

On Canonical Partition of Edge Set

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(Abstract)

Let E be the edge set of a nonseparable graph G . For each edge e , let $t(e)$ be the number of trees which contain e , and $\bar{t}(e)$ be the number of trees which does not contain e . Here, let E_+ , E_0 , E_- be edge sets which satisfy $t(e) < \bar{t}(e)$, $t(e) = \bar{t}(e)$, $t(e) > \bar{t}(e)$, respectively, then we have the tripartition (E_+, E_0, E_-) of E . We call this tripartition the canonical partition of edge set E .

This paper presents properties in connection with the structure of graphs which satisfy $E = E_0$, $E_+ = E_- = \emptyset$. G is called to have the complementary tree structure when E is partitioned into E_1 , E_2 and each E_i is the edge set of a tree of G . [Theorem 1] Let $(\emptyset, E, \emptyset)$ be the canonical partition of E , then G have the complementary tree structure.

[Example 1] The canonical partitions of E of Graphs L_2 , K_4 and $K_{3,4}$ shown in Fig. 1 are all $(\emptyset, E, \emptyset)$, and they also have the complementary tree structures.

[Theorem 2] Let G_1, G_2 be graphs whose canonical partitions are $(\emptyset, E_1, \emptyset)$ and $(\emptyset, E_2, \emptyset)$, respectively. If we delete any edges $e_i = (u_i, v_i) \in E_i$ ($i=1,2$) and connect u_1 and u_2 , v_1 and v_2 , respectively, then the canonical partition of the obtained graph G is $(\emptyset, E, \emptyset)$, where $E = E_1 \cup E_2 - \{e_1, e_2\}$. (END)

Fig. 2 explains the manner obtaining G from G_1 and G_2 by the procedure stated in Theorem 2.

The converse of this theorem is stated as follows.

[Theorem 3] Let G be the graph shown in Fig. 2(b) which is connected by two vertices u and v , and its canonical partition is $(\emptyset, E, \emptyset)$. If we delete $E - E_i'$ and add an edge e_i between u and v , then the canonical partition of the obtained graph G_i is $(\emptyset, E_i, \emptyset)$, respectively, where $E_i' = E_i - \{e_i\}$ ($i=1,2$).

[Theorem 4] Let G be the graph whose canonical partition is $(\emptyset, E, \emptyset)$. The canonical partition of any graph G' which contains G as a partial subgraph is not $(\emptyset, E', \emptyset)$. Also, the canonical partition of G'' which is contained in G as a partial subgraph is not $(\emptyset, E'', \emptyset)$. (END)

The following conjecture is of interest.

[Conjecture] The canonical partition of edge set E of three connected graph G is $(\emptyset, E, \emptyset)$ if and only if G is K_4 or $K_{3,4}$.

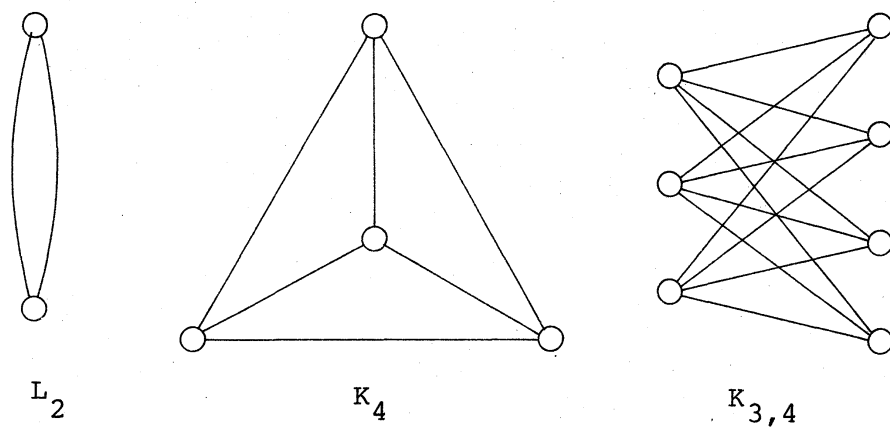


Fig. 1

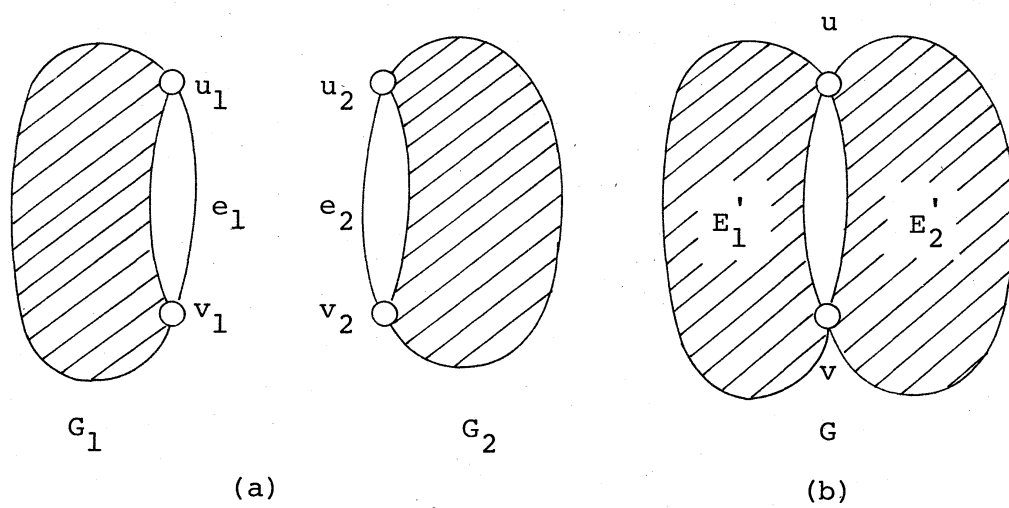


Fig. 2