

Unsteady aerodynamic force modelling for 3-DoF-galloping of four-bundled conductors

Short title: **Unsteady aerodynamic force modelling**

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Abstract

Galloping of overhead transmission lines is characterised by large-amplitude, low-frequency, vertical–horizontal–torsional three degree-of-freedom (DoF) oscillations, which can potentially induce interphase short circuits and fatigue in the conductors and support structures. Numerical analyses are useful for estimating the galloping response of transmission lines, and unsteady aerodynamic forces should be evaluated for accurate simulation. In this study, the unsteady aerodynamic force modelling of four-bundled conductors is investigated based on aerodynamic force measurement test results. Three types of unsteady aerodynamic force measurement tests were performed: under constant torsional angular velocity and under torsional and vertical 1-DoF sinusoidal oscillations. Evaluating the dependence of the aerodynamic forces on the various motions enabled an improved unsteady aerodynamic force model to be developed, considering the effect of angular velocity. First,

two-variable aerodynamic coefficients were proposed as a function of the angle of attack and non-dimensional angular velocity. These coefficients were determined via tests performed at constant torsional angular velocities; results with different combinations of wind speed and angular velocity confirmed that the non-dimensional angular velocity can be used to define the unsteady aerodynamic forces. Then, the validity of the two-variable aerodynamic force formulation was confirmed for predicting torsional or vertical 1-DoF oscillations, by comparing the unsteady aerodynamic force obtained via the sinusoidal forced-vibration tests and via the constant angular-velocity tests. Finally, results of the time-history analyses using the two-variable aerodynamic coefficients were compared with those of previous wind-response measurement tests to obtain the best unsteady aerodynamic force model for simulating large-amplitude 3-DoF galloping. These results indicate that the unsteady aerodynamic forces for 3-DoF-galloping of four-bundled conductors can be evaluated using the relative angle of attack, the rate of change of the relative angle of attack, and the relative wind speed, using two-variable aerodynamic coefficients measured under constant angular velocity conditions.

Keywords

Galloping, overhead transmission line, four-bundled conductor, wind tunnel test, unsteady aerodynamic force

1. Introduction

Large-amplitude, low-frequency, wind-induced vibrations occasionally occur on overhead transmission lines subjected to ice and snow accretion and strong winds. Such vibrations are called ‘galloping’, and they are problematic because they can induce interphase short circuits and fatigue in the conductors, the cross-arms of the towers, and insulators of the transmission line (CIGRE, 2007; Farzaneh, 2008). The galloping of overhead

transmission lines is characterised by large-amplitude, low-frequency, and vertical-horizontal-torsional three-degrees-of-freedom (3-DoF) coupled oscillations. Anti-galloping devices are installed on existing lines to prevent accidents caused by galloping. Further, the towers are designed to maintain a specific distance between conductors for the construction of new lines. Thus, solutions for the galloping problem should focus on not only the conditions under which it occurs, but also the oscillation amplitudes under various structural conditions.

Field observations of full-scale transmission lines have been conducted to investigate the actual characteristics of ice accretion and to analyse the dynamic response of the lines to galloping (Morishita et al., 1984; Yukino et al., 1995; Lilien et al., 1998; Mitsuzuka 2007; Matsumiya et al., 2012). However, obtaining several sets of galloping data is difficult because climatic conditions that cause galloping may occur rarely during the observation period. Therefore, observations of transmission lines with the addition of plastic imitation ice have been conducted to increase the chances of obtaining galloping data (Lilien and Vinogradov 2002; Van Dyke and Laneville, 2008). In addition to understanding the actual phenomena via field observations, numerical analysis methods for simulating the galloping of full-scale overhead transmission lines have been developed (Yu et al., 1993; Desai et al., 1996; Wang and Lillien, 1998; Shimizu et al., 1998, 2005; Liu et al., 2009). These analysis methods have focused on the development of structural modelling; the ice accretion shape and wind speed distribution are obtained as input conditions from other investigations. Further, the aerodynamic forces acting on the lines are considered as quasi-steady aerodynamic forces (Den Hartog, 1956; Parkinson and Smith, 1964) that are measured using steady-state aerodynamic coefficients that depend on the ice accretion shape. Many wind-tunnel tests and numerical fluid analyses have been conducted to obtain the steady-state aerodynamic coefficients (Chabart and Lilien, 1998; Shimizu et al., 2004; Propplewell, 2005; Matsumiya et al., 2011; Ishihara and Oka 2018).

Actual aerodynamic forces may differ from the assumed quasi-steady aerodynamic forces during large-amplitude motion. Kimura et al. (1999) and Phuc et al. (2004) performed aerodynamic force measurement tests under large-amplitude forced torsional vibrations, and they proposed that the unsteady aerodynamic forces depend on the torsional velocity of the four-bundled conductor, which is not considered in normal quasi-steady aerodynamic theory. To overcome this limitation, an improvement was achieved by formulating the quasi-steady aerodynamic forces of a four-bundled conductor during large-amplitude galloping independently for each sub-conductor, instead of formulating it for the whole bundle (Matsumiya et al., 2018). To validate this analysis, Matsumiya et al. (2018) developed an experimental technique for the elastic support of a sectional model in a wind tunnel. Using this technique, large-amplitude, low-frequency, vertical-horizontal-torsional 3-DoF galloping motion can be simulated physically. In previous wind-response measurement tests for conductor galloping, the oscillation amplitude was limited to a comparatively small range because of equipment limitations (Chabart and Lilien, 1998; Kimura et al., 1998; Waris et al., 2008). The results of time-history analysis using the quasi-steady aerodynamic forces with the improvement showed good agreement with the sectional model test results (Matsumiya et al., 2018). Hence it was shown that aerodynamic forces caused by torsional velocities can be formulated using the individual sub-conductor aerodynamic force model. This formulation is equivalent to one that considers the aerodynamic coefficients on the overall bundle as functions of the relative angle of attack and non-dimensional torsional velocity. Time-history response analyses that consider the aerodynamic forces attributed to the torsional velocity can hence be performed using two-variable-dependent aerodynamic coefficients.

In this study, aerodynamic force measurement tests were performed using servomotors with a range of input waveforms to investigate the unsteady aerodynamic force characteristics of four-bundled conductors and evaluate the use of two-variable aerodynamic coefficients for simulating various motions. First, the two-

variable aerodynamic coefficients (i.e., functions of the angle of attack and non-dimensional angular velocity) were measured from tests performed at constant torsional angular velocity. The dependence of the lift, drag and moment coefficients on the torsional angular velocity was found to be consistent with that from comparing with the values calculated from the quasi-steady aerodynamic force for each sub-conductor. Then, the unsteady aerodynamic force coefficients used for flutter analysis on bridge decks (Scanlan and Tomko, 1971) were measured, varying the oscillation amplitudes in torsional or vertical 1-DoF sinusoidal forced-vibration tests. Unsteady aerodynamic coefficients (equivalent to flutter derivatives) were calculated using the two-variable aerodynamic coefficients by simulating forced oscillations, which were then compared with the measured unsteady aerodynamic force coefficients from the sinusoidal motion tests, to confirm the validity of the aerodynamic force formulation using the two variables. The best modelling of unsteady aerodynamic forces for simulating large-amplitude 3-DoF galloping was achieved by comparing the results of the time-history analysis using the aerodynamic coefficients obtained in this study and those of the previous study using wind-response measurement tests (Matsumiya et al., 2018).

The remainder of this paper is organised as follows: Section 2 presents the conditions and relevant results of the wind-response measurement tests using the four-bundled conductor section model, reported in Matsumiya et al. (2018). Section 3 presents the unsteady aerodynamic force measurement test setup under three motion conditions — constant torsional angular velocity and with torsional and vertical 1-DoF sinusoidal oscillations. The measured aerodynamic coefficient definitions are also presented in Section 3. Section 4 presents the unsteady aerodynamic force measurement test results. The two-variable aerodynamic coefficients obtained from the constant torsional angular velocity tests are presented and compared with the calculated values from the quasi-steady aerodynamic force for each sub-conductor. Additionally, the unsteady aerodynamic coefficients obtained from the tests with torsional and vertical sinusoidal oscillations are presented

and compared with the calculated values from the two-variable aerodynamic coefficients. Section 5 presents the discussion of the unsteady aerodynamic force modelling for vertical-horizontal-torsional-3-DoF motion.

Finally, Section 6 presents the conclusions.

2. Wind-response measurement tests for validating the quasi-steady aerodynamic force formulation

The results of wind-response measurement tests performed previously (Matsumiya et al., 2018) were used to validate the unsteady aerodynamic force modelling for simulating large-amplitude 3-DoF galloping of a four-bundled conductor. This previous research indicated the importance of aerodynamic forces caused by torsional velocities for simulating the wind-response test results and the applicability of two-variable aerodynamic coefficients. These coefficients are defined as functions of the angle of attack and non-dimensional torsional velocity, and can be evaluated from equivalence with the quasi-steady aerodynamic formulation for individual sub-conductors.

2.1 Experimental setup and equations of motion for time-history analysis

Fig. 1 shows the elastic support method of the section model for physically simulating galloping of a four-bundled conductor in an open-circuit blow-down type wind tunnel. This method is characterised by the support from multiple long elastic cords with low rigidity. This unique support method allows the section model to vibrate with large amplitudes at low frequencies in the vertical, horizontal, and torsional directions. The non-linear governing equations for the associated 3-DoF of motion were derived, and it was verified that the time-history analyses for simulating the wind-response measurement tests can be performed using these equations (Matsumiya et al., 2018). The non-linear equations of motion, using the parameters shown in Fig.1 (a), are given as

$$\begin{aligned}
m\ddot{y} + 2m\omega_{y0}\zeta_y(A_y)\dot{y} + R_y(y, z, \theta) &= F_y, \\
m\ddot{z} + 2m\omega_{z0}\zeta_z(A_z)\dot{z} + R_z(y, z, \theta) &= F_z, \\
I\ddot{\theta} + 2I\omega_{\theta0}\zeta_\theta(A_\theta)\dot{\theta} + R_\theta(y, z, \theta) &= F_\theta,
\end{aligned} \tag{1}$$

where y , z , and θ denote the vertical displacement, horizontal displacement and torsional angle, respectively and the dots represent differentiation with respect to time. ω_{q0} ($q = y, z, \theta$) represents the circular natural frequency for each direction when the amplitude is infinitesimally small. ζ_q ($q = y, z, \theta$) represents the structural damping ratio for each direction, which was assumed to be a function of the corresponding oscillation amplitude A_q ($q = y, z, \theta$). Each function was determined by free vibration tests at a wind speed of 0 m/s, as described below. R_q ($q = y, z, \theta$) represents the restoring force for each direction, and each one was theoretically derived as a non-linear function of the displacements and torsional angle, by considering the geometry for large deflections and the corresponding tension change in the elastic cords. For the verified formulations of R_q , please refer to Matsumiya et al. (2018). F_q ($q = y, z, \theta$) represents the aerodynamic force (or moment) in each direction. The aerodynamic forces (and moment) are non-linear functions that depend on the motion of the body. Proposing the best model of these forces is the goal of the present study.

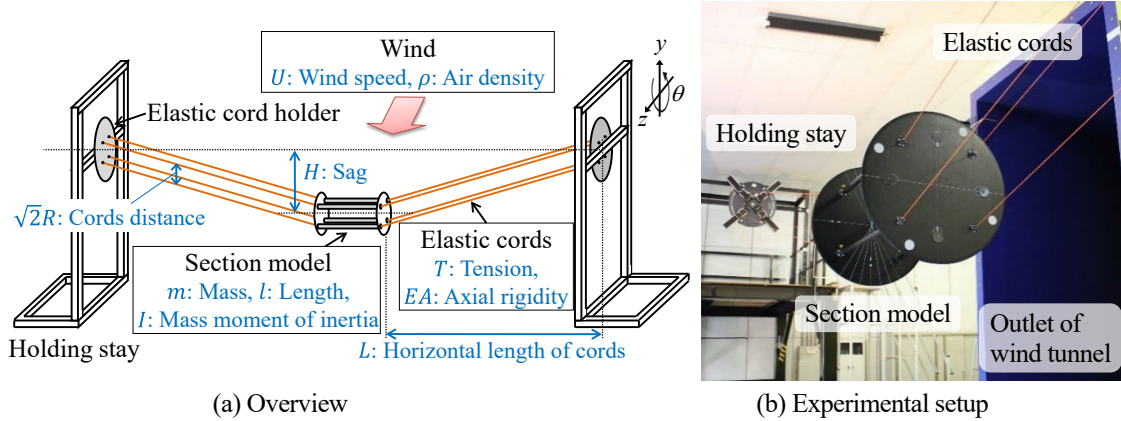


Fig. 1 Elastic support method for the section model (Matsumiya et al., 2018)

Fig. 2 shows the cross-section of the model of the four-bundled conductor with a simulated ice-accretion shape, with a sharp triangular tip, on each sub-conductor. The dimensions of the model were identical to those of aluminium conductor steel-reinforced (ACSR) conductors with a nominal cross-sectional area of

410 mm². The mass and mass moment of inertia of the model, per unit length (including the effective contributions from the endplates), were also identical to the values for actual conductors. The test conditions are presented in Table 1. Although the natural frequencies in these tests depend on the oscillation amplitudes, they can be accurately estimated by the governing equations described in Eqs. (1) and the test conditions shown in Table 1, as shown in Fig. 3 (Matsumiya et al., 2018). The natural frequencies calculated theoretically, with the assumption of infinitesimally small amplitudes, are listed in Table 2. The natural frequencies — f_{y0} , f_{z0} , and $f_{\theta0}$ ($= \frac{\omega_{q0}}{2\pi}$, $q = y, z, \theta$) — were set to correspond to those of the first asymmetric modes, in which galloping is likely to occur, on actual transmission lines that have a span length of approximately 250–300 m.

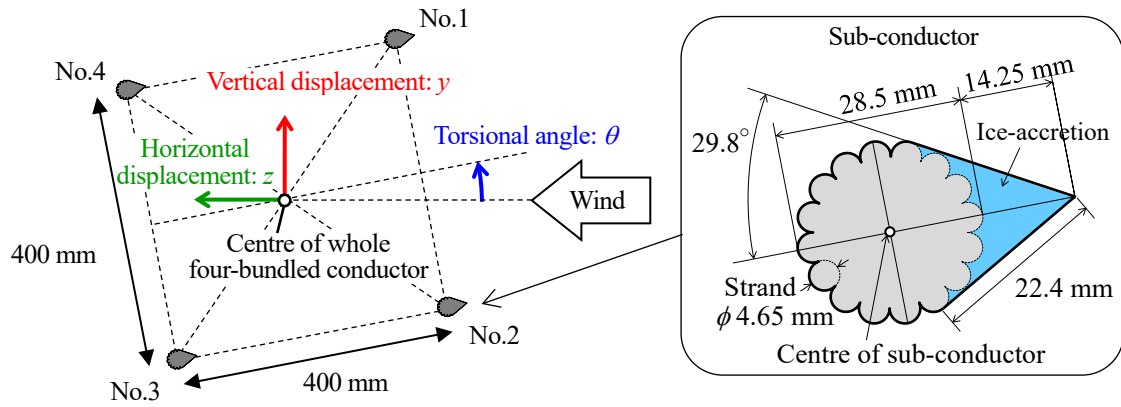


Fig. 2 Cross-sections of the ice-accreted four-bundled conductor model (Based on Matsumiya et al. (2018))

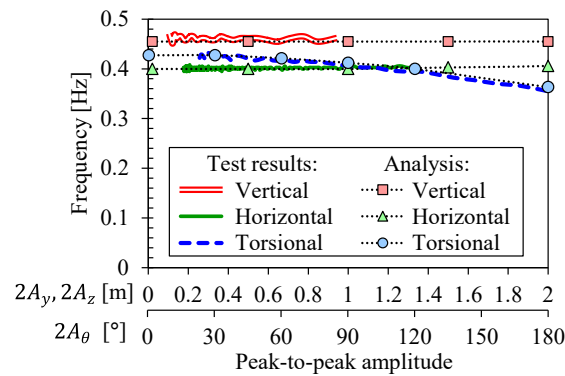


Fig.3 Natural frequencies depending on the oscillation amplitudes (Matsumiya et al., 2018)

Table 1 Test conditions (Matsumiya et al., 2018)

Parameter	Value
Mass of model: m	6.692 kg
Mass moment of inertia: I	0.535 kg·m ²
Model length: l	1.000 m
Spacing of sub-conductors: B	0.400 m
Diameter of sub-conductors: D	0.0285 m
Horizontal length of cords: L	6.255 m
Spacing of cords: $\sqrt{2}R$	0.400 m
Axial rigidity of each cord: EA	176.4 N
Sag of cords: H	1.560 m
Tension of each cord: T	33.9 N
Wind speed: U	10.2 m/s
Air density: ρ	1.20 kg/m ³

Table 2 Theoretical natural frequencies (Matsumiya et al., 2018)

Direction	Value (Hz)
Vertical frequency: f_{y0}	0.456
Horizontal frequency: f_{z0}	0.399
Torsional frequency: $f_{\theta0}$	0.428

2.2 Structural damping modelling

The structural damping ratio ζ_q , for vibrations in each direction, changes with oscillation amplitude. Fig. 4 shows the characteristics of the structural damping ratios, which were measured using free vibration tests at a wind speed of 0 m/s and setup torsional angle (mean torsional angle) $\theta_0 = 0^\circ$. In accordance with the definition in Eqs. (1), the amplitude-dependent damping ratios were calculated, considering the circular natural frequency ω_{q0} as a constant value in the damping term, which corresponds to the theoretical infinitesimal-amplitude frequencies shown in Table 2. The damping ratios ζ_{qm} change with amplitude $A_q (q = y, z, \theta)$, and they can be linearly approximated by fitting the function

$$\zeta_{qm}(A_q) = \zeta_{q0} + \zeta_{q1}A_q, \quad (2)$$

where ζ_{q0} and $\zeta_{q1} (q = y, z, \theta)$ are the coefficients identified by the free vibration tests at a wind speed of 0 m/s. Compared to the previous study (Matsumiya et al., 2018), the modelling of the structural damping in this study was much improved by allowing for the contribution from aerodynamic damping from the model moving through the still air in the free vibration tests. This aerodynamic damping component depends on the oscillation

amplitude, and it becomes zero when the amplitude is zero. The aerodynamic damping contribution to the total measured damping can be theoretically estimated using quasi-steady aerodynamic theory and equating the energy dissipation in one vibration cycle to that from equivalent viscous damping. Furthermore, the relative angle of attack of each sub-conductor, with the model oscillating in a single degree of freedom at a wind speed of 0 m/s, has only one constant value for the positive velocity part of the vibration cycle and another constant value for the negative velocity part. The aerodynamic damping contribution to the measured damping, ζ_{qw} ($q = y, z, \theta$), can hence be obtained from the following equations:

$$\begin{aligned} \int_0^{\frac{2\pi}{\omega_y}} (2m\omega_{y0}\zeta_{yw}\dot{y} \times \dot{y})dt &= \int_0^{\frac{\pi}{\omega_y}} \left(\frac{1}{2}\rho\dot{y}^2 4DlC_{Ds}(-90^\circ) \times \dot{y} \right) dt - \int_{\frac{\pi}{\omega_y}}^{\frac{2\pi}{\omega_y}} \left(\frac{1}{2}\rho\dot{y}^2 4DlC_{Ds}(90^\circ) \times \dot{y} \right) dt, \\ \int_0^{\frac{2\pi}{\omega_z}} (2m\omega_{z0}\zeta_{zw}\dot{z} \times \dot{z})dt &= \int_0^{\frac{\pi}{\omega_z}} \left(\frac{1}{2}\rho\dot{z}^2 4DlC_{Ds}(180^\circ) \times \dot{z} \right) dt - \int_{\frac{\pi}{\omega_z}}^{\frac{2\pi}{\omega_z}} \left(\frac{1}{2}\rho\dot{z}^2 4DlC_{Ds}(0^\circ) \times \dot{z} \right) dt, \\ \int_0^{\frac{2\pi}{\omega_\theta}} (2I\omega_{\theta0}\zeta_{\theta w}\dot{\theta} \times \dot{\theta})dt &= \int_0^{\frac{\pi}{\omega_\theta}} \left(\frac{1}{2}\rho \left(\frac{B}{\sqrt{2}} \right)^3 \dot{\theta}^2 Dl(C_{Ds}(-135^\circ) + C_{Ds}(-45^\circ) + C_{Ds}(45^\circ) + C_{Ds}(135^\circ)) \times \dot{\theta} \right) dt \\ &\quad - \int_{\frac{\pi}{\omega_\theta}}^{\frac{2\pi}{\omega_\theta}} \left(\frac{1}{2}\rho \left(\frac{B}{\sqrt{2}} \right)^3 \dot{\theta}^2 Dl(C_{Ds}(45^\circ) + C_{Ds}(135^\circ) + C_{Ds}(-135^\circ) + C_{Ds}(-45^\circ)) \times \dot{\theta} \right) dt. \end{aligned}$$

Here, the aerodynamic force caused by the motion of the model at $U = 0$ m/s and $\theta_0 = 0^\circ$ is assumed to be the same as the quasi-steady aerodynamic force, which is calculated for each sub-conductor using the steady aerodynamic force coefficients of a single conductor with the same ice accretion shape. C_{Ds} denotes the drag coefficient of a single ice-accreted sub-conductor, the values of which were taken from measurements by Matsumiya et al. (2011). Assuming that the motion is sinusoidal and noting that the ice accretion shape is symmetric and that the non-linearity only causes a small change in the natural frequency with amplitude (Fig. 3), the aerodynamic damping component ζ_{qw} ($q = y, z, \theta$) can be estimated as

$$\begin{aligned}
\zeta_{yw} &= \zeta_{yw1} A_y, \quad \zeta_{zw} = \zeta_{zw1} A_z, \quad \zeta_{\theta w} = \zeta_{\theta w1} A_\theta, \\
\zeta_{yw1} &= \frac{16\rho D l C_{Ds}(90^\circ) \omega_y(A_y)}{3m \omega_{y0}} \approx \frac{16\rho D l C_{Ds}(90^\circ)}{3m}, \\
\zeta_{zw1} &= \frac{8\rho D l (C_{Ds}(0^\circ) + C_{Ds}(180^\circ)) \omega_z(A_z)}{3m \omega_{z0}} \approx \frac{8\rho D l (C_{Ds}(0^\circ) + C_{Ds}(180^\circ))}{3m}, \\
\zeta_{\theta w1} &= \frac{2\sqrt{2}\rho B^3 D l (C_{Ds}(45^\circ) + C_{Ds}(135^\circ)) \omega_\theta(A_\theta)}{3I \omega_{\theta0}} \approx \frac{2\sqrt{2}\rho B^3 D l (C_{Ds}(45^\circ) + C_{Ds}(135^\circ))}{3I},
\end{aligned}
\tag{4}$$

where $\omega_q(A_q)$ ($q = y, z, \theta$) is the circular frequency of the free vibrations at amplitude A_q .

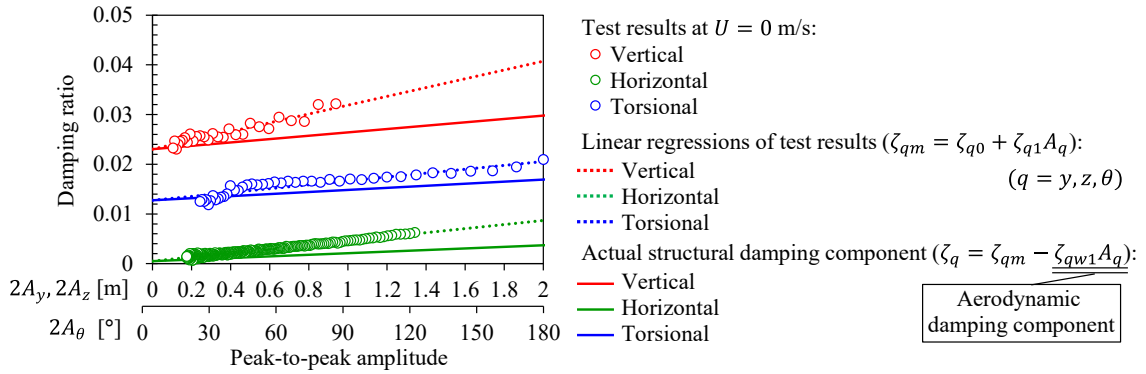


Fig. 4 Structural damping coefficients (linear regressions ζ_{qm} and corrected values ζ_q)

Since ζ_{q1} includes the aerodynamic damping component ζ_{qw1} , the actual structural damping ratio

ζ_q can be expressed as

$$\zeta_q(A_q) = \zeta_{qm} - \zeta_{qw} = \zeta_{q0} + (\zeta_{q1} - \zeta_{qw1})A_q. \tag{5}$$

Table 3 shows the components of the damping coefficients for motion in each direction. Also, the corrected structural damping ratios ζ_q — used in the time-history analysis in this study — are shown in Fig. 4, along with the damping ratios directly from the measurements, ζ_{qm} , as used in the time-history analysis in the previous study (Matsumiya et al., 2018).

Table 3 Components of structural damping ratios

	Fitted intercept		Fitted slope		Aerodynamic damping slope
ζ_{y0}	2.30×10^{-2}	ζ_{y1} [1/m]	1.77×10^{-2}	ζ_{yw1} [1/m]	1.08×10^{-2}
ζ_{z0}	0.05×10^{-2}	ζ_{z1} [1/m]	0.82×10^{-2}	ζ_{zw1} [1/m]	0.50×10^{-2}
$\zeta_{\theta0}$	1.28×10^{-2}	$\zeta_{\theta1}$ [1/°]	8.66×10^{-5}	$\zeta_{\theta w1}$ [1/°]	3.99×10^{-5}

2.3 Methods of wind-response measurement tests

Free vibration tests for a constant wind speed of 10.2 m/s were performed to obtain wind-response data. The mean torsional angle $\bar{\theta}$ was varied for each test by rotating the elastic cord holders on the holding stays (Fig. 1). Because large-amplitude galloping is non-linear, the occurrence of these vibrations can depend on the initial conditions. In a non-linear oscillating system, there can be several stable steady-state solutions with different amplitudes (including zero amplitude). In general, there can also be several unstable solutions which, along with the initial conditions, relate to which stable solution is obtained (Matsumiya et al., 2021). Although the discussion about unstable solutions is important, only the stable steady-state solutions have been focused on in this study. The dynamic response was measured in two ways for each angle of attack: after disengaging the model from the stationary position held softly against the wind, and after applying a large initial displacement. To focus on the oscillations with larger steady-state amplitude, only the results with large initial displacement are shown in this study, for which the initial displacements in the vertical, horizontal, and torsional directions were approximately -0.5 m, 0.5 m, and 45° , respectively, relative to the static equilibrium position in the wind. Because the galloping oscillations observed in the tests had a steady amplitude and only one predominant frequency, the total (peak-to-peak) amplitudes were calculated as $2\sqrt{2}$ times the standard deviation of the displacements. Other details of the wind-response measurement tests are provided in the previous paper (Matsumiya et al., 2018).

2.4 Methods of time-history analysis using quasi-steady aerodynamic forces

Matsumiya et al. (2018) applied two different formulations of quasi-steady aerodynamic forces on four-bundled conductors, as shown in Fig. 5. One formulation considers all the sub-conductors as one group, as shown in Fig.

222 5 (a), and the quasi-steady aerodynamic forces for the whole four-bundled conductor (L_{qf} : lift force, D_{qf} : drag
223 force, M_{qf} : aerodynamic moment) are expressed as

$$\begin{aligned} L_{qf} &= \frac{1}{2} \rho U_r^2 4Dl (C_L(\alpha_r) \cos \phi_r + C_D(\alpha_r) \sin \phi_r), \\ D_{qf} &= \frac{1}{2} \rho U_r^2 4Dl (-C_L(\alpha_r) \sin \phi_r + C_D(\alpha_r) \cos \phi_r), \\ M_{qf} &= \frac{1}{2} \rho U_r^2 4DBl C_M(\alpha_r). \end{aligned} \quad (6)$$

225 Here C_D , C_L , and C_M are, respectively, the steady aerodynamic coefficients for lift, drag, and aerodynamic
226 moment acting on the whole four-bundled conductor: α_r and U_r are the relative angle of attack and relative
227 wind speed, respectively, at the midpoint of all four sub-conductors, and are expressed as follows:

$$\alpha_r = \theta + \phi_r, \quad \phi_r = \tan^{-1} \left(\frac{-\dot{y}}{U - \dot{z}} \right), \quad U_r = \sqrt{(-\dot{y})^2 + (U - \dot{z})^2}. \quad (7)$$

229 The other formulation considers each sub-conductor independently, as shown in Fig. 5 (b), and the
230 total quasi-steady aerodynamic forces (L_{qs} : lift force, D_{qs} : drag force, M_{qs} : aerodynamic moment) are
231 expressed as

$$\begin{aligned} L_{qs} &= \sum_{i=1}^4 L_{qsi}, \quad D_{qs} = \sum_{i=1}^4 D_{qsi}, \quad M_{qs} = \sum_{i=1}^4 M_{qsi} + \frac{B}{\sqrt{2}} (L_{qs1} - D_{qs2} - L_{qs3} + D_{qs4}) \cos \left(\frac{\pi}{4} + \theta \right) \\ &\quad + \frac{B}{\sqrt{2}} (D_{qs1} + L_{qs2} - D_{qs3} - L_{qs4}) \sin \left(\frac{\pi}{4} + \theta \right), \\ L_{qsi} &= \frac{1}{2} \rho U_{ri}^2 D l (C_{Li}(\alpha_{ri}) \cos \phi_{ri} + C_{Di}(\alpha_{ri}) \sin \phi_{ri}), \\ D_{qsi} &= \frac{1}{2} \rho U_{ri}^2 D l (-C_{Li}(\alpha_{ri}) \sin \phi_{ri} + C_{Di}(\alpha_{ri}) \cos \phi_{ri}), \\ M_{qsi} &= \frac{1}{2} \rho U_{ri}^2 D^2 l C_{Mi}(\alpha_{ri}). \end{aligned} \quad (8)$$

234 Here C_{Di} , C_{Li} , and C_{Mi} ($i = 1 - 4$) are, respectively, the steady aerodynamic coefficients for lift, drag, and
235 aerodynamic moment acting on each sub-conductor: α_{ri} and U_{ri} ($i = 1 - 4$) are the relative angle of attack
236 and relative wind speed, respectively, for each sub-conductor, and are expressed as follows:

$$\begin{aligned}
\alpha_{ri} &= \theta + \phi_{ri}, \quad \phi_{ri} = \tan^{-1} \left(\frac{U_{yi}}{U_{zi}} \right), \quad U_{ri} = \sqrt{U_{yi}^2 + U_{zi}^2}, \\
U_{y1} &= -\dot{y} - \frac{B}{\sqrt{2}} \dot{\theta} \cos \left(\frac{\pi}{4} + \theta \right), \quad U_{z1} = U - \dot{z} - \frac{B}{\sqrt{2}} \dot{\theta} \sin \left(\frac{\pi}{4} + \theta \right), \\
U_{y2} &= -\dot{y} - \frac{B}{\sqrt{2}} \dot{\theta} \sin \left(\frac{\pi}{4} + \theta \right), \quad U_{z2} = U - \dot{z} + \frac{B}{\sqrt{2}} \dot{\theta} \cos \left(\frac{\pi}{4} + \theta \right), \\
U_{y3} &= -\dot{y} + \frac{B}{\sqrt{2}} \dot{\theta} \cos \left(\frac{\pi}{4} + \theta \right), \quad U_{z3} = U - \dot{z} + \frac{B}{\sqrt{2}} \dot{\theta} \sin \left(\frac{\pi}{4} + \theta \right), \\
U_{y4} &= -\dot{y} + \frac{B}{\sqrt{2}} \dot{\theta} \sin \left(\frac{\pi}{4} + \theta \right), \quad U_{z4} = U - \dot{z} - \frac{B}{\sqrt{2}} \dot{\theta} \cos \left(\frac{\pi}{4} + \theta \right).
\end{aligned} \tag{10}$$

The difference between the two formulations is that the aerodynamic forces caused by torsional velocities are only captured in the individual sub-conductor aerodynamic force formulation. In contrast, when the torsional velocity is zero, the aerodynamic forces calculated from Eqs. (6) are identical to those calculated using Eqs. (8). The aerodynamic coefficients of the sub-conductors and the whole four-bundled conductor were measured using surface-pressure measurement tests (Matsumiya et al., 2011). Therefore, the aerodynamic coefficients of each sub-conductor include the influence of the flow interference between the sub-conductors in the steady state, in the same manner as the coefficients of the whole four-bundled conductor.

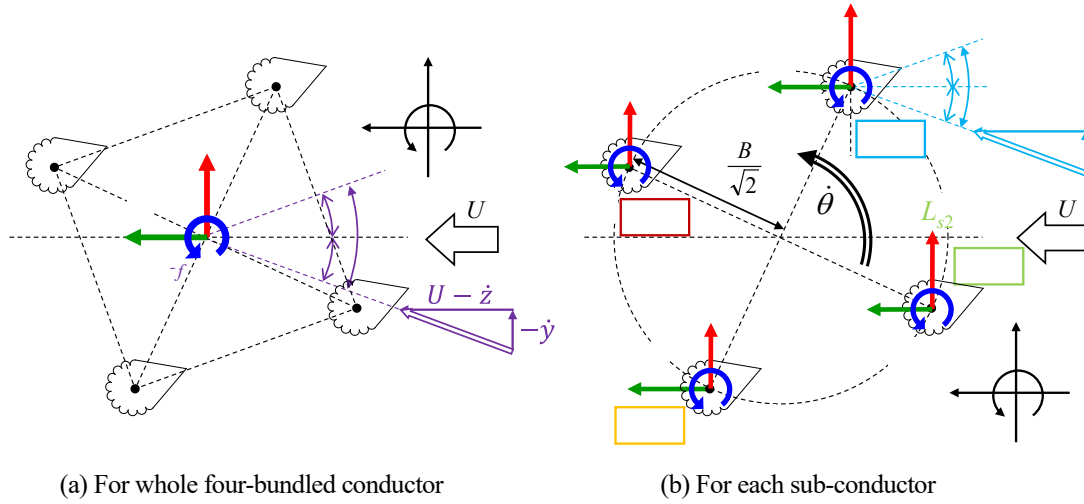


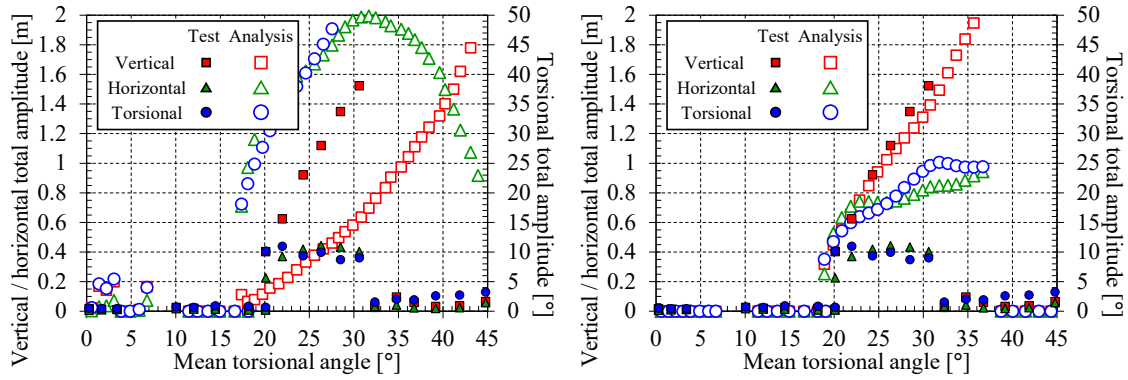
Fig. 5 Two different quasi-steady aerodynamic force formulations (Based on Matsumiya et al. (2018))

Time-history response analyses using the two quasi-steady aerodynamic force formulations were performed based on Eqs. (1). The structural damping ratios were set as ζ_q instead of ζ_{qm} (see Section 2.2), but the resulting difference in the calculated response amplitudes was small under the test conditions listed in

Table 1. The details of the time-history analysis method are provided in the previous paper (though using ζ_{qm}) (Matsumiya et al., 2018).

2.5 Comparison of the results of wind-response measurement tests with those of time-history analyses

Fig. 6 shows the total amplitudes of the wind-response measurement tests compared with those obtained from the time-history analyses using the two quasi-steady aerodynamic force formulations. The results of the time-history analysis using the quasi-steady aerodynamic forces for each sub-conductor (Fig. 6 (b)) shows better agreement with the test results than those using the quasi-steady aerodynamic forces for the entire four-bundled conductor (Fig. 6 (a)). Therefore, the quasi-steady aerodynamic forces of a four-bundled conductor during large-amplitude galloping should be formulated independently for each sub-conductor, rather than for all sub-conductors as a group (Matsumiya et al., 2018). The different results obtained using the two formulations is attributed to the aerodynamic forces caused by the torsional velocities of the four-bundled conductors. However, because there remain some discrepancies between the torsional and horizontal amplitudes obtained from the tests and those obtained from the time-history analysis, further improvement of the aerodynamic force modelling is required. Therefore, the unsteady aerodynamic force characteristics of four-bundled conductors are investigated in this study.



(a) Quasi-steady aerodynamic forces formulated for whole four-bundled conductor

(b) Quasi-steady aerodynamic forces formulated for each sub-conductor

Fig. 6 Comparison of total amplitudes from wind-response tests and time-history analyses using the two different quasi-steady aerodynamic force formulations ($U = 10.2$ m/s, with initial displacement)

2.6 Aerodynamic coefficient as a function of the relative angle of attack and non-dimensional torsional velocity

Another important conclusion of the previous study (Matsumiya et al., 2018) was that the aerodynamic force formulation using quasi-steady theory for the individual sub-conductors is equivalent to the formulation that uses aerodynamic coefficients defined as functions of the relative angle of attack and non-dimensional torsional velocity. The transformed formulation has the same form as the normal quasi-steady aerodynamic force formulation, in Eqs. (6), except that the aerodynamic coefficients are defined as functions of the non-dimensional torsional velocity $B\dot{\theta}/U_r$ as well as the relative angle of attack α_r :

$$\begin{aligned}
 L_{qs} &= \frac{1}{2} \rho U_r^2 4Dl \left(C_{Lqs}^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \cos \phi_r + C_{Dqs}^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \sin \phi_r \right), \\
 D_{qs} &= \frac{1}{2} \rho U_r^2 4Dl \left(-C_{Lqs}^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \sin \phi_r + C_{Dqs}^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \cos \phi_r \right), \\
 M_{qs} &= \frac{1}{2} \rho U_r^2 4BDl C_{Mqs}^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right).
 \end{aligned} \tag{11}$$

Here, C_{Lqs}^* , C_{Dqs}^* , and C_{Mqs}^* denote the two-variable coefficients of lift, drag, and aerodynamic moment, respectively. They can be calculated from the quasi-steady aerodynamic forces for each sub-conductor using the steady aerodynamic coefficients of the sub-conductors. Fig. 7 shows the calculated two-variable aerodynamic coefficients of the four-bundled conductor, along with the steady aerodynamic coefficients of the four-bundled

conductor. C_{Mqs}^* varies considerably based on the non-dimensional torsional velocity whereas the variations in C_{Lqs}^* and C_{Dqs}^* with torsional velocity are relatively small.

In this study, the unsteady aerodynamic forces were measured for different imposed motions of the four-bundled conductor, defining the aerodynamic coefficients as functions of two variables (angle of attack and torsional angular velocity). They are then compared with the calculated values in Fig. 7. It is noted that, if the unsteady aerodynamic forces can be defined in a similar form to Eqs. (11), it is possible to perform time history response analysis considering aerodynamic nonlinearity. It should be noted that the definitions of the two-variable aerodynamic coefficients are different from the unsteady aerodynamic force coefficients used as flutter derivatives for linear instability analysis of bridge decks (Scanlan and Tomko, 1971). The relationships between them are described in Section 3.

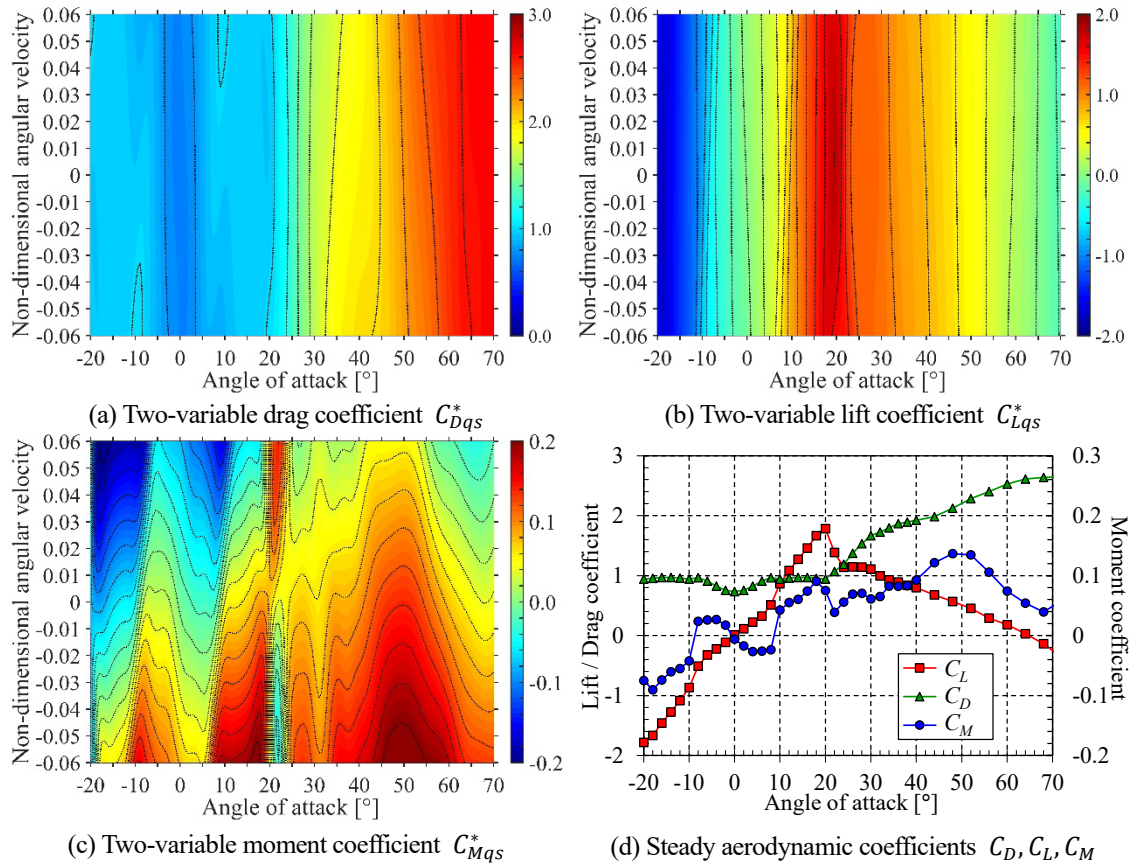


Fig. 7 Aerodynamic coefficients of four-bundled conductor calculated from quasi-steady aerodynamic forces on each sub-conductor (Matsumiya et al., 2011, 2018)

3. Setup and conditions of unsteady aerodynamic force measurement tests

In this study, three types of unsteady aerodynamic force measurement tests were performed to investigate the unsteady aerodynamic force characteristics of four-bundled conductors and evaluate two-variable aerodynamic coefficients for simulating the forces under various motions. Two different types of rotational motion and one type of vertical motion were used. In the two rotational motion tests, one employed constant torsional angular velocities and the other sinusoidal torsional oscillations. Only sinusoidal oscillations were used for the vertical motion tests. Various motions, with different oscillation amplitudes, frequencies and rotational speeds were easily applied using servomotors.

3.1 Common conditions—wind tunnel and section model

The unsteady aerodynamic force measurement tests were performed in a blow-down-type wind tunnel at the Central Research Institute of Electric Power Industry, Japan. The closed measurement section setting for this study was 2.50 m high and 1.04 m wide. The turbulent intensity was less than 0.5%.

The cross-section of the model of the four-bundled conductor was identical to that used in the previous steady aerodynamic measurement tests (Matsumiya et al., 2011) and the previous wind-response measurement tests (Matsumiya et al., 2018), as shown in Fig. 2. The sectional model had dimensions identical to the ACSR410 mm² conductor, and the span length l was 1.0 m. The model was configured with roughness representing the strands. However, the twist of the strands, as on actual electricity conductors, was not modelled to ensure two-dimensional characteristics, as in the previous experiments. The model was supported by three-component-loadcells (Nissho Electric Works Co., Ltd., LMC-3501-200 N, rated load: 200 N, rated moment: 20 N·m) at the centre of the four-bundled conductor via circular endplates (thickness: 5 mm; diameter: 800 mm),

which were installed on both ends of the model. The various unsteady aerodynamic forces were measured using these loadcells; hence these tests were force balance measurement tests.

3.2 Experimental setup for torsional motion

3.2.1 Rotary device

The rotary device used for the unsteady aerodynamic force measurement is shown in Fig. 8. In this device, a servomotor (YASKAWA Electric Co., SGMAV-02A DAH761, rated power: 200 W, rated speed: 3000 rpm, rated torque: 0.637 N·m, reduction ratio: 1/33) was used as the rotator, and the input waveform could be conveniently controlled from a PC. This facilitated the easy evaluation of the dependence of the unsteady aerodynamic force coefficients on the oscillation amplitude under sinusoidal torsional vibrations. Furthermore, the excitation tests could be performed using not only sinusoidal waveforms but also arbitrary waveforms as inputs. Therefore, unsteady aerodynamic forces at constant angular velocities could be measured with appropriate waveforms. There were two servomotors, one at either end of the model. The servomotor controller could output the torsional angle of each motor, as well as its velocity and acceleration signal.

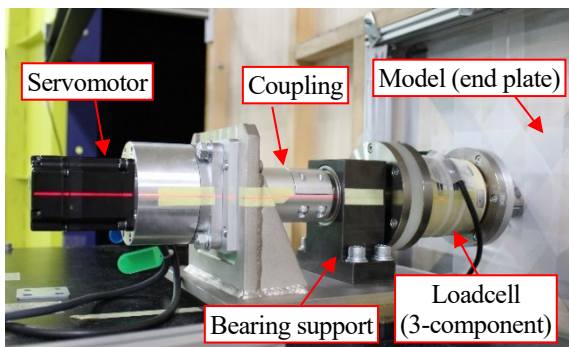


Fig. 8 Rotary device for unsteady aerodynamic force measurement

3.2.2 Unsteady aerodynamic force measurement tests with constant torsional angular velocity

Although the servomotor could rotate unidirectionally, constant torsional angular velocity tests were performed with reciprocating motions instead of using unidirectional rotations to avoid entangling the cables. Fig. 9 shows the time series of an example waveform and the Lissajous figure of all the waveforms used in the constant angular velocity tests. The measurement range of the torsional angle (angle of attack) θ ranged from -20° to 70° . In the reciprocating motions, the range of constant angular velocity was from -25° to 75° , and the transient range was determined to smoothly turn back depending on the angular velocity. In a preliminary experiment, it was confirmed that the transient range was sufficiently long to not affect the measurements in the measurement range. Table 4 shows the conditions of the unsteady aerodynamic force measurement tests with constant angular velocity. Because the averaged data were obtained from 100 ensemble data, the measurement time depended on the angular velocity. The sampling frequency was set such that the data could be measured at 0.1° intervals of the angle of attack. After the force due to self-weight and the inertial force were subtracted using the results of measurements without wind, the values of the unsteady aerodynamic forces, averaged over 100 cycles with respect to the angle of attack, were obtained for each angular velocity.

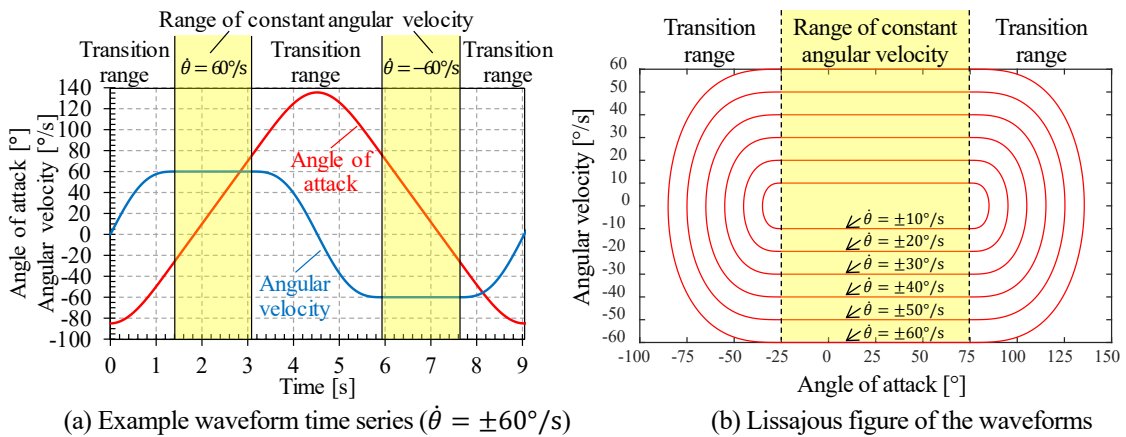


Fig. 9 Motions for unsteady aerodynamic force measurement with constant angular velocity

Table 4 Conditions of unsteady aerodynamic force measurement tests with constant angular velocity

Measurement range of angle of attack (torsional angle), θ	$-20 - 70^\circ$
Constant angular velocity, $\dot{\theta}$	$2.5-60^\circ/\text{s}$ (at $2.5^\circ/\text{s}$ intervals)
Wind speed, U	6.8, 8.5, and 10.3 m/s

343

344 In this study, the two-variable aerodynamic coefficients were calculated from the values of lift L_{uc} ,
 345 drag D_{uc} , and aerodynamic moment M_{uc} , measured under the constant-angular-velocity $\dot{\theta}$ condition and
 346 averaged with respect to the angle of attack, which can be expressed as

$$\begin{aligned}
 L_{uc}(\theta, \dot{\theta}) &= \frac{1}{2} \rho U^2 4DlC_L^* \left(\theta, \frac{B\dot{\theta}}{U} \right), \\
 D_{uc}(\theta, \dot{\theta}) &= \frac{1}{2} \rho U^2 4DlC_D^* \left(\theta, \frac{B\dot{\theta}}{U} \right), \\
 M_{uc}(\theta, \dot{\theta}) &= \frac{1}{2} \rho U^2 4BDlC_M^* \left(\theta, \frac{B\dot{\theta}}{U} \right).
 \end{aligned} \tag{5}$$

348 Here, C_L^* , C_D^* , and C_M^* denote the lift, drag, and moment coefficients, respectively, defined as functions of
 349 both the angle of attack θ and non-dimensional angular velocity $B\dot{\theta}/U$.

350

351 3.2.3 Unsteady aerodynamic force measurement tests under sinusoidal torsional motion

352 Unsteady aerodynamic force measurement tests using a sinusoidal torsional waveform ($\theta = \theta_0 + \tilde{\theta}$, $\tilde{\theta} =$
 353 $A_\theta \sin \omega_G t$; ω_G : circular frequency; t : time) were performed by varying the mean torsional angle (angle of
 354 attack) θ_0 and torsional amplitude A_θ . Table 5 lists the conditions of the unsteady aerodynamic force
 355 measurement tests with sinusoidal torsional excitation. The wind speed and vibration frequency were set
 356 considering the conditions of previous wind-response measurement tests, and the sampling frequency was 1000
 357 Hz. The phase-averaged data were obtained from 100 cycles, and the force due to self-weight and the inertial
 358 force were subtracted using the results of measurements without wind.

359 In this study, four unsteady aerodynamic force coefficients—used as flutter derivatives with respect
 360 to torsional motion for flutter analysis on bridge decks (Scanlan and Tomko, 1971)—were obtained from the
 361 unsteady aerodynamic force measurement tests using sinusoidal torsional waveforms. Considering only the
 362 fluctuating components at the motion frequency, the unsteady aerodynamic force coefficients
 363 $L_{\theta I}^*$, $L_{\theta R}^*$, $M_{\theta I}^*$, and $M_{\theta R}^*$ are defined using the lift and aerodynamic moment, which can be expressed as

$$364 \quad \begin{aligned} L_{uf} &= A_{L_{uf}} \sin(\omega_G t - \Phi_{L_{uf}}) = \frac{1}{2} \rho (4D)^2 l \omega_G^2 \left(L_{\theta R}^* B \tilde{\theta} + L_{\theta I}^* \frac{B \dot{\tilde{\theta}}}{\omega_G} \right), \\ M_{uf} &= A_{M_{uf}} \sin(\omega_G t - \Phi_{M_{uf}}) = \frac{1}{2} \rho (4D)^2 B l \omega_G^2 \left(M_{\theta R}^* B \tilde{\theta} + M_{\theta I}^* \frac{B \dot{\tilde{\theta}}}{\omega_G} \right). \end{aligned} \quad (6)$$

365 Here, L_{uf} and M_{uf} denote the fluctuating components of the lift and aerodynamic moment at the motion
 366 frequency; $A_{L_{uf}}$ and $A_{M_{uf}}$ denote their amplitudes, and $\Phi_{L_{uf}}$ and $\Phi_{M_{uf}}$ denote the phase differences from
 367 the torsional displacement. The four unsteady aerodynamic force coefficients can be obtained by using the one-
 368 period integral of the product of the unsteady aerodynamic force or moment and the displacement or velocity.
 369 In this study, they were obtained by phase-averaged values of lift L_{us} and aerodynamic moment M_{us} ,
 370 measured under sinusoidal torsional motion, as

$$371 \quad \begin{aligned} L_{\theta I}^* &= \frac{\int_0^{\frac{2\pi}{\omega_G}} L_{us}(t) \dot{\tilde{\theta}}(t) dt}{\frac{1}{2} \rho (4D)^2 B l A_{\theta}^2 \omega_G^2 \pi}, L_{\theta R}^* = \frac{\int_0^{\frac{2\pi}{\omega_G}} L_{us}(t) \tilde{\theta}(t) dt}{\frac{1}{2} \rho (4D)^2 B l A_{\theta}^2 \omega_G \pi}, \\ M_{\theta I}^* &= \frac{\int_0^{\frac{2\pi}{\omega_G}} M_{us}(t) \dot{\tilde{\theta}}(t) dt}{\frac{1}{2} \rho (4DB)^2 l A_{\theta}^2 \omega_G^2 \pi}, M_{\theta R}^* = \frac{\int_0^{\frac{2\pi}{\omega_G}} M_{us}(t) \tilde{\theta}(t) dt}{\frac{1}{2} \rho (4DB)^2 l A_{\theta}^2 \omega_G \pi}. \end{aligned} \quad (7)$$

Table 5 Conditions of unsteady aerodynamic force measurement tests with sinusoidal torsional excitation

Mean torsional angle, θ_0	0, 8, 16–40° (at 2° intervals)
Torsional amplitude, A_{θ}	2–20° (at 2° intervals)
Wind speed, U	10.4 m/s
Vibration frequency, f_G	0.44 Hz

3.3 Experimental setup for vertical motion

Unsteady aerodynamic forces were measured under sinusoidal vertical excitation ($y = A_y \sin \omega_G t$) in addition to torsional motions. The actuator for the vertical motion is shown in Fig. 10. In this system, the rotational motion controlled by a servomotor (YASKAWA Electric Co., SGM7A-15A6A6C, rated power: 1.5 kW, rated speed: 3000 rpm, rated torque: 4.9 N·m) was converted to vertical motion via a ball-screw-type actuator. The waveform of the vertical motion could be controlled conveniently from a PC. The unsteady aerodynamic forces were measured by varying the amplitude of the sinusoidal vertical oscillations. Tables 6 and 7 show the conditions of the unsteady aerodynamic force measurement tests with sinusoidal vertical excitation. The constant torsional angle (angle of attack) of the model was set at 22.5° , which is slightly larger than the stall angle. At this angle the Den Hartog (1956) criterion was satisfied and vertical 1-DoF galloping would occur. Both the vibration frequency and wind speed were varied for each oscillation amplitude. The sampling frequency was set at 1000 Hz, and phase-averaged data were obtained from 100 cycles. The force due to self-weight and the inertial force were subtracted using the results of measurements without wind, to obtain only the unsteady aerodynamic forces from the measurement data.

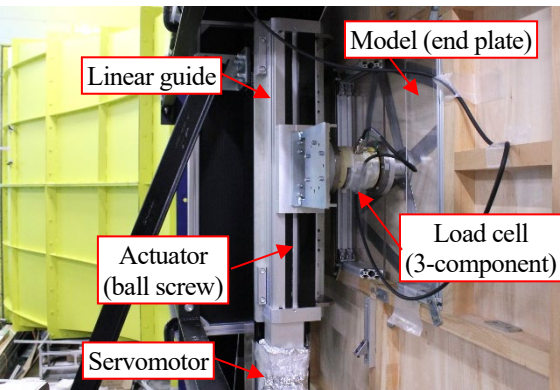


Fig. 10 Actuator for unsteady aerodynamic force measurement with vertical motion

Table 6 Conditions of unsteady aerodynamic force measurement tests with sinusoidal vertical excitation

Constant torsional angle, θ_0	22.5°
Vertical amplitude, A_y	5–180 mm (at 5 mm intervals)

Table 7 Conditions of oscillation frequencies and wind speeds for the sinusoidal vertical excitation tests

Vibration frequency, f_G	Wind speed, U
0.44 Hz	2.5, 5.2, and 9.7 m/s
1.00 Hz	5.2, 7.6, and 10.3 m/s

In this study, the unsteady aerodynamic force coefficients L_{yI}^* , L_{yR}^* , M_{yI}^* , and M_{yR}^* for vertical motion are defined using the components of the lift and aerodynamic moment L_{uf} and M_{uf} at the motion frequency, which can be expressed as

$$\begin{aligned} L_{uf} &= A_{L_{uf}} \sin(\omega_G t - \Phi_{L_{uf}}) = \frac{1}{2} \rho (4D)^2 l \omega_G^2 \left(L_{yR}^* y + L_{yI}^* \frac{\dot{y}}{\omega_G} \right), \\ M_{uf} &= A_{M_{uf}} \sin(\omega_G t - \Phi_{M_{uf}}) = \frac{1}{2} \rho (4D)^2 B l \omega_G^2 \left(M_{yR}^* y + M_{yI}^* \frac{\dot{y}}{\omega_G} \right). \end{aligned} \quad (8)$$

However, since the influence of L_{yR}^* and M_{yR}^* on the aerodynamic instability is small (Matsumoto et al., 2010, Matsumoto, 2013), this study only uses L_{yI}^* and M_{yI}^* . The unsteady aerodynamic force coefficients L_{yI}^* and M_{yI}^* were obtained by phase-averaged values of lift L_{us} and aerodynamic moment M_{us} , measured under sinusoidal vertical motion, as

$$L_{yI}^* = \frac{\int_0^{\frac{2\pi}{\omega_G}} L_{us}(t) \dot{y}(t) dt}{\frac{1}{2} \rho (4D)^2 l A_y^2 \omega_G^2 \pi}, M_{yI}^* = \frac{\int_0^{\frac{2\pi}{\omega_G}} M_{us}(t) \dot{y}(t) dt}{\frac{1}{2} \rho (4D)^2 l A_y^2 \omega_G^2 \pi}. \quad (9)$$

4. Results of the unsteady aerodynamic force measurement tests

4.1 Two-variable aerodynamic coefficients obtained from tests with constant angular velocities

Figs. 11, 12, and 13 show the two-variable aerodynamic coefficients obtained from the measurement tests performed with constant angular velocity at wind speeds of $U = 6.8, 8.5$, and 10.3 m/s. The vertical axis represents the non-dimensional angular velocity $B\dot{\theta}/U$, so the physical angular velocity differs depending on the figure (wind speed) for the same non-dimensional angular velocity. Further, there may be some minor errors in the measurements, indicated by the figures having slightly jagged contour lines. As shown in these figures, C_M^* varies considerably with the non-dimensional angular velocity, whereas variations in C_L^* and C_D^* are relatively small, as previously found for the estimates based on the quasi-steady aerodynamic forces on the individual sub-conductors, in Fig. 7. Changes in the coefficients in relation to the non-dimensional angular

velocity are almost the same for the tests at different wind speeds. For a more detailed comparison, Fig. 14 compares the values at certain non-dimensional angular velocities, $B\dot{\theta}/U = 0.03$ and -0.03 . Because the interval of angular velocity in the tests was $2.5^\circ/\text{s}$, interpolated values are shown in the figures. The results with different combinations of wind speed and angular velocity show good agreement. Thus, the results indicate that the non-dimensional parameter $B\dot{\theta}/U$ is appropriate for modelling the unsteady aerodynamic forces.

Furthermore, the variation of the coefficients is similar between the values measured in the constant angular velocity tests (Figs. 11–13) and the calculated values obtained from the quasi-steady aerodynamic forces for each sub-conductor (Fig. 7). However, certain differences exist, and the experimental results give smoother variations than the calculated values, especially for C_M^* . The calculated values at the non-dimensional angular velocities of $B\dot{\theta}/U = 0.03$ and -0.03 are also shown in Fig. 14. The differences between the experimental and calculated results are relatively large around angles of attack where the coefficients change rapidly, such as for C_L^* and C_M^* around $\theta = 20^\circ$, which corresponds to the stalling angle. Time-history analyses were performed to clarify the influence of these discrepancies on the galloping response, and the results are presented in Section 5.

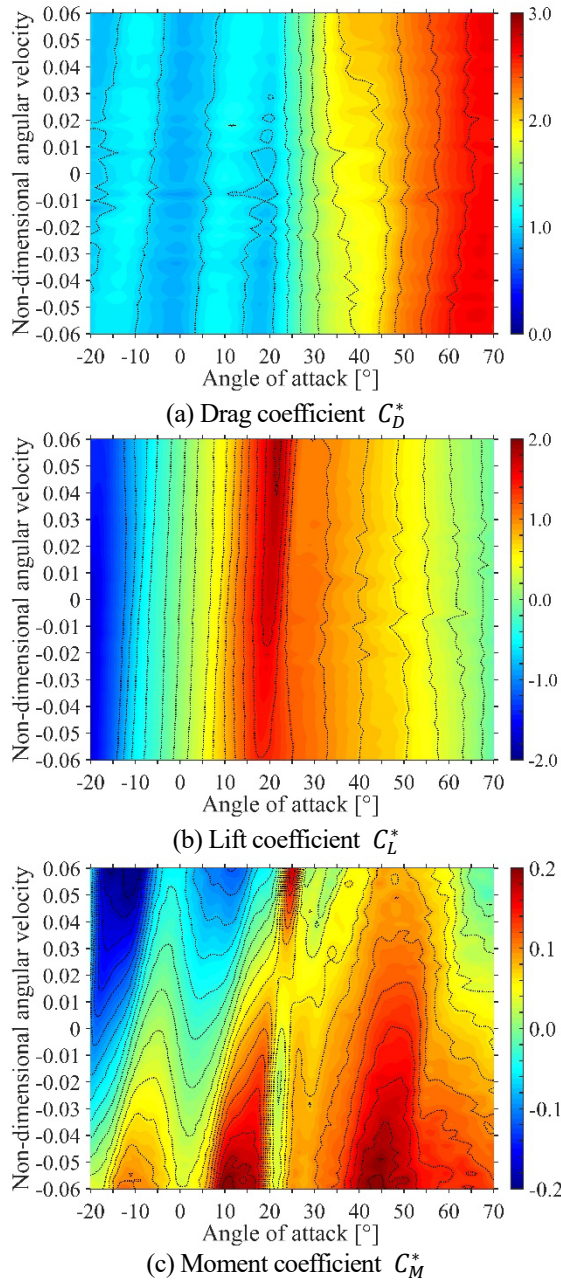


Fig. 11 Two-variable aerodynamic coefficients obtained from measurement tests performed with constant angular velocity ($U = 6.8$ m/s)

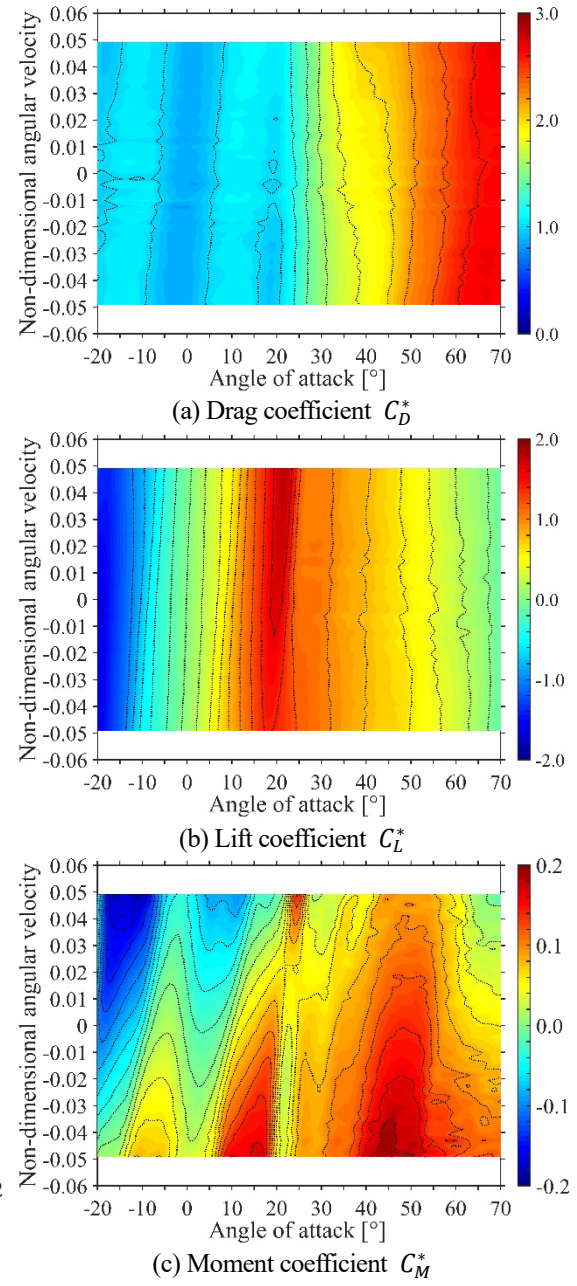


Fig. 12 Two-variable aerodynamic coefficients obtained from measurement tests performed with constant angular velocity ($U = 8.5$ m/s)

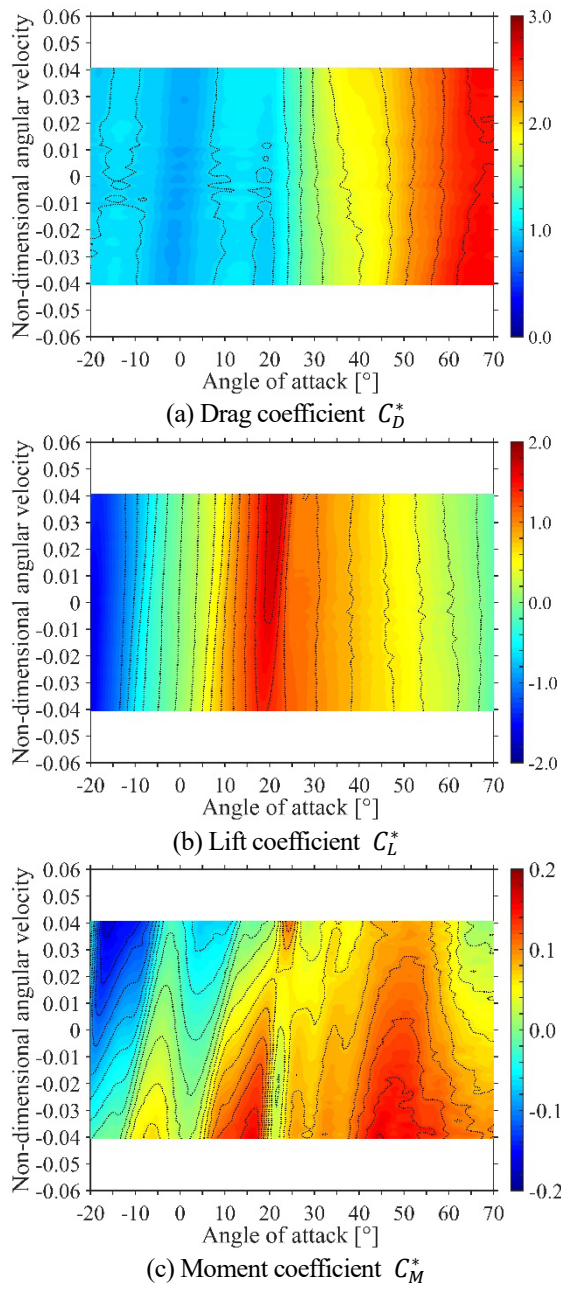


Fig. 13 Two-variable aerodynamic coefficients obtained from measurement tests performed with constant angular velocity ($U = 10.3 \text{ m/s}$)

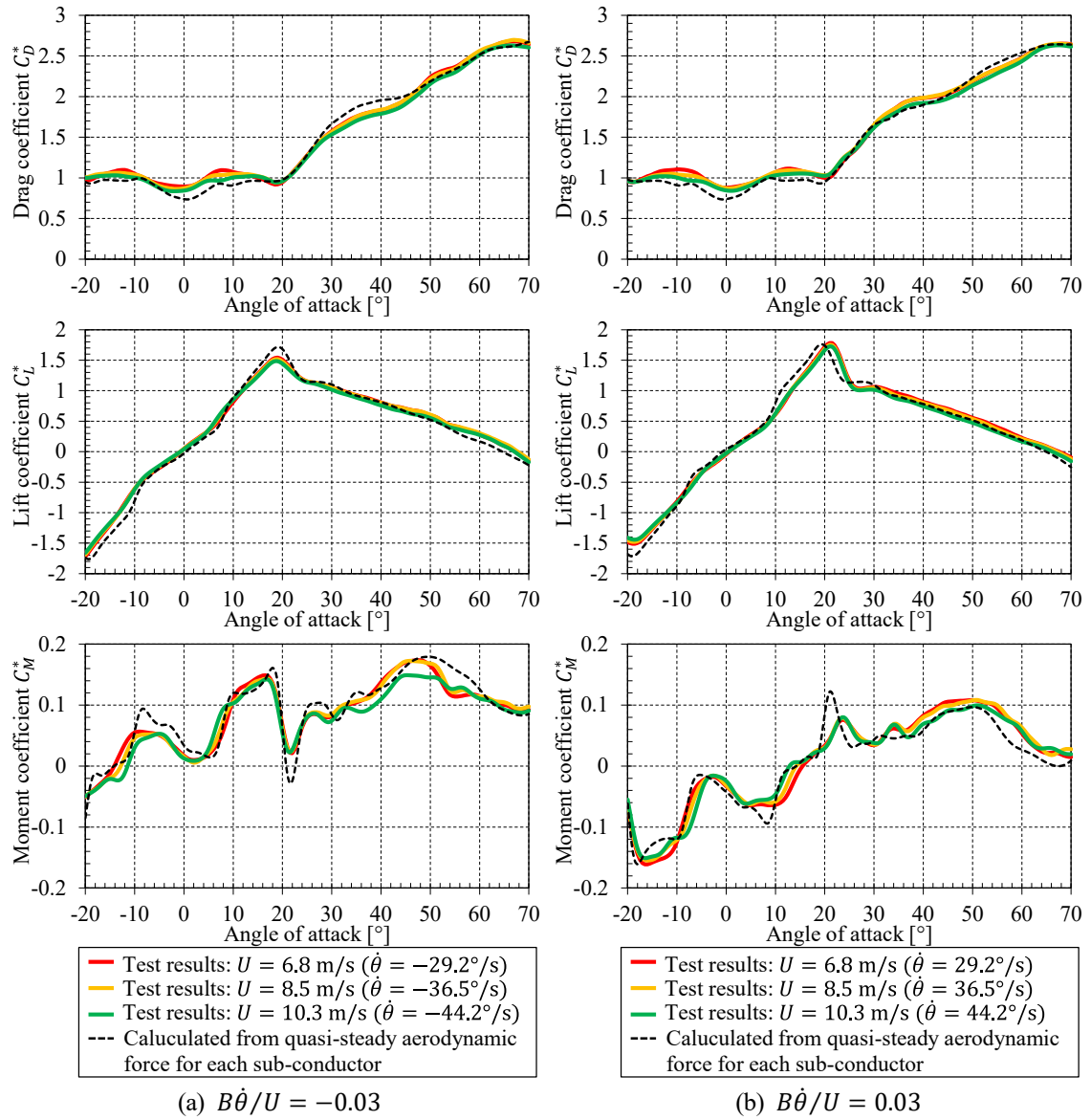


Fig. 14 Comparison of two-variable aerodynamic coefficients obtained at different wind speeds

4.2 Unsteady aerodynamic coefficients obtained from torsional forced vibration tests

Fig. 15 shows the unsteady aerodynamic coefficients obtained during the torsional forced-vibration tests. The role of each unsteady aerodynamic coefficient on aerodynamic instabilities has been investigated using linear instability analyses such as flutter analysis (Matsumoto et al., 2010, Matsumoto, 2013). $M_{\theta I}^*$ represents the aerodynamic damping term for torsional oscillation, and torsional 1-DoF instability (torsional flutter) may occur when $M_{\theta I}^*$ indicates a positive value. $M_{\theta R}^*$ represents the aerodynamic stiffness term for torsional

oscillation, and the frequency of torsional 1-DoF oscillations becomes large when $M_{\theta R}^*$ indicates a negative value. $L_{\theta I}^*$ and $L_{\theta R}^*$ represent aerodynamic coupling terms, and coupled flutter in a vertical-torsional-2-DoF system may occur depending on their combination. Although the changes in the unsteady aerodynamic coefficients with mean torsional angle are significant in the case of small oscillation amplitudes, such as $A_\theta = 2^\circ$, the changes become small for larger oscillation amplitudes.

By using the angle of attack and non-dimensional angular velocity at each time step for assumed sinusoidal oscillations ($\theta = \theta_0 + A_\theta \sin \omega_G t$), the values of the unsteady aerodynamic forces can be calculated using Eqs. (12) and Figs. 11–13. Then, the unsteady aerodynamic force coefficients can be calculated using Eqs. (14). Fig. 16 shows the calculated unsteady-aerodynamic force coefficients, using the measured two-aerodynamic coefficients C_L^* , C_D^* , and C_M^* presented in Fig. 13. Although there are some discrepancies between the measured and calculated unsteady aerodynamic coefficients in the case of small oscillation amplitudes, such as in $L_{\theta I}^*$ for $A_\theta = 2^\circ$, the results demonstrate good agreement (Figs. 15 and 16). Because the unsteady aerodynamic coefficients obtained from the tests with different motions, i.e., sinusoidal oscillations and constant angular velocity, show good agreement, the validity of the aerodynamic force formulation using two variables, as described in Eqs. (12), is confirmed for evaluating torsional 1-DoF oscillations.

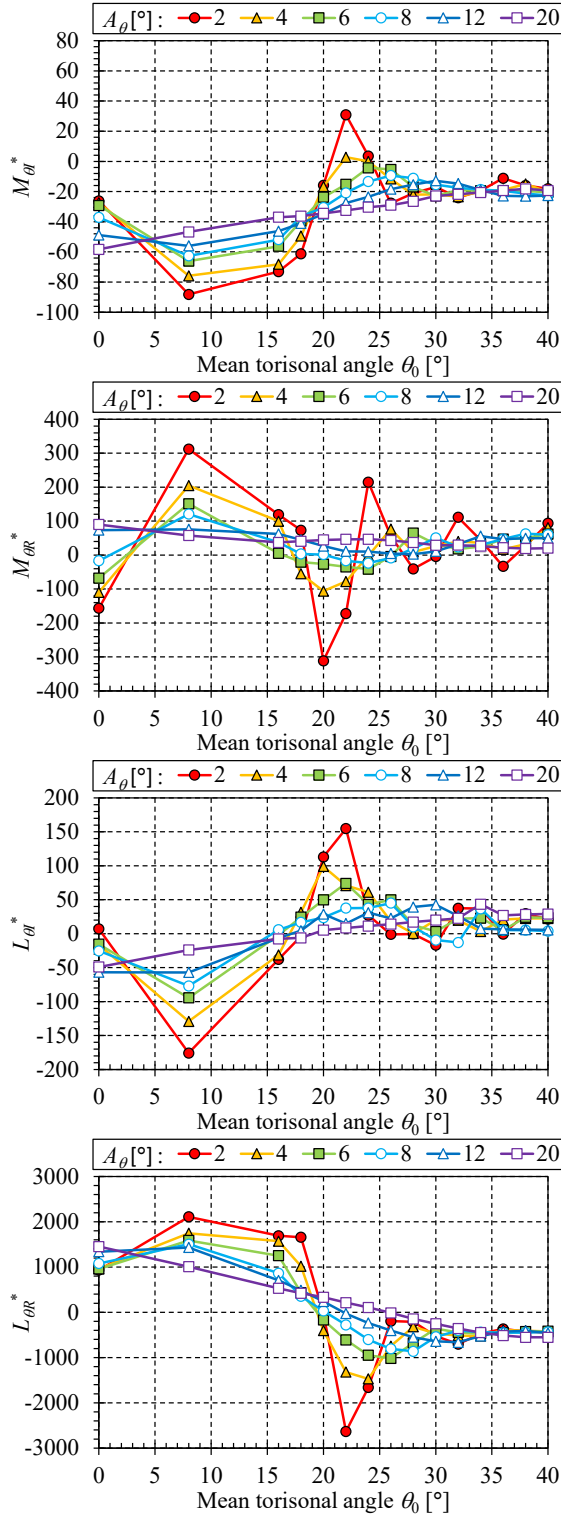


Fig. 15 Unsteady aerodynamic force coefficients
obtained during forced-vibration tests
($U/fB = 59.4$)

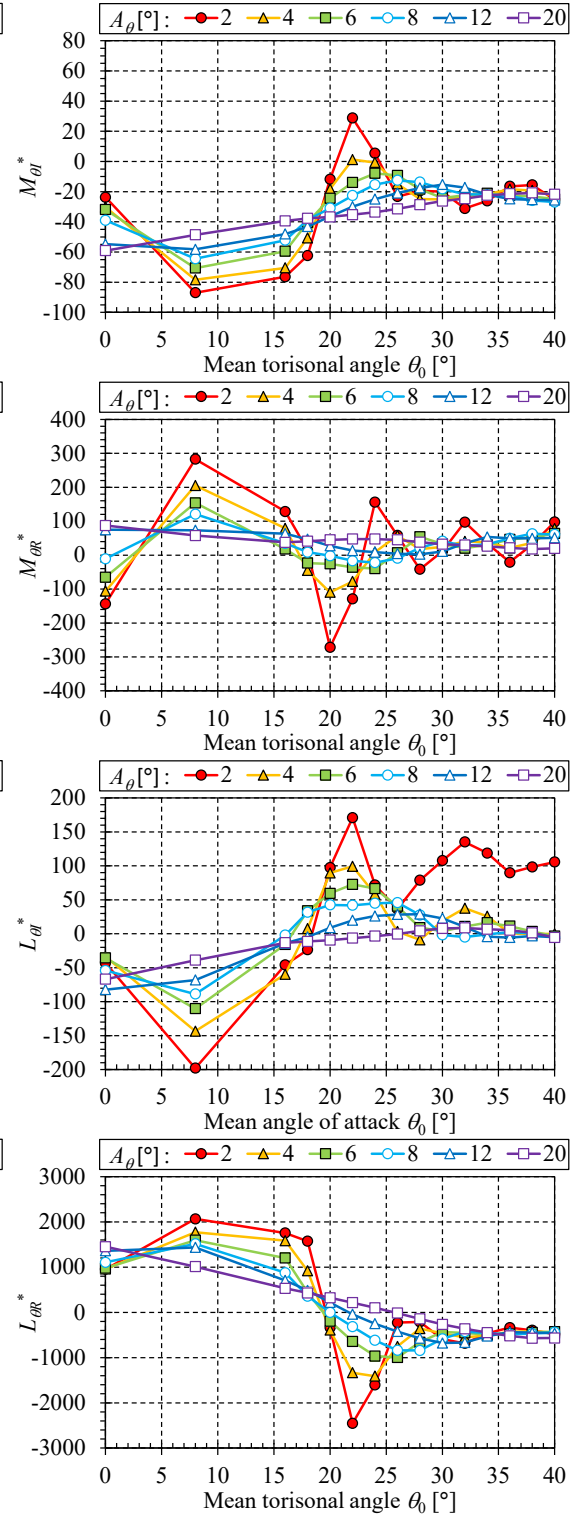


Fig. 16 Unsteady aerodynamic force coefficients
calculated from C_D^* , C_L^* , and C_M^* , shown in Fig. 13
($U/fB = 59.4$)

4.3 Unsteady aerodynamic coefficients obtained from vertical forced vibration tests

Unsteady aerodynamic force coefficients were measured in the vertical 1-DoF sinusoidal forced-vibration tests. Meanwhile, to obtain the unsteady aerodynamic coefficients for vertical motion ($y = A_y \sin \omega_G t$) from the two-variable aerodynamic coefficients, the unsteady lift force and moment $L_{us\dot{\theta}}$ and $M_{us\dot{\theta}}$, respectively, are calculated based on the formulation shown in Eqs. (11) as

$$\begin{aligned} L_{us\dot{\theta}} &= \frac{1}{2} \rho U_r^2 4Dl (C_L^*(\alpha_r, 0) \cos \phi_r + C_D^*(\alpha_r, 0) \sin \phi_r), \\ M_{us\dot{\theta}} &= \frac{1}{2} \rho U_r^2 4BDl C_M^*(\alpha_r, 0), \\ \alpha_r &= \phi_r, \quad \phi_r = \tan^{-1} \left(\frac{-\dot{y}}{U} \right), \quad U_r = \sqrt{(-\dot{y})^2 + U^2}. \end{aligned} \quad (17)$$

Because the torsional velocity $\dot{\theta}$ is zero in the vertical 1-DoF oscillations, the two-variable aerodynamic coefficients are equal to the normal static aerodynamic coefficients, i.e., $C_L^*(\alpha_r, 0) = C_L(\alpha_r)$, $C_D^*(\alpha_r, 0) = C_D(\alpha_r)$, and $C_M^*(\alpha_r, 0) = C_M(\alpha_r)$. Therefore, Eqs. (17) correspond to the formulation of normal quasi-steady aerodynamic forces. The unsteady aerodynamic force coefficients L_{yI}^* and M_{yI}^* can then be calculated using Eqs. (16).

Figs. 17 and 18 show the relationships between the oscillation amplitude and unsteady aerodynamic coefficients L_{yI}^* and M_{yI}^* for the oscillation frequencies of $f_G = 0.44$ and 1.0 Hz. L_{yI}^* represents the aerodynamic damping term for vertical oscillations. Vertical 1-DoF instability (1-DoF galloping) may occur and the oscillation amplitude increases when L_{yI}^* indicates a positive value. Because the constant torsional angle was set at 22.5° where the Den Hartog criterion for vertical 1-DoF galloping (Den Hartog, 1956) was satisfied, as previously mentioned, L_{yI}^* indicates a positive value except for the case with low wind speed and large oscillation amplitude. M_{yI}^* represents an aerodynamic coupling term and coupled flutter in a vertical-torsional-2DoF system may occur depending on its combination with the other coupling terms, such as $L_{\theta I}^*$ and $L_{\theta R}^*$ (Matsumoto et al., 2010, Matsumoto, 2013). The absolute values of L_{yI}^* and M_{yI}^* generally

become small as the oscillation amplitude becomes large. This is because the variation of the relative angle of attack during the oscillations increases as the amplitude increases, and the coefficients are less sensitive to local changes in the aerodynamic forces.

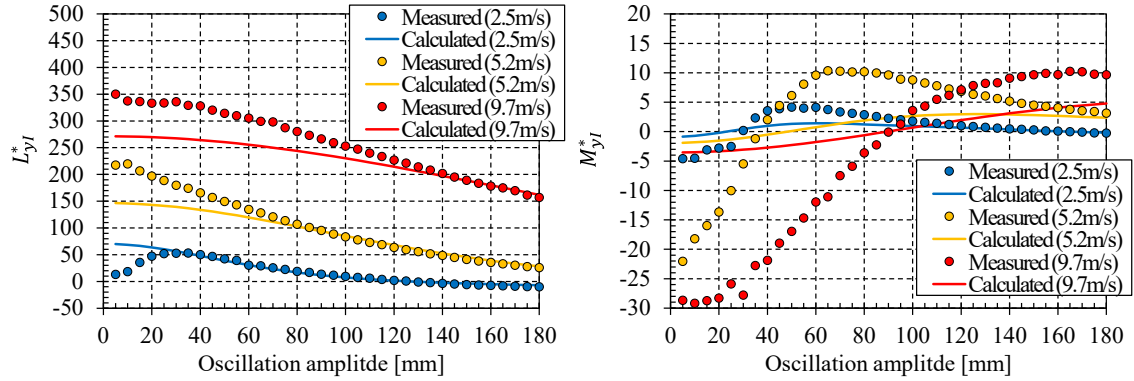


Fig. 17 Comparison of the unsteady aerodynamic coefficients obtained from vertical forced vibration tests with those calculated from the two-variable aerodynamic coefficients shown in Fig. 13 ($f_G = 0.44$ Hz)

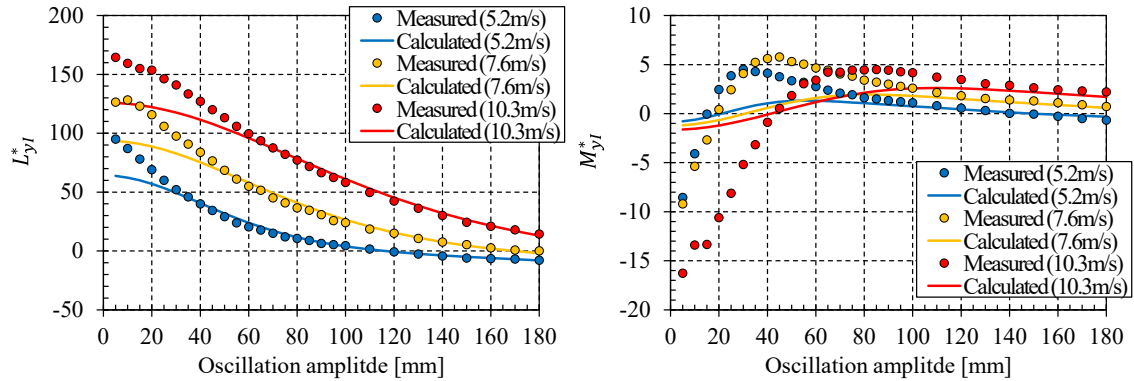


Fig. 18 Comparison of unsteady aerodynamic coefficients obtained from vertical forced vibration tests with those calculated from two-variable aerodynamic coefficients shown in Fig. 13 ($f_G = 1.0$ Hz)

Figs. 17 and 18 compare the unsteady aerodynamic coefficients obtained from the vertical forced vibration tests to those calculated from the two-variable aerodynamic coefficients (Fig. 13), although the angular velocity terms are set to zero. The results from the two methods show good agreement when the oscillation amplitude is large; however, there are some discrepancies for smaller oscillation amplitudes. In the case of lower wind speeds or the higher oscillation frequency, the two sets of results agree down to lower oscillation amplitudes. Fig. 19 shows the relationships between the oscillation amplitude and the maximum relative angle of attack caused by the vertical motion. These results suggest that the unsteady aerodynamic

coefficients can be estimated using Eqs. (17) if the variation in the relative angle of attack is larger than a certain threshold value. The threshold value for L_{yI}^* is approximately 2° , while that for M_{yI}^* is approximately 4° . The discrepancy between the calculated results and the directly measured results might be caused by the phase-averaging process (ensemble averaging) for obtaining the two-variable aerodynamic coefficients. This process could result in a smoothing of the variation of the aerodynamic force coefficient in the direction of the angle of attack. It can be concluded that the results from the two methods are in good agreement, since this study is concerned with relatively large-amplitude oscillations. However, it is necessary to examine the measurement conditions for obtaining the two-variable aerodynamic coefficients (e.g., sampling frequency and ensemble number) and their ensemble process in the future, to improve the accuracy of the calculated results when the variation in the relative angle of attack is small.

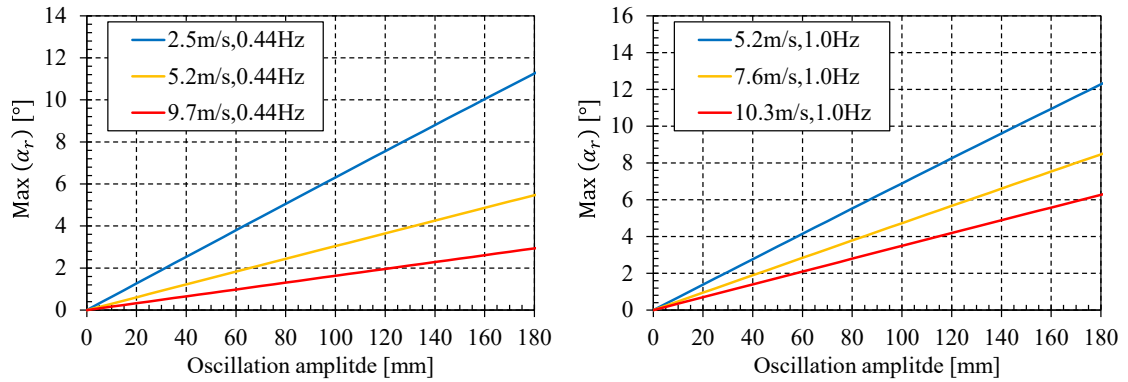


Fig. 19 Relationships between oscillation amplitude and maximum relative angle of attack caused by vertical motion

Because Eqs. (17) use $\dot{\theta} = 0$ as the angular velocity in the two-variable aerodynamic coefficients according to the formulation shown in Eqs. (11), these equations correspond to the normal quasi-steady aerodynamic force model. For the angular velocity dependency, another approach is to use the rate of change of the relative angle of attack, $\dot{\alpha}_r$, which is non-zero for vertical oscillations. It may be natural that the two-variable aerodynamic coefficients are functions of the relative angle of attack α_r and $\dot{\alpha}_r$, instead of functions of α_r and $\dot{\theta}$. The unsteady lift force and moment using $\dot{\alpha}_r$, i.e., $L_{us\dot{\alpha}}$ and $M_{us\dot{\alpha}}$, can be obtained as

$$L_{us\dot{\alpha}} = \frac{1}{2} \rho U_r^2 4Dl \left(C_L^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \cos \phi_r + C_D^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \sin \phi_r \right),$$

$$M_{us\dot{\alpha}} = \frac{1}{2} \rho U_r^2 4BDl C_M^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right).$$
(18)

Table 8 summarises the effect of $\dot{\alpha}_r$ on the calculated unsteady aerodynamic coefficients L_{yl}^* and M_{yl}^* . This table compares the calculated unsteady aerodynamic coefficients using the different formulations of the unsteady lift force and moment (Eqs. (17) and (18), respectively). The values obtained from these two formulations are almost identical, making it difficult to identify any differences graphically; hence, they are not shown in a figure. The effect of $\dot{\alpha}_r$ on L_{yl}^* and M_{yl}^* is very small, and these unsteady aerodynamic coefficients can be estimated via both formulations. Hence, it is difficult to evaluate which formulation is better from the vertical or torsional 1-DoF sinusoidal vibration tests, and it is necessary to evaluate these formulations from the test results via multi-DoF motion.

Table 8 Comparison of unsteady aerodynamic coefficients calculated from two-variable aerodynamic coefficients C_L^* , C_D^* , and C_M^*

f_G	U	A_y	$\max\left(\frac{B\dot{\alpha}_r}{U_r}\right)$	Using $L_{us\dot{\theta}}, M_{us\dot{\theta}}$ L_{yl}^*	M_{yl}^*	Using $L_{us\dot{\alpha}}, M_{us\dot{\alpha}}$ L_{yl}^*	M_{yl}^*
0.44 Hz	2.5 m/s	50 mm	0.024	39.0	1.34	39.0	1.32
0.44 Hz	5.2 m/s	50 mm	0.006	127.2	0.11	127.2	0.10
0.44 Hz	9.7 m/s	50 mm	0.002	260.3	-2.29	260.3	-2.29
0.44 Hz	5.2 m/s	180 mm	0.020	27.9	2.46	28.0	2.37
0.44 Hz	9.7 m/s	180 mm	0.005	161.9	4.83	161.9	4.78
1.0 Hz	5.2 m/s	50 mm	0.029	31.9	1.30	31.9	1.28
1.0 Hz	7.6 m/s	50 mm	0.014	66.8	1.17	66.8	1.16
1.0 Hz	10.3 m/s	50 mm	0.007	104.1	0.55	104.1	0.54
1.0 Hz	10.3 m/s	180 mm	0.026	13.0	1.70	13.0	1.69

5. Unsteady aerodynamic force modelling for 3-DoF-galloping of four-bundled conductors

This section presents a time-history analysis method for vertical-horizontal-torsional-3-DoF motion, developed for two-variable aerodynamic coefficients. Further, the time-history analysis for simulating the wind-response

measurement tests for the large-amplitude 3-DoF oscillations described in Section 2, and the best modelling method for 3-DoF-galloping of four-bundled conductors, are also discussed in this section.

5.1 Time-history analysis using two-variable aerodynamic coefficients

Two different models of the unsteady aerodynamic forces are considered for the vertical-horizontal-torsional-3-DoF system, by focusing on the angular velocity used in the two-variable aerodynamic coefficients. The first model uses α_r , $\dot{\theta}$, and U_r at each time step, based on the formulation shown in Eqs. (11). The unsteady aerodynamic forces using $\dot{\theta}$, denoted $L_{u\dot{\theta}}$, $D_{u\dot{\theta}}$ and $M_{u\dot{\theta}}$, can be obtained as

$$\begin{aligned} L_{u\dot{\theta}} &= \frac{1}{2} \rho U_r^2 4Dl \left(C_L^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \cos \phi_r + C_D^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \sin \phi_r \right), \\ D_{u\dot{\theta}} &= \frac{1}{2} \rho U_r^2 4Dl \left(-C_L^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \sin \phi_r + C_D^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right) \cos \phi_r \right), \\ M_{u\dot{\theta}} &= \frac{1}{2} \rho U_r^2 4BDl C_M^* \left(\alpha_r, \frac{B\dot{\theta}}{U_r} \right). \end{aligned} \quad (19)$$

Further, α_r and U_r can be calculated using Eqs. (7). The other model uses α_r , its rate of change $\dot{\alpha}_r$, and U_r at each time step. The unsteady aerodynamic forces using $\dot{\alpha}_r$, denoted $L_{u\dot{\alpha}_r}$, $D_{u\dot{\alpha}_r}$ and $M_{u\dot{\alpha}_r}$, can be obtained as

$$\begin{aligned} L_{u\dot{\alpha}_r} &= \frac{1}{2} \rho U_r^2 4Dl \left(C_L^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \cos \phi_r + C_D^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \sin \phi_r \right), \\ D_{u\dot{\alpha}_r} &= \frac{1}{2} \rho U_r^2 4Dl \left(-C_L^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \sin \phi_r + C_D^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right) \cos \phi_r \right), \\ M_{u\dot{\alpha}_r} &= \frac{1}{2} \rho U_r^2 4BDl C_M^* \left(\alpha_r, \frac{B\dot{\alpha}_r}{U_r} \right). \end{aligned} \quad (20)$$

α_r and U_r are still as in Eqs. (7), while $\dot{\alpha}_r$ can be calculated as

$$\dot{\alpha}_r = \dot{\theta} + \dot{\phi}_r = \dot{\theta} - \frac{\ddot{y}(U - \dot{z}) + \dot{y}\ddot{z}}{\dot{y}^2 + (U - \dot{z})^2}. \quad (21)$$

Two sets of time-history response analyses for simulating the wind-response measurement tests described in Section 2 were performed using Eqs. (19) and Eqs. (20), respectively, as the external forces F_y , F_z , and F_θ in Eqs. (1). Because Eqs. (20) include the accelerations $\ddot{y}$ and $\ddot{z}$, an implicit method is

necessary to perform time-history analysis. The Newmark-beta method and Newton–Raphson method were used to solve the non-linear differential equations in this study, whereas the fourth-order Runge–Kutta method was used in the previous study (Matsumiya et al., 2018). The time-history response analyses were performed for a series of values of θ_0 , which corresponds to the torsional angle without wind and the rotational angle of both elastic cord holders, as shown in Fig. 1. The responses were obtained after applying large initial displacements from the static equilibrium position, as in the wind-response measurement tests. First, the values of the displacements y_s, z_s and θ_s , for the static equilibrium position in the wind, were calculated. Then, the dynamic analyses were performed, wherein the initial displacements were set as $(y_{t=0}, z_{t=0}, \theta_{t=0}) = (y_s - 0.5 \text{ m}, z_s + 0.5 \text{ m}, \theta_s + 45^\circ)$. The initial displacements imposed in the dynamic analyses were hence the same as those applied in the wind-response measurement tests. The corrected structural damping ratios ζ_q (Eq. (5)), excluding the aerodynamic damping component in still air, were used in Eqs. (1). However, under the test conditions listed in Table 1, there was little difference in the response amplitudes found if ζ_{qm} (Eq. (2)) was used instead. Because ζ_q is a function of the response amplitude A_q , ζ_q was updated in the analysis periodically, at intervals of 30 s, in response to changes in the amplitude. The time step in these analyses was set as 0.02 s, and the total simulated time of each run was 600 s, by which time the response amplitudes and ζ_q had almost reached a steady state in all cases.

5.2 Effect of the rate of change of the relative angle of attack on 3-DoF-galloping of four-bundled conductors

Fig. 20 shows the results of the time-history analyses using the two different formulations, compared with the results of the wind-response measurement tests described in Section 2. The measured two-aerodynamic coefficients C_L^*, C_D^* , and C_M^* presented in Fig. 13 were used in these analyses. As shown in Fig. 20 (a), which shows the results of the time-history analysis using Eqs. (19), the discrepancy between the results of the tests

and the analysis remains comparable to the results of the time-history analysis using the quasi-steady aerodynamic forces on each sub-conductor (Eqs. (8)–(10), Fig. 6 (b)). This latter approach is equivalent to time-history analysis using Eqs. (19) and the calculated two-variable aerodynamic coefficients C_{Lqs}^* , C_{Dqs}^* , and C_{Mqs}^* (Fig. 7), based on the quasi-steady aerodynamic forces on each sub-conductor.

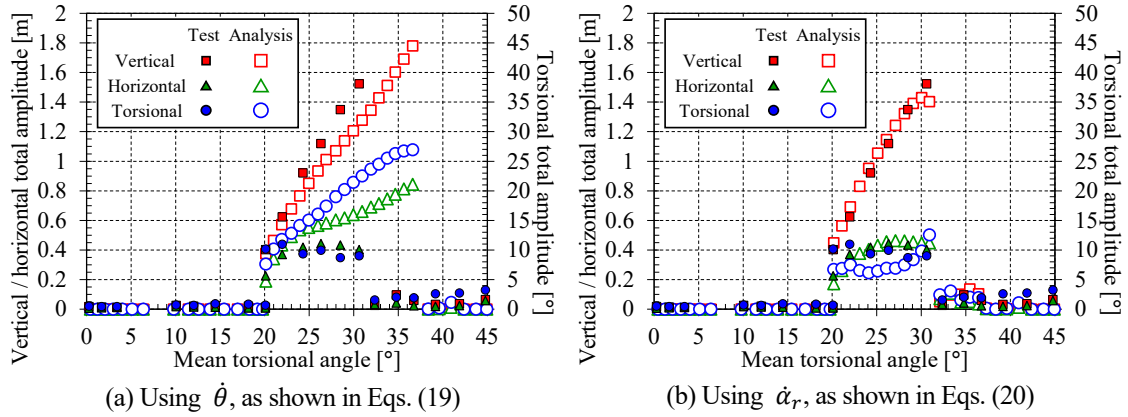


Fig. 20 Comparison of peak-to-peak amplitudes from wind response measurement tests and time-history analyses using two-variable aerodynamic coefficients ($U = 10.2$ m/s, with initial displacement)

In contrast, as shown in Fig. 20 (b), the results of the time-history analysis using Eqs. (20) agree very well with the test results. The estimates of the horizontal and torsional amplitudes and the angle range where galloping occurs with certain initial conditions are much closer to the measured values, while the vertical amplitude is also estimated more accurately than from Eqs. (19). These results indicate that the unsteady aerodynamic force for 3-DoF-galloping of four-bundled conductors should be modelled using Eqs. (20). As an example, Fig. 21 shows the time series of the response for a mean torsional angle $\bar{\theta} = 30.7^\circ$ ($\theta_0 = 28^\circ$) from the time-history analysis using Eqs. (20). There is a significant difference between $\bar{\theta}$ and $\dot{\alpha}_r$. Fig. 22 shows the time series of the unsteady aerodynamic forces corresponding to the oscillations in Fig. 21. These figures compare the two sets of unsteady aerodynamic forces, $L_{u\bar{\theta}}$, $D_{u\bar{\theta}}$, $M_{u\bar{\theta}}$ and $L_{u\dot{\alpha}_r}$, $D_{u\dot{\alpha}_r}$, $M_{u\dot{\alpha}_r}$, calculated from Eqs. (19) and Eqs. (20), respectively. The unsteady aerodynamic moment is affected by the difference in the angular velocity used in the analysis. Because of the resulting change in the torsional

amplitude, the vertical and horizontal amplitudes are also changed, giving better agreement with the test results.

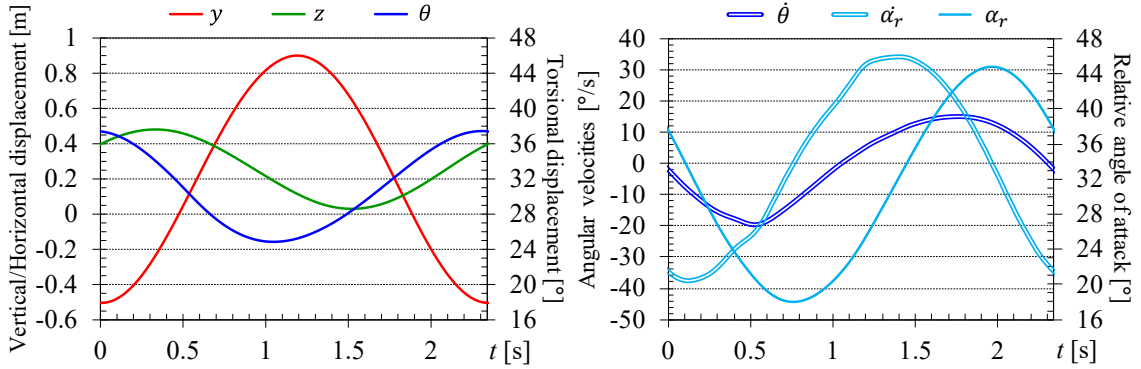


Fig. 21 Time series of response from time-history analysis using Eqs. (20) ($\theta_0 = 28^\circ, \bar{\theta} = 30.7^\circ$)

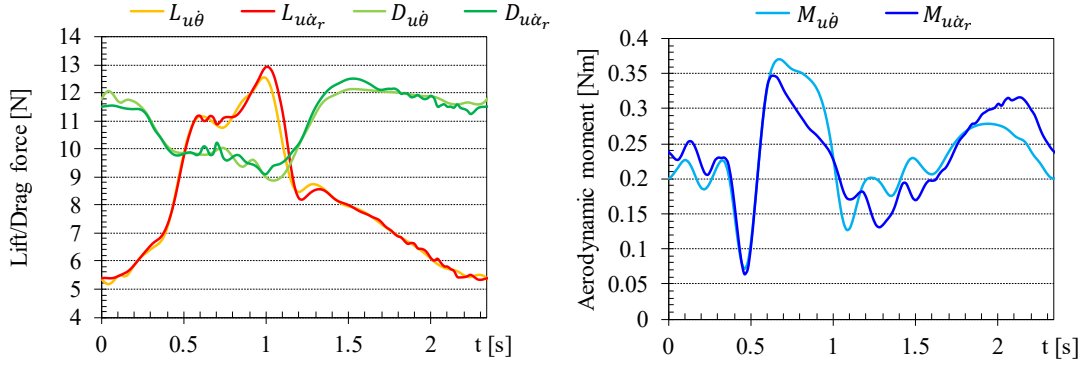


Fig. 22 Time series of two sets of unsteady aerodynamic forces ($\theta_0 = 28^\circ, \bar{\theta} = 30.7^\circ$)

These results indicate that the unsteady aerodynamic forces for 3-DoF-galloping of four-bundled conductors can be evaluated using α_r , $\dot{\alpha}_r$, and U_r , with two-variable aerodynamic coefficients measured under constant angular velocities. Note that $\dot{\alpha}_r$ should be used for the modelling the unsteady aerodynamic forces using two-variable aerodynamic coefficients in the 3-DoF system, whereas $\dot{\theta}$ can be used for the torsional 1-DoF system. The two-variable aerodynamic coefficients can be measured in wind tunnel tests under constant angular velocities, as developed in this study. Alternatively, because the discrepancies between the measured coefficients and the calculated values from the quasi-steady aerodynamic forces on each sub-conductor are not large, the unsteady aerodynamic forces for 3-DoF galloping can be estimated using the calculated two-variable aerodynamic coefficients.

In future work, we plan to measure the two-variable aerodynamic coefficients under other

experimental conditions and verify the validity of the modelling of the unsteady aerodynamic forces proposed in this study for other situations. If the discrepancies between the measured two-variable aerodynamic coefficients and the calculated values from the quasi-steady aerodynamic forces on each sub-conductor are sufficiently small, the time-history analysis using $\dot{\alpha}_r$ could be carried out using the results of only static aerodynamic force measurement tests, though the forces on each individual sub-conductor are necessary. Therefore, the discrepancies between the measured two-variable aerodynamic coefficients and the calculated values from the quasi-steady aerodynamic forces on each sub-conductor, and their impact on the estimated amplitudes, need to be clarified, under various conditions, including for various ice accretion shapes and sub-conductor diameters and spacings. Furthermore, in order to clarify the applicable range of the unsteady aerodynamic force model using $\dot{\alpha}_r$ with two-variable aerodynamic coefficients, it is important that the accuracy of the time-history analysis using the proposed model be checked using the results of wind response measurement tests at various wind speeds, oscillation amplitudes and frequencies. Further, the applicability of the proposed unsteady aerodynamic force model to galloping of single conductors, in addition to multi-bundled conductors, should be explored.

6. Conclusions

Unsteady aerodynamic force measurement tests were performed on four-bundled conductors to develop the best unsteady aerodynamic force model for simulating large-amplitude 3-DoF galloping. Because the measurement systems were developed using servomotors as excitation devices, different types of motion under different conditions could be used. Three different types of imposed motion were used: constant angular velocity with various wind speeds and angular velocities, and torsional and vertical sinusoidal oscillations with various wind speeds and oscillation amplitudes and frequencies. The results of the tests with constant angular

605 velocities indicated that the unsteady aerodynamic forces depend on the angle of attack and non-dimensional
606 angular velocity. Two-variable aerodynamic coefficients were hence obtained from these tests, consistently
607 from tests performed at different wind speeds. Furthermore, the measured unsteady aerodynamic force
608 coefficients under sinusoidal forced-vibration tests showed good agreement with the calculated values obtained
609 via numerical simulations using the two-variable aerodynamic coefficients. Hence the validity of the
610 aerodynamic force model using the two variables was confirmed for evaluating torsional or vertical 1-DoF
611 oscillations. Finally, by comparing the results of time-history analysis using the two-variable aerodynamic
612 coefficients obtained in this study and the results of previous wind-response measurement tests, the best model
613 of unsteady aerodynamic forces for simulating large-amplitude 3-DoF galloping was proposed. The unsteady
614 aerodynamic forces for 3-DoF-galloping of four-bundled conductors can be evaluated with the two-variable
615 aerodynamic coefficients, using the relative angle of attack, the rate of change of the relative angle of attack
616 (rather than the angular velocity of the conductor) and the relative wind speed.

617 Using the best model proposed in this study, all of the vertical, horizontal, and torsional amplitudes in
618 the time-history analysis are in good agreement with those previously obtained from wind-response
619 measurement tests. The development of the two-variable aerodynamic coefficient measurement system using
620 the proposed model is an important outcome of this study, in addition to proposing the unsteady aerodynamic
621 force model. In the future, we plan to continue to measure the two-variable aerodynamic coefficients under
622 other experimental conditions and identify the application range of the proposed model using the results from
623 other wind-response tests. Moreover, one of the reasons why the two-variable aerodynamic coefficients can be
624 adapted for galloping of four-bundled conductors might be that the target range of non-dimensional wind speed
625 is very large. Accordingly, we will continue to evaluate the applicability of the proposed unsteady aerodynamic
626 force model using the rate of change of the relative angle of attack and the two-variable aerodynamic

coefficients for aerodynamic instabilities of various structures and identify the application range of the proposed model.

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