

FREQUENCY MIXING OF ELASTIC WAVES AT A NONLINEAR SPRING-TYPE INTERFACE

Shiro Biwa, Takuya Takahashi

Kyoto University, Kyoto, Japan

email: biwa@kuaero.kyoto-u.ac.jp

Nonlinear frequency mixing of elastic waves, i.e., the generation of sum- and difference-frequency components by the interaction of two waves of different frequencies, is currently attracting active attention in the field of nondestructive testing as a means to characterize closed defects in structural components. In this study, a nonlinear spring-type interface model, characterized by two linear stiffness parameters and three nonlinearity parameters, is used to analyse the frequency mixing of two monochromatic plane longitudinal waves incident on an interface between two linear elastic bodies. A perturbation technique is used to linearize the problem of analyzing the reflection/transmission of the fundamental components and the generation/radiation of the sum- and difference-frequency components at the interface. The propagation directions of these nonlinearly generated components are derived analytically from the phase matching condition which is different from that for the material nonlinearity. Experimental results are also presented for the frequency mixing of ultrasonic waves at a contacting interface of metallic blocks.

Keywords: nondestructive testing, contact acoustic nonlinearity, frequency mixing

1. Introduction

Ultrasonic nondestructive testing of structural components often encounters a difficulty in receiving clear echo signals from closed defects [1, 2]. To overcome this difficulty, nonlinear ultrasonic methods have been attracting attention as a sensitive means to characterize closed defects as they focus on the nonlinear spectral components generated by the opening/closure of such defects when subjected to elastic waves with high amplitudes [3, 4]. To clarify the physical background for these methods, nonlinear acoustic responses of contacting solid-solid interfaces have been a subject of intensive investigation. For instance, it has been shown that the higher harmonic components generated by a narrow-band incident wave varies sensitively with the state of contacting interfaces [5-10]. The acoustic nonlinearity of contacting interfaces includes, in addition to higher harmonic generation, the frequency mixing phenomenon, i.e., the generation of sum- and difference-frequency components due to the nonlinear interaction of two waves of different frequencies [11]. The frequency mixing has some merits for nondestructive testing as it employs the frequency components different from the higher harmonics of incident waves and facilitates discrimination of the defect-induced nonlinearity.

The frequency mixing of elastic waves due to the inherent stress-strain nonlinearity of materials has been known since long in the field of physical acoustics [12], and recently applied to the evaluation of material degradation such as fatigue damage [13]. In contrast, the frequency mixing at contacting interfaces is a relatively new topic which currently attracts more attention. In this study, a nonlinear spring-type interface model [11, 14], characterized by two linear stiffness parameters and three nonlinearity parameters, is used to analyse the frequency mixing of two monochromatic plane elastic waves with

different frequencies which are incident on the interface with different incident angles. A perturbation technique is used to linearize the problem, by first analysing the reflection and transmission of the fundamental frequency components at the interface and then solving for the generation and radiation of the sum- and difference-frequency components. A similar approach has been used to analyse the second-harmonic generation at imperfect interfaces [5, 6]. In this paper, the propagation directions of these nonlinearly generated components are derived analytically from the phase matching condition which is different from that for the material nonlinearity. Some relevant experimental results are also presented.

2. Theoretical analysis

Two semi-infinite isotropic and linear elastic bodies of identical properties (density ρ , longitudinal wave velocity c_L , transverse wave velocity c_T) are assumed to be in contact with each other at their flat boundary located at $x = 0$ in the Cartesian coordinate system. At the interface, the stresses (normal stress σ and shear stress τ) are given as nonlinear functions of the relative displacements of the two surfaces (ξ and s in the normal and tangential directions, respectively). Considering only the quadratic nonlinearity, the interfacial stresses are given by [14]

$$\sigma = K_N(\xi - \beta\xi^2 - \gamma s^2), \quad \tau = K_T(s - \chi\xi s), \quad (1)$$

where the relative displacements are defined as

$$\xi(y, t) = u_x(0^+, y, t) - u_x(0^-, y, t), \quad s(y, t) = u_y(0^+, y, t) - u_y(0^-, y, t). \quad (2)$$

In the above expressions, K_N and K_T are the normal and tangential stiffnesses, and β , γ and χ are the parameters representing the quadratic nonlinearity of the interface. It is noted that among the three nonlinearity parameters, β is the only parameter responsible for the interfacial nonlinearity for normally incident longitudinal waves [6, 14].

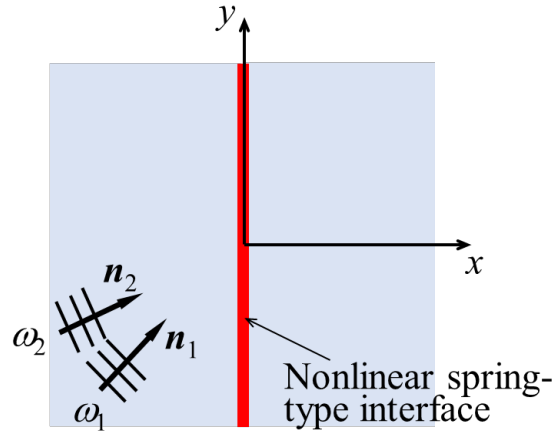


Figure 1: Interaction of two plane longitudinal waves with a nonlinear spring-type interface.

In the present analysis, the contacting elastic bodies are subjected to the incidence of two plane monochromatic longitudinal waves with different angular frequencies (ω_1 and ω_2 , assumed to satisfy $\omega_1 > \omega_2$ without loss of generality) and different propagation directions (represented by unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, assumed to lie in the x - y plane) as shown in Fig. 1. The displacement components of the incident waves are given by

$$\mathbf{u}^{\text{inc}}(x, y, t) = \sum_{J=1}^2 A_J \begin{pmatrix} n_{Jx} \\ n_{Jy} \end{pmatrix} \cos\{k_J(n_{Jx}x + n_{Jy}y) - \omega_J t + \alpha_J\}, \quad (3)$$

where $\mathbf{n}_J = (n_{Jx}, n_{Jy})$, A_J are the amplitudes, $k_J = \omega_J/c_L$ are the wave numbers and α_J are the phase constants of two waves ($J = 1, 2$). The interaction of these waves with the interface is governed by Navier's equations satisfied in each body and the nonlinear boundary conditions in Eq. (1).

A perturbation analysis [14] can be employed to obtain an approximate solution to the above problem which describes the nonlinear frequency mixing at the interface. Namely, the solution is assumed to be the sum of the linear part $\bar{\mathbf{u}}(x, y, t)$ (denoted with a superposed bar) which represents the reflection/transmission of two incident waves at the interface and the perturbation $\hat{\mathbf{u}}(x, y, t)$ (denoted with a superposed hat) which corresponds to the nonlinearly generated frequency components, i.e., $\mathbf{u}(x, y, t) = \bar{\mathbf{u}}(x, y, t) + \hat{\mathbf{u}}(x, y, t)$. Only the outline of the analysis is described here (the detailed formulation will be presented elsewhere). First, the linear part is analysed with Navier's equations and the linearized boundary conditions $\bar{\sigma} = K_N \bar{\xi}$ and $\bar{\tau} = K_T \bar{s}$ by neglecting the quadratic terms in Eq. (1). This problem can be solved separately for the two waves in the frequency domain in a straightforward manner and yields the reflected and transmitted plane longitudinal and transverse waves of the frequencies ω_1 and ω_2 . The resulting relative displacements $\bar{\xi}$ and $\bar{s}$ at the interface contain the two frequency components, i.e.

$$\begin{aligned} \bar{\xi}(y, t) &= \sum_{J=1}^2 A_J |M_J| \cos(k_J n_{Jy} y - \omega_J t + \alpha_J + \varphi_J), \\ \bar{s}(y, t) &= \sum_{J=1}^2 A_J |N_J| \cos(k_J n_{Jy} y - \omega_J t + \alpha_J + \psi_J), \end{aligned} \quad (4)$$

where M_J and N_J ($J = 1, 2$) are complex-valued parameters determined by the solution to the linear part, and φ_J and ψ_J are their arguments, respectively. Subsequently, the perturbation is analysed with Navier's equations and the boundary conditions

$$\hat{\sigma} = K_N(\hat{\xi} - \beta \bar{\xi}^2 - \gamma \bar{s}^2), \quad \hat{\tau} = K_T(\hat{s} - \chi \bar{\xi} \bar{s}), \quad (5)$$

with the source terms $-K_N(\beta \bar{\xi}^2 + \gamma \bar{s}^2)$ and $-K_T \chi \bar{\xi} \bar{s}$ transferred from the linear part. These quadratic terms contain the static ($\omega = 0$), the second-harmonic ($\omega = 2\omega_1$ and $\omega = 2\omega_2$), the sum-frequency ($\omega = \omega_1 + \omega_2$) and the difference-frequency ($\omega = \omega_1 - \omega_2$) components, e.g.,

$$\begin{aligned} \bar{\xi}^2(y, t) &= \sum_{J=1}^2 \frac{1}{2} A_J^2 |M_J|^2 \{1 + \cos(2k_J n_{Jy} y - 2\omega_J t + 2\alpha_J + 2\varphi_J)\} \\ &+ A_1 A_2 |M_1| |M_2| [\cos\{k_y^{\text{sum}} y - (\omega_1 + \omega_2)t + \alpha_1 + \alpha_2 + \varphi_1 + \varphi_2\} \\ &+ \cos\{k_y^{\text{diff}} y - (\omega_1 - \omega_2)t + \alpha_1 - \alpha_2 + \varphi_1 - \varphi_2\}], \end{aligned} \quad (6)$$

where

$$\begin{aligned} k_y^{\text{sum}} &= \frac{\omega_1 n_{1y} + \omega_2 n_{2y}}{c_L} = k_1 n_{1y} + k_2 n_{2y}, \\ k_y^{\text{diff}} &= \frac{\omega_1 n_{1y} - \omega_2 n_{2y}}{c_L} = k_1 n_{1y} - k_2 n_{2y}. \end{aligned} \quad (7)$$

The source terms $-K_N(\beta\bar{\xi}^2 + \gamma\bar{s}^2)$ and $-K_T\chi\bar{\xi}\bar{s}$ in Eq. (5) thus emit the second-harmonic, sum- and difference-frequency components into each body, which can be solved similarly to the linear part.

The above expressions imply that the sum- and difference-frequency waves are emitted as plane longitudinal and transverse waves with the y -component of the wave vector given by k_y^{sum} and k_y^{diff} , respectively. Equation (7) represents the phase matching condition for the frequency mixing at the interface, which differs from that for the material nonlinearity, commonly expressed in the form $\mathbf{k}^{\text{sum(diff)}} = \mathbf{k}_1 \pm \mathbf{k}_2$ [12]. The y -components of the unit vectors representing the propagation directions of the longitudinal waves of the mixed frequencies are given by

$$n_y^{\text{sum}} = \frac{\omega_1 n_{1y} + \omega_2 n_{2y}}{\omega_1 + \omega_2}, \quad n_y^{\text{diff}} = \frac{\omega_1 n_{1y} - \omega_2 n_{2y}}{\omega_1 - \omega_2} \quad (8)$$

and those of the transverse waves are given by $(c_T/c_L)n_y^{\text{sum}}$ and $(c_T/c_L)n_y^{\text{diff}}$, respectively. The corresponding angles of the propagation direction from the x -axis are given by

$$\sin \theta_{\text{sum}} = |n_y^{\text{sum}}|, \quad \sin \theta_{\text{diff}} = |n_y^{\text{diff}}|, \quad (9)$$

if the right-hand sides of the above expressions are not greater than unity (otherwise, the wave will be evanescent). Furthermore, the amplitudes of the generated mixed frequency components can be obtained by seeking for the plane wave fields for the boundary conditions in Eq. (5).

As a special case, the interaction of two plane longitudinal waves with close frequencies and symmetric incident angles as shown in Fig. 2 is considered, i.e.,

$$\omega_1 = \omega_{\text{ave}} + \Delta\omega, \quad \omega_2 = \omega_{\text{ave}} - \Delta\omega, \quad n_{1x} = n_{2x} = \cos\theta_0, \quad n_{1y} = -n_{2y} = \sin\theta_0, \quad (10)$$

where ω_{ave} and $\Delta\omega$ are the average and the half of the difference of two frequencies, respectively, and θ_0 is the incident angle of two waves. In this case, Eq. (8) reduces to

$$n_y^{\text{sum}} = \frac{\Delta\omega}{\omega_{\text{ave}}} \sin \theta_0, \quad n_y^{\text{diff}} = \frac{\omega_{\text{ave}}}{\Delta\omega} \sin \theta_0. \quad (11)$$

The above expressions imply that the sum-frequency components travel into both sides in a nearly perpendicular direction to the interface, and the difference-frequency components are expected to become evanescent ones for a small value of the incident angle.

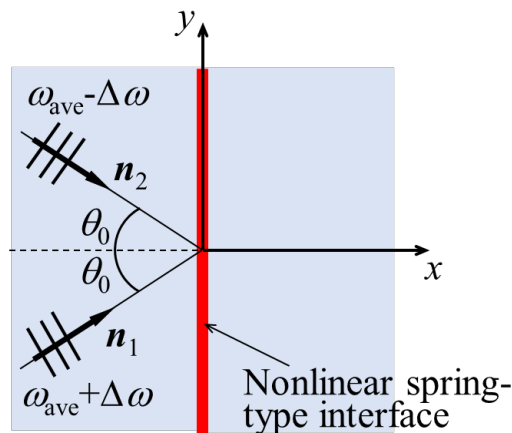


Figure 2: Two incident longitudinal waves incident on the interface with symmetric directions.

Some numerical results are presented using the material parameters of an aluminum alloy. Figure 3 shows the angles of the propagation direction of the sum-frequency longitudinal and transverse waves

from the normal direction to the interface as functions of the incident angle θ_0 in the case where $\Delta\omega/\omega_{\text{ave}} = 0.1$ and 0.4 and the interfacial parameters are given by $K_N/(\rho c_L \omega_{\text{ave}}) = 0.6$ and $K_T/K_N = 0.3$. When the difference of the frequencies of two incident waves is small, the propagation angles of the sum-frequency waves remain quite small. Figure 4 shows the amplitude of the sum-frequency longitudinal and transverse waves normalized by $\beta A_1 A_2$ as a function of the incident angle θ_0 when $\gamma = \chi = 0$. It is seen in Fig. 4 that the transverse wave is relatively weak as compared to the longitudinal wave for the entire range of the incident angle. The sum-frequency longitudinal wave has a higher amplitude for a smaller incident angle and for a smaller frequency difference. In the case where $\gamma = \chi = 0$, it can be shown that the sum-frequency longitudinal and transverse waves are emitted to both sides of the interface with equal amplitudes.

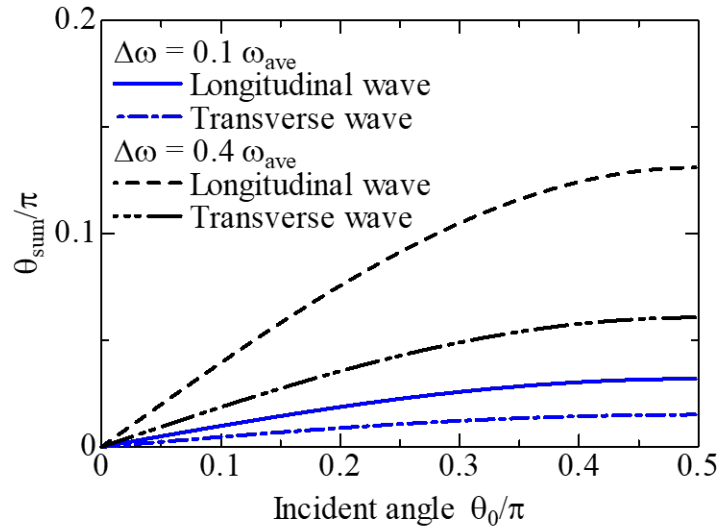


Figure 3: Dependence of the angles of propagation of the sum-frequency waves from the normal to the interface on the incident angle.

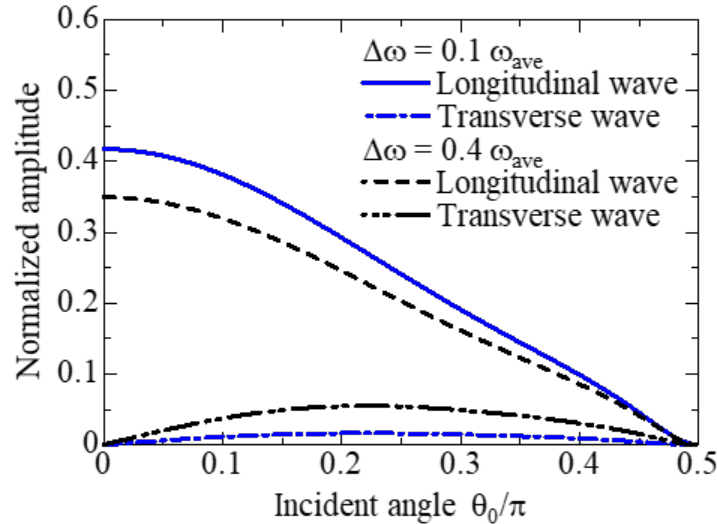


Figure 4: Dependence of the amplitudes of the sum-frequency waves on the incident angle.

3. Experimental results

The nonlinear frequency mixing at an interface between solids is also examined here from an experimental point of view. To this purpose, polished surfaces of circular area (diameter 35 mm) of two aluminium alloy blocks were mated and compressed to each other, and the longitudinal tone-burst waves (frequencies 1.1 MHz and 0.9 MHz) were emitted to the contacting interface with the symmetric angle of 45 degrees by piezoelectric transducers (Panametrics, nominal center frequency 1 MHz) as shown in Fig. 5, using a RITEC RAM-5000 high-power amplifier with embedded pulse generators. The voltage amplitudes of the excitation signals applied to both transducers were varied to change the amplitudes of the incident waves. The receiving piezoelectric transducers (Panametrics, nominal center frequency 2.25 MHz) were set on the reflection and transmission sides of the interface in the perpendicular direction (denoted as R and T in Fig. 5), since the theory outlined above indicates that the sum-frequency waves are emitted in a nearly normal direction from the interface ($\theta_{\text{sum}}/\pi = 0.025$, i.e., around 4.5 degrees from the perpendicular direction). The emitting and receiving transducers were attached to the blocks in circular cylindrical cavities arranged on their surfaces with the same distance (38 mm) to the interface. The recorded waveforms were analysed by the fast Fourier transform to extract the amplitude of the sum-frequency components.

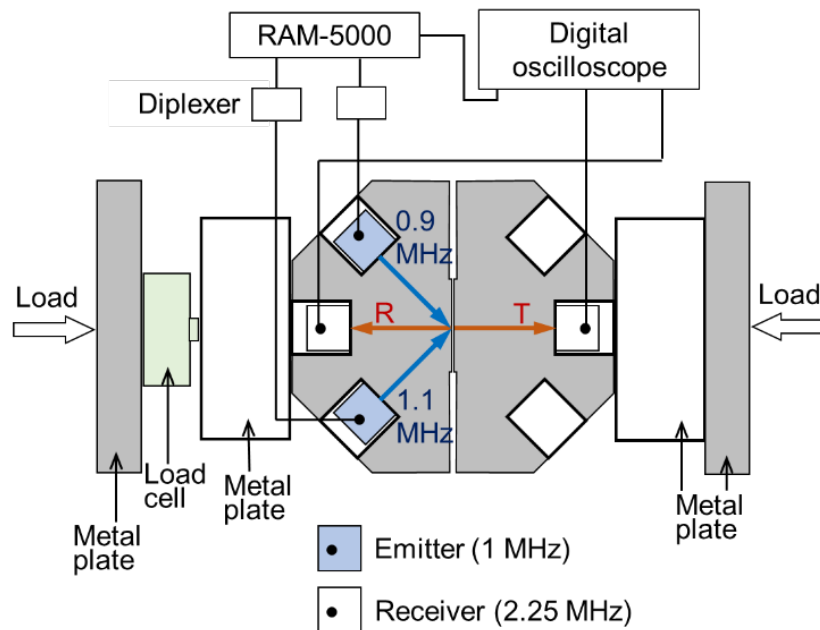


Figure 5: Schematic illustration of the experimental setup.

Figure 6 shows the amplitude spectra of the waveforms recorded by the transducer on the transmission side when the excitation voltage amplitude V_1 applied to the transducer emitting the 1.1 MHz wave was varied from 150 V to 350 V (including the case it was switched off) while the corresponding amplitude V_2 for the 0.9 MHz wave was fixed as 350 V. The nominal contact pressure applied to the blocks was 0.41 MPa. It is seen that the amplitude of the sum-frequency (2 MHz) component in the recorded signal grows almost proportionally when the excitation voltage amplitude V_1 is increased. The same trend was confirmed when V_2 was varied while keeping V_1 fixed.

Figure 7 shows the variation of the spectral amplitudes of the sum-frequency component observed on the reflection and transmission sides with the excitation voltage amplitude V_1 when V_2 was fixed as

350 V. It is shown in Fig. 7 that the sum-frequency component is contained with very similar magnitude in the signals recorded on both sides. This is in qualitative agreement with the theoretical result shown above for the case when $\gamma = \chi = 0$.

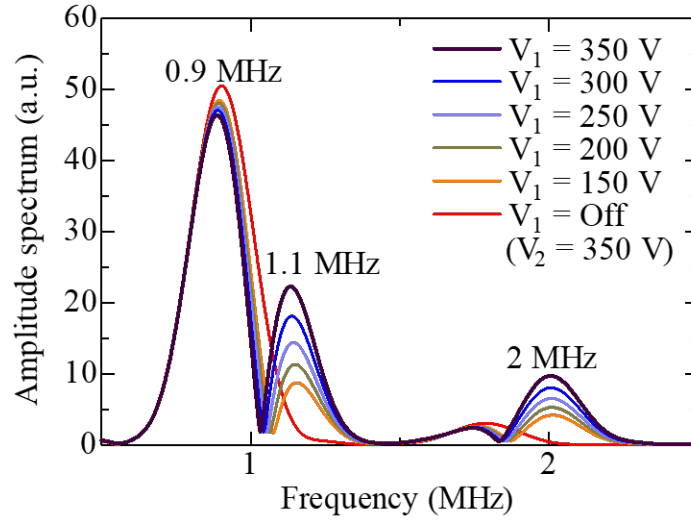


Figure 6: Amplitude spectra of the recorded waveforms.

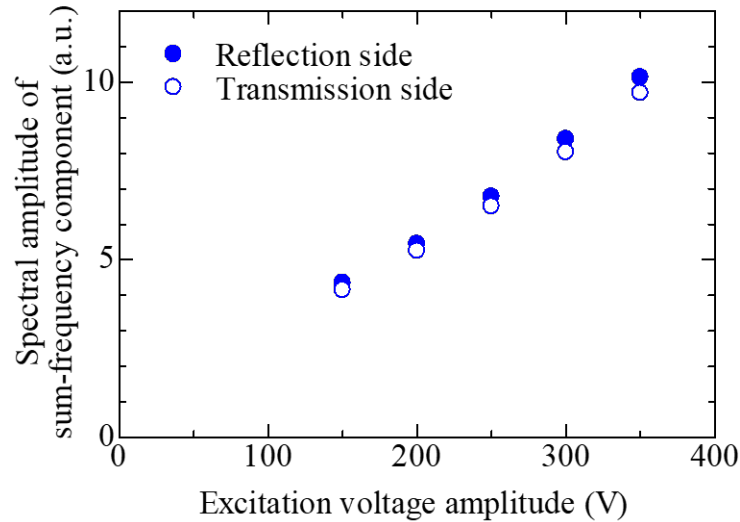


Figure 7: Variation of the spectral amplitude of the sum-frequency component recorded on both sides of the interface with the excitation voltage amplitude for the 1.1 MHz incident wave (the corresponding voltage amplitude for the 0.9 MHz wave was fixed as 350 V).

4. Conclusion

The frequency mixing of two monochromatic plane longitudinal waves with different frequencies incident on a nonlinear spring-type interface with different incident angles has been analysed. A perturbation technique has been used to linearize the problem, and the propagation directions of the nonlinearly generated sum- and difference-frequency components have been derived analytically from the

phase matching condition. Experimental results have also been shown for the generation of the sum-frequency component of ultrasonic longitudinal waves at a contacting interface of aluminum alloy blocks. More detailed theoretical analysis and experimental study will be presented in separate publications.

Acknowledgments

This work was supported by JSPS KAKENHI No. JP16H04235, and has subsequently been supported by JSPS KAKENHI No. JP19H02026.

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