

## Energy Straggling of 6.74 MeV Protons in Cu

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Energy straggling of 6.74 MeV protons in Cu foils has been measured. Foils are commercially prepared and approximately  $3.9 \text{ mg/cm}^2$  thick. Measurements have been made for two different thicknesses, using one foil and two foils. The straggling results are compared with theoretical predictions of Vavilov. The energy straggling greater than theoretical predictions is considered to be the effect of foil non-uniformity. It has been shown that the energy loss measurement with non-uniform target gives the same results as the measurement with uniform target, supposing the mean thickness of non-uniform target is equal to the thickness of uniform one.

### 1 INTRODUCTION

Anomalous energy straggling of 5.486 MeV alpha particles in Al has been reported by Sykes and Harris.<sup>1)</sup> A similar result was obtained by Comfort *et al.*<sup>2)</sup> To investigate this anomaly several authors<sup>3,4,5)</sup> have studied the straggling of alpha particles in Al and other metal foils. One of the origin of anomalous straggling is microscopic non-uniformity of foils. Sofield *et al.*<sup>6)</sup> have examined the thickness uniformity of commercially produced rolled Al foils and evaporated Al foils with an areal resolution of about  $5 \times 10^{-6} \text{ cm}^2$  by proton backscattering method. They concluded that the evaporated foils were uniform to better than 1% but the rolled foils had 5–10% non-uniformities of their mean thickness. These studies indicate that anomalous energy straggling can be accounted for by the non-uniformity of foils.

Stopping power of metal foils for MeV proton has been measured to an accuracy of about 0.5% by Andersen *et al.*<sup>7)</sup> and Ishiwari *et al.*<sup>8,9)</sup> Although the uniformity of vacuum evaporated foil seems to be much better than commercially rolled one, usually rolled foils are used in stopping power measurements. It is suspected that rolled foils will give rise to anomalous energy straggling. It is desirable to obtain the information of foil uniformity by measuring energy straggling and to examine the effect of non-uniformity on measured stopping power values.

In the present work the stopping power and energy straggling of 6.74 MeV protons in rolled Cu foils have been measured. Straggling distributions have been compared with theoretical predictions of Vavilov.<sup>10)</sup> In the estimation of straggling width the theories of Bohr<sup>11)</sup> and Bethe-Livingston<sup>12)</sup> are also considered.

### 2 EXPERIMENTAL METHOD

A schematic diagram of the experimental arrangement is given in Fig. 1. Details

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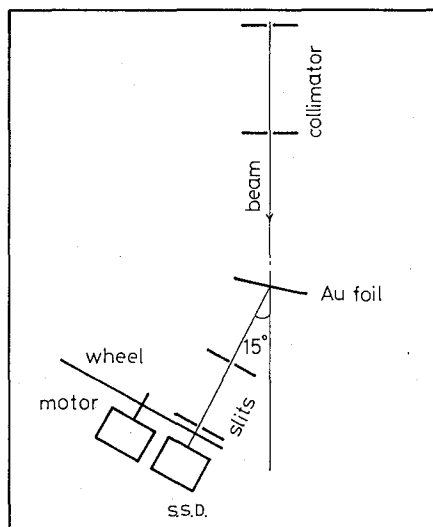


Fig. 1. Schematic of the experimental arrangement.

are similar to those described in Ref. 9. A proton beam from Kyoto University cyclotron was focussed onto the Au foil of  $180 \mu\text{g}/\text{cm}^2$ , the normal of which was at the angle of  $7.5^\circ$  with respect to the beam direction. The beam scattered at  $15^\circ$  by the Au foil was collimated by the double slit system. In front of a surface barrier silicon detector was mounted the rotating wheel with two windows, one was open and the other was covered with sample Cu foil. Hence straggled and non-straggled protons could be measured under the same condition.

The pulses from detector were amplified with a low noise amplifier. To expand a part of energy spectrum was used a biased amplifier, then pulses were fed into a 400 channel pulse height analyzer. Before and after each run energy calibration of detector-amplifier system was performed with a precision pulser.

Foils were commercially prepared rolled ones. Their thicknesses were determined as weight/area. In this experiment two foils were used, one was  $3.891 \text{ mg}/\text{cm}^2$  and the other was  $3.900 \text{ mg}/\text{cm}^2$ . Two different thicknesses were attained to use one foil or both foils. The measurements were made twice for each thickness. From the energy spectrum stopping power as well as energy straggling was obtained.

### 3 RESULTS

The measurements were made for two different Cu thicknesses of  $3.891 \text{ mg}/\text{cm}^2$  and  $7.791 \text{ mg}/\text{cm}^2$ . A typical spectrum of the straggled and non-straggled protons is shown in Fig. 2. In Fig. 3 are shown the energy-loss spectra. The attached error bars indicate statistical uncertainties only. The theoretical fittings of Vavilov are given as solid curves. The theoretical distributions have been corrected for the finite resolution of the detection system and the energy spread of the beam scattered by Au foil. Assuming that the energy distribution of protons which passed through Au foil could be described by the Vavilov theory, a Gaussian resolution function was determined so as to reproduce the non-straggled proton peak. The

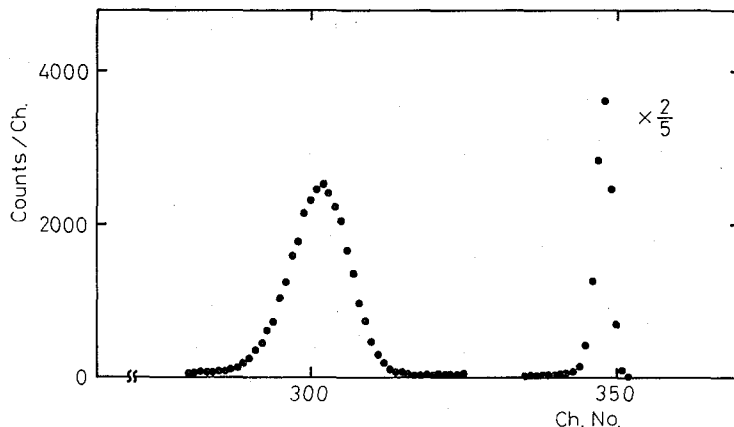


Fig. 2. Typical spectrum of straggled and non-straggled protons for 7.791 mg/cm<sup>2</sup> Cu sample at  $E_p=6.740$  MeV.

resultant standard deviation was 6.76 keV. For straggled distribution, a larger standard deviation was needed to obtain a good fit to the experiment. Standard deviations that gave the best fit were searched, using chi-square fitting criteria. The resultant standard deviations were 13.01 and 17.05 keV for 3.891 and 7.791 mg/cm<sup>2</sup> target respectively. The larger standard deviation was needed for the thicker sample experiment. This means that the addition of one foil causes a certain additional standard deviation. This amount can be deduced from three resolution functions, including one for non-straggled protons. From its quadratic sum rule the additional standard deviations were calculated as 11.11 keV for 3.891 mg/cm<sup>2</sup> foil and 11.02 keV for 3.900 mg/cm<sup>2</sup> foil. The nearly equal additional standard deviations for nearly equal thicknesses were interpreted as indicating that these were due to the non-uniformity of foils.

Figure 4 shows the comparison between the experimental straggling and the calculations using the Bohr, the Bethe-Livingston, and the Vavilov theories. In this figure are compared the straggling widths from which the finite spread arising from the detection system and the beam is removed. To calculate the straggling from the Bethe-Livingston theory ionization potential for each shell was evaluated from the table of Sternheimer.<sup>13)</sup>

Stopping power was obtained from the energy difference between straggled and non-straggled proton peaks, and attributed to the mean proton energy within the foil, i.e.,  $E' = E_0 - \Delta E/2$ . Where  $E_0$  and  $\Delta E$  are incident proton energy and energy loss. The stopping powers were shown in Fig. 5 together with those of Andersen *et al.*<sup>14)</sup> as a solid curve and Ishiwari *et al.*<sup>15)</sup> These stopping powers were in good agreement.

#### 4 DISCUSSION

When the relative energy loss of incident protons is small and the variation of proton energy over the thickness of the sample foil can be ignored, the Vavilov theory gives a good fit to the observed straggling distribution. This was tested by Kolata

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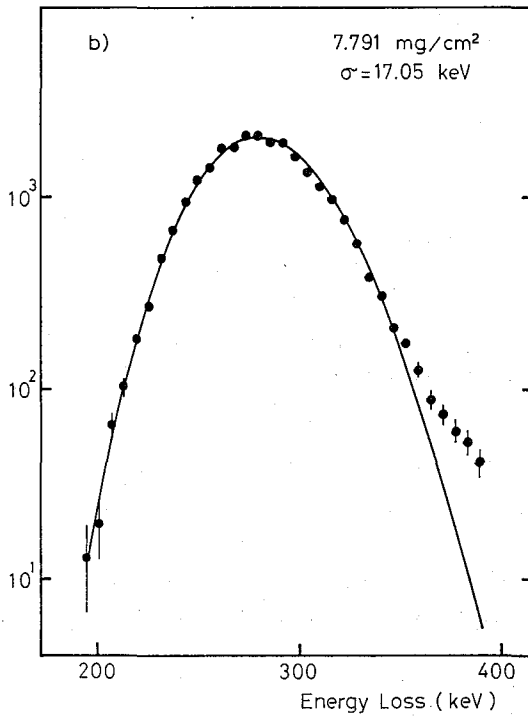
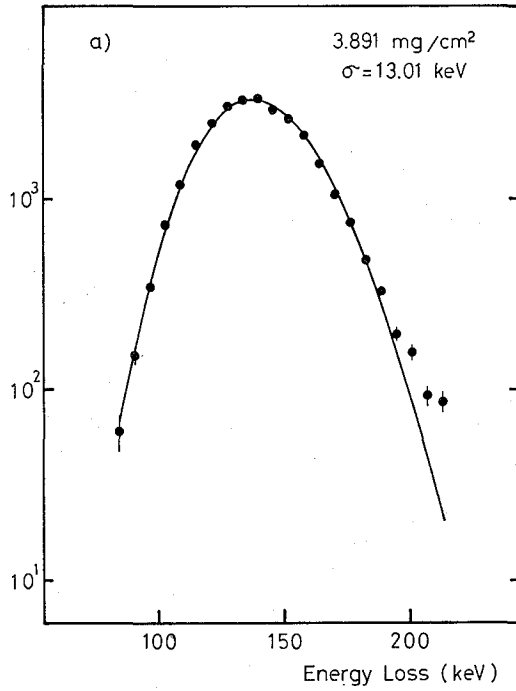


Fig. 3. Straggling distributions for 6.74 MeV protons. Error bars indicate statistical uncertainties only. The solid curves are the fittings of the Vavilov theory. The values of  $\sigma$  are the standard deviations of searched Gaussian resolution functions.

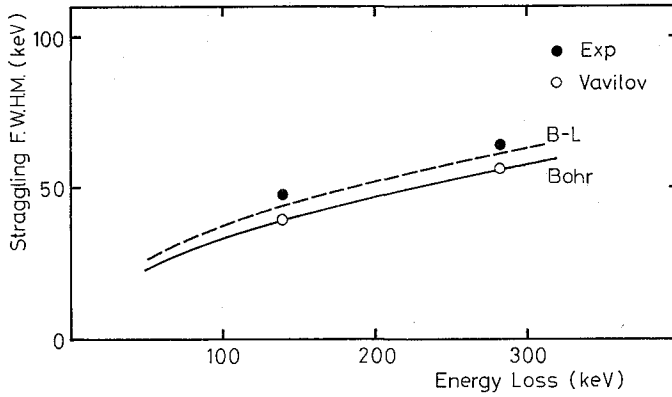


Fig. 4. Energy straggling as a function of energy loss. The predictions of the Bohr (solid curve) and the Bethe-Livingston (dashed curve) theories are also shown. Open circles indicate the predictions of the Vavilov theory.

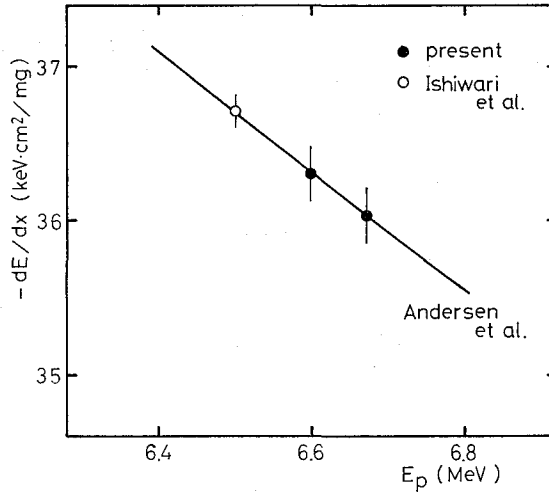


Fig. 5. Stopping powers for protons as a function of incident energy. The solid curve is deduced from the data of Andersen *et al.* Open circle is the new Nara datum.

*et al.*<sup>16)</sup> and it was found that the agreement between experimental results and the prediction of the theory was excellent when the relative energy loss was less than 5%. It is the case of the present work. Anomalous energy straggling of alpha particles has been discussed<sup>1,2)</sup> and measurements for the targets with good uniformity<sup>3,4,5)</sup> have shown that the straggling distribution is well reproduced by the theories of Bohr or Bethe-Livingston.

The need of the resolution functions with additional standard deviations and their regularity with foil thickness indicate the non-uniformity of the sample foils.

It is of interest that the Vavilov and the Bohr theories give almost equal straggling width. It is known that for the parameter  $\kappa = \xi/E_{max} \gg 1$  Vavilov distribution becomes Gaussian and for  $\kappa \ll 1$  it becomes Landau<sup>17)</sup> type, where  $\xi$  is directly related

to the mean energy loss and  $E_{\max}$  is the maximum energy loss of incident particle in a single collision. In this experiment the values of  $\kappa$  are 1.30 and 2.60. Though measured distribution shows a slight asymmetry, Gaussian approximation seems to be fairly good. The Bethe-Livingston theory gave the larger straggling width than the Bohr and the Vavilov theories. This was considered to be due to the different treatment of atomic electrons in the sample foil. In the Bethe-Livingston theory the electron binding effect was considered, while in the Bohr and the Vavilov theories electrons were treated as free.

We discuss next the effect of target non-uniformity on stopping power. For simplicity the following approximations are assumed. First, the distribution of target thickness is Gaussian. Second, the mean proton energy after traversing the target of thickness  $t$  is given by the linear function of  $t$ . To evaluate this energy the stopping powers of Andersen *et al.*<sup>14)</sup> were used. In the present case this approximation is correct within 0.1 keV. Third, proton straggling is described by the Bohr or the Bethe-Livingston theory. Under these conditions has been calculated the mean energy of protons which passed through the non-uniform target. The result shows that the non-uniform target gives the same stopping power as the uniform one which has the same thickness as the mean thickness of non-uniform target. As for the distribution of foil thickness, if only it is symmetric with respect to its mean value, the above result holds (Appendix). Though the distribution of foil thickness is not known correctly and straggling shows slight asymmetry, these effects are considered to be small. Hence we can obtain the correct stopping power with non-uniform target. The present data are in good agreement with those of Andersen *et al.*<sup>14)</sup> and Ishiwari *et al.*<sup>15)</sup>

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#### APPENDIX

We will show that the non-uniform target gives the same stopping power that the uniform target does under the conditions assumed in the text. The thickness distribution of the target,  $T(t)$ , is given by

$$T(t) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-\frac{(t-\bar{t})^2}{2\sigma_t^2}\right]. \quad (1)$$

Where  $\bar{t}$  and  $\sigma_t$  are the mean target thickness and the standard deviation respectively. When protons go through the target of thickness  $t$ , their energy distribution,  $P(E, t)$ , is given by

$$P(E, t) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left[-\frac{(E-\bar{E}(t))^2}{2\sigma_s^2}\right]. \quad (2)$$

Where  $\bar{E}(t)$  is the mean proton energy. The standard deviation  $\sigma_s$  is directly related to the straggling width. From the assumptions the equations

$$\bar{E}(t) = a \cdot t + b \quad (3)$$

$$\sigma_s^2 = k \cdot t \quad (4)$$

are satisfied. Where  $a$ ,  $b$ , and  $k$  are constants. The mean energy of protons which traversed the non-uniform target is calculated by

$$\langle E \rangle = \int dt \int dE E \cdot T(t) \cdot P(E, t).$$

Using Eqs. (1), (2), (3), and (4) we can execute the integral. The result is

$$\langle E \rangle = a \cdot \bar{t} + b$$

In the case of the uniform target we can use  $T(t) = \delta(t - \bar{t})$  instead of Eq. (1) and obtain

$$\langle E \rangle = a \cdot \bar{t} + b.$$

Thus we can reach the same result. This can be shown if  $T(t)$  satisfies the more general relation

$$T(t) = T(2\bar{t} - t).$$

Therefore, it is not necessary that  $T(t)$  is Gaussian but enough that  $T(t)$  is symmetric with respect to  $\bar{t}$ .

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