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DOCTORAL THESIS

Topographic Relief Correlated Monte Carlo 3D Radiative Transfer Simulator for Forests

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Declaration of Authorship

I, Sheng-ye JIN, declare that this thesis titled, ' Topographic Relief Correlated Monte Carlo 3D Radiative Transfer Simulator for Forests ' and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Date:

"I am an investigator by inclination. I feel a great thirst for knowledge and an impatient eagerness to advance, also satisfaction at each progressive step."

Immanuel Kant

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Abstract

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Topographic Relief Correlated Monte Carlo 3D Radiative Transfer Simulator for Forests

by Sheng-ye JIN

Changing of the environment performs a critical factor which affects the life of the beings, both short and long termly, on the earth. Human activities of industry and agriculture accelerate the changing since the Industrial Revolution. Detecting and monitoring the change become meaningful to detect the global changing. There is no doubt that remote sensing (RS) technique is the best option to grasp the activities on the Earth.

As one of the most important environmental component on the Earth, the forest, which not only provides the wood resources but also prevents the desertification and so on, plays a key role to reflect the environmental change. The leaf organs convert solar flux into chemical energy via photosynthesis, which is a process generating the oxygen being indispensable to most of the bionic. Since the utilization of fossil fuels impacts atmosphere and climate, forest is a critical factor of the carbon circuiting of global ecosystems. Monitoring forests come to be a major task of environmental changing studies. Remote sensing shows great potential with the development of forestry investigation by using remotely sensed data.

Understanding the physical processes that affect electromagnetic waves within forests is a key to a better analysis of global environmental change by remote sensing. Studies with the understandings of the radiative transfer equation (RTE) are the fundamental of the radiative transfer (RT) models. Since the boundary condition problem of RTE has been overcome, RTE becomes a fast and convenient way to compute the canopy radiation budget. However, with the request of 3-D heterogeneous canopy RT model, RTE method, dealing with the homogeneous RT progress, is not suitable to solve such kind of issues. Monte Carlo ray-tracing method, as a branch of computer simulation methods, is a candidate for its good performance to estimate the radiative transfer budget.

In this study, a new paradigm of Monte Carlo ray-tracing method based 3-D vector model (TRCMCRTS) for estimating a 3-D open forest bidirectional reflectance factor (BRF) is developed. Different from other Monte Carlo canopy RT models, this model takes into account rugged terrain conditions when estimating the bidirectional distribution of the area reflection. The topographic relief, which is a difficult issue to take account in Monte Carlo method, is well indicated by using bilinear interpolation of the DEM data of the forest covered area.

In Chapter 1, the author demonstrates the connection between global warming and vegetation, and the necessities of monitoring vegetation globally. Then, he is trying to present the background of the remote sensing of vegetation, and trying to highlight the importance of the radiative transfer modeling is a key approach to illustrate the interaction between photon and vegetation which is the physical fundamental of the remote sensing data processing. To overcome the difficulties of the traditional RTE modeling methods based on radiative transfer equation, Monte Carlo ray-tracing method becomes a expectable candidate for simulating the radiative regime. As a description of the whole study, the structure of the thesis is also expressed at the end of this chapter.

Related literatures are summarized in Chapter 2. In this part, the author attempts

to comprehensively provide the studies about vegetation radiative transfer modeling by Monte Carlo method. In order to determine the necessary parameters, like LAD (leaf area density) and leaf angle distribution, of the radiative transfer models, reviews are expressed as well.

Mathematical and physical fundamentals are introduced in Chapter 3, including the geometry of the Monte Carlo ray-tracing algorithms and the numerical methods for accelerating the calculation. Applications for retrieving LAD and leaf inclination angle distribution function are consequently presented.

Algorithm of TRCMCRTS and simulation of open forest radiative regime are indicated in Chapter 4. Both simplified and complicated ground conditions for the radiative transfer simulation are introduced. In addition, for generating random forests, a 2-D circle packing algorithm is employed. The validation of the Monte Carlo model is by comparing TRCMCRTS with well-performed FLiES model with the same parameters input. Results of both models indicate high identities.

In Chapter 5, methods for the retrieving of the canopy structural parameters like leaf area index (LAI) and leaf area density (LAD), which are difficult to directly and rapidly determined, is also investigated and validated. Both of the methods, the using of close-range photograph and laser scanner point cloud data (PCD), obtained the good correlation with the field investigations. Especially, new method of estimating leaf normal distribution with point cloud data by using scientific computer geometrical algorithms.

Conclusions of the MCRT modeling and canopy structural parameters retrieving are summarized in Chapter 6; some suggestions about MC modeling and canopy parameters retrieving are given as well. According to the discussions about the study, future work plans are presented as well.

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Abbreviations

APAR	Absorbed PAR
AVHRR	Advanced V ery H igh R esolution R adiometer
BIC	B ayesian I nformation C riterion
BRDF	B idirectional R eflectance D istribution F unction
BRF	B idirectional R eflectance F actor
CDF	C umulative D istribution F unction
CGAL	C omputational G eometry A lgorithms L ibrary
CPU	C entral P rocessing U nit
CUDA	C ompute U nified D evice A rchitecture
DEM	D igital E levation M odel
ELAI	E ffective L eaf A rea I ndex
EM	E xpectation M aximum
ENVI	E xelis V isual I nformation S olutions
ERTS	E arth R esources T echnology S atellite
ESA	E uropean S pace A gency
FLiES	F orest L ight E nvironmental S imulator
GFLOPS	G iga F loating-point O perations P er S econd
GMM	G aussian M ixture M odel
GPU	G raphics P rocessing U nit
IFOV	I ntantaneous F ield O f V iew
IKONOS	is a commercial high-resolution earth observation satellite.
LAI	L eaf A rea I ndex
LAD	L eaf A rea D ensity
LiDAR	L ight D etection A nd R anging
LND	L eaf N ormal D istribution
MC	M onte C arlo

MCRT	M onte C arlo R adiative T ransfer
MODIS	M oderate Resolution I maging S pectroradiometer
NASA	N ational A eronautics and S pace A dministration (USA)
NDVI	N ormalized D ifference V egetation I ndex
PCA	P rinciple C omponents A nalysis
PCD	P oint C loud D ata
PDF	P robability D istribution F unction
PPFD _{leaf}	P hotosynthetic P hoton F lux D ensity at leaf surface
RAMI	R Adiation Transfer M odel I ntercomparison
RNG(or)	R andom N umber G eneration / R andom N umber G enerator
ROMC	R AMI O n-line M odel C hecker
RS	R emote S ensing
RT	R adiative T ransfer
SAIL	S cattering by A rbitrarily I nclined L eaves
SVM	S upport V ector M achine
TLS	T erritorial L aser S canner
TOA	T op O f the A tmosphere
TOC	T op O f the C anopy
TRCMCRTS	T opographic R elief C orrelated M onte C arlo R adiative T ransferring S imulator
VR	V ariance R eduction

Dedicated to my wife and our daughter.

Chapter 1

Introduction

1.1 Global Warming and Vegetation

Global warming, which is mainly caused by greenhouse gases from combustion of the fossil fuels, hasten the deterioration of the environment drastically (see FIGURE 1.1). Longwave infrared radiation emitted by the Earth surface is absorbed partially and returned to the land by greenhouse gases, generally by water vapor and carbon dioxide. This is so called greenhouse effect, which is one of the key reasons of the climate change. Since the Industry Revolution, the global temperature growing and the increased fossil fuels usage correlated obviously (see FIGURE 1.2). To reduce the greenhouse gases therefore becomes an important issue of improving the environment. As a natural consumer of the carbon dioxide, plant is a key factor of reducing the quantity of the CO₂. Plant leaves take in carbon dioxide from the air and release moisture to the air as part of the photosynthesis process. The release of moisture through evapotranspiration causes the air to cool. With increasing atmospheric carbon dioxide, one can expect to see a change in plant carbon exchange rates and water relations. This may result in reduced evaporation rates, thus amplifying the summer continental warming. Without plants, the ground and air would become warmer, exacerbating the problem. On the other hand, the response of the vegetation is also a factor of the impact from the climate change. Phenological studies represent that it is a factor that mediates vegetation feedbacks to

the atmosphere and climate system through a diverse set of processes and mechanisms [1–6].

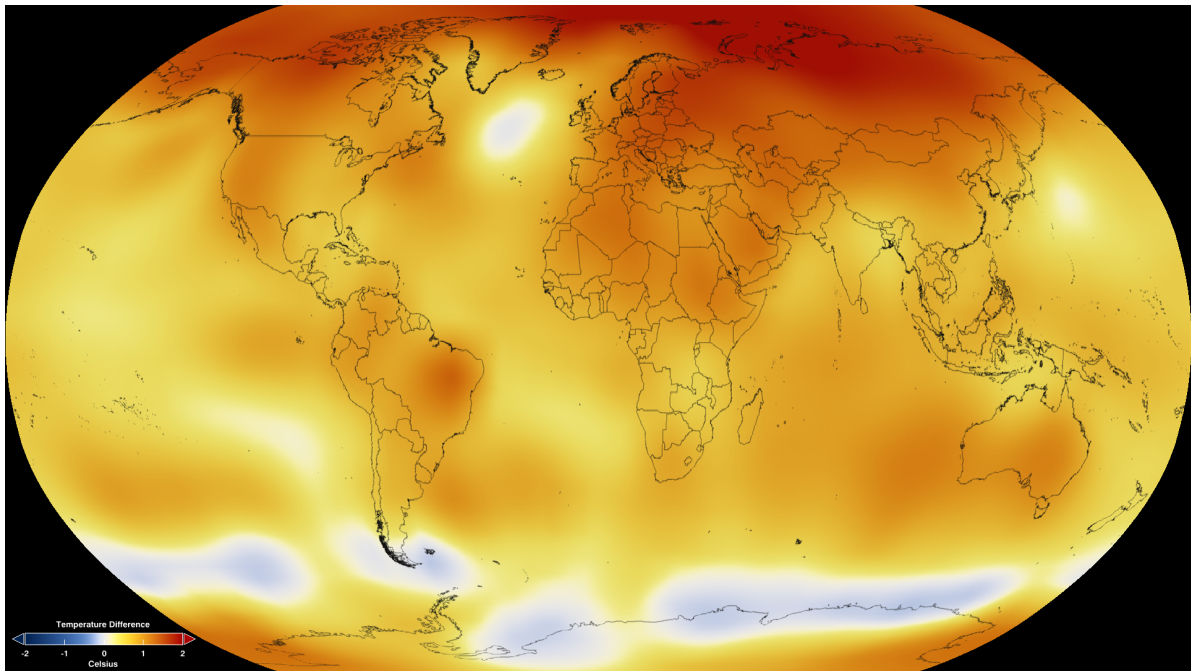


FIGURE 1.1: Map in Robinson projection represents the global temperatures five year averaged from 2011 to 2015. Scale bar is in degree Celsius from -2 to 2 [7].

1.2 Remote Sensing of Vegetation

Remote sensing is, broadly but logically speaking, the collection of the information about an object without physically contacting with it. A restricted definition is by focusing one's attention on the Earth's surface and atmosphere, viewed from air-borne or satellite-platform via electromagnetic radiation. The explosive growth in the availability of remote sensing technology during last decades extends our knowledge of both nature and urban society about the Earth, as well as the objects of astrospace. Since the launch of the Landsat Multispectral Scanner System (MMS) in 1972, the number and diversity of the RS systems are increased dramatically.

The passive remote sensing in regions from $0.4 \mu\text{m}$ to $14 \mu\text{m}$, or from visible to infrared band, requests sensors which measure radiation reflected or emitted from the landscape, atmosphere, and clouds. The visible, near infrared and short-wave infrared

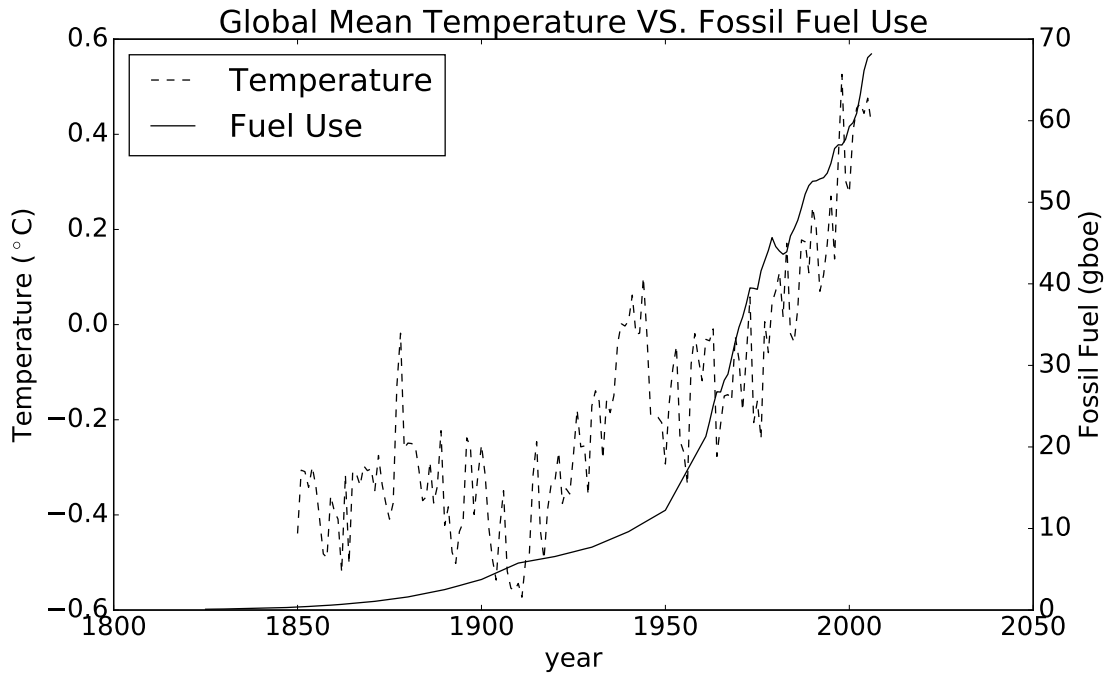


FIGURE 1.2: Consumption of fossil fuels, together with the curve of global mean temperatures in degree Celsius ($^{\circ}\text{C}$). Fossil fuel consumption is measured in gigabarrels of oil equivalent (gboe) per year, shown on the vertical scale on the right side of the graph. These data were principally from British Petroleum's Statistical Review of World Energy (2007) for oil and gas, and the History Database of the Environment, a publication of The Netherlands, for coal. Predictions for 2007 and later are based on my personal Hubbert analyses of coal, natural gas, conventional oil, and extra-heavy oil [8].

are in regions from $0.4\ \mu\text{m}$ to about $3\ \mu\text{m}$, which are caused by the sun's illumination at the Earth's surface. The mid-wave infrared (3 to $4\ \mu\text{m}$ and 4.5 to $5\ \mu\text{m}$) regions are the transmission zone from solar-reflective to thermal radiation. Above $5\ \mu\text{m}$ self-emitted thermal radiation from the Earth dominates. Since the cause of the formation of the thermal radiation does not directly by the solar illumination, this phenomena can be detected as microwave brightness temperature [9].

The term "active remote sensing" refers to the detecting that sensors generating the microwave radiation to illuminate the surface of the object. Because the sensors' detecting geometry can be selected and controlled, and the illuminated microwave energy is self-generated, they are so-called active sensors. All the active microwave sensors' wavelength, on board in a satellite or an airplane, operated in the range from $7.5\ \text{mm}$ to $1000\ \text{mm}$ [10]. Therefore, their performance is not affected by the cloud layer, weather

conditions and time of a day. In particular, with the development of the synthetic aperture radar (SAR), which is an imaging technology by receiving the echoed beam from the object surface, a new era of microwave remote sensing starts.

Massive information detected by the remote sensor extended our vision which make the understanding of the Earth and the Universe wider and quicker. It also requires us to develop the application of data storage and processing [11]. The signals detected by the sensors are immediately converted into digital numbers and transmitted to dedicated receiving stations, where these data are heavily processed. The effective exploitation of remote sensing data to reliably generate useful, pertinent information hinges on the availability and performance of specific tools and techniques of data analysis and interpretation. A variety of mathematical models can be used for this purpose; they are implemented as computer codes that read the data or intermediary products and ultimately lead to the generation of products and services usable in specific applications.

Inner information of the object retrieving, which might be the most challenging objective, needs to be achieved from the presentation of the RS data as well. Remote sensing instruments on space platforms are sophisticated detectors recording the occurrence of elementary events, for instance the absorption of photons or electromagnetic waves, as variations in electrical currents or voltages. Users, on the other hand, require information on events and processes occurring in the geophysical environment (e.g., atmospheric pollution, oceanic currents, terrestrial net productivity, etc.) Extracting useful environmental information from RS data observed in space is the main challenge of the remote sensing scientists. One superior characteristic of the new generation RS is the comprehensive utility of quantitative numerical models for evaluating the landscape parameters [12].

With the increasingly request of the studies about global changing, remote sensing technique indeed plays an irreplaceable role of the environmental detecting and monitoring. The vegetation, which perhaps the most important component of the environment, becomes a key object of RS studies. Remote sensing technology, in the sense of the collecting of information by a sensor remotely located from the object, are growingly

providing a critical component of the vegetation studies. In spite of the systematical application for vegetation studies might be much earlier than RS techniques, particular advantages of RS include the fact that it is non-contact and non-destructive and the ease with which observations are readily extrapolated to the larger scales that are of particular interest in vegetation studies [13]. In order to build up a quantitative functional presentation of the processes and mechanisms of how photon interacts with materials of landscape, physical models based on radiative transferring theory open up a possibility to researchers.

It is without a doubt that forest should be one of the most concerned to be detected objects of RS. In spite of the earth observation RS began in 1970s, aerial photogrammetry and photo interpretation had been developed 40 years earlier. Airborne sensor is used for identifying small-grain cereal crops and diseases, as well as other aims of plant science during 1950s [14]. Remote sensing technique, which is widely utilized for crop estimation, biotic stress assessment, plant breeding planning, plant physiology strategies, biochemistry progress decomposing, precession agriculture etc., becomes an irreplaceable application of plant sciences [15–23].

Taking the interactive radiative transferring progress of forest for instance, when solar radiation reaches the top of the canopy, it is scattered and reflected, and its direction and spectral composition are altered in a complex procedure by the forest structural manner. For the purpose of RS, part of this reflected radiation is intercepted and captured by the sensor [24]. However, lack of the understanding of the physical progress of the radiation transferring with in the vegetation canopy drives one to model the physical progress of the interaction between detected objects and radiation. In order to evolve a quantitative radiative transfer modeling of forest, people adopt to approximate experimental data via empirical or semi-empirical formula on the foundation of a simplified treatment of the general regularities of the attenuation of radiation in plants [25]. This is an acceptable way if there are huge amount RS data of the area, therefore the model has simplified form for avoiding the complicated RT equation solving. The advantage of this method is conveniently calculating and coincident with ground data. However, the empirical formula cannot provide a physical explanation of the RT progress inside

and outside the canopy. Huge amount of RS data are not available for everywhere as well. Physical model has been proposed increase for precisely explaining the RT regime rather than empirical model since 1980. Different from the empirical model, it expresses the RT progress physically and mathematically. In the NASA Earth Vision project by 2020, environmental change prediction is the principal objective. The reliable predicting depends on the physical dynamic model of land surface processes whose variables must be quantitatively estimated from the earth RS data [12].

1.3 Radiative Transfer Equation (RTE)

Since space-borne instruments can only measure the properties of electromagnetic waves emitted or scattered by the Earth, scientists need first to understand where these waves originate from, how they interact with the environment, and how they propagate towards the sensor. To this effect, they develop models of radiation transfer, assuming that everything is known about the sources of radiation and the environment, and calculate the properties of the radiation field as the sensor should measure them. This is the so-called direct problem. In practice, one does acquire the measurements from the satellite, and would like to know what is going on in the environment: This is the inverse problem, which is much more complicated. Knowing the value of the electromagnetic measurements gathered in space, how can we derive the properties of the environment that were responsible for the radiation to reach the sensor. In solar domain, radiation transfer models are tools to represent the scattering and absorption of radiation by scattering elements/centers. They should satisfy energy conservation. Rather than discussing the solving of approximate equations applicable to realistic geophysical situations, we will concentrate on exact and light mathematical treatment of a simplified physical system. The characteristic one most concerns about a beam of photon is the intensity I_λ which is the radiance at the point defined by

$$I_\lambda = \frac{dE_\lambda}{\cos\theta dA d\Omega d\lambda dt}, \quad (1.1)$$

where dE_λ is differential amount of radiant energy within a time interval dt and spectral wavelength interval $d\lambda$, $\cos\theta dA$ is the effective area intercepting beam [26]. $d\Omega$ is the solid angle element which can be expressed by zenith angle θ and azimuth angle ϕ in polar coordinates

$$d\Omega = \sin\theta d\theta d\phi. \quad (1.2)$$

In the early stage, transfer theory is concerned with the transfer of radiation through a star's atmosphere layer which is defined as a turbid medium [27]. Therefore, the radiative transfer equation could be presented by a Fredholm's integro-differential equation

$$\Omega \cdot \nabla I(\mathbf{r}, \Omega) = -\sigma(\mathbf{r})I(\Omega, \mathbf{r}) + \sigma_s(\mathbf{r}) \int_{4\pi} I(\Omega', \mathbf{r}) \Phi(\Omega', \Omega, \mathbf{r}) d\Omega', \quad (1.3)$$

without considering polarization effects and self-emitting of the medium. The beam radiance decreases in the medium in terms of absorption and scattering affection. These kind of affections is characterized by the extinction coefficient σ of the medium (FIGURE 1.3). The σ is determined by the characteristic of the medium, the wavelength λ and the position $\mathbf{r}(x, y, z)$ if the medium holds the heterogeneous nature. The extinction caused by absorption σ_a and scattering σ_s consist of the σ , which

$$\sigma = \sigma_a + \sigma_s. \quad (1.4)$$

In particular, if the medium was divided into non-overlapping components, the extinction coefficients satisfy the associative law, which

$$\sigma_a = \sum_i \sigma_a^{(i)}, \quad \sigma_s = \sum_i \sigma_s^{(i)}.$$

Here $\sigma_a^{(i)}$ and $\sigma_s^{(i)}$ are the absorption and scattering coefficient of the i th component of the medium. Scattering affection caused on a radiance beam $I(\Omega', \mathbf{r})$ could be presented

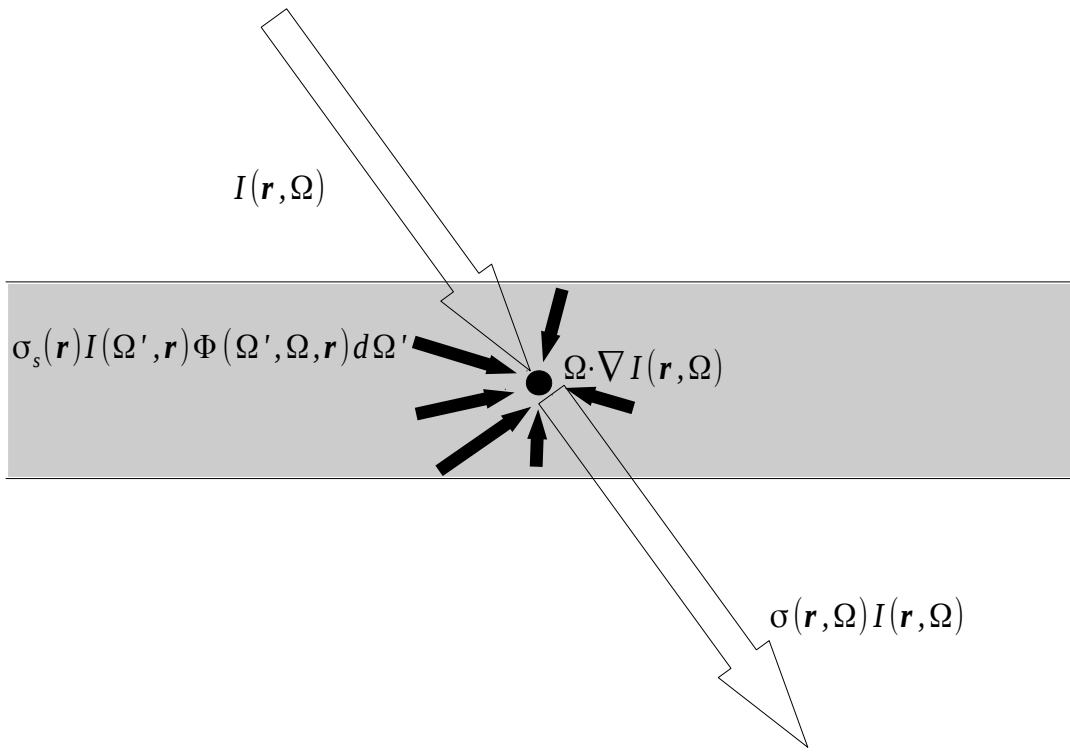


FIGURE 1.3: Mechanism sketch of the radiative transfer equation.

by

$$\sigma_s(\mathbf{r})I(\Omega', \mathbf{r})\Phi(\Omega', \Omega, \mathbf{r})d\Omega'.$$

Here $\Phi(\Omega', \Omega, \mathbf{r})$ is named a phase function, which $\Phi(\Omega', \Omega, \mathbf{r})d\Omega$ means the probability of the photon from Ω' scattered to Ω :

$$\int_{4\pi} \Phi(\Omega', \Omega, \mathbf{r})d\Omega = 1. \quad (1.5)$$

The turbid medium radiative transfer problem is developed remarkably in various of subjects, a classical way of finding the solution of the RT problems in vegetation canopies using the analogy of turbid layer in past decades. The canopy layer is dimensionally finite medium, therefore, the boundary conditions of canopy depend on the incident radiation field and on the reflection properties of the boundaries (FIGURE 1.4). The medium is essentially divided into atmospheric and canopies, respectively. Conditions

at the adjacent between them varies according to the conditions of medium boundaries. Generally, bidirectional reflectance factor (BRF) $R(\Omega', \Omega)$ is defined by

$$I = \frac{1}{\pi} R(\Omega', \Omega) I' |\cos \theta'|, \quad (1.6)$$

where $\Omega'(\theta', \phi')$ is the incident direction, I and I' are reflected and incident radiance, respectively. The boundary problems could mainly separated into three phases which are detailed discussed in [28].

- (i) The transferring progress in atmosphere which is adjacent to the top of the canopy layer (FIGURE 1.4 (i)).

$$I(0, \Omega) = \pi I_0 \delta(\Omega - \Omega_0), \quad \cos \theta < 0, \quad (1.7a)$$

$$I(z_1, \Omega) = \frac{1}{\pi} \int_{2\pi^-} R_{\text{cnp}}(\Omega', \Omega) I(z_1, \Omega') |\cos \theta'| d\Omega', \quad \cos \theta > 0, \quad (1.7b)$$

where, Ω_0 is the initialized incident direction, I_0 is the incident intensity and R_{cnp} is the canopy BRF, $\delta()$ is Dirac delta function.

- (ii) The canopy layer is considered as turbid medium which is ideally adjoining with atmosphere layer without any inter-diffusion (FIGURE 1.4 (ii)). Therefore the boundary conditions of canopy layer are

$$I(z_1, \Omega) = I^*(\Omega), \quad \cos \theta < 0, \quad (1.8a)$$

$$I(z_2, \Omega) = \frac{1}{\pi} \int_{2\pi^-} R_{\text{soil}}(\Omega', \Omega) I(z_2, \Omega') |\cos \theta'| d\Omega', \quad \cos \theta > 0. \quad (1.8b)$$

The incident radiation $I(z, \Omega)$ might be extincted in the atmosphere layer. Here I^* is addressed for setup the extinction

$$I^*(\Omega) = I_d(\Omega) + \pi I_0 \exp\left(-\frac{\tau_{\text{atm}}}{|\cos \theta_0|}\right) \delta(\Omega - \Omega_0), \quad \cos \theta < 0, \quad (1.9)$$

where I_d is diffuse sky radiance, τ_{atm} is the optical thickness of atmosphere.

- (iii) In FIGURE 1.4 (iii), the downer boundary of atmosphere layer is considered as a reflecting boundary. Photon escaping from the top of canopy layer is reflected by atmosphere. In this case, only the first boundary condition (Equation 1.8a) is specified as

$$I^*(\Omega) = \frac{1}{\pi} \int_{2\pi^+} R_{atm}(\Omega', \Omega) |\cos \theta'| I(z_2, \Omega') d\Omega' + I_d(\Omega) + \pi I_0 \exp\left(-\frac{\tau_{atm}}{|\cos \theta_0|}\right) \delta(\Omega - \Omega_0), \quad \cos \theta < 0, \quad (1.10)$$

where $R_{atm}(\Omega', \Omega)$ is BRF of atmosphere.

The boundary-value problems are overcome by iterative methods by setting

$$I(z_1, \Omega) = f_{z_1}(\Omega), \quad \cos \theta < 0, \quad (1.11a)$$

$$I(0, \Omega) = f_0(\Omega), \quad \cos \theta > 0, \quad (1.11b)$$

where f_{z_1} and f_0 are given functions. A question is, therefore, addressed that the discretized media does not continuous. The reflecting characteristic, which is imposed for numerical computing, seems destructs the consistency of the medium at each slice boundaries. In particular, for simulating the 3-D complex conditions of the RT regime, RTE methods obviously paralyzed. Monte Carlo method, therefore, becomes alternate candidate for solving the complicated conditions.

1.4 Monte Carlo Radiative Transfer (MCRT) Modeling for Vegetation Canopy

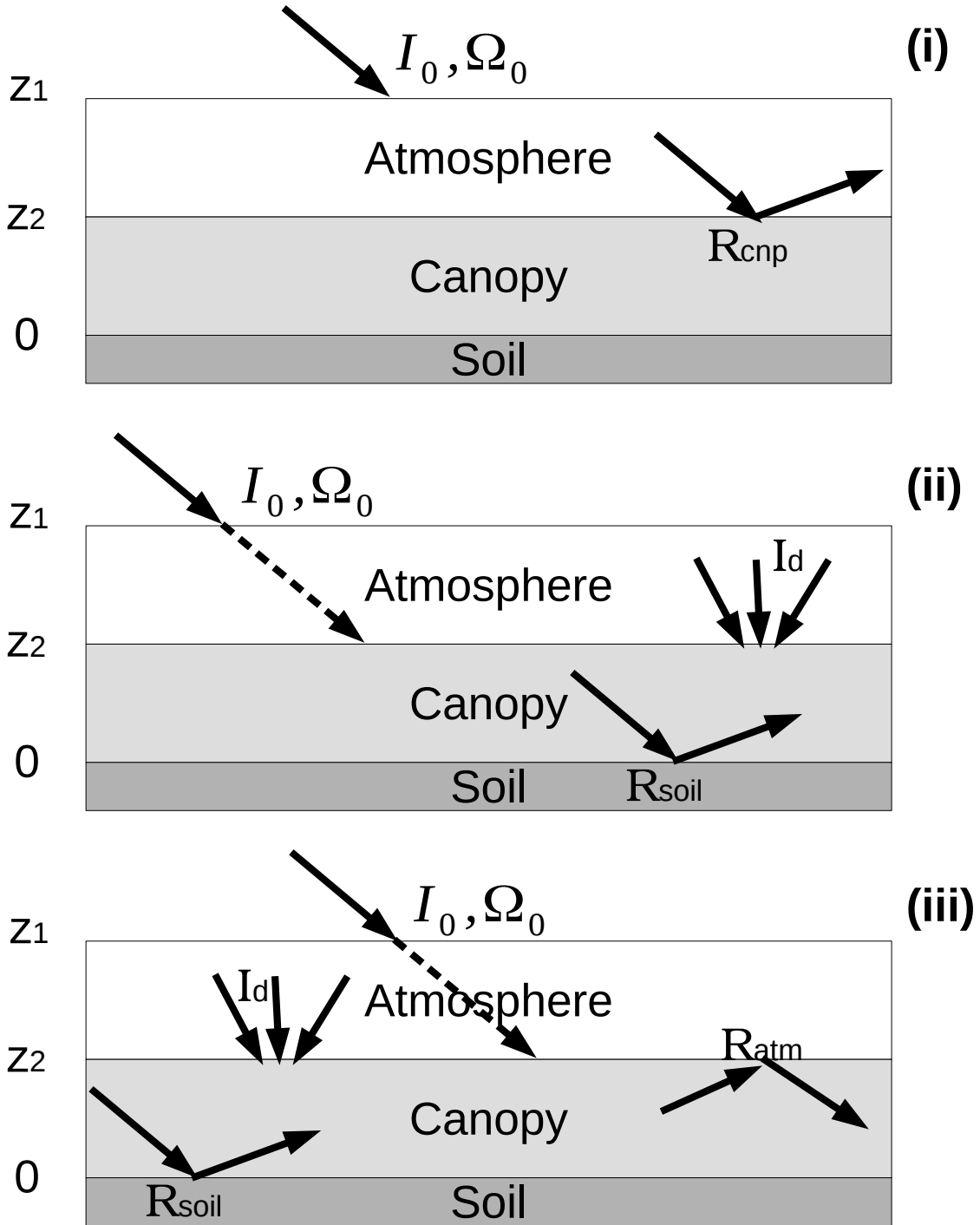


FIGURE 1.4: Sketch of the boundary conditions [29]. (i) Radiation transfer in atmosphere, here the top of canopy is considered as reflecting surface, (ii) transferring in canopy layer, incident radiance is including the atmospheric attenuated and diffused radiance, and (iii) bottom of the atmosphere is considered as reflecting surface which resends radiation back to canopy layer.

1.4.1 Why Monte Carlo?

Most of the solved RTE methods of the transfer theory either are concerned with atmospheres (or medium) stratified in plane parallel layers, or they can be reduced to equations occurring in the theory of plane parallel media, or they can be solved by methods used for such media, and we shall therefore confine our attention to the plane case [30]. This means the canopy, the media, is treated as a set of statistical ensembles defined over a volume of averaged materials. However, when we dealing with transferring progress of the medium with inhomogeneous characteristics, or when we dealing with accurate computation of the radiation distribution over a complex media configuration, it is improper to use the integro-differential equations. Propagation intensity has to be successively calculated by taking account of heterogeneity of the medium. In such cases, computational simulation by Monte Carlo method is an expectable option.

The fundamental of MCRT modeling could be addressed as so-called *light transport* equation. At a given position $\mathbf{r}_{\text{cnp}}(x, y, z)$ within canopy, the exiting radiance $I(\mathbf{r}_{\text{cnp}}, \Omega)$ consists of out-going radiance $I_o(\mathbf{r}_{\text{cnp}}, \Omega)$ and an integral of in-coming radiance $I_i(\mathbf{r}_{\text{cnp}}, \Omega')$ from entire 4π space:

$$I(\mathbf{r}_{\text{cnp}}, \Omega) = I_o(\mathbf{r}_{\text{cnp}}, \Omega) + \int_{4\pi} I_i(\mathbf{r}_{\text{cnp}}, \Omega') \mathcal{R}_{\text{cnp}}(\Omega', \Omega, \mathbf{r}_{\text{cnp}}) d\Omega', \quad (1.12)$$

where $\mathcal{R}_{\text{cnp}}(\Omega', \Omega, \mathbf{r}_{\text{cnp}})$ is the canopy BRDF [31]. This is a parallel canopy layer configuration. However, as Booth [32] mentioned in his paper, Monte Carlo particle transport method solves not only the averaged configuration transport equation issues, but also dealing with the more complex situation like the correlation between particles. The particle correlation issue is ignored by most transport equation users. Monte Carlo transport method should not be imposed on numerically solving the RTE, but also considered as a analog of physical transport progress of particles. In particular, if we discussing Monte Carlo ray-tracing method, most of the conditions we are thinking a physical progress rather than a RTE. Therefore, even without a RTE in this case,

an transferring progress can still be simulated by correctly particle interaction event configuring.

1.4.2 Issues of Canopy MCRT Modeling

Monte Carlo methods have advantages, however, they can be improved by solving existed models' issues:

- (i) To raise the computing speed of the MC model. Hardware development is beyond all doubt which accelerates the calculation much more than the past decades. With the utilizing of quantum computational techniques and new materials for manufacturing the CPU, the computing speed updating every twenty four months as Moore's law addressed [33]. The improvement of the software for MC methods is attractive either. Parallel computing is a well known approach for hastening MC method [34, 35]. A new strategy for reducing the consuming time of computing is to use GPU parallel computing approaches [36–38]. Algorithms for variance reduction (VR) of Monte Carlo methods are key issues to every MC model. As a consistent issue of MC simulation, VR is a key step to reduce the computational cost. Theoretical analysis of the VR algorithms still a critical issue, though various of VR algorithms have been published and effective under corresponding issues [39–41].
- (ii) To improve the Monte Carlo method algorithm. Apart from (i), which was considering hardware and compiling technology, algorithms of MCRT hold the potential to be improved as well. Conventional application for increasing the efficiency of MC method is by reducing the variance of the estimation with various approaches for instance, importance sampling, stratification, conditional Monte Carlo, Latin hypercube sampling, and quasi Monte Carlo *et al.*, detailed discussions are presented in [42].
- (iii) To simulate more complex systems. The complexity of the 3-D scene in MCRT modeling should approximate reality as much as possible. Considering about

topographic effects is one aspect we should pay attention. Models provided RT simulation concentrated on canopy-photon interactions for simplifying the complex procedure of the topographic affections. Scattering caused by topographic reliefs can not be neglected when ground inclination is big enough to affect the scattering directions.

In this study, the author only focusing on the analog code development of the canopy radiative transfer progress by considering the microscopically physical interaction between photon and the materials, rather than discussing the issues of Monte Carlo method as most of the vegetation canopy MCRT model did.

1.5 Scientific Objective of the Study

The aim of this study is to develop a new MCRT code considering about the affection of the topographic reliefs to the BRF estimation for a forest. For the application of the model, the approaches for precisely retrieving the canopy structural parameters, will be developed as well. An L-system based 3-D tree model is generated as the discrete canopy approximation for the future MCRT modeling.

1.5.1 Topographic Relief Approximation for Forest MCRT Modeling

In spite of many outstanding MCRT models performing actively in vegetation MCRT simulation [43, 44], topographic relief affection has not been discussed enough. Flat ground situation though holds prior state, it is difficult, however, to find a perfectly flat in a natural born forest. In MC ray-tracing methods, RT theoretical studies seems separating from the 3-D scene modeling [32], but the solution of the complex ground condition for ray-tracing algorithm still is a challenging issue. In this study, the author aims to develop a vector based photon tracing model with the terrain relief factor in

order to assess how does the topographic parameter affect the bidirectional reflecting distribution of the forest.

1.5.2 LAI/LAD Retrieving for RT Models

Physical RT model holds critical meaning to meteorology, phenology and then also on RS technique. Remote sensor's calibration needs an precise estimation of the RT budget by using physical models with accurate input of the parameters. Reliable estimation of the input parameters is surely raised. Apart from others, canopy LAI and LAD are the key parameters for the radiative regime estimation by RT models. Canopy LAI/LAD which impact the light-canopy interaction very much, are key parameters input of the model for simulating the RT regime of a proper area. LAI and LAD are the key variables which are used to model the RT processes. According to Equation 3.14, 3.24, 3.15, 3.22, the connection of the attenuation of the incident beam and canopy LAI/LAD could be concluded. They determine the size of the plant-atmosphere interface and thus play the critical role in the exchange of energy and mass between canopy and atmosphere [45]. Therefore, to obtain LAI or LAD effectively and accurately by using feasible approaches, by retrieving optical or laser scanned data, are reported for decades. The detailed discussion will be listed in the Chapter 2.3. In this study, the author tries to retrieve the LAD and effective LAI from optical and terrestrial laser sensor. According to the optical application a fractal 3-D tree model was created as well. Both of the methods has been validated and good results were obtained.

1.6 Structure of the Dissertation

As a sequent of this chapter's introduction, related scientific literatures concern with this study will be reviewed in the second chapter. Then, theoretical fundamental of MCRT modeling and canopy parameter retrieving will be addressed in the third chapter. MCRT model description and examples of forest BRF estimation will be presented in the fourth chapter. Fifth chapter will discuss the canopy architectural parameter retrieving

by using both photogrammetric application and point cloud data. Finial chapter will address the conclusion of this study.

Chapter 2

Related Scientific Studies

As a derivative of computational technology, MC simulation develops simultaneously. People tried to utilize the method on various of mathematical, physical, engineering fields. MC application is robust, unbiased and reliable which is used as calibration of the other modeling methods.

Canopy structural parameters are key features not only for vegetation RT modeling but also meaningful for the agriculture and forestry studies. Traditional applications for data collecting are mainly by artificial works which are time-consuming and conditional limited. With the development of the sensing technology people tried to retrieve the parameters from remotely sensed data. This is more advanced and effective than artificial method.

In this chapter, the author will give a detailed review of the related works about vegetation MCRT simulation and canopy parameter retrieving during past decades, respectively.

2.1 Various of Radiative Transfer Models

The components of the vegetation RS system are constituted by the following parts [24], which could be sketched the physical progress as FIGURE 2.1:

1. The source of the radiation, generally the sun. This part includes the spectral intensity I_0 and the incident geometry (θ_0, φ_0) , where θ_0 is the zenith angle and φ_0 is the azimuth angle, respectively.
2. Atmosphere, this is the first object of the earth that interacts with the solar radiation. The particle components, such as the aerosol particles, water vapor, ice particles, ozone *et al.*, of the atmosphere cause the wavelength dependent scattering and absorption characteristics.
3. Vegetation canopy, this is the main part we concern about. The canopy structural parameters (geometrical patterns and positions) of vegetation, phytological components (leaves, petioles, branches, trunk), environmental parameters. These parameters impact the radiation distribution within and outside the canopy.
4. Ground and soil, for the parameters input to the model should consider the topography, roughness, absorption, reflectance, moisture, bulk density.
5. Sensors, the observation geometry zenith θ_v , azimuth φ_v , height, IFOV, spectral sensitivity, aperture and calibration should be considered.

The solar intensity with the initial parameters: wavelength (λ_0), incident zenith (θ_0), and azimuth (φ_0). It first penetrates the atmosphere, roughly 29% of the total solar energy is reflected back into space by clouds, atmospheric aerosols. About 23% of incoming solar energy is absorbed in the atmosphere by water vapor, dust, and ozone [46]. The abide fraction interacts with the earth's surface, only 7% will return to the space [47], which is caught by the IFOV of the spacial sensor. With different object one studies, the emphasis on the physical progress is manifold. However, the core of the physical modeling of the radiatively transferring is invariable, that is to accurately simulate the progress of setting the parameters of the whole RT regime. In the following section, different types of the physics based canopy RT models including our MCRT methods will be introduced individually. Because of the development of RT studies of astronomy, people tried to utilize the applications on the modeling of RS RT regime.

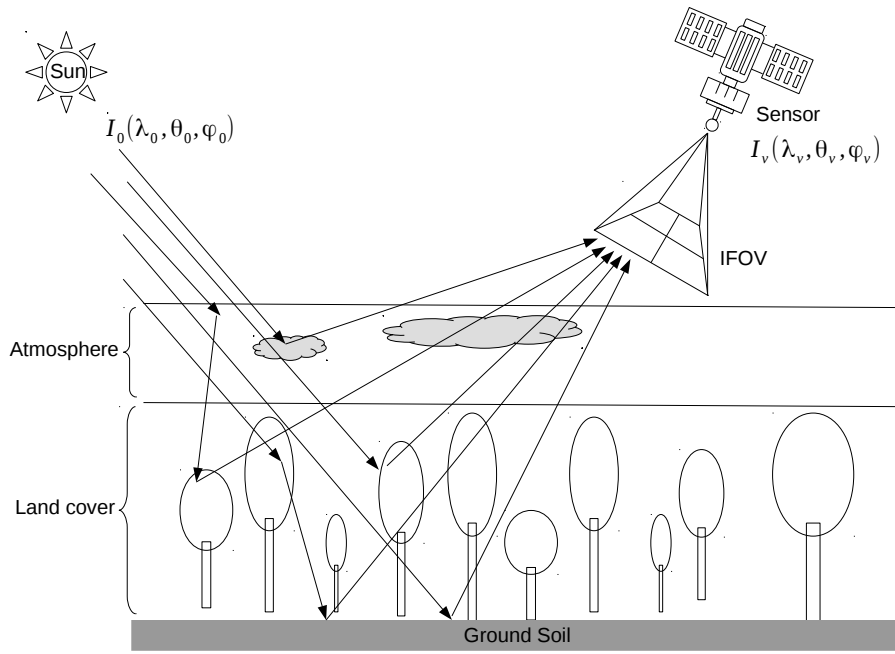


FIGURE 2.1: Sketch of the physical principal of RS observation. I_0 is the solar intensity reached on the top of atmosphere; λ_0 is the wavelength of I_0 ; θ_0, φ_0 are the incident zenith and azimuth individually; θ_v, φ_v and H_v are the sensor viewing zenith, azimuth and height, respectively.

2.1.1 Geometrical Model

The radiation from the canopy is mainly from direct solar intensity and the scattered radiation from the sky, when we put the weather condition as sunny or cloud less. Air scattered radiation is about 10% – 15%. If we concern about an open forest condition (distances between trees are clearly identified from RS image), the brightness of the scene will be composed by the bright and dark area. With these conditions, Li and Strahler (1985, 1986, 1986, 1988) published their innovative model which discussed about RT regime of a scattered forest. The model is running with the following assumptions:

1. The object has Lambertian feature.
2. The weather should be sunny or the scattering of air is low.
3. Multi-scattering is ignored.
4. Pattern of the canopy is regular.

The model was improved by using Boolean theorem for dense forest conditions [51]. However the geometrical model was still confront with problems: first, the Boolean theorem assumed the center of canopy followed uniform distribution which is difficult to find in nature. Second, canopy was simplified that the inner structure, which holds affection on RT modeling, was not considered. To overcome these issues, Chen *et al.* published their model with multi-scaled canopy settings. The canopy gap size distributions between and within tree canopies are used to describe the hotspot size and shape. The description was first close to the forest BRDF attributes [52]. The advantages of this application is to consider the canopy structures from following different scales:

- For larger scale than canopy the model, the spatial distribution is considered as the coefficient of the BRDF.
- The pattern of the canopy is also a considerable aspect for Solar-Canopy-Sensor geometry relations so that the BRDF relatively changes.
- The inner structure of the canopy like branches is considered for the effect on the BRDF.
- The pattern of leaf is also an effect of the BRDF change.

The scale of the object in the model is a progress from macro to micro. According to the meaning of multi-scale, the former geometrical model is on the second scale level.

2.1.2 RT Equation Based Model

The RT theory is firstly developed for studying the radiative regime within the planet atmosphere. Interaction between radiation and atmosphere is expressed by derivative formulas, with the boundary conditions definition, the intensity of within the atmosphere can be solved. If we take the canopy as continuous media like atmosphere, the interaction between light and vegetation can be considered similarly as the atmospheric condition. For RT modeling in vegetation, scattering and absorption are considered as well. However, due to the dimension of the leaf is much larger than gas molecular, the

feature of the scattering and absorption will be different from the atmosphere condition. Scattering from leaf is correlated with the orientation of the leaf and the structure of the leaf, so as absorption. The mathematical expression of the continuous canopy should be determined at first. Using statistical distribution function to define the canopy feature is a common accepted application [53–57]. These models of leaf spatial feature are useful to determine the extinction coefficient of the RT equation and the phase function of the light transferring within the canopy.

In order to solve the RT equation, several applications have been developed. The components, in Suit model, (leaf, flower, branch) of the canopy are projected on vertical and horizontal plane, the projection area is used for estimating the scattering, absorption and transmission effect [58]. This model found the solution of the RT equation, but the result of the BRDF was beyond the ground truth for the simplifying of the projection. SAIL model has the same RT equation pattern with Suit model. The difference between them is SAIL model uses arbitrarily inclined leaves instead of projecting on vertical and horizontal plane [59]. SAIL model has better performance than Suit model, however, none of them express the hotspot effect. Hot spot effect is an important feature of vegetation BRDF. To express this characteristic, one solution is to build a statistical model of the leaf gap and the light transfer direction [60, 61]. Another is to refer the overlapping effect from geometrical models [62, 63].

2.1.3 Computational Simulation Model

With the development of the computational technology of last decades, lots of scientific problems impeded by the lack of the capability of calculation were conquered. One capability of computer is executing calculation repeatedly and rapidly. Therefore people propose to put the complicated integral problems into discrete progress with the help of statistic theory to find a solution. Radiosity method was presented as a new procedure for realistic image synthesis by Goral et al. (1984), which is well-utilized in thermal engineering. It describes an equilibrium radiation energy balance within a given enclosure that contains reflecting surfaces. It models the interaction between light

and Lambertian-like surfaces. Then it calculates the outgoing diffuse, the radiosity, on all surfaces, taking multiple reflections and transmissions into account. Borel et al. (1991) developed an optical remote sensing radiosity model for 3-D structured surfaces, which concluded that the radiosity is a valuable tool to model the light transferring in vegetated region. Moreover, radiosity modeling as well as a tool to evaluate bidirectional reflectance properties of individual leaf based canopy structures. Compared to MC methods, radiosity-based models performed more suitable to efficiently estimate light distribution within an explicitly described structure [66, 67].

The Monte Carlo method for the study of the canopy light regime was first used by Japanese researchers [68, 69]. It is such a robust, adapted application for complex integral formula fixing, which requires large number of independent samples for obtaining a convergence sequence which provides a reliable average solution [70]. MC method is widely used for simulating the BRF/BRDF of the RT regime, related works will be listed in consequent section.

2.2 Canopy MCRT Simulation Methods

Radiative transfer in heterogeneous environments is simulated as a random Markov's chain of collisions between photons and objects. Generally MC methods can be used in estimating the mean statistical features of this process [71]. Since 1960s many researchers put efforts on simulating interactions between solar radiation and vegetation canopy by MC application. These studies covered a great range of the scale of the simulated scene from the BRDF calculation of a wide area of forest to the single leaf reflection feature simulation.

2.2.1 Simplified Conditions of the Canopy MCRT

Tanaka (1969) adopted MC method to estimated the effective sunlit leaf area of tobacco plants. For effective leaf area calculation three different solar altitudes were used. Total

solar radiant received by all leaflets was not so different in three cases. Effective sunlit area varies with the changing of the solar altitude. The results indicated that this method will be promising to estimate the solar radiant accepted by many kinds of plant or community models. With low capability of the computing speed, the first beginning of the MC method met with low efficiency issues. Therefore some assumptions had been set for simplifying the complexity. Canopy were considered as infinite which extended in horizontal directions. Leaves inclination angles were set in fixed angles. The penetration function was greater for clumped area densities than for random leaf distributions. This was coincide with the natural realities [73–75].

Szwarcbaum and Shaviv developed an MC model for evaluating the radiation field inside plant canopies. The model can be expanded to include additional special features. Numerical examples of the radiation field inside a assuming canopy and some features of the albedo were given [76]. Gerstl and his colleagues proposed a simple 3-D MC ray-tracing model and a 2-D analytical model to estimate the angular distribution of the hotspot as a function of leaf area [77]. Ross and Marshak (1988) modeled canopy RT by MC method. In order to overcome the low efficiency of the photon utilizing in BRDF simulation, they assume that the photon was not absorbed by the canopy. Moreover the photon was forced in the direction of observation after each interaction. In practice, the backward MC methods could be described in the following three types [79]: the first and most simple method is backward emission/absorption (EMABS). It is based on a random emission of photons in the grid box of interest and a simple backward tracing. Since only those photons which cross the grid box boundaries contribute to the heating rate, this approach behaves poorly for large optical thickness which are common in the thermal spectral range. For this reason, the second method, the optimized backward emission/absorption (EMABS_OPT) utilized a variance reduction technique to improve the distribution of the photons in a way that more photons are started close to the grid box edges and thus contribute to the result which reduces the uncertainty. The third method, net flux divergence (DENET), uses the flux divergence approach where -in backward MC methods - all photons contribute to the result, but in particular for small optical thickness the noise becomes large.

Kimes and Smith (1980) developed a MC model for estimating the canopy spectral reflectance, transmittance, and absorption. A good agreement of the model calculations. Kimes *et al.* (1980) proved that when the distribution of leaves inclination angles like erectophile canopy shows more efficiency for absorbing the solar radiation. Cooper and Smith (1985) simulated vegetated soil RT regime by MC method. The result of the model showed that the irregularities will impact reflectance only when they were much larger than wavelength. Comparing well with experiment in the sense that they exhibit the same trends, results from MC method are well agreed with the investigation. When shadowing occurs, the reflectance pattern from a rough soil is skewed with a peak in the direction of the source because of back-scattering from the surface. Oker-Blom (1984) illustrated the penumbral effect by the incident of direct solar radiation both in the canopy of the coniferous tree and in the forest using MC simulation.

The early stage of the canopy MCRT modeling shows that the application can be used for the RT regime calculation like the numerical applications as well. Moreover, MC method has advantages which can overcome obstacles that the others cannot fix. The optimistic aspect are: MC method put inner structure into light-canopy interaction so that the simulation will be more accurate than numerical methods. The pattern of the leaves and canopy are considered, that means the canopy will no longer like turbid media. Hot-spot effect also can be expressed by MC method. The drawbacks of the MC is time-consuming, especially at the elementary stage.

2.2.2 Advanced Canopy MCRT Studies

Monte Carlo Ray-tracing methods are based on a sampling of photon trajectories within the scene, which are useful for a wide variety of radiation transfer issues. The key to effective use of ray tracing methods is the application of effective sampling schemes [84]. Victor and Alexander (1990a, 1990b) calculated canopy reflectance using the radiative transfer equation which was solved by MC method. The scales of leaf were taken into account so that the hot-spot effect can be estimated. The model only considered about the plane canopy community, so that the model was available for homogeneously planted

agriculture area. According to the model they tentatively retrieved canopy parameters (coefficients of leaf reflectance, transmittance, and soil albedo) and geometrical canopy parameters such as leaf area index (LAI), leaf normal distribution (LND [87], in some literatures [88, 89] it is also named as leaf angle distribution or leaf inclination angle distribution). A 3D maize canopy model RT simulation was published by Espana *et al.* (1999). The simulations were performed in the principal plane, with the sun altitude 45° . In order to simplify the condition, the diffuse radiation was ignored. Comparing the results with SAIL model, their MC model was more sensitive comparing with previous models.

Govaerts *et al.* (1994, 1996, 1998) systematically developed a MC ray-tracing model of 3-D heterogeneous canopy RT regime simulation, named *Raytran*, of both natural and man made canopy scene. The emphasized the ability of Monte Carlo ray-tracing techniques to describe the transfer of radiation in scenes of arbitrary complexity. Raytran did not rely on any variance reduction methods, but only used parallelization technique. Comparisons with laboratory measurements thus show the capability of Raytran to reproduce actual observed reflectance factors of a fully documented target, even in the case of non-Lambertian surfaces.

North (1996) developed one dimensional forest ray tracing MC model, and tested his model on a coniferous forest. It was available in this model that the accurate simulation of multiple scattering within the canopy, between separated canopies, trunks and ground surface. The influence of the anisotropic sky radiance field is modeled by decoupling the incoming field from the reflectance calculation. Bidirectional reflectance of a coniferous forest was simulated, results showed strong anisotropic was caused by the shadow effects.

García-Haro *et al.* (2002a, 2002b) developed a model for light interaction for calculating spectral and bidirectional reflectance from discontinuous canopies approximated by an arbitrary configuration of plants. The model is detailed enough to deal with realistic remote sensing scenarios offered by coarse-scale pixels. Simulated faster images provide an ideal frame to validate automated image interpretation methods such

as inversion techniques, vegetation indices, spectral mixture analysis, and statistical methods, allowing to investigate the potential of such methods to produce vegetation related parameters from spectral mixtures in the presence of significant multiple scattering. The model provides a fast and efficient strategy to derive hyperspectral images at appropriate spatial resolutions over a wide range of ecosystems.

Widlowski *et al.* (2006) discussed simulated impact on canopy absorption. Detailed radiative transfer simulations in 3-D coniferous forest environments are presented to show how the magnitude of local net horizontal fluxes can reach multiple times the incident solar radiation at the top of the canopy (TOC) level. Furthermore, the probability distribution functions (PDFs) of these local net horizontal fluxes are skewed toward negative values, in particular when the radiative regime is dominated by single-scattering interactions and geometric shading is prominent. For forest domains that are smaller than about $30 \times 30 \text{ m}^2$, horizontal radiation transport will still affect the domain-averaged canopy absorption values and thus a spatially exhaustive sampling of the true local absorption may be more appropriate.

Kobayashi and Iwabuchi (2008) combined one dimensional atmospheric radiative transfer MC model and 3-D forest MCRT model (Forest Light Environmental Simulator, FLiES) to get the impact from atmospheric properties and canopy structure on the photosynthetic active radiation (PAR) at the top of canopy. After the interaction of light with atmospheric particles such as aerosols and clouds, several types of change occur simultaneously in the incident photosynthetic active radiation (PAR) at the TOC. Coupled simulation helps in understanding of the realistic light environment and canopy photosynthesis under various atmospheric conditions and various canopy structures. The BRF simulation at the top of the atmosphere (TOA) would be useful to evaluate the atmospheric effect on BRF as well as the effect of 3-D canopy structure. Also the simulation results showed difference in 3-D distributions of APAR in canopy under various atmospheric, canopy, and surface conditions.

Greiner *et al.* (2009) simulated multiple photon scattering in sugar maple tree canopies. The results of both grounds and receiver planes with the assumed view zenith

80°. As the simulation results, a Gaussian distribution fit the range and cross-range cross sections of the simulated and measured beam footprints with great accuracy. The standard deviations of the best-fit Gaussian distributions were of the same order in both simulation and experiment. Two factors were obtained as the mismatched one is the experimental errors, another is the simulation results represent an average over billions of realization of the canopy.

North *et al.* (2010) presented a method for a model the interaction of waveform LiDAR with a 3-D vegetation canopy. A sensitivity analysis showed expected effects of canopy parameters on the waveform, and illustrated potential for retrieval of the canopy properties of fractional cover and leaf area, in addition to the tree height. Where canopy and ground pulses can be separated, an index derived from the waveform showed theoretical retrieval of vertically projected plant area index with correlation coefficient was 0.87.

Iio *et al.* (2011) described a 3-D leaf distribution of a individual canopy on a small voxel by using a ray-tracing program based on a turbid medium analogy to simulate photosynthetic photon flux density at leaf surface ($\text{PPFD}_{\text{leaf}}$). Good agreement was obtained between measured and estimated values of $\text{PPFD}_{\text{leaf}}$ for evaluating canopy photosynthesis.

Kuester *et al.* (2014) presented the description of a spectral, spatial and temporal simulation system (HySimCaR) that can be used to model the bidirectional canopy reflectance and the BRDF of canopies for three cereal types using MC method. Canopy architecture was described by detailed plant representatives linked with spectral information. The plants were built with a structural whole plant simulation tool based on botanical knowledge that is able to describe plant architecture geometrically at different phenological stages.

Accelerating of the simulation speed is one of the major issue of MCRT modeling. With the development of the computational technology a new application based on GPU has been developed [103]. With more computing cores, the peak performance of GPUs, measured in giga floating-point operations per second (GFLOPS), has been

steadily increasing than CPU [104]. People therefore employed GPU programming technology for canopy MCRT simulation and the results showed the performance of the GPU programming will enhance the efficiency of the calculating [105–107].

Su *et al.* 2014 propose a GPU based massively parallel approach for a two-layers vegetation-ground 3-D vector Monte Carlo model. With the NVIDIA Corporation parallel computing platform CUDA (Compute Unified Device Architecture), the simulation speed is impressively accelerated with 213-fold (on the *NVIDIA GTX 480* GPU), and 291-fold (on the *NVIDIA GTX 590* GPU), respectively.

A MC forward ray-tracing method based 3-D vector model is proposed by Kallel and Gastellu-Etchegorry [109]. Both Lambertian reflectance and specular reflection are considered in the model. A new approach, in particular, to estimate the condition on leaf orientation to produce reflection is proposed, and its probability to occur is found to be low, but when reflection happen, the corresponding radiance vector is very high. Such a phenomenon increases dramatically the MC variance and produces an irregular reflectance distribution function with a number of peaks corresponding to the specular effect. Experimental results confirm that in forward direction, canopy polarizes horizontally light, and in inclined forward direction, diagonal polarization can be observed.

2.3 Canopy LAI/LAD Retrieving

The structure of canopy is the spatial distribution of the photosynthetic tissues of a plant serve both as solar energy collector and interacting with air. Stems and branches support these exchange surfaces in such a way that radiative and convective exchange can occur in an efficient manner. Canopy structure affects radiative and convective exchange of the plant community, so information about canopy structure is necessary for modeling these processes. Therefore, it can be important in determining the physical environment of other organisms within the plant community.

2.3.1 Models of Leaf Normal Distribution

For simplifying the model, leaf normal zenith and azimuth are considered as independent with each other, and the azimuth is taken as randomly distributed. Therefore the LND is only considering about the zenith of the leaf normal in this study. De Wit (1966) gave six types of leaf inclination angle distribution functions, and *G-function* will be easily obtained by computing the integral of them in the range $[0, \pi/2]$. And he simplified the distribution functions were independent from the depth z .

- *planophile*, the horizontal leaves are the most frequent, that is,

$$g(\theta_L) = \frac{2}{\pi}(1 - \cos 2\theta_L), \quad (2.1)$$

- *erectophile*, the vertical leaves are the most frequent, that is,

$$g(\theta_L) = \frac{2}{\pi}(1 + \cos 2\theta_L), \quad (2.2)$$

- *plagiophile*, oblique leaves are the most frequent,

$$g(\theta_L) = \frac{2}{\pi}(1 - \cos 4\theta_L), \quad (2.3)$$

- *extremophile*, the oblique leave are the least frequent,

$$g(\theta_L) = \frac{2}{\pi}(1 + \cos 4\theta_L), \quad (2.4)$$

- *spherical*, the relative frequency of leaf angle is the same as for surface elements of a sphere,

$$g(\theta_L) = \sin \theta_L, \quad (2.5)$$

- *uniform*, proportion of leaf angle is the same at any arbitrary angle,

$$g(\theta_L) = \frac{2}{\pi}. \quad (2.6)$$

The azimuth angle (ϕ_L) was taken as randomly distributed.

Goel and Strebel (1984) employed two-parameter Beta distribution to express the leaf angle distribution.

$$g(t) = \frac{1}{\mathcal{B}(\mu, \nu)} (1-t)^{\mu-1} t^{\nu-1} \quad (2.7)$$

where $t = 2\theta/\pi$, $\mathcal{B}(\mu, \nu)$ is the Beta function

$$\mathcal{B}(\mu, \nu) = \int_0^1 (1-x)^{\mu-1} x^{\nu-1} dx = \frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)}, \quad (2.8)$$

where Γ is the Gamma function, μ and ν are related to the mean $\bar{t}$ by,

$$\mu = (1 - \bar{t}) \left(\frac{\sigma_0^2}{\sigma_t^2} - 1 \right), \quad (2.9a)$$

$$\nu = \bar{t} \left(\frac{\sigma_0^2}{\sigma_t^2} - 1 \right), \quad (2.9b)$$

where σ_0^2 and σ_t^2 were the maximum standard deviation and variance of t , respectively, and defined as,

$$\sigma_0^2 = \bar{t}(1 - \bar{t}), \quad (2.10a)$$

$$\sigma_t^2 = \text{var}(t). \quad (2.10b)$$

The mentioned above leaf angle distribution models were utilized for canopy structural investigation, Pisek *et al.* (2013) concluded that a spherical leaf angle distribution is not a valid assumption for most of the studied tree species in temperate and boreal ecoclimatic regions. Which means for actual investigation of the particular region, leaf angle distribution should be retrieved from field collection data rather than the model utilizing. Therefore, the applications of leaf angle distribution retrieving and moreover the LAI retrieving should be developed.

2.3.2 Methods of Parameter Retrieving

Both direct and indirect methods are aiming to obtain accurate LAI estimation. Directly destruction of the canopy will get true LAI, however, this is low productivity and costly. With the development of the remote sensor and retrieving algorithm, direct methods accounting for less proportion than indirect methods. RS data is advanced for wide region LAI retrieving but for accurate LAI result, ground based optical and laser sensor will be more appropriate. Furthermore, the information obtained from digital hemispherical photographs is only two dimensional, thus, terrestrial laser scanner is an ideal instrument to complement photography by permanently recording the three-dimensional structural information. According to the leaf spatial distribution of the canopy, overlapping and clumping effects, it is difficult to get true LAI despite removing partial affections based on the canopy gap theory. Another problem of LAI retrieving is to remove the branches and trunks.

Direct Methods LAI is first defined as one-sided of photosynthetic tissue per unit ground horizontal area [112]. At the beginning of the LAI studies the canopy structural parameters are measured directly. It is accurate, however, time consuming and destructive to plants [113]. Only lab experimental investigation for assessing the LAI calculation for manually collecting the leaf area. This application is phasing out with the development of the retrieving algorithms from RS data.

Indirect Methods Theoretically it is possible to retrieve the leaf area from the canopy using optical sensed data. Since 1960s people adopted to indirectly retrieve canopy structural parameters from optical images [114–116]. Several ground-based commercial instruments for LAI collection were testified, and the advantages and drawbacks of each instrument were presented as well; moreover the potential of the laser instruments for LAI measurement was pointed out acutely [117]. Landsat TM data and ground-based measurements of LAI were correlated for estimating effective LAI, which was the product of LAI and clumping index. Some issues have been discussed for the LAI estimation, which were correlated to the LAI retrieving from Landsat images in spite of the limitations of TM image [118].

The development of the optical sensor resolution improves LAI measurement accuracy and mathematical modeling. An application with spectral and textural information for mapping the LAI of different vegetation types, was presented by using IKONOS images [119]. Moreover when the land cover was stratified, spectral and textural relations were considerably better for mapping LAI spatial variability for different crop types. Encouraging results in LAI estimation by using optical data were obtained via data fusion process for high spatial resolution images of the LAI. For achieving reliable results over a large area, LAI measurement with auxiliary variables such as NDVI computed from Landsat images were combined [120]. However, due to the assumption of randomly located foliage elements within the canopy, LAI obtained from gap fraction [121] theory was not the actual LAI [122], thus effective LAI was proposed to describe the result more accurately, which is obtained by considering the clumping index affection.

Recent studies propose to utilize laser scanner for LAI estimation since it holds good performance at range detection and the consistency of the precision. Morsdorf (2006) *et al.* evaluated the potential of deriving effective LAI proxy by estimating a fraction of first and last echo types inside the canopy. Based on the regression results, LAI has been computed for a region, and compared qualitatively to the result based on imaging spectrometry, revealing similar spatial patterns and ranges of values. Effective LAI has been calculated by referring empirical model from airborne LiDAR data. Due to the narrow scan angle of the LiDAR, the parameters of the model have been estimated from the

field investigation [124]. An accurate application has been developed for calculating leaf area density (LAD) and cumulative LAI profiles of small trees [125]. This voxel based 3-D modeling application is implemented in the study. This model allows the LAD and LAI of trees, which have complex spatial distributed foliage, to be computed by direct counting of the beam-contact frequency in each layer using a point-quadrat method. The non-photosynthetic tissue has to be removed as redundancy in the application so that the damage to the plant cannot be avoided. A computational geometry based application has been published for retrieving the effective leaf area using terrestrial laser scanner (TLS) [126]. Due to the high sampling density of TLS, this method can be taken as a calibration tool for airborne laser scanning and optical sensing. PCD were divided into slides, and the leaf area within each slide has been calculated, but the non-photosynthetic structure still cannot be removed. Non-photosynthetic tissue removing is a challenging issue of the laser scanner utilized LAI estimation study, especially for the canopies which are not dense.

Chapter 3

Theoretical Fundamentals

This chapter presents theoretical principles of the study that two aspects are illustrated in this part. One is the physical and mathematical basic of the MCRT modeling. The other is the fundamentals related with LAD and LAI retrieving from the terrestrial sensor collected data.

3.1 Basic of Monte Carlo Ray-tracing

3.1.1 Random Number Generator (RNGor)

Physical progress of radiative transfer in Monte Carlo Ray-tracing method is represented by simulating the motion of photon within the scene. Therefore, the randomness of the photon trajectory is the key issue in order to analog the radiation transferring of the reality. In this part an introduction to a relatively new algorithm, which is named *Xorshift*, to generate random numbers is given. *Xorshift* random number generators are a class of pseudorandom number generators that the subsequent number is by repeatedly taking the exclusive or of a number with a bit-shifted version of the current number itself. The algorithm is impressively fast on modern computer's architectures. Prove of the randomness is provided in [127] explicitly. FIGURE 3.1 shows the scheme of how the bitwise operator works in C/C++ language.

Here shows sample codes of the in C/C++ language RNGor for 64-bit machine in our model, which typically provided 200 million per second random numbers, where $a = 12$, $b = 25$, $c = 27$, respectively. The constant of “2685821657736338717” is a number which provide good lattice structure for RNGor at $2^{64} - 1$ repeating periodic, similar numbers could be used like “1181783497276652981”, “4292484099903637661”, “7664345821815920749”, “1865811235122147685” and “1803442709493370165” [128].

```

1  uint64_t xorshift64(void){
2      x ^= (x >> a);
3      x ^= (x << b);
4      x ^= (x >> c);
5      return x * UINT64_C(2685821657736338717);
6  }

```

3.1.2 Geometry of Photon Transferring

The coordinates of the model was defined following Cartesian coordinate system right hand rule, and original point was at the downer left corner point of the cube which contained the objects of the 3-D scene. The directional normal vector $\mathbf{n}$ was set by defining z axes as 0° of the zenith $\theta \in [0^\circ, 180^\circ]$ (clockwise), x axes as 0° of the azimuth $\phi \in [0^\circ, 360^\circ]$ (counterclockwise) (FIGURE 3.2). Then a normalized form of $\mathbf{n}$ was obtained by

$$\mathbf{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}, \quad (3.1)$$

where $\|\mathbf{n}\| = 1$. In ray-tracing algorithm, ray was mathematically defined into parametric line equation, for a given initial particle transfered from $\mathbf{r}_0(x_0, y_0, z_0)$ to $\mathbf{r}(x, y, z)$

could be defined as

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{n}_0, \quad (3.2)$$

where $\mathbf{r}_0 \in \mathbb{R}^3$ is the initial photon position, $\mathbf{n}_0 \in [0, 1]^3$ is the directional vector (see Equation 3.1), $t(t \geq 0)$ is the travelling distance of the photon, where $\mathbf{n}_0(n_{0x}, n_{0y}, n_{0z})$ is the directional vector, which is defined by Equation 3.1, t is the distance from $\mathbf{r}_0$ to $\mathbf{r}$ (FIGURE 3.3).

3.1.2.1 Ray-Ellipsoid Intersection

The ellipsoid model for expressing the canopy was defined in the 3D scene as

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{a^2} + \frac{(z - z_c)^2}{b^2} = 1, \quad (3.3)$$

where the center is (x_c, y_c, z_c) , horizontal and vertical axis are a and b , respectively. Intersection between line and ellipsoid could be obtained by substituting Equation 3.2 to Equation 3.3, then we obtain a quadratic equation which forms as

$$At^2 + Bt + C = 0, \quad (3.4)$$

where the parameters A , B , and C are

$$A = \frac{n_x^2}{a^2} + \frac{n_y^2}{a^2} + \frac{n_z^2}{b^2}, \quad (3.5a)$$

$$B = \frac{2n_x(x_0 - x_c)}{a^2} + \frac{2n_y(y_0 - y_c)}{a^2} + \frac{2n_z(z_0 - z_c)}{b^2}, \quad (3.5b)$$

$$C = \frac{(x_0 - x_c)^2}{a^2} + \frac{(y_0 - y_c)^2}{a^2} + \frac{(z_0 - z_c)^2}{b^2} - 1. \quad (3.5c)$$

The discriminant of Equation 3.4 is

$$\Delta = B^2 - 4AC. \quad (3.6)$$

If $\Delta > 0$, there are two intersections Z_1, Z_2 which can be calculated by substituting Equation 3.7 into Equation 3.2.

$$t = \frac{-B \pm \sqrt{\Delta}}{2A}. \quad (3.7)$$

Here we only need the minimum positive t as the intersection between line and canopy. If $\Delta = 0$, there is only one intersection Z_1 , and there is no intersection when $\Delta < 0$ (FIGURE 3.4).

3.1.2.2 Ray-Cone Intersection

Cone model for expressing canopy, which contains profile surface and bottom, was determined by

$$(x - x_c)^2 + (y - y_c)^2 = \left(\frac{a}{h}\right)^2 (z - z_c)^2, \quad (3.8a)$$

$$z = z_c - h, (x - x_c)^2 + (y - y_c)^2 < a^2. \quad (3.8b)$$

Here the cone apex is (x_c, y_c, z_c) , Equation 3.8a is cone side, Equation 3.8b is cone bottom (FIGURE 3.5).

The intersection solving is similar with ellipsoid, here only the parameters of quadratic equation are shown as

$$A = n_x^2 + n_y^2 - \left(\frac{a}{h}\right)^2 n_z^2, \quad (3.9a)$$

$$B = 2n_x(x_0 - x_c) + 2n_y(y_0 - y_c) + 2\left(\frac{a}{h}\right)^2 n_z(z_0 - z_c), \quad (3.9b)$$

$$C = (x_0 - x_c)^2 + (y_0 - y_c)^2 - \left(\frac{a}{h}\right)^2 (z_0 - z_c)^2. \quad (3.9c)$$

The solution is determined by Equation 3.7, here an additional condition should be set as $z_c - h \leq z \leq z_c$ for constraining the intersection on the side of cone.

3.1.2.3 Ray-Cylinder Intersection

To express trunk and cylindrical canopy object, a cylinder model was defined as

$$(x - x_c)^2 + (y - y_c)^2 = a^2, z \in [z_{c1}, z_{c2}], \quad (3.10a)$$

$$(x - x_c)^2 + (y - y_c)^2 \leq a^2, z = z_{c1} \text{ or } z = z_{c2}. \quad (3.10b)$$

Equation 3.10a is cylinder side surface, and Equation 3.10b is top and bottom of the cylinder. Parameters of the quadratic equation are

$$A = n_x^2 + n_y^2, \quad (3.11a)$$

$$B = 2n_x(x_0 - x_c) + 2n_y(y_0 - y_c), \quad (3.11b)$$

$$C = (x_0 - x_c)^2 + (y_0 - y_c)^2 - a^2. \quad (3.11c)$$

The intersection point's z-coordinate must be in $[z_{c1}, z_{c2}]$. Conditions that the ray hitting on the cap or bottom should be carefully considered.

3.1.2.4 Ray-Bilinear Surface Intersection

Ground was assumed as a bilinear surface $B(u, v)$ interpolated from DEM grid:

$$B(u, v) = uv\mathbf{a} + u\mathbf{b} + v\mathbf{c} + \mathbf{d}, \quad (u, v) \in [0, 1]^2, \quad (3.12)$$

where $\mathbf{a} = \mathbf{p}_{11} - \mathbf{p}_{10} - \mathbf{p}_{01} + \mathbf{p}_{00}$, $\mathbf{b} = \mathbf{p}_{10} - \mathbf{p}_{00}$, $\mathbf{c} = \mathbf{p}_{01} - \mathbf{p}_{00}$, and $\mathbf{d} = \mathbf{p}_{00}$; $\mathbf{p}_{ij}$ is DEM grid point coordinates with $i \in \{0, 1\}$, $j \in \{0, 1\}$. Ray-bilinear intersection algorithm was by substituting Equation 3.2 into Equation 3.12 [129]. The local surface normal $\mathbf{n}_g$ of the ground on the point $\mathbf{p}_g$ is important to the scattering direction of the photon, which is determined by

$$\mathbf{n}_g = \mathbf{n}_x \times \mathbf{n}_y, \quad (3.13)$$

where $\mathbf{n}_x = \mathbf{p}_x - \mathbf{p}_g$, $\mathbf{n}_y = \mathbf{p}_y - \mathbf{p}_g$, $\mathbf{p}_x$ and $\mathbf{p}_y$ are two neighbor points of $\mathbf{p}_g$ along the positive direction of x and y axis (FIGURE 3.7).

3.1.3 Ray Exponential Attenuation in Canopy

Solar beam is traditionally considered as uncharged ionizing radiations, which lose the energy exponentially in relatively few interactions with medium. The ratio, $T(\theta_0, \phi_0, \Delta l)$, of direct beam passing through canopy is expressed as

$$T(\theta_0, \phi_0, \Delta l) = e^{-\sigma_c(\theta_0, \phi_0, \mathbf{r}_c)\Delta l}. \quad (3.14)$$

where σ_c is canopy extinction coefficient θ_0, ϕ_0 are incident zenith and azimuth, Δl is free path length within the medium. Some monographs also named $T(\theta_0, \phi_0, \Delta l)$ as gap

fraction or gap frequency [117, 130, 131]. σ_c is conventionally simplified to independent from the varying of azimuth, therefore the extinction effect is mainly depend on the incident zenith

$$\sigma_c(\theta_0, \mathbf{r}_c) = \frac{G(\theta_0)u(\mathbf{r}_c)}{\cos \theta_0}, \quad (3.15)$$

where $\mathbf{r}_c$ is the coordinates within canopy, $u(\mathbf{r}_c)$ is canopy leaf area density (LAD, m^2/m^3). $G(\theta_0)$ is mean projection leaf area on the plane perpendicular to the zenith, which is conservatively named as *G-function* [25, 132]. The optical thickness of canopy is therefore by integrating Equation 3.15 on traveling length L

$$\tau_c = \int_0^L \sigma_c(\theta_0, \mathbf{r}_c) d\Delta l. \quad (3.16)$$

3.1.4 Leaf Scattering Phase Function

MCRT method analog the radiance uses huge amount of photon, with counting the weight reducing and scattering of each photon to estimate the radiation budget of the regime. An issue must be deliberated is the outgoing direction after photon interacting with materials, which is characterized by phase function. The phase function we used in this model is expressed as [133],

$$\Phi(\Omega', \Omega) = \frac{2}{G(\Omega')} \int_{2\pi} g(\Omega_L) |\Omega' \cdot \Omega| \phi(\Omega', \Omega, \Omega_L) d\Omega_L, \quad (3.17)$$

where $\Omega_L(\theta_L, \phi_L)$ is the unit vector stands for the leaf normal, $g(\Omega_L)$ is leaf normal distribution function (see Equation 3.25), Ω' and Ω are incident vector and outgoing vector, respectively. $\Omega' \cdot \Omega$ is the inner product of Ω' and Ω . $\phi(\Omega', \Omega, \Omega_L)$ is single leaf scattering phase function within canopy [25]

$$\phi(\Omega', \Omega, \Omega_L) = r_L |\Omega_L \cdot \Omega| (\omega_L \pi)^{-1}, \quad (\Omega_L \cdot \Omega)(\Omega_L \cdot \Omega') < 0, \quad (3.18a)$$

$$\phi(\Omega', \Omega, \Omega_L) = t_L |\Omega_L \cdot \Omega| (\omega_L \pi)^{-1}, \quad (\Omega_L \cdot \Omega)(\Omega_L \cdot \Omega') > 0, \quad (3.18b)$$

where ω_L , r_L and t_L are single leaf albedo, reflectance and transmittance, respectively. Furthermore, we define the leaf albedo determined by

$$\omega_L = r_L + t_L. \quad (3.19)$$

For other materials, like soil and wooden structures, Lambertian scattering was supposed as well. FIGURE 3.8 shows the LUT generated when $\chi = 1.0$, the phase function varies according to the changing of incident zenith θ_0 , observation zenith θ and relative azimuth ϕ .

3.2 Fundamental of Canopy Structural Parameter Retrieving

BRF estimation using an existing RT model needs parameters of the physics progress. Most parameters of MCRT method, which were mentioned in previous section, can be obtained by laboratory measurement. However, parameters like LND function and LAD/LAI, which convey the structural information of the canopies, have to be investigated in field with a number of samples. In this section, principles of terrestrial sensor based methods of retrieving LND and LAD/LAI are presented.

3.2.1 Canopy Structural Parameters

3.2.1.1 Leaf Area Density (LAD) and Leaf Area Index (LAI)

Considering about a unit volume surround a point within canopy, an equation

$$u(\mathbf{r}_c) = \frac{1}{V(\mathbf{r}_c)} \sum_{i=1}^n a_i, \quad (3.20)$$

is defined as the LAD (u) around a point ($\mathbf{r}_c$) within a unit volume (V), where n is the number of the leaves, a_i is the single leaf area. Because the leaf distribution within the canopy is not uniform and single leaf area varies as well, the leaf area density is a function of $\mathbf{r}_c$. LAD is a key structural parameter for both individual plant and a communicate, according to its definition, it is measured in m^2/m^3 . If canopy is divided into several layers, each layer's total leaf area can be calculated by a polar integral, the entire canopy leaf area therefore can be computed by [25]

$$A_{\text{cnp}} = \int u(\mathbf{r}_c) d\mathbf{r}_c = \int_{z_1}^{z_2} dz \int_0^{2\pi} d\alpha \int_0^{a_p} u(z, \alpha, a) a da, \quad (3.21)$$

where A_{cnp} is the total area of leaves, $u(z, \alpha, a)$ is another way to take Cartesian coordinates into depth-polar coordinates, which is equivalent with $u(\mathbf{r}_c)$. z_1 and z_2 are different depth within canopy. If integration is only between the depth z_1 and z_2 , and we obtained the total foliage area within a cylinder of unit cross-section constructed about a vertical with coordinates (α, a) which is

$$L(\alpha, a, \Delta z) = \int_{z_1}^{z_2} u(\alpha, a, z') dz'. \quad (3.22)$$

$L(\alpha, a, \Delta z)$ is called downward cumulative LAI at point (α, a) within the depth range (z_1, z_2) . As an important vegetation biophysical parameter, LAI is a dimensionless variable and a ratio of one-side leaf area to unit horizontal ground area. LAI can be correlated to vegetation-gas exchange processes like photosynthesis, evaporation, transpiration, rainfall interception, and carbon flux [134–141]. The LAI estimation can provide a quantified dynamic change information in productivity and climate impacts

on forest ecosystems. Moreover, LAI can be employed as an impacts of stress in forests, it can thus be utilized to examine relationships between environmental stress factors and forest insect damage [142].

3.2.1.2 Definitions of LND and *G-function*

The characterization of plant canopy by statistical parameters has been discussed in detail by Ross (1981). He gives a definition of a $g(z, \Omega)$ function for expressing the distribution of the leaf inclination angle per unit solid angle $\frac{1}{2\pi}g(z, \Omega)$, which is satisfied to the equation

$$\frac{1}{2\pi} \int_{2\pi} g(z, \Omega_L) d\Omega_L = 1, \quad (3.23)$$

where z means the vertical depth from the top of the canopy, $\Omega_L(\theta_L, \phi_L)$ is the leaf normal vector (see FIGURE 3.9); thus we obtain $d\Omega_L = \sin \theta_L d\theta_L d\phi_L$. The $g(z, \Omega_L)$ function is for computing the projection of the foliage area, *G-function*, which is defined by a integral of the LND function $g(z, \Omega_L)$,

$$G(z, \Omega) = \frac{1}{2\pi} \int_{2\pi} g(z, \Omega_L) |\Omega \cdot \Omega_L| d\Omega_L, \quad (3.24)$$

where

$$\Omega \cdot \Omega_L = \cos \theta \cos \theta_L + \sin \theta \sin \theta_L \cos(\phi - \phi_L).$$

$\Omega(\theta, \phi)$ is the observation direction [58, 143]. The *G-function* can be explained as the mean effective leaf area on both affection of observation geometry Ω and leaf normal Ω_L .

3.2.1.3 Campbell's Model

Campbell (1990) developed a LND function based on the assumption that the angular distribution of leaves in a canopy is similar to the distribution of area on the surface of a prolate or oblate ellipsoid, the function is defined by,

$$g(\theta_L) = \frac{2\chi^3 \sin \theta_L}{\Lambda(\cos^2 \theta_L + \chi^2 \sin^2 \theta_L)^2}, \quad (3.25)$$

where χ is the ratio of the horizontal axis to the vertical axis of an ellipsoid; Λ was a parameter determined by χ , as

$$\Lambda = \begin{cases} \chi + \frac{\sin^{-1} \varepsilon}{\varepsilon}, \varepsilon = \sqrt{1 - \chi^2} & \chi < 1 \\ 2 & \chi = 1 \\ \chi + \frac{\ln[(1+\varepsilon)/(1-\varepsilon)]}{2\varepsilon\chi}, \varepsilon = \sqrt{1 - \chi^{-2}} & \chi > 1 \end{cases} \quad (3.26)$$

An analytical form of *G-function* is then provided [145]

$$G(\theta_0) = \frac{1}{\Lambda} \sqrt{\cos^2 \theta_0 + \chi^{-2} \sin^2 \theta_0}. \quad (3.27)$$

As shown in FIGURE 3.10, $G(\theta_0)$ presents a continuous variation according to $\chi \in (1.0, 11.0)$, which means a look-up-table, for $G(\theta_0)$ interpolation according to χ , could be created without high density so that could save the computational resources.

Especially, Equation 3.25 becomes to spherical (uniform) distribution when $\chi = 1$. The cumulative density function, $\mathcal{C}(\Theta)$, is defined as

$$\mathcal{C}(\Theta) = \int_0^\Theta g(\theta_L) d\theta_L, \quad \Theta \in \left[0, \frac{\pi}{2}\right]. \quad (3.28)$$

3.2.2 LAD Retrieving by Gap Fraction Method

For estimating the illuminated flux of the forest, the structural parameters, like LAI, LAD, LND *et al.*, of the vegetation are the key parameters for calculating the ratio of the survival photon particles passing through the media (Equation 3.14). Extinction coefficient σ_c of Equation 3.14 is related with *G-function*. LAD/LAI expresses the density of the canopy media. The LAD $u(\mathbf{r}_c)$ could be retrieved by substituting Equation 3.15 to 3.14:

$$u(\mathbf{r}_c) = -\frac{\ln(T(\theta_0, \phi_0, \Delta l)) \cos \theta_0}{G(\theta_0) \Delta l}. \quad (3.29)$$

LAI could be subsequently calculated from Equation 3.22. Equation 3.29 therefore provides a possibility to retrieve the LAD by extracting the gap fraction, that is $T(\theta_0, \phi_0, \Delta l)$, from optical image of the tree on viewing direction $\Omega_0(\theta_0, \phi_0)$.

3.2.3 LND and LAI Retrieving by Using PCD

PCD can be considered as the aggregation of multivariate distributed clusters according to the object location in Cartesian space. If the point clusters in a sub-space (a voxel), can be distinguished, the PCD can be classified into several clusters, which hold significance for leaf area retrieval. Leaf area was achieved by using computer geometrical scientific libraries. For clustering the points we chose the clustering analysis packages from *R* platform based on Gaussian mixture model, which is an iterative expectation-maximization (EM) processing method for determining maximum likelihood via a statistical model.

3.2.3.1 Simplified Curvature Estimation

PCD give the average Cartesian or polar coordinates of the object surface patch. The curvature of a point can therefore be estimated by considering the normals of its neighboring points. Assume there is a point set P_N^* which contains N points, whose element

p_i^* is the 3-D coordinates (x, y, z) of the Cartesian space. The amount of the point is determined by both vertical and horizontal scanning intervals. The set of neighboring points P_n is determined by the searching radius r centering on the query point p_q , which is $P_n = \{p_i : \|p_q - p_i\|_2 \leq r\}, (i = 1, 2, \dots, n)$. A simplified curvature c_q of point p_q is given by [146]:

$$c_q = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, (\lambda_1 \leq \lambda_2 \leq \lambda_3), \quad (3.30)$$

where $\lambda_j (j = 1, 2, 3)$ is the eigenvalue given by $\mathbf{Cov} \cdot \mathbf{x}_j = \lambda_j \cdot \mathbf{x}_j$, $\mathbf{Cov}$ is the covariance matrix of P_n determined by $\mathbf{Cov} = \frac{1}{n} \sum_{i=1}^n (p_i - \bar{p})(p_i - \bar{p})^T$, $\mathbf{x}_j$ is the eigenvector of $\mathbf{Cov}$. $\bar{p}$ is the center of P_n , which is given by:

$$\bar{p} = \frac{1}{n} \sum_{i=1}^n p_i. \quad (3.31)$$

3.2.3.2 Gaussian Mixture Model (GMM)

Voluminous point amount and complexity are the principle characteristic of PCD. PCD carry the Cartesian coordinates from the patch of an object interacting with the laser impulse by receiving objects surface echo. Points therefore correspond to heterogeneous distributions, like multivariate clusters, of discrete objects within a sample space according to their positions. Exploring constructional information is one of the major purposes for PCD users. We used the *mclust* package of the *R* platform to cluster the point set observations for reducing the complexity of the PCD. It provides functions for model-based clustering (covariance parameterization and number of clusters selected via Bayesian information criterion (BIC)), which gives the maximized log-likelihood for the number of components in the data. In general, the bigger value of BIC, the more reliable for the number of clusters becomes [147]. It attempted to iterate group sets of objects according to a rule stating that objects within the same cluster were more optimized to each other than those in other groups. The detailed algorithm explanation

was presented by the articles [148, 149]. For the detail background knowledge about GMM is indicated in A.1.

3.2.3.3 Leaf Normal Retrieving

This study assumed that one leaf is a fitting plane retrieved from point cluster which was obtained by *mclust* (see section 3.2.3.2). We chose the principal component analysis routine from CGAL to calculate the plane [150]. Suppose the plane equation we computed is: $Ax + By + Cz + D = 0$. The orientation of the plane can be calculated by:

$$\theta_L = \begin{cases} \arccos(\frac{C}{\|n_p\|_2}) & : C \geq 0 \\ \pi - \arccos(\frac{C}{\|n_p\|_2}) & : C < 0 \end{cases}, \quad (3.32)$$

$$\phi_L = \arccos(\frac{A}{\sqrt{A^2 + B^2}}),$$

where $\|n_p\|_2 = \sqrt{A^2 + B^2 + C^2}$, θ_L and ϕ_L are the zenith and azimuth angle of the plane normal, respectively. Leaf normal calculation is a key step to retrieve the leaf area we discuss in following section.

3.2.3.4 Effective Leaf Area Calculation:

In order to find the geometrical boundary of the cluster, the 3-D convex hull function of CGAL for generating the convex hull, which contained all the points of the cluster, was selected [151]. Assume the convex hull contains m points as vertices, the leaf area can be approximately calculated as the topological closure of the points that vertices projected on the fitting plane which we computed from section 3.2.3.3. Suppose $P_V = \{p_k(x_k, y_k, z_k) : k = 1, 2, \dots, m\}$ are the vertices projection on the fitting plane, the center $\bar{p}_v(x_v, y_v, z_v)$ of P_V can be calculated by (3.31). If the points are ordered subsequently,

according to the triangulation mesh function of CGAL, the leaf area could be computed by accumulating the triangles:

$$S = \sum_{l=1}^m s_l, \quad (3.33)$$

where s_l is the triangle area constructed by $\bar{p}_v$, p_k and p_{k+1} . When $k = m$, $p_{k+1} = p_1$. Those points inside the closure are skipped. The effective leaf area S^* can be computed by considering the projection coefficient [25]:

$$S^* = |\Omega' \cdot \Omega_L| S, \quad (3.34)$$

where

$$\Omega' \cdot \Omega_L = \cos \theta' \cos \theta_L + \sin \theta' \sin \theta_L \cos(\phi' - \phi_L),$$

$\Omega'(\theta', \phi')$ and $\Omega_L(\theta_L, \phi_L)$ are view direction and leaf normal, respectively.

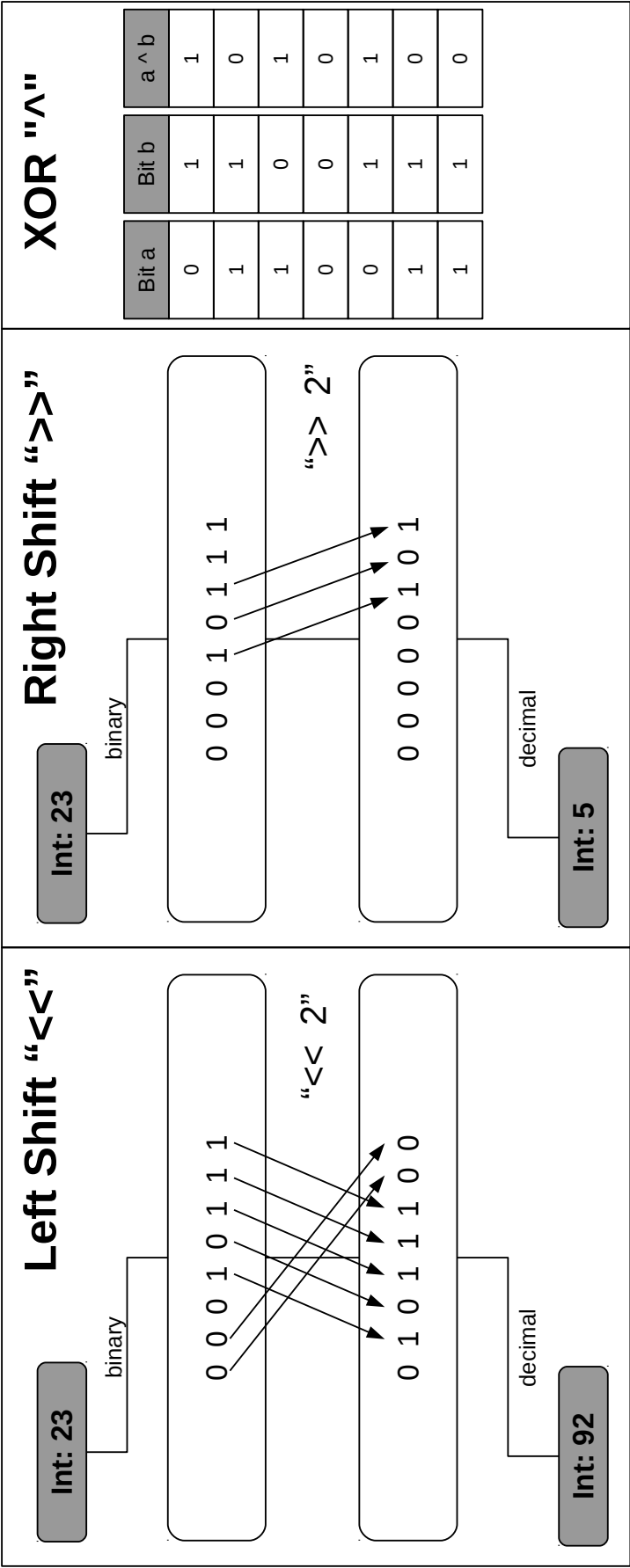


FIGURE 3.1: Rules of “XOR” operator (“^”) and bitwise shift operator “<<” and “>>” in C/C++ language.

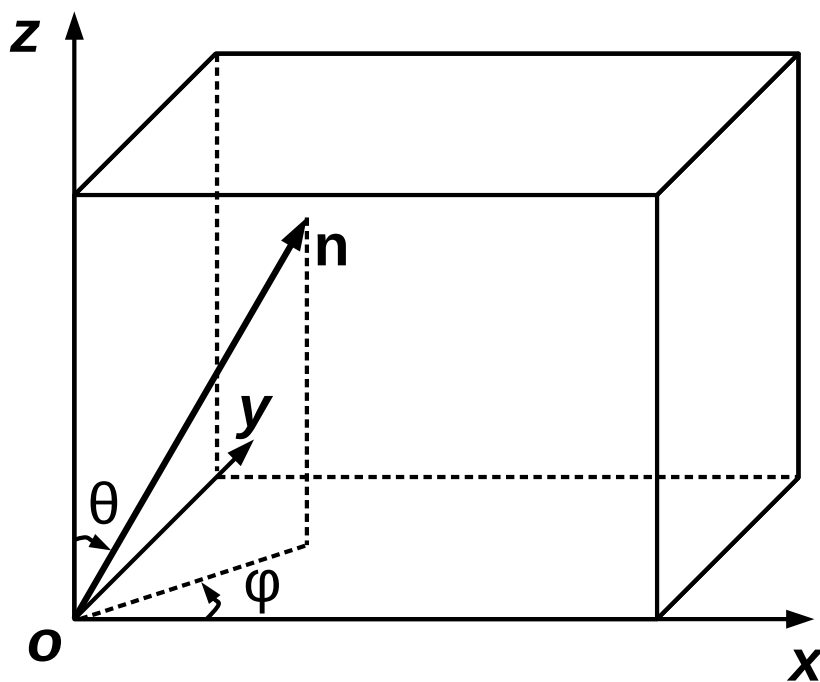


FIGURE 3.2: Cartesian coordinate system of the model. θ is the zenith (clockwise) started from z axes, and ϕ is the azimuth (counterclockwise) started from x axes.

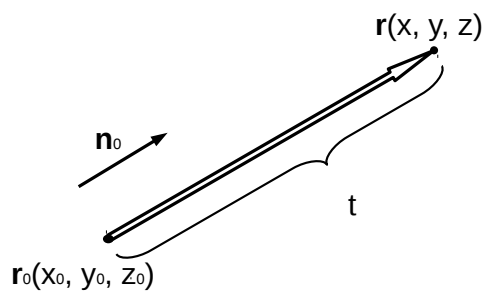


FIGURE 3.3: Mechanism of the vectorial geometry from $\mathbf{r}_0$ to $\mathbf{r}$, $\mathbf{n}_0$ is the directional vector, and t is the traveling distance.

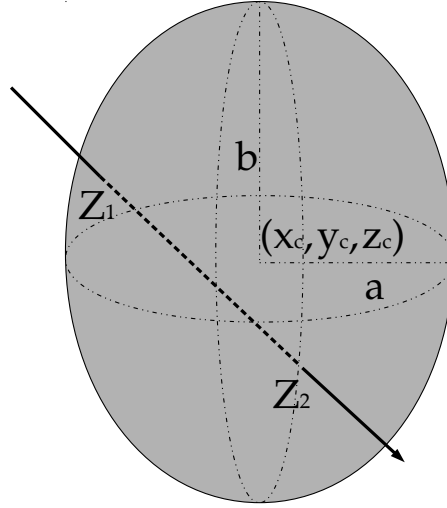


FIGURE 3.4: Conditions of a light intersect with an rotational symmetry ellipsoid, which are none intersection, one point (Z_1) and two points (Z_1, Z_2), here we set the horizontal radius of ellipsoid as a , vertical axis as b .

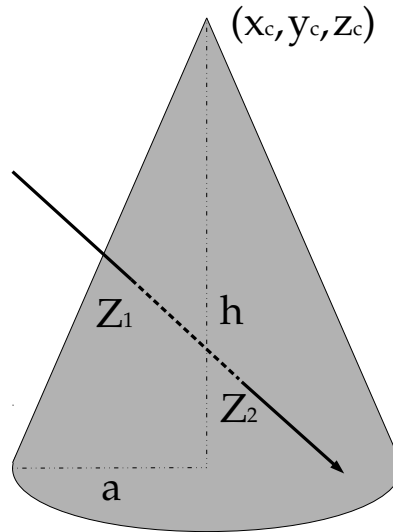


FIGURE 3.5: Conditions of a light intersect with a cone, which are none intersection, and two points (Z_1, Z_2), here we set the radius of cone as a , height as h .

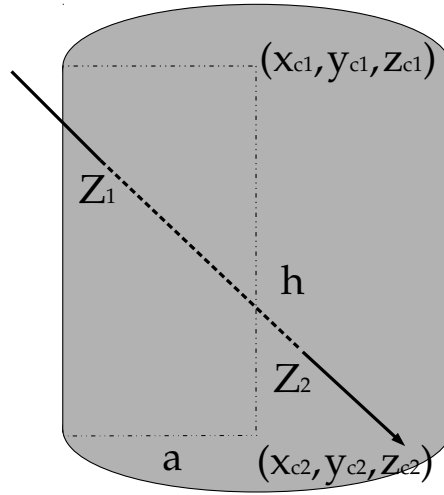


FIGURE 3.6: Conditions of a light intersect with a cylinder, which are none intersection, and two points (Z_1, Z_2) , here we set the radius of cylinder as a , height as h .

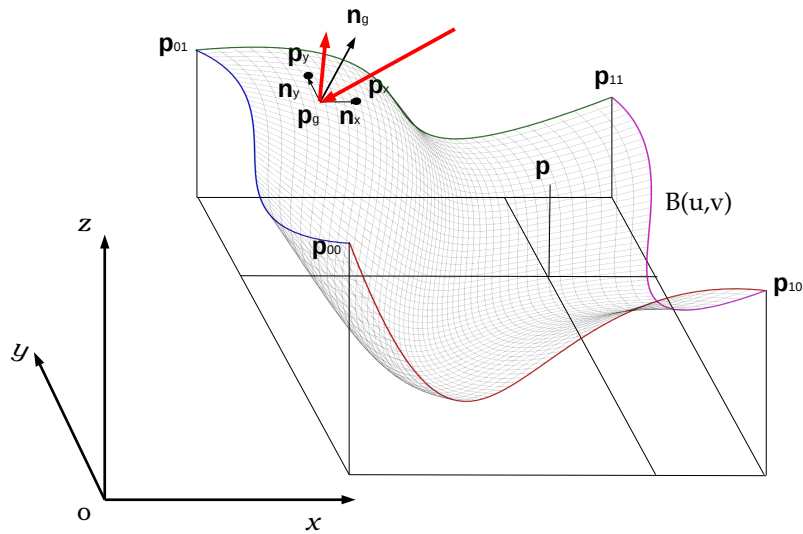


FIGURE 3.7: Bilinear Surface $B(u,v)$ is determined by linear interpolating the four corner points $p_{ij}(i, j = 0, 1)$. Local normal n_g from the point p_g of the bilinear surface is determined by the vector n_x and n_y of the neighbor points p_x and p_y .

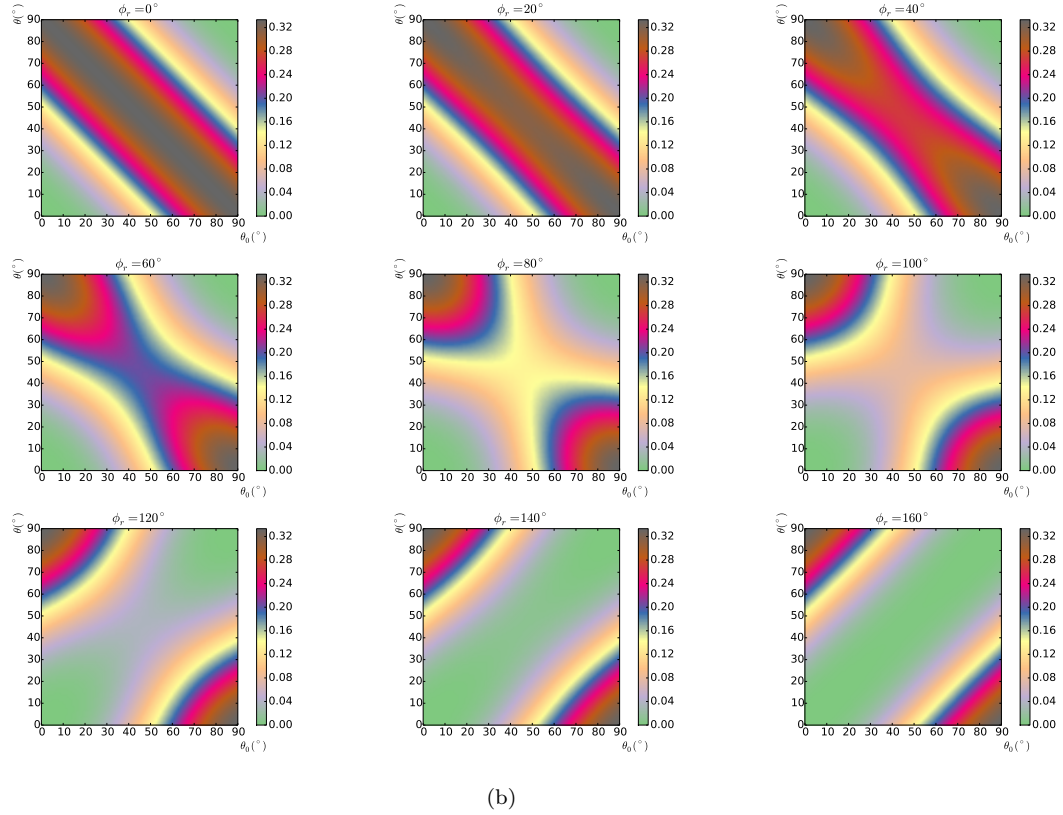
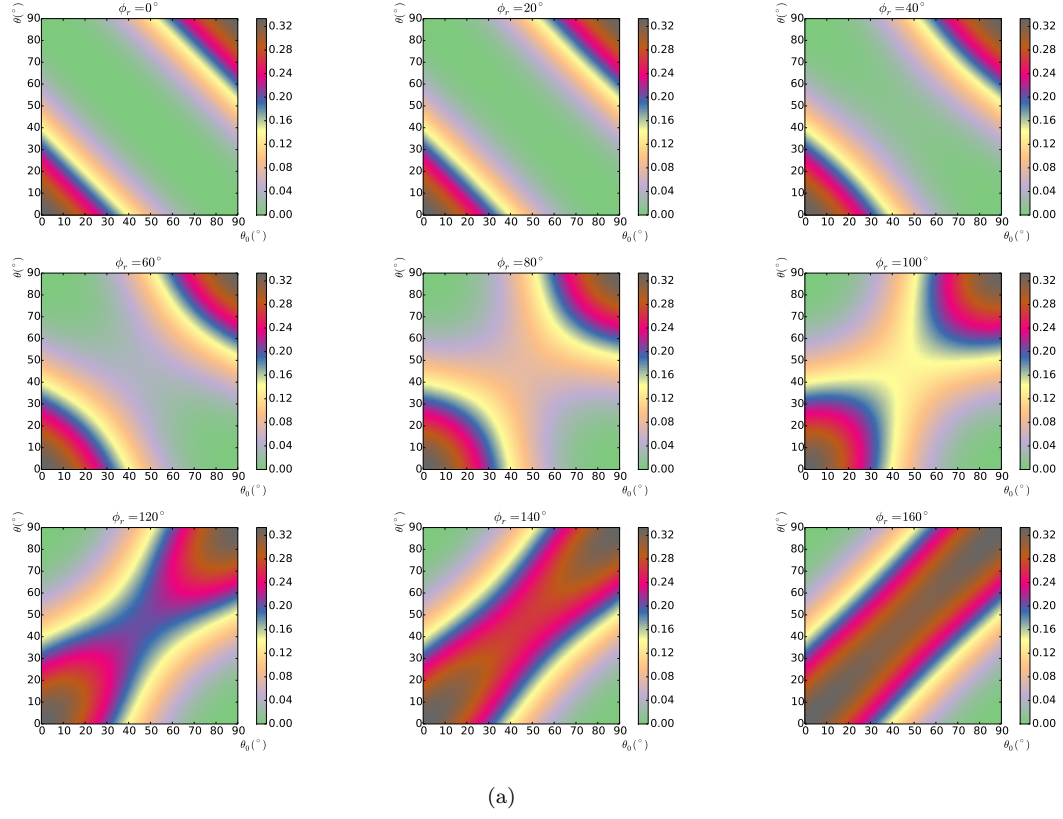


FIGURE 3.8: Single leaf scattering phase function LUTs when $\chi = 1.0$, varying with incident angle θ_0 , observing angle θ , and relative azimuth angle ϕ_r . (a) LUTs for reflecting condition according to Equation 3.18a, and (b) LUTs for transmitting conditions according to Equation 3.18b.

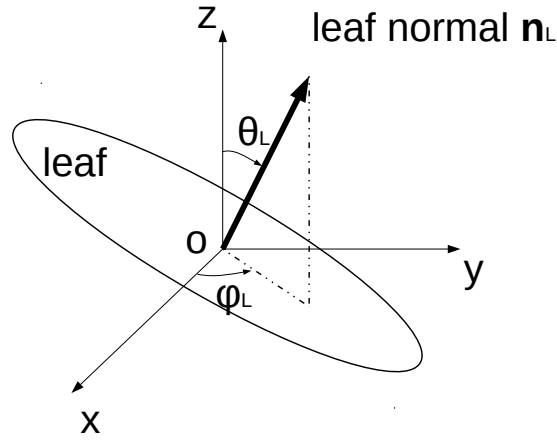


FIGURE 3.9: Definition of leaf normal ($\mathbf{n}_L$), which is perpendicular to the leaf plane L. θ_L ϕ_L are leaf normal zenith and azimuth. red dash dot line on XOY plane is leaf normal projection.

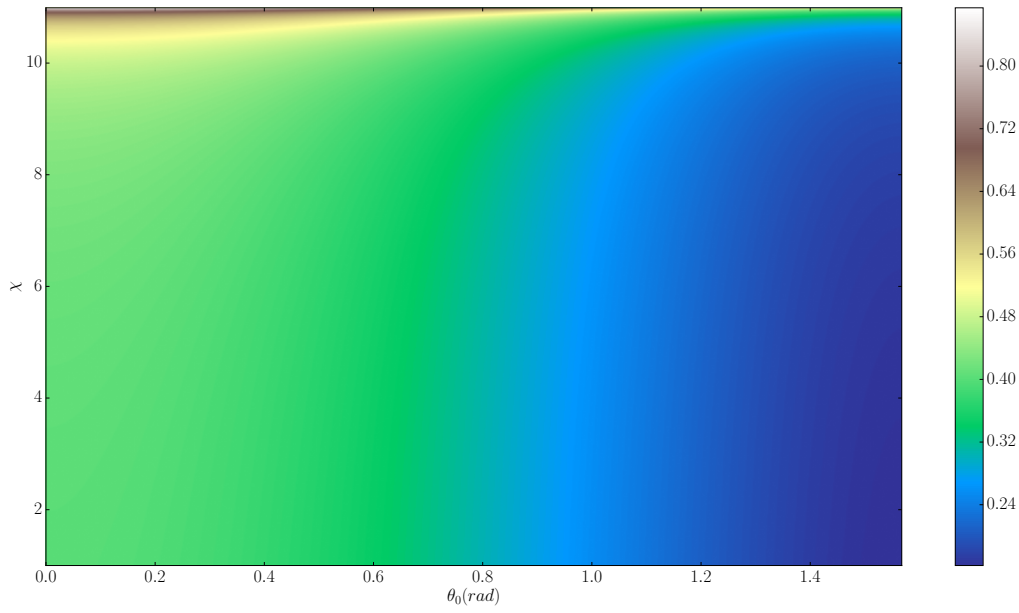


FIGURE 3.10: Look-up-table generated according to Equation 3.27 with χ varying in the range (1.0, 11.0) and θ_0 from 0 to $\frac{\pi}{2}$.

Chapter 4

Canopy MCRT Model Description and Examples of BRF Simulation

This chapter deals with the 3-D heterogeneous open forest BRF simulation. A Monte Carlo ray-tracing algorithm based vector BRF model, named TRCMCRTS (Topographic Relief Correlated Monte Carlo Radiative Transferring Simulator), is introduced. As indicated in Chapter 1, this model mainly focuses on the RT balance estimation without atmospheric conditions. TRCMCRTS ray-tracing algorithm is presented in this chapter with indication of the scattering and LEM scheme, for variance reduction the conventional Russian roulette method is employed. The relief of ground in the forest is simulated other than most Monte Carlo ray-tracing models. Algorithms of the model is presented in detail, and the BRF results comparing with FLiES model [98] is proposed as validation of TRCMCRTS. The TRCMCRTS codes are developed in C++ language (ISO 2011 standard), and testified on Linux (*Ubuntu 14.04 LTS*) operating system.

4.1 Introduction of TRCMCRTS Algorithm

TRCMCRTS was developed basing on Monte Carlo ray-tracing progress, which simulated the motion of each photon sample from entering to escaping the 3D open forest

scene, or being absorbed by the materials. The forest in this model was composed of individual trees heterogeneously distributed in an area with relief topography. In terms of RS observation, the canopy consists of particle leaves and branches formed as regular geometrical volumes. In order to take account of topographic effects, the forest scene was divided into voxel matrix according to the DEM grid of the area. Each voxel contains trees and ground objects. A tree object included canopy (leaf), branch (stem) and trunk objects. A ground object held the understory grass turbid layer and soil. The RT simulations were carried out with regard to a reference plane which was higher than the maximum height of the top of the canopy. Only those photons entering and escaping through this reference plane were to be counted in this model (FIGURE 4.1). In the following contexts we describe the physical foundations of photon-material interaction events.

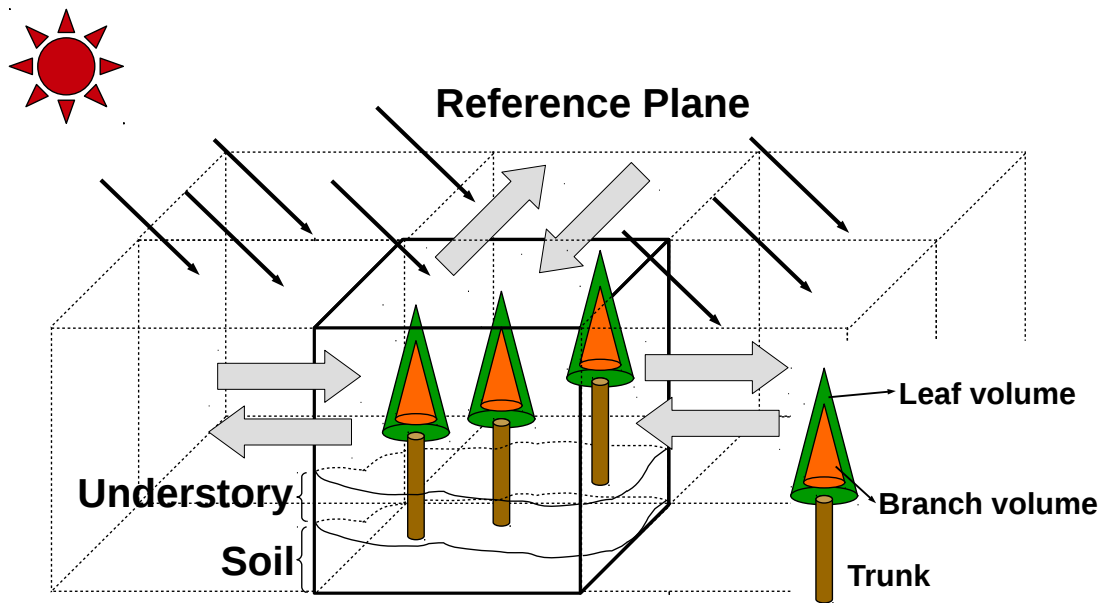


FIGURE 4.1: 3-D scene definition and the components of an individual tree.

Photon tracing in the forest scene was implemented at voxel scale, and was designed according to the following steps.

- (i) Initialize a new photon

- (ii) Trace photon in voxels:
 - (a) determine if it interacts with materials (trees, grass, soil)
 - (b) apply single leaf or Lambertian scattering
 - (c) check if absorption occurs (photon died)
- (iii) Update photon position (Equation 3.2)
- (iv) If the photon passes through the reference plane, go to (i)
- (v) Else run into next voxel and go to (ii)

The total 3-D forest scene was contained by a virtual cube, when a photon was hitting on the cube's wall, it was moved to the opposite of the wall with the same moving direction. If a photon was absorbed at (c), a new photon was then generated and shot to the scene continuously from a random location at the top of the scene. The free path Δl within the turbid medium was determined by a random number $\rho \in [0, 1]$ and Equation 3.14:

$$\tau(\Delta l) = -\ln \rho, \quad (4.1)$$

where

$$\tau(\Delta l) = \sigma_c(\theta_0, \mathbf{x})\Delta l.$$

The direction of travel after scattering was randomly determined by rejection method [39]. For effective BRF sampling, local estimation method (LEM) processing and Russian roulette method were employed as well.

4.1.1 Scattering

4.1.1.1 Lambertian Scattering

An Lambertian scattering (Lambert's cosine law) characteristic was assumed for photon scattering events that occurred on soil or trunk structure of the trees, which supposed

the radiant intensity from an ideal diffuse surface was directly proportional to the cosine of the angle θ between the direction of the incident light and the local normal,

$$I = I_0 \cos \theta d\Omega dA. \quad (4.2)$$

Here I_0 is incident intensity, I is the scattered radiation intensity from the surface, θ is the zenith, dA is the area element and $d\Omega$ is the solid angle element (see FIGURE 4.3), when zenith is zero, reflecting intensity I is equivalent to the incident I_0 . Lambert's cosine law in its reversed form (Lambertian reflection) implies that the apparent brightness of a Lambertian surface is proportional to the cosine of the angle between the surface normal and the direction of the incident light. The weight of photon was therefore determined by Equation 4.10 after scattering from soil or trunk surface.

4.1.1.2 Leaf Scattering

Leaf Inclination Angle Generation The Leaf inclination angle, θ_L , was determined by inverting the cumulative density $\mathcal{C}(\Theta)$ (see Equation 3.28)

$$\theta_L = \mathcal{C}^{-1}(\rho_\theta). \quad (4.3)$$

Here $\rho_\theta \in [0, 1]$ is a random number generated from the algorithm given by Section 3.1.1. The azimuth of leaf normal vector, which was assumed independent from zenith, ϕ_L was then determined by

$$\phi_L = 2\pi\rho_\phi. \quad (4.4)$$

Leaf normal vector $\mathbf{n}_L$ is then constructed by Equation 3.1.

Vector Rotation The motion directional vector $\mathbf{n}$ rotation caused by scattering events was determined by: the increased zenith $\Delta\theta$ and $\Delta\phi$ were randomly determined by rejection method. New direction $\mathbf{n}'$ was then calculated by

$$\mathbf{n}' = \cos(\Delta\theta) + \sin(\Delta\theta)\mathcal{M}. \quad (4.5)$$

Here, $\mathcal{M}$ is determined by

$$\mathcal{M} = \frac{1}{\sin\theta} \begin{bmatrix} n_x n_z & -n_y \\ n_y n_z & n_x \\ -\sin^2\theta & 0 \end{bmatrix} \begin{bmatrix} \cos(\Delta\phi) \\ \sin(\Delta\phi) \end{bmatrix}, \quad (4.6)$$

where the $\mathbf{n}(\theta, \phi)$ is given by Equation 3.1 as well as the definition of n_x , n_y and n_z . In particular, when $\sin\theta \rightarrow 0$, $\mathbf{n}'$ is given by

$$\mathbf{n}' = \text{sgn}(n_z) \begin{pmatrix} \sin(\Delta\theta) \cos(\Delta\phi) \\ \sin(\Delta\theta) \sin(\Delta\phi) \\ \cos(\Delta\theta) \end{pmatrix}. \quad (4.7)$$

Here, $\text{sgn}()$ is the sign function of a real number:

$$\text{sgn}(x) := \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}.$$

Single Leaf Scattering Scheme Photon scattering direction of Monte Carlo method was randomly decided. Here presents a scheme step by step determining the scattering direction. The leaf was assumed as bi-Lambertian surface. Firstly, randomly select a leaf normal Ω_L by the cumulative density function (Equation 3.28). Then, to use rejection method checking Ω_L is acceptable or not. Picking a random number $\xi \in [0, 1]$,

if $|\Omega \cdot \Omega_L| < \xi$, the Ω_L was re-generated; else the Ω_L was accepted. Finally, another random number ξ' was used to determine whether the photon reflected or transmitted by checking, if

$$\frac{r_L}{\omega_L} < \xi',$$

the photon transmitted, otherwise reflected. Scattering direction $\mathbf{n}'(\theta', \phi')$ was determined by

$$\theta' = \cos^{-1}(\sqrt{\xi_{\theta'}})$$

and

$$\phi' = 2\pi\xi_{\phi'},$$

where $\xi_{\theta'}$ and $\xi_{\phi'}$ are two random numbers. In a reflection case, the $\mathbf{n}'$ was accepted when $\Omega' \cdot \Omega_L < 0$, otherwise $-\mathbf{n}'$ was picked. Similar determination was employed in a transmission case, if $\Omega' \cdot \Omega_L \geq 0$, $\mathbf{n}'$ was accepted otherwise $-\mathbf{n}'$ was selected.

4.1.2 Effective BRF Photon Sampling

4.1.2.1 LEM Application

TRCMCRTS was implemented by counting the photons through the reference plane running into the observing direction. The productivity of the photon sampling was low, if only collecting the survival photons to the defined observation direction. Marchuk et al. (1980) provide an effective method, the local estimation method (LEM), to accelerate the converging of the MCRT method which is used in atmospheric optics. Photon contribution $C_s(\Omega, \Omega_i)$ to the observation direction was calculated by

$$C_s(\Omega, \Omega_i) = \frac{w_s}{4\pi\mu} \Phi(\Omega, \Omega_i) \exp\left(-\sum_{j=1}^n \tau'_j\right), \quad (4.8)$$

where w_s is the single photon weight when a scattering event occurred, $\mu = |\cos \theta|$, weight updates (w'_s) depends on the weight of last scattering event which follows

$$w'_s = \omega_m w_s, \quad (4.9)$$

that ω_m is material surface albedo, $\Omega(\theta, \phi)$, $\Omega_i(\theta_i, \phi_i)$ are photon scattering direction and the i th observation direction, respectively. In particular, when scattering is isotropic the Equation 4.9 becomes

$$w'_s = \frac{w_s}{\pi}. \quad (4.10)$$

j is the canopy number which the ray traveled through towards Ω_i , τ'_j is the hot spot effect corrected canopy optical thickness [152]:

$$\tau'_j = \frac{\tau_j \tan \alpha}{\tan \alpha + H(\Omega, \Omega_i)}, \quad (4.11)$$

where α is the half of the angle between Ω and Ω_i , τ_j is the j th canopy optical thickness, $H(\Omega, \Omega_i)$ is the exponential hot spot coefficient that,

$$H(\Omega, \Omega_i) = \frac{1}{4} r_l u_m (G(\Omega) + G(\Omega_i)), \quad (4.12)$$

where r_l is average leaf radius, u_m is the mean LAD of the canopy. BRF ($R(\Omega, \Omega_i)$) of the MCRT method is calculated by

$$R(\Omega, \Omega_i) = \frac{\pi}{N w_0} \sum_{I=0}^N \sum_{J=0}^S C_{IJ}(\Omega, \Omega_i), \quad (4.13)$$

where w_0 is the initialized photon weight, N is the total number of the photon, S is the total scattering times of each photon, $C_{IJ}(\Omega, \Omega_i)$ is the contribution of each photon at each scattering event to the i th observation direction.

4.1.2.2 Russian Roulette Method

Russian roulette is one of the most widely used variance reduction techniques. Approximately 50% of computational time is consumed to track secondary and higher-order scattered photons. It is possible to remove a part of these photons by Russian roulette method [153]. Conventionally, users determine a threshold w_{th} empirically for Russian roulette. The larger threshold is picked, the lower accuracy is obtained. Here we defined

$$w_{th} = 0.1\omega^2,$$

where ω is the albedo of the materials. Russian roulette is an efficient application to accelerate the convergence of the MC simulation, however, it introduces additional source of variance for the varying photon weights. It is therefore a method needs to be mathematically proved [154].

4.2 MCRT Model BRF Results

4.2.1 Simple Albedo Simulation for Random Open Forests

For a rough prediction of the total reflectance estimation of an open forest area without investigation in the field, a random open forest model is therefore required. A random open forest generator was developed for generating a randomly 2-D distributed open forest by developing a circle packing algorithm.

4.2.1.1 2-D Circle Packing for Open Forest Generation

A 2-D circle packing algorithm based random open forest model was developed in order to escape from the manually forest setting. As an important step in MCRT simulation, 3-D forest generation was to locate each tree of the forest to provide the RT regime. In order to simplify the algorithm, we did not consider the canopy overlapping conditions.

The forest was generated according to the input coverage percentage and tree amount. The radius of each canopy radius from nadir view was randomly generated as well as the center of the canopy. Overlapping between circles was therefore inevitable. Those overlapped circles were moved away from each other along the line between the two circle center points. Then, check the circles whether they were located out of the scene boundary. Move the outside circle(s) to the loop back position(s) of the scene. FIGURE 4.5 shows a scheme of the circle packing algorithm. FIGURE 4.5 (a) indicates when the overlapped circles C_1 and C_2 were detected, moving them oppositely along their centers-connection line until they are separated to each other. If C_1 is out of the square upper boundary with length d (FIGURE 4.5 (b)), C_1 is moved to its opposite position of the square which the distance between circle and square downer boundary is d (FIGURE 4.5 (c)). The progress will be repeated by N times, which is a big number to make the canopy circle not to overlap with each other. The progress repeated N times, where N is a big number. The geometrical parameters of individual tree we followed the regressive model from Ishikawa and Ito (1989). FIGURE 4.7 shows examples of random forests generated by the circle packing algorithm.

4.2.1.2 Reflectance of Random Open Forests

The algorithm of a simplified reflectance simulation using random open forest model is shown in FIGURE 4.6. For the simplicity of the estimation, the 3-D scene was not divided into voxel matrix; the relief of the land was no considered as well. A photon, initialized (w_i) on the top of scene randomly located at $\mathbf{r}_i$ with moving direction Ω_i , intersection with canopy or trunk or soil objects was checked. The photon weight is updated (w'_i) at each scattering event and will be checked by Russian Roulette method (if $w_i < w_{th}$) to determine if the photon died or not. When photon run out of scene, a new photon will be generate, else the photon will keep on moving.

FIGURE 4.8 (a) shows total reflectance of generated forests (FIGURE 4.7). Spectral wavelength is proposed as 450, 550, 650, and 800 nm, respectively. Leaf reflectance and transmittance for each band are shown in FIGURE 4.1. LAD of the tree is 2.0, SZA is 20

degrees. The leaf angle distribution parameter χ is set to be 1.0. Area green coverage is 35%. Reflectance from trees of single-scattering is shown in FIGURE 4.8 (b). As FIGURE 4.1 shows, total reflectance does not change much at blue (450 nm) and green (550 nm) bands; obvious reflectance differences start from red (650 nm) band and so as to near-infrared band (800 nm). The single-scattered reflectance from the canopies is stable at blue, green and red bands. Identifiable difference comes at near-infrared band.

TABLE 4.1: Parameters for the random forests reflectance simulation

band	450 nm	550 nm	650 nm	800 nm
canopy leaf reflectance	0.038	0.110	0.046	0.502
canopy leaf transmittance	0.017	0.038	0.021	0.190
soil albedo	0.085	0.115	0.149	0.268
solar zenith (°)	20			
LAD	2.0			
mean leaf radius (m)	0.1			
leaf angle distribution	$\chi = 1.0$			
area green coverage	35%			

4.2.2 TRCMCRTS Validation by Comparing with FLiES

In order to validate TRCMCRTS, a comparison between our model and FLiES was executed for a 100×100 m² open forest area. Individual tree canopies were set as spherical whose center height varied from 11.0 m to 19.0 m above ground. FIGURE 4.10 shows the comparison between TRCMCRTS and FLiES with 3×10^5 photon samples and the same input parameters (given TABLE 4.2) in the red and infrared bands with solar zeniths at 20 and 50 degrees. The results show a high correlation (> 0.99) for BRF estimation. Canopies location distributed as FIGURE 4.9 shows. The spheres stands for individual canopies with the same radius as 10.0 m. Centers height varies in the range from 11.0 to 18.0 m. Other than the simplified algorithm in Section 4.2.1, more complicated algorithm was developed which is shown in FIGURE 4.2

TABLE 4.2: Parameters for the two model simulations

band	red	near infrared
canopy leaf reflectance	0.046	0.50
canopy leaf transmittance	0.021	0.45
soil albedo	0.15	0.20
solar zenith ($^{\circ}$)	20, 50	
LAD	2.0	
Tree count	15	
canopy sphere radius (m)	10.0	
mean leaf radius (m)	0.1	
leaf angle distribution	$\chi = 1.0$	

4.2.3 BRF Simulation with Topographic Relief

To demonstrate the effect of the topographic relief on the BRF estimation, six different ground relief conditions were examined. In each case the solar beam was incident from the west ($\theta_0 = 20^{\circ}$, $\phi_0 = 180^{\circ}$). The topographic relief was 0.0 to 1.5 m above the ground (see FIGURE 4.11). Other parameters were set as given in TABLE 4.2.

FIGURE 4.12 shows the BRF estimations for the six different topographic conditions shown in FIGURE 4.11 (a) to (f). FIGURE 4.12 (a) shows the total BRF of the forest, which varies according to ground relief. Conditions c, d, and f show significant changes in response due to the altered topography. FIGURE 4.12 (b) shows the BRF contributions from the canopy: with the same parameters for the trees (conditions a to f), the BRF contribution from the canopy is reasonably stable. The BRF contributions from the ground are shown in FIGURE 4.12 (c), which indicates that this is the source of the variations of BRF in FIGURE 4.12 (a). Conditions a and b, which had a bowl-shaped area located symmetrically around the east-west line, show symmetrical responses at a viewing angle around the $\pm 60^{\circ}$. Conditions c and d, which had slopes symmetrical about the north-south line, present symmetrical responses with the variation of observation angle. Condition e gives only a small effect because the ground relief has symmetrical appearance for front light and back light conditions. Condition f, which is condition e rotated clockwise by 90° , produces a strong contribution to the back light condition. The peak of the BRF caused by the hot spot effect is the same for all six conditions.

4.2.4 BRF Simulation with Canopy LAD Variations

In order to demonstrate the LAD affection to the BRF, simulation with a increasing setting of the canopy LAD was executed. The LAD settings varied from 0.5 to 3.0 with 0.5 span. Leaf reflectance and transmittance were 0.50 and 0.45, respectively. Soil albedo was 0.20, and canopy sphere radius was 10.0 m. The mean of the leaf radius is 0.1 m, leaf angle distribution $g(\theta_L)$ is set to be the spherical distribution with $\chi = 1.0$. FIGURE 4.13 shows the near-infrared band BRF simulation results with SZA at 20° and 50° , respectively. It shows a decelerated growing when LAD value increased at principle-plane BRF at the hot-spot, which means the BRF will not unlimitedly increase with LAD. Both FIGURE 4.13 (a) and (c) presented cross points of the curve, situation in (c) is more obviously, meaningfully that there will be a view angle with the proper SZA is independent to the LAD. Which means, for the spherical leaf angle distribution canopy, if SZA is fixed, there will be a view angle on principle-plane that all the LADs cause the same BRF value. Such kind of phenomenon was also illustrated in Gastellu-Etchegorry's (1999). On the 'left' side of the cross points of FIGURE 4.13 (a) and (c), BRF from lower LAD will higher than those from higher LAD. On the 'right' side, contrary feature is obtained. For cross-plane BRF results, on SZA at 20° BRF value shows obviously decelerated with the increasing of the LAD value in the range of $(-40^\circ, 40^\circ)$. The characteristic is not very clear at SZA is 50° .

4.3 Summary and Discussion

For open forests, stable reflectance at blue and green bands indicated the spatial distribution of trees did not affect reflectance significantly. For estimating the blue and green bands reflectance of the open forests, precise locations of the trees were not necessary. With the same LAD and green coverage, differences of the reflectance at red and near-infrared bands came from multiple scattered radiance.

Comparison between TRCMCRTS and FLiES showed that TRCMCRTS performs identical BRF estimation ability as FLiES. Furthermore, unlike the published canopy

MCRT codes, TRCMCRTS responded sensitively with the relief of the ground. In spite of the affection from topographic reliefs impacting the BRF distribution on some observation geometry, the hot spot (peak point of the BRF plot) was the same between with and without topography reliefs. LEM for BRF estimations were especially well adapted to situations that involve detectors with high resolutions or with narrow IFOV. One condition should be configured that the detector can be put outside to the RT regime for simplifying the conditions.

LAD can be retrieved by interpolating the BRF value at a given view. A wider LAD-BRF look-up-table can be generated according to the simulation shown in Section 4.2.4. User should avoid the cross points (see FIGURE 4.13 (a) and (c)) when retrieving the LADs since it is impossible to derive which LAD is exactly corresponding to the BRF value. On the other hand, the variance of BRF according to the LAD changing indicates that the canopy structural parameter like LAD, directly affects BRF. Therefore, in order to obtain accurate BRF, LAD should be determined precisely. However, direct measuring of LAD costs labor and time, and thus a more efficient way is necessary to be investigated. Detailed retrieving approaches for LAD, LAI and leaf angle distribution function from both photogrammetry and point cloud data will be illustrated in next chapter.

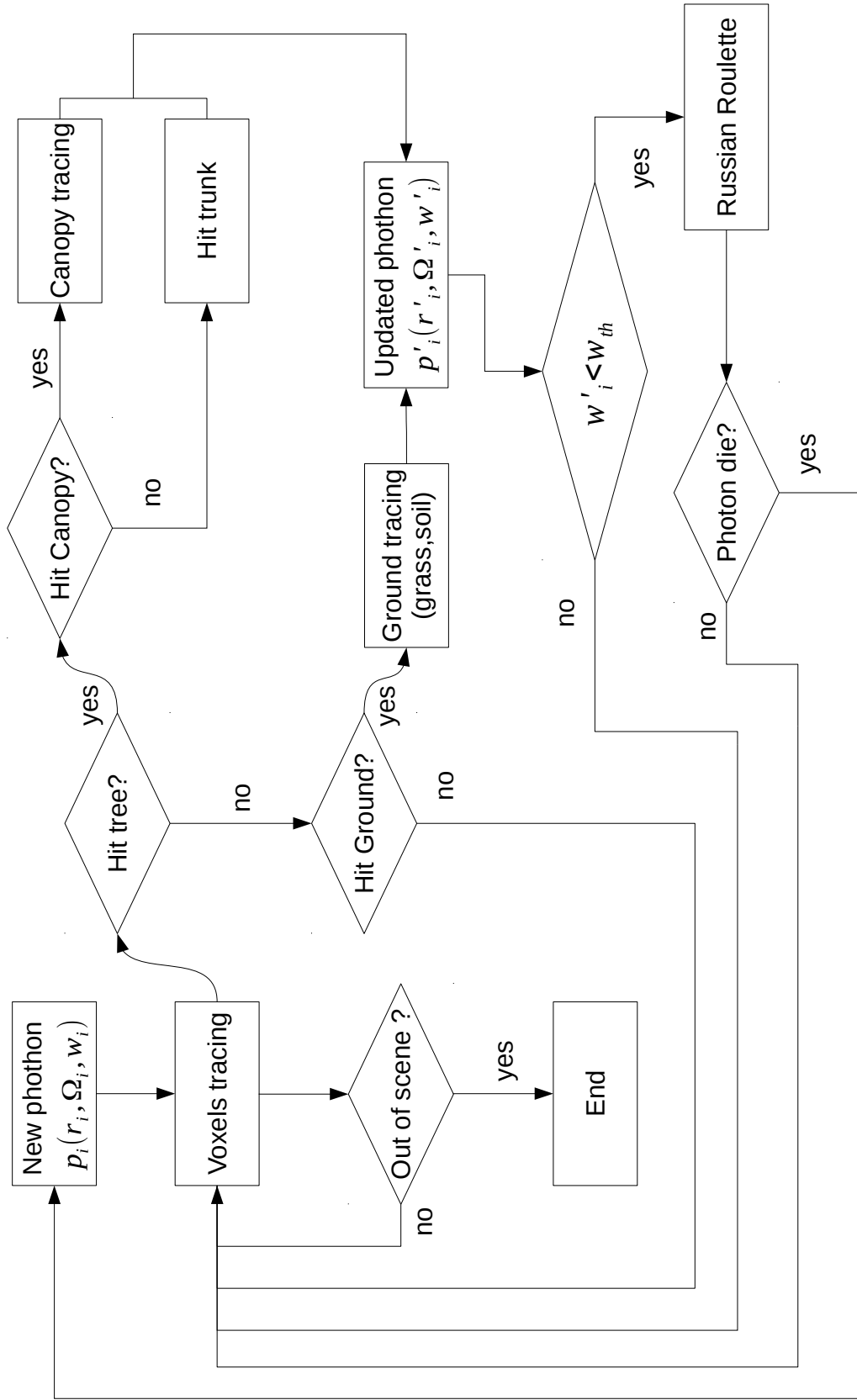


FIGURE 4.2: Flowchart of ray-tracing algorithm of the BRF simulation. Initial a photon ($p_i(\mathbf{r}_i, \Omega_i, w_i)$) on the top plane of the 3-D scene, then tracing p_i with in each voxel object. If the updated photon weight w'_i less than the threshold w_{th} , the Russian Roulette function will be executed to determine the photon tracing will be continued or not. If the photon run out of the scene, the ray-tracing is ended. Otherwise the photon will keep on moving.

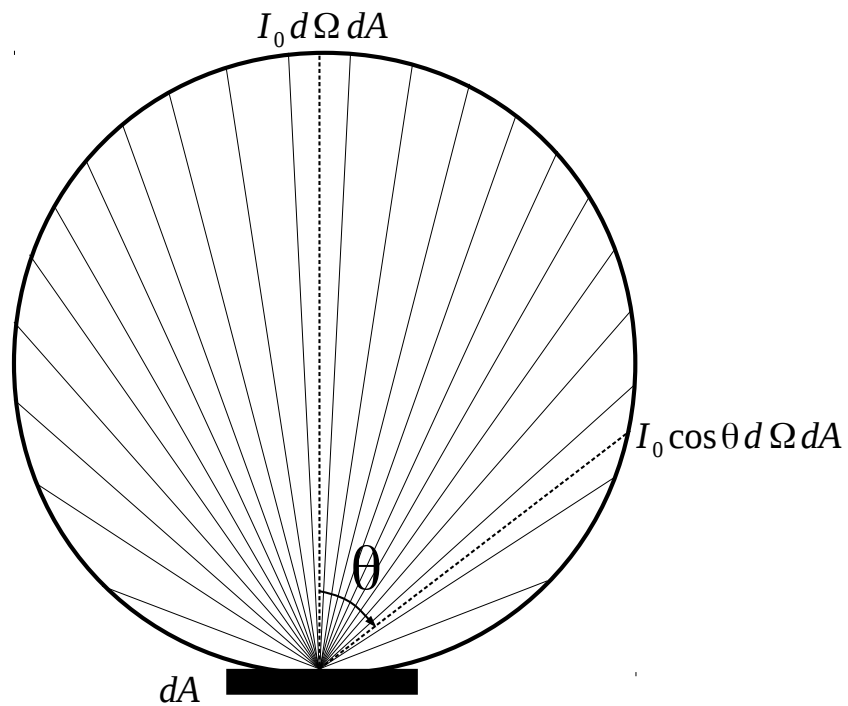


FIGURE 4.3: Lambert's cosine law sketch. Incident intensity is I_0 , θ is the zenith, dA is the area element and $d\Omega$ is the element of the solid angle around dA .

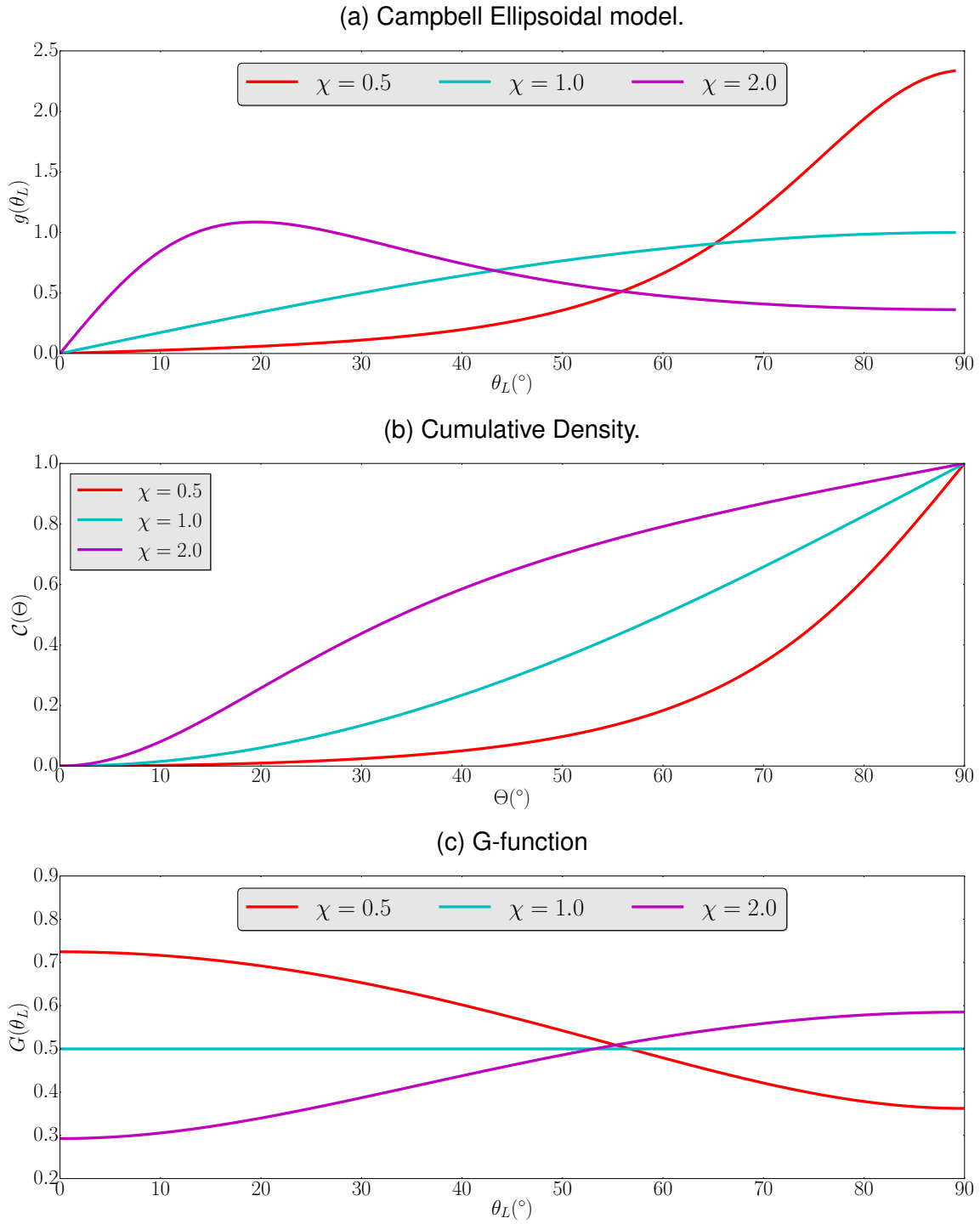


FIGURE 4.4: Examples of LUTs from Campbell's Model. (a) Campbell's model with $\chi = 0.5, 1.0$ and 2.0 , respectively, (b) cumulative density according to Equation 3.25, and (c) G -function generated according to Equation 3.27.

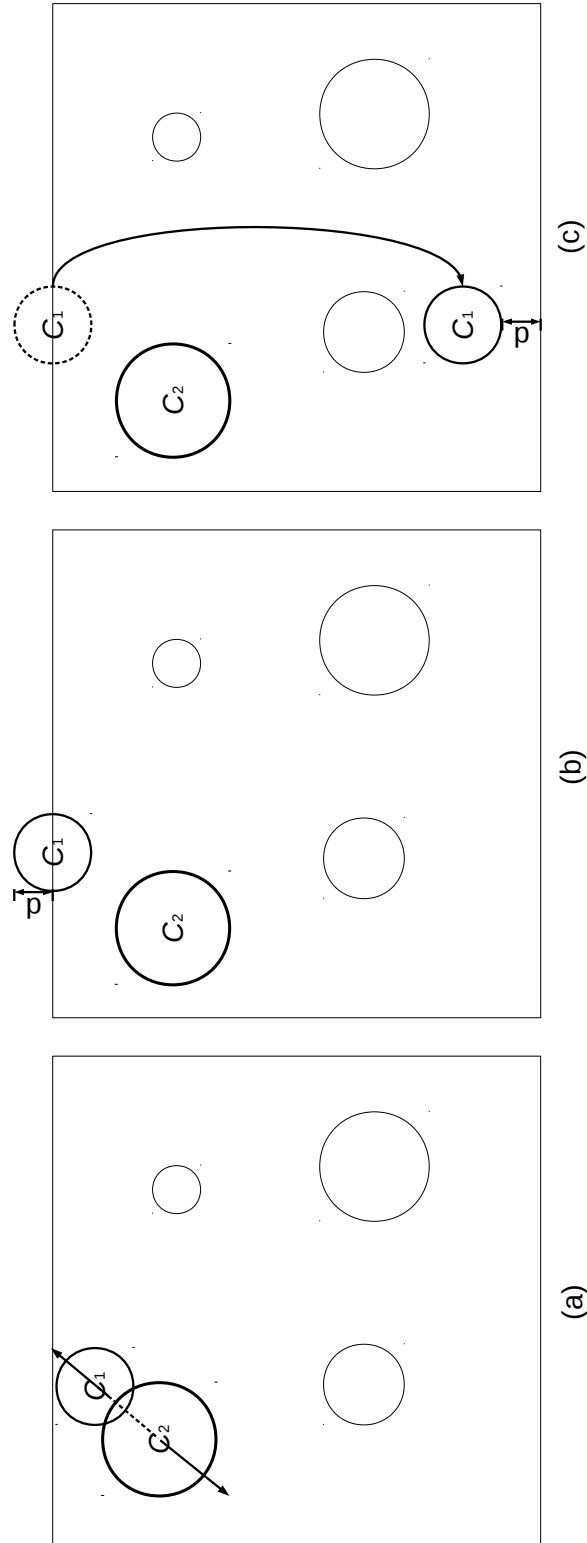


FIGURE 4.5: Circle packing algorithm. (a) The overlapped circles C_1 and C_2 are moving oppositely along their centers-connection line until they are separated to each other. (b) C_1 is out of the square upper boundary with length d . (c) C_1 is moved to its looped position of the square which the distance between circle and square downer boundary is d . The progress will be repeated by N times, which is a big number to make the canopy circle not to overlap with each other.

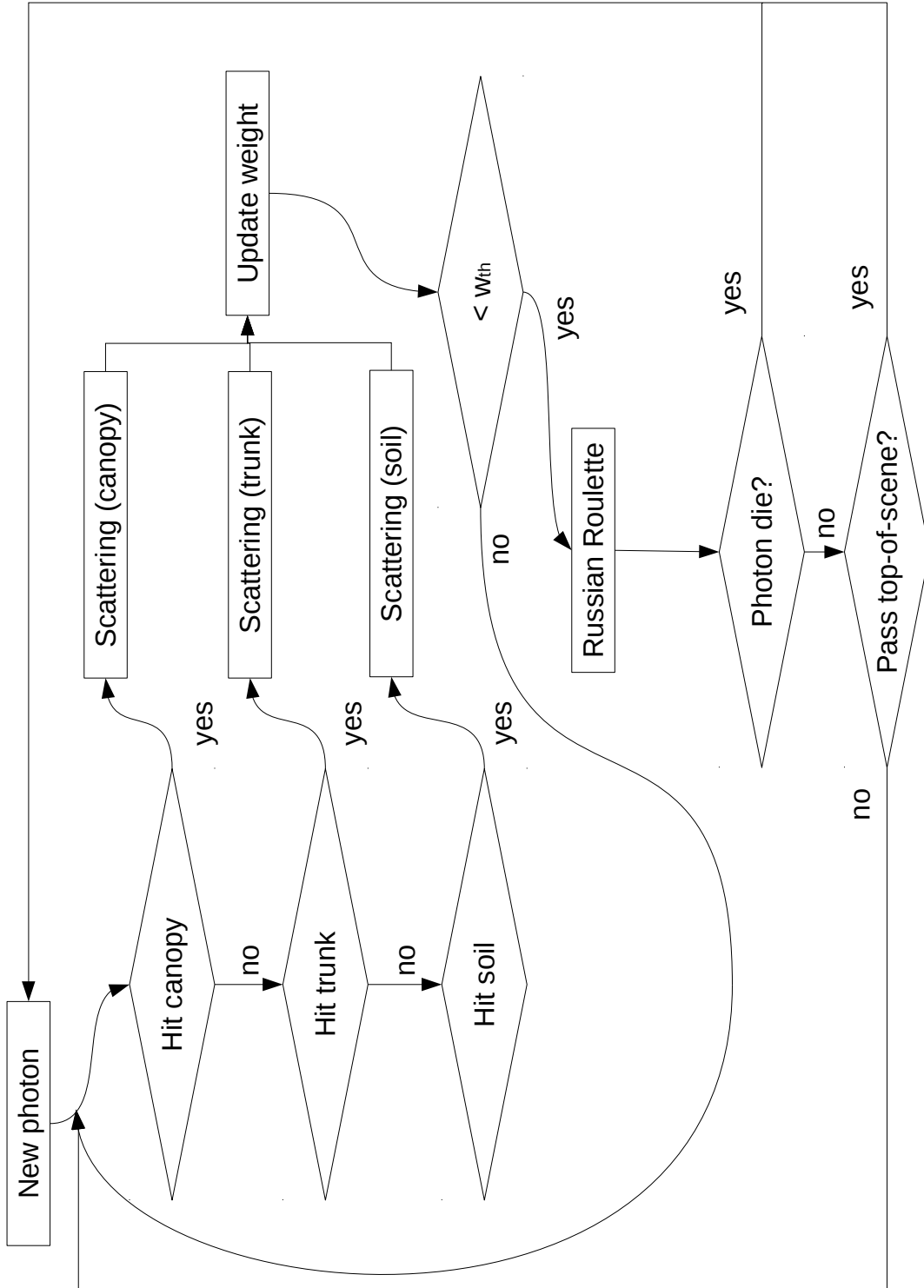


FIGURE 4.6: Flowchart of the MC ray-tracing algorithm of the reflectance simulation. 3-D scene includes canopy, trunk and soil. A photon, initialized (w_i) on the top of scene randomly located, intersects with canopy or trunk or soil at each interactions with materials. The photon weight is updated (w'_i) at each scattering event and will be checked by Russian Roulette method (if $w'_i < w_{th}$) to reduce the variance. When photon run out of scene, a new photon will be generate, else the photon will keep on moving.

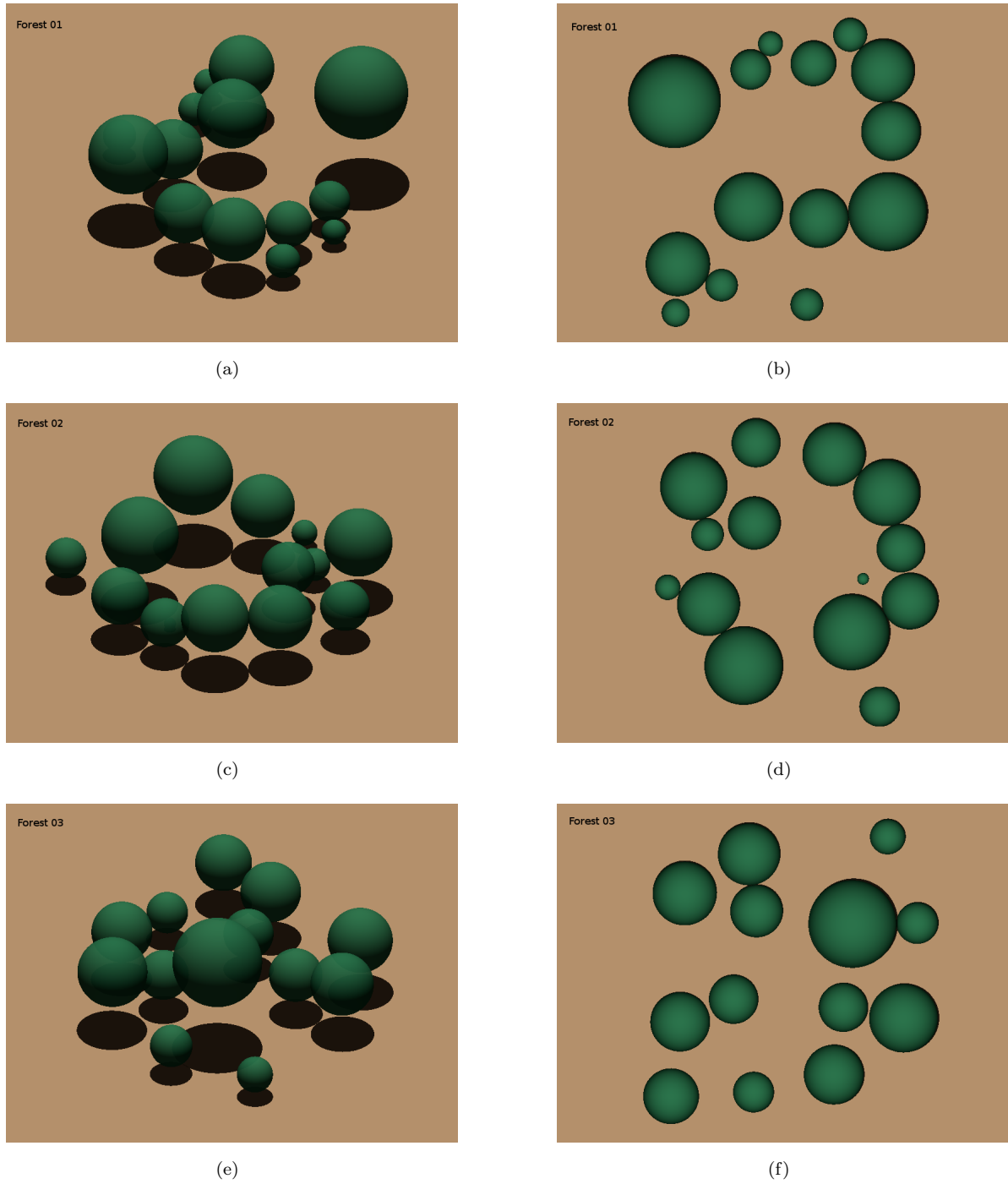


FIGURE 4.7: Randomly located trees for area albedo estimation. Three forests with 35% area coverage generated randomly according to circle packing algorithm for a $100 \times 100 \text{ m}^2$ area. Side view (first column graphs) and vertical view (second column graphs) are listed.

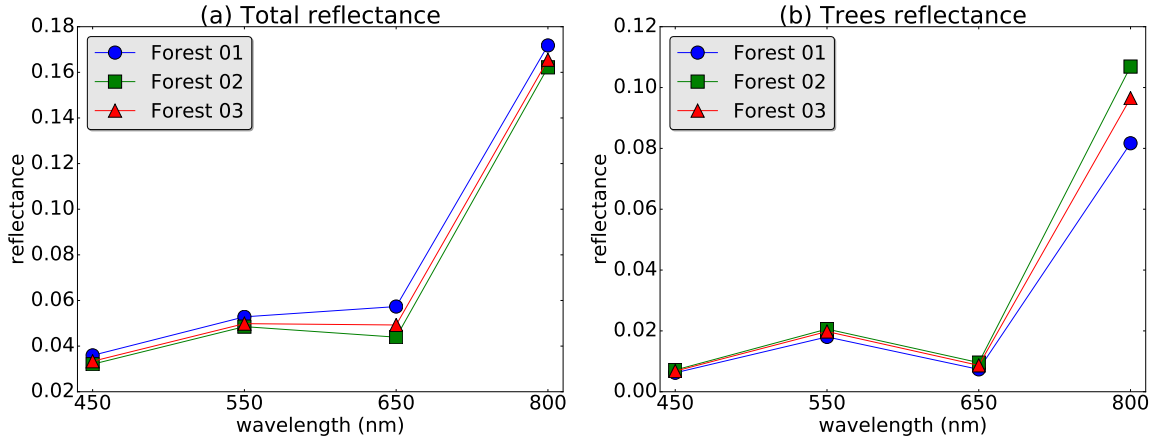


FIGURE 4.8: Reflectance simulation according to the random forests of FIGURE 4.7. SZA is 20 degrees, spectral wavelength is set as 450, 550, 650, and 800 nm, respectively. (a) total reflectance from the area, and (b) single-scattered reflectance from trees.

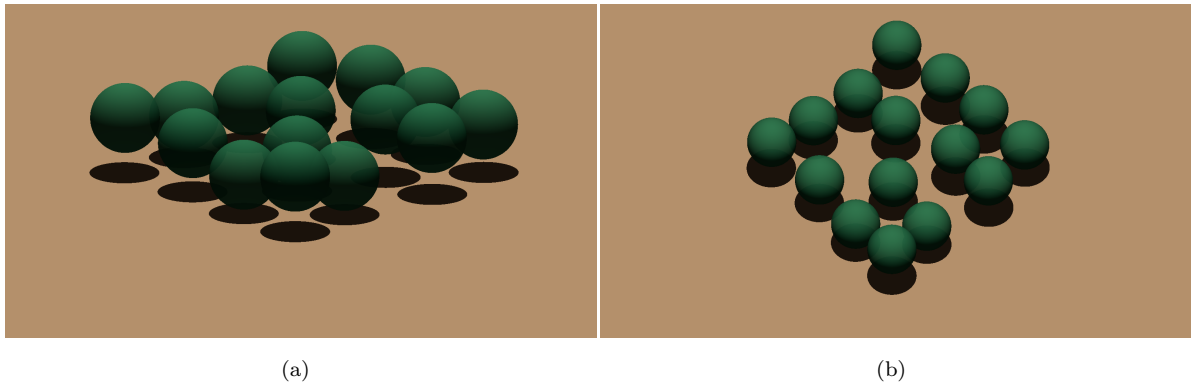


FIGURE 4.9: 3-D scene generated for BRF estimation, the spheres stands for individual canopies with the same radius as 10.0 m. Centers height varies in the range from 11.0 to 18.0 m. (a) and (b) show the different view sight of the area.

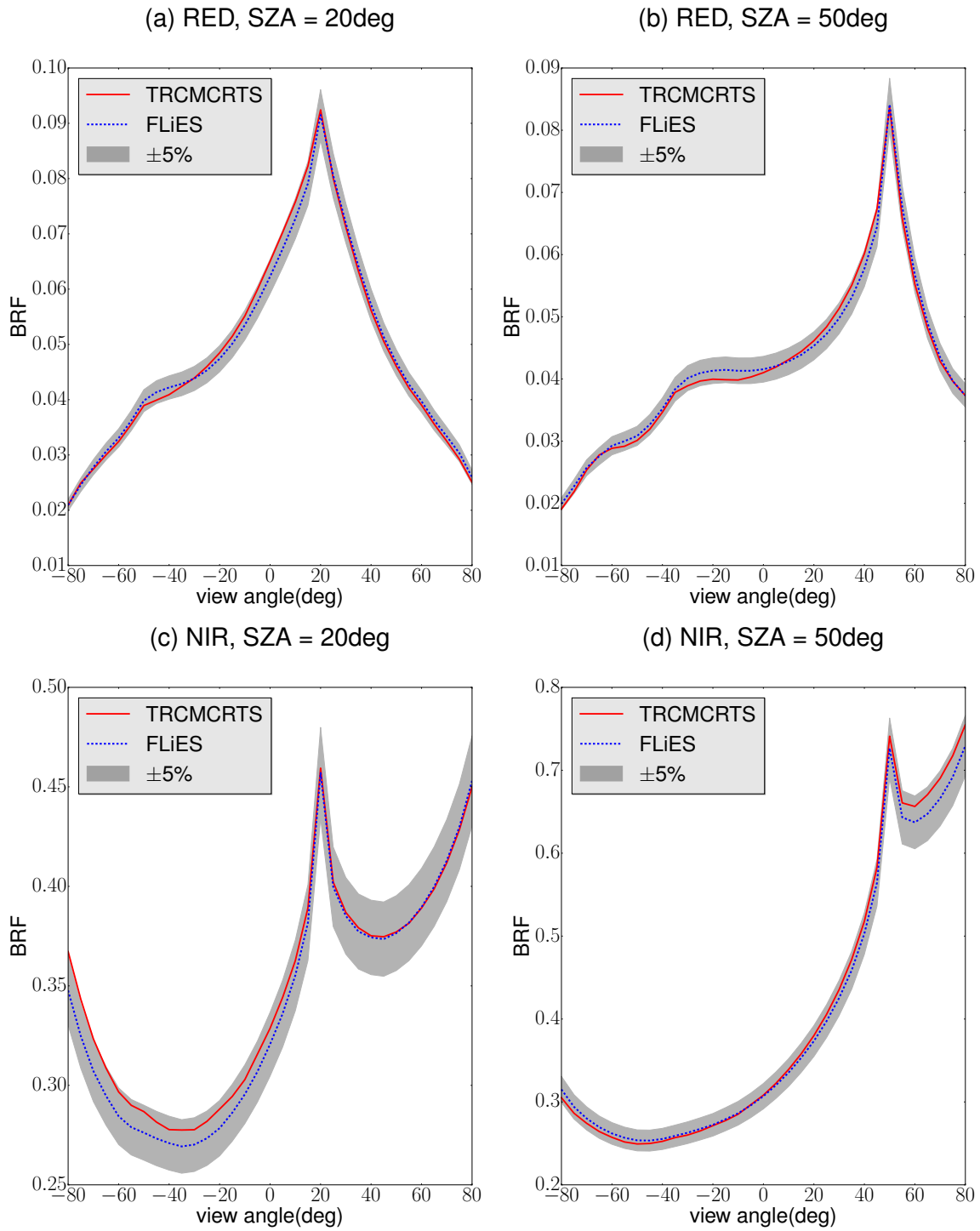


FIGURE 4.10: Comparison simulated BRF results for TRCMCRTS and FLiES on the principle plane. Figures (a) and (b) show results for the red band with solar zenith angles (SZAs) of 20° and 50° , respectively; (c) and (d) show the results for the near-infrared band with the same SZAs as (a) and (b), respectively. The gray area is the $\pm 5\%$ error area of the FLiES results.

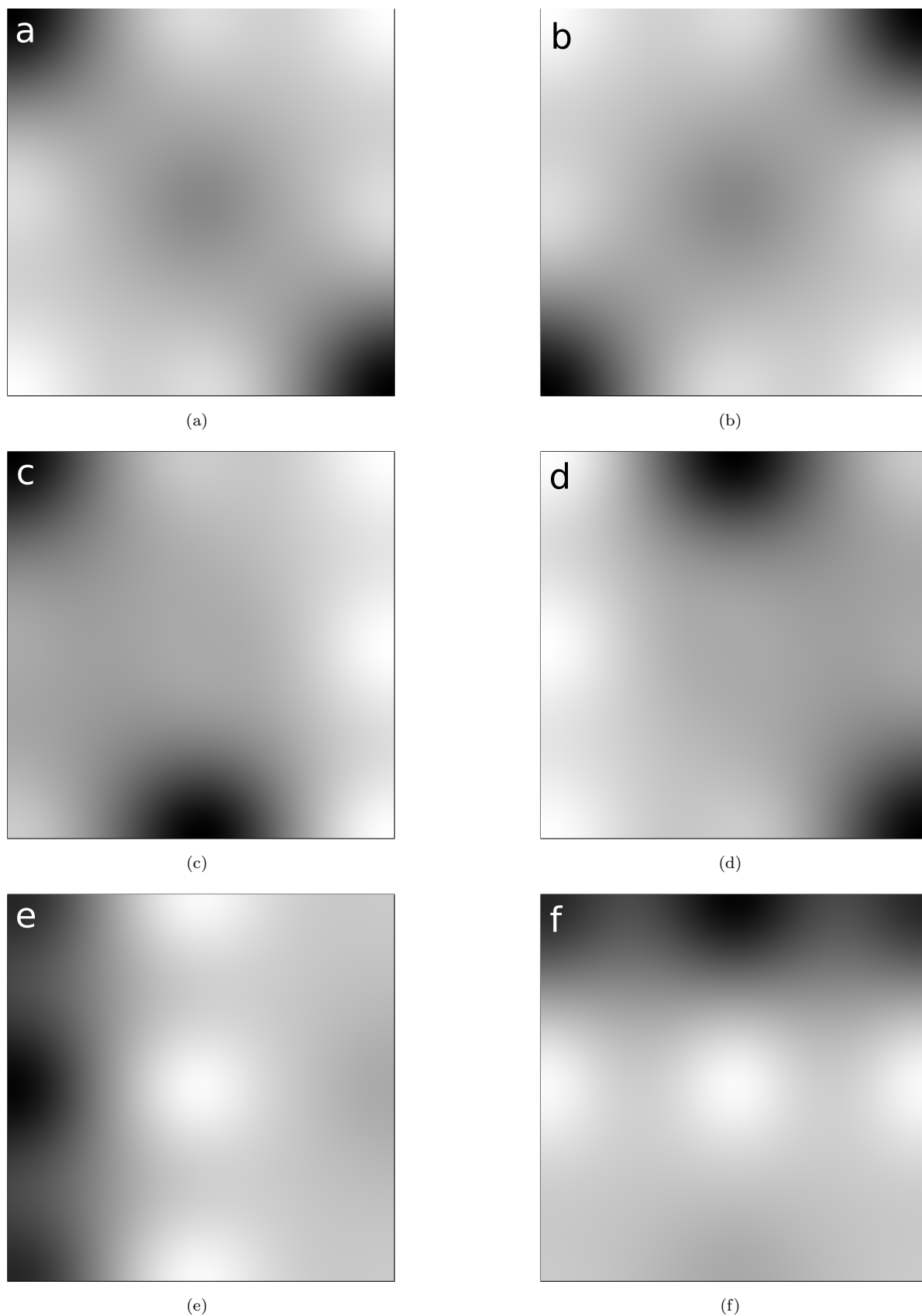


FIGURE 4.11: Topographic conditions of a $100 \times 100 \text{ m}^2$ area used to demonstrate the effect of terrain relief on the BRF. Solar direction is from the west ($\theta_0 = 20^\circ$, $\phi_0 = 180^\circ$) in all cases. Gray scale color (from black to white) shows the linear increasing of the elevation from 0.0 to 1.5 m.

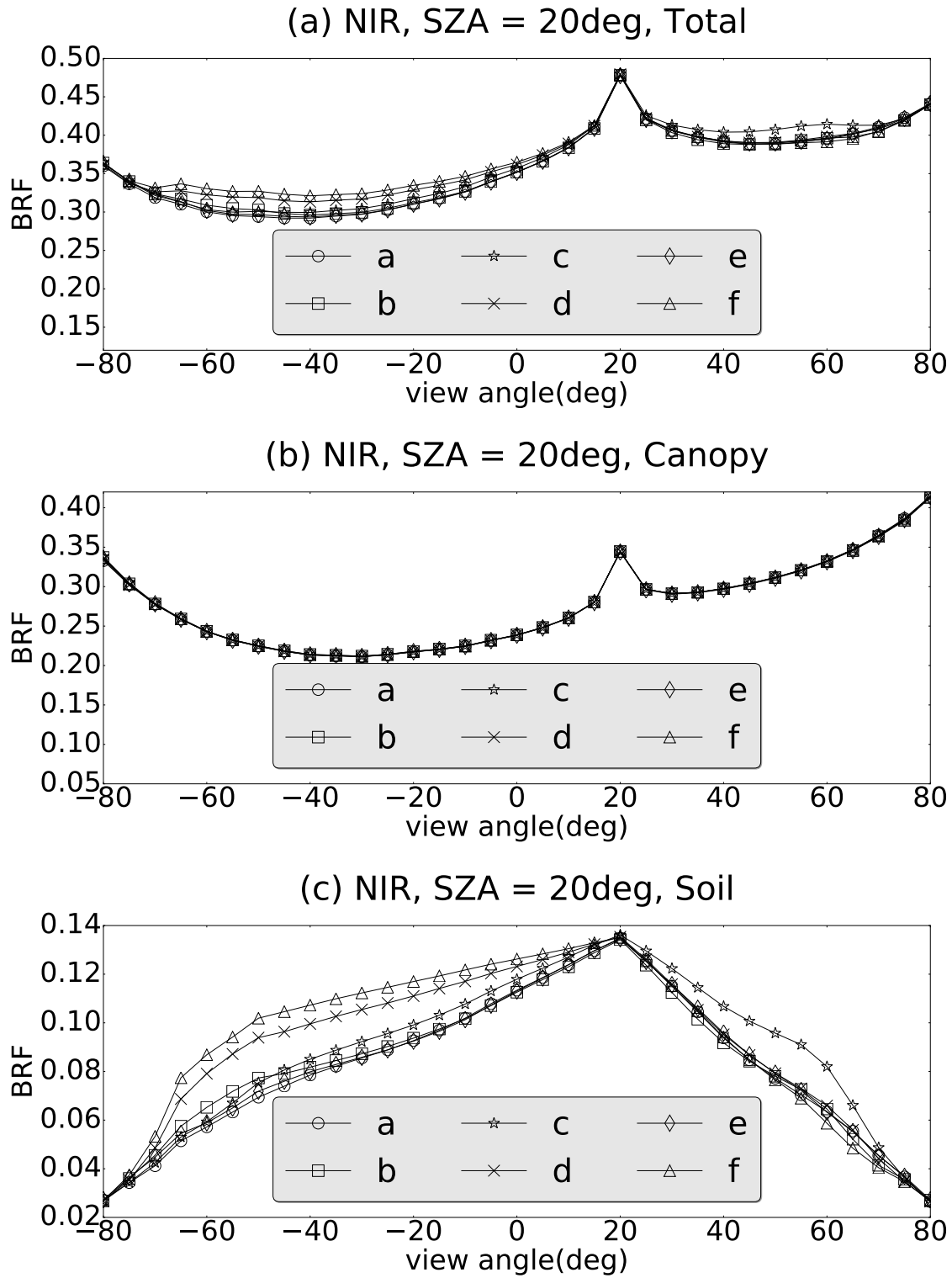


FIGURE 4.12: BRF for the six different topographic conditions in FIGURE 4.11; other parameters are those given TABLE 4.2. (a) total BRF of the 3-D scene, (b) canopy contribution, and (c) soil contribution.

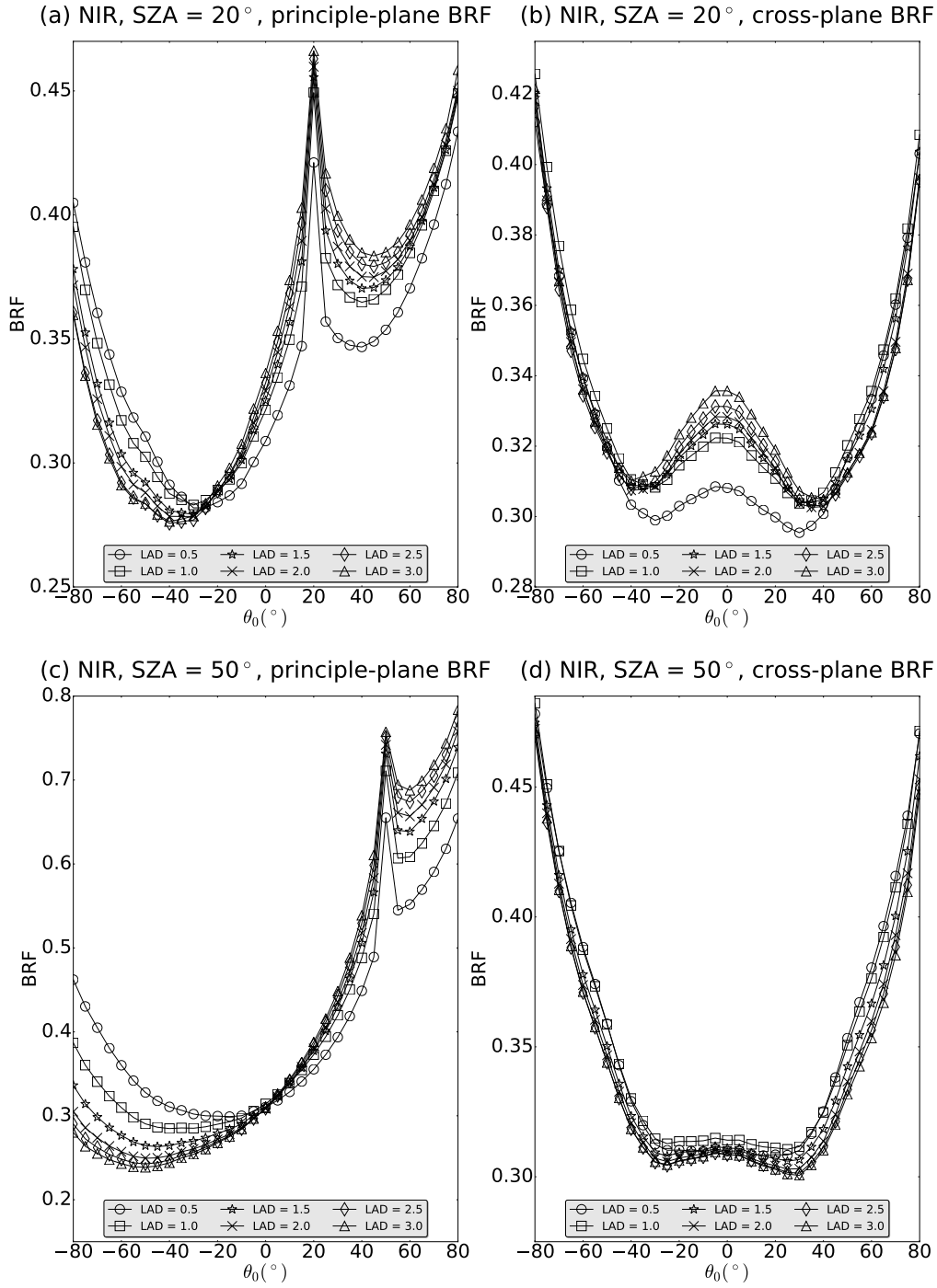


FIGURE 4.13: Simulated BRFs with different LADs and SZAs. From (a) to (d) the LAD varies from 0.5 to 3.0 with 0.5 span. SZA = 20° in (a) and (b); SZA = 50° in (c) and (d). BRFs of principle-plane ($\phi_0 = 0^\circ$) in (a) and (c), cross-plane ($\phi_0 = 90^\circ$) in (b) and (d) with band near-infrared settings.

Chapter 5

Canopy Structural Parameter Retrieving for MCRT Model Application

Applications of canopy structural parameters retrieving is presented in this chapter. Experiments on an isolated tree are tested by using two different strategies; one is to use photographing and the other is to employ the PCD from TLS. Results from digital camera are validated by *LAI-2000* instrument. Sample tree's LND function is obtained according to the field leaf normal collection, an L-System algorithm based 3-D tree model is generated in order to input to MCRT model. Investigations for checking PCD processing method is executed. Field leaf normal collection is held in order to testify the result from PCD.

5.1 LAD Retrieving Using Digital Camera

An algorithm was developed to retrieve LAD from ground based optical photographs taken from eight directions using a commercial digital camera. Then, the canopy gap fraction was extracted. Leaf normal zenith samples were collected in the field for getting

the leaf normal distribution function and deriving the *G-function* by Equation 3.24. The sample tree's LAD was calculated by Equation 3.29. *LAI-2000* instrument was selected as the standard data source provider for evaluating the result.

5.1.1 Data Collections and Processing

A well-isolated broad leaf tree (*Cinnamomum camphora*, FIGURE 5.1(a)) was selected in Kyoto University Katsura Campus of Kyoto Prefecture, Japan (N 34°58'55.61", E 135°40'43.54"). *SONY DSLR-A300* camera was used as the sensor and the focus length was constantly set to 35 mm, eight directions (N, S, E, W, NE, NW, SE and SW) were chosen to take photos for the camphor tree. The horizontal distance between tree and camera was set to make sure that the entire canopy was captured within the IFOV of the camera on each direction. Ten pictures were taken on each direction with recording the camera zenith (θ_0). The light path Δl was measured directly in field, which is also could be obtained by using photogrammetric methods. For extracting the leaf area more effectively, a blue screen was put closely behind the tree. For getting the orthography photographs, all the pictures were calibrated using the *Image Master Calib* software, which is developed by *TOPCON* corporation. In order to estimate the leaf angle distribution canopy was divided into ten layers for collecting the leaf normal zenith (θ_L) which is named the leaf inclination angle as well. 1018 samples were collected for the total canopy and the cumulative distribution, on each layer the author took about 100 samples using inclinometer. Most of the data are in the range 20° to 80° (FIGURE 5.2(b)) and the mean value of the leaf inclination angles is 43.24°.

5.1.2 Results and Validating by Comparing with *LAI-2000* Collection

According to the field raw data collections of the leaf normal, a numerical function $g^*(x)$ was then obtained by retrieving the density curve (FIGURE 5.3) of collected leaf

inclination angles

$$g^*(\theta_L) = \sum_{i=0}^N p_i \theta_L^{N-i}, \quad (5.1)$$

where N is positive integer number; and p_i is polynomial fitting coefficient shown in TABLE 5.2 which we set $N = 8$. Then G -function $G(\theta_0)$ can be calculated by solving integral of Equation 5.1 in terms of Equation 3.24. Gap fractions were extracted from calibrated pictures by *ENVI* software, each direction an average gap fraction ($T(\theta_0, \phi_0, \Delta l)$) was calculated by using SVM method (FIGURE 5.4). In order to assess the retrieving result, LAD data were collected on five directions from the bottom of the canopy next to the trunk (see FIGURE 5.5) by using *LAI-2000* instrument. Light free path of the five θ_0 was calculated by estimating the photographs taken in the field. Parameters for calculating eight directions LAD are listed in TABLE 5.1. The mean value of eight directions photographs LAD was 0.54, G -function results were calculated from each direction shown stable around 0.59.

TABLE 5.1: LAD calculation result and the input parameters.

Direction	$\theta_0(^{\circ})$	Δl	$G(\theta_0)$	$T(\theta_0, \phi_0, \Delta l)$	LAD(m ² /m ³)
N	101.3	4.10	0.587	0.30	0.49
NW	100.6	3.49	0.589	0.32	0.55
W	97.1	3.74	0.595	0.28	0.57
SW	92.2	3.87	0.599	0.31	0.51
S	87.9	4.10	0.599	0.24	0.58
SE	97.7	3.49	0.594	0.27	0.64
E	97.0	3.74	0.595	0.32	0.51
NE	99.8	3.87	0.590	0.36	0.45

TABLE 5.2: Parameters of Equation 5.1.

i	0	1	2	3	4	5	6	7	8
p_i	30.24	-201.77	537.59	-724.81	517.95	-190.36	33.38	-1.34	0.18

The light free path of *LAI-2000* measurements and LAD collected data is shown in TABLE 5.3, where zero value means there was no leaf intersecting light on the zenith direction. Besides the average of the five collections can be obtained as 0.63, an adjusted

value of LAD finally calculated as 0.52 according to [157] which concludes that the *LAI-2000* measurement is 18% higher than directly collection from the canopy.

TABLE 5.3: LAD collected from five θ_0 directions.

No.	Light free path (m)					LAD
	7°	23°	38°	53°	68°	
1	3.802	3.307	2.099	0	0	0.72
2	3.324	2.547	2.283	0	0	0.66
3	3.437	2.106	1.539	0	0	0.62
4	3.407	3.305	0	0	0	0.20
5	3.326	2.748	1.601	1.502	0	0.98

5.1.3 3-D L-System Tree Generation

For generating 3-D discrete tree model, L-System is a good application which explains the internal canopy structure in detail [158]. First, the main tree frame (FIGURE 5.7(a)) was generated, and on N1 and N2 node there was one mother branch. The height of N1 was 57.2 cm above ground. The height of N2 was 66.4 cm. On N3 node there were ten mother branches, and the height of N3 was 137.1 cm. On fourth node (N4) there are three mother branches, and the height of N4 was 155.4 cm. The core of the tree model was the recursive son-branches growing function of the L-System algorithm. Dimension of the tree is defined as two to four [159]. On first depth there was one son-branch or there were two son-branches. From the next depth, the number of the son-branches on a node was set from three to five. There were eight to eighteen leaves on a shoot that the number also randomly chosen; the leaf normals were determined randomly according to the cumulative distribution of FIGURE 5.2(a). Leaf radius was random number within range 4.0 to 5.0 cm. FIGURE 5.7(b) and 5.7(c) are indicating the profile and top view of the L-System tree model. The perspective cylinder which containing the tree is the volume we used to calculate the LAD. The LAD was 0.54 in our tree model which indicated the 3-D tree model can be characterized with given parameters collected in the field.

5.2 LAI/LAD and LND Retrieving Using PCD

Laser scanning technique can provide accurate surface echoed coordinates with densely scanned intervals. To utilize the density based statistical algorithm for analyzing the voluminous amount of the 3-D points data is one of the subjects of the laser scanning applications. Computational geometry also provides some mature applications for point cloud data (PCD) processing and analyzing. Author here investigated the feasibility of a new application for retrieving the effective LAI of an isolated broad leaf tree. Simplified curvature was calculated for each point in order to remove those non-photosynthetic tissues. Then PCD were discretized into voxel, and clustered by using Gaussian mixture model. Subsequently the area of each cluster was calculated by employing the computational geometry applications.

5.2.1 PCD Collection and Processing

The sample tree was selected as the same isolated tree as shown in FIGURE 5.1(a). We set six directions around the tree for collecting data. Raw data were assigned to an assumed coordinate system, in which the original point was set to the first station (*ScanPos001* see FIGURE 5.1(b)). Cartesian coordinates (x, y, z) units were in meters (m). Both vertical and horizontal scanning intervals were set to 0.02° . Raw data points amount was over 9.7×10^6 (FIGURE 5.8(a)).

Effective leaf area retrieving algorithm was designed as follows (FIGURE 5.9). Simplified curvature of each point was calculated, see Equation 3.30, and the searching radius (r) was set to 4.0 cm according to the average long axis of the leaves. Points whose $c_q > T_1$ ($T_1 = 0.21$) were taken as D_1 , which were mostly from leaves, and points whose $c_q \leq T_1$ were taken as D_2 (FIGURE 5.8(c)). In D_2 , there were points from leaves as well, which should be restored to D_1 . Hence the D_2 points were detected within a sphere (the radius is $T_2 = 0.01$ m), which centered at each point of D_1 . The point(s) in D_2 located in each sphere were merged to D_1 as leaf data (FIGURE 5.8(d)). After curvature estimation (FIGURE 5.8(b)), 76.13% data were rejected according to the threshold,

many of them were from leaves. Therefore, the author employs the method to restore the excessively removed points. The result of rejected data and leaf data were shown in FIGURE 5.8(c) and 5.8(d), individually. The leaf data accordingly increased to 52.33% and the opposite data were 47.67%. The leaf data were effectively restored. The leaf data were voxelized with each lattice size $8 \times 8 \times 8$ cm (FIGURE 5.11(a)), which can contain the maximum leaf size. The data within each voxel were clustered according to section 3.2.3.2. Those clusters containing less than thirty ($T_3 = 30$) points were rejected due to the possibility of low reliability.

The algorithm of data processing within a voxel is indicated in FIGURE 5.10. The initial condition of the data in the voxel is shown in FIGURE 5.10(a), which is out-of-order arranged. After *mclust* function (see section 3.2.3.2) processing, points are distinguished into four clusters: *c1*, *c2*, *c3* and *c4* (FIGURE 5.10(b)). Picking one (*c3*) of them as an example for retrieving the cluster area, red plane (FIGURE 5.10(c)) is obtained by using PCA function of CGAL (see section 3.2.3.3). First, a convex hull of this cluster is generated by using 3-D convex hull generation function from CGAL, then the vertices of this convex hull are obtained. Vertices are projected on the fitting plane of the cluster and those points within the convex hull are removed. Finally the area of this cluster is calculated by accumulating the triangles which are constructed from each two vertices and the center (red point in FIGURE 5.10(d)).

5.2.2 LND Function Retrieving and Validating

All clusters' orientation normal in each voxel were estimated by using PCA method, then the results were taken as the leaf normal (FIGURE 5.11(b)). FIGURE 5.6 (a) shows comparison between LND function retrieved from PCD and LND function from field collected data. FIGURE 5.6 (b) presents the quantilequantile plot between the measurement data and retrieved data. The mean and standard deviation of collected data were 55.53° and 20.04, individually. The retrieved leaf inclination angle mean and standard deviation were 58.32° and 20.81, respectively. The mean shift is 2.79° , and the standard deviation of the shift is 0.77. Namely, the LNDs obtained by the field

collection and retrieved from PCD were very close to each other. With a correlation coefficient of 0.96, the results indicated a good performance and reliable. The leaf area were calculated for each cluster according to Section 3.2.3.4, and then the accumulated area for entire canopy was calculated. Finally the ELAI was computed according to proposed 6×6 observation directions (see TABLE 5.4).

5.2.3 Canopy ELAI Results and Validation

The volume of sample tree's canopy was calculated as $V = 33.918 \text{ m}^3$ from 3-D convex hull algorithm as well. For the light travel length we assumed the canopy was contained by a sphere, according the maximum and minimum coordinates of the PCD, we calculated the effective LAI by

$$\begin{aligned}
 ELAI &= \frac{D\eta}{V}, \\
 D &= \frac{\Delta x + \Delta y + \Delta z}{3}, \\
 \eta &= \sum_{m=1}^I \sum_{n=1}^J S_{mn}^*, \\
 \Delta x &= x_{\max} - x_{\min}, \\
 \Delta y &= y_{\max} - y_{\min}, \\
 \Delta z &= z_{\max} - z_{\min},
 \end{aligned} \tag{5.2}$$

where D is the average light traveling distance, η is the effective leaf area of the entire canopy, m is the index of the voxel (from 1 to I), n is the index of the cluster within the n th voxel (from 1 to J), S_{mn}^* is the effective area of each cluster within a voxel. Parameters were set to $\Delta x = 4.64 \text{ m}$, $\Delta y = 4.61 \text{ m}$, and $\Delta z = 4.79 \text{ m}$, respectively.

The effective LAI of the tree was calculated from 6×6 assumed observation directions (TABLE 5.4). It was indicated that our calculation can precisely provide effective LAI estimation according to the variation of the observation directions. Because of the lack of the resource we cannot accurately measure the total leaf area of the tree we selected. We assessed the application using an indoor plant (FIGURE 5.12(a)) and marked

down nine well-isolated leaves before this plant was scanned. We manually measured the area of the leaves to compare with the calculation results. A high correlation coefficient up to 98.28% was achieved (FIGURE 5.12(b)). Especially, for viewing zenith θ_0 at 0° , ELAI was the equivalence to the conventional LAI.

TABLE 5.4: The ELAI (m^2/m^2) results which were calculated from 6×6 assumed observation directions.

$\theta_0 \backslash \phi_0$	0°	30°	60°	90°	120°	150°
0°	2.929	2.929	2.929	2.929	2.929	2.929
15°	2.929	2.985	3.013	3.008	2.980	2.966
30°	2.938	3.046	3.093	3.069	2.994	2.980
45°	2.976	3.055	3.102	3.065	2.966	2.971
60°	3.088	3.018	3.041	3.022	2.966	2.994
75°	3.177	2.990	2.938	3.027	3.027	3.083

5.3 Summary of Canopy Structural Parameters Retrieving

Photographing method is convenient and low-cost method in forestry close-range investigation especially dealing with isolated tree. One can use photogrammetric applications directly measure the leaf sample's inclination angle (θ_L), if the azimuth (ϕ_L) is ignored. Time consuming is also a disadvantage of the method when the leaf amount of canopy is huge. However, in terms of high resolution of the digital camera, the application is a solution that one does not have to directly collect leaf samples in the field. Camera needs to be well calibrated and viewing zenith θ_0 showed be recorded. Three directions at least of the photographing are recommended, in terms of a back ground screen for easily extracting the leaves. Good light conditions is necessary in order to well distinguishing the leave and branches especially on the inner space of the canopy. Artificial light source may be helpful when solar light can not reach the target canopy. Fish eye lens camera is also a good candidate if one investigating a wide forest area.

Lindenmayer system (L-system) generated plants are widely used not only in computer graphics art but also in biological and even mathematical investigations of the dimensions of the plants. It is consequently used in RT model for generating the discrete structural plants. L-System can simulate the pattern of the nature plants in a proper way fitting with modern computational algorithms. Without considering the computational resources costs L-System is a good candidate for simulating the growth and the shape of nature plants than the discrete heterogeneous scene canopy model [160].

Curvature is a common parameter during PCD processing in computational geometry. Different from the Gaussian curvature and mean curvature, the simplified curvature we utilized in this study is the approximation of the change of curvature in a neighborhood P_n centered at p_q , and it is invariant under rescaling. It can be computed conveniently as well. Small values indicate that the neighborhood points P_n are on the plane tangent to the surface [146]. This simplified curvature was an effective parameter for identifying whether the point was on trunk or leaf by using a properly defined search radius r . In our study we defined the searching radius as the average long axis of the leaves. However, we did not discuss how the searching radius affects the curvature. Different searching radius settings would definitely change the point normal result, which was determined by the surface complexity of the object. Smaller searching radius corresponded higher curvature fluctuation, and bigger searching radius causes that the distinction of leaf points and non-photosynthetic tissue points was not obvious. A future study for investigating the optimized searching radius setting method needs to be investigated so that to develop this application into an unsupervised function.

The clustering strategy in the study was to first discrete the PCD into voxel matrices, for reducing the computational complexity of the PCD. The voxel size was set according to the maximum single-leaf size observed in the canopy. Other applications for determining the voxel size should be considered in line with different leaf patterns. We then clustered the points within each voxel using GMM method. Estivill-Castro points out that one cannot precisely define clusters, so the clustering results almost certainly contain errors [161]. They show, in some cases, weak correlations and may generate

small amount points clusters. Therefore we removed those clusters which held less than thirty points. It caused the loss of leaf area we retrieved from PCD, but the reducing area of thirty points is very small according to the scanning resolution in our case. However, when the scanning interval and range are big the reducing of the leaf points need to be cautiously considered. Our validation results showed a correlation coefficient of 98.28%, indicating that the indoor experiment presented high accuracy. Yet with the condition of wild plant and forest, the application should be demonstrated in several conditions. According to the pattern of the canopy in this study, which was approximately taken as a sphere, therefore the light travel distance was accordingly a constant in this case, for different conditions the light travel distance should be calculated individually.

Our study utilized dense PCD with millimeter-level accuracy, allowing reliable object spatial information. There were at least three directional collections under similar conditions so that the whole canopy can be well and completely scanned, with minimal disturbance from wind and beam absorption by the canopy. The detection distance was not suggested to be large. PCD needed to be calibrated with georeferences for validation, and a local GPS coordinate system should be used to measure angles. For precision of the TLS, our results can be taken as a validation of optical sensors. Branch filtering can be considered by determining whether the object surface was closed like a branch or open like a leaf.

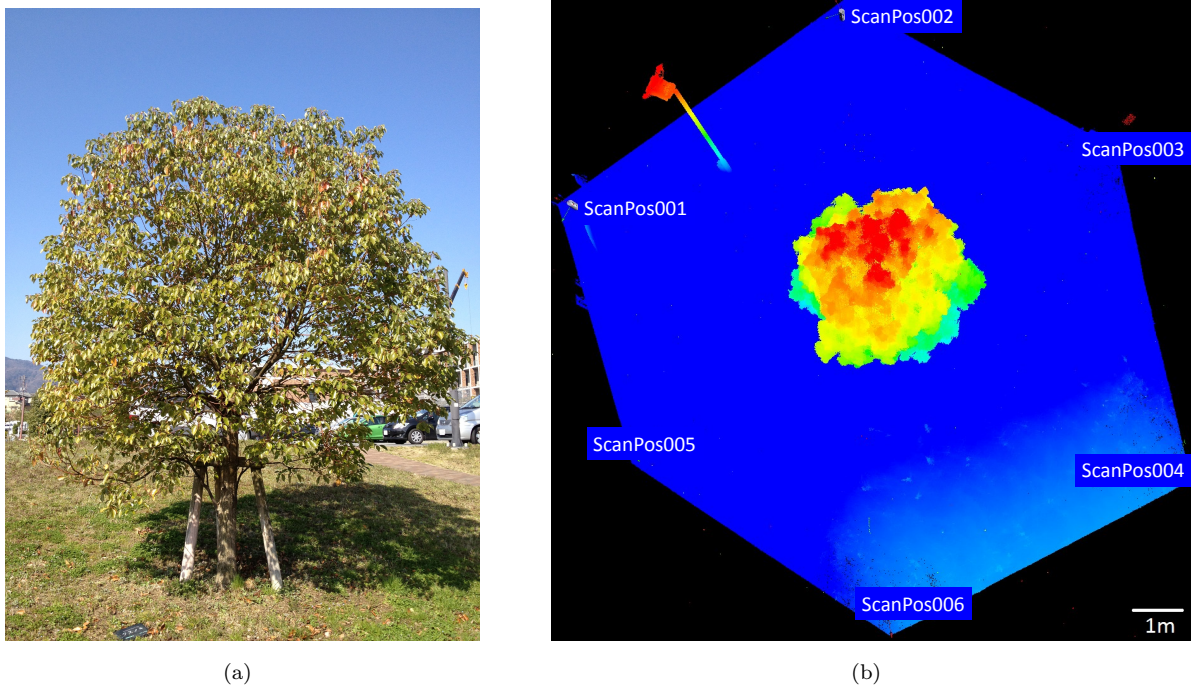


FIGURE 5.1: *Cinnamomum Camphora* tree and stations of PCD collection. (a) *Cinnamomum Camphora* tree used in this study ($N34^{\circ}58'55.61''$, $E135^{\circ}40'43.54''$) and (b) six stations for laser scanning, *ScanPos001* is taken as assumed original point of the coordinate system we used.

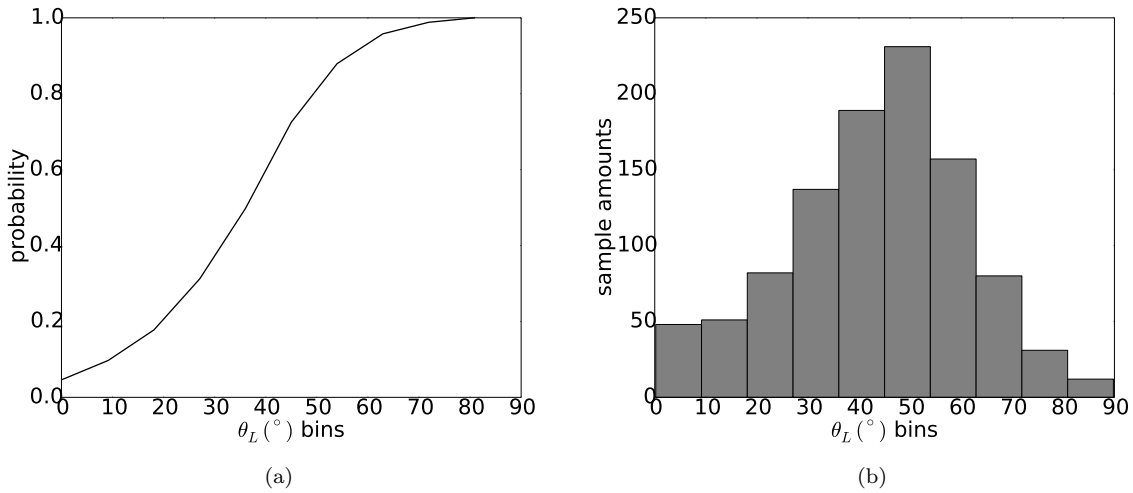


FIGURE 5.2: Leaf inclination angles cumulative distribution and histogram. (a) leaf inclination angles cumulative distribution, and (b) histogram of leaf inclination angles of the whole canopy.

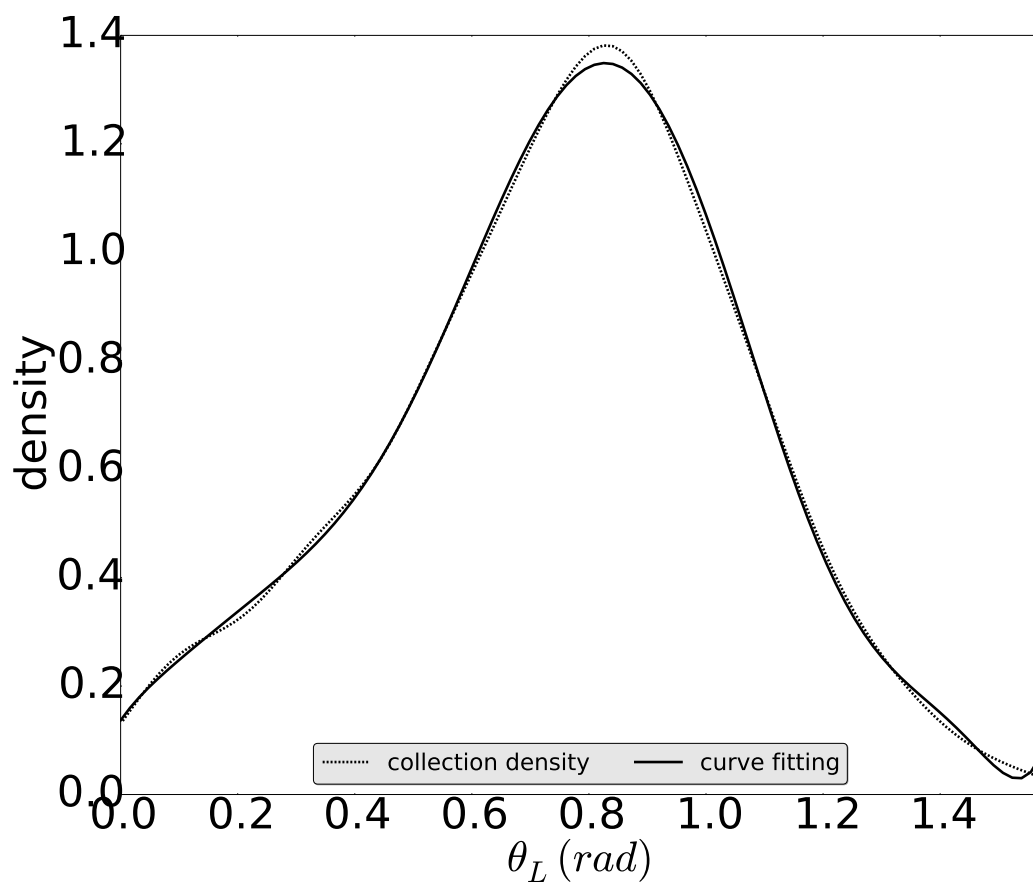
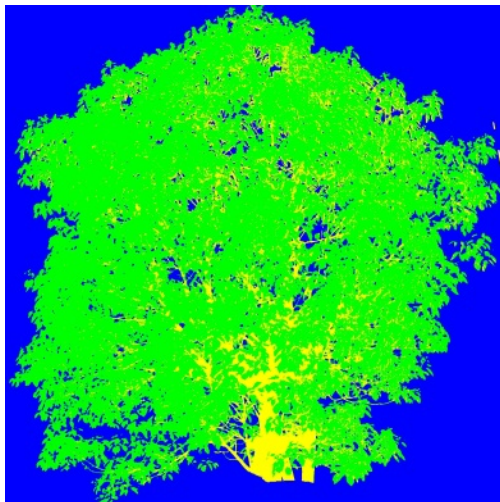
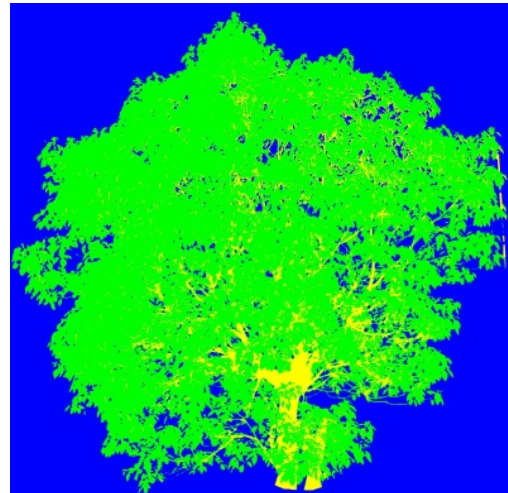


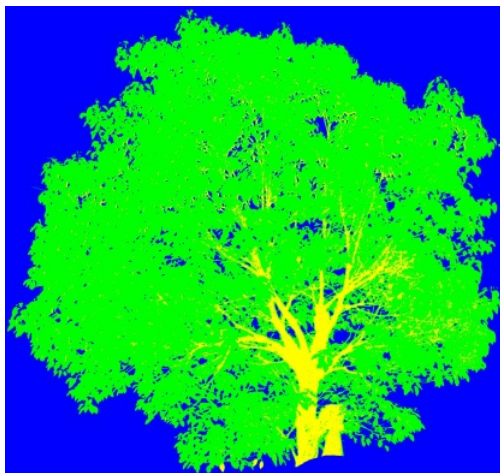
FIGURE 5.3: Density plots (blue dash line) of the collected raw data of the leaf normals, where a polynomial function (Equation 5.1) of leaf normal distribution function is generated based on the fitted (green solid) line.



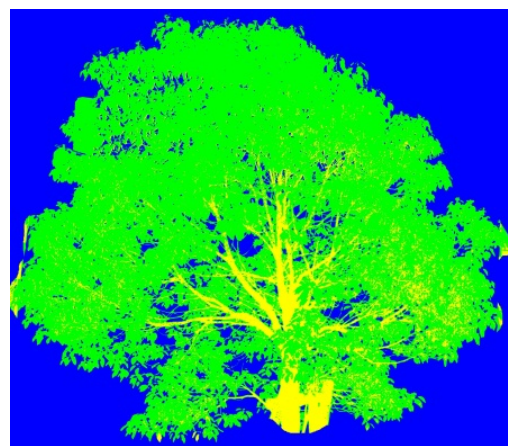
(a)



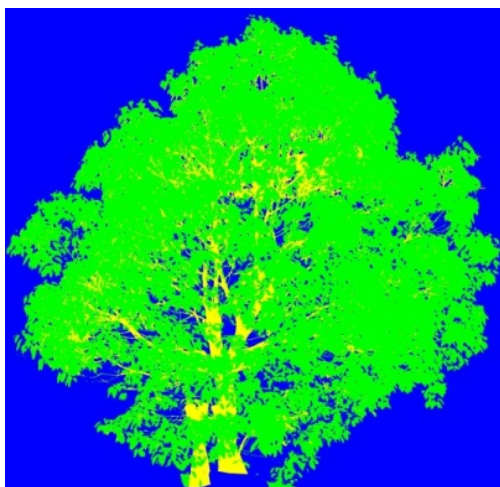
(b)



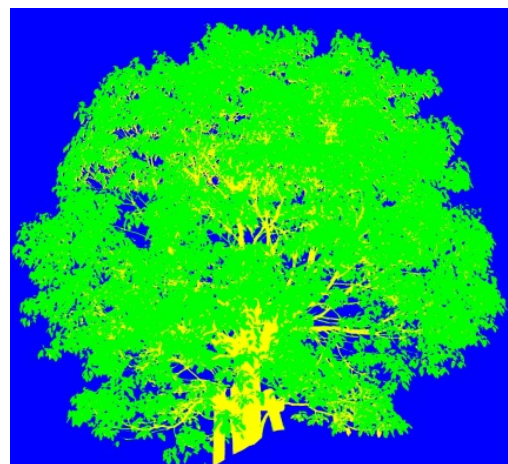
(c)



(d)



(e)



(f)

FIGURE 5.4: Classification examples for gap fraction extraction which are obtained from the photographs taken surrounding the tree. Green colored pixels are the leaves, yellow pixels are wooden structures, blue parts are the rest pixels of the photographs.

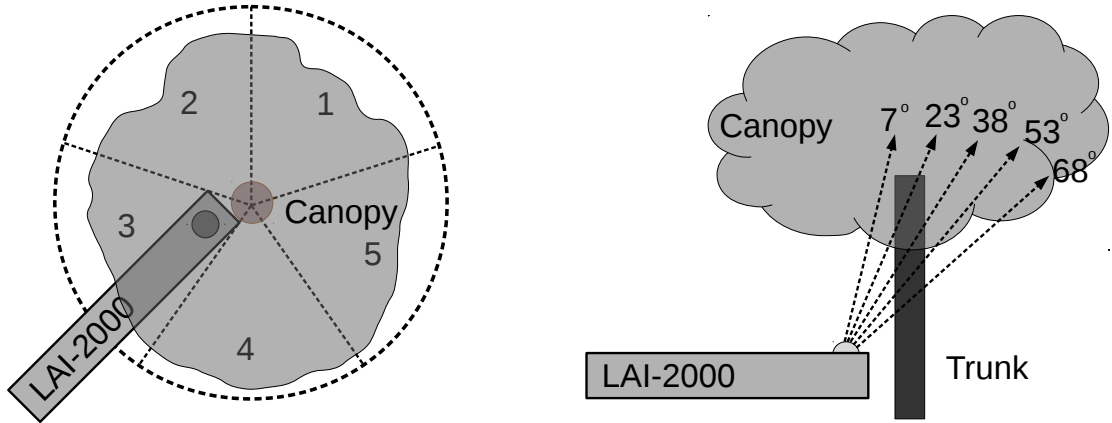


FIGURE 5.5: Sketch of *LAI-2000* observation from the bottom of the canopy on five directions. Each measurement samples five zenith angles ($\theta_0 = \{7^\circ, 23^\circ, 38^\circ, 53^\circ, 68^\circ\}$) from the fish-eye lens.

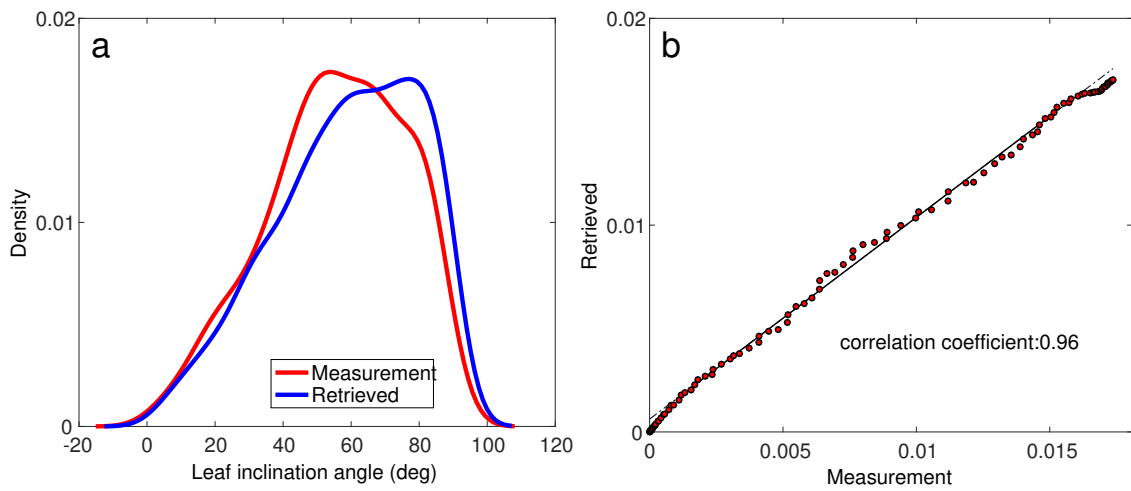
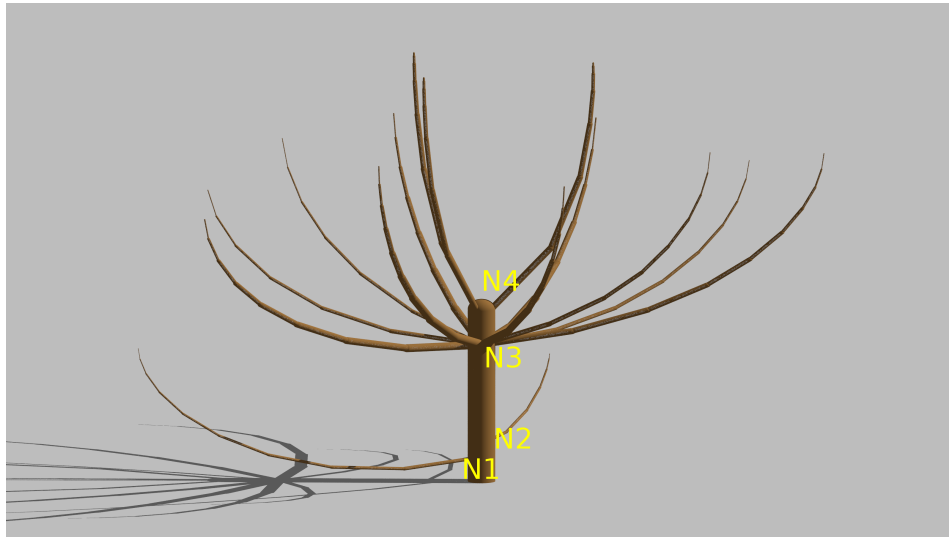
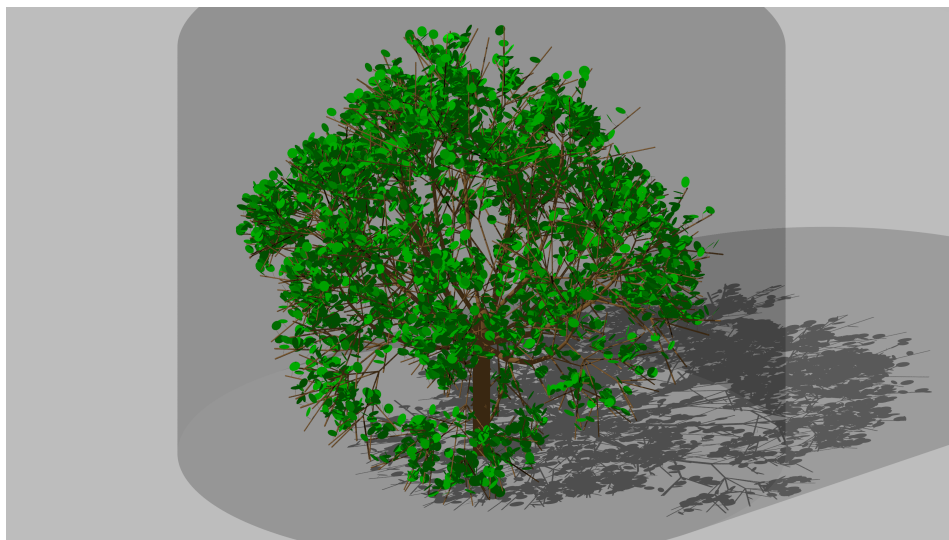


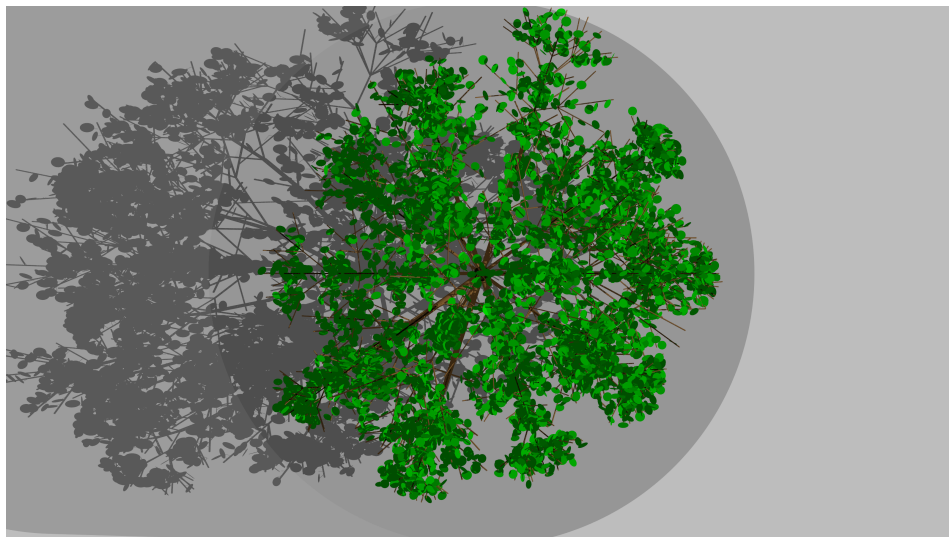
FIGURE 5.6: LND function comparison and validation. (a) LND function comparison between field measurements and PCA method retrieved results, and (b) quantile-quantile plot of measurements and retrieved results, which holds a correlation coefficient of 0.96.



(a)



(b)



(c)

FIGURE 5.7: Tree model generated by L-System. (a) Tree frame, (b) profile view of the tree model, and (c) top view of the tree model.

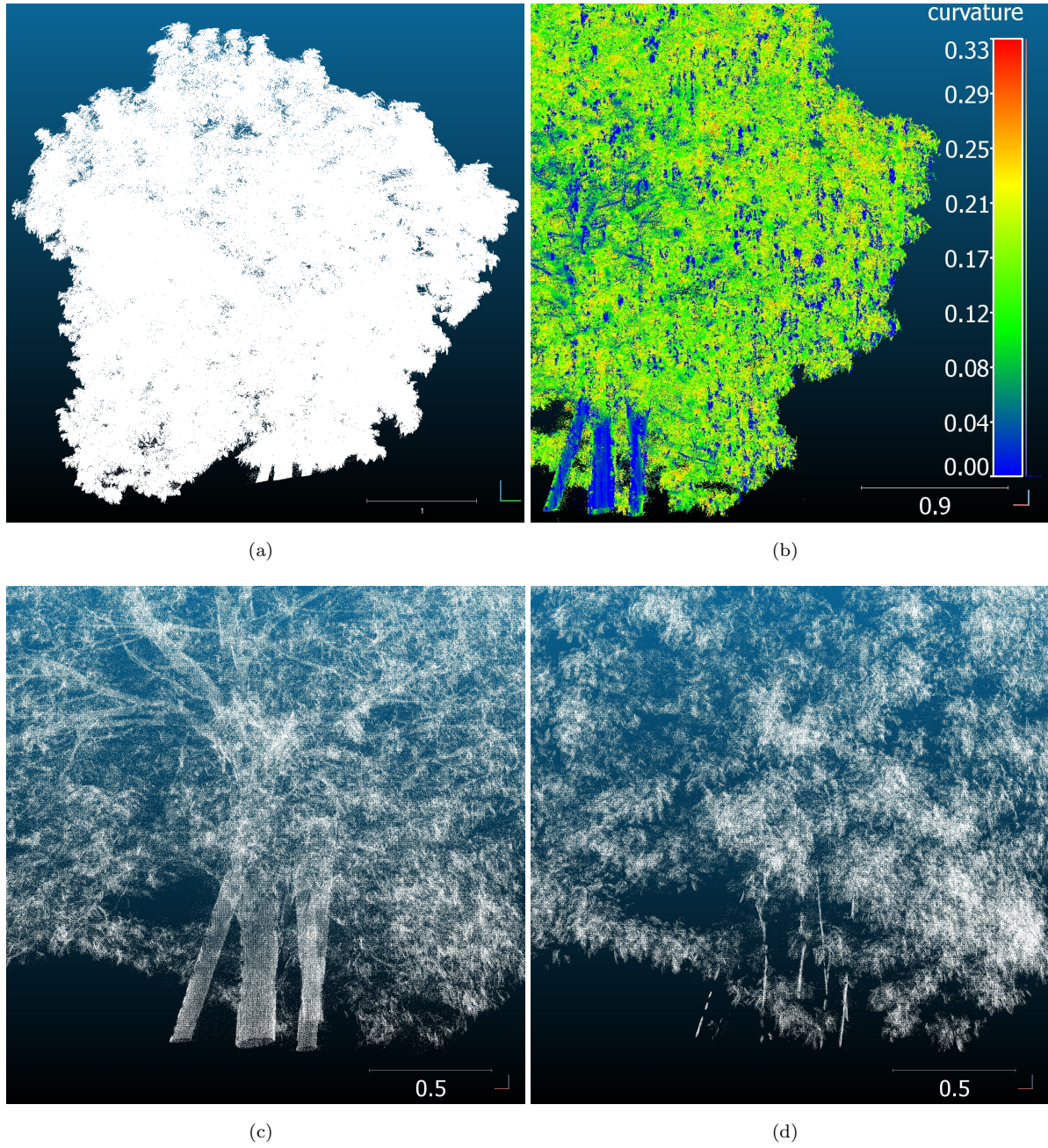


FIGURE 5.8: PCD of the sample tree with curvature estimation. (a) Raw point cloud data of the tree with scale bar 1.0 m, (b) curvature estimation of the raw data, scale bar shows in 0.9 m, color varies according to the curvature, which ranged from 0.00 to 0.33, and (c) Data (D_2) contain branches after curvature filtering. The scale bar shows in 0.5 m. (d) Leaf data after merging leaf points from D_2 and data D_1 ; the scale bar shows in 0.5 m (see section 5.2.1).

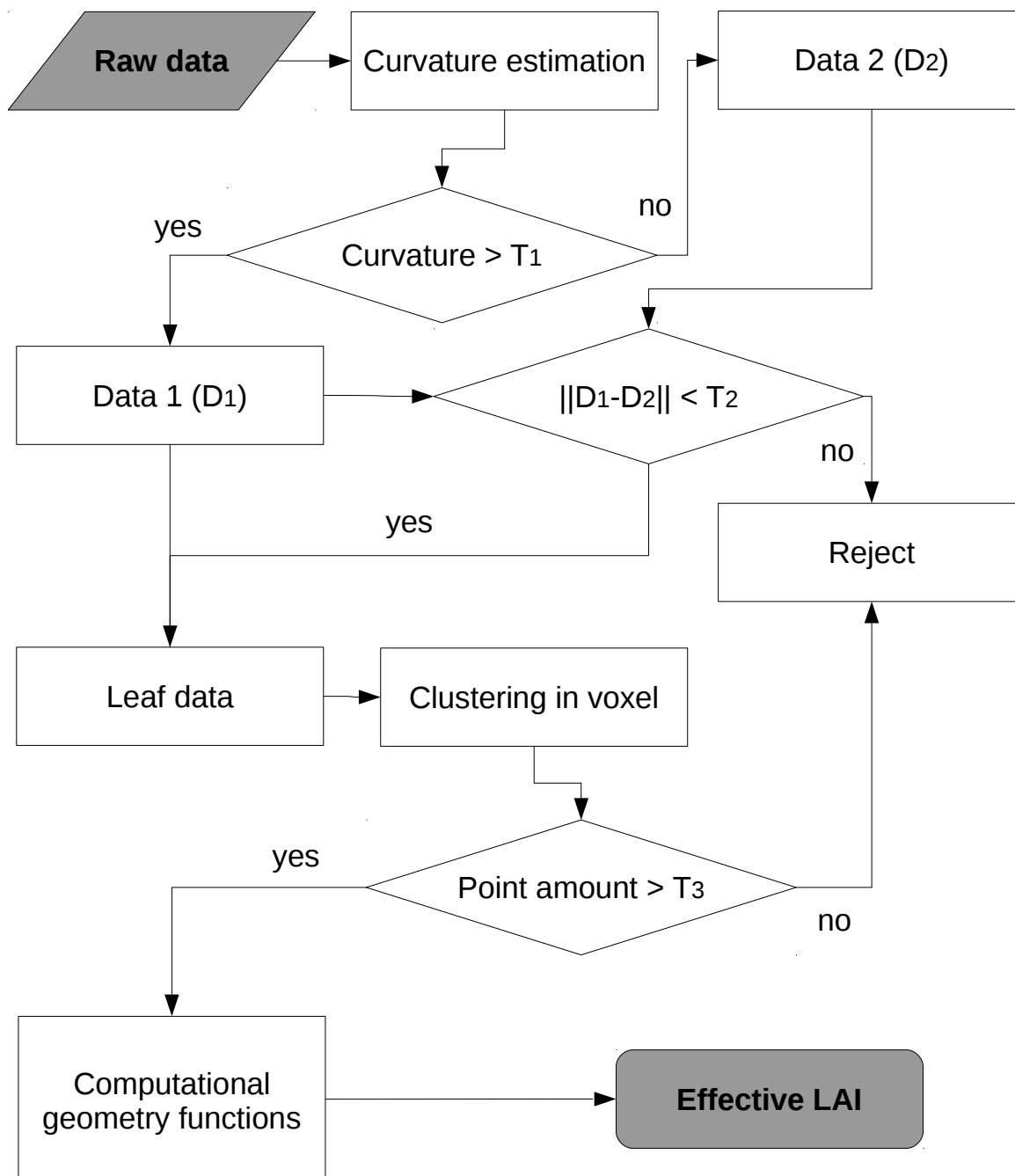


FIGURE 5.9: Data processing algorithm of the effective LAI retrieving from PCD.

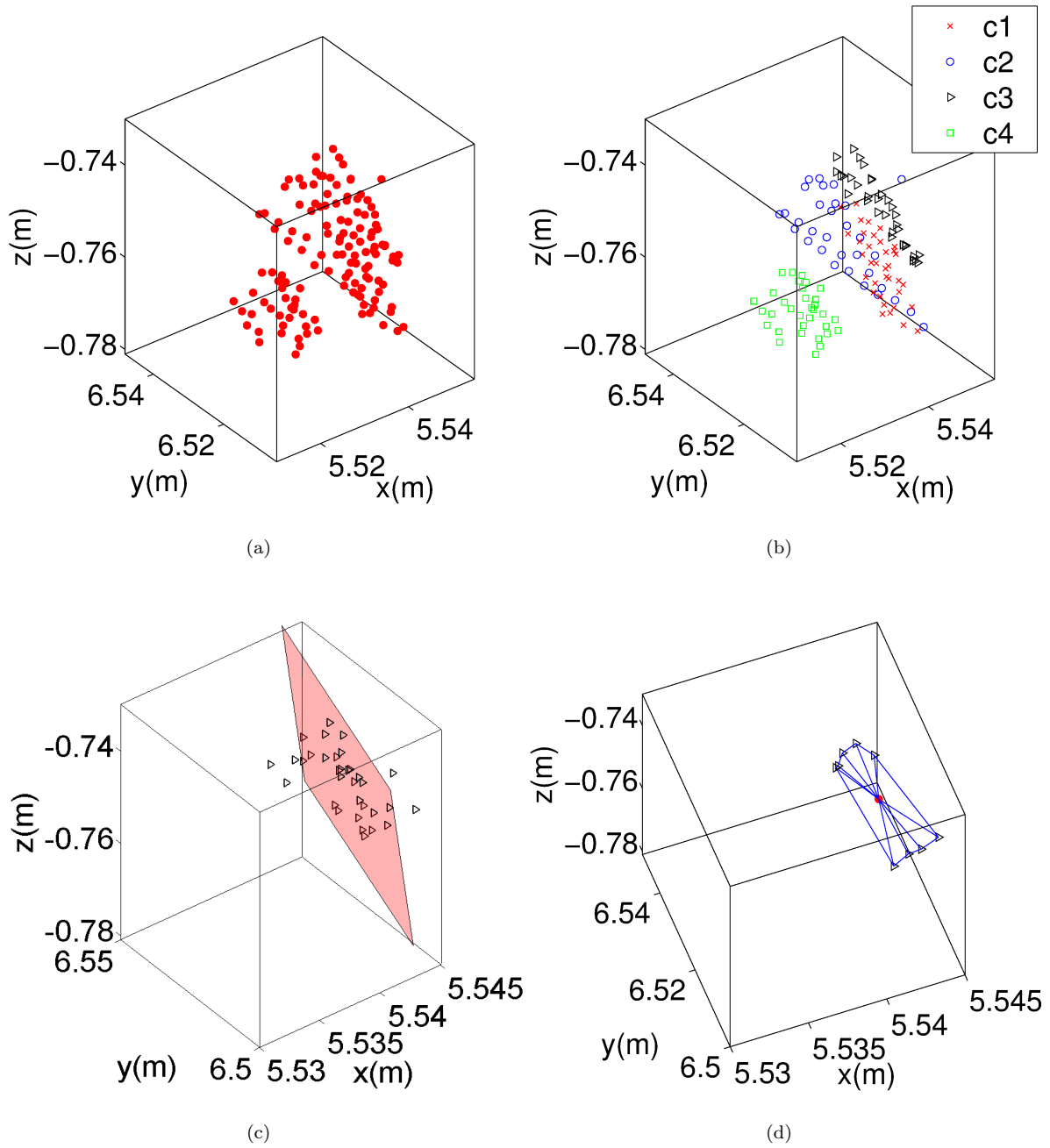


FIGURE 5.10: Leaf area estimation within a voxel. (a) Point data with in a voxel, (b) after *mclust* clustering, the data were indicated into four subsets ($c1$, $c2$, $c3$, $c4$) each symbol indicates one cluster, (c) one cluster ($c3$) from (b), red plane is the fitting plane retrieved from PCA algorithm, and (d) vertices of the cluster are obtained by convex hull generation, then project these vertices on PCA fitting plane. Finally the area of this cluster is calculated by accumulating the triangles which construct from each two projected vertices and the center.

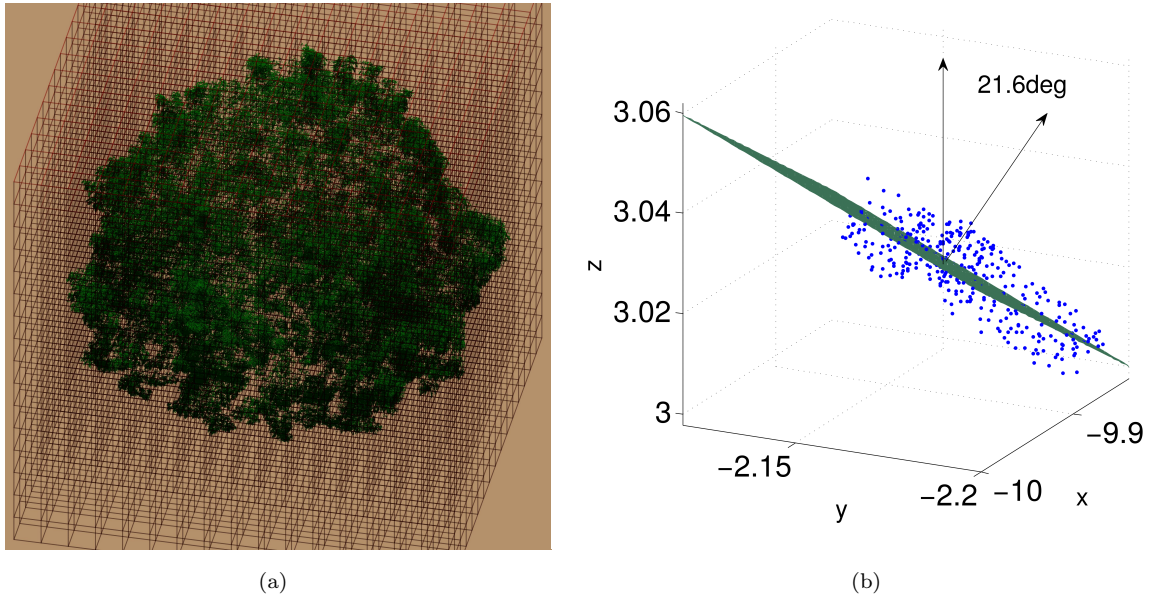


FIGURE 5.11: Voxelization and PCA estimation. (a) Voxelization scheme which divides the PCD into $8 \times 8 \times 8$ cm voxel matrix, and (b) retrieve the leaf points cluster normal by PCA method which shows the leaf normal zenith angle is 21.6° by using Equation 3.32.

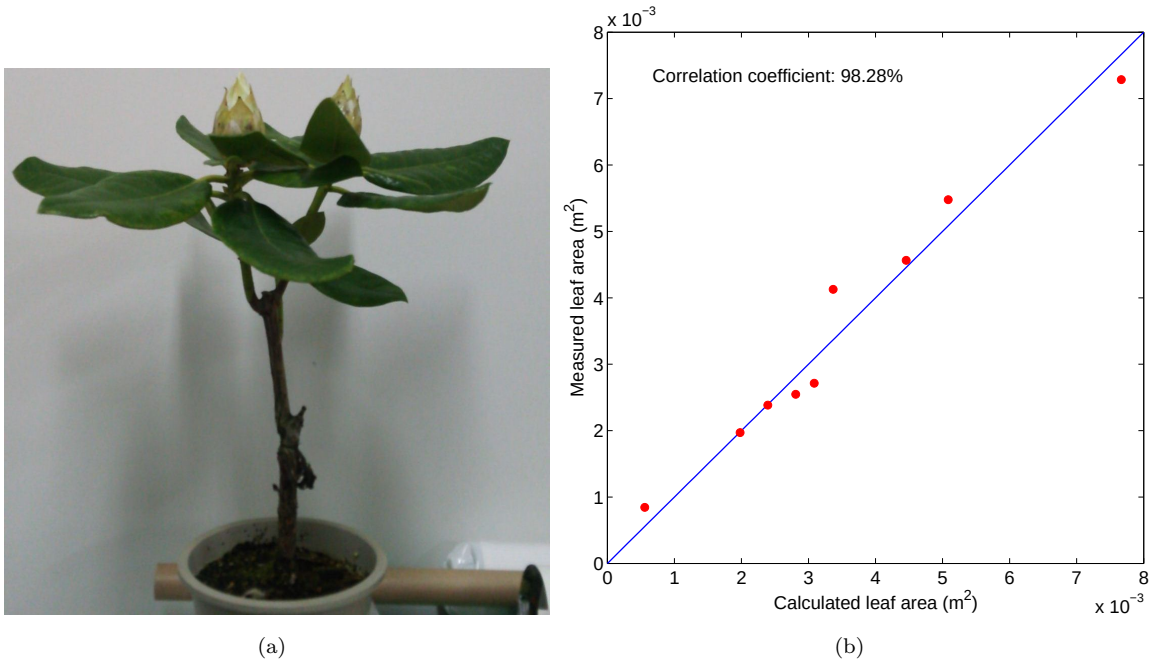


FIGURE 5.12: Indoor Experiment for testing PCD application. (a) The picture of the indoor plant for validation, and (b) comparison between measurements and calculated each leaf area of the plant, which have a correlation coefficient of 98.28%.

Chapter 6

Conclusions

A 3-D vector MCRT model, TRCMCRTS, for complex topographic relief forest bidirectional reflectance assessment was developed. Preliminary results indicated that the model provided similar BRF results in the red and infrared bands as the FLiES model. The paradigm presented responses according to the changing topographic reliefs reasonably. In order to estimate an open forest BRF of a unknown canopy structural parameters, applications were developed by using both close-range photograph and PCD. Good correlation between field collected and the retrieved canopy structural parameters was obtained. In order to provide an overall conclusion of the study, the author presents summaries of each chapter.

6.1 Summary

In the first chapter, the author drew out the necessity of the vegetation remote sensing from the illustration of the issue of global warming. British Petroleum's Statistical Review of World Energy (2007) for oil and gas, and the History Database of the Environment, a publication of The Netherlands, for coal showing that there is a positive correlation between the increasing of the global temperature growing and fossil fuels usage. Which indicated the critical evidence of the global warming caused by greenhouse gases. To overcome the issue, vegetation is a key factor of the natural greenhouse gases

consumer. Therefore, the monitoring and assessing of the vegetation on the Earth have to be achieved effectively and comprehensively by RS technology. To deeply understand the RS data from remote sensor, a mathematical and physical based model is important and effective way. In spite of traditional numerical method though holds the advantages of the RT modeling, ill-posed integral-differential equation issues still difficult to solve. The Monte Carlo method therefore becomes a expectable candidate to solve the problems.

Comprehensive literature reviews were presented in this chapter. Main methods of the MCRT simulation of the vegetation were listed. Advantages and issues need to be improved of the studies were illustrated. The micro-topographic reliefs under the vegetation cover does not discussed by these models. Besides of the MCRT modeling, the author also investigated monographs of the canopy structural parameters retrieving from close-range detected optical and laser scanned PCD. This is a key step of the application of the RT models. The author found that the use of Gaussian mixture model to clustering the PCD within voxel matrix is a novel approach, which provides high accuracy and precession.

The fundamental of MCRT modeling and canopy parameters retrieving of the study was represented in this chapter. The algorithm of the Monte Carlo ray-tracing model was introduced, in order to accelerating the computational speed, numerical methods of local estimation was employed. For solving the micro-relief issue of the ray-tracing, the topography conditions were approximated by the bilinear interpolation of the DEM. Local ground normal vector, which is the key of the ground ray-tracing algorithm, were utilized in this model. Both optical and TSL sensor based applications were investigated for retrieving the structural parameters of the tree canopy.

Chapter 4 is the description of the TRCMCRTS model. Here, the author provided the entire introduction of the definition of the model structure and gave several examples to show the performance of the model. The main components of the model including the scattering events definitions, effective photon sampling during the ray-tracing and the 3-D scene discretizing. In order to testify the model, several executions with different

parameter settings were given. For generating an green coverage defined random forest, the author also provided an algorithm based on 2-D circle packing program. The areal albedo of the open forest, if the canopy are with the same leaf angle distribution, does not change much for the same area green coverage. In another word, the area albedo does not sensitive to the spatial distribution of the individual canopy. Evaluation of the reliability of TRCMCRTS was by comparing the BRF result with FLiES. Since the topographic conditions is not supported by the FLiES, the validation of TRCMCRTS with topography conditions can not be assessed. However, the comparison between TRCMCRTS and FLiES with flat ground shows the ray-tracing algorithm developed in this study can estimate forest BRF with reliable results. In order to evaluate the responsibility of the TRCMCRTS performance with different input ground conditions, the author tried six different land relief settings; the sensitivities of the TRCMCRTS to the change of the ground relief was also considered, therefore, the DEM reliefs did not turbulent very much. In other word, if the TRCMCRTS response to such small variation of the ground reliefs, it can be considered as a micro-topographic oriented simulator, and of course can estimate the conditions with relatively large scale change like slope, valley etc. Results showed that it was reliable and correlated that the BRFs various with the micro-topographic settings.

In order to fix the issues of difficulties for determining canopy structural parameters like LAI, LAD and leaf angle distribution functions, the author tried to explore the approaches in Chapter 5. Ground based close-range sensor data were used for retrieving the canopy parameters. Besides the parameters retrieving, an L-system algorithm based tree model was developed as well. Leaf angle distribution of the L-system tree model was highly correlated with the field investigation. The L-system can be employed for the further discrete canopy modeling, so that the tree model was prepared as the advanced canopy to the MCRT model which can approximate the real trees with the conditions of user defined leaf angle distribution function. The PCD retrieving method is another new challenging application of the study. The trunk and branch with geometrical cylindrical patten structures of the tree were removed from the raw PCD by using the curvature estimation. Then, by using the Gaussian mixture model based *R* package *mclust*, the

clusters within a voxel were mathematically identified, which means the area of each single leaf can be accurately calculated. Finally, the leaf inclination angle was estimated by using PCA for each cluster in the voxel. This algorithm holds a potential to many fields like precision agriculture, prescriptive planting etc.

6.2 Considerations and Future Plans

Canopy in this study was assumed as the hybrid model like the turbid medium which is contained by geometric solid. 3-D spatial distribution of the trees was expressed by this assumption in an forest area, however, it was still different from real trees. For the coming version of the TRCMCRTS, the author plans to build up L-system based tree model to approximate the real single tree. It is acknowledged that such kind of operation will be a challenge to the model and hardware both, but is a target that geology model to overcome. The topographic approximation in this research is mainly based on bilinear interpolation of DEM, and the local normal of the point which photon hitting on the ground is based on Equations 3.12 and 3.13, here the neighboring points along x and y axes are determined by a constant distance to the ground point e.g. 10 cm. It is necessary to discuss further to find an optimized distance. If the local normal, with in a pixel, can be represented by the center point's, that will be greatly reduce the computational complexity. Further validation of the TRCMCRTS in the field is planned. The model could be improved by including atmospheric and polarization modules. Computational accelerating approaches, for instance the parallel computing methods like OpenMP and GPU-accelerated computing, are considered to be employed for decreasing the computational time.

The canopy structural parameter retrieving from terrestrial and areal sensed data is an important topic which can reduce labor and cost. In the research, a GMM based clustering method combined with the curvature filtering application was developed for accurately estimating leaf angle distribution and effective LAI and also the individual canopy LAD estimation. In this study the voxel size of the PCD was set to the average long axis of leaf, which was considered as reasonable but not further discussed if it is

possible to the bigger voxel size. Voxel size will directly affect the GMM clustering, which means the computational time consuming. If an optimized voxel size can be found mathematically, this approach is more acceptable. Furthermore, the method dealing with needle which might be a different approach was not discovered in this study, the author will contentiously investigate on it.

Appendix A

Mathematical Foundations

A.1 Gaussian Mixture Model

A.1.1 Introduction of GMM

For the massive quality of PCD, it is unwise to use supervised application to determine the number of classes within a space. That means classification method is not utilized in this study. Another option is clustering algorithm, which is the unsupervised application. K-Means clustering is a method originally from signal processing, which attempts partition n observations into K clusters [162]. One drawback of K-Means clustering is the amount of cluster should be defined before processing, this drives the method is difficult to utilize in PCD clustering. Because PCD of canopy is divided into a vast number of small voxels, which means it is impossible to define the number of cluster voxel by voxel. Actually, K-Means algorithm and its associated expectation-maximization algorithm is a particular case of a Gaussian mixture model, specifically, the limit of taking all covariances as diagonal, equal, and small. Generally it is easy to generalize a K-Means issue into a Gaussian mixture model [163]. According to Reynolds (2009), a

Gaussian mixture model is a weighted sum of M component Gaussian densities as given by the equation,

$$P(\mathbf{x}|\boldsymbol{\beta}) = \sum_{i=1}^M w_i \mathcal{G}(\mathbf{x}|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i), \quad (\text{A.1})$$

where $\mathbf{x}$ is a D -dimensional continuous-valued data vector, w_i , $i = 1, \dots, M$, are the mixture weights, and $\mathcal{G}(\mathbf{x}|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, $i = 1, \dots, M$, are the component Gaussian densities. Each of the density is a D -variate Gaussian function of the form,

$$\mathcal{G}(\mathbf{x}|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i) = \frac{1}{\sqrt{(2\pi)^D \det(\boldsymbol{\Sigma}_i)}} \exp \left\{ -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} (\mathbf{x} - \boldsymbol{\mu}_i) \right\}, \quad (\text{A.2})$$

with mean vector $\boldsymbol{\mu}_i$ and covariance matrix $\boldsymbol{\Sigma}_i$. Weights satisfy the condition $\sum_{i=1}^M w_i = 1$. The parameter $\boldsymbol{\beta}$ in Equation A.1 is represented by

$$\boldsymbol{\beta} = \{w_i, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i\}, \quad i = 1, \dots, M. \quad (\text{A.3})$$

A.1.2 Expectation Maximum (EM) Algorithm

The coming issue is how to determine weight w_i and cluster number M . The motive of maximum likelihood estimation is for a subset of the data $X = \mathbf{x}_1, \dots, \mathbf{x}_T$, the GMM likelihood, assuming independence of $\mathbf{x}_i$, can be expressed like

$$P(X|\boldsymbol{\beta}) = \prod_{t=1}^T P(\mathbf{x}_t|\boldsymbol{\beta}). \quad (\text{A.4})$$

However, this maximum likelihood can not be estimate directly. Therefore we can iteratively using a special case of the EM algorithm to estimate the maximum likelihood [165]. The basic meaning of EM algorithm is, beginning with an initial parameter $\boldsymbol{\beta}$, to estimate a new $\boldsymbol{\beta}'$, such that $P(X|\boldsymbol{\beta}') \geq P(X|\boldsymbol{\beta})$. The new $\boldsymbol{\beta}'$ therefore becomes the

initial of the next iteration until the user defined convergence threshold is reached. On each step of iteration, the following variable will be re-calculated,

$$w'_i = \frac{1}{T} \sum_{t=1}^T P^*(i|\mathbf{x}_t, \boldsymbol{\beta}), \quad (\text{A.5a})$$

$$\boldsymbol{\mu}'_i = \frac{\sum_{t=1}^T P^*(i|\mathbf{x}_t, \boldsymbol{\beta}) \mathbf{x}_t}{\sum_{t=1}^T P^*(i|\mathbf{x}_t, \boldsymbol{\beta})}, \quad (\text{A.5b})$$

$$\sigma'^2_i = \frac{\sum_{t=1}^T P^*(i|\mathbf{x}_t, \boldsymbol{\beta}) x_t^2}{\sum_{t=1}^T P^*(i|\mathbf{x}_t, \boldsymbol{\beta})} - \mu'^2_i. \quad (\text{A.5c})$$

where σ_i^2 , x_t , and μ_i refer to arbitrary elements of the clusters $\boldsymbol{\sigma}_i^2$, $\mathbf{x}_t$, and $\boldsymbol{\mu}_i$, respectively. The posteriori probability $P^*(i|\mathbf{x}_t, \boldsymbol{\beta})$ of the i th component is given by

$$P^*(i|\mathbf{x}_t, \boldsymbol{\beta}) = \frac{w_i \mathcal{G}(\mathbf{x}_t | \mu_i, \Sigma_i)}{\sum_{k=1}^M w_k \mathcal{G}(\mathbf{x}_t | \mu_k, \Sigma_k)}. \quad (\text{A.6})$$

A.1.3 Bayesian Information Criterion (BIC)

To set the convergence threshold, the BIC was selected in this study. Schwarz (1978) derived BIC to serve as an asymptotic approximation to a transformation of the Bayesian posterior probability of a candidate model. The computation of BIC is based on the empirical log-likelihood and does not require the specification of priors. According to Bayes theorem,

$$p(\Xi_i | y_1, \dots, y_n) = \frac{p(y_1, \dots, y_n | \Xi_i) p(\Xi_i)}{p(y_1, \dots, y_n)}, \quad (\text{A.7})$$

where $p(y_1, \dots, y_n | \Xi_i)$ is called the marginal likelihood of the cluster Ξ_i . Here we assume the maximum of the posterior probability of a cluster is the same as the maximum of

the marginal likelihood

$$\begin{aligned} p(y_1, \dots, y_n | \Xi_i) &= \int_{\beta} L(\beta_i | y_1, \dots, y_n) g_i(\beta_i) d\beta_i \\ &= \int_{\beta} D(y_1, \dots, y_n | \beta_i) g_i(\beta_i) d\beta_i, \end{aligned} \tag{A.8}$$

where β_i is defined in Equation A.3 as the parameter of the cluster Ξ_i , $L(\beta_i | y_1, \dots, y_n)$ is the likelihood function, $D(\mathbf{y} | \beta_i)$ is density of $\mathbf{y} = \{y_1, \dots, y_n\}$, and $g_i(\beta_i)$ is the probability distribution function according to β_i . For the full derivation of the BIC, Bhat and Kumar (2010) gave a detailed expression. Therefore, BIC could be expressed

$$\log p(\mathbf{y} | \Xi_i) = \log L(\beta_i | \mathbf{y}) - \frac{M}{2} \log n. \tag{A.9}$$

The right-hand side of Equation A.9 is the BIC estimation for a cluster Ξ_i . n is the number of the sample. For large n , BIC performs good clustering estimations.

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