

**MAGNETIC SUSPENSION DEVICES
USING TUNED LCR CIRCUITS**

**BY
SHIRO HAGIHARA**

OCTOBER, 1978



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ABSTRACT

This thesis deals with the performance of a magnetic suspension device using tuned LCR circuits. The device under consideration contains an electromagnet which is the inductive part of a series resonance circuit and suspends a body magnetically without mechanical contact. Owing to the inherent characteristics of the circuit, the average magnetic attractive-force is controlled passively and hence there needs no servo controller to achieve the restorative action. The device is described by nonlinear differential equations of nonautonomous type. The method of averaging is applied to those equations and an autonomous system is obtained. Both steady-state and dynamic operations of the device are studied by the analysis of the autonomous system.

The text consists of six chapters. The first chapter is an introductory one in which various methods of magnetic suspension are reviewed with emphasis on the method using tuned circuits. The contents of the text are outlined. Chapter 2 deals with the ideal single-axis suspension device which suspends a body by an electromagnet against the gravitational force. Static and dynamic stability conditions are obtained. The theoretical results are confirmed by an analog-computer analysis. In Chapter 3, the effect of leakage flux on the stability of the single-axis suspension device is investigated. It is shown that there exists an upper limit in the supply frequency of the tuned circuit to achieve a stable operation. The theoretical results are confirmed by experiments. Chapter 4 deals with a quasi-periodic oscillation which occurs in the magnetic suspension system considered in Chapter 3. When the mechanical or the electrical damping of the system is insufficient, the system becomes unstable and the quasi-periodic oscillation occurs: the sinusoidal variation of the magnetic flux in the electromagnet becomes unstable and the amplitude and phase of the magnetic flux vary slowly with time even in the steady state. The suspended body also oscillates. The amplitude and frequency of the quasi-periodic oscillation are analytically computed by using the method of harmonic balance, and compared with the experimental results. In Chapter 5, the effect of magnetic saturation of the electromagnet is investigated. It is analytically shown that the effect of magnetic saturation results in an increase in the equivalent spring constant of the magnetic suspension. For a great degree of the magnetic saturation this effect, however, makes the system unstable and the quasi-periodic oscillation occurs in the system even if the mechanical damping is sufficiently large. The

generation of this kind of oscillation is verified by experiments. Chapter 6 is concerned with the performance of the push-pull type suspension device which supports a body by two electromagnets. Formulas for the equivalent spring constant and the stability conditions are derived for the convenience of the design. The basic characteristics of the device are confirmed by experiments.

PREFACE

Magnetic suspension devices are used as a means of eliminating friction in mechanical bearings. The possibilities of suspension by electromagnetic fields have been widely explored in various application fields including ultracentrifuges, contamination-free melting of metals, ultrahigh-speed trains, measuring instrument pivots, and so forth.

This paper deals with the study of the performance of a magnetic suspension device which utilizes tuned LCR circuits. The device has an electromagnet which is the inductive part of a series resonance circuit. The basic principle of operation of the device relies on the variation of inductance of the electromagnet with the distance between the magnet and a ferromagnetic material to be suspended. A stable suspension is achieved as follows. If the suspended object is moved away from the suspending magnet, then the resulting decrease in the inductance of the electromagnet causes the resonance circuit into a tuned condition. This causes the current flow to increase. The average magnetic force increases and restores the object to its original position. Thus a statically stable force-distance characteristic is obtained for a limited range of the distance from the suspending magnet.

This method is significant in that it achieves suspension by applying relatively simple circuitry and using a surprisingly small number of components. The device is, for instance, applicable to the suspension of the gyro rotors of inertial guidance systems for space vehicles.

It is known, however, that a statically stable characteristic is not always dynamically stable. A theory dealing with the dynamic behavior as well as the steady-state operation of the device must be established for the development of the device. There appears, however, to be very little work done especially on the dynamic behavior of the system.

This paper investigates the single-axis suspension device with an electromagnet and also the push-pull type suspension device with two electromagnets. The equations which describe the system are nonlinear differential equations and are analyzed by using the method of averaging. Analytical formulas characterizing the device (e.g. equivalent spring constant) are derived. Stability criteria are given for the operation of the device. The effects of leakage flux and magnetic saturation on the performance of the device are considered. The theoretical results are confirmed by computer analysis and also by experiments.

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LIST OF PRINCIPAL SYMBOLS

a :	coefficient of cubic term of magnetic saturation curve
A :	cross section of air gap
B :	nondimensional supply voltage
B_0 :	nondimensional mechanical force
C :	capacitance
e_L :	instantaneous coil voltage with voltage drop due to coil resistance
E :	maximum value of supply voltage
E_e :	effective value of supply voltage
E_L :	effective value of coil voltage with voltage drop due to coil resistance
E_{L0} :	effective value of coil voltage without voltage drop due to coil resistance
f :	supply frequency
f_m :	nondimensional magnetic attractive force
$\bar{f}_m$:	time average of f_m
F_d :	mechanical disturbance force
F_m, F_{m1}, F_{m2} :	magnetic attractive forces
$\bar{F}_m$:	time average of F_m
g :	gravitational acceleration
h :	gap distance of vertical-axis suspension device or displacement of push-pull type suspension device
h_1, h_2 :	gap distances of push-pull type suspension device
h_0, h_{10}, h_{20} :	steady-state values of h, h_1 , and h_2 , respectively
h_c :	gap distance at center of push-pull type suspension device
H_n :	base quantity of gap distance
i, i_1, i_2 :	instantaneous coil currents
i_c, i_{c1}, i_{c2} :	instantaneous capacitor currents
i_R, i_{R1}, i_{R2} :	instantaneous leakage currents of capacitors
I :	effective value of coil current i
k_1 :	nondimensional damping coefficient of tuned circuit due to parallel resistance of capacitor
k_2 :	nondimensional coefficient of viscous mechanical damping

k_m :	coefficient of viscous mechanical damping
k_s :	nondimensional damping coefficient of tuned circuit due to coil resistance
L :	coil inductance
M :	mass of suspended body
n :	number of turns of coil
r, r_1, r_2 :	amplitudes of u, u_1 , and u_2 , respectively
r_0, r_{10}, r_{20} :	steady-state values of r, r_1 , and r_2 , respectively
r_x, r_y, r_v :	amplitudes of x, y , and v , respectively, for self-excited oscillation
R :	parallel resistance of capacitor
R_s :	coil resistance (total series resistance of tuned circuit)
$\mathcal{R}, \mathcal{R}_1, \mathcal{R}_2$:	total reluctances of magnetic circuits
$\mathcal{R}_c$:	reluctance of iron portions
$\mathcal{R}_l$:	leakage reluctance
S :	equivalent spring constant
S_0 :	nondimensional equivalent spring constant
t :	time
u, u_1, u_2 :	nondimensional magnetic fluxes
v :	nondimensional gap distance of vertical-axis suspension device or non-dimensional displacement of push-pull type suspension device
v_1, v_2 :	nondimensional gap distances of push-pull type suspension device
v_0, v_{10}, v_{20} :	steady-state values of v, v_1 , and v_2 , respectively
v_c :	nondimensional gap distance at center of push-pull type suspension device
v_d :	unidirectional component of v for self-excited oscillation
$\dot{v}$:	derivative $dv/d\tau$
$\dot{v}_0$:	steady-state value of $\dot{v}$
x, x_1, x_2 :	sine components of u, u_1 , and u_2 , respectively
x_0, x_{10}, x_{20} :	steady-state values of x, x_1 , and x_2 , respectively
x_d :	unidirectional component of x for self-excited oscillation
x_x, x_y :	sine components of x and y , respectively, for self-excited oscillation
y, y_1, y_2 :	cosine components of u, u_1 , and u_2 , respectively
y_0, y_{10}, y_{20} :	steady-state values of y, y_1 , and y_2 , respectively
y_d :	unidirectional component of y for self-excited oscillation
y_x, y_y :	cosine components of x and y , respectively, for self-excited oscillation

α :	nondimensional coefficient of cubic term of magnetic saturation curve
β :	nondimensional angular frequency of self-excited oscillation
ϵ :	small parameter
ξ :	variation of v
η, η_1, η_2 :	variations of y, y_1 , and y_2 , respectively
μ_0 :	permeability of air gap
ν :	nondimensional supply angular frequency
ξ, ξ_1, ξ_2 :	variations of x, x_1 , and x_2 , respectively
ρ :	nondimensional leakage permeance
σ :	reluctance of air gap for unit gap length
τ, τ_n :	nondimensional time
ϕ, ϕ_1, ϕ_2 :	magnetic fluxes generated by coils
ϕ_i :	magnetic flux passing through suspended body
ϕ_l :	leakage magnetic flux
Φ_n :	base quantity of magnetic flux
ψ :	variation of $\dot{v}$
ω :	supply angular frequency
ω_0 :	base quantity of angular frequency

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CHAPTER 1

INTRODUCTION

1.1 Outline of Magnetic Suspension Devices

There are numerous cases in which gravitational force is undesirable. The rotation and horizontal translation of bodies, for instance, are hindered by friction caused by gravity. Many attempts have been made to compensate for gravitational forces in such cases.

As one of the compensating forces, magnetic force is widely used. The possibility of levitation by magnetic fields has been widely explored. Boerdijk [6] has discussed the general technical feasibility of various methods of levitation. As for magnetic suspension a very large number of applications have been suggested which include frictionless bearings for spindles rotating at a high speed in synthetic fiber industry or in ultra high-speed centrifuges [3, 4], suspension of metals in space for melting them without using a crucible [11, 46, 71, 74], contactless suspension of articles for paint and plastic coating or of models in wind tunnels, and high-speed magnetic suspension trains [1, 7, 20, 44, 45, 49, 50]. Furthermore, a number of magnetic bearings for space applications have been studied [12, 13, 16-19, 29, 42, 48, 53, 54, 62-67, 69, 70], and continuous efforts are still being made to improve the performance of magnetic suspension bearings. Some of these are already in use, and others may be very challenging in the near future. The magnetic force for suspension may be repulsive or attractive, and the magnetic field may be unidirectional or alternating. Stability is always an important problem in the design of practical systems.

Contactless magnetic suspension is subject to the fundamental restriction of Earnshaw's theorem [14], which states that stable suspension can never be achieved in stationary magnetic fields. A permanent magnet, for example, cannot be supported in a stable state by other permanent magnets alone, but requires some mechanical means to guarantee the stability. Braunbek [8] has developed this theorem. He showed that the instability occurs when the relative permeability of materials to be suspended is greater than unity. Stable suspension in stationary magnetic fields is therefore achieved by using diamagnetics or superconductors. Such suspension systems usually involve expensive apparatus. If suitable time-varying magnetic fields are used for suspension, the restriction mentioned

above is eliminated. Stable suspension by time-varying magnetic fields can be achieved either by sensing the position of the body and controlling the force fields by servo action, or by making use of an inherent characteristic of the excitation system of the force fields. The servo-controlled suspension may be called of an "active type", whereas a suspension that is inherently stable owing to the characteristics of the excitation system may be called of a "passive type".

For the active, or the servo-controlled suspension, the apparatus generally comprises the following elements:

- (1) an electric source for generating the field,
- (2) the body to be suspended,
- (3) a sensing element for detecting the position of the body,
- (4) a device which controls the source according to the output of the sensing element in such a manner that the position of the body is stabilized.

In the papers hitherto published, the magnetic field is generated by an electromagnet, in most cases, using direct current. The suspended body contains a ferromagnetic material. The sensing element is formed by a photocell and a beam of light partly intercepted by the body [2, 73, pp. 192-200]. Inductive [4, 54, 70] or capacitive [58] probes are also used. The undesirable oscillation of the suspended body caused by external forces can be eliminated by servo design [21].

For the passive, or the self-stabilizing suspension, three practical systems have been suggested. The first system is concerned with a method of heating and melting metals without using a crucible. The metals are levitated in space by an electromagnetic field [9, 11, 41, 46, 60, 71, 74]. The force of suspension results from the eddy current induced in a conducting body (melting metal) by an alternating magnetic field. The eddy current reacts on the excitation system to produce a force which may be used for suspension. The second system was proposed as a method of suspending trains [1, 7, 20, 32, 44, 45, 49, 50], and uses the magnetic repulsive force between the superconductive magnets on the moving train and the repelling currents induced in the track by the magnets. The third system was suggested as a means of eliminating friction in mechanical bearings of inertial guidance gyros for space vehicles [13, 16-19, 42, 64-67]. This uses tuned LCR circuits to control the force-distance characteristics between the electromagnet and the suspended body of a ferromagnetic material. Unlike the first two systems of the passive suspension, this system utilizes the magnetic attractive force.

This paper deals with the passive suspension system using tuned LCR circuits. Its

details will be described in the following sections.

1.2 Magnetic Suspension Device Using Tuned LCR Circuits

The method of magnetic suspension using tuned LCR circuits is significant in achieving a suspension system by applying relatively simple circuitry and using a surprisingly small number of components. It is considered to be superior to the servo-controlled suspension methods in terms of reliability since no external sensing device is required. Furthermore, with respect to the power-to-weight ratio the tuned suspension method is also found to be far more superior to the eddy current methods investigated by Laithwaite [41].

An example of the magnetic suspension device is shown in Fig. 1.1. The electromagnet coil of inductance L and the capacitor C form a tuned circuit driven by an alternating voltage $E \sin \omega t$. For simplicity of explanation, it is assumed that the suspended body moves only in the vertical direction. If the coil is excited simply by a direct or alternating current, the equilibrium point where the magnetic attractive force balances the gravitational force will be unstable. In this case the body will either cling to the pole face or fall down by even a slight disturbance. However, if the coil is tuned with a capacitor C as shown in Fig. 1.1, the amplitude of the coil current varies as the coil inductance L changes with change of gap distance h ; therefore the system can be self-stabilizing. Figure 1.2 shows schematically the variation of the inductance L , the effective value of the coil current I , and the average magnetic force $\bar{F}_m$ as the gap distance h changes. The gravitational force Mg acting on the suspended body is also shown by a horizontal straight line. The intersections, P_1 and P_2 , of the two curves $\bar{F}_m$ and Mg are the equilibrium points where the upward average magnetic force $\bar{F}_m$ balances the downward gravitational force Mg . Point P_1 is stable, because a small increment in h from h_1 results in an increase of the net restoring force; on the contrary, point P_2 is unstable. Thus over a limited range of distances from the suspending magnet a stable force-distance characteristic is obtained.

Point P_1 is statically stable, but is not always dynamically stable. There is a possibility that the suspended body may begin to oscillate with negative damping, although a static analysis of the average forces shows that the system is stable. Some type of stabilization (including the application of an external damping) is therefore necessary to achieve a stable operation.

Slater [59] and Knobel [40] have described an analogous suspension method which uses no electromagnetic force but uses instead a electrostatic force. The suspended body is

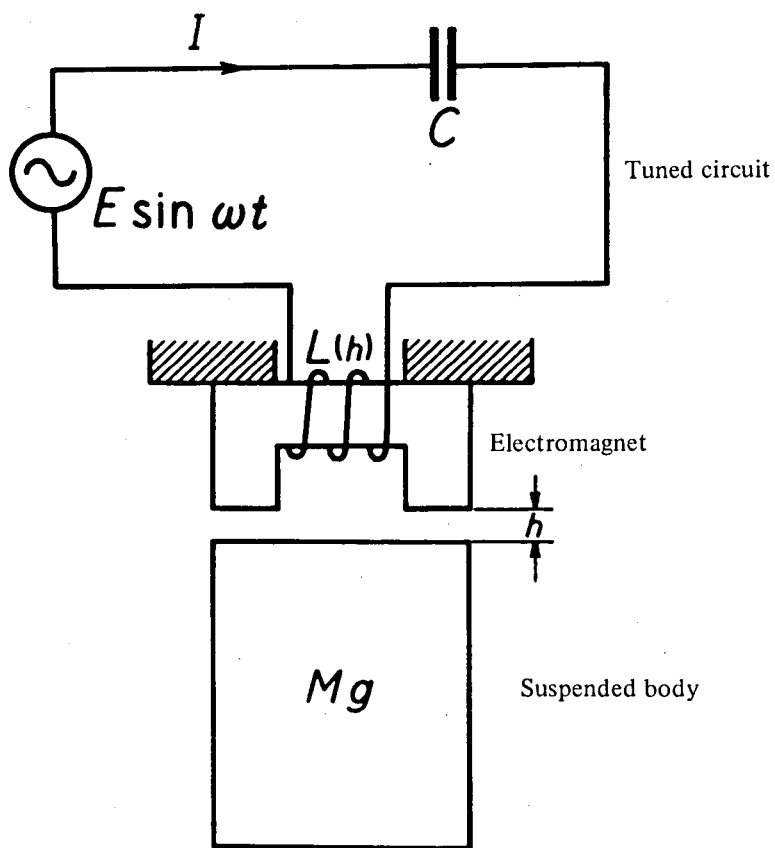


Fig. 1.1 Magnetic suspension device using a tuned LCR circuit.

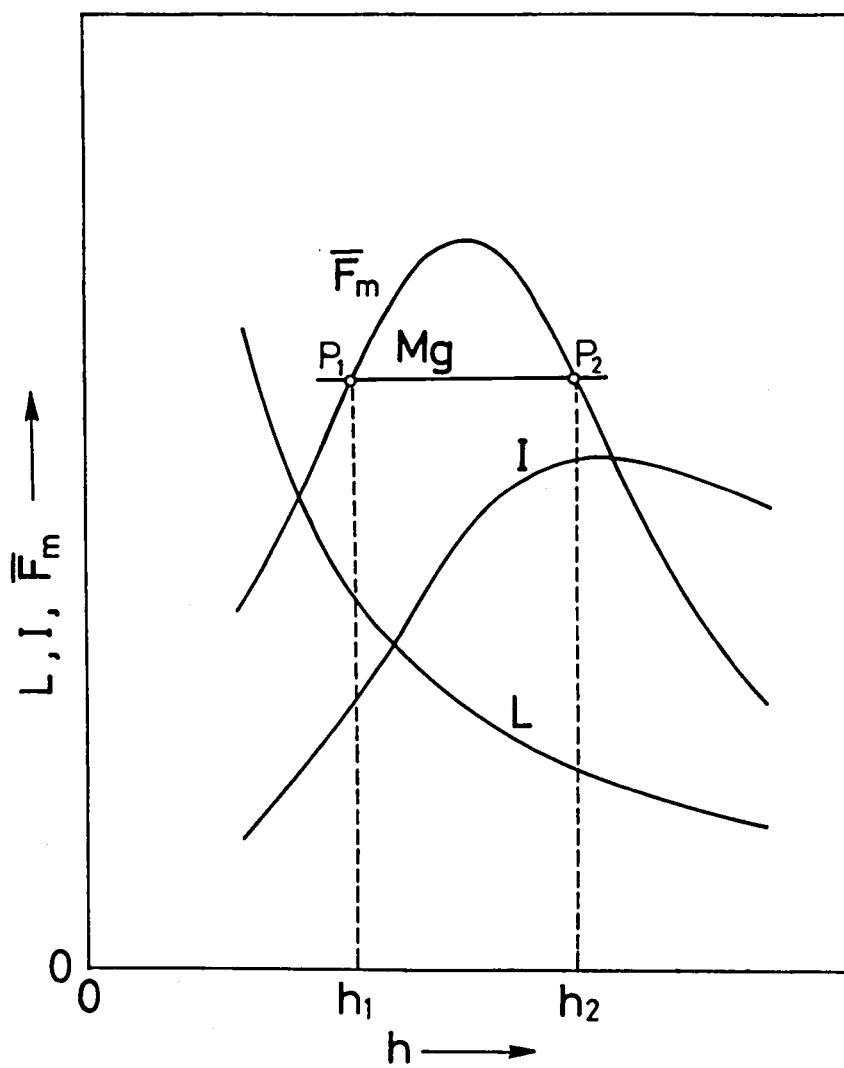


Fig. 1.2 Relationship between h and $L, I, \bar{F}_m$

made of conducting materials (e.g. beryllium), and is suspended between electrodes which are connected through inductor coils to an alternating current source, thus producing a resonance effect very similar to our model. The necessity of introducing damping to the suspension system is mentioned in their papers. Knobel also suggests a stabilizing method which makes use a saturable reactor control. This method is applicable to our model. However, they do not give an explanation about the dynamic behavior of the suspended body.

Kaplan [36] has demonstrated experimentally the occurrence of an unstable oscillation in the system illustrated in Fig. 1.1. He has discussed the dynamic instability [73, pp. 192-200] of the system by comparison with electromechanical parametric oscillators [5, 25, 26, 28, 31, 43, 52, 56, 61, 72].

In connection with the jumping-ring experiment using a tuned LCR circuit [33], Jayawant and Rea have also reported that such a system tends to be self-oscillatory. They proposed a number of stabilizing schemes and did an analytical study including an analog-computer analysis [34]. Jayawant and Kaplan [35, 37] have achieved the stable suspension without using mechanical damper (e.g. oil-dashpot damper). They used the saturable reactor control which is connected in series with the main electromagnet. This method was originally proposed by Knobel [40]. However, the dynamic behavior of the system was not fully investigated at that time.

For the application of the tuned suspension device to inertial guidance gyros, the suspended element is usually floated in a suitable fluid [13, 16-19, 42, 64-67], which serves also as a damping element, and thus the suspension problem is considerably reduced. Hence, there has been very little attention given to the dynamic instability. On the other hand, the existence of self-sustained oscillations was recognized as early as 1954 from experimental results reported by Gilinson, Jr., et al. [18].

Thus, on the analytical side, there appears to be very little work done on the dynamic stability and the quantitative aspects of dynamic behavior of the system. The dynamic analysis is complicated because the equations of motion of the electromechanical system are described by nonlinear differential equations. Parente [47] has analyzed the differential equations by the method of functional analysis. He has obtained an analytical criterion for static stability, assuming no leakage flux of the electromagnet. He has also discussed the possibility of the dynamic instability, but the analysis is not suitable for numerical computation. Kaplan [38] has given an approximate representation of the system equations which is suitable for digital computation. He has shown that the dynamic

behavior calculated from the approximate representation of the system agrees well with that obtained in experimental models. However, he does not discuss the concrete criterion for the elimination of static and dynamic instability. Recently he has analyzed a mechanical method for dynamic stabilization [39]. Button and Murgatroyd have proposed a circuit for achieving dynamic stability [10].

1.3 Description of the Contents

This thesis is devoted to the study of the performance of a magnetic suspension device which uses tuned LCR circuits. The problem is limited to suspension along single axis only thus avoiding the complications that accompany suspension with respect to all direction. However, the principal features of single axis suspension are a great aid to the understanding of multiaxis designs and more complex applications [17].

The thesis consists of six chapters, including this introductory chapter. Chapter 2 to 5 are concerned with both static and dynamic operation of the single-axis suspension device which suspends a body by an electromagnet against the gravitational force. Chapter 6 deals with the push-pull type suspension device which supports a body by two electromagnets working in push-pull. Complementary remarks are provided in three appendices to the text.

Chapter 2 deals with an ideal single-axis (vertical axis) suspension device with emphasis on its dynamic characteristics. To discuss the basic characteristics of the device, the leakage flux and the magnetic saturation are neglected. An oil-dashpot damper is used for this ideal model. The damping force generated by the dashpot is proportional to the velocity of the suspended body. Fundamental equations which describe the electro-mechanical system are derived. The system is described by two differential equations of the second order; one is the equation dealing with the magnetic flux of the electromagnet connected in series with the capacitor and the alternating voltage source, and the other is the equation of motion of the body to be suspended.

The system is assumed to operate in a state of near equilibrium. The magnetic flux is therefore assumed to take the form of sinusoidal oscillation of which the amplitude and phase angle vary slowly with time. The suspended body is also assumed to move slowly. Under these assumptions, the method of averaging is applied to the fundamental equations which are nonautonomous and an autonomous system of the equations is derived. The gap-distance characteristic of the device is obtained from the state of equilibrium of this autonomous system. By analogy of the restorative action of the device to that of a spring,

the equivalent spring constant of the device is defined, and its characteristic is obtained. The stability of the states of equilibrium is discussed by solving variational equations which characterize small deviations from the states of equilibrium. The variational equations are linear differential equations with constant coefficients. Therefore, the stability conditions are obtained by making use of the Routh-Hurwitz criterion. Two conditions for stability are obtained. One is identical with the condition that the equivalent spring constant be positive; therefore, this is the condition for static stability. The other is the condition for dynamic stability. To verify the theoretical analysis, an analog-computer analysis is carried out for the fundamental equations of the nonautonomous type, and the solution shows a satisfactory agreement with the theoretical result. It is shown that, in the dynamically unstable region, quasi-periodic oscillations occur. The suspended body oscillates slowly and periodically in the steady state, and the amplitude and phase of the magnetic flux also vary accordingly.

In Chapter 3, the effect of leakage flux on the stability of the single-axis suspension device is studied. The fundamental equations are derived and analyzed by the same method as mentioned above. The static and dynamic stability conditions which take into account the effect of leakage flux are derived. By numerical examples the effect of leakage flux on these stability conditions is considered. The theoretical analysis is confirmed by experiments.

Chapter 4 deals with quasi-periodic oscillations which occur in the nonautonomous system in which the damping of the mechanical or electrical system is insufficient. It is shown that a stable limit cycle exists in the autonomous system corresponding to the occurrence of the quasi-periodic oscillation. The amplitude and frequency of the self-excited oscillation correlated with the limit cycle are determined by using the method of harmonic balance. The numerical analysis is carried out for the same parameters as those of the experimental model. The theoretical results are compared with the solutions of the nonautonomous equations directly obtained from digital computer and experimental results.

In Chapter 5, the effect of magnetic saturation is studied. The saturation curve of the core of the electromagnet is assumed to be a cubic function of the magnetic flux. The fundamental equations are derived and analyzed by the same method as that mentioned in Chapter 2. It is shown that quasi-periodic oscillations may occur in the system due to the magnetic saturation, the mechanism of the occurrence being different from that discussed in Chapter 4. The occurrence of the oscillation is confirmed by experiments.

Chapter 6 deals with the performance of the push-pull type suspension device. Two

electromagnets are used to support a body. The body can hence move only between the two electromagnets. The characteristic of each electromagnet is assumed to be equal. The leakage flux is taken into account, but the effects of magnetic saturation are neglected. The system is described by three second-order differential equations; two of them are the equations for the magnetic fluxes of the electromagnets, and the third is the equation of motion of the body to be supported. These equations are, as we have done in Chapter 2, analyzed by using the method of averaging. The steady-state solution and the formula for the equivalent spring constant are obtained. The stability of the steady-state solution is investigated. The basic characteristics of this type of suspension device are confirmed by experiments.

As has been mentioned earlier, three appendices are annexed to the text. Appendix I shows the transfer-function analysis of the system which is obtained from linearization of the autonomous system derived in Chapter 2. The time solution of the suspended body motion is obtained by using the Laplace-transform method. Appendix II indicates the procedure for determination of the reluctances in the magnetic circuit of the experimental model used in Chapters 3 and 4. Appendix III expresses the steady-state characteristics of the suspension device derived in Chapter 3 using the device parameters.

CHAPTER 2

VERTICAL-AXIS SUSPENSION DEVICE HAVING ONE ELECTROMAGNET

2.1 Introduction

In this chapter, we consider the performance of an ideal model to obtain a physical concept and a first approximation to a theory of the self-stabilizing suspension device. The schematic diagram of the ideal model is shown in Fig. 2.1. The suspended body is considered to move along the vertical axis only. In this model, we assume that (1) all the flux produced by the current passes through the suspended body of ferromagnetic material, (2) winding resistance is negligible, (3) the magnetic material is lossless, so that it has no influence on effective resistance, (4) the permeability of the magnetic material is infinite, (5) damping force generated by the dashpot is linearly proportional to velocity, (6) magnetic saturation is negligible. The gap in the suspension is very short compared with the cross-sectional dimensions, being usually a few millimeters, so that fringing effect generally is negligible within the accuracy with which computation can predict performance. At the relatively low supply frequency, around 400 hertz, the influence of magnetic materials such as eddy currents and hysteresis can be made negligible by use of high-permeability, low-loss, thinly laminated ferromagnetic material or by use of ferrites.

As mentioned in the preceding chapter, this suspension device is not always dynamically stable, although it is statically stable; the suspended body may begin to oscillate with negative damping [34-36]. However, this kind of instability has not been fully investigated because of the difficulty of the dynamic analysis of nonlinear differential equations which describe the system.

In this chapter, the dynamic analysis of the equations is carried out by using the method of averaging [22]. First, the fundamental equations which describe the system are derived for this ideal model. Then, these equations are transformed into an autonomous system by using the method of averaging under the assumption that the system is operating in a state of near equilibrium. The gap distance characteristic of the device is obtained from the states of equilibrium of this autonomous system. By analogy of the restorative

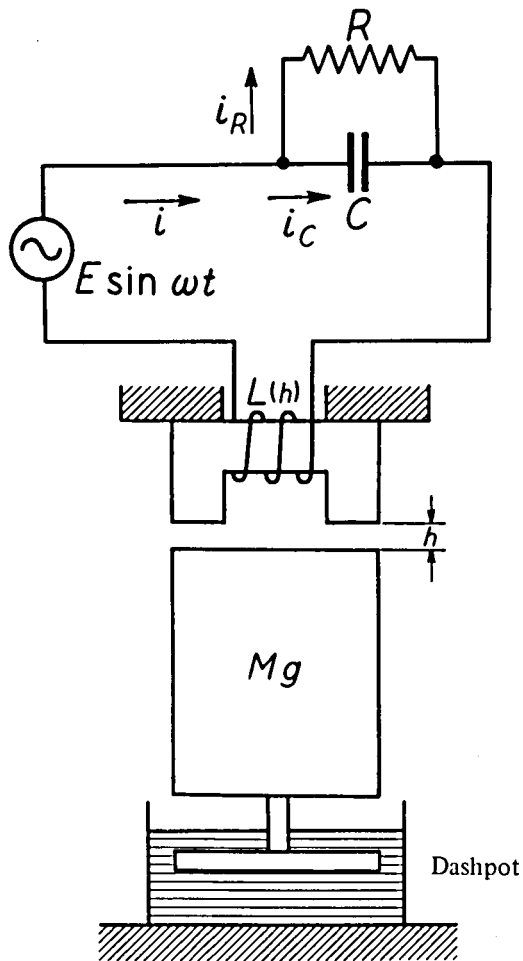


Fig. 2.1 Magnetic suspension device using a tuned LCR circuit (ideal model).

action of the device to that of a spring, the equivalent spring constant of the device is defined, and its characteristic is also obtained. The stability of the states of equilibrium is discussed by making use of the Routh-Hurwitz criterion for variational equations which characterize small deviations from the states of equilibrium. Two conditions for stability are obtained. One is identical with the condition that the equivalent spring constant is positive, so that it is a condition for static stability. The other is a condition for dynamic stability. To verify the theoretical analysis, an analog-computer analysis is carried out for the fundamental equations. It is confirmed that in the dynamically unstable region quasi-periodic oscillations occur: The suspended body oscillates slowly and periodically in the steady state, and the amplitude and phase of the magnetic flux also vary slowly.

2.2 Fundamental Equations

Following the notations in Fig. 2.1, the circuit equations may be written as follows:

$$\begin{aligned} n \frac{d\phi}{dt} + Ri_R &= E \sin \omega t \\ Ri_R &= \frac{1}{C} \int i_C dt \\ i &= i_R + i_C \end{aligned} \quad (2.1)$$

where ϕ is the magnetic flux linking the winding, and n is the number of turns of the coil wound around the core of the electromagnet. For the magnetic circuit in Fig. 2.1, an analogous electrical circuit is shown in Fig. 2.2, in which $\mathcal{R}$ is the reluctance of the magnetic path. Since we neglect fringing in the gaps, leakage flux, and reluctance of the cores, the reluctance $\mathcal{R}$ is given by

$$\mathcal{R} = \frac{2h}{\mu_0 A} \quad (2.2)$$

where h is the air-gap distance between the electromagnet and the suspended body, A is the cross section of the air gap and μ_0 is the permeability of the air gap. The equation for the magnetic circuit illustrated in Fig. 2.2 is given by

$$ni = \mathcal{R}\phi \quad (2.3)$$

The dynamic behavior of the suspended body is represented by

$$M \frac{d^2 h}{dt^2} + k_m \frac{dh}{dt} + F_m = Mg + F_d \quad (2.4)$$

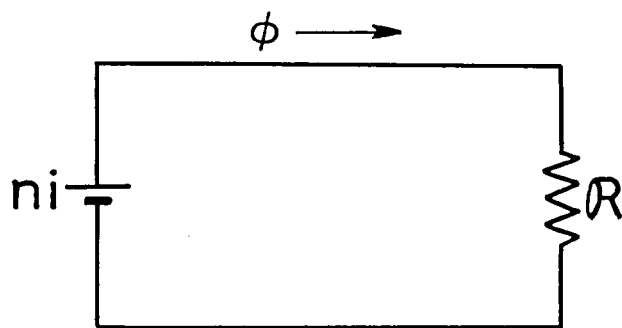


Fig. 2.2 Electrical equivalent of the magnetic circuit in Fig. 2.1.

where M is the mass of the suspended body, k_m is the coefficient of the viscous mechanical damping generated by the dashpot, F_m is the magnetic attractive force, F_d is the external disturbance force exerted on the suspended body, and g is the gravitational acceleration. Magnetic attractive force F_m in the vertical direction is given by [15]

$$F_m = \frac{1}{2} \phi^2 \frac{\partial R}{\partial h} = \frac{1}{\mu_0 A} \phi^2 \quad (2.5)$$

In order to take frequency variation into account, we put $\omega = \nu \omega_0$, where ω_0 is a fixed base frequency and ν is a variable parameter. Furthermore, we introduce nondimensional variables u and v , defined by

$$\phi = \Phi_n u \quad h = H_n v \quad (2.6)$$

where Φ_n and H_n are appropriate base quantities of the flux and the gap distance, respectively. For convenience of computation, we define H_n and Φ_n by

$$H_n = \frac{\mu_0 A n^2 \omega_0^2 C}{2} \quad \Phi_n^2 = \mu_0 A \omega_0^2 M H_n \quad (2.7)$$

We retain the variables ϕ and h , and eliminate i , i_R , i_C and F_m in Eqs. (2.1) and (2.4), respectively. Thereupon by using the nondimensional quantities defined by Eqs. (2.6), we obtain

$$\begin{aligned} \frac{d^2 u}{d\tau^2} + k_1 \frac{du}{d\tau} + uv &= B \cos \nu \tau \\ \frac{d^2 v}{d\tau^2} + k_2 \frac{dv}{d\tau} + u^2 &= B_0 \end{aligned} \quad (2.8)$$

where

$$\begin{aligned} \tau &= \omega_0 t - \frac{1}{\nu} \tan^{-1} \frac{k_1}{\nu} & k_1 &= \frac{1}{\omega_0 C R} & k_2 &= \frac{k_m}{\omega_0 M} \\ B &= \frac{E}{n \omega_0 \Phi_n} \sqrt{\nu^2 + k_1^2} & B_0 &= \frac{Mg + F_d}{\omega_0^2 M H_n} \end{aligned}$$

Equations (2.8) are the fundamental equations of the suspension device to be analyzed in the following sections. These equations are nonautonomous since the time τ appears explicitly in them.

2.3 Analysis of the Fundamental Equations

2.3.1 Derivation of an Autonomous System by Using the Method of Averaging

Equations (2.8) are both nonlinear differential equations having nonlinear terms uv and u^2 . There exist in general no methods capable of yielding exact solutions of such

equations. Hence we consider an approximate solution of the system (2.8) under the condition that the system is operating at a point of near equilibrium, where the time average of the nondimensional magnetic force u^2 is approximately balanced with the nondimensional mechanical load force B_0 .

In order to consider the movement of the suspended body in an interval of one period of the external force, we rewrite Eqs. (2.8) as

$$\begin{aligned}\frac{du}{d\tau_n} &= \dot{u} \\ \frac{d\dot{u}}{d\tau_n} &= \frac{B_0}{\nu^2} \cos \tau_n - \frac{k_1}{\nu} \dot{u} - uv_n\end{aligned}\quad (2.9)$$

and

$$\frac{d^2 v_n}{d\tau_n^2} + \frac{k_2}{\nu} \frac{dv_n}{d\tau_n} + \frac{u^2}{\nu^4} = \frac{B_0}{\nu^4} \quad (2.10)$$

where

$$v_n = \frac{v}{\nu^2} \quad \tau_n = \nu\tau \quad (2.10a)$$

In Eqs. (2.9) and (2.10), we introduce new variables $x(\tau_n)$ and $y(\tau_n)$ defined by

$$\begin{aligned}x(\tau_n) &= u(\tau_n) \sin \tau_n + \dot{u}(\tau_n) \cos \tau_n \\ y(\tau_n) &= u(\tau_n) \cos \tau_n - \dot{u}(\tau_n) \sin \tau_n\end{aligned}\quad (2.11)$$

so that

$$\begin{aligned}u(\tau_n) &= x(\tau_n) \sin \tau_n + y(\tau_n) \cos \tau_n \\ \dot{u}(\tau_n) &= x(\tau_n) \cos \tau_n - y(\tau_n) \sin \tau_n\end{aligned}\quad (2.12)$$

Substituting Eqs. (2.12) into (2.9) yields

$$\begin{aligned}\frac{dx}{d\tau_n} \sin \tau_n + \frac{dy}{d\tau_n} \cos \tau_n &= 0 \\ \frac{dx}{d\tau_n} \cos \tau_n - \frac{dy}{d\tau_n} \sin \tau_n &= f_1(x, y, v_n, \tau_n)\end{aligned}\quad (2.13)$$

where

$$\begin{aligned}f_1(x, y, v_n, \tau_n) &= \frac{1}{\nu^2} [\nu^2 (1 - v_n) x + \nu k_1 y] \sin \tau_n \\ &\quad + \frac{1}{\nu^2} [-\nu k_1 x + \nu^2 (1 - v_n) y + B] \cos \tau_n\end{aligned}$$

Solving Eqs. (2.13) for the derivatives $dx/d\tau_n$ and $dy/d\tau_n$ gives us

$$\begin{aligned}\frac{dx}{d\tau_n} &= f_1(x, y, v_n, \tau_n) \cos \tau_n \\ \frac{dy}{d\tau_n} &= -f_1(x, y, v_n, \tau_n) \sin \tau_n\end{aligned}\quad (2.14)$$

Thus, Eqs. (2.9) are transformed into (2.14) by the use of the new variables $x(\tau_n)$ and $y(\tau_n)$.

Furthermore, we introduce in Eq. (2.10) new variable w_n defined by

$$w_n = \frac{\nu}{k_2} \cdot \frac{dv_n}{d\tau_n} + v_n \quad (2.15)$$

Inserting Eqs. (2.12) and (2.15) into (2.10) yields

$$\begin{aligned} \frac{dv_n}{d\tau_n} &= \frac{k_2}{\nu} (w_n - v_n) \\ \frac{dw_n}{d\tau_n} &= \epsilon f_2(x, y, \tau_n) \end{aligned} \quad (2.16)$$

where

$$\begin{aligned} f_2(x, y, \tau_n) &= 1 - \frac{1}{2B_0} [r^2 + (y^2 - x^2) \cos 2\tau_n + 2xy \sin 2\tau_n] \\ r^2 &= x^2 + y^2 \\ \epsilon &= \frac{B_0}{\nu^3 k_2} \end{aligned} \quad (2.16a)$$

Since the first equation of (2.16) is a linear differential equation, its particular solution is given by

$$\begin{aligned} v_n &= \frac{k_2}{\nu} \exp\left(-\frac{k_2}{\nu} \tau_n\right) \int w_n \exp\left(\frac{k_2}{\nu} \tau_n\right) d\tau_n \\ &= w_n - \exp\left(-\frac{k_2}{\nu} \tau_n\right) \int \frac{dw_n}{d\tau_n} \exp\left(\frac{k_2}{\nu} \tau_n\right) d\tau_n \end{aligned} \quad (2.17)$$

By using the first equation of (2.16), the derivative of this particular solution is given by

$$\frac{dv_n}{d\tau_n} = \frac{k_2}{\nu} \exp\left(-\frac{k_2}{\nu} \tau_n\right) \int \frac{dw_n}{d\tau_n} \exp\left(\frac{k_2}{\nu} \tau_n\right) d\tau_n \quad (2.18)$$

Letting ϵ_m denote the maximum value of $dw_n/d\tau_n$ during one period 2π of the external force, that is,

$$\left| \frac{dw_n}{d\tau_n} \right| \leq \epsilon_m$$

then we obtain from Eq. (2.18)

$$\left| \frac{dv_n}{d\tau_n} \right| \leq \epsilon_m \quad (2.19)$$

This shows that the maximum value of $dv_n/d\tau_n$ during one period of the external force is the same order as that of $dw_n/d\tau_n$. If ϵ in the second equation of (2.16) is sufficiently small, the derivative $dw_n/d\tau_n$ becomes sufficiently small, and hence ϵ_m also becomes sufficiently small. Therefore we may conclude that if ϵ is sufficiently small, both the

maximum values of $dv_n/d\tau_n$ and $dw_n/d\tau_n$ during one period of the external force become sufficiently small. Consequently, v_n and w_n in Eqs. (2.16) vary slowly with time τ_n . As can be seen from the definition of ϵ given by Eqs. (2.16a), this states that when the supply frequency ν is high or the viscous damping coefficient k_2 is large, or the mechanical load force B_0 is weak, the suspended body moves slowly during one period 2π of the external force. From the physical point of view, this is a plausible conclusion.

After the following consideration we may expect that when v_n varies slowly with time τ_n , x and y in Eqs. (2.14) also vary slowly with time τ_n : If v_n in Eqs. (2.9) is a constant, then Eqs. (2.9) become linear differential equations with constant coefficients. The steady-state solution for the variables u and $\dot{u}$ takes the form of Eqs. (2.12). But in this case the components x and y become constants of which values are determined by the value of v_n . However, if v_n is not a constant but a function which varies slowly with time τ_n , x and y may also vary slowly with time τ_n near the steady state. As will be shown in Sec. 2.5, we can verify this consideration by an analog-computer analysis of the system (2.8). Experimental observations also show that this consideration is legitimate (e.g. see Figs. 3.13 through 3.15, and 4.7).

It may therefore be considered that $x(\tau_n)$, $y(\tau_n)$, $v_n(\tau_n)$, and $w_n(\tau_n)$ on the right-hand sides of Eqs. (2.14) and (2.16) remain approximately constant during one period 2π . Thereupon, by using the method of averaging [27, pp. 24-28], we can transform Eqs. (2.14) and (2.16) into an autonomous system:

$$\begin{aligned}\frac{dx}{d\tau_n} &= \frac{1}{2\pi} \int_0^{2\pi} f_1(x, y, v_n, \tau_n) \cos \tau_n d\tau_n \\ \frac{dy}{d\tau_n} &= -\frac{1}{2\pi} \int_0^{2\pi} f_1(x, y, v_n, \tau_n) \sin \tau_n d\tau_n \\ \frac{dv_n}{d\tau_n} &= \frac{k_2}{2\pi\nu} \int_0^{2\pi} (w_n - v_n) d\tau_n \\ \frac{dw_n}{d\tau_n} &= \frac{\epsilon}{2\pi} \int_0^{2\pi} f_2(x, y, \tau_n) d\tau_n\end{aligned}\tag{2.20}$$

The integration is to be performed with respect to the explicitly appearing τ_n in the integrants. Performing the integration gives us

$$\begin{aligned}
\frac{dx}{d\tau_n} &= \frac{1}{2\nu^2} [-\nu k_1 x + \nu^2 (1 - v_n) y + B] \\
\frac{dy}{d\tau_n} &= -\frac{1}{2\nu^2} [\nu^2 (1 - v_n) x + \nu k_1 y] \\
\frac{dv_n}{d\tau_n} &= \frac{k_2}{\nu} (w_n - v_n) \\
\frac{dw_n}{d\tau_n} &= \epsilon \left(1 - \frac{r^2}{2B_0}\right)
\end{aligned} \tag{2.21}$$

If these equations are compared with the exact equations (2.14) and (2.16), it is seen that the equations of the first approximation are obtained from the exact equations by averaging the latter equations over the period 2π , thus retaining only the first terms and eliminating all the other terms of the Fourier series in the right sides of Eqs. (2.14) and (2.16). Using Eqs. (2.10a), (2.15), and (2.16a), we rewrite Eqs. (2.21) as follows:

$$\begin{aligned}
\frac{dx}{d\tau} &= \frac{1}{2\nu} [-\nu k_1 x + (\nu^2 - v) y + B] \equiv X(x, y, v) \\
\frac{dy}{d\tau} &= -\frac{1}{2\nu} [(\nu^2 - v) x + \nu k_1 y] \equiv Y(x, y, v) \\
\frac{dv}{d\tau} &= \dot{v} \equiv V(x, y, v) \\
\frac{d\dot{v}}{d\tau} &= B_0 - k_2 \dot{v} - \frac{r^2}{2} \equiv \dot{V}(x, y, \dot{v})
\end{aligned} \tag{2.22}$$

In contrast to Eqs. (2.8), Eqs. (2.22) are autonomous since the time τ does not appear in the equations. The results which are obtained from Eqs. (2.22) may be considered to be legitimate as far as we deal with the dynamic behavior of the system (2.8) near the equilibrium state.

2.3.2 Steady-State Solutions

We consider the steady state in which $x(\tau)$, $y(\tau)$, $v(\tau)$, and $\dot{v}(\tau)$ in Eqs. (2.22) are constant, so that

$$\frac{dx}{d\tau} = 0 \quad \frac{dy}{d\tau} = 0 \quad \frac{dv}{d\tau} = 0 \quad \frac{d\dot{v}}{d\tau} = 0 \tag{2.23}$$

Substituting these conditions into Eqs. (2.22) leads to

$$\begin{aligned}
 \nu k_1 x_0 - (\nu^2 - v_0) y_0 &= B \\
 (\nu^2 - v_0) x_0 + \nu k_1 y_0 &= 0 \\
 \dot{v}_0 &= 0 \\
 B_0 - k_2 \dot{v}_0 - \frac{1}{2} r_0^2 &= 0
 \end{aligned} \tag{2.24}$$

with

$$r_0^2 = x_0^2 + y_0^2$$

Equations (2.24) determine the components x_0 , y_0 , v_0 , and $\dot{v}_0$ of the steady-state solution.

From the first two equations of (2.24), we obtain

$$\begin{aligned}
 x_0 &= \frac{\nu k_1}{(\nu^2 - v_0)^2 + \nu^2 k_1^2} B \\
 y_0 &= -\frac{\nu^2 - v_0}{(\nu^2 - v_0)^2 + \nu^2 k_1^2} B
 \end{aligned} \tag{2.25}$$

Equations (2.25) and the fourth equation of (2.24) lead to

$$r_0^2 = \frac{B^2}{(\nu^2 - v_0)^2 + \nu^2 k_1^2} = 2B_0 \tag{2.26}$$

Solving the above equation for v_0 gives us

$$v_0 = \nu^2 \mp \sqrt{\frac{B^2}{2B_0} - \nu^2 k_1^2} \tag{2.27}$$

Two equilibrium gap-distances given by Eq. (2.27) correspond to h_1 and h_2 in Fig. 1.2. For a statically stable equilibrium gap-distance h_1 , the negative sign should be taken in Eq. (2.27). Physically speaking, v_0 should take a real-valued positive value and therefore the condition for the existence of the statically stable equilibrium point is represented by

$$\nu^2 (\nu^2 + k_1^2) > \frac{B^2}{2B_0} > \nu^2 k_1^2 \tag{2.28}$$

Hence the amplitude of the supply voltage B should be chosen so as to satisfy the above inequality. If B is larger than the above-stated upper bound, the suspended body adheres to the electromagnet. If B is smaller than the lower bound, the suspended body drops downward.

2.3.3 Equivalent Spring Constant

Near the equilibrium point, the slope of the curve of the time average force $\bar{F}_m$ versus h , i.e.,

$$S = \left(\frac{\partial \bar{F}_m}{\partial h} \right)_0 \tag{2.29}$$

is called the equivalent spring constant of the device (by analogy of the restorative action to that of a spring), where $(\frac{\partial \bar{F}_m}{\partial h})_0$ denotes the value of $\frac{\partial \bar{F}_m}{\partial h}$ at the equilibrium point. We define here the nondimensional equivalent spring constant by

$$S_0 = (\frac{\partial \bar{u}^2}{\partial v})_0 = \frac{1}{2} (\frac{\partial r^2}{\partial v})_v = v_0 \quad (2.30)$$

where $\bar{u}^2$ denotes the time average of u^2 . From Eqs. (2.5) through (2.7), we have

$$\frac{\partial \bar{F}_m}{\partial h} = \frac{\Phi_n^2}{\mu_0 A H_n} \cdot \frac{\partial \bar{u}^2}{\partial v} = \omega_0^2 M \frac{\partial \bar{u}^2}{\partial v} \quad (2.31)$$

$$\therefore \frac{\Phi_n^2}{\mu_0 A H_n} = \omega_0^2 M \quad (2.32)$$

Hence, the relationship between S and S_0 is

$$S = \omega_0^2 M S_0 \quad (2.33)$$

Equations (2.26) and (2.30) give the following equivalent spring constant:

$$S_0 = \frac{\nu^2 - v_0}{[(\nu^2 - v_0)^2 + \nu^2 k_1^2]^2} B^2 \quad (2.34)$$

If the device has achieved a statically stable suspension, then the magnetic attractive force must, on the average, act so as to restore the body to the equilibrium point. Thus a necessary condition for the statically stable suspension is that the equivalent spring constant S_0 be positive:

$$S_0 > 0 \quad \text{or} \quad \nu^2 - v_0 > 0 \quad (2.35)$$

As will be shown in the following section, the static stability condition (2.35) can also be derived from the linearized variational equations of Eqs. (2.22). Elimination of v_0 in Eqs. (2.34) by using (2.27) gives

$$S_0 = \pm \frac{4B_0^2}{B^2} \sqrt{\frac{B^2}{2B_0} - \nu^2 k_1^2} \quad (2.36)$$

The positive sign in Eq. (2.36) is taken if the negative sign is taken in Eq. (2.27) and vice versa. According to Eq. (2.36), the smaller of the two equilibrium points derived from Eq. (2.27) is stable, as physically suggested by the curves in Fig. 1.2.

2.3.4 Stability Investigation

In the preceding section, the condition for static stability is derived from the physical point of view which states that the equivalent spring constant should be positive. The

condition is, however, the only necessary condition for the stable suspension. As suggested by many authors [18, 34-37, 47], there is a possibility that the suspended body may begin to oscillate with negative damping, although the equivalent spring constant is positive.

In this section, we seek the necessary and sufficient conditions for stability of the equilibrium states determined by Eqs. (2.25) and (2.27). The steady-state solution, i.e., the equilibrium state of the system (2.22) is correlated with the singular point $(x_0, y_0, v_0, \dot{v}_0)$ in the $x, y, v, \dot{v}$ phase space. If the singular point is stable, the corresponding steady-state solution is also stable; if not, it is unstable. The stability of the singular point is studied by the behavior of integral curves near that singular point. To this end we consider small variations ξ, η, ζ , and ψ , respectively, from the steady-state values $x_0, y_0, v_0, \dot{v}_0$:

$$\begin{aligned} x &= x_0 + \xi & y &= y_0 + \eta \\ v &= v_0 + \zeta & \dot{v} &= \dot{v}_0 + \psi \end{aligned} \quad (2.37)$$

If ξ, η, ζ , and ψ approach zero with increase of time τ , the steady-state solution is stable. Substituting Eqs. (2.37) into (2.22), we obtain

$$\begin{aligned} \frac{d\xi}{d\tau} &= a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14}\psi \\ \frac{d\eta}{d\tau} &= a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24}\psi \\ \frac{d\zeta}{d\tau} &= a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34}\psi \\ \frac{d\psi}{d\tau} &= a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44}\psi \end{aligned} \quad (2.38)$$

with

$$\begin{aligned} a_{11} &= \left(\frac{\partial X}{\partial x} \right)_0 = -\frac{1}{2} k_1 \\ a_{12} &= \left(\frac{\partial X}{\partial y} \right)_0 = \frac{1}{2\nu} (\nu^2 - v_0^2) \\ a_{13} &= \left(\frac{\partial X}{\partial v} \right)_0 = -\frac{1}{2\nu} y_0 \\ a_{14} &= \left(\frac{\partial X}{\partial \dot{v}} \right)_0 = 0 \\ a_{21} &= \left(\frac{\partial Y}{\partial x} \right)_0 = -\frac{1}{2\nu} (\nu^2 - v_0^2) \end{aligned} \quad (2.39)$$

$$\begin{aligned}
a_{22} &= \left(\frac{\partial Y}{\partial y} \right)_0 = -\frac{1}{2} k_1 \\
a_{23} &= \left(\frac{\partial Y}{\partial v} \right)_0 = \frac{1}{2\nu} x_0 \\
a_{24} &= \left(\frac{\partial Y}{\partial \dot{v}} \right)_0 = 0 \\
a_{31} &= \left(\frac{\partial V}{\partial x} \right)_0 = 0 \\
a_{32} &= \left(\frac{\partial V}{\partial y} \right)_0 = 0 \\
a_{33} &= \left(\frac{\partial V}{\partial v} \right)_0 = 0 \\
a_{34} &= \left(\frac{\partial V}{\partial \dot{v}} \right)_0 = 1 \\
a_{41} &= \left(\frac{\partial \dot{V}}{\partial x} \right)_0 = -x_0 \\
a_{42} &= \left(\frac{\partial \dot{V}}{\partial y} \right)_0 = -y_0 \\
a_{43} &= \left(\frac{\partial \dot{V}}{\partial v} \right)_0 = 0 \\
a_{44} &= \left(\frac{\partial \dot{V}}{\partial \dot{v}} \right)_0 = -k_2
\end{aligned} \tag{2.39}$$

where $\left(\frac{\partial X}{\partial x} \right)_0, \dots, \left(\frac{\partial \dot{V}}{\partial \dot{v}} \right)_0$ stand for $\frac{\partial X}{\partial x}, \dots, \frac{\partial \dot{V}}{\partial \dot{v}}$ at $x = x_0, y = y_0, v = v_0$, and $\dot{v} = \dot{v}_0$, respectively. The characteristic equation of the system (2.38) is given by

$$\begin{vmatrix}
a_{11} - \lambda & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} - \lambda & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} - \lambda & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44} - \lambda
\end{vmatrix} = 0$$

or

$$\lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \tag{2.40}$$

where

$$a_1 = -(a_{11} + a_{22} + a_{33} + a_{44})$$

$$\begin{aligned}
 a_2 = & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{14} \\ a_{41} & a_{44} \end{vmatrix} \\
 & + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{24} \\ a_{42} & a_{44} \end{vmatrix} + \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \\
 a_3 = & - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} - \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} \\
 & - \begin{vmatrix} a_{11} & a_{13} & a_{14} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} - \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} \\
 a_4 = & \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}
 \end{aligned} \tag{2.41}$$

The steady-state solution is stable, provided that the real parts of λ 's are negative. The stability conditions are given by the use of the Routh-Hurwitz criterion [51, 27, pp.71 – 74], i.e.,

$$\begin{aligned}
 a_1 > 0 \quad a_2 > 0 \quad a_3 > 0 \quad a_4 > 0 \\
 a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0
 \end{aligned} \tag{2.42}$$

Substituting Eqs. (2.39) into (2.41) gives us

$$\begin{aligned}
 a_1 &= k_1 + k_2 \\
 a_2 &= k_1 k_2 + \frac{1}{4\nu^2} [(\nu^2 - v_0)^2 + \nu^2 k_1^2] \\
 a_3 &= \frac{k_2}{4\nu^2} [(\nu^2 - v_0)^2 + \nu^2 k_1^2] \\
 a_4 &= \frac{r_0^2}{4\nu^2} (\nu^2 - v_0)
 \end{aligned} \tag{2.43}$$

By virtue of Eqs. (2.43), we see that the first three conditions of (2.42) are always fulfilled. Therefore the stability conditions are reduced to

$$a_4 > 0 \quad a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0 \quad (2.44)$$

When Eqs. (2.43) are substituted into these two conditions and appropriate reorderings are made by using Eq. (2.26), we obtain

$$\nu^2 - v_0 > 0 \quad (2.45)$$

and

$$B^2 k_1 k_2 \left[(k_1 + k_2) k_2 + \frac{B^2}{8\nu^2 B_0} \right] - 4B_0^2 (k_1 + k_2)^2 (\nu^2 - v_0) > 0 \quad (2.46)$$

These are the necessary and sufficient conditions for stability of the steady-state solution. The condition (2.45) gives the same static stability condition as the inequality (2.35). By making use of Eq. (2.26), we may write this condition as

$$\frac{d(B^2/2B_0)}{dv_0} < 0 \quad (2.47)$$

Hence the vertical tangency of the characteristic curve ($B^2/2B_0$ vs. v_0 curve) results at the stability limit of (2.47). On the other hand, the condition (2.46) represents a dynamic stability condition which should be satisfied in order for the solution to be convergent. We see that the dynamic stability condition (2.46) is affected not only by the damping coefficient k_1 of the electrical system but also by the damping coefficient k_2 of the mechanical system.

We may express the stability conditions (2.45) and (2.46) in terms of the equivalent spring constant S_0 by making use of Eqs. (2.26) and (2.34):

$$\frac{k_1 k_2}{(k_1 + k_2)^2} \left[(k_1 + k_2) k_2 + \frac{B^2}{8\nu^2 B_0} \right] > S_0 > 0 \quad (2.48)$$

Inequality (2.48) indicates that the equivalent spring constant should be positive in order to keep the system stable although it may become unstable if the equivalent spring constant is too large.

2.3.5 Complementing Remarks for Dynamic Analysis

We transformed the nonautonomous system (2.8) which is described by nonlinear differential equations into the autonomous system (2.22). However, the autonomous system is also described by nonlinear differential equations. There are no general analytical

techniques for solving these equations, although a numerical solution can always be found for specific numerical values. Nevertheless much useful information can be obtained by the use of simplifying approximations. One of these approximate methods is linearization of the equations.

In the preceding section, we have used the linearization technique and obtained the variational equations (2.38) which are reduced to linear differential equations with constant coefficients. All the well-known and highly developed analytical techniques for solving this type of equations can then be used. As an example of the analysis, the transfer-function analysis of the linearized system is shown in the Appendix I.

The dynamic performance around the stable equilibrium point can be investigated either by calculating the transient response of the gap distance ξ (small increment of v) to step load variation δB_0 from the linearized system in Appendix I, or by studying the root locus of the characteristic equation (2.40).

2.4 Numerical Examples

The gap distance characteristic calculated by using Eq. (2.27) for $k_1 = 0.4$ is illustrated in Fig. 2.3. The thick lines $p = 0$ and $q = 0$ are the loci of the characteristic at the static and dynamic stability limits, respectively, and the surface of the characteristic (v_0 versus $B^2/2B_0$ and ν^2) bordered by the thick lines shows the gap distance in the stable suspension. The stability limits are given by the surfaces

$$p \equiv \nu^2 - v_0 = 0$$

$$q \equiv \frac{B^2}{2B_0} k_1 k_2 [(k_1 + k_2) k_2 + \frac{1}{4\nu^2} \cdot \frac{B^2}{2B_0}] - 2B_0 (k_1 + k_2)^2 (\nu^2 - v_0) = 0$$

As can be easily seen, the gap distance characteristic and the static stability limit are determined without the values of k_2 and B_0 by using the parameter $B^2/2B_0$. However, the dynamic stability limit is not determined without these values, and in the figure, the case for $k_2 = 2.0$ and $B_0 = 0.333$ is illustrated. The thick lines on the surface of the characteristic are projected on the $(B^2/2B_0)\nu^2$ -plane, and the stable region on this plane is represented by hatching. Reviewing this stable region, we see that it is possible to change a dynamically unstable equilibrium point to a stable equilibrium point by increasing supply frequency ν . This agrees with the results which are obtained by Jayawant and Rea from an analog-computer analysis [34].

The equivalent spring constant characteristic which corresponds to the gap distance characteristic shown in Fig. 2.3 is calculated by using Eq. (2.36) and illustrated in Fig. 2.4.

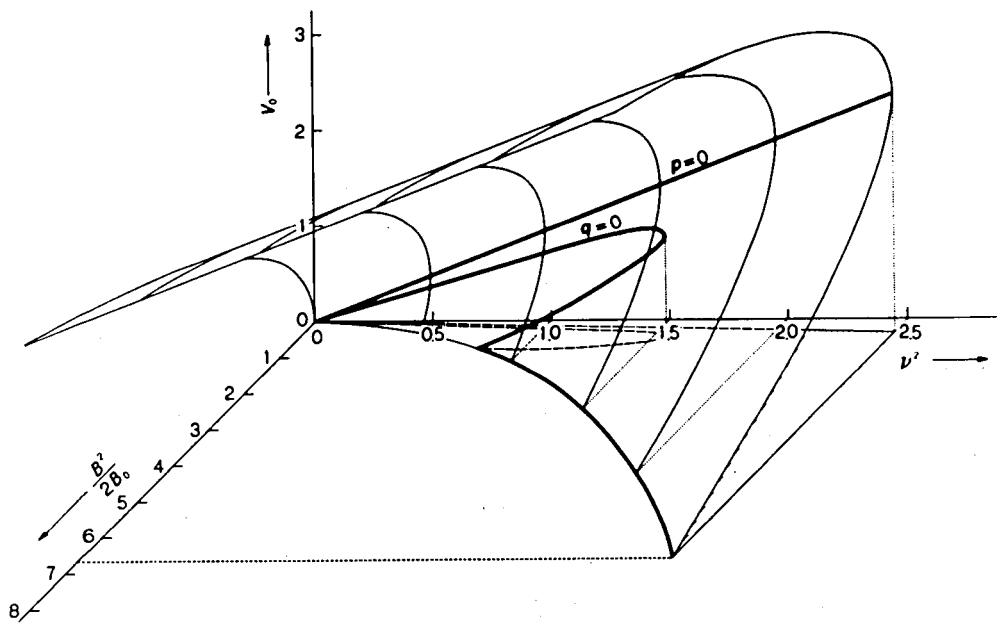


Fig. 2.3 Gap distance characteristic.

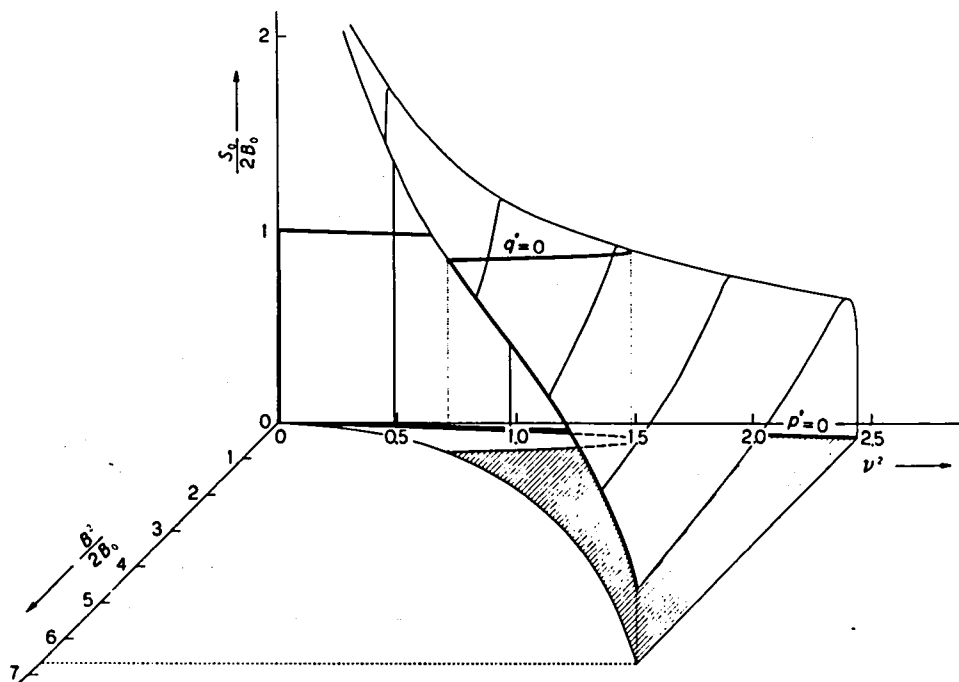


Fig. 2.4 Equivalent spring constant characteristic.

In this case the stability limits are given by

$$p' \equiv \frac{S_0}{2B_0} = 0$$

$$q' \equiv \frac{k_1 k_2}{2B_0 (k_1 + k_2)^2} [(k_1 + k_2) k_2 + \frac{1}{4\nu^2} \cdot \frac{B^2}{2B_0}] - \frac{S_0}{2B_0} = 0$$

Figure 2.5a and b shows the response characteristics for the system parameters as given by

$$\nu = 1.0 \quad k_2 = 2.0 \quad B_0 = 0.2$$

The thick lines in the figure show the stable characteristic curves. The system is unstable if $v_0 > 1$ or, equivalently, if $S_0 < 0$. The hatched area shows the dynamically unstable region where the condition (2.46) is not satisfied. As k_1 is varied continuously from 0.8 to 0 keeping v_0 equal to 0.5 by changing B , the stable equilibrium point approaches to the dynamically unstable region as indicated by the arrow.

The root loci of the characteristic equation (2.40) corresponding to the variations of the system parameters k_1 and B are shown in Fig. 2.6. At the point of stability limit where the real part of the characteristic root vanishes, we have $k_1 = 0.322$ and $B = 0.376$. The transient response curves of ξ (small variation of v) for $\delta B_0 = 0.04$ (small variation of B_0) are obtained from the linearized system of (2.22) (see Appendix I) and shown in Fig. 2.7. Curves in Figs. 2.5 through 2.7 indicate that the stability of the system worsens as k_1 which is the electrical system damping decreases although the equivalent spring constant S_0 increases.

2.5 Analog-Computer Analysis

The theoretical results obtained in the preceding sections are compared with the solutions obtained by using an analog computer. Figure 2.8 shows the block diagram of a computer setup for the solution of Eqs. (2.8). The symbols in the figure follow the conventional notation. By using this setup, we obtain the transient response curves as illustrated in Figs. 2.9 and 2.10, which are corresponding to the curves in Fig. 2.7 for $k_1 = 0.8$ and 0.6, respectively. The step input disturbance $\delta B_0 = 0.04$ is applied at time $\tau = 10\pi$. The successive points on the curves show the instants when $\tau = 2n\pi$, n being 1, 2, 3, . . . While the steady-state solution of $v(\tau)$ contains a double frequency component in addition to the DC components, the response curves in Figs. 2.9 and 2.10 agree very well with those in Fig. 2.7 as far as their DC components are concerned; in the theoretical analysis, the AC components are neglected because the right-hand sides of Eqs. (2.16) are

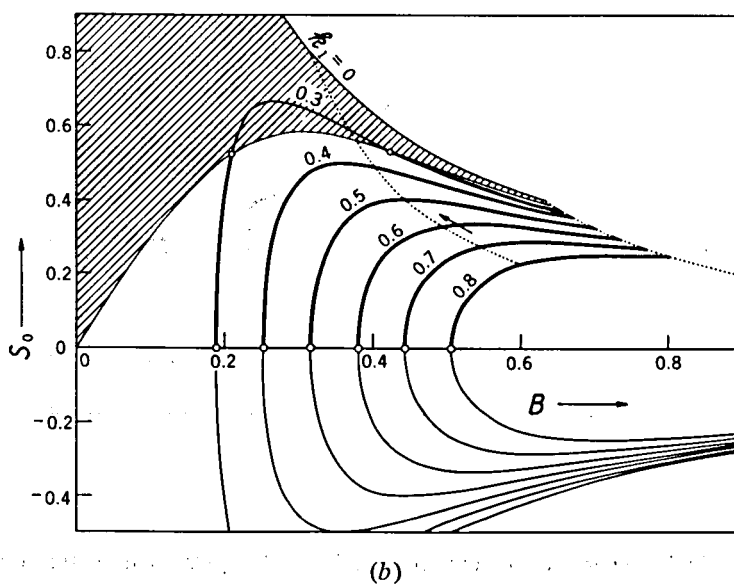
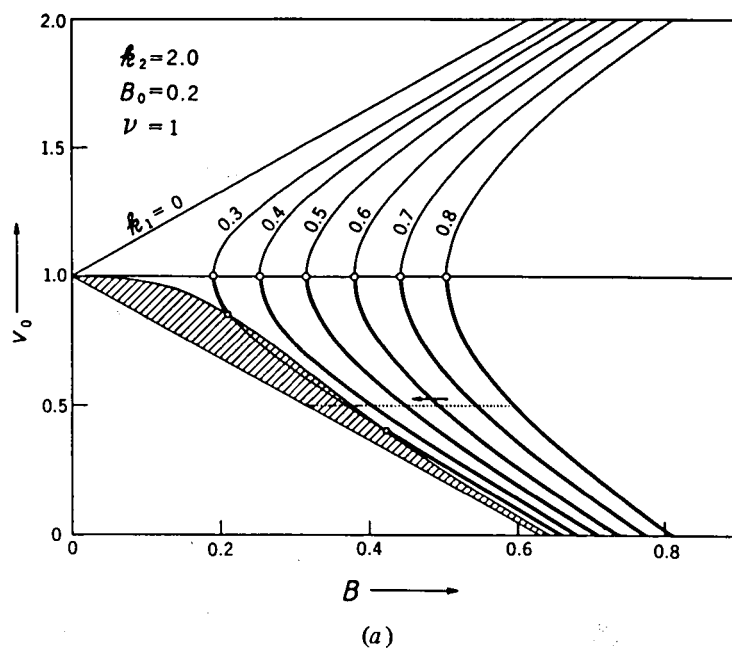


Fig. 2.5 Response characteristics. (a) Gap distance characteristic. (b) Equivalent spring constant characteristic.

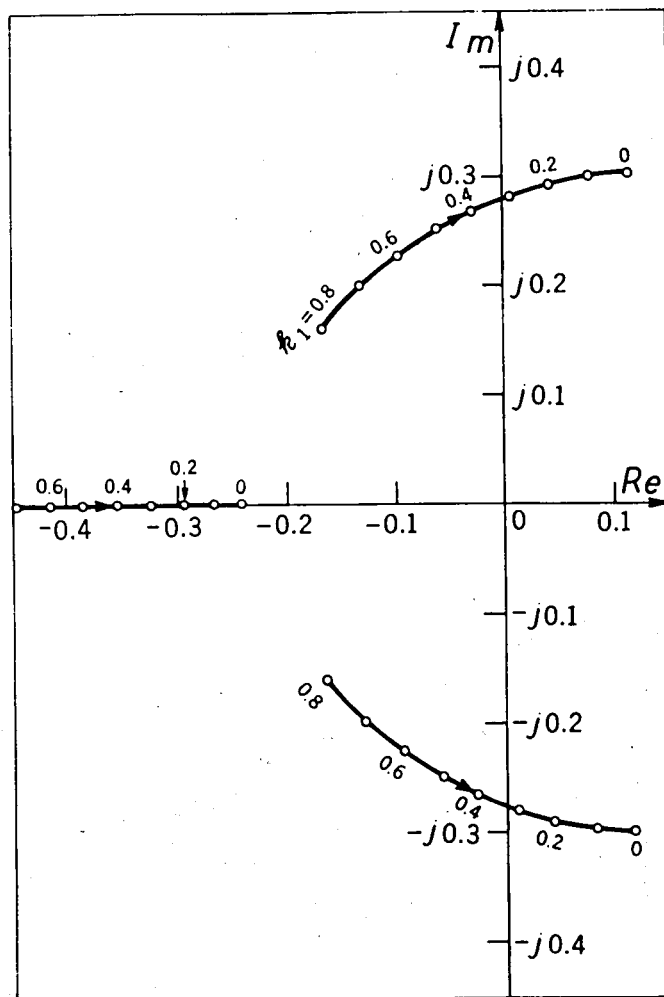


Fig. 2.6 Root loci of characteristic equation (2.40). The system parameters are the same as those for $v_0 = 0.5$ in Fig. 2.5.

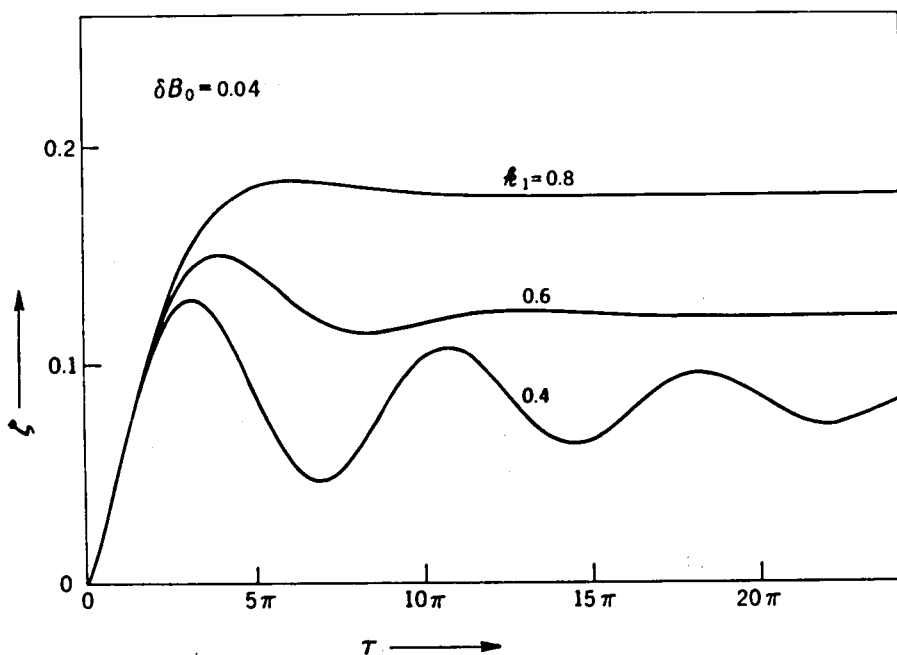


Fig. 2.7 Transient response curves for the step input disturbance $\delta B_0 = 0.04$.
The system parameters are the same as those for $v_0 = 0.5$ in Fig. 2.5.

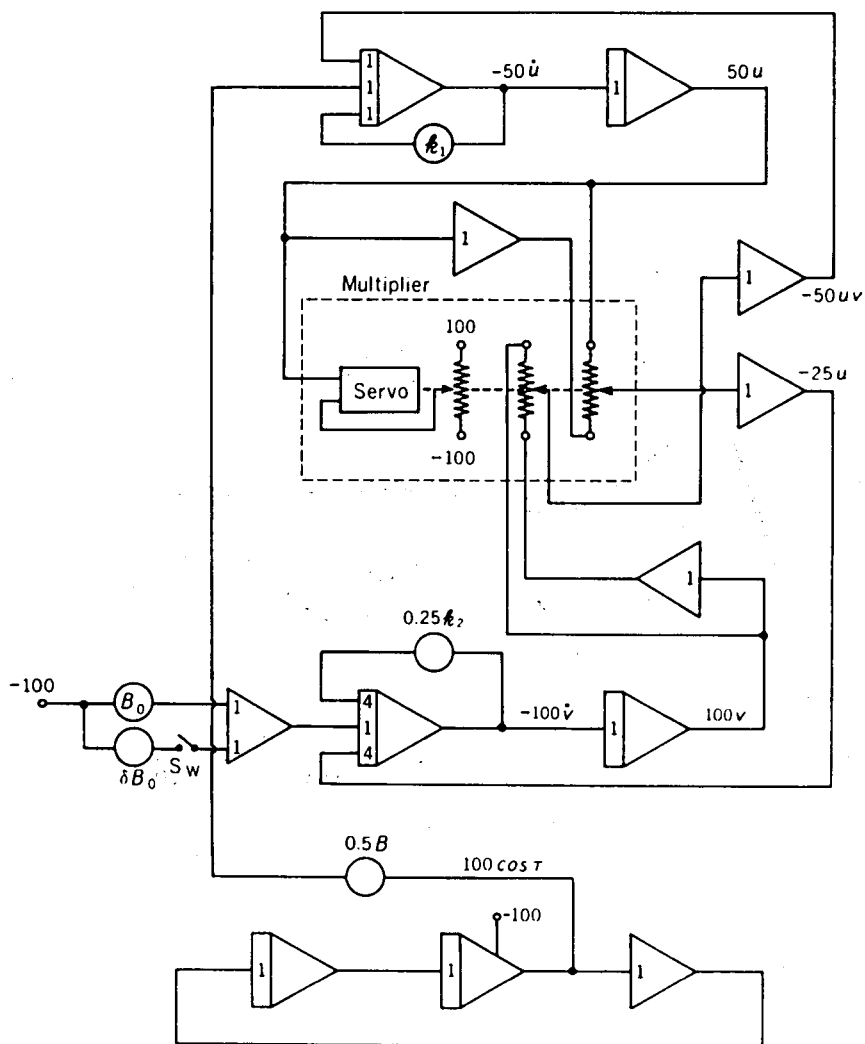


Fig. 2.8 Computer block diagram for Eqs. (2.8).

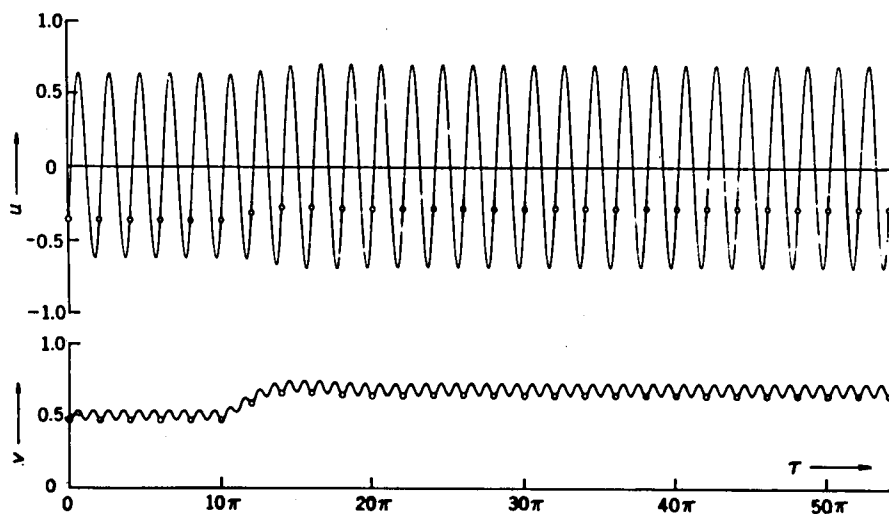


Fig. 2.9 Transient response curve obtained by analog-computer analysis corresponding to the curve for $k_1 = 0.8$ in Fig. 2.7.

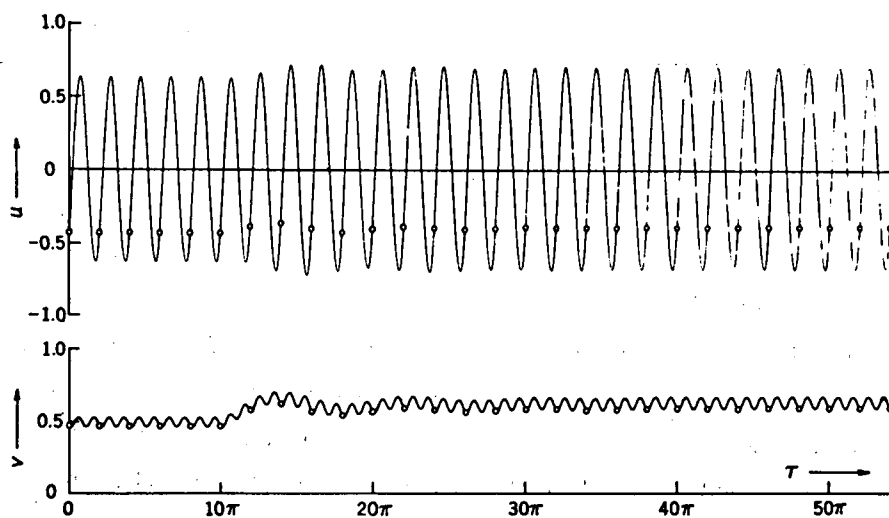


Fig. 2.10 Transient response curve obtained by analog-computer analysis corresponding to the curve for $k_1 = 0.6$ in Fig. 2.7.

averaged.

As k_1 and B are decreased keeping v_0 equal to 0.5, the steady-state solution tends to become unstable and a quasi-periodic oscillation [27, pp. 262-282] occurs when $k_1 = 0.388$ and $B = 0.400$; the theoretical stability limit values of k_1 and B as derived from the condition (2.46) are equal to 0.322 and 0.376, respectively. As the values of k_1 and B are decreased further, the amplitude of $v(\tau)$ increases although it never diverges. A sub-harmonic oscillation may also occur depending on the values of k_1 and B . The minimum value of $v(\tau)$ becomes negative and the system becomes unstable when $k_1 = 0.242$ and $B = 0.351$.* In practice, the suspended body strikes repeatedly against the electromagnet and it finally drops downward.

The waveforms of the quasi-periodic oscillation for $k_1 = 0.3$ and $B = 0.369$ are shown in Fig. 2.11.

For the case shown in Fig. 2.5, if the mechanical damping k_2 is reduced from 2.0 to 0.8, the characteristic curve for $k_1 = 0.6$ has the dynamically unstable portion. However, as mentioned in the preceding section, this portion can be stabilized by increasing the supply frequency ν . For example, with $k_1 = 0.6$, $k_2 = 0.8$, and $B_0 = 0.2$, although the system is unstable for $\nu = 1$, it can be stabilized for $\nu = 2$. Figure 2.12 shows the transient response curve of this stabilized system with $B = 1.475$. In this figure, the step input $\delta B_0 = 0.04$ is applied at $\tau = 5\pi$. The characteristic roots for Eq. (2.40) are, in this case, given by

$$\begin{aligned}\lambda_{1,2} &= -0.160 \pm j0.364 \\ \lambda_{3,4} &= -0.540 \pm j0.157\end{aligned}$$

2.6 Concluding Remarks

In this chapter, we have analyzed the dynamic stability of the magnetic suspension device using a tuned LCR circuit. The behavior of this system is described by nonlinear differential equations with two degrees of freedom and they are solved by the method of averaging. The results obtained for the static stability and the equivalent spring constant agree with Parente's results [47]. Further, the effects of system parameters on the dynamic stability have also been clarified. Hence, it has been found that the system may

* When $v(\tau)$ becomes negative the restoring term uv in the first equation of (2.8) also becomes negative, and hence $u(\tau)$ diverges.

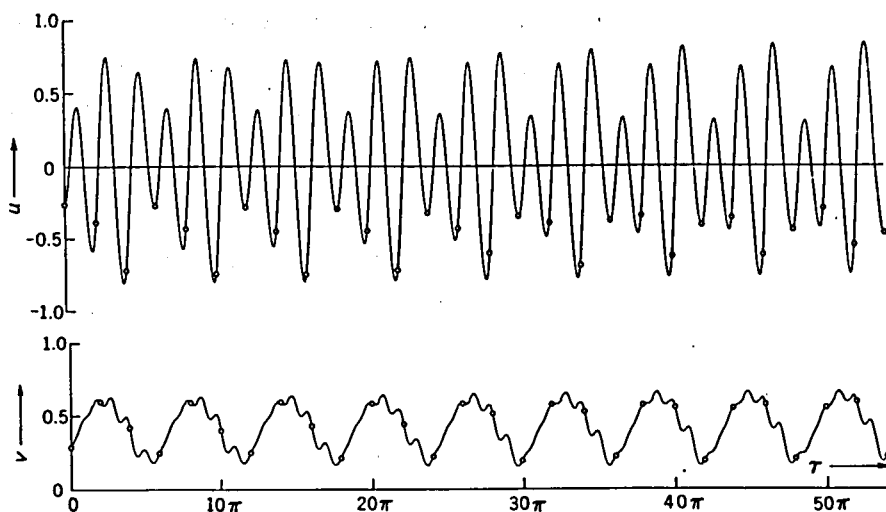


Fig. 2.11 Waveforms of a quasi-periodic oscillation obtained by analog-computer analysis, the system parameters being $\nu = 1.0$, $k_1 = 0.3$, $k_2 = 2.0$, $B_0 = 0.2$, and $B = 0.369$.

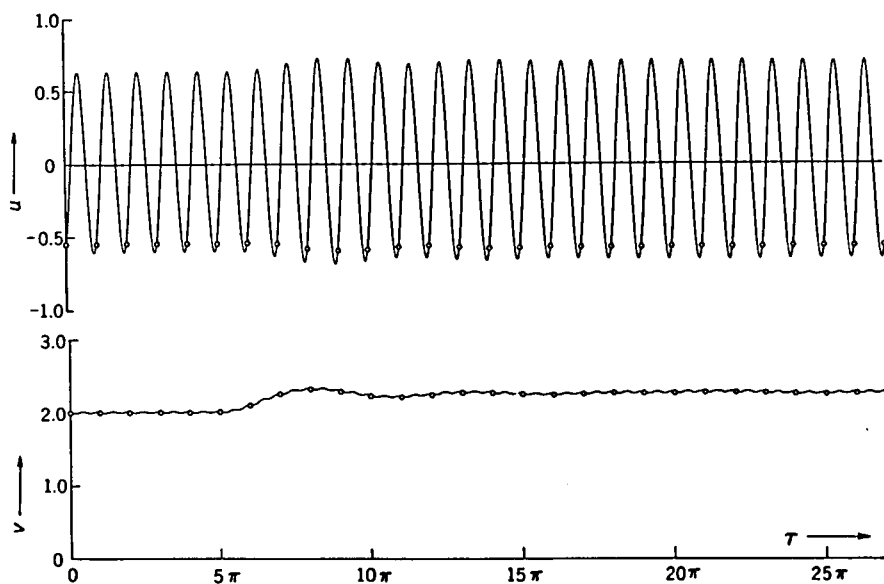


Fig. 2.12 Transient response curve obtained by analog computer analysis, the system parameters being $\nu = 2.0$, $k_1 = 0.6$, $k_2 = 0.8$, $B_0 = 0.2$, and $B = 1.475$.

become unstable if the equivalent spring constant is too large although it should be positive in order to keep the system stable. The calculated transient response curves agree very well with those obtained by analog simulation. In this sense, the proposed procedure of the theoretical analysis would be useful in designing the magnetic suspension device using a tuned LCR circuit.

It has also been confirmed that a quasi-periodic oscillation may occur in the dynamically unstable region predicted by the analysis. A more detailed analysis of the quasi-periodic oscillation will be carried out in Chapter 4.

CHAPTER 3

EFFECT OF LEAKAGE FLUX

3.1 Introduction

In Chapter 2, the equivalent spring constant and the static and dynamic stability conditions of the single-axis magnetic suspension device are derived on the basis of the ideal model shown in Fig. 2.1. To adapt this ideal model to a realistic device, the effects of departures from the idealizing assumptions must be examined. A more complete theory must include the effects of the winding resistance, the reluctance of the magnetic materials, and the leakage flux.

The winding resistance acts as a damping on the electrical system of the device. As shown in Fig. 2.1, we have already introduced the damping of the electrical system for the ideal model by considering the resistor R paralleled with the capacitor C . Furthermore, the winding resistance is generally small. Therefore, we may consider that the effect of the winding resistance essentially gives only slightly different characteristics from those obtained in the preceding chapter.

The reluctance of magnetic materials is connected in series with that of the gap distance ($\mathcal{R}$) in the magnetic circuit of Fig. 2.2. Hence by the effect of the reluctance of magnetic materials, the practical gap distance is slightly decreased from that obtained by using the ideal model. Thus the reluctance of magnetic materials does not produce an important effect upon the performance of the device.

On the other hand, although the leakage flux which does not pass through the suspended body is a small fraction of the total flux, it produces an important effect on the stability of the suspension device. It decreases the magnetic attractive force and the equivalent spring constant.

Parente [47] has neglected the leakage flux in the analysis of the system describing the device. For the estimation of the magnetic attractive force, Takizawa, Otsuki, and Suzuki [64] have taken into account the effects of the flux which leaks from the sides of the pole pair, but assumed that all the flux passes through the suspended body. Therefore, they have still neglected the component of the flux which does not pass through the suspended body.

This chapter focuses attention upon the effect of the leakage flux on the stability of the device [23]. Figure 3.1 shows the realistic model of the single-axis magnetic suspension device to be considered in this chapter. In this model, the winding resistance, the reluctance of the magnetic materials, and the leakage flux are taken into account. The fundamental equations for this realistic model are derived and analyzed by a procedure analogous to that of the preceding chapter. Theoretical results are verified by experiments.

3.2 Fundamental Equations

With the notations of Fig. 3.1, the equations for the circuit are written as follows:

$$\begin{aligned} n \frac{d\phi}{dt} + R_s i + R i_R &= E \sin \omega t \\ R i_R &= \frac{1}{C} \int i_C dt \\ i &= i_R + i_C \end{aligned} \quad (3.1)$$

where ϕ is the total flux linking the winding coil, and n is the number of turns of the coil. As shown in the figure, we consider a leakage flux ϕ_l which is a portion of the total flux ϕ and does not enter into the suspended body.

The analogous electrical circuit for the magnetic circuit is shown in Fig. 3.2, in which ϕ_i is the magnetic flux which passes through the suspended body, $\mathcal{R}_i$ and $\mathcal{R}_l$ are the reluctances of the magnetic paths of ϕ_i and ϕ_l , respectively. The equations for the magnetic circuit are therefore given by

$$\begin{aligned} ni &= \mathcal{R}_i \phi_i = \mathcal{R}_l \phi_l & \phi &= \phi_i + \phi_l \\ \text{or} & & & \\ ni &= \mathcal{R} \phi & \mathcal{R} &= \frac{\mathcal{R}_i \mathcal{R}_l}{\mathcal{R}_i + \mathcal{R}_l} \end{aligned} \quad (3.2)$$

We assume that the reluctance $\mathcal{R}_l$ is constant and $\mathcal{R}_i$ is nearly proportional to the gap distance h , i.e.,

$$\mathcal{R}_i = \sigma h + \mathcal{R}_c \quad (3.3)$$

where σ and $\mathcal{R}_c$ are constant, $\mathcal{R}_c$ being the reluctance of the magnetic materials.

The equation of motion of the suspended body is given by

$$M \frac{d^2 h}{dt^2} + k_m \frac{dh}{dt} + F_m = Mg \quad (3.4)$$

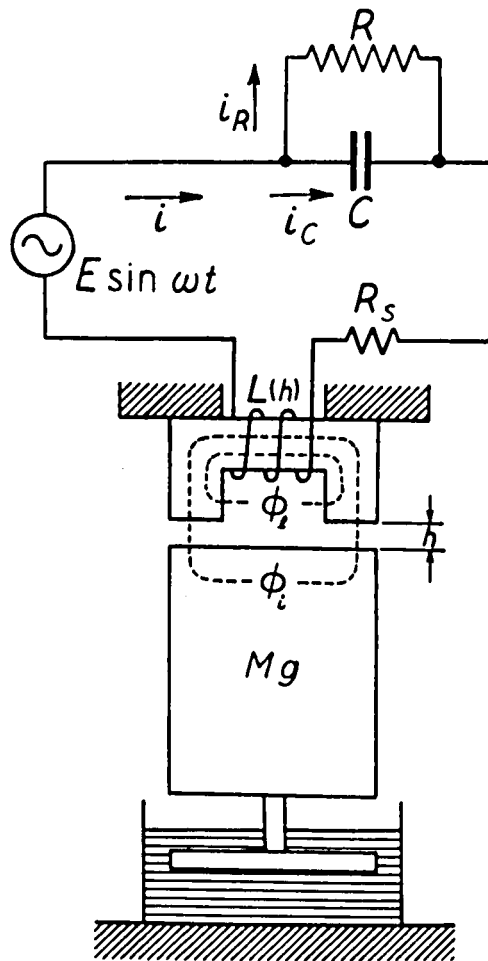


Fig. 3.1 Magnetic suspension device using a tuned LCR circuit (realistic model).

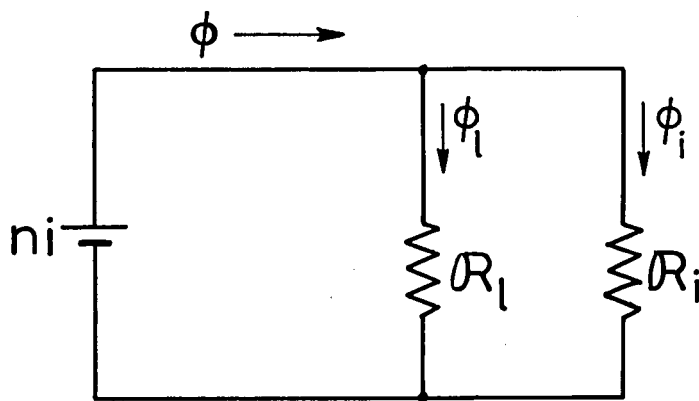


Fig. 3.2 Electrical equivalent of the magnetic circuit in Fig. 3.1.

where M is the mass of the suspended body, k_m is the viscous damping coefficient of the dashpot, F_m is the magnetic attractive force and g is the acceleration of gravity. Magnetic attractive force F_m is given by

$$F_m = \frac{1}{2} \phi^2 \frac{\partial \mathcal{R}}{\partial h} = \frac{\sigma \phi^2}{2 [1 + (\sigma h + \mathcal{R}_c)/\mathcal{R}_l]^2} \quad (3.5)$$

By a procedure analogous to that of Sec. 2.2, we introduce nondimensional frequency ν , magnetic flux u and gap distance v , defined by

$$\omega = \omega_0 \nu \quad \phi = \Phi_n u \quad h + \frac{\mathcal{R}_c}{\sigma} = H_n v \quad (3.6)$$

where ω_0 , Φ_n , and H_n are base quantities of ω , ϕ , and h , respectively. For the convenience of computation, we define H_n and Φ_n by

$$H_n = \frac{n^2 \omega_0^2 C}{\sigma (1 + R_s/R)} \quad \Phi_n^2 = \frac{2 \omega_0^2 M H_n}{\sigma} \quad (3.7)$$

Eliminating i , i_R , i_C , and F_m in Eqs. (3.1) and (3.4), and using Eqs. (3.6) we obtain

$$\begin{aligned} \frac{d^2 u}{d\tau^2} + k_1 \frac{du}{d\tau} + k_s \frac{d}{d\tau} \left(\frac{uv}{1 + \rho v} \right) + \frac{uv}{1 + \rho v} &= B \cos \nu \tau \\ \frac{d^2 v}{d\tau^2} + k_2 \frac{dv}{d\tau} + \left(\frac{u}{1 + \rho v} \right)^2 &= B_0 \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} \tau &= \omega_0 t - \frac{1}{\nu} \tan^{-1} \frac{k_1}{\nu} & k_1 &= \frac{1}{\omega_0 C R} & k_2 &= \frac{k_m}{\omega_0 M} \\ k_s &= \frac{\omega_0 C R_s}{1 + R_s/R} & \rho &= \frac{\sigma H_n}{\mathcal{R}_l} \\ B &= \frac{E}{n \omega_0 \Phi_n} \sqrt{\nu^2 + k_1^2} & B_0 &= \frac{g}{\omega_0^2 H_n} \end{aligned} \quad (3.8a)$$

Equations (3.8) are the fundamental equations for the case where the leakage flux is considered. If these equations are compared with Eqs. (2.8), it is seen that two parameters k_s and ρ are added. The parameter k_s is the damping coefficient of the tuned circuit resulted from the series resistance R_s , and ρ implies the nondimensional leakage permeance of the electromagnet.

3.3 Analysis of the Fundamental Equations

3.3.1 Derivation of an Autonomous System by Using the Method of Averaging

By a procedure analogous to that of Sec. 2.3, we derive an autonomous system from the nonautonomous system (3.8) by using the method of averaging.

In Eqs. (3.8), we introduce new variables $x(\tau)$ and $y(\tau)$ defined by

$$\begin{aligned} x(\tau) &= u(\tau) \sin \nu \tau + \frac{1}{\nu} \dot{u}(\tau) \cos \nu \tau \\ y(\tau) &= u(\tau) \cos \nu \tau - \frac{1}{\nu} \dot{u}(\tau) \sin \nu \tau \end{aligned} \quad (3.9)$$

so that

$$\begin{aligned} \dot{u}(\tau) &= x(\tau) \sin \nu \tau + y(\tau) \cos \nu \tau \\ \dot{u}(\tau) &= \nu x(\tau) \cos \nu \tau - \nu y(\tau) \sin \nu \tau \end{aligned} \quad (3.10)$$

where

$$\dot{u}(\tau) = \frac{du}{d\tau}$$

Substituting Eqs. (3.10) into (3.8) and solving for the derivatives $dx/d\tau$ and $dy/d\tau$ leads to

$$\begin{aligned} \frac{dx}{d\tau} &= f_1(x, y, v, \dot{v}, \tau) \cos \nu \tau \\ \frac{du}{d\tau} &= -f_1(x, y, v, \dot{v}, \tau) \sin \nu \tau \\ \frac{dv}{d\tau} &= \dot{v} \\ \frac{d\dot{v}}{d\tau} &= f_2(x, y, v, \dot{v}, \tau) \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} f_1(x, y, v, \dot{v}, \tau) &= \frac{1}{\nu} \left\{ \left[\nu^2 - \frac{v}{1 + \rho v} - \frac{k_s \dot{v}}{(1 + \rho v)^2} \right] x \right. \\ &\quad \left. + \nu \left(k_1 + \frac{k_s v}{1 + \rho v} \right) y \right\} \sin \nu \tau + \frac{1}{\nu} \left\{ -\nu \left(k_1 + \frac{k_s v}{1 + \rho v} \right) x \right. \\ &\quad \left. + \left[\nu^2 - \frac{v}{1 + \rho v} - \frac{k_s \dot{v}}{(1 + \rho v)^2} \right] y + B \right\} \cos \nu \tau \\ f_2(x, y, v, \dot{v}, \tau) &= B_0 - k_2 \dot{v} - \frac{r^2 + (y^2 - x^2) \cos 2\nu \tau + 2xy \sin 2\nu \tau}{2(1 + \rho v)^2} \\ r^2 &= x^2 + y^2 \end{aligned}$$

As mentioned in Sec. 2.3, if the nondimensional number ϵ defined by Eq. (2.16a) is sufficiently small, the variables $x(\tau)$, $y(\tau)$, $v(\tau)$, and $\dot{v}(\tau)$ in the right sides of Eqs. (3.11) may be considered to vary slowly with time τ . Hence, by using the method of averaging, we can write Eqs. (3.11), to a first approximation, as

$$\begin{aligned}\frac{dx}{d\tau} &= -\frac{1}{T} \int_0^T f_1(x, y, v, \dot{v}, \tau) \cos \nu \tau d\tau \\ \frac{dy}{d\tau} &= -\frac{1}{T} \int_0^T f_1(x, y, v, \dot{v}, \tau) \sin \nu \tau d\tau \\ \frac{dv}{d\tau} &= \frac{1}{T} \int_0^T \dot{v} d\tau \\ \frac{d\dot{v}}{d\tau} &= \frac{1}{T} \int_0^T f_2(x, y, v, \dot{v}, \tau) d\tau\end{aligned}\tag{3.12}$$

where

$$T = \frac{2\pi}{\nu}$$

Performing the integrations of Eqs. (3.12) yields an autonomous system:

$$\begin{aligned}\frac{dx}{d\tau} &= \frac{1}{2\nu} (-A_1 x + A_2 y + B) \equiv X(x, y, v, \dot{v}) \\ \frac{dy}{d\tau} &= -\frac{1}{2\nu} (A_2 x + A_1 y) \equiv Y(x, y, v, \dot{v}) \\ \frac{dv}{d\tau} &= \dot{v} \equiv V(\dot{v}) \\ \frac{d\dot{v}}{d\tau} &= B_0 - k_2 \dot{v} - \frac{r^2}{2(1 + \rho v)^2} \equiv \dot{V}(x, y, v, \dot{v})\end{aligned}\tag{3.13}$$

where

$$\begin{aligned}A_1 &= \nu \left(k_1 + \frac{k_s v}{1 + \rho v} \right) \\ A_2 &= \nu^2 - \frac{v}{1 + \rho v} - \frac{k_s \dot{v}}{(1 + \rho v)^2}\end{aligned}\tag{3.13a}$$

3.3.2 Steady-State Solutions

The steady-state solutions for which the components $x(\tau)$, $y(\tau)$, $v(\tau)$, and $\dot{v}(\tau)$ are constant are determined by

$$\begin{aligned} X(x_0, y_0, v_0, \dot{v}_0) &= 0 \\ Y(x_0, y_0, v_0, \dot{v}_0) &= 0 \\ V(\dot{v}_0) &= 0 \\ \dot{V}(x_0, y_0, v_0, \dot{v}_0) &= 0 \end{aligned} \quad (3.14)$$

where the subscript 0 is used to designate the values of x , y , v , and $\dot{v}$ for the steady-state solutions.

From the first two equations of (3.14), we obtain

$$\begin{aligned} x_0 &= \frac{\nu k_{10} B}{(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2} \\ y_0 &= \frac{(v_{10} - \nu^2) B}{(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2} \end{aligned} \quad (3.15)$$

where

$$\begin{aligned} v_{10} &= \frac{v_0}{1 + \rho v_0} \\ k_{10} &= k_1 + k_s v_{10} \end{aligned} \quad (3.15a)$$

Consequently, we obtain

$$r_0^2 = x_0^2 + y_0^2 = \frac{B^2}{[(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2]^2} \quad (3.16)$$

Further, the last two equations of (3.14) give us

$$\begin{aligned} \dot{v}_0 &= 0 \\ \frac{r_0^2}{2(1 + \rho v_0)^2} &= B_0 \end{aligned} \quad (3.17)$$

Substituting Eqs. (3.16) into (3.17) and solving for v_0 gives us

$$\begin{aligned} v_0 &= \frac{1}{K} \left\{ \nu^2 [1 - \rho \nu^2 - k_1(k_s + \rho k_1)] \right. \\ &\quad \left. \pm \sqrt{\frac{KB^2}{2B_0} - \nu^2(k_1 + \nu^2 k_s)} \right\} \end{aligned} \quad (3.18)$$

where

$$K = (1 - \rho \nu^2)^2 + \nu^2(k_s + \rho k_1)^2 \quad (3.18a)$$

The gap distance v_0 has physical meanings only when

$$v_0 > \frac{R_c}{\sigma H_n}$$

since the practical gap distance h in Eqs. (3.6) must be positive from the physical point of view.

Eq. (3.18) shows that there exist two equilibrium points where the time average of the magnetic attractive force balances the mechanical load force.

3.3.3 Equivalent Spring Constant

As mentioned in Sec. 2.3.3, we define the nondimensional equivalent spring constant S_0 by

$$S_0 = \left(\frac{\partial \bar{f}_m}{\partial v} \right)_{v=v_0} = \frac{\partial \bar{f}_{m0}}{\partial v_0} \quad (3.19)$$

where $\bar{f}_{m0}$ denotes the time average of the magnetic attractive force:

$$\bar{f}_{m0} = \frac{r_0^2}{2(1 + \rho v_0)^2} \quad (3.20)$$

By substituting Eqs. (3.16) into (3.20) and performing the differentiation of Eq. (3.19), we obtain the equivalent spring constant as

$$S_0 = \frac{(\nu^2 - v_{10})(1 - \rho\nu^2) - \nu^2 k_{10}(k_s + \rho k_1)}{(1 + \rho v_0)^3 [(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2]^2} B^2 \quad (3.21)$$

The static stability is guaranteed by

$$S_0 > 0 \quad (3.22)$$

As can be easily seen from Eq. (3.21), if the leakage flux exists or the leakage permeance ρ is not zero, the equivalent spring constant S_0 decreases with the increase in the supply frequency ν and becomes negative for a high frequency. Hence there exists a range of the supply frequency to achieve the stable suspension. From Eq. (3.21), the upper limit of the supply frequency ν_m for which the equivalent spring constant S_0 becomes zero at the gap distance $v_0 = 0$ is given by

$$\nu_m = \sqrt{\frac{1 - k_1(k_s + \rho k_1)}{\rho}} \quad (3.23)$$

The equivalent spring constant S_0 is also expressed by

$$S_0 = \pm \frac{4B_0^2}{B^2} \sqrt{\frac{KB^2}{2B_0} - \nu^2(k_1 + \nu^2 k_s)^2} \quad (3.24)$$

where v_0 in Eq. (3.21) is eliminated by making use of Eqs. (3.18). The positive sign in Eq. (3.24) is taken if the negative sign is taken in Eq. (3.18) and vice versa.

3.3.4 Stability Investigation

The steady-state solutions given by Eqs. (3.15) through (3.18) are maintained only when they are stable. The stability of the steady-state solutions is studied by solving the variational equations derived from Eqs. (3.13) which are reduced to linear differential equations with constant coefficients. By a procedure analogous to that of Sec. 2.3.4, the characteristic equation of the variational equations is given by

$$\lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \quad (3.25)$$

where

$$\begin{aligned} a_1 &= k_{10} + k_2 \\ a_2 &= k_{10} k_2 + \frac{1}{4\nu^2} [(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2] - \frac{\rho r_0^2}{(1 + \rho v_0)^3} \\ a_3 &= \frac{k_2}{4\nu^2} [(\nu^2 - v_{10})^2 + \nu^2 k_{10}^2] - \frac{k_s (v_{10} + \nu^2) r_0^2}{4\nu^2 (1 + \rho v_0)^4} \\ &\quad - \frac{\rho k_{10} r_0^2}{(1 + \rho v_0)^3} \\ a_4 &= \frac{r_0^2}{4\nu^2 (1 + \rho v_0)^3} [(\nu^2 - v_{10})(1 - \rho \nu^2) - \nu^2 k_{10} (k_s + \rho k_1)] \end{aligned} \quad (3.26)$$

The stability conditions are given by the Routh-Hurwitz criterion:

$$\begin{aligned} a_1 &> 0 & a_2 &> 0 & a_3 &> 0 & a_4 &> 0 \\ a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 &> 0 \end{aligned} \quad (3.27)$$

By virtue of Eqs. (3.26), the condition $a_1 > 0$ is always fulfilled. The condition $a_2 > 0$ is also fulfilled from the practical point of view since ρr_0^2 is sufficiently small. The condition $a_3 > 0$ is fulfilled if all the other conditions are satisfied, and this condition is not necessary.

Substituting Eqs. (3.26) into the fourth and fifth conditions of (3.27) and using Eqs. (3.16) and (3.17) gives us

$$(\nu^2 - v_{10})(1 - \rho \nu^2) - \nu^2 k_{10} (k_s + \rho k_1) > 0 \quad (3.28)$$

and

$$D > 0 \quad (3.29)$$

where k_{10} and v_{10} are defined by Eqs. (3.15a), and D is defined by

$$\begin{aligned}
 D = & (B^2 k_2 - 4B_0 B_k) \left[(k_{10} k_2 - \frac{2\rho B_0}{1 + \rho v_0}) (k_{10} + k_2) + \frac{B^2 k_{10}}{8\nu^2 B_0 (1 + \rho v_0)^2} \right. \\
 & + \frac{B_k}{2\nu^2 (1 + \rho v_0)^2} \left. \right] - 4B_0^2 (k_{10} + k_2)^2 [(\nu^2 - v_{10})(1 - \rho\nu^2) \\
 & - \nu^2 k_{10} (k_s + \rho k_1)] \\
 B_k = & B_0 \left[k_s (\nu^2 + v_{10}) + 4\rho\nu^2 k_{10} (1 + \rho v_0) \right]
 \end{aligned} \tag{3.30}$$

These are the conditions for stability of the steady-state solution for the case where the leakage flux and the series resistance are taken into account. Evidently the condition of (3.28) is identical with the static stability condition $S_0 > 0$ of (3.22). On the other hand, as mentioned in Sec. 2.3.4, the condition of (3.29) is the dynamic stability condition.

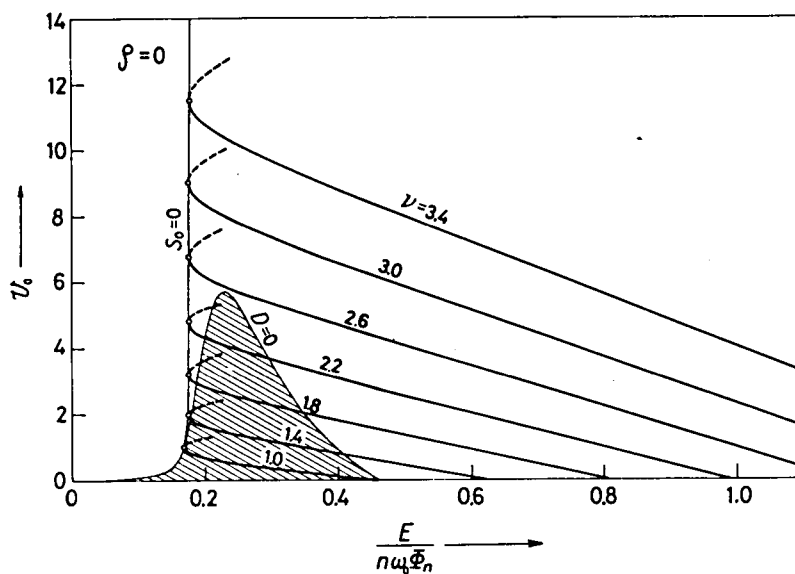
3.4 Numerical Examples

In order to investigate the effect of the leakage flux, a numerical analysis of the response characteristic is carried out for the system parameters as given by

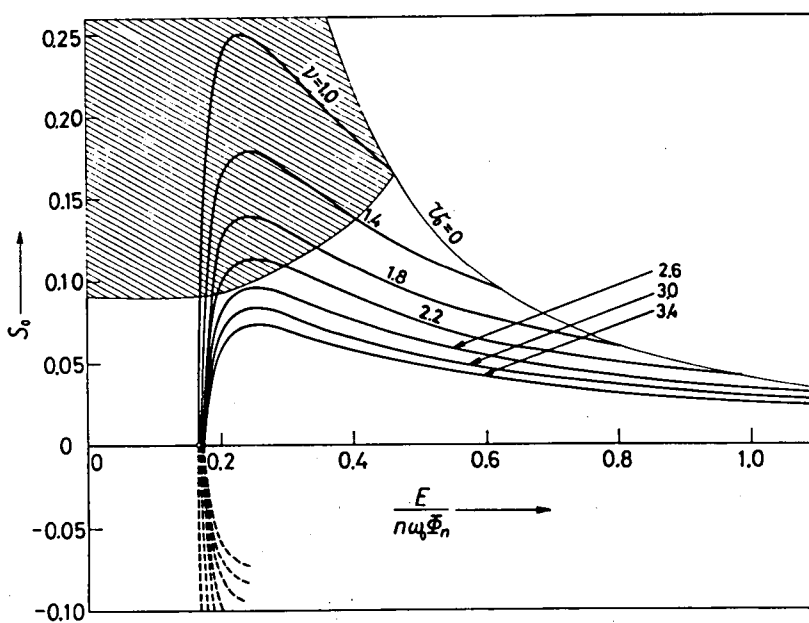
$$k_1 = 0.4 \quad k_s = 0 \quad k_2 = 0.4 \quad B_0 = 0.1$$

With these values of the parameters, the response characteristics represented by Eqs. (3.18) and (3.24) are calculated against the supply voltage $E/(n\omega_0\Phi_n)$ with the supply frequency ν as a parameter. Figure 3.3 shows the response characteristics for the case in which the leakage flux is neglected or the leakage permeance $\rho = 0$; and Figure 3.4 shows the case in which $\rho = 0.2$. In these figures, the broken lines show the statically unstable responses for which the condition of (3.28) is not fulfilled. The hatched area shows the dynamically unstable region where the condition of (3.29) is not fulfilled. The curve $S_0 = 0$ in Figs. 3.3a and 3.4a shows the boundary of the static stability, and the curve $v_0 = 0$ in Figs. 3.3b and 3.4b shows the relationship between S_0 and $E/(n\omega_0\Phi_0)$ for $v_0 = 0$.

If the response characteristics shown in Fig. 3.4 are compared with those in Fig. 3.3, it is seen that, owing to the effect of the leakage flux, the gap distance v_0 increases and the equivalent spring constant S_0 decreases remarkably with the increase of the supply frequency ν ; and for $\nu \geq 2.2$ a stable suspension can never be achieved. Thus the leakage flux makes the system statically unstable especially in the case where the supply frequency ν is high. On the other hand, it makes the system dynamically stable; for example, the dynamically unstable portion of the response for $\nu = 1.8$ in Fig. 3.3 disappears in Fig. 3.4.



(a)



(b)

Fig. 3.3 Response characteristics for the case where the leakage flux is ignored.

(a) Gap distance characteristic.

(b) Equivalent spring constant characteristic.

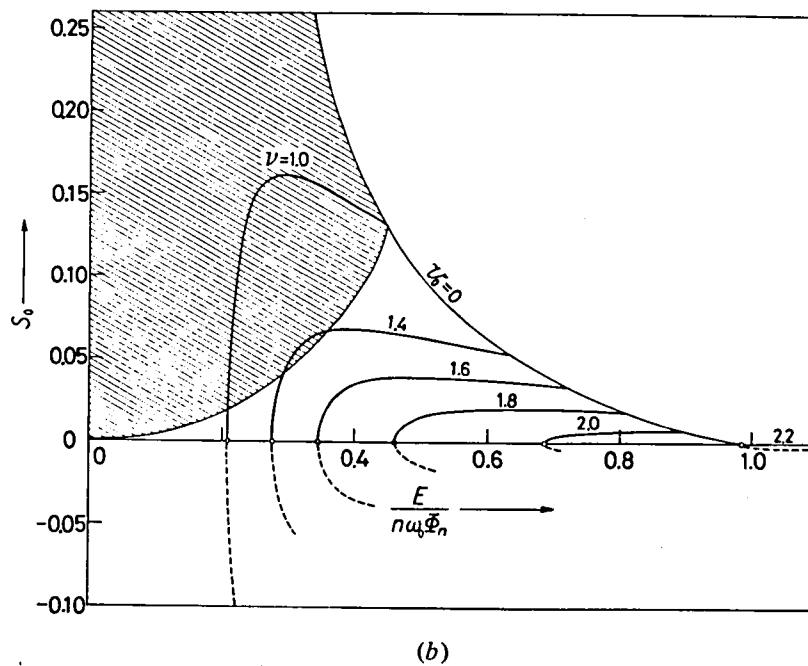
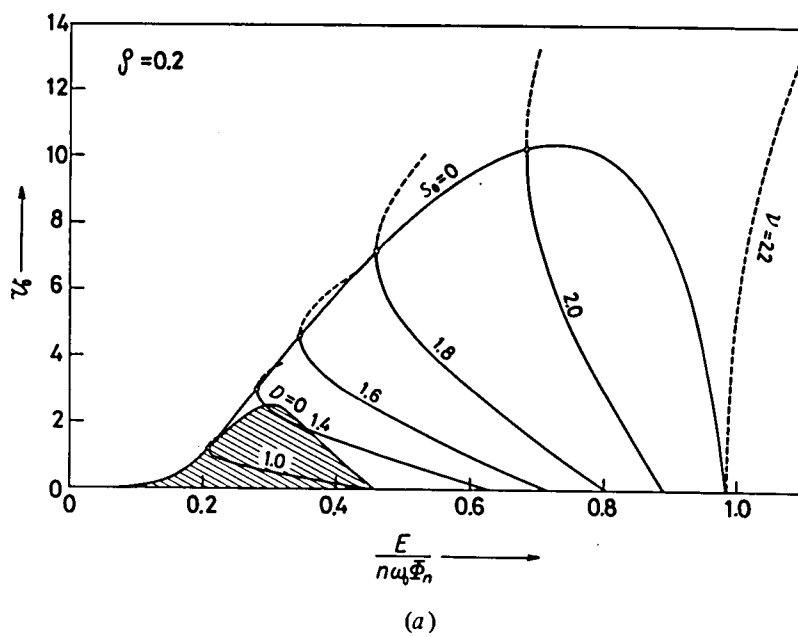


Fig. 3.4 Response characteristics for the case where the leakage flux is considered.

(a) Gap distance characteristic.

(b) Equivalent spring constant characteristic.

The boundary curves of the stable suspension in Figs. 3.3 and 3.4 are plotted on the $E/(n\omega_0\Phi_n)$ and ν coordinate plane and are shown in Figs. 3.5 and 3.6, respectively. Hence, when the supply voltage $E/(n\omega_0\Phi_n)$ and the frequency ν of the external force are given at any point inside these regions the stable suspension can be achieved. As can be easily seen, the stable region is extremely decreased by the effect of the leakage flux.

3.5 Experimental Investigation

In this section some experimental results are shown in order to compare them with the foregoing analysis. The outline of the experimental model is shown in Fig. 3.7. To avoid the effects of eddy current and hysteresis, the electromagnet and the suspended body are made of 0.35mm silicon steel laminations or the so-called 'cut core'. The suspended body's motion is limited by a vertical guide. The oil-dashpot damper is not used in this model since the friction force between the guide and the shaft of the suspended body acts as a damping force. The numerical parameters of the experimental model are as follows:

$$\begin{aligned} n &= 400 \text{ turn} & C &= 10.59\mu\text{F} \\ R &= 252.3 \Omega & R_s &= 2.57 \Omega \\ M &= 0.3395 \text{ kg} \end{aligned}$$

To measure the gap distance between the electromagnet and the suspended body, the inductive probe is used.

The response characteristics like those of Fig. 3.4 are experimentally sought. The results are shown in Fig. 3.9 by using the notations in Fig. 3.8. The equivalent spring constant S is obtained from

$$S = \frac{\Delta M}{\Delta h_0}$$

where Δh_0 is the increment of the gap distance caused by an additional weight of $\Delta M = 74$ g. The coil current I and the coil voltage E_{L0} against the gap distance h_0 are plotted in Figs. 3.10 and 3.11, respectively. The coil voltage E_{L0} is obtained from

$$E_{L0} = \sqrt{E_L^2 - I^2 R_s^2}$$

where E_L , I , and R_s are measured values (see Fig. 3.8). The inductance L of the coil at a gap distance h_0 is given by

$$L = \frac{E_{L0}}{2\pi f I}$$

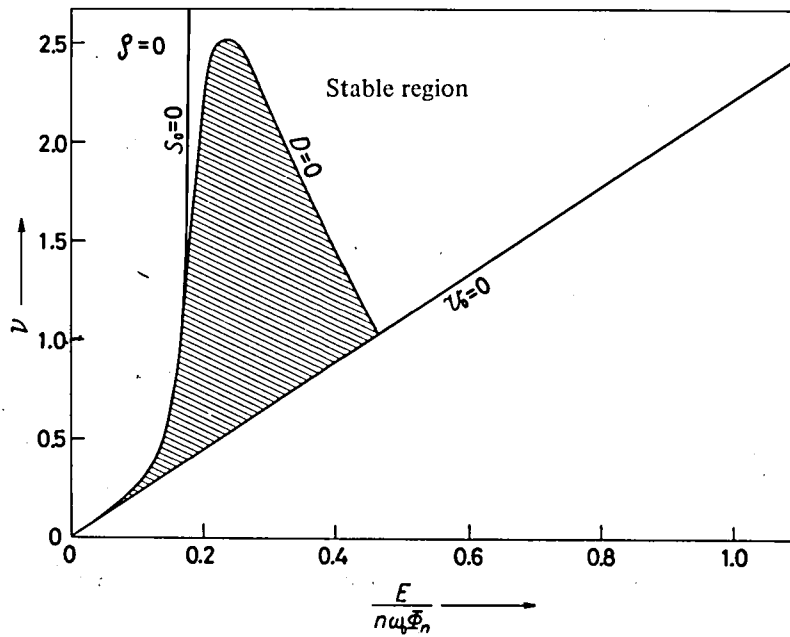


Fig. 3.5 Region in which stable suspension is obtained. The leakage flux is ignored.

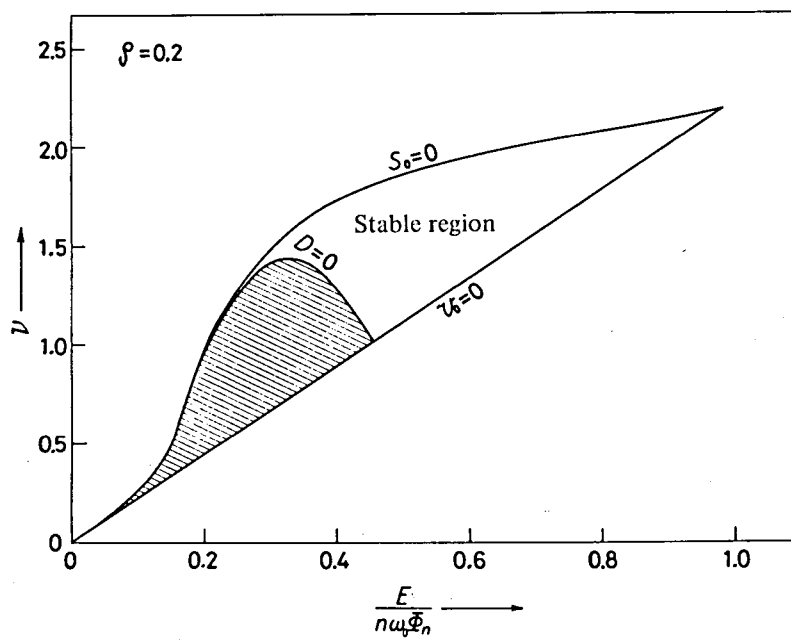


Fig. 3.6 Region in which stable suspension is obtained. The leakage flux is considered.

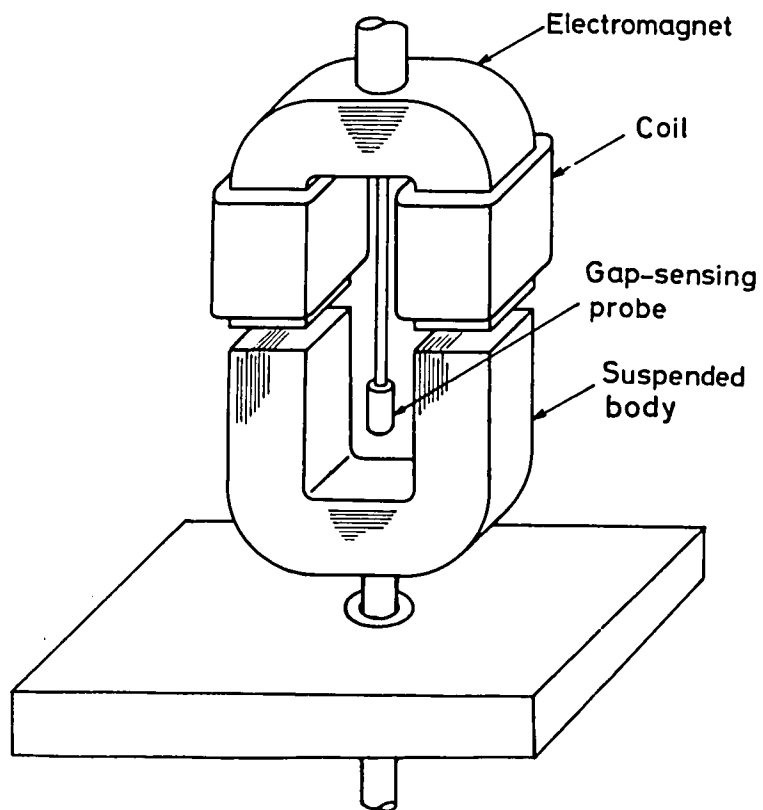


Fig. 3.7 Experimental model.

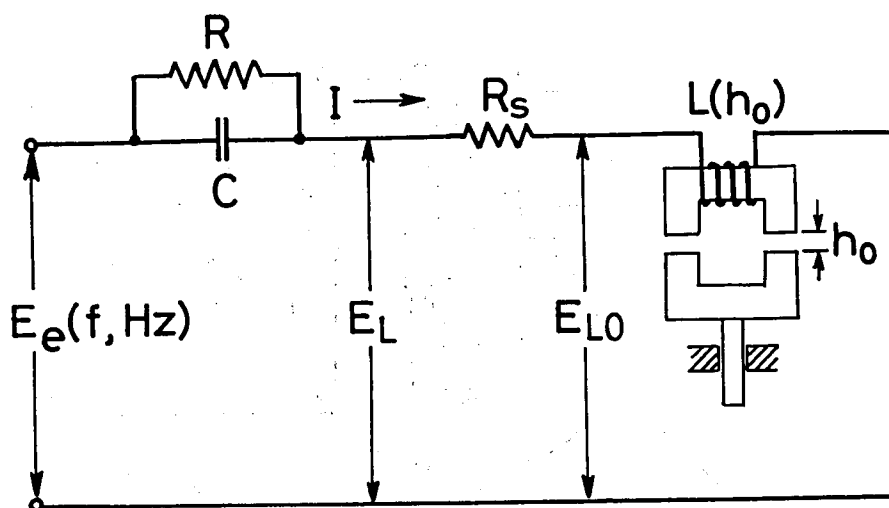
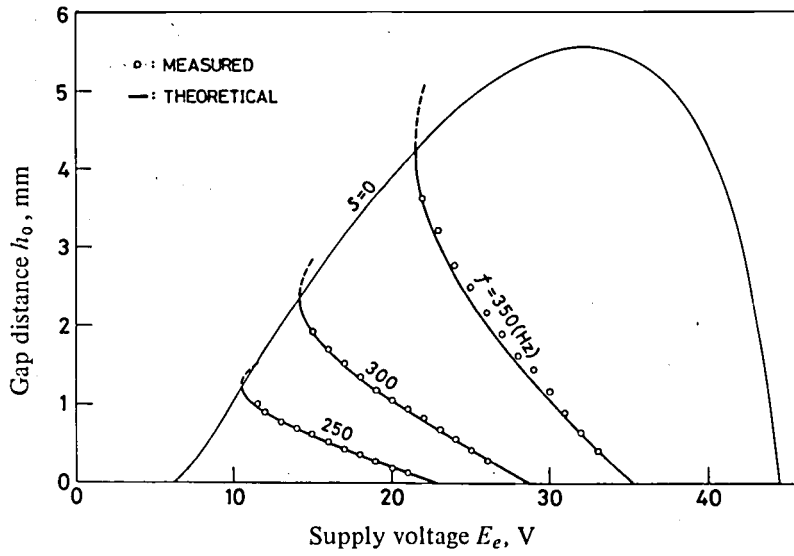
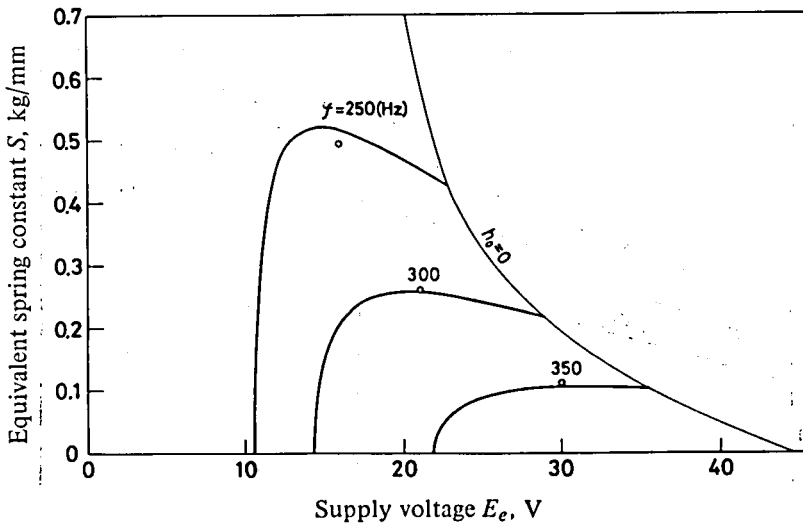


Fig. 3.8 Tuned circuit of the magnetic suspension device.



(a)



(b)

Fig. 3.9 Response characteristics for the experimental model.

(a) Gap distance characteristic.

(b) Equivalent spring constant characteristic.

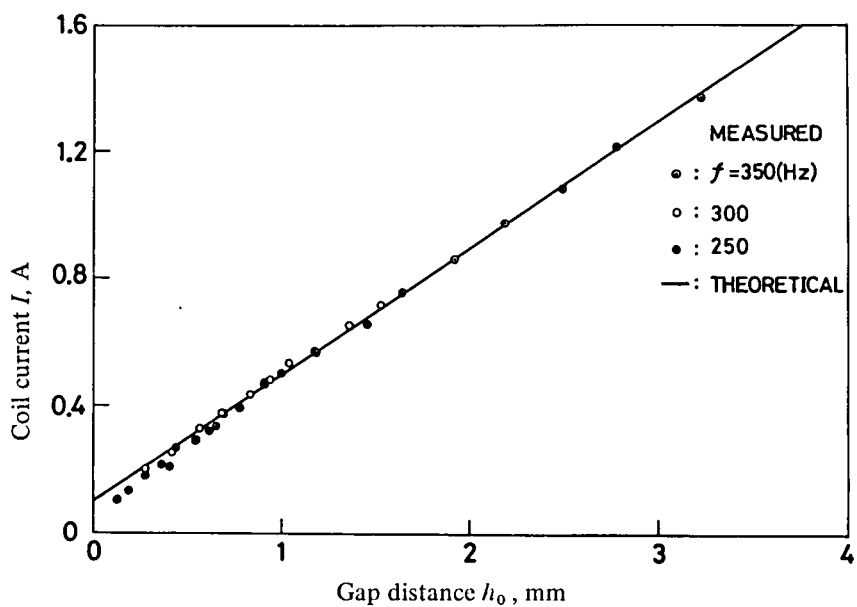


Fig. 3.10 Coil current versus gap distance.

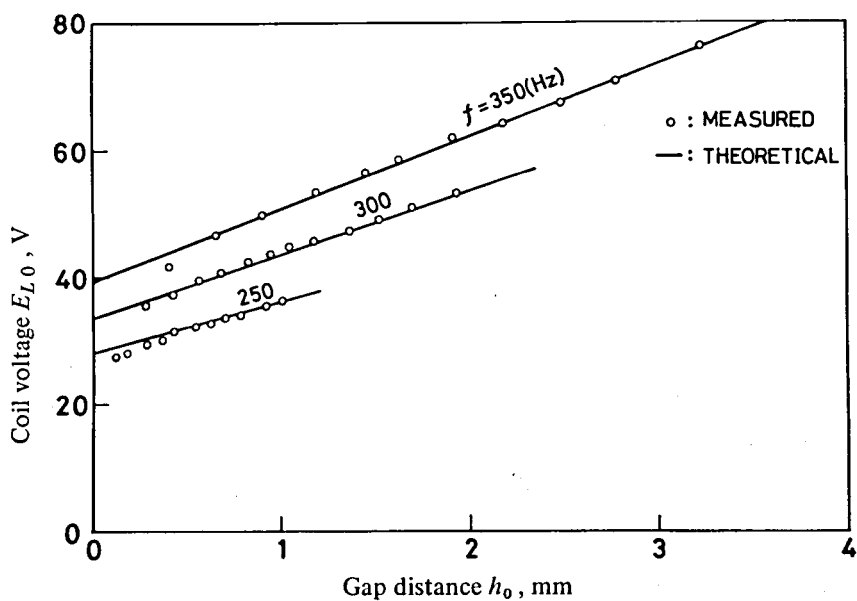


Fig. 3.11 Coil voltage versus gap distance.

The results are shown in Fig. 3.12.

As shown in Appendix II, the parameters σ , $\mathcal{R}_c$ and $\mathcal{R}_l$ in Eqs. (3.2) and (3.3) are determined from the experimental results of Figs. 3.10 and 3.11: We have

$$\sigma = 3.877 \times 10^9 \text{ AT/W}_b \cdot \text{m}$$

$$\mathcal{R}_c = 0.95 \times 10^6 \text{ AT/W}_b$$

$$\mathcal{R}_l = 12.33 \times 10^6 \text{ AT/W}_b$$

Thereupon the theoretical results corresponding to the experimental results can be calculated by using these values and are shown in Figs. 3.9 through 3.12. In Appendix III, we express the theoretical values of h_0 , S , I , E_{L0} , and L by using the parameters of the experimental model. The theoretical analysis shows that the stable suspension can never be achieved if the supply frequency f is beyond 425 Hz. This is verified by the experiments.

The transient response curve like that of Fig. 2.9 is experimentally obtained for a step decrease of 74 g in the suspended mass, and shown in Figs. 3.13 through 3.15. The step decrease in the suspended mass is obtained by using a small DC magnet of 74 g as follows: We first attach the DC magnet to the suspended body by its magnetic force, and then drop it down instantaneously by switching off the current of the DC magnet. We see that the amplitudes of the current I and voltage E_L , and the gap distance h vary slowly with time, as we expected in Sec. 2.3.1.

3.6 Concluding Remarks

We have studied the effect of the leakage flux on the stability of the single-axis suspension device using a tuned circuit. The leakage flux under consideration is a component of the total flux which does not pass through the suspended body. It decreases the magnetic attractive force and the equivalent spring constant. The nonlinear differential equations which describe the system are derived and analyzed by using the method of averaging.

The curves which show the relations of the gap distance and the equivalent spring constant to the supply voltage are obtained theoretically. The static and dynamic stability conditions are derived. It has been found that, as the supply frequency increases, the magnetic restoring force decreases remarkably due to the effect of the leakage flux. There exists the upper limit of the supply frequency to achieve the stable suspension.

On the other hand, the leakage flux stabilizes the system dynamically, since the dynamic instability results from the magnetic restoring force being too strong. The theoretical results are confirmed by experiments.

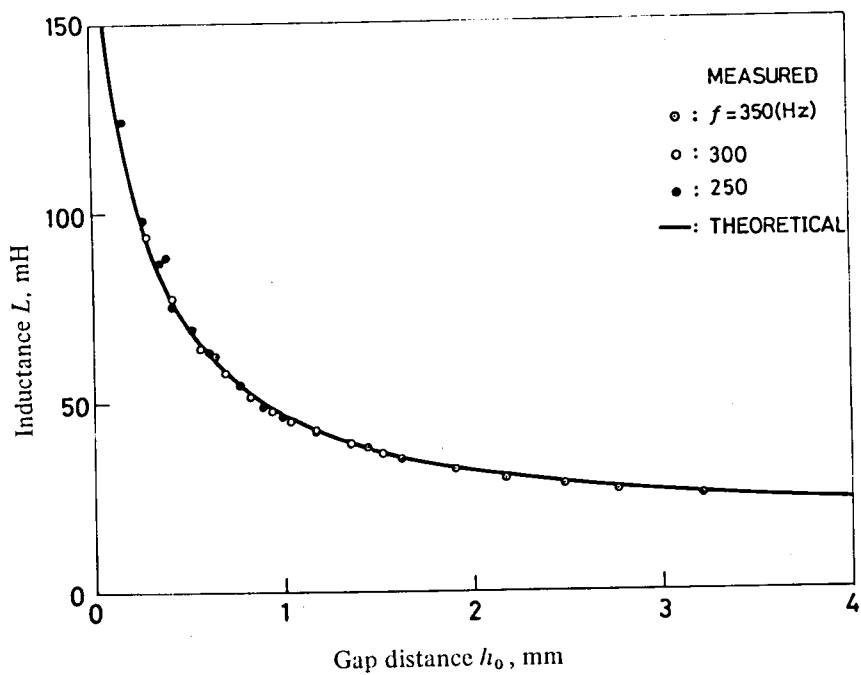


Fig. 3.12 - Coil inductance versus gap distance.

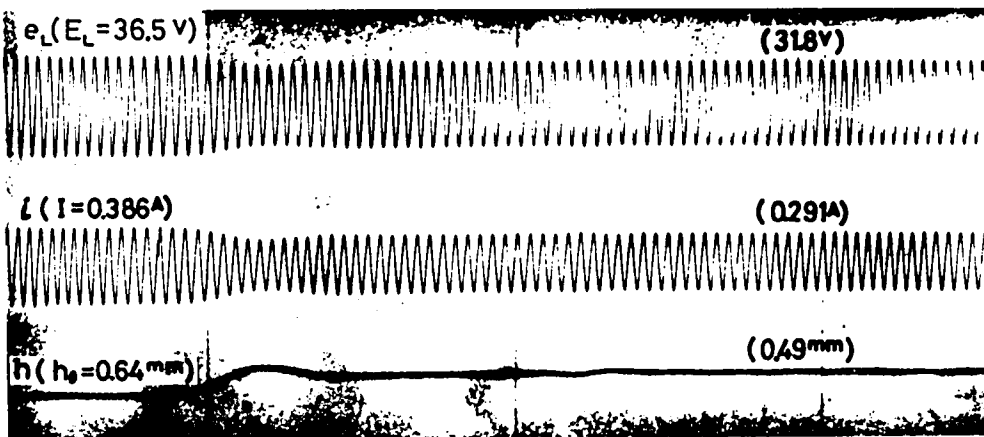


Fig. 3.13 Transient response curve obtained by experiments ($f = 250 \text{ Hz}$, $E_e = 16 \text{ V}$).

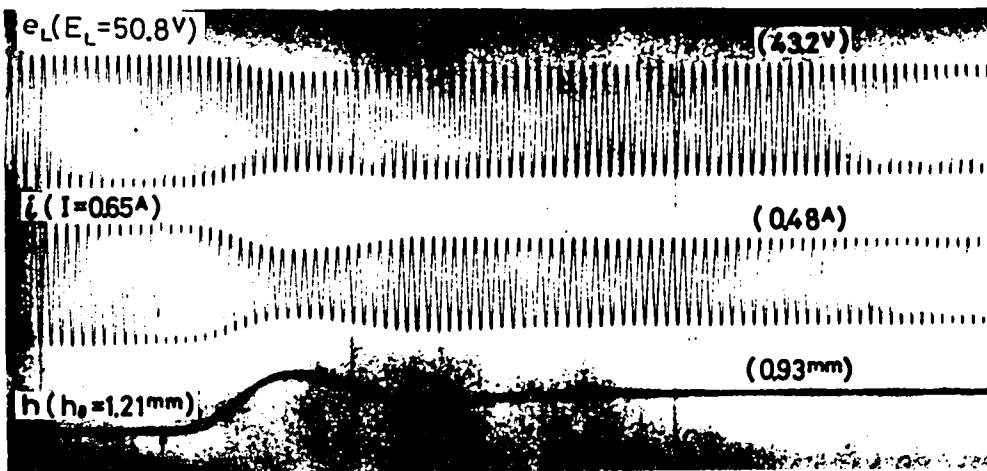


Fig. 3.14 Transient response curve obtained by experiments ($f = 300 \text{ Hz}$, $E_e = 21 \text{ V}$).

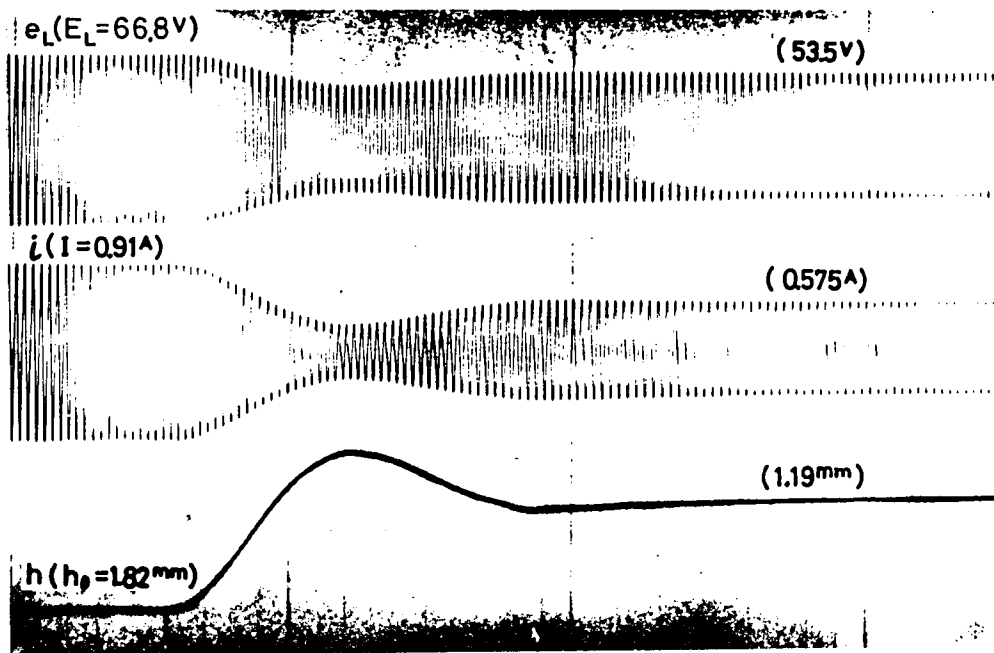


Fig. 3.15 Transient response curve obtained by experiments ($f = 350$ Hz, $E_e = 30$ V).

CHAPTER 4

QUASI-PERIODIC OSCILLATIONS

4.1 Introduction

From the analog-computer analysis in Sec. 2.5, we have predicted that quasi-periodic oscillations may occur in the magnetic suspension device, provided that the system is dynamically unstable. The suspended body oscillates slowly and periodically after some transients have died out. The amplitude and phase of the magnetic flux also vary slowly and periodically. Furthermore, the ratio between the period for amplitude variation and the period of the external force is in general irrational, and thus there is no periodicity in the quasi-periodic oscillation.

As investigated by many authors [27, pp. 262-282, pp. 309-338, 30, 55, 57, 68], these kinds of oscillations also occur in a magnetic amplifier circuit, a logical circuit with parametric excitation, and a resonant nonlinear control circuit. However, the oscillations in the magnetic suspension device have not been fully investigated. In this case, the quasi-periodic oscillations can be utilized to operate reciprocating devices such as small pumps, compressors, or fans. These devices are operated at low speeds, without the use of gearing or mechanical linkages or other means of speed reduction, directly by alternating current from supply lines of 60 Hz or higher frequency. A "parametric motor" proposed by Stockman [5, 25, 61] may also utilize these kinds of oscillations to obtain a low speed rotation. Though the efficiency of these devices is quite low, for small devices for which energy economy is not an objective, the disadvantages are offset by the simplicity of construction and operation and by the fact that no switching devices, either mechanical or electronic, are needed. Therefore the drive design can be rough for long life without servicing or repairs. For a fixed electrical frequency, the mechanical frequency can be controlled by adjustment of the electric-circuit parameters, and a particular frequency can be encouraged by introduction of a spring and mechanical tuning.

In this chapter, we obtain the amplitude and frequency characteristics of these kinds of oscillations [24]. First, we show that the quasi-periodic oscillations occur in the nonautonomous system (3.8) corresponding to the occurrence of self-excited oscillations in

the autonomous system (3.13). Second, by using the method of harmonic balance we determine the amplitude and frequency of the self-excited oscillations. The theoretical results are verified by digital simulations and experiments.

4.2 Analysis of the Quasi-Periodic Oscillations

As mentioned in Sec. 3.2, the dynamic behavior of the magnetic suspension device under consideration is described by the nonautonomous system:

$$\begin{aligned}\frac{d^2 u}{d\tau^2} + k_1 \frac{du}{d\tau} + k_s \frac{d}{d\tau} \left(\frac{uv}{1 + \rho v} \right) + \frac{uv}{1 + \rho v} &= B \cos \nu \tau \\ \frac{d^2 v}{d\tau^2} + k_2 \frac{dv}{d\tau} + \left(\frac{u}{1 + \rho v} \right)^2 &= B_0\end{aligned}\quad (4.1)$$

Introducing new variables $x(\tau)$ and $y(\tau)$ defined by

$$\begin{aligned}x(\tau) &= u \sin \nu \tau + \frac{1}{\nu} \frac{du}{d\tau} \cos \nu \tau \\ y(\tau) &= u \cos \nu \tau - \frac{1}{\nu} \frac{du}{d\tau} \sin \nu \tau\end{aligned}\quad (4.2)$$

and using the method of averaging, the nonautonomous system (4.1) is approximated to an autonomous system:

$$\begin{aligned}\frac{dx}{d\tau} &= \frac{1}{2\nu} (-A_1 x + A_2 y + B) \equiv X(x, y, v, \dot{v}) \\ \frac{dy}{d\tau} &= -\frac{1}{2\nu} (A_2 x + A_1 y) \equiv Y(x, y, v, \dot{v}) \\ \frac{dv}{d\tau} &= \dot{v} \equiv V(\dot{v}) \\ \frac{d\dot{v}}{d\tau} &= B_0 - k_2 \dot{v} - \frac{r^2}{2(1 + \rho v)^2} \equiv \dot{V}(x, y, v, \dot{v})\end{aligned}\quad (4.3)$$

where

$$\begin{aligned}A_1 &= \nu \left(k_1 + \frac{k_s v}{1 + \rho v} \right) \\ A_2 &= \nu^2 - \frac{v}{1 + \rho v} - \frac{k_s \dot{v}}{(1 + \rho v)^2} \\ r^2 &= x^2 + y^2\end{aligned}$$

The singular point $(x_0, y_0, v_0, \dot{v}_0)$ of the autonomous system (4.3) is obtained from

$$\begin{aligned} X(x_0, y_0, v_0, \dot{v}_0) &= 0 \\ Y(x_0, y_0, v_0, \dot{v}_0) &= 0 \\ V(\dot{v}_0) &= 0 \\ \dot{V}(x_0, y_0, v_0, \dot{v}_0) &= 0 \end{aligned} \quad (4.4)$$

A periodic solution of the nonautonomous system (4.1) is then correlated with the singular point $(x_0, y_0, v_0, \dot{v}_0)$ in the $x, y, v, \dot{v}$ phase-space, which fixes the amplitude and phase of the periodic solution. If the singular point is stable, the corresponding periodic solution is also stable; if not, it is unstable. As we have obtained in Sec. 3.3. 4, the conditions for stability of the singular point are given as follows:

$$(\nu^2 - v_{10})(1 - \nu^2 \rho) - \nu^2 k_{10}(k_s - k_1 \rho) > 0 \quad (4.5)$$

$$D > 0 \quad (4.6)$$

where

$$\begin{aligned} v_{10} &= \frac{v_0}{1 + \rho v_0} \\ k_{10} &= k_1 + k_s v_{10} \\ D &= (B^2 k_2 - 4B_0 B_k) \left[(k_{10} k_2 - \frac{2\rho B_0}{1 + \rho v_0})(k_{10} + k_2) \right. \\ &\quad \left. + \frac{B^2 k_{10}}{8\nu^2 B_0 (1 + \rho v_0)^2} + \frac{B_k}{2\nu^2 (1 + \rho v_0)^2} \right] - 4B_0^2 (k_{10} + k_2)^2 \\ &\quad [(\nu^2 - v_{10})(1 - \rho \nu^2) - \nu^2 k_{10}(k_s + \rho k_1)] \\ B_k &= B_0 [k_s (\nu^2 + v_{10}) + 4\rho \nu^2 k_{10} (1 + \rho v_0)] \end{aligned} \quad (4.7)$$

The conditions (4.5) and (4.6) are the static and dynamic stability conditions, respectively.

When the autonomous system (4.3) has a stable limit cycle in the $x, y, v, \dot{v}$ space, the representative point whose coordinates are $x(\tau), y(\tau), v(\tau)$, and $\dot{v}(\tau)$ keeps on moving along the limit cycle with increasing time τ ; in other words, a quasi-periodic oscillation occurs in the nonautonomous system (4.1) since the amplitude and phase of the oscillation are modulated.

In order to explain the occurrence of these kinds of oscillations, a numerical analysis of the system (4.3) is carried out by using a digital computer. The system parameters are given by

$$\begin{aligned} \nu &= 2.0 & k_1 &= 0.121 & k_s &= 0.049 & k_2 &= 0.30 \\ \rho &= 0.09 & B_0 &= 0.064 & B &= 0.419 \end{aligned}$$

These values are corresponding to those for the experimental model which will be shown in Sec. 4.3. After a sufficiently long period of time τ , the representative point of the system (4.3) moves along the limit cycle as illustrated in Fig. 4.1. The waveforms of the self-excited oscillation corresponding to the limit cycle are shown in Fig. 4.2 by the thick-line curves. Also plotted by the fine-line curves in the figure are the waveforms of the corresponding quasi-periodic oscillation which are obtained from the numerical solutions of Eqs. (4.1). The waveforms of x and y illustrated by the fine-line curves are obtained by inserting the numerical solutions u and $du/d\tau$ into Eqs. (4.2). We see that the waveforms of x , y , v of the self-excited oscillation and those of the corresponding quasi-periodic oscillation are in good agreement.

To analyze the amplitude and frequency of the self-excited oscillation by using the method of harmonic balance, we assume

$$\begin{aligned} x &= x_d + x_x \sin \beta\tau + y_x \cos \beta\tau \\ y &= y_d + x_y \sin \beta\tau + y_y \cos \beta\tau \\ v &= v_d + r_v \sin \beta\tau \end{aligned} \quad (4.8)$$

where x_d , x_x , y_x , y_d , x_y , y_y , v_d , r_v and β are constant to be determined. As shown in Fig. 4.2, terms of harmonics higher than the second are certain to be presented but are ignored to this order of approximation.

Substituting Eqs. (4.8) into (4.3) and equating the constant terms and the coefficients of the terms containing $\sin \beta\tau$ and $\cos \beta\tau$ separately to zero yields

$$\begin{aligned} B_a x_d + \frac{1}{2} B_b x_x - \nu\beta A_b y_x - C_a y_d - \frac{1}{2} C_b x_y + \frac{1}{2} C_c y_y &= A_a B \\ B_b x_d + (B_a + \frac{1}{2} B_c) x_x - \nu\beta (2A_a + A_c) y_x - C_b y_d - (C_a + \frac{1}{2} C_d) x_y &= A_b B \\ \nu\beta (2A_a - A_c) x_x + (B_a - \frac{1}{2} B_c) y_x + C_c y_d - (C_a - \frac{1}{2} C_d) y_y &= 0 \\ C_a x_d + \frac{1}{2} C_b x_x - \frac{1}{2} C_c y_x + B_a y_d + \frac{1}{2} B_b x_y - \nu\beta A_b y_y &= 0 \\ C_b x_d + (C_a + \frac{1}{2} C_d) x_x + B_b y_d + (B_a + \frac{1}{2} B_c) x_y - \nu\beta (2A_a + A_c) y_y &= 0 \\ C_c x_d - (C_a - \frac{1}{2} C_d) y_x - \nu\beta (2A_a - A_c) x_y - (B_a - \frac{1}{2} B_c) y_y &= 0 \\ r_d^2 + \frac{1}{2} r_x^2 + \frac{1}{2} r_y^2 - \beta^2 A_b r_v &= 2A_a B_0 \\ 2(x_d x_x + y_d x_y) - \beta^2 (2A_a + A_c) r_v &= 2A_b B_0 \\ 2(x_d y_x + y_d y_y) + k_2 \beta (2A_a - A_c) r_v &= 0 \end{aligned} \quad (4.9)$$

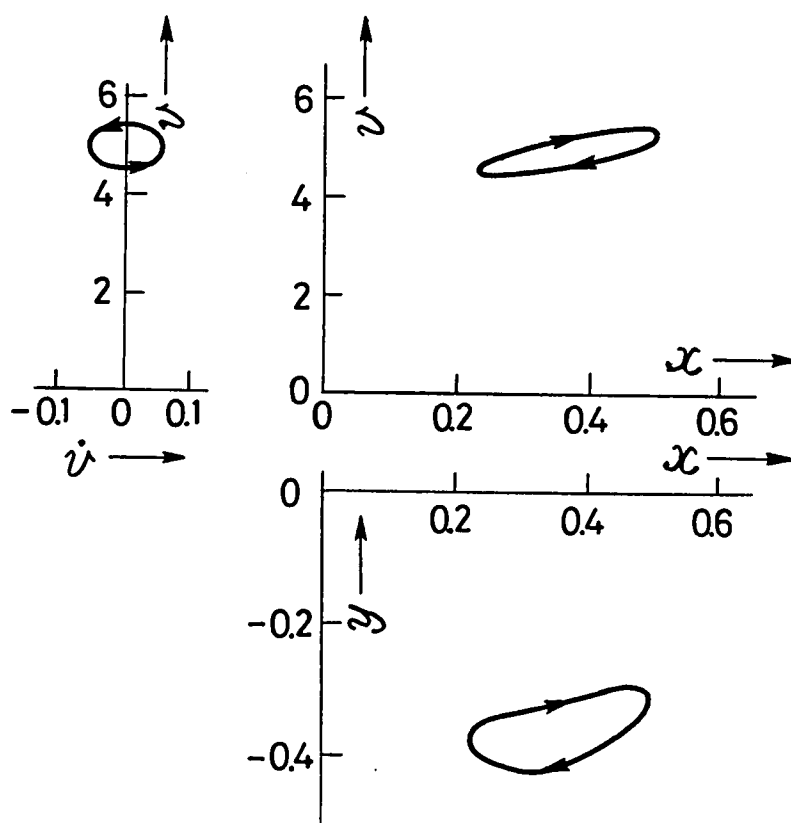


Fig. 4.1 Limit cycle representing a quasi-periodic oscillation.

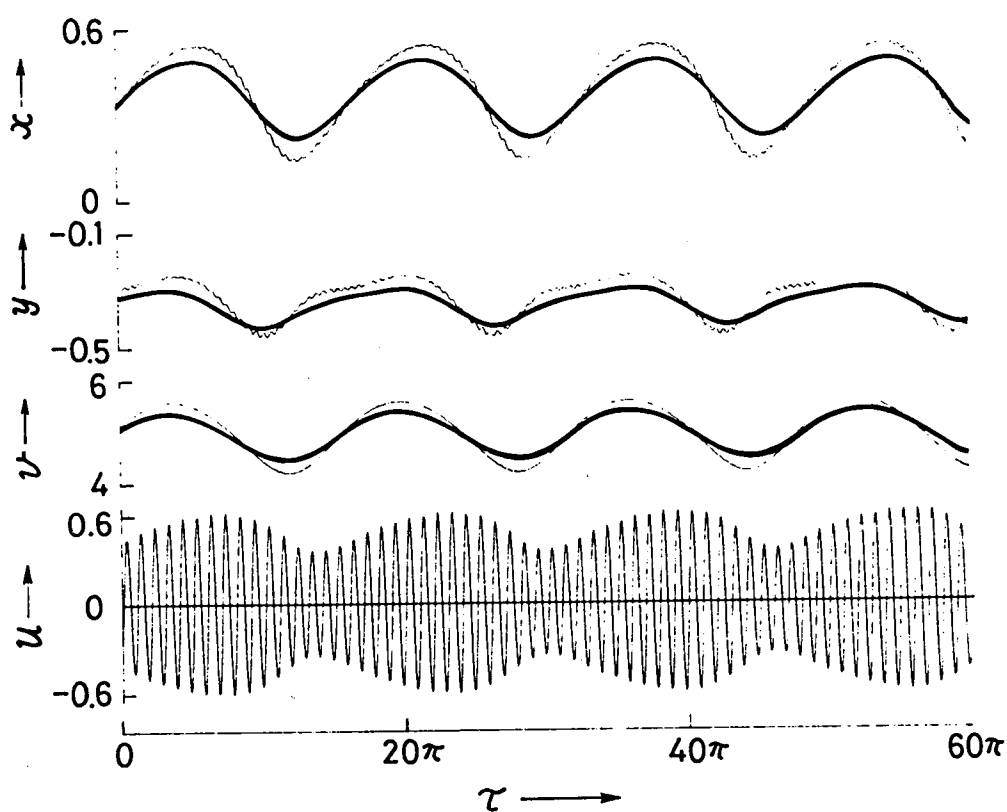


Fig. 4.2 Waveforms of the self-excited oscillation and the corresponding quasi-periodic oscillation.

where

$$\begin{aligned}
A_d &= (1 + \rho v_d)^2 + \frac{1}{2} \rho^2 r_v^2 \\
A_b &= 2\rho r_v (1 + \rho v_d) \\
A_c &= \frac{1}{2} \rho^2 r_v^2 \\
B_d &= \nu [k_1 A_d + k_s (v_d + \rho v_d^2 + \frac{1}{2} \rho r_v^2)] \\
B_b &= \nu [k_1 A_b + k_s r_v (1 + 2\rho v_d)] \\
B_c &= \nu (k_1 A_c + \frac{1}{2} \rho k_s r_v^2) \\
C_d &= \nu^2 A_d - v_d (1 + \rho v_d) - \frac{1}{2} \rho r_v^2 \\
C_b &= \nu^2 A_b - r_v (1 + 2\rho v_d) \\
C_c &= k_s \beta r_v \\
C_d &= \nu^2 A_c - \frac{1}{2} \rho r_v^2 \\
r_d^2 &= x_d^2 + y_d^2 \\
r_x^2 &= x_x^2 + y_x^2 \\
r_y^2 &= x_y^2 + y_y^2
\end{aligned} \tag{4.10}$$

As can be seen, there exists such a solution for Eqs. (4.9) that all the amplitudes of the oscillatory terms are zero, i.e.,

$$r_x = r_y = r_v = 0$$

In this case the constant terms x_d , y_d , and v_d are identical with x_0 , y_0 , and v_0 , respectively, which are obtained from Eqs. (4.4). However, if there exists another solution in which r_x , r_y , r_v are not zero, the autonomous system (4.3) may have a stable limit cycle; if not, it may have no limit cycle.

The response characteristic represented by Eqs. (4.9) is calculated against the supply voltage B for the same values of the system parameters as those in Fig. 4.2, that is,

$$\begin{aligned}
\nu &= 2.0 & k_1 &= 0.121 & k_s &= 0.049 & k_2 &= 0.30 \\
\rho &= 0.09 & B_0 &= 0.064
\end{aligned} \tag{4.11}$$

The result is illustrated in Fig. 4.3. The statically and dynamically unstable regions show the regions in which the singular point obtained from Eqs. (4.4) does not fulfill the stability conditions (4.5) and (4.6), respectively. To show the effects of the leakage flux, the case for $\rho = 0$ in Eqs. (4.11) is also calculated and the result is shown in Fig. 4.4.

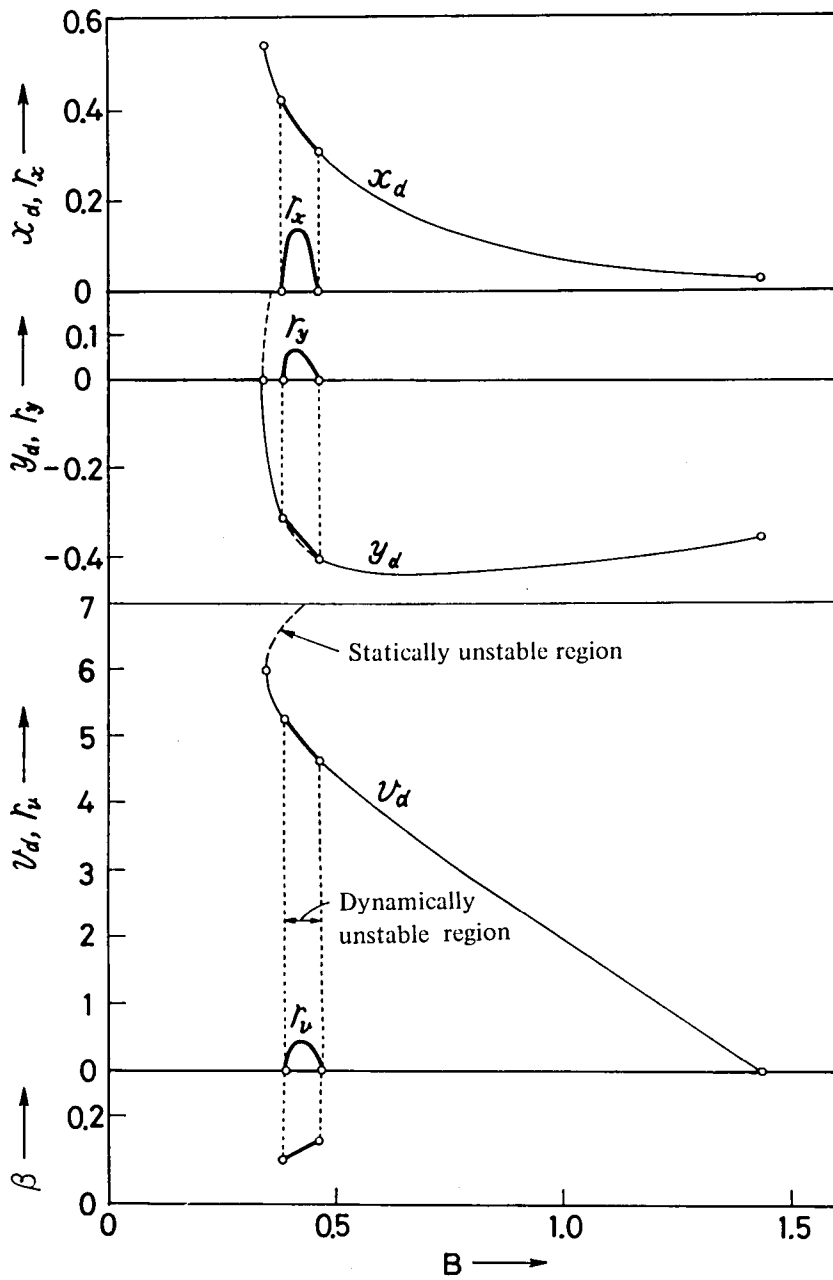


Fig. 4.3 Response characteristic, the system parameters being $\nu = 2.0$, $k_1 = 0.121$, $k_s = 0.049$, $k_2 = 0.3$, $\rho = 0.0901$, $B_0 = 0.064$, $B = 0.419$.

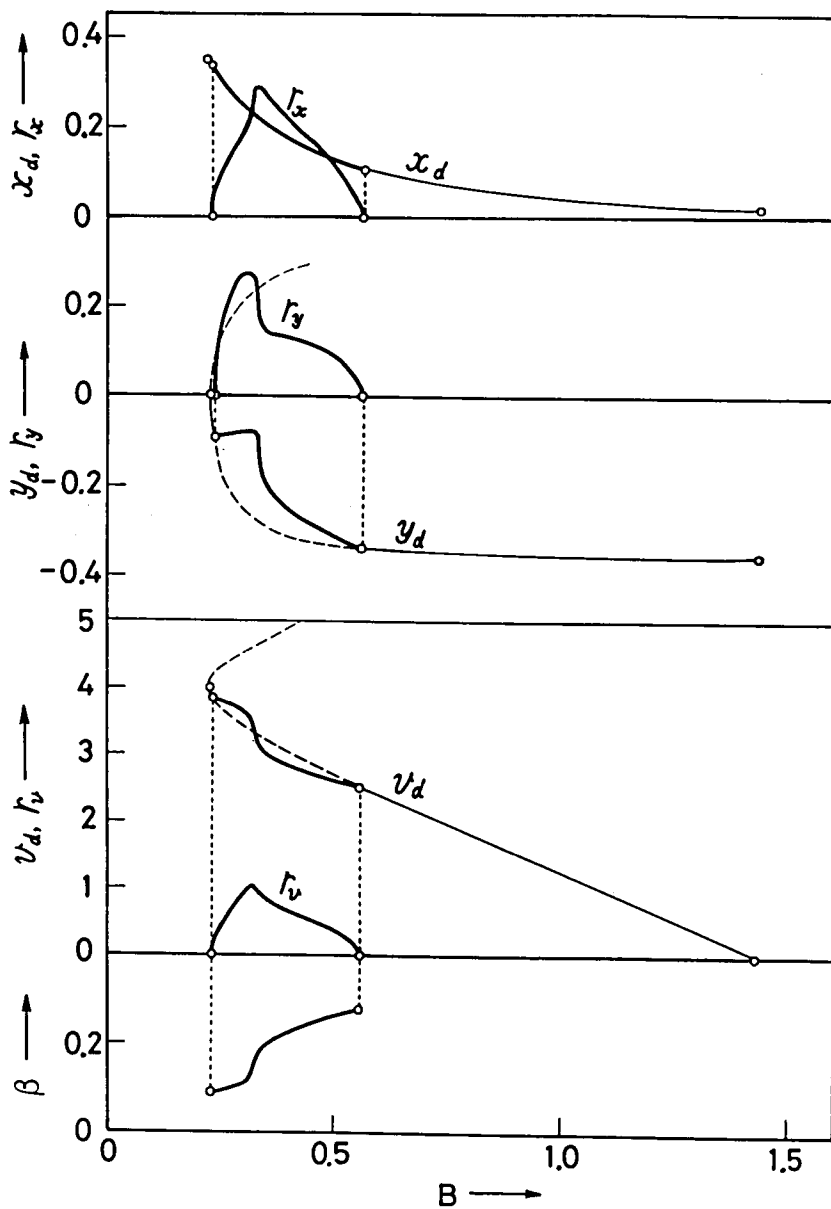


Fig. 4.4 Response characteristic, the system parameters being the same as those in Fig. 4.3, but $\rho = 0$.

From the results we can verify that in the dynamically unstable region determined by the condition (4.6), the system (4.3) has a stable limit cycle; in other words, a quasi-periodic oscillation occurs in the system (4.1).

4.3 Experimental Investigation

By making use of the experimental model shown in Fig. 3.7, we shall describe an experiment concerned with the quasi-periodic oscillation and compare the result with the foregoing analysis. As shown in Fig. 4.5, the mechanical structure of the experimental model is somewhat different from that shown in Fig. 3.7; a sliding ball bearing and an oil dashpot damper are used in the present experiment, since the waveforms of the quasi-periodic oscillation are greatly affected by nonlinearities of mechanical friction of the vertical guide. The parameters of the experimental model are as follows:

$$\begin{array}{ll}
 n = 400 \text{ turn} & C = 10.59 \mu\text{F} \\
 R = 1000 \Omega & R_s = 5.97 \Omega \\
 M = 0.36 \text{ kg} & k_m = 84.82 \text{ N}\cdot\text{s/m} \\
 \sigma = 4.17 \times 10^9 \text{ AT/W}_b\cdot\text{m} & \mathcal{R}_c = 0 \text{ AT/W}_b \\
 \mathcal{R}_l = 11.5 \times 10^6 \text{ AT/W}_b & f = 250 \text{ Hz} \\
 \omega = 1570.8 \text{ rad/s} &
 \end{array}$$

where the constants for the magnetic circuit σ , $\mathcal{R}_c$, and $\mathcal{R}_l$ are obtained by a procedure similar to that mentioned in Sec. 3.5. If we put $\omega_0 = 785.4 \text{ rad/s}$ ($f_0 = 125 \text{ Hz}$), the corresponding nondimensional parameters defined by Eqs. (3.8a) are given by Eqs. (4.11).

Proceeding as in Sec. 3.5, we seek the gap distance characteristic of the experimental model in relation to the supply voltage $E_e (=E/\sqrt{2})$. The results are shown in Fig. 4.6. Quasi-periodic oscillations occur in the range of E_e as indicated in the figure; the gap distance h varies slowly and periodically to the extent of the area surrounded by the curve a . The theoretical area where h varies can be obtained from the values of v_d and r_v in Fig. 4.3 and is also illustrated by the curve b in Fig. 4.6. The nondimensional quantities in Fig. 4.3 and the practical values in Fig. 4.6 are related by

$$E_e = 18.1 B \text{ (V)} \quad h_0 = 0.249 v_0 \text{ (mm)}$$

The observed waveforms of the quasi-periodic oscillations are shown in Fig. 4.7. The amplitude of the oscillation varies periodically, thus presenting a type of beat oscillation. The frequency of the amplitude variation becomes higher as E_e increases. The theoretical

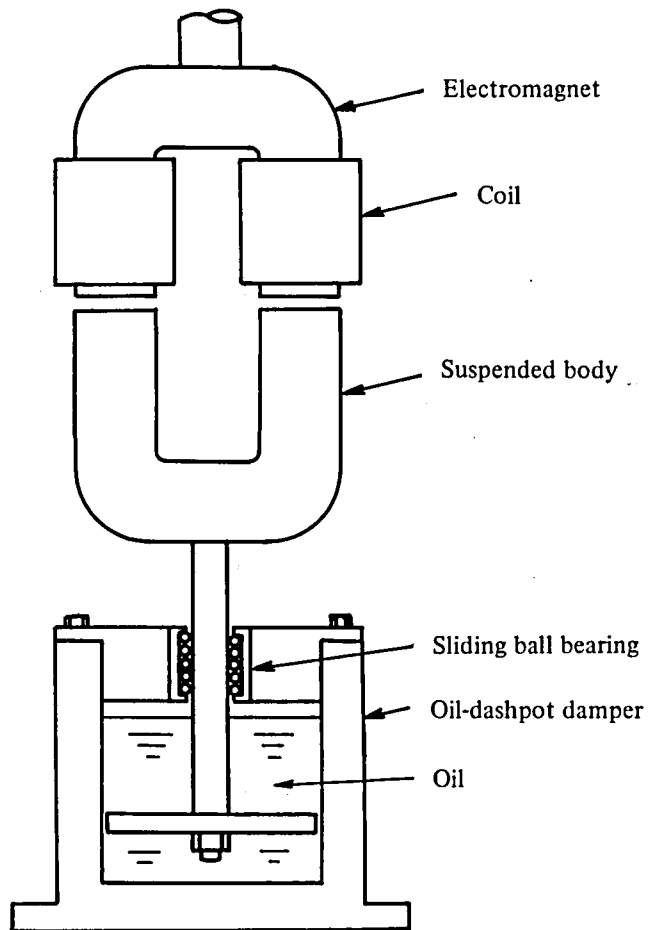


Fig. 4.5 Illustration of experimental model with oil-dashpot damper.

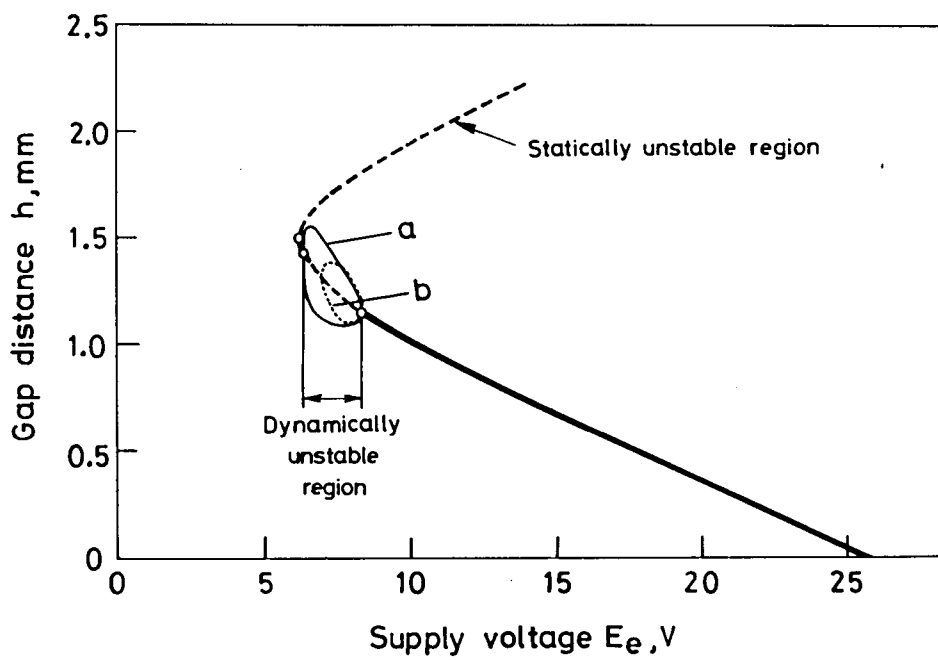
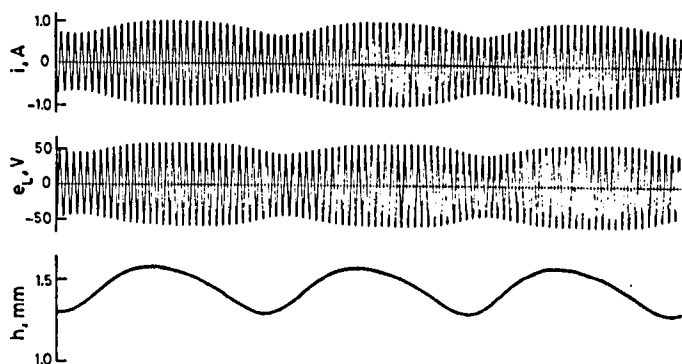
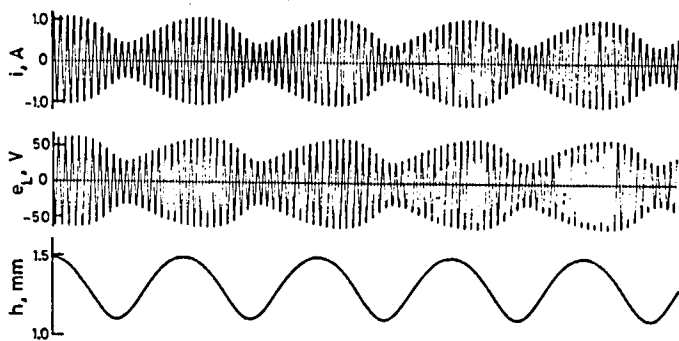


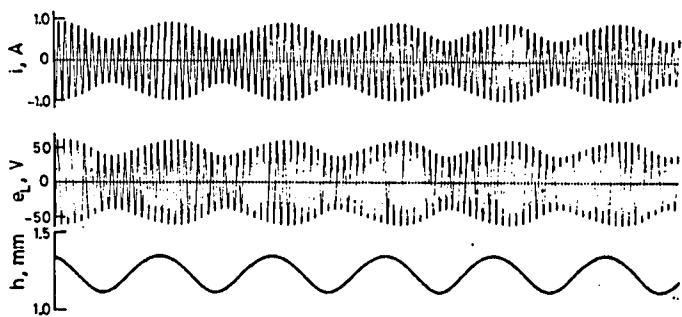
Fig. 4.6 Gap distance characteristic for the experimental model ($f = 250$ Hz).



(a) $E_e = 6.4 \text{ V}$



(b) $E_e = 6.9 \text{ V}$



(c) $E_e = 7.6 \text{ V}$

Fig. 4.7 Waveforms of the quasi-periodic oscillation obtained by experiment ($f = 250 \text{ Hz}$).

waveforms shown in Fig. 4.2 correspond to the oscillogram (c), where the time scales are related by

$$t = \frac{4}{\pi} \tau \text{ (ms)}$$

In the preceding analysis the circuit equations were set-up with respect to the magnetic flux in the core and were represented by the nondimensional variable u . The voltage e_L is directly related to the flux u , since the time derivative of the flux is proportional to e_L provided that the winding resistance is neglected.

The theoretical results are generally in good agreement with the experiments.

4.4 Concluding Remarks

Quasi-periodic oscillations which occur in the single-axis suspension device have been investigated. The suspended body oscillates, and the amplitude and phase of the magnetic flux vary slowly and periodically in the steady state.

The nonautonomous system which describes the dynamic behavior of the device is approximated to an autonomous system. A periodic oscillation is then correlated with a singular point of the autonomous system, and a quasi-periodic oscillation, if it exists, is correlated with a limit cycle.

It has been shown that when the singular point is dynamically unstable, the autonomous system has a limit cycle and a self-excited oscillation occurs in the autonomous system; corresponding to this, the quasi-periodic oscillation occurs in the nonautonomous system. The amplitude and frequency of the self-excited oscillation are determined by the method of harmonic balance. The numerical analysis is carried out for the same parameters as those of the experimental model. The theoretical results are compared with the solutions of the exact nonautonomous equations directly obtained by a digital computer and the experimental results, and are found to be in satisfactory agreement with them.

CHAPTER 5

EFFECT OF MAGNETIC SATURATION

5.1 Introduction

As shown in Chapters 3 and 4, the realistic model illustrated in Fig. 3.1 satisfactorily represents the static and dynamic performance of the practical device. The mathematical representation of this model is based on the assumption that the reluctance of the gap is so dominant in the magnetic circuit that the reluctance of the magnetic material may be neglected or remain constant. To make this assumption valid practically, the flux density in the cores of the electromagnet and the suspended body must be in the unsaturated portion of the magnetization curve. However, if miniaturization is an objective, one may encounter a problem caused by the magnetic saturation.

In this chapter, we consider the effect of magnetic saturation in the iron portions of the magnetic circuit. For convenience of analysis, the magnetization curve of the magnetic material is assumed to be expressed by a cubic function of the magnetic flux. The fundamental equations which describe the device are derived, and analyzed by the method of averaging. The effect of magnetic saturation on the static and dynamic performance of the device is considered with varying the degree of magnetic saturation in the iron portions of the magnetic circuit. It is demonstrated that quasi-periodic oscillations occur due to the effect of magnetic saturation. The mechanism of the oscillations is different from that discussed in Chapter 4. It is found that the quasi-periodic oscillations occur even if the mechanical damping of the device is large. The occurrence of these oscillations is verified by experiments.

5.2 Fundamental Equations

We derive the fundamental equations which take into account the effect of magnetic saturation by using the same procedure as in Sec. 3.2. We assume that the magnetic saturation occurs in the core of the electromagnet. Then, the net magnetomotive force ni given by Eq. (3.2) is decreased by $\mathcal{F}(\phi)$ due to the effect of magnetic saturation:

$$ni - \mathcal{F}(\phi) = \frac{\mathcal{R}_I \mathcal{R}_I}{\mathcal{R}_I + \mathcal{R}_I} \phi \quad (5.1)$$

In order to facilitate the computation, we assume that the saturation curve of the saturable core is represented by a cubic function of the flux ϕ , that is,

$$\mathcal{F}(\phi) = a\phi^3 \quad (5.2)$$

where a is a constant characterizing the core [27, PP. 114]. Substituting Eq. (5.2) into (5.1) yields

$$ni = \frac{\mathcal{R}_i \mathcal{R}_l}{\mathcal{R}_i + \mathcal{R}_l} \phi + a\phi^3 \quad (5.3)$$

In the procedure in Sec. 3.2, if we use Eq. (5.3) instead of (3.2), we obtain the fundamental equations of nonautonomous type with the effect of magnetic saturation as follows:

$$\begin{aligned} \frac{d^2 u}{d\tau^2} + k_1 \frac{du}{d\tau} + k_s \frac{d}{d\tau} \left(\frac{uv}{1 + \rho v} + \alpha u^3 \right) \\ + \frac{uv}{1 + \rho v} + \alpha u^3 = B \cos \nu \tau \\ \frac{d^2 v}{d\tau^2} + k_2 \frac{dv}{d\tau} + \left(\frac{u}{1 + \rho v} \right)^2 = B_0 \end{aligned} \quad (5.4)$$

where

$$\alpha = \frac{a\Phi_n^2}{\sigma H_n}$$

The nondimensional parameter α physically indicates the degree of the magnetic saturation of the core.

Following the same procedure as that in Sec. 3.3.1, we introduce new variables $x(\tau)$ and $y(\tau)$ defined by

$$\begin{aligned} x(\tau) &= u(\tau) \sin \nu \tau + \frac{1}{\nu} \dot{u}(\tau) \cos \nu \tau \\ y(\tau) &= u(\tau) \cos \nu \tau - \frac{1}{\nu} \dot{u}(\tau) \sin \nu \tau \end{aligned}$$

so that

$$\begin{aligned} u(\tau) &= x(\tau) \sin \nu \tau + y(\tau) \cos \nu \tau \\ \dot{u}(\tau) &= \nu x(\tau) \cos \nu \tau - \nu y(\tau) \sin \nu \tau \end{aligned} \quad (5.5)$$

Then, substituting Eqs. (5.5) into (5.4) and solving for the derivatives $dx/d\tau$ and $dy/d\tau$ and further using the method of averaging, we obtain an autonomous system:

$$\begin{aligned}
\frac{dx}{d\tau} &= \frac{1}{2\nu} (-A_1 x + A_2 y + B) \equiv X(x, y, v, \dot{v}) \\
\frac{dy}{d\tau} &= -\frac{1}{2\nu} (A_2 x + A_1 y) \equiv Y(x, y, v, \dot{v}) \\
\frac{dv}{d\tau} &= \dot{v} \equiv V(\dot{v}) \\
\frac{d\dot{v}}{d\tau} &= B_0 - k_2 \dot{v} - \frac{r^2}{2(1+\rho v)^2} \equiv \dot{V}(x, y, v, \dot{v})
\end{aligned} \tag{5.6}$$

where

$$\begin{aligned}
A_1 &= \nu [k_1 + k_s (\frac{v}{1+\rho v} + \frac{3}{4} \alpha r^2)] \\
A_2 &= \nu^2 - \frac{v}{1+\rho v} - \frac{k_s \dot{v}}{(1+\rho v)^2} - \frac{3}{4} \alpha r^2 \\
r^2 &= x^2 + y^2
\end{aligned} \tag{5.6a}$$

5.3 Steady-State Solutions and Equivalent Spring Constant

The steady-state solutions for which the components $x(\tau)$, $y(\tau)$, $v(\tau)$, and $\dot{v}(\tau)$ are constant are determined by

$$\begin{aligned}
X(x_0, y_0, v_0, \dot{v}_0) &= 0 & Y(x_0, y_0, v_0, \dot{v}_0) &= 0 \\
V(\dot{v}_0) &= 0 & \dot{V}(x_0, y_0, \dot{v}_0, \dot{v}_0) &= 0
\end{aligned} \tag{5.7}$$

where the subscript 0 is used to designate the values of x , y , v , and $\dot{v}$ for the steady-state solutions. We readily obtain from the third equation of (5.7)

$$\dot{v}_0 = 0 \tag{5.8}$$

Eliminating x_0 and y_0 in Eqs. (5.7) gives the following relations to determine

r_0 ($=\sqrt{x_0^2 + y_0^2}$) and v_0 :

$$\begin{aligned}
(A_1^2 + A_2^2) r_0^2 &= B^2 \\
\frac{r_0^2}{2(1+v_0)^2} &= B_0
\end{aligned} \tag{5.9}$$

The components x_0 and y_0 of the amplitude r_0 are given by

$$x_0^2 = \frac{r_0^2}{1 + (A_2/A_1)^2} \quad y_0^2 = \frac{r_0^2}{1 + (A_1/A_2)^2} \tag{5.10}$$

The equivalent spring constant S_0 which is defined by Eq. (3.19) is found to be

$$S_0 = \frac{r_0^2}{J(1 + \rho v_0)^4} [A_2 - \nu k_s A_1 - \rho(1 + \rho v_0)J] \quad (5.11)$$

where

$$J = A_1^2 + A_2^2 - \frac{3}{2} \alpha r_0^2 (A_2 - \nu k_s A_1) \quad (5.11a)$$

Equation (5.11) shows that the value of S_0 increases with the increase in α because the value of J in the denominator decreases. This shows that the equivalent spring constant S_0 increases due to the effect of magnetic saturation.

5.4 Conditions for Stability

The steady-state solutions given by Eqs. (5.8) through (5.10) are maintained only when they are stable. By a procedure analogous to that of Sec. 3.3.4, we make use of the Routh-Hurwitz criterion to obtain the conditions for stability of the steady-state solutions. The result is

$$\begin{aligned} a_1 > 0 \quad a_2 > 0 \quad a_3 > 0 \quad a_4 > 0 \\ a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0 \end{aligned} \quad (5.12)$$

where

$$\begin{aligned} a_1 &= k_{11} + k_2 \\ a_2 &= k_{11} k_2 + \frac{J}{4\nu^2} - \frac{\rho r_0^2}{(1 + \rho v_0)^3} \\ a_3 &= \frac{k_2 J}{4\nu^2} - \frac{k_s r_0^2 (2\nu^2 - A_2)}{4\nu^2 (1 + \rho v_0)^4} - \frac{\rho k_{11} r_0^2}{(1 + \rho v_0)^3} \\ a_4 &= \frac{r_0^2}{4\nu^2 (1 + \rho v_0)^4} [A_2 - \nu k_s A_1 - \rho(1 + \rho v_0)J] \end{aligned} \quad (5.12a)$$

and

$$k_{11} = k_1 + k_s \left(\frac{v_0}{1 + \rho v_0} + \frac{3}{2} \alpha r_0^2 \right) \quad (5.12b)$$

However, as a brief criterion for the stability, the static stability condition is available though it is the only necessary condition for stable suspension. It states that $S_0 > 0$.

From Eq. (5. 11) we obtain*

$$A_2 - \nu k_s A_1 - \rho (1 + \rho v_0) J > 0$$

(5. 13)

and

$$J > 0$$

We see that the first condition of (5. 13) guarantees $a_4 > 0$, and the second condition of (5. 13) is a necessary condition for $a_3 > 0$. For $\alpha = 0$, the second condition of (5. 13) is always fulfilled. Hence this condition is an additional condition brought by the effect of magnetic saturation, and for a large value of α , there is a case in which this condition is not fulfilled while the first condition of (5. 13) is satisfied. In this case, the system becomes statically unstable and quasi-periodic oscillations may occur even if the mechanical damping of the system is large. The occurrence of these kinds of oscillations will be discussed in detail in the following section.

5.5 Quasi-Periodic Oscillations Caused by Magnetic Saturation

5.5.1 Generation of Quasi-Periodic Oscillations

It has been shown in the preceding chapter that quasi-periodic oscillations may arise in the magnetic suspension system when the steady-state solutions are dynamically unstable. However, this kind of oscillation may also occur when the steady-state solution as determined by Eqs. (5. 9) becomes statically unstable owing to the effect of magnetic saturation. In order to explain the occurrence of the oscillation due to the effect of magnetic saturation, we assume that the viscous mechanical damping k_2 is large, and hence $d\dot{v}/d\tau$ in Eqs. (5. 6) can be neglected near the steady state. Under this assumption, Eqs. (5. 6) may be approximated to

* One may consider that the condition for $S_0 > 0$ is also given by

$$A_2 - \nu k_s A_1 - \rho (1 + \rho v_0) J < 0$$

and

$$J < 0$$

But these two conditions are never fulfilled simultaneously. To show this, let us assume that these two conditions are fulfilled simultaneously. Then we obtain

$$A_2 - \nu k_s A_1 < \rho (1 + \rho v_0) J < 0$$

Thus the value $A_2 - \nu k_s A_1$ must at least be negative. However, if this value is negative, the value of J fixed by Eq. (5. 11a) must be positive. This result conflicts with the assumption that $J < 0$.

$$\begin{aligned}
\frac{dx}{d\tau} &= \frac{1}{2\nu} (-A_1 x + A_2 y + B) \\
\frac{dy}{d\tau} &= -\frac{1}{2\nu} (A_2 x + A_1 y) \\
\frac{dv}{d\tau} &= \frac{1}{k_2} [B_0 - \frac{r^2}{2(1+\rho v)^2}]
\end{aligned} \tag{5.14}$$

Since k_2 is large, it is evident that $dv/d\tau \ll dx/d\tau$ and $dv/d\tau \ll dy/d\tau$, so that the representative point is first governed by

$$\begin{aligned}
\frac{dx}{d\tau} &= \frac{1}{2\nu} (-A_1 x + A_2 y + B) \\
\frac{dy}{d\tau} &= -\frac{1}{2\nu} (A_2 x + A_1 y)
\end{aligned}$$

and approaches the characteristic curve defined by $dx/d\tau = 0$ and $dy/d\tau = 0$, or

$$(A_1^2 + A_2^2) r^2 = B^2 \tag{5.15}$$

During this transient $v(\tau)$ is held nearly constant. After this period $dx/d\tau$, $dy/d\tau$, and $dv/d\tau$ will all be of the same order in magnitude. In Fig. 5.1 is shown the characteristic curve (5.15) for

$$\nu = 2.0 \quad k_1 = 0.121 \quad k_s = 0.049 \quad \rho = 0.09 \quad \alpha = 10.0 \quad B = 0.35$$

Also plotted in the figure is the curve represented by

$$\frac{dv}{d\tau} = \frac{1}{k_2} [B_0 - \frac{r^2}{2(1+\rho v)^2}] = 0 \tag{5.16}$$

for $B_0 = 0.064$. The intersections P_1 and P_2 of these curves represent equilibrium states, since these points are satisfied by Eqs. (5.9). However, these equilibrium states are statically unstable; the points P_1 and P_2 do not fulfill the second and the first condition of (5.13), respectively. As suggested in the preceding section, the point P_1 is the unstable equilibrium point resulting from the effect of magnetic saturation. Around this point, a limit cycle occurs as follows. Since $dv/d\tau$ is negative in the region above the curve (5.16) and positive below the curve, the representative point around the point P_1 will gradually move in the direction of the arrows with increasing τ . Hence, discontinuous jumps occur at the limiting points Q and R , and the representative point keeps on moving near the limit cycle represented by the thick line in the figure.

As expected from the third equation of (5.14), the period required for the representative point to complete one revolution along the limit cycle decreases with the increase in k_2 . A more concrete example of the limit cycle will be given in what follows.

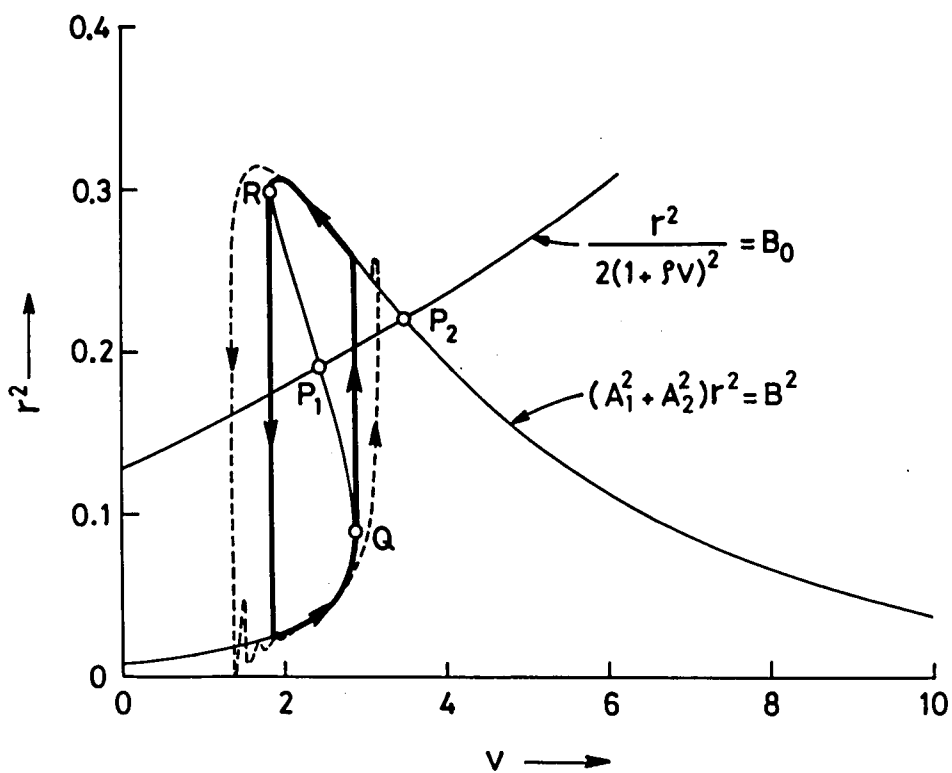


Fig. 5. 1 Limit cycle with discontinuities due to magnetic saturation.

5.5.2 Numerical Analysis

A numerical analysis of the system (5. 6) is carried out for the same values of the system parameters as those in Fig. 5. 1, that is,

$$\begin{aligned} \nu &= 2.0 & k_1 &= 0.121 & k_s &= 0.049 & k_2 &= 3.0 & \rho &= 0.09 \\ \alpha &= 10.0 & B_0 &= 0.064 & B &= 0.35 \end{aligned}$$

These values are nearly corresponding to those of the experimental model which will be shown in the following section. After a sufficiently long period of the time τ , the representative point moves along the limit cycle as illustrated in Fig. 5. 2. The projection of the limit cycle on the $r^2 v$ plane is shown by dashed line in Fig. 5. 1. The waveforms of the oscillation corresponding to the limit cycle are shown in Fig. 5. 3. The period of the oscillation is $\pi \times 105.3 \dots$; this period decreases continuously as k_2 increases. It is clear from Eqs. (5. 5) that, when x and y vary periodically, the amplitude and phase of the oscillating flux u also vary periodically; in other words, a quasi-periodic oscillation occurs in the system described by Eqs. (5. 4).

To compare the validity of the analysis, a numerical solution of Eqs. (5. 4) is also calculated by using a digital computer and shown in Fig. 5. 4. The waveforms of x and y are obtained by inserting the numerical solutions u and $du/d\tau$ into Eqs. (5. 5). These curves show satisfactory agreement with the theoretical result illustrated in Fig. 5. 3.

5.5.3 Experimental Investigation

In this section we observe the quasi-periodic oscillation which occurs due to the effect of magnetic saturation, and we compare the results obtained from experiments with those of the foregoing analysis. In that analysis we have assumed that the magnetic saturation occurs in the core of the electromagnet. However, here we perform experiments for a case where the magnetic saturation occurs in the core of the suspended body. To this case is also applicable the foregoing analysis because the magnetic flux for suspension flows into the electromagnet and the suspended body in series.

The outline of the experimental model is shown in Fig. 5.5. The same electromagnet as that in Fig.3. 7 is used for this model. The parameters of the experimental model are as follows:

$$\begin{aligned} n &= 400 \text{ turn} & C &= 10.59 \mu\text{F} \\ R &= 1000 \Omega & R_s &= 5.97 \Omega \\ M &= 0.235 \text{ kg} & k_m &= 1537.2 \text{ N} \cdot \text{s/m} \\ f &= 250 \text{ Hz} \end{aligned}$$

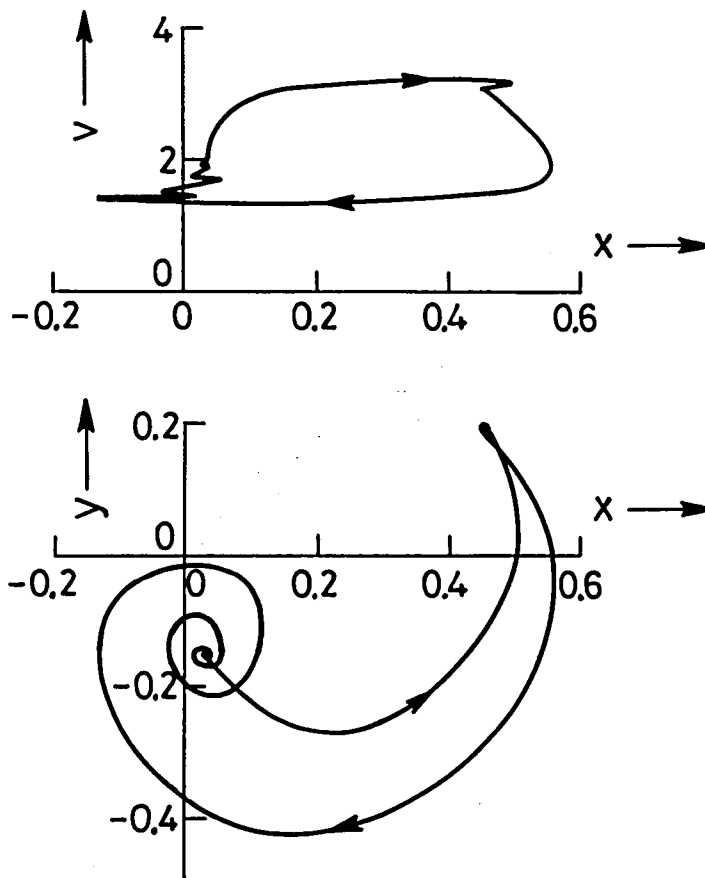


Fig. 5. 2 Limit cycle in the autonomous system (5. 6).

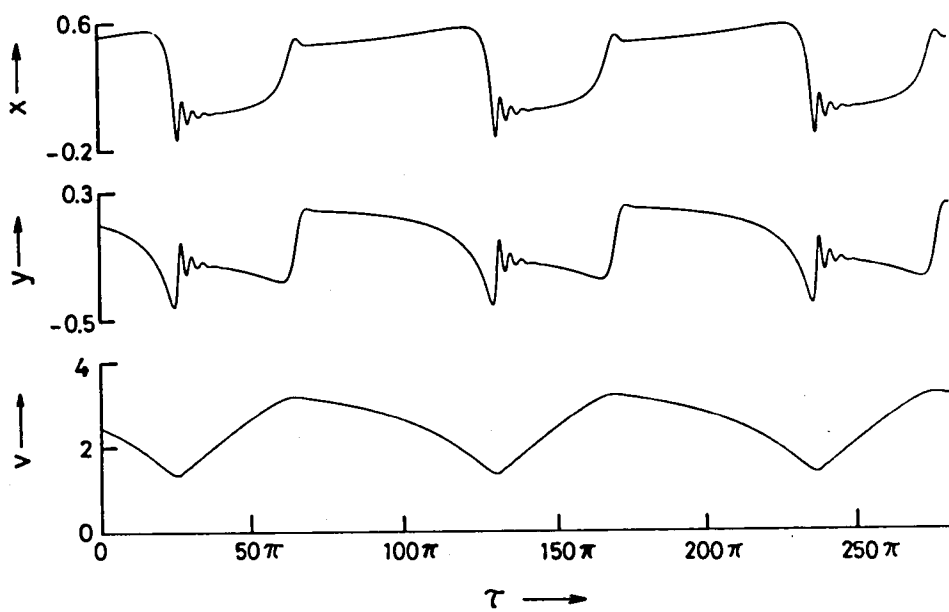


Fig. 5. 3 Waveforms of the self-excited oscillation correlated with the limit cycle of Fig. 5. 2.

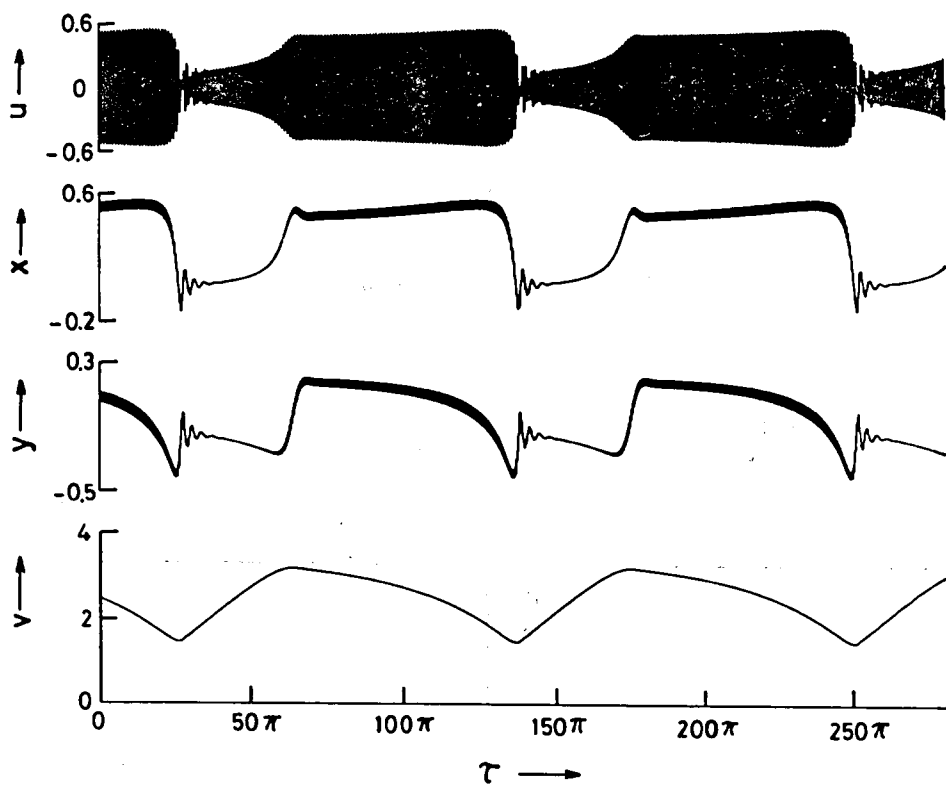


Fig. 5. 4 Waveforms of a quasi-periodic oscillation in the nonautonomous system (5. 4). The system parameters are the same as in Fig. 5. 3.

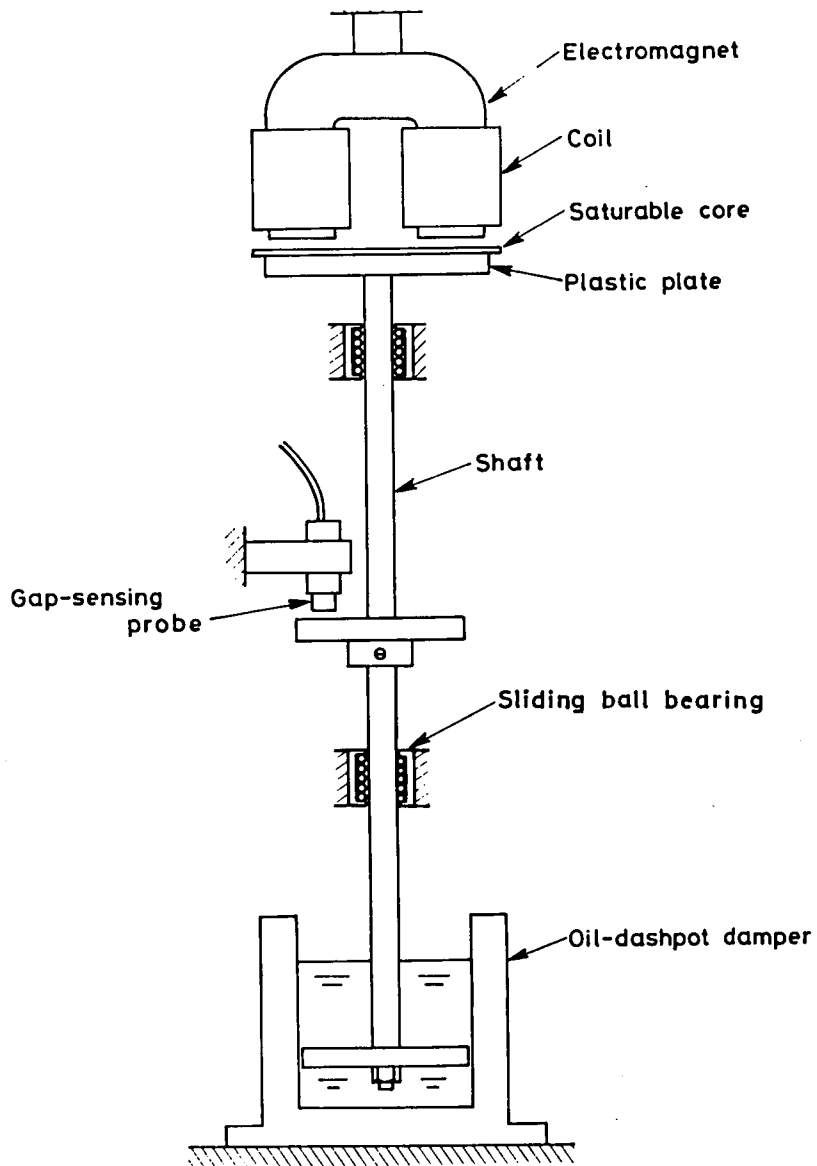


Fig. 5.5 Experimental model.

The saturable core used for the suspended body is made of 3 sheets of silicon steel. The size of one sheet is 65 x 25 x 0.35 mm. The magnetization curve of the electromagnet for the gap distance $h = 0$ is shown in Fig. 5.6. In the foregoing analysis, we have approximated this curve to the cubic function represented by Eq. (5. 2).

Figure 5.7 shows the static relationship between the gap distance h and the coil voltage E_L with the supply voltage E_e as a parameter. We obtain these curves by shifting the suspended body vertically against the magnetic attractive force exerted on it. The thick line in the figure is a relation for $E_e = 4.7$ V. In this case, as the gap distance h is increased, the coil voltage E_L jumps with the accompanying increase in the magnitude. When the experiment is reversed, i. e., with decreasing h , it is found that the same curve is not retraced completely, but the jump from the higher to the lower voltage takes place at a lower value of h than before. The voltage E_L is directly related to the amplitude r of the nodimensional flux u since the time derivative of the flux ϕ is proportional to E_L . Thus we see that the jumps like those in Fig. 5. 1 take place due to the effect of magnetic saturation.

The observed waveforms of the quasi-periodic oscillation are shown by the oscillogram in Fig. 5. 8. The detailed waveforms of the oscillation are also shown by the oscillograms (a) and (b) in Fig. 5. 9, in which the time scale is expanded. The limit cycle on the E_L h plane correlated with the quasi-periodic oscillation is shown by the thick line in Fig. 5. 10, where the static characteristic curve illustrated in Fig. 5. 7 is also shown by the dashed line.

5. 6 Concluding Remarks

The effect of magnetic saturation on the static and dynamic performance of the vertical-axis suspension device has been studied. The saturation characteristic of the iron portions in the magnetic circuit is represented by a cubic function of the magnetic flux. The fundamental equations are analyzed by using the method of averaging. It has been shown that with increase in the degree of magnetic saturation, the equivalent spring constant increases. However, if this degree exceeds a limit, the sign of the equivalent spring constant changes and the system becomes statically unstable. In this case a quasi-periodic oscillation occurs in the system even if the mechanical damping of the system is large. The occurrence of this kind of oscillation has been verified by experiments.

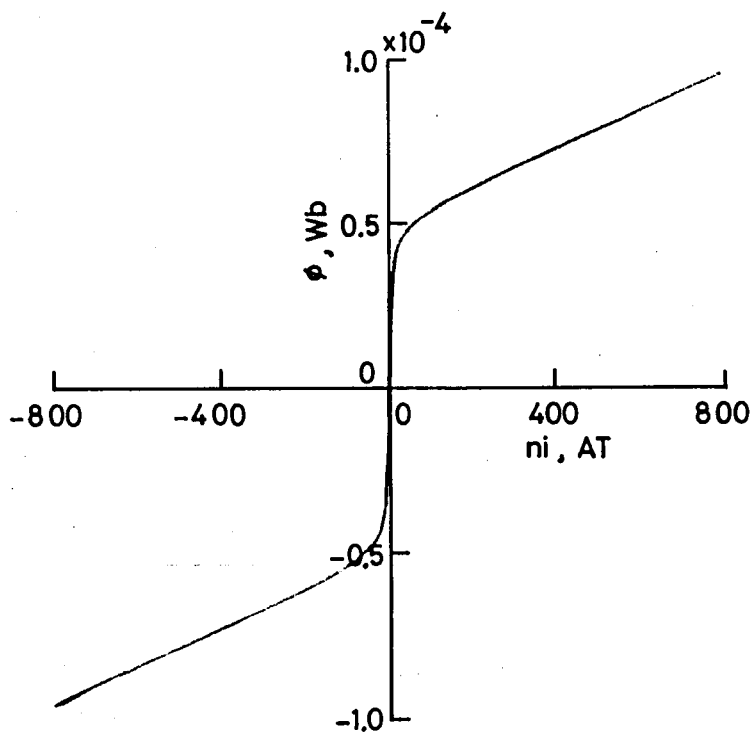


Fig. 5. 6 Magnetization curve of the electromagnet with the saturable core (suspended body) in its flux path, the gap distance being zero.

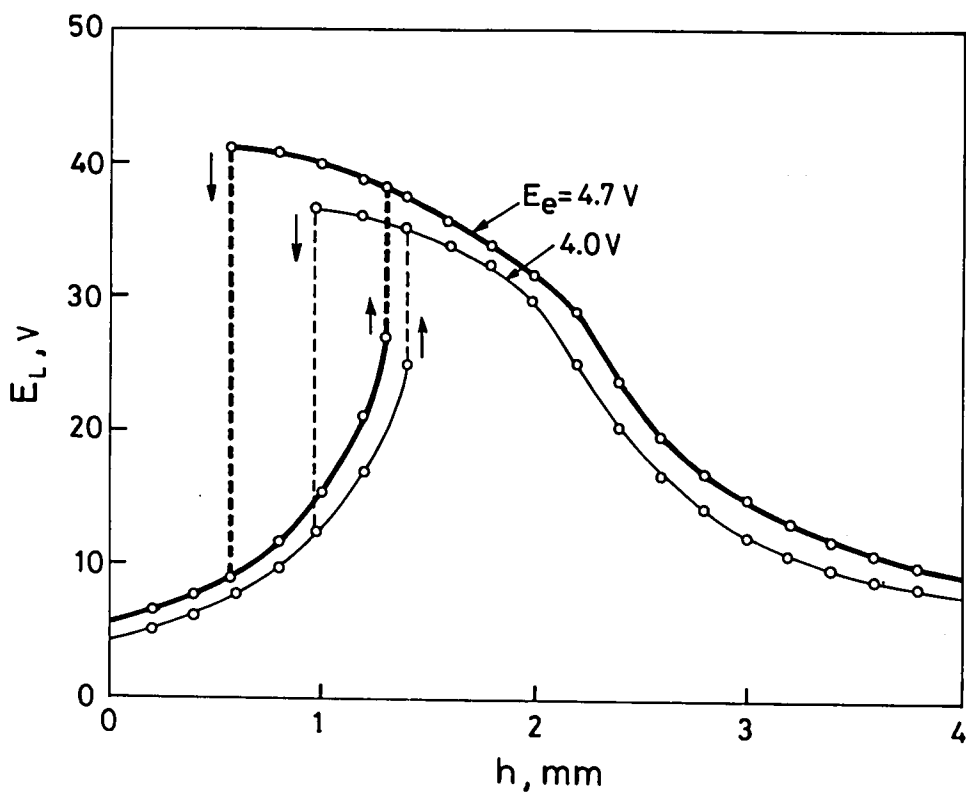


Fig. 5.7 Discontinuities of the coil voltage due to the effect of the magnetic saturation.

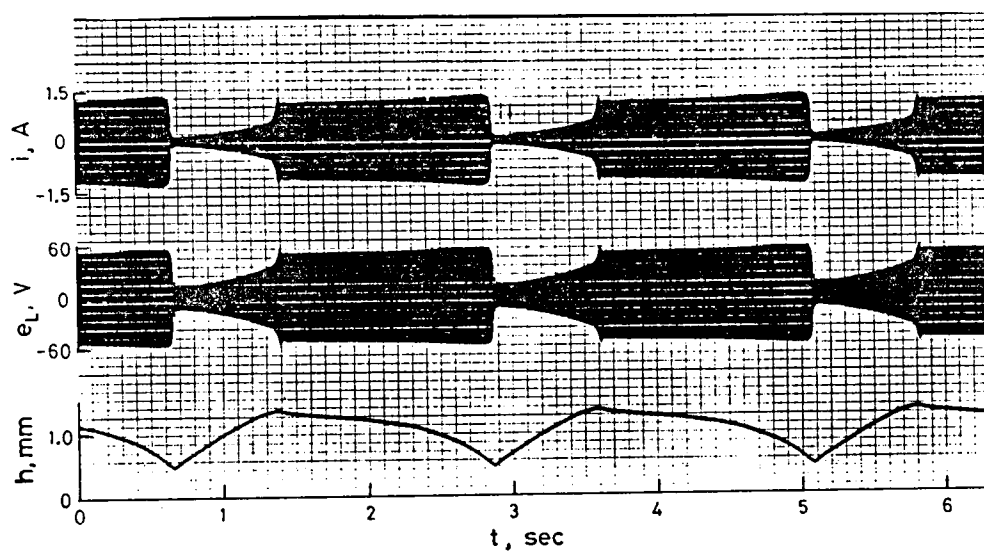
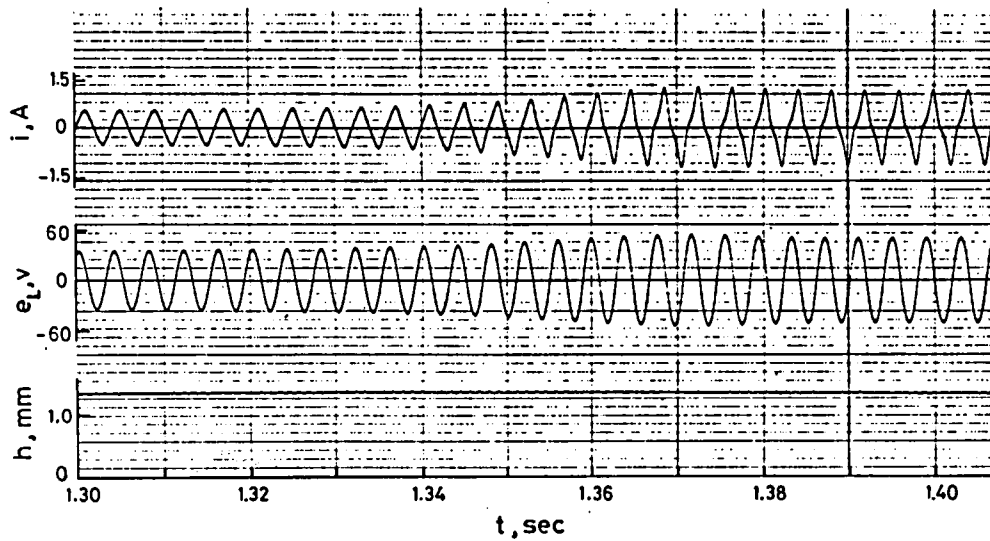
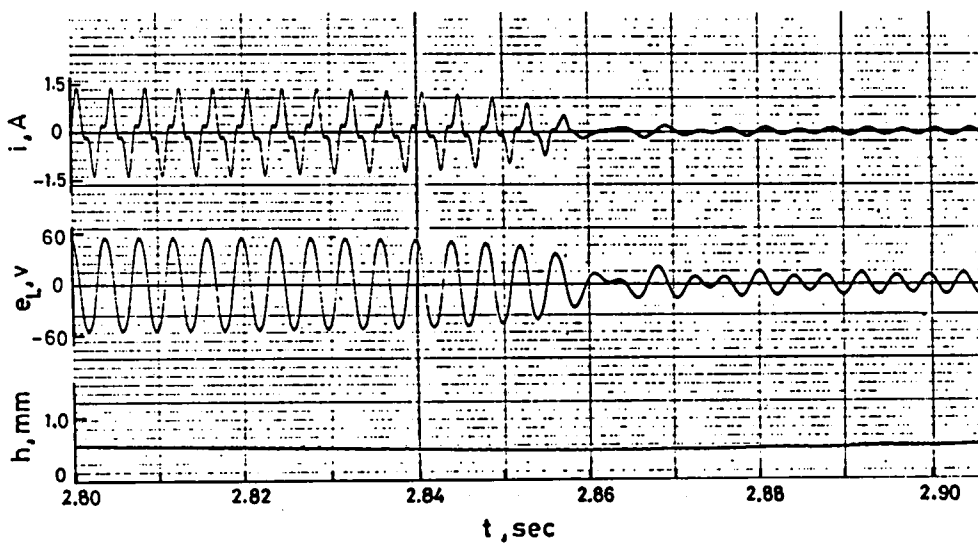


Fig. 5. 8 Waveforms of a quasi-periodic oscillation obtained by experiment ($E_e = 4.7 \text{ V}$).



(a) $t = 1.3 \sim 1.4$ sec



(b) $t = 2.8 \sim 2.9$ sec

Fig. 5. 9 Detailed waveforms of the oscillogram in Fig. 5. 8.

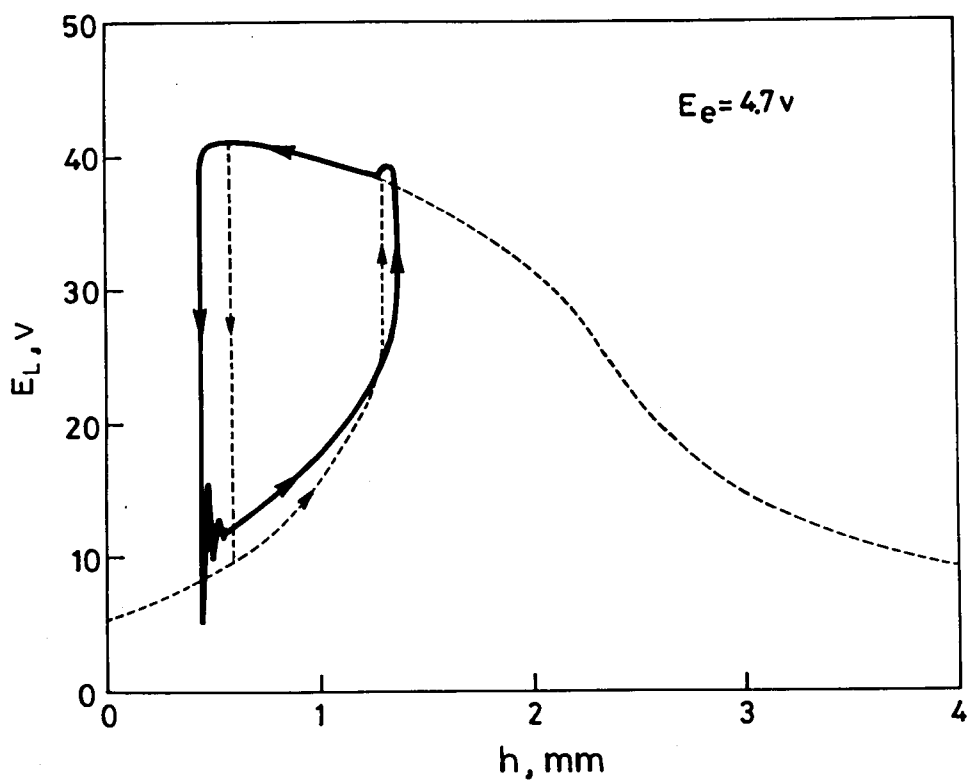


Fig. 5.10 Limit cycle in the hE_L plane obtained by experiment.

CHAPTER 6

PUSH-PULL TYPE SUSPENSION DEVICE HAVING TWO ELECTROMAGNETS

6.1 Introduction

To use magnetic suspension for centering when displacement may occur in a single axis, two electromagnets are used, as illustrated in Fig. 6.1. This type of suspension may be called a push-pull type. The restorative action of the push-pull type suspension device is achieved as follows. If the suspended body is moved toward the right, then the resultant change in inductance detunes the circuit on the right but tunes the circuit on the left. This causes the current flow on the right to decrease and the current flow on the left to increase. This imbalance in currents produces a net magnetic force to the left, which restores the body to center.

This chapter deals with the performance of this push-pull type suspension device in which the leakage flux is considered but the effects of magnetic saturation are ignored. The magnetic circuits for the two electromagnets are assumed not to be coupled, so that they can be solved independently. The fundamental equations are described by three second-order differential equations; two of them are the equations for the magnetic fluxes in the magnetic circuits, and the other is the equation of motion of the body to be supported. The method of averaging is applied to these equations and the formulas for the equivalent spring constant and the stability conditions are derived. These formulas are nearly the same as those for the vertical-axis suspension device with one electromagnet. However, with respect to design, the characteristics of the push-pull type suspension device are different from those of the vertical-axis suspension device. The basic characteristics of the push-pull type suspension device are verified by experiments.

6.2 Fundamental Equations

As we have shown in Sec. 3.2, the electric and magnetic circuit equations for the electromagnets in Fig. 6.1 may be expressed as follows:*

* Throughout this chapter, the subscript j designates $j = 1, 2$.

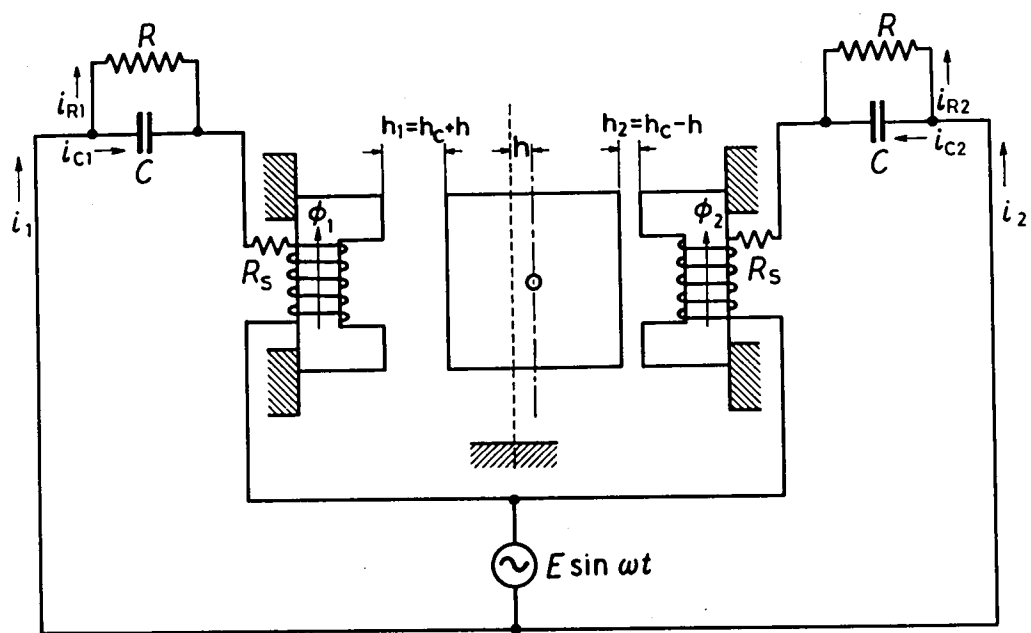


Fig. 6. 1 Push-pull type suspension device.

$$\begin{aligned}
n \frac{d\phi_j}{dt} + R_s i_j + R i_{Rj} &= E \sin \omega t \\
R i_{Rj} &= \frac{1}{C} \int i_{Cj} dt \\
i_j &= i_{Rj} + i_{Cj} \\
n i_j &= \mathcal{R}_j \phi_j \quad (j = 1, 2)
\end{aligned} \tag{6.1}$$

where

$$\begin{aligned}
\mathcal{R}_j &= \frac{\mathcal{R}_{ij} \mathcal{R}_l}{\mathcal{R}_{ij} + \mathcal{R}_l} & \mathcal{R}_{ij} &= \sigma h_j + \mathcal{R}_c \\
h_1 &= h_c + h & h_2 &= h_c - h
\end{aligned} \tag{6.1a}$$

The equation of motion of the suspended body is described by

$$M \frac{d^2 h}{dt^2} + k_m \frac{dh}{dt} + F_{m1} - F_{m2} = F_d \tag{6.2}$$

where F_{m1} , F_{m2} are the magnetic attractive forces of the electromagnets, and F_d is the external disturbance force. Referring to Eq. (3.5), we readily obtain

$$F_{mj} = \frac{\sigma \phi_j^2}{2[1 + (\sigma h_j + \mathcal{R}_c)/\mathcal{R}_l]^2} \quad (j = 1, 2) \tag{6.3}$$

By a procedure analogous to that of Sec. 3.2, Eqs. (6.1) and (6.2) are transformed to

$$\begin{aligned}
\frac{d^2 u_j}{d\tau^2} + k_1 \frac{du_j}{d\tau} + k_s \frac{d}{d\tau} \left(\frac{u_j v_j}{1 + \rho v_j} \right) + \frac{u_j v_j}{1 + \rho v_j} &= B \cos \nu \tau \\
\frac{d^2 v}{d\tau^2} + k_2 \frac{dv}{d\tau} + \left(\frac{u_1}{1 + \rho v_1} \right)^2 - \left(\frac{u_2}{1 + \rho v_2} \right)^2 &= B_0 \\
v_1 &= v_c + v & v_2 &= v_c - v
\end{aligned} \tag{6.4}$$

where

$$\phi_j = \Phi_n u_j \quad h_j + \frac{\mathcal{R}_c}{\sigma} = H_n v_j \quad \omega = \nu \omega_0 \quad B_0 = \frac{F_d}{\omega_0^2 M H_n}$$

and τ , k_1 , k_2 , k_s , ρ , B are defined, as before, by Eqs. (3.8a). The base quantities H_n and Φ_n may be fixed by Eqs. (3.7).

Following the same procedure as that of Sec. 3.3.1, we introduce new variables $x_j(\tau)$ and $y_j(\tau)$ defined by

$$\begin{aligned}
x_j(\tau) &= u_j(\tau) \sin \nu \tau + \frac{1}{\nu} \frac{du_j}{d\tau} \cos \nu \tau \\
y_j(\tau) &= u_j(\tau) \cos \nu \tau - \frac{1}{\nu} \frac{du_j}{d\tau} \sin \nu \tau
\end{aligned} \tag{6.5}$$

Then, substituting Eqs. (6.5) into (6.4) and solving for the derivatives $dx_j/d\tau$ and $dy_j/d\tau$ and further using the method of averaging, we obtain an autonomous system:

$$\begin{aligned}\frac{dx_j}{d\tau} &= \frac{1}{2\nu} (-A_{1j} \dot{x}_j + A_{2j} \dot{y}_j + B) \equiv X_j(x_j, y_j, v, \dot{v}) \\ \frac{dy_j}{d\tau} &= -\frac{1}{2\nu} (A_{2j} \dot{x}_j + A_{1j} \dot{y}_j) \equiv Y_j(x_j, y_j, v, \dot{v}) \quad (j = 1, 2) \\ \frac{dv}{d\tau} &= \dot{v} \equiv V(\dot{v}) \\ \frac{d\dot{v}}{d\tau} &= B_0 - k_2 \dot{v} - \frac{r_1^2}{2(1+\rho v_1)^2} + \frac{r_2^2}{2(1+\rho v_2)^2} \equiv \dot{V}(x_1, y_1, x_2, y_2, v, \dot{v})\end{aligned}\tag{6.6}$$

where

$$\begin{aligned}r_j^2 &= x_j^2 + y_j^2 \\ A_{1j} &= \nu \left(k_1 + \frac{k_s v_j}{1+\rho v_j} \right) \\ A_{2j} &= \nu^2 - \frac{v_j}{1+\rho v_j} - \frac{k_s \dot{v}_j}{(1+\rho v_j)^2}\end{aligned}\tag{6.6a}$$

6.3 Steady-State Solutions and Equivalent Spring Constant

The steady-state solutions for which the components $x_j(\tau)$, $y_j(\tau)$, $v(\tau)$, and $\dot{v}(\tau)$ are constant are determined by

$$\begin{aligned}X_j(x_{j0}, y_{j0}, v_0, \dot{v}_0) &= 0 & Y_j(x_{j0}, y_{j0}, v_0, \dot{v}_0) &= 0 \\ V(\dot{v}_0) &= 0 & \dot{V}(x_{10}, y_{10}, x_{20}, y_{20}, v_0, \dot{v}_0) &= 0\end{aligned}\tag{6.7}$$

where the subscript 0 is used to designate the values of x_j , y_j , v , and $\dot{v}$ for the steady-state solutions. From the third equation of (6.7) we obtain

$$\dot{v}_0 = 0\tag{6.8}$$

Eliminating x_{j0} and y_{j0} in Eqs. (6.7) gives the following relations to determine r_{j0} ($=\sqrt{x_{j0}^2 + y_{j0}^2}$) and v_0 , that is,

$$\begin{aligned}(A_{1j}^2 + A_{2j}^2) r_{j0}^2 &= B^2 \quad (j = 1, 2) \\ \frac{r_{10}^2}{2(1+\rho v_{10})^2} - \frac{r_{20}^2}{2(1+\rho v_{20})^2} &= B_0\end{aligned}\tag{6.9}$$

where

$$v_{10} = v_c + v_0 \quad v_{20} = v_c - v_0$$

The components x_{j0}, y_{j0} of the amplitude r_{j0} are given by

$$x_{j0}^2 = \frac{r_{j0}^2}{1 + (A_{2j}/A_{1j})^2} \quad y_{j0}^2 = \frac{r_{j0}^2}{1 + (A_{1j}/A_{2j})^2} \quad (6.10)$$

The equivalent spring constant due to one electromagnet is given by Eq. (3.21). Hence for the push-pull type under consideration it is given by

$$S_0 = \sum_{j=1}^2 \frac{(\nu^2 - v_{j00})(1 - \rho\nu^2) - \nu^2 k_{j00}(k_s + \rho k_1)}{(1 + \rho v_{j0})^3 [(\nu^2 - v_{j00})^2 + \nu^2 k_{j00}^2]} B^2 \quad (6.11)$$

where

$$v_{j00} = \frac{v_{j0}}{1 + \rho v_{j0}} \quad k_{j00} = k_1 + k_s v_{j00} \quad (6.11a)$$

For a special case of $B_0 = 0$, Eqs. (6.9) through (6.11) lead to

$$\begin{aligned} v_{10} &= v_{20} = v_c & v_0 &= 0 \\ A_{11} &= A_{12} = \nu k_{c0} \equiv A_1 \\ A_{21} &= A_{22} = \nu^2 - v_{c0} \equiv A_2 \\ r_{10}^2 &= r_{20}^2 = \frac{B^2}{A_1^2 + A_2^2} \equiv r_0^2 \\ x_{10}^2 &= x_{20}^2 = \frac{r_0^2}{1 + (A_2/A_1)^2} \equiv x_0^2 \\ y_{10}^2 &= y_{20}^2 = \frac{r_0^2}{1 + (A_1/A_2)^2} \equiv y_0^2 \end{aligned} \quad (6.12)$$

and

$$S_0 = \frac{2[(\nu^2 - v_{c0})(1 - \rho\nu^2) - \nu k_{c0}(k_s + \rho k_1)]}{(1 + \rho v_c)^3 [(\nu^2 - v_{c0})^2 + \nu^2 k_{c0}^2]} B^2 \quad (6.13)$$

where

$$v_{c0} = \frac{v_c}{1 + \rho v_c} \quad k_{c0} = k_1 + k_s v_{c0} \quad (6.13a)$$

6.4 Conditions for Stability

The steady-state solution given by Eqs. (6.8) through (6.10) is maintained only when it is stable. As we have done in Sec. 2.3.4, the stability of the solution is tested by the behavior of small variations ξ_j, η_j ($j=1, 2$), ζ , and ψ defined by

$$\begin{aligned} x_j &= x_{j0} + \xi_j & y_j &= y_{j0} + \eta_j \\ v &= v_0 + \zeta & \dot{v} &= \dot{v}_0 + \psi \end{aligned} \quad (6.14)$$

Substituting Eqs. (6. 14) into (6. 6) gives the following variational equations.

$$\begin{aligned}
 \frac{d\xi_1}{d\tau} &= a_{11}\xi_1 + a_{12}\eta_1 + a_{13}\xi_2 + a_{14}\eta_2 + a_{15}\zeta + a_{16}\psi \\
 \frac{d\eta_1}{d\tau} &= a_{21}\xi_1 + a_{22}\eta_1 + a_{23}\xi_2 + a_{24}\eta_2 + a_{25}\zeta + a_{26}\psi \\
 \frac{d\xi_2}{d\tau} &= a_{31}\xi_1 + a_{32}\eta_1 + a_{33}\xi_2 + a_{34}\eta_2 + a_{35}\zeta + a_{36}\psi \\
 \frac{d\eta_2}{d\tau} &= a_{41}\xi_1 + a_{42}\eta_1 + a_{43}\xi_2 + a_{44}\eta_2 + a_{45}\zeta + a_{46}\psi \\
 \frac{d\zeta}{d\tau} &= a_{51}\xi_1 + a_{52}\eta_1 + a_{53}\xi_2 + a_{54}\eta_2 + a_{55}\zeta + a_{56}\psi \\
 \frac{d\psi}{d\tau} &= a_{61}\xi_1 + a_{62}\eta_1 + a_{63}\xi_2 + a_{64}\eta_2 + a_{65}\zeta + a_{66}\psi
 \end{aligned} \tag{6. 15}$$

with

$$\begin{aligned}
 a_{11} &= \left(\frac{\partial X_1}{\partial x_1} \right)_0 = -\frac{A_{11}}{2\nu} \\
 a_{12} &= \left(\frac{\partial X_1}{\partial y_1} \right)_0 = \frac{A_{12}}{2\nu} \\
 a_{13} &= \left(\frac{\partial X_1}{\partial x_2} \right)_0 = 0 \\
 a_{14} &= \left(\frac{\partial X_1}{\partial y_2} \right)_0 = 0 \\
 a_{15} &= \left(\frac{\partial X_1}{\partial v} \right)_0 = -\frac{y_{10} + \nu k_s x_{10}}{2\nu(1 + \rho v_{10})^2} \\
 a_{16} &= \left(\frac{\partial X_1}{\partial \dot{v}} \right)_0 = -\frac{k_s y_{10}}{2\nu(1 + \rho v_{10})^2} \\
 a_{21} &= \left(\frac{\partial Y_1}{\partial x_1} \right)_0 = -\frac{A_{12}}{2\nu} \\
 a_{22} &= \left(\frac{\partial Y_1}{\partial y_1} \right)_0 = -\frac{A_{11}}{2\nu} \\
 a_{23} &= \left(\frac{\partial Y_1}{\partial x_2} \right)_0 = 0 \\
 a_{24} &= \left(\frac{\partial Y_1}{\partial y_2} \right)_0 = 0 \\
 a_{25} &= \left(\frac{\partial Y_1}{\partial v} \right)_0 = \frac{x_{10} - \nu k_s y_{10}}{2\nu(1 + \rho v_{10})^2}
 \end{aligned} \tag{6. 15a}$$

$$a_{26} = \left(\frac{\partial Y_1}{\partial \dot{v}} \right)_0 = \frac{k_s x_{10}}{2\nu(1 + \rho v_{10})^2}$$

$$a_{31} = \left(\frac{\partial X_2}{\partial x_1} \right)_0 = 0$$

$$a_{32} = \left(\frac{\partial X_2}{\partial y_1} \right)_0 = 0$$

$$a_{33} = \left(\frac{\partial X_2}{\partial x_2} \right)_0 = -\frac{A_{21}}{2\nu}$$

$$a_{34} = \left(\frac{\partial X_2}{\partial y_2} \right)_0 = -\frac{A_{22}}{2\nu}$$

$$a_{35} = \left(\frac{\partial X_2}{\partial v} \right)_0 = \frac{y_{20} + \nu k_s x_{20}}{2\nu(1 + \rho v_{20})^2}$$

$$a_{36} = \left(\frac{\partial X_2}{\partial \dot{v}} \right)_0 = \frac{k_s y_{20}}{2\nu(1 + \rho v_{20})^2}$$

$$a_{41} = \left(\frac{\partial Y_2}{\partial x_1} \right)_0 = 0$$

$$a_{42} = \left(\frac{\partial Y_2}{\partial y_1} \right)_0 = 0$$

$$a_{43} = \left(\frac{\partial Y_2}{\partial x_2} \right)_0 = -\frac{A_{22}}{2\nu}$$

(6. 15a)

$$a_{44} = \left(\frac{\partial Y_2}{\partial y_2} \right)_0 = -\frac{A_{21}}{2\nu}$$

$$a_{45} = \left(\frac{\partial Y_2}{\partial v} \right)_0 = -\frac{x_{20} - \nu k_s y_{20}}{2\nu(1 + \rho v_{20})^2}$$

$$a_{46} = \left(\frac{\partial Y_2}{\partial \dot{v}} \right)_0 = -\frac{k_s x_{20}}{2\nu(1 + \rho v_{20})^2}$$

$$a_{51} = \left(\frac{\partial V}{\partial x_1} \right)_0 = 0$$

$$a_{52} = \left(\frac{\partial V}{\partial y_1} \right)_0 = 0$$

$$a_{53} = \left(\frac{\partial V}{\partial x_2} \right)_0 = 0$$

$$a_{54} = \left(\frac{\partial V}{\partial y_2} \right)_0 = 0$$

$$a_{55} = \left(\frac{\partial V}{\partial v} \right)_0 = 0$$

$$\begin{aligned}
a_{56} &= \left(\frac{\partial V}{\partial \dot{v}} \right)_0 = 1 \\
a_{61} &= \left(\frac{\partial \dot{V}}{\partial x_1} \right)_0 = - \frac{x_{10}}{(1 + \rho v_{10})^2} \\
a_{62} &= \left(\frac{\partial \dot{V}}{\partial y_1} \right)_0 = - \frac{y_{10}}{(1 + \rho v_{10})^2} \\
a_{63} &= \left(\frac{\partial \dot{V}}{\partial x_2} \right)_0 = \frac{x_{20}}{(1 + \rho v_{20})^2} \\
a_{64} &= \left(\frac{\partial \dot{V}}{\partial y_2} \right)_0 = \frac{y_{20}}{(1 + \rho v_{20})^2} \\
a_{65} &= \left(\frac{\partial \dot{V}}{\partial v} \right)_0 = \frac{\rho r_{10}^2}{(1 + \rho v_{10})^3} + \frac{\rho r_{20}^2}{(1 + \rho v_{20})^3} \\
a_{66} &= \left(\frac{\partial \dot{V}}{\partial \dot{v}} \right)_0 = -k_2
\end{aligned} \tag{6.15a}$$

where $\left(\frac{\partial X_1}{\partial x_1} \right)_0, \dots, \left(\frac{\partial \dot{V}}{\partial \dot{v}} \right)_0$ denote the values of $\frac{\partial X_1}{\partial x_1}, \dots, \frac{\partial \dot{V}}{\partial \dot{v}}$ at $x_j = x_{j0}, y_j = y_{j0}$ ($j = 1, 2$), $v = v_0$, and $\dot{v} = \dot{v}_0$, respectively.

The characteristic equation of the system (6. 15) is given by

$$\begin{vmatrix}
a_{11} - \lambda & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
a_{21} & a_{22} - \lambda & a_{23} & a_{24} & a_{25} & a_{26} \\
a_{31} & a_{32} & a_{33} - \lambda & a_{34} & a_{35} & a_{36} \\
a_{41} & a_{42} & a_{43} & a_{44} - \lambda & a_{45} & a_{46} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} - \lambda & a_{56} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} - \lambda
\end{vmatrix} = 0 \tag{6. 16}$$

The steady-state solution is stable provided that the real parts of λ 's are negative.

For the special case of $B_0 = 0$, the amplitude and phase of the steady-state solution are given by Eqs. (6. 12). In this case the characteristic equation (6. 16) is reduced to

$$\begin{vmatrix}
a_{11} - \lambda & a_{12} \\
a_{21} & a_{22} - \lambda
\end{vmatrix}
\begin{vmatrix}
a_{33} - \lambda & a_{34} & a_{35} & a_{36} \\
a_{43} & a_{44} - \lambda & a_{45} & a_{46} \\
a_{53} & a_{54} & a_{55} - \lambda & a_{56} \\
a_{63} & a_{64} & a_{65} & a_{66} - \lambda
\end{vmatrix} = 0$$

or

$$(\lambda^2 + a_1 \lambda + a_2) (\lambda^4 + b_1 \lambda^3 + b_2 \lambda^2 + b_3 \lambda + b_4) = 0 \tag{6. 17}$$

where

$$\begin{aligned}
 a_1 &= k_{c0} \\
 a_2 &= \frac{1}{4\nu^2} (A_1^2 + A_2^2) \\
 b_1 &= k_{c0} + k_2 \\
 b_2 &= k_{c0} k_2 + \frac{1}{4\nu^2} (A_1^2 + A_2^2) - \frac{2\rho r_0^2}{(1 + \rho v_c)^3} \\
 b_3 &= \frac{k_2}{4\nu^2} (A_1^2 + A_2^2) - \frac{k_s r_0^2}{(1 + \rho v_c)^4} - \frac{2\rho k_{c0} r_0^2}{(1 + \rho v_c)^3} \\
 b_4 &= \frac{r_0^2}{2\nu^2 (1 + \rho v_c)^3} [(\nu^2 - v_{c0})(1 - \rho\nu^2) - \nu^2 k_{c0}(k_s + \rho k_1)]
 \end{aligned} \tag{6.17a}$$

By making use of the Routh-Hurwitz criterion, we obtain the stability conditions of the steady-state solution for $B_0 = 0$:

$$\begin{aligned}
 a_1 > 0 \quad a_2 > 0 \quad b_1 > 0 \quad b_2 > 0 \quad b_3 > 0 \\
 b_4 > 0 \quad D \equiv b_1 b_2 b_3 - b_1^2 b_4 > 0
 \end{aligned} \tag{6.18}$$

From Eqs. (6.17a) it can be seen that the first three conditions of (6.18) are always fulfilled. The condition $b_3 > 0$ is not necessary since if all the other conditions are satisfied, this condition is guaranteed. The condition $b_2 > 0$ may almost always be fulfilled because in practice ρ is sufficiently small. The condition $b_4 > 0$ is identical with the condition for static stability $S_0 > 0$, and $D > 0$ is the condition for dynamic stability.

6.5 Numerical Examples

A numerical analysis of the equivalent spring constant for the push-pull type suspension device is carried out for the parameters as given by

$$k_1 = 0.15 \quad k_s = 0.1 \quad k_2 = 0.05 \quad \rho = 0.1$$

with v_c , ν , B_0 , and $E/(n\omega_0\Phi_n)$ as variable parameters.

First, we consider the case for which the external disturbance force $B_0 = 0$. In this case, as shown by Eqs. (6.12), the steady-state displacement $v_0 = 0$. The equivalent spring constant S_0 is calculated from Eq. (6.13), and the stability conditions are given by conditions (6.18). The results are shown in Fig. 6.2 for $E/(n\omega_0\Phi_n) = 0.2$. The thick-line curve in the figure is the locus of the characteristics at the dynamic stability limit $D = 0$. The dynamically unstable characteristics are illustrated by the dashed-line curves. The thick-line curve is projected on

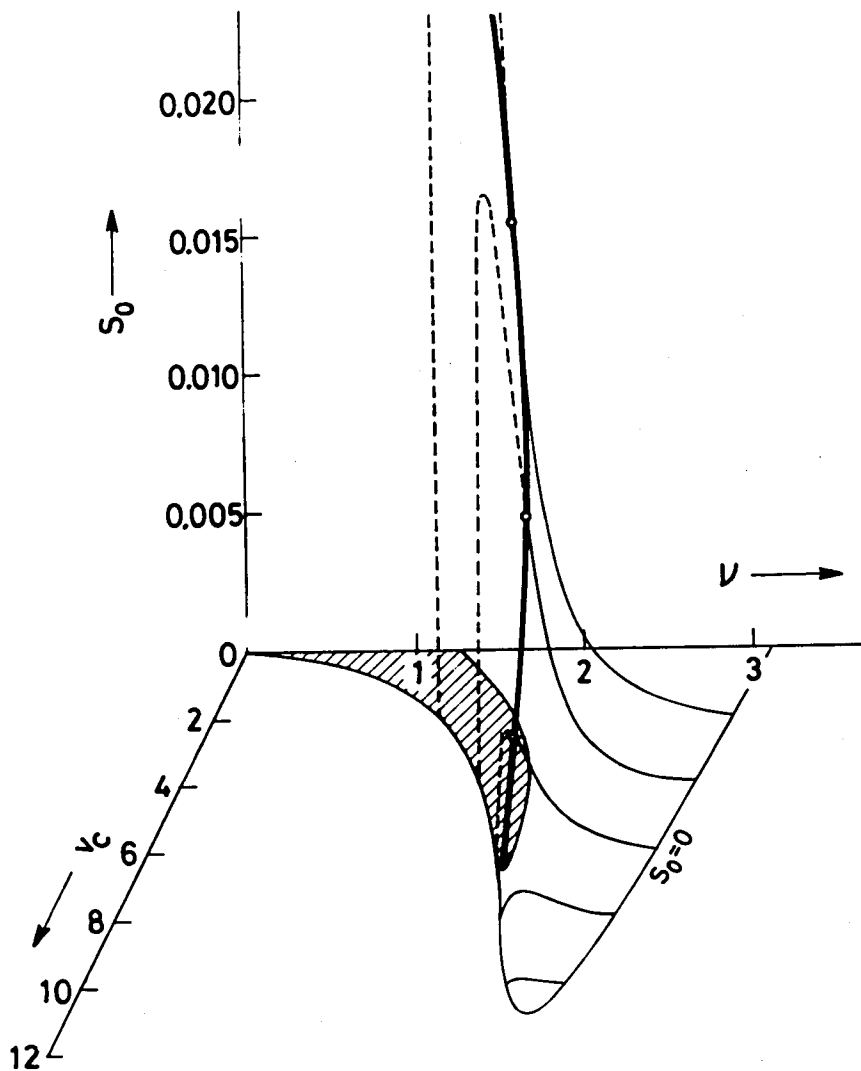


Fig. 6. 2 Characteristic of the equivalent spring constant for $E/(n\omega_0\Phi_n) = 0.2$, the system parameters being $k_1 = 0.15$, $k_s = 0.1$, $k_2 = 0.05$, $\rho = 0.1$.

the νv_c plane, and the dynamically unstable region on this plane is shown by hatching. The static stability limit on this plane is also illustrated by the curve $S_0 = 0$ in the figure. Hence, when the supply frequency ν and the gap distance at the center v_c are chosen inside the region for $S_0 > 0$ except the hatched area, the stable suspension can be achieved. Similarly, Fig. 6.3 shows the characteristic of S_0 versus $E/(n\omega_0\Phi_n)$ and ν for $v_c = 6.0$.

Second, we consider the case in which $B_0 \neq 0$. In this case, as determined by Eqs. (6.9), the steady-state displacement v_0 varies within $\pm v_c$ as B_0 varies. The corresponding equivalent spring constant S_0 which is given by Eq. (6.11) also varies with the variation of v_0 .

Figure 6.4 illustrates the limits of the statically stable region on the νv_c plane obtained from Eq. (6.11) for the same values of the system parameters k_1 , k_s , and ρ as those in Fig. 6.2. In the figure the symbols S_c and S_e denote the equivalent spring constants when the body is supported at the center ($v_0 = 0$) and at the ends of the position between two electromagnets ($v_0 = \pm v_c$), respectively. The curve $S_c = 0$ shows the static stability limit for $v_0 = 0$. Hence this curve is identical with the curve $S_0 = 0$ in Fig. 6.2. The curve $S_e = 0$ indicates the static stability limit for $v_0 = \pm v_c$. The curve $S_c = S_e$ represents the limit on which the equivalent spring constant for $v_0 = 0$ is equal to that for $v_0 = \pm v_c$. As shown in the figure, the statically stable region bordered by the curve $S_c = 0$ may be divided into three regions:

$$\begin{aligned} \text{region I:} \quad & S_c > 0 \quad S_e < 0 \\ \text{region II:} \quad & S_c > S_e > 0 \\ \text{region III:} \quad & S_e > S_c > 0 \end{aligned}$$

The examples of the characteristics of v_0 and S_0 where ν and v_c are chosen inside each region are shown in Figs. 6.5, 6.6, and 6.7. In these figures, the hatched areas show the unstable regions in which the real part of λ of the characteristic equation (6.16) is positive. We see in Fig. 6.5 that, as the external disturbance force B_0 increases, the gap distance v_0 also increases but the equivalent spring constant S_0 decreases. At the point where S_0 becomes zero, v_0 jumps to $6.0 (= v_c)$; the suspended body strikes against the electromagnet. Thus, when ν and v_c are chosen inside the region I of Fig. 6.4, statically stable suspension is achieved only near the center.

6.6 Experimental Investigation

Experiments are performed to confirm the basic characteristics of the push-pull type suspension device. The experimental model is shown in Fig. 6.8. The suspended body is supported by the shaft with the knife-edge, as shown in the figure. Hence the movement of

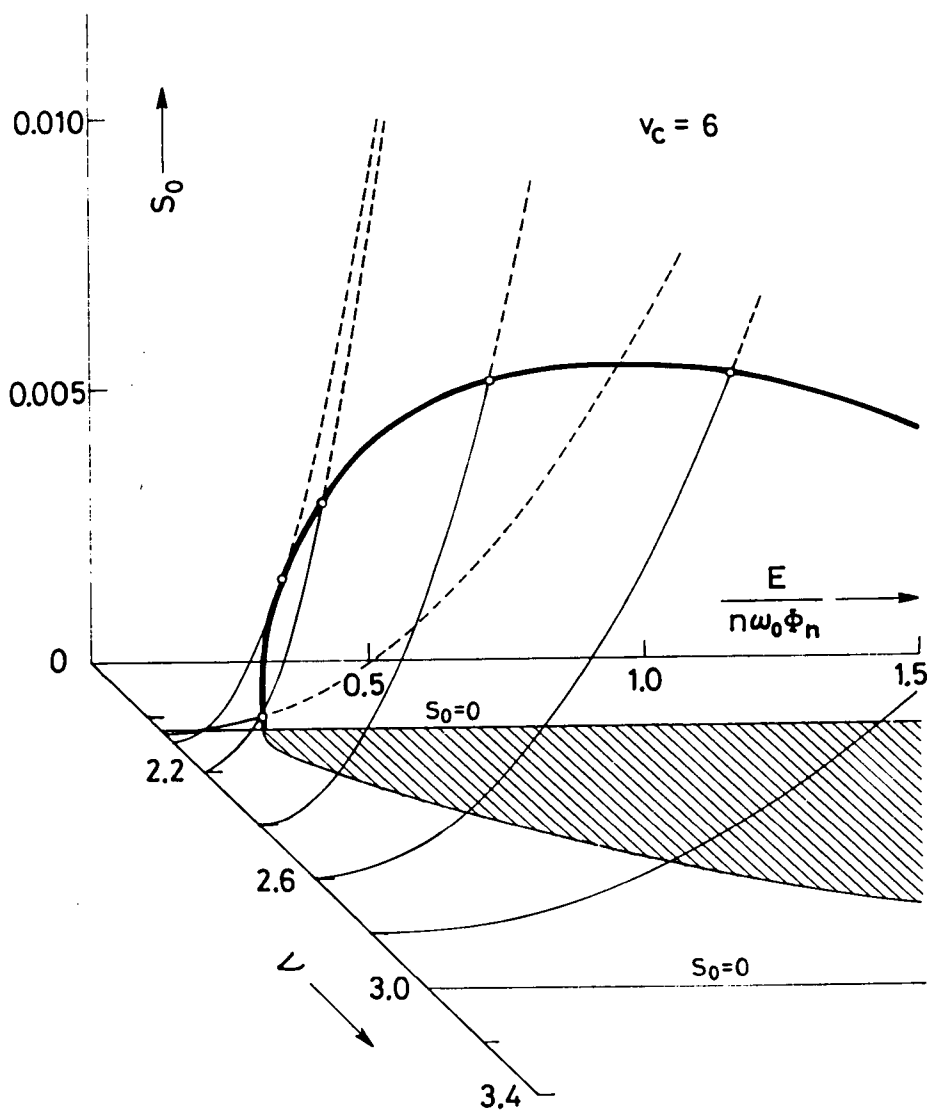


Fig. 6.3 Characteristic of the equivalent spring constant for $v_c = 6.0$, the system parameters k_1 , k_s , k_2 , and ρ being the same as those in Fig. 6. 2.

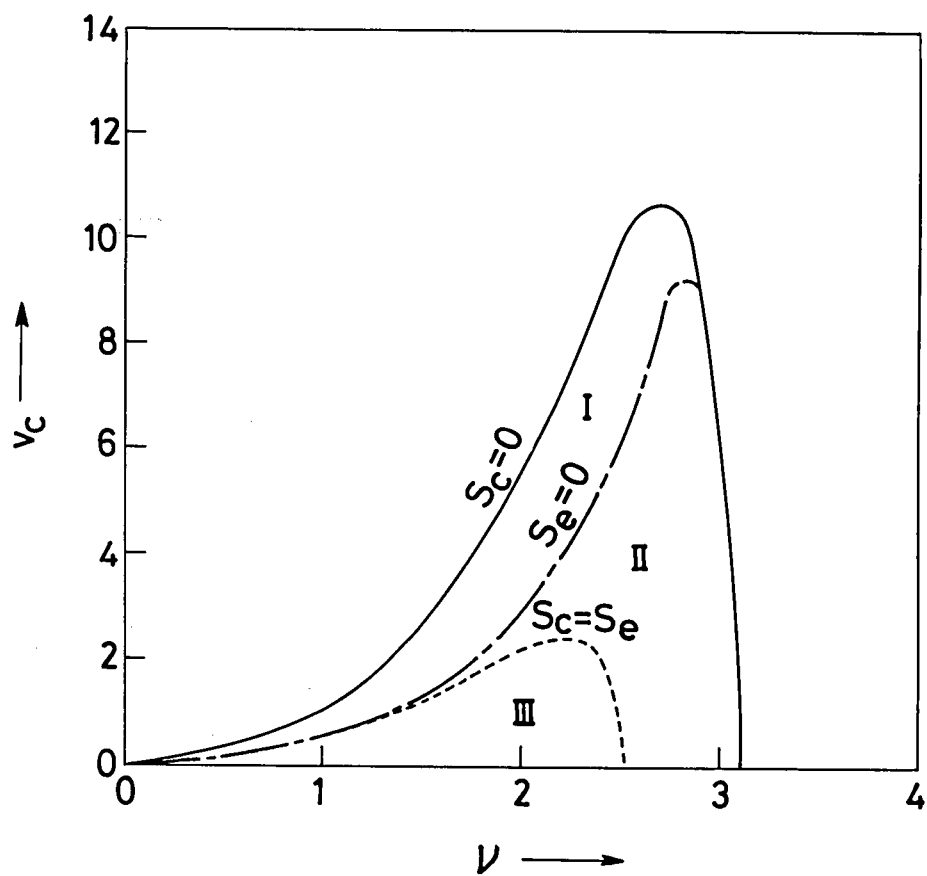


Fig. 6.4 Statically stable region when the suspended body is supported at the center (curve $S_c = 0$) and at the ends of the displacement axis (curve $S_e = 0$), the system parameters k_1 , k_s , and ρ being the same as those in Fig. 6.2

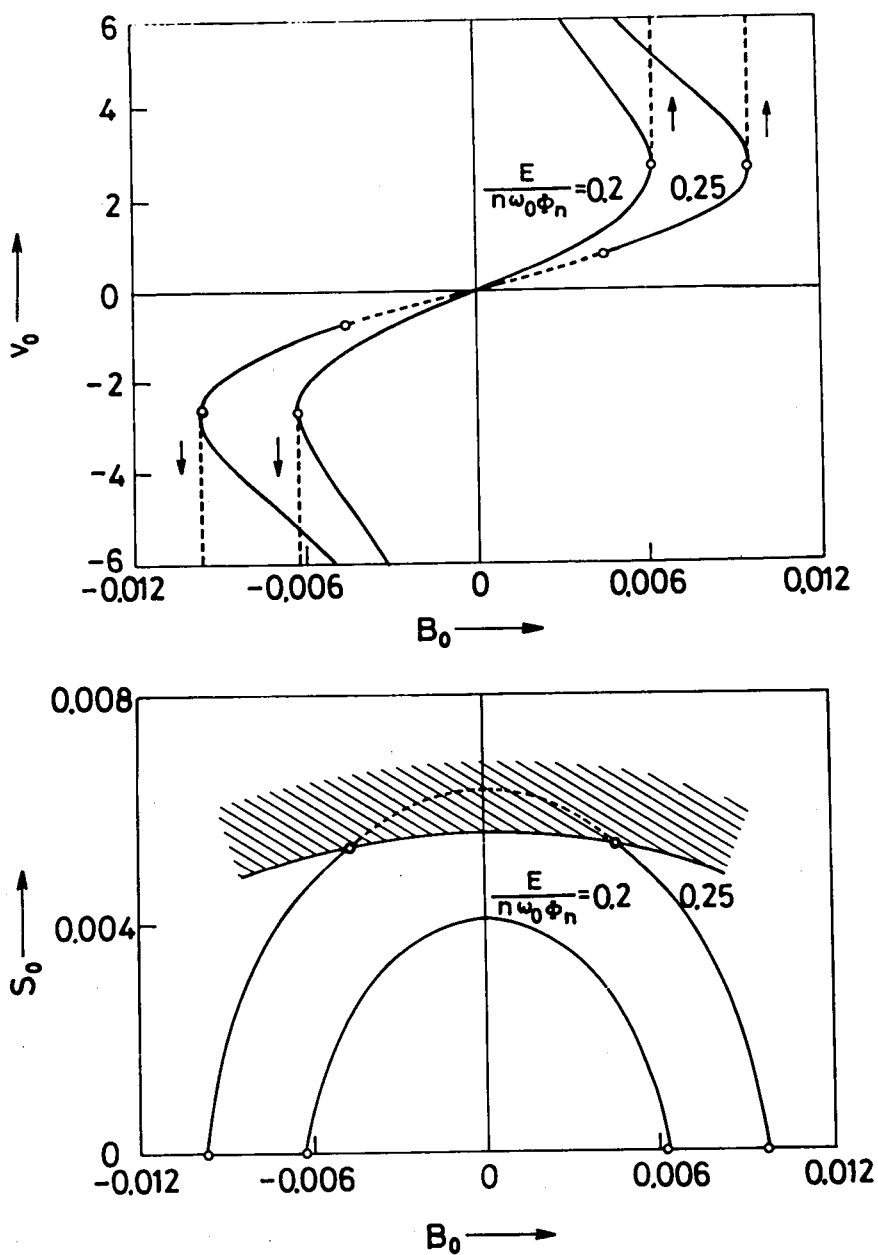


Fig. 6. 5 Response characteristics for $\nu = 2.2$, $v_c = 6.0$ (region I in Fig. 6. 4).

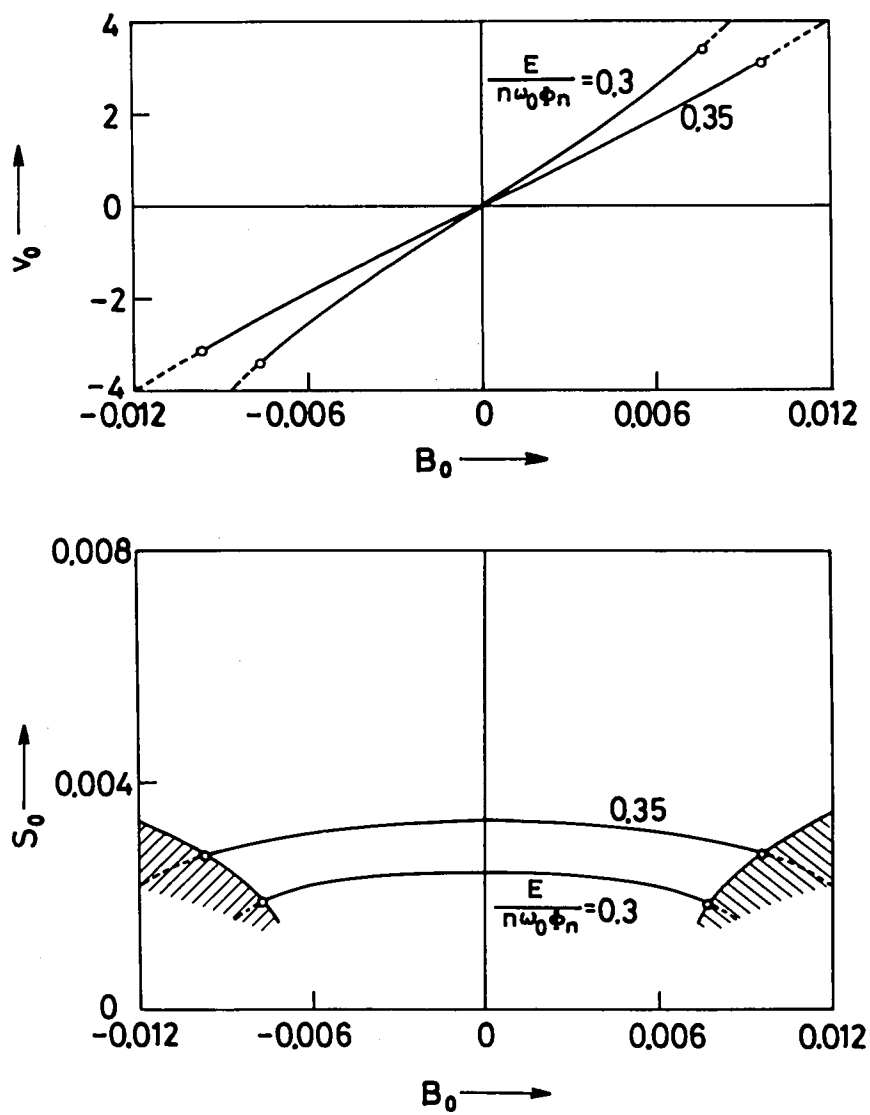


Fig. 6.6 Response characteristics for $\nu = 2.5$, $v_c = 4.0$ (region II in Fig. 6.4).

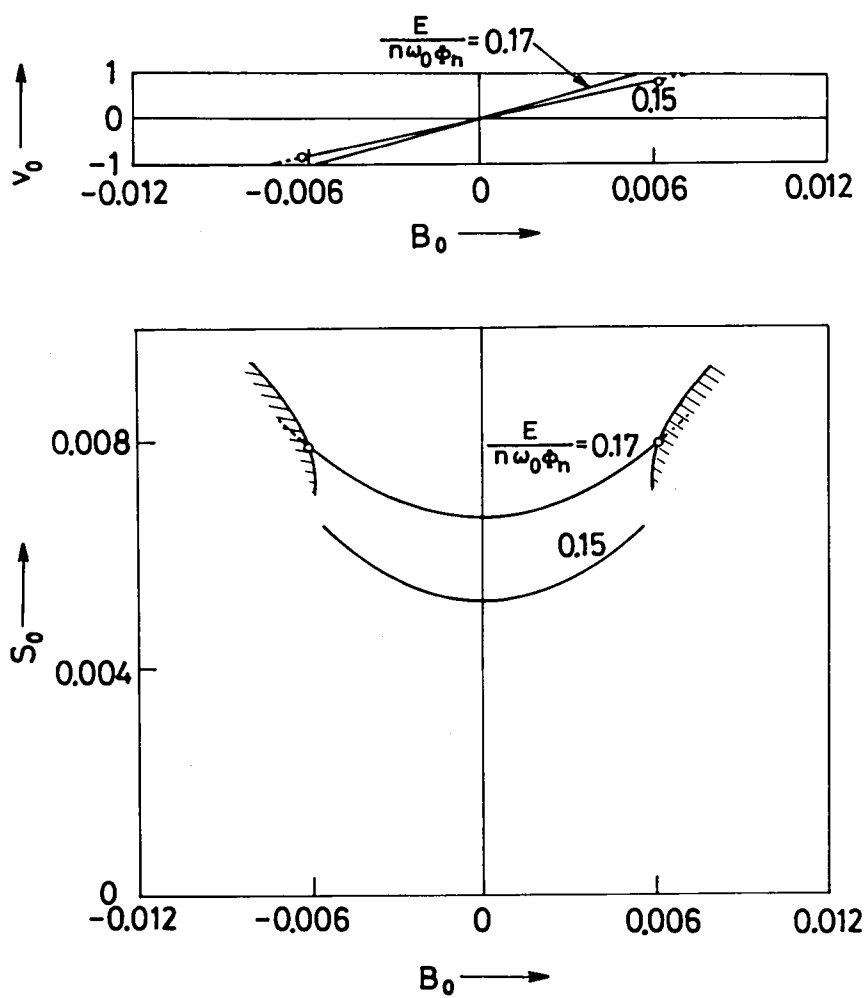


Fig. 6. 7 Response characteristics for $\nu = 1.8, v_c = 1.0$ (region III in Fig. 6.4).

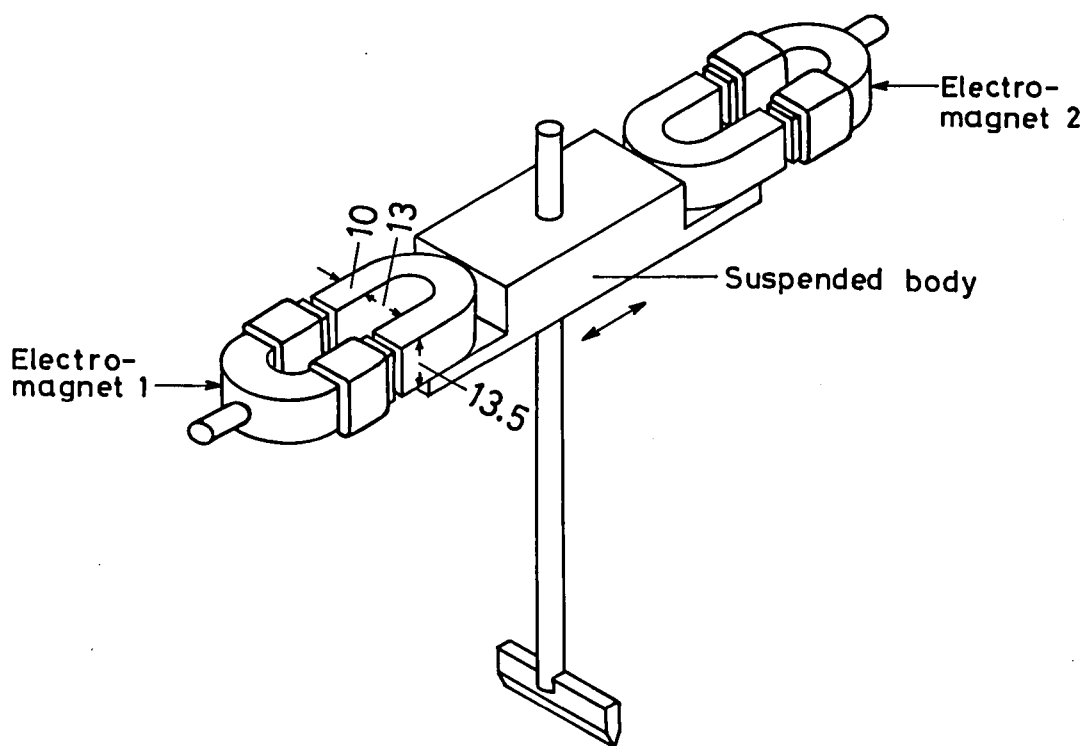


Fig. 6. 8 Experimental model.

the body is limited to one direction as shown by the arrow. The damping force is generated by using a mechanical friction, but the method is not shown in the figure. The parameters of the experiment are as follows:

$$\begin{aligned}
 n &= 600 \text{ turn} \\
 R_{s1} &= 11.97 \, \Omega & R_{s2} &= 12.22 \, \Omega \\
 C_1 &= 10.57 \, \mu\text{F} & C_2 &= 9.99 \, \mu\text{F} \\
 R_1 &= 1000 \, \Omega & R_2 &= 1000 \, \Omega \\
 M &= 0.508 \text{ kg (without the mass of the shaft)}
 \end{aligned}$$

where the subscripts 1 and 2 denote the parameters for the electromagnets 1 and 2, respectively, in Fig. 6.8.

The statically stable region like that in Fig. 6.4 for $S_c = 0$ is experimentally obtained and illustrated in Fig. 6.9. In the figure f denotes the supply frequency. The stable region and the characteristic of the equivalent spring constant S like those in Fig. 6.3 are also experimentally obtained for $h_c = 1.0$ mm, and shown in Figs. 6.10 and 6.11, respectively. In these figures, E_e denotes the effective value of the supply voltage.

6.7 Concluding Remarks

Performance of the push-pull type suspension device with two electromagnets has been investigated. The fundamental equations which describe the device are analyzed by using the method of averaging. It has been shown that when the suspended body is centered, the equivalent spring constant is twice that of a suspension device having one electromagnet. As the body departs from the central position, the equivalent spring constant increases or decreases depending on the system parameters. There is a case in which the equivalent spring constant is positive when the body is centered, but is negative when it is at the ends of the displacement axis. In this case, the stable suspension can not be achieved over the range of displacement of the suspended body. More detailed investigation for stability including the dynamic stability is performed on the basis of the variational equations with constant coefficients, as we have done in the preceding chapters.

The basic characteristics of the push-pull type suspension device are verified by experiments.

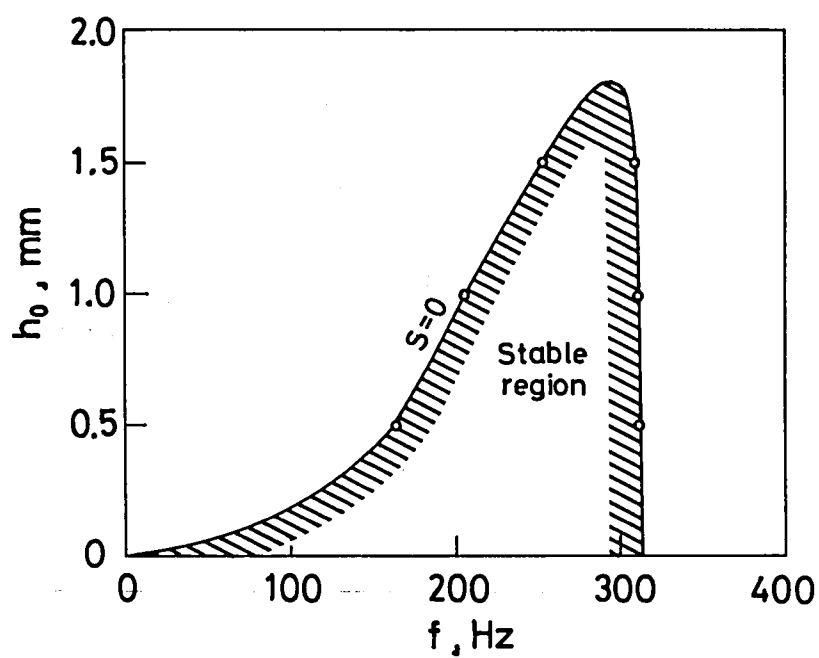


Fig. 6. 9 Statically stable region when the suspended body is supported at the center.

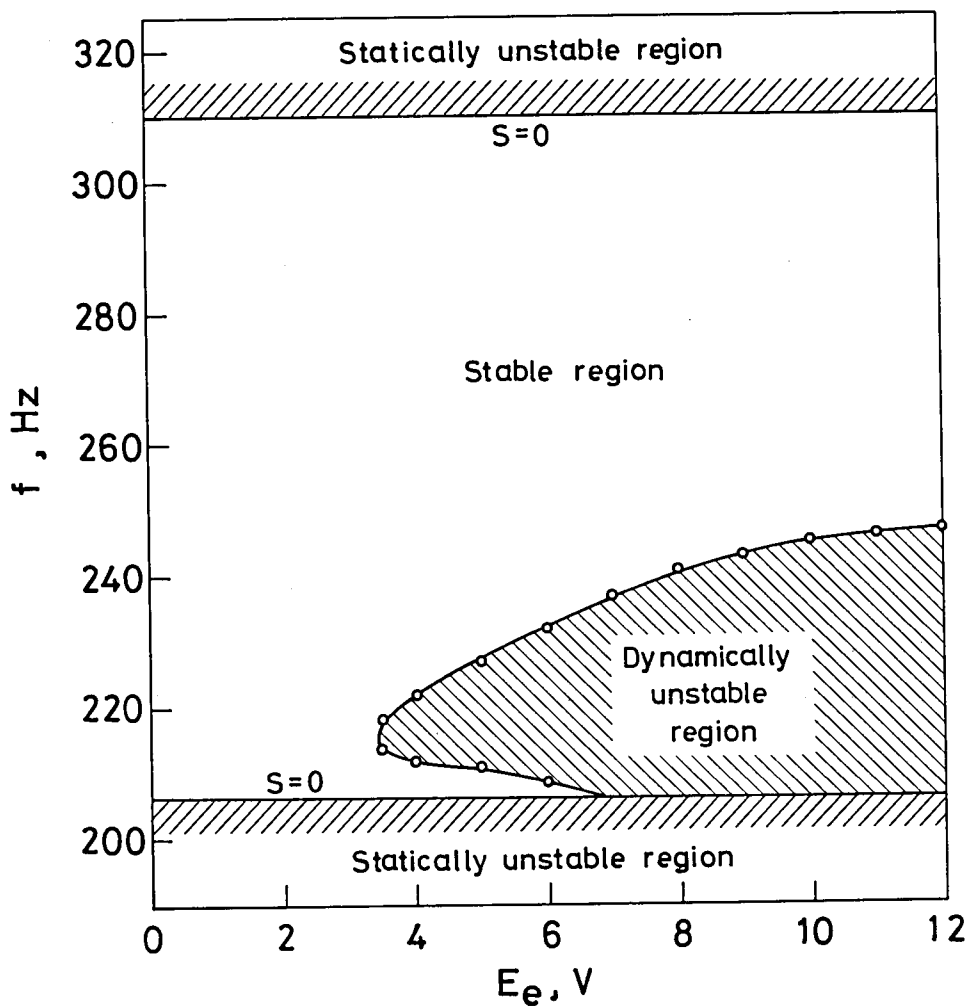


Fig. 6. 10 Region in which stable suspension is achieved ($h_c = 1.0$ mm).

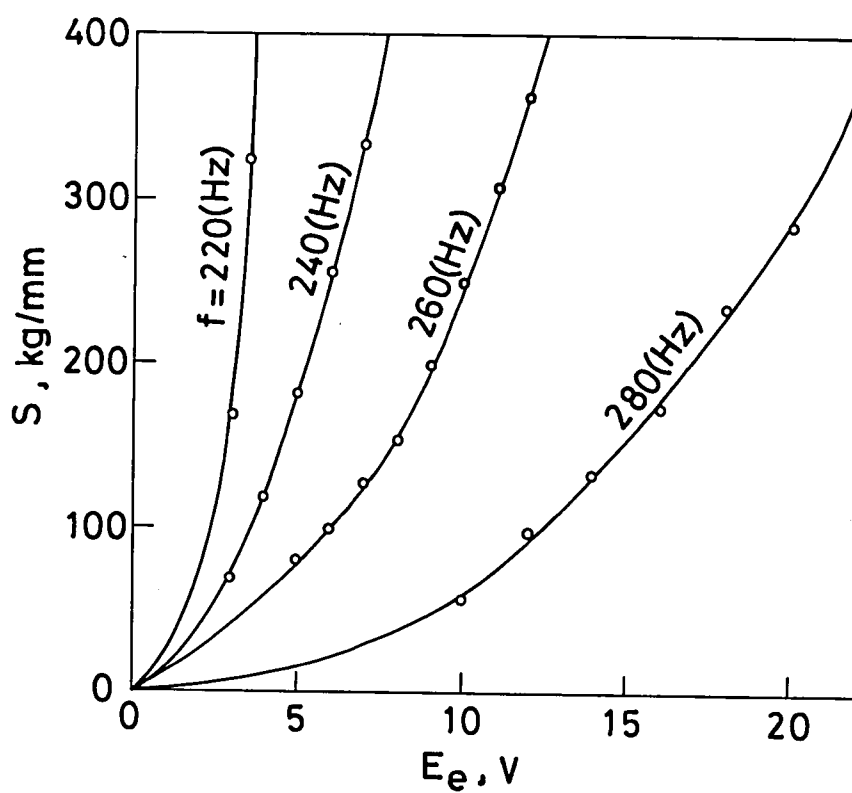


Fig. 6. 11 Characteristic of the equivalent spring constant ($h_c = 1.0$ mm).

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APPENDIX I

TRANSFER-FUNCTION ANALYSIS OF THE LINEARIZED SYSTEM

We investigate the dynamic behavior of the system (2. 22) near the equilibrium point by introducing the linearized system. To this end we introduce the variations ξ , η , ζ , and ψ defined by Eqs. (2. 37). These variations are assumed to be caused by a small disturbance force δB_0 . By a procedure analogous to that of Sec. 2. 3. 4, the variational equations are written as

$$\begin{aligned}\frac{d\xi}{d\tau} &= a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14}\psi \\ \frac{d\eta}{d\tau} &= a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24}\psi \\ \frac{d\zeta}{d\tau} &= a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34}\psi \\ \frac{d\psi}{d\tau} &= a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44}\psi + \delta B_0\end{aligned}\tag{I. 1}$$

where $a_{11}, \dots, a_{44}$ are given by Eqs. (2. 39). Equations (I. 1) are the linear differential equations with constant coefficients. Substituting Eqs. (2. 39) into (I. 1) and applying the Laplace transformation with zero initial conditions leads to

$$\begin{aligned}(p + \frac{1}{2}k_1)\Xi - \frac{1}{2\nu}(\nu^2 - v_0)H + \frac{1}{2\nu}y_0Z &= 0 \\ \frac{1}{2\nu}(\nu^2 - v_0)\Xi + (p + \frac{1}{2}k_1)H - \frac{1}{2\nu}x_0Z &= 0 \\ pZ - \Psi &= 0 \\ x_0\Xi + y_0H + (p + k_2)\Psi &= B_d\end{aligned}\tag{I. 2}$$

where p is the Laplace-transform operator, Ξ, H, Z, Ψ , and B_d denote the transformed variables of ξ, η, ζ, ψ , and δB_0 , respectively. The corresponding block diagram is shown in Fig. A1.

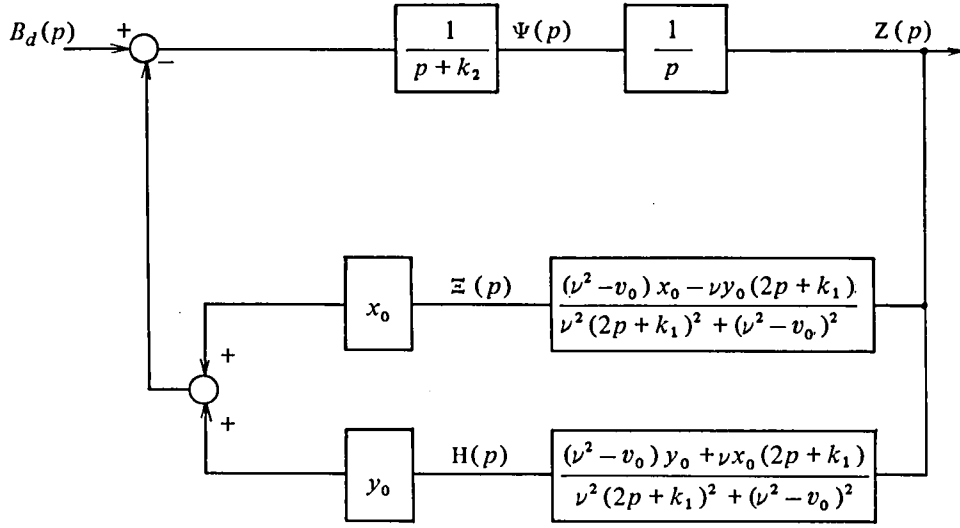


Fig. A1 Block diagram of the linearized system.

The transformed output $Z(p)$ is given by

$$Z(p) = \frac{G_1(p)}{1 + G_1(p)G_2(p)} B_d(p) \quad (\text{I. 3})$$

where

$$\begin{aligned} G_1(p) &= \frac{1}{p(p+k_2)} \\ G_2(p) &= \frac{(\nu^2 - v_0)r_0^2}{4\nu^2(p^2 + k_1p + p_0)} \\ p_0 &= \frac{1}{4\nu^2} [(\nu^2 - v_0)^2 + \nu^2 k_1^2] \\ r_0^2 &= x_0^2 + y_0^2 \end{aligned} \quad (\text{I. 4})$$

The characteristic equation of this linearized system is given by

$$1 + G_1(p)G_2(p) = 0$$

or

$$p^4 + a_1p^3 + a_2p^2 + a_3p + a_4 = 0 \quad (\text{I. 5})$$

where a_1, a_2, a_3, a_4 are identical with those given by Eqs. (2. 43). Thus the same stability conditions as in Sec. 2. 3. 4 are obtained.

Since the Laplace transform of a unit step input is $B_d(p) = 1/p$, the step response of $Z(p)$ for the unit input is given by

$$Z(p) = \frac{G_1(p)}{1 + G_1(p) G_2(p)} \cdot \frac{1}{p} \quad (\text{I. 6})$$

Substituting Eqs. (I. 4) into (I. 6) gives

$$Z(p) = \frac{p^2 + k_1 p + p_0}{p(p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4)} \quad (\text{I. 7})$$

If the roots of the characteristic equation (I. 5) are found to be p_1, p_2, p_3 , and p_4 , then Eqs. (I. 7) can be written as

$$Z(p) = \frac{p^2 + k_1 p + p_0}{p(p - p_1)(p - p_2)(p - p_3)(p - p_4)} \quad (\text{I. 8})$$

As shown by numerical examples in Sec. 2. 4, the characteristic equation (I. 5) contains almost always simple roots only. In this case, Eq. (I. 8) can be expanded in partial fractions as follows:

$$Z(p) = \frac{A_0}{p} + \frac{A_1}{p - p_1} + \frac{A_2}{p - p_2} + \frac{A_3}{p - p_3} + \frac{A_4}{p - p_4} \quad (\text{I. 9})$$

where

$$\begin{aligned} A_0 &= \frac{p_0}{a_4} \\ A_1 &= \frac{p_1^2 + k_1 p_1 + p_0}{p_1(p_1 - p_2)(p_1 - p_3)(p_1 - p_4)} \\ A_2 &= \frac{p_2^2 + k_1 p_2 + p_0}{p_2(p_2 - p_1)(p_2 - p_3)(p_2 - p_4)} \\ A_3 &= \frac{p_3^2 + k_1 p_3 + p_0}{p_3(p_3 - p_1)(p_3 - p_2)(p_3 - p_4)} \\ A_4 &= \frac{p_4^2 + k_1 p_4 + p_0}{p_4(p_4 - p_1)(p_4 - p_2)(p_4 - p_3)} \end{aligned} \quad (\text{I. 10})$$

The inverse Laplace transform of Eq. (I. 9) yields the time solution

$$\begin{aligned} \xi(\tau) &= A_0 + A_1 \exp(p_1 \tau) + A_2 \exp(p_2 \tau) + A_3 \exp(p_3 \tau) \\ &\quad + A_4 \exp(p_4 \tau) \end{aligned} \quad (\text{I. 11})$$

When complex conjugate roots exist, Eq. (I. 11) is rewritten using the trigonometric function. For example, if p_1 and p_2 are complex conjugate, that is,

$$p_1 = p_r + j p_i, \quad p_2 = p_r - j p_i \quad (\text{I. 12})$$

then A_1 and A_2 are also complex conjugate:

$$A_1 = A_r + jA_i, \quad A_2 = A_r - jA_i \quad (\text{I. 13})$$

Hence we obtain

$$A_1 \exp(p_1 \tau) + A_2 \exp(p_2 \tau) = 2 (A_r \cos p_i \tau - A_i \sin p_i \tau) \exp(p_r \tau) \quad (\text{I. 14})$$

In Eq. (I. 11), the first term A_0 shows the steady-state gap distance ξ_0 due to the unit step disturbance-force ($\delta B_0 = 1$). By substituting a_4 and p_0 which are given, respectively, by Eqs. (2. 43) and (I. 4) into A_0 in Eqs. (I. 10), and using Eqs. (2. 26) and (2. 34), we can rewrite

$$A_0 = \frac{1}{S_0} \quad (\text{I. 15})$$

This shows that the steady-state gap distance due to the unit step disturbance-force decreases with the increase of the equivalent spring constant S_0 . From the physical point of view, this is a plausible conclusion.

APPENDIX II

RELUCTANCES OF THE MAGNETIC CIRCUIT FOR THE EXPERIMENTAL MODEL

In this appendix, we obtain the reluctances σ , $\mathfrak{R}_c$, and $\mathfrak{R}_l$ of the magnetic circuit for the experimental model shown in Fig. 3. 7. As shown in Fig. 3. 10, we experimentally obtain the relationship between the gap distance h_0 and the coil current I for the experimental model. Furthermore in Fig. 3. 11 is shown the relationship between the gap distance h_0 and the coil voltage E_{L0} . These relations are given by Eqs. (III. 6) and (III. 7) in Appendix III, that is,

$$I = a_I h_0 + b_I \quad (\text{II. 1})$$

$$E_{L0} = \omega (a_E h_0 + b_E) \quad (\text{II. 2})$$

where

$$a_I = \frac{1}{n} \sqrt{2 \sigma Mg} \quad (\text{II. 3})$$

$$b_I = \frac{\mathfrak{R}_c}{n} \sqrt{\frac{2Mg}{\sigma}} \quad (\text{II. 4})$$

$$a_E = \frac{n}{\mathfrak{R}_l} \sqrt{2 \sigma Mg} \quad (\text{II. 5})$$

$$b_E = n \left(1 + \frac{\mathfrak{R}_c}{\mathfrak{R}_l} \right) \sqrt{\frac{2Mg}{\sigma}} \quad (\text{II. 6})$$

Hence if we obtain these constants a_I , b_I , a_E , and b_E from the experimental results in Figs. 3. 10 and 3. 11, we can obtain the reluctances σ , $\mathfrak{R}_c$, and $\mathfrak{R}_l$ from Eqs. (II. 3) through (II. 5) as follows:

$$\begin{aligned} \sigma &= \frac{n^2 a_I^2}{2Mg} \\ \mathfrak{R}_c &= nb_I \sqrt{\frac{\sigma}{2Mg}} = \frac{n^2 a_I b_I}{2Mg} \\ \mathfrak{R}_l &= \frac{n}{a_E} \sqrt{2 \sigma Mg} = \frac{n^2 a_I}{a_E} \end{aligned} \quad (\text{II. 7})$$

As can be easily seen, the reluctance $\mathfrak{R}_l$ may also be obtained from Eq. (II. 6). However, since in general $\mathfrak{R}_c/\mathfrak{R}_l \ll 1$, a large error would be included in the value $\mathfrak{R}_l$ obtained from Eq. (II. 6). Hence this equation is not suitable for estimation of the value of $\mathfrak{R}_l$.

We obtain the constants a_I , b_I , a_E , and b_E by using the method of least-squares. The constants a_I and b_I are obtained as follows. If m points data exist for the relationship between I and h_o , the summation of error squared is given by

$$e_r = \sum_{j=1}^m (I_j - a_I h_{oj} - b_I)^2 \quad (\text{II. 8})$$

where I_j and h_{oj} are the experimental data of I and h_o , respectively. Following the method of least-squares, the constants a_I and b_I are obtained from $\partial e_r / \partial a_I = 0$ and $\partial e_r / \partial b_I = 0$, that is,

$$\begin{aligned} c_1 a_I + c_2 b_I &= c_3 \\ c_2 a_I + m b_I &= c_4 \end{aligned} \quad (\text{II. 9})$$

where

$$\begin{aligned} c_1 &= \sum_{j=1}^m h_{oj}^2 & c_2 &= \sum_{j=1}^m h_{oj} \\ c_3 &= \sum_{j=1}^m I_j h_{oj} & c_4 &= \sum_{j=1}^m I_j \end{aligned} \quad (\text{II. 10})$$

Solving Eqs. (II. 9) for a_I and b_I gives us

$$\begin{aligned} a_I &= \frac{m c_3 - c_2 c_4}{m c_1 - c_2^2} \\ b_I &= \frac{c_1 c_4 - c_2 c_3}{m c_1 - c_2^2} \end{aligned} \quad (\text{II. 11})$$

By the same procedure as that mentioned above, the constants a_E and b_E are obtained from

$$\begin{aligned} a_E &= \frac{m c_5 - c_2 c_6}{m c_1 - c_2^2} \\ b_E &= \frac{c_1 c_6 - c_2 c_5}{m c_1 - c_2^2} \end{aligned} \quad (\text{II. 12})$$

where c_1 and c_2 are defined by Eqs. (II. 10), and

$$c_5 = \sum_{j=1}^m \frac{E_{L0j}}{\omega} h_{oj} \quad c_6 = \sum_{j=1}^m \frac{E_{L0j}}{\omega} \quad (\text{II. 13})$$

The values E_{L0j} and h_{oj} are the experimental data of E_{L0} and h_o , respectively.

Substituting the constants a_I , b_I , and a_E obtained from Eqs. (II. 11) and (II. 12) into Eqs. (II. 7), we can obtain the values of reluctances σ , $\mathcal{R}_c$, and $\mathcal{R}_l$ for the experimental model.

APPENDIX III

STEADY-STATE CHARACTERISTICS OF THE VERTICAL-AXIS SUSPENSION DEVICE EXPRESSED IN TERMS OF DEVICE PARAMETERS

In chapter 3, we obtain the steady-state characteristics of the suspension device which are expressed by using the nondimensional parameters. In this appendix, those characteristics are expressed in terms of the device parameters.

The instantaneous magnetic flux in the steady state can be expressed as

$$\phi = \Phi_m \sin(\omega t - \theta) \quad (\text{III. 1})$$

where Φ_m and θ are constant. The instantaneous magnetic force due to this magnetic flux is given by Eq. (3. 5). Its time average is, therefore, given by

$$\bar{F}_m = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} F_m dt = \frac{\sigma \omega \int_0^{2\pi/\omega} \phi^2 dt}{4\pi[1 + (\sigma h_0 + \mathcal{R}_c)/\mathcal{R}_l]^2} \quad (\text{III. 2})$$

Substituting Eq. (III. 1) into (III. 2), we obtain

$$\bar{F}_m = \frac{\sigma \Phi_m^2}{4[1 + (\sigma h_0 + \mathcal{R}_c)/\mathcal{R}_l]^2} \quad (\text{III. 3})$$

Since in the equilibrium state $\bar{F}_m$ balances the weight of the suspended body Mg , the maximum value Φ_m of the magnetic flux must be

$$\Phi_m = \sqrt{\frac{4Mg}{\sigma}} [1 + (\sigma h_0 + \mathcal{R}_c)/\mathcal{R}_l] \quad (\text{III. 4})$$

From Eqs. (3. 2), the coil current is given by

$$i = \frac{\mathcal{R}_l \mathcal{R}_l}{n (\mathcal{R}_l + \mathcal{R}_l)} \Phi_m \sin(\omega t - \theta) \quad (\text{III. 5})$$

where the reluctance $\mathcal{R}_l$ is given by Eq. (3. 3).

Substituting Eq. (III. 4) into (III. 5), we obtain the effective value of the coil current:

$$I = \frac{1}{n} \sqrt{2\sigma Mg} \left(h_0 + \frac{\mathcal{R}_c}{\sigma} \right) \quad (\text{III. 6})$$

On the other hand, the coil voltage is given by

$$e_{L_0} = n \frac{d\phi}{dt} = n\omega \Phi_m \cos(\omega t - \theta)$$

Consequently, the effective value of the coil voltage is

$$E_{L_0} = n\omega \sqrt{\frac{2Mg}{\sigma}} [1 + (\sigma h_0 + R_c)/R_l] \quad (\text{III. 7})$$

The coil inductance L is, from its definition,

$$L = \frac{n\phi}{i} = n^2 \left(\frac{1}{R_l} + \frac{1}{\sigma h_0 + R_c} \right) \quad (\text{III. 8})$$

The dimensional gap distance h_0 and the corresponding equivalent spring constant S are obtained from Eqs. (3. 18) and (3. 24) by using Eqs. (3. 6), (3.7), (3. 8a) and (2. 33).

The results are as follows:

$$h_0 = \frac{n^2}{\sigma (Z_1 + Z_2)} \left[Z_2 \mp \sqrt{\frac{\sigma E^2}{4n^2 Mg} (Z_1 + Z_2) - \omega^2 Z_1} \right] - \frac{R_i}{\sigma} \quad (m) \quad (\text{III. 9})$$

$$S = \pm \frac{8M^2 g^2}{E^2} \sqrt{\frac{\sigma E^2}{4n^2 Mg} (Z_1 + Z_2) - \omega^2 Z_1} \quad (N/m) \quad (\text{III. 10})$$

where

$$Z_1 = \left(R_s + \frac{R}{1 + \omega^2 C^2 R^2} \right)^2$$

$$Z_2 = \omega^2 \left(\frac{n^2}{R_l} - \frac{CR^2}{1 + \omega^2 C^2 R^2} \right)^2$$

The positive sign in Eq. (III. 10) is taken if the negative sign is taken in Eq. (III. 9) and vice versa.