

OPTIMAL CONTROL FOR SEMILINEAR ABSTRACT EQUATIONS OF PARABOLIC TYPE

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1. INTRODUCTION

In the preceding paper [8], the authors studied the optimal control problems for the Keller-Segel equations. In that paper we showed the existence of optimal control and the first order necessary condition by formulating the Keller-Segel equations as a semilinear abstract equation. Many papers have already been published to study the control problems for nonlinear parabolic equations. In the books Ahmed [1] and Barbu [2], some general frameworks are given for handling the semilinear parabolic equations with monotone perturbations. In [1] the nonlinear terms are monotone functions with linear growth, and in [2] they are generalized to the multivalued maximal monotone operators determined by lower semicontinuous convex functions. Papageorgiou [7] and Casas et al. [3] have studied some quasilinear parabolic equations of monotone type. This note is the generalization of [8] as a semilinear abstract equation of non-monotone type.

Notations. $\mathbb{R}$ denotes the sets of real numbers. Let I be an interval in $\mathbb{R}$. $L^p(I; \mathcal{H})$, $1 \leq p \leq \infty$, denotes the L^p space of measurable functions in I with values in a Hilbert space $\mathcal{H}$. $\mathcal{C}(I; \mathcal{H})$ denotes the space of continuous functions in I with values in $\mathcal{H}$. Let $\mathcal{D}(I)$ denote the space of $\mathcal{C}^\infty$ -functions with compact support on I and $\mathcal{D}'(I)$ denote the space of distributions on I . For simplicity, we shall use a universal constant C to denote various constants which are determined in each occurrence in a specific way by δ, M , and so forth. In a case when C depends also on some parameter, say θ , it will be denoted by C_θ .

2. THE FORMULATION OF PROBLEM

Let $\mathcal{V}$ and $\mathcal{H}$ be two separable real Hilbert spaces with dense and compact embedding $\mathcal{V} \hookrightarrow \mathcal{H}$. Identifying $\mathcal{H}$ and its dual $\mathcal{H}'$ and denoting the dual space of $\mathcal{V}$ by $\mathcal{V}'$, we have $\mathcal{V} \hookrightarrow \mathcal{H} \hookrightarrow \mathcal{V}'$. We denote the scalar product of $\mathcal{H}$ by $(\cdot, \cdot)$ and the norm by $|\cdot|$. The duality product between $\mathcal{V}'$ and $\mathcal{V}$ which coincides with the scalar product of $\mathcal{H}$ on $\mathcal{H} \times \mathcal{H}$ is denoted by $\langle \cdot, \cdot \rangle$, and the norms of $\mathcal{V}$ and $\mathcal{V}'$ by $\|\cdot\|$ and $\|\cdot\|_*$, respectively. $\mathcal{U} = L^2(0, T; \mathcal{V}')$ and $\mathcal{U}_{ad}$ is closed, bounded and convex subset of $\mathcal{U}$.

We consider the following Cauchy problem

$$(E) \quad \begin{cases} \frac{dY}{dt} + AY = F(Y) + U(t), & 0 < t \leq T, \\ Y(0) = Y_0 \end{cases}$$

in the space $\mathcal{V}'$. Here, A is the positive definite self-adjoint operator of $\mathcal{H}$ defined by a symmetric sesquilinear form $a(Y, \tilde{Y})$ on $\mathcal{V}$, $\langle AY, \tilde{Y} \rangle = a(Y, \tilde{Y})$, which satisfies

$$(a.i) \quad |a(Y, \tilde{Y})| \leq M \|Y\| \|\tilde{Y}\|, \quad Y, \tilde{Y} \in \mathcal{V},$$

$$(a.ii) \quad a(Y, Y) \geq \delta \|Y\|^2, \quad Y \in \mathcal{V}$$

with some δ and $M > 0$. A is also a bounded operator from $\mathcal{V}$ to $\mathcal{V}'$. $F(\cdot)$ is a given continuous function from $\mathcal{V}$ to $\mathcal{V}'$ satisfying

(f.i) For each $\eta > 0$, there exists an increasing continuous function $\phi_\eta : [0, \infty) \rightarrow [0, \infty)$ such that

$$\|F(Y)\|_* \leq \eta \|Y\| + \phi_\eta(|Y|), \quad Y \in \mathcal{V};$$

(f.ii) For each $\eta > 0$, there exists an increasing continuous function $\psi_\eta : [0, \infty) \rightarrow [0, \infty)$ such that

$$\|F(\tilde{Y}) - F(Y)\|_* \leq \eta \|\tilde{Y} - Y\| + (\|\tilde{Y}\| + \|Y\| + 1) \psi_\eta(|\tilde{Y}| + |Y|) |\tilde{Y} - Y|, \quad \tilde{Y}, Y \in \mathcal{V}.$$

$U(\cdot) \in L^2(0, T; \mathcal{V}')$ is a given function and $Y_0 \in \mathcal{H}$ is an initial value.

We then obtain the following result (For the proof, see Ryu and Yagi [8]).

Theorem 2.1. *Let (a.i), (a.ii), (f.i), and (f.ii) be satisfied. Then, for any $U \in L^2(0, T; \mathcal{V}')$ and $Y_0 \in \mathcal{H}$, there exists a unique weak solution*

$$Y \in H^1(0, T(Y_0, U); \mathcal{V}') \cap \mathcal{C}([0, T(Y_0, U)]; \mathcal{H}) \cap L^2(0, T(Y_0, U); \mathcal{V})$$

to (E), the number $T(Y_0, U) > 0$ is determined by the norms $\|U\|_{L^2(0, T; \mathcal{V}')}$ and $|Y_0|$.

In this section we are concerned with the following problem

$$(P) \quad \text{Minimize } J(U),$$

where the cost functional $J(U)$ is of the form

$$J(U) = \int_0^S \|DY(U) - Y_d\|^2 dt + \gamma \int_0^S \|U\|_*^2 dt, \quad U \in \mathcal{U}_{ad}.$$

Here, $Y(U)$, $U \in \mathcal{U}_{ad}$, is the weak solution of (E) and is assumed to exist on a fixed interval $[0, S]$. D is a bounded operator from $\mathcal{V}$ into $\mathcal{V}$ and Y_d is a fixed element of $L^2(0, S; \mathcal{V})$. γ is a nonnegative constant.

Remark. Let $Y_0 \in \mathcal{H}$ be fixed. By Theorem 2.1, for $U \in \mathcal{U}_{ad}$, $Y(U)$ exists on the interval $[0, T(U)]$ with $T(U) > 0$ depending on $\|U\|_{L^2(0, T; \mathcal{V}')}$. Hence, $0 < S \leq \inf\{T(U); U \in \mathcal{U}_{ad}\}$.

We prove the following theorem.

Theorem 2.2. *There exists an optimal control $\bar{U} \in \mathcal{U}_{ad}$ for (P) such that*

$$J(\bar{U}) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

Proof. The proof can be carried out in the same way as that of Theorem 2.1 (see [8, Theorem 2.1]). As it is standard (cf. [2, Chap. 5, Proposition 1.1] and [6, Chap. III, Theorem 15.1]), we will only sketch.

Let $\{U_n\} \subset \mathcal{U}_{ad}$ be a minimizing sequence such that $\lim_{n \rightarrow \infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U)$. Since $\{U_n\}$ is bounded, we can assume that $U_n \rightarrow \bar{U}$ weakly in $L^2(0, S; \mathcal{V}')$. For simplicity, we will write Y_n instead of the solution $Y(U_n)$ of (E) corresponding to U_n ,

$$\begin{cases} \frac{dY_n}{dt} + AY_n = F(Y_n) + U_n(t), & 0 < t \leq S, \\ Y_n(0) = Y_0. \end{cases}$$

Taking the scalar product of the equation and Y_n , we obtain that

$$\frac{1}{2} \frac{d}{dt} |Y_n(t)|^2 + \langle AY_n(t), Y_n(t) \rangle = \langle F(Y_n(t)), Y_n(t) \rangle + \langle U_n(t), Y_n(t) \rangle.$$

Then, from (a.ii) and (f.i),

$$\frac{1}{2} \frac{d}{dt} |Y_n(t)|^2 + \delta \|Y_n(t)\|^2 \leq \eta \|Y_n(t)\|^2 + \{\phi_\eta(|Y_n(t)|) + \|U_n(t)\|_*\} \|Y_n(t)\|.$$

With some increasing, locally Lipschitz continuous function $\phi: [0, \infty) \rightarrow [0, \infty)$, it follows that

$$(2.1) \quad \begin{cases} \frac{d}{dt} |Y_n(t)|^2 + \delta \|Y_n(t)\|^2 \leq \phi(|Y_n(t)|^2) + \frac{8}{\delta} \|U_n(t)\|_*^2, & 0 < t \leq S, \\ |Y_n(0)|^2 = |Y_0|^2. \end{cases}$$

Let $z_n(t) = |Y_n(t)|^2 - \frac{8}{\delta} \int_0^t \|U_n(s)\|_*^2 ds$, $0 \leq t \leq S$. Since $\int_0^S \|U_n(s)\|_*^2 ds \leq C$, it follows that

$$\frac{dz_n}{dt} \leq \phi(z_n + 8C\delta^{-1}).$$

On the other hand, let $z(t)$ be a solution to the ordinary differential equation

$$\begin{cases} \frac{dz}{dt} = \phi(z + 8C\delta^{-1}), & 0 \leq t \leq S, \\ z(0) = |Y_0|^2. \end{cases}$$

Then, by the theorem of comparison, $z_n(t) \leq z(t)$ for all $0 \leq t \leq S$. Hence, $|Y_n(t)|^2 \leq \|z\|_{C([0, S])} + 8C\delta^{-1}$.

The sequence $\{Y_n\}$ is thus bounded in $L^\infty(0, S; \mathcal{H})$. As a consequence, it follows from (2.1) that $\{Y_n\}$ is bounded in $L^2(0, S; \mathcal{V})$ also. Moreover, from (f.i), $\{dY_n/dt\}$ is

bounded in $L^2(0, S; \mathcal{V}')$. Therefore, choosing a subsequence if necessary, we can assume that

$$\begin{aligned} Y_n &\rightarrow \bar{Y} \quad \text{weakly in } L^2(0, S; \mathcal{V}), \\ \frac{dY_n}{dt} &\rightarrow \frac{d\bar{Y}}{dt} \quad \text{weakly in } L^2(0, S; \mathcal{V}'). \end{aligned}$$

Since $\mathcal{V}$ is compactly embedded in $\mathcal{H}$, it is shown by [5, Chap. 1, Theorem 5.1] that

$$(2.2) \quad Y_n \rightarrow \bar{Y} \quad \text{strongly in } L^2(0, S; \mathcal{H}).$$

Let us verify that $\bar{Y}$ is a solution to (E) with the control $\bar{U}$. Let $\xi \in \mathcal{D}(0, S)$ and $V \in \mathcal{V}$, and put $\Phi(t) = \xi(t)V$. Then,

$$\begin{aligned} \int_0^S \langle Y'_n(t), \Phi(t) \rangle dt + \int_0^S \langle AY_n(t), \Phi(t) \rangle dt \\ = \int_0^S \langle F(Y_n(t), \Phi(t)) \rangle dt + \int_0^S \langle U_n(t), \Phi(t) \rangle dt. \end{aligned}$$

Let here n tend to infinity. It is then observed from (f.ii) that

$$\begin{aligned} \int_0^S |\langle F(Y_n(t) - \bar{Y}(t), \Phi(t)) \rangle| dt &\leq \eta \int_0^S \|Y_n(t) - \bar{Y}(t)\| \|\Phi(t)\| dt \\ &+ \int_0^S (\|Y_n(t)\| + \|\bar{Y}(t)\| + 1) \psi_\eta(|Y_n(t)| + |\bar{Y}(t)|) \|Y_n(t) - \bar{Y}(t)\| \|\Phi(t)\| dt, \end{aligned}$$

where $\eta > 0$ is arbitrary. From (2.2) it is seen that $\int_0^S \langle F(Y_n), \Phi(t) \rangle dt$ converges to $\int_0^S \langle F(\bar{Y}(t)), \Phi(t) \rangle dt$ as $n \rightarrow \infty$. Therefore, we obtain that

$$\begin{aligned} \int_0^S \langle \bar{Y}'(t), \Phi(t) \rangle dt + \int_0^S \langle A\bar{Y}(t), \Phi(t) \rangle dt \\ = \int_0^S \langle F(\bar{Y}(t), \Phi(t)) \rangle dt + \int_0^S \langle \bar{U}(t), \Phi(t) \rangle dt. \end{aligned}$$

This then shows that $\bar{Y}(t)$ satisfies the equation of (E) for almost all $t \in (0, S)$. In a similar way it is also shown that $\bar{Y}(0) = Y_0$, note from [4, Chap. XVIII, Theorem 1] that $\bar{Y} \in \mathcal{C}([0, S]; \mathcal{H})$. Hence, $\bar{Y}$ is the unique solution to (E) with the control $\bar{U}$; that is, $\bar{Y} = Y(\bar{U})$.

Since $Y_n - Y_d$ is weakly convergent to $\bar{Y} - Y_d$ in $L^2(0, S; \mathcal{V})$, we have:

$$\min_{U \in \mathcal{U}_{ad}} J(U) \leq J(\bar{U}) \leq \lim_{n \rightarrow \infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

Hence, $J(\bar{U}) = \min_{U \in \mathcal{U}_{ad}} J(U)$. $\square$

3. FIRST ORDER NECESSARY CONDITION

In this section, we show the first order necessary condition for the Problem (P). We denote the scalar products in $\mathcal{V}$ and $\mathcal{V}'$ by $\langle \cdot, \cdot \rangle_{\mathcal{V}}$ and $\langle \cdot, \cdot \rangle_{\mathcal{V}'}$, respectively. In order to the necessary conditions of optimality, we need some additional assumptions:

(f.iii) The mapping $F(\cdot) : \mathcal{V} \rightarrow \mathcal{V}'$ is Fréchet differentiable and for each $\eta > 0$, there exists an increasing continuous functions $\mu_\eta, \nu : [0, \infty) \rightarrow [0, \infty)$ such that

$$|\langle F'(Y)Z, P \rangle| \leq \begin{cases} \eta \|Z\| \|P\| + (\|Y\| + 1)\mu_\eta(|Y|)\|Z\| \|P\|, & Y, Z, P \in \mathcal{V}, \\ \eta \|Z\| \|P\| + (\|Y\| + 1)\mu_\eta(|Y|)\|Z\| \|P\|, & Y, Z, P \in \mathcal{V}, \\ \nu(|Y|)\|Z\| \|P\|, & Y, Z, P \in \mathcal{V}. \end{cases}$$

(f.iv) $F'(\cdot)$ is continuous from $\mathcal{H}$ into $\mathcal{L}(\mathcal{V}, \mathcal{V}')$.

Proposition 3.1. *Let (a.i), (a.ii), (f.i), (f.ii), (f.iii), and (f.iv) be satisfied. The mapping $Y : \mathcal{U}_{ad} \rightarrow H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ is Gâteaux differentiable with respect to U . For $V \in \mathcal{U}_{ad}$, $Y'(U)V = Z$ is the unique solution in $H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ of the problem*

$$(3.1) \quad \begin{cases} \frac{dZ}{dt} + AZ - F'(Y)Z = V(t), & 0 < t \leq S, \\ Z(0) = 0. \end{cases}$$

Proof. Let $U, V \in \mathcal{U}_{ad}$ and $0 \leq h \leq 1$. Let Y_h and Y be the solutions of (E) corresponding to $U + hV$ and U , respectively.

Step 1. $Y_h \rightarrow Y$ strongly in $\mathcal{C}([0, S]; \mathcal{H})$ as $h \rightarrow 0$. Let $W = Y_h - Y$. Obviously, W satisfies

$$(3.2) \quad \begin{cases} \frac{dW}{dt} + AW - (F(Y_h(t)) - F(Y(t))) = hV(t), & 0 < t \leq S, \\ W(0) = 0. \end{cases}$$

Taking the scalar product of the equation (3.2) with W , we obtain that

$$\frac{1}{2} \frac{d}{dt} |W(t)|^2 + \langle AW(t), W(t) \rangle = \langle F(Y_h(t)) - F(Y(t)), W(t) \rangle + \langle hV(t), W(t) \rangle.$$

Using (a.ii) and (f.ii), we have

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} |W(t)|^2 + \delta \|W(t)\|^2 \\ & \leq \frac{\delta}{2} \|W(t)\|^2 + (\|Y_h(t)\|^2 + \|Y(t)\|^2 + 1) \psi_{\frac{\delta}{4}}(|Y_h(t)| + |Y(t)|)^2 |W(t)|^2 \\ & \quad + 4h^2 \delta^{-1} \|V(t)\|_*^2. \end{aligned}$$

Therefore,

$$\begin{aligned}
 (3.3) \quad & \frac{1}{2}|W(t)|^2 + \frac{\delta}{2} \int_0^t \|W(s)\|^2 ds \\
 & \leq \int_0^t (\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1) \psi_{\frac{\delta}{4}}(|Y_h(s)| + |Y(s)|)^2 |W(s)|^2 ds \\
 & \quad + 4h^2 \delta^{-1} \int_0^S \|V(s)\|_*^2 ds.
 \end{aligned}$$

Using Gronwall's lemma, we obtain that

$$|W(t)|^2 \leq Ch^2 \|V\|_{L^2(0,S;\mathcal{V}')}^2 e^{\int_0^S (\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1) \psi_{\frac{\delta}{4}}(|Y_h(s)| + |Y(s)|)^2 ds}$$

for all $t \in [0, S]$. Hence, $Y_h \rightarrow Y$ strongly in $\mathcal{C}([0, S]; \mathcal{H})$ as $h \rightarrow 0$.

Step 2. $\frac{Y_h - Y}{h} \rightarrow Z$ strongly in $H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ as $h \rightarrow 0$. We rewrite the problem (3.2) in the form

$$(3.4) \quad \begin{cases} \frac{d}{dt} \frac{Y_h - Y}{h} + A \frac{Y_h - Y}{h} - \frac{F(Y_h) - F(Y)}{h} = V(t), & 0 < t \leq S, \\ \frac{Y_h - Y}{h}(0) = 0. \end{cases}$$

On the other hand, we consider the linear problem (3.1). From (a.i), (a.ii), (f.i), (f.ii), and (f.iii), we can easily verify that (3.1) possesses a unique weak solution $Z \in H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$ on $[0, S]$ (cf. [4, Chap. XVIII, Theorem 2]). Define $F'_h = \int_0^1 F'(Y + \theta(Y_h - Y)) d\theta$. Then $\widetilde{W} = \frac{Y_h - Y}{h} - Z$ satisfies

$$(3.5) \quad \begin{cases} \frac{d\widetilde{W}(t)}{dt} + A\widetilde{W}(t) - F'_h \widetilde{W}(t) = (F'_h - F'_0)Z(t), & 0 < t \leq S, \\ \widetilde{W}(0) = 0. \end{cases}$$

Taking the scalar product of the equation of (3.5) with $\widetilde{W}$, we obtain that

$$\begin{aligned}
 & \frac{1}{2} \frac{d}{dt} |\widetilde{W}(t)|^2 + \langle A\widetilde{W}(t), \widetilde{W}(t) \rangle \\
 & = \langle F'_h \widetilde{W}(t), \widetilde{W}(t) \rangle + \langle (F'_h - F'_0)Z(t), \widetilde{W}(t) \rangle. \\
 & \leq \frac{\delta}{2} \|\widetilde{W}(t)\|^2 + (\|Y(t)\|^2 + \|Y_h(t) - Y(t)\|^2 + 1) \mu(|Y_h|^2 + |Y|^2) |\widetilde{W}(t)|^2 \\
 & \quad + \frac{4}{\delta} \|(F'_h - F'_0)Z(t)\|_*^2,
 \end{aligned}$$

where $\mu : [0, \infty) \rightarrow [0, \infty)$ is some increasing continuous function. Therefore,

$$\begin{aligned}
 (3.6) \quad & |\widetilde{W}(t)|^2 + \delta \int_0^t \|\widetilde{W}(s)\|^2 ds \\
 & \leq \int_0^t (\|Y(s)\|^2 + \|Y_h(s)\|^2 + 1) \mu(|Y_h|^2 + |Y|^2) |\widetilde{W}(s)|^2 ds \\
 & \quad + \frac{8}{\delta} \|(F'_h - F'_0)Z(t)\|_{L^2(0,S;\mathcal{V}')}^2.
 \end{aligned}$$

From (f.iii), we have $\|F'_h Z(t)\|_* \leq C\|Z(t)\|$, $t \in [0, S]$. Since $Y_h \rightarrow Y$ strongly in $\mathcal{H}$, it follows from (f.iv) that

$$F'_h Z(t) \rightarrow F'_0 Z(t) \quad \text{strongly in } \mathcal{V}' \text{ a.e..}$$

By the dominated convergence theorem, we have

$$\|(F'_h - F'_0)Z(t)\|_{L^2(0,S;\mathcal{V}')}^2 \rightarrow 0 \quad \text{as } h \rightarrow 0.$$

Using Gronwall's lemma, it follows from (3.6) that $\frac{Y_h - Y}{h}$ is strongly convergent to Z in $H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$. $\square$

With the aid of this proposition, we can easily show the first order necessary condition.

Theorem 3.2. *Let $\bar{U}$ be an optimal control of (P) and let $\bar{Y} \in L^2(0, S; \mathcal{V}) \cap \mathcal{C}([0, S]; \mathcal{H}) \cap H^1(0, S; \mathcal{V}')$ be the optimal state, that is $\bar{Y}$ is the solution to (E) with the control $\bar{U}(t)$. Then, there exists a unique solution $P \in L^2(0, S; \mathcal{V}) \cap \mathcal{C}([0, S]; \mathcal{H}) \cap H^1(0, S; \mathcal{V}')$ to the linear problem*

$$(3.7) \quad \begin{cases} -\frac{dP}{dt} + AP - F'(\bar{Y})^* P = D^* \Lambda(D\bar{Y} - Y_d), & 0 \leq t < S, \\ P(S) = 0 \end{cases}$$

in $\mathcal{V}'$, where $\Lambda : \mathcal{V} \rightarrow \mathcal{V}'$ is a canonical isomorphism; moreover,

$$\int_0^S \langle \Lambda P + \gamma \bar{U}, V - \bar{U} \rangle_{\mathcal{V}'} dt \geq 0 \quad \text{for all } V \in \mathcal{U}_{ad}.$$

Proof. Since J is Gâteaux differentiable at $\bar{U}$ and $\mathcal{U}_{ad}$ is convex, it is seen that

$$J'(\bar{U})(V - \bar{U}) \geq 0 \quad \text{for all } V \in \mathcal{U}_{ad}.$$

On the other hand, we verify that

$$(3.8) \quad J'(\bar{U})(V - \bar{U}) = \int_0^S \langle DY(\bar{U}) - Y_d, DZ \rangle_{\mathcal{V}} dt + \gamma \int_0^S \langle \bar{U}, V - \bar{U} \rangle_{\mathcal{V}'} dt$$

with $Z = Y'(\bar{U})(V - \bar{U})$. Let P be the unique solution of (3.7) in $H^1(0, S; \mathcal{V}') \cap \mathcal{C}([0, S]; \mathcal{H}) \cap L^2(0, S; \mathcal{V})$. From (a.i), (a.ii), (f.i), (f.ii), and (f.iii), we can guarantee that such a solution P exists (cf. [4, Chap. XVIII, Theorem 2]). Thus, in view of Proposition 3.1 the first integral in the right hand side of (3.8) is shown to be

$$\begin{aligned} \int_0^S \langle DY(\bar{U}) - Y_d, DZ \rangle_{\mathcal{V}} dt &= \int_0^S \langle D^* \Lambda(DY(\bar{U}) - Y_d), Z \rangle dt \\ &= \int_0^S \langle -\frac{dP}{dt} + AP - F'(\bar{Y})^* P, Z \rangle dt = \int_0^S \langle P, \frac{dZ}{dt} + AZ - F'(\bar{Y})Z \rangle dt \\ &= \int_0^S \langle \Lambda P, V - \bar{U} \rangle_{\mathcal{V}'} dt. \end{aligned}$$

Hence,

$$\int_0^S \langle \Lambda P + \gamma \bar{U}, V - \bar{U} \rangle_{V'} dt \geq 0, \quad \text{for all } V \in \mathcal{U}_{ad}. \quad \square$$

Remark. Note that our result covers that of [8, 9] when the sensitivity function $\chi(\rho)$ is linear function of ρ , $\chi(\rho) = b\rho$ (b being a positive constant). Furthermore, since all assumptions of our abstract result are satisfied when $\chi(\rho) = \frac{b\rho}{1+\rho}$, our result is also applied in this case.

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