

## Numerical Data of Complex Capacitance and Conductance for the Circular Arc Rule in Dielectric Relaxations

Naokazu KOIZUMI\*, Tetsuya HANAI\* and Eiji IKADA\*\*

*Received July, 17, 1971*

The analytical expressions of the complex capacitance and the complex conductance have been derived for the circular arc rule on dielectric relaxation. Numerical data of the normalized complex capacitance and conductance as a function of frequency are tabulated over the wide range of the parameter for the distribution of relaxation times to facilitate the analysis of data on dielectric relaxation. For general cases of the circular arc rule, neither the real nor the imaginary part of the complex conductance tend to zero or finite values but they increase infinitely at high frequencies and give complicated loci dissimilar to circular arcs in the complex plane.

### I. INTRODUCTION

It is well known that the frequency dependence of the dielectric constants and the loss factors for a number of dielectric materials is represented satisfactorily by the so-called circular arc rule which was proposed by Cole and Cole.<sup>1,2)</sup>

Some incorrect interpretations and treatments, however, are sometimes made for lack of the consideration on the complex conductivity associated with the dielectric relaxation which is given by the circular arc rule. For example Pauly and Packer<sup>3)</sup> assumed that the conductivity tends to a finite value at high frequencies for the dielectric dispersion which is characterized by a circular arc. Such an incomplete consideration led them to a doubtful interpretation that the further increase in the conductivity at higher frequencies far beyond the critical frequency implies the beginning of another dielectric dispersion.

An incomplete analysis of the complex conductivity for the circular arc rule results in an incorrect deduction that the locus of the complex conductivity in the complex plane would also give a circular arc. Following such a deduction Pauly *et al.*<sup>3,4)</sup> suggested a false relation that the critical frequency may be the geometical mean of loss maximum frequencies in the complex dielectric constant plane and the complex conductivity plane.

In the present paper, the analytical expressions of the complex dielectric constant for the circular arc rule and the corresponding complex conductivity are derived. Numerical values of normalized dielectric constants and normalized conductivities are calculated from these expressions to discuss the behavior in

\* 小泉 直一, 花井 哲也: Laboratory of Dielectrics, Institute for Chemical Research, Kyoto University, Uji, Kyoto.

\*\* 筏 英之: Faculty of Engineering, Kobe University, Kobe.

the complex planes. The interpretation and the treatment which were made in the references<sup>3,4)</sup> for the frequency dependence of the conductivity are also discussed.

## II. THE EXPRESSIONS OF THE COMPLEX CAPACITANCE AND THE COMPLEX CONDUCTANCE

In this paper complex capacitances and conductances are used instead of complex dielectric constants and conductivities for brevity.

### A. The General Relations between the Relaxation Terms and the Measured Capacitance and Conductance

The complex capacitance  $C^*$  and the complex conductance  $G^*$  for the whole system observed are written in the form

$$C^* = \bar{C}' - j\bar{C}'' + C_\infty + \frac{1}{j\omega}G_0 \quad (1)$$

and

$$G^* = \bar{G}' + j\bar{G}'' + j\omega C_\infty + G_0 \quad (2)$$

where  $\bar{C}'$  and  $\bar{C}''$  are the real and the imaginary part of the complex capacitance associated with a dielectric dispersion,  $\bar{G}'$  and  $\bar{G}''$  the real and the imaginary part of the corresponding complex conductance,  $C_\infty$  the capacitance at infinite frequency,  $G_0$  the dc conductance,  $\omega$  the angular frequency, and  $j = \sqrt{-1}$ .

Since we have a relation

$$G^* = j\omega C^*, \quad (3)$$

it follows from Eqs. 1 and 2 that

$$\bar{G}'' = \omega \bar{C}' \quad (4)$$

and

$$\bar{G}' = \omega \bar{C}''. \quad (5)$$

The capacitance  $C'$  and the conductance  $G'$  measured are related to  $C^*$  and  $G^*$  with the relations

$$C^* = C' + \frac{1}{j\omega}G' \quad (6)$$

and

$$G^* = G' + j\omega C'. \quad (7)$$

Equating Eq. 6 to Eq. 1, we have

$$\bar{C}' = C' - C_\infty \quad (8)$$

and

$$\bar{C}'' = \frac{1}{\omega}(G' - G_0). \quad (9)$$

Insertion of Eqs. 8 and 9 into Eqs. 4 and 5 gives

$$\bar{G}' = G' - G_0 \quad (10)$$

and

$$\bar{G}'' = \omega(C' - C_\infty). \quad (11)$$

Thus it is possible to evaluate  $\bar{C}'$ ,  $\bar{C}''$ ,  $\bar{G}'$ , and  $\bar{G}''$  from the measured  $C'$  and  $G'$  by use of Eqs. 8 to 11.

## B. The Expressions for the Circular Arc Rule

According to Cole and Cole<sup>1)</sup> the circular arc rule is given by

$$\bar{C}' - j\bar{C}'' = (C_0 - C_\infty) \frac{1}{1 + (j\omega\tau)^\beta} \quad ; \quad 1 \gg \beta > 0, \quad (12)$$

where  $C_0$  is the static capacitance,  $\tau$  the most probable relaxation time, and  $\beta$  denotes the parameter for the distribution of relaxation times.

If we put

$$\omega\tau = e^x, \quad (13)$$

then we have

$$(\omega\tau)^\beta = e^{\beta x} \quad (14)$$

and

$$x = \ln(\omega\tau). \quad (15)$$

If the right of Eq. 12 is separated into the real and the imaginary part, and if Eq. 14 is used to derive the expressions with respect to  $x$ , then the following equations are obtained.

$$\frac{\bar{C}'}{C_0 - C_\infty} = \frac{C' - C_\infty}{C_0 - C_\infty} = \frac{1 + (\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right)}{1 + 2(\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right) + (\omega\tau)^{2\beta}} \quad , \quad (16)$$

$$= \frac{e^{-\beta x} + \cos\left(\frac{\pi}{2}\beta\right)}{e^{\beta x} + e^{-\beta x} + 2\cos\left(\frac{\pi}{2}\beta\right)} \quad , \quad (17)$$

$$= \frac{1}{2} \left[ 1 - \frac{\sinh \beta x}{\cosh \beta x + \cos\left(\frac{\pi}{2}\beta\right)} \right] \quad . \quad (18)$$

$$\frac{\bar{C}''}{C_0 - C_\infty} = \frac{\frac{1}{\omega}(G' - G_0)}{C_0 - C_\infty} = \frac{(\omega\tau)^\beta \sin\left(\frac{\pi}{2}\beta\right)}{1 + 2(\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right) + (\omega\tau)^{2\beta}} \quad , \quad (19)$$

$$= \frac{\sin\left(\frac{\pi}{2}\beta\right)}{e^{\beta x} + e^{-\beta x} + 2\cos\left(\frac{\pi}{2}\beta\right)} \quad , \quad (20)$$

$$= \frac{\sin\left(\frac{\pi}{2}\beta\right)}{2 \left[ \cosh \beta x + \cos\left(\frac{\pi}{2}\beta\right) \right]} \quad . \quad (21)$$

### Numerical Data of Complex Capacitance and Conductance

By use of Eqs. 16 to 21 the equations for  $\bar{G}'$  and  $\bar{G}''$  are derived as follows.

$$\frac{\bar{G}'\tau}{C_0-C_\infty} = \frac{\bar{C}'\omega\tau}{C_0-C_\infty} = \frac{\omega\tau(\omega\tau)^\beta \sin\left(\frac{\pi}{2}\beta\right)}{1+2(\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right) + (\omega\tau)^{2\beta}} \quad , \quad (22)$$

$$= \frac{e^x \sin\left(\frac{\pi}{2}\beta\right)}{e^{\beta x} + e^{-\beta x} + 2\cos\left(\frac{\pi}{2}\beta\right)} \quad , \quad (23)$$

$$= \frac{\sin\left(\frac{\pi}{2}\beta\right)}{2} \cdot \frac{\cosh x + \sinh x}{\cosh \beta x + \cos\left(\frac{\pi}{2}\beta\right)} \quad . \quad (24)$$

$$\frac{\bar{G}''\tau}{C_0-C_\infty} = \frac{\bar{C}''\omega\tau}{C_0-C_\infty} = \frac{\omega\tau\left[1 + (\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right)\right]}{1+2(\omega\tau)^\beta \cos\left(\frac{\pi}{2}\beta\right) + (\omega\tau)^{2\beta}} \quad , \quad (25)$$

$$= \frac{e^x \left[e^{-\beta x} + \cos\left(\frac{\pi}{2}\beta\right)\right]}{e^{\beta x} + e^{-\beta x} + 2\cos\left(\frac{\pi}{2}\beta\right)} \quad , \quad (26)$$

$$= \frac{\cosh x + \sinh x}{2} \left[ 1 - \frac{\sinh \beta x}{\cosh \beta x + \cos\left(\frac{\pi}{2}\beta\right)} \right] \quad . \quad (27)$$

### III. THE RESULTS OF NUMERICAL CALCULATION OF THE COMPLEX CAPACITANCE AND THE COMPLEX CONDUCTANCE

The values of  $\frac{\bar{C}'}{C_0-C_\infty}$ ,  $\frac{\bar{C}''}{C_0-C_\infty}$ ,  $\frac{\bar{G}'\tau}{C_0-C_\infty}$ , and  $\frac{\bar{G}''\tau}{C_0-C_\infty}$  were calculated for respective values of  $\beta$  by use of Eqs. 16 to 27. In Table 1 the numerical values of  $\frac{\bar{C}'}{C_0-C_\infty}$  and  $\frac{\bar{C}''}{C_0-C_\infty}$  are given over the range of  $\log_{10}(\omega\tau)$  of 0 to 3 for values of  $\beta$  of 1 to 0.05 in a floating point form. The five significant digits denote the mantissa in which the decimal point is just to the left of the most significant digit. One digit with a sign to the right of the mantissa denotes the exponent which specifies the position of the decimal point by a sign and the digit. Thus a number such as 0.012345 or  $0.12345 \times 10^{-1}$  is written in the form 12345-1. Evidently Eq. 20 is an even function of  $x$ . Equation 17 gives an odd function of  $x$  with respect to a coordinate system whose origin is located at  $\ln(\omega\tau)=0$  and  $\frac{\bar{C}'}{C_0-C_\infty}=0.5$ . Thus the values of  $\bar{C}'$  and  $\bar{C}''$  at  $\ln(\omega\tau)<0$  are readily obtained from those at  $\ln(\omega\tau)>0$  which are given in Table 1.

Numerical values of  $\frac{\bar{G}'\tau}{C_0-C_\infty}$  and  $\frac{\bar{G}''\tau}{C_0-C_\infty}$  are given in Table 2. The frequency dependence of the complex capacitance and conductance is shown graphically in Figs. 1, 2, 3, and 4 for several representative values of  $\beta$ . The plots of the values in the complex capacitance and conductance planes are shown in Figs. 5 and 6.

Table 1. Numerical Data of Real and Imaginary Part of the Complex Capacitance for the Circular Arc Rule. In the Table the numerical format designated as 12345-1 denotes the number 0.012345 or  $0.12345 \times 10^{-1}$ .

| $\log_{10} \omega \tau$ | $\omega \tau$ | $\beta=1.00$   |                | $\beta=0.99$   |                | $\beta=0.98$   |                | $\beta=0.97$   |                |
|-------------------------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                         |               | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                         |               | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                     | 1.0000        | 50000-0        | 50000-0        | 50000-0        | 49221-0        | 50000-0        | 48453-0        | 50000-0        | 47698-0        |
| 0.1                     | 1.2589        | 38686-0        | 48703-0        | 38965-0        | 47988-0        | 39235-0        | 47281-0        | 39497-0        | 46583-0        |
| 0.2                     | 1.5849        | 28475-0        | 45129-0        | 28962-0        | 44580-0        | 29437-0        | 44032-0        | 29900-0        | 43485-0        |
| 0.3                     | 1.9953        | 20076-0        | 40057-0        | 20669-0        | 39715-0        | 21252-0        | 39368-0        | 21826-0        | 39016-0        |
| 0.4                     | 2.5119        | 13681-0        | 34364-0        | 14288-0        | 34218-0        | 14890-0        | 34062-0        | 15486-0        | 33896-0        |
| 0.5                     | 3.1623        | 90909-1        | 28748-0        | 96508-1        | 28754-0        | 10211-0        | 28749-0        | 10771-0        | 28733-0        |
| 0.6                     | 3.9811        | 59351-1        | 23628-0        | 64187-1        | 23736-0        | 69066-1        | 23834-0        | 73986-1        | 23923-0        |
| 0.7                     | 5.0119        | 38287-1        | 19189-0        | 42292-1        | 19355-0        | 46366-1        | 19515-0        | 50506-1        | 19667-0        |
| 0.8                     | 6.3096        | 24503-1        | 15461-0        | 27734-1        | 15654-0        | 31045-1        | 15844-0        | 34435-1        | 16028-0        |
| 0.9                     | 7.9433        | 15602-1        | 12393-0        | 18165-1        | 12592-0        | 20811-1        | 12790-0        | 23539-1        | 12985-0        |
| 1.0                     | 10.000        | 99010-2        | 99010-1        | 11916-1        | 10094-0        | 14008-1        | 10286-0        | 16180-1        | 10477-0        |
| 1.1                     | 12.589        | 62700-2        | 78935-1        | 78450-2        | 80718-1        | 94909-2        | 82510-1        | 11209-1        | 84309-1        |
| 1.2                     | 15.849        | 39653-2        | 62846-1        | 51937-2        | 64453-1        | 64848-2        | 66077-1        | 78406-2        | 67717-1        |
| 1.3                     | 19.953        | 25056-2        | 49993-1        | 34632-2        | 51415-1        | 44752-2        | 52858-1        | 55435-2        | 54323-1        |
| 1.4                     | 25.119        | 15824-2        | 39748-1        | 23293-2        | 40988-1        | 31228-2        | 42252-1        | 39648-2        | 43541-1        |
| 1.5                     | 31.623        | 99900-3        | 31591-1        | 15824-2        | 32661-1        | 22053-2        | 33757-1        | 28695-2        | 34878-1        |
| 1.6                     | 39.811        | 63056-3        | 25103-1        | 10869-2        | 26019-1        | 15766-2        | 26961-1        | 21014-2        | 27927-1        |
| 1.7                     | 50.119        | 39795-3        | 19945-1        | 75562-3        | 20724-1        | 11413-2        | 21528-1        | 15564-2        | 22355-1        |
| 1.8                     | 63.096        | 25113-3        | 15845-1        | 53196-3        | 16505-1        | 83618-3        | 17187-1        | 11652-2        | 17891-1        |
| 1.9                     | 79.433        | 15847-3        | 12587-1        | 37935-3        | 13143-1        | 61974-3        | 13719-1        | 88090-3        | 14317-1        |
| 2.0                     | 100.00        | 99990-4        | 99990-2        | 27400-3        | 10465-1        | 46426-3        | 10951-1        | 67187-3        | 11455-1        |
| 2.1                     | 125.89        | 63091-4        | 79428-2        | 20038-3        | 83330-2        | 35118-3        | 87400-2        | 51644-3        | 91644-2        |
| 2.2                     | 158.49        | 39810-4        | 63093-2        | 14827-3        | 66349-2        | 26794-3        | 69755-2        | 39966-3        | 73315-2        |
| 2.3                     | 199.53        | 25118-4        | 50117-2        | 11091-3        | 52828-2        | 20598-3        | 55670-2        | 31108-3        | 58649-2        |
| 2.4                     | 251.19        | 15849-4        | 39810-2        | 83775-4        | 42061-2        | 15938-3        | 44428-2        | 24331-3        | 46915-2        |
| 2.5                     | 316.23        | 99990-5        | 31622-2        | 63824-4        | 33489-2        | 12401-3        | 35455-2        | 19110-3        | 37528-2        |
| 2.6                     | 398.11        | 63075-5        | 25119-2        | 48997-4        | 26663-2        | 96935-4        | 28295-2        | 15060-3        | 30019-2        |
| 2.7                     | 501.19        | 39805-5        | 19953-2        | 37855-4        | 21228-2        | 76062-4        | 22580-2        | 11902-3        | 24011-2        |
| 2.8                     | 630.96        | 25120-5        | 15849-2        | 29409-4        | 16901-2        | 59879-4        | 18019-2        | 94270-4        | 19206-2        |
| 2.9                     | 794.33        | 15850-5        | 12589-2        | 22949-4        | 13456-2        | 47258-4        | 14380-2        | 74813-4        | 15362-2        |
| 3.0                     | 1000.0        | 10000-5        | 10000-2        | 17978-4        | 10713-2        | 37380-4        | 11475-2        | 59461-4        | 12288-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.96$   |                | $\beta=0.95$   |                | $\beta=0.94$   |                | $\beta=0.93$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 46953-0        | 50000-0        | 46220-0        | 50000-0        | 45496-0        | 50000-0        | 44784-0        |
| 0.1                   | 1.2589       | 39752-0        | 45894-0        | 40000-0        | 45213-0        | 40241-0        | 44539-0        | 40475-0        | 43874-0        |
| 0.2                   | 1.5849       | 30353-0        | 42940-0        | 30795-0        | 42397-0        | 31227-0        | 41856-0        | 31648-0        | 41317-0        |
| 0.3                   | 1.9953       | 22389-0        | 38658-0        | 22943-0        | 38295-0        | 23487-0        | 37928-0        | 24022-0        | 37557-0        |
| 0.4                   | 2.5119       | 16077-0        | 33721-0        | 16663-0        | 33537-0        | 17243-0        | 33343-0        | 17817-0        | 33142-0        |
| 0.5                   | 3.1623       | 11330-0        | 28706-0        | 11889-0        | 28670-0        | 12447-0        | 28622-0        | 13004-0        | 28565-0        |
| 0.6                   | 3.9811       | 78943-1        | 24002-0        | 83936-1        | 24072-0        | 88961-1        | 24132-0        | 94016-1        | 24182-0        |
| 0.7                   | 5.0119       | 54712-1        | 19813-0        | 58982-1        | 19950-0        | 63312-1        | 20080-0        | 67702-1        | 20203-0        |
| 0.8                   | 6.3096       | 37904-1        | 16208-0        | 41452-1        | 16383-0        | 45077-1        | 16552-0        | 48778-1        | 16716-0        |
| 0.9                   | 7.9433       | 26349-1        | 13177-0        | 29243-1        | 13367-0        | 32220-1        | 13554-0        | 35281-1        | 13737-0        |
| 1.0                   | 10.000       | 18432-1        | 10668-0        | 20765-1        | 10858-0        | 23181-1        | 11046-0        | 25680-1        | 11233-0        |
| 1.1                   | 12.589       | 13002-1        | 86113-1        | 14871-1        | 87921-1        | 16816-1        | 89730-1        | 18841-1        | 91540-1        |
| 1.2                   | 15.849       | 92628-2        | 69372-1        | 10753-1        | 71040-1        | 12314-1        | 72720-1        | 13947-1        | 74411-1        |
| 1.3                   | 19.953       | 66702-2        | 55808-1        | 78572-2        | 57313-1        | 91065-2        | 58837-1        | 10420-1        | 60379-1        |
| 1.4                   | 25.119       | 48573-2        | 44853-1        | 58023-2        | 46189-1        | 68017-2        | 47548-1        | 78577-2        | 48930-1        |
| 1.5                   | 31.623       | 35770-2        | 36024-1        | 43297-2        | 37196-1        | 51295-2        | 38393-1        | 59785-2        | 39614-1        |
| 1.6                   | 39.811       | 26629-2        | 28920-1        | 32630-2        | 29937-1        | 39036-2        | 30981-1        | 45866-2        | 32050-1        |
| 1.7                   | 50.119       | 20027-2        | 23208-1        | 24818-2        | 24085-1        | 29955-2        | 24988-1        | 35455-2        | 25917-1        |
| 1.8                   | 63.096       | 15205-2        | 18620-1        | 19035-2        | 19372-1        | 23158-2        | 20148-1        | 27592-2        | 20949-1        |
| 1.9                   | 79.433       | 11641-2        | 14936-1        | 14708-2        | 15577-1        | 18023-2        | 16241-1        | 21602-2        | 16928-1        |
| 2.0                   | 100.00       | 89797-3        | 11979-1        | 11438-2        | 12524-1        | 14106-2        | 13089-1        | 16998-2        | 13676-1        |
| 2.1                   | 125.89       | 69718-3        | 96068-2        | 89449-3        | 10068-1        | 11095-2        | 10548-1        | 13435-2        | 11047-1        |
| 2.2                   | 158.49       | 54431-3        | 77036-2        | 70287-3        | 80924-2        | 87633-3        | 84984-2        | 10658-2        | 89223-2        |
| 2.3                   | 199.53       | 42697-3        | 61771-2        | 55451-3        | 65042-2        | 69457-3        | 68468-2        | 84811-3        | 72053-2        |
| 2.4                   | 251.19       | 33625-3        | 49529-2        | 43893-3        | 52274-2        | 55212-3        | 55157-2        | 67666-3        | 58183-2        |
| 2.5                   | 316.23       | 26568-3        | 39712-2        | 34840-3        | 42011-2        | 43994-3        | 44432-2        | 54102-3        | 46980-2        |
| 2.6                   | 398.11       | 21049-3        | 31839-2        | 27718-3        | 33762-2        | 35125-3        | 35791-2        | 43334-3        | 37931-2        |
| 2.7                   | 501.19       | 16714-3        | 25527-2        | 22094-3        | 27131-2        | 28090-3        | 28829-2        | 34760-3        | 30625-2        |
| 2.8                   | 630.96       | 13296-3        | 20466-2        | 17638-3        | 21803-2        | 22494-3        | 23221-2        | 27915-3        | 24725-2        |
| 2.9                   | 794.33       | 10593-3        | 16408-2        | 14098-3        | 17520-2        | 18033-3        | 18703-2        | 22440-3        | 19961-2        |
| 3.0                   | 1000.0       | 84498-4        | 13154-2        | 11280-3        | 14079-2        | 14469-3        | 15064-2        | 18053-3        | 16114-2        |

| $\log_{10} \omega \tau$ | $\omega \tau$ | $\beta=0.92$   |                | $\beta=0.91$   |                | $\beta=0.90$   |                | $\beta=0.89$   |                |
|-------------------------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                         |               | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                         |               | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                     | 1.0000        | 50000-0        | 44081-0        | 50000-0        | 43388-0        | 50000-0        | 42704-0        | 50000-0        | 42029-0        |
| 0.1                     | 1.2589        | 40703-0        | 43216-0        | 40925-0        | 42566-0        | 41141-0        | 41923-0        | 41351-0        | 41287-0        |
| 0.2                     | 1.5849        | 32060-0        | 40780-0        | 32462-0        | 40245-0        | 32854-0        | 39712-0        | 33238-0        | 39182-0        |
| 0.3                     | 1.9953        | 24547-0        | 37181-0        | 25063-0        | 36802-0        | 25570-0        | 36419-0        | 26069-0        | 36033-0        |
| 0.4                     | 2.5119        | 18385-0        | 32931-0        | 18947-0        | 32713-0        | 19503-0        | 32487-0        | 20053-0        | 32253-0        |
| 0.5                     | 3.1623        | 13560-0        | 28497-0        | 14113-0        | 28420-0        | 14665-0        | 28332-0        | 15215-0        | 28236-0        |
| 0.6                     | 3.9811        | 99098-1        | 24223-0        | 10420-0        | 24254-0        | 10933-0        | 24276-0        | 11448-0        | 24287-0        |
| 0.7                     | 5.0119        | 72149-1        | 20317-0        | 76652-1        | 20423-0        | 81207-1        | 20521-0        | 85813-1        | 20610-0        |
| 0.8                     | 6.3096        | 52555-1        | 16875-0        | 56405-1        | 17027-0        | 60329-1        | 17173-0        | 64324-1        | 17312-0        |
| 0.9                     | 7.9433        | 38424-1        | 13917-0        | 41651-1        | 14092-0        | 44960-1        | 14264-0        | 48352-1        | 14432-0        |
| 1.0                     | 10.000        | 28263-1        | 11419-0        | 30930-1        | 11602-0        | 33682-1        | 11783-0        | 36521-1        | 11962-0        |
| 1.1                     | 12.589        | 20945-1        | 93348-1        | 23131-1        | 95152-1        | 25400-1        | 96950-1        | 27753-1        | 98740-1        |
| 1.2                     | 15.849        | 15653-1        | 76111-1        | 17435-1        | 77819-1        | 19294-1        | 79532-1        | 21233-1        | 81250-1        |
| 1.3                     | 19.953        | 11800-1        | 61938-1        | 13248-1        | 63512-1        | 14767-1        | 65101-1        | 16359-1        | 66704-1        |
| 1.4                     | 25.119        | 89724-2        | 50333-1        | 10148-1        | 51757-1        | 11387-1        | 53202-1        | 12690-1        | 54666-1        |
| 1.5                     | 31.623        | 68787-2        | 40860-1        | 78324-2        | 42131-1        | 88417-2        | 43425-1        | 99089-2        | 44743-1        |
| 1.6                     | 39.811        | 53140-2        | 33145-1        | 60879-2        | 34266-1        | 69104-2        | 35412-1        | 77838-2        | 36584-1        |
| 1.7                     | 50.119        | 41337-2        | 26871-1        | 47621-2        | 27851-1        | 54328-2        | 28857-1        | 61478-2        | 29890-1        |
| 1.8                     | 63.096        | 32354-2        | 21775-1        | 37461-2        | 22626-1        | 42933-2        | 23503-1        | 48790-2        | 24406-1        |
| 1.9                     | 79.433        | 25460-2        | 17639-1        | 29615-2        | 18375-1        | 34083-2        | 19134-1        | 38884-2        | 19919-1        |
| 2.0                     | 100.00        | 20128-2        | 14286-1        | 23511-2        | 14918-1        | 27164-2        | 15573-1        | 31101-2        | 16252-1        |
| 2.1                     | 125.89        | 15976-2        | 11567-1        | 18734-2        | 12108-1        | 21721-2        | 12671-1        | 24954-2        | 13255-1        |
| 2.2                     | 158.49        | 12724-2        | 93647-2        | 14973-2        | 98262-2        | 17418-2        | 10307-1        | 20074-2        | 10809-1        |
| 2.3                     | 199.53        | 10162-2        | 75806-2        | 11998-2        | 79732-2        | 14001-2        | 83837-2        | 16184-2        | 88127-2        |
| 2.4                     | 251.19        | 81344-3        | 61358-2        | 96343-3        | 64688-2        | 11276-2        | 68180-2        | 13071-2        | 71841-2        |
| 2.5                     | 316.23        | 65244-3        | 49660-2        | 77503-3        | 52479-2        | 90969-3        | 55442-2        | 10574-2        | 58557-2        |
| 2.6                     | 398.11        | 52414-3        | 40190-2        | 62439-3        | 42571-2        | 73487-3        | 45081-2        | 85645-3        | 47725-2        |
| 2.7                     | 501.19        | 42162-3        | 32524-2        | 50363-3        | 34531-2        | 59431-3        | 36653-2        | 69443-3        | 38894-2        |
| 2.8                     | 630.96        | 33953-3        | 26319-2        | 40664-3        | 28009-2        | 48108-3        | 29799-2        | 56354-3        | 31696-2        |
| 2.9                     | 794.33        | 27366-3        | 21297-2        | 32859-3        | 22718-2        | 38972-3        | 24226-2        | 45765-3        | 25828-2        |
| 3.0                     | 1000.0        | 22073-3        | 17233-2        | 26570-3        | 18425-2        | 31591-3        | 19695-2        | 37187-3        | 21046-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.88$   |                | $\beta=0.87$   |                | $\beta=0.86$   |                | $\beta=0.85$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 41364-0        | 50000-0        | 40706-0        | 50000-0        | 40058-0        | 50000-0        | 39417-0        |
| 0.1                   | 1.2589       | 41555-0        | 40658-0        | 41755-0        | 40036-0        | 41949-0        | 39421-0        | 42138-0        | 38812-0        |
| 0.2                   | 1.5849       | 33613-0        | 38654-0        | 33979-0        | 38128-0        | 34337-0        | 37605-0        | 34687-0        | 37084-0        |
| 0.3                   | 1.9953       | 26558-0        | 35644-0        | 27039-0        | 35252-0        | 27511-0        | 34858-0        | 27975-0        | 34461-0        |
| 0.4                   | 2.5119       | 20597-0        | 32013-0        | 21135-0        | 31765-0        | 21666-0        | 31510-0        | 22191-0        | 31248-0        |
| 0.5                   | 3.1623       | 15763-0        | 28129-0        | 16308-0        | 28013-0        | 16851-0        | 27888-0        | 17391-0        | 27755-0        |
| 0.6                   | 3.9811       | 11965-0        | 24289-0        | 12483-0        | 24282-0        | 13002-0        | 24264-0        | 13522-0        | 24237-0        |
| 0.7                   | 5.0119       | 90468-1        | 20691-0        | 95170-1        | 20764-0        | 99915-1        | 20827-0        | 10470-0        | 20882-0        |
| 0.8                   | 6.3096       | 68389-1        | 17445-0        | 72523-1        | 17571-0        | 76723-1        | 17690-0        | 80990-1        | 17802-0        |
| 0.9                   | 7.9433       | 51825-1        | 14594-0        | 55381-1        | 14752-0        | 59017-1        | 14905-0        | 62733-1        | 15053-0        |
| 1.0                   | 10.000       | 39445-1        | 12138-0        | 42457-1        | 12312-0        | 45555-1        | 12482-0        | 48741-1        | 12649-0        |
| 1.1                   | 12.589       | 30191-1        | 10052-0        | 32715-1        | 10229-0        | 35327-1        | 10404-0        | 38028-1        | 10577-0        |
| 1.2                   | 15.849       | 23252-1        | 82971-1        | 25354-1        | 84692-1        | 27541-1        | 86411-1        | 29813-1        | 88127-1        |
| 1.3                   | 19.953       | 18025-1        | 68318-1        | 19767-1        | 69943-1        | 21589-1        | 71577-1        | 23491-1        | 73218-1        |
| 1.4                   | 25.119       | 14062-1        | 56149-1        | 15503-1        | 57650-1        | 17016-1        | 59167-1        | 18603-1        | 60699-1        |
| 1.5                   | 31.623       | 11036-1        | 46083-1        | 12226-1        | 47446-1        | 13481-1        | 48830-1        | 14803-1        | 50235-1        |
| 1.6                   | 39.811       | 87103-2        | 37781-1        | 96922-2        | 39003-1        | 10732-1        | 40249-1        | 11832-1        | 41520-1        |
| 1.7                   | 50.119       | 69093-2        | 30948-1        | 77195-2        | 32033-1        | 85809-2        | 33145-1        | 94958-2        | 34282-1        |
| 1.8                   | 63.096       | 55051-2        | 25335-1        | 61739-2        | 26291-1        | 68876-2        | 27273-1        | 76484-2        | 28282-1        |
| 1.9                   | 79.433       | 44035-2        | 20730-1        | 49558-2        | 21566-1        | 55472-2        | 22429-1        | 61800-2        | 23318-1        |
| 2.0                   | 100.00       | 35342-2        | 16955-1        | 39905-2        | 17683-1        | 44808-2        | 18436-1        | 50073-2        | 19215-1        |
| 2.1                   | 125.89       | 28447-2        | 13863-1        | 32219-2        | 14493-1        | 36286-2        | 15148-1        | 40668-2        | 15827-1        |
| 2.2                   | 158.49       | 22953-2        | 11332-1        | 26073-2        | 11876-1        | 29448-2        | 12443-1        | 33096-2        | 13032-1        |
| 2.3                   | 199.53       | 18559-2        | 92611-2        | 21140-2        | 97294-2        | 23942-2        | 10218-1        | 26980-2        | 10728-1        |
| 2.4                   | 251.19       | 15031-2        | 75676-2        | 17168-2        | 79692-2        | 19495-2        | 83897-2        | 22027-2        | 88298-2        |
| 2.5                   | 316.23       | 12192-2        | 61829-2        | 13961-2        | 65266-2        | 15895-2        | 68873-2        | 18004-2        | 72659-2        |
| 2.6                   | 398.11       | 99005-3        | 50511-2        | 11366-2        | 53445-2        | 12973-2        | 56533-2        | 14731-2        | 59781-2        |
| 2.7                   | 501.19       | 80478-3        | 41262-2        | 92625-3        | 43761-2        | 10598-2        | 46398-2        | 12064-2        | 49181-2        |
| 2.8                   | 630.96       | 65471-3        | 33703-2        | 75539-3        | 35829-2        | 86638-3        | 38077-2        | 98859-3        | 40456-2        |
| 2.9                   | 794.33       | 53299-3        | 27528-2        | 61643-3        | 29333-2        | 70871-3        | 31246-2        | 81062-3        | 33276-2        |
| 3.0                   | 1000.0       | 43414-3        | 22483-2        | 50330-3        | 24013-2        | 58003-3        | 25640-2        | 66500-3        | 27369-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.84$   |                | $\beta=0.83$   |                | $\beta=0.82$   |                | $\beta=0.81$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 38784-0        | 50000-0        | 33159-0        | 50000-0        | 37541-0        | 50000-0        | 36931-0        |
| 0.1                   | 1.2589       | 42322-0        | 38210-0        | 42502-0        | 37614-0        | 42677-0        | 37024-0        | 42848-0        | 36439-0        |
| 0.2                   | 1.5849       | 35029-0        | 36566-0        | 35364-0        | 36050-0        | 35691-0        | 35536-0        | 36011-0        | 35026-0        |
| 0.3                   | 1.9953       | 28430-0        | 34062-0        | 28878-0        | 33660-0        | 29317-0        | 33257-0        | 29749-0        | 32852-0        |
| 0.4                   | 2.5119       | 22710-0        | 30981-0        | 23223-0        | 30707-0        | 23729-0        | 30427-0        | 24229-0        | 30142-0        |
| 0.5                   | 3.1623       | 17928-0        | 27612-0        | 18462-0        | 27461-0        | 18992-0        | 27301-0        | 19519-0        | 27133-0        |
| 0.6                   | 3.9811       | 14043-0        | 24201-0        | 14564-0        | 24155-0        | 15085-0        | 24100-0        | 15607-0        | 24035-0        |
| 0.7                   | 5.0119       | 10953-0        | 20928-0        | 11439-0        | 20965-0        | 11929-0        | 20993-0        | 12422-0        | 21012-0        |
| 0.8                   | 6.3096       | 85319-1        | 17906-0        | 89711-1        | 18003-0        | 94162-1        | 18092-0        | 98672-1        | 18173-0        |
| 0.9                   | 7.9433       | 66528-1        | 15195-0        | 70402-1        | 15331-0        | 74353-1        | 15461-0        | 78380-1        | 15585-0        |
| 1.0                   | 10.000       | 52013-1        | 12812-0        | 55373-1        | 12971-0        | 58821-1        | 13125-0        | 62355-1        | 13276-0        |
| 1.1                   | 12.589       | 40818-1        | 10749-0        | 43698-1        | 10918-0        | 46669-1        | 11085-0        | 49732-1        | 11249-0        |
| 1.2                   | 15.849       | 32172-1        | 89838-1        | 34621-1        | 91540-1        | 37160-1        | 93232-1        | 39790-1        | 94912-1        |
| 1.3                   | 19.953       | 25476-1        | 74865-1        | 27545-1        | 76515-1        | 29702-1        | 78167-1        | 31947-1        | 79819-1        |
| 1.4                   | 25.119       | 20267-1        | 62245-1        | 22010-1        | 63803-1        | 23834-1        | 65372-1        | 25742-1        | 66951-1        |
| 1.5                   | 31.623       | 16195-1        | 51660-1        | 17659-1        | 53103-1        | 19198-1        | 54564-1        | 20815-1        | 56042-1        |
| 1.6                   | 39.811       | 12995-1        | 42815-1        | 14223-1        | 44132-1        | 15519-1        | 45472-1        | 16886-1        | 46833-1        |
| 1.7                   | 50.119       | 10467-1        | 35445-1        | 11496-1        | 36634-1        | 12587-1        | 37848-1        | 13741-1        | 39087-1        |
| 1.8                   | 63.096       | 84588-2        | 29318-1        | 93212-2        | 30381-1        | 10238-1        | 31471-1        | 11213-1        | 32587-1        |
| 1.9                   | 79.433       | 68564-2        | 24234-1        | 75789-2        | 25177-1        | 83498-2        | 26147-1        | 91718-2        | 27145-1        |
| 2.0                   | 100.00       | 55720-2        | 20020-1        | 61773-2        | 20851-1        | 68253-2        | 21710-1        | 75186-2        | 22596-1        |
| 2.1                   | 125.89       | 45384-2        | 16531-1        | 50455-2        | 17261-1        | 55902-2        | 18017-1        | 61749-2        | 18799-1        |
| 2.2                   | 158.49       | 37035-2        | 13646-1        | 41285-2        | 14283-1        | 45865-2        | 14945-1        | 50796-2        | 15633-1        |
| 2.3                   | 199.53       | 30272-2        | 11261-1        | 33834-2        | 11815-1        | 37684-2        | 12393-1        | 41844-2        | 12996-1        |
| 2.4                   | 251.19       | 24777-2        | 92901-2        | 27763-2        | 97714-2        | 31001-2        | 10274-1        | 34510-2        | 10800-1        |
| 2.5                   | 316.23       | 20304-2        | 76630-2        | 22807-2        | 80793-2        | 25530-2        | 85156-2        | 28490-2        | 89727-2        |
| 2.6                   | 398.11       | 16654-2        | 63198-2        | 18753-2        | 66791-2        | 21044-2        | 70566-2        | 23540-2        | 74532-2        |
| 2.7                   | 501.19       | 13671-2        | 52115-2        | 15432-2        | 55208-2        | 17358-2        | 58467-2        | 19464-2        | 61900-2        |
| 2.8                   | 630.96       | 11230-2        | 42970-2        | 12707-2        | 45628-2        | 14327-2        | 48436-2        | 16103-2        | 51401-2        |
| 2.9                   | 794.33       | 92303-3        | 35427-2        | 10469-2        | 37707-2        | 11832-2        | 40122-2        | 13330-2        | 42679-2        |
| 3.0                   | 1000.0       | 75900-3        | 29207-2        | 86287-3        | 31159-2        | 97749-3        | 33232-2        | 11038-2        | 35433-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.80$   |                | $\beta=0.79$   |                | $\beta=0.78$   |                | $\beta=0.77$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 36327-0        | 50000-0        | 35731-0        | 50000-0        | 35141-0        | 50000-0        | 34557-0        |
| 0.1                   | 1.2589       | 43015-0        | 35861-0        | 43178-0        | 35288-0        | 43337-0        | 34721-0        | 43492-0        | 34159-0        |
| 0.2                   | 1.5849       | 36324-0        | 34517-0        | 36631-0        | 34012-0        | 36931-0        | 33508-0        | 37224-0        | 33008-0        |
| 0.3                   | 1.9953       | 30173-0        | 32445-0        | 30590-0        | 32037-0        | 30999-0        | 31628-0        | 31401-0        | 31217-0        |
| 0.4                   | 2.5119       | 24722-0        | 29851-0        | 25210-0        | 29556-0        | 25691-0        | 29255-0        | 26166-0        | 28949-0        |
| 0.5                   | 3.1623       | 20043-0        | 26957-0        | 20563-0        | 26773-0        | 21079-0        | 26582-0        | 21592-0        | 26383-0        |
| 0.6                   | 3.9811       | 16128-0        | 23961-0        | 16649-0        | 23878-0        | 17170-0        | 23786-0        | 17689-0        | 23686-0        |
| 0.7                   | 5.0119       | 12918-0        | 21021-0        | 13417-0        | 21022-0        | 13918-0        | 21013-0        | 14422-0        | 20996-0        |
| 0.8                   | 6.3096       | 10324-0        | 18246-0        | 10786-0        | 18311-0        | 11253-0        | 18368-0        | 11725-0        | 18416-0        |
| 0.9                   | 7.9433       | 82482-1        | 15703-0        | 86657-1        | 15813-0        | 90905-1        | 15917-0        | 95223-1        | 16014-0        |
| 1.0                   | 10.000       | 65975-1        | 13421-0        | 69682-1        | 13562-0        | 73474-1        | 13697-0        | 77352-1        | 13827-0        |
| 1.1                   | 12.589       | 52887-1        | 11410-0        | 56134-1        | 11567-0        | 59474-1        | 11721-0        | 62908-1        | 11871-0        |
| 1.2                   | 15.849       | 42514-1        | 96576-1        | 45331-1        | 98222-1        | 48245-1        | 99848-1        | 51254-1        | 10145-0        |
| 1.3                   | 19.953       | 34283-1        | 81468-1        | 36711-1        | 83112-1        | 39233-1        | 84749-1        | 41852-1        | 86377-1        |
| 1.4                   | 25.119       | 27736-1        | 68537-1        | 29818-1        | 70129-1        | 31991-1        | 71725-1        | 34256-1        | 73323-1        |
| 1.5                   | 31.623       | 22511-1        | 57535-1        | 24290-1        | 59042-1        | 26155-1        | 60561-1        | 28107-1        | 62090-1        |
| 1.6                   | 39.811       | 18327-1        | 48215-1        | 19843-1        | 49617-1        | 21439-1        | 51038-1        | 23117-1        | 52475-1        |
| 1.7                   | 50.119       | 14962-1        | 40350-1        | 16253-1        | 41637-1        | 17616-1        | 42946-1        | 19055-1        | 44278-1        |
| 1.8                   | 63.096       | 12247-1        | 33729-1        | 13344-1        | 34898-1        | 14507-1        | 36093-1        | 15739-1        | 37313-1        |
| 1.9                   | 79.433       | 10047-1        | 28170-1        | 10980-1        | 29223-1        | 11971-1        | 30303-1        | 13025-1        | 31410-1        |
| 2.0                   | 100.00       | 82597-2        | 23510-1        | 90512-2        | 24451-1        | 98960-2        | 25420-1        | 10797-1        | 26418-1        |
| 2.1                   | 125.89       | 68020-2        | 19609-1        | 74740-2        | 20446-1        | 81934-2        | 21310-1        | 89632-2        | 22203-1        |
| 2.2                   | 158.49       | 56101-2        | 16347-1        | 61805-2        | 17087-1        | 67931-2        | 17855-1        | 74505-2        | 18650-1        |
| 2.3                   | 199.53       | 46332-2        | 13622-1        | 51172-2        | 14275-1        | 56387-2        | 14953-1        | 62001-2        | 15658-1        |
| 2.4                   | 251.19       | 38307-2        | 11348-1        | 42415-2        | 11921-1        | 46853-2        | 12518-1        | 51645-2        | 13140-1        |
| 2.5                   | 316.23       | 31703-2        | 94512-2        | 35188-2        | 99520-2        | 38965-2        | 10476-1        | 43055-2        | 11024-1        |
| 2.6                   | 398.11       | 26258-2        | 78695-2        | 29215-2        | 83065-2        | 32429-2        | 87649-2        | 35920-2        | 92455-2        |
| 2.7                   | 501.19       | 21763-2        | 65514-2        | 24272-2        | 69317-2        | 27006-2        | 73318-2        | 29985-2        | 77524-2        |
| 2.8                   | 630.96       | 18048-2        | 54532-2        | 20177-2        | 57835-2        | 22503-2        | 61319-2        | 25043-2        | 64992-2        |
| 2.9                   | 794.33       | 14975-2        | 45385-2        | 17830-2        | 48248-2        | 18759-2        | 51276-2        | 20926-2        | 54477-2        |
| 3.0                   | 1000.0       | 12430-2        | 37769-2        | 13961-2        | 40246-2        | 15643-2        | 42873-2        | 17491-2        | 45657-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.76$   |                | $\beta=0.75$   |                | $\beta=0.74$   |                | $\beta=0.73$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 33980-0        | 50000-0        | 33409-0        | 50000-0        | 32844-0        | 50000-0        | 32285-0        |
| 0.1                   | 1.2589       | 43643-0        | 33603-0        | 43791-0        | 33052-0        | 43936-0        | 32505-0        | 44077-0        | 31964-0        |
| 0.2                   | 1.5849       | 37511-0        | 32510-0        | 37792-0        | 32014-0        | 38068-0        | 31521-0        | 38337-0        | 31031-0        |
| 0.3                   | 1.9953       | 31796-0        | 30805-0        | 32184-0        | 30392-0        | 32566-0        | 29979-0        | 32940-0        | 29565-0        |
| 0.4                   | 2.5119       | 26635-0        | 28639-0        | 27097-0        | 28324-0        | 27554-0        | 28005-0        | 28005-0        | 27682-0        |
| 0.5                   | 3.1623       | 22100-0        | 26176-0        | 22605-0        | 25963-0        | 23105-0        | 25743-0        | 23602-0        | 25516-0        |
| 0.6                   | 3.9811       | 18208-0        | 23576-0        | 18725-0        | 23457-0        | 19241-0        | 23330-0        | 19756-0        | 23195-0        |
| 0.7                   | 5.0119       | 14927-0        | 20969-0        | 15435-0        | 20933-0        | 15944-0        | 20888-0        | 16454-0        | 20833-0        |
| 0.8                   | 6.3096       | 12202-0        | 18455-0        | 12683-0        | 18486-0        | 13169-0        | 18508-0        | 13659-0        | 18522-0        |
| 0.9                   | 7.9433       | 99611-1        | 16103-0        | 10407-0        | 16185-0        | 10859-0        | 16259-0        | 11317-0        | 16325-0        |
| 1.0                   | 10.000       | 81313-1        | 13951-0        | 85358-1        | 14069-0        | 89485-1        | 14181-0        | 93694-1        | 14286-0        |
| 1.1                   | 12.589       | 66435-1        | 12017-0        | 70055-1        | 12159-0        | 73768-1        | 12296-0        | 77575-1        | 12428-0        |
| 1.2                   | 15.849       | 54361-1        | 10303-0        | 57567-1        | 10458-0        | 60871-1        | 10609-0        | 64275-1        | 10757-0        |
| 1.3                   | 19.953       | 44568-1        | 87993-1        | 47383-1        | 89594-1        | 50299-1        | 91177-1        | 53317-1        | 92740-1        |
| 1.4                   | 25.119       | 36616-1        | 74920-1        | 39074-1        | 76514-1        | 41631-1        | 78103-1        | 44290-1        | 79684-1        |
| 1.5                   | 31.623       | 30150-1        | 63629-1        | 32286-1        | 65174-1        | 34518-1        | 66724-1        | 36848-1        | 68277-1        |
| 1.6                   | 39.811       | 24879-1        | 53929-1        | 26730-1        | 55397-1        | 28671-1        | 56878-1        | 30705-1        | 58371-1        |
| 1.7                   | 50.119       | 20572-1        | 45632-1        | 22171-1        | 47005-1        | 23855-1        | 48397-1        | 25626-1        | 49808-1        |
| 1.8                   | 63.096       | 17043-1        | 38558-1        | 18423-1        | 39827-1        | 19880-1        | 41120-1        | 21419-1        | 42435-1        |
| 1.9                   | 79.433       | 14145-1        | 32545-1        | 15333-1        | 33706-1        | 16592-1        | 34893-1        | 17928-1        | 36106-1        |
| 2.0                   | 100.00       | 11757-1        | 27444-1        | 12779-1        | 28497-1        | 13867-1        | 29579-1        | 15024-1        | 30689-1        |
| 2.1                   | 125.89       | 97862-2        | 23125-1        | 10665-1        | 24075-1        | 11604-1        | 25053-1        | 12605-1        | 26061-1        |
| 2.2                   | 158.49       | 81556-2        | 19473-1        | 89112-2        | 20325-1        | 97204-2        | 21205-1        | 10586-1        | 22115-1        |
| 2.3                   | 199.53       | 68040-2        | 16390-1        | 74531-2        | 17149-1        | 81504-2        | 17937-1        | 88988-2        | 18754-1        |
| 2.4                   | 251.19       | 56816-2        | 13788-1        | 62391-2        | 14463-1        | 68396-2        | 15166-1        | 74862-2        | 15896-1        |
| 2.5                   | 316.23       | 47481-2        | 11596-1        | 52267-2        | 12194-1        | 57438-2        | 12817-1        | 63021-2        | 13468-1        |
| 2.6                   | 398.11       | 39708-2        | 97491-2        | 43815-2        | 10277-1        | 48265-2        | 10829-1        | 53084-2        | 11406-1        |
| 2.7                   | 501.19       | 33226-2        | 81944-2        | 36750-2        | 86587-2        | 40579-2        | 91460-2        | 44737-2        | 96573-2        |
| 2.8                   | 630.96       | 27816-2        | 68862-2        | 30839-2        | 72938-2        | 34133-2        | 77229-2        | 37719-2        | 81744-2        |
| 2.9                   | 794.33       | 23297-2        | 57859-2        | 25890-2        | 61430-2        | 28722-2        | 65200-2        | 31815-2        | 69177-2        |
| 3.0                   | 1000.0       | 19519-2        | 48606-2        | 21742-2        | 51729-2        | 24177-2        | 55035-2        | 26843-2        | 58532-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.72$   |                | $\beta=0.71$   |                | $\beta=0.70$   |                | $\beta=0.69$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 31731-0        | 50000-0        | 31183-0        | 50000-0        | 30640-0        | 50000-0        | 30102-0        |
| 0.1                   | 1.2589       | 44215-0        | 31427-0        | 44350-0        | 30895-0        | 44483-0        | 30368-0        | 44612-0        | 29845-0        |
| 0.2                   | 1.5849       | 38602-0        | 30543-0        | 38860-0        | 30057-0        | 39114-0        | 29574-0        | 39363-0        | 29093-0        |
| 0.3                   | 1.9953       | 33308-0        | 29150-0        | 33670-0        | 28734-0        | 34026-0        | 28318-0        | 34375-0        | 27902-0        |
| 0.4                   | 2.5119       | 28449-0        | 27355-0        | 28888-0        | 27025-0        | 29321-0        | 26691-0        | 29748-0        | 26353-0        |
| 0.5                   | 3.1623       | 24094-0        | 25282-0        | 24581-0        | 25042-0        | 25065-0        | 24796-0        | 25544-0        | 24544-0        |
| 0.6                   | 3.9811       | 20268-0        | 23051-0        | 20780-0        | 22899-0        | 21289-0        | 22739-0        | 21796-0        | 22571-0        |
| 0.7                   | 5.0119       | 16966-0        | 20770-0        | 17479-0        | 20698-0        | 17992-0        | 20616-0        | 18506-0        | 20526-0        |
| 0.8                   | 6.3096       | 14153-0        | 18526-0        | 14650-0        | 18522-0        | 15151-0        | 18508-0        | 15655-0        | 18485-0        |
| 0.9                   | 7.9433       | 11782-0        | 16384-0        | 12253-0        | 16434-0        | 12729-0        | 16475-0        | 13211-0        | 16508-0        |
| 1.0                   | 10.000       | 97982-1        | 14385-0        | 10235-0        | 14476-0        | 10679-0        | 14560-0        | 11132-0        | 14637-0        |
| 1.1                   | 12.589       | 81474-1        | 12554-0        | 85465-1        | 12675-0        | 89548-1        | 12790-0        | 93722-1        | 12899-0        |
| 1.2                   | 15.849       | 67780-1        | 10902-0        | 71385-1        | 11042-0        | 75091-1        | 11178-0        | 78898-1        | 11310-0        |
| 1.3                   | 19.953       | 56439-1        | 94280-1        | 59665-1        | 95794-1        | 62998-1        | 97279-1        | 66437-1        | 98731-1        |
| 1.4                   | 25.119       | 47052-1        | 81255-1        | 49920-1        | 82813-1        | 52894-1        | 84355-1        | 55978-1        | 85878-1        |
| 1.5                   | 31.623       | 39279-1        | 69831-1        | 41814-1        | 71383-1        | 44455-1        | 72930-1        | 47204-1        | 74470-1        |
| 1.6                   | 39.811       | 32837-1        | 59872-1        | 35068-1        | 61381-1        | 37402-1        | 62896-1        | 39841-1        | 64413-1        |
| 1.7                   | 50.119       | 27489-1        | 51234-1        | 29447-1        | 52675-1        | 31503-1        | 54129-1        | 33660-1        | 55595-1        |
| 1.8                   | 63.096       | 23044-1        | 43771-1        | 24757-1        | 45128-1        | 26563-1        | 46504-1        | 28465-1        | 47897-1        |
| 1.9                   | 79.433       | 19342-1        | 37345-1        | 20838-1        | 38608-1        | 22421-1        | 39894-1        | 24094-1        | 41203-1        |
| 2.0                   | 100.00       | 16253-1        | 31826-1        | 17559-1        | 32991-1        | 18944-1        | 34182-1        | 20413-1        | 35399-1        |
| 2.1                   | 125.89       | 13673-1        | 27098-1        | 14810-1        | 28163-1        | 16020-1        | 29257-1        | 17308-1        | 30380-1        |
| 2.2                   | 158.49       | 11512-1        | 23054-1        | 12502-1        | 24023-1        | 13559-1        | 25021-1        | 14686-1        | 26049-1        |
| 2.3                   | 199.53       | 97016-2        | 19601-1        | 10562-1        | 20476-1        | 11484-1        | 21382-1        | 12470-1        | 22318-1        |
| 2.4                   | 251.19       | 81817-2        | 16655-1        | 89295-2        | 17443-1        | 97328-2        | 18261-1        | 10595-1        | 19109-1        |
| 2.5                   | 316.23       | 69044-2        | 14146-1        | 75538-2        | 14853-1        | 82536-2        | 15588-1        | 90070-2        | 16353-1        |
| 2.6                   | 398.11       | 58298-2        | 12010-1        | 63935-2        | 12641-1        | 70027-2        | 13300-1        | 76604-2        | 13988-1        |
| 2.7                   | 501.19       | 49249-2        | 10193-1        | 54140-2        | 10755-1        | 59440-2        | 11344-1        | 65179-2        | 11960-1        |
| 2.8                   | 630.96       | 41621-2        | 86492-2        | 45864-2        | 91482-2        | 50473-2        | 96722-2        | 55477-2        | 10222-1        |
| 2.9                   | 794.33       | 35188-2        | 73371-2        | 38866-2        | 77792-2        | 42873-2        | 82448-2        | 47235-2        | 87349-2        |
| 3.0                   | 1000.0       | 29759-2        | 62229-2        | 32946-2        | 66137-2        | 36428-2        | 70264-2        | 40228-2        | 74621-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.68$   |                | $\beta=0.67$   |                | $\beta=0.66$   |                | $\beta=0.65$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 29570-0        | 50000-0        | 29042-0        | 50000-0        | 28519-0        | 50000-0        | 28001-0        |
| 0.1                   | 1.2589       | 44739-0        | 29327-0        | 44862-0        | 28813-0        | 44984-0        | 28303-0        | 45103-0        | 27796-0        |
| 0.2                   | 1.5849       | 39606-0        | 28615-0        | 39845-0        | 28140-0        | 40080-0        | 27666-0        | 40309-0        | 27195-0        |
| 0.3                   | 1.9953       | 34719-0        | 27486-0        | 35057-0        | 27069-0        | 35389-0        | 26652-0        | 35716-0        | 26235-0        |
| 0.4                   | 2.5119       | 30169-0        | 26013-0        | 30585-0        | 25669-0        | 30995-0        | 25322-0        | 31399-0        | 24973-0        |
| 0.5                   | 3.1623       | 26019-0        | 24285-0        | 26489-0        | 24022-0        | 26955-0        | 23753-0        | 27416-0        | 23478-0        |
| 0.6                   | 3.9811       | 22300-0        | 22395-0        | 22803-0        | 22212-0        | 23303-0        | 22021-0        | 23800-0        | 21823-0        |
| 0.7                   | 5.0119       | 19020-0        | 20427-0        | 19534-0        | 20319-0        | 20049-0        | 20203-0        | 20563-0        | 20077-0        |
| 0.8                   | 6.3096       | 16163-0        | 18454-0        | 16672-0        | 18413-0        | 17185-0        | 18363-0        | 17699-0        | 18304-0        |
| 0.9                   | 7.9433       | 13699-0        | 16533-0        | 14192-0        | 16548-0        | 14689-0        | 16555-0        | 15192-0        | 16553-0        |
| 1.0                   | 10.000       | 11591-0        | 14706-0        | 12058-0        | 14767-0        | 12532-0        | 14821-0        | 13013-0        | 14865-0        |
| 1.1                   | 12.589       | 97986-1        | 13002-0        | 10234-0        | 13098-0        | 10678-0        | 13187-0        | 11131-0        | 13269-0        |
| 1.2                   | 15.849       | 82806-1        | 11436-0        | 86815-1        | 11557-0        | 90925-1        | 11673-0        | 95136-1        | 11782-0        |
| 1.3                   | 19.953       | 69985-1        | 10015-0        | 73640-1        | 10153-0        | 77406-1        | 10286-0        | 81281-1        | 10415-0        |
| 1.4                   | 25.119       | 59173-1        | 87379-1        | 62481-1        | 88855-1        | 65902-1        | 90302-1        | 69439-1        | 91718-1        |
| 1.5                   | 31.623       | 50064-1        | 76000-1        | 53037-1        | 77518-1        | 56126-1        | 79019-1        | 59333-1        | 80502-1        |
| 1.6                   | 39.811       | 42389-1        | 65931-1        | 45049-1        | 67446-1        | 47823-1        | 68957-1        | 50715-1        | 70460-1        |
| 1.7                   | 50.119       | 35921-1        | 57069-1        | 38291-1        | 58550-1        | 40772-1        | 60036-1        | 43368-1        | 61524-1        |
| 1.8                   | 63.096       | 30466-1        | 49307-1        | 32570-1        | 50730-1        | 34782-1        | 52166-1        | 37105-1        | 53613-1        |
| 1.9                   | 79.433       | 25861-1        | 42533-1        | 27725-1        | 43883-1        | 29691-1        | 45252-1        | 31763-1        | 46637-1        |
| 2.0                   | 100.00       | 21969-1        | 36641-1        | 23617-1        | 37908-1        | 25361-1        | 39198-1        | 27206-1        | 40510-1        |
| 2.1                   | 125.89       | 18677-1        | 31530-1        | 20131-1        | 32708-1        | 21676-1        | 33913-1        | 23314-1        | 35143-1        |
| 2.2                   | 158.49       | 15889-1        | 27107-1        | 17171-1        | 28193-1        | 18536-1        | 29310-1        | 19989-1        | 30454-1        |
| 2.3                   | 199.53       | 13526-1        | 23285-1        | 14654-1        | 24282-1        | 15860-1        | 25309-1        | 17147-1        | 26367-1        |
| 2.4                   | 251.19       | 11521-1        | 19988-1        | 12513-1        | 20897-1        | 13576-1        | 21838-1        | 14715-1        | 22810-1        |
| 2.5                   | 316.23       | 98177-2        | 17148-1        | 10690-1        | 17974-1        | 11626-1        | 18831-1        | 12633-1        | 19720-1        |
| 2.6                   | 398.11       | 83702-2        | 14705-1        | 91356-2        | 15451-1        | 99605-2        | 16229-1        | 10849-1        | 17038-1        |
| 2.7                   | 501.19       | 71388-2        | 12604-1        | 78104-2        | 13277-1        | 85362-2        | 13980-1        | 93201-2        | 14714-1        |
| 2.8                   | 630.96       | 60908-2        | 10799-1        | 66796-2        | 11404-1        | 73178-2        | 12038-1        | 80089-2        | 12701-1        |
| 2.9                   | 794.33       | 51981-2        | 92505-2        | 57142-2        | 97926-2        | 62749-2        | 10362-1        | 68839-2        | 10960-1        |
| 3.0                   | 1000.0       | 44375-2        | 79217-2        | 48895-2        | 84064-2        | 53820-2        | 89170-2        | 59183-2        | 94547-2        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.64$   |                | $\beta=0.63$   |                | $\beta=0.62$   |                | $\beta=0.61$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 27488-0        | 50000-0        | 26979-0        | 50000-0        | 26474-0        | 50000-0        | 25973-0        |
| 0.1                   | 1.2589       | 45219-0        | 27294-0        | 45333-0        | 26796-0        | 45445-0        | 26302-0        | 45554-0        | 25811-0        |
| 0.2                   | 1.5849       | 40535-0        | 26726-0        | 40756-0        | 26260-0        | 40973-0        | 25796-0        | 41187-0        | 25334-0        |
| 0.3                   | 1.9953       | 36037-0        | 25818-0        | 36353-0        | 25401-0        | 36663-0        | 24984-0        | 36969-0        | 24568-0        |
| 0.4                   | 2.5119       | 31798-0        | 24621-0        | 32192-0        | 24266-0        | 32580-0        | 23910-0        | 32963-0        | 23550-0        |
| 0.5                   | 3.1623       | 27873-0        | 23199-0        | 28325-0        | 22914-0        | 28773-0        | 22625-0        | 29217-0        | 22330-0        |
| 0.6                   | 3.9811       | 24294-0        | 21618-0        | 24786-0        | 21405-0        | 25275-0        | 21186-0        | 25761-0        | 20961-0        |
| 0.7                   | 5.0119       | 21076-0        | 19944-0        | 21590-0        | 19802-0        | 22102-0        | 19652-0        | 22613-0        | 19493-0        |
| 0.8                   | 6.3096       | 18216-0        | 18235-0        | 18735-0        | 18158-0        | 19255-0        | 18072-0        | 19777-0        | 17977-0        |
| 0.9                   | 7.9433       | 15699-0        | 16542-0        | 16210-0        | 16522-0        | 16726-0        | 16492-0        | 17245-0        | 16454-0        |
| 1.0                   | 10.000       | 13500-0        | 14902-0        | 13994-0        | 14930-0        | 14495-0        | 14949-0        | 15002-0        | 14959-0        |
| 1.1                   | 12.589       | 11592-0        | 13344-0        | 12062-0        | 13410-0        | 12540-0        | 13469-0        | 13027-0        | 13520-0        |
| 1.2                   | 15.849       | 99447-1        | 11886-0        | 10386-0        | 11983-0        | 10837-0        | 12073-0        | 11297-0        | 12156-0        |
| 1.3                   | 19.953       | 85266-1        | 10540-0        | 89362-1        | 10659-0        | 93568-1        | 10772-0        | 97885-1        | 10880-0        |
| 1.4                   | 25.119       | 73092-1        | 93099-1        | 76863-1        | 94441-1        | 80753-1        | 95741-1        | 84762-1        | 96995-1        |
| 1.5                   | 31.623       | 62659-1        | 81962-1        | 66107-1        | 83397-1        | 69678-1        | 84802-1        | 73375-1        | 86175-1        |
| 1.6                   | 39.811       | 53726-1        | 71953-1        | 56861-1        | 73432-1        | 60121-1        | 74893-1        | 63508-1        | 76334-1        |
| 1.7                   | 50.119       | 46083-1        | 63012-1        | 48919-1        | 64496-1        | 51880-1        | 65974-1        | 54969-1        | 67442-1        |
| 1.8                   | 63.096       | 39543-1        | 55067-1        | 42099-1        | 56527-1        | 44778-1        | 57990-1        | 47583-1        | 59453-1        |
| 1.9                   | 79.433       | 33946-1        | 48038-1        | 36243-1        | 49452-1        | 38659-1        | 50877-1        | 41197-1        | 52310-1        |
| 2.0                   | 100.00       | 29155-1        | 41842-1        | 31213-1        | 43194-1        | 33386-1        | 44563-1        | 35677-1        | 45948-1        |
| 2.1                   | 125.89       | 25051-1        | 36398-1        | 26892-1        | 37677-1        | 28841-1        | 38979-1        | 30903-1        | 40301-1        |
| 2.2                   | 158.49       | 21535-1        | 31627-1        | 23178-1        | 32827-1        | 24923-1        | 34053-1        | 26776-1        | 35304-1        |
| 2.3                   | 199.53       | 18520-1        | 27455-1        | 19984-1        | 28572-1        | 21544-1        | 29719-1        | 23206-1        | 30894-1        |
| 2.4                   | 251.19       | 15933-1        | 23813-1        | 17236-1        | 24847-1        | 18629-1        | 25913-1        | 20117-1        | 27009-1        |
| 2.5                   | 316.23       | 13713-1        | 20640-1        | 14871-1        | 21592-1        | 16113-1        | 22577-1        | 17443-1        | 23594-1        |
| 2.6                   | 398.11       | 11805-1        | 17879-1        | 12834-1        | 18752-1        | 13940-1        | 19657-1        | 15128-1        | 20596-1        |
| 2.7                   | 501.19       | 10166-1        | 15479-1        | 11079-1        | 16276-1        | 12063-1        | 17106-1        | 13123-1        | 17968-1        |
| 2.8                   | 630.96       | 87570-2        | 13395-1        | 95662-2        | 14120-1        | 10441-1        | 14877-1        | 11386-1        | 15667-1        |
| 2.9                   | 794.33       | 75449-2        | 11588-1        | 82618-2        | 12245-1        | 90390-2        | 12934-1        | 98810-2        | 13655-1        |
| 3.0                   | 1000.0       | 65019-2        | 10021-1        | 71366-2        | 10615-1        | 78265-2        | 11240-1        | 85760-2        | 11896-1        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.60$   |                | $\beta=0.59$   |                | $\beta=0.58$   |                | $\beta=0.57$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 25476-0        | 50000-0        | 24984-0        | 50000-0        | 24495-0        | 50000-0        | 24010-0        |
| 0.1                   | 1.2589       | 45662-0        | 25324-0        | 45767-0        | 24840-0        | 45871-0        | 24360-0        | 45972-0        | 23883-0        |
| 0.2                   | 1.5849       | 41396-0        | 24874-0        | 41602-0        | 24417-0        | 41803-0        | 23962-0        | 42002-0        | 23508-0        |
| 0.3                   | 1.9953       | 37270-0        | 24151-0        | 37566-0        | 23735-0        | 37857-0        | 23319-0        | 38144-0        | 22903-0        |
| 0.4                   | 2.5119       | 33342-0        | 23189-0        | 33715-0        | 22825-0        | 34083-0        | 22460-0        | 34446-0        | 22093-0        |
| 0.5                   | 3.1623       | 29656-0        | 22032-0        | 30091-0        | 21729-0        | 30521-0        | 21422-0        | 30947-0        | 21111-0        |
| 0.6                   | 3.9811       | 26244-0        | 20728-0        | 26724-0        | 20490-0        | 27201-0        | 20245-0        | 27675-0        | 19994-0        |
| 0.7                   | 5.0119       | 23124-0        | 19327-0        | 23633-0        | 19152-0        | 24141-0        | 18970-0        | 24648-0        | 18780-0        |
| 0.8                   | 6.3096       | 20300-0        | 17872-0        | 20824-0        | 17759-0        | 21349-0        | 17637-0        | 21875-0        | 17506-0        |
| 0.9                   | 7.9433       | 17768-0        | 16406-0        | 18294-0        | 16348-0        | 18824-0        | 16282-0        | 19356-0        | 16206-0        |
| 1.0                   | 10.000       | 15514-0        | 14960-0        | 16032-0        | 14952-0        | 16556-0        | 14935-0        | 17085-0        | 14908-0        |
| 1.1                   | 12.589       | 13521-0        | 13562-0        | 14023-0        | 13596-0        | 14533-0        | 13621-0        | 15050-0        | 13637-0        |
| 1.2                   | 15.849       | 11768-0        | 12232-0        | 12247-0        | 12299-0        | 12737-0        | 12359-0        | 13235-0        | 12411-0        |
| 1.3                   | 19.953       | 10231-0        | 10981-0        | 10685-0        | 11076-0        | 11150-0        | 11164-0        | 11625-0        | 11245-0        |
| 1.4                   | 25.119       | 88892-1        | 98200-1        | 93142-1        | 99352-1        | 97514-1        | 10045-0        | 10201-0        | 10148-0        |
| 1.5                   | 31.623       | 77198-1        | 87511-1        | 81150-1        | 88807-1        | 85231-1        | 90058-1        | 89443-1        | 91262-1        |
| 1.6                   | 39.811       | 67027-1        | 77750-1        | 70677-1        | 79139-1        | 74463-1        | 80496-1        | 78386-1        | 81817-1        |
| 1.7                   | 50.119       | 58190-1        | 68897-1        | 61545-1        | 70336-1        | 65037-1        | 71755-1        | 68670-1        | 73150-1        |
| 1.8                   | 63.096       | 50519-1        | 60914-1        | 53589-1        | 62368-1        | 56796-1        | 63813-1        | 60145-1        | 65244-1        |
| 1.9                   | 79.433       | 43864-1        | 53749-1        | 46662-1        | 55191-1        | 49596-1        | 56633-1        | 52671-1        | 58072-1        |
| 2.0                   | 100.00       | 38091-1        | 47345-1        | 40633-1        | 48754-1        | 43309-1        | 50171-1        | 46123-1        | 51593-1        |
| 2.1                   | 125.89       | 33084-1        | 41643-1        | 35388-1        | 43002-1        | 37821-1        | 44376-1        | 40388-1        | 45763-1        |
| 2.2                   | 158.49       | 28741-1        | 36580-1        | 30824-1        | 37878-1        | 33032-1        | 39197-1        | 35368-1        | 40534-1        |
| 2.3                   | 199.53       | 24974-1        | 32097-1        | 26854-1        | 33326-1        | 28852-1        | 34581-1        | 30974-1        | 35859-1        |
| 2.4                   | 251.19       | 21704-1        | 28136-1        | 23398-1        | 29292-1        | 25204-1        | 30476-1        | 27128-1        | 31689-1        |
| 2.5                   | 316.23       | 18867-1        | 24643-1        | 20391-1        | 25723-1        | 22020-1        | 26835-1        | 23761-1        | 27977-1        |
| 2.6                   | 398.11       | 16404-1        | 21568-1        | 17773-1        | 22572-1        | 19241-1        | 23610-1        | 20814-1        | 24680-1        |
| 2.7                   | 501.19       | 14265-1        | 18864-1        | 15494-1        | 19794-1        | 16815-1        | 20758-1        | 18235-1        | 21755-1        |
| 2.8                   | 630.96       | 12407-1        | 16491-1        | 13508-1        | 17348-1        | 14696-1        | 18239-1        | 15977-1        | 19165-1        |
| 2.9                   | 794.33       | 10793-1        | 14409-1        | 11779-1        | 15196-1        | 12846-1        | 16018-1        | 13999-1        | 16874-1        |
| 3.0                   | 1000.0       | 93897-2        | 12584-1        | 10273-1        | 13305-1        | 11230-1        | 14060-1        | 12268-1        | 14849-1        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.56$   |                | $\beta=0.55$   |                | $\beta=0.54$   |                | $\beta=0.53$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 23528-0        | 50000-0        | 23050-0        | 50000-0        | 22576-0        | 50000-0        | 22105-0        |
| 0.1                   | 1.2589       | 46072-0        | 23409-0        | 46169-0        | 22939-0        | 46266-0        | 22471-0        | 46360-0        | 22007-0        |
| 0.2                   | 1.5849       | 42197-0        | 23037-0        | 42389-0        | 22608-0        | 42577-0        | 22161-0        | 42762-0        | 21716-0        |
| 0.3                   | 1.9953       | 38426-0        | 22488-0        | 38704-0        | 22073-0        | 38978-0        | 21658-0        | 39248-0        | 21244-0        |
| 0.4                   | 2.5119       | 34805-0        | 21724-0        | 35158-0        | 21353-0        | 35508-0        | 20981-0        | 35852-0        | 20608-0        |
| 0.5                   | 3.1623       | 31368-0        | 20796-0        | 31786-0        | 20477-0        | 32198-0        | 20155-0        | 32607-0        | 19829-0        |
| 0.6                   | 3.9811       | 28145-0        | 19737-0        | 28612-0        | 19475-0        | 29076-0        | 19207-0        | 29536-0        | 18933-0        |
| 0.7                   | 5.0119       | 25153-0        | 18583-0        | 25656-0        | 18378-0        | 26158-0        | 18166-0        | 26657-0        | 17947-0        |
| 0.8                   | 6.3096       | 22401-0        | 17367-0        | 22928-0        | 17219-0        | 23454-0        | 17062-0        | 23981-0        | 16897-0        |
| 0.9                   | 7.9433       | 19891-0        | 16120-0        | 20429-0        | 16026-0        | 20969-0        | 15922-0        | 21511-0        | 15809-0        |
| 1.0                   | 10.000       | 17619-0        | 14872-0        | 18158-0        | 14826-0        | 18701-0        | 14771-0        | 19249-0        | 14706-0        |
| 1.1                   | 12.589       | 15574-0        | 13644-0        | 16105-0        | 13641-0        | 16643-0        | 13629-0        | 17188-0        | 13607-0        |
| 1.2                   | 15.849       | 13743-0        | 12455-0        | 14260-0        | 12489-0        | 14785-0        | 12515-0        | 15320-0        | 12531-0        |
| 1.3                   | 19.953       | 12111-0        | 11318-0        | 12608-0        | 11384-0        | 13116-0        | 11441-0        | 13634-0        | 11490-0        |
| 1.4                   | 25.119       | 10662-0        | 10245-0        | 11136-0        | 10336-0        | 11621-0        | 10419-0        | 12119-0        | 10494-0        |
| 1.5                   | 31.623       | 93787-1        | 92413-1        | 98263-1        | 93508-1        | 10287-0        | 94543-1        | 10761-0        | 95514-1        |
| 1.6                   | 39.811       | 82447-1        | 83098-1        | 86649-1        | 84335-1        | 90994-1        | 85524-1        | 95481-1        | 86660-1        |
| 1.7                   | 50.119       | 72447-1        | 74516-1        | 76369-1        | 75851-1        | 80439-1        | 77149-1        | 84660-1        | 78407-1        |
| 1.8                   | 63.096       | 63639-1        | 66660-1        | 67282-1        | 68054-1        | 71076-1        | 69423-1        | 75026-1        | 70763-1        |
| 1.9                   | 79.433       | 55891-1        | 59504-1        | 59259-1        | 60926-1        | 62782-1        | 62334-1        | 66462-1        | 63723-1        |
| 2.0                   | 100.00       | 49079-1        | 53017-1        | 52184-1        | 54441-1        | 55441-1        | 55860-1        | 58856-1        | 57271-1        |
| 2.1                   | 125.89       | 43095-1        | 47160-1        | 45947-1        | 48564-1        | 48950-1        | 49972-1        | 52109-1        | 51381-1        |
| 2.2                   | 158.49       | 37840-1        | 41889-1        | 40453-1        | 43257-1        | 43213-1        | 44638-1        | 46127-1        | 46026-1        |
| 2.3                   | 199.53       | 33226-1        | 37160-1        | 35614-1        | 38480-1        | 38146-1        | 39819-1        | 40826-1        | 41173-1        |
| 2.4                   | 251.19       | 29176-1        | 32928-1        | 31354-1        | 34191-1        | 33671-1        | 35478-1        | 36131-1        | 36786-1        |
| 2.5                   | 316.23       | 25620-1        | 29149-1        | 27604-1        | 30350-1        | 29720-1        | 31578-1        | 31974-1        | 32831-1        |
| 2.6                   | 398.11       | 22500-1        | 25782-1        | 24303-1        | 26916-1        | 26232-1        | 28080-1        | 28295-1        | 29274-1        |
| 2.7                   | 501.19       | 19760-1        | 22787-1        | 21398-1        | 23852-1        | 23154-1        | 24949-1        | 25038-1        | 26080-1        |
| 2.8                   | 630.96       | 17356-1        | 20126-1        | 18841-1        | 21122-1        | 20438-1        | 22152-1        | 22155-1        | 23217-1        |
| 2.9                   | 794.33       | 15245-1        | 17766-1        | 16590-1        | 18693-1        | 18041-1        | 19656-1        | 19605-1        | 20654-1        |
| 3.0                   | 1000.0       | 13392-1        | 15674-1        | 14608-1        | 16534-1        | 15925-1        | 17431-1        | 17348-1        | 18364-1        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.52$   |                | $\beta=0.51$   |                | $\beta=0.50$   |                | $\beta=0.48$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 21637-0        | 50000-0        | 21172-0        | 50000-0        | 20711-0        | 50000-0        | 19796-0        |
| 0.1                   | 1.2589       | 46453-0        | 21545-0        | 46544-0        | 21086-0        | 46634-0        | 20630-0        | 46809-0        | 19727-0        |
| 0.2                   | 1.5849       | 42945-0        | 21273-0        | 43124-0        | 20832-0        | 43301-0        | 20393-0        | 43645-0        | 19519-0        |
| 0.3                   | 1.9953       | 39513-0        | 20831-0        | 39775-0        | 20417-0        | 40033-0        | 20005-0        | 40538-0        | 19181-0        |
| 0.4                   | 2.5119       | 36193-0        | 20233-0        | 36529-0        | 19857-0        | 36861-0        | 19479-0        | 37512-0        | 18721-0        |
| 0.5                   | 3.1623       | 33012-0        | 19500-0        | 33412-0        | 19167-0        | 33808-0        | 18832-0        | 34589-0        | 18152-0        |
| 0.6                   | 3.9811       | 29993-0        | 18654-0        | 30447-0        | 18370-0        | 30897-0        | 18082-0        | 31788-0        | 17489-0        |
| 0.7                   | 5.0119       | 27155-0        | 17721-0        | 27650-0        | 17488-0        | 28144-0        | 17248-0        | 29124-0        | 16750-0        |
| 0.8                   | 6.3096       | 24507-0        | 16723-0        | 25033-0        | 16542-0        | 25559-0        | 16352-0        | 26608-0        | 15949-0        |
| 0.9                   | 7.9433       | 22056-0        | 15686-0        | 22601-0        | 15555-0        | 23149-0        | 15414-0        | 24247-0        | 15106-0        |
| 1.0                   | 10.000       | 19801-0        | 14631-0        | 20356-0        | 14546-0        | 20915-0        | 14452-0        | 22044-0        | 14235-0        |
| 1.1                   | 12.589       | 17738-0        | 13576-0        | 18295-0        | 13535-0        | 18858-0        | 13484-0        | 20000-0        | 13351-0        |
| 1.2                   | 15.849       | 15862-0        | 12538-0        | 16413-0        | 12535-0        | 16972-0        | 12523-0        | 18112-0        | 12468-0        |
| 1.3                   | 19.953       | 14163-0        | 11530-0        | 14701-0        | 11561-0        | 15250-0        | 11583-0        | 16376-0        | 11597-0        |
| 1.4                   | 25.119       | 12629-0        | 10562-0        | 13150-0        | 10621-0        | 13684-0        | 10672-0        | 14785-0        | 10747-0        |
| 1.5                   | 31.623       | 11249-0        | 96416-1        | 11750-0        | 97246-1        | 12264-0        | 97999-1        | 13333-0        | 99259-1        |
| 1.6                   | 39.811       | 10011-0        | 87739-1        | 10489-0        | 88757-1        | 10982-0        | 89708-1        | 12010-0        | 91396-1        |
| 1.7                   | 50.119       | 89034-1        | 79619-1        | 93562-1        | 80782-1        | 98248-1        | 81889-1        | 10809-0        | 83921-1        |
| 1.8                   | 63.096       | 79134-1        | 72070-1        | 83404-1        | 73338-1        | 87838-1        | 74563-1        | 97208-1        | 76863-1        |
| 1.9                   | 79.433       | 70303-1        | 65090-1        | 74310-1        | 66430-1        | 78486-1        | 67738-1        | 87360-1        | 70237-1        |
| 2.0                   | 100.00       | 62434-1        | 58669-1        | 66179-1        | 60051-1        | 70097-1        | 61412-1        | 78466-1        | 64050-1        |
| 2.1                   | 125.89       | 55430-1        | 52788-1        | 58918-1        | 54187-1        | 62580-1        | 55575-1        | 70443-1        | 58299-1        |
| 2.2                   | 158.49       | 49200-1        | 47420-1        | 52439-1        | 48816-1        | 55851-1        | 50210-1        | 63215-1        | 52975-1        |
| 2.3                   | 199.53       | 43663-1        | 42539-1        | 46663-1        | 43915-1        | 49832-1        | 45297-1        | 56709-1        | 48064-1        |
| 2.4                   | 251.19       | 38744-1        | 38113-1        | 41515-1        | 39455-1        | 44453-1        | 40811-1        | 50858-1        | 43549-1        |
| 2.5                   | 316.23       | 34375-1        | 34109-1        | 36930-1        | 35408-1        | 39647-1        | 36727-1        | 45600-1        | 39410-1        |
| 2.6                   | 398.11       | 30497-1        | 30496-1        | 32848-1        | 31744-1        | 35356-1        | 33016-1        | 40876-1        | 35625-1        |
| 2.7                   | 501.19       | 27055-1        | 27241-1        | 29215-1        | 28433-1        | 31526-1        | 29653-1        | 36636-1        | 32171-1        |
| 2.8                   | 630.96       | 24001-1        | 24315-1        | 25982-1        | 25447-1        | 28108-1        | 26610-1        | 32831-1        | 29027-1        |
| 2.9                   | 794.33       | 21291-1        | 21689-1        | 23106-1        | 22758-1        | 25059-1        | 23862-1        | 29417-1        | 26169-1        |
| 3.0                   | 1000.0       | 18886-1        | 19334-1        | 20547-1        | 20340-1        | 22339-1        | 21383-1        | 26356-1        | 23575-1        |

| $\log_{10} \omega \tau$ | $\omega \tau$ | $\beta=0.46$     |                  | $\beta=0.45$     |                  | $\beta=0.44$     |                  | $\beta=0.42$     |                  |
|-------------------------|---------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                         |               | $\bar{C}'$       | $\bar{C}''$      | $\bar{C}'$       | $\bar{C}''$      | $\bar{C}'$       | $\bar{C}''$      | $\bar{C}'$       | $\bar{C}''$      |
|                         |               | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ | $C_0 - C_\infty$ |
| 0.0                     | 1.0000        | 50000-0          | 18893-0          | 50000-0          | 18446-0          | 50000-0          | 18001-0          | 50000-0          | 17119-0          |
| 0.1                     | 1.2589        | 46978-0          | 18833-0          | 47061-0          | 18390-0          | 47142-0          | 17949-0          | 47302-0          | 17074-0          |
| 0.2                     | 1.5849        | 43980-0          | 18653-0          | 44144-0          | 18223-0          | 44305-0          | 17794-0          | 44620-0          | 16941-0          |
| 0.3                     | 1.9953        | 41029-0          | 18359-0          | 41270-0          | 17949-0          | 41507-0          | 17540-0          | 41973-0          | 16723-0          |
| 0.4                     | 2.5119        | 38147-0          | 17959-0          | 38459-0          | 17576-0          | 38768-0          | 17193-0          | 39374-0          | 16424-0          |
| 0.5                     | 3.1623        | 35354-0          | 17461-0          | 35730-0          | 17112-0          | 36103-0          | 16760-0          | 36838-0          | 16050-0          |
| 0.6                     | 3.9811        | 32665-0          | 16879-0          | 33098-0          | 16568-0          | 33529-0          | 16252-0          | 34379-0          | 15609-0          |
| 0.7                     | 5.0119        | 30095-0          | 16226-0          | 30577-0          | 15956-0          | 31056-0          | 15679-0          | 32007-0          | 15110-0          |
| 0.8                     | 6.3096        | 27654-0          | 15516-0          | 28175-0          | 15288-0          | 28695-0          | 15052-0          | 29731-0          | 14561-0          |
| 0.9                     | 7.9433        | 25349-0          | 14762-0          | 25901-0          | 14577-0          | 26453-0          | 14383-0          | 27558-0          | 13971-0          |
| 1.0                     | 10.000        | 23184-0          | 13978-0          | 23758-0          | 13836-0          | 24335-0          | 13684-0          | 25493-0          | 13351-0          |
| 1.1                     | 12.589        | 21162-0          | 13178-0          | 21750-0          | 13076-0          | 22343-0          | 12964-0          | 23540-0          | 12709-0          |
| 1.2                     | 15.849        | 19282-0          | 12372-0          | 19877-0          | 12308-0          | 20478-0          | 12234-0          | 21700-0          | 12053-0          |
| 1.3                     | 19.953        | 17540-0          | 11571-0          | 18135-0          | 11542-0          | 18739-0          | 11503-0          | 19973-0          | 11392-0          |
| 1.4                     | 25.119        | 15932-0          | 10783-0          | 16523-0          | 10786-0          | 17124-0          | 10779-0          | 18357-0          | 10732-0          |
| 1.5                     | 31.623        | 14454-0          | 10017-0          | 15034-0          | 10048-0          | 15627-0          | 10069-0          | 16850-0          | 10080-0          |
| 1.6                     | 39.811        | 13098-0          | 92765-1          | 13664-0          | 93319-1          | 14245-0          | 93781-1          | 15449-0          | 94409-1          |
| 1.7                     | 50.119        | 11858-0          | 85675-1          | 12407-0          | 86436-1          | 12972-0          | 87112-1          | 14150-0          | 88193-1          |
| 1.8                     | 63.096        | 10726-0          | 78929-1          | 11255-0          | 79861-1          | 11801-0          | 80719-1          | 12948-0          | 82188-1          |
| 1.9                     | 79.433        | 96952-1          | 72547-1          | 10202-0          | 73618-1          | 10728-0          | 74624-1          | 11837-0          | 76421-1          |
| 2.0                     | 100.00        | 87576-1          | 66543-1          | 92419-1          | 67721-1          | 97459-1          | 68846-1          | 10814-0          | 70912-1          |
| 2.1                     | 125.89        | 79063-1          | 60920-1          | 83669-1          | 62179-1          | 88479-1          | 63395-1          | 98729-1          | 65676-1          |
| 2.2                     | 158.49        | 71343-1          | 55677-1          | 75708-1          | 56992-1          | 80283-1          | 58274-1          | 90081-1          | 60720-1          |
| 2.3                     | 199.53        | 64350-1          | 50806-1          | 68474-1          | 52155-1          | 72810-1          | 53482-1          | 82146-1          | 56048-1          |
| 2.4                     | 251.19        | 58021-1          | 46295-1          | 61907-1          | 47660-1          | 66006-1          | 49013-1          | 74874-1          | 51659-1          |
| 2.5                     | 316.23        | 52299-1          | 42131-1          | 55950-1          | 43497-1          | 59814-1          | 44858-1          | 68216-1          | 47550-1          |
| 2.6                     | 398.11        | 47128-1          | 38298-1          | 50552-1          | 39650-1          | 54186-1          | 41007-1          | 62127-1          | 43714-1          |
| 2.7                     | 501.19        | 42459-1          | 34777-1          | 45662-1          | 36105-1          | 49074-1          | 37445-1          | 56562-1          | 40143-1          |
| 2.8                     | 630.96        | 38245-1          | 31550-1          | 41237-1          | 32846-1          | 44432-1          | 34159-1          | 51481-1          | 36825-1          |
| 2.9                     | 794.33        | 34443-1          | 28598-1          | 37232-1          | 29854-1          | 40221-1          | 31133-1          | 46843-1          | 33750-1          |
| 3.0                     | 1000.0        | 31014-1          | 25903-1          | 33611-1          | 27114-1          | 36402-1          | 28353-1          | 42613-1          | 30905-1          |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.40$   |                | $\beta=0.38$   |                | $\beta=0.36$   |                | $\beta=0.35$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 16246-0        | 50000-0        | 15382-0        | 50000-0        | 14526-0        | 50000-0        | 14101-0        |
| 0.1                   | 1.2589       | 47457-0        | 16208-0        | 47607-0        | 15350-0        | 47754-0        | 14499-0        | 47826-0        | 14077-0        |
| 0.2                   | 1.5849       | 44928-0        | 16095-0        | 45227-0        | 15254-0        | 45518-0        | 14419-0        | 45662-0        | 14003-0        |
| 0.3                   | 1.9953       | 42426-0        | 15908-0        | 42869-0        | 15096-0        | 43301-0        | 14286-0        | 43514-0        | 13881-0        |
| 0.4                   | 2.5119       | 39966-0        | 15652-0        | 40546-0        | 14878-0        | 41113-0        | 14102-0        | 41392-0        | 13714-0        |
| 0.5                   | 3.1623       | 37559-0        | 15331-0        | 38266-0        | 14605-0        | 38961-0        | 13871-0        | 39303-0        | 13502-0        |
| 0.6                   | 3.9811       | 35216-0        | 14952-0        | 36041-0        | 14280-0        | 36854-0        | 13596-0        | 37255-0        | 13249-0        |
| 0.7                   | 5.0119       | 32948-0        | 14519-0        | 33878-0        | 13909-0        | 34799-0        | 13280-0        | 35255-0        | 12959-0        |
| 0.8                   | 6.3096       | 30761-0        | 14042-0        | 31785-0        | 13497-0        | 32802-0        | 12928-0        | 33307-0        | 12634-0        |
| 0.9                   | 7.9433       | 28663-0        | 13527-0        | 29767-0        | 13050-0        | 30869-0        | 12544-0        | 31419-0        | 12280-0        |
| 1.0                   | 10.000       | 26659-0        | 12981-0        | 27830-0        | 12575-0        | 29004-0        | 12134-0        | 29593-0        | 11900-0        |
| 1.1                   | 12.589       | 24752-0        | 12413-0        | 25977-0        | 12077-0        | 27212-0        | 11701-0        | 27833-0        | 11499-0        |
| 1.2                   | 15.849       | 22945-0        | 11829-0        | 24210-0        | 11561-0        | 25494-0        | 11251-0        | 26143-0        | 11080-0        |
| 1.3                   | 19.953       | 21238-0        | 11236-0        | 22532-0        | 11035-0        | 23854-0        | 10788-0        | 24524-0        | 10648-0        |
| 1.4                   | 25.119       | 19631-0        | 10640-0        | 20943-0        | 10502-0        | 22290-0        | 10317-0        | 22977-0        | 10207-0        |
| 1.5                   | 31.623       | 18122-0        | 10047-0        | 19441-0        | 99684-1        | 20805-0        | 98417-1        | 21503-0        | 97600-1        |
| 1.6                   | 39.811       | 16711-0        | 94617-1        | 18027-0        | 94375-1        | 19398-0        | 93657-1        | 20103-0        | 93112-1        |
| 1.7                   | 50.119       | 15392-0        | 88880-1        | 16699-0        | 89135-1        | 18067-0        | 88925-1        | 18775-0        | 88636-1        |
| 1.8                   | 63.096       | 14165-0        | 83295-1        | 15453-0        | 83996-1        | 16812-0        | 84251-1        | 17518-0        | 84199-1        |
| 1.9                   | 79.433       | 13024-0        | 77891-1        | 14288-0        | 78987-1        | 15630-0        | 79662-1        | 16331-0        | 79827-1        |
| 2.0                   | 100.00       | 11965-0        | 72691-1        | 13200-0        | 74131-1        | 14520-0        | 75180-1        | 15212-0        | 75542-1        |
| 2.1                   | 125.89       | 10985-0        | 67710-1        | 12186-0        | 69445-1        | 13478-0        | 70823-1        | 14159-0        | 71361-1        |
| 2.2                   | 158.49       | 10078-0        | 62961-1        | 11242-0        | 64944-1        | 12502-0        | 66608-1        | 13170-0        | 67302-1        |
| 2.3                   | 199.53       | 92408-1        | 58452-1        | 10365-0        | 60638-1        | 11590-0        | 62545-1        | 12242-0        | 63375-1        |
| 2.4                   | 251.19       | 84687-1        | 54185-1        | 95504-1        | 56534-1        | 10738-0        | 58645-1        | 11373-0        | 59591-1        |
| 2.5                   | 316.23       | 77574-1        | 50161-1        | 87957-1        | 52635-1        | 99430-1        | 54912-1        | 10559-0        | 55957-1        |
| 2.6                   | 398.11       | 71029-1        | 46377-1        | 80969-1        | 48943-1        | 92024-1        | 51352-1        | 97992-1        | 52478-1        |
| 2.7                   | 501.19       | 65010-1        | 42829-1        | 74504-1        | 45456-1        | 85131-1        | 47966-1        | 90894-1        | 49157-1        |
| 2.8                   | 630.96       | 59482-1        | 39511-1        | 68530-1        | 42172-1        | 78721-1        | 44754-1        | 84275-1        | 45995-1        |
| 2.9                   | 794.33       | 54406-1        | 36414-1        | 63012-1        | 39086-1        | 72766-1        | 41714-1        | 78106-1        | 42992-1        |
| 3.0                   | 1000.0       | 49750-1        | 33530-1        | 57920-1        | 36192-1        | 67237-1        | 38844-1        | 72362-1        | 40147-1        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.34$   |                | $\beta=0.32$   |                | $\beta=0.30$   |                | $\beta=0.25$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 13678-0        | 50000-0        | 12838-0        | 50000-0        | 12004-0        | 50000-0        | 99456-1        |
| 0.1                   | 1.2589       | 47898-0        | 13656-0        | 48038-0        | 12819-0        | 48174-0        | 11989-0        | 48504-0        | 99371-1        |
| 0.2                   | 1.5849       | 45803-0        | 13589-0        | 46081-0        | 12764-0        | 46354-0        | 11944-0        | 47012-0        | 99114-1        |
| 0.3                   | 1.9953       | 43724-0        | 13478-0        | 44138-0        | 12672-0        | 44543-0        | 11869-0        | 45524-0        | 98689-1        |
| 0.4                   | 2.5119       | 41668-0        | 13324-0        | 42213-0        | 12545-0        | 42748-0        | 11765-0        | 44045-0        | 98098-1        |
| 0.5                   | 3.1623       | 39643-0        | 13131-0        | 40313-0        | 12385-0        | 40973-0        | 11633-0        | 42577-0        | 97346-1        |
| 0.6                   | 3.9811       | 37654-0        | 12899-0        | 38443-0        | 12192-0        | 39221-0        | 11475-0        | 41122-0        | 96437-1        |
| 0.7                   | 5.0119       | 35709-0        | 12633-0        | 36608-0        | 11970-0        | 37499-0        | 11292-0        | 39683-0        | 95377-1        |
| 0.8                   | 6.3096       | 33811-0        | 12335-0        | 34814-0        | 11721-0        | 35808-0        | 11086-0        | 38262-0        | 94173-1        |
| 0.9                   | 7.9433       | 31968-0        | 12009-0        | 33063-0        | 11447-0        | 34154-0        | 10858-0        | 36862-0        | 92834-1        |
| 1.0                   | 10.000       | 30182-0        | 11658-0        | 31360-0        | 11150-0        | 32540-0        | 10611-0        | 35484-0        | 91368-1        |
| 1.1                   | 12.589       | 28457-0        | 11287-0        | 29709-0        | 10835-0        | 30967-0        | 10347-0        | 34131-0        | 89783-1        |
| 1.2                   | 15.849       | 26795-0        | 10898-0        | 28111-0        | 10503-0        | 29439-0        | 10068-0        | 32804-0        | 88090-1        |
| 1.3                   | 19.953       | 25200-0        | 10496-0        | 26569-0        | 10158-0        | 27958-0        | 97756-1        | 31504-0        | 86299-1        |
| 1.4                   | 25.119       | 23672-0        | 10084-0        | 25084-0        | 98027-1        | 26525-0        | 94730-1        | 30234-0        | 84419-1        |
| 1.5                   | 31.623       | 22212-0        | 96657-1        | 23658-0        | 94393-1        | 25142-0        | 91619-1        | 28994-0        | 82460-1        |
| 1.6                   | 39.811       | 20820-0        | 92440-1        | 22291-0        | 90707-1        | 23809-0        | 88444-1        | 27786-0        | 80434-1        |
| 1.7                   | 50.119       | 19497-0        | 88220-1        | 20984-0        | 86993-1        | 22528-0        | 85225-1        | 26610-0        | 78349-1        |
| 1.8                   | 63.096       | 18240-0        | 84022-1        | 19736-0        | 83275-1        | 21297-0        | 81981-1        | 25467-0        | 76217-1        |
| 1.9                   | 79.433       | 17051-0        | 79871-1        | 18547-0        | 79572-1        | 20119-0        | 78729-1        | 24358-0        | 74047-1        |
| 2.0                   | 100.00       | 15926-0        | 75787-1        | 17416-0        | 75904-1        | 18991-0        | 75486-1        | 23283-0        | 71848-1        |
| 2.1                   | 125.89       | 14864-0        | 71790-1        | 16342-0        | 72288-1        | 17914-0        | 72266-1        | 22242-0        | 69630-1        |
| 2.2                   | 158.49       | 13863-0        | 67893-1        | 15324-0        | 68738-1        | 16887-0        | 69083-1        | 21236-0        | 67401-1        |
| 2.3                   | 199.53       | 12921-0        | 64110-1        | 14360-0        | 65267-1        | 15909-0        | 65949-1        | 20264-0        | 65169-1        |
| 2.4                   | 251.19       | 12036-0        | 60452-1        | 13449-0        | 61885-1        | 14979-0        | 62874-1        | 19327-0        | 62941-1        |
| 2.5                   | 316.23       | 11205-0        | 56926-1        | 12588-0        | 58603-1        | 14095-0        | 59868-1        | 18423-0        | 60725-1        |
| 2.6                   | 398.11       | 10427-0        | 53538-1        | 11776-0        | 55426-1        | 13256-0        | 56937-1        | 17554-0        | 58528-1        |
| 2.7                   | 501.19       | 96972-1        | 50292-1        | 11011-0        | 52360-1        | 12461-0        | 54089-1        | 16718-0        | 56355-1        |
| 2.8                   | 630.96       | 90150-1        | 47191-1        | 10291-0        | 49410-1        | 11708-0        | 51328-1        | 15914-0        | 54211-1        |
| 2.9                   | 794.33       | 83773-1        | 44235-1        | 96137-1        | 46578-1        | 10996-0        | 48659-1        | 15143-0        | 52101-1        |
| 3.0                   | 1000.0       | 77817-1        | 41425-1        | 89776-1        | 43866-1        | 10323-0        | 46085-1        | 14403-0        | 50031-1        |

| $\log_{10}\omega\tau$ | $\omega\tau$ | $\beta=0.20$   |                | $\beta=0.15$   |                | $\beta=0.10$   |                | $\beta=0.05$   |                |
|-----------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                       |              | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    | $\bar{C}'$     | $\bar{C}''$    |
|                       |              | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ | $C_0-C_\infty$ |
| 0.0                   | 1.0000       | 50000-0        | 79192-1        | 50000-0        | 59179-1        | 50000-0        | 39351-1        | 50000-0        | 19645-1        |
| 0.1                   | 1.2589       | 48820-0        | 79149-1        | 49125-0        | 59161-1        | 49421-0        | 39346-1        | 49712-0        | 19644-1        |
| 0.2                   | 1.5849       | 47641-0        | 79020-1        | 48250-0        | 59107-1        | 48842-0        | 39330-1        | 49423-0        | 19642-1        |
| 0.3                   | 1.9953       | 46466-0        | 78806-1        | 47376-0        | 59018-1        | 48263-0        | 39304-1        | 49135-0        | 19639-1        |
| 0.4                   | 2.5119       | 45294-0        | 78508-1        | 46504-0        | 58893-1        | 47685-0        | 39267-1        | 48847-0        | 19635-1        |
| 0.5                   | 3.1623       | 44127-0        | 78126-1        | 45633-0        | 58734-1        | 47107-0        | 39220-1        | 48559-0        | 19629-1        |
| 0.6                   | 3.9811       | 42967-0        | 77663-1        | 44766-0        | 58539-1        | 46530-0        | 39163-1        | 48271-0        | 19622-1        |
| 0.7                   | 5.0119       | 41815-0        | 77121-1        | 43902-0        | 58311-1        | 45954-0        | 39095-1        | 47983-0        | 19613-1        |
| 0.8                   | 6.3096       | 40672-0        | 76501-1        | 43042-0        | 58048-1        | 45380-0        | 39017-1        | 47695-0        | 19603-1        |
| 0.9                   | 7.9433       | 39539-0        | 75807-1        | 42185-0        | 57753-1        | 44806-0        | 38929-1        | 47408-0        | 19592-1        |
| 1.0                   | 10.000       | 38417-0        | 75041-1        | 41334-0        | 57425-1        | 44234-0        | 38831-1        | 47121-0        | 19580-1        |
| 1.1                   | 12.589       | 37308-0        | 74207-1        | 40488-0        | 57066-1        | 43663-0        | 38723-1        | 46833-0        | 19566-1        |
| 1.2                   | 15.849       | 36213-0        | 73307-1        | 39647-0        | 56675-1        | 43094-0        | 38605-1        | 46546-0        | 19551-1        |
| 1.3                   | 19.953       | 35132-0        | 72346-1        | 38813-0        | 56255-1        | 42527-0        | 38477-1        | 46260-0        | 19535-1        |
| 1.4                   | 25.119       | 34066-0        | 71327-1        | 37985-0        | 55806-1        | 41962-0        | 38340-1        | 45973-0        | 19518-1        |
| 1.5                   | 31.623       | 33016-0        | 70254-1        | 37164-0        | 55329-1        | 41399-0        | 38193-1        | 45687-0        | 19499-1        |
| 1.6                   | 39.811       | 31984-0        | 69131-1        | 36351-0        | 54825-1        | 40838-0        | 38037-1        | 45401-0        | 19479-1        |
| 1.7                   | 50.119       | 30969-0        | 67961-1        | 35545-0        | 54296-1        | 40279-0        | 37872-1        | 45115-0        | 19458-1        |
| 1.8                   | 63.096       | 29972-0        | 66750-1        | 34748-0        | 53741-1        | 39724-0        | 37698-1        | 44830-0        | 19435-1        |
| 1.9                   | 79.433       | 28995-0        | 65500-1        | 33959-0        | 53164-1        | 39170-0        | 37516-1        | 44545-0        | 19412-1        |
| 2.0                   | 100.00       | 28037-0        | 64217-1        | 33179-0        | 52564-1        | 38620-0        | 37324-1        | 44260-0        | 19387-1        |
| 2.1                   | 125.89       | 27099-0        | 62903-1        | 32409-0        | 51943-1        | 38072-0        | 37124-1        | 43976-0        | 19360-1        |
| 2.2                   | 158.49       | 26181-0        | 61564-1        | 31648-0        | 51302-1        | 37528-0        | 36916-1        | 43692-0        | 19333-1        |
| 2.3                   | 199.53       | 25284-0        | 60203-1        | 30896-0        | 50642-1        | 36986-0        | 36700-1        | 43408-0        | 19304-1        |
| 2.4                   | 251.19       | 24408-0        | 58825-1        | 30155-0        | 49966-1        | 36448-0        | 36477-1        | 43125-0        | 19274-1        |
| 2.5                   | 316.23       | 23553-0        | 57432-1        | 29424-0        | 49273-1        | 35914-0        | 36245-1        | 42843-0        | 19243-1        |
| 2.6                   | 398.11       | 22719-0        | 56028-1        | 28704-0        | 48565-1        | 35382-0        | 36006-1        | 42561-0        | 19211-1        |
| 2.7                   | 501.19       | 21907-0        | 54617-1        | 27994-0        | 47844-1        | 34855-0        | 35761-1        | 42279-0        | 19177-1        |
| 2.8                   | 630.96       | 21116-0        | 53203-1        | 27296-0        | 47111-1        | 34331-0        | 35508-1        | 41998-0        | 19143-1        |
| 2.9                   | 794.33       | 20347-0        | 51787-1        | 26608-0        | 46367-1        | 33811-0        | 35248-1        | 41717-0        | 19107-1        |
| 3.0                   | 1000.0       | 19598-0        | 50375-1        | 25932-0        | 45613-1        | 33295-0        | 34982-1        | 41437-0        | 19070-1        |

Table 2. Numerical Data of Real and Imaginary Part of the Complex Conductance for the Circular Arc Rule.

| $\ln \omega \tau$ | $\omega \tau$ | $\beta=1.00$                        |                                      | $\beta=0.90$                        |                                      | $\beta=0.80$                        |                                      |
|-------------------|---------------|-------------------------------------|--------------------------------------|-------------------------------------|--------------------------------------|-------------------------------------|--------------------------------------|
|                   |               | $\frac{\bar{G}'\tau}{C_0-C_\infty}$ | $\frac{\bar{G}''\tau}{C_0-C_\infty}$ | $\frac{\bar{G}'\tau}{C_0-C_\infty}$ | $\frac{\bar{G}''\tau}{C_0-C_\infty}$ | $\frac{\bar{G}'\tau}{C_0-C_\infty}$ | $\frac{\bar{G}''\tau}{C_0-C_\infty}$ |
|                   |               |                                     |                                      |                                     |                                      |                                     |                                      |
| -4.5              | 0.01111       | 0.00012                             | 0.01111                              | 0.00019                             | 0.01108                              | 0.00028                             | 0.01101                              |
| -4.0              | 0.01832       | 0.00034                             | 0.01831                              | 0.00049                             | 0.01823                              | 0.00069                             | 0.01807                              |
| -3.5              | 0.03020       | 0.00091                             | 0.03017                              | 0.00126                             | 0.02994                              | 0.00168                             | 0.02954                              |
| -3.0              | 0.04979       | 0.00247                             | 0.04966                              | 0.00322                             | 0.04906                              | 0.00404                             | 0.04809                              |
| -2.5              | 0.08209       | 0.00669                             | 0.08154                              | 0.00818                             | 0.07992                              | 0.00959                             | 0.07761                              |
| -2.0              | 0.1353        | 0.01799                             | 0.1329                               | 0.02048                             | 0.1287                               | 0.02230                             | 0.1234                               |
| -1.5              | 0.2231        | 0.04743                             | 0.2126                               | 0.04975                             | 0.2022                               | 0.05006                             | 0.1910                               |
| -1.0              | 0.3679        | 0.1192                              | 0.3240                               | 0.1143                              | 0.3027                               | 0.1063                              | 0.2832                               |
| -0.5              | 0.6065        | 0.2689                              | 0.4434                               | 0.2378                              | 0.4153                               | 0.2075                              | 0.3929                               |
| -0.25             | 0.7788        | 0.3775                              | 0.4848                               | 0.3254                              | 0.4642                               | 0.2786                              | 0.4484                               |
| +0.00             | 1.000         | 0.5000                              | 0.5000                               | 0.4270                              | 0.5000                               | 0.3633                              | 0.5000                               |
| +0.25             | 1.284         | 0.6225                              | 0.4848                               | 0.5365                              | 0.5188                               | 0.4594                              | 0.5448                               |
| +0.5              | 1.649         | 0.7311                              | 0.4434                               | 0.6465                              | 0.5198                               | 0.5640                              | 0.5808                               |
| +1.0              | 2.718         | 0.8808                              | 0.3240                               | 0.8445                              | 0.4814                               | 0.7851                              | 0.6260                               |
| +1.5              | 4.482         | 0.9526                              | 0.2126                               | 0.9993                              | 0.4206                               | 1.005                               | 0.6451                               |
| +2.0              | 7.389         | 0.9820                              | 0.1329                               | 1.118                               | 0.3642                               | 1.217                               | 0.6540                               |
| +2.5              | 12.18         | 0.9933                              | 0.08154                              | 1.215                               | 0.3220                               | 1.423                               | 0.6649                               |
| +3.0              | 20.09         | 0.9975                              | 0.04966                              | 1.300                               | 0.2944                               | 1.628                               | 0.6844                               |
| +3.5              | 33.12         | 0.9991                              | 0.03017                              | 1.381                               | 0.2786                               | 1.839                               | 0.7152                               |
| +4.0              | 54.60         | 0.9997                              | 0.01831                              | 1.460                               | 0.2716                               | 2.061                               | 0.7581                               |
| +4.5              | 90.02         | 0.9999                              | 0.01111                              | 1.540                               | 0.2711                               | 2.299                               | 0.8129                               |
| +5.0              | 148.4         | 0.9999                              | 0.00672                              | 1.623                               | 0.2752                               |                                     |                                      |
| +6.0              | 403.4         |                                     |                                      | 1.797                               | 0.2928                               |                                     |                                      |
| +7.0              | 1096.6        |                                     |                                      | 1.988                               | 0.3185                               |                                     |                                      |

| $\ln \omega \tau$ | $\omega \tau$ | $\beta=0.70$   |                 | $\beta=0.60$   |                 | $\beta=0.40$   |                 | $\beta=0.20$   |                 |
|-------------------|---------------|----------------|-----------------|----------------|-----------------|----------------|-----------------|----------------|-----------------|
|                   |               | $\bar{G}'\tau$ | $\bar{G}''\tau$ | $\bar{G}'\tau$ | $\bar{G}''\tau$ | $\bar{G}'\tau$ | $\bar{G}''\tau$ | $\bar{G}'\tau$ | $\bar{G}''\tau$ |
|                   |               | $C_0-C_\infty$ | $C_0-C_\infty$  | $C_0-C_\infty$ | $C_0-C_\infty$  | $C_0-C_\infty$ | $C_0-C_\infty$  | $C_0-C_\infty$ | $C_0-C_\infty$  |
| -4.5              | 0.01111       | 0.00041        | 0.01088         | 0.00056        | 0.01066         | 0.00083        | 0.00973         | 0.00072        | 0.00795         |
| -4.0              | 0.01832       | 0.00094        | 0.01778         | 0.00121        | 0.01731         | 0.00159        | 0.01559         | 0.00124        | 0.01272         |
| -3.5              | 0.03020       | 0.00214        | 0.02890         | 0.00258        | 0.02793         | 0.00300        | 0.02481         | 0.00212        | 0.02029         |
| -3.0              | 0.04979       | 0.00482        | 0.04667         | 0.00545        | 0.04471         | 0.00559        | 0.03924         | 0.00360        | 0.03231         |
| -2.5              | 0.08209       | 0.01070        | 0.07455         | 0.01129        | 0.07077         | 0.01025        | 0.06155         | 0.00610        | 0.05133         |
| -2.0              | 0.1353        | 0.02315        | 0.1171          | 0.02283        | 0.1103          | 0.01853        | 0.09567         | 0.01029        | 0.08135         |
| -1.5              | 0.2231        | 0.04831        | 0.1795          | 0.04466        | 0.1682          | 0.03288        | 0.1472          | 0.01727        | 0.1286          |
| -1.0              | 0.3679        | 0.09589        | 0.2656          | 0.08392        | 0.2500          | 0.05720        | 0.2239          | 0.02884        | 0.2027          |
| -0.5              | 0.6065        | 0.1783         | 0.3747          | 0.1502         | 0.3598          | 0.09746        | 0.3367          | 0.04791        | 0.3188          |
| -0.25             | 0.7788        | 0.2361         | 0.4360          | 0.1970         | 0.4261          | 0.1262         | 0.4109          | 0.06164        | 0.3994          |
| +0.00             | 1.000         | 0.3064         | 0.5000          | 0.2548         | 0.5000          | 0.1625         | 0.5000          | 0.07919        | 0.5000          |
| +0.25             | 1.284         | 0.3893         | 0.5652          | 0.3248         | 0.5816          | 0.2080         | 0.6065          | 0.1016         | 0.6255          |
| +0.5              | 1.649         | 0.4845         | 0.6301          | 0.4084         | 0.6706          | 0.2649         | 0.7336          | 0.1302         | 0.7821          |
| +1.0              | 2.718         | 0.7085         | 0.7559          | 0.6201         | 0.8712          | 0.4227         | 1.064           | 0.2131         | 1.220           |
| +1.5              | 4.482         | 0.9703         | 0.8754          | 0.8971         | 1.103           | 0.6604         | 1.526           | 0.3469         | 1.899           |
| +2.0              | 7.389         | 1.264          | 0.9937          | 1.246          | 1.369           | 1.012          | 2.166           | 0.5618         | 2.948           |
| +2.5              | 12.18         | 1.588          | 1.119           | 1.676          | 1.680           | 1.522          | 3.048           | 0.9056         | 4.565           |
| +3.0              | 20.09         | 1.946          | 1.259           | 2.199          | 2.047           | 2.253          | 4.256           | 1.453          | 7.050           |
| +3.5              | 33.12         | 2.345          | 1.422           | 2.831          | 2.485           | 3.288          | 5.905           | 2.319          | 10.86           |
| +4.0              | 54.60         | 2.794          | 1.614           | 3.594          | 3.014           | 4.738          | 8.149           | 3.686          | 16.71           |
| +4.5              | 90.02         | 3.302          | 1.842           | 4.517          | 3.657           | 6.755          | 11.20           | 5.834          | 25.63           |

# Numerical Data of Complex Capacitance and Conductance

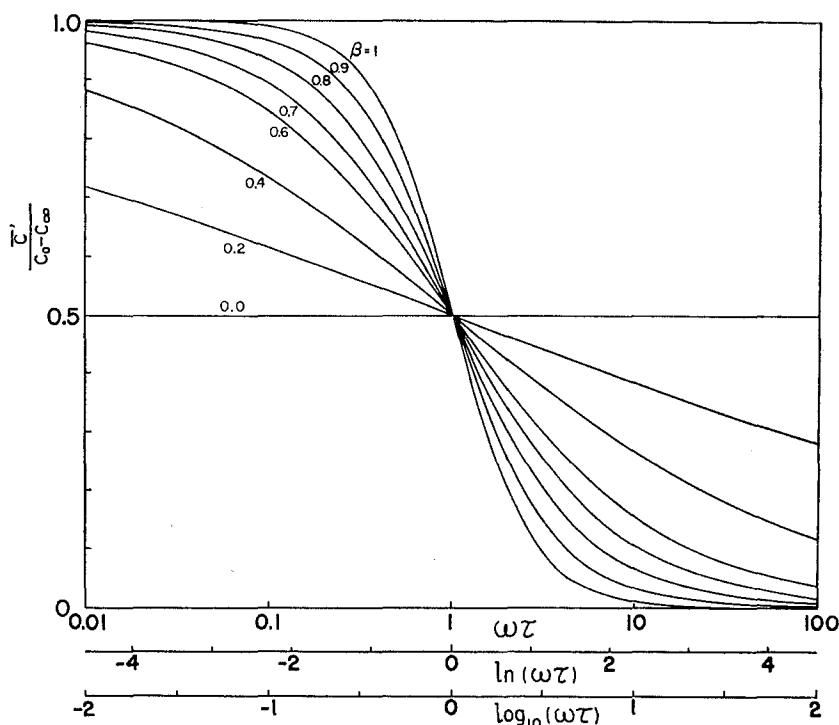


Fig. 1. The real part of the complex capacitance for the circular arc rule as a function of frequency. Numbers beside the curves denote the values of  $\beta$ .

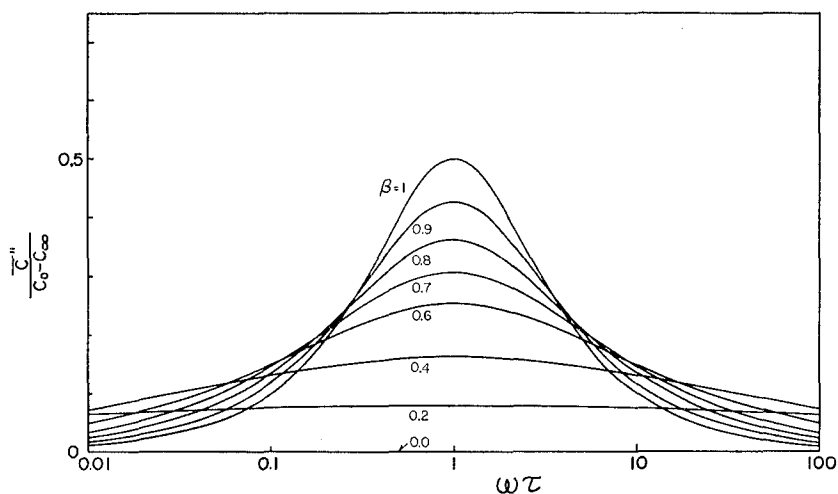


Fig. 2. The imaginary part of the complex capacitance for the circular arc rule as a function of frequency. Numbers beside the curves denote the values of  $\beta$ .

## IV. DISCUSSIONS

### A. The Real and the Imaginary Part of the Complex Capacitance

The behavior of the complex capacitance given in Table 1 and shown in

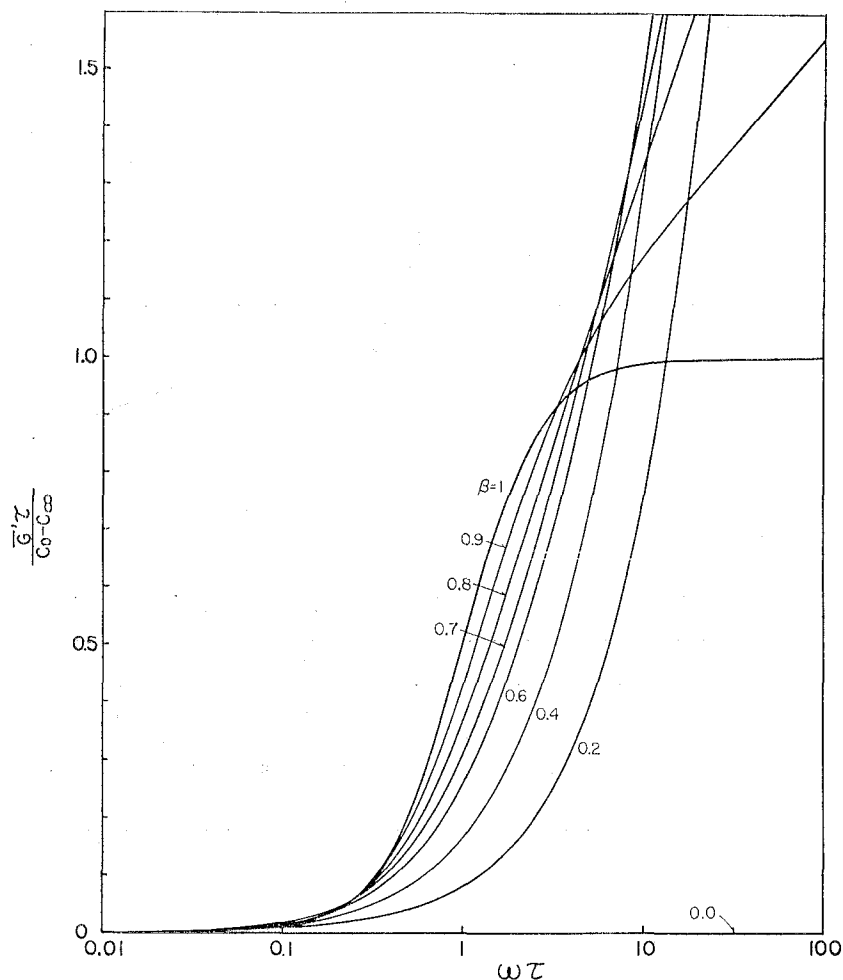


Fig. 3. The real part of the complex conductance for the circular arc rule as a function of frequency. Numbers beside the curves denote the value of  $\beta$ .

Figs. 1, 2 and 5 is the same as has been known so far. The numerical data of complex capacitance are useful for accurate determination of the distribution parameter  $\beta$  in the system with a wide distribution of relaxation times<sup>5)</sup> and also for facilitating the analysis of the system with multiple relaxation regions.<sup>6)</sup>

#### B. The Frequency Dependence of the Real and the Imaginary Part of the Complex Conductance

As seen in Fig. 3, the real part  $G'$  of the complex conductance for  $\beta=1$  increases markedly in the  $\omega\tau$ -range of 0.1 to 10, and tends to  $(C_0 - C_\infty)/\tau$  at higher frequencies. In the case of  $\beta < 1$ , the conductance increases infinitely at higher frequencies.

The frequency dependence of the imaginary part  $G''$  of the complex conductance, which has been discussed very little so far, is seen in Fig. 4. In the

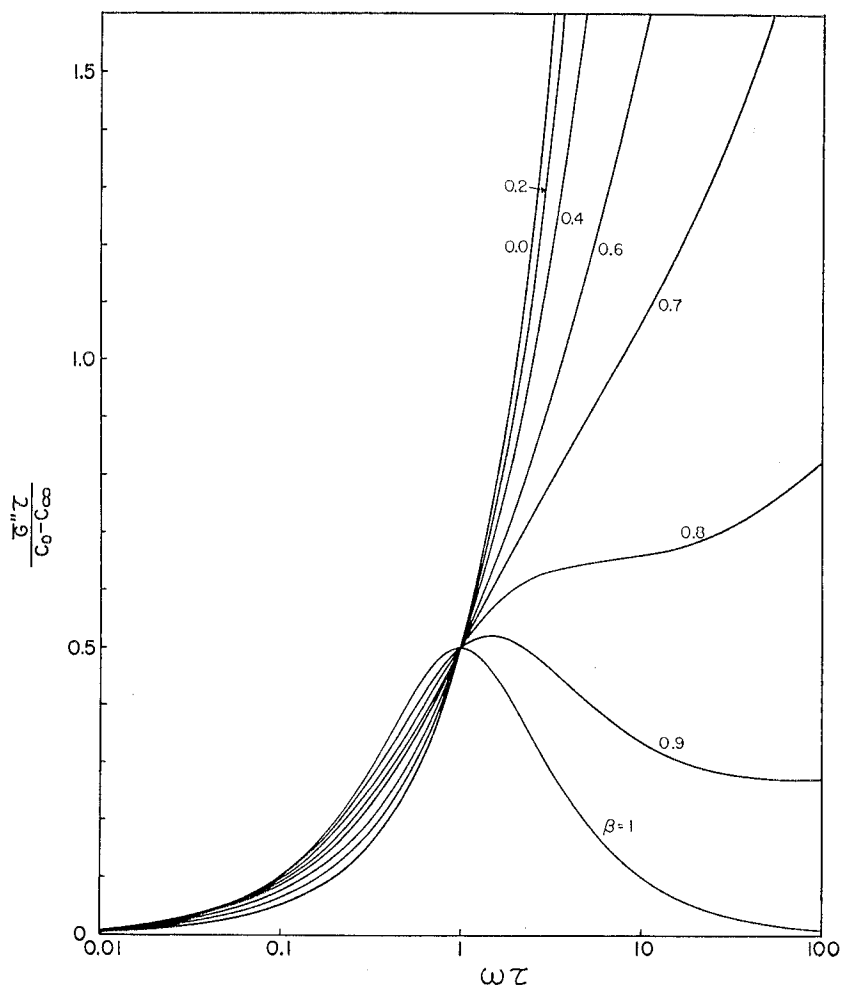


Fig. 4. The imaginary part of the complex conductance for the circular arc rule as a function of frequency. Numbers beside the curves denote the value of  $\beta$ .

case of  $\beta=1$ ,  $G''$  attains its maximum value  $(C_0 - C_\infty)/2\tau$  at  $\omega\tau=1$ , and decreases monotonously to zero at higher frequencies. The curve of  $G''$  against  $\ln(\omega\tau)$  is symmetrical with respect to an axis  $\omega\tau=1$ . In the case of  $\beta<1$ , it is worthy of being noted that  $G''$  does not tend to zero at the higher frequency side. For  $\beta=0.9$ ,  $G''$  shows a maximum value at  $\omega\tau=1.46$ , then decreases temporarily to a certain minimum value with increasing frequency, and finally increases infinitely at the higher frequency side. For  $\beta<0.810$ ,  $G''$  increases with increasing frequency over the whole range of frequency.

### C. The Complex Plane Plots of Conductance

Figure 6 shows the complex plane plots of the complex conductance for respective values of  $\beta$ . The plot for  $\beta=1$  gives a semicircle. For  $\beta<1$  the complex conductance plots deviate from a semicircle. For  $\beta=0.9$  the imaginary part of the admittance plot shows a maximum value at the frequency given by  $\omega\tau=$

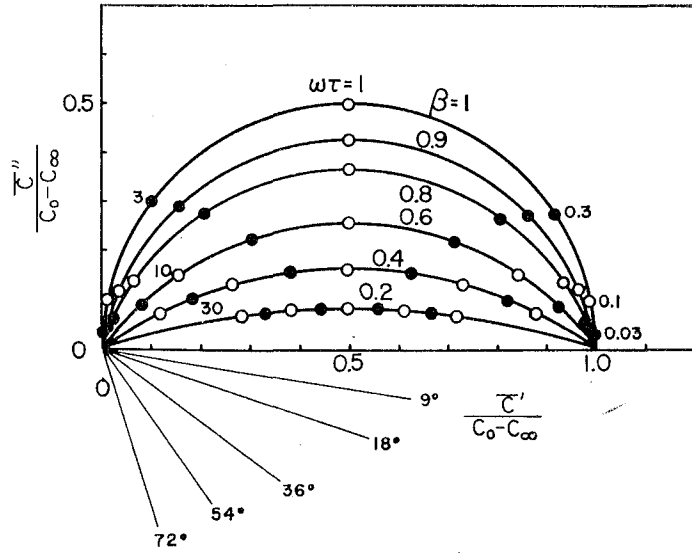


Fig. 5. Complex plane plots of the complex capacitance for the circular arc rule. Numbers beside the curves indicate the values of  $\omega\tau$  and  $\beta$ .

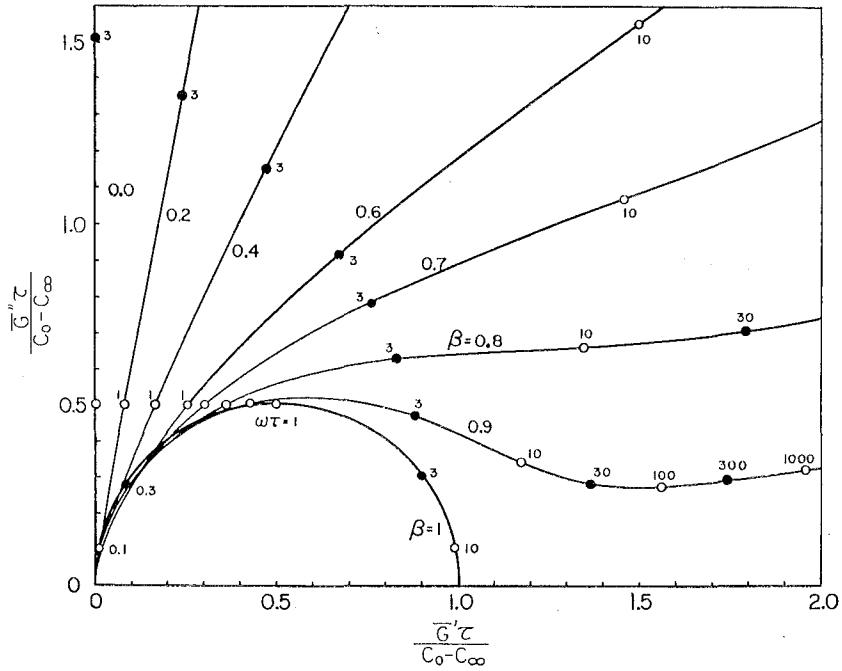


Fig. 6. Complex plane plots of the complex conductance associated with the circular arc rule. Numbers beside the curves indicate the values of  $\omega\tau$  and  $\beta$ .

1.46. At higher frequencies the plots deviate markedly from a semicircle. In the case of  $\beta < 0.810$ ,  $\bar{G}''$  increases monotonously with the increase of  $\bar{G}'$ , resulting in the remarkable deviation of complex conductance plots from a semicircle or a circular arc.

#### D. Some Comments on the Analysis of the Data Characterized by the Circular Arc Rule

It is readily seen in Fig. 3 that the conductance increases infinitely at higher frequencies so far as  $\beta < 1$  which corresponds to the cases of circular arcs.

Pauly *et al.* reported the dielectric dispersions of suspensions of guinea pig heart mitochondria<sup>3)</sup> and of rat liver mitochondria.<sup>4)</sup> The main dispersion of the dielectric constant observed by them gave a circular arc in the complex conductance plane, and were understood reasonably to be attributed to the heterogeneous structure characteristic of a suspension of mitochondrial globules covered with less conductive membranes. Moreover they observed a further rise of the conductivity at the frequencies about ten times as high as the critical frequency of the main dispersion. They suggested that this rise of the conductivity is the beginning of another dielectric dispersion due to the heterogeneous structure inside the mitochondrial globules. According to Fig. 3, however, such a rise of the conductivity at higher frequencies ( $\omega\tau > 10$ ) is readily interpretable as the property concomitant with the circular arc rule without any other dispersion mechanism.

In the papers aforementioned<sup>3,4)</sup> the critical frequency for the conductivity or the conductance dispersion was not identical with that for the dielectric or the capacitance dispersion. In view of this discrepancy Pauly *et al.* suggested that a reasonable approximation for the experimental determination of the critical frequency  $f_0$  may be the geometrical mean of  $f_{0\epsilon}$  and  $f_{0\kappa}$ , that is,

$$f_0 = \sqrt{f_{0\epsilon} \cdot f_{0\kappa}} \quad (28)$$

where  $f_{0\epsilon}$  and  $f_{0\kappa}$  are the frequencies at which the dielectric constant  $\epsilon(f)$  and the conductivity  $\kappa(f)$  are given by

$$\frac{\epsilon(f_{0\epsilon}) - \epsilon_\infty}{\epsilon_0 - \epsilon_\infty} = \frac{1}{2} \quad (29)$$

and

$$\frac{\kappa(f_{0\kappa}) - \kappa_0}{\kappa_\infty - \kappa_0} = \frac{1}{2} \quad (30)$$

respectively.

In the case of the circular arc rule, however,  $\kappa_\infty$  does not necessarily take finite values at high frequencies as seen in Fig. 3. It thus turns out impossible to determine the value of  $f_{0\epsilon}$  by means of Eq. 30 and to evaluate  $f_0$  from Eq. 28. For the systems showing a circular arc it is concluded that the critical frequency  $f_0$  should be taken to be identical with  $f_{0\epsilon}$  determined by Eq. 29 regardless of the conductivity dispersion.

#### REFERENCES

- (1) K. S. Cole and R. H. Cole, *J. Chem. Phys.*, **9**, 341 (1941).
- (2) K. S. Cole and R. H. Cole, *J. Chem. Phys.*, **10**, 98 (1942).
- (3) H. Pauly and L. Packer, *J. Biophys. Biochem. Cytology*, **7**, 603 (1960).
- (4) H. Pauly, L. Packer and H. P. Schwan, *J. Biophys. Biochem. Cytology*, **7**, 589 (1960).
- (5) Naokazu Koizumi and Shinichi Yano, *Bull. Inst. Chem. Res., Kyoto Univ.*, **47**, 320 (1969).
- (6) Eiji Ikada, *Bull. Inst. Chem. Res., Kyoto Univ.*, **45**, 352 (1967).