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ON THE COEFFICIENTS OF THE RIEMANN MAPPING FUNCTION FOR THE EXTERIOR OF THE MANDELBROT SET

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ABSTRACT. We consider the family of rational maps of the complex plane given by $P_{d,c}(z):=z^{d}+c$ where $c\in \mathbb{C}$ is a parameter and $d\in \mathbb{N}\setminus\{1\}$. The generalized Mandelbrot set is the set of all $c\in \mathbb{C}$ such that the forward orbit of 0 under $P_{d,c}$ is bounded. Let f_{d} : $\mathbb{D}\rightarrow \mathbb{C}\setminus\{1/z:z\in \mathcal{M}_{d}\}$ and Ψ_{d} : $\hat{\mathbb{C}}\setminus\overline{\mathbb{D}}\rightarrow\hat{\mathbb{C}}\setminus \mathcal{M}_{d}$ be the Riemann mapping functions and let their expansions be $f_{d}(z)=z+\sum_{m=2}^{\infty}a_{d,m}z^{m}$ and $\Psi_{d}(z)=z+\sum_{m=0}^{\infty}b_{d,m}z^{-m}$, respectively. We investigate several properties of the coefficients $a_{d,m}$ and $b_{d,m}$. In this paper, we concentrate on the zero coefficients of f_{d}. Detailed statements and proofs will be presented in [13].

1. INTRODUCTION

Let $\mathbb{D}$ be the open unit disk, $\mathbb{D}^{*}$ the exterior of the closed unit disk, $\mathbb{C}$ the complex plane and $\hat{\mathbb{C}}$ the Riemann sphere. Furthermore let $G\subsetneq \mathbb{C}$ be a simply connected domain with $0\in G$ and $G'\subsetneq\hat{\mathbb{C}}$ be a simply connected domain with $\infty\in G'$ which has more than one boundary point. In particular, there exist unique conformal mappings f : $\mathbb{D}\rightarrow G$ such that $f(0)=0$, $f'(0)>0$ and g : $\mathbb{D}^{*}\rightarrow G'$ with $g(\infty)=\infty$, $\lim_{z\rightarrow\infty}g(z)/z>0$. We call f and g the normalized Riemann mapping function of G and G'.

Let $c\in \mathbb{C}$, $n\in \mathbb{N}\cup\{0\}$ and $P_{c}(z):=z^{2}+c$. We denote the n-th iteration of P_{c} by $P_{c}^{\circ n}$ which is defined inductively by $P_{c}^{\circ n+1}=P_{c}\circ P_{c}^{\circ n}$ with $P_{c}^{\circ 0}(z)=z$. For each fixed c, the *filled-in Julia set* of $P_{c}(z)$ consists of those values z, which remain bounded under iteration. The boundary of the filled-in Julia set is called the *Julia set*. The *Mandelbrot set* $\mathcal{M}$ is the set of all parameters $c\in \mathbb{C}$ for which the Julia set of $P_{c}(z)$ is connected. It is known that $\mathcal{M}=\{c\in \mathbb{C}:\{P_{c}^{\circ n}(0)\}_{n=0}^{\infty}$ is bounded$\}$ is compact and is contained in the closed disk of radius 2 with center 0. Furthermore, $\mathcal{M}$ is connected. We want to note, that there is an important conjecture which states that $\mathcal{M}$ is locally connected (see [2]).

Douady and Hubbard demonstrated the connectedness of the Mandelbrot set by constructing a conformal isomorphism Φ : $\hat{\mathbb{C}}\setminus \mathcal{M}\rightarrow \mathbb{D}^{*}$. If the inverse map $\Phi^{-1}(z)=:\Psi(z)=z+\sum_{m=0}^{\infty}b_{d,m}z^{-m}$ extends continuously to the unit circle, then the Mandelbrot set is locally connected, according to Carathéodory's continuity theorem. This is a motivation of our study.

Jungreis presented an method to compute the coefficients b_{m} of $\Psi(z)$ in [7]. Several detailed studies of b_{m} are given in [1, 3, 4, 9]. An analysis of the dynamics of $P_{d,c}(z):=z^{d}+c$ with an integer $d\geq 2$ is presented in [15]. The generalized Mandelbrot set is defined as $\mathcal{M}_{d}:=\{c\in \mathbb{C}:\{P_{d,c}^{\circ n}(0)\}_{n=0}^{\infty}$ is bounded $\}$, which is the connected locus of the Julia set of $P_{d,c}$ (see [10]). $\mathcal{M}_{d}$ is also connected, compact and contained in the closed disk of radius $2^{1/(d-1)}$ (see [8, 15]). Constructing the normalized Riemann mapping function $\Psi_{d}(z)=z+\sum_{m=0}^{\infty}b_{d,m}z^{-m}$ of $\hat{\mathbb{C}}\setminus \mathcal{M}_{d}$, Yamashita [15] analyzed the coefficients $b_{d,m}$.

In addition, Ewing and Schober studied the coefficients a_m of the Taylor series expansion of the function $f(z) := 1/\Psi(1/z)$ at the origin in [5]. The function f is the normalized Riemann mapping function of the exterior of the reciprocal of the Mandelbrot set $\mathcal{R} := \{1/z : z \in \mathcal{M}\}$. If f has a continuous extension to the boundary, the Mandelbrot set is locally connected.

In [14], we investigated properties of the coefficients $a_{d,m}$ of the normalized Riemann mapping function $f_d(z) = z + \sum_{m=2}^{\infty} a_{d,m} z^m$ for the exterior of the reciprocal of the generalized Mandelbrot set $\mathcal{R}_d := \{1/z : z \in \mathcal{M}_d\}$ and $b_{d,m}$. In this paper, we present several properties of $a_{d,m}$. In particular, we concentrate on the zero-coefficients.

2. Computation of the Coefficients $b_{d,m}$ and $a_{d,m}$

In this section, we present a method how to compute the coefficients $a_{d,m}$ and $b_{d,m}$ with $d \geq 2$. First we recall the construction of the inverse map of the normalized Riemann mapping function of $\widehat{\mathbb{C}} \setminus \mathcal{M}_d$ (see [1, 2, 7, 15]).

Theorem 1. *The map $\Phi_d : \widehat{\mathbb{C}} \setminus \mathcal{M}_d \to \mathbb{D}^*$ defined as*

$$\Phi_d(z) := z \prod_{k=1}^{\infty} \left(1 + \frac{z}{P_{d,z}^{\circ k-1}(z)^d}\right)^{\frac{1}{d^k}}$$

is a conformal isomorphism which satisfies $\Phi_d(z)/z \to 1 (z \to \infty)$.

We set $\Psi_d := \Phi_d^{-1}$ which is the normalized Riemann mapping function of $\widehat{\mathbb{C}} \setminus \mathcal{M}_d$. It follows immediately that $f_d(z) := 1/\Psi_d(1/z)$ is the normalized Riemann mapping function of $\mathbb{C} \setminus \mathcal{R}_d$. $\Psi_d(z)$ has the following property.

Proposition 2. *Let $n \in \mathbb{N} \cup \{0\}$ and $A_{d,n}(c) := P_{d,c}^{\circ n}(c)$. Then*

$$A_{d,n}(\Psi_d(z)) = z^{d^n} + O(1/z^{d^{n+1}-d^n-1}) \text{ as } z \to \infty.$$

This proposition leads to the next method, given by Jungreis in [7], to compute $b_{d,m}$.

Let $j \in \mathbb{N}$ be fixed. Assume that the values of $b_{d,0}, b_{d,1}, \ldots, b_{d,j-1}$ are known. Set $\hat{\Psi}_d(z) := z + \sum_{i=0}^{j} b_{d,i} z^{-i}$. Take $n \in \mathbb{N}$ large enough such that $j \leq d^{n+1} - 3$ is satisfied. Considering the definition of $A_{d,m}$ and the multinominal theorem, we obtain

$$\begin{aligned} A_{d,n}(\hat{\Psi}_d(z)) &= z^{d^n} + (d^n b_{d,0} + C) z^{d^n-1} \\ &\quad + \sum_{i=1}^{j} (d^n b_{d,i} + q_{d,n,i-1}(b_{d,0}, b_{d,1}, \ldots, b_{d,i-1})) z^{d^n-i-1} + O(z^{d^n-j-2}) \end{aligned}$$

as $z \to \infty$, where C is a constant, and $q_{d,n,i-1}(b_{d,1}, b_{d,2}, \cdots, b_{d,i-1})$ is a polynomial of $b_{d,1}, b_{d,2}, \cdots, b_{d,i-1}$ which has integer coefficients. According to Proposition 2, the coefficients of z^{d^n-j-1} are zero. The desired $b_{d,j}$ is the solution of the algebraic equation

$$d^n b_{d,j} + q_{d,n,i-1}(b_{d,1}, b_{d,2}, \cdots, b_{d,j-1}) = 0.$$

Considering $a_{d,m} = -b_{d,m-2} - \sum_{j=2}^{m-1} a_{d,j} b_{d,m-1-j}$ for $m \in \mathbb{N} \setminus \{1\}$, we get $a_{d,m}$. In addition, we obtain the following lemma.

Lemma 3. *The coefficients $a_{d,m}$ and $b_{d,m}$ are d-adic rational numbers.*

Building a program to compute the exact values of $b_{2,m}$ and $a_{2,m}$ by using the C programing language with multiple precision arithmetic library GMP [6], we get the first 30000 exact values of $a_{2,m}$. Some of these values (numerator, exponent of 2 for the denominator) are presented in Table 1 of Section 5.

3. Coefficient Formula

In this section, we introduce a generalization of the coefficient formula presented in [5].

Theorem 4. *Let $n \in \mathbb{N}$, $2 \leq m \leq d^{n+1}-1$ and r sufficiently large. Then*

$$ma_{d,m} = \frac{1}{2\pi i}\int_{|w|=r} P_{d,w}^{\circ n}(w)^{m/d^n}\frac{\mathrm{d}w}{w^2}.$$

This formula shows that $a_{d,m}$ is the coefficient of degree 1 of the Laurent series expansion of $P_{d,w}^{\circ n}(w)^{m/d^n}$ at ∞. Using `Mathematica`, we calculate the exact values of $a_{3,m}$, $a_{4,m}$, $a_{5,m}$, $a_{6,m}$ and $a_{7,m}$. Part of these values (numerator, exponent of each factor for the denominator) are presented in Tables 2, 3, 4, 5 and 6 of Section 5. In these tables, we omit the zero coefficients indicated in Corollary 6.

The next lemma follows from this theorem. Let $C_j\left(a\right)$ be the general binomial coefficient, i.e. for a real number a and $|x|<1$ it is $(1+x)^a = \sum_{j=0}^{\infty} C_j\left(a\right)x^j$.

Lemma 5. *Let $n, N \in \mathbb{N}$, $2 \leq m \leq d^{n+1}-1$ and $1 \leq N \leq n$. We obtain that $ma_{d,m}$ is the coefficient of w in the Laurent series of the expression*

$$\sum_{j_1=0}^{\infty}\cdots\sum_{j_N=0}^{\infty} C_{j_1}\left(\frac{m}{d^n}\right)C_{j_2}\left(\frac{m}{d^{n-1}}-dj_1\right)C_{j_3}\left(\frac{m}{d^{n-2}}-d^2j_1-dj_2\right)$$
$$\cdots C_{j_N}\left(\frac{m}{d^{n-N+1}}-d^{N-1}j_1-d^{N-2}j_2-\cdots-dj_{N-1}\right)$$
$$\times w^{j_1+\cdots+j_n}P_{d,w}^{\circ n-N}(w)^{m/d^{n-N}-d^Nj_1-d^{N-1}j_2-\cdots-dj_N}.$$

Setting $N=n$ and considering $P_{d,w}^{\circ 0}(w)=w$ leads to the next corollary.

Corollary 6. *Let $n \in \mathbb{N}$ and $2 \leq m \leq d^{n+1}-1$. Then*

$$ma_{d,m} = \sum C_{j_1}\left(\frac{m}{d^n}\right)C_{j_2}\left(\frac{m}{d^{n-1}}-dj_1\right)C_{j_3}\left(\frac{m}{d^{n-2}}-d^2j_1-dj_2\right)$$
$$\cdots C_{j_n}\left(\frac{m}{d}-d^{n-1}j_1-d^{n-2}j_2-\cdots-dj_{n-1}\right),$$

where the sum is over all non-negative indices $j_1,\ldots,j_n$ such that $(d^n-1)j_1+(d^{n-1}-1)j_2+(d^{n-2}-1)j_3+\cdots+(d-1)j_n = m-1$.

4. Zero Coefficients

Ewing and Schober proved the following theorem concerning these coefficients for $d=2$.

Theorem 7 (see [5]). *For any integers k and ν satisfying $k \geq 1$ and $2^\nu \geq k+1$, let $m=(2k+1)2^\nu$. Then $a_{2,m}=0$.*

It is unknown whether the converse is true. They reported that their computation of 1000 terms of $a_{2,m}$ has not produced a zero-coefficient besides those indicated in the theorem [5]. The next statement is a generalization of the above.

Theorem 8. *Suppose the positive integers k,ν satisfy $\nu \geq 1, 2 \leq k \leq d^{\nu+1}-1$ and $k \not\equiv 0 (mod\ d)$. Then $a_{d,m}=0$ for $m=kd^\nu$.*

For $d = 3$, if m is even, then $a_{d,m} = 0$. In addition, when $d = 4$, if $m \not\equiv 1 (\bmod 3)$, then $a_{d,m} = 0$. This phenomena is caused by the rotation symmetry of the generalized Mandelbrot set (see [8, 15]). We gave a short proof in [13].

Corollary 9. *Suppose $d \geq 3$ and $m \not\equiv 1 (\bmod d - 1)$. Then $a_{d,m} = 0$.*

Furthermore there are other zero-coefficients for $d \geq 3$. For example, $d = 3$ and $m = 39$. Some of these can be determined as follows:

Theorem 10. *Suppose $d \geq 3$ and the positive integers k, ν satisfy $\nu \geq 1, 2 \leq k \leq 2(d^{\nu+1} - 1), k \not\equiv 0 (mod\ d)$ and $k \not\equiv -1 (mod\ d)$. Then $a_{d,m} = 0$ for $m = kd^{\mu}$.*

5. Tables

m	Numerator	Exponent of 2
2	1	1
3	1	3
4	1	2
5	15	7
6	0	0
7	81	10
8	1	3
9	1499	15
10	1	5
11	16551	18
12	0	0
13	-19557	22
14	7	8
15	1026129	25
16	1	4
17	78558483	31
18	7	9
19	496067595	34
20	0	0
21	-506111055	38
22	135	12
23	66414150615	41
24	0	0
25	402782136143	46
26	683	16
27	-7661205650557	49
28	0	0
29	159606082621811	53
30	159	14
31	1420861495703249	56
32	1	5
33	11880246651163725l	63
34	6147	20
35	978823547108164723	66
36	7	10
37	11679916854812498869	70
38	136987	23
39	87928513714596704251	73
40	0	0

TABLE 1. The coefficients of f_2

m	Numerator	Exponent of 3
3	1	1
5	1	2
7	2	4
9	1	2
11	52	6
13	155	8
15	0	0
17	2657	10
19	29533	13
21	0	0
23	-69655	15
25	2969930	17
27	1	3
29	23095973	19
31	56696777	21
33	10	6
35	2343898963	23
37	24995524274	26
39	0	0
41	115000492832	28
43	3201040250650	30
45	0	0
47	-6747874422283	32
49	27156979500091	34
51	206	9
53	1754740271356126	36
55	39359185743143624	40
57	104	10
59	664202023454689654	42
61	8022885267816295453	44
63	0	0
65	-7391510296706161637	46
67	221780172965492286820	48
69	998	11
71	-2686651941493059666679	50
73	-32087457055180397296552	53
75	8788	14
77	746925320310443260300229	55
79	6876851947180179910150669	57
81	1	4
83	1248557981800212552239446495	59
85	6374377631178572693579374 78	61
87	39127	15
89	99424739177213541521956607 08	63
91	120356314540026798358102260334	66
93	17849	15
95	238821046435703298297129023039	68
97	1063773779833556053746882813278 6	70
99	0	0
101	-1037073520049114848277411278959 1	72
103	111719030172930182970912859124588	74
105	9614018	19
107	1486830360462347400629819537969302 6	76
109	43289223140475405083713767665492127 5	80
111	808906	19

TABLE 2. The coefficients of f_3

m	Numerator	Exponent of 2
4	1	2
7	3	5
10	1	5
13	15	11
16	1	4
19	2995	16
22	93	12
25	59451	23
28	0	0
31	7405653	28
34	17127	20
37	102177851	34
40	0	0
43	-1017988077	39
46	2092125	27
49	716781072211	47
52	0	0
55	-8057836991135	52
58	-107583317	36
61	2910453741726705	58
64	1	6
67	91893393031048069	63
70	37808167947	43
73	1318087272305007215	70
76	231	15
79	44491312477272873591 3	75
82	15183120823331	51
85	5638826034225284751059	81
88	0	0
91	313435297799410921771475	86
94	2446791012271421	58
97	11845011179826719081484 0195	95
100	0	0
103	-1301193230791636493236184615	100
106	664048285923294771	68
109	51211345120475652876034366 0597	106
112	0	0
115	-352042307049021932694979765 4607	111
118	-45727887792645710401	75
121	506212175722490985695107186905045	118
124	165891	25
127	5879684164307162716544942248791 6363	123
130	2309263552422315210245 7	83
133	397055491958203159505945410084345677	129
136	715	19
139	7843132991039880577064297511264057 5077	134
142	5449594290814991549012715	90
145	79806241738863875692831897287343964 65431	142
148	0	0
151	911522998008107567568371725308259355 97981	147
154	1498244827443611355653020543	99
157	391068031699780588181706969998341410 46385285	153
160	0	0
163	-27213280152884716837462079140894594129 9571111	158
166	-5015072798157096341615114953	106

TABLE 3. The coefficients of f_4

m	Numerator	Exponent of 5
5	1	1
9	2	2
13	4	3
17	7	4
21	44	6
25	1	2
29	12272	8
33	36603	9
37	85256	10
41	669768	12
45	0	0
49	112321771	14
53	388257398	15
57	1032056524	16
61	9125770814	18
65	0	0
69	-81246358698	20
73	5215736042762	21
77	13061209292514	22
81	120874136987029	24
85	0	0
89	-1223557557246132	26
93	-6414828347025054	27
97	274979536551155328	28
101	8963521300059176051	31
105	0	0
109	-89389483729234487652	33
113	-486246831892374376053	34
117	-1649151316991870622151	35
121	391483254035866680450124	37
125	1	3
129	10243115362133254704701388	39
133	27491557925964752563813559	40
137	56011891263226276862420782	41
141	366148195561395087109540258	43
145	1596	8
149	182444599361456314269533021049	45
153	614013623811293037508175596984	46
157	1566340171549905562720996254608	47
161	13129024868901786766016022008219	49
165	0	0
169	1240651330101237709943531838913108	51
173	11090312466240819735580402303782236	52
177	29483003113410510802951827213615633	53
181	272457560896503207828646458948743729	55
185	0	0
189	-2621807240948417080067307939236241968	57
193	65028369153591069630335027153700993403	58
197	684198297180196449153739641357729566871	59
201	25735645302412330165611933363120519759738	62
205	0	0
209	-264976014648208932338860141790274161278099	64
213	-1425266395615462618015450915405906612862477	65
217	18210411808639514012847109021885181438394584	66
221	1091692220547900332047427749010983590042017202	68

TABLE 4. The coefficients of f_5

m	Numerator	Exponent of 2	3
6	1	1	1
11	5	3	2
16	5	1	4
21	5	7	1
26	7	1	6
31	8645	10	8
36	1	2	2
41	44166115	15	10
46	96545	2	13
51	20051	18	2
56	224695	2	15
61	682050153785	22	17
66	0	0	0
71	510189065505655	25	19
76	412426453	2	21
81	120083275	31	2
86	2394396445	3	23
91	491447396125243274 15	34	26
96	0	0	0
101	-2801171227435232984071	38	28
106	52774878534565	5	30
111	2597391412505	41	5
116	19211930633005	2	32
121	11377817781313153019908138 15	46	34
126	0	0	0
131	-363487493366526490964863567 45	49	36
136	-1219849324263 1315	1	40
141	36377488424879315	53	6
146	6524104154298 2158265	8	42
151	605308061182790006814937684652 84389	56	44
156	0	0	0
161	-3262783829004206164807576200526 5809365	63	46
166	-4934686005225577895375	7	48
171	-2600834225025 06625	66	2
176	2963646870280316029705	0	50
181	2023043180367055898049277990728006 4385147543	70	53
186	0	0	0
191	-615519080443710081786835807335058560652341635	73	55
196	-287514846503900576853 3865	2	57
201	-250734186268353826949737	78	7
206	-1714693662743917675142615131	9	59

TABLE 5. The coefficients of f_6

m	Numerator	Exponent of 7
7	1	1
13	3	2
19	10	3
25	33	4
31	102	5
37	276	6
43	3828	8
49	1	2
55	4886892	10
61	24323193	11
67	106806036	12
73	412326959	13
79	1338614628	14
85	21663929508	16
91	0	0
97	15881452467207	18
103	88458814741695	19
109	430684532453670	20
115	1827731386205295	21
121	6473496513847509	22
127	113543753471947366	24
133	0	0
139	-2397245366530485621	26
145	399981602588647434254	27
151	1896566348461207903576	28
157	8353830782381410698123	29
163	31069977330243000729086	30
169	573571363151516572431860	32
175	0	0
181	-13382662008145137285729374	34
187	-117342656009013997691001647	35
193	11347420366217549394264329589	36
199	41815800807425247397362906544	37
205	153373107599284780595599688311	38
211	2885412977789314774479022653216	40
217	0	0
223	-71706622150248358307439522992655	42
229	-651008699388212278045484479088121	43
235	-4098721239127187521462669622469906	44
241	345927006962224035750738278739011930	45
247	866140781977152053621693275086815604	46
253	15245220943627210103007062650905532493	48
259	0	0
265	-380306789601016566873834419097402398703	50
271	-3523387908136747198365040697162766512823	51
277	-22724723781272885854678843859430404875092	52
283	-117794971375196444565305062061434092520623	53
289	11051541173662752530186346243508710914942760	54
295	684816309855195490889404171649532724071846121	57
301	0	0
307	-14547727532901231765679437172717656067150007195	59
313	-134126312225852378807562286473816429601253045251	60
319	-872404263082910091131629789120394545746805945905	61
325	-4590488276809319231550940780999336662495098259853	62
331	-19440010418342626606123045335588177346374583978436	63

TABLE 6. The coefficients of f_7

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