

Stationary solutions of a hydrodynamic model for semiconductors

Dedicated to professor Toshitaka Nagai on the occasion of his sixties birthday

By

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Abstract

The present paper concerns the existence and the asymptotic stability of a stationary solution to a hydrodynamic model for semiconductors. Moreover, we prove the non-existence of the stationary solution. Precisely, the existence and the stability are discussed under the assumption that the boundary voltage is sufficiently small. On the other hand, unless this assumption, we are able to construct an example which does not admit the stationary solution in classical sense.

§ 1. Hydrodynamic model

To analyze the flow of electrons in semiconductor devices, several kinds of models are proposed. Especially, a hydrodynamic model, which is derived by Bløtekjær [2], is often used in the numerical device simulation and attracts interests of not only engineers but also mathematicians. The present paper concerns the model and study the existence and the asymptotic stability of a stationary solution as well as the non-existence in classical sense.

For the readers' reference, we refer several mathematical results on this model. The books [7, 13] give general introduction of semiconductor physics and discuss the derivation of the model. Degond and Markowich in [3] establish the unique existence of

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the stationary solution over the one-dimensional bounded domain. Precisely, they show that for given electric current, there exists a certain value of boundary voltage such that the stationary solution exists. The engineering experiments, however, aim to measure electric current for given voltage on the boundary. Therefore, it is desirable to show the existence of the stationary solution for the given voltage. This problem has been solved in the authors' previous paper [14]. Namely, it proves the unique existence of the stationary solution for the given voltage as far as it is sufficiently small. This result is also discussed in Section 2 with details. On the other hand, due to the hyperbolic property of the model, we cannot expect the existence of the stationary solution in classical sense for the large voltage. In fact, we give an example in Section 3. This example shows the non-existence of the stationary solution. Here we still have possibilities that the model admits the stationary solution in weak sense. This problem has been considered in several papers [1, 4, 9] under settings other than the present paper. The existence of the weak stationary solution, which satisfies the equation in distribution sense, is shown for the large voltage by Gamba [4]. Ascher, Markowich, Pietra and Schmeiser in [1] and Rosini in [9] construct the piece wise smooth stationary solution.

Lastly, we mention several results on the stability analysis on the stationary solution. Li, Markowich and Mei in [8] show the stability over the one-dimensional bounded domain under the assumption that the doping profile is flat. This assumption is, however, too strict to cover the real devices. In fact, the doping profile of $n^+ - n - n^+$ diode does not satisfy this assumption (see [5]). Guo and Strauss in [6] extend the result in [8] to cover the non-flat doping profile. In these researches, it is assumed that the electric current in the stationary solution is sufficiently small, although this fact should be derived from the smallness of voltage from physical point of view. In fact, the authors in [14] estimate the current with respect to the voltage and show the stability theorem under the smallness hypotheses on the voltage. This result is briefly discussed in Section 2. Also see [10, 17, 19].

The hydrodynamic model is given by the system of three equations

$$\begin{aligned} (1.1a) \quad & \rho_t + (\rho u)_x = 0, \\ (1.1b) \quad & j_t + (\rho u^2 + p(\rho))_x = \rho \phi_x - \rho u, \\ (1.1c) \quad & \phi_{xx} = \rho - D, \end{aligned}$$

where $x \in \Omega := (0, 1)$ is a spatial variable and $t > 0$ is a time variable. The unknown functions ρ , u and ϕ stand for electron density, electron velocity and electrostatic potential, respectively. Here the product $j := \rho u$ means electric current. As we study isothermal and/or isentropic flow, the pressure p is a function of electron density ρ :

$$(1.2) \quad p = p(\rho) = \rho^\gamma,$$

where the constant γ is supposed to satisfy $\gamma \geq 1$. The doping profile D is a distribution of density of positively ionized impurities in semiconductor devices, which is not an unknown function but a given one. We assume it satisfies $D \in \mathcal{B}^0(\overline{\Omega})$ and

$$(1.3) \quad 0 < c \leq D(x)$$

for some $c > 0$. The initial and the boundary data to (1.1) are imposed as

$$(1.4) \quad (\rho, u)(0, x) = (\rho_0, u_0)(x),$$

$$(1.5) \quad \rho(t, 0) = \rho_l > 0, \quad \rho(t, 1) = \rho_r > 0,$$

$$(1.6) \quad \phi(t, 0) = 0, \quad \phi(t, 1) = \phi_r > 0,$$

where ρ_l , ρ_r and ϕ_r are positive constants. Here the explicit formula of the potential is obtained by solving (1.1c) with (1.6) as

$$(1.7) \quad \begin{aligned} \phi(t, x) &= \Phi[\rho](t, x) \\ &:= \int_0^x \int_0^y (\rho - D)(t, z) dz dy + \left(\phi_r - \int_0^1 \int_0^y (\rho - D)(t, z) dz dy \right) x. \end{aligned}$$

This initial boundary value problem is considered in the region where the subsonic condition (1.8a) and positivity of density (1.8b) hold. Namely,

$$(1.8a) \quad \inf_{x \in \Omega} (p'(\rho) - u^2) > 0,$$

$$(1.8b) \quad \inf_{x \in \Omega} \rho > 0.$$

Precisely, assuming the initial data satisfies the conditions

$$(1.9) \quad \inf_{x \in \Omega} (p'(\rho_0) - u_0^2)(x) > 0, \quad \inf_{x \in \Omega} \rho_0(x) > 0,$$

then we shall construct the solution satisfying (1.8). Note that the subsonic condition is equivalent to that a characteristic speed λ_1 of the hyperbolic equations (1.1a), (1.1b) is negative and another characteristic λ_2 is positive, that is,

$$(1.10) \quad \lambda_1 := u - \sqrt{p'(\rho)} < 0, \quad \lambda_2 := u + \sqrt{p'(\rho)} > 0.$$

Hence, the subsonic condition means that two boundary conditions (1.5), (1.6) are sufficient and necessary for the well-posedness of this initial boundary value problem.

In Section 2, we introduce the unique existence and the asymptotic stability of the stationary solution $(\tilde{\rho}, \tilde{u}, \tilde{\phi})$, which are proved in [14]. Here the stationary solution is a solution to (1.1), independent of time t , belonging to the function space $C(\overline{\Omega}) \cap C^2(\Omega)$:

$$(1.11a) \quad (\tilde{\rho}\tilde{u})_x = 0,$$

$$(1.11b) \quad (\tilde{\rho}\tilde{u}^2 + p(\tilde{\rho}))_x = \tilde{\rho}\tilde{\phi}_x - \tilde{\rho}\tilde{u},$$

$$(1.11c) \quad \tilde{\phi}_{xx} = \tilde{\rho} - D$$

with the boundary condition

$$(1.12) \quad \tilde{\rho}(0) = \rho_l > 0, \quad \tilde{\rho}(1) = \rho_r > 0,$$

$$(1.13) \quad \tilde{\phi}(0) = 0, \quad \tilde{\phi}(1) = \phi_r > 0.$$

To study its asymptotic stability, it is necessary, from the above observation on the characteristics, that the stationary solution satisfies (1.8a). In the proof, the strength of the boundary data

$$(1.14) \quad \delta := |\rho_r - \rho_l| + |\phi_r|$$

plays an essential role.

On the other hand, we do not expect, from the hyperbolicity of the hydrodynamic model, that the classical stationary solution exists for the large voltage ϕ_r . One of the main purposes of the present paper is to give an example of the non-existence. It is discussed in Section 3.

Notation. For a nonnegative integer $l \geq 0$, $H^l(\Omega)$ denotes the l -th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_l$. We note $H^0 = L^2$ and $\|\cdot\| := \|\cdot\|_0$. $C^k([0, T]; H^l(\Omega))$ denotes the space of the k -times continuously differentiable functions on the interval $[0, T]$ with values in $H^l(\Omega)$. Moreover $\mathfrak{X}$ denotes the function spaces

$$\mathfrak{X}_i^j([0, T]) := \bigcap_{k=0}^i C^k([0, T]; H^{j+i-k}(\Omega)), \quad \mathfrak{X}_i([0, T]) := \mathfrak{X}_i^0([0, T])$$

for nonnegative integers $i, j \geq 0$. For a nonnegative integer $k \geq 0$, $\mathcal{B}^k(\overline{\Omega})$ denotes the space of the functions whose derivatives up to k -th order are continuous and bounded over $\overline{\Omega}$, equipped with the norm $|\cdot|_i$.

§ 2. Existence and stability of stationary solution

This section is devoted to summarizing the result in [14], that is, the existence and the stability of stationary solution $(\tilde{\rho}, \tilde{u}, \tilde{\phi})$ to the problem (1.11)–(1.13). As we only give outline of the proofs, the readers are referred to [14] for the details.

The existence of the stationary solution satisfying the conditions (1.8) is stated in

Lemma 2.1. *Let the doping profile and the boundary conditions satisfy (1.3), (1.5) and (1.6). For an arbitrary ρ_l , there exists a constant δ_0 such that if $\delta \leq \delta_0$, then stationary problem (1.11)–(1.13) has a unique solution $(\tilde{\rho}, \tilde{u}, \tilde{\phi}) \in \mathcal{B}^2(\Omega)$ satisfying (1.8).*

Proof. The existence of the stationary solution is shown by the Schauder fixed-point theorem. On the other hand, the uniqueness is proved by the maximum principle. For the details, see the authors' paper [14]. $\square$

The above theorem asserts the stationary solution exists as far as δ is sufficiently small. On the contrary, without this assumption, we can construct the example of non-existence. It is discussed in Section 3.

The asymptotic stability of the stationary solution is summarized in

Theorem 2.2. *Let $(\tilde{\rho}, \tilde{u}, \tilde{\phi})$ be stationary solution of (1.11)–(1.13). Suppose the initial data $(\rho_0, u_0) \in H^2(\Omega)$ and the boundary data ρ_l , ρ_r and ϕ_r satisfy (1.5), (1.6) and (1.9). Assume the compatibility condition*

$$\rho_0(0) = \rho_l, \quad \rho_0(1) = \rho_r, \quad (\rho_0 u_0)_x(0) = (\rho_0 u_0)_x(1) = 0$$

holds. Then there exists a constant δ_0 such that if $\delta + \|(\rho_0 - \tilde{\rho}, u_0 - \tilde{u})\|_2 \leq \delta_0$, the initial boundary value problem (1.1) and (1.4)–(1.6) has a unique solution (ρ, u, ϕ) satisfying $(\rho - \tilde{\rho}, u - \tilde{u}, \phi - \tilde{\phi}) \in \mathfrak{X}_2([0, \infty)) \times \mathfrak{X}_2([0, \infty)) \times \mathfrak{X}_2^2([0, \infty))$. Moreover, it verifies the decay estimate

$$(2.1) \quad \|(\rho - \tilde{\rho}, u - \tilde{u})(t)\|_2 + \|(\phi - \tilde{\phi})(t)\|_4 \leq C \|(\rho_0 - \tilde{\rho}, u_0 - \tilde{u})\|_2 e^{-\alpha t},$$

where C and α are certain positive constants, independent of time t .

Proof. We establish the unique existence of the time local solution by using an iteration method similarly as in [11, 12]. In order to construct the time global solution, it is sufficient to derive an a-priori estimate

$$(2.2) \quad \|(\psi, \eta)(t)\|_2^2 + \|\omega(t)\|_4^2 + \int_0^t \|(\psi, \eta)(\tau)\|_2^2 + \|\omega(\tau)\|_4^2 d\tau \leq C \|(\psi_0, \eta_0)\|_2^2,$$

$$\psi := \rho - \tilde{\rho}, \quad \eta := u - \tilde{u}, \quad \omega := \phi - \tilde{\phi}.$$

Here we give a brief sketch of the derivation of (2.2). To this end, we employ an energy form

$$(2.3) \quad \mathcal{E} := \frac{1}{2} \rho (u - \tilde{u})^2 + \Psi(\rho, \tilde{\rho}) + \frac{1}{2} \left\{ (\phi - \tilde{\phi})_x \right\}^2,$$

$$\Psi(\rho, \tilde{\rho}) := \int_{\tilde{\rho}}^{\rho} h(\xi) - h(\tilde{\rho}) d\xi, \quad h(\xi) := \int_1^{\xi} \frac{p'(\zeta)}{\zeta} d\zeta.$$

It follows from the equations (1.1) and (1.11) that the energy form (2.3) verifies

$$(2.4) \quad \mathcal{E}_t + \tilde{\rho} \eta^2 = R_{1x} + R_2,$$

$$R_1 := \omega \omega_{xt} + \omega(\rho \phi - \tilde{\rho} \tilde{\phi}) - \{h(\rho) - h(\tilde{\rho})\}(\rho u - \tilde{\rho} \tilde{u}) + \{h(\rho) - h(\tilde{\rho})\} \psi \tilde{u},$$

$$R_2 := - \left\{ \frac{1}{2} (u^2 - \tilde{u}^2)(\rho u - \tilde{\rho} \tilde{u}) \right\}_x - \psi \eta u + (\rho u - \tilde{\rho} \tilde{u})_x \eta \tilde{u}$$

$$+ \left\{ \frac{1}{2} (u^2 - \tilde{u}^2)_x - \omega_x + \eta \right\} \psi \tilde{u} - \{h(\rho) - h(\tilde{\rho})\} (\psi \tilde{u})_x.$$

Integrating (2.4) yields the basic estimate. Moreover, we derive the higher order estimates by applying the energy method to the system of the equations for the perturbation (ψ, η, ω) . Then the a-priori estimate (2.2) is obtained by combining the basic and the higher order estimates. Hence, the continuation argument combining the time local existence and the a-priori estimate yields the existence of the time global solution in Theorem 2.2. Lastly, applying the Gronwall inequality, we have the decay estimate (2.1). $\square$

§ 3. Non-existence of stationary solution

In this section, it is shown that the boundary value problem (1.11)–(1.13) does not have any stationary solutions $(\tilde{\rho}, \tilde{u}, \tilde{\phi})$ belonging to $C(\overline{\Omega}) \cap C^2(\Omega)$ unless the boundary voltage ϕ_r is sufficiently small. As far as we know, it is the first example of the non-existence. Hereafter we study the isothermal flow, that is,

$$(3.1) \quad \gamma = 1, \quad p(\tilde{\rho}) = \tilde{\rho}.$$

We begin the detailed discussion with proving

Lemma 3.1. *Let $(\tilde{\rho}, \tilde{u}, \tilde{\phi}) \in C(\overline{\Omega}) \cap C^2(\Omega)$ be a classical solution to the boundary value problem (1.11)–(1.13). Then the function $\tilde{\rho}$ satisfies the positivity (1.8b).*

Proof. We show $\tilde{\rho} > 0$ by contradiction. Suppose that there exists a point $x_0 \in [0, 1]$ such that $\tilde{\rho}(x_0) \leq 0$. Let $x_* := \inf\{x > 0; \tilde{\rho}(x) = 0\}$. It is apparent that $0 < x_* < 1$ and $\tilde{\rho}(x_*) = 0$ due to the boundary condition (1.12). Notice that $\tilde{\rho}\tilde{u}(x) = 0$ holds for an arbitrary $x \in [0, 1]$ owing to (1.11a). Then substituting $\tilde{\rho}\tilde{u} = 0$ in (1.11b), dividing the result by $\tilde{\rho}$ and integrating the resulting equation over the domain $(0, x_*)$, we reach a contradiction

$$-\infty = \log \tilde{\rho}(x_*) - \log \rho_l = \tilde{\phi}(x_*).$$

Hence the positivity (1.8b) holds. $\square$

Letting $\tilde{j} := \tilde{\rho}\tilde{u}$, we rewrite the system (1.11) with (3.1) as

$$(3.2a) \quad \tilde{j}_x = 0,$$

$$(3.2b) \quad \left(1 - \frac{\tilde{j}^2}{\tilde{\rho}^2}\right) \tilde{\rho}_x = \tilde{\rho}\tilde{\phi}_x - \tilde{j},$$

$$(3.2c) \quad \tilde{\phi}_{xx} = \tilde{\rho} - D.$$

Once we show the non-existence of the classical solution to (1.12), (1.13) and (3.2), the non-existence of the classical solution to (1.11)–(1.13) immediately follows.

Divide the equation (3.2b) by $\tilde{\rho}$ and differentiate the result to get

$$(3.3) \quad \left\{ \left(\frac{1}{\tilde{\rho}} - \frac{\tilde{j}^2}{\tilde{\rho}^3} \right) \tilde{\rho}_x + \frac{\tilde{j}}{\tilde{\rho}} \right\}_x = \tilde{\rho} - D,$$

where we have also used (3.2c).

In Section 2, we have shown the existence of the stationary solution under assuming the smallness $\delta = |\rho_l - \rho_r| + |\phi_r| \ll 1$. Without this assumption, the stationary solution does not exist in general. In the next theorem showing non-existence, we do not suppose ϕ_r is so small. Precisely, we take $\phi_r = 1$ and $\rho_l = \rho_r$.

Theorem 3.2. *Let doping profile D be a constant. Assume that $\rho_l = \rho_r \neq D$ and $\phi_r = 1$. Then the boundary value problem (1.12), (1.13) and (3.2) does not admit any classical solutions $(\tilde{\rho}, \tilde{j}, \tilde{\phi}) \in C(\bar{\Omega}) \cap C^2(\Omega)$.*

Proof. We show, by contradiction, the case $D < \rho_l$ only since another case is proved more easily. Suppose that the problem (1.12), (1.13) and (3.2) has a classical solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$. Dividing the equation (3.2b) by $\tilde{\rho}$ and integrating the result over the domain $(0, 1)$, we have

$$(3.4) \quad \int_0^1 \frac{\tilde{j}}{\tilde{\rho}} dx = 1$$

as $\tilde{j}$ is a constant. It together with the mean value theorem implies that there exists a certain point x_0 such that $\tilde{j}/\tilde{\rho}(x_0) = 1$. Notice that $\tilde{\rho}$ is not constant. In fact, if so, (3.3) means $\tilde{\rho} = D$, which violates the assumption $\rho_l = \rho_r \neq D$. Hence, we can find certain points $x_1, x_2 \in (0, 1)$ such that

$$(3.5) \quad \frac{\tilde{j}}{\tilde{\rho}}(x_1) < 1, \quad \frac{\tilde{j}}{\tilde{\rho}}(x_2) > 1.$$

These inequalities mean the classical solution $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ has to traverse from the subsonic region to the supersonic region.

Now we claim that $\tilde{\rho}$ attains its maximum at the boundary $x = 0, 1$. In fact, if $\tilde{\rho}$ attains the maximum at a certain point $y_1 \in (0, 1)$, it follows that

$$\tilde{\rho}(y_1) > \rho_l > D, \quad \tilde{\rho}_x(y_1) = 0, \quad \tilde{\rho}_{xx}(y_1) \leq 0, \quad \left(1 - \frac{\tilde{j}^2}{\tilde{\rho}^2} \right)(y_1) > 0.$$

Evaluating (3.3) at $x = y_1$ and substituting the above inequalities yield

$$(3.6) \quad 0 \geq \left\{ \left(\frac{1}{\tilde{\rho}} - \frac{\tilde{j}^2}{\tilde{\rho}^3} \right) \tilde{\rho}_x + \frac{\tilde{j}}{\tilde{\rho}} \right\}_x (y_1) = (\tilde{\rho} - D)(y_1) > 0,$$

which is a contradiction.

As $\tilde{\rho}$ takes the maximum on the boundary, we see that $\tilde{j}/\tilde{\rho}(x) > \tilde{j}/\rho_l$ holds for an arbitrary $x \in (0, 1)$. Hence, if $\tilde{j}/\rho_l \geq 1$, then $\tilde{j}/\tilde{\rho}(x) > 1$ for an arbitrary $x \in (0, 1)$. It apparently contradicts (3.5). Consequently, we have shown

$$(3.7) \quad \tilde{j} < \rho_l.$$

Owing to (3.7) together with (3.4), we can find certain points $z_1, z_2 \in (0, 1)$ such that $z_1 < z_2$,

$$(3.8) \quad \frac{\tilde{j}}{\tilde{\rho}}(z_1) = \frac{\tilde{j}}{\tilde{\rho}}(z_2) = 1$$

and $\tilde{j}/\tilde{\rho}(z) > 1$ holds for an arbitrary $z \in (z_1, z_2)$ since the solution traverses from the subsonic region to the supersonic region.

Hereafter, we divide the proof into the two cases, $\tilde{j} \leq D$ and $D < \tilde{j}$.

Case: $\tilde{j} \leq D$. Evaluate (3.2b) at $x = z_1$ and $x = z_2$, take a difference between these two results and then use (3.8) to get

$$0 = \tilde{\phi}_x(z_1) - \tilde{\phi}_x(z_2) = \int_{z_2}^{z_1} \tilde{\rho} - D \, dx < 0,$$

where we have also used the formula (1.7), and $\tilde{\rho} < \tilde{j} \leq D$ in deriving the last inequality. It is a contradiction.

Case: $D < \tilde{j}$. We define, for a constant $\tilde{j}$, the Lyapunov function

$$(3.9) \quad L(\tilde{\rho}, \tilde{\phi}_x) := \frac{1}{2} \left(\tilde{\phi}_x - \frac{\tilde{j}}{\tilde{\rho}} \right)^2 + \frac{(D - \tilde{\rho})^2 \tilde{j}^2}{2\tilde{\rho}^2 D} - \tilde{\rho} + D \log \tilde{\rho} + D - D \log D,$$

which is a slight modification of the Lyapunov function in [9] (also see [1]). By differentiating $L(\tilde{\rho}(x), \tilde{\phi}_x(x))$ with respect to x , we have

$$\frac{dL}{dx}(\tilde{\rho}(x), \tilde{\phi}_x(x)) = \frac{\tilde{j}(\tilde{\rho}_x)^2}{\tilde{\rho}^3} \left(1 - \frac{\tilde{j}^2}{\tilde{\rho}^2} \right) (x) < 0$$

for an arbitrary $x \in (z_1, z_2)$. Then integration of this inequality over the interval (z_1, z_2) yields a contradiction,

$$0 = L(\tilde{j}, 1) - L(\tilde{j}, 1) = L(\tilde{\rho}(z_2), \tilde{\phi}_x(z_2)) - L(\tilde{\rho}(z_1), \tilde{\phi}_x(z_1)) < 0.$$

Here we have also used the identity $\phi_x(z_1) = \phi_x(z_2) = 1$, which follows from (3.2b) together with (3.8).

Consequently, the boundary value problem (1.12), (1.13) and (3.2) does not admit any classical solutions $(\tilde{\rho}, \tilde{j}, \tilde{\phi})$ in $C(\overline{\Omega}) \cap C^2(\Omega)$. $\square$

Remark. Note that the function L , defined in (3.9), is non-negative in the supersonic region where $\tilde{j}^2/\tilde{\rho}^2 > 1$ holds. Moreover $L(\tilde{\rho}, \tilde{\phi}_x) = 0$ if and only if $(\tilde{\rho}, \tilde{\phi}_x) = (D, \tilde{j}/D)$.

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