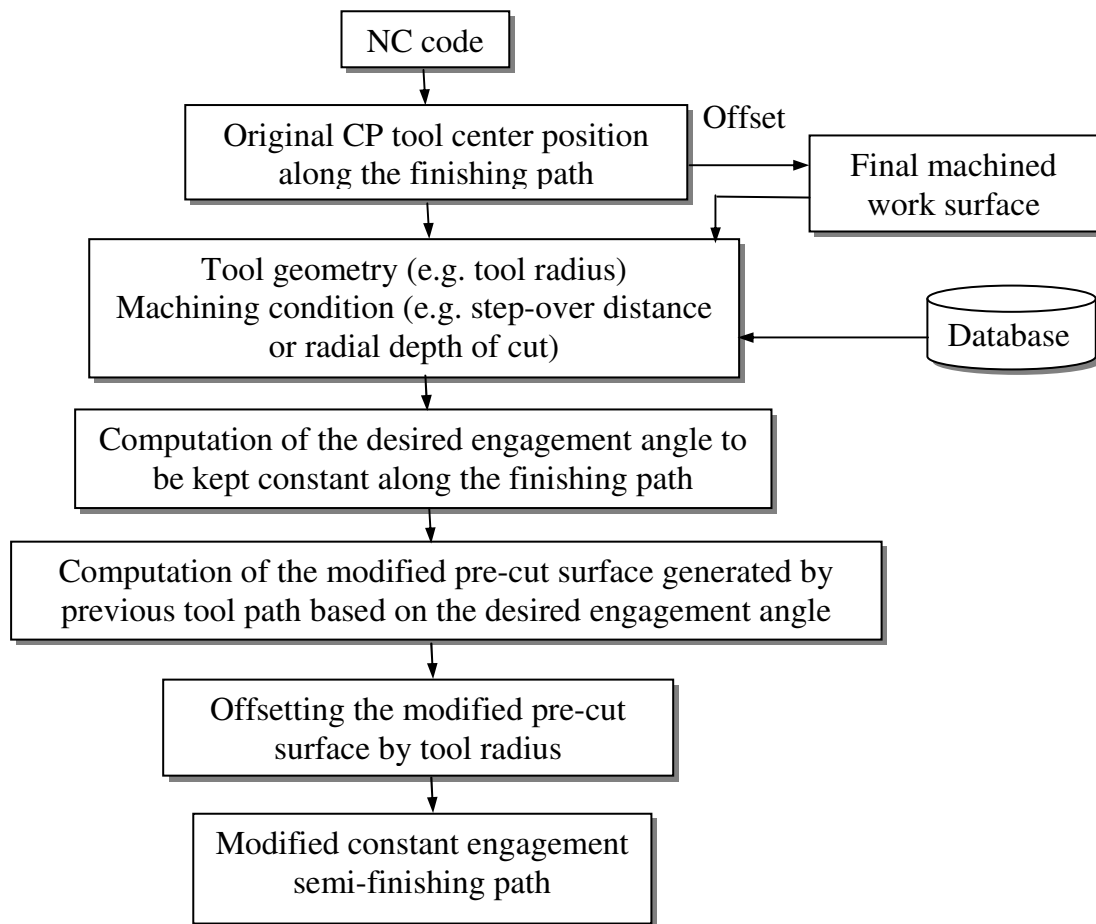


Figure 3-2 Simplified flowchart of the proposed algorithm for backward tool path modification

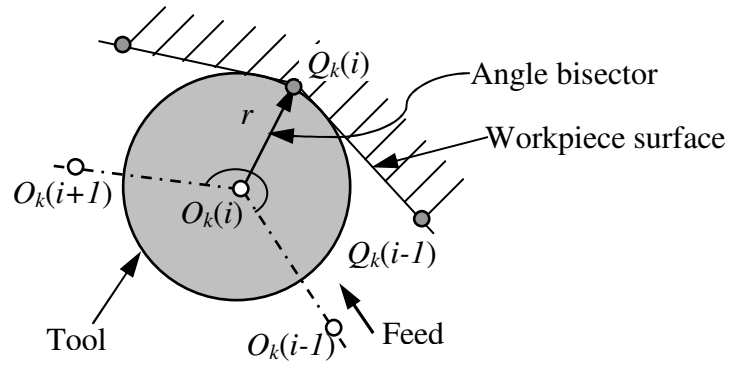


Figure 3-3 Parallel offset of the tool center location, $O_k(i)$ by the distance, r

3.2.2 An Illustrating Example of Tool Path Modification

For more clear understanding of tool path modification by the proposed algorithm, a simple illustrating example on a simple contour consisting of a concave and a convex arc is demonstrated. As can be seen in Fig. 3-4, both Path 1 and Path 2 correspond to conventional original contour parallel offset tool paths. Path 1 is applied to semi-finishing while Path 2 is a finishing path. By applying the proposed algorithm for backward tool path modification as illustrated earlier, Path 1 is modified into Path 1-a such that the cutting engagement angle with the workpiece along the Path 2 is regulated at a desired level. Cutter radius of 5.0 mm and step-over distance (i.e. radial depth of cut) of 1.0 mm are considered for parallel offsetting of tool paths to compute the modified semi-finishing path (Path 1-a).

It can also be seen from Fig. 3-4 that, along the concave arc, the semi-finishing path is modified to go “closer” toward the finishing path, which consequently results in smaller cutting engagement with the workpiece in the machining of the finishing path (Recall Fig. 2-6). On the contrary, along the convex arc, the semi-finishing path goes “farther” from the finishing path, indicating a larger cutting engagement in the machining of the finishing path. Hence, the resulting action is to regulate the cutting engagement angle always at a desirable level along the final workpiece contour on the machining of the finishing path.

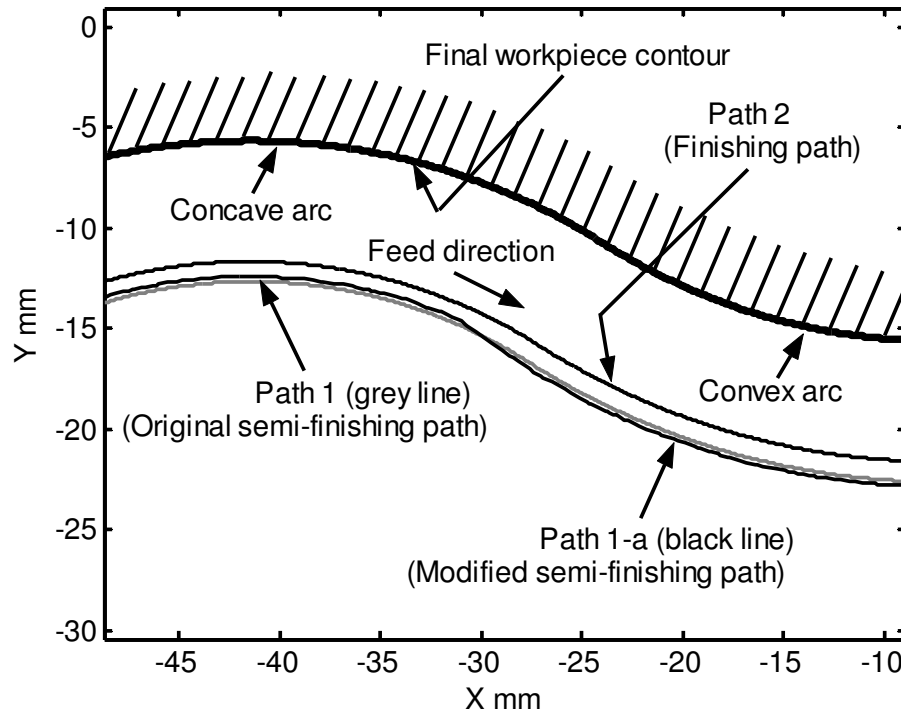


Figure 3-4 An illustrating example of tool path modification by the proposed algorithm

3.2.3 Remarks

Remark 1:

It must be emphasized that the proposed backward path modification algorithm focuses on the finishing process by using a straight end mill. In the machining of three-dimensional (3D) geometry, the finish path is in many cases executed with a ball or filleted end-mill. This may obscure the practical usefulness of the proposed approach. Even in such a case, we claim the proposed approach is of a practical value for the following reasons: (1) In die/mold machining, it is often the case that roughing and semi-finish processes are machined by using a straight end mill, followed by a finishing process by using a ball end mill (note that in this paragraph the terms “semi-finish process” and “finishing process” are used in a different meaning from previous sections). Since the radial depth of cut in a ball end mill process is generally very small, when the leftover volume from the semi-finish process significantly varies, it easily causes the variation in the final geometric error of the workpiece surface.

By applying the proposed approach to the semi-finishing process by a straight end mill, it can be expected that the semi-finishing process leaves more uniform surface error, which consequently improves the geometric accuracy of the final surface. (2) The engagement in a ball end mill process is determined by more complex three-dimensional interaction between the tool and the previous workpiece surface, and thus the straightforward extension of the proposed algorithm to a ball end mill process will be difficult. However, when the entire process assumes the 2D machining, it is in practice reasonable to limit the modification only on the XY plane, although, in this case, the cutting engagement cannot be strictly regulated. In such a case, the proposed algorithm for backward tool path modification presented in this dissertation can be extended to a ball end mill process in a straightforward manner.

Remark 2:

It must be noted that it is not always possible to generate the modified semi-finish path to regulate the engagement angle at the desired level by using the present approach. For example, when the tool center trajectory of the finishing path contains a square corner as shown in Fig. 3-5(a), the profile of cutting engagement angle, $\alpha_{en}(i)$, has a sudden jump, and becomes discontinuous, near the corner. Geometrically, if the semi-finish surface is given as illustrated in Fig. 3-5(b), the engagement angle can be regulated constant. It is, however, generally not possible to obtain the semi-finish path that generates this surface. This issue, caused by the discontinuity of the desired semi-finish path, will occur when the finishing path is not smooth. It must be emphasized that the proposed approach can be applied only to a smooth finish path, i.e. the case where the directional difference of the vectors $O(i)-O(i-1)$ and $O(i+1)-O(i)$ is smaller than some threshold value for any i .

In practical applications, it is often the case that the finishing path contains unsmooth corners. On such a corner, the cutting force, and consequently

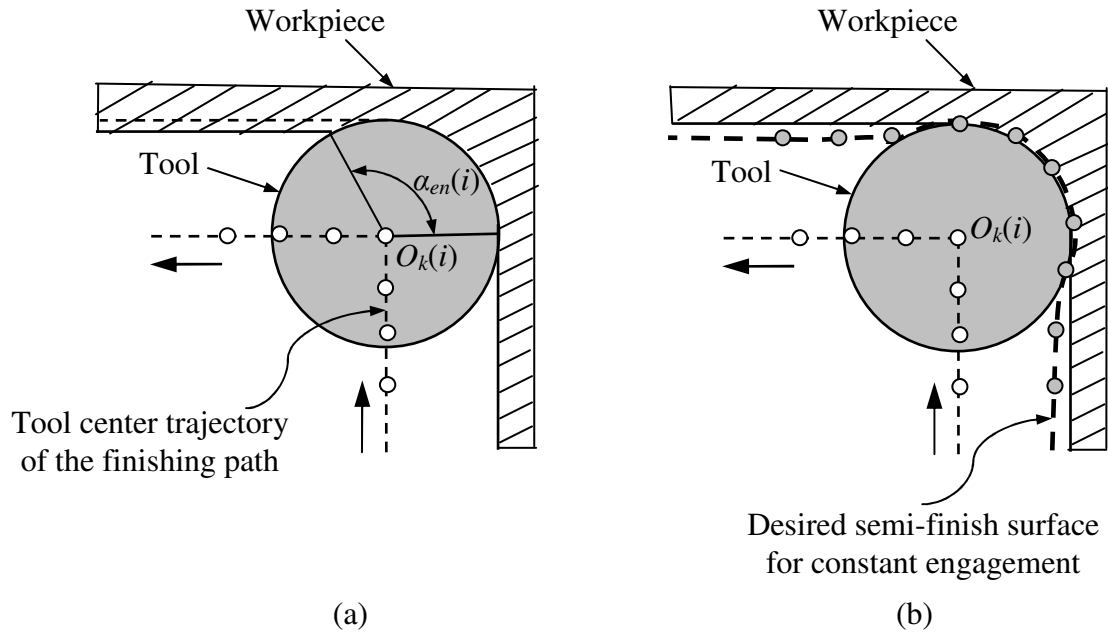


Figure 3-5 An illustrative example where constant engagement over the finishing path is not geometrically possible (a) Finishing path with a square corner, (b) Semi-finish surface for constant engagement

the geometric accuracy of the machined surface, are determined dominantly by the feedrate and the machine's contouring error. Under such a condition, it is extremely difficult to determine the "optimal" engagement angle such that the variation in cutting force can be minimized. Therefore, we consider that the regulation of the engagement angle on such an unsmooth corner would not contribute much to the improvement of machining accuracy, and thus that the limited applicability of the present approach is not of a practical importance.

3.3 Cutting Force Prediction Model and Feedrate Scheduling

In case studies which will be presented in Section 3.4, the control performance of cutting force with the proposed tool path modification approach will be compared to that with a feedrate scheduling approach. Hence, this subsection describes a cutting force prediction model based on which feedrates

are scheduled to regulate the cutting force at a desired level in two-dimensional contour end milling.

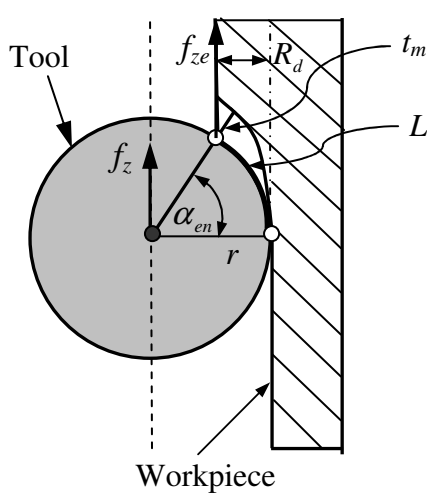
As a simpler approach to cutting force regulation in contour machining, process planning based on a cutting force prediction model has been widely studied. That is, if an accurate cutting force model is known, one can schedule or optimize the feedrate in priori such that the cutting force is regulated at a desired level. Because of its simplicity, a model-based feed-forward control approach has an inherent advantage over feedback control approaches to handle time delay of monitoring and control systems. In practice, many consider this model-based approach, which can be seen as a feed-forward control approach, to be more feasible and practical than the adaptive control approaches. A number of examples of research works on this model-based feed-forward control approach are briefly illustrated in the literature review section (Section 2.4.1) of Chapter 2.

In this section, we adopt a simple cutting force prediction model for the cutting geometry of 2D end milling proposed by Otsuka et al. [Otsuka et al., 2001]. Based on the cutting force prediction model, the procedure to schedule feedrate to regulate the cutting force at a desired level is also described.

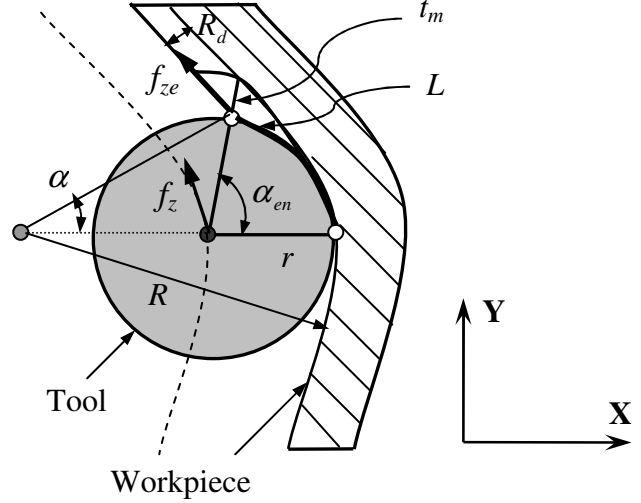
Figures 3-6(a) and (b) depict the 2D end milling geometry on a straight and a circular path respectively. In the figures, t_m and L respectively denote the maximum undeformed chip thickness and the arc length of cutting engagement (which is proportional to the engagement angle). Otsuka et al. [Otsuka et al., 2001] developed a simple mathematical “response surface” model to predict the cutting force by using these two process variables. According to the model, the resultant cutting force, F_{XY} on the XY plane (i.e. $F_{XY} = \sqrt{F_X^2 + F_Y^2}$) can be estimated by the following equation.

$$F_{XY} = C_0 + C_1 X_1 + C_2 X_2 + C_3 X_1^2 + C_4 X_2^2 + C_5 X_1 X_2 \quad (3-3)$$

where, X_1 and X_2 respectively indicates the maximum undeformed chip thickness, t_m and the arc length of the cutting engagement, L , normalized by their central values, and can be expressed as:



(a) On a straight path



(b) On a circular path

Figure 3-6 Schematics of the cutting geometry of 2D end milling process

$$X_1 = \frac{t_m - t_{m0}}{dt_m} \quad (3-4)$$

$$X_2 = \frac{L - L_0}{dL} \quad (3-5)$$

and, $C_0, C_1, C_2, C_3, C_4, C_5$ are the cutting force co-efficients which can be identified by cutting experiments.

From the geometry of end milling processes shown in Fig. 3-6, the following equation can be derived.

$$L = r \cdot \alpha_{en} \quad (3-6)$$

$$t_m = f_{ze} \cdot \sin(\alpha_{en} - \alpha) \quad (3-7)$$

$$(R - R_d)^2 = (R - r)^2 + r^2 + 2 \cdot (R - r) \cdot r \cdot \cos \alpha_{en} \quad (3-8)$$

$$\sin \alpha = \frac{r \cdot \sin \alpha_{en}}{R - R_d} \quad (3-9)$$

$$f_{ze} = f_z \cdot \frac{R - R_d}{R - r} \quad (3-10)$$

where, f_{ze} : feed per tooth at the tool center (mm/tooth), f_e : feed per tooth at the engagement point (mm/tooth), α_{en} : the cutting engagement angle (rad), α : the

angle between the directions of f_z and f_{ze} (rad), R : the arc radius of work surface (mm), R_d : radial depth of cut (mm), and r : the tool radius (mm).

When R , R_d , and r are given along a tool path trajectory, the optimum feedrate, f_z^* , to regulate the cutting force at a given desired level, F^* , can be determined uniquely by using Eqs. (3-3) ~ (3-10).

The performance of such a feedrate scheduling approach to regulate the cutting force completely depends on the accuracy of the prediction model. For an accurate prediction of the cutting force, the process model should contain various machining conditions that are quite difficult to model, such as tool geometry, tool material, tool coating, tool wear, workpiece material, and temperature.

However, in the sense that the cutting force prediction model shown in Eq. (3-3) is based on two parameters that define the geometry of an undeformed chip, it is analogous to well-known mechanistic models such as the one proposed by Yellowsley [Yellowsley, 1985]. Although the above model shown in Eq. (3-3) does not offer much insight into physical cutting mechanism, the practical validity of this second-order “response surface” model (Eq. (3-3)) has been verified in [Kakino et al., 2000 and Otsuka et al., 2001] by extensive cutting experiments over a wide range of tool path geometry (particularly on free-form curves), when six coefficients (C_0 , C_1 , C_2 , C_3 , C_4 , C_5) in Eq. (3-3) are properly identified for the given tool and workpiece.

Hence, in this study, as a conventional approach to regulate the cutting force, feedrate scheduling based on the above cutting force prediction model will be applied in case studies (Section 3.4) to experimentally validate and compare the performance of the proposed algorithm for backward tool path modification.

3.4 Experimental Validation

In this section, several case studies on different contour geometries of the workpieces made of different work-materials using straight end mills are demonstrated to experimentally justify the effectiveness of the proposed

algorithm for backward tool path modification. Experimental results in terms of cutting force, geometric error of the machined surface are investigated and compared with those for conventional approaches (e.g. contour parallel path, feedrate control scheme).

3.4.1 Case Study I: An Illustrative Example

In this case study, the proposed approach is applied to a simple corner geometry to illustrate its effectiveness. In this experiment, the desired cutting engagement angle, α_{en}^* , is determined as follows: assuming the machining along a linear path with the given tool radius, r , and the step-over distance (i.e. radial depth of cut), s , of original contour parallel paths, the cutting engagement angle, α_{en} is computed. This value is then used as the desired cutting engagement angle, α_{en}^* , for the tool path modification by the proposed algorithm. In other words, the proposed backward tool path modification scheme is applied such that the engagement angle along the finishing path does not deviate from the level under which the linear path in the finishing is cut.

For the purpose of applying the proposed algorithm for tool path modification, a simple contour geometry of the workpiece with its dimensions as shown in Fig. 3-7 is selected in the case study I. Figure 3-8 shows the modified constant engagement (CE) semi-finishing path (Path 1-a) generated by the proposed algorithm on the contour geometry along with the original contour parallel (CP) semi-finishing (Path 1) and finishing (Path 2) paths. First NC programs to produce the final geometry of the contour are generated by commercial CAM software, and the CP finishing offset path is extracted from NC programs. Then, by using the proposed algorithm for backward tool path modification, the modified CE semi-finishing path is generated as can be seen in Fig. 3-8.

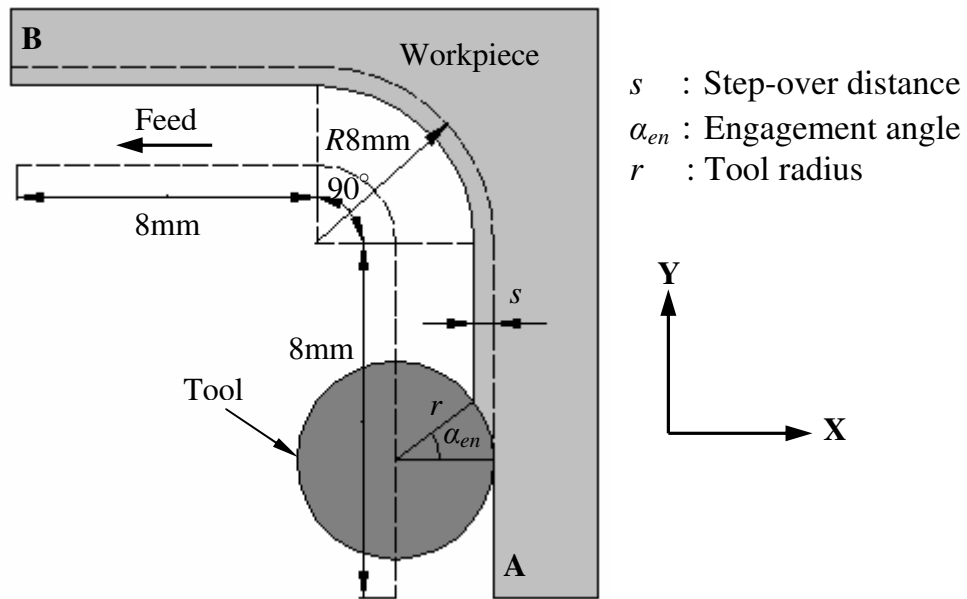


Figure 3-7 Geometry of the workpiece contour

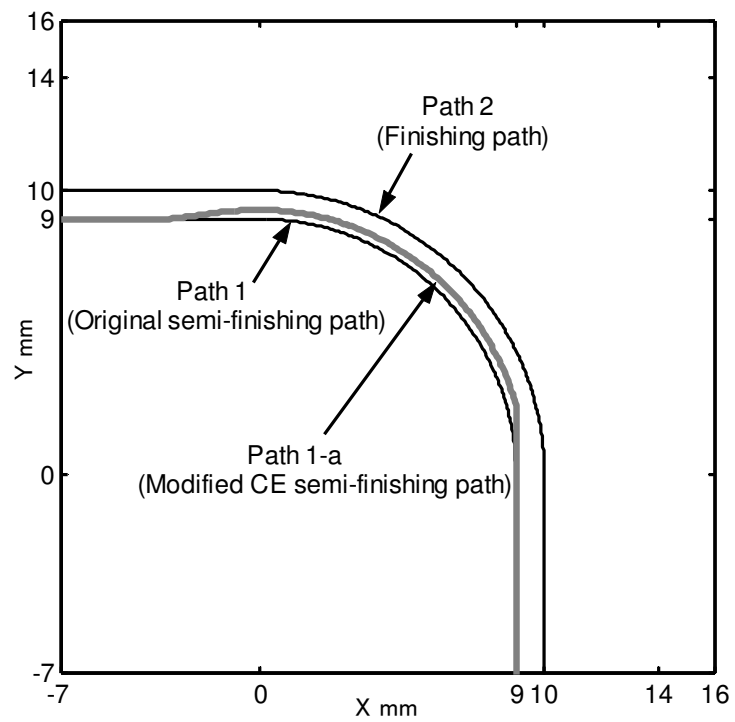


Figure 3-8 Modified CE semi-finishing too path generated by the proposed algorithm

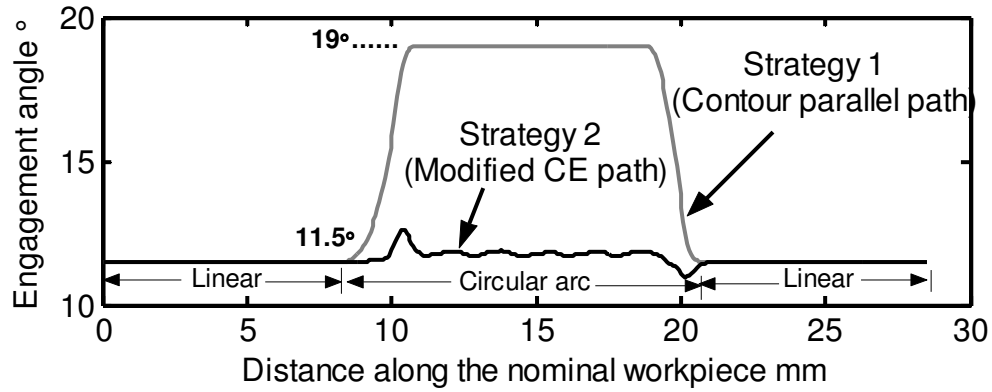


Figure 3-9 Simulated engagement angle profiles for the contour parallel path and modified CE tool path generated by the proposed algorithm

Further, for the above given contour geometry, Fig. 3-9 compares the simulated profiles of the engagement angle with an original CP path and the modified CE tool path generated by the proposed algorithm on the finishing path. It is distinctly evident from simulated results that while an original CP path shows a sudden jump (from 11.5° to 19.0°) in engagement angle at the circular arc part, the modified CE tool path is able to regulate the engagement angle almost at a desired constant level (11.5°) on the machining of the finishing path. The little discontinuity in the constant engagement angle profile at the entry and exit points between linear and circular arc could be due to the computational errors in simulation.

3.4.1.1 Experimental Details

Machining experiments are carried out to validate the effectiveness of the proposed approach. A three axis vertical high speed machining center (VCN-410A by Yamazaki Mazak Corp.) is used for the cutting tests. The basic specification of the machining centers is summarized in Table 3-1. A simple contour geometry as shown in Fig. 3-7 is used as the test workpiece. The machining conditions used in the tests are summarized in Table 3-2. For comparison of cutting performance, three machining strategies are adopted.

Strategy 1 (Contour parallel path) represents the case where conventional contour parallel paths are applied to both the semi-finishing (Path 1) and the finishing (Path 2). The feedrate is kept constant all the way.

Strategy 2 (Modified CE path) features the case where the proposed modified CE tool path is applied to the semi-finishing (Path 1-a), as presented in previous section. The finishing path (Path 2) is the same as that in Strategy 1. The feedrate is also maintained constant along the paths.

Strategy 3 (Feedrate control) features the case where variable feedrate is applied to the finishing path, as shown in Table 3-2, such that the cutting force is maintained constant throughout the finishing path. The geometry of the semi-finishing and finishing paths is the same as those in Strategy 1. The feedrate along the semi-finishing path is constant. Note that, for the given contour geometry, variable feedrates are determined by the cutting force prediction model and the procedure as described in Section 3.3.

In all the strategies, an (Al,Ti)N-coated sintered carbide straight end mill (diameter: 10 mm ($r=5\text{mm}$), 4 flutes) is used in the cutting test. The workpiece material is hardened steel, JIS SKD61 (HRC53).

Table 3-1 Specifications of the machining center

Parameters	Value
Spindle speed (min^{-1}) (max.)	15000
Rapid traverse (mm/min) (max.)	50000
Feed rate (mm/min) (max.)	8000
Acceleration G (max.)	X-axis: 0.50 Y-axis: 0.50 Z-axis: 0.75

Table 3-2 Machining conditions used in the experiments for case study I

Spindle speed (min^{-1})	4,775
Axial depth of cut (mm)	5
Feed per tooth (mm/tooth)	Strategies 1 and 2: 0.04 Strategy 3: 0.04 in the linear part 0.015 in the circular part
Step-over distance, s (mm)	0.10
Coolant	Dry air
Cutting direction	Down cut

3.4.1.2 Results and Discussion

During the machining tests, the resultant cutting force on the XY plane, F_{XY} is measured by a dynamometer (Kistler's 9272). It must be also noted that all the cutting force profiles presented in this dissertation are digitally filtered by using a low-pass filter of the cut-off frequency of 5 Hz. The profiles show the average of the variation in the cutting force for one rotation of the tool.

Figure 3-10 shows the comparison of the cutting force on the finishing path for three machining strategies. It is seen from the figure that while machining under Strategy 1 (Contour parallel path) indicates a large increase in the cutting force at the circular arc, by using the proposed modified CE path (Strategy 2), the variation of the cutting force is suppressed significantly within 17N. The variation of cutting force on machining under Strategy 3 (Feedrate control) is reduced, but it is larger than that under Strategy 2. This can be attributable to the lack of the machine's servo-control performance to follow the commanded feedrate to regulate the desired cutting force.

Figure 3-11 shows the geometric error profiles of the machined surface with respect to the nominal surface trajectory measured by a coordinate measuring machine (CMM). The geometric error (i.e. machined surface error) can be defined as the normal distance between the machined surface trajectory

and the nominal surface trajectory. For a clear comparison, the same error profiles drawn from Fig. 3-11 starting from “A” to “B” along the nominal surface trajectory are shown in Fig. 3-12. From Figs. 3-11 and 3-12, it is clearly seen, machining under conventional contour paths (Strategy 1) shows a left-over of the maximum depth of $14.7\mu\text{m}$ at the circular corner arc. However, by applying the proposed modified CE path (Strategy 2), the geometric surface error at the corner is significantly reduced. Further, as shown in both figures, machining under feedrate control (Strategy 3) is also effective to improve the machining accuracy. However, its effectiveness is more strongly influenced by the machine’s servo control performance. For example, geometric surface errors observed at an entry and exit of the corner arc in Fig. 3-11(c) are caused by the servo delay in velocity control. To validate it, a comparison of measured and commanded feedrate under Strategy 3 in finishing is shown in Fig. 3-13, indicating that an error between commanded and measured feedrates becomes significantly large especially at an entry and an exit of line and arc parts. In high-speed machining, and/or when a machine does not have sufficient velocity control performance, this problem possibly becomes more critical.

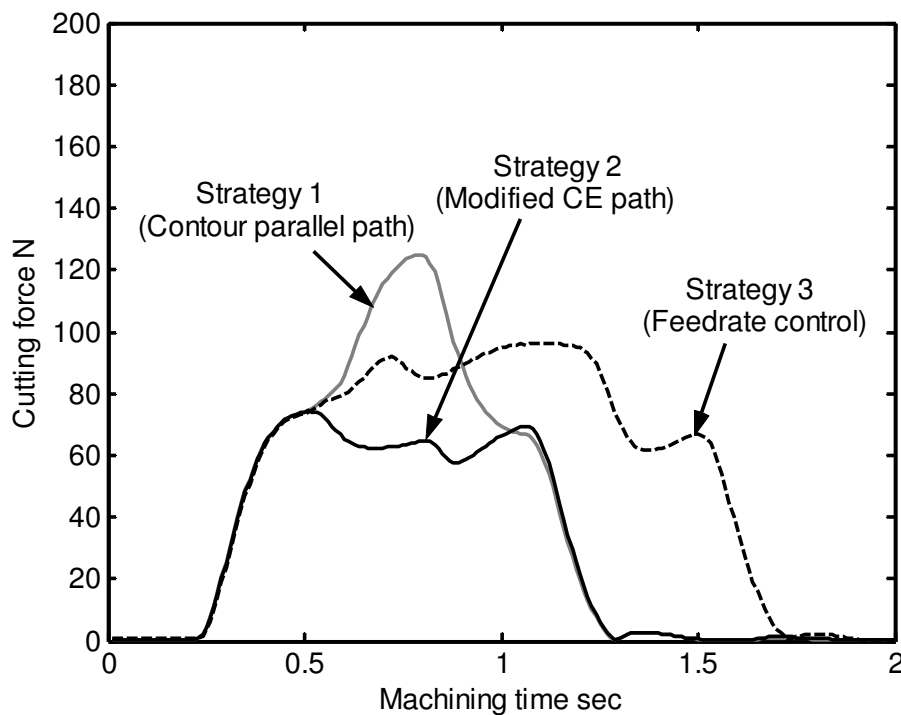
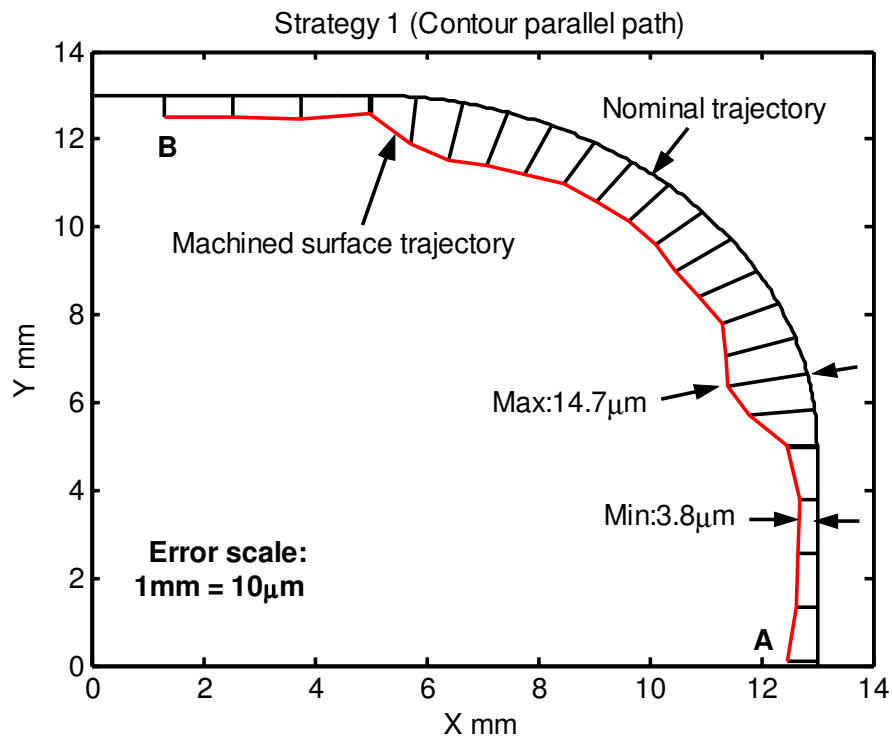
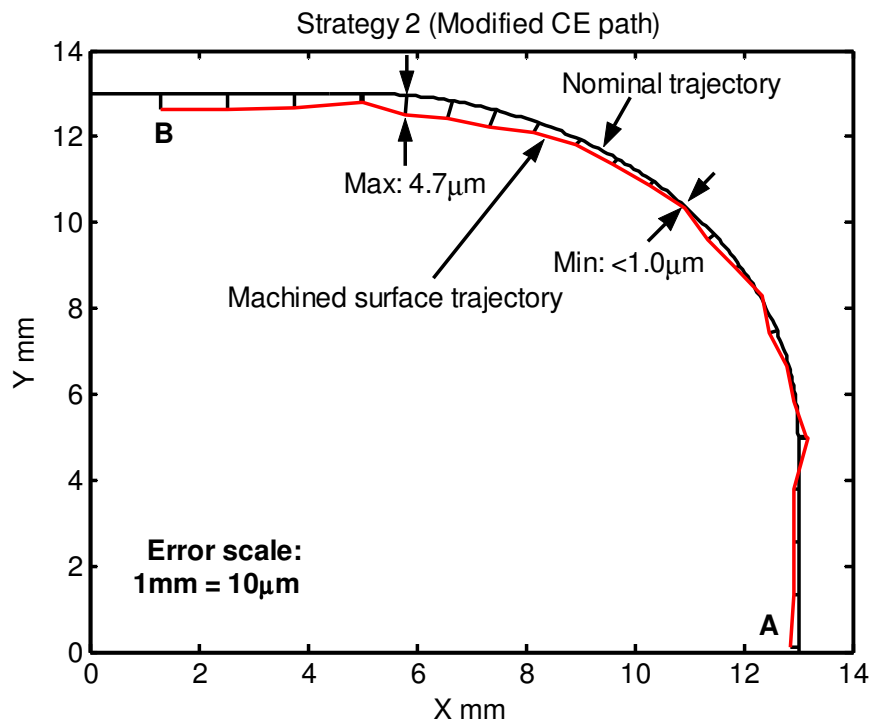


Figure 3-10 Comparison of cutting forces in finishing under three strategies

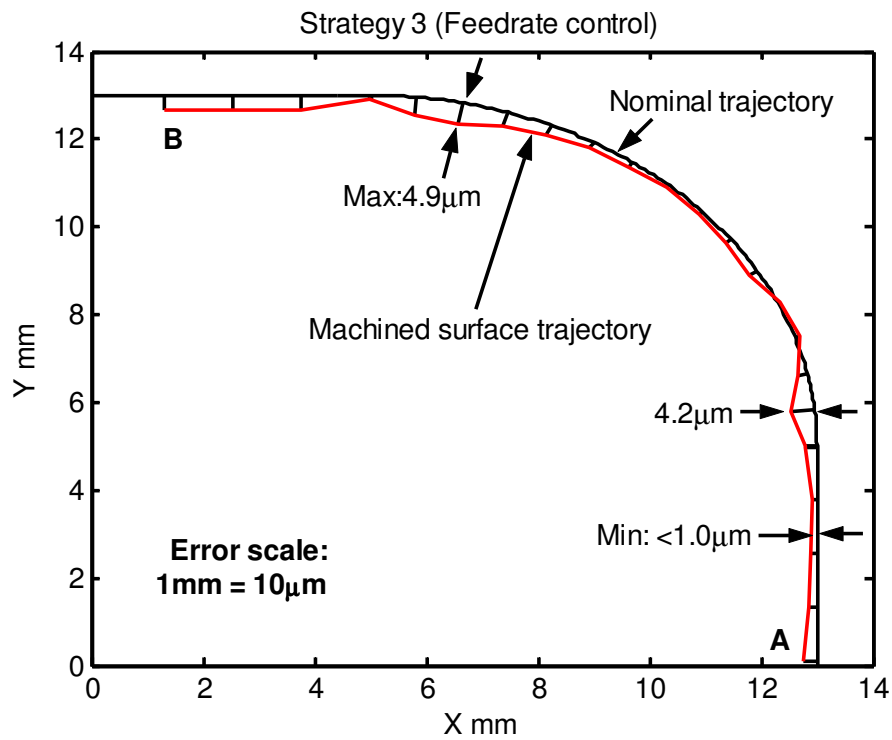


(a) Strategy 1 (Contour parallel path)



(b) Strategy 2 (Modified CE path)

Figure 3-11 Geometric error profiles of the machined surface with respect to its nominal trajectory for three strategies



(c) Strategy 3(Feedrate control)

Figure 3-11 Geometric error profiles of the machined surface with respect to its nominal trajectory for three strategies

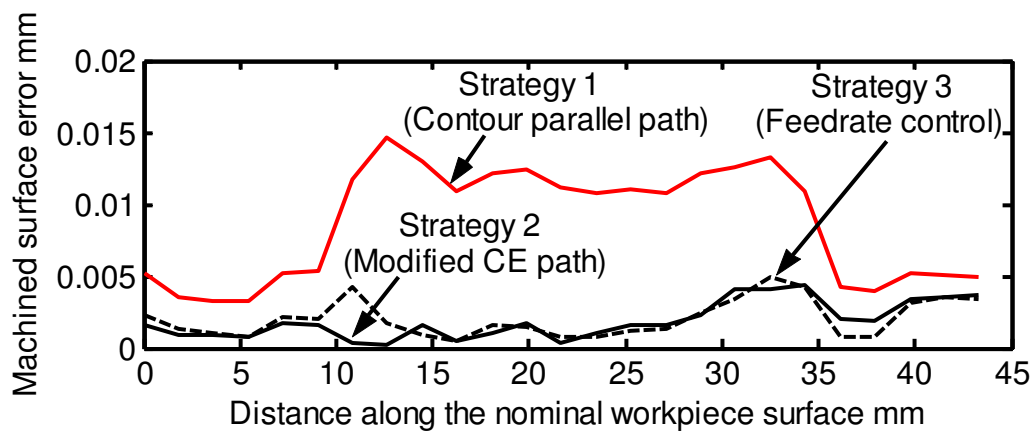


Figure 3-12 Profiles of the machined surface error for three strategies

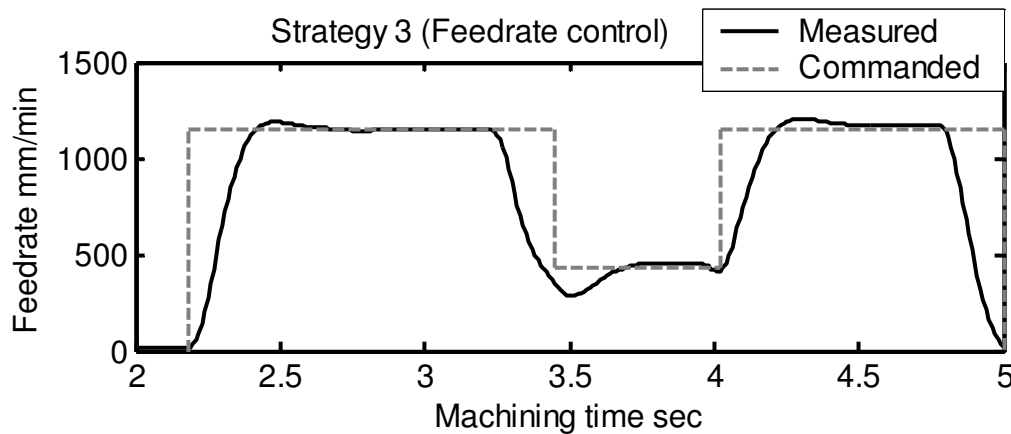


Figure 3-13 Comparison of measured and commanded feedrate under Strategy 3 (Feedrate control)

3.4.2 Case Study II: Carbon Steel Workpiece Case

In Case Study II, the proposed tool path modification approach is applied to the pocketing process of a more complex geometry. A pocket made of different circular arc geometries shown in Fig. 3-14 is used as the test workpiece. The workpiece material is the carbon steel (JIS S50C, size: 70mm×40mm×20mm) without any hardening. Figure 3-15 shows the modified CE semi-finishing path generated by the proposed algorithm along with contour parallel paths for machining the pocket geometry. Spiraling-out tool paths starting from machining the inner part until the final contour of the pocket are applied. The effect of path modification on the trajectory of the semi-finishing path can be seen in the magnified view of the tool paths in Fig. 3-15(b). The modified CE semi-finishing path consists of total 1544 line segments of the length 0.15 mm. The computer implementation of the proposed algorithm for backward tool path modification is carried out by using MATLAB by Mathworks, Inc. on a desktop PC of a 2.66 GHz Intel(R) processor. The computation time required from processing the G-code of the finishing path until the generation of the modified CE semi-finishing tool path is less than a second.

For the given tool radius and the step-over distance (i.e. radial depth of cut), a comparison of simulated engagement angle profiles with an original contour parallel path and the modified CE semi-finishing path generated by the

proposed algorithm in the finishing path is shown in Fig. 3-16. The computation of the simulated engagement angle profiles starts from the point “A” (as indicated in Fig. 3-14) of the pocket contour while the tool moves in a counter-clockwise direction along the contour. Simulated profiles of engagement angle indicate that by applying the modified CE tool path to the semi-finishing, the cutting engagement angle is able to be regulated at a desired constant level on the machining of the finishing path. The desirable constant engagement angle to be regulated in the finishing path is determined by the same procedure as in Case Study I.

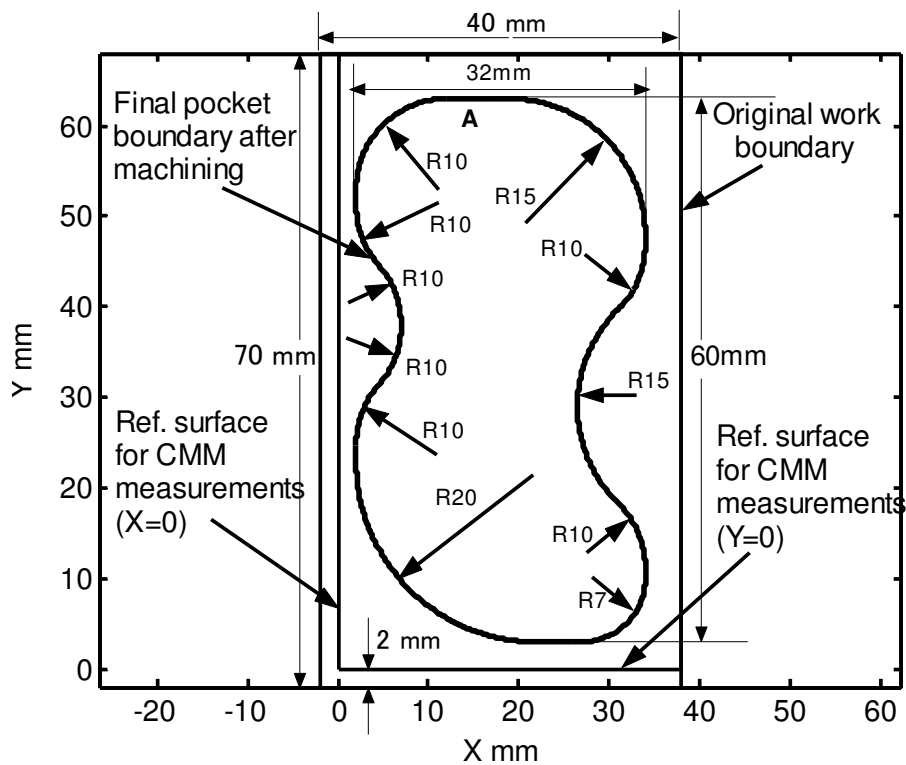
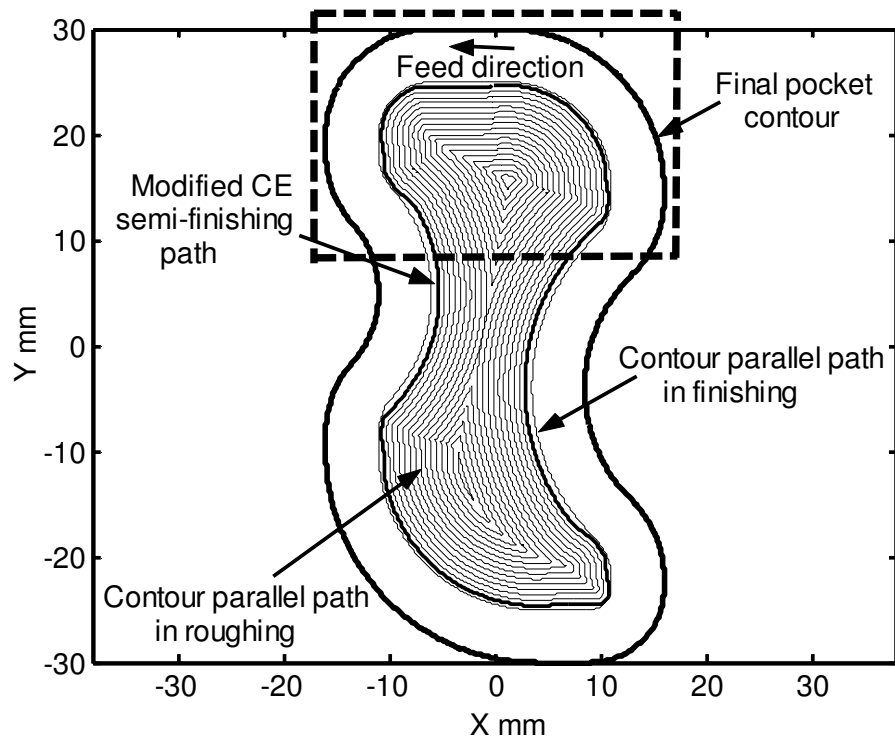
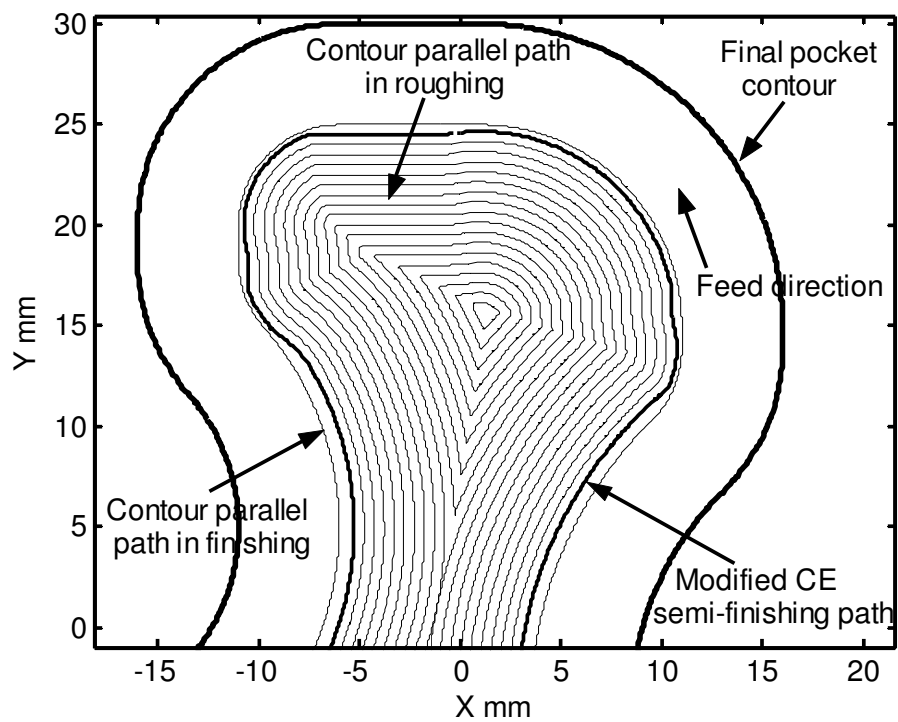


Figure 3-14 Geometry of the pocket to be machined



(a) Modified CE semi-finishing path generate by the proposed algorithm



(b) Magnified view of the tool paths in the rectangular box in (a)

Figure 3-15 Modified CE semi-finishing path generated on the pocket contour by the proposed algorithm along with contour parallel paths

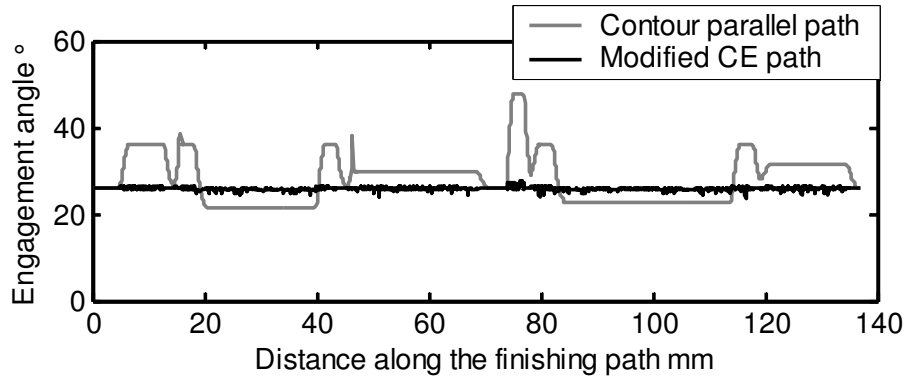


Figure 3-16 Profiles of simulated engagement angle generated by the proposed algorithm and the contour parallel path

3.4.2.1 Experimental Details

The cutting experiments are carried out on a three-axis high speed vertical machining center (VCN-410A by Yamazaki Mazak Corp.), as is used in the Case Study I. For comparison of the cutting performance, the whole cutting tests are categorized into the following three machining strategies.

Strategy 1 (Contour parallel path) represents the case where conventional contour parallel paths are applied throughout the machining (i.e. from roughing to finishing paths). The feedrate is kept constant all the way.

Strategy 2 (Modified CE path) features the case where the proposed modified CE tool path is applied to the semi-finishing, the path prior to the finishing operation as is shown in Fig. 3-15. All the paths except for the semi-finishing path are the same as those in Strategy 1. The feedrate is also maintained constant along the paths.

Strategy 3 (Feedrate control) features the case where variable feedrate is applied to the finishing path while the feedrate is varied such that the cutting force is maintained constant throughout the finishing path. The roughing and semi-finishing paths are the same as those in Strategy 1. Note that, for the given contour geometry, variable feedrates are determined by the cutting force prediction model and the procedure as described in Section 3.3. The varied feedrate profile will be shown in Section 3.4.2.3 (see Fig. 3-22).

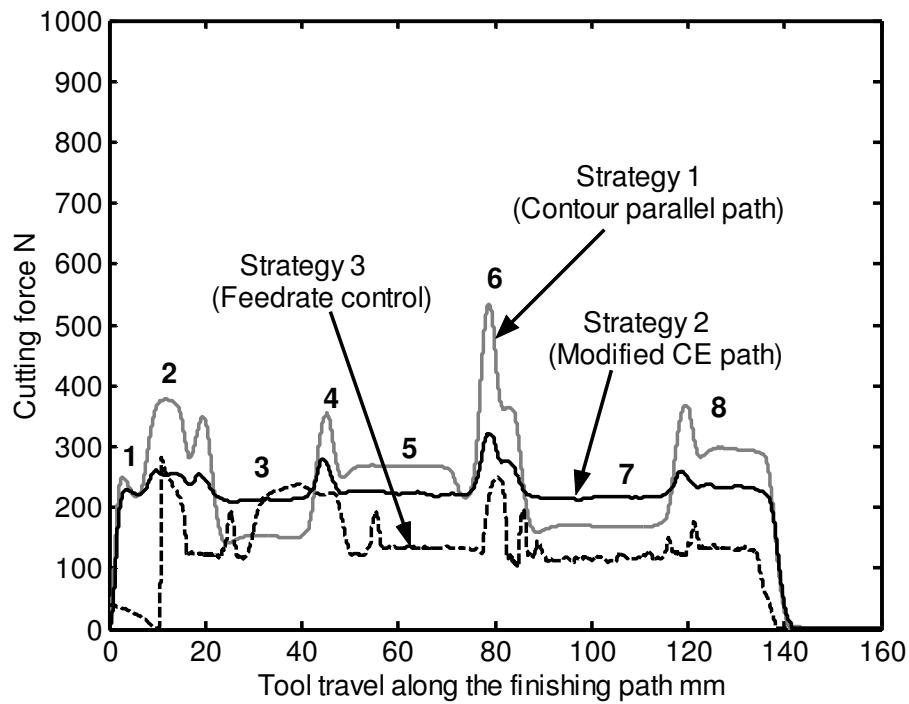
Table 3-3 Machining conditions used in the experiments for case study II

Spindle speed (min-1)	2800
Axial depth of cut (mm)	10
Feedrate (mm/min)	Strategy 1 and 2: 1000 Strategy 3: Variable
Step-over distance (mm)	0.50
Coolant	Dry air
Cutting direction	Down cut

Throughout the experiments, an (Al,Ti)N-coated sintered carbide radius end mill (tool radius: 10 mm, 4 flutes and tool extension: 35mm) is used. The machining conditions used in the experiments are illustrated in Table 3-3. It is to be noted that before the semi-finishing path, contour parallel tool paths with the step-over distance (i.e. radial depth of cut) of 0.50mm are used in three strategies for rough machining. During each cutting test, the cutting force is measured by using a piezoelectric four-component dynamometer, 9272 by Kistler Instrument Corp. After the finishing path, the geometric error of the final machined workpiece contour with respect to its nominal geometry is measured by a CMM, UPMC850 by Carl Zeiss.

3.4.2.2 Improvement of Machining Accuracy by the Modified CE Tool Path

Figure 3-17 depicts a comparison of cutting forces measured during the machining of the finishing path for Strategies 1, 2 and 3. This section first discusses the comparison between Strategy 1 (Contour parallel path) and Strategy 2 (Modified CE path) (the comparison with Strategy 3 (Feedrate control) will be discussed in Section 3.4.2.3). It can be distinctly observed that by applying the modified constant engagement (CE) tool path (Strategy 2), the cutting force variation is significantly reduced. Notably, a greater reduction in maximum cutting force variation of about 75% can be revealed at the very sharp

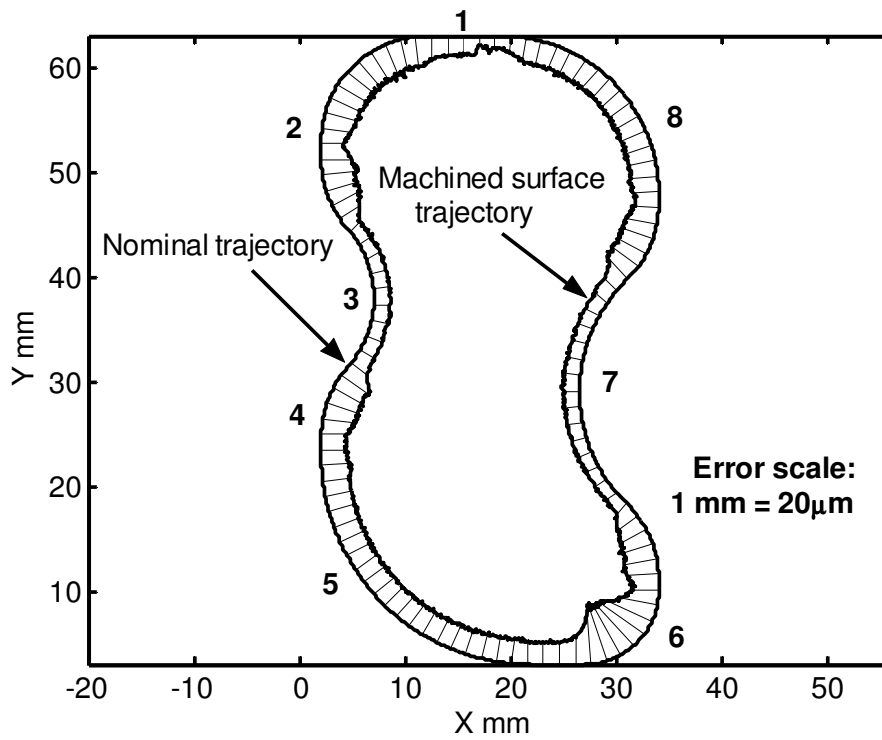


* The numbers 1~8 above the graphs indicate the corner name of the contour, corresponding to those in Fig. 3-18

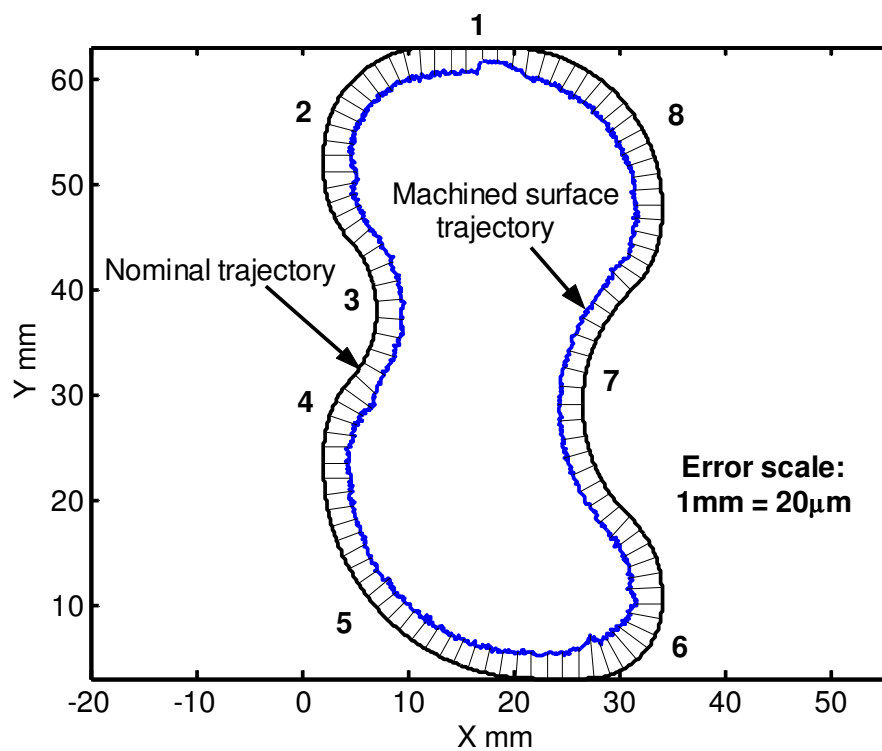
Figure 3-17 Comparison of the cutting forces in finishing under Strategies 1~3 (for the case with step-over distance of 0.50 in finishing)

arc (as indicated by “6” in Fig. 3-17) when compared with those in contour parallel path (Strategy 1).

Figure 3-18 illustrates geometric error profiles of the machined workpiece contour with respect to its reference trajectory measured. Figure 3-19 shows the same profiles drawn with the distance along the workpiece surface from the starting point indicated by “1” in Fig. 3-18. It is seen that, in terms of variation of machined surface error, the modified constant engagement tool path (Strategy 2) shows an improved machining accuracy compared to that in contour parallel path (Strategy 1). The variation in machined surface error is significantly less while the case with contour parallel paths reveals a distinct variation in machined surface error as is observed in Fig. 3-19. The mean value of the surface error over each of six arcs is summarized in Fig. 3-20. By applying the proposed modified constant engagement (CE) tool path (Strategy 2), a remarkable reduction in mean surface error can be observed at concave corners of smaller radius of curvature

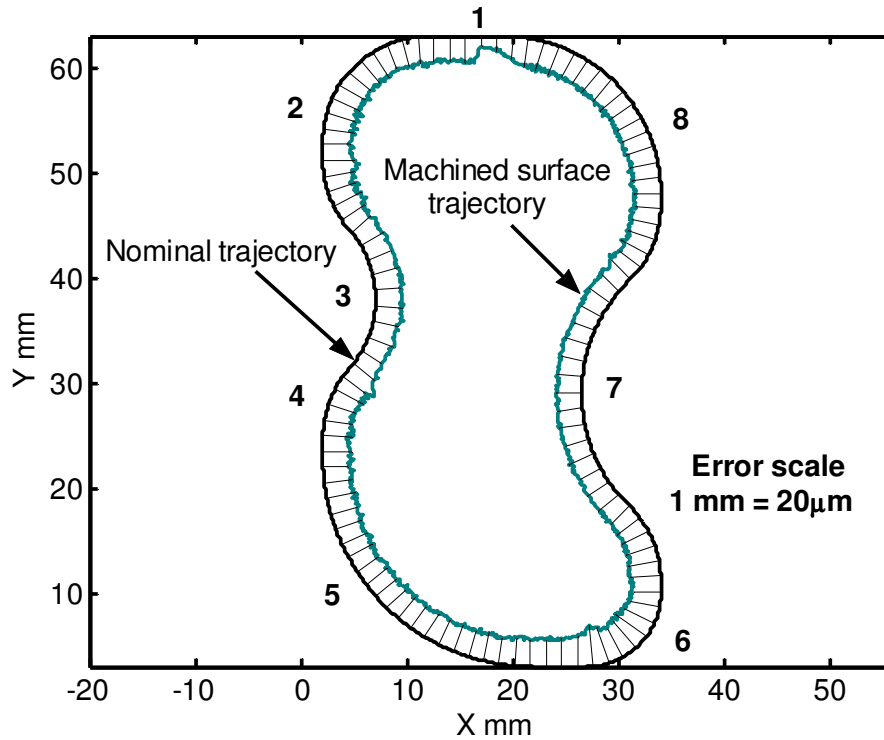


(a) Strategy 1 (Contour parallel path)



(b) Strategy 2 (Modified CE path)

Figure 3-18 Machined surface trajectories with respect to the nominal trajectory for Strategies 1~3



(c) Strategy 3 (Feedrate control)

Figure 3-18 Machined surface trajectories with respect to the nominal trajectory for Strategies 1~3

(as indicated by “2” and “6” in Fig. 3-20) while comparing with that for contour parallel path (Strategy 1). The maximum variation of surface error along the whole workpiece contour is reduced by approximately 70% when comparing with that for contour parallel path (Strategy 1). In this study, the maximum machined surface error variation is defined as the difference between the maximum machined surface error (i.e. peak of the error profile) and the minimum machined surface error (i.e. valley of the error profile).

In order to validate the effectiveness of the proposed scheme in another machining condition, another set of machining test on the same workpiece contour is conducted for the case where the step-over distance, s , is 0.1 mm in the finishing path. All the roughing paths are subject to the same step-over distance as the previous experiments, namely $s = 0.5$ mm. In practical pocketing processes, the step-over distance for the final finishing path is often significantly smaller than that in roughing paths, in order to improve the geometric

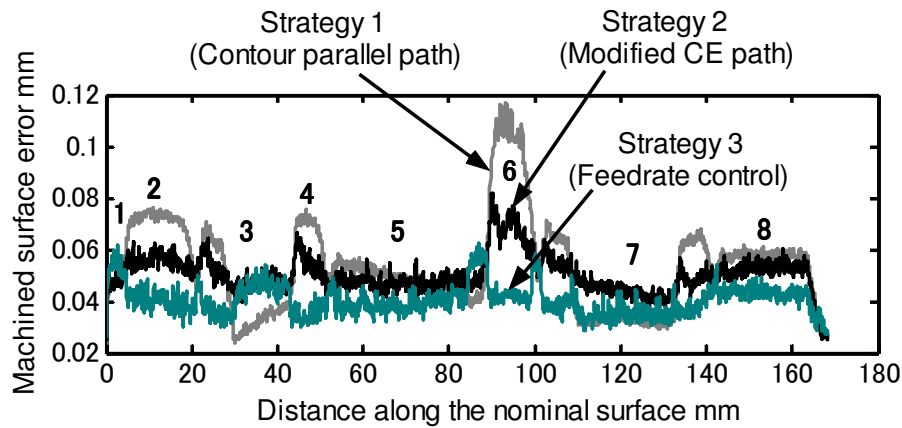
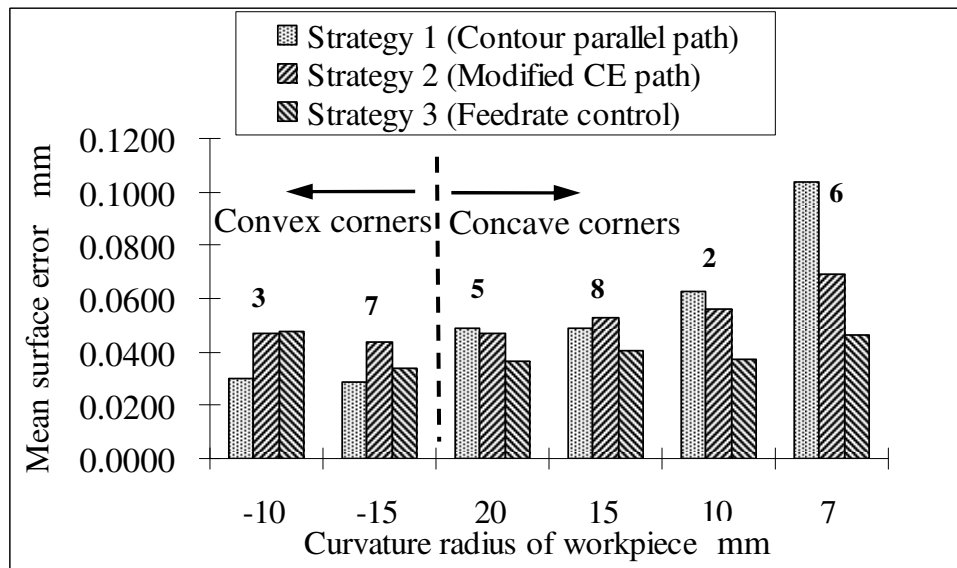


Figure 3-19 Geometric error profiles of the machined surface for Strategies 1~3



* The numbers on the top of graphs indicate the corner name, corresponding to those in Fig. 3-18

Figure 3-20 Mean surface error profiles with respect to the curvature radius of the workpiece contour for Strategies 1~3

accuracy of the finished workpiece. This condition corresponds to such a case. All the other machining conditions are the same as those shown in Table 3-3.

Figure 3-21 compares the geometric error profiles of the machined workpiece contour measured by the CMM. As can be easily predicted, in all cases, the geometric error is averagely smaller throughout the entire path compared to that for the previous case with $s = 0.5\text{mm}$, since the tool is subject to

smaller cutting force and thus smaller deflection. Even in such a case, it is evident that the proposed modified constant engagement (CE) tool path (Strategy 2) significantly reduces the variation of the geometric error than the contour parallel path case (Strategy 1), particularly in sharp corners such as “2” and “6”.

3.4.2.3 Comparison with the Feedrate Control Scheme

In Strategy 3 (Feedrate control), the original contour parallel path with variable feed rate is applied to the finishing path, while the feed rate is varied such that the cutting force is maintained constant throughout the finishing path. In this paper, the feed rate profile is determined off-line based on the cutting force prediction model described in Section 3.3, such that the cutting force in the XY plane is regulated at the desired constant level, which is given as the “predicted” cutting force under which the machining of a straight path with the radial depth of cut of 0.5mm and the feed rate of 1,000 mm/min is conducted.

In Fig. 3-17, the cutting force measured during the machining of the finishing path under feedrate control scheme (Strategy 3) is also shown. The geometric error profiles of the machined workpiece contour under Strategy 3 are also shown in Figs. 3-18(c), 3-19, 3-20, and 3-21.

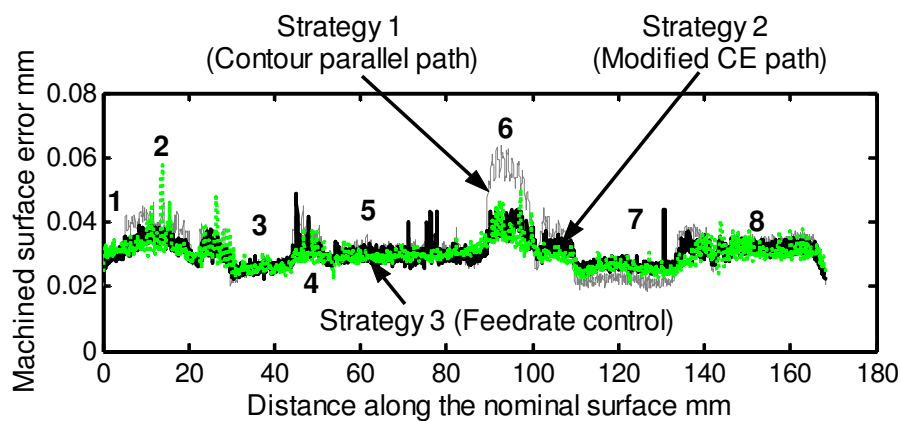


Figure 3-21 Machined surface error profiles for Strategies 1~3 (for the case with step-over distance of 0.10mm on the finishing path)

Figure 3-22 shows the comparison between commanded and measured feedrate profiles during the machining of the finishing path under Strategy 3 for the case with $s=0.5\text{mm}$ in the finishing path. The actual feed rate is measured by monitoring the signal of rotary encoders installed in the feed drives of the machining center. It is found that there is more distinct difference between commanded and measured feedrate profile, due to the limitation of feed rate control performance of the machine's servo controllers. In Fig. 3-17, the cutting force is smaller under Strategy 3 throughout the entire finishing path, simply because the actual feedrate is much smaller than the cases under other two strategies. As a natural consequence, the geometric error of the machined workpiece is also smaller in the most part, as can be observed in Figs. 3-18, 3-19, and 3-20. However, it provides a significantly longer machining time as is shown in Fig. 3-23. In the practical implementation, a critical drawback of any feedrate control schemes is a sort of uncertainty in the feed rate control performance of servo controllers. In this particular experiment, significantly large feedrate error

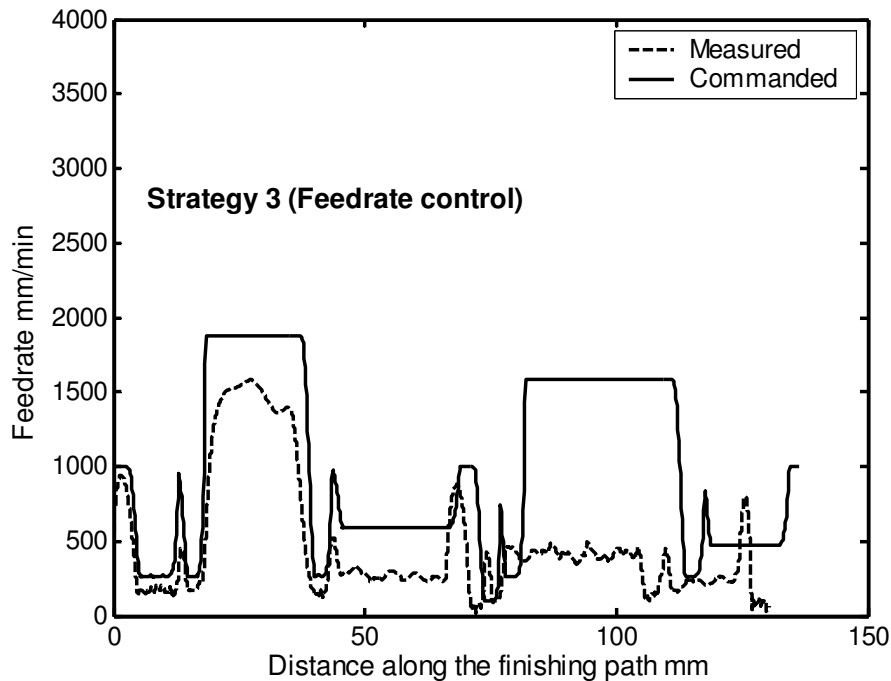


Figure 3-22 Comparison between commanded and measured feedrate for Strategy 3 (for the case with the step-over distance of 0.5mm on the finishing path)

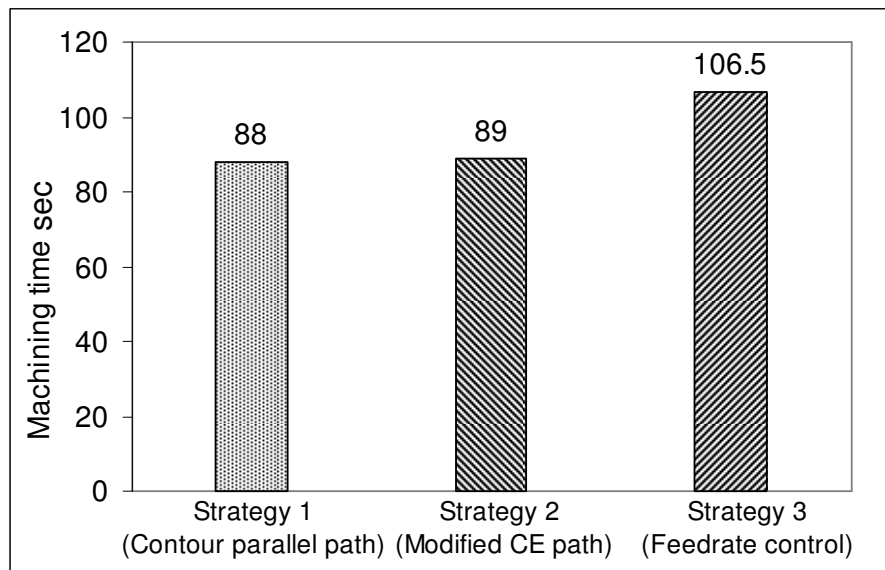


Figure 3-23 Comparison of machining time (from roughing to finishing) for Strategies 1~3 (for the case with the step-over distance of 0.5mm on the finishing path)

resulted in smaller geometric error of the machined workpiece, with a significant sacrifice of the machining time. In many commercial servo controllers, the feedrate error tends to be larger particularly when the tool path is given as a series of small segments with varying feedrate. The uncertainty in feedrate control performance is one of critical issues, which makes it difficult to implement a feedrate control scheme to a finishing path in practical applications.

Another possible issue with a feedrate control scheme is the unsmoothness of the feedrate profile. As can be observed in Fig. 3-22, the commanded feedrate rapidly changes at the connection of arcs of different curvature. This is caused by the discontinuity of the curvature of the reference tool location trajectory, and this problem often occurs in any feedrate control schemes that are based on the idea of constant material removal rate. The rapid change in the feedrate often results in the deterioration of surface finish, which is naturally not favorable in a finishing process.

In contrary, the tool path modification scheme proposed in this research work achieves a constant cutting force without changing the feedrate over the finishing path. Hence, it can be inferred that the proposed tool path modification

scheme can improve the machining accuracy in a more stable, reliable manner in practical applications than feedrate control schemes.

3.4.3 Case Study III: Hardened Steel Workpiece Case

In early 90's, a sintered carbide end mill with an (Al,Ti)N coating was introduced into the manufacturing of dies and molds, which made it possible to directly machine pre-hardened steel of the hardness up to HRC60. By first performing heat treatment on raw steel and then machining it by using this tool, die/mold making process can be significantly simplified, eliminating the needs for grinding or EDM (electric discharge machining) processes after the heat treatment. In this case study, in order to validate its effectiveness, the proposed algorithm for backward tool path modification is applied to the machining of a core contour workpiece made of hardened steel material (JIS, SKD61 with HRC53).

Figure 3-24 shows the geometry of the core contour workpiece consisting of different circular arcs (e.g. concave and convex). Similarly, by the applying the proposed algorithm, the semi-finishing path is modified in a way such that the cutting engagement angle is regulated at a desired constant level in finishing path. Figure 3-25 shows the modified CE semi-finishing path along with conventional contour parallel tool paths. The effect of path modification with the desired constant engagement angle on the trajectory of the semi-finishing tool path can be noticed in the magnified view of tool paths in Fig. 3-25(b). The modified semi-finish path consists of total 1,978 line segments of the length 0.1mm. The computer implementation of the proposed algorithm for tool path modification is carried out by using MATLAB by Mathworks, Inc. on the same desktop PC as in Case Study II. The computation time required from processing the G-code of the finishing path until the generation of the modified CE semi-finishing tool path is less than one second.

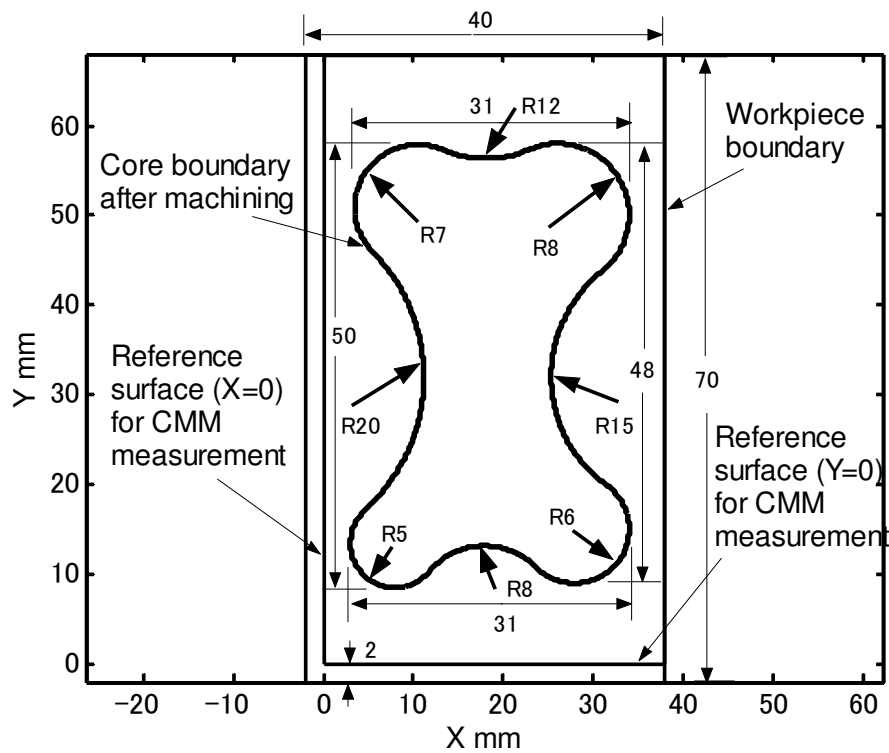
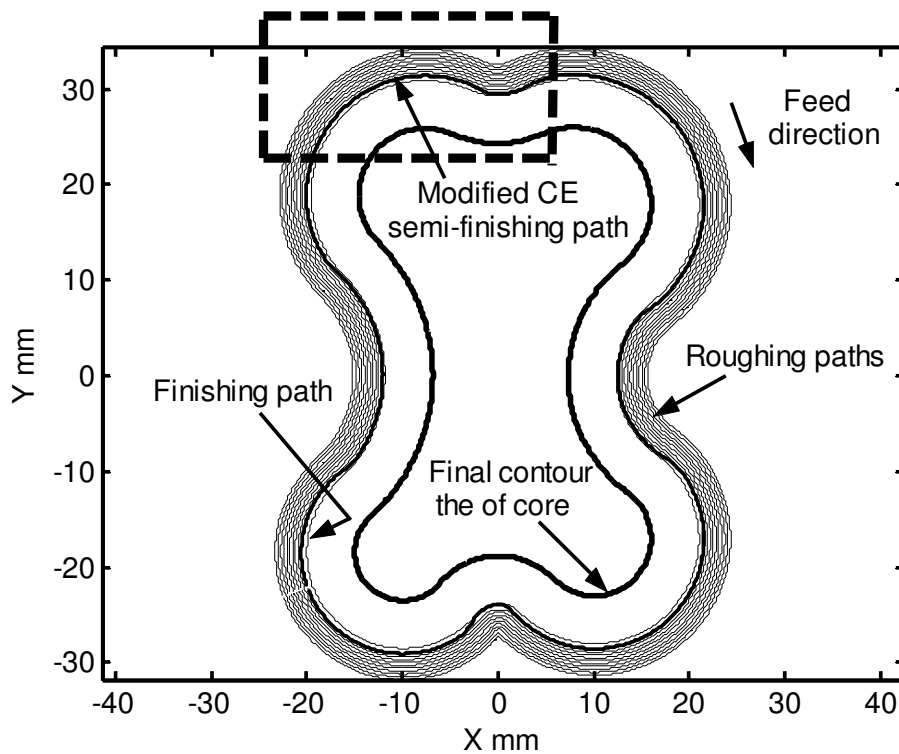
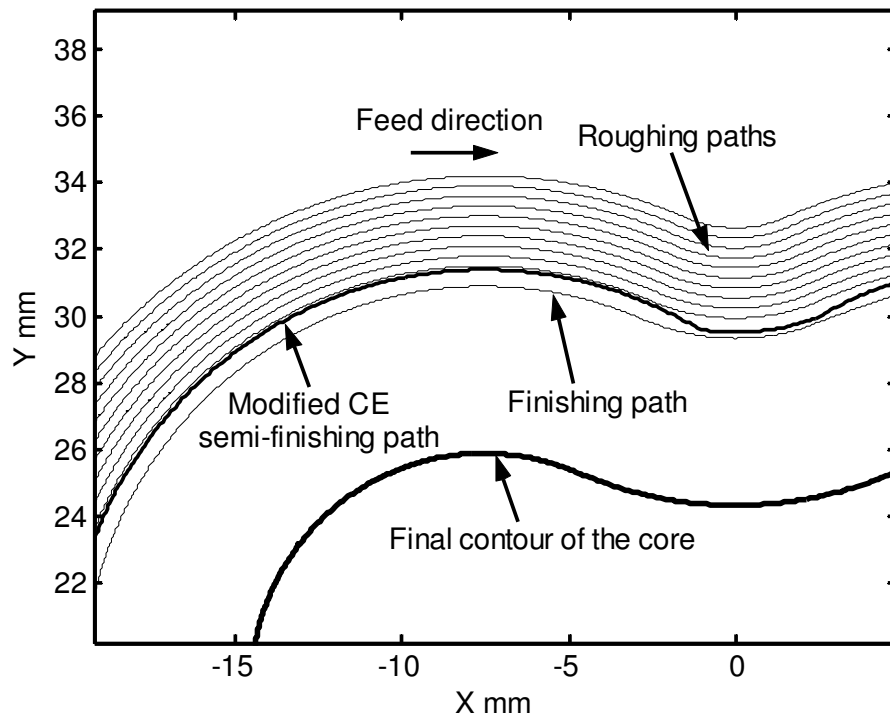


Figure 3-24 Geometry of the core contour



(a) Modified CE tool path generated by the proposed algorithm

Figure 3-25 Modified CE tool path generated by the proposed algorithm along with contour parallel tool paths



(b) Magnified view of the tool paths in the rectangular box in (a)

Figure 3-25 Modified CE tool path generated by the proposed algorithm along with contour parallel tool paths

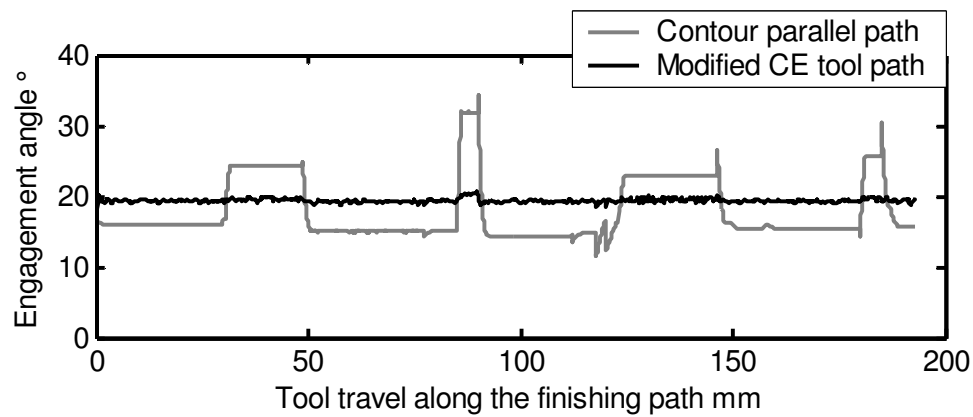


Figure 3-26 Simulated profiles of engagement angle of a contour parallel path and the modified CE path generated by the proposed algorithm

Further, Figure 3-26 compares the simulated profiles of engagement angle with an original contour parallel path and the modified CE semi-finishing path generated by the proposed algorithm on the finishing path. The computation of engagement angle profiles starts from the circular arc “A” (as marked in Fig. 3-

24) of the core contour while the tool moves in a clockwise direction along the contour. It is clearly evident from the simulated results that while an original contour parallel path shows a significant variation in engagement angle, the modified CE path is able to regulate the engagement angle at a desired level on the machining of the finishing path.

3.4.3.1 Experimental Details

The machining tests are conducted on the same machining center as used in Case Study II. The detailed cutting conditions used in the tests are summarized in Table 3-4. For comparison of the cutting performance, two machining strategies are adopted in this case study.

Strategy 1 (Contour parallel path) represents the case where conventional contour parallel tool paths are applied throughout the machining (i.e. from roughing to finishing operation). The feedrate is kept constant throughout.

Strategy 2 (Modified CE path) features the case where the proposed modified CE tool path is applied the semi-finishing. Note that, in this case also,

Table 3-4 Cutting conditions used in experiments

Cutting tool	(Al,Ti)N-coated sintered tungsten carbide Radius end mill, Diameter: 10 mm No. of flutes: 6, Tool extension: 35 mm
Workpiece	Hardened steel (JIS SKD 61) with HRC53 Size: 70mm × 40mm × 20mm
Cutting parameters	Spindle speed: 4772 min ⁻¹ Feedrate: 1200 mm/min Step-over distance: 0.3 mm Axial depth of cut: 5.0 mm
Cutting direction	Down cut
Coolant	Oil mist

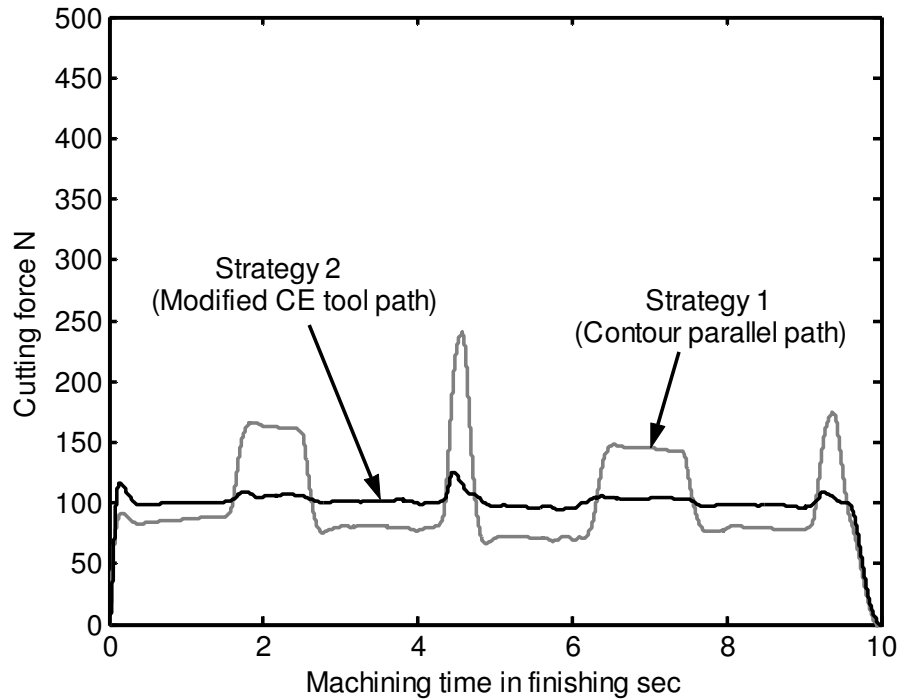


Figure 3-27 Comparison of cutting force between a contour parallel path (Strategy 1) and the modified CE path (Strategy 2)

all the paths except for the semi-finishing path are the same as those in Strategy 1. The feedrate is maintained constant throughout on all the roughing and finishing paths.

3.4.3.2 Results and Discussion

Figure 3-27 depicts a comparison of cutting forces measured by a dynamometer (Kistler's 9272) during machining of the finishing path for two strategies. It is seen from Fig. 3-27 that the proposed modified constant engagement (CE) tool path (Strategy 2) reveals a reduced variation of cutting force. While comparing with contour parallel path (Strategy 1), it reduces maximum variation of cutting force by about 83%.

It is to be noted that since the proposed approach modifies the semi-finish path to regulate cutting engagement in finishing, the variation in cutting force on the semi-finishing will be often larger. Figure 3-28 compares measured cutting force profiles along the semi-finish path under Strategy 1 (Contour parallel path)

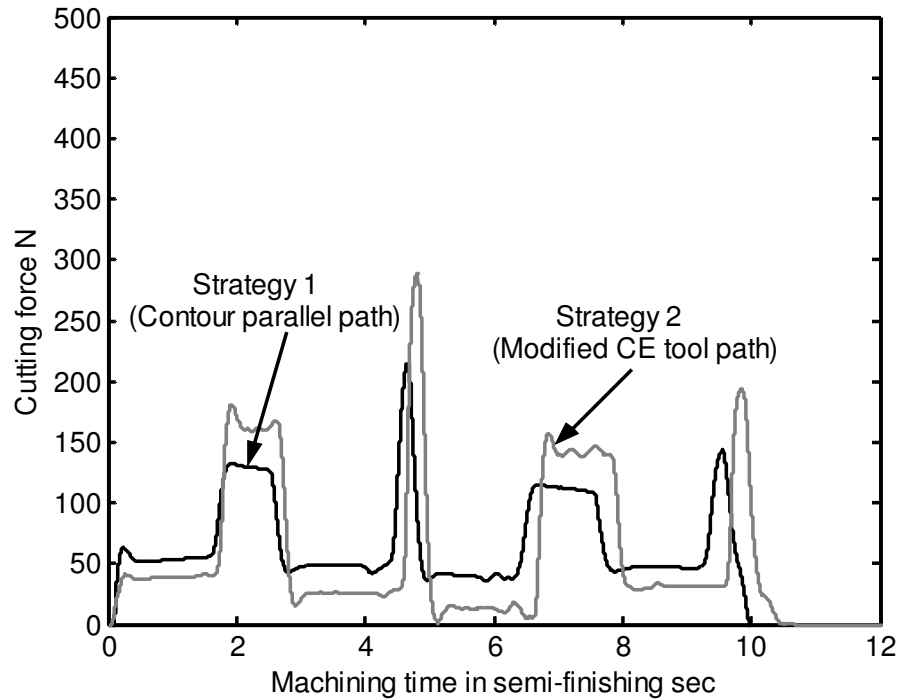
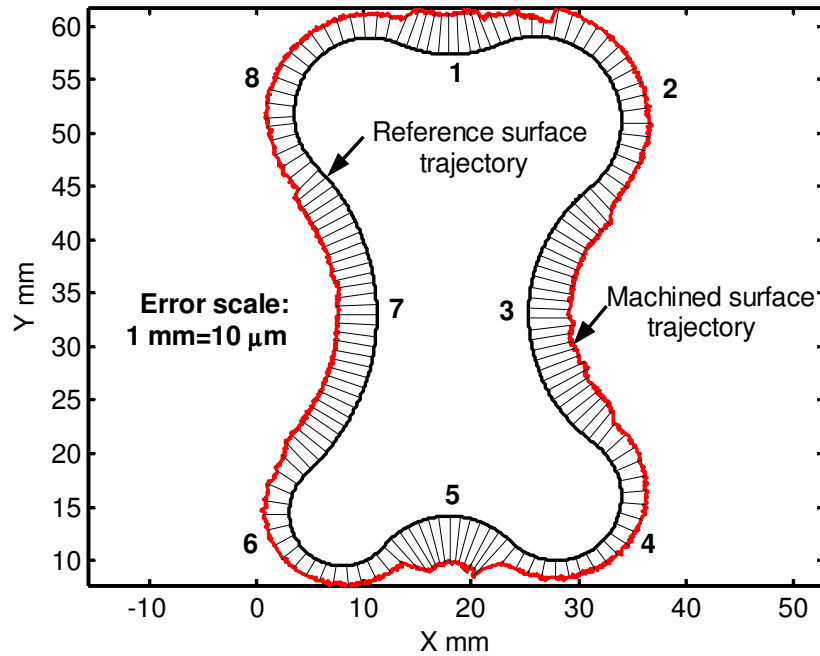


Figure 3-28 Cutting force profiles in semi-finishing under Strategies 1 and 2

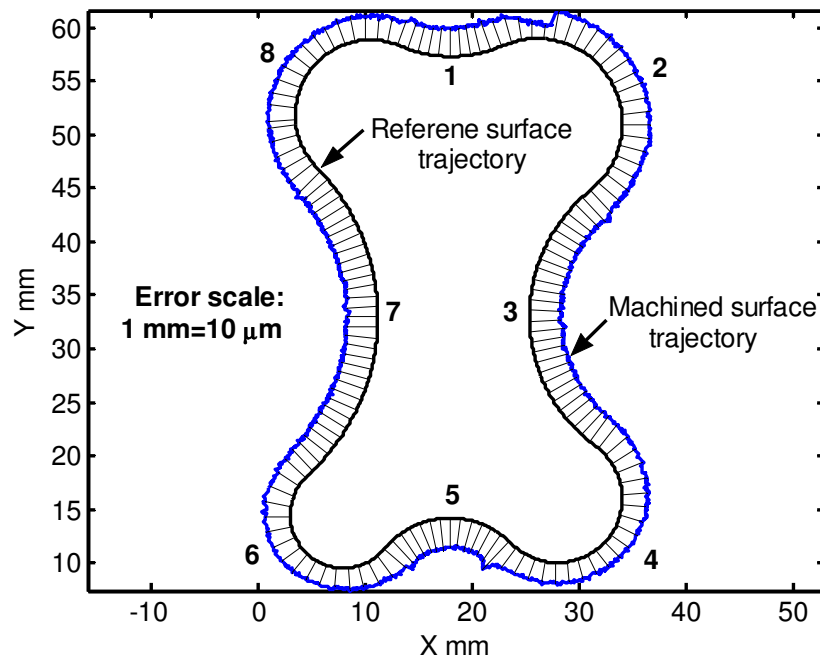
and Strategy 2 (modified CE tool path). From Fig. 3-28, it is found that the variation in cutting force along the semi-finish path is larger in Strategy 2. The difference in the cutting force is about 103N at maximum. From the static stiffness of this tool, the tool deflection caused by the cutting force 103N is estimated to be about 6 μ m. Although large variations in the tool deflection on the semi-finish path may potentially change the engagement angle on the finishing path, it can be assumed that the variation in the tool deflection on the semi-finish path is sufficiently small compared to the radial depth of cut on the finishing path. The fact that the variation in the cutting force on the finishing path is significantly suppressed in Strategy 2 (as shown in Fig. 3-27) shows the validity of this assumption in the experiments.

Figure 3-29 shows the geometric error profiles of the machined surface with respect to reference surface trajectory measured by a coordinate measuring machine (CMM) (UPMC 850 by Carl Zeiss Inc.). The same error profiles drawn from Fig. 3-29 with the distance along the reference surface from the starting point indicated by “8” are shown in Fig. 3-30. From the figures, it can be revealed that by applying modified CE tool path (Strategy 2), the variation of

machined surface error is significantly reduced, when compared with that in contour parallel tool path (Strategy 1). The maximum variation of machined surface error is reduced by 75%.

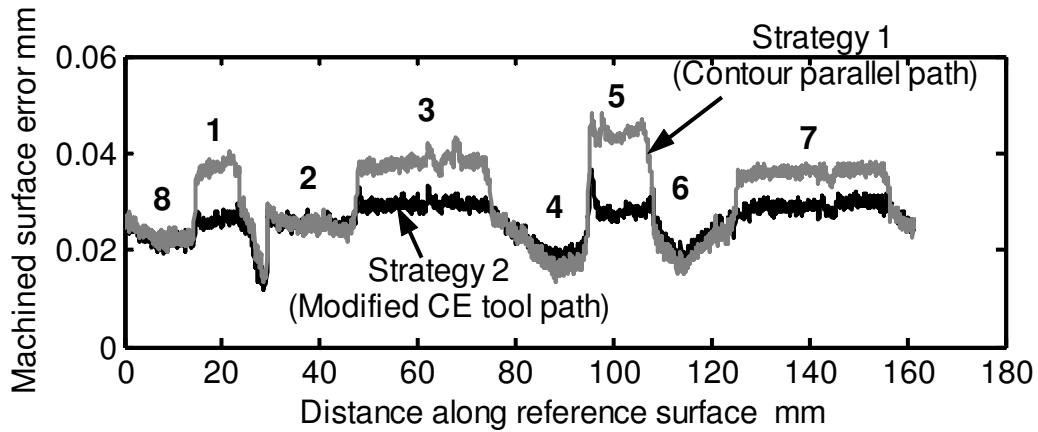


(a) Strategy 1 (Contour parallel path)



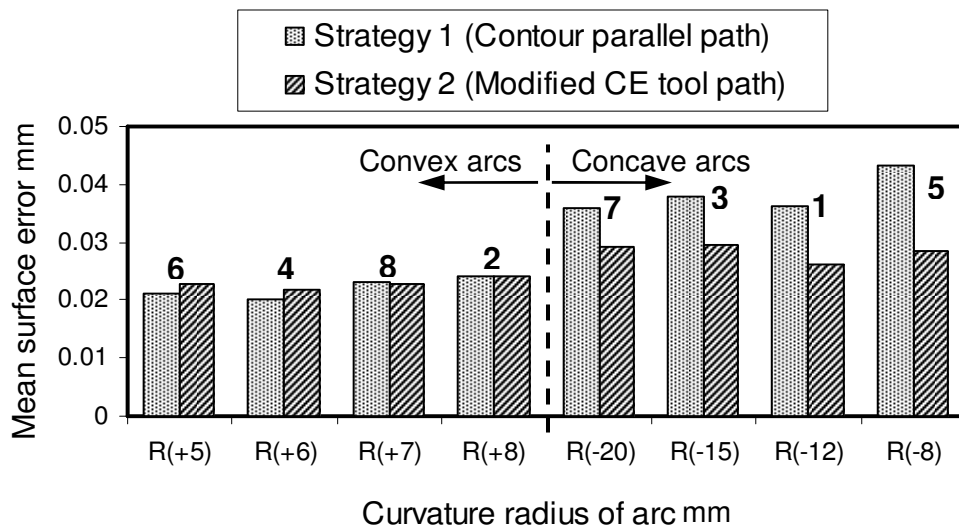
(b) Strategy 2 (Modified CE tool path)

Figure 3-29 Machined surface error trajectories of the core workpiece in finishing under Strategies 1 and 2



* The numbers on top of graphs correspond to the corner names, same as those indicated in Fig. 3-29

Figure 3-30 Geometric error profiles of the machined core contour with respect to distance along its reference surface



* The numbers on top of graphs correspond to the corner names, same as those in Fig. 3-29

Figure 3-31 Mean surface geometric error profiles with respect to curvature radius of the core workpiece (R (+): convex arc, R (-): concave arc))

The mean values of machined surface error for each of eight corners along the core contour are presented in Fig. 3-31. From Fig. 3-31, it is evident that by applying modified CE tool path, a significant reduction of mean machined

surface error is observed especially at concave circular arcs, indicating an overall uniform error level along the contour of the machined core workpiece.

3.5 Conclusion

In this chapter, an algorithm for backward tool path modification to generate an offset tool path such that a desirable constant engagement angle is regulated through the finishing path is proposed. By applying the proposed approach, the cutting force on the machining of finishing path can be maintained at a constant level, which naturally improves the geometric accuracy of the machined surface. Case studies with cutting experiments on different contour geometries such as a simple corner, the pocket and the core made of different workpiece materials are carried out to demonstrate the effectiveness of the proposed scheme. From the machining results obtained from case studies, the following conclusions can be drawn.

a) Comparing with an original contour parallel path, by applying the modified constant engagement (CE) tool path generated by the proposed algorithm, the maximum variation of the cutting force in finishing is significantly reduced (by 75%~83%), which consequently results in a significant reduction in geometric error of machined surface (by 62%~75%).

b) A critical problem with a feedrate control scheme for constant cutting force regulation is that it is often the case that a sufficient feedrate control performance cannot be achieved due to the performance limitation of servo-controllers of the machine tools. Furthermore, the unsmoothness of the optimized feedrate profile due to the discontinuity of the curvature of the tool path trajectory often causes a critical problem with the surface quality of the machined workpiece. In practical applications, the proposed algorithm for tool path modification, thus, has a crucial advantage over feedrate control schemes.

Chapter 4

Regulation of Cutting Engagement under Feedrate Scheduling

4.1 Introduction

In the previous chapter, an algorithm for backward tool path modification is applied such that the cutting engagement angle is kept constant throughout the finishing path, which consequently regulates the cutting force at a desirable constant level when the finishing path is machined under a constant feedrate.

However, in 2D contour machining, feedrate at the actual cutting point varies even when the feedrate at the tool center is kept constant. The variation in feed per tooth at the cutting point naturally causes the variation in the width of cutter marks generated on the machined surface, which often deteriorates the surface quality. To address it, a scheme to regulate the feedrate at tool center such that the feedrate at the cutting point is kept constant has been well known [Nishida et al., 2004], and it is implemented in some latest commercial CAM software. The idea is that by maintaining the feedrate at the cutting point constant, the width of cutter marks on the machined surface will be ideally constant, which potentially will contribute to the improvement of the surface quality. However, a constant feedrate at the cutting point generally does not keep the cutting force constant.

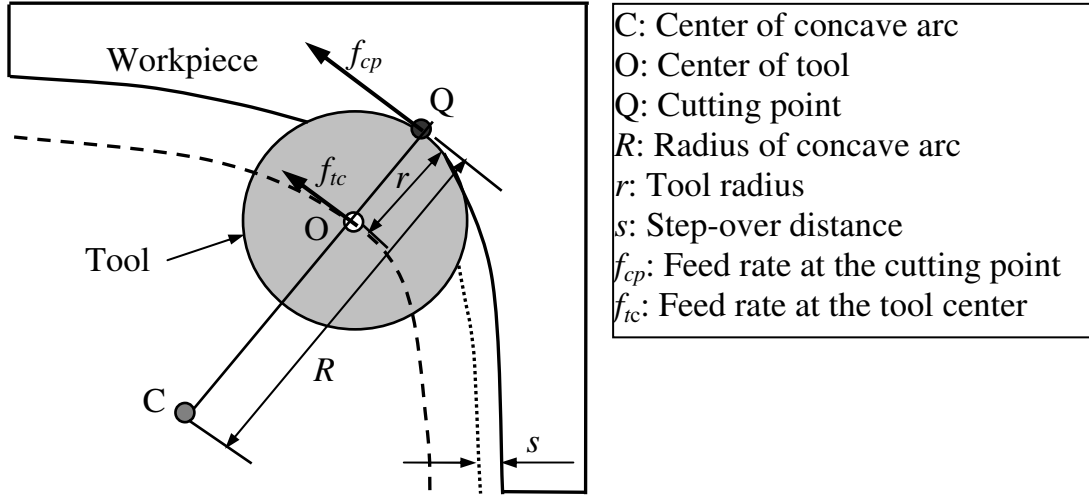
To overcome this problem, this chapter proposes a new scheme to regulate cutting engagement angle under feedrate scheduling/control. As has been discussed in Section 3.4.2.3, the performance of any feedrate scheduling schemes can be significantly limited by the lack of sufficient feedrate control performance of servo controllers. Under the condition that servo controllers have sufficient feedrate control performance, however, it well known that a feedrate scheduling scheme can be effective to improve the surface finish. Variable feedrate at the

tool center is applied such that feed per tooth at the actual cutting point is regulated at a desirable constant level in the finishing path. Then, by applying the algorithm for backward tool path modification as illustrated in Chapter 3, the semi-finishing path is modified with an optimized cutting engagement angle such that an expected cutting force is regulated efficiently in the finishing path. By applying both the feedrate optimization and the tool path modification, it can be expected that both the geometric accuracy and the surface quality of the machined workpiece will be improved.

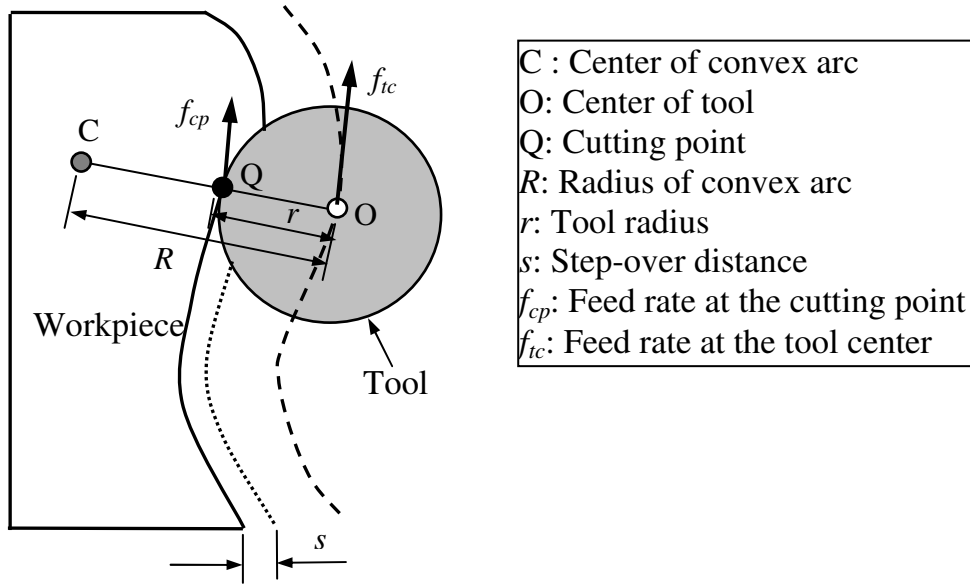
The remainder of this chapter is organized as follows. Section 4.2 describes the definition of feedrate at the actual cutting point and its regulation in 2D contour machining. An optimization of the cutting engagement angle to regulate the cutting force at a desirable level in finishing is illustrated in Section 4.3. By using the algorithm for backward tool path modification, Section 4.4 describes the modification of the semi-finishing path with the optimized cutting engagement angle. Case studies with cutting experiments to validate the effectiveness of the proposed scheme are shown in Section 4.5. Lastly, Section 4.6 concludes the chapter with a brief summary.

4.2 Definition of Feedrate at the Cutting Point and Its Regulation

Depending on the geometry of the workpiece contour to be machining, the feedrate at the cutting point can be different from that at the tool center. Figure 4-1 defines the feedrate at the cutting point and the feedrate at the tool center for concave and convex arc milling. From the figure, it can be easily seen that, in concave arc milling (Fig. 4-1(a)), the feedrate at the cutting point, f_{cp} , is larger than that at the tool center, f_{tc} , whereas, the phenomenon is reverse in convex arc milling (Fig. 4-1(b)). Hence, in order to keep the feedrate at the cutting point, f_{cp} , at a constant level, we have varied the feedrate at the tool center, f_{tc} . Assume that the geometry of newly generated workpiece contour such as the curvature



(a) Concave arc milling



(b) Convex arc milling

Figure 4-1 Definition of feedrate at the cutting point and feedrate at the tool center in 2D contour end milling

radius, $R(i)(i=1,...,N_k)$ along the trajectory of the finishing path, $O_k(i) \in \mathcal{R}^2(i=1,...,N_k)$, is given. Thus, variable feedrate rate at the tool center, $f_{tc}^*(i)(i=1,...,N_k)$, for concave and convex arc milling can be optimized as follows.

$$\text{For concave arc, } f_{tc}^*(i) = f_{cp} \cdot \frac{(R(i) - r)}{R(i)} \quad (4-1)$$

$$\text{For convex arc, } f_{tc}^*(i) = f_{cp} \cdot \frac{(R(i) + r)}{R(i)} \quad (4-2)$$

where, r is the tool radius. The desired feedrate at the cutting point, f_{cp} , is chosen from the machining database or recommended by the industry.

4.3 Optimization of Cutting Engagement Angle for Constant Cutting Force Regulation

Although maintaining a constant feedrate at the cutting point will generate uniform cutter marks on the machined surface, potentially improving the surface finish, it does not necessarily keep the cutting force at a constant value. However, in contour machining, regulating cutting force at a desired level is an important concern to reduce the tool deflection, and thus to enhance the geometric accuracy of the machined surface as has been discussed in Chapter 3. Hence, by regulating the cutting force at a desirable constant level while simultaneously keeping the feedrate at the cutting point at a constant level, it can be expected that both geometric accuracy and the surface quality of the machined workpiece will be improved.

When the feedrate along the finishing tool path trajectory is given by $f_{tc}^*(i) (i = 1, \dots, N_k)$, first a profile of the desired engagement angle is computed such that the cutting force is regulated at the given desired level. We here assume that a kinematic model to predict the cutting force from given cutting conditions is available, on which the computation of the desired engagement angle is based. In this study, we adopt the cutting force prediction model developed by Otsuka et al. [Otsuka et al., 2001]. The fundamental equation of the model is already illustrated in Section 3.3. Assuming that the feedrate at the tool center is given, in terms of the relationship between the cutting force and the engagement angle, Otsuka's cutting force prediction model (Eq. (3-3)) can be rewritten as follows.

$$\hat{F} = \beta_0 + \beta_1 \sin \alpha_{en}(i) + \beta_2 \alpha_{en}(i) + \beta_3 (\sin \alpha_{en}(i))^2 + \beta_4 (\alpha_{en}(i))^2 + \beta_5 \alpha_{en}(i) \cdot \sin \alpha_{en}(i) \quad (4-3)$$

where $\hat{F}$ denotes the predicted cutting force on the X-Y plane, $\alpha_{en}(i)$ is the engagement angle, and $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$, and β_5 are constant coefficients which must be identified in advance for the given tool and the workpiece by cutting experiments as shown in [Otsuka et al., 2001]. As the Eq. (4-3) is a nonlinear equation, a trust-region method for nonlinear optimization [Byrd, 1988] is adopted to solve the above equation to obtain a profile of the optimized cutting engagement angle, $\alpha_{en}^*(i)$ ($i=1, \dots, N_k$) along the tool path trajectory for the given desired cutting force level.

4.4 Tool path modification with optimized cutting engagement angle

Once the optimized cutting engagement angle to regulate the desired cutting force in finishing is determined, the next step is to modify the semi-finishing path with the optimized engagement angle. Hence, by using the proposed algorithm for backward tool path modification as described in Chapter 3, the trajectory of modified semi-finishing path, $O_{k-1}(i) \in \mathfrak{R}^2$ ($i=1, \dots, N_k$) is generated such that the cutting engagement angle is maintained at $\alpha_{en}^*(i)$ along the trajectory of finishing path, $O_k(i) \in \mathfrak{R}^2$ ($i=1, \dots, N_k$).

4.5 Experimental Verification

To apply the algorithm for tool path modification with the optimized engagement angle, the same core contour as the one used in Case Study III (Section 3.4.3) (see Fig. 3-24) is selected. To realize the given contour geometry and the proposed tool path modification on it, here the geometry of the same core contour workpiece is reposted and shown in Fig. 4-2. Figure 4-3 shows the modified semi-finishing path generated by the algorithm, along with the finishing path on the core workpiece contour, and the effect of path modification on the

trajectory of semi-finishing path can be noticed in the magnified view of the tool paths (see Fig. 4-3(b)). To be noted again is that, in the machining along the finishing path trajectory, the optimized variable feedrate at the tool center, $f_{tc}^*(i)$, is maintained to keep constant feedrate at the cutting point.

4.5.1 Experimental Details

Machining experiments on the core workpiece shown in Fig. 4-2 are carried out in order to verify the significance of the proposed scheme. A three-axis vertical machining center (Mori Seiki GV503) is used in cutting tests. The major specifications of the machining center used are summarized in Table 4-1. Cutting conditions used during the experiments are shown in Table 4-2.

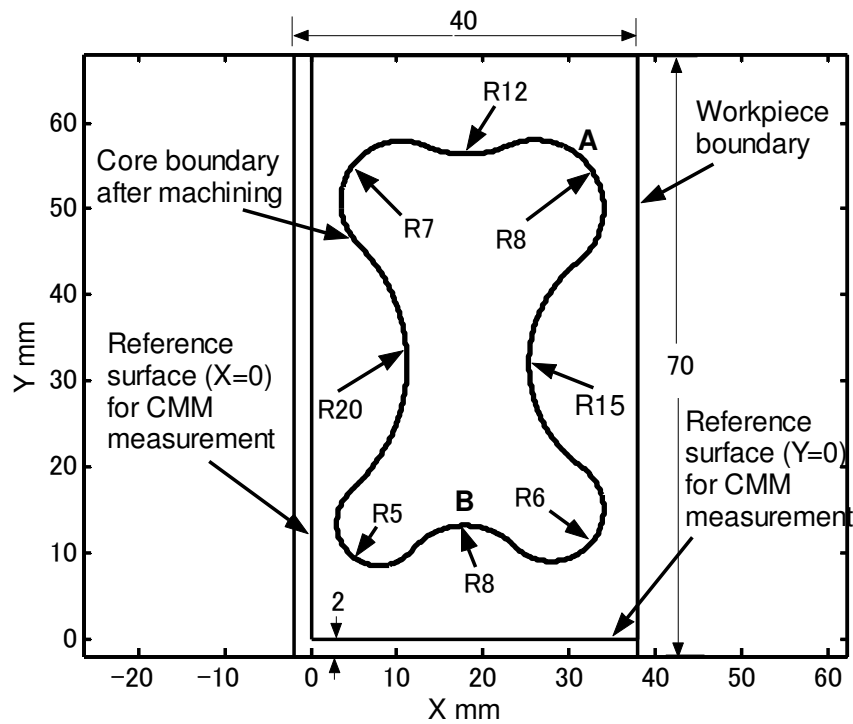
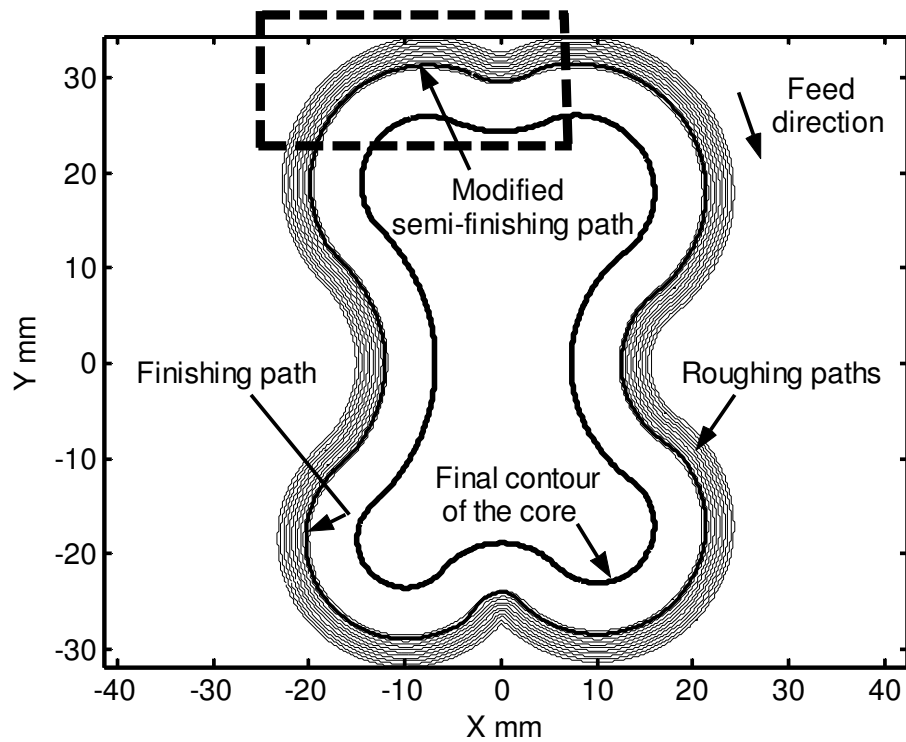
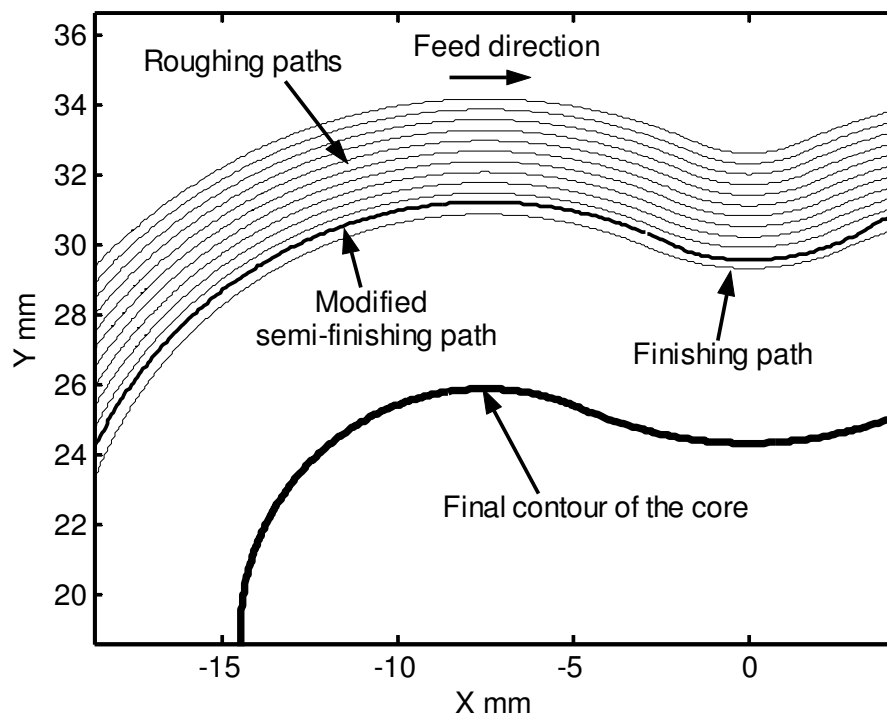


Figure 4-2 Geometry of the core contour used as the test workpiece (reposted)



(a) Modified semi-finishing path along with contour parallel paths



(b) Magnified view of the tool paths in the rectangular box in (a)

Figure 4-3 Modified semi-finishing tool paths generated by the proposed algorithm for tool path modification with the optimized cutting engagement angle

Table 4-1 Specifications of the machining center (GV503 by Mori Seiki)

Spindle	Spindle speed (min^{-1})	200-20,000
	Power, 15min/cont (KW)	22/18.5
	Tool inference	7/24 Taper 40
Feed drive	Max. rapid traverse (mm/min)	33,000
	Max. feedrate (mm/min)	10,000
	Max. acceleration (G)	X-axis: 0.67, Y-axis: 0.64 Z-axis: 0.56
CNC servo system	64-bit CPU (RISC processor) + High gain servo amplifier	

Table 4-2 Cutting conditions used in the machining tests

Cutting tool	:	(Al,Ti)N-coated sintered tungsten carbide Radius end mill, Diameter: 10 mm No. of flutes: 6, Tool extension: 35 mm
Workpiece	:	Hardened steel (JIS SKD 61) with HRC53 Size: 70mm × 40mm × 20mm
Cutting parameters	:	Spindle speed: 4772 min^{-1} Step-over distance: 0.3 mm Axial depth of cut: 5.24 mm
Cutting direction	:	Down cut
Coolant	:	Oil mist

For comparison of the cutting performance, three machining strategies are adopted in this case study.

Strategy 1 (Contour parallel path) represents the case where contour parallel tools with a constant feedrate of 1200 mm/min are applied throughout the machining (i.e. from roughing to finishing).

Strategy 2 (Feedrate control) is the case where a contour parallel tool path with variable feedrate at tool center is applied to the finishing in order to keep a constant feedrate of 1200 mm/min at the actual cutting point. As computed by using Eqs.(4-1) and (4-2) for the geometry of the core contour, the variable feedrate at tool center in the finishing path ranges from 450 mm/min to 2400 mm/min (see Table 4-3 for the detailed feedrate profile). Note that, in Strategy 2, all the tool paths are the same as original contour parallel paths.

Finally, Strategy 3 (Feedrate control with modified tool path) designates the case where the tool path modification with the optimized engagement angle is applied to the semi-finishing while a contour parallel tool path with variable feedrate at the tool center (same as in Strategy 2) is applied to the finishing to regulate the cutting force at a desired level.

For the given geometry of the core contour workpiece shown in Fig. 4-2, Table 4-3 summarizes the computed results of optimized variable feedrate at tool center to keep constant feedrate at cutting point, and the optimized cutting engagement angle to modify the semi-finishing path to regulate the desired cutting force in finishing for the case of Strategy 3 (Feedrate control with modified tool path). First, to keep the constant feedrate at the cutting point, f_{cp} , a profile of variable feedrate at the tool center, f_{tc}^* along the finishing path is calculated by using Eqs.(4-1) and (4-2). The cutting force under the variable feedrate at the tool center, $\hat{F}_{xy}$ is predicted by using the Otsuka's cutting force prediction model [Otsuka et al., 2001]. Then, in order to maintain a constant cutting force, F^* in the finishing path, a profile of optimized cutting engagement angle, α_{en}^* is computed by using the relationship between the predicted cutting force and the engagement angle in Eq. (4-3). The semi-finishing path is then modified with the computed optimized engagement angle. It is to be noted that the desired constant cutting force to regulated in the finishing can be determined under the case when the cutting tool machines a linear segment of workpiece with a given tool radius and step-over distance.

Table 4-3 Computed values of the optimized variable feedrate at tool center and the optimized engagement angle for path modification in semi-finishing

Corner geometry (curvature radius)	Feedrate at cutting point, f_{cp} (mm/min)	Feedrate at tool center, f_{tc}^* (mm/min)	Predicted cutting force under variable feed at tool center, $\hat{F}_{xy}$ (N)	Desired cutting force, F^* (N)	Optimized engagement angle, α_{en}^* ($^{\circ}$)
R(-12)	1200	700	165	145.47	23.16
R(+8)		1950	132.5		16.98
R(-15)		800	160		22.38
R(+6)		2200	132		16.26
R(-8)		450	190		25.56
R(+5)		2400	128		15.74
R(-20)		900	158		21.69
R(+7)		2057	130		16.66

* R(-): concave arc radius, R(+): convex arc radius

4.5.2 Results and discussion

Figure 4-4 illustrates a comparison of cutting forces measured by a dynamometer (Kistler's 9257B) in finishing. The definition of cutting forces measured in the cutting tests is as same as that described in case studies of Chapter 3. From the figure, it is seen that contour parallel path (Strategy 1) shows a drastic variation of cutting force. Cutting test under a feedrate control with keeping constant feedrate at the cutting point (Strategy 2) also reveals a variation of cutting force. However, machining under a feedrate control with modified tool path generated by the proposed algorithm (Strategy 3) reduces the variation of cutting force by about 80% and 44% (at maximum) while comparing with those under Strategy 1 and Strategy 2 respectively. In addition, Strategy 2 and 3 improve cutting time by about 13% with respect to that in Strategy 1 due to the feedrate control. Note that, in Fig. 4-4, there is a little jump of cutting forces at the beginning of the cut for all the cases. This can be attributed to the fact that

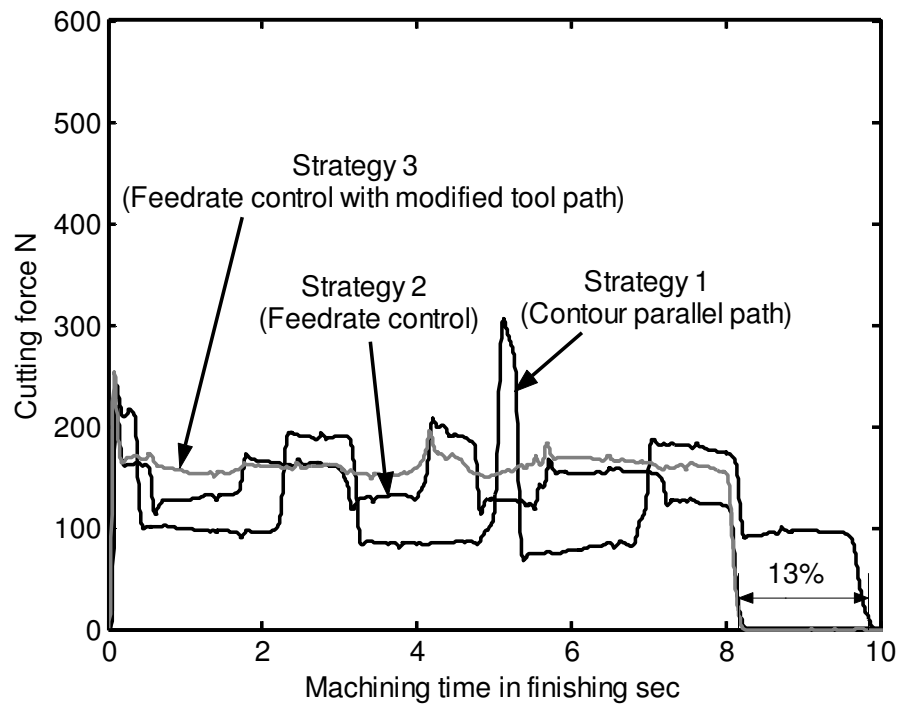


Figure 4-4 Comparison of cutting forces in finishing for strategies 1~3

at the beginning of cut, the tool approaches to the workpiece in a direction perpendicular to the contour, which causes a sudden rise in cutting force since the tool undergoes the full immersion cutting there for a short time. This process is ignored in all the strategies.

Figure 4-5 shows profiles of commanded and measured feedrates on the machining of finishing path under Strategies 2(feedrate control) and 3(feedrate control with modified tool path). It indicates that, in actual machining, the commanded variable feedrate at tool center to keep constant feedrate at the cutting point is regulated almost appropriately by the servo controller of the machine tool. In this experiment, the measured feedrate profile is given through linear encoders on X and Y-directional feed drives.

Surface profiles of machined core workpiece are measured by a contour form measurement system (Mitsutoyo's SV-C500). Measurements of surface profiles are taken at about 2.6 mm depth from the top face of the core workpiece in axial direction (i.e. about half of the axial depth of cut). Surface measurement results shown in Fig. 4-6(a) indicate that, in the machining under contour parallel path with constant feedrate at the tool center (Strategy 1), cutting marks are dense

at convex arc as marked by “A” in Fig. 4-2, while they are wide at concave arc as marked by “B” in Fig. 4-2. In contrary, in the cutting under the feedrate control with modified tool path (Strategy 3), cutting marks are likely to span more evenly at both convex and concave arcs (Fig. 4-6(b)). This result manifests that, when the feedrate at the tool center is constant and thus the feedrate at the cutting point varies depending on the geometry of the tool path (Strategy 1), the density of cutter marks varies significantly. By keeping constant feedrate at the cutting point (Strategy 3), the surface quality can be improved. Note that in Fig. 4-6(b), the amplitude of cutting mark peaks on the convex arc (“A”) is slightly larger in Strategy 3 than that in Strategy 1. This might happen due to the fact that the average cutting force on the convex arc (“A”) is larger in Strategy 3 than that in Strategy 1 due to the feedrate control.

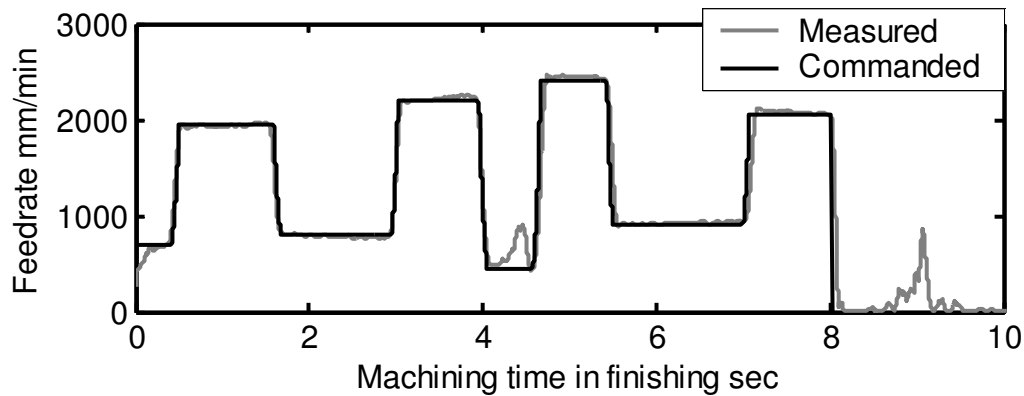
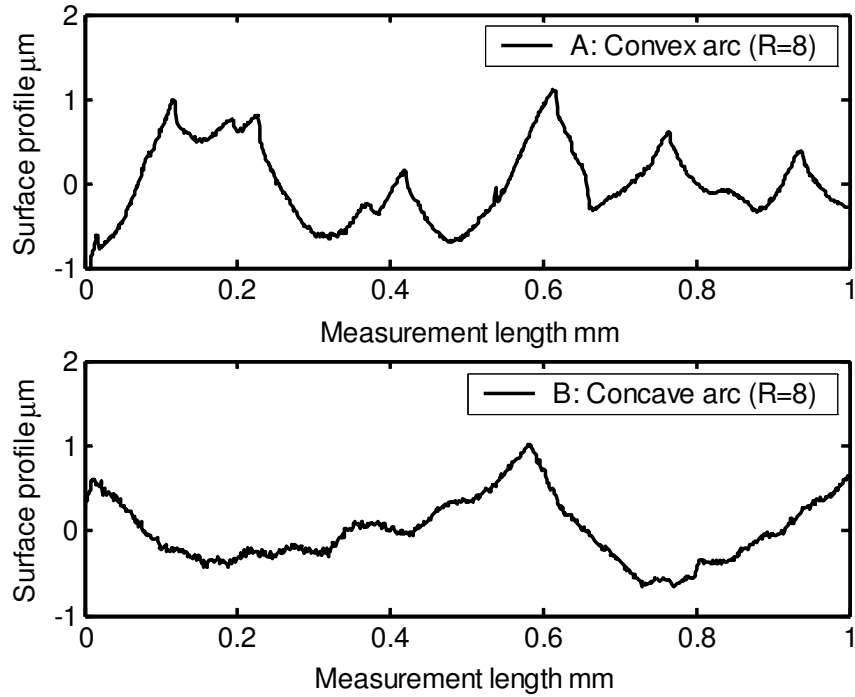
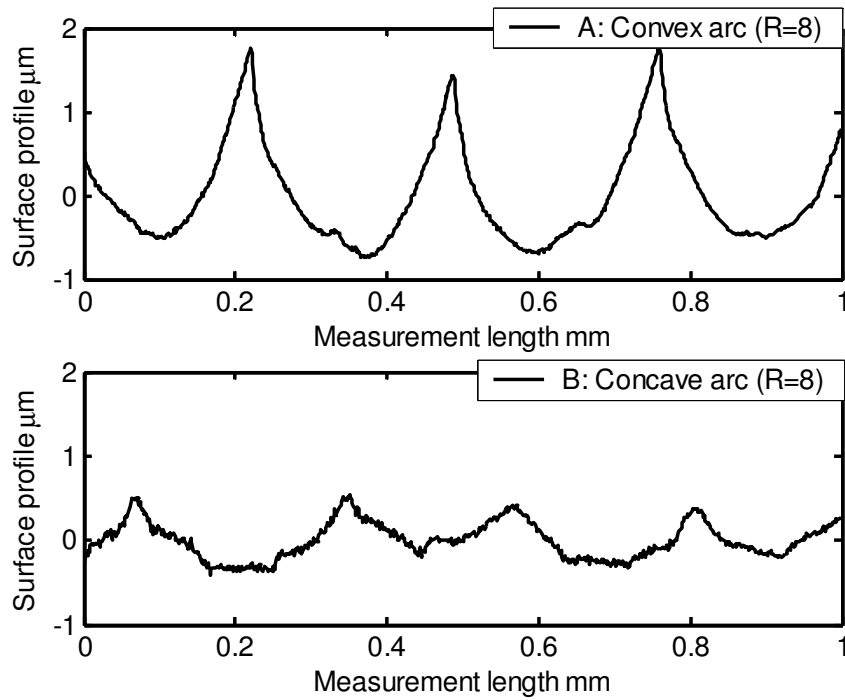


Figure 4-5 Profiles of measured and commanded feedrate in finishing under Strategy 2 (Feedrate control)



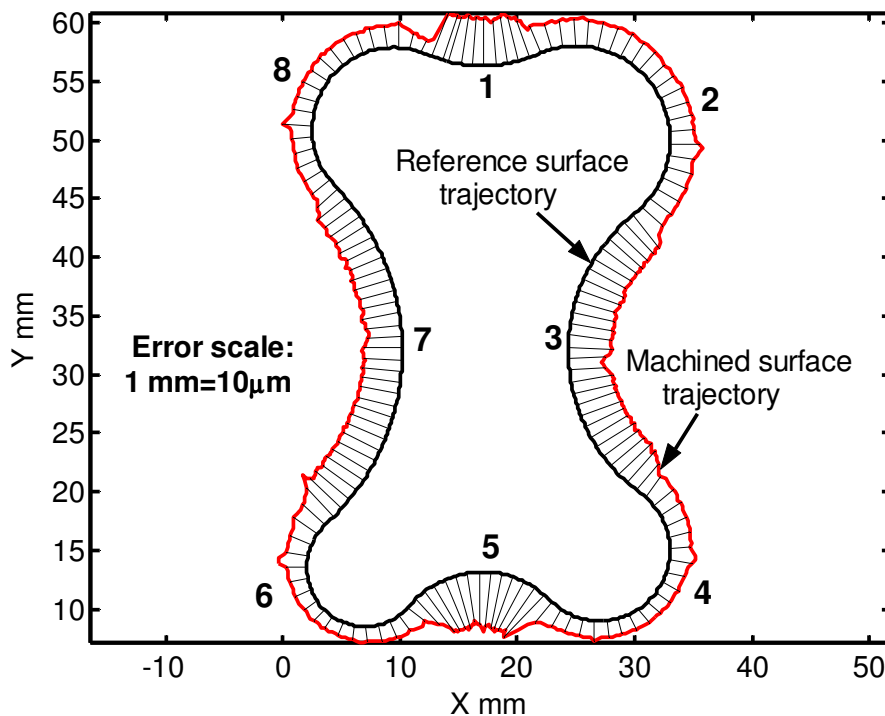
(a) Contour parallel path (Strategy 1)



(b) Feedrate control with modified tool path (Strategy 3)

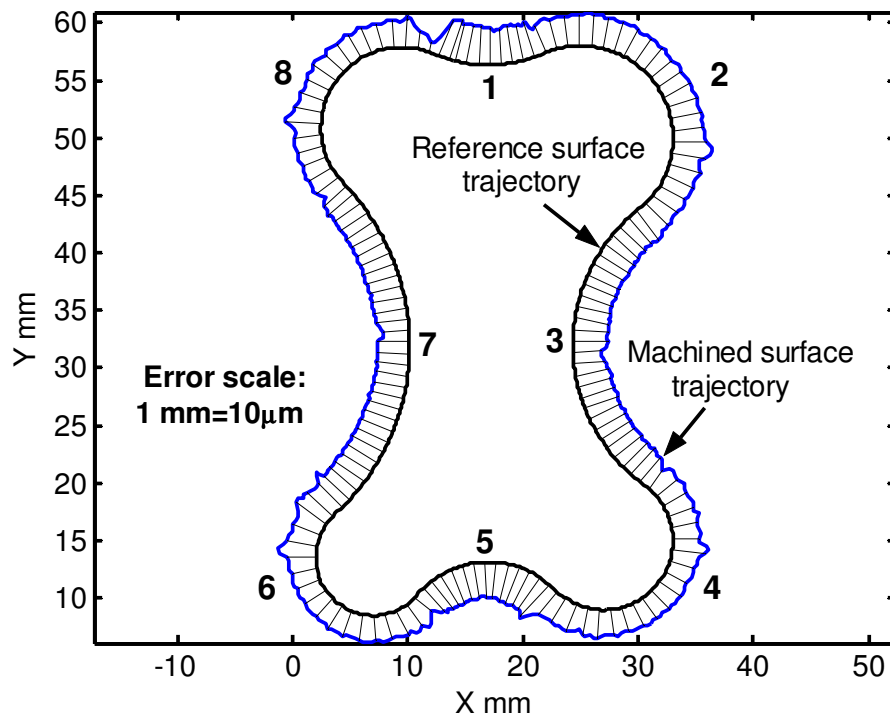
Figure 4-6 Cutter mark profiles of a convex (marked by “A” in Fig. 4-2) and a concave arc (marked by “B” in Fig. 4-2) of curvature radius of 8mm of the machined core

Machined surface trajectories of the core workpiece for three strategies measured by a CMM (Leitz's PMM 866) are shown in Fig. 4-7. Figure 4-8 describes machined surface geometric error profiles with respect to the distance along the reference surface in the finishing. From Figs. 4-7 and 4-8, it is seen that, in the machining under feedrate control with the modified tool path (Strategy 3), machined surface geometric error is more constant along the reference trajectory of the core workpiece while comparing with those under Strategies 1 and 2. Neglecting larger error at entry/exit points of the cutting near the point “1”, the feedrate control with the modified tool path (Strategy 3) reduces the maximum machined surface error variation by about 57.5% and 19.3% compared to those under the conventional contour parallel path (Strategy 1) and the feedrate control (Strategy 2) respectively.

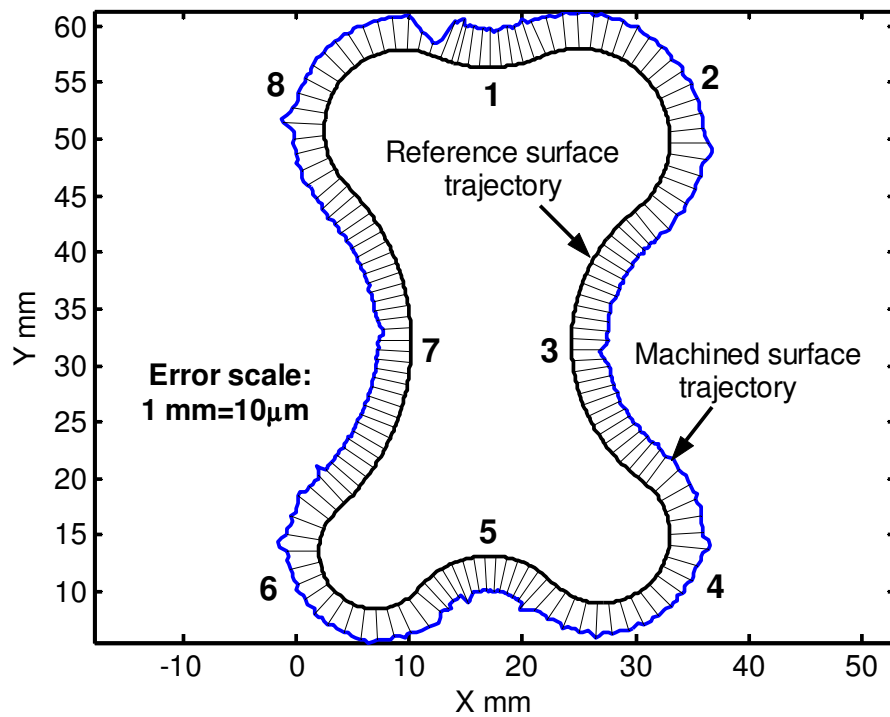


(a) Contour parallel path (Strategy 1)

Figure 4-7 Machined surface trajectories with respect to the reference surface of the core workpiece



(b) Feedrate control (Strategy 2)

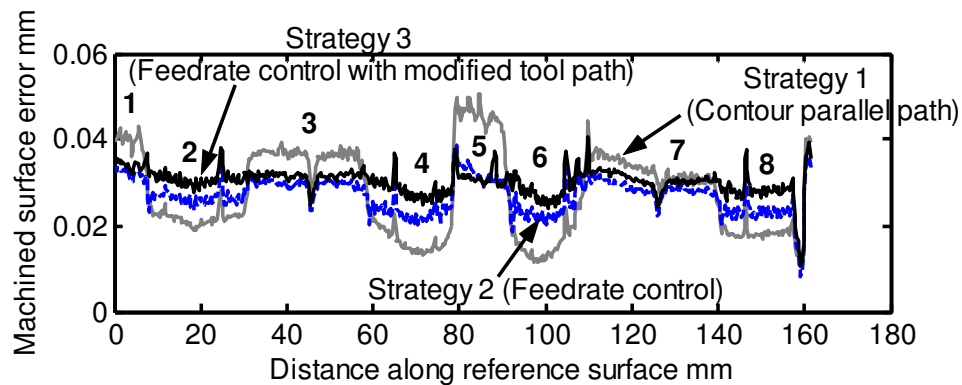


(c) Feedrate control with modified tool path (Strategy 3)

Figure 4-7 Machined surface trajectories with respect to the reference surface of the core workpiece

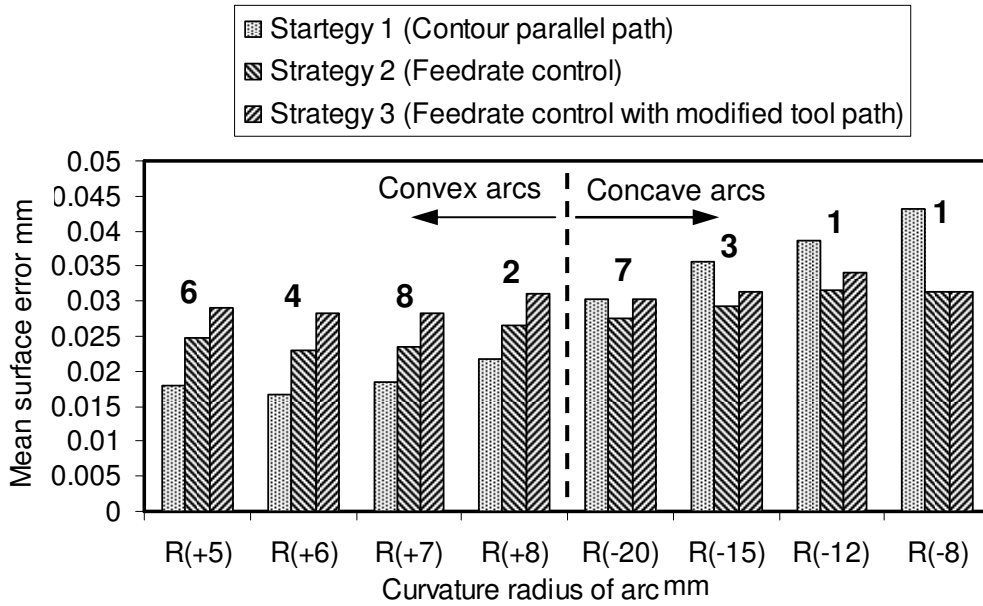
Note that, in Fig. 4-8, it can be observed that the machining under contour parallel path (Strategy 1) produces the least machined surface error on convex arcs. The intention of applying the modified semi-finish path (and the feedrate control) is to reduce the variation in the cutting force over the entire finishing path, which leads to the *increase* of cutting force at convex arcs (see also Fig. 4-4). This naturally results in larger tool deflection, and consequently, larger machined surface error at convex arcs. In this study, we are more interested in obtaining the least variation of machined surface error (i.e. uniform geometric surface) over the entire finishing surface. In Strategy 3, the semi-finish path and the feedrate are modified such that the cutting force at convex arcs is increased, and that at concave arcs is decreased.

Further, the mean values of machined surface geometric error for each of eight corners of the core contour are drawn in Fig. 4-9. Results from the figure verify more uniform geometric error achieved under the feedrate control with the modified tool path (Strategy 3) developed by the proposed algorithm.



* The numbers on top of graphs correspond to the corner names, same as those in Fig. 4-7

Figure 4-8 Machined surface geometric error profiles with respect to distance along the reference surface of the core workpiece



* The numbers on top graphs correspond to the corner names, same as those in Fig. 4-7

Figure 4-9 Mean surface geometric error profiles with respect to curvature radius of the core workpiece (R(+): convex arc, R(-): concave arc)

4.6 Conclusion

In this chapter, the algorithm for tool path modification to regulate cutting engagement is applied to a feedrate control scheme, where feedrate at the tool center is varied such that a constant feedrate at the cutting point is maintained. While a constant feedrate at the cutting point does not regulate constant cutting force, it has been shown that by applying feedrate control with modified tool path, a desired cutting force can be regulated more accurately and efficiently, and hence, the variation of machined surface geometric error is minimized and the surface quality is improved. Results from experimental verification show that compared to contour parallel path and feedrate control, the proposed scheme (feedrate control with modified tool path) reduces variation of cutting force by about 80.4% and 44% (at maximum) respectively. Consequently, geometric surface error variation is reduced by about 57.5% and 19.3% (at maximum) respectively, revealing an improved accuracy on the machined contour.

Chapter 5

Modeling and Identification of Kinematic Errors

On 5-Axis Machining Centers

5.1 Introduction

In Chapter 2 and 3, we have mainly proposed the forward and backward tool path modification approaches respectively such that the cutting engagement is regulated at a desired constant level in two-dimensional contour end milling. An approach to a constant cutting engagement regulation with the backward tool path modification under the feedrate scheduling is shown in Chapter 4. The ultimate expectation with the approaches is to suppress the variation of the cutting force and tool deflection, and hence, consequently enhance the geometric accuracy of the machined surface in 2D contour milling. Case studies with experimental verification using straight end mills are shown in the previous chapters demonstrating the significance of the proposed tool path modification approaches.

While NC tool path modification approaches, developed by considering the physical geometry of the cutting process, have been able to suppress the variation in cutting force and hence improve the geometric accuracy of the machined surface in 3-axis machining, various errors sources discussed in Chapter 1, which always exist in the machining centers, must be taken into account to improve the machine's motion accuracy and hence, the geometric accuracy of the machined surface.

Today's manufacturing industries are facing an increasing demand for machined components with geometric complexity and high dimensional accuracy. In an aim to meet such tremendous demands, recently, as multi-axis machining centers, 5-axis machining centers are extensively used in various machining. With two more axes of rotational degree of freedom than its 3-axis counterpart,

5-axis machining centers can provide cutting tools with an improved directional coordination for cutting free-form surfaces. Therefore, 5-axis machining centers offer notable benefits that include increased material removal rate (MRR), enhanced surface accuracy, and reduced effective machining time.

However, it is well known that a 5-axis machining center generally has many error sources that can cause significant errors on the tool position and orientation with respect to workpiece, hence leading to a geometric inaccuracy of the machined workpiece surface in actual cutting operation. Among such error sources, kinematic errors due to geometric inaccuracies of structural components in a 5-axis machining center are known to be potentially one of the dominant error sources. Kinematic errors are primarily resulted from the imperfection in manufacturing processes starting from making unit components until assembling them to produce the final structure of the machine. On such a multi-axis machining center, it is technically more difficult to eliminate the assembly error of a rotary drive, which potentially causes the translational error or the squareness error between linear and rotary axes, than on conventional 3-axis machines that have only orthogonal linear axes. In order to produce an accurate machined part, it is important, to a great extent, to realize accurate tool position and orientation in 5-axis machining. Hence, kinematic errors in the machining centers have to be modeled and identified, and then their influence on the machining accuracy must be predicted to improve the geometric accuracy of machined part.

With this motivation, the present chapter focuses on modeling and identification of kinematic errors on 5-axis machining centers. Kinematic errors associated with rotary and linear axes of a 5-axis machining center with tilting rotary table type are defined. A procedure to identify kinematic errors practically by using a telescoping double ball bar (DBB) measuring device is also illustrated. Case studies with simulation and experiments are shown to demonstrate the capability of the DBB measurements and an identification procedure to estimate the kinematic errors in the machining center of the interest.

The remainder of the chapter is structured as follows. Section 5.2 reviews past research works on modeling and identification of kinematic errors on 5-axis machining centers. Common configurations of 5-axis machining centers and definitions of kinematic errors associated with rotary and linear axes in the machining centers are described briefly in Section 5.3. Section 5.4 presents a kinematic model of a 5-axis machining center with its kinematic errors. A theory and modeling of DBB measurement method to identify the kinematic errors are described in Section 5.5. With the aid of simulation, an identification procedure to estimate kinematic errors is also presented in the section. In Section 5.6, case studies with experiment are shown to validate the DBB measurement and identification procedure for kinematic errors. Some problems associated with the DBB measurement in identifying kinematic errors are addressed in Section 5.7. Finally, the conclusion of the chapter is shown in Section 5.8.

5.2 Literature Review: Modeling and Identification of Kinematic Errors on 5-Axis Machining Centers

Numerous error origins affect the tool position and orientation with respect to workpiece in 5-axis machining. Among the key factors that affect the accuracy of this relative position and orientation are kinematic errors due to geometric inaccuracies in mechanical components in the machining centers and thermal errors. Geometric inaccuracies originate from the manufacturing and assembly defects of the different parts of the machining center [Cecil and Sutherland, 1998].

In the past, a number of research works has been carried out to realize kinematic errors, and then to simulate and improve motion accuracy of the 5-axis machining centers. These works attempted mainly at first to build up an error model with kinematic errors and then to predict motion accuracy of the machining centers. Considering the machine tools as the rigid body kinematics, generally homogeneous transformation matrices (HTMs) are used to build up the

direct kinematic error model of the machining centers since they can easily accommodate both link and motion error modeling [Hocken et al., 1977 and Donmez et al., 1986]. By using a matrix summation approach, Lin and Shen [Lin and Shen, 2003] introduced modeling of the errors in the 5-axis machine tools. Kiridena and Ferreira [Kiridena and Ferreira, 1993] used the Denavit-Hartenberg (D-H) method to construct the HTMs, and developed a model showing the effects of kinematic errors on the accuracy of the cutting tool position relative to the workpiece on a 5-axis machine tool. By using small angle approximations for inaccurate links and joints, Srivastava et al. [Srivastava et al., 1995] proposed a volumetric error model of kinematic and thermal errors in a 5-axis machine tool, which is later used to demonstrate that the practical compensation of some kinematic error is possible to improve motion accuracy of the machine tool. Eman et al., [Eman et al., 1987] and Soons et al. [Soons et al., 1992] presented a generalized methodology for modeling the errors due to inaccuracies in geometry, finite stiffness, and thermal deformations of multi-axis machine tools of arbitrary configurations. Suh et al. [Suh et al., 1998] investigated the error modeling and measurement for a 5-axis machine tool with the tilting rotary table type. Very recently, Rahman et al. [Rahman, 2000] first investigated modeling and measurement of multi-axis machine tools with kinematic errors, and then, compensated some of kinematic errors by modifying NC programs in a post-processor or creating an extra NC program processor.

While modeling of errors in 5-axis machining centers have been widely studied, measurement and identification of kinematic errors particular to 5-axis machining centers need to be addressed. The telescoping double ball bar (DBB) specified in ISO 230-1 [ISO 230-1, 1996] as a measuring instrument has been mainly used for the circular tests of the conventional 3-axis machining center. Recently, this DBB measuring device has been applied to identify kinematic errors particular to 5-axis machining centers [Sakamoto and Inasaki, 1994, Kakino et al., 1994, Mayer et al., 1999, Abbaszaheh et al., 2002, and Tsutsumi and Saito, 2004]. There have been also approaches proposed in the literature using other measurement devices. For example, Lei and Hsu [Lie and Hsu, 2002]

applied a 3D probe ball device to identify some of kinematic errors. For a 5-axis machine tool, Bringmann and Knapp [Bringmann and Knapp, 2006] recently showed the “R-Test” measurement device, where three dimensional displacement of the tool center is measured by using a sphere attached to the spindle tip, and four displacement sensors pointed to the center of the sphere, and then the machine’s kinematic errors are identified based on its kinematic model.

As a test method describing the inspection of motion accuracy of a 5-axis machine tool, ISO 10791-1 [ISO 10791-1] defines measurement schemes to evaluate kinematic errors using displacement sensors and artifacts for 5-axis machine configurations having rotary or swivel spindle heads. Later ISO 10791-6 [ISO 10791-6] specifies method for accuracy evaluation of 5-axis machines of the same type by considering the effect of feedrate and speed of the motion axes. For 5-axis machine configurations having a tilting rotary table, ISO 230-7 [ISO 230-7, 2005] was recently published in 2005 developing a measurement scheme of kinematic errors of rotary axes.

Compared to other measurement methods mentioned above, the notable benefit of using DBB measuring device is that the method is quicker, faster and easy-to-use to identify kinematic errors of most multi-axis machine tools. Here, it is to be noted that, in this research work, we adopt the DBB measuring method to identify the kinematic errors in 5-axis machining center of the interest.

5.3 5-Axis Machining Centers and Kinematic Errors

5.3.1 Configuration of 5-Axis Machining Centers

Generally a 5-axis machining center has three linear axes and two controllable rotary axes. There are different types of 5-axis machining centers currently available in the machine tool manufacturing market. Besides some with special configuration, 5-axis machining centers with three types of configuration are very commonly used in the industries [Tsutsumi and Saito, 2004]. The first is

a universal head type with two rotary axes, which is often applied to the manufacturing of aerospace components or dies/molds. The second is a tilting rotary table type with two rotary axes, which is applied for the machining of small precise components. And the third is a kind of combination of first and second types with a swivel head and a rotary table.

In this research work, a 5-axis machining center with the tilting rotary table type (second type), or ZX/YAC configuration [JIS B6310: 2003] is considered as the target. Figure 5-1 shows its basic configuration. As can be seen in Fig.5-1, the machine contains three linear-axis drives (X, Y, Z) for generating linear motions in X, Y, and Z directions and two rotary-axis drives (A,C) for generating rotary motions on the tilting-rotary table about X and Z axes respectively.

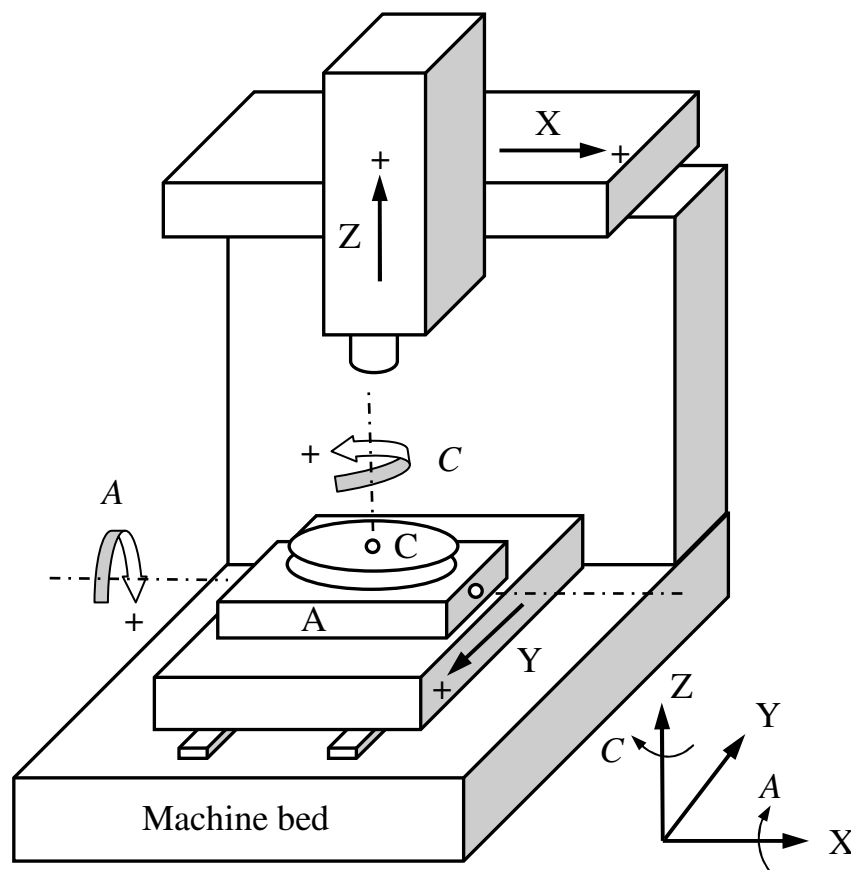


Figure 5-1 Configuration of a 5-axis machining center of tilting rotary table type

5.3.2 Definitions of Kinematic Errors in a 5-Axis Machining Center

5.3.2.1 Kinematic Errors Associated With Rotary Axes

The kinematic errors inherent to a 5-axis machining center can be determined by considering the configuration of the machining center according to the shape generation theory [Inasaki et al., 1997]. Inasaki et al. [Inasaki et al., 1997] pointed out that there are total thirteen kinematic errors inherent to the configuration of a 5-axis machining center as shown in Fig. 5-1. Namely, eight kinematic errors (α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , δy_{CA}) are associated with rotary axes and five kinematic errors (γ_{XY} , α_{YZ} , β_{ZX} , α_{ZS} , β_{ZS}) are associated with linear axes of the machining center. The suffixes X, Y, Z, A, and C indicate the name of coordinate axes, and the suffix S indicates the spindle axis. This subsection illustrates the definition of the eight kinematic errors associated with rotary axes. The definitions of kinematic errors are as follows.

α_{AY} is the angular error of A-axis with respect to Y-axis about X-axis. Ideally when $A=0^\circ$ (no tilt), the titling-rotary table should be horizontal. However, if the table is tilted about X-axis, it makes the angular error, α_{AY} . Similarly, β_{AY} and γ_{AY} are the angular errors of A-axis about Y and Z-axes respectively. β_{CA} is the angular error of C-axis with respect to A-axis about X-axis. Ideally, the center lines of C-axis and A-axis should be perpendicular to each other. However, if the C-axis is tilted with respect to A-axis about Y-axis, it makes the angular error, β_{CA} .

On the other hand, δx_{AY} , δy_{AY} , and δz_{AY} are linear shifts of A-axis with respect to Y-axis (or the origin of machine, XYZ) in X, Y and Z directions respectively. δy_{CA} is the linear shift of C-axis with respect to A-axis in Y-direction. Ideally, the center lines of the C-axis and A-axis should cross a point perpendicularly. However if there is a displacement between C and A-axes along Y-direction, it makes the linear shift, δy_{CA} .

5.3.2.2 Kinematic Errors Associated With Linear Axes

γ_{YX} , α_{ZY} , and β_{ZX} are defined as the orthogonality or squareness errors between X and Y axes, Y and Z axes, and Z and X axes respectively. On the other hand, α_{ZS} is defined as the squareness error between the center line of spindle axis and Z-axis on Y-Z plane. And, β_{ZS} is the squareness error between the center line of the spindle axis and Z-axis on Z-X plane.

5.3.2.3 Kinematic Errors to Be Identified

In this study, it is assumed that the center line of spindle axis is absolutely parallel with the Z-axis movement. Hence, in this study, eight kinematic errors associated with rotary axes (α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , δy_{CA}) and three kinematic errors associated with linear axes (γ_{YX} , α_{ZY} , β_{ZX}) are of the interest and to be identified practically on the target 5-axis machining center.

5.4 Modeling of 5-Axis Machining Center with Kinematic Errors

The kinematic model of the machine configuration in Fig. 5-1 is given in details in [Tsutsumi and Saito, 2004]. In this paper, the objective to employ the kinematic model of the machining center is to calculate the tool center location relative to the workpiece under the existence of kinematic errors. This section briefly reviews this calculation based on the kinematic model. Coordinate systems for the machine configuration shown in Fig.5-1 are illustrated in Fig.5-2.

First, the location of the tool center in the reference frame is calculated with kinematic errors. Here, the reference frame (X_F - Y_F - Z_F) is the coordinate system fixed to the machine frame or bed as shown in Fig. 5-2. When the command position of the tool center is given by $(\hat{X}_c, \hat{Y}_c, \hat{Z}_c)$, the actual tool center location in the reference frame under the influence of the machine's kinematic errors, ${}^r p_t \in \mathbb{R}^4$, is given as follows. First three elements of ${}^r p_t$ represent X, Y,

and Z coordinates and its fourth element is one. Note that the left-side superscript r denotes the vector defined in the reference frame.

$${}^r p_t = {}^r T_z {}^z T_x p_t^0 \quad (5-1)$$

where $p_t^0 = [0 \ 0 \ 0 \ 1]^T$. ${}^r T_z \in \mathbb{R}^{4 \times 4}$ is a homogeneous transformation matrix (HTM) [Slocum, 1992] representing the motion of the Z axis in the reference frame. Similarly, ${}^z T_x \in \mathbb{R}^{4 \times 4}$ denotes a HTM representing the motion of the X axis with respect to the Z axis. They are respectively given by:

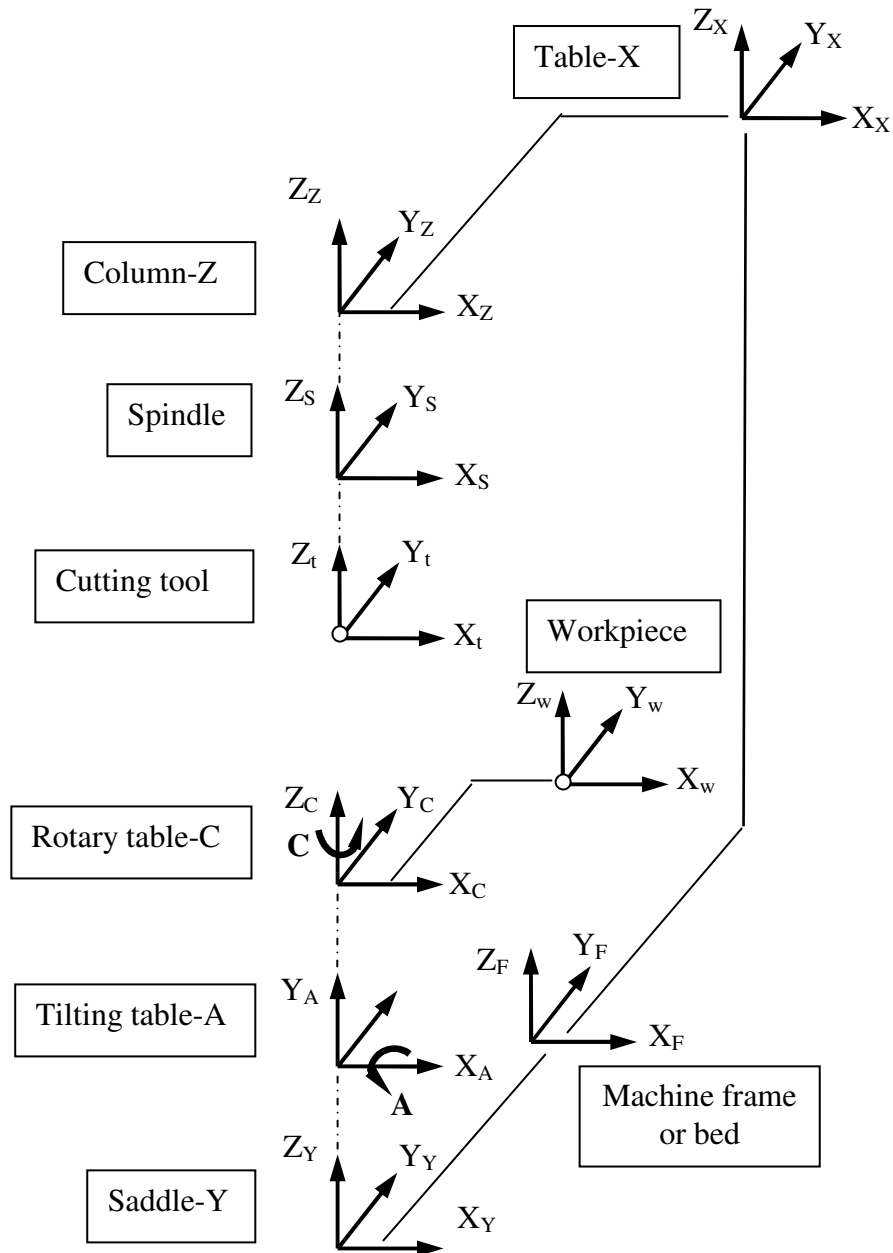


Figure 5-2 Coordinate systems for the kinematic chain of a 5-axis machining center with a tilting rotary table [Tsutsumi and Saito, 2004]

$${}^rT_z = D^4(\alpha_{ZY})D^5(\beta_{ZX})D^3(\hat{Z}_c) \quad (5-2)$$

$${}^zT_x = D^1(\hat{X}_c) \quad (5-3)$$

where, α_{ZY} and β_{ZX} are kinematic errors defined in Section 5.3.2. $D^{1-6}(\ast) \in \Re^{4 \times 4}$ represent the HTMs, where $D^1(x)$, $D^2(y)$, and $D^3(z)$ respectively represent the HTM for linear motions in the X, Y, and Z directions, and $D^4(a)$, $D^5(b)$, and $D^6(c)$ respectively represent the HTM for angular motions about X, Y and Z axes (For the definition of HTMs, see Appendix I).

Then, the transformation from the workpiece frame to the reference frame is considered. Note that the workpiece frame (X_w - Y_w - Z_w) is attached to the workpiece as shown in Fig. 5-2. The HTM representing the transformation from the workpiece frame to the reference frame is given by:

$${}^rT_w = {}^rT_y {}^yT_a {}^aT_c \quad (5-4)$$

where, the HTMs, aT_c , yT_a , and rT_y are respectively given as:

$${}^aT_c = D^2(\delta y_{CA})D^5(\beta_{CA})D^6(\hat{C}) \quad (5-5)$$

$${}^yT_a = D^1(\delta x_{AY})D^2(\delta y_{AY})D^3(\delta z_{AY})D^4(\alpha_{AY})D^5(\beta_{AY})D^6(\gamma_{AY})D^4(\hat{A}) \quad (5-6)$$

$${}^rT_y = D^6(\gamma_{YX})D^2(-\hat{Y}_c) \quad (5-7)$$

where, α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , δy_{CA} , and γ_{YX} are kinematic errors as defined in Section 5.3.2. $\hat{A}$ and $\hat{C}$ are the tool orientation angles about X and Z axes respectively represented in the workpiece frame.

Hence, the tool center location in the workpiece frame, ${}^w p_t \in \Re^4$, can be given as follows. Note that the left-side superscript w denotes the vector defined in the workpiece frame.

$${}^w p_t = ({}^rT_w)^{-1} \cdot {}^r p_t \quad (5-8)$$

Therefore, the error in the tool tip position due to kinematic errors expressed in the workpiece frame, ${}^w \delta p_t$, can be given as follows:

$${}^w \delta p_t = {}^w p_t - {}^w p_t^* \quad (5-9)$$

Similarly, the error in the vector representing the orientation of tool expressed in the workpiece frame, ${}^w \delta v_t$, can be given as follows:

$${}^w\delta v_t = {}^w v_t - {}^w v_t^* \quad (5-10)$$

where, the vectors, ${}^w p_t^*$ and ${}^w v_t^*$ represent the nominal position and orientation of the tool in the workpiece frame, which can be obtained by setting all kinematic errors zero in the above error model.

5.5 Identification of Kinematic Errors by Using DBB measurements

5.5.1 DBB measuring Device

Despite of having some other methods to the measurement of kinematic errors as reviewed in the literature in Section 5.2, in this research work, a DBB measurement method is applied to identify the kinematic errors of the target machining center. DBB method was first reported by Bryan [Bryan, 1982], and was applied to CMM (Coordinate Measuring Machine). The methodology to identify error sources in NC machines based on the DBB method was proposed by Kakino et al. [Kakino et al., 1986], and is now widely accepted in today's industry. Recently, this method as specified in ISO 230-1 has been reliably applied to measure and identify kinematic errors particular to the 5-axis machining center [Sakamoto and Inasaki, 1994, Tsutsumi and Saito, 2003].

Generally a DBB device consists of two high-precision balls connected by a telescoping bar, and the distance between two balls is measured by a linear encoder installed on the telescoping bar [Renishaw]. Figure 5-3 shows a typical DBB measuring device. One ball of the telescoping ball-bar is fixed magnetically to a socket mounted on the spindle and the other ball with magnetic socket is placed on the work-table. The DBB device measures the contouring error profile as the machine is traversing along a circular trajectory. When there exists a contouring error, the distance between two balls is changed and measured by the linear encoder in the telescoping bar. The measured distance is then sent to a

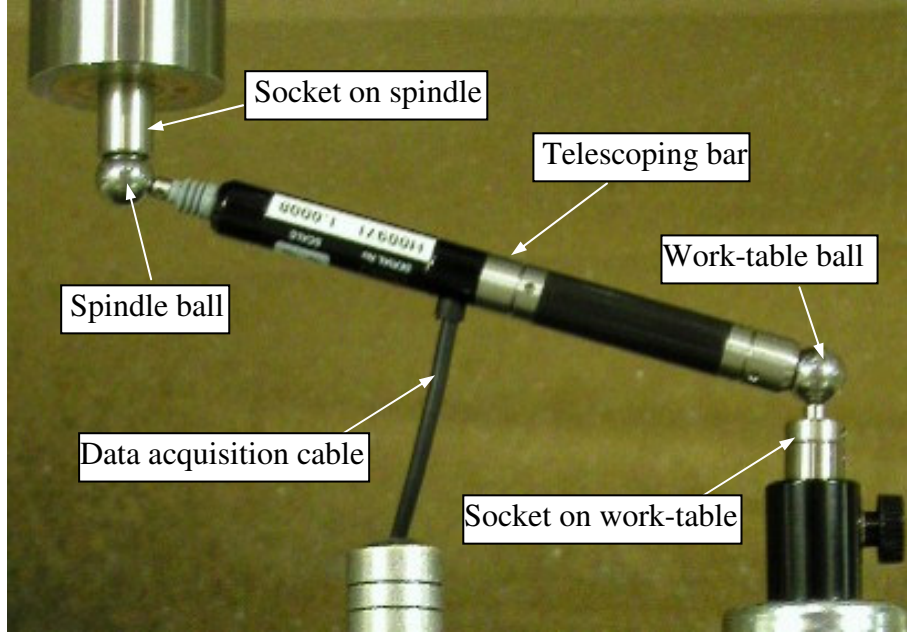


Figure 5-3 DBB measuring device [Renishaw, 2007]

personal computer and the error analysis is performed. A DBB measuring device provides a quick and effective way to measure the machine's contouring error.

5.5.2 Modeling of DBB Measurements

As is mentioned earlier, the DBB method gives the relative distance between the spindle-side ball and the work-table-side ball while the nominal length of the ball bar is kept constant. This section briefly discusses the relationship between the location of two balls and the DBB measurement.

For a circular motion trajectory of a specific DBB measurement pattern, consider the command tool position in the reference frame, $(\hat{X}_c, \hat{Y}_c, \hat{Z}_c)$ and orientation, $(\hat{A}_c, \hat{C}_c)$. First, the actual position of the spindle-side ball in the reference frame, ${}^r p_s = [X_s \ Y_s \ Z_s \ 1]^T$ can be calculated as follows:

$${}^r p_s = {}^r T_z {}^z T_x p_s^0 \quad (5-11)$$

where, $p_s^0 = [0 \ 0 \ 0 \ 1]^T$, and ${}^r T_z \in \mathbb{R}^{4 \times 4}$, ${}^z T_x \in \mathbb{R}^{4 \times 4}$ are the HTMs, which contain kinematic errors and are already defined explicitly in Eqs. (5-2) and (5-3) respectively.

Then, under the influence of kinematic errors, the actual position of the work-table side ball in the reference frame, ${}^r p_w = [X_w \ Y_w \ Z_w \ 1]^T$ can be calculated as follows:

$${}^r p_w = {}^r T_y {}^y T_a {}^a T_c p_w^0 \quad (5-12)$$

where, $p_w^0 = [X_{w0} \ Y_{w0} \ Z_{w0} \ 1]^T$ is an initial position of the table-side ball on the work-table, and ${}^r T_y \in \mathbb{R}^{4 \times 4}$, ${}^y T_a \in \mathbb{R}^{4 \times 4}$, ${}^a T_c \in \mathbb{R}^{4 \times 4}$ are the HTMs and already defined in Eqs. (5-5) ~ (5-7).

Now, with respect to the nominal length of the DBB bar, R_0 , the relative displacement between the actual positions of spindle-side ball and the table-side ball, ΔR , can be determined as follows:

$$\Delta R = \sqrt{\{(X_s - X_w)^2 + (Y_s - Y_w)^2 + (Z_s - Z_w)^2\}} - R_0 \quad (5-13)$$

Then this relative displacement between two balls will give contouring error profiles for a circular motion trajectory of a specific DBB measurement.

5.5.3 DBB Measurement Patterns to Identify Kinematic Errors Associated With Linear Axes

In this research work, a number of DBB tests with specific measurement patterns proposed by Tsutsumi and Saito [Tsutsumi and Saito, 2003] are employed to identify kinematic errors of the target 5-axis machining center. The remainder of this section will review Tsutsumi and Saito's procedure to identify kinematic errors on 5-axis machining centers.

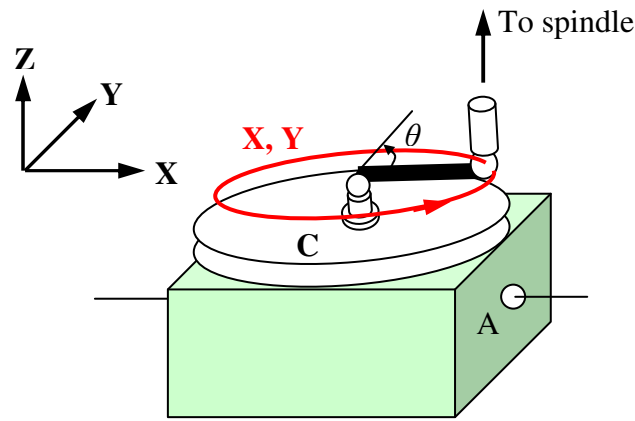
First, three DBB measurements are considered to identify three squareness errors (γ_{YX} , α_{ZY} , and β_{ZX}) associated with linear axes (X, Y, Z) [Kakino et al., 1986]. During the measurement, two linear-axis motions are simultaneously controlled. Figure 5-4 shows the DBB measurement pattern on X-Y, Y-Z and Z-X planes to identify squareness errors, γ_{YX} , α_{ZY} , and β_{ZX} respectively. In each measurement, the table-side ball is placed at the center of the work-table.

To illustrate the effect of kinematic errors on the DBB trajectory, we present a simple simulation based on the model described in Section 5.5.2. Figure

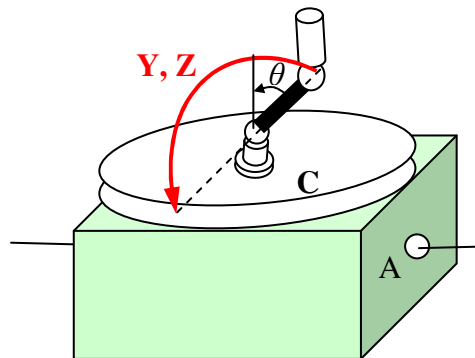
5-5 shows the simulated DBB displacement trajectories with respect to the nominal trajectory for three measurement patterns when each kinematic error is set to $+0.005^\circ$. The nominal length of DBB bar is kept 100 mm. The nominal DBB trajectory of a full circular motion ($0^\circ \sim 360^\circ$) is considered in simulation. As is clear from those simulation results, Kakino et al. [Kakino et al., 1986] showed that the relationship between the squareness errors, γ_{yx} , α_{zy} , β_{zx} and radial error, $\Delta R(\theta)$ of the trajectory can be written as follows:

$$(\gamma_{yx}, \alpha_{zy}, \beta_{zx}) = \frac{2\Delta R(\theta)}{R \sin 2\theta} \quad (5-14)$$

where, R is the nominal length of the DBB bar, and θ is the angular location of the spindle-side balls (see Fig. 5-5) of the reference DBB trajectory.

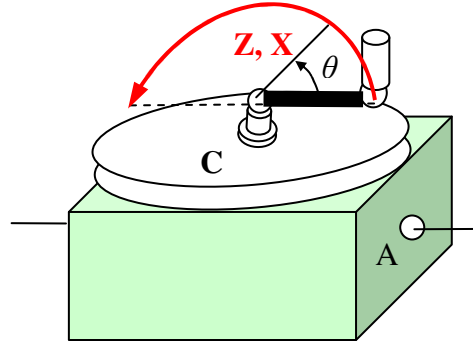


(a) DBB measurement on X-Y plane



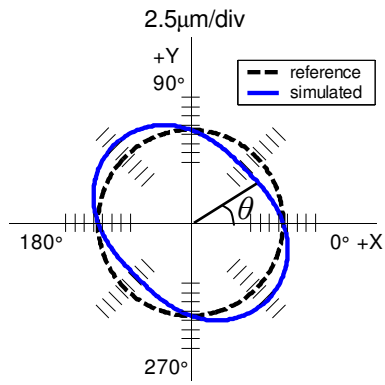
(b) DBB measurement on YZ plane

Figure 5-4 DBB measurement patterns on X-Y, Y-Z, and Z-X planes

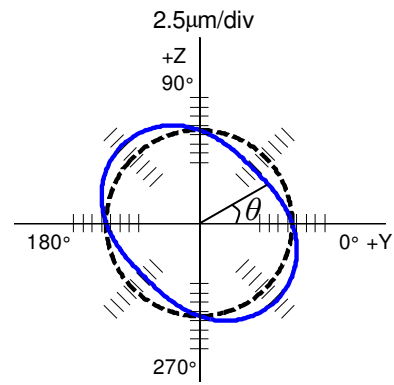


(c) DBB measurement on ZX plane

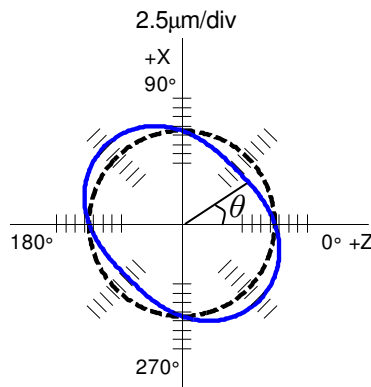
Figure 5-4 DBB measurement patterns on X-Y, Y-Z, and Z-X planes



(a) $\gamma_{YX} = 0.005^\circ$



(b) $\alpha_{ZY} = 0.005^\circ$



(c) $\beta_{ZX} = 0.005^\circ$

Figure 5-5 Relationship between squareness errors in linear axes (X, Y, and Z) and simulated DBB displacement trajectories

5.5.4 DBB Measurement Patterns to Identify Kinematic Errors Associated With Rotary Axes

Four DBB measurements are conducted by simultaneously controlling three-axis motion where two linear and one rotary axis motion exists, to estimate kinematic errors (α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , and δy_{CA}) associated with rotary axes (A,C) in the 5-axis machining center. In order to illustrate the influence of individual kinematic error on the DBB displacement trajectory, by using the modeling of DBB measurement as described in Section 5.5.2, simulation of the DBB measurements is carried out. In simulation, the nominal length of the DBB bar is kept at 100mm, while each linear kinematic error of +10 μ m and angular kinematic error of +0.005° are maintained throughout. Please note that angular motion in the counter-clock wise direction when viewed from the positive direction of the reference axis is defined to be the positive direction. The following section will describe all the DBB measurement patterns and the relationship between the kinematic errors and simulated DBB displacement trajectories.

5.5.4.1 DBB Measurement Pattern 1

In this DBB measurement pattern, the telescoping ball bar is set on the work-table in parallel with the axis line of the C-axis and the table-side ball is placed in a position away from the center of the work-table. During the measurement, two linear axis motions on X-Y, and a rotary motion on C-axis are simultaneously controlled as can be shown in Figure 5-6.

Figure 5-7 shows the simulated DBB displacement trajectories under the influence of each kinematic error for the DBB measurement pattern 1. DBB displacement trajectories are drawn with the reference to the angular location of the table-side ball, θ (the configuration in Fig. 5-6 shows the start of the measurement, and in this state, it is assumed that $\theta = 0$). The angular location of the spindle-side ball is indicated by the angle, θ as shown in Figs. 5-6 and 5-7. It

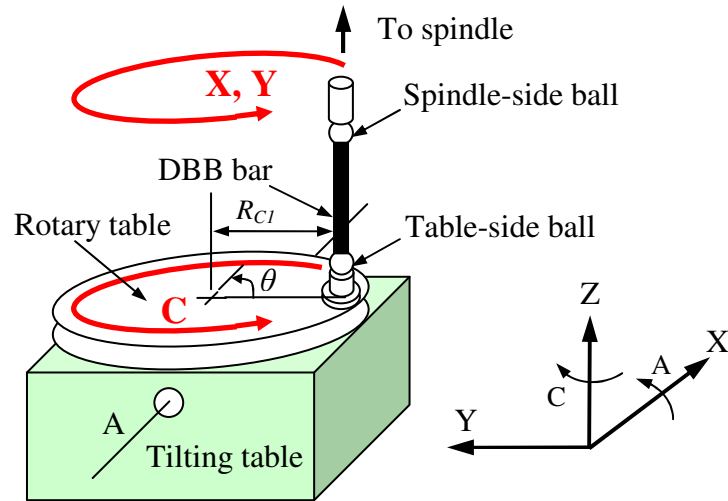
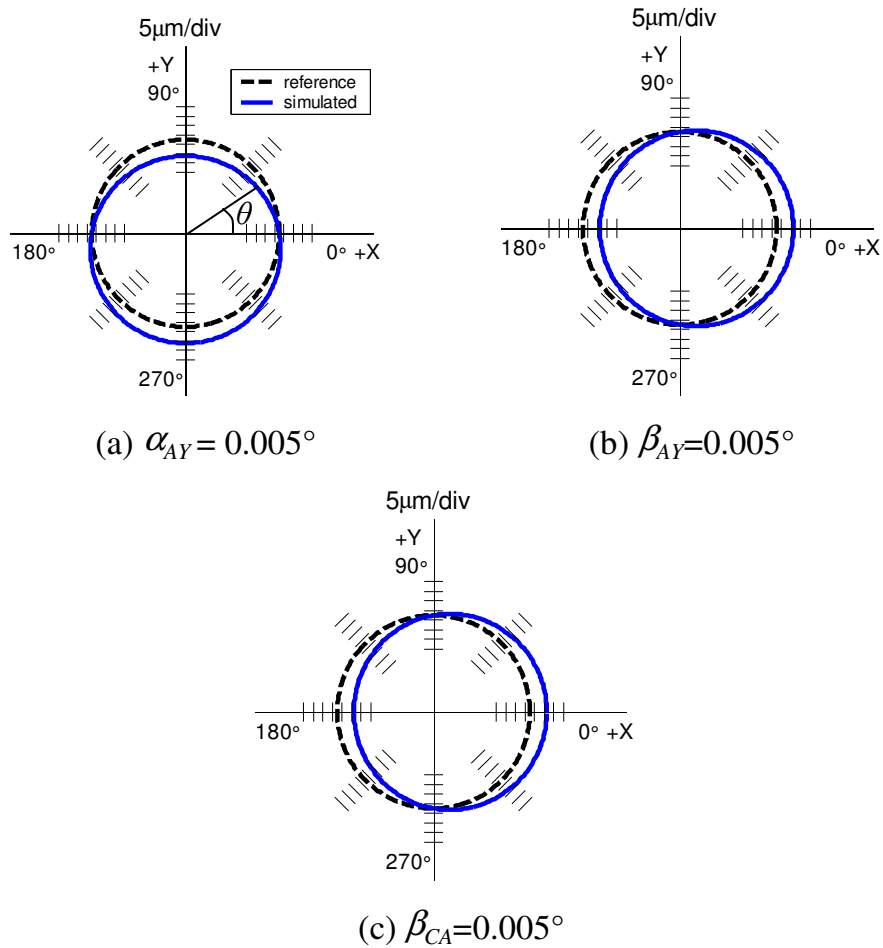


Figure 5-6 DBB measurement pattern 1(synchronous motion of X, Y, and C)



* Simulated DBB trajectories for other kinematic errors are not shown here because they do not have effect on the trajectories with a center shift

Figure 5-7 Relationship between kinematic errors and simulated DBB displacement trajectories for DBB measurement pattern 1

is seen from Fig. 5-7, only kinematic errors, α_{AY} , β_{AY} , and β_{CA} , have effects on the DBB displacement trajectory, showing a center shift of the DBB trajectory from its nominal trajectory. The center shift is defined as the deviation of center location of the measured DBB displacement trajectory from that of its nominal trajectory.

The relationship between the resultant center shift in X and Y directions (e_{x1} and e_{y1}) and the kinematic errors associated with DBB measurement pattern 1 (α_{AY} , β_{AY} , and β_{CA}) can be given as follows:

$$e_{x1} = R_{C1}\beta_{AY} + R_{C1}\beta_{CA} \quad (5-15)$$

$$e_{y1} = -R_{C1}\alpha_{AY} \quad (5-16)$$

where, R_{C1} is the distance between the center of the table-side ball and the center line of C-axis in radial direction of C-axis.

5.5.4.2 DBB Measurement Pattern 2

In this measurement pattern, the ball bar is set in parallel with the axis line of A-axis and the table-side ball is placed at the center of the work-table. Two linear-axis motions on Y-Z plane and a rotary motion on A-axis are simultaneously controlled as shown in Fig. 5-8. Figure 5-9 shows simulated DBB displacement trajectories under the influence of individual kinematic errors (β_{AY} , γ_{AY} , γ_{XY} , and β_{XX}), which cause the trajectory with a center shift for this measurement pattern. In this case, simulated DBB displacement trajectories are drawn with the reference to the angular location of the spindle-side ball, θ (the configuration in Fig. 5-8 shows the start of the measurement, and in this state, it is assumed that $\theta=90^\circ$).

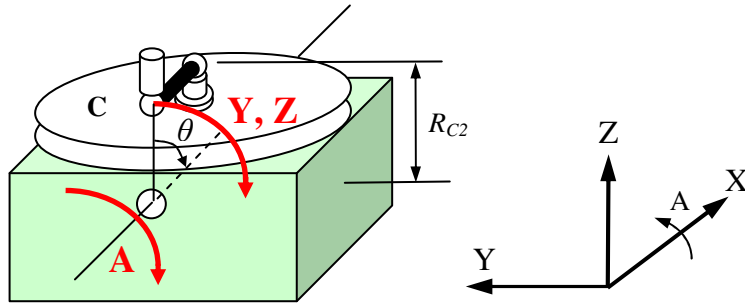
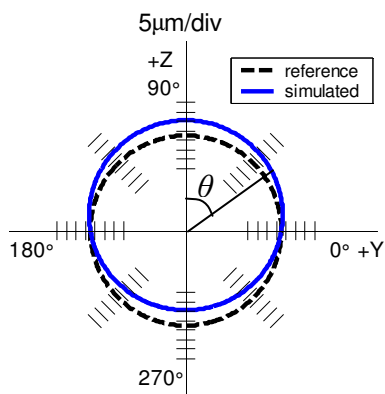
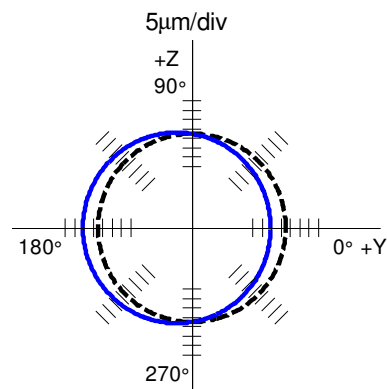


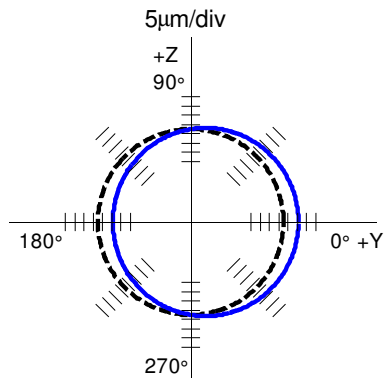
Figure 5-8 DBB measurement pattern 2 (synchronous motion of Y, Z, and A)



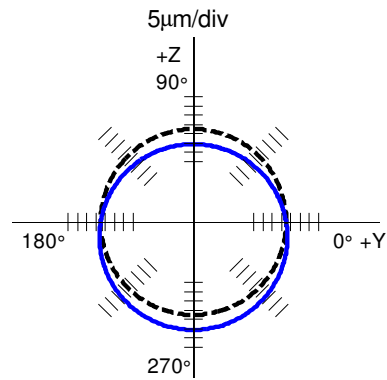
(a) $\beta_{AY} = 0.005^\circ$



(b) $\gamma_{AY} = 0.005^\circ$



(c) $\gamma_{YX} = 0.005^\circ$



(d) $\beta_{ZX} = 0.005^\circ$

Figure 5-9 Relationship between kinematic errors and simulated DBB displacement trajectories for the DBB measurement pattern 2

The relationship between the resultant center shift (e_{y2} and e_{z2}) in Y and Z directions and kinematic errors associated with the DBB measurement pattern 2 (β_{AY} , γ_{AY} , γ_{YX} , and β_{ZX}) can be written by the following equations.

$$e_{y2} = -R_{C2}\gamma_{AY} + R_{C2}\gamma_{YX} \quad (5-17)$$

$$e_{z2} = R_{C2}\beta_{AY} - R_{C2}\beta_{ZX} \quad (5-18)$$

where, R_{C2} is the distance between the center line of the table-side ball and the center line of A-axis in the radial direction of A-axis.

5.5.4.3 DBB Measurement Pattern 3

In this measurement pattern, the DBB bar is set in the radial direction of the A-axis and the table-side ball is placed at the center of the work-table, as shown in Fig. 5-10. Two linear motions on Y-Z plane and a rotary motion on A-axis are simultaneously controlled. Figure 5-11 shows simulated DBB displacement trajectories under the influence of individual kinematic errors (δy_{AY} and δz_{AY}), which cause the trajectory with a center shift for this specific measurement pattern. Here, the DBB displacement trajectories are drawn with the reference to the angular location of the table-side ball, θ (the configuration in Fig. 5-10 shows the start of the measurement, and in this state, it is assumed that $\theta = 90^\circ$).

The relationship between the resultant center shift (e_{y3} and e_{z3}) in Y and Z directions and kinematic errors associated with the DBB measurement pattern 3 (δy_{AY} and δz_{AY}) can be written by the following equations.

$$e_{y3} = -\delta y_{AY} \quad (5-19)$$

$$e_{z3} = -\delta z_{AY} \quad (5-20)$$

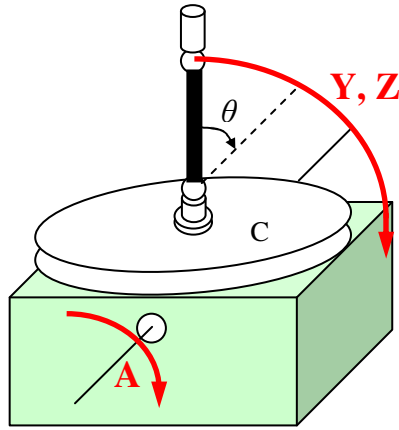


Figure 5-10 DBB measurement pattern 3 (synchronous motion of Y, Z, and A)

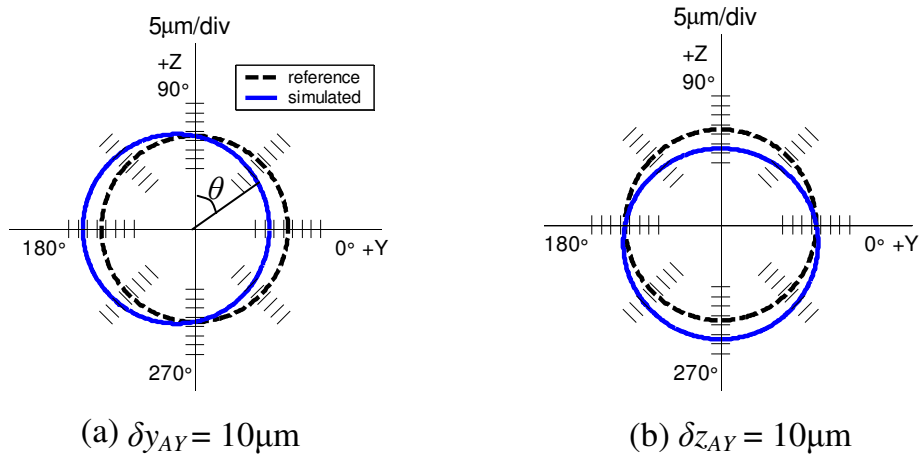


Figure 5-11 Relationship between kinematic errors and simulated DBB displacement trajectories for the DBB measurement pattern 3

5.5.4.4 DBB Measurement Pattern 4

The DBB bar is set in the radial direction of the C-axis and the table-side ball is placed in a position away from the center of the work-table. Figure 5-12 shows the measurement pattern. During the measurement, two linear axis motions on X-Y, and a rotary motion on C-axis are simultaneously controlled as can be shown in Figure 5-12. The aim of this measurement is modify the origin of X and Y axes. Figure 5-13 shows simulated DBB displacement trajectories under the influence of individual kinematic errors (δx_{AY} , δy_{AY} , and δy_{CA}), which cause the trajectory with a center shift for this specific measurement pattern.

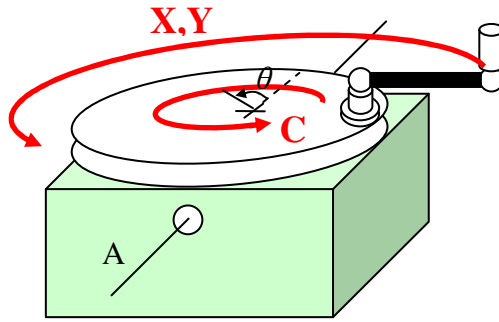
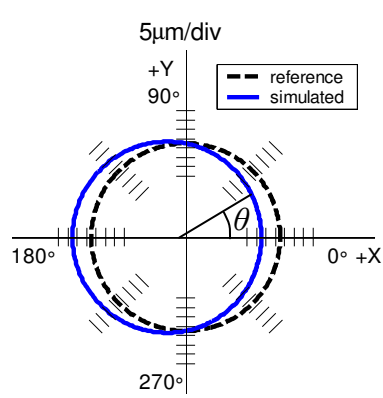
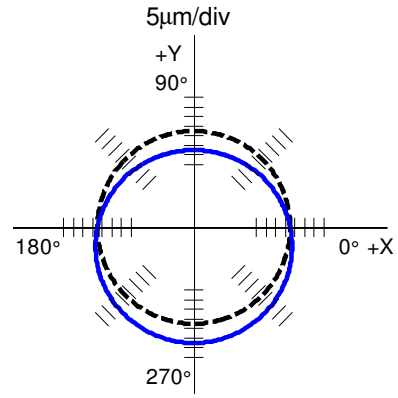


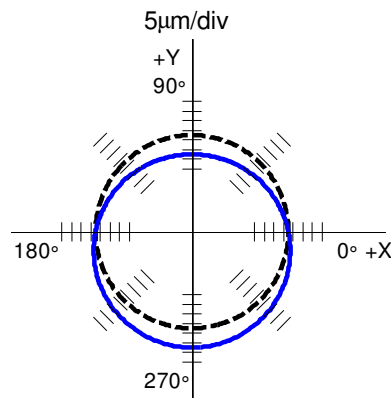
Figure 5-12 DBB measurement pattern 4 (synchronous motion of X, Y, and C)



(a) $\delta x_{AY} = 10\mu\text{m}$



(b) $\delta y_{AY} = 10\mu\text{m}$



(c) $\delta y_{CA} = 10\mu\text{m}$

Figure 5-13 Relationship between kinematic errors and simulated DBB displacement trajectories for the DBB measurement pattern 4

DBB displacement trajectories are drawn with the reference to the angular location of the table-side ball, θ (the configuration in Fig. 5-12 indicate the start of the measurement, and in this state, it is assumed that $\theta = 0$).

The relationship between the resultant center shift in X and Y directions (e_{x4} and e_{y4}) and kinematic errors associated with the DBB measurement pattern 4 (δx_{AY} , δy_{AY} , and δy_{CA}) can be written by the following equations.

$$e_{x4} = -\delta x_{AY} \quad (5-21)$$

$$e_{y4} = -\delta y_{AY} - \delta y_{CA} \quad (5-22)$$

5.5.5 Procedure for Identifying Kinematic Errors

For the given nominal length of the ball bar, R , and radial error, ΔR , obtained from the DBB displacement trajectories on X-Y, Y-Z, and Z-X planes, first three squareness errors associated with linear axes, namely, γ_{YX} , α_{ZY} , and β_{ZX} , can be calculated by using the Eq. (5-14) as defined in Section 5.5.3.

Then, eight kinematic errors associated with rotary axes, namely, α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , and δy_{CA} , can be estimated by the following procedure.

From the relationship between kinematic errors and center shift of the trajectory in the DBB measurement pattern 1 as described in Section 5.5.4, kinematic error, α_{AY} can be calculated as:

$$\alpha_{AY} = -\frac{e_{y1}}{R_{C1}} \quad (5-23)$$

From DBB measurement pattern 2, β_{AY} and γ_{AY} can be calculated as:

$$\beta_{AY} = \frac{1}{R_{C2}}(R_{C2}\beta_{ZX} + e_{z2}) \quad (5-24)$$

$$\gamma_{AY} = -\frac{1}{R_{C2}}(e_{y2} - R_{C2}\gamma_{YX}) \quad (5-25)$$

From DBB measurement pattern 3, δy_{AY} and δz_{AY} can be calculated as:

$$\delta y_{AY} = -e_{y3} \quad (5-26)$$

$$\delta z_{AY} = -e_{z3} \quad (5-27)$$

From DBB measurement patter 4, δx_{AY} and δy_{CA} can be calculated as:

$$\delta x_{AY} = -e_{x4} \quad (5-28)$$

$$\delta y_{CA} = -(e_{y4} + \delta y_{AY}) \quad (5-29)$$

Finally, from DBB measurement pattern 1, β_{CA} can be calculated as:

$$\beta_{CA} = \frac{1}{R_{C1}}(e_{x1} - R_{C1}\beta_{AY}) \quad (5-30)$$

Hence, by using the above equations, total eleven kinematic errors existing in the 5-axis machining center can be identified.

5.6 Case Studies

Actual DBB measurement tests with the measurement patterns described in Section 5.5.3 and Section 5.5.4 are carried out to practically identify total eleven kinematic errors in a commercial 5-axis machining center with the configuration shown in Fig. 5-1.

First, three DBB measurement tests on X-Y, Y-Z, and Z-X planes are conducted to identify squareness errors associated with linear axes. The nominal length of the DBB bar is kept at 100mm and the feedrate of 1000mm/min are maintained in all the three measurement tests. Circular arc of the DBB displacement trajectory ranges from $\theta = 0^\circ$ to $+360^\circ$ for measurement on X-Y plane, from $\theta = -10^\circ$ to $+190^\circ$ for measurements on Y-Z and Z-X planes (see Fig. 5-4). DBB measurements are taken in both the counter-clock wise (CCW) and clock wise (CW) directions.

In order to estimate values of kinematic errors, at first, from the measured DBB displacement trajectory in CCW direction, the radial errors along the circular trajectory of the DBB measurement are calculated. By using the Eq. (5-14), a profile of squareness errors along the trajectory is computed and, then, an average value of them is taken. Similarly, a value of squareness error is

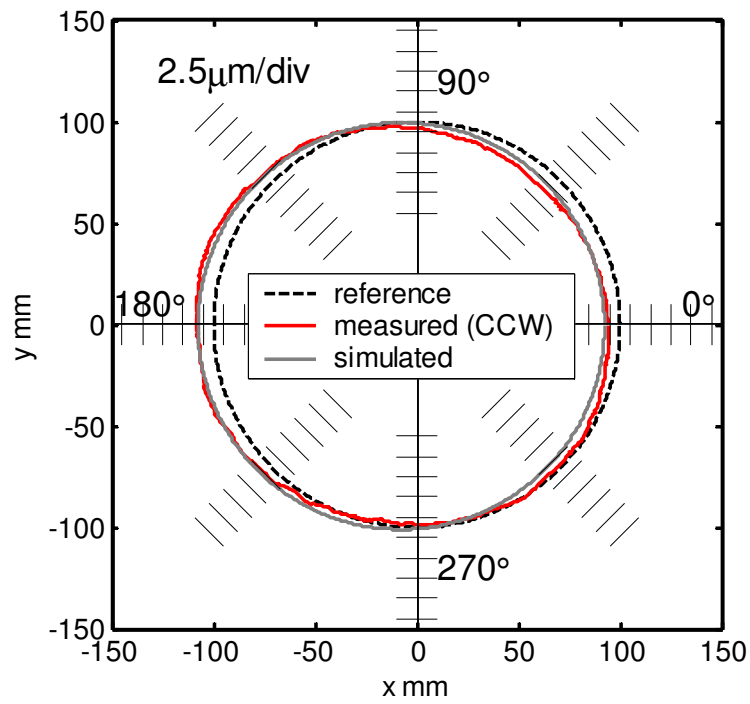
Table 5-1 Identified three squareness errors associated with linear axes

Kinematic error	Value
γ_{yx} (between X and Y axes)	-0.0034°
α_{zy} (between Y and Z axes)	+0.0054°
β_{zx} (between Z and X axes)	+0.0037°

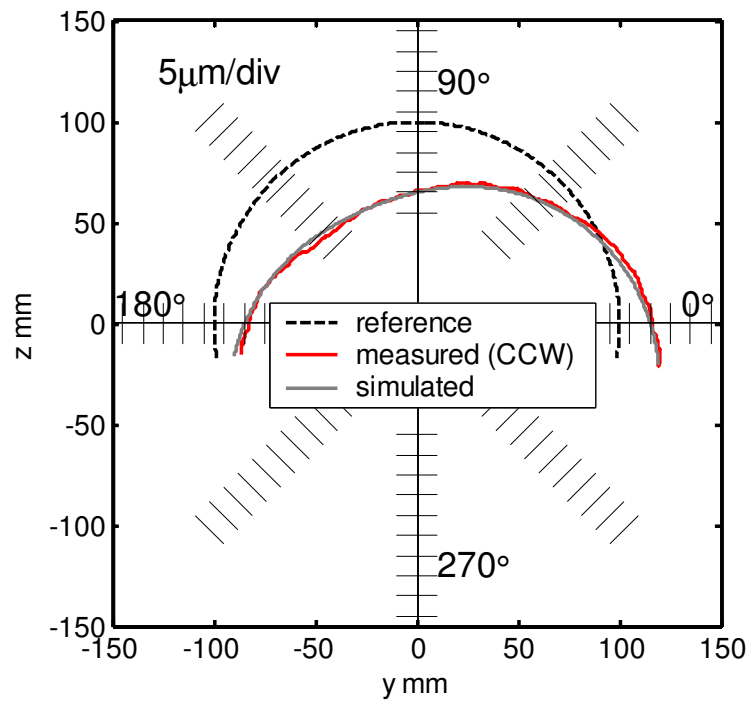
calculated for the measured trajectory in CW direction. Finally, taking average of squareness error values for measurements in CCW and CW directions will give the actual estimated value of squareness error between any two linear axes. Table 5-1 summarizes three squareness errors identified from DBB tests.

Then, another four DBB tests with the measurement patterns 1~4 as described in Section 5.5.4 are carried out to identify eight kinematic errors associated with rotary axes of the machining center. The nominal length of the DBB bar is kept at 100mm. Feedrates of the spindle-side ball used in the tests for DBB measurement patterns 1~4 are 1023.5mm/min, 512mm/min, 512mm/min, and 510mm/min respectively. In DBB measurement pattern 1 and 4, C-axis motion starts from $\theta = 0^\circ$ to 360° . On the other hand, in DBB measurement pattern 2 and 3, A-axis motion starts from $\theta = -100^\circ$ to $+100^\circ$. The reference to the A-axis and C-axis motions for these four DBB measurement patterns is already defined in Section 5.5.4. DBB measurements are taken in both the CCW and CW directions.

Figure 5-14 shows measured and simulated DBB displacement trajectories with respect to the reference trajectory for four measurement patterns. In the DBB displacement trajectories, the error is magnified by 4000, 2000, 2000, and 2000 times for DBB measurement pattern 1, 2, 3, and 4 respectively. Figure 5-14 only shows DBB error trajectories measured in CCW direction. First, the center shifts of the measured DBB displacement trajectories with respect to the reference trajectory in CCW and CW directions for the measurement patterns 1~4 are calculated. An average value of center shifts for each measurement in both CCW and CW directions is taken. Values of center shift obtained from the

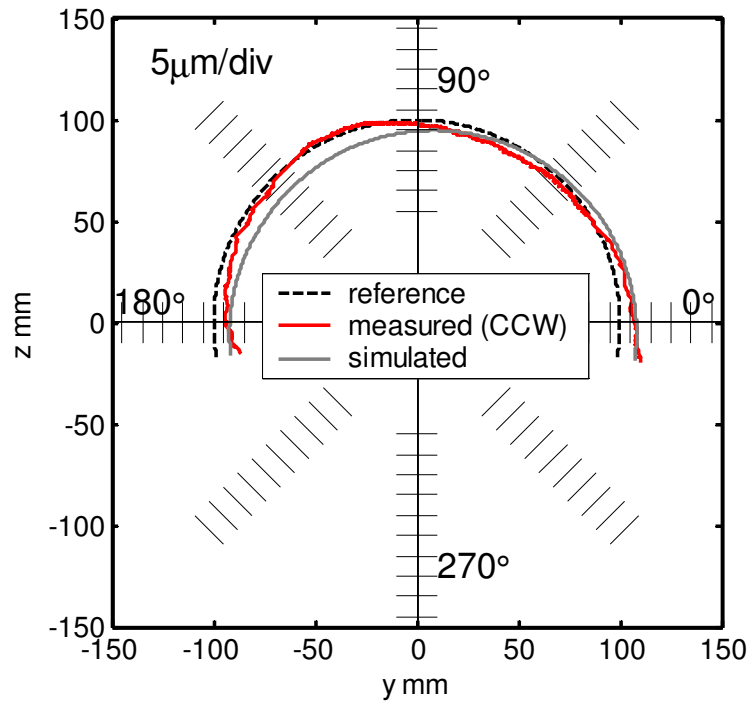


(a) DBB measurement pattern 1

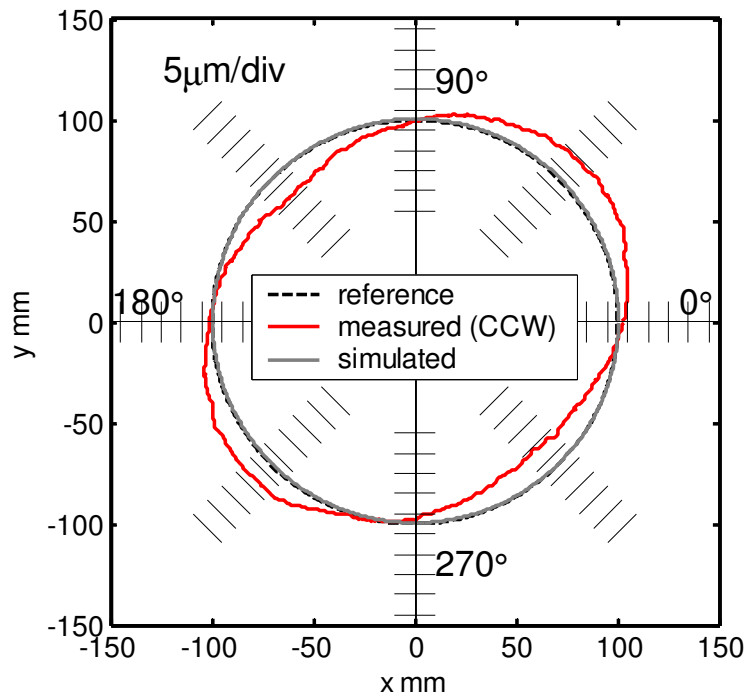


(b) DBB measurement pattern 2

Figure 5-15 Measured and simulated DBB displacement trajectories for measurement patterns 1~4



(c) DBB measurement pattern 3



(d) DBB measurement pattern 4

Figure 5-15 Measured and simulated DBB displacement trajectories for measurement patterns 1~4

measured displacement trajectories for DBB measurement patterns 1~4 are summarized in Table 5-2. Then, by putting these values of center shift in Eqs. (5-23)~(5-30) defined in Section 5.5.4, eight kinematic errors associated with rotary axes, namely, α_{AY} , β_{AY} , γ_{AY} , β_{CA} , δx_{AY} , δy_{AY} , δz_{AY} , and δy_{CA} are estimated. Values of R_{C1} and R_{C2} in Eqs. (5-23) ~ (5-25), and (5-30) are kept at 100mm and 93mm respectively. Table 5-3 summarizes values of eight kinematic errors identified from DBB tests. Note that simulated DBB displacement trajectories shown in Fig. 5-14 are drawn by using the modeling of DBB measurement (as described in Section 5.5.2) with identified eight kinematic errors. A good agreement between simulated and measured DBB trajectories justifies the identification of kinematic errors in the target 5-axis machining center.

Table 5-2 Values of center shift of the measured displacement trajectories for DBB measurement patterns 1~4

Center shift	DBB pattern 1 (e_{x1} , e_{y1})	DBB pattern 2 (e_{y2} , e_{z2})	DBB pattern 3 (e_{y3} , e_{z3})	DBB pattern 4 (e_{x4} , e_{y4})
CCW (μm)	(-1.8, -0.3)	(8.5, -17.3)	(1.5, -1.0)	(0.30, 0.8)
CW (μm)	(-2.0, 0.0)	(6.6, -17.7)	(6.4, -4.7)	(0.10, -0.20)
Average (μm)	(-1.9, -0.15)	(7.55, -17.5)	(3.95, -2.85)	(0.20, 0.30)

Table 5-3 Identified eight kinematic errors associated with rotary axes

Kinematic error	Value
α_{AY}	+0.0001°
β_{AY}	-0.0071°
γ_{AY}	-0.0081°
β_{CA}	+0.006°
δx_{AY}	-0.20μm
δy_{AY}	- 4.0μm
δz_{AY}	+2.8μm
δy_{CA}	+3.7μm

5.7 Problems with the DBB measurement in Identifying Kinematic Errors

There are several problems associated with the DBB measurement to identify the kinematic errors in a 5-axis machining center. Major drawbacks of the DBB measurement are that during the measurement, DBB usually captures the actual data of the machine tool, which is resulted from multiple error sources. Errors involved in linear axes (X, Y, and Z) cannot be separated from those involved in rotary axes. For example, in DBB measurement patterns 1~3 shown in Section 5.5.4 to identify kinematic errors associated with rotary axes, both linear and rotary axis motions exist simultaneously. The equations to identify kinematic errors (Eqs. (5-15) ~ (5-20)) assume that there is no contouring error in the motion of linear axes, except for their squareness errors, γ_{YX} , α_{ZY} , and β_{XZ} . In reality, the DBB displacement trajectories may contain the effect of other error sources for linear axes, such as linear positioning errors and straightness errors, which may exaggerate the DBB measurements, and hence making it difficult to

practically identify the exact values of kinematic errors only associated with rotary axes.

Due to the limitation of the physical configuration of the machining center, during the DBB measurement, A-axis does not complete a full rotation of 360° (see also DBB displacement trajectories for measurement pattern 2 and 3 shown in Fig. 5-14). This may cause errors in estimating the actual center shift from the DBB displacement trajectories, as well as errors in the identification of actual kinematic errors. Further, because of the physical limitation of the DBB length, it is practically not possible to get the information about the whole working area of the machining center to be measured.

Another problem with the DBB measurement is the error in setting up the DBB balls during the measurement. This error usually causes slight eccentricity of the balls relative to the circular trajectory, which affects the measured DBB displacement trajectories, and causes errors in estimating kinematic errors. Especially, this set-up error may impose significant effect in the case of measurement pattern 4, while it does not affect that much the DBB trajectories for the measurement pattern 1~3, shown in Section 5.5.4.

The above discussion about problems associated with the DBB measurement implies that identified kinematic errors shown in Tables 5-1 and 5-3 may not be the perfect values.

5.8 Conclusion

In this chapter, kinematic errors in the 5-axis machining center with tilting rotary table are defined. A kinematic model of the machining center with the kinematic errors considered is briefly described. A telescoping DBB measuring method, a quickest and efficient way to the measurement of the machining center, is applied to identify kinematic errors existing in the target 5-axis machining center. Modeling of the DBB measurements and a procedure to identify kinematic errors are briefly discussed. Case studies with actual DBB tests on the

target 5-axis machining center are demonstrated to practically estimate total eleven kinematic errors associated with linear and rotary axes in the machining center. Although there are some problems with the DBB measurement process, simulated and measured DBB displacement trajectories show a good match, verifying the identification of kinematic errors in the 5-axis machining center.

Chapter 6

Prediction and Compensation of Machining Geometric Errors on 5-Axis Machining Centers

6.1 Introduction

In the previous chapter, kinematic errors associated with linear and rotary axes of a 5-axis machining center are defined and modeled. By applying a DBB method, total eight kinematic errors in the machining center of the configuration depicted in Fig. 5-1 are practically identified.

In order to enhance machine's performance, the evaluation of the effect of kinematic errors on the machining geometric errors is very important. As is well known in the machine tool industry, machining of a tilted cone frustum as specified in NAS979 [NAS979, 1969] standard is a widely accepted as a final performance test for 5-axis machining centers. A critical issue with this machining test is, however, that the influence of the machine's error sources on the geometric accuracy of the machined cone frustum is not fully understood by machine tool builders, and thus it is quite difficult to find causes of machining errors. Also this NAS979 standard does not specify the location and the geometry of the cone frustum workpiece to be machined.

To address these issues, this chapter presents a simulator of machining geometric errors in 5-axis machining by considering the effect of identified kinematic errors as shown in Chapter 5 on the three-dimensional interference of the tool and the workpiece. By using an error model of the machining center with identified kinematic errors as described in Chapter 5 and considering the location and the geometry of the workpiece, machining geometric errors with respect to the nominal geometry of the workpiece are predicted and evaluated. In an aim to improve geometric accuracy of the machined surface, an error compensation for tool position and orientation is also presented.

The remainder of the chapter is organized as follows. Section 6.2 briefly reviews literature on evaluation of machining accuracy on 5-axis machining centers. Machining configuration for the tilted cone frustum as specified in NAS979 standard and computation of cutting tool paths in the reference frame for machining the cone frustum are presented in Section 6.3. By considering the effect of identified kinematic errors, a simulation procedure to compute the machining geometric errors on the cone frustum is described in Section 6.4. As simulation examples, Section 6.5 illustrates the sensitivity of individual kinematic error on machining geometric errors. An error compensation scheme to improve machining geometric accuracy is presented in Section 6.6. In Section 6.7, case studies with cutting experiments by using a straight end mill are shown to demonstrate the prediction and compensation of machining geometric errors on 5-axis machining centers. Lastly, Section 6.8 concludes the chapter with a brief summary.

6.2 Literature Review: Evaluation of Machining Accuracy on 5-Axis Machining Centers

As has been reviewed in Section 5.2 of Chapter 5, there are many research works, and some standards, describing the method to measure kinematic errors on a 5-axis machining center. Although it is crucial to evaluate each kinematic error by using such a measurement, typical machine tool users concern more the accuracy of a 5-axis machine when it performs actual machining. NAS 979 [NAS 979, 1969] describes the evaluation of the machining accuracy of a 5-axis machine by the machining of a tilted cone frustum. Since it is only standard describing a machining test by 5-axis machines well known in the machine tool industry, a machining test of a tilted cone frustum is widely accepted to many machine tool builders as a final performance test for 5-axis machines. The Boeing Company, a world-leading aero-parts manufacturer, has specified this

standard test method as an acceptance condition of 5-axis machining centers for the manufacturing of aero-components [Boeing, 1999].

A critical issue with this machining test is, however, that the influence of the machine's error sources on the geometric accuracy of the machined cone frustum is not fully understood by machine tool builders. Therefore, it is generally very difficult for machine tool builders to find causes of geometric errors of the machined cone frustum. Unlike conventional 3-axis machining centers, it is difficult to intuitively understand the effect of the machine's motions errors on the machining accuracy in 5-axis machines. Another critical issue with NAS 979 standard is that it does not refer to the location and setting angle of the workpiece to be machined. On 5-axis machines, the influence of kinematic errors of each axis on the overall positioning error of the tool with respect to the workpiece may significantly differ depending on the position of each axis.

While there have been many research works on modeling, measurement, and evaluation of 5-axis machine tools as reviewed in Section 5.2 of Chapter 5, no or little work is available in literature, focusing on the explicit analysis on the effect of kinematic errors in 5-axis machining centers on the machining geometric accuracy. In particular, it is of a practical importance to exploit the effect of kinematic errors on the geometric error of a cone frustum machined by using a 5-axis machine, as a basis to establish a diagnosis methodology of errors sources from machining results or to implement a compensation scheme of machining geometric errors. Although some latest papers [Matsushita, 2007 and Yumiza et al., 2007] discuss the effect of kinematic errors on the machining error of a cone frustum, they only consider the displacement at the tool tip in the simulation, and thus do not consider the three-dimensional interference of the tool and the workpiece.

The present research work proposes a simulator of machining errors in 5-axis machining by considering the effect of kinematic errors on the three-dimensional interference of the tool and the workpiece. The effect of center location and the inclination angle of the workpiece on the machining geometric errors are considered in the present simulator. Such a simulator is crucial to

quantitatively evaluate the significance of each kinematic error on the machine's overall machining accuracy. In order to enhance the geometric accuracy on machined workpiece surface, an error compensation for tool position and orientation is also presented. Finally, as an example, the machining of the tilted cone frustum by using a straight end mill, as described in the standard NAS979, is considered in case studies to experimentally verify the prediction and compensation of machining errors in 5-axis machining.

6.3 Machining of Cone Frustum

6.3.1 Machining configuration

As is mentioned earlier in Section 6.2, National Aerospace Standard Association specifies a tilted cone frustum as the test workpiece for evaluating contouring accuracy of the multi-axis machining center (NAS979) [NAS979, 1969]. In this standard, after machining, the test part is measured to evaluate the circularity error of circumferences, the concentricity error of top and bottom circumferences, and the angular error of side surface of the machined cone frustum, showing an overall contouring performance of the machining center.

In this research work, a tilted cone frustum as specified in the standard NAS979 is adopted as the test part to evaluate machining accuracy of the machining center with kinematic errors. Hence, the machining simulation on the tilted cone frustum will be carried out. It must be emphasized that the simulation scheme of machining accuracy presented in the following sections is not limited to the machining of a cone frustum. However, since the machining of a cone frustum is widely accepted as a practical machining performance test in the industry, this research only presents the evaluation of machining accuracy of a cone frustum as an application example of practical interest.

Figure 6-1 shows the machining configuration and parameters of the titled cone frustum to be considered in the present simulation study. In Fig. 6-1, D , ψ ,

and φ are defined as the diameter of the cone frustum machining tool path in the workpiece frame, the taper angle, and the tilt or inclination angle of the cone frustum respectively, while (C_x, C_y, C_z) is the center location of the workpiece on the work-table. H_t and H_b are respectively the heights of the cone frustum and the base cylinder on which the cone frustum is placed.

6.3.2 Tool Path for Machining Cone Frustum

Based on the machining configuration and parameters of the titled cone frustum shown in Fig. 6-1, the tool paths required to machine the cone frustum are computed. This section presents the computation of a profile of tool position and orientation in the reference frame for the machining of the tilted cone frustum on a 5-axis machining center. In this section, we assume the followings in the machining of a tilted cone frustum: (1) the side surface of a cone frustum is machined by using a straight end mill by a single finishing path (as depicted in Fig. 6-1), (2) the tool is tilted toward the center of the cone frustum throughout the finishing.

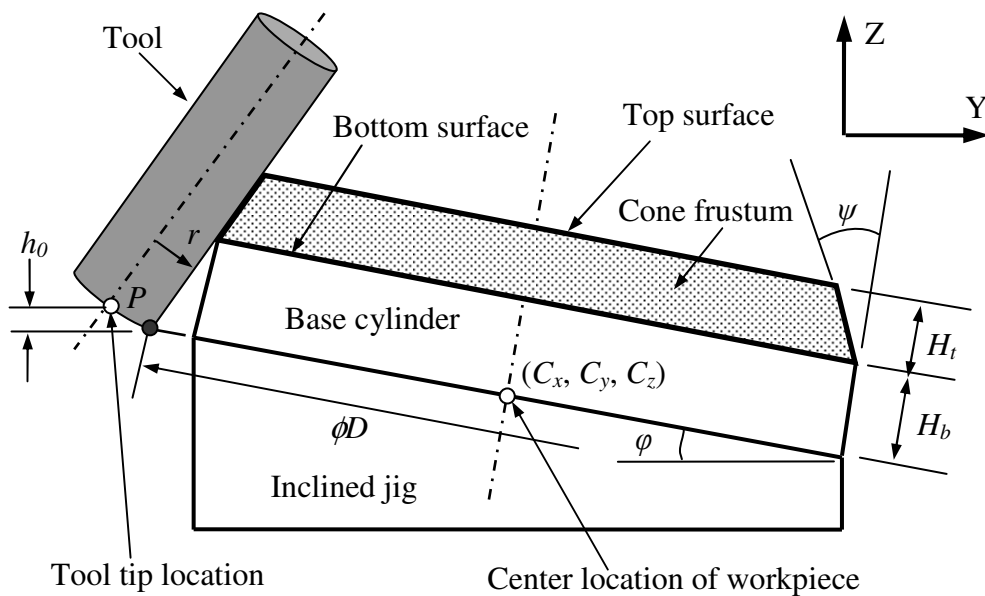


Figure 6-1 Machining configuration for the tilted cone frustum

(a) *Coordinates of tool tip position before tilting the cone frustum ($\varphi = 0$)*

First, when the cone frustum is not tilted (i.e. ($\varphi = 0$)), the profile of tool tip position in the workpiece frame, ${}^w p_{t,0}(X_{w0}, Y_{w0}, Z_{w0})$ can be written as:

$$\begin{cases} X_{w0}(j) = R_w \cos \theta(j) \\ Y_{w0}(j) = R_w \sin \theta(j) \\ Z_{w0}(j) = h_0 \end{cases} \quad (6-1)$$

where, R_w is the radius of the cone frustum machining tool path trajectory, and based on the machining configuration depicted in Fig. 6-1, it can be expressed as:

$$R_w = \frac{D}{2} + r \cos \psi \quad (6-2)$$

and, $h_0 = \frac{r}{\sin \psi}$, r is the tool radius, and $\theta(j)=0 \sim 2\pi$ is the angular step of the tool path trajectory.

(b) *Coordinate of the tool tip position after tilting the cone frustum*

In order to obtain the coordinate of tool position for the tilted cone frustum, consider a unit tilting vector, $\bar{v}(a_x, a_y, a_z)$ acting perpendicularly at the center of the top face of the cone frustum. Here, the unit tilting vector, $\bar{v}(a_x, a_y, a_z)$ defines the orientation of the tilted cone frustum on X-Y-Z plane, and is the function of the tilt angles of the cone frustum, φ and γ about X and Y axes respectively. Figure 6-2 shows the unit tilting vector, $\bar{v}(a_x, a_y, a_z)$ acting in perpendicular direction on the top face of the tilted cone. Hence, this unit tilting vector $\bar{v}(a_x, a_y, a_z)$ can be obtained as follows:

$$\bar{v} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \varphi \sin \gamma \\ \sin \varphi \\ \cos \varphi \cos \gamma \end{bmatrix} \quad (6-3)$$

Then, by using the unit tilting vector, $\bar{v}(a_x, a_y, a_z)$, a tilting unit transformation matrix, $T_{ilt} \in \Re^{3 \times 3}$ is constructed and can be written as:

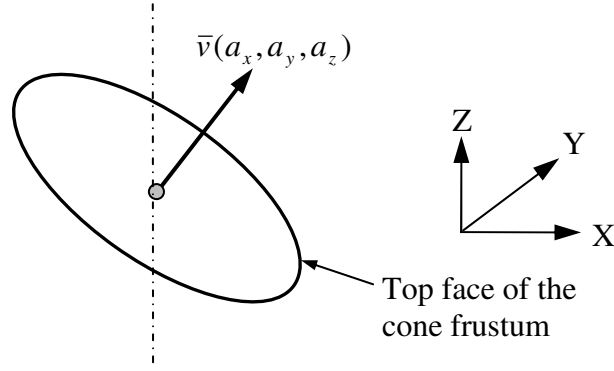


Figure 6-2 The unit tilting vector, $\bar{v}(a_x, a_y, a_z)$

$$T_{tilt} = \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \quad (6-4)$$

where, $\alpha = \tan^{-1} \left(\frac{\sqrt{a_x^2 + a_y^2}}{a_z} \right)$ and $\beta = \tan^{-1} \left(\frac{a_y}{a_x} \right)$.

Now, by considering the center location of the cone frustum workpiece, (C_x, C_y, C_z) and making the inverse of the matrix, T_{tilt} , the profile of the tool tip position in the workpiece frame for the tilted cone frustum, ${}^w p_t(X_w, Y_w, Z_w)$ can be obtained as:

$$\begin{bmatrix} X_w(j) \\ Y_w(j) \\ Z_w(j) \end{bmatrix} = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} + (T_{tilt})^{-1} \begin{bmatrix} X_{w0}(j) \\ Y_{w0}(j) \\ Z_{w0}(j) \end{bmatrix} \quad (6-5)$$

(c) Tool orientation

Once the coordinate of the tool tip position is determined, then the orientation of the tool for the cone frustum machining tool path is required to calculate. The orientation of the tool is defined as the angular position of the axis-line of the cutting tool, and is often expressed in three-dimensional vector form, namely, (i, j, k) . Now, the tool orientation vector, ${}^w O_{t,0}(i_{w0}, j_{w0}, k_{w0})$ in the workpiece frame for the tool center trajectory before tilting the cone frustum can be written as:

$$\begin{bmatrix} i_{w0}(j) \\ j_{w0}(j) \\ k_{w0}(j) \end{bmatrix} = \begin{bmatrix} -\cos\theta(j)\sin\psi \\ -\sin\theta(j)\sin\psi \\ -\cos\psi \end{bmatrix} \quad (6-6)$$

Similarly, the tool orientation vector, ${}^wO_t(i_w, j_w, k_w)$ for the tilted cone frustum can be obtained as:

$$\begin{bmatrix} i_w(j) \\ j_w(j) \\ k_w(j) \end{bmatrix} = (T_{tilt})^{-1} \begin{bmatrix} i_{w0}(j) \\ j_{w0}(j) \\ k_{w0}(j) \end{bmatrix} \quad (6-7)$$

Since, in a 5-axis machining, the tool orientation is determined by the rotation of the rotary table-C and the tilting table-A, the rotational angles, $C(j)$ and $A(j)$ made by both tables C and A respectively can be calculated as follows:

$$C(j) = -\tan^{-1}\left(\frac{i_w(j)}{j_w(j)}\right) \quad (6-8)$$

$$A(j) = \cos^{-1}(k_w(j)) \quad (6-9)$$

(d) Coordinate transformation

In actual machining on a 5-axis machining center, any position of tool tip in the workpiece frame must be transformed into the reference or machine frame, allowing CNC control unit to drive machine's axes to obtain the desired motion of tool tip. Define the tool tip position in the reference or machine frame, ${}^r p_t(X_r, Y_r, Z_r)$. Hence, the tool tip position in the workpiece frame, ${}^w p_t(X_w, Y_w, Z_w)$ can be transformed into that in the reference frame by the following equation.

$$\begin{bmatrix} X_r(j) \\ Y_r(j) \\ Z_r(j) \\ 1 \end{bmatrix} = {}^rT_w \begin{bmatrix} X_w(j) \\ Y_w(j) \\ Z_w(j) \\ 1 \end{bmatrix} \quad (6-10)$$

In the above equation, ${}^rT_w \in \mathfrak{R}^{4 \times 4}$ is the HTM and can be expressed as:

$${}^rT_w = D^4(A(j))D^6(C(j)) \quad (6-11)$$

where $C(j)$ and $A(j)$ are the rotational angles made by the rotary table-C and the

tilting table-A respectively for machining the tilted taper cone, and $D^4(*), D^6(*) \in \mathfrak{R}^{4 \times 4}$ are the HTMs as defined in Appendix I.

6.4 Simulation of Machining Accuracy of a 5-Axis Machining Center with Its Kinematic Errors

By using the kinematic model of the machining center presented in Chapter 5, for any given commanded tool position and orientation, one can compute the actual tool position and orientation given in the workpiece frame by considering the effect of kinematic errors. For the computation of machining geometric errors, the three-dimensional interference of the tool and the workpiece must be computed. Therefore, an error in the tool orientation must be evaluated as well as an error in the tool position.

For the simulation of the workpiece geometry machined by an end mill, many commercial CAM or machining simulation software adopt a discrete modeling of the workpiece using e.g. the Z-map model [Anderson, 1978] or the voxel model [Walstra et al., 1994]. In this paper, the objective of applying the machining simulation is to evaluate the geometric accuracy of the machined workpiece along some given reference trajectories. For example, in the machining of a cone frustum, geometric errors of top and bottom circular surfaces (including the circularity error of the circumference, the concentricity error, the flatness error, the parallelism error and so on) are of sole interest. Therefore, it is sufficient to construct the discretized model of workpiece geometry only on the region of interest, which makes it easy to minimize the simulation error due to the discretization by reducing the size of each unit model. In the case of a cone frustum, circumferences of top and bottom surfaces are discretized into a set of points, and the interference of the tool and the segment, which starts from each point on circumferences to the direction normal to the frustum surface, is geometrically calculated to compute the geometric error of the machined surface.

First assume that a set of points that defines the nominal geometry of the region of interest, $W_i \in \mathfrak{R}^3 (i = 1, \dots, N_w)$ is given, as well as a set of unit vectors normal to the nominal geometry, $v_{w,i} \in \mathfrak{R}^3 (i = 1, \dots, N_w)$. In the case of a cone frustum, W_i and $v_{w,i}$ can be given as illustrated in Fig. 6-3. Also assume that the tool is a straight end mill, and is modeled simply as a cylinder of the tool radius, r . Suppose that the commanded trajectory of tool tip location in the workpiece frame is given by ${}^w \hat{p}_j \in \mathfrak{R}^3 (j = 1, \dots, N_p)$ and the commanded tool orientation vector in the workpiece frame is given by $\hat{v}_{t,j} \in \mathfrak{R}^3 (j = 1, \dots, N_p)$. The detailed algorithm for the computation of geometric error of the machined surface is illustrated as follows.

(1) For given j , compute the actual tool center position, ${}^w p_j \in \mathfrak{R}^3$, and the tool orientation vector, $v_{t,j} \in \mathfrak{R}^3$, using the kinematic model presented in Chapter 5 with given kinematic errors.

(2) Define the plane, Θ_j , that is parallel to both of the tool orientation vector, $v_{t,j}$ and the tool motion direction vector, $M_{t,j} = {}^w p_{j+1} - {}^w p_j$, and is apart by the distance, r from the tool center, ${}^w p_j$, as illustrated in Fig. 6-4. The plane, Θ_j , can be represented by the following equation.

$$\Theta_j = \{q \in R^3 : q = {}^w p_j + {}^w \xi_j + (\alpha \cdot v_{t,j} + \beta \cdot M_{t,j})\} \quad (6-12)$$

where, ${}^w \xi_j \in \mathfrak{R}^3$ is the vector that perpendicular to both $v_{t,j}$ and $M_{t,j}$, and has the length of r . $\alpha, \beta \in \mathfrak{R}$ are parameters.

(3) Find a set of W_i 's that is within some threshold distance from ${}^w p_j$. For all these W_i 's, compute the intersection point, $q_i \in \mathfrak{R}^3$, of the line, $W_i + \gamma \cdot v_{w,i}$, where $\gamma \in R$ is a parameter, and the plane, Θ_j . When the intersection, $q_i \in \mathfrak{R}^3$, is given in the above equation with $0 \leq \beta \leq 1$, compute the distance, $d_{j,i}$, as follows:

$$d_{j,i} = \|W_i - q_i\| \cdot \text{sign}(\gamma) \text{ (or } d_{j,i} = \gamma) \quad (6-13)$$

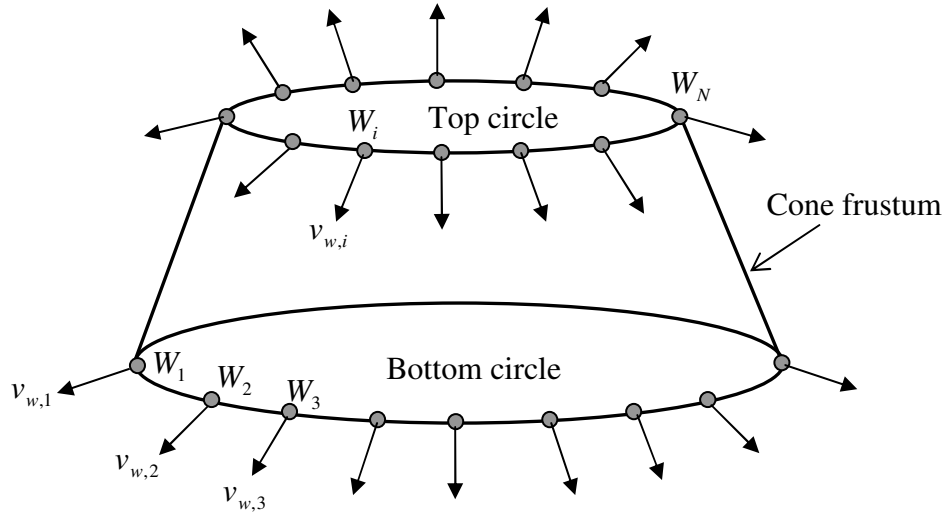


Figure 6-3 Nominal geometry of bottom and top surfaces of the cone frustum and their normal vectors

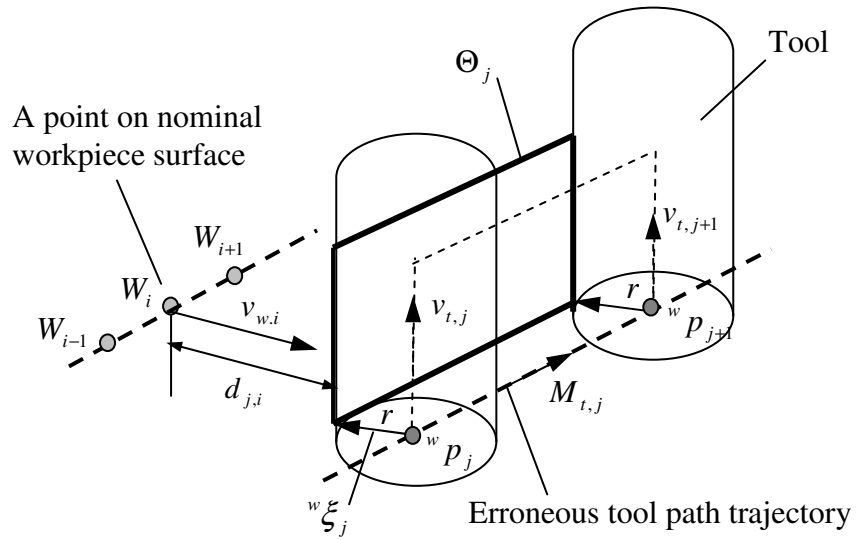


Figure 6-4 Algorithm for the computation of the machining geometric errors

(4) Make $j = j + 1$, and repeat (1) ~ (3) till $j = N_p$

(5) For all i 's, take $d_i = \min_{j=1, \dots, N_p} d_{j,i}$. This gives the actual machined surface at the nominal workpiece point, W_i , in the direction of $v_{w,i}$.

6.5 Sensitivity of Individual Kinematic Error on Machining Geometric Errors: Simulation Examples

To illustrate the simulation of machining geometric error by applying the algorithm presented in the previous section, this section presents simple simulation examples. The geometry and the setup of the workpiece are summarized in Table 6-1 (Note that this setup is the same as “Condition III” in experimental case studies that will be presented in Section 6.5). To illustrate the effect of each kinematic error on the geometry error of the machined cone frustum, one kinematic error is set to either $+0.005^\circ$ (in case of angular errors) or $+10\mu\text{m}$ (in case of linear errors), and all the other kinematic errors are set to zero. The simulation of machining geometric error is repeated with total 11 kinematic errors.

Figure 6-5 illustrates the simulated machining error trajectories of bottom and top surfaces of the cone frustum with the effect of individual kinematic error. From the simulated machining error trajectories, it is seen that some kinematic errors have similar effect on the machining error trajectories in terms of their shapes and some do not have any effect on the machining geometric errors. It is found that the machining error trajectories with the effect of kinematic errors, β_{AY} , β_{CA} , and δx_{AY} show a similar shape. Similarly, δy_{AY} , δy_{CA} , and α_{YZ} also show a similar effect in terms of shape on the machining error trajectories. On the other hand, γ_{AY} and δz_{AY} do not have any effect on the machining errors. Notably, compared to other kinematic errors, squareness error between X and Y axes, γ_{XY} shows a significant effect on the machining error trajectories. A

summary of circularity and concentricity errors of simulated machining error trajectories for bottom and top surfaces of the cone frustum is presented in Table 6-2. The circularity error is defined as the difference between maximum and minimum values of the machining errors of the machined cone frustum, whereas the concentricity error is the difference between center locations of the machining error trajectories of bottom and top surfaces of the machined cone frustum. A general procedure to calculate these two errors is given as follows:

When a radial error trajectory, $\Delta R(\theta_i)(i = 1, \dots, N)$, for a circular reference trajectory of the radius, r_{ref} , is given, first it is mapped into a trajectory on the reference plane:

$$p(\theta_i) = \begin{bmatrix} (r_{ref} + \Delta R(\theta_i)) \cos \theta_i \\ (r_{ref} + \Delta R(\theta_i)) \sin \theta_i \end{bmatrix} \quad (i = 1, \dots, N) \quad (6-14)$$

Then, the “optimal center”, $O^* \in \Re^2$, and the “optimal radius”, $r^* \in \Re$, are computed by solving the following minimization problem:

$$\min_{O^*, r^*} \sum_{i=1}^N (\|p(\theta_i) - O^*\| - r^*)^2 \quad (6-15)$$

In practice, this nonlinear optimization problem can be solved by using, for example, an algorithm implemented in Optimization Toolbox of Matlab, Mathworks Inc. [Mathwaorks, 2007]. Based on this optimal center location, O^* , the modified radial error trajectory is given by:

$$\Delta R^*(\theta_i) = \|p(\theta_i) - O^*\| - r_{ref} \quad (i = 1, \dots, N) \quad (6-16)$$

The circularity error, $E_{circ} \in \Re$, is defined by:

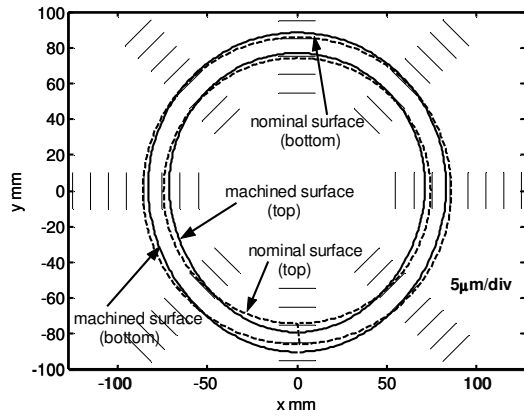
$$E_{circ} = \max_i (\Delta R^*(\theta_i)) - \min_i (\Delta R^*(\theta_i)) \quad (6-17)$$

By using the optimum center locations of the machining error trajectories for the bottom and top surfaces of the machined cone frustum, $O_{bottom}^* \in \Re^2$, and $O_{top}^* \in \Re^2$, in this dissertation, the concentricity error, $E_{conc} \in \Re$, is defined as follows:

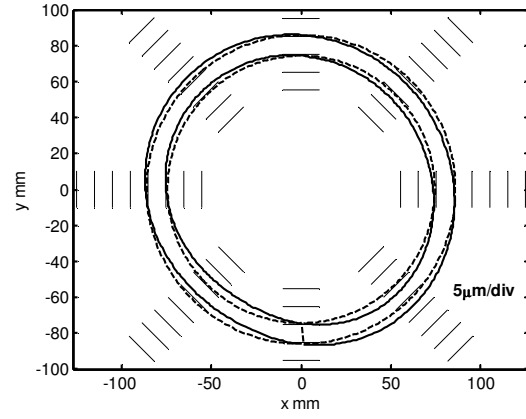
$$E_{conc} = \|O_{bottom}^* - O_{top}^*\| \quad (6-18)$$

Table 6-1 Machining parameters of the cone frustum used in simulation

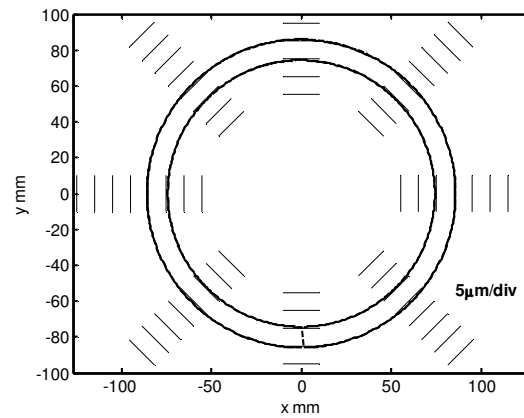
Parameter	Value
Diameter of the cone frustum machining tool path, D (mm)	206
Tilt angle, φ ($^{\circ}$)	75
Taper angle, ψ ($^{\circ}$)	30
Center location of workpiece (C_x, C_y, C_z) (mm)	(0,-100,93)
Height of cone frustum workpiece, H_t (mm)	20
Height of the base cylinder, H_b (mm)	30



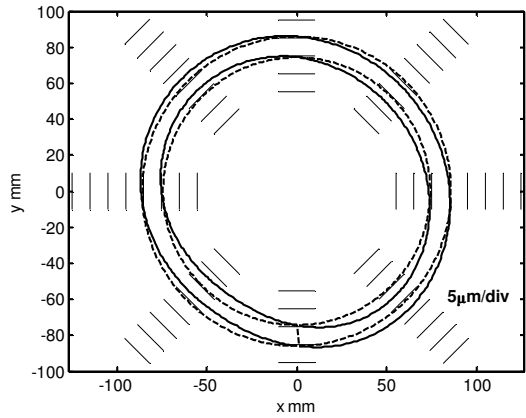
(a) $\alpha_{AY} = 0.005^{\circ}$



(b) $\beta_{AY} = 0.005^{\circ}$

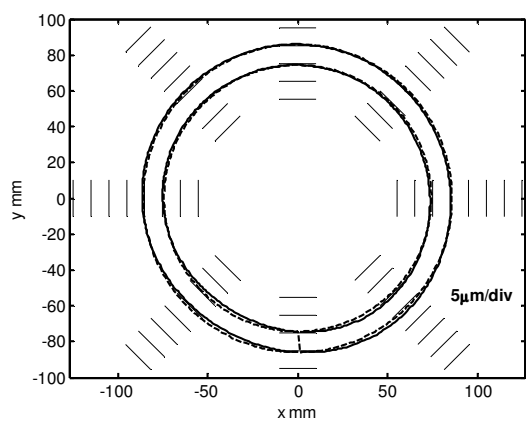


(c) $\gamma_{AY} = 0.005^{\circ}$

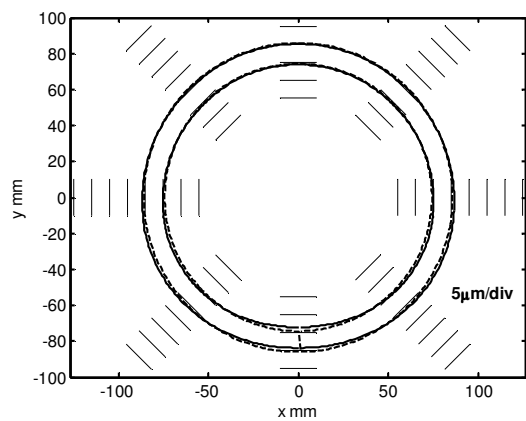


(d) $\beta_{CA} = 0.005^{\circ}$

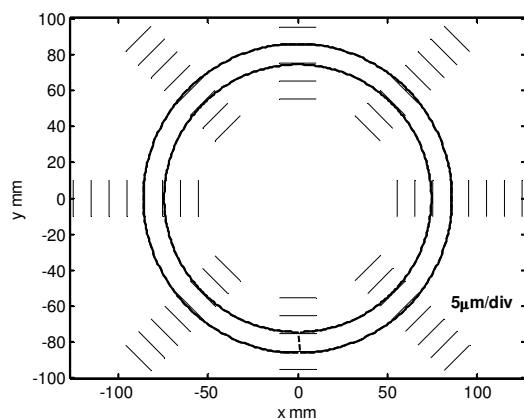
Figure 6-5 Influence of individual kinematic error on machining error trajectories of bottom and top surfaces of the cone frustum



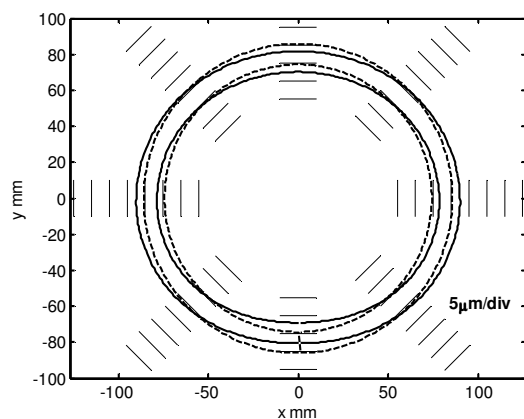
(e) $\Delta x_{AY} = 10\mu\text{m}$



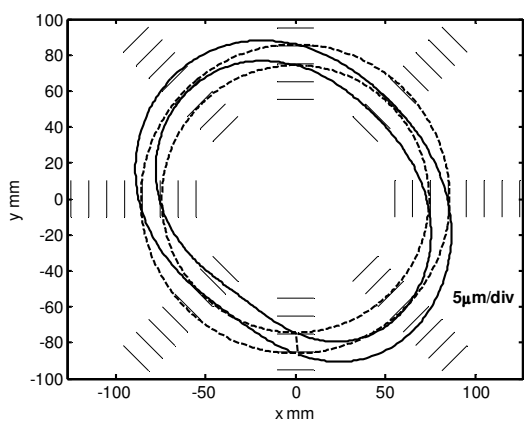
(f) $\Delta y_{AY} = 10\mu\text{m}$



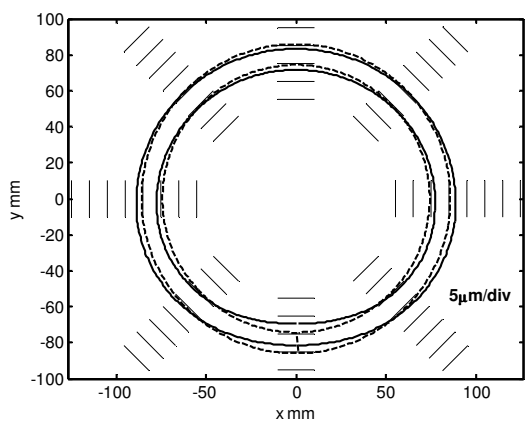
(g) $\Delta x_{AY} = 10\mu\text{m}$



(h) $\Delta y_{CA} = 10\mu\text{m}$

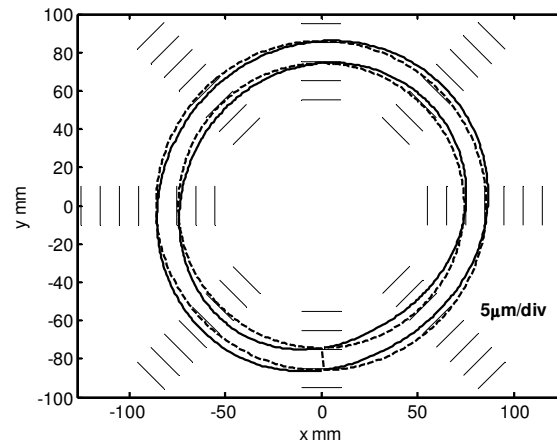


(i) $\gamma_{YX} = 0.005^\circ$



(j) $\alpha_{ZY} = 0.005^\circ$

Figure 6-5 Influence of individual kinematic error on machining error trajectories of bottom and top surfaces of the cone frustum



$$(k) \beta_{zx} = 0.005^\circ$$

Figure 6-5 Influence of individual kinematic error on machining error trajectories of bottom and top surfaces of the cone frustum

Table 6-2 Summary of simulated machining geometric errors

Kinematic error	Circularity error (μm)		Concentricity error (μm)
	Bottom surface	Top surface	
(a) $\alpha_{AY} = 0.005^\circ$	3.90	4.20	1.70
(b) $\beta_{AY} = 0.005^\circ$	4.20	4.50	1.80
(c) $\gamma_{AY} = 0.005^\circ$	≈ 0	≈ 0	0.012
(d) $\beta_{CA} = 0.005^\circ$	4.60	5.70	0.44
(e) $\delta x_{AY} = 10\mu\text{m}$	1.90	1.90	0.0017
(f) $\delta y_{AY} = 10\mu\text{m}$	1.80	1.80	≈ 0
(g) $\delta z_{AY} = 10\mu\text{m}$	≈ 0	≈ 0	≈ 0
(h) $\delta y_{CA} = 10\mu\text{m}$	4.80	4.80	≈ 0
(i) $\gamma_{XY} = 0.005^\circ$	12.60	12.60	0.0019
(j) $\alpha_{YZ} = 0.005^\circ$	3.90	4.20	1.70
(k) $\beta_{ZX} = 0.005^\circ$	4.20	4.50	1.80

6.6 Compensation of Machining Geometric Errors

In order to enhance the geometric accuracy of the machined surface, an error compensation for tool position and orientation is presented. When kinematic errors are accurately identified and thus the error in the tool position and orientation can be estimated by using the kinematic model, an error compensation can be applied to reach the ideal position and orientation by modifying the reference trajectory. In the literature, many compensation schemes have been presented. For examples, Srivastava et al. [Srivastava et al., 1995] presented an error compensation technique for 5-axis machine tools, which calculates the motion required for the five axes to reach the ideal position and orientation. Neural networking method has been applied to compensate the errors resulting from temperature variation and 5-axis motion [Veldhuis and Elbestawi, 1995]. Mahbubur et al. [Mahbubur et al., 1997] also discussed a compensation mechanism for position errors of 5-axis machine tools with the Newton-Raphson method, and the mechanism was then applied in a post-processor to enhance the accuracy of the 5-axis machine tools. Very recently, Hsu and Wang [Hsu and Wang, 2007] presented a decouple error compensation method to improve motion accuracy of the 5-axis machine tools.

In this research work, we adopt the following simple error compensation scheme to cancel the effect of kinematic errors. Figure 6-6 shows the concept of the error compensation. When the commanded tool position and orientation in the workpiece frame are respectively given by ${}^w\hat{p}_t$ and ${}^w\hat{v}_t$, compute the actual tool position and orientation in the workpiece frame, ${}^w p_t$ and ${}^w v_t$, under the influence of kinematic errors. Then, by simply modifying the commanded tool position and orientation as follows, the effect of kinematic errors can be cancelled, under the assumption that the error induced by kinematic errors is sufficiently small.

First, the errors in tool position, ${}^w\delta p_t$, and orientation, ${}^w\delta v_t$, between erroneous and commanded tool path trajectories are respectively computed as:

$${}^w\delta p_t = {}^w p_t - {}^w\hat{p}_t \quad (6-19)$$

$${}^w\delta v_t = {}^w v_t - {}^w\hat{v}_t \quad (6-20)$$

Then, the compensated tool center position, ${}^w\tilde{p}_t$, and orientation, ${}^w\tilde{v}_t$, in the workpiece frame is calculated by simply canceling the errors in tool position and orientation from those for the nominal tool path trajectory as follows:

$${}^w\tilde{p}_t = {}^w\hat{p}_t - {}^w\delta p_t \quad (6-21)$$

$${}^w\tilde{v}_t = {}^w\hat{v}_t - {}^w\delta v_t \quad (6-22)$$

By using the procedure for coordinate transformation described in Section 6.3.2(c), the coordinate of the compensated tool center position, ${}^r\tilde{p}_t$ and orientation, ${}^r\tilde{v}_t$ in the reference frame (i.e. the machine or global frame) can be computed.

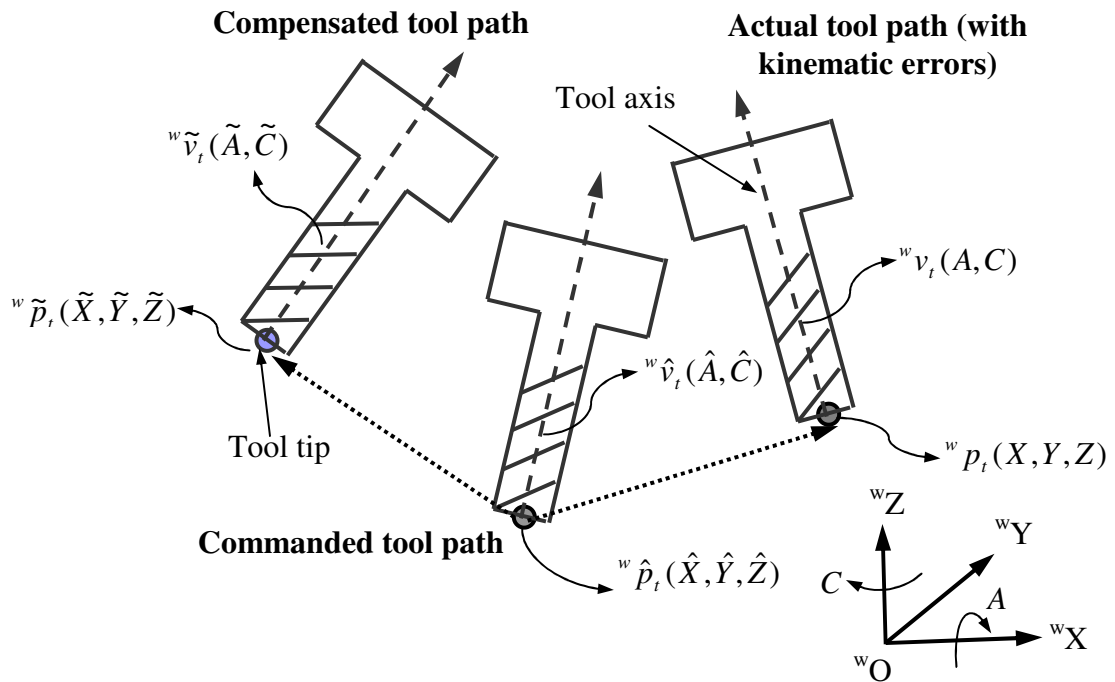


Figure 6-6 Concept of the error compensation

6.7 Case Studies

In order to justify the prediction and compensation of machining geometric errors on a 5-axis machining center with its kinematic errors, case studies with cutting experiments on the cone frustum workpiece are demonstrated. The machining tests are conducted by using the same 5-axis machining center as the one used in the case study presented in Section 5.6 of Chapter 5.

6.7.1 Machining Conditions

Simulation of machining geometric errors and, then the cutting experiments of a cone frustum workpiece are conducted with three different conditions on the center location and the geometry (e.g. taper angle) of the workpiece. As per the configuration of the machining shown in Fig. 6-1, the machining conditions (namely, Condition I, II, and III) of the cone frustum used in both simulation and experiments are summarized in Table 6-3. During cutting tests in all the cases, a sintered carbide straight end mill of the radius 10mm is used. The workpiece material is the aluminum alloy. The side surface of the cone frustum is finished by a single path under the following cutting conditions: the feedrate of 1000mm/min, the spindle speed of 5000min^{-1} (in Condition I) and 5500min^{-1} (in Condition II and III) and the radial depth of cut of 0.01mm.

6.7.2 Cutting Tool Paths

By using the procedure to compute tool paths described in Section 6.3.2, coordinates of tool path trajectory for machining the cone frustum as per three conditions (Condition I, II, and III) shown in Table 6-3 are calculated. The tool path trajectory here defines tool position (X , Y , and Z) and orientation (A and C) in the reference or the global frame during cutting. For all the three conditions, the velocity of the tool motion with respect to the workpiece is 1,000 mm/min, and the velocity of each axis is calculated accordingly.

Figures 6-7, 6-8, and 6-9 show the calculated profiles of tool position (X , Y , and Z) and orientation (A and C), and an equivalent feedrate (F') of the tool path trajectory in the reference frame with respect to the angle of rotation of tool path for finish machining under Condition I, II, and III respectively, which are to be used in simulation and actual cutting experiments. In Figs. 6-7~6-9, the angle of rotation of tool path is defined as the angular step of the tool path trajectory in the workpiece frame, where the tool starts to move from a given reference point on the tool path until the machining path ends to complete 360° . Details of calculation of the equivalent feedrate for the tool path trajectory are illustrated in Appendix II.

Table 6-3 Machining conditions of the titled cone frustum used in simulation and experiments

Parameter of the cone frustum	Condition I	Condition II	Condition III
Diameter of the cone frustum- machining tool path, ϕD (mm) }	194	210	206
Tilt angle, φ ($^\circ$)	7.5	15	75
Taper angle, ψ ($^\circ$)	15	30	30
Center location of workpiece- (C_x, C_y, C_z) (mm) }	(0,-100,43)	(0,-100,53)	(0,-103,93)
Height of the cone frustum, H_t (mm)	20	20	20
Height of the base cylinder, H_b (mm)	30	30	30

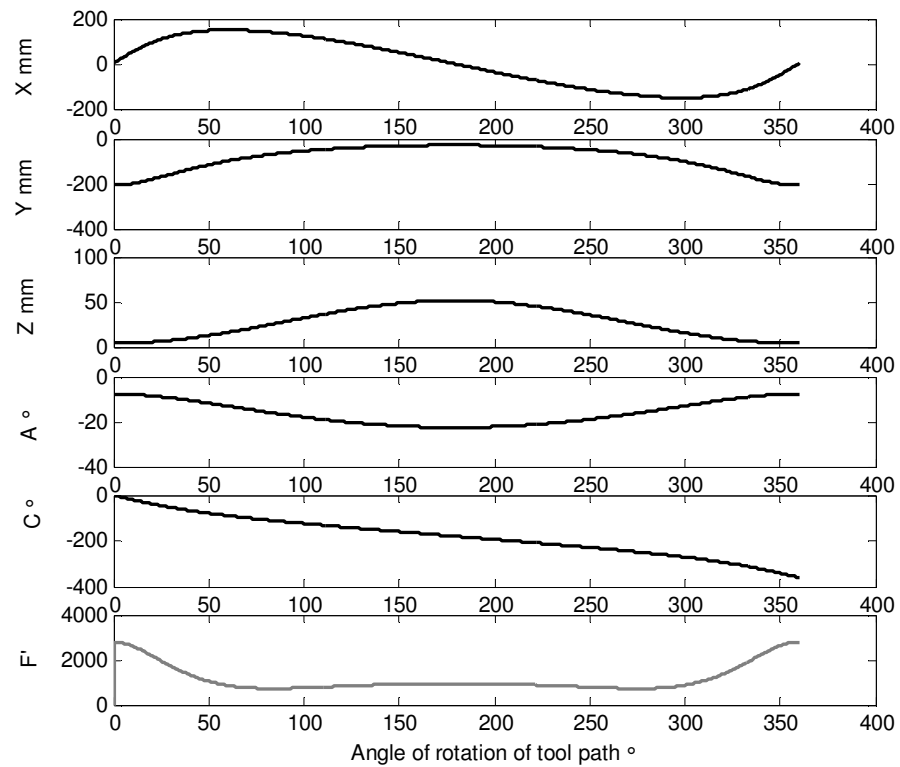


Figure 6-7 Coordinates of tool path and an equivalent feedrate profile for machining under Condition I

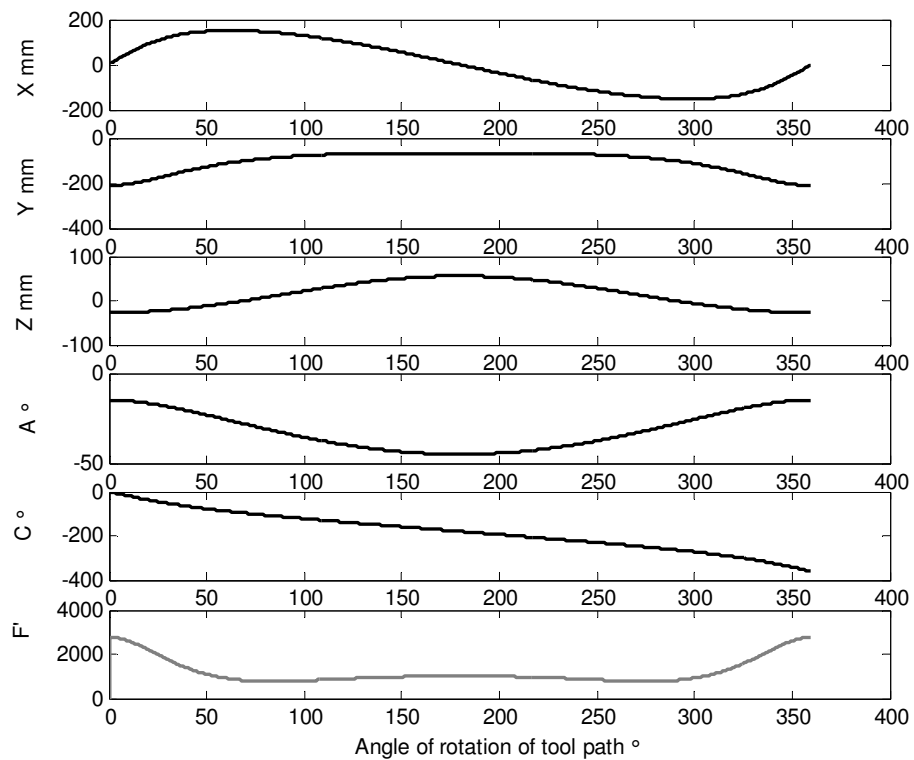


Figure 6-8 Coordinates of tool path and an equivalent feedrate profile for machining under Condition II

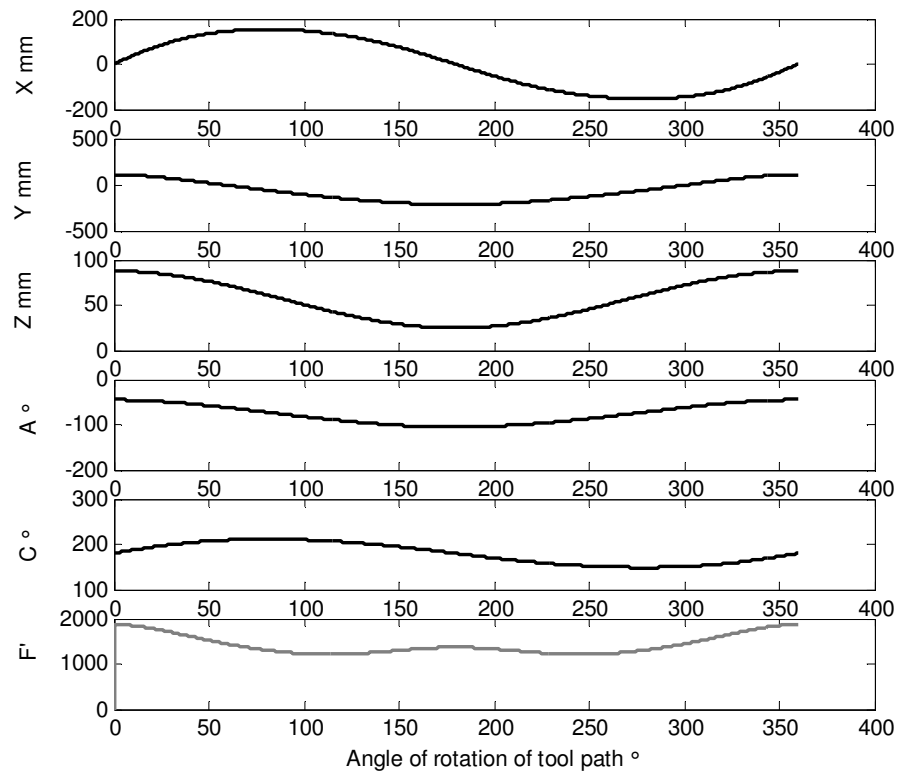


Figure 6-9 Coordinates of tool path and an equivalent feedrate profile for machining under Condition III

6.7.3 Simulation and Experimental Results

Using the above machining conditions and cutting tool paths shown in Section 6.7.1 and 6.7.2 respectively, both simulation and cutting experiments under the influence of identified kinematic errors on the target 5-axis machining center are conducted. In simulations, kinematic errors identified in the case study presented in Section 5.6 of Chapter 5 (see Tables 5-1 and 5-3) are assumed. After the finish machining on the cone frustum, surface measurements taken by a roundness measuring system (Talycenta1000 by Taylor Hobson, Ltd.) are carried out to measure machining geometric errors on bottom and top surfaces of the machined cone frustum workpiece. The bottom and top surfaces here are defined as the set of points on the circumference of bottom and top circles of the machined cone frustum respectively (see Fig. 6-1). Figure 6-10 shows a photo of the measurement of surfaces of the machined cone frustum.

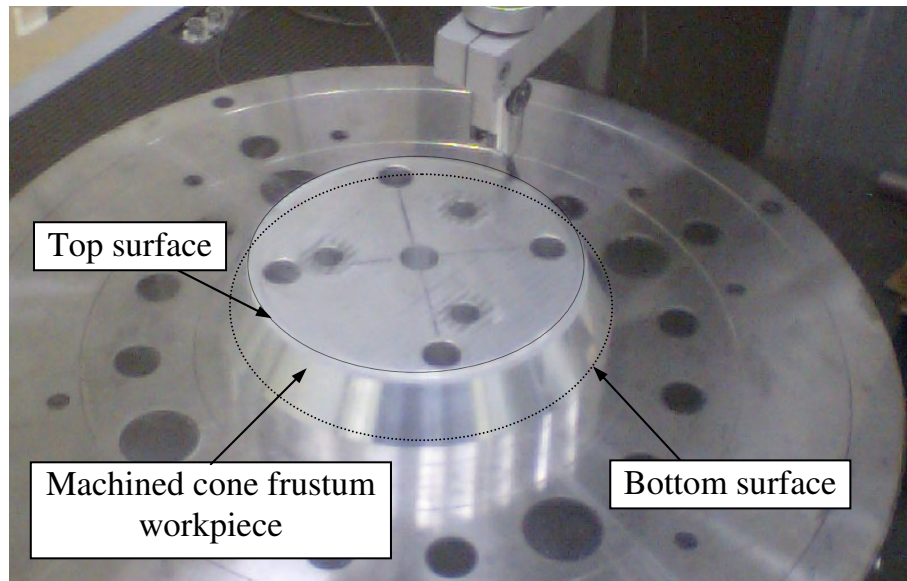


Figure 6-10 Machined cone frustum workpiece

Figures 6-11, 6-12, and 6-13 show simulated and measured machining error trajectories with respect to the nominal trajectory for the bottom and top surfaces of the machined cone frustum workpiece. The errors are magnified by 2000 times on both top and bottom surface trajectories. The simulated error trajectories are computed by using the simulator presented in Section 6.4 and identified kinematic errors shown in Tables 5-1 and 5-3. From the figures, we can observe in all the conditions that rough geometries of measured and simulated machining errors trajectories well agree, verifying the effectiveness of the prediction of machining errors.

Notice that machining error trajectories in Condition III significantly differ from those in Condition I and II, which is mostly attributable to the difference in the tilt angle of the workpiece, ϕ . It clearly shows that, in the 5-axis machining, the effect of kinematic errors on the machine's positioning error may significantly differ depending on the location, the orientation, and the geometry of the workpiece. Even in Condition I, where circularity errors are as small as $7.1\sim 9.3\mu\text{m}$, simulated trajectories still show a good match with measured trajectories. This indicates that the machine's kinematic errors have notable effect on the machining geometric accuracy even under such a condition.

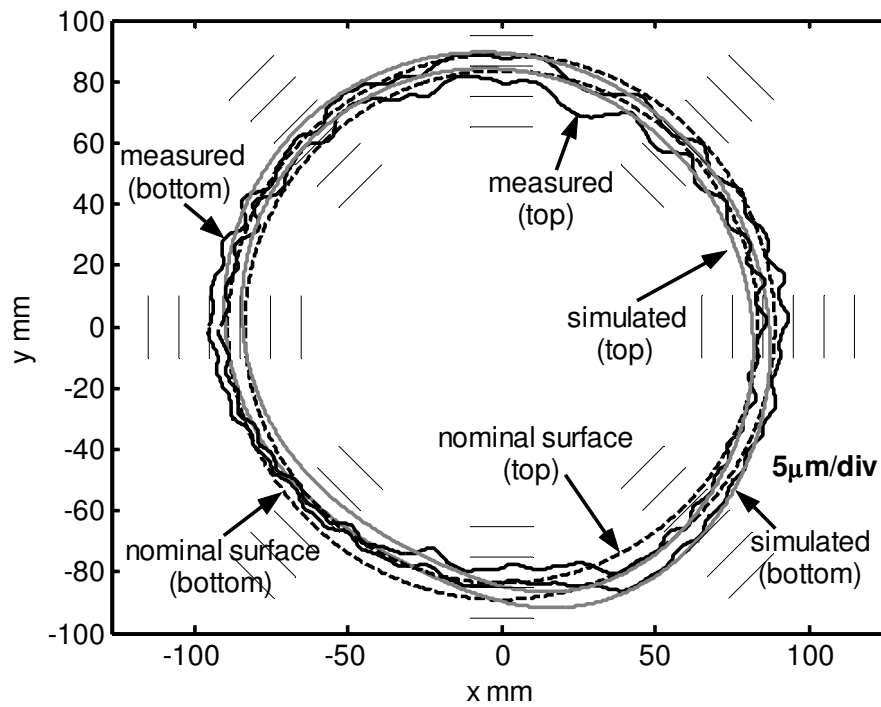


Figure 6-11 Simulated and measured machining error trajectories of the machined cone frustum surface in finishing under Condition I

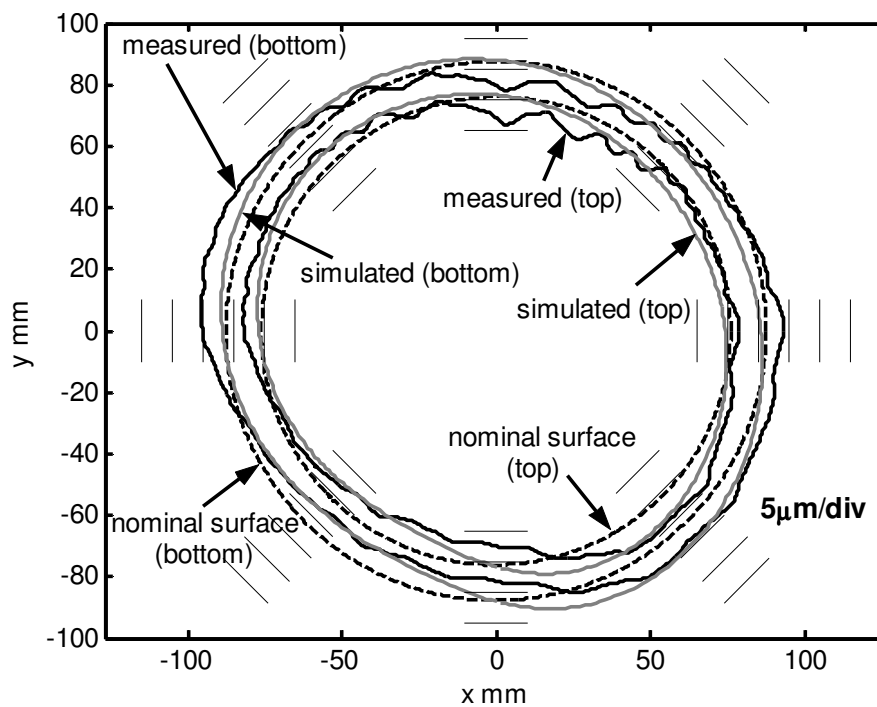


Figure 6-12 Simulated and measured machining error trajectories of the machined cone frustum surface in finishing under Condition II

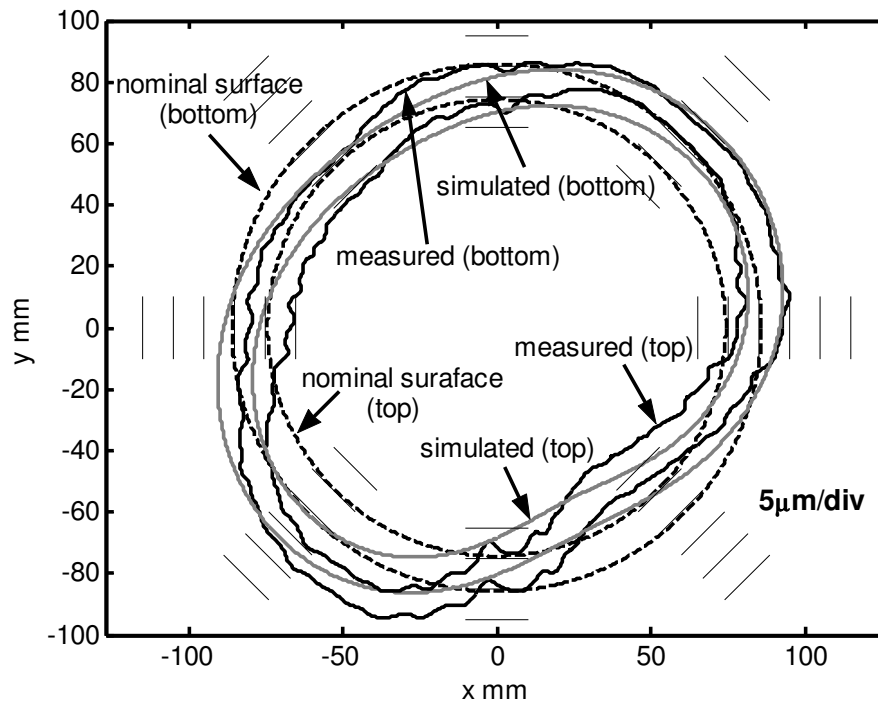


Figure 6-13 Simulated and measured machining error trajectories of the machined cone frustum surface in finishing under Condition III

A comparison of the circularity error and the concentricity error obtained from the simulated and measured machining error trajectories of the machined cone frustum under Condition I, II, and III is summarized in Table 6-4. It is seen that circularity errors of measured trajectories become larger on both top and bottom surfaces in the order of Condition I-II-III. The tendency in simulated trajectories well matches with this. As are observed in Figs. 6-11~6-13, rough geometries of simulated trajectories also well match with measured ones. However, there are still large differences in terms of circularity and concentricity errors between simulated and measured trajectories of the machined cone frustum. This could be mostly attributable to higher frequency vibrations observed in measured trajectories, which are not included in the kinematic model used in the present simulation. Simulation errors may be also attributable to other causes, such as static or dynamic motion errors of linear or rotary axes, or the tool vibration due to the cutting force. The inclusion of these possible error sources into simulation will be left for our future research.

Table 6-4 Summary of circularity and concentricity errors of the machined cone frustum in finishing under Condition I, II and III

	Condition I		Condition II		Condition III	
	Simu.	Meas.	Simu.	Meas.	Simu.	Meas.
Circularity error at bottom (μm)	6.3	7.1	7.2	9.3	13.3	17.3
Circularity error at top (μm)	5.8	9.3	6.7	8.8	13.0	19.5
Concentricity error (μm)	0.85	1.1	1.0	8.4	3.8	1.6

* Simu.: simulated, Meas.: measured

6.7.4 Compensation Results

In order to justify its effectiveness, the error compensation scheme presented in Section 6.6 is demonstrated to the case study under Condition III. Cutting experiments for error compensation is carried out using the same cutting conditions as in the machining under Condition III. Figure 6-14 shows the compensated machining error trajectories for both bottom and top surfaces of the machined cone frustum. Compared to Fig. 6-13 (machining results before compensation), it is seen that by applying the error compensation scheme, circularity errors are reduced from $17.3\mu\text{m}$ to $10.1\mu\text{m}$ at bottom and $19.5\mu\text{m}$ to $11\mu\text{m}$ at top surfaces of the machined cone frustum, and hence machining geometric accuracy is improved significantly.

6.8 Conclusion

This chapter presents a scheme to predict and compensate the machining geometric errors on a 5-axis machining center with its kinematic errors. By using

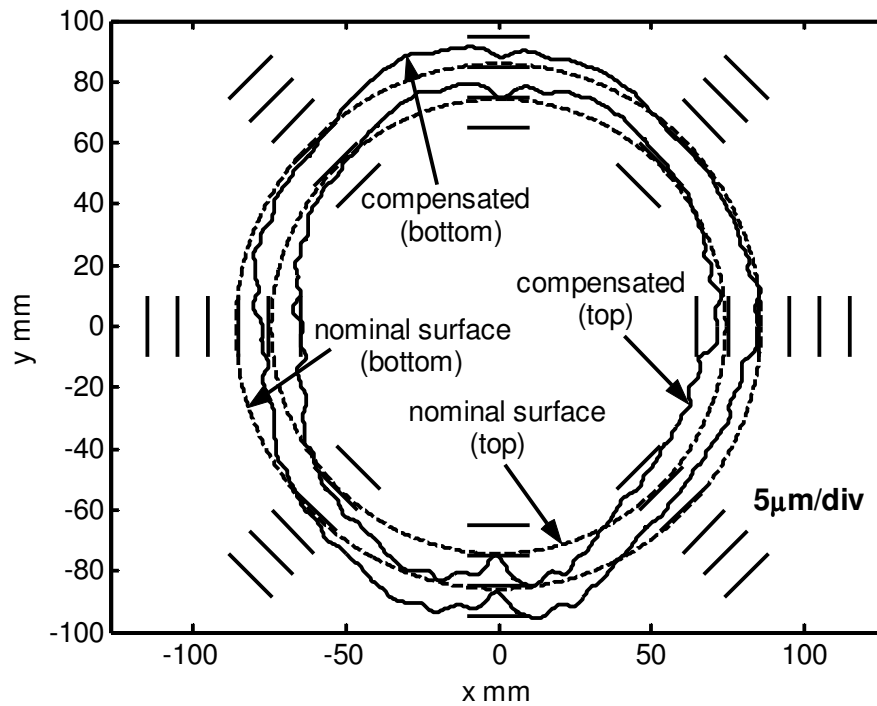


Figure 6-14 Compensated machining error trajectories of the machined cone frustum in finishing under Condition III

an error model with the kinematic errors, three-dimensional interference of the tool and the workpiece is calculated along the tool path to simulate the machining geometric error under the influence of the machine's kinematic errors. As an application example, the present scheme is applied to the machining of a tilted cone frustum, which is described in the standard NAS979 and is widely accepted in the machine tool industry as a machining performance test for 5-axis machines. Case studies with cutting experiments on the target 5-axis machining center verify the effectiveness of the prediction of machining errors in 5-axis machining. Further, experiments for error compensation show a significant improvement in machining accuracy in terms of reduction of circularity errors of the machined workpiece surface.

Chapter 7

Conclusions and Recommendations for Future Work

7.1 Conclusions

Due to the tremendous increasing requirements of products with the tight tolerance limit and high geometric accuracy, high speed CNC end milling is becoming popular, and being carried out to machine these parts to meet the demands. However, the geometric accuracy of the machined components is greatly influenced by many error sources which include errors in the machine tool system itself and the errors due to machining process. Among errors due to the machine tool system, kinematic errors due to geometric inaccuracies in mechanical structures of the machine tools, that cause the machine's motion errors, and hence the geometric errors of the machined surface, are more dominant. Especially, during free-form surface machining on multi-axis machines, like 5-axis machining centers, these kinematic errors cause significant deviation of tool tip position and orientation, which consequently results in the geometric inaccuracy on the machined surface. On the other hand, in 3-axis machining, significant factors among errors due to the machining process, which affect the machining geometric accuracy, are the cutting force variation and tool deflection. While other cutting parameters are fixed for any 2D end milling, depending on the cutting geometry, cutting tool path patterns such as contour parallel offset path used play an important role to the variation in cutting force and tool deflection.

Therefore, by considering the effect of error sources due to both the machine tool system and the cutting process on the machining geometric accuracy, the present research work has presented tool path modification approaches for the improvement of the machining geometric accuracy in 3-axis and 5-axis machining processes.

The following are the main contributions achieved in each chapter of this dissertation.

Part – I: Tool path modification for cutting engagement regulation in 3-axis machining

⟨ *Forward tool path modification* ⟩

- (1) Conventional contour parallel offset tool path used in 2D contour end milling, and cutting problems associated it, such as variation in cutting engagement and cutting force, and tool deflection, are described and addressed.
- (2) A comprehensive and brief literature on research works attempted to overcome cutting problems with contour parallel offset tool paths is reviewed. While the performance of feedrate scheduling scheme to regulate the cutting force is limited by the velocity control performance of the servo-controller of machine tools, tool path modification approaches developed based on the physical geometry of the cutting process has been more promising considering the application to a finishing path to improve the geometric accuracy of the machined surface.
- (3) With this aim, an algorithm to generate a constant engagement tool path by the forward tool path modification is proposed. An illustrating example of forward tool path modification and cutting force regulation by it on a simple corner is shown.
- (4) A critical issue with the proposed forward tool path modification approach is that, for spiral-out tool paths geometry, the outermost tool path is subject to the largest modification, hence leaving excessive material at the corners. Therefore, while the approach can be justified in roughing, it cannot be applied to the finishing process to improve the machining geometric accuracy.

〈 *Backward tool path modification* 〉

- (5) In order to overcome issues with the forward tool path modification, an algorithm for the backward tool path modification to generate an offset tool path such that a desirable constant engagement angle is regulated through the finishing path is proposed. By applying the proposed backward tool path modification approach, the cutting force on the machining of finishing path can be ideally maintained at a constant level, which naturally improves the geometric accuracy of the machined surface.
- (6) Case studies with cutting experiments on different contour geometries made of different workpiece materials are carried out, and machining results are compared with contour parallel paths and feedrate control scheme to demonstrate the significance of the proposed scheme. Comparing with an original contour parallel path, by applying the modified constant engagement tool path generated by the proposed algorithm for backward tool path modification, the maximum variation of the cutting force in finishing is significantly reduced (by 75%~83%), which consequently results in a significant reduction in geometric error of machined surface (by 62%~75%).
- (7) A critical problem with a feedrate control scheme for constant cutting force regulation is that it is often the case that a sufficient feedrate control performance cannot be achieved due to the performance limitation of servo-controllers of the machine tools. In practical applications to a finishing path, the proposed algorithm for backward tool path modification, thus, has a crucial advantage over feedrate control schemes.

〈 *Regulation of Cutting engagement under feedrate scheduling* 〉

- (8) The algorithm for backward tool path modification to regulate cutting engagement is applied to a feedrate control scheme, where feedrate at the tool center is varied such that a constant feedrate at

the cutting point is maintained. While a constant feedrate at the cutting point does not regulate constant cutting force, it has been shown that by applying feedrate control with modified tool path, a desired cutting force can be regulated more accurately and efficiently, and hence, the variation of machined surface geometric error is minimized and the surface quality is improved.

- (9) Results from experimental verification show that compared to contour parallel path and feedrate control, the proposed scheme (feedrate control with modified tool path) reduces variation of cutting force by about 80.4% and 44% (at maximum) respectively. Consequently, the variation in geometric surface error is reduced by about 57.5% and 19.3% (at maximum) respectively, revealing an improved geometric accuracy on the machined contour.

Part – II: Tool path modification to compensate kinematic errors on 5-axis machining centers

⟨ Modeling and identification of kinematic errors on 5-axis machining centers ⟩

- (10) Kinematic errors or geometric errors in a 5-axis machining center with tilting rotary table are defined. A kinematic model of the machining center with the kinematic errors considered is briefly described.
- (11) A telescoping DBB measuring method, a quick and efficient way to the measurement of the machining center, is applied to identify kinematic errors existing in the target 5-axis machining center. Modeling of the DBB measurements and a procedure to identify kinematic errors are briefly discussed.
- (12) Case studies with actual DBB tests on the target 5-axis machining center are demonstrated to practically estimate total eleven

kinematic errors associated with linear and rotary axes in the machining center. Although there are some problems with the DBB measurement process, simulated and measured DBB displacement trajectories show a good match, verifying the identification of kinematic errors in the 5-axis machining center.

〈 Prediction and compensation of machining geometric errors on 5-axis machining centers〉

- (13) A scheme to predict and compensate the machining geometric errors on a 5-axis machining center with its kinematic errors is presented. By using the error model of the machining center with kinematic errors, three-dimensional interference of the tool and the workpiece is calculated along the tool path to simulate the machining geometric error under the influence of the machine's kinematic errors.
- (14) As an application example, the present scheme is applied to the machining of a tilted cone frustum, which is described in the standard NAS979 and is widely accepted in the machine tool industry as a machining performance test for 5-axis machines.
- (15) For the improvement of the machining geometric accuracy in 5-axis machining, an error compensation for tool position and orientation is presented.
- (16) Case studies with cutting experiments on the target 5-axis machining center verify the effectiveness of the prediction of machining geometric errors in terms of circularity and concentric errors of the machine workpiece in 5-axis machining.
- (17) Experiments for error compensation by modifying the trajectory of tool position and orientation such that the effect of kinematic errors is canceled, show a significant improvement in machining geometric accuracy in terms of reduction of circularity errors of the machined workpiece surface in 5-axis machining.

7.2 Recommendations for Future Work

The development of any research work is a continuous process. Though we have accomplished some works in this dissertation, there are still more ideas and opportunities to extend and develop this research work for achieving the desired objectives. Hence, the following is a brief description of the topics that can be extended from this research work.

- (1) The algorithm for tool path modification to regulate the cutting engagement and hence the cutting force, presented in this dissertation is applied to two-dimensional contour end milling with a straight end mill. In practice, there are many cases where free-form surfaces are finished by contour end milling with a ball end mill. In such cases, the proposed algorithm for tool path modification must be extended to the ball end milling geometry by computing the cutting engagement or the interference between tool and workpiece in three-dimensional space.
- (2) It has been shown that the present simulator is able to predict the machining errors on the cone frustum (as specified in NAS 979) with the effect of kinematic errors on 5-axis machining centers. A sensitivity analysis of the effect of individual kinematic errors on the machining errors is also illustrated. This could further be used to establish a reliable methodology in order to identify kinematic errors more accurately by evaluating measured machining error of the cone frustum. As has been discussed in Section 6.2, a methodology to diagnose error sources from machining error trajectories has not been developed, and is strongly needed by machine tool builders.
- (3) The present simulator to predict machining geometric errors of the cone frustum workpiece on 5-axis machining centers includes only the effect of kinematic or geometric errors in the machine tool.

Other error sources such as positioning error and dynamics of linear and rotary axes, chatter vibration on cutting tool, thermal deformation may also cause machining geometric errors. Therefore, all these error sources can be incorporated in the present simulator to better predict, evaluate, and compensate the machining geometric errors in 5-axis machining.

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Appendices

Appendix I: Homogeneous Transformation Matrix (HTM)

The 4x4 Homogeneous Transformation Matrices (HTMs) [Slocum, 1992], $D^1(x)$, $D^2(y)$, and $D^3(z)$ for purely any linear motion in X, Y, and Z directions respectively can be defined as follows:

$$D^1(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D^2(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D^3(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where x , y , and z are linear displacements in X, Y, and Z directions respectively.

Similarly, the HTMs, $D^4(a)$, $D^5(b)$, and $D^6(c)$ for purely any angular motion about X, Y, and Z axes respectively can be defined as follows:

$$D^4(a) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) & 0 \\ 0 & \sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D^5(b) = \begin{bmatrix} \cos(b) & 0 & \sin(b) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(b) & 0 & \cos(b) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D^6(c) = \begin{bmatrix} \cos(c) & -\sin(c) & 0 & 0 \\ \sin(c) & \cos(c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, a , b , and c are angular motions about X, Y, and Z axes respectively.

Appendix II: Equivalent Feedrate Calculation on 5-Axis Machining Centers

In 5-axis machining, feedrate in linear-axis drives must be synchronized with that of rotary axes. Hence, an equivalent feedrate is calculated when a feedrate that defines the velocity of tool with respect to the workpiece, is given. The equivalent feedrate of a tool path trajectory for the three machining conditions (namely, Condition I, II, and III) presented in Section 6.6 of Chapter 6 can be determined as follows. First, differential values of each of five coordinates for a tool path trajectory, $(X(i), Y(i), Z(i), A(i), C(i)) \in (i = 1, \dots, N_t)$, are calculated as:

$$\begin{cases} dX(i) = X(i) - X(i-1) \\ dY(i) = Y(i) - Y(i-1) \\ dZ(i) = Z(i) - Z(i-1) \\ dA(i) = A(i) - A(i-1) \\ dC(i) = C(i) - C(i-1) \end{cases}$$

An equivalent differential length, $dL(i)$ of all coordinates along the whole tool path trajectory, and an equivalent segment length of the tool path, dl are calculated as:

$$dL(i) = \sqrt{(dX^2(i) + dY^2(i) + dZ^2(i) + dA^2(i) + dC^2(i))}$$

$$dl = R \cdot s$$

where, R and s are the radius and the segment length of the tool path trajectory in the workpiece frame. Then, the equivalent feedrate profile, $F'(i)$ for the tool path trajectory which will be used in actual machining operation, is calculated as:

$$F'(i) = (dL(i)/dl) \cdot F$$

where, F is the given feedrate defining the velocity of tool with respect to workpiece.