

Errata to “Internal Waves in Lake Biwa (I) — The Responses of the Thermocline to Wind Action —”

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Corrections of some numerical errors appearing in the paper “Internal Waves in Lake Biwa (I)” presented in the Bulletin, Vol. 19, Feb. 1970

Errata

Page 26, line 3 change $\frac{B}{2r} = 6.6$ to $\frac{B}{2r} = 42.0$

Page 26, line 7 change about 2.2 to about 6.6

Fortunately, in spite of these errors the fundamental results of this study remain unaffected.

Internal Waves in Lake Biwa (I)

—The Responses of the Thermocline to the Wind Action—

By Seiichi KANARI

(Manuscript received October 31, 1969)

Abstract

In the recent preliminary observation of the internal waves in Lake Biwa, the longitudinal inclination of the thermocline due to the wind with the north component and the initial stage of the free oscillation after the wind has ceased, are observed. The period of the first swing was about 56 hours and this period just coincides with the one calculated for a simplified two layer lake.

Another small periodic change with the period of 10 hr was also observed. And it is concluded that this oscillation corresponds to the transverse internal seiches under geostrophic effects.

1. Introduction

In many lakes, the vertical gradients of density are associated with only the

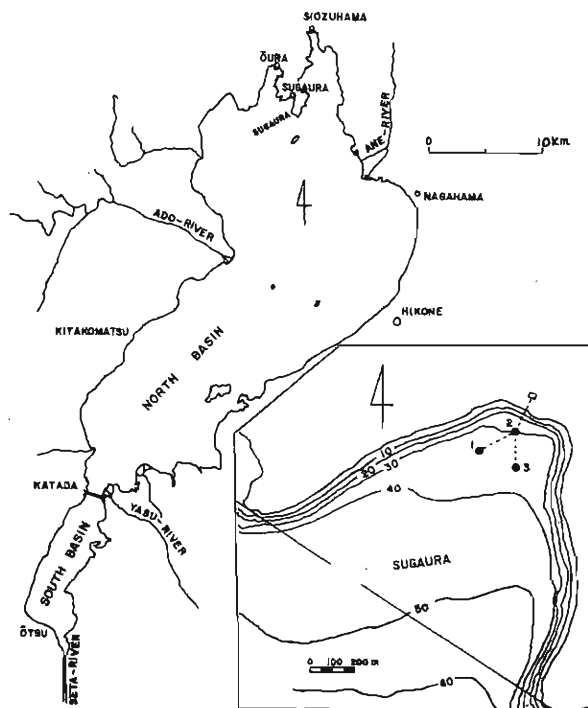


Fig. 1 Chart of Lake Biwa and Sugaura Bay. The positions of the three subsurface buoy-stations are shown by black circles.

vertical gradients of temperature, and are particularly large in the thermocline-layer. Consequently, internal oscillations involve oscillations of temperature at a fixed depth. From observations of these oscillations of temperature, and from a knowledge of the vertical gradients, it is possible to calculate the amplitude of the oscillations of any particular layer of water. We had a plan for the observation of the internal waves in Lake Biwa using three multi-layered subsurface buoy-stations in 1969. For this purpose, the preliminary observation was carried out during the period from September 17 to 23, 1968, with three single-layered subsurface buoy-stations at Sugaura in the northern part of Lake Biwa.

During the preliminary observation, we confirmed the fact that the longitudinal internal seiches in Lake Biwa are induced by wind with a northerly component.

It has been supposed that the longitudinal internal seiches in Lake Biwa may be induced by wind²⁾. However, this has not been confirmed up to the recent day.

Sugaura-bay (Fig. 1) is situated at the north end of Lake Biwa, and is open towards the south end of the lake. The mean depth of this bay is about 34 meters, and the maximum depth (50 m) is found at the mouth of this bay, which is the same as the mean depth of the whole north basin.

This bay is the most suitable for buoy-stations (Our buoy-stations need electric cables between the buoy and the recorder on the shore.), because large depths can be obtained at short distances off the shore.

The three stations were set in this bay with intervals of 200 meters from each other. Their situations are shown in Fig. 1, by the black circles with the station-number 1, 2 and 3.

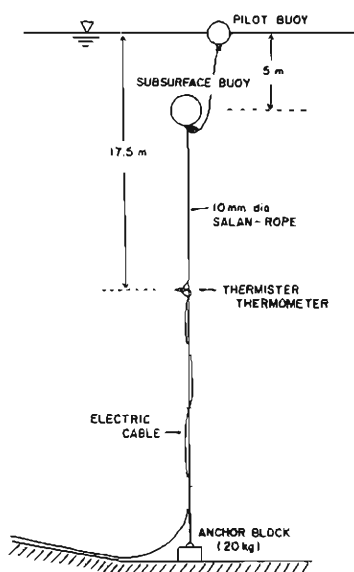


Fig. 2 Schematic structure of the subsurface buoy-stations used in Lake Biwa.

Of course, the original purposes of the observation with the three subsurface buoy-station were (1) observation of the progressive internal waves, (2) observation of the standing internal waves and (3) observation of the continuance of the progressive internal waves. However, in this paper we will treat the problem concerning the purpose (2) only. The problems concerning (1) and (3) will be discussed in a future article.

2. Method of Observation

The structure of the buoy-station is shown in Fig. 2. The subsurface buoy is 5 m below the surface, and a thermister thermometer is fixed at the depth of 17.5 m, where the thermocline was found at the time of the buoy-stations setting. The thermister elements at each station are connected with three independent bridge circuits, and their

unbalanced out-put voltages were recorded successively on a three channel recorder. The time interval between adjacent measurements at one station is about 15 sec; therefore the period of the measurable fluctuation can not be shorter than 30 sec.

The range of the temperature measurement is from 9°C to 30°C, and the temperature measurement is executed with an accuracy of $\pm 0.1^\circ\text{C}$ in this range.

3. Results of Observation

The temperature fluctuations observed from September 17 to 23, 1968 are shown in Fig. 3 together with the records of the wind direction and wind speed obtained at Hikone and the Ado-river. During this period, the circuit of station No. 3 was in trouble, so the record is not shown in Fig. 3. The full line (Station No. 1) and the dotted line (Station No. 2) were plotted by reading with intervals of 30 minutes from the continuous records.

As shown in Fig. 3, the maximum temperature change appeared during the time 1^h00^m to 18^h00^m on September 21, and its magnitude reaches 9°C.

Other small periodic changes with an amplitude of 3°C to 4°C are seen all over the record. Its period is about 9 to 10 hours. At a glance, it is clear that the periodic changes with the period shorter than 6 hours can not have any correlation with the wind speed and wind direction shown in Fig. 3 because of the rough plot of the wind with three hours interval. Therefore we cut out the comparatively short period fluctuations by means of the running-mean of 8.5 hours interval. The resulting curves are shown in Fig. 3 by the broken line (Station No. 1) and the dotted broken line (Station No. 2) respectively.

Now let us pay attention to the strong wind that blew on September 21, and the calm weather on September 22 and 23.

The longitudinal motions of the lake water may be affected mainly by the

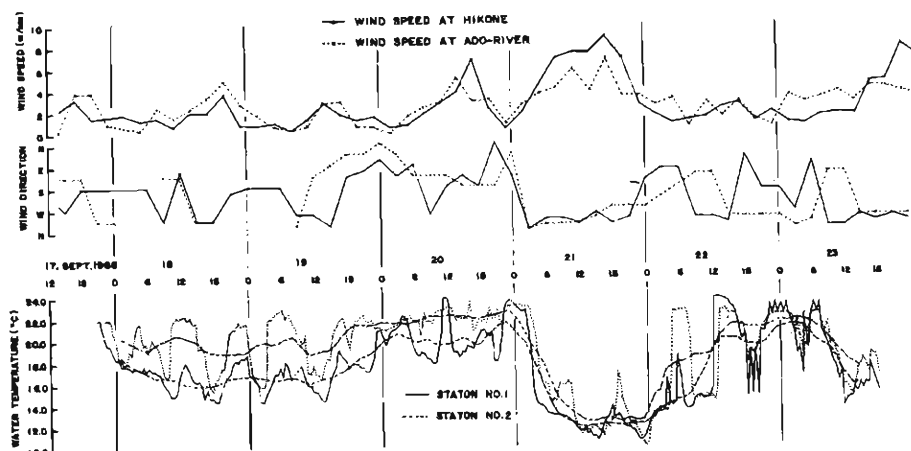


Fig. 3 Variations of the wind speed and wind direction at Hikone and the Ado-river, and water temperature variations obtained from the records of subsurface buoy-stations No. 1 and No. 2 (full line and dotted line). The broken line and dotted broken line show the result of the running mean with the time interval of 8.5 hr.

wind of the N-S component. Consequently, we decomposed the wind speed into the N-S component and E-W component respectively, and compared the smoothed temperature variation during the period from September 21 to 23, with the N-S component of the wind velocity during the same period.

The broken line in Fig. 4 show the smoothed temperature variation and wind velocity of the N-S component respectively, in which the wind velocity is averaged between the weather stations at Hikone and the Ato-viver, and the temperature variation is also averaged between Station No. 1 and Station No. 2.

4. Longitudinal Internal Seiche Due to Wind Actions

For the situation as shown in Fig. 4, we suppose that because of the large wind stresses produced by the north component of the wind along the surface of the lake over about 18 hours, a large amount of lake water in the upper layer was transported towards the south, and water in the lower layer towards the north. Accordingly, the depth of the thermocline will become shallower at the north end and deeper at the south end of the lake.

It is clear that such an inclination of the thermocline will produce a drop in water temperature at the north end of the lake. The full lines in Fig. 4 show such a condition schematically. In spite of the lower velocity of wind during the period from 18^h00^m 21st to 12^h00^m 23rd, the water temperature is going to rise to its maximum at 22^h00^m 22nd.

It may be considered that the temperature change on and after 18^h00^m on September 21st was caused by the longitudinal internal seiches due to the wind, and the length of the time between the minimum temperature and the maximum temperature corresponds approximately with half the period of the longitudinal

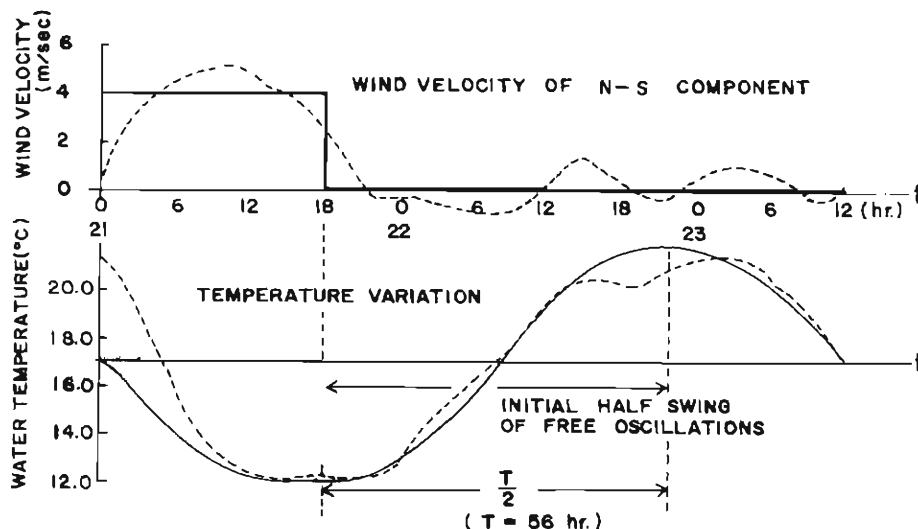


Fig. 4 Variation of the wind velocity of the N-S component (upper panel) and variation of the water temperature at the thermocline-layer observed in Sugaura Bay (lower panel). The full lines show the schematic change of the wind velocity of the N-S component and temperature variation.

internal seiches.

From the schematic curve in Fig. 4, we obtain the Period $T=56$ hours of the longitudinal internal seiche as twice the time difference between the minimum temperature and the maximum temperature, if the vertical temperature gradient is invariant.

The 24 hr-observation of the vertical temperature distribution was made at Shiozu Bay on September 19, 1968, and we obtained the vertical density distribution curve as shown in Fig. 5. Obviously the sharp interface exists at the depth of 17.5 m, so we can consider the density structure of this lake as two layer at that time. From the curve in Fig. 5, we obtained a density difference between the upper layer and the lower layer as follows:—

$$\begin{aligned}\Delta\rho &= \frac{1}{h'} \int_h^{h+h'} \rho(z) dz - \frac{1}{h} \int_0^h \rho(z) dz \\ &= 2.211 \times 10^{-3} \text{ (g/cm}^3\text{)}\end{aligned}$$

where h , h' denote the thickness of the upper layer and the lower layer respectively.

The fundamental period of the longitudinal and transverse internal seiches in the two layered rectangular lake are given by the well-known formulas²⁾:—

$$T_l = 2L \sqrt{\frac{\rho(h+h')}{g\Delta\rho \cdot hh'}} \quad (1)$$

$$T_t = 2B \sqrt{\frac{\rho(h+h')}{g\Delta\rho \cdot hh'}} \quad (2)$$

The meaning of the symbols in the formulae are shown in Table 1.

Table 1. Equivalent size of north basin of Lake Biwa.

	Latitude	Length	Mean breadth	Mean depth	Depth of thermocline
Symbol	φ	L	B	$h+h'$	h
Equivalent size	$35^\circ 20'$	50 km	12 km	50 m	17.5 m

When the motion of lake water is under geostrophic effects, it has been said that the period of the internal seiches, T_0 is given by the following formula:—

$$\frac{1}{T_p^2} = \frac{1}{T_l^2} + \frac{1}{T_p^2} \quad (3)$$

where T_p denotes the period of half a pendulum-day.

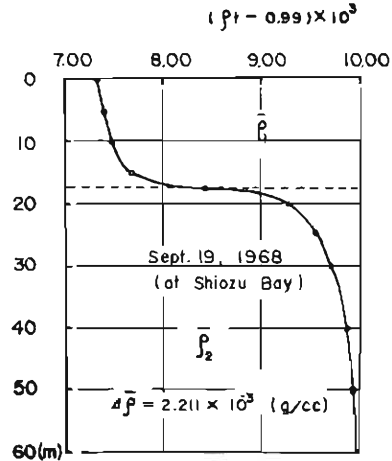


Fig. 5 Vertical density distribution at Shiozu Bay calculated from the vertical temperature distribution curve.

By using the values given in Table 1, we evaluated the periods of the longitudinal and transverse internal seiches and the period of transverse internal seiche under geostrophic effects in Lake Biwa, with the formulae of (1), (2) and (3). The results are shown in Table 2, together with the period of the inertia circulation in Lake Biwa.

As shown in Table 2, the theoretical period of the longitudinal internal seiche without the geostrophic effects coincides with that of the rough estimation in Fig. 4, and we may conclude that the large temperature variations with long periods in Fig. 3 are caused by the longitudinal internal seiche due to the wind.

The maximum velocity of water in the lower layer induced by such an internal seiche, will occur in the middle part of the lake and it will be estimated in the following form:—

$$|U'_{max}| = \frac{2L}{h'T_i} |\zeta'_0| \quad (4)$$

where ζ'_0 is the amplitude of the internal oscillation of the thermocline at both ends of the lake. As described in section 3, the displacement of the thermocline is about 10 m. Then, ζ'_0 takes the value of 5 m. Therefore, we obtain $|U'_{max}| = 8$ cm/sec.

Table 2. Periods of various internal seiches in Lake Biwa.

	Longitudinal	Transverse	Under geostrophic	Period of inertia
Symbol	T_l	T_t	T_g	T_p
Calculated period	55.83 hr	13.33 hr	11.20 hr	20.7 hr
Observed period	56 hr	—	9.5–10.5 hr	—

5. Internal Seiche under Geostrophic Effects

Fig. 6 shows the power spectrum (full line) of the temperature variation, obtained by Fourier's analysis with the fundamental period of 112 hr. In the figure, the higher harmonics are shown by the arrows. Except for the significant period of 56 hr, the amplitudes of every harmonic uniformly decrease with the increasing frequency, in which only one harmonic component of the period of 28 hr has an extremely low amplitude. The dotted line shows a decreasing curve of the power of these higher harmonics, and we consider that the peaks on and below this curve are insignificant as regards the real motions of the lake water.

The dominant peaks exist at the frequencies of 0.018 c/hr (56 hr) and 0.095 to 0.105 c/hr (10.5 to 9.5 hr). The former obviously just corresponds to the longitudinal internal seiche without geostrophic effects. What kind of motion can be considered for the later peak? The only possibility will be found in the transverse internal seiche under geostrophic effects. In contrast to the longitudinal internal seiche, in which the inertia circle may be restricted by the width of the lake, the transverse internal seiche has no restriction towards the longitudinal direction. Consequently, the assumption,²⁾ with which the formula (3) has been derived, will be satisfied. The calculated period is shown in

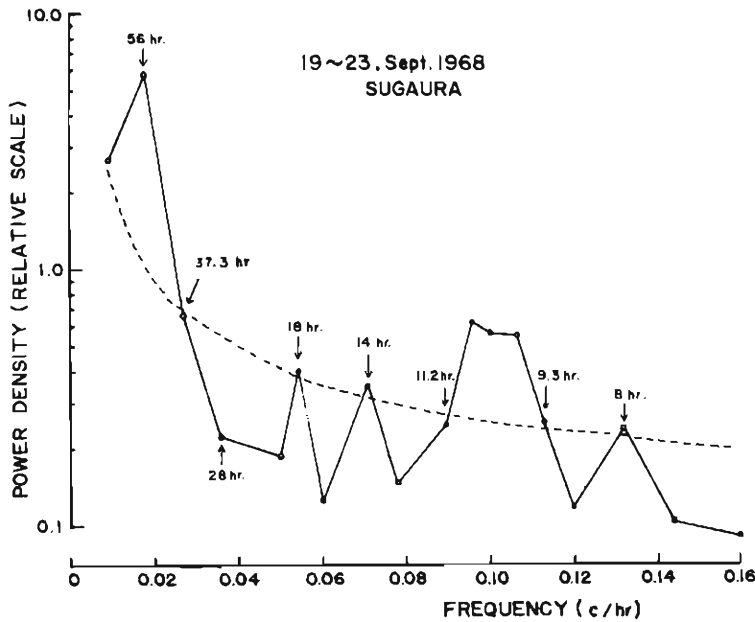


Fig. 6 Power spectrum of the temperature variation, obtained from Fourier's analysis method with the fundamental period of 112 hr. The higher harmonics are shown by the arrows.

Table 2 as T_θ . In this case, the maximum velocity in the lower layer is also estimated by the following formula:—

$$|U'_{\theta \max}| = \frac{2B}{g'T_\theta} |\zeta'_{\theta 0}| \quad (5)$$

taking the values of $|\zeta'_{\theta 0}|$ (The amplitude of the transverse internal oscillation of the thermocline on both sides of the lake.) = 2.5 m, $T_\theta = 10$ hr, $B = 12$ km and $h' = 32.5$ m. Then, we obtain $|U'_{\theta \max}| = 5$ cm/sec, which is almost comparable to the maximum velocity of the longitudinal internal seiche without geostrophic effects.

6. Concluding Remarks

From the preliminary investigation of internal waves in Lake Biwa, we observed large temperature variations in the thermocline-layer. These variations involve two types of dominant periodic changes of water temperature. One has a period of 56 hr and the other has a period of 10 hr. It was found that the periodic change of the period of 56 hr corresponds to the longitudinal internal seiche of the north basin, caused by the northerly component of the wind, which had blown over the lake during an interval of about 24 hr, and that the periodic change of the period of about 10 hr corresponds to the transverse internal seiche under geostrophic effects (This period does not agree so well with the calculated one).

In section 5, we calculated the maximum velocity of water in the lower layer

and we obtained $U'_0 = 5$ cm/sec. This means that the ratio of the mean breadth to the diameter of the inertia circle is:—

$$\frac{B}{2r} = 6.6$$

where r is the radius of the inertia circle.

In formula (3), if we take the period of longitudinal internal seiche for T_l , then the ratio of the mean breadth and the diameter of the inertia circle³⁾ becomes about 2.2, which is not so large. Smallness of this ratio suggests that the motion of the longitudinal internal seiche under geostrophic effects cannot be free from the width of the lake. Consequently, we concluded that the coexistence of the longitudinal internal seiche without geostrophic effects and the transverse internal seiche under geostrophic effects may be consistent, and, that in narrow lakes of great length, formula (3) should not be applied to the longitudinal internal seiche, but applied to the transverse internal seiche only.

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