

Proton-Induced Deuteron Knockout (p, pd) on ^{12}C and ^{16}O at 226 MeV

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 The proton-induced deuteron knockout reaction (p, pd) provides a unique opportunity to exploit proton–neutron correlations and the deuteron cluster structure of nuclei. Direct deuteron knockout experiments on ^{12}C and ^{16}O under quasi-free scattering conditions have been carried out under the ONOKORO project using a 226-MeV proton beam at the Research Center for Nuclear Physics, Osaka, Japan. Deuterons from the knockout reaction, acting as clusters, were unambiguously detected at the focal plane of the Large Acceptance Spectrometer. Outgoing protons, following the knockout of clusters, were detected at the focal plane of the Grand Raiden spectrometer to correctly reconstruct the expected events. (p, pd) reactions are established successfully, and excitation energy spectra for the residual nuclei ^{10}B and ^{14}N are obtained. Results showed that a large difference in transition strength toward the low-lying energy level of residual nuclei, including the ground state, was found in comparison to other studies. Theoretical calculations using the Proton-Induced KnockOut reaction calculation for Exclusive processes (PIKOE) package based on the distorted-wave impulse approximation are conducted to deduce the triple differential cross section of the (p, pd) reaction, and experimental spectroscopic factors of the deuteron cluster are obtained by normalizing the experimental cross section to the theoretical computation. Consistent spectroscopic factors in comparison to shell-model expectations are obtained for transitions involving the orbital angular momentum $L = 0$ of the cluster. The present work demonstrates that the (p, pd) reactions at 226 MeV as a spectroscopic tool have an advantage in examining spectroscopic aspects of deuteron clustering and the mechanism to form a proton–neutron pair in the spin triplet state with negligible interference of final-state interactions and refraction effects.

Subject Index C30, D04, D11, D13, D22

1. Introduction

1.1. Proton-induced deuteron cluster knockout reaction (p, pd)

The proton-induced deuteron knockout reaction (p, pd) is a powerful tool to explore proton–neutron (pn) correlations in nuclei in the $T = 0, S = 1$ channel, where T and S indicate isospin and spin, respectively. It may provide the most direct information about the deuteron cluster structure of nuclei as well [1]. The tensor interaction driven by one-pion exchange is known to cause prominent deuteron-like correlation in finite nuclei [2]. Therefore, knocking out

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Table 1. Experimental conditions for several quasi-free deuteron knockout reactions on ^{12}C and ^{16}O .

Target	Reaction	E_{beam} [MeV]	Angle θ_p [degree]	Angle θ_d [degree]	Reference
^{16}O	(p, pd)	101.3	40.1	−40	Samanta et al. [6]
^{16}O	(p, pd)	75	49	−40, −37.5, −35	Grossiord et al. [4]
^{12}C	(p, pd)	75	49	−37, −33, −29	Grossiord et al. [4]
^{12}C	(p, pd)	58	36, 42	−31.6, −30	Descroix et al. [7]
^{12}C	(p, pd)	670	147	6.5	Erö et al. [5]
^{12}C	(p, pd)	392	124.8–145.2	8.6	Terashima et al. [9]
^{12}C	$(e, e'd)$	466	θ'_e not specified	not specified	Ent et al. [8]

deuterons as a cluster (or correlated pn pair) from light nuclei has been carried out for many years [3–8]. Most of the previous studies utilized (p, pd) reactions [3–7] except $(e, e'd)$ experiments performed by Ent et al. [8]. Although they detected deuterons and scattered probes (proton or electron) in coincidence to establish deuteron knockout data, there remained practical difficulties such as poor energy resolution, discernment of quasi-free scattering (QFS) conditions, and selectivity of transitions related to the orbital angular momentum L in the residual nucleus. (p, pd) reactions under QFS conditions can be realized by two methods. The first is a direct knockout of deuterons, where the outgoing deuteron is the identical system as in the target nucleus, and the second is the formation of deuterons by neutron pick-up processes as an indirect process. The former is the desired mechanism in this article. The latter has been investigated by Erö et al. for ^{12}C [5] and Terashima et al. for ^{16}O [9]. Their measurements of (p, pd) accomplished QFS conditions by choosing appropriate combinations of scattering angles of protons θ_p and those of deuterons θ_d .

Table 1 shows details of several experiments of deuteron knockout reactions on ^{12}C and ^{16}O with different probes and energies that have been conducted so far. For “direct” knockout reactions, the beam energy was about 100 MeV [6] or less [4,7]. The refraction effect in the elementary process of the (p, pd) reaction might be significant at these energies below 100 MeV [10]. According to Ref. [10], theoretical calculations of the $^{16}\text{O}(p, pd)$ reaction at about 100 MeV with the asymptotic momentum approximation (AMA) [11,12] show a large discrepancy from the experimental result, whereas calculations with the local semi-classical momentum approximation (LSCA) [13,14] could reproduce the experimental data well. The latter approach takes into account the propagation of the distorted waves under nuclear potentials, i.e. the refraction of the reaction particles is necessary to reproduce the experimental data well. Furthermore, there is no large difference between AMA- and LSCA-based DWIA calculations for $^{16}\text{O}(p, pd)$ at 250 MeV. This means that it is highly recommended to perform the (p, pd) experiment at sufficiently high energy, e.g. around 250 MeV, to minimize the impacts of the refraction effect throughout the reaction process. Erö et al. [5] and Terashima et al. [9] employed a high-enough energy of the proton so that refraction effects are considerably reduced at lower energies. However, their experiments observed scattered deuterons at forward angles θ_d , keeping the QFS condition with protons scattered to backward angles. These types of (p, pd) reactions are governed by a neutron pickup picture, not the direct knockout of a deuteron, and therefore the kinematics completely differ from direct deuteron knockout.

As aforementioned, a remarkable QFS off a deuteron by electrons ($e, e'd$) on ^{12}C was conducted by Ent et al. [8] and they suggested a “deuteron integration” mechanism to form a deuteron in nuclei among the pn correlations. This suggestion is extremely interesting since a transition to an isospin-forbidden state by a direct deuteron knockout process is reasonably described. Furthermore, they stated that such isospin-forbidden transitions might take place in a hadron-induced deuteron knockout reaction. This point of view with a probe dependence would be a fascinating aspect of deuteron knockouts.

The kinematics for direct (p, pd) knockouts under QFS conditions has flexible options for an experimental realization. Because the final state consists of a scattered proton, a knocked-out deuteron, and a residual nucleus, the number of kinematical degrees of freedom is 12 in the final state, taking into account the four-momentum of each participating particle. After considering conservation laws for energy and momentum and the rotational symmetry around the beam axis, the number of degrees of freedom is reduced to 5, whereas two-body kinematics gives 2 [15]. This kinematical feature gives a flexibility which makes it possible to separate the kinematical conditions of the elementary process from the orbital motion of the knocked-out particle at a fixed beam energy [15]. Offsetting this beneficial aspect, there are complexities in the kinematics attributed to the off-shell effect and the infeasibility of measuring the missing momentum k_{miss} [15].

Important kinematical quantities for (p, pd) knockout are: the deuteron separation energy S_d , the missing momentum $\vec{k}_{\text{miss}}$, and the momentum transfer $\vec{q}$. The separation energy S_d of the (p, pd) reaction can be described as

$$S_d = m_R + m_d - m_A, \quad (1)$$

where m_R, m_d, m_A are the mass of the residual nucleus, the knocked-out deuteron, and the target nucleus, respectively. Note that the residual nucleus can be in an excited level, and S_d can be regarded as the binding energy of a deuteron in the target nucleus A . The missing momentum $\vec{k}_{\text{miss}}$ can also be described as $\vec{k}_R$, the momentum of the residual nucleus in normal kinematics, and it is impossible to measure $\vec{k}_R$ directly, because of its smallness. It is deeply related to the momentum of the deuteron cluster $\vec{k}_F$ in the target nucleus in the initial state. Under QFS conditions where the residual nucleus does not interact or has a negligibly small interaction with the incoming protons, outgoing protons, and knocked-out deuterons, $\vec{k}_F$ can be regarded as $-\vec{k}_{\text{miss}}$. These conditions can be realized when the momentum transfer $|\vec{q}|$ is reasonably large, i.e. when there is a high incident energy of the probe above the average Fermi momentum of deuterons in the target nucleus. In order to satisfy QFS conditions, scattering angles are expected to be large for (p, pd) knockout reactions, whereas small-angle scatterings are preferred for two-body transfer (p, pd) reactions.

1.2. Theoretical models for cluster knockout reactions

Theoretical approaches to describe the cluster knockout reactions have also been developed over the decades [11,16–19]. At the very beginning of using theoretical calculations to describe cluster knockout reactions, the plane-wave impulse approximation (PWIA) was employed; however, it requires a radial cutoff parameter to simulate the effects of distortion by the residual nucleus on the incident and emitted particle wave functions. Such an arbitrary procedure involves vagueness as to the validity of the resultant nuclear structure information [16]. Eventually, the distorted-wave impulse approximation (DWIA) with a proper treatment of

distortion effects was developed for nucleon and cluster knockout [16,17]. For decades, DWIA has been the workhorse model for describing (p, pN) reactions, where N indicates a nucleon. DWIA treats the reaction as a quasi-elastic collision: The incident proton scatters from a single bound nucleon much as it would in free space, the only differences being that the struck nucleon carries its momentum in the initial state and the two-body interaction is evaluated off-the-energy-shell. Both the incoming proton and the two outgoing nucleons are propagated with one-body optical potentials generated by the target and by the residual nucleus. Under these conditions, the driving force of the reaction is an effective nucleon–nucleon (NN) interaction, t_{NN} , which resums the ladder series of the bare NN force; this distinguishes DWIA from a simple distorted-wave Born approximation (DWBA) [20]. The NN transition amplitude, which is needed to reproduce free NN observables, is obtained by sandwiching t_{NN} between initial and final NN plane waves. In this impulse picture, collective many-body effects on t_{NN} are ignored and the off-shell matrix is replaced by its on-shell counterpart [20]. Those simplifications make it possible to use NN amplitudes derived from phase-shift analyses, or equivalently a parameterized t_{NN} fitted to elastic NN data, directly in calculations of (p, pN) reactions [20].

Building on the DWIA formalism, Chant and Roos wrote a code called THREEDEE [16,17]. Over time, the code has been steadily upgraded: It now offers multiple global optical potential parameterizations, a variety of t_{NN} interactions, and even includes a module for density-dependent t_{NN} . Thanks to these developments, THREEDEE has become the favorite analysis package for nearly everyone who extracts cross sections from (p, pN) experiments performed in normal kinematics [20]. Recently, Ogata, Yoshida, and Chazono have developed a DWIA framework code called PIKOE (Proton-Induced KnockOut reaction calculation for Exclusive processes) [20], which calculates quadruple differential cross sections (QDX) as well as triple differential cross sections (TDX) in both inverse and normal kinematics without any external conversion process for the kinematics [20]. The most remarkable difference compared to THREEDEE is that the inputs, such as nucleon–nucleon interaction and nucleon–nucleus potentials, are expected to be provided by external files while maintaining user-friendliness [20]. For a deuteron knockout reaction, the TDX for (p, pd) can be obtained by introducing a global deuteron optical potential and a pd scattering cross section, instead of t_{NN} . The differential cross section of the pd elastic scattering was obtained by fitting the experimental results from 5 to 800 MeV with respect to the scattering energy [10]. The detailed application of PIKOE will be discussed below.

2. Experiment

As introduced in the previous section, proton-induced deuteron knockouts on ^{16}O and ^{12}C have been conducted to probe the deuteron cluster structure in nuclei and its spectroscopic aspects. This work aims to establish an experimental methodology for proton-induced deuteron knockout under QFS conditions free from final-state interaction (FSI) in normal kinematics. The experiment has been carried out at the West-South (WS) course of the Research Center for Nuclear Physics (RCNP), the University of Osaka, under the ONOKORO project.¹

Protons are accelerated by the coupled cyclotrons up to 226 MeV and transported to the high-resolution beam line, WS course. Figure 1 shows a schematic view of the experiment. A proton

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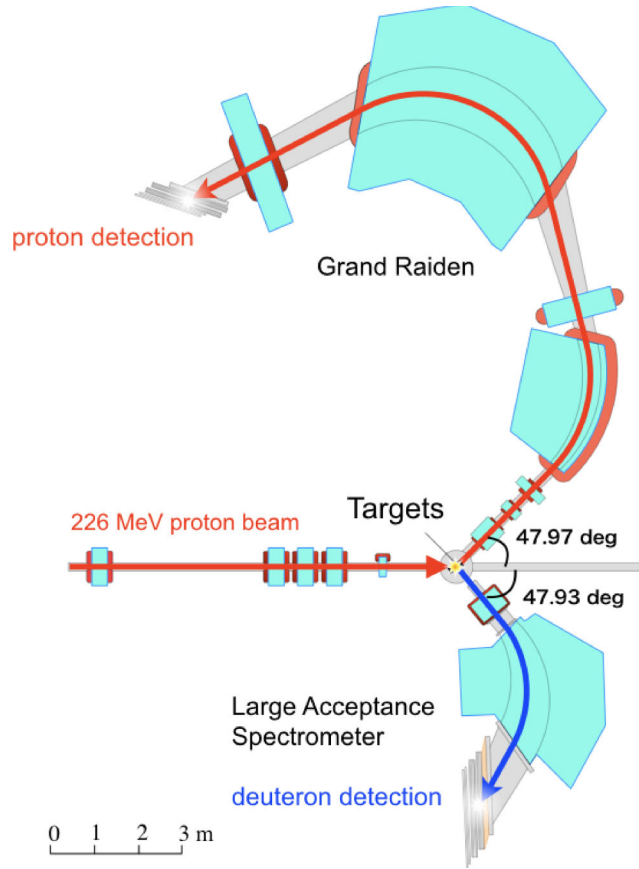


Fig. 1. A schematic view of the double-arm spectrometer in the experimental setup on the xz -plane. The beam direction is defined as the z -axis. A more detailed sketch of the configuration can be found in Ref. [21].

beam impinged on the targets set in the scattering chamber. Outgoing particles, protons, and deuterons were analyzed by two magnetic spectrometers: Grand Raiden (GR) for analyzing protons and the Large Acceptance Spectrometer (LAS) for deuterons. The angular setup for GR was 47.97° and that for the LAS was 47.93° . These setup values were determined by reaction kinematics where the center-of-momentum angle θ_{CM} of the elementary pd scattering process is equal to 75° , fulfilling the condition that the momentum $|\vec{k}_F|$ of the deuteron in the target nucleus is zero. This means that the kinematics focuses on a recoilless, i.e. $|\vec{k}_R| \approx 0$ reaction keeping the QFS condition and a high momentum transfer. The typical beam current during the experiment was about 100 pA, and it was continuously monitored by both a Faraday cup and a beam line polarimeter (BLP). The BLP contains four pairs of plastic scintillation counters (PLs), and it detects elastically scattered protons and the recoiled protons from an analyzer target in coincidence. The number of counts in the BLP can be directly converted to the beam current by applying a proper calibration factor. A 10.7-mg/cm²-thick natural carbon foil was set on the target ladder as a ^{12}C target. And a Mylar($\text{C}_{10}\text{H}_8\text{O}_4$) sheet with a thickness of 10.5 mg/cm² was employed as an oxygen target. This can be realized by subtracting events originating from the carbon components contributing to the entire yield of Mylar(p, pd) events. Eventually, both $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ reactions can be examined in series. The detectors at each focal plane consist of double-layered PLs and two vertical drift-type multiwire drift chambers (VDCs). PLs

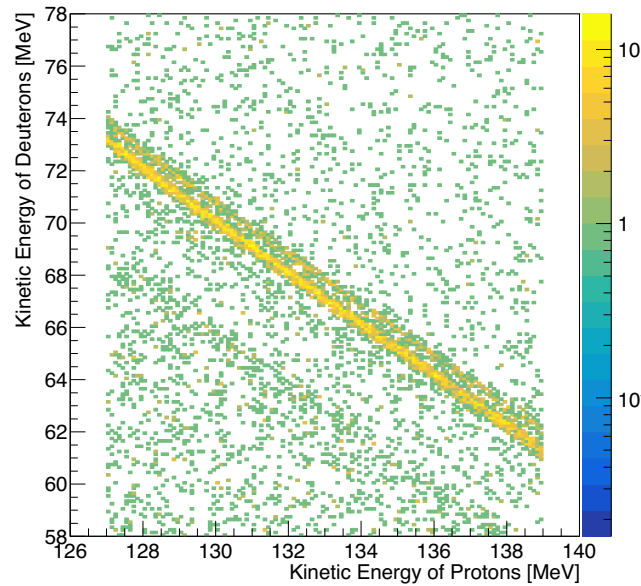


Fig. 2. Correlation between the kinetic energy of outgoing protons and that of knocked-out deuterons from $^{12}\text{C}(p, pd)$ reactions.

measure the timing and energy loss of outgoing particles, whereas VDCs measure the angle and position of them at each focal plane. Then, the kinetic energies of the outgoing particles T_p and T_d can be determined by the magnetic rigidity $B\rho$ at each spectrometer. $B\rho$ of the central orbit for the outgoing proton $B\rho^{\text{GR}}$ was set to 1.7234 Tm, and that for the deuteron cluster $B\rho^{\text{LAS}}$ was set to 1.7245 Tm. The energy acceptance for the (p, pd) reaction was determined by the horizontal acceptance of GR for proton detection, as 127–139 MeV.

3. Results

Particle identifications (PIDs) for protons and deuterons are unambiguously achieved by using the time-of-flight (ToF) and energy loss ΔE of outgoing particles [22]. Accidentally coincident events, which mainly originate from coincidences of independent (p, p) and (p, d) reactions, are properly estimated and subtracted by coincidence time analysis [22]. After PIDs and removal of accidental coincidences, (p, pd) events were established for each target nucleus. Figures 2 and 3 show the correlation between the kinetic energy of outgoing protons and that of the knocked-out deuterons from $^{12}\text{C}(p, pd)$ and Mylar(p, pd) reactions, respectively. One observes that there are clear loci which correspond to the energy levels of residual nuclei from (p, pd) reactions.

There is a mixing of components $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ in Fig. 3 originating from the molecular structure of the Mylar target. Since the experimental setups for $^{12}\text{C}(p, pd)$ and Mylar(p, pd) reactions are identical and the separation energies for the energy levels of the residual nuclei ^{10}B and ^{14}N are noticeably different, there is no difficulty in getting rid of the $^{12}\text{C}(p, pd)$ component from the excitation energy spectrum of Mylar(p, pd). Figure 4 shows the excitation energy E_{ex} spectrum of the $^{12}\text{C}(p, pd)$ reaction. The $3_{\text{g.s.}}^+$ ground state of the residual nucleus ^{10}B and the 1_1^+ state at 0.718 MeV are clearly seen. The 0^+ state at 1.74 MeV and the 1_2^+ state at 2.154 MeV are also distinguishable but relatively weak. Note that the transition to the 0^+ state

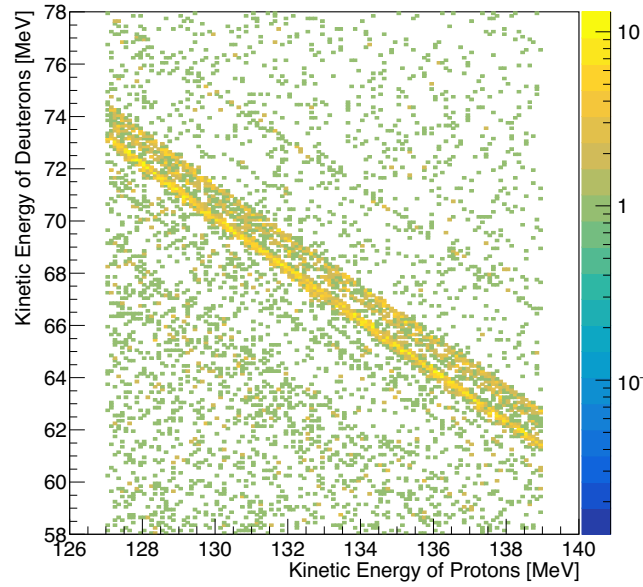


Fig. 3. Correlation between the kinetic energy of outgoing protons and that of knocked-out deuterons for the Mylar(p, pd) reaction.

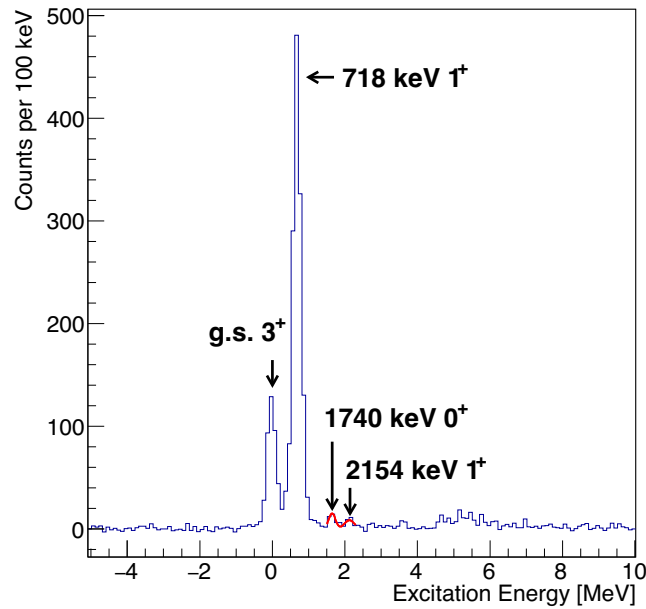


Fig. 4. Excitation energy spectrum of $^{12}\text{C}(p, pd)$ at 226 MeV. Red curves are only for eye-guides to 1.74 MeV 0^+ and 2.154 MeV 1_2^+ states.

via a direct deuteron knockout process is prohibited by isospin conservation, in principle. This topic will be discussed below.

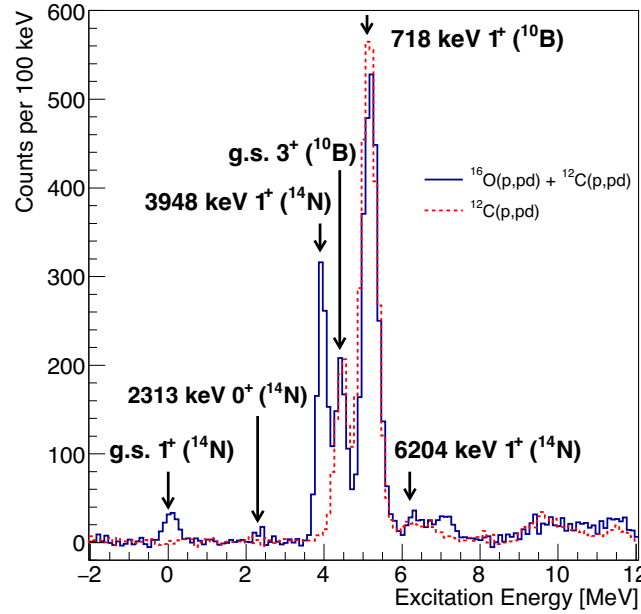


Fig. 5. Excitation energy spectrum of Mylar(p, pd) (solid line) together with a scaled spectrum of $^{12}\text{C}(p, pd)$ (dashed line) at 226 MeV.

The excitation energy spectrum of the Mylar(p, pd) reaction is shown in Fig. 5. As already mentioned, it contains both $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ events. From Fig. 5, the achieved energy resolution for the (p, pd) reaction is evaluated as 297 keV full width at half maximum (FWHM) by fitting the 1_2^+ peak of the Mylar(p, pd) reaction. Above the region of $E_{\text{ex}} \geq 4$ MeV in Fig. 4, ^{10}B still has known energy levels; however, their structure includes multiple states very close to each other. Therefore, discrimination and specification of single individual states above $E_{\text{ex}} = 4$ MeV are beyond our analysis capabilities, considering the energy resolution of about 0.3 MeV.

The excitation energies are calculated by subtracting the separation energy of the ground state of ^{14}N , 20.735 MeV, from those of the excited states. The portion of the ^{12}C molecule in the Mylar molecule, the thickness of both targets, beam currents, tracking efficiencies at each focal plane ϵ_{GR} , ϵ_{LAS} , and the data acquisition efficiency ϵ_{DAQ} should be taken into account to deduce the portion of $^{12}\text{C}(p, pd)$ events in the Mylar(p, pd) spectrum. The ground state ($1_{\text{g.s.}}^+$) and the 1_2^+ state at 3.948 MeV of ^{14}N are clearly seen in Fig. 5. The 0^+ state at 2.31 MeV, which cannot be allowed by a direct deuteron knockout process, is also identified in the spectrum, but with very minor strength. This is similar to the result of $^{12}\text{C}(p, pd)$ and will be discussed shortly in the next section.

4. Discussion

4.1. Transition ratios

One of the most remarkable differences compared to other (p, pd) reactions reported by Refs. [4–8] is that there are extremely low transition rates to the 0^+ state of the residual nuclei since this is a $T = 1$ transition, which is hindered by isospin conservation for a direct deuteron knockout process. Ent et al. [8] demonstrated this phenomenon in 1989 with a $^{12}\text{C}(e, e'd)$ experiment using a 466-MeV electron beam. The authors focused on the strongly populated 0^+ state of ^{10}B at 1.74 MeV and claimed that the formation of a deuteron in the

Table 2. Comparison of transition ratios to the strongest 1^+ state for $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ reactions.

$^{12}\text{C}(p, pd) ^{10}\text{B}$	$R_{\text{g.s.}}^{3^+_{1^+}/1^+_{1^+}}$	$R^{0^+}/1^+_{1^+}$	$R^{1^+}_{1^+}/1^+_{1^+}$
	0.271(0.0101)	0.0236(0.0027)	0.0365(0.0033)
$^{16}\text{O}(p, pd) ^{14}\text{N}$	$R_{\text{g.s.}}^{1^+_{1^+}/1^+_{1^+}}$	$R^{0^+}/1^+_{1^+}$	$R^{1^+}_{1^+}/1^+_{1^+}$
	0.151(0.0139)	0.0392(0.0067)	0.0593(0.0084)

nucleus is ascribed to “deuteron integration,” a mechanism that combines pn pairs in a relative $T = 1$ state into deuterons ($T = 0$) [8]. They also mentioned the possibility of the occurrence of the same mechanism in hadron-induced deuteron knockout reactions, where a hadronic probe changes a pn pair in a $T = 1$ state into the deuteron [8]. However, the results of this work tell us that transition rates to the 0^+ state via a direct deuteron knockout (p, pd) are tremendously low. Table 2 shows comparisons of transition ratios to the strongest 1^+ state for $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ reactions. The yield ratio $R^{0^+}/1^+_{1^+} = Y(0^+)/Y(1^+_{1^+})$ of the 0^+ state to the strongest 1^+ state for $^{12}\text{C}(p, pd) ^{10}\text{B}$ is 0.0236(0.0027) and $R^{0^+}/1^+_{1^+}$ for $^{16}\text{O}(p, pd) ^{14}\text{N}$ is 0.0392(0.0067). This might be interpreted as a direct deuteron “cluster” knockout from ^{12}C and ^{16}O with negligibly small effects from other reaction mechanisms related to the formation of pn pairs, such as deuteron integration. The statistical errors are considered as the error for the ratios.

The energy of the proton beam in the present work is 226 MeV, where the momentum transfer q is higher than 2.6 fm^{-1} , high enough to keep the QFS condition. Moreover, the experimental setup was optimized to observe recoilless knockout reactions with $k_R \approx 0$. Transitions dominated by the orbital angular momentum $L = 0$ state are more strongly populated than $L = 2$ transitions. That is the reason why the yield of the $3^+_{\text{g.s.}}$ ground state of ^{10}B is significantly suppressed as compared to that of the $1^+_{1^+}$ state at 0.718 MeV. For other $^{12}\text{C}(p, pd)$ experiments at different beam energies below 100 MeV [4,5], much higher transition strengths to the ground state of ^{10}B are observed in the excitation energy spectra. In addition to this, a higher ratio of $R^{0^+}/1^+_{1^+}$ had been measured at 75 MeV, which is explained by a QFS mechanism on pn pairs in a spin singlet $S = 0$, $T = 1$ state induced from a spin-flip in the final state [4]. Similar results were found for $^{16}\text{O}(p, pd) ^{14}\text{N}$ at different energies [4,6] where both ratios $R^{0^+}/1^+_{1^+}$ and $R^{1^+}_{1^+}/1^+_{1^+}$ are much higher than in the present work. Samanta et al. [6] also mentioned an emergence of a weak 0^+ state at 2.313 MeV in the excitation energy spectrum of $^{16}\text{O}(p, pd)$. This should be the consequence of a multistep or a single-step spin-flip process converting a spin singlet $S = 0$ pn pair to a spin triplet $S = 1$ deuteron during the reaction process [6].

The discrepancies of the ratios $R^{0^+}/1^+_{1^+}$ and $R^{1^+}_{1^+}/1^+_{1^+}$ can be explained by differences in the kinematics of the experiments. First, the beam energy of 226 MeV used in this work ensures much-reduced refraction effects of the deuteron [10] compared to Ref. [6] where the beam energy is 101.3 MeV. Second, the selectivity of the kinematics in our work is better adapted to select $L = 0$ transitions than in Ref. [6], although the angular coverage is much narrower. A weaker population of the 1^+ ground state of ^{14}N is reasonable because this transition is known to be dominated by $L = 2$ components [9,23]. The reason that both transition ratios $R^{1^+}_{1^+}/1^+_{1^+}$ for $^{16}\text{O}(p, pd)$ and $R^{1^+}_{1^+}/1^+_{1^+}$ for $^{12}\text{C}(p, pd)$ are also suppressed might be explained by the relative strength of the reaction cross section for $L = 0$ in Ref. [23], where the transition

magnitudes of the strongest 1^+ show much higher degrees by a factor of ≈ 4 for $^{12}\text{C}(p, pd)$ and ≈ 9 for $^{16}\text{O}(p, pd)$. Although the ratios $R^{1^+_{3/2}/1^+_{1/2}}$ and $R^{1^+_{3/2}/1^+_{1/2}}$ do not agree well between the experiment and the shell-model predictions, a qualitative understanding may still be valid. Hence, a direct deuteron knockout (p, pd) at 226 MeV with a double-arm spectrometer in normal kinematics could be regarded as a “cleaner” probe to investigate the spectroscopic feature of deuteron clustering in a nucleus than other probes or energies introduced so far, considering a significantly suppressed yield of deuterons in $S = 0$ originating from spin-flipped pn pairs and an excellent excitation energy resolution. One thing should be mentioned: The results of the $^{16}\text{O}(p, pd)$ experiment at 392 MeV by Terashima et al. showed a significantly low $R^{0^+/1^+}$ value which is similar to the present work, although they focused on (p, pd) reactions dominated by a neutron pickup process [9]. Both experiments explored the pn pair at large relative momenta caused by the tensor force at sufficiently high incident energies, but with different kinematics. Therefore, it is reasonably conjectured that a largely diminished $R^{0^+/1^+}$ ratio in the (p, pd) reaction can be accomplished by using a high enough incident energy of protons above 200 MeV.

4.2. Spectroscopic results

In order to deduce spectroscopic factors of the deuteron cluster from the (p, pd) reaction data, theoretical calculations using PIKOE [20], based on the DWIA, are employed to obtain TDX_{th} for (p, pd) reactions. The experimental TDX_{exp} is defined as

$$\frac{d^3\sigma_{\text{exp.}}}{d\Omega_p d\Omega_d dE_p} = \frac{e \cdot A \cdot N_{(p,pd)} / (\epsilon_{\text{VDC}}^{\text{LAS}} \cdot \epsilon_{\text{VDC}}^{\text{GR}} \cdot \epsilon_{\text{DAQ}})}{Q_{\text{beam}} N_{\text{Avo}} \cdot t \cdot a \cdot \Delta\Omega_{\text{GR}} \Delta\Omega_{\text{LAS}} \Delta E_p} [\mu\text{b} \cdot \text{MeV}^{-1} \cdot \text{sr}^{-2}], \quad (2)$$

where e is the elementary charge $1.602 \times 10^{-19} \text{C}$, A the atomic mass number of the target nucleus, t the target thickness in g/cm^2 , a the abundance of the specific target nucleus, N_{Avo} the Avogadro number 6.022×10^{23} , Q_{beam} the charge of the beam in units of C , and ϵ_i denotes the efficiencies of tracking and data acquisition. $N_{(p,pd)}$ is the yield of the (p, pd) reaction, $\Delta\Omega_i$ are solid angles for outgoing particles, and ΔE_p is the energy acceptance determined by protons. TDXs are calculated by PIKOE using the experimental acceptance for each energy level of the residual nucleus, assuming a unit spectroscopic factor (S -factor). The calculations also provide the cross sections integrated over the experimental acceptance σ_{th} for each state. As previously mentioned, data from the elastic p - d scattering are employed in fitting the elementary process [10]. Moreover, the EDAD1 global Dirac phenomenological optical potential [24–26] is employed as the proton–nucleus optical potential, and the global deuteron–nucleus optical potential of Ref. [27] is adopted to compute TDX_{th} and its integration in the PIKOE framework. Then a normalization of the experimental cross section $\sigma_{\text{exp.}}$ (the integrated TDX along the experimental acceptance) to the theoretical cross section σ_{th} can be deduced as an experimental spectroscopic factor of the deuteron cluster Σ_d defined as

$$\Sigma_d \equiv \frac{\sigma_{\text{exp.}}}{\sigma_{\text{th}}}. \quad (3)$$

Furthermore, in order to figure out the configuration mixing for each level of the residual nuclei, nuclear shell-model calculations have been performed by using the shell-model code KSHELL [28]. It provides not only the configuration mixing of the levels, but also the two-nucleon transfer amplitudes (TNAs) [29,30] between the initial and final states. They are the reduced shell–model matrix elements of a pair–creation operator that transfers two specified

Table 3. Deduced TDXs, experimental spectroscopic factors Σ_d , and shell-model spectroscopic factors for the (p, pd) reaction on ^{12}C and ^{16}O . The unit of TDX is $\mu\text{b} \cdot \text{MeV}^{-1} \cdot \text{sr}^{-2}$ and that of the integrated TDX, σ , is μb . The uncertainty of TDX_{exp} is addressed in Table 4. Values of TDX_{th} are calculated and averaged over the experimental acceptance. CK(1970) indicates spectroscopic factors for $L = 0$ given in the Table 2 of Cohen and Kurath [23]. The radius parameter r_0 and diffuseness a_0 for the bound-state wave function are 1.41 fm and 0.65 fm in the PIKOE calculations, respectively.

Target	E_{ex} [MeV]	J^π	TDX_{exp}	TDX_{th}	$\sigma_{\text{exp.}} [\mu\text{b}]$	$\sigma_{\text{th.}} [\mu\text{b}]$	Σ_d ($\equiv \frac{\sigma_{\text{exp.}}}{\sigma_{\text{th.}}}$)	CK(1970) ($L = 0$)
^{12}C	0.00	3^+	1.87	0.23	0.00108	0.0001	-	-
^{12}C	0.72	1^+	6.90	6.59	0.00398	0.0025	1.59	1.19
^{12}C	1.74	0^+	0.16	-	0.00009	-	-	ND
^{12}C	2.15	1^+	0.25	6.30	0.00014	0.0024	0.06	0.27
^{16}O	0.00	1^+	1.00	6.04	0.00057	0.0023	0.25	0.01
^{16}O	2.31	0^+	0.26	-	0.00015	-	-	ND
^{16}O	3.95	1^+	6.59	5.20	0.00379	0.0020	1.90	1.75
^{16}O	6.20	1^+	0.39	4.78	0.00022	0.0018	0.12	0.20

valence-orbit nucleons from an initial state $|J_A\rangle$ and leaves the residual nucleus in a final state $|J_B\rangle$. Note that the initial state of the deuteron knockout reaction does not coincide with $|J_A\rangle$, but $|J_B\rangle$. The TNA is given by

$$\text{TNA}(B, A, j_a, j_b, J) = \frac{\langle J_B | A^\dagger(j_a, j_b, J) | J_A \rangle}{\sqrt{2J_B + 1}}, \quad (4)$$

where $A^\dagger$ is the two-particle creation operator of rank J ,

$$A^\dagger(j_a, j_b, J, M) = \frac{[a_{j_a}^\dagger \times a_{j_b}^\dagger]_M^{(J)}}{\sqrt{1 + \delta_{j_a j_b}}}. \quad (5)$$

The TNAs are normalized such that the sum of their squares over all states in the $(N - 2)$ nucleon system is $N(N - 1)/2$ for the removal of two nucleons in the model space [29,30]. The TNAs can be converted to a spectroscopic amplitude S_{nLJ} by applying the two-nucleon coefficient of fractional parentage (2c.f.p.) [23,31], and the squared sum of the spectroscopic amplitude $\sum |S_{nLJ}|^2$ will give the spectroscopic factor [31].

The shell configurations are employed to determine the radial node n_r and orbital angular momentum L , which are used as input parameters of PIKOE. It should be noted that the 1^+ state of the residual allows both $L = 0$ and $L = 2$ transitions. The theoretical estimates for 0^+ and 1^+ states are presumed to be $L = 0$ transitions, considering the experimental setups. The implemented interaction for the KSHELL computation is the Cohen–Kurath interaction where the model space is the p -shell [32]. Eventually, spectroscopic factors Σ_d are derived from the TNAs as 1.80 for the 1_2^+ state of $^{16}\text{O}(p, pd)$, and 1.22 for the 1_1^+ state of $^{12}\text{C}(p, pd)$, respectively. These spectroscopic factors are very close to the values, 1.75 and 1.19, respectively, deduced by Cohen and Kurath [23].

Table 3 shows the results of theoretical calculations and comparison to the experimental values. The radius parameter r_0 and diffuseness a_0 used to construct the Woods–Saxon potential between the deuteron cluster and the residual core are taken from Ref. [6], with $r_0 = 1.41$ fm and $a_0 = 0.65$ fm for the (p, pd) reaction, to allow a direct comparison of

Table 4. The uncertainties of the $\text{TDX}_{\text{exp.}}$ for each transition. $\sigma_{\text{sys.}}$ is the systematic error, which originates from the uncertainty of the target thickness, and $\sigma_{\text{stat.}}$ is the statistical error, which comes from the yield of the events. The unit of TDX and the errors is $\mu\text{b} \cdot \text{MeV}^{-1} \cdot \text{sr}^{-2}$.

Target	$E_{\text{ex}}[\text{MeV}]$	J^π	$\text{TDX}_{\text{exp.}}$	$\sigma_{\text{sys.}}$	$\sigma_{\text{stat.}}$
^{12}C	0.00	3^+	1.87	0.19	0.06
^{12}C	0.72	1^+	6.90	0.69	0.12
^{12}C	1.74	0^+	0.16	0.02	0.02
^{12}C	2.15	1^+	0.25	0.03	0.02
^{16}O	0.00	1^+	1.00	0.10	0.09
^{16}O	2.31	0^+	0.26	0.03	0.04
^{16}O	3.95	1^+	6.59	0.66	0.22
^{16}O	6.20	1^+	0.39	0.04	0.05

spectroscopic factors. These parameters are consistent with values derived from electron-scattering data and proton binding energies, making them most suitable for describing proton removal [6]. Nevertheless, they suffice for our purposes because the resultant cluster wave functions have root-mean-square radii which are close to or slightly larger than the empirical nuclear radii [6]. The depth of the Woods–Saxon potential is tuned to reproduce the deuteron separation energies by solving the Schrödinger equation for the bound-state wave function with given r_0 and a_0 . This approach can be justified because the knockout reactions are peripheral and probe the tail of the cluster removal wave function due to the absorption at low impact parameters [33]. Note that if the radius parameter r_0 varies by 0.1 fm, $\text{TDX}_{\text{th.}}$ change approximately 28%, whereas a 10% deviation of the optical potential, especially the imaginary part affecting the distortion, results in changes of around 5% of $\text{TDX}_{\text{th.}}$

The experimental spectroscopic factor Σ_d for the 1^+ state at 3.95 MeV in the $^{16}\text{O}(p, pd)$ reaction is deduced as $1.90 \pm 0.19 \pm 0.06$, which is in better agreement with the shell-model expectation of 1.75 [23]. The agreement is better than is the case in Ref. [6], where the minimum recoil momentum of the residue is about 41 MeV/c. Note that the experimental TDX contains a systematic error $\sigma_{\text{sys.}}$ of around 10%, which originated from the uncertainty of the target thickness, and a statistical error $\sigma_{\text{stat.}}$ involved by the yield of the events. The first term of the error in Σ_d corresponds to $\sigma_{\text{sys.}}$ and the second term is $\sigma_{\text{stat.}}$. Table 4 shows the $\sigma_{\text{sys.}}$ and $\sigma_{\text{stat.}}$ for each (p, pd) channel. Each of Σ_d in Table 3 can be interpreted as an upper limit, because a possible mixing of the $L = 2$ component always exists, according to the shell-model expectation [23]. In addition, a limited angular coverage of the cluster momentum $\vec{k}_F$ in terms of outgoing proton energy E_p causes an insufficient comparison to the theoretical calculation for ensuring the $L \neq 0$ determination. The experimental spectroscopic factor Σ_d for the $L = 0$ dominant 1^+ state in the $^{12}\text{C}(p, pd)$ reaction at 0.72 MeV is deduced as $1.59 \pm 0.16(\sigma_{\text{sys.}}) \pm 0.03(\sigma_{\text{stat.}})$, and this value also has to be regarded as an upper limit, considering possible mixing of the $L = 2$ component. The DWIA calculations for $^{12}\text{C}(p, pd)$ and $^{16}\text{O}(p, pd)$ show the distinct feature that the $L = 0$ transition strengths corresponding to the 0^+ and 1^+ levels are almost equivalently estimated, while the $L = 2$ transition to the 3^+ state (the ground state of ^{10}B) is heavily suppressed. This can be explained by the kinematical selectivity for $L = 0$ in this work. It should be noted that the experimental spectroscopic factors of other 1^+ states at 2.15 MeV for $^{12}\text{C}(p, pd)$ and at 6.02 MeV for $^{16}\text{O}(p, pd)$, where $L = 2$ components dominate, disagree with the

shell-model predictions. Detailed analysis of those states will require future measurements at finite momenta.

Although DWIA has worked well for α -cluster knockout [22,34], which has a small size and a large binding energy, its validity has been considered to be less certain for knocking out a fragile cluster such as the deuteron. The present work demonstrates that the $L = 0$ dominant 1^+ states at 0.72 MeV for $^{12}\text{C}(p, pd)$ and at 3.95 MeV for $^{16}\text{O}(p, pd)$ with 226-MeV protons can be reliably described by a direct deuteron cluster knockout picture with deuteron formation probabilities consistent with shell-model calculations. The conclusion is fairly consistent with that in Ref. [34] where the breakup of deuterons, reformation of pn pairs, and so on, are evaluated to be reasonably small at intermediate energies, and do not significantly affect the cross section. Theoretical models considering the breakup of deuterons, reformation of pn pairs, and so on, to describe our experimental results properly are definitely required.

5. Summary

Proton-induced deuteron cluster knockout reactions (p, pd) on ^{12}C and ^{16}O at 226 MeV beam energy were investigated to explore deuteron clustering with a hadron probe. The behavior of the excitation energy spectrum is significantly different from the case of a lepton probe, $^{12}\text{C}(e, e'd)$ in particular. There is much less transition strength to the 0^+ state at 1.74 MeV in ^{10}B , which is not allowed by a direct deuteron knockout process, whereas ($e, e'd$) shows a comparable transition strength to the ground state. Similarly, the excitation energy spectrum of the $^{16}\text{O}(p, pd)$ reaction shows almost negligible contribution of the 0^+ state at 2.31 MeV. This tendency is related to the isospin character of the proton–neutron pair [9] and can be interpreted as a probe dependence, including a change with the initial momentum. Therefore, the most suitable probe to knock out a deuteron from the nucleus under QFS conditions might be a high-energy proton.

The incident energy is also a crucial variable of the (p, pd) reaction. Reference [10] investigates the effects of the refraction effect due to the optical potential, and shows that it is safe to perform the (p, pd) experiment at sufficiently high energy, typically more than 200 MeV, to minimize the impacts of the refraction effect throughout the reaction process. Thus, the incident energy of this work, 226 MeV, is high enough to carry out the (p, pd) experiment.

The recoilless condition employed in the present work corresponds to the selection of the orbital angular momentum $L = 0$. It is accomplished by using a double-arm spectrometer set by specific scattering angles satisfying the desired kinematics. Owing to the high momentum resolving power of the spectrometers, an energy resolution of 297 keV in FWHM is achieved, and this enables us to discriminate low-lying energy levels in the residual nuclei. Eventually, spectroscopic analyses can be unambiguously performed by choosing the transition component of interest in the energy spectrum. $L = 0$ transitions to the strongest 1^+ state for each residual nucleus are analyzed by comparing theoretical calculations of cross sections in the DWIA formalism with the PIKOE framework. Ultimately, experimental spectroscopic factors of the deuteron cluster Σ_d are deduced by comparing the experimental cross section $\sigma_{\text{exp.}}$ to the theoretical cross section $\sigma_{\text{th.}}$. Experimental spectroscopic factors Σ_d for the 1^+ states populated in the $^{12}\text{C}(p, pd)$ reaction at 0.72 MeV and in the $^{16}\text{O}(p, pd)$ reaction at 3.95 MeV are obtained as 1.59 and 1.90, respectively. Those values are consistent with the shell-model expectation of Ref. [23]; however, an extraction of the $L = 0$ component for a 1^+ state might be affected by a slight mixing of the $L = 2$ component. That is the reason why Σ_d for the 1^+ state in (p, pd)

reactions is considered as an upper limit, although the contribution of $L = 2$ is anticipated to be very small (less than 4%) [4,9,23,31]. Evaluation of the $L = 2$ components requires additional measurements of the momentum distribution of the deuteron, whereas the current work has been done under a single kinematical setup.

Results of the present work demonstrate that proton-induced deuteron cluster knockout at 226 MeV has the advantage of deducing the experimental spectroscopic factor Σ_d of pn pairs in $T = 0$ for various target nuclei. DWIA calculations to estimate the integrated triple differential cross section for each transition using PIKOE have been carried out and successfully contributed to deducing the experimental spectroscopic factors.

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Data availability

The data of this study have been presented in the main text.

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